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ACTL3141 Assignment
Mortality of Chilean Pensioners

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Descriptive Analysis
The descriptive analysis aims to gain a better understanding of the profile of the annuitants in the Chilean Mortality dataset.

According to the Chilean Mortality dataset, there are a total of 739,241 female Chilean annuitants and 552,786 male Chilean annuitants. From this distribution, 1,178,361 annuitants were healthy whereas 113,666 annuitants were disabled. The table below showcases the percentage of healthy people by gender:

Percentage of healthy male annuitants	Percentage of healthy female annuitants
87%	95%

The higher proportion of disability in male annuitants at old ages may suggest that there are more males in the Chilean workforce, and that males partake in more risk-taking behaviour in general assuming that the cause of the disability was mostly due to work-related injuries. This insight is supported by data from *gender data* which states that the labour force participation rate is 20% higher in males than females in Chile (World Bank Gender Data Portal, n.d.). Furthermore, although not specific to Chile, 'men had higher rates of physical injury claims than women, but this was mostly attributable to occupational factors' (Janneke Berecki-Gisolf et al, 2015) was claimed in the research paper on '*Gender differences in occupational injury incidence*'.

Secondly, when viewing the distribution of the age of males and females at the start of the observation period (Appendix 1.1), we notice a sharp peak in the distribution of each sex at the age of 60 and 65 in the distribution of females and males respectively. These peaks occur at the age of retirement (i.e. age 60 for females and 65 for males). This occurs because the Chilean Pension system offers retirees the option to purchase an annuity at the age of retirement (Ferreiro Yazigi et al. 2003), therefore in both genders, most people decide to purchase an annuity at the age of retirement as illustrated by the peaks in the distribution.

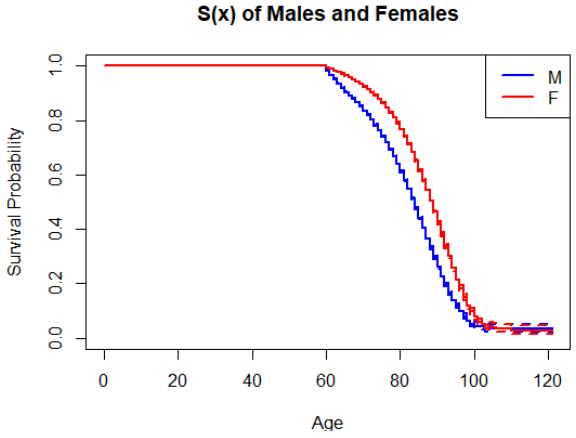
Thirdly, the chart illustrating the proportion of beneficiaries to main annuitants by gender (Appendix 1.2) suggests that the overwhelming beneficiaries are female whereas the majority of main annuitants are male. This gives us insight about the distribution of gender in the Chilean workforce being mostly composed of men as main annuitants receive annuities from their superannuation and life insurance, whereas beneficiaries are mostly composed of females as the men in the workforce will most likely choose their spouse to be entitled to benefits. This claim is also supported by the gender data stated previously which suggest a higher labour participation rate in males than females in Chile.

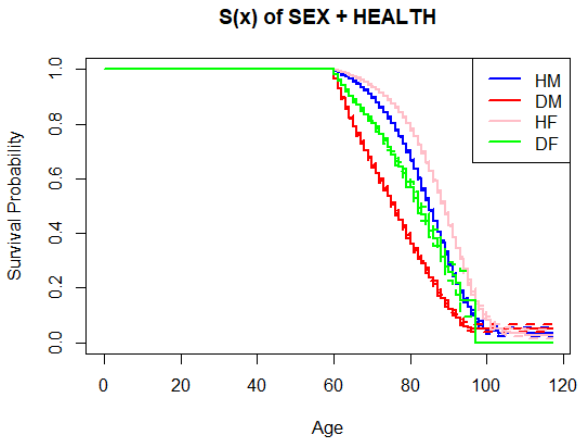
Finally, the distribution of the age of death by gender (Appendix 1.3) indicates that males have a higher death count, and the deaths occur at earlier ages when compared to the age of death of females. This is a statistic commonly found in most countries and Chile is no different. An interesting observation can be made around the age of 87, where the death count in females rises above the death count of males. Upon further speculation, this is attributed to the fact that there are more females than males at those ages and hence the death count of females is higher, regardless of the higher male mortality rate.

Survival Analysis
The survival analysis aims to understand the impact of the covariates (i.e. sex, person type and health) on the survival of Chilean annuitants. In order to conduct this investigation, we first look at the survival of individuals qualitatively using non-parametric methods such as the Kaplan-Meier estimator and look

to solidify our claims using more quantitative methods using semi-parametric estimation which involve Cox Regression.

To begin with, we conduct survival analysis using the Kaplan-Meier estimator. Kaplan-Meier relies on multiple key assumptions about the data being analysed. Firstly, the data must only contain independent censoring (i.e. the censoring in non-informative). In the dataset, there are a total of 1,159,685 total censors, 934 of which occurred before the last day of observation. Therefore, since 99.92% of censors occurred due to the termination of the investigation and not due to the decision of the annuitant, we can assume that almost all censors were independent. Secondly, Kaplan Meier estimation assumes that lives are independent and therefore the survival of one person should not influence the survival of another. This assumption is challenged as research claimed by Katsiferis et al suggests that men who lost their spouse were 70% more likely to die than similarly aged men who did not lose their spouse in the following year; this statistic was 27% for women (Katsiferis et al, 2023). However, since this dependency is only between pairs of data at most, the Kaplan-Meier estimates will still be useful for survival analysis.





We first analyse the survival of individuals by gender. From the plot of the Kaplan-Meier survival function by gender on the left, it is clear that the mortality rates in Chilean males are significantly higher than females, suggesting that gender is a significant factor and qualitatively supports the use of different life tables by sex.

Now that we understand that at least qualitatively, the sex of the individual is significant in determining the survival of Chilean annuitants, we now proceed to test the effect of health status on the survival of the Chilean population. From the survival plot illustrating the effects of sex and health of individuals on survival probability on the left, it can be seen that for both males and females, a health status of ‘healthy’ drastically increases the probability of survival compared to ‘disabled’. Those with the highest likelihood of survival are healthy females, followed by healthy males, disabled females and disabled males. Therefore, we can qualitatively conclude that health status has a significant impact on survival of Chilean annuitants.

Finally, we look at the impact on survival by person type by analysing the survival functions of healthy females by person type (Appendix 2.1). It is clear that healthy female main annuitants have a slightly higher survival probability than healthy female beneficiaries, however this distinction seems to be very small and may not be significant and requires quantitative analysis to come to a convincing conclusion.

Cox Regression was used to conduct further quantitative survival analysis to study the impact of each covariate on the survival of Chilean annuitants. Cox regression has multiple key assumptions, the most important being the proportional hazards assumption. To test this assumption, we use the Schoenfeld's test which has a null hypothesis that states that hazards are proportional and an alternate hypothesis that hazards are not proportional. Unfortunately, the test has a p-value less than 0.05 (Appendix 2.2), suggesting that the proportional hazard assumption does not hold. However, since our analysis is guided by our qualitative observations using Kaplan-Meier, using Cox Regression will still be useful in conducting survival analysis with higher confidence.

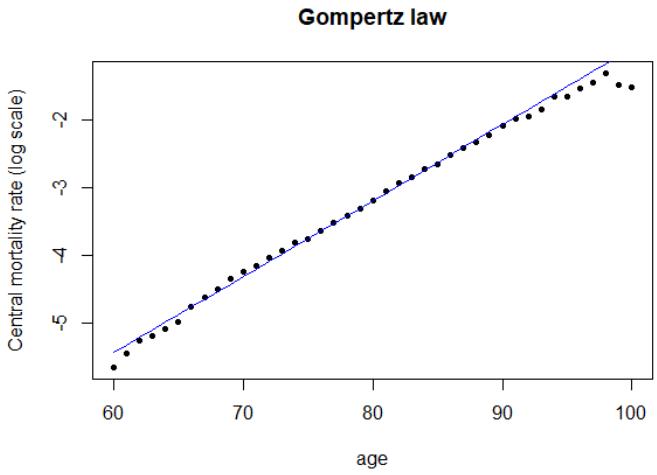
According to the cox regression analysis, all covariates are statistically significant. Furthermore, the score test (equivalent to a log rank test which is used to compare survival functions) suggests that the survival functions of Chilean annuitants by each covariate differ statistically. Therefore, combining the strong qualitative analysis conducted using Kaplan-Meier with the results from Cox Regression, we can conclude that each covariate has a significant impact on the survival of Chilean annuitants and therefore supports the idea of having 5 different life tables.

To conclude, the significant impact of gender on the survival of annuitants as shown in the Kaplan-Meier chart on the previous page and the results from Cox Regression support the idea of dividing the life tables by gender. Similarly, the significance in the health status of individuals on the survival of Chilean annuitants support the idea of further dividing the life tables by health status, resulting in a life table for healthy males, disabled males, healthy females, disabled females. Finally, the results from Cox regression suggest that person type had a significant impact in the survival of annuitants, which align with the slight difference in the survival functions of healthy females by person type using Kaplan-Meier estimates. Therefore, it is also justified that a separate life table be used to capture this result. As a result, the survival analysis supports the idea of using 5 life tables. Further investigation is required to conclude whether more than 5 life tables are necessary. This investigation will require the use of Kaplan-Meier estimates to plot survival functions of healthy males, disabled females and disabled males by person type to see if person type has any significant impact on the aforementioned individuals. This is outside the scope of this report.

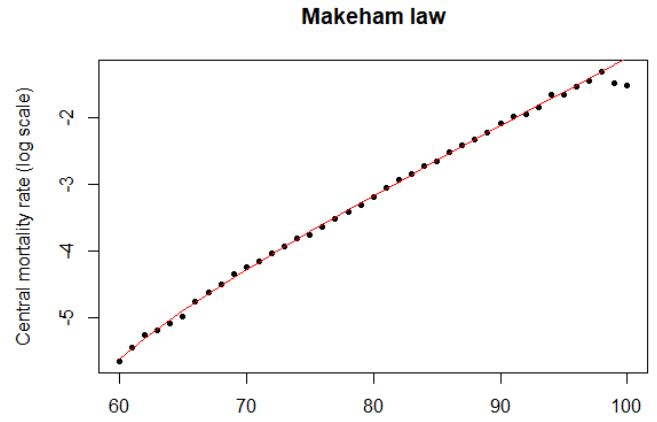
To complete our survival analysis, we look at the results from the Cox regression (Appendix 2.3) to view how each covariate impacts the survival of an individual. The coefficient for a male individual is positive in the Cox regression, whereas being healthy or a main annuitant have negative coefficients. This suggests that males have a higher mortality compared to females, healthy individuals have a lower mortality compared to disabled individuals and main annuitants have a lower mortality compared to beneficiaries. All the results from the Cox regression align with our qualitative analysis using Kaplan-Meier estimates.

Graduation

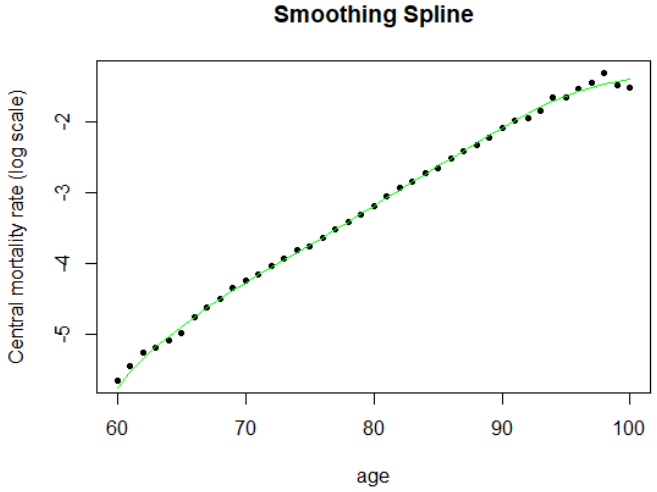
Our graduation aims to construct a unisex life table for healthy Chilean annuitants that is smooth and adheres well with the crude rates.



The first choice for graduating crude rates is through utilising Gompertz law (Appendix 3.1). This is because it is common for mortality rates to show log-linear behaviour at older ages. From the Gompertz graduation as illustrated on the chart on the left, the model is smooth and able to capture the trend of the mortality rates of healthy Chilean annuitants very well in the middle, but fails to capture the changes at the extremities (i.e. ages 60 and 100). We can add an additional term to the Gompertz model, known as the Makeham model, as it may allow for better adherence.



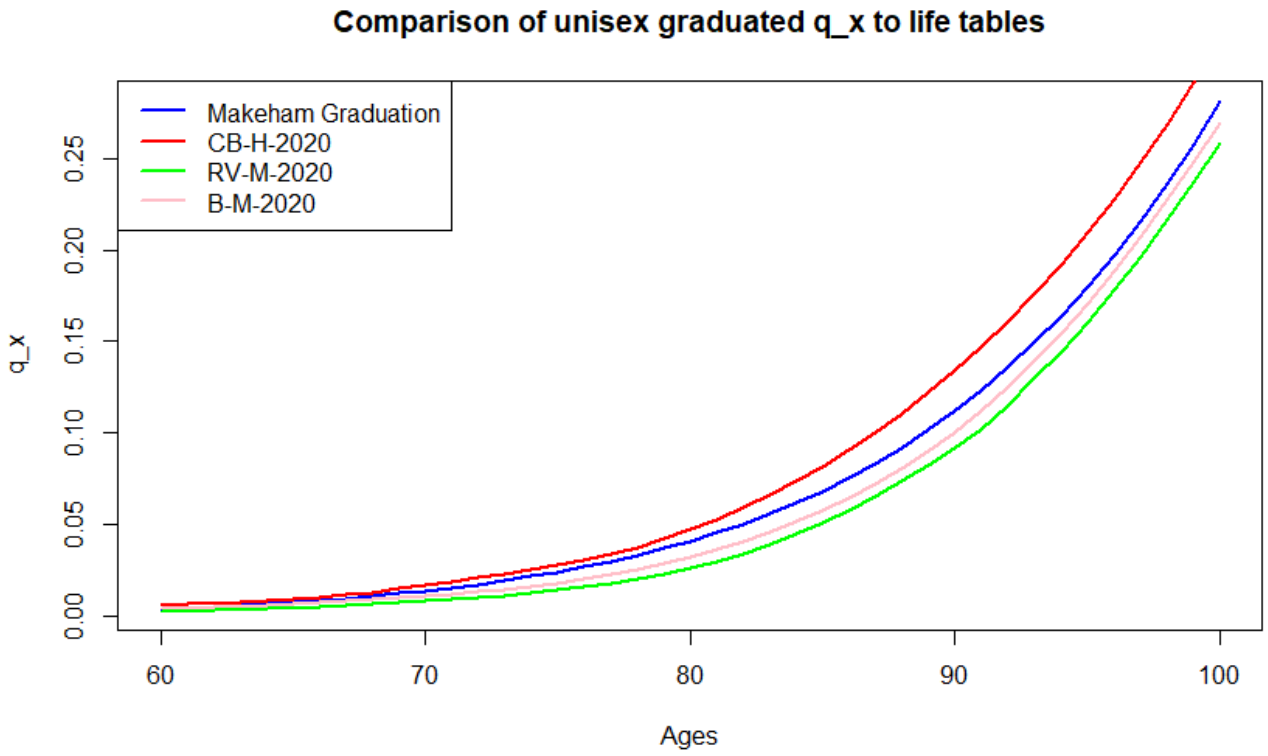
Qualitatively, graduating using the Makeham law (Appendix 3.2) seems to be the better choice as the model is still smooth, but is able to capture the extremities much better. The only concern is that the Makeham model fails to adhere well to the crude rates at ages 99 and 100, but this may also be good as those crude rates seem to be outliers. Nevertheless, we can attempt to adhere to these outliers using a smoothing spline to try and improve adherence without sacrificing too much smoothness.



Lastly, the smoothing spline (Appendix 3.3) seems to adhere well to the crude rates, sacrificing the linear trend a little to adhere to the crude rates at ages 99 and 100. It is difficult to conclude on which model is the best choice to graduate the unisex life tables of healthy Chilean annuitants, so it is important to test the adherence of these models.

According to the results and analysis of the adherence tests in Appendix 3.4, the makeham and smoothing spline models have the best adherence to the crude rates. However, the best graduation method for creating a unisex life table for healthy Chilean annuitants is the Makeham model as the smoothing spline adheres too much to the crude rates at ages 99 and 100 which I consider to be outliers. This produces a strange behaviour, resulting in a mortality probability curve that seems contradictory to the average human life as it starts to increase at a decreasing rate at very old ages. See appendix 3.5 for the comparison of the smoothing spline graduated mortality probabilities against the healthy Chilean life tables.

We now compare our selected Makeham graduation model with the Chilean life tables of healthy annuitants.



The Makeham graduated mortality probabilities are smooth and follow the same exponential trend as the Chilean life tables. The graduated mortality probability lies below the mortality probabilities of healthy males (CB-H-2002) and lies above the mortality probability of healthy females who are either main annuitants (RV-M-2002) or beneficiaries (B-M_2002). This makes sense because the graduated model is a unisex life table that contains both healthy males and healthy females, and therefore the aggregate curve will lie somewhere between the healthy male life table (highest mortality) and the healthy female life table (lowest mortality).

Ethical implication of unisex annuity pricing

We now discuss from an ethical perspective whether insurers in Chile should be allowed to continue the use of gender as a rating criteria for setting annuity prices. We aim to discuss this issue using ethical frameworks which involve utilitarianism, deontology and the ethical framework provided by Dobrin (2009).

Dobrin (2009) emphasises the importance of identifying stakeholders and understanding their impacts in ethical decision-making. The decision to use gender as a factor in pricing annuities in Chile affects insurance companies, policyholders, transgender and non-binary individuals, financial advisors, and the working class, along with wider society. Insurance companies need accurate pricing for financial sustainability, potentially involving gender as a factor. Policyholders' premiums and payments are directly affected by gender-based pricing, influencing their financial planning and retirement income. Transgender and non-binary individuals may face discrimination due to pricing misalignment with their gender identity. Financial advisors' recommendations for retirement planning are influenced by gender-based pricing. The working class, comprising the majority of annuity users, is significantly impacted as annuities are a key income source in retirement.

Utilitarianism advocates for actions that maximise overall benefit, considering both direct and indirect impacts. In the context of gender-based pricing for annuities in Chile, this entails evaluating not only immediate financial effects but also societal and cultural consequences. While such pricing can promote actuarial fairness and sustain insurance business, it may exacerbate economic disparities and reinforce gender stereotypes. Additionally, it can lead to discrimination against LGBTQ+ individuals whose gender identity differs from their assigned sex at birth. Thus, while gender-based pricing may offer financial benefits, its societal implications warrant careful consideration.

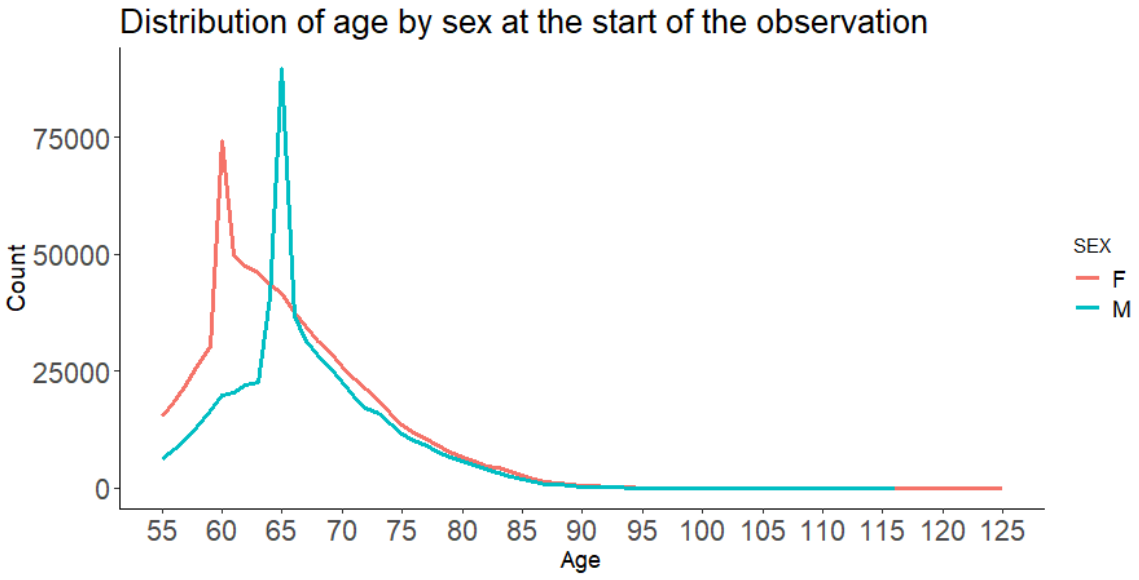
From a deontological perspective, the morality of an action is determined by adherence to a set of rules and principles rather than its consequences. The Universal Declaration of Human Rights asserts equality without distinction of sex, suggesting that gender-based pricing violates this principle. However, exceptions exist in laws such as the Anti Discrimination Act of 1977, permitting gender-based pricing if based on statistical and actuarial data and deemed reasonable. Thus, the impact on stakeholders varies depending on the moral and ethical principles followed.

After analysing various ethical frameworks, I advocate for the elimination of gender-based pricing for annuities in Chile. From a utilitarian perspective, the detrimental effects such as exacerbating economic disparities and perpetuating gender inequality outweigh the benefits of actuarial fairness. Moreover, maintaining fairness in pricing and profitability for insurance companies is achievable without relying on gender. By implementing pre-screenings based on individual health, lifestyle, risk, and longevity decisions, gender-neutral pricing can be achieved, ensuring both fairness and sustainability in annuity products.

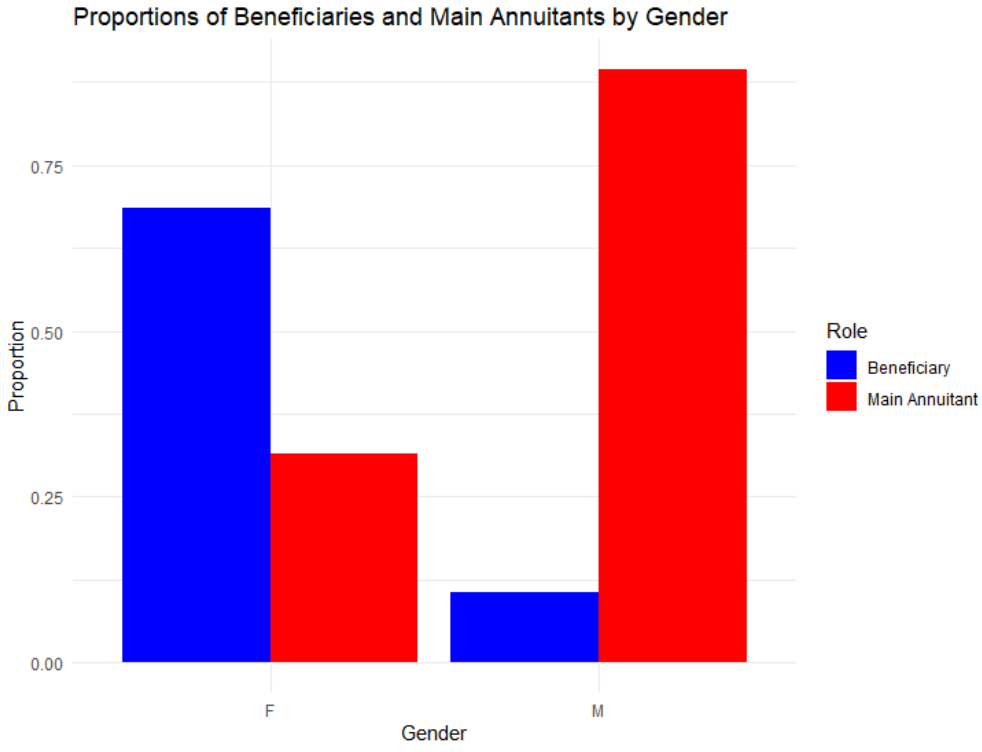
Appendix

1 Descriptive Analysis

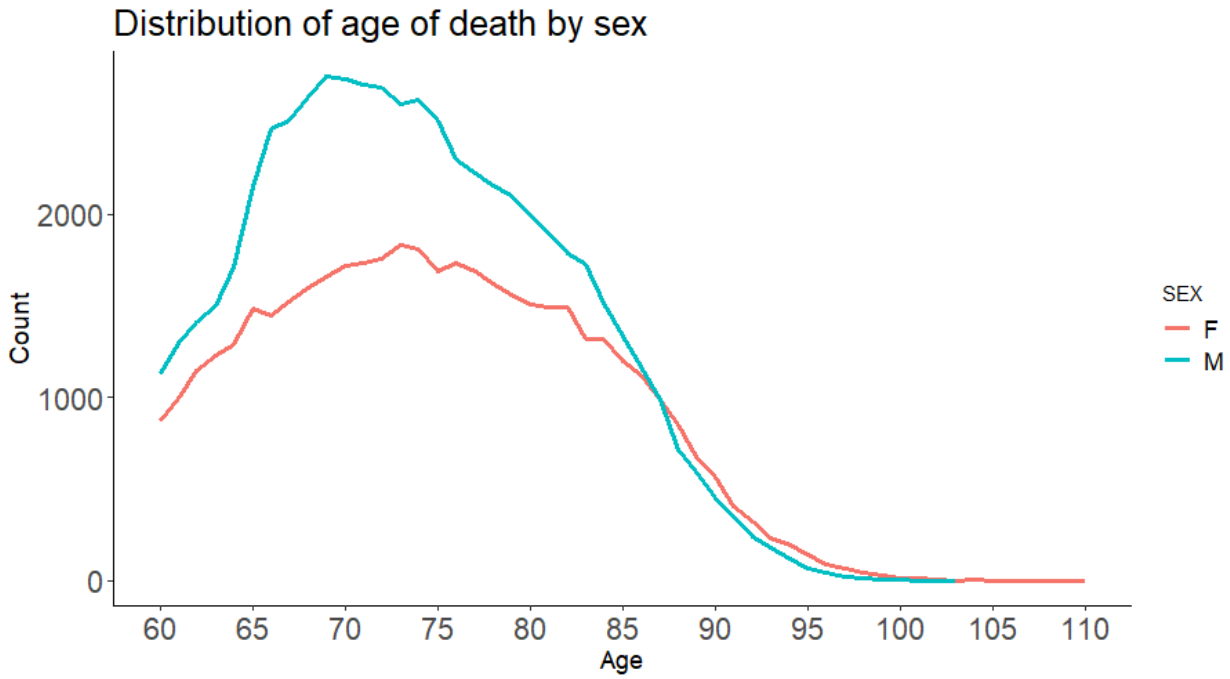
1.1 Distribution of age by sex at the start of the observation



1.2 The proportion of beneficiaries and main annuitants by sex

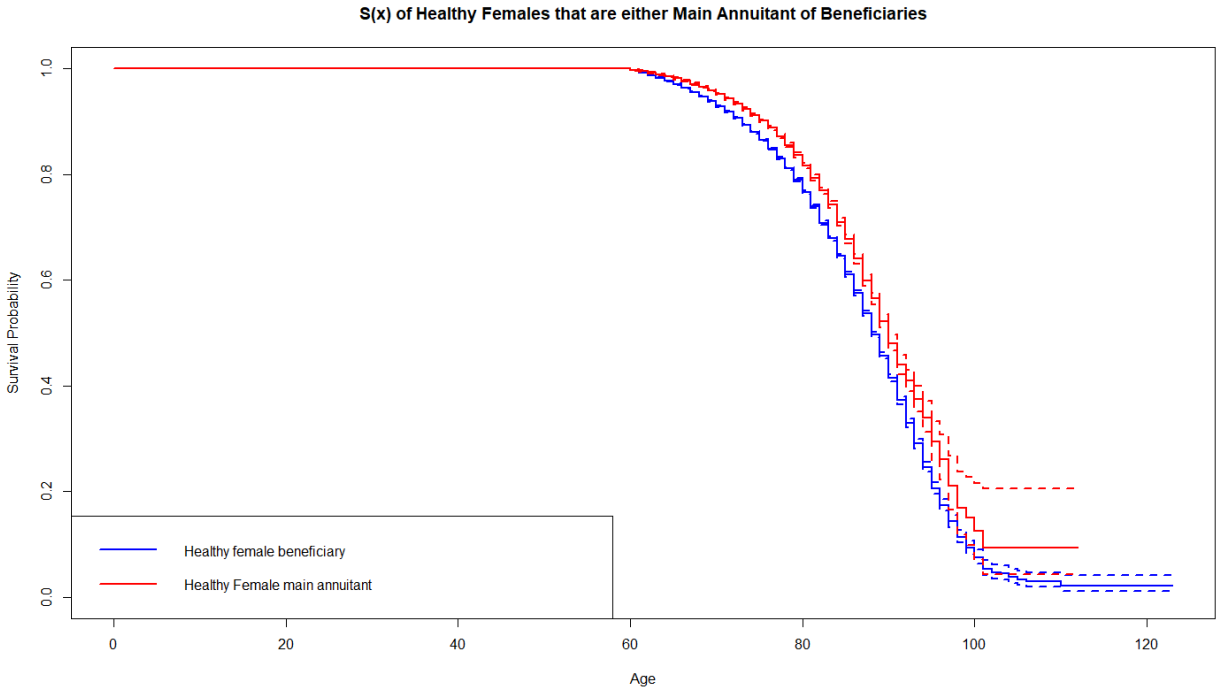


1.2 Distribution of the age of death by sex



2 Survival Analysis

2.1 Survival function of healthy females by person type



2.2 Cox Regression Schoenfeld Test Results

	Chi-squared Test Statistic	**Degrees of freedom**	**P-value**
Sex	396	1	<2e-16
Health	1709	1	<2e-16
Person Type	448	1	<2e-16
GLOBAL	1806	3	<2e-16

2.3 Results of Cox Regression

	Coefficient	**Standard Error**	**Z**	**P-value**
SEX: M	0.544311	0.009077	59.97	**<2e-16**
HEALTH: Healthy	-1.037410	0.009739	-106.52	**<2e-16**
PERSON_TYPE: Main annuitant	-0.134535	0.009765	-13.78	**<2e-16**

	Statistic	**Degrees of Freedom**	**P-value**
Concordance	0.62	N/A	N/A
Likelihood Ratio Test	16979	3	<2e-16
Wald Test	20177	3	<2e-16
Score (logrank) Test	21956	3	<2e-16

3 Graduation

3.1 Results of Gompertz graduation model

model: $\mu_x \sim exp\{b_0 + b_1 x\}$

b_0	b_1
-12.1596	0.1121

Weighted residual sum-of-squares: 257.9

3.2 Results of Makeham graduation model

model: $\mu_x \sim A + exp\{b_0 + b_1 x\}$

A	b_0	b_1
-0.00223	-11.21592	0.10114

Weighted residual sum-of-squares: 92.18

3.3 Results of Smoothing Spline graduation model

Smoothing Parameter spar= 0.65 lambda= 0.000555823
Equivalent Degrees of Freedom (Df): 6.921289
Penalised Criterion (RSS): 0.002796501
GCV: 9.872622e-05

3.4 Results of the adherence tests

3.4.1 Chi-Squared Test

	Statistic	**P-Value**	**Degrees of Freedom**
Gompertz	235.6083	0	39
Makeham	88.73136	6.149e-06	38
Smoothing Spline	76.44722	4.31279e-05	34

All three models show a poor overall adherence to the crude rates as all p-values are less than 0.05. The Makeham model makes a significant improvement from the Gompertz model and this was also observed from their graphs as the Makeham was better able to capture the extremities. However, the best performing model is the smoothing spline, and has slightly better adherence than the Makeham model.

3.4.2 Standard Deviations Test

	Statistic	P-Value	Degrees of Freedom
Gompertz	26.375	7.958e-06	3
Makeham	5.5374	3	0.1364
Smoothing Spline	0.83471	3	0.8411

The Gompertz model has a p-value less than 0.05 suggesting that deviations are not normally distributed. Hence, there are excessively large or excessively small deviations. After further investigation of the distribution of the deviations as shown in the table below, it is clear that there are too large deviations, suggesting overgraduation.

(-Inf,-1]	(-1,0]	(0,1]	(1, Inf]
17	9	5	10

Both the Makeham and Smoothing Spline models show no sign of over or under graduation as their p-values are greater than 0.05, suggesting that their standardised deviations are normally distributed.

3.4.3 Signs Test

	P-Value
Gompertz	0.1173
Makeham	1
Smoothing Spline	1

The sign test checks for the balance of positive and negative standardised deviations as a good model that adheres well to the data is expected to be equally above and below the crude rates. All three models have p-values that are greater than 0.05, suggesting that they have a good balance of positive and negative deviations. However, we must still check to see if these positive and negative deviations occur randomly, have any correlation at consecutive ages and the magnitude of the deviations overall.

We first check the magnitude of the bias of deviation in either the positive or negative direction using the cumulative deviations test.

3.4.4 Cumulative Deviations Test

	Test Statistic	**P-Value**
Gompertz	0.8754104	0.3813506
Makeham	0.3126102	0.7545768
Smoothing Spline	0.7715756	0.4403658

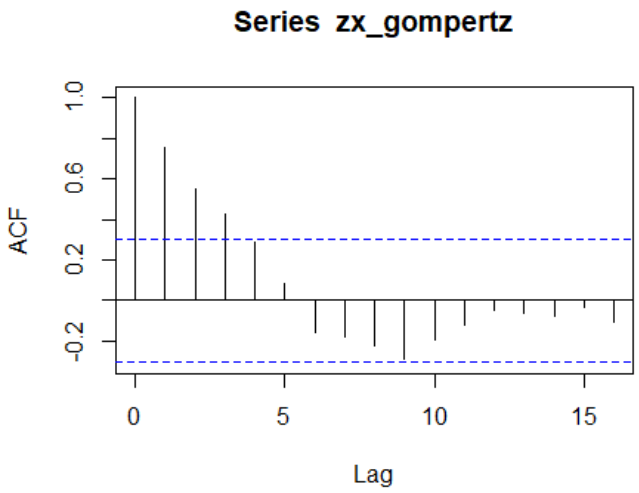
Judging from the test statistics which are all between 0 and 1, and the p values of the models which are all greater than 0.05, we can conclude that all three models do not have a large negative or positive bias. However, the weakness of this test is that big negative bias at certain ages may cancel out with large positive bias at other age groups. Therefore, we must still check that groupings of the sign of the deviations are random and that signs are uncorrelated at consecutive age groups.

3.4.5 Grouping of signs test

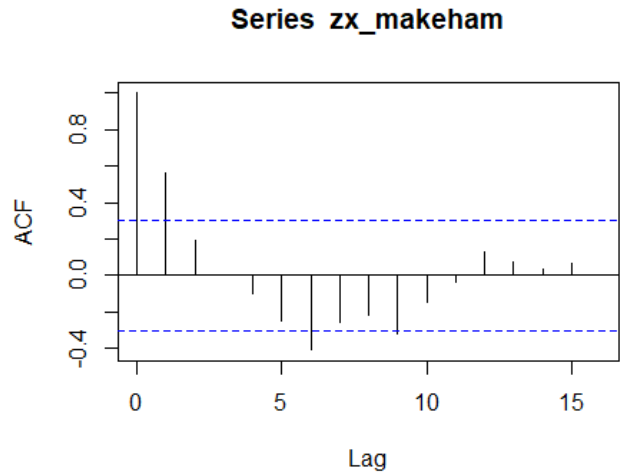
	Test Statistic	**P-Value**
Gompertz	5	0.001603956
Makeham	6	0.003903025
Smoothing Spline	7	0.02091142

The grouping of signs test suggests that the Gompertz and Makeham models have negative and positive deviations that do not occur at random and may be correlated at certain ages. This is a sign of overgraduation. The smoothing spline also fails this test at a significance level of 0.05, but it is important to address that the smoothing spline performs better than the two other models, as the test suggests that negative and positive deviations occur at random for a significance level of 0.02.

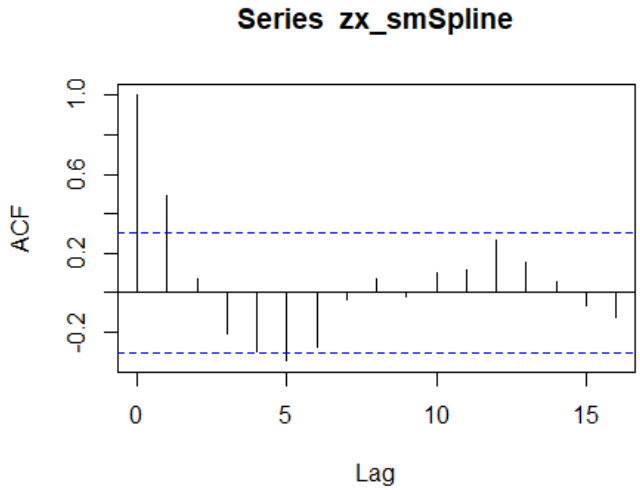
3.4.6 Serial Correlation



In the Gompertz model, the serial correlations start to be contained within the confidence interval at the 4th lag, suggesting that standardised deviations at consecutive ages are not independent.

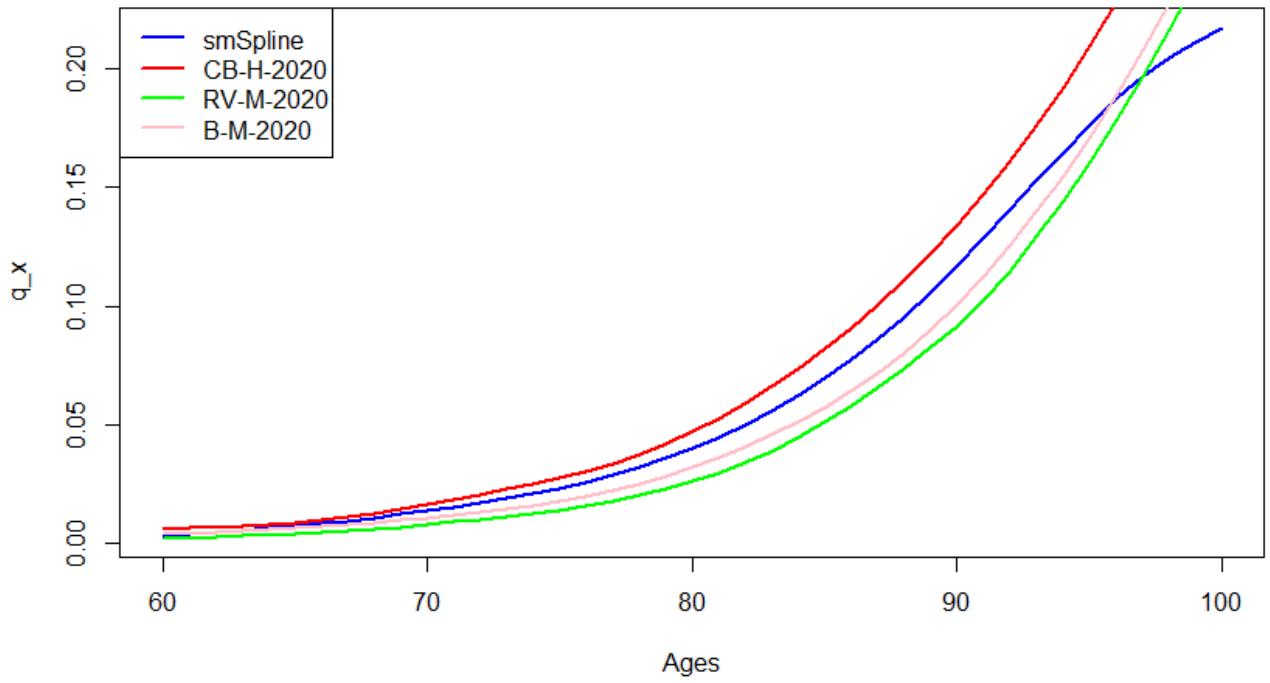


The serial correlations mostly stay within the confidence interval after the first lag. In addition, some lags in the future are also outside the confidence interval, suggesting that deviations may not be independent at consecutive ages.



Showing similar properties to the autocorrelation function of the makeham model, suggesting that standardised deviations may not be independent at consecutive ages.

3.5 Comparison of smoothing spline graduation to the Healthy Chilean Life Tables



The behaviour of the mortality probability after the age of 95 seems unnatural and is the result of overfitting the outlier crude rates at the ages of 99 and 100.

References

Berecki-Gisolf, J., Smith, P.M., Collie, A. and McClure, R.J. (2015). Gender differences in occupational injury incidence. *American Journal of Industrial Medicine*, [online] 58(3), pp.299–307. doi:https://doi.org/10.1002/ajim.22414.

Ferreiro Yazigi, Alejandro, Osvaldo Macías M., Jorge Mastrángelo, Marcia Miranda T., José Luis Ruiz V., Marcia Salinas B., and Dagoberto Valenzuela C. 2003. The Chilean Pension System. Translated by Linda Craddock. 4th ed. Santiago, Chile: Superintendency of Pension Fund Administrators. www.safp.cl.

Katsiferis, A., Bhatt, S., Mortensen, L.H., Mishra, S. and Westendorp, R.G.J. (2023). Sex differences in health care expenditures and mortality after spousal bereavement: A register-based Danish cohort study. PLOS ONE, 18(3), p.e0282892. doi:https://doi.org/10.1371/journal.pone.0282892.

World Bank Gender Data Portal. (n.d.). *Chile*. [online] Available at: https://genderdata.worldbank.org/countries/chile/#:~:text=In%20Chile%2C%20the%20labor%20force.