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A More Efficient Low-pass Filter

G3TMG compares Zolotarev quasi-elliptic low pass-band characteristics with the classical Chebyshev design in terms of the expected performance for lumped-element circuits, and shows a universal table of values that can be used in filters for Amateur Radio bands.

A comparison between two synthesized quasi-elliptic low-pass filters — one possessing a Chebyshev and the other a Zolotarev pass-band characteristic — is carried out in terms of the expected performance for lumped-element circuits. Two examples of the Zolotarev filter type have been constructed and tested using different, but appropriate, inductor techniques. Also, a universal table of values, which can be used for most of the Amateur Radio bands, is for the first time made available.

Introduction

A radio transmitter/amplifier combination can be a powerful source of interference. It is particularly important to protect against harmonic emission. As a consequence the output stages for both driver and amplifier equipments are most often followed by filters that provide harmonic suppression — usually for HF — using a lumped element low-pass network. When consideration is given to a modern transceiver, which allows moderately high power operation from 1.8 – 50 MHz (6 octaves), it is clearly necessary to have many switched analog output filters within the equipment — one thing that the Software Defined Radio concept hasn't yet solved.

Using stock low-pass designs[1,2] with well-matched pass bands from dc to a predefined cut-off frequency f_c, is clearly not efficient since more than half of the low-pass bandwidth is of no practical value — the required signal frequency must lie above $f_c/2$ to provide any harmonic attenuation from the filters transition edge or ultimate stop-band.

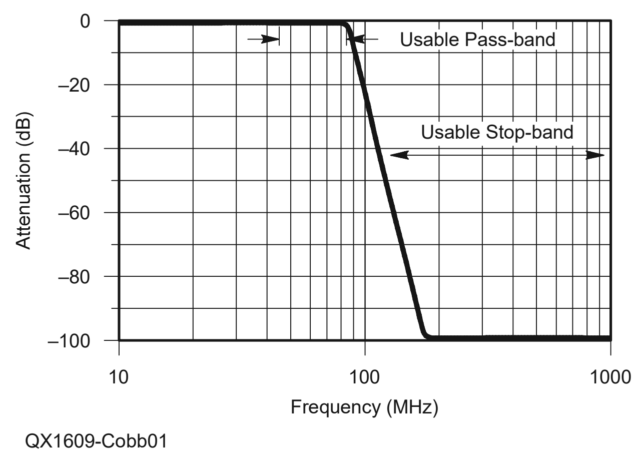


Figure 1 — Generic low-pass filter.

Figure 1 shows a generic low-pass filter. This is somewhat of an over simplification but that's the way it appears because the Amateur Radio bands have a small fractional bandwidth.

Available Low-pass Solutions

It is known that N^{th} order Cauer or Elliptic low-pass filters provide the fastest pass-band to stop- band roll-off rate together with a predefined minimum attenuation across the entire stop-band. They have (N-1)/2 or N/2 finite frequency transmission zeros, for even or odd orders respectively, periodically placed across the entire stop-band. An example 7th order response, shown in Figure 2, is based on the 4 m band — say 70 – 71 MHz for the sake of argument — being coincident with the second reflection zero left of the cut-off frequency so as to reduce reflection loss. Additionally, a single real frequency transmission zero (most adjacent to the transition region) is arranged to coincide with the 4 m signal 2nd harmonic of 140 MHz.

This approach needs 10 components but, except for the first transmission zero, the far out zeros do not actually contribute greatly to the transition region roll-off rate and, as can be seen, does not provide for the production of a zero at, or near, the 3rd harmonic frequency of 210 MHz. Also, and importantly, the inductance ratio within the realized network is in the region of 25:1

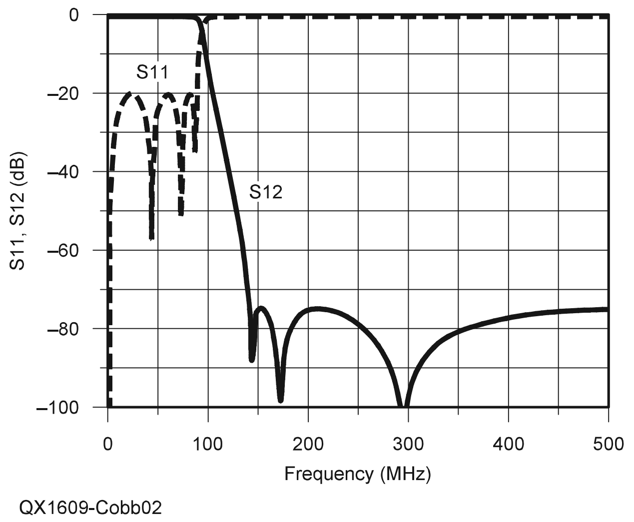


Figure 2 — A loss-less 7th order Elliptic response.

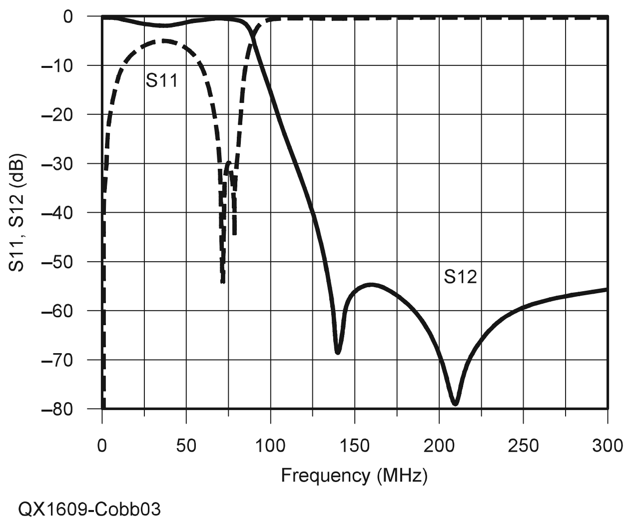


Figure 3 — A 5th order response from *OptLowpass* program, tweaked and with losses.

which, for the higher HF bands, makes it quite difficult to realize the additional zeros due to mutual coupling and unintentional resonance issues. Although Elliptic designs may in many cases be impractical, one might still consider the out-of-band performance to be the state-of-the-art target.

To keep the insertion loss to a minimum, the lowest order filter together with the widest pass-band width, and the smallest number of components, is the usual approach. Generally, the solution is most often the use of a low-pass filter with Chebyshev pass-band characteristics — equi-ripple response — whose order can be further minimized when a single transmission zero is placed immediately adjacent to the filters transition region. The article[3] by Ed Wetherhold, W3NQN, on this subject summarizes the work of Jim Tonne, W4ENE (formerly WB6BLD), on what has become known as the CWAZ (Chebyshev With Added Zero) low-pass filter. These filters are more often known as quasi- or pseudo-elliptic types in modern parlance.

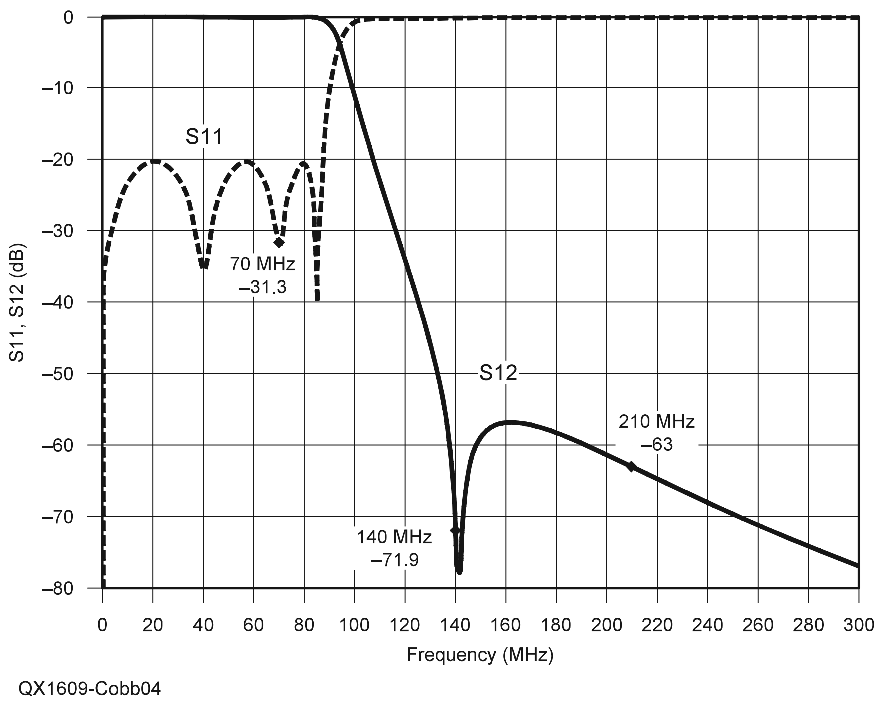


Figure 4 — A 7th order CWAZ transmission response [after: Wetherhold, with losses].

Recently, Jim has further provided a useful solution to the harmonic rejection problem with his *OptLowpass* design program[4] that combines the described Elliptic stop-band characteristic and a matched region only in the top half of the low-pass pass-band where it is needed. This program is currently restricted to a 5th order response but nevertheless may be considered optimal as it demonstrates firstly, a significant improvement in the general stop-band rejection when compared to the standard Chebyshev design of the same order and secondly, the provided stop-band zeros happen to nearly coincide with the 2nd and 3rd harmonic frequencies. With a little adjustment — by using trimmer capacitors in the reject resonators in reality — improvements can be made by precisely tuning the stop-band notches. Typically then, Jim's solution provides for, in principle, around 65 dB of 2nd and 75 dB of 3rd harmonic protection and 54 dB generally elsewhere with just 7 components — see Figure 3. This is potentially a very good performance with a cost of about 0.2 dB insertion loss based on practical solenoid inductors. In practice however, it is difficult to achieve the 3rd harmonic notch because the required resonating capacitance is only 7.5 pF with an inductance of nearly 80 nH. A shielded solenoid inductor of this value, designed to provide an unloaded Q of 200 (~15 mm diameter), is found to have a self capacitance of between 3 and 4 pF

— a significant proportion of the desired resonating capacitance. Compensating for this is a little on the tricky side but somewhat easier than trying to tune the ideal Elliptic!

Suppose that more stop-band attenuation, in general, is required. Well, we could try a 7th order CWAZ low-pass filter as previously described by Ed Wetherhold. Calculating the transfer function based on scaled prototype values for the 70 MHz example (with tweaks) provides for a response as shown in Figure 4. Clearly, the rejection has improved for the 2nd harmonic as compared to the circuit produced by *OptLowpass* at the cost of one more component — now 8. The 3rd harmonic band is clearly worse at 63 dB, as we might have expected with no coincident zero, but the general rejection is better, monotonically improving beyond the 3rd harmonic frequency. Taking a leaf out of Jim Tonne's book, the rejection could still further be improved by providing a matched region only in the top half of the low-pass pass-band. How might this be done and what would be the cost?

It turns out that Jim has discovered, intentionally or otherwise, the characteristic that has become known as the Zolotarev function. This function is similar to the ubiquitous Chebyshev characteristic which has a *y*-valued unit amplitude cyclic behavior between *x*-axis values of -1 and 1 radian. In the Zolotarev case, which has an extra parameter, the first cycle about the origin can have a magnitude greater than 1. Figure 5 compares the two functions.

This issue was researched more than 40 years ago by Ralph Levy[5] who determined a method of incorporating this characteristic into low-pass filter designs for, in principle, any order. The mathematics therein is not trivial but the basic outcome is that even ordered functions are analytic and can be generated by modifying the root locations of a conventional Chebyshev polynomial function in a predetermined geometric fashion. For these to be realizable however, it is necessary for the source and load to be unequally valued, the ratio depends on the bandwidth compression factor chosen.

Odd-ordered functions — more appropriate for low-pass filters — are unfortunately not so easily dealt with, requiring some rather knotty mathematical techniques. Nevertheless, these are of greater value since the resulting network source and load are always equal. Because of this, a simpler design methodology for any odd ordered function has been recently developed and fully described previously in *QEX*.[6] The mathematics will not be described here but the Zolotarev With Added Zero (ZWAZ) will first be compared to that of the CWAZ, by way of example, using essentially the same electrical circuit. Second, a couple of practical examples will demonstrate the achievable performance using typical and appropriate inductor construction techniques.

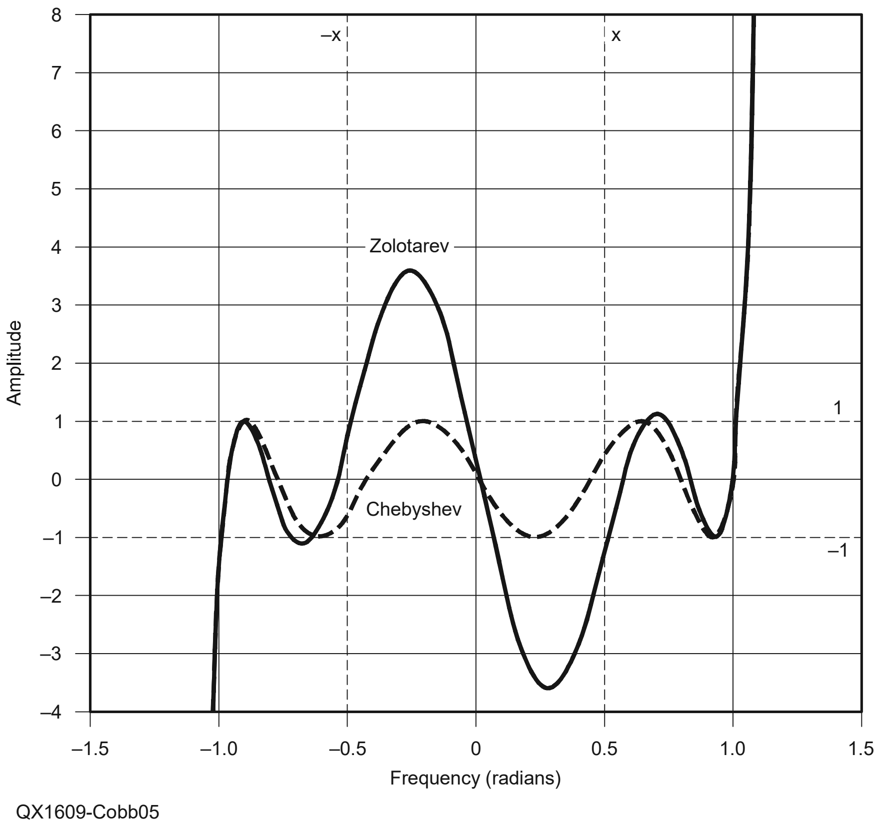


Figure 5 — A 7th order Chebyshev and Zolotarev low-pass function.

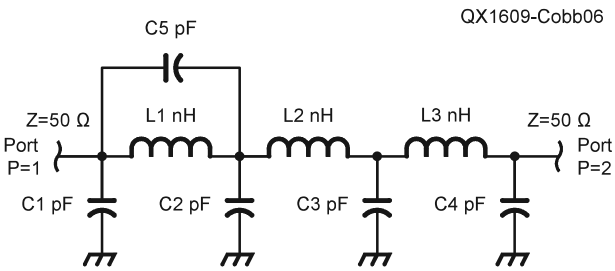


Figure 6 — Basic circuit configuration for analysis. Element values are in Table 2.

Approximation and Synthesis of 7th Order ZWAZ Filter

To create a 7th order ZWAZ filter, we first generate a 7th order CWAZ polynomial filter function whose characteristic pole and zero singularities are then mapped into the Zolotarev domain using a freely chosen fractional bandwidth compression factor providing a new polynomial function. Prototype lumped element component values are then extracted from the equivalent admittance polynomial and scaled in both frequency and impedance so as to produce the necessary component values.

Two filters using the common network configuration (Figure 6), have been selected for comparison, one with Chebyshev and the other with Zolotarev pass-band characteristics with a chosen bandwidth compression factor of 35%. It is common practice, particularly for high power filter designs, to select the cut-off frequency based on minimizing the in-band insertion-loss, by positioning the transmission band center to be coincident with one of the filter reflection zeros. Therefore, both filter types will have distinctly different cut-off frequencies because the reflection zeros will be differently distributed. However, both have the same pass-band equi-ripple value of 0.044 dB corresponding to a maximum in-band return loss of 20 dB. The values determined for each design are shown in Table 1.

Observe that the ZWAZ design has the property of low inductance values with a smaller variation, at approximately 1/2, and some 10% less respectively, when compared with the CWAZ design. This is a distinct practical advantage when consideration is given to parasitic issues such as self resonance, mutual coupling and Q so that realization of the far out rejection can be more easily achieved.

Analysis and Comparison

The networks so described can be analyzed in any one of the many freely available circuit simulators — *QUCS*, *Ansoft Designer SV*, *Elsie*, etc. Assuming that good quality capacitors would be the norm, the main contributor to in-band loss would be the inductors whether realized using air cored solenoid or ferrite cored toroidal coils. Toroids may be preferred particularly for the lower bands so that adequately small parasitic coupling within the network can be achieved without shielding. Shielded solenoids can also provide sufficiently low couplings for the higher frequency bands. To this end, a practical value of Q=200 for solenoids and Q=150 for toroids have been used for the simulations presented here.

Figure 7 shows the transmission/reflection behavior for the Chebyshev valued network, the equi-ripple performance being demonstrated by the familiar multi-lobed, equal amplitude return loss characteristic across the full passband. It should be noted that with the transmission zero placed at the 2nd harmonic frequency, a stop band non-harmonically related lobe amplitude of 58 dB and approximately 64 dB of attenuation at the 3rd harmonic (210 MHz) is achieved. Also, the second reflection zero corresponds closely to the nominal signal band (70 MHz) exactly as the Wetherhold CWAZ design showed previously in Figure 4.

Figure 8 shows the simulated transmission/reflection behavior for the Zolotarev valued network. Here we can clearly see the effect of incorporating the Zolotarev pole/zero distribution such that the equi-ripple performance is limited to a portion in the

Table 1
Element values for the two filter types: (*C*, pF; *L*, nH, frequency, MHz).

Type	*C1*	*C2*	*C3*	*C4*	*C5*	*L1*	*L2*	*L3*	*fc*
CWAS	26.9	65.6	73.6	38.1	13.5	95.8	154.7	135.7	84.8
ZWAZ	64.0	196.0	225.8	77.25	21.4	60.6	52.6	76.8	81.5

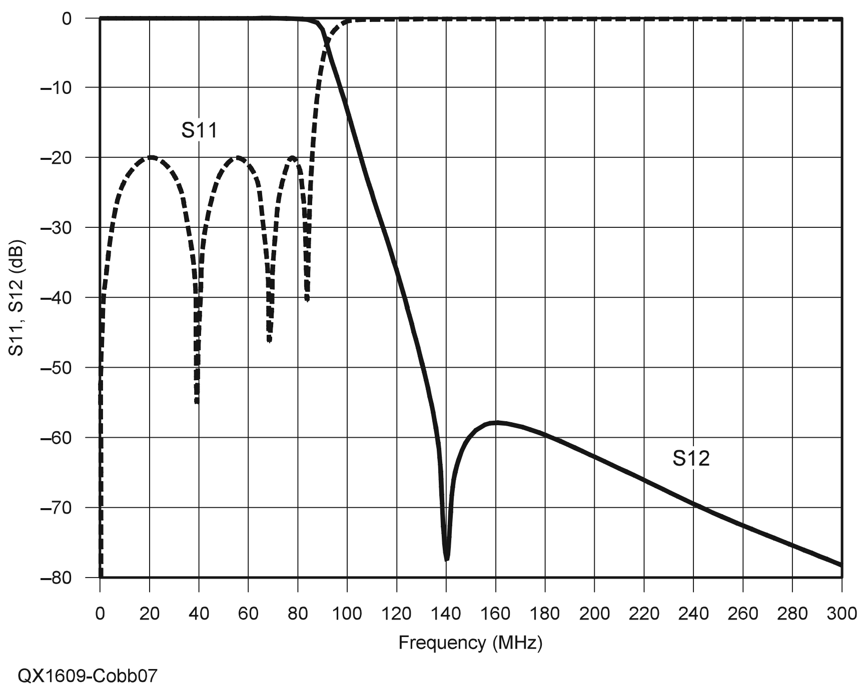


Figure 7 — Chebyshev transmission/reflection response.

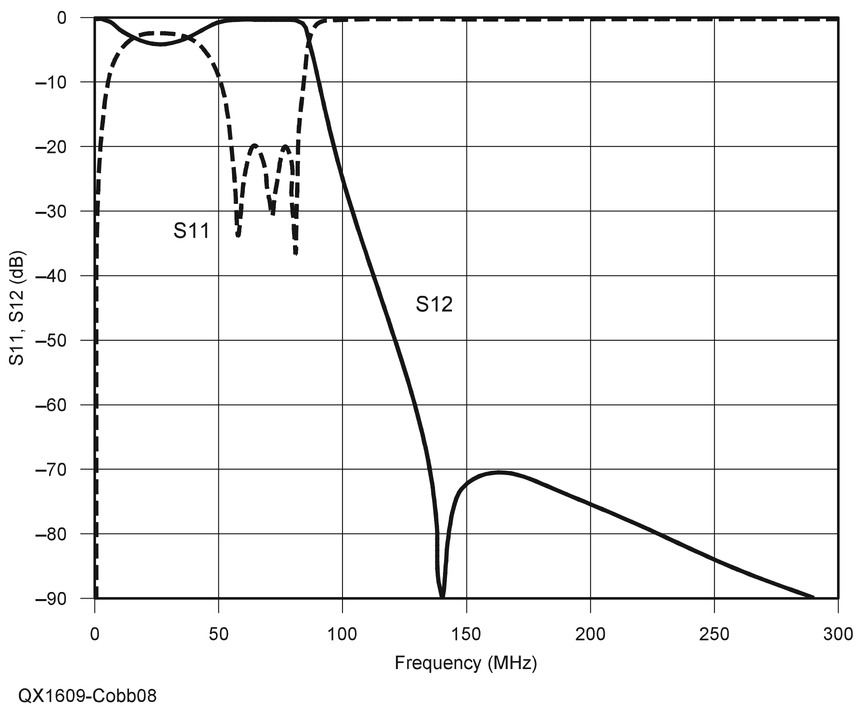


Figure 8 — Zolotarev transmission/reflection response.

upper 35% of the pass-pass. It is also seen that, with the single transmission zero placed at the 2nd harmonic frequency, a stop- band non-harmonically related lobe amplitude of 71 dB together with approximately 77 dB of attenuation at the 3rd harmonic is provided. Also, observe that the second reflection zero also corresponds to the desired 70 MHz band and that attenuation monotonically increases beyond the stop-band lobe unlike the ideal Elliptic filter.

As one might expect, there is a price to be paid for this apparent performance improvement. Figure 9 shows the comparative pass-band insertion-loss for both filter types examined together with the transition/stop-band attenuation characteristics.

From the design data, it is possible to quantify some practical aspects of realizability. For example, as mentioned earlier in respect of problems relating to the design realization of an ideal Elliptic filter, the inductance ratio for both Chebyshev and Zolotarev filters are considerably better. Additionally, the average value of the required inductors is also important as it is an indicator of the probable parasitic issue — larger inductors generally have larger self capacitance. To this end, these values are summarized in Table 2.

Since the Zolotarev approach is perceived as having significant benefits, the necessary component values have been calculated for most of the Amateur Radio bands and made available for the first time in Table 3.

Table 2
Ratio and average values for inductive elements.

Parameter	*CWAZ*	*ZWAZ*
Inductance ratio	1.614	1.460
Average inductance (nH)	128.7	63.3

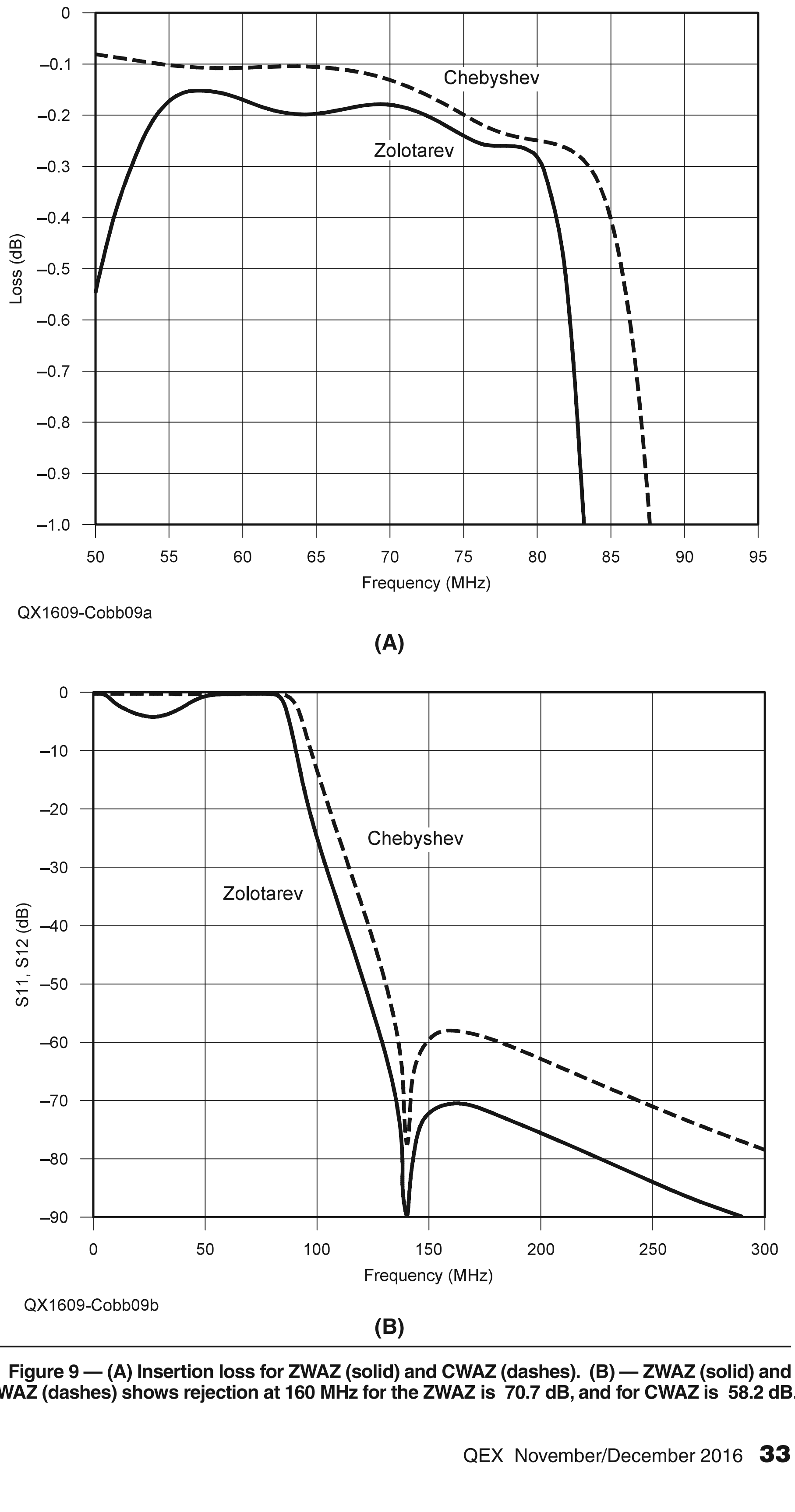


Figure 9 — (A) Insertion loss for ZWAZ (solid) and CWAZ (dashes). (B) — ZWAZ (solid) and CWAZ (dashes) shows rejection at 160 MHz for the ZWAZ is 70.7 dB, and for CWAZ is 58.2 dB.

Practical Testing of ZWAZ Designs

To demonstrate realizability, two design examples in different bands, have been constructed and tested against predicted performance. Component values have been selected from Table 3 and built using one of two commonly used inductor techniques. Both builds used ceramic surface mount capacitors so that the desired values were made up of parallel combinations of 2% tolerance parts. Some allowance was made for stray or parasitic capacitance.

In the first instance, a design chosen for the original 70 MHz band of interest was realized using the typical VHF technique of shielded air-cored solenoid inductors with a designed Q of 200 (Figures 10a and 10b). The measured result for transmission loss, 0.28 dB, is close to the target function, 0.19 dB, although the cut-off frequency is in error by approximately -1.5 MHz due to component tolerances and tuning inaccuracies. Good stop-band lobe definition with a peak attenuation ~71 dB is achieved with quite simple mechanical screening arrangements.

In the second instance, a design chosen for 60 m operation uses toroidal inductors with a designed Q of ~200. In this case,

Table 3
ZWAZ component values for the 2 to160 m Amateur Radio bands: (*C*, pF; *L*, nH; frequency, MHz).

Band, m	*f0*	*C1*	*C2*	*C3*	*C4*	*C5*	*L1*	*L2*	*L3*	*fc*
160	1.9	2360	7240	8340	2850	790.7	2240	1940	2840	2.21
80	3.65	1230	3770	4340	1490	411.6	1170	1010	1480	4.24
60	5.3	847.7	2600	2990	1020	283.4	802.7	696.7	1020	6.15
40	7.1	632.8	1940	2230	763.8	211.6	599.2	520.1	759.3	8.24
30	10.1	444.8	1360	1570	536.9	148.7	421.2	365.6	533.8	11.7
20	14.2	316.4	969.0	1120	381.9	105.8	299.6	260.0	379.7	16.5
17	18.1	248.2	760.2	875.8	299.6	83.0	235.0	204.0	297.9	21.0
15	21.2	211.9	649.0	747.7	255.8	70.9	200.7	174.2	254.3	24.6
12	24.2	185.7	568.6	655.0	224.1	62.1	175.8	152.6	222.8	28.1
10	29	154.9	474.5	546.6	187.0	51.8	146.7	127.3	185.9	33.7
6	51	88.1	269.8	310.8	106.3	29.5	83.4	72.4	105.7	59.2
4	70.2	64	196	225.8	77.2	21.4	60.6	52.6	76.8	81.5
2	145	31	95	109.3	37.4	10.4	29.3	25.5	37.2	168.3

the frequency response has little error with respect to that of the target function. However, the measured transmission loss of 0.88 dB is not acceptable in comparison with the target function value of 0.15 dB. The cores used are Amidon T68-2 (red) iron powder types, which, from the manufacturer's data, claims a considerably better Q than the 50 or so, implied from the measured result.

Subsequently, the inductor Q's were measured individually and found to be 250 or greater mitigating the choice of core type. The problem was ultimately traced to some of the capacitors. All of the 1000 pF, 1206 style surface mount components used were found to have a poor Q value of between 40 or 50 when measured at 5 MHz. As a consequence all of the capacitors were replaced by the more usual Silver Mica types having measured Q's of >600 at the test frequency.

This filter was re-measured after re-assembly and found to be now totally compliant with the simulated data with a more usable insertion-loss value of 0.17 dB – see Fig 11(A) and (B). Also note that the stop-band attenuation lobe is in good agreement with the predicted value of ~71 dB. Impressive is the fact that the stop-band attenuation exceeds the measurement noise floor (~85 dB) to well over 30 MHz without additional inductor shielding.

Conclusion

Two low-pass filter types (CWAZ and ZWAZ) have been synthesized and compared as an evaluation of a preferred output filter for power amplifiers in general. The ZWAZ filter is demonstrated to be significantly better, by design and realization, in terms of both harmonic and general stop-band attenuation, as compared to the CWAZ filter. In fact, the transition rate and the ultimate stop-band attenuation beyond the 3rd harmonic frequency all exceed that of the ideal Elliptic filter.

The two test samples generally show good agreement with the target design

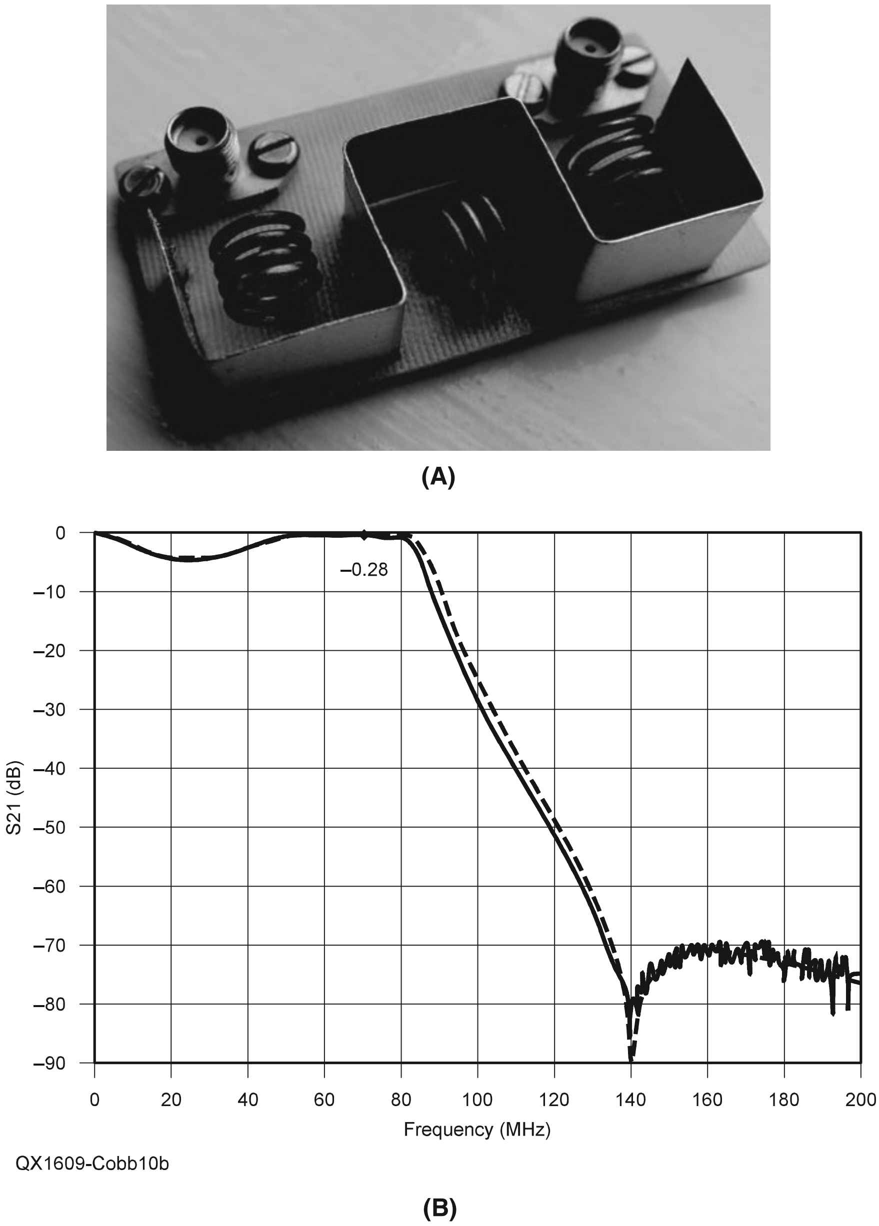


Figure 10 — (A) A 4 m band Zolotarev low-pass filter test piece construction. (B) — The 4 m band Zolotarev low-pass filter S21 test result – measured (solid), predicted (dashed). [Gary Cobb, G3TMG, photo]

characteristic without significant deviation the synthesized component values. The HF filter sample illustrates the need not only to ensure that the inductors are of adequate design in terms of achievable Q but proper consideration should be given to selecting good quality capacitors for the operational frequency chosen.

Gary Cobb, G3TMG, has held the same call sign since he was first licensed in 1964. He holds a BA degree in Mathematics, an MSc in Microwave Physics, and is a Life Member of the IEEE. His early professional career development took place in the defense industry where he designed high resolution microwave antenna interferometry systems. Later, research activities moved toward the realization of adaptive arrays for ship-borne radio communications in a Navy environment. Gary spent the last 20 years in military and commercial satellite payload engineering, developing output multiplexers, multi-port amplifiers and filter techniques appropriate for high-power geostationary systems. Now retired, Gary operates CW and SSB on HF and VHF amateur bands with a special interest in sporadic-E during the summer months.

Notes

[1]Radio Society of Great Britain, *Radio Communications Handbook*, all editions since 1938. Available from your ARRL dealer or the ARRL Bookstore, ARRL item no. 2040. Telephone 860-594-0355, or toll-free in the US 888-277-5289; **www.arrl.org/shop**; **pubsales@arrl.org**.

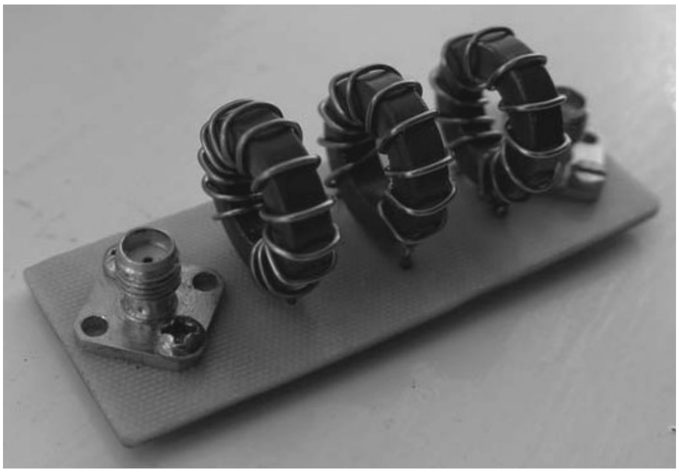
[2](In all editions since 1926),*The ARRL Handbook for Radio Communications,* 2016 Edition. Available from your ARRL dealer or the ARRL Bookstore, ARRL item no. 0413 (Hardcover 0420). Telephone 860-594-0355, or toll-free in the US 888-277-5289; **www.arrl.org/shop**; **pubsales@arrl.org**.

[3]E. Wetherhold, W3NQN, "Second-Harmonic-Optimized Low-Pass Filters", *QST*, Feb 1999, pp 44-46.

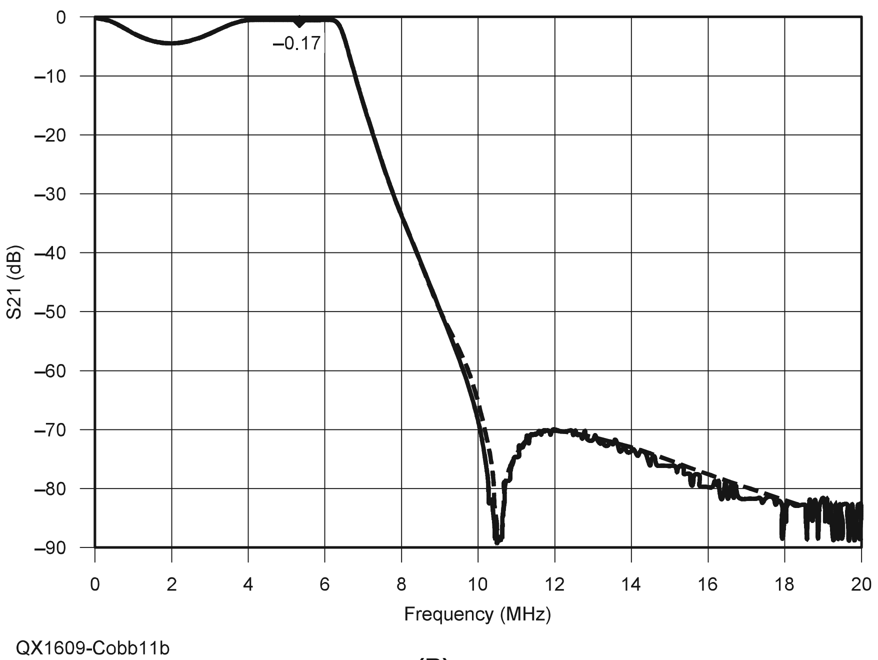
[4]J. Tonne, W4ENE, *OptLowpass*, **www.TonneSoftware.com**.

[5]R. Levy, "Generalized Rational Function Approximation in Finite Intervals Using Zolotarev Functions", *IEEE Trans, MTT*, Vol 18, Dec, 1970, pp 1051-1064.

[6]G. Cobb, G3TMG, "Zolotarev Low-pass Filter Design", *QEX*, Jul/Aug, 2016, pp 23-29.



(A)



(B)

Figure 11 — (A) A 60 m band Zolotarev low-pass filter test piece construction. (B) — The 60 m band Zolotarev low-pass filter S21 test result – measured (solid), predicted (dashed). [Gary Cobb, G3TMG, photo]