Illumination Modeling and Normalization for Face Recognition

Haitao Wang[1], Stan Z. Li[2], Yangsheng Wang[1], Weiwei Zhang[1]

[1]*Institute of Automation*
Chinese Academy of Sciences
Beijing, 100080, China
{htwang, wys, wwzhang}@nlpr.ia.ac.cn

[2]*Microsoft Research Asia*
Beijing Sigma Center,
Beijing, 100080, China
szli@Microsoft.com

Abstract

In this paper, we present a general framework for face modeling under varying lighting conditions. First, we show that a face lighting subspace can be constructed based on three or more training face images illuminated by non-coplanar lights. The lighting of any face image can be represented as a point in this subspace. Second, we show that the extreme rays, i.e. the boundary of an illumination cone, cover the entire light sphere. Therefore, a relatively sparsely sampled face images can be used to build a face model instead of calculating each extremely illuminated face image. Third, we present a face normalization algorithm, illumination alignment, i.e. changing the lighting of one face image to that of another face image. Experiments are presented.

1. Introduction

As face recognition techniques advance, more researchers have focused on challenging issues arising from illumination and pose. Varying illumination is one of the most difficult problems and has received much attention [1-4][7-25] in recent years. It is know that image variation due to lighting changes is larger than that due to different personal identity.

Because lighting direction changes alter the relative gray scale distribution of face image, the traditional histogram equalization method used in image processing [30] and face detection [31] for image normalization only transfers the holistic image gray scale distribution from one to another. This processing ignores the face-specific information and can not normalize these gray level distribution variations. To deal with this problem, researchers have made many breakthroughs in recent years.

Adini [1] has compared different face representations, such as edge map, image intensity derivatives, and images convolved with 2D Gabor-like filters, under lighting direction changes. Their results demonstrated that none of these algorithms were robust to variations due to light direction changes. The main drawback of this kind of approaches is that the most valuable information, gray value, is discarded and person's discriminative information in face image is weakened in perusing so called "illumination invariant features".

The quotient Image method [3][4] proposed by Shashua and Tammy provided an invariant approach to deal with the illumination variation. Under assumption of Lambertian model without shadow and ideal class, they deduced that the image ratio (Quotient image) between a test image and linear combination of three non-coplanar illuminated images reflects only the texture information, which is illumination free. The main contribution of this method is that it provided very simple and practical approach for robust recognizing face under varied lighting directions with only 2D images. It has been used in some image based rendering papers [5][6]. A drawback is that this method failed in case of shadow [3][4].

The illumination Cone method [7][8][9] theoretically explained the property of face image variations due to light direction changes. In this algorithm, both self-shadow and cast-shadow were considered and its experiment results outperformed most existing methods. The main drawbacks of illumination cone are the computational cost and the strict requirement of seven input images per person.

Terence Sim et al [16] proposed a statistical algorithm for modeling face. This method used abundant training face images illuminated differently in which the general shape of the face and its integrability are statistically implied. These images have enough information to construct face lighting space.

Recently two researchers, Ramamoorthi [10-12] and Basri [13-15], independently developed spherical harmonic representation in face images variation under various lighting conditions. This original representation explained why images of an object under different lighting conditions can be described by low dimensional subspace in some empirical experiments [10] [11]. The implementation of this algorithm requires knowledge of face's albedos and surface normals, however no practical method was provided in their original work [10-15].

The above approaches have a common characteristic that they all analyze face images in 3D space instead of 2D image space. They are based on a more general face representation depicted in Figure 1. In fact, such a representation has been used by computer graphics for Image-based-rendering (IBR) for a long time. However, the face analysis problem is an inverse procedure of IBR, i.e. decomposing the image into three parts, 3D shape, texture and lighting.

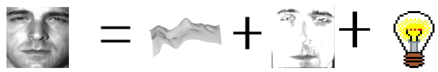


Figure 1. A general face representation

The rest of this paper is organized as follow. In section 2, we present a face lighting subspace model and show how the illumination of a face image can be easily estimated. Section 3 is the implementation of our illumination alignment approach in global and local face lighting space. The experiment results are discussed and analyzed in section 4. Finally, we make a conclusion and suggest the future work in section 5.

2. Face and Lighting Modeling

2.1 Notions for Face and lighting Subspaces

Let us first look at the following experiment shown in Figure 2. Fig.2(a) shows ten persons' frontal face images under 12 different illumination directions. 2(b) shows the ten persons' face manifolds (trajectories) in a 3-D face PCA subspace where the PCA is derived based on all training examples of all persons. It is clear that all the manifolds have very similar shape. 2(c) is similar to 2(b) but the PCA is derived based on face images of only one person. The red curve (if the pdf/ps file is viewed) is the person self manifold and the other nine trajectories are the projections in that person's PCA subspace.

The above shows that lighting subspace constructed from one person's face images can represent face lighting conditions of face images with different identities. This trait gives us an opportunity to estimate the lighting conditions of a person by using a subspace representation derived from face images of another person. Shashua[3][4] explicitly employed this characteristic in their quotient image method. Sim [16] also implicitly used this trait in their statistical model.

Now, we put forward concepts of face lighting space, general face model and general face imaging model, to further describe the face illumination analysis.

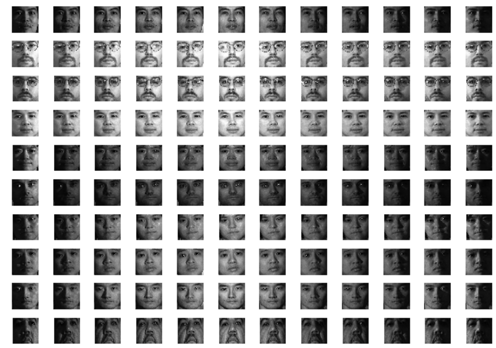
For a face image $\boldsymbol{I}_i$ with n pixels, it lies in a subspace of R^n. Let $\boldsymbol{I}$ represent this space.

$$I \subset R^n$$

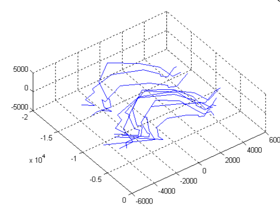
Face images with the same pose under different lighting conditions form a subspace of $\boldsymbol{I}$. Let $\boldsymbol{F}$ represent this space.

$$F \subset I\,,\; F \subset R^m\,,\; m < n$$

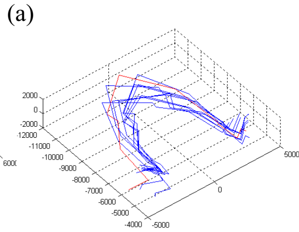
All our following concepts and theories are based on the subspace $\boldsymbol{F}$.



(a)



(b)



(c)

Figure 2 (a) face images of 10 persons under 12 different lighting conditions. (b) face manifolds (consisting of "trajectory" of each person under the varying lighting) in a 3-D PCA subspace derived from training examples of *all* persons. (c) face manifolds (consisting of "trajectory" of each person under the varying lighting) in a 3-D PCA subspace derived from training examples of *one* person.

Concept 1: Face Lighting Space

Face lighting space L is a d-dimensional subspace constructed from face images space F under different lightings.

$$L = T(F)\,, L \subset F\,,\;\; L \subset R^d\,,\; d < m$$

where T is a transformation function from face image space to face lighting space.

For any two face image I_1 and I_2 under the same lighting with different identities, we have

$$l = T(I_1) = T(I_2)$$

where l is their projections into lighting space $l \in L$.

In this space, only face image's lighting condition instead of face identity information is concerned.

This definition imply that human face have similar 3D structure and the lighting condition in one person images can be estimated by the other person's subspace.

According to this definition, face images of different persons under the same lighting conditions are overlapping points in this space. The difference between face lighting space and light space[1] is that face lighting space is d-dimensional ($d \geq 3$) space and light space is 3-dimensional space. A point in the lighting space reflects a face image's lighting conditions including shadow and shading information and not just a light source that illuminates the face.

Concept 2: General Face Model

A general face model M is the function of face 3D shape and texture.

$$M = f(D, T), \tag{2}$$

where D is face 3D shape and T is the face texture.

Concept 3: General Face Imaging Model

$$I = M \bullet l \tag{3}$$

where • represents dot product. This concept means that any face images I can be represented by the dot product of face model M and lighting conditions l.

Our face model and face-imaging model are very generic ones without any constrains, which can be adapted to various models according to different application environments. In the spherical harmonic analysis, M can be expressed in spherical harmonic images [11-15] and l becomes the spherical harmonic light. The simplest model M is composed of face images from the same pose under different lightings. In this case, a point l in the lighting space L reflects the relative intensity ratio of different point lighting conditions in these face images. Our face model can be further simplified to Lambertian Model as in the case quotient image, where l becomes the point light source[2].

2.2. Appearance-Based Illumination Modeling

Belhumeur etc. [7] have proved that under the assumption of Lambertian model and convexity of face 3D shape, face image variations due to lighting changes lie in a high dimensional convex space, called illumination cone. In fact, the face space F under various lightings is also approximately convex even if there is no any assumption of imaging model.

[1] Let set the S is a point light source. It can also be rewritten to $\|S\|(\theta, \Phi)$ form in sphere coordinates, where $\|S\|$ is the intensity of S, $0<\theta<\pi$ and $0<\phi<2\pi$ are azimuth angle and the polar angle respectively. If lighting intensity is not considered, points in unit sphere (Light sphere) can represent light directions.

[2] Face images in daily life are usually illuminated by area light source instead of an ideal point light source.

(a) Original Images

(b) 10 equally sampled points along the line segment between the two images of (a)

Figure 3 the convexity of face space F under different lightings

To test a space is convex, we must check that any point in the line segment between two points in the space also belongs to the space. Figure 3. (a) displays two real face images with shading and shadow (cast shadow and attached shadow). The images in Fig.3 (b) are ten equally distributed points between the two points of 3.(a) in face space. And all the ten images belong to the same person as the two points in 3. (a).

To build face model in a very high dimensional subspace, Belhumeur etc. [7] employed extreme ray concept. According to their definition, the extreme ray (Figure 4) is defined as

$$S_{ij} = n_i \times n_j \tag{4}$$

where n_i, n_j are two non-collinear Lambertian surface normals of face.

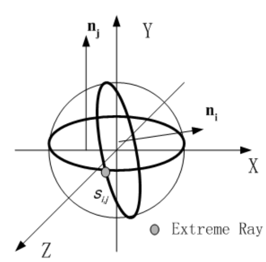


Figure 4 Extreme ray S_{ij} from two surface normals (n_i, n_j) in the light space

For a p-pixel-face image with q ($q<p$) independent surface normals, there at most $(q-1)q+2$ extreme rays[7]. From the above equation, we can see that the face space and the face lighting space are determined uniquely by face's surface normals.

If all the extreme rays are consider in recognition, the computation expense will be very high. In fact, this space can be further compressed by Principle Component Analysis (PCA) [9] in the construction of illumination cone. Therefore a more practical way is needed to model face.

To propose our new idea, we first present Lemma 1.

Lemma 1: All cross products of two surface normals in a unit hemisphere $\boldsymbol{H_0}$ form a unit sphere $\boldsymbol{S}$.

Proof: (Omitted due to length constraint).

For a general analysis, a human face can be roughly represented as a hemisphere or has all the surface normals that a hemisphere has if face surface is considered as continuous or piecewise continuous surface and face image's resolution is infinite.

Theorem 1: Extreme ray directions form the whole unit light sphere.

Proof: According to Lemma 1 and the assumption of face hemisphere representation, all extreme rays consist a unit light sphere.

The calculation of all these rays will require high computational expense and are very impractical for real application.

Theorem 2: Face lighting space $\boldsymbol{L}$ is convex for a given face.

Proof: Let $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ are face images with illumination conditions $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$ respectively in the face lighting space $\boldsymbol{L}$, and $\boldsymbol{l_3}$ is another point in the space, which is sampled among line segment between $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$, then

$l_3 = \alpha l_1 + (1-\alpha) l_2$, where 0< α <1.

Left multiplying face model $\boldsymbol{M}$ on both side of the above equation, we get

$$M \bullet l_3 = \alpha M \bullet l_1 + (1-\alpha) M \bullet l_2$$

$I_3 = \alpha I_1 + (1-\alpha) I_2$, where 0< α <1.

$\because I_1, I_2 \in F$ and convexity of face image space $\boldsymbol{F}$ under different lightings,

$$\therefore I_3 \in F$$

$$\therefore l_3 \in L$$

In this theorem, we do not assume any constrains on face model and light.

This result is also consistent with our daily experience. No matter how extreme the lighting directions of two images from the same person are, their convex combination is still a face image of the same person. Even if all kinds of shadow, self and cast, are considered, this theory is still valid (shown in Figure 3).

According to theorem 1 and 2, similarity of all human face 3D structure, convexity of face space under different lightings, a practical and simple approach of modeling face is constructing subspace from sparsely sampled face images under different lighting conditions. More variations in lighting changes among these sampled images will ensure more accurate lighting space. If no computational expense is considered, these images can be directly used as coordinate axes of the lighting space $\boldsymbol{L}$. The unknown lighting conditions $\boldsymbol{l}$ of input face image can be estimated or interpolated by convex combination of known lighting conditions due to the convexity of face lighting space.

3. Illumination Alignment

In this section, we present a method for illumination alignment, i.e. relighting the illumination of a face to any designated illumination.

3.1 Ideas

The following assumptions are made:

(1) Images in face database (training set) have little or no shadow and the test image for identification or verification can be in any illumination conditions.

(2) All face images have been geometrically aligned.

Condition (1) can be satisfied by most applications cases. And condition (2) limits our focus only on face illumination problem. Our recognition approach is a synthetic one based on illumination alignment.

Illumination alignment is a procedure in face lighting space that maps all face images in the database into the same lighting condition as that of test image.

Let $\boldsymbol{I_i}$ is the i-th image in face database and $\boldsymbol{l}$ is lighting conditions of input unknown identity image, then

$$\widehat{I}_i = P_l(I_i), \qquad (5)$$

where $\boldsymbol{P_l}$ is mapping function and $\widehat{I}_i$ is the mapped image according to the lighting $\boldsymbol{l}$. By illumination alignment we can compare face images under same lighting conditions shown in Figure 8, where Fig. 5(a) is the input image and Fig. 5 (b) to (f) are face images in face database and their corresponding mapped images according to lighting condition of input image. In some application without 3D model, the mapping direction is one way. Because the test image may contain large shadow, no 3D shape and albedos can be deduced from these regions.

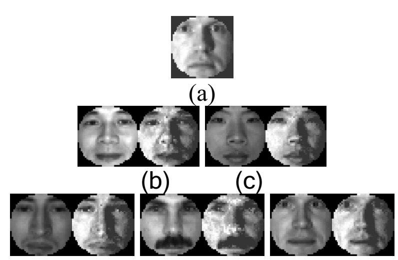


Figure 5 (a) the input image, (b)(c)(d)(e)(f) the face image in training set and its corresponding synthetic image according to illumination effects of input (a)

Two kinds of algorithms can be derived from the illumination alignment, global lighting space method and individual lighting space method.

Global lighting space is constructed from face images with the same pose (e.g. frontal) under different lighting conditions. Due to calculation efficiency, we can compress these face images into a subspace. In fact, many researchers have done a large amount of work in this field.

Before Ramamoorthi [10-12] and Basri [13-15]'s theoretic analysis, many researchers [21-23] had found similar results in empirical experiments that the face images under different lighting conditions lie in a very low dimensional subspace. Ramamoorthi [10-12] and Basri [13-15] explained this phenomenon by spherical harmonic analysis. According to their analysis, nine-dimensional subspace can account for 99% the image's energy.

But all their results are deduced from Lambertian Model without cast shadow. Because face is not perfect convex, even small illumination direction variation affects the subspace dimensionality severely. Figure 6 shows the energy accumulative distribution in eigen-subspace. In Fig. 6. (b), the first 6 eigen-values consist only 93% the total image energy in the first 23 images of Fig.9(a). Moreover in Fig. 6(c) less than 80% energy is included in the first-9-eigen-vector subspace built from all the 64 images of Fig. 6(a).

Therefore more than nine-dimensional subspace is need to model face variation under varied lighting conditions for a real face recognition system.

After building face lighting space, we can estimate the face lighting conditions and transform the face images in the face database into the same lighting conditions as the input one. Sim [16] used statistical model for this transformation and Shashua [3][4] employ a very simple and practical image ratio method to map the face images into different lighting conditions.

The highlight of global lighting space approach is that it requires only one face image in the face database for recognition system. This method is based on a fact that all human faces have similar 3D shape.

In individual lighting space, we build each person's lighting space according to his 3D information, 3D shape and albedos. Spherical harmonic representation [10-15] and illumination cone method [7-9] are two theoretical methods, which have been proved to outperform other approaches including global lighting space methods. The excellent performance is due to the fact that each person has its own 3D model. Illumination cone method utilized 3D face model to synthesize densely sampled face images illuminated by extreme rays and spherical harmonic method made use of 3D face model to create the orthogonal subspace composed by harmonic images. The lighting estimation in individual lighting space is the same as that in global lighting space. Illumination alignment in individual lighting space is relatively simple and can be achieved by corresponding computer graphic synthetic methods.

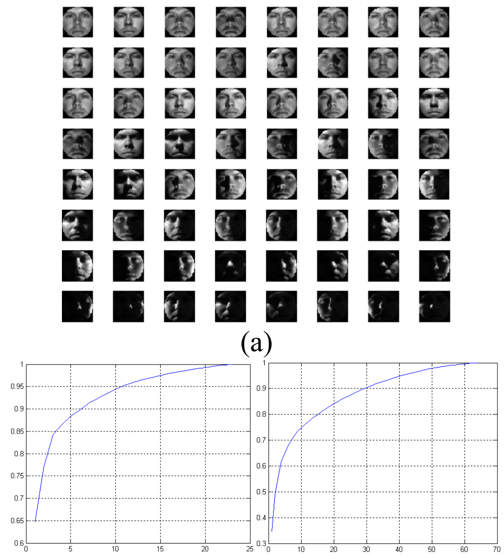


Figure 6. (a) face image (b) energy accumulative distribution for the first 23 images (c) energy accumulative distribution for the first 23 images

The main drawback of individual lighting space is that they depend on face 3D model. In illumination cone method, 7 images of each person in the face database are needed to precisely build a 3D face model from 2D images. In spherical harmonic approach, face 3D model is assumed as known parameters.

3.2. Illumination Alignment

3.2.1. Based on Global Lighting Space From One-Person's Images

In this algorithm, we construct the lighting space ***L*** from face images of one person under different lighting conditions (the training set). This lighting space, called global lighting space, is universal in light estimation and illumination alignment for all face images in the face database. After estimating the lighting conditions of input image by this global lighting space, we map the face images in the face database into the same lighting conditions as that of the input-unknown-identity face image by image ratio technique [3][4].

In our experiment, we choose one person's face images in Yale B Face database with the frontal pose but systemically sampled lighting conditions for building global lighting space. Because face images lie in a low dimension subspace [10-15], we compress these images by Singular Value Decomposition (SVD). This linear transformation does not change the convexity of face image space and face lighting space.

Let $B = [b_1, b_2, ..., b_K]$ be the compressed lighting K-dimensional subspace. Like the texture mapping in quotient image, we have the image ratio for each face in the database by the ratio of $\boldsymbol{Y_i}$ and $\boldsymbol{B \bullet l}$,

$$Q_i = \frac{Y_i}{Bl}, \quad i=1, 2, .., N \tag{6}$$

$$f(x) = \min \| y_t - B \bullet l_t \| \tag{7}$$

where $\boldsymbol{l}$ is the estimated lighting conditions by minimizing equation (7), N is the number faces in the database, $\boldsymbol{y_t}$ is the input image for recognition and its estimated lighting is $\boldsymbol{l_t}$,.

After estimating the lighting $\boldsymbol{l_t}$, we can align all face images in the database into the same lighting conditions as that of the input image by

$$Y_{syn_i} = Q_i \otimes B \bullet l_t, \quad i=1, 2, .., N \tag{8}$$

The equation (8) is the illumination alignment process and the face model is $Q_i \otimes B$, where the $\otimes$ represents the pixel-by-pixel multiplication. After this alignment, the recognition is carried out among the input image $\boldsymbol{Y_t}$ and the $\boldsymbol{Y_{syni}}$, and $\boldsymbol{Y_t}$ belongs to class $\boldsymbol{i}$ if it has shortest distance to $\boldsymbol{Y_{syni}}$. Though our algorithm has a similar form as that of quotient image algorithm, our subspace is more complex and lighting $\boldsymbol{l_t}$ is not real light. Our estimated lighting $\boldsymbol{l_i}$ contains shading and shadow information instead of only shading information in quotient image method.

3.2.2. Based on Individual Lighting Space From Each Person's Images

In this algorithm, we need three images with equal intensity, non-collinear light source directions and small shadow, we can build an individual lighting subspace by spherical harmonic representation.. According to the spherical harmonic approach, we can build 9-dimensional subspace for each person. It is well known that Signaler Value Decomposition (SVD) analysis can only recover face surface normals with ambiguity [27][32]. A practical approach we proposed in [26] makes use of the similarity of 3D human face shape to estimate the ambiguity matrix. Then face images in the database are aligned to the same lighting conditions as that of the input one. Because this lighting space is based on individual images for face database, we call it individual lighting space. In this algorithm, the face model $\boldsymbol{M}$ itself is the face individual lighting space $\boldsymbol{L}$.

After estimating the albedo and surface normals of face, we align all the face images into the same lighting condition $\boldsymbol{l}$ of input image.

To estimate the lighting $\boldsymbol{l_t}$ of the input image $\boldsymbol{y_t}$, we minimize the energy function,

$$f(a) = \min \| y_t - H_i \bullet l_t \|, \text{i=1,2..,N} \tag{9}$$

where $H_i = [h_{i1}, h_{i2}, ...h_{in}]$ is made of the harmonic images from each face in the database and N is the number faces in the database . The face model $\boldsymbol{M_i}$ is the harmonic images $\boldsymbol{H_i}$, which is the polynomial form of face 3D surface normals and albedos. The aligned image is synthesized by

$$Y_{syni} = H_i \bullet l_t, \quad i=1, 2, .., N \tag{10}$$

The recognition is the same as in global lighting space.

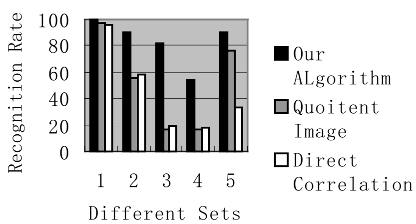
4. Experiments and Discussion

Two kinds of experiments, recognition and synthesis, have been done to verify our new approach.

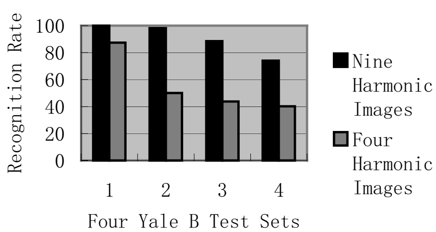
Our recognition is carried out on 5 sets. Set 1 to Set 4 is from Yale B face database [27] and Set 5 is from MIT face database [29]. The training set of quotient image algorithm is from Yale face database and part of Yale B face database. Only frontal images with lighting variation are selected and manually cropped and aligned.

Figure 7 is the recognition results of global lighting space and individual lighting space. In global lighting space algorithm Fig. 7(a), our algorithm can strikingly improve the recognition rate by the illumination alignment compared with direct correlation method, except in Yale B Set 1, in which the illumination variety is relatively small. The results of Yale B Set 1 also demonstrate that our algorithm do not affect the recognition rate under normal near frontal illumination condition. Even in the extremely illuminated sets, such as Yale B Set 3 and 4, our algorithm still can correctly recognize most of the images. Compared with quotient image method, our algorithm has higher recognition rate in all test sets. The recognition rate of quotient image algorithm decreases strikingly with the increasing lighting variation.

It is clear in local lighting space method shown in Fig. 7(b) that our estimated model (nine harmonic images) has almost perfect recognition rate in Set 1 and Set 2, which is comparable to that of the complex illumination cone [21]. Even in the case of four harmonic images, this model still demonstrates its robustness for lighting changes in Set 3 and Set 4. The decreasing of recognition rate from Set 1 to Set 4 indicates that the representation effects deteriorate with the increasing azimuth or elevation. This result tells us that neglecting the cast shadow makes the spherical representation only valid when the azimuth and elevation is relatively small. Because more images (three images for each face) were used in local lighting space algorithm, its recognition results were better than that of global lighting space method. In most real applications, it is difficult to get three images with the same lighting intensity and non-collinear lighting directions. Therefore global lighting space method is more practical for face recognition system.



(a) Results on global lighting space



(b) Results on individual lighting space

Figure 7 Recognition results

Figure 8 displays the lighting transformation results of Lena and Mona Lisa. Though the geometric alignment is not perfect due to pose problem, the transformation results are still reasonable.

We also did some experiments (Figure 9) on nature lighting conditions, where the lighting variation is due to curtain and different lamps turning on and off. The first column of Fig. 9 is the input face images for mapping. Column 2 to 8 and Row 1, 3 and 5 are real face images and Column 2 to 8 and Row 2, 4 and 6 are synthetic images according to the lighting conditions of the above images by our global lighting space algorithm. In this case, the light source is not point source and our synthetic results are still reasonable and realistic.

Figure 8 Face image-synthesizing results of Lena and Mona Lisa

Figure 9 Nature lighting image-synthesizing results

5. Conclusion and Future Work

The main contribution of this paper is that we proposed a very general framework for analyzing and modeling face images under varied lighting conditions. In this paper, we introduced the concepts of face lighting space, general face model and general face imaging model for face modeling under varied lighting conditions. Then illumination alignment approach is proposed for face recognition.

Experiment results showed that our algorithms could render reasonable face images and effectively improve the face recognition rate under varied lighting conditions.

Though we deduced an approach that lighting space can be built from sparsely sampled images with different lighting, how to construct an optimal global lighting space from these images and whether a global lighting space constructed from one person's images is better than multi-persons' images is still an open issue.

In addition, for a practical system, illumination and pose are two problems that have to be faced concurrently. Therefore the potential approach is to solve these problems at the same time, such as in morphable method [17-19]. How to modeling 3D face parameters only from 2D images will be the most appealing focus in future face recognition research.

Reference

[1] Yael Adnin, Yael Moses and Shimon Ullman, "Face recognition: The problem of compensating for changes in illumination direction", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 19, No. 7, 1997, pp712-732.

[2] T. Sim, S. Baker, and M. Bsat, "The CMU Pose, Illumination, and Expression (PIE) Database**",** Proceedings of the IEEE International Conference on Automatic Face and Gesture Recognition, May, 2002

[3] Amnon Shashua, and Tammy Riklin-Raviv, "The quotient image: Class-based re-rendering and recognition with varying illuminations", Transactions on Pattern Analysis and Machine Intelligence, Vol. 23, No. 2, pp129-139, 2001

[4] T. Riklin-Raviv and A. Shashua. "The Quotient image: Class based recognition and synthesis under varying illumination". In Proceedings of the 1999 Conference on Computer Vision and Pattern Recognition, pages 566--571, Fort Collins, CO, 1999.

[5] Zicheng Liu , Ying Shan , Zhengyou Zhang, Expressive expression mapping with ratio images, Proceedings of the 28th annual conference on Computer graphics and interactive techniques, pp 271-276

[6] Zhen Wen, Zicheng Liu, Tomas Huang, "Face Relighting with Radiance Environment Maps", MSR-TR-2002-111

[7] P. N. Belhumeur, David J. Kriegman, "What is the set of Images of an Object Under All possible Lighting Conditions?", IEEE conf. On Computer Vision and Pattern Recognition", 1996

[8] Athinodoros S. Georghiades and Peter N. Belhumeur, "Illumination cone models for recognition under variable lighting : Faces", CVPR, 1998

[9] Athinodoros S. Georghiades and Peter N. Belhumeur, "From Few to many: Illumination cone models for face recognition under variable lighting and pose", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 23, No. 6, pp 643-660, 2001

[10] Ravi Ramamoorthi, Pat Hanrahan, " On the relationship between radiance and irradiance: determining the illumination from images of a convex Lambertian object", J. Opt. Soc. Am., Vol. 18, No. 10, 2001

[11] Ravi Ramamoorthi, "Analytic PCA Construction for Theoretical Analysis of Lighting Variability in Images of a Lambertian Object", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 24, No. 10, 2002-10-21

[12] Ravi Ramamoorthi and Pat Hanrahan, "An Efficient Representation for Irradiance Environment Maps", SIGGRAPH 01, pages 497--500, 2001

[13] Ronen Basri, David Jacobs, "Lambertian Reflectance and Linear Subspaces", NEC Research Institute Technical Report 2000-172R

[14] Ronen Basri and David Jacobs, Lambertian Reflectance and Linear Subspaces, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, forthcoming

[15] Ronen Basri and David Jacobs, Photometric Stereo with General, Unknown Lighting, *IEEE Conf. on Computer Vision and Pattern Recognition (CVPR)*, Kauai, Vol. II: 374-381, 2001

[16] Terence Sim, Takeo Kanade, "Illuminating the Face", CMU-RI-TR-01-31, Sept. 28, 2001

[17] Blanz, V. and Romdhani, S. and Vetter, T. "Face Identification across Different Poses and Illuminations with a 3D Morphable Model", Proc. of the 5th Int. Conference on Automatic Face and Gesture Recognition", pp 202-207, 2002

[18] S. Romdhani, V. Blanz, and T. Vetter, "Face Identification by Fitting a 3D Morphable Model using Linear Shape and Texture Error Functions", Computer Vision - ECCV 2002, May 2002, LNCS 2353, pp. 3-19

[19] Blanz, V. and Vetter, T., "Face Recognition Based on Fitting a 3D Morphable Model" IEEE Transactions on Pattern Analysis and Machine Intelligence, accepted, 2003

[20] Phong, B. T., "Illumination for Compouter-Generated Images", 1975, CACM, 18(6), pp 311-317

[21] R. Epstein, A. L. Yuille and P.N. Bellumeur, " Learning Object Reorientations form Lighting Variation", in Object Rep. in Computer Vision II (J. Ponce, A. Zisserman, and M. Hebert, eds.), pp.179--199, Springer-Verlag, 1996

[22] P. Hallanan, "A Low-Demensional Representation of Human Faces for Arbitary Lighting Conditions", IEEE Conf. On Computer Vision and Pattern Recognition, pp 995-99, 1994

[23] A. Yuille, D. Snow, R. Epstein, P. Belhumeur, "Determining Generative Models of Objects Under Varying Illumination: Shape and Albedo from Multiple Images Using SVD and Integrability, International Journal of Computer Vision, 35(3), pp203--222, 1999

[24] R. Epstein, P. Hallanan, A. L. Yuille, "5±2 Eigenimages Suffice: An Empirical Investigation of Low-Dimensional Lighting Models", IEEE Conf. Workshop on Physics-Based Vision, pp 108-116, 1995.

[25] Haitao Wang, Yangsheng Wang, "RECOGNIZING FACE IMAGES UNDER DIFFERENT LIGHTING CONDITIONS", International Conference on Acoustic, Speech and Signal Processing (ICASSP), 2003, accepted

[26] Haitao Wang, Yangsheng Wang, "FACE REPRESENTATION UNDER DIFFERENT ILLUMINATION CONDITIONS", IEEE International Conference on Multimedia & Expo (ICME), 2003, accepted

[27] P. N. Belhumeur, David J. Kriegman, A. L. Yuille, " The Bas-Belief Ambiguity", CVPR, 1997, pp 1060-1066

[28] Georghiades, A.S. and Belhumeur, P.N. and Kriegman, D.J., "From Few To Many: Generative Models For Recognition Under Variable Pose and Illumination", IEEE Int. Conf. on Automatic Face and Gesture Recognition,2000, pp277-284

[29] M. Turk and A. Pentland, ``Eigenfaces for recognition," Journal of Cognitive Neuroscience, Vol. 3, No. 1, 1991.

[30] A. Jain, "*Fundamentals of Digital Image Processing"*, Prentice-Hall, 1986, pp 241 - 243.

[31] Henry A. Rowley, Shumeet Baluja, and Takeo Kanade. Neural networkbased face detection. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 20(1):23–38, January 1998.

[32] Haitao Wang, Yangsheng Wang, Hong Wei, "Face Representation and Reconstruction under Different Illumination Conditions", 7th International Conference On Information Visualisation IV03, accepted.

0-7695-2010-3/03 $ 17.00 © 2003 IEEE