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Assignment # 2

FA21-BEE-241

Aden Fazal -3C

Q(1) a) $\lim_{(x,y)\to(2,1)} = \dfrac{x^2-2xy}{x^2-4y^2}$

$= \dfrac{(2)^2-2(2)(1)}{(2)^2-4(1)^2}$

$= 4-4$

$x^2 = 4y^2 \Rightarrow \boxed{x=2y}$

$\boxed{y=\dfrac{x}{2}}$

So Now

$\dfrac{x^2/16 - 2(x)(x/4)}{x^2 - 4(\frac{x^2}{16})} = \dfrac{\frac{x^2}{16}-\frac{x^2}{2}}{x^2-\frac{x^2}{4}} = \dfrac{(x^2-8x^2)\,4}{4\cancel{16}(4x^2-x^2)}$

$= \dfrac{-7x^2}{4(3x^2)} = \boxed{-7/12}$

b) $\lim_{(x,y)\to(0,0)} \dfrac{x-4y}{6y+7x}$

$6y = 7x$

$y = \dfrac{7x}{6}$ putting the limit

$\dfrac{x-4(\frac{7x}{6})}{6(\frac{7x}{6})+7x} = \dfrac{x-\frac{28x}{6}}{14x}$

$= \dfrac{6x-28x}{6(14x)} = \boxed{-\dfrac{11}{52}}$

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(c) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$

let $y=mx$

$$\frac{x^2-m^6x^6}{xm^3x^3}$$

$$= \frac{\cancel{x^2}(1-m^6x^3)}{\cancel{x^2}(x^2m^3)}$$

$$\left[\frac{1-m^6x^3}{x^2m^3}\right]$$

D) $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3-2e^2y}{6x+2y-3z}$

$$\frac{(-1)^2-4(0)e^2}{6(-1)+2(0)-3(4)} = \frac{-1-0}{-6+2-12}$$

$$= \frac{1}{16}$$

Q(2)

(a) $f(x,y) = \cos\left(\frac{x}{y}\right)$ $\vec{V} = (3,-4)$

$$f\nabla = \frac{\partial}{\partial x}\left(\cos\frac{x}{y}\right)i + \frac{\partial}{\partial y}\left(\cos(x/y)\right)j$$

$$= -\frac{1}{y}\sin(x/y)\,i + x(-\sin x/y)\left(-\frac{1}{y^2}\right)j$$

$$= -\frac{1}{y}\sin\left(\frac{x}{y}\right)i + \frac{x}{y}\sin(x/y)\,j$$

$$\frac{3i-4j}{\sqrt{9+16}} = \frac{3}{\sqrt{25}}i - \frac{4}{\sqrt{25}}j$$

$$D_{\vec{u}}f = \frac{-3}{5y}\sin\left(\frac{x}{y}\right) + \frac{4x}{5y^2}\sin(x/y)$$

$$D_{\vec{u}}f = \frac{1}{5y}\sin(x/y)\left(\frac{4x}{y}-3\right)$$

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(b) $f(x,y,z) = x^2y^3 - 4xz \quad ; \vec{v} = (-1,2,0)$

$$\nabla f = (2y^3x\,i + (-4z)i) + 3y^2x^2j^3 - 4xk$$

$$\vec{v} = \frac{-i + 2j + 0k}{\sqrt{1+4}} \Rightarrow \frac{-1}{\sqrt{5}}i + \frac{2}{\sqrt{5}} + 0k$$

$$D_{\vec{u}}f = \frac{-1}{\sqrt{5}}(2y^2x - 4z) + \frac{2}{\sqrt{5}}(3y^3x^2) + 0$$

Q(3)

$$f(x,y,z) = 4x^2 - y^2e^{3xz}$$

$$\nabla f = (4 - y^2e^{3xz}(3x))\,i - (2ye^{3xz})j + -(y^2e^{3xz} - 3x)k$$

$$\nabla f_{(3,-1,0)} = 4 - (-1)^2e^{3(3)(0)}(0)(3)i - 2(-1)e^{3(3\times3\times0)}j - (-1)e^{(3)(3)(0)} \cdot 3(3)\hat{k}$$

$$= (4-0)i - 2j - 9k$$

$$v = (-1,4,2)$$

$$v = \frac{-1j + 4j + 2k}{\sqrt{1+16+4}} \Rightarrow \frac{-1}{\sqrt{21}}i + \frac{4}{\sqrt{21}}j + \frac{2}{\sqrt{21}}k$$

$$\frac{-1}{\sqrt{21}}(4) \quad \frac{-2(4)}{\sqrt{21}} \quad \frac{-9(2)}{\sqrt{21}}$$

$$\frac{-4}{\sqrt{21}} \quad \frac{-8}{\sqrt{21}} \quad \frac{-18}{\sqrt{21}} \longrightarrow \frac{-4-8-18}{\sqrt{21}} = \frac{-32}{\sqrt{21}}$$

Q[4].

(a) $f(x,y) = \sqrt{x^2+y^3}$ at $(-2,3)$

$$\nabla f = \frac{1}{2}(x^2+y^3)^{-1/2}(2x)i + \frac{1}{2}(x^2+y^3)^{-1/2}(2y^2)j$$

$$\nabla f(-2,3) = (4+9)^{-1/2}(-2)i + \frac{1}{2}(4+3)^{-1/2}(3)(9)j$$

$$= \frac{2}{\sqrt{13}}i + \frac{27}{2\sqrt{13}}j$$

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(b) $f(x,y,z) = e^{2x}\cos(y-2z)$ at $(4,-2,0)$

$\nabla f = e^{2x}\cdot 2\cos(y-2x)i + e^{2x}(-\sin(y-2x)j$
$+ e^{2x}(-\sin(y-2x)(-2)k$

$\nabla f(4,-2,0) = e^{4x}\cdot 2\cos(-2-0)\hat{i} + e^{4x}(-\sin(-2))j$
$+ e^{4x}(-\sin(-2)(-2)$

$[\nabla f(4,-2,0) = e^{4x}(-2\cos 2\hat{i} - \sin(-2)\hat{j} + 2\sin(-2)\hat{k})]$

Q5

(a) $\vec{F} = x^2y\,i - (z^3-3x)j + 4y^2\hat{k}$

$\nabla = 2xy\,i - 0j + 0\hat{k}$

Div $f = \nabla f\cdot f$

$(2xy\,i)\cdot(x^2y\,i - (z^3-3x)j + 4y^2k)$

~~Df~~ Div $f = 2x^3y^2$

Curl $\nabla f \times f$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & z^3-3x & 4y^2 \end{vmatrix}$$

$= (8y+3z^2)i - (0-0)j) + (3x^2-x^2)\hat{k}$

$[= (8y+3z^2)i - 0j + (2x^2)k]$

(b) $F = (3x+2z^2)i + \frac{x^3y^2}{z}j - (z-7x)\hat{k}$

Div $= \nabla f\cdot f$

$(\frac{\partial}{x}i + \frac{\partial}{y}j + \frac{\partial}{\partial z}k)\cdot((3x+2z^2 + \frac{x^3y^2}{z}j - (z-7x)\hat{k})$

$= 2 + \frac{2x^3y}{z} - (1)$

$= 2 + \frac{2x^3y}{z} - 1$

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Curve = $\nabla f \times f$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x+2z^2 & \frac{x^3y^2}{z} & 1-7x \end{vmatrix} = \left(0+\frac{x^3y^2}{z^2}\right)i - (7-4x)j + \left(\frac{3x^2y^2}{z} - 0\right)\hat{k}$$

$$\frac{x^3y^2}{z^2}\, i - (7-4z)j + \left(\frac{3x^2y^2}{z}\right)\hat{k}$$

Q6:-

(a) $\vec{F} = \left(4y^2 + \frac{3x^2y}{z^2}\right)i + \left(8xy + \frac{x^3}{z^2}\right)j + \left(11 - \frac{2x^3}{z^2}y\right)\hat{k}$

Vectors are Conservative only if :-

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z^2}, \quad \frac{\partial N}{\partial x} = 8x + \frac{3x^2}{z^2}$$

$$\frac{\partial P}{\partial x} = -\frac{6x^3y}{z^3} \qquad \frac{\partial M}{\partial z} = \frac{-6x^2y}{z}$$

$$\frac{\partial M}{\partial z} = -\frac{6x^2y}{z}$$

hence Vector is Conservative

(b) $\vec{F} = \underset{M}{6x}\, i + \underset{N}{(2x - y^2)}j + \underset{P}{(6z - x^3)}k$

$$\frac{\partial M}{\partial y} = 0 \quad ; \quad \frac{\partial N}{\partial x} = 2 \qquad \frac{\partial N}{\partial z} = 0$$

$$\frac{\partial P}{\partial y} = 0 \qquad \frac{\partial M}{\partial z} = 0 \qquad \frac{\partial P}{\partial x} = -3x^2$$

since $\partial P/\partial x$ & $\partial N/\partial x$ are Not Equal

it is Non - Conservative.

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Q7:-

(a) $z = \frac{x^2 - w}{y^4}$; $x = t^3 + 7$; $y = \cos(2t)$

$w = 4t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial z}{\partial w} \cdot \frac{\partial w}{\partial t}$$

$$= \left(\frac{2x}{y^4} \cdot 3t^2\right) + 4\left(\frac{1}{y^5}(x^2 - w)\right) \cdot (-\sin 2t)(2) + -\frac{1}{y^4} \cdot 4$$

$$= \frac{6xt^2}{y^4} + \frac{8\sin 2t(x^2 - w) - 2}{y} - \frac{4}{y^4}$$

$$= \frac{2}{y^4}\left(3xt^2 + \frac{4\sin 2t(x^2 - w)}{y} - 2\right)$$

(b) $z = x^2y^4 - 2y$, $y = \sin(x^2)$

$$\frac{dz}{dt} = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$(4y^3x^2 - 2) \cdot \cos(x) \cdot 2x$$

$$= 8y^3x^3\cos(x^2) - 4x\cos x^2)$$

(c) Find $\frac{dy}{dx}$ for $x^2y^4 - 3 = \sin(xy)$

$$\frac{d}{dx}(x^2y^4 - 3) = \frac{d}{dx}(\sin(xy))$$

$$(2xy^4) + 4y^3\frac{d}{dx}y\,x^2 = \cos(xy)\left(1y + \frac{dy}{dx}x\right)$$

$$2xy^4 + 4y^3\frac{dy}{dx}x^2 = y\cos xy + \frac{dy}{dx}x\cos xy$$

$$2xy^4 - y\cos xy = x\frac{dy}{dx}\cos xy - 4y^3x^2\frac{dy}{dx}$$

$$2xy^4 - y\cos xy \cdot \frac{dy}{dx}(x\cos xy - 4y^3x^2)$$

$$\frac{dy}{dx} = \frac{2xy^4 - y\cos xy}{x\cos xy - 4y^3x^2}$$