M T W T F S

DATE: __/__/____

Assignment # 1
Multivariable Calculus
FA21 - BEE - 241
Aden fazal

(1) a) $4\hat{i} + 4\hat{j} + \hat{k} \rightarrow A$, $-4\hat{i} + 3\hat{j} - 4\hat{k} \rightarrow B$

$4\hat{i} - \hat{j} - 2\hat{k} \rightarrow C$ are position Vectors

Answer:

$A(4, 4, 1)$

$B(-4, 3, -4)$

$C(4, -1, -2)$

$\vec{AB} = \vec{OA} - \vec{OB}$

$$\begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix}$$

$\rightarrow \vec{AB} = -8\hat{i} + 7\hat{j} - 5\hat{k}$

$\vec{AC} = \vec{OC} - \vec{OA} \Rightarrow \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} - \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$

$\rightarrow \vec{AC} = 3\hat{j} - 3\hat{k}$

$n = \vec{AB} \times \vec{AC}$

$$\begin{bmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -5 \\ 3 & -3 \end{bmatrix} i - \begin{bmatrix} -8 & -5 \\ 0 & -3 \end{bmatrix} j + \begin{bmatrix} -8 & 7 \\ 0 & 3 \end{bmatrix} k$$

$n = -6i - 24j - 24\hat{k}$

Simplyify $\Rightarrow$ $\boxed{+i + 4j + 4\hat{k}}$

Equation of plane $r \cdot n = d$

$d = a \cdot n$

$(4i - 4j + k) \cdot (i + 4j + 4\hat{k})$

$\boxed{= -8}$

$r(i + 4j + 4\hat{k}) = -8$

$(xi + yj + z\hat{k}) \cdot (\hat{i} + 4j + 4\hat{k}) = -8$

Answer:- $\boxed{x + 4y + 4z + 8 = 0}$

M T W T F S DATE: __/__/__

(b) Find perpendicular Distance from O to ABC

ABC ⇒ $x+4y+4z+8=0$

$$\frac{d}{|n|} = \frac{+8}{\sqrt{1^2+4^2+4^2}} = \frac{8}{3.3} = \boxed{1.39}$$

(c) Point D → position vector $2\hat{i}+3\hat{j}-3\hat{k}$

~~In OD ⇒ $r = a \times \lambda b$~~

~~$r =$~~

ABC plane ⇒ $x+4y+4z+8=0$

OD ⇒ $2\hat{i}+3j-3\hat{k}$

$$D = \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$$

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix} \Rightarrow \begin{bmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} = -8$$

$$= 2\lambda + \cancel{12\lambda} - \cancel{12\lambda} = -8$$

$$\frac{2\lambda}{2} = \frac{-8}{2}$$

$$\begin{bmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{bmatrix} = \begin{bmatrix} -8 \\ -12 \\ 12 \end{bmatrix} \qquad \boxed{\lambda = -4}$$

⇒ $\boxed{\text{point} = (-8, -12, 12)}$

Q2

(a) A → $7i+4j-k$ B ⇒ $11i+3j$ C ⇒ $2i+6j+3k$

D ⇒ $2i+7j+\lambda k$

$$\vec{AB} = \vec{OA} - \vec{OB} \qquad \vec{CD} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 7 \\ \lambda \end{bmatrix}$$

$$\begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 11 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} \qquad r_2 = \begin{bmatrix} 0 \\ 1 \\ \lambda-3 \end{bmatrix}$$

$$r_1 = \underbrace{\begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix}}_{\vec{OA}} + \lambda \underbrace{\begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}}_{\vec{AB}}$$

M T W T F S DATE: __/__/__

$$b_1 \times b_2 = \begin{bmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & 1 & \lambda-3 \end{bmatrix}$$

$$i\left(-(\lambda-3)-1\right) - \hat{j}\left(4(\lambda-3)\right) + \hat{k}(4)$$

$$= (2-\lambda)i - (4\lambda-12)j + 4\hat{k}$$

$$(a_2 - a_1) = \overset{\vec{OC}}{\begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}} - \overset{\vec{OA}}{\begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix}} = \begin{bmatrix} -5 \\ 2 \\ 4 \end{bmatrix}$$

$$|b_1 \times b_2| = \sqrt{(2-\lambda)^2 + (4\lambda-12)^2 + (4)^2}$$

$$= \sqrt{17\lambda^2 - 100\lambda + 164}$$

$$(b_1 \times b_2) \cdot (a_2 - a_1) = \left[(2-\lambda)i - (4\lambda-12)j + 4\hat{k}\right] \cdot (-5\hat{i} + 2j + 4\hat{k})$$

$$\Rightarrow -5(2-\lambda) - 2(4\lambda-12) + 16$$

$$10 + 5\lambda - 8\lambda + 24 + 16$$

$$= \boxed{30 - 3\lambda}$$

$$d = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$3 = \frac{30 - 3\lambda}{\sqrt{17\lambda^2 - 100\lambda + 164}}$$

$$(3)^2 = \frac{(30-3\lambda)^2}{\left(\sqrt{17\lambda^2 - 100\lambda + 164}\right)^2} \Rightarrow 9 = 900 + 9\lambda^2 - 180\lambda$$

$$9(17\lambda^2 - 100\lambda + 164) = 900 + 9\lambda^2 - 180\lambda$$

$$153\lambda^2 - 900\lambda + 1476 - 900 - 9\lambda^2 + 180\lambda = 0$$

$$144\lambda^2 - 720\lambda + 576 = 0$$

factorize $\Rightarrow x_1 = 4$ or $x_2 = 1$

put in eq

$$\lambda^2 - 5\lambda + 4 = 0 \Rightarrow (4)^2 - 5(4) + 4 = 0$$

$$0 = 0$$

M T W T F S DATE: __/__/__

Now Put 1

$\Rightarrow (1)^2 - 5 + 4 = 0$

$0 = 0$

Both points hence Satisfy the Equation.

(b)

i) $\vec{AD} = \vec{OD} - \vec{OA}$

$$\begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix}$$

$$\pi_1 = (7i + 4j - k) + s(4i + j + k) + t(-5i + 3j + 2k)$$

ii) $\pi_2 = \vec{r_2} = \vec{OA} + \lambda\vec{AB} + \mu\vec{AD}$

when $\lambda = 4$

$$\vec{AD} = \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$\pi_2 = r_2 \quad \vec{OA}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ +1 \end{bmatrix} + \mu \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$x = 7 + 4\lambda - 5\mu$

$y = 7 - \lambda + 4\mu$

$z = -1 + \lambda + 5\mu$

Eq $\Rightarrow$ (ii) & Eq (3)

$y = 4 - \lambda + 3\mu$

$z = 1 + \lambda + 5\mu$

$y + z = 3 + 8\mu \rightarrow$ (iv)

Now $y = 4 - \lambda + 3\mu$

$\times 4$

$4y = 16 - 4\lambda + 12\mu \rightarrow$ (v)

Eq 1 & Eq V

$x + 4y = 23 + 7\mu$

$$\frac{x+4y-23}{7}=u$$

put the Values in Eq (IV)

$$y+z=3+8(x+4y-23)$$

$$y+z=\frac{3+8x+32y-184}{7}$$

Multiply by 7

$$7(y+z)=21+8x+32y-184$$

$$7y+7z=21+8x+32y-184$$

$$-8x-32y+7y+7z=-184$$

$$\Rightarrow \boxed{8x+25y-7z=184}$$

(c)

$$x_1=r_1=\vec{OA}+\lambda\vec{AB}+\mu\vec{AD}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7\\4\\-1\end{bmatrix}+\lambda\begin{bmatrix}4\\-1\\1\end{bmatrix}+\mu\begin{bmatrix}-5\\3\\2\end{bmatrix}$$

$$x=7+4\lambda-5\mu$$

$$y=4-\lambda+3\mu$$

$$x+4y=23+7\mu$$

$$\frac{x+4y-23}{7}=\mu$$

$$z=-1+\lambda+2\mu$$

$$y=4-\lambda+3\mu$$

$$z+y=3+5\mu$$

$$z+y=3+5\left(\frac{x+4y-23}{7}\right)$$

$$z+y=\frac{3+5x+20y-115}{7}$$

$$7(z+y)=21+5x+20y-115$$

$$\Rightarrow -5x-13y+7z+94=0$$