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Assignment # 1
Multivariable Calculus
FA21 - BEE - 241
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(1) $4\hat{i}+4\hat{j}+\hat{k} \to A$, $-4\hat{i}+3\hat{j}-4\hat{k} \to B$
a) $4\hat{i}-\hat{j}-2\hat{k} \to C$ are position Vectors

Answer: A (4, 4, 1)
B (-4, 3, -4)
C (4, -1, -2)

$\vec{AB} = \vec{OA} - \vec{OB}$

$\begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix}$

$\to \vec{AB} = -8\hat{i}+7\hat{j}-5\hat{k}$

$\vec{AC} = \vec{OC} - \vec{OA} \Rightarrow \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} - \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$

$\to \vec{AC} = 3\hat{j}-3\hat{k}$

$n = \vec{AB} \times \vec{AC}$ $\begin{bmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{bmatrix}$

$= \begin{bmatrix} 7 & -5 \\ 3 & -3 \end{bmatrix} i - \begin{bmatrix} -8 & -5 \\ 0 & -3 \end{bmatrix} j + \begin{bmatrix} -8 & 7 \\ 0 & 3 \end{bmatrix} k$

$n = -6i - 24j - 24\hat{k}$

Simplify $\Rightarrow \boxed{+i + 4\hat{j} + 4\hat{k}}$

$d = a.n$

$(4i-4j+k).(i+4j+4\hat{k})$

$\boxed{= -8}$

Equation of plane $r.n = d$

$r(\hat{i}+4\hat{j}+4\hat{k}) = -8$

$(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+4\hat{j}+4\hat{k}) = -8$

Answer:- $\boxed{x+4y+4z+8=0}$

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(b) Find perpendicular Distance from O to ABC

ABC ⇒ $x+4y+4z+8=0$

$$\frac{d}{|n|} = \frac{+8}{\sqrt{1^2+4^2+4^2}} = \frac{8}{3.3} = \boxed{1.39}$$

(c) Point D → position vector $2\hat{i}+3\hat{j}-3\hat{k}$

~~line OD ⇒ $r = a+\lambda b$~~

$r = ($

ABC plane ⇒ $x+4y+4z+8=0$

OD ⇒ $2\hat{i}+3j-3\hat{k}$

$$D = \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$$

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix} \Rightarrow \begin{bmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} = -8$$

$$= 2\lambda + \cancel{12\lambda} - \cancel{12\lambda} = -8$$

$$\frac{2\lambda}{2} = \frac{-8}{2}$$

$$\begin{bmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{bmatrix} = \begin{bmatrix} -8 \\ -12 \\ 12 \end{bmatrix} \qquad \boxed{\lambda = -4}$$

⇒ $\boxed{\text{point} = (-8, -12, 12)}$

Q2

(a) A ⇒ $7i+4j-k$ B ⇒ $11i+3j$ C ⇒ $2i+6j+3k$

D ⇒ $2i+7j+\lambda k$

$$\vec{AB} = \vec{OA} - \vec{OB}$$

$$\begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 11 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{CD} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 7 \\ \lambda \end{bmatrix}$$

$$r_2 = \begin{bmatrix} 0 \\ 1 \\ \lambda-3 \end{bmatrix}$$

$$r_1 = \overset{\vec{OA}}{\begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix}} + \lambda \overset{\vec{AB}}{\begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}}$$

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$$b_1 \times b_2 = \begin{bmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & 1 & \lambda-3 \end{bmatrix}$$

$$i\left(-(\lambda-3)-1\right) - \hat{j}\left(4(\lambda-3)\right) + \hat{k}(4)$$

$$= (2-\lambda)i - (4\lambda-12)j + 4\hat{k}$$

$$\vec{OC} - \vec{OA}$$

$$(a_2 - a_1) = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 4 \end{bmatrix}$$

$$|b_1 \times b_2| = \sqrt{(2-\lambda)^2 + (4\lambda-12)^2 + (4)^2}$$

$$= \sqrt{17\lambda^2 - 100\lambda + 164}$$

$$(b_1 \times b_2) \cdot (a_2 - a_1) = \left[(2-\lambda)i - (4\lambda-12)j + 4\hat{k}\right] \cdot (-5\hat{i} + 2j + 4\hat{k})$$

$$\Rightarrow -5(2-\lambda)i - 2(4\lambda-12)j + 16\hat{k}$$

$$10 + 5\lambda - 8\lambda + 24 + 16$$

$$= \boxed{30 - 3\lambda}$$

$$d = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$3 = \frac{30 - 3\lambda}{\sqrt{17\lambda^2 - 100\lambda + 164}}$$

$$(3)^2 = \frac{(30-3\lambda)^2}{\left(\sqrt{17\lambda^2 - 100\lambda + 164}\right)^2} \Rightarrow 9 = 900 + 9\lambda^2 - 180\lambda$$

$$9(17\lambda^2 - 100\lambda + 164) = 900 + 9\lambda^2 - 180\lambda$$

$$153\lambda^2 - 900\lambda + 1476 - 900 - 9\lambda^2 + 180\lambda = 0$$

$$144\lambda^2 - 720\lambda + 576 = 0$$

factorize $\Rightarrow x_1 = 4$ or $x_2 = 1$

put in eq

$$\lambda^2 - 5\lambda + 4 = 0 \Rightarrow (4)^2 - 5(4) + 4 = 0$$

$$0 = 0$$

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$\pi_1 \cdot \pi_2$

$Q = \cos^{-1}(\pi_1 \pi_2)$

$$\begin{bmatrix} 5 \\ 13 \\ -7 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 25 \\ -7 \end{bmatrix}$$

$= 40 + 325 + 49 = 414$

$|n_1| = \sqrt{(5)^2 + (13)^2 + (-7)^2} = 9\sqrt{3}$

$|n_2| \sqrt{8^2 + 25^2 + (-7)^2} = 3\sqrt{82}$

$$\cos^{-1}\left(\frac{414}{9\sqrt{3} \times 3\sqrt{82}}\right)$$

$= \boxed{Q = 12.15^\circ}$

Q3

a) find value of t

$l_1 = \quad ti + j \quad \& \quad -2j \; -j$

$l_2 = \quad j - tk \quad \& \quad -2j \; +k$

Shortest d between L_1 & L_2 is $\sqrt{21}$

$r_1 = \vec{OA} + \lambda \vec{AB}$

$r_2 = \vec{OA} + \mu \vec{AB}$

$$r_1 = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix} \qquad r_2 = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + u \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

Normal vector

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$(b_1 \times b_2) = \begin{vmatrix} i & j & k \\ -2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix} = -\hat{i} + 2\hat{j} + 4\hat{k}$$

$= \sqrt{(-1)^2 + (2)^2 + (4)^2}$

$= \sqrt{21}$

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Now Put 1

$\Rightarrow (1)^2 - 5 + 4 = 0$

$0 = 0$

Both points hence satisfy the Equation.

(b)

i) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$$\begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix}$$

$$\pi_1 = (7i + 4j - k) + s(4i + j + k) + t(-5i + 3j$$

ii) $\pi_2 = \overrightarrow{r_2} = \overrightarrow{OA} + \lambda\overrightarrow{AB} + 4\overrightarrow{AD}$

when $\lambda = 4$

$$\overrightarrow{AD} = \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$\pi_2 = r_2 \quad \overrightarrow{OA}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ +1 \end{bmatrix} + 4\begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$x = 7 + 4\lambda - 5u$

$y = 7 - \lambda + 43$

$z = -1 + \lambda + 54$

Eq $\Rightarrow$ (ii) & Eq (3)

$y = 4 - \cancel{\lambda} + 3u$

$z = 1 + \cancel{\lambda} + 5u$

$y + z = 3 + 8u \rightarrow$ (iv)

Now $y = 4 - \lambda + 3u$

$\times 4$

$4y = 16 - 4\lambda + 12u \rightarrow$ (v)

Eq 1 & Eq V

$x + 4y = 23 + 7u$

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$$\frac{x+4y-23}{7}=u$$

put the Values in Eq (iv)

$$y+z=3+8\left(\frac{x+4y-23}{7}\right)$$

$$y+z=3+\frac{8x+32y-184}{7}$$

Multiply by 7

$$7(y+z)=\cancel{3}+21+8x+32y-184$$

$$7y+7z=21+8x+32y-184$$

$$-8x-32y+7y+7z=-184$$

$$\Rightarrow \boxed{8x+25y-7z=184}$$

(c)

$$x_1=r_1=\vec{OA}+\lambda\vec{AB}+u\vec{AD}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7\\4\\-1\end{bmatrix}+\lambda\begin{bmatrix}4\\-1\\1\end{bmatrix}+u\begin{bmatrix}-5\\3\\2\end{bmatrix}$$

$$x=7+4\lambda-5u$$

$$4(y=4-\lambda+3u)$$

$$x+4y=23+7u$$

$$\frac{x+4y-23}{7}=u$$

$$z=-1+\lambda+2u$$

$$y=4-\lambda+3u$$

$$z+y=3+5u$$

$$z+y=3+5\left(\frac{x+4y-23}{7}\right)$$

$$z+y=3+\frac{5x+20y-115}{7}$$

$$7(z+y)=21+5x+2y-115$$

$$\Rightarrow -5x-13y+7z+94=0$$

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[redacted]

$$|a| = \sqrt{1^2 + \left(\frac{21}{5}\right)^2} = 4.3$$

$$|b| = \sqrt{(5)^2 + (6)^2 + (7)^2} = 10.49$$

$$\theta = \cos^{-1}\left(\frac{23.4}{4.3 \times 10.49}\right)$$

$$\theta = \cos^{-1}\left(\frac{23.4}{45.11}\right)$$

$$\boxed{\theta = 59.34^\circ}$$

(D) $\pi_1 \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix}, \pi_2 \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

Dot prodect $= 21 - 6 = -27$

$$|a| = \sqrt{(-21/5)^2 + (1)^2} = 4.3$$

$$|b| = \sqrt{(5)^2 + (-6)^2 + (7)^2} = 10.49$$

$$\theta = \cos^{-1}\left(\frac{-27}{4.3 \times 10.49}\right) = \theta = 126.78$$

$$\theta = 180^\circ - 126.76 = \boxed{53.23^\circ}$$

(Q5). Mid point

$$\left(\frac{-2-6}{2}\right), \left(\frac{-1-3}{2}\right)$$

$$\left(\frac{-8}{2}, \frac{-4}{2}\right)$$

$$= (-4, -2)$$

Eq $\rightarrow$ $(x+h)^2 + (y-k)^2 = r^2$

$$(x+4)^2 + (y+2)^2 = r^2$$

$$= (-2+4)^2 + (-1+2)^2 = r^2$$

$$2^2 + 1^2 = r^2$$

$$5 = r^2$$

$$\sqrt{5} = r$$

Eq $= (x+4)^2 + (y+2)^2 = 5$

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Q5

b) Equation of Circle

$(x-h)^2 + (y-k)^2 = r^2$

16 $x = 0$ $y = b$

at point $(4, 0)$

$(4)^2 + (-b)^2 = (r)^2$

$16 + b^2 = r^2$

at point $(0, 2)$

$(0)^2 + (2-b)^2 = r^2$

$16 + b^2 = 4 - 4b + b^2$

$16 + \cancel{b^2} - \cancel{b^2} - 4 + 4b = 0$

$12 + 4b = 0$

$4b = -12$

$\boxed{b = -3}$

put the values in Eq (1)

$r^2 = (4)^2 + (-3)^2$

$\boxed{r^2 = \pm 5}$

(part c)

$y^2 = 100x$

$y^2 = 4ax$

$a = 25$

Eq of directrix $= x = -a$

$x = -25$

(Part D)

$x^2 = 24y$

$x^2 = 4ay$

$4a = 24$

$a = 8$

so focus $= (a, 0)$

$(8, a)$

Eq of directrix $\boxed{x = -8}$

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$(a_2 - a_1) = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} - \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix}$

$= -t\hat{i} + t\hat{k}$

$D = \dfrac{(-\hat{i} + 2\hat{j} + 4\hat{k}) \cdot (-t\hat{i} + t\hat{k})}{\sqrt{21}}$

$(\sqrt{21})(\sqrt{21}) = (-i + 2j + 4k) \cdot (-ti + tk)$

$21 = 5t + 4 \, / \cancel{\sqrt{21}}$

$\frac{21}{5} = \frac{5t}{5}$

$\boxed{\frac{21}{5} = t}$

(b) $\vec{r_1} = \frac{21}{5} i + j + \lambda(2\hat{i} - j)$

$\vec{r_2} = j - \frac{21}{5} \hat{k} + u(-2j + \hat{k})$

$\pi_1 = r = \vec{OA} + \lambda \vec{AB} + u\vec{AC}$

$\boxed{r = \frac{-21}{5} i + j + \lambda(-2i - j) + u(-2j + k)}$

(c) from L_2 & π_2 find Acute Angle..

$\lambda_2 = 5x - 6y + 5z = 0$

$\lambda_2 = \frac{x-0}{0} \quad \lambda_2 = \frac{y-1}{2}$

$\lambda_2 = \frac{2-4 \cdot 2}{1}$

from l_2 direction Vector $= \{0, -2, 1\}$

π_2 Normal Vector $= (5, -6, 7)$

$\cos\theta = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{|a||b|}$

$(a \cdot b) = \begin{bmatrix} 0 \\ 1 \\ 21/5 \end{bmatrix} \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$= -6 + 29.4 = 23.4$

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(part e) $\left(\frac{x}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{5^2} + \frac{y^2}{4^2}$$

$$\Rightarrow a = 5$$

$$b = 4$$

$$c = \sqrt{a^2 - b^2} = \sqrt{(5)^2 - (4)^2}$$

$$c = \pm 3$$

$$F_1 = (3, 0)$$

$$F_2 = (-3, 0)$$

length Of Major $2a$

$$2(5) = 10$$

(Part f)

Major Axis $= 10$

Minor " $= 8$

$2a = 10$

$a = 5$

$2b = 8$

$b = 4$

<u>Equation</u>

$$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$