FAETERJ-Rio
Cálculo I
Professor DSc. Wagner Zanco

Solução dos Exercícios 2.25

a) $f(x) = \sin x.\cos x$
b) $f(x) = e^{(\sin x)}$
c) $f(x) = \frac{\sin x}{x}$
d) $f(x) = \sin x.e^{x}$
e) $f(x) = ln(x).e^{x}$
f) $f(x) = \sin(2x^2 - 1)$
g) $f(x) = \sin\sqrt{x}$
h) $f(x) = e^{2x}.\cos(3x)$
i) $f(x) = \sec x$
j) $f(x) = \text{cossec}\, x$
k) $f(x) = 7tg\ (2x^2 - 1)$
l) $f(x) = \sin(x^4)$
m) $f(x) = sin^4\ x$
n) $f(x) = \ln(\cos x)$
o) $f(x) = 5\sec(2x^2 - 1)$

a) $f(x) = \sin x.\cos x$

$$f'(x) = sinx.(-sinx) + cosx.cosx$$
$$= -sin^2x + cos^2x$$

b) $f(x) = e^{(\sin x)}$

$$u = sinx;\ \ u' = cosx$$
$$f'(x) = e^{(u)'}.u'$$
$$= e^{u}.cosx$$
$$= cosx.e^{sinx}$$

c) $f(x) = \frac{\sin x}{x}$

$$Dx\frac{f(x)}{g(x)} = \frac{g(x).Dxf(x) - f(x).Dxg(x)}{g(x)^2}$$

$$f'(x) = \frac{x.cosx - sinx.(1)}{x^2}$$

$$= \frac{x.cosx - sinx}{x^2}$$

$$= \frac{x.cosx}{x^2} - \frac{sinx}{x^2}$$

$$= \frac{cosx}{x} - \frac{sinx}{x^2}$$

d) $f(x) = \sin x.e^x$

$$f'(x) = \sin x.e^x + cosx.e^x$$
$$= e^x(sinx + cosx)$$

e) $f(x) = ln(x).e^x$

$$f'(x) = ln(x).e^x + \frac{1}{x}e^x$$
$$= e^x(\ln(x) + \frac{1}{x})$$

f) $f(x) = \sin(2x^2 - 1)$

$$u = 2x^2 - 1;\ u' = 4x$$
$$f'(x) = sin(u)'.u'$$
$$= \cos(u).4x$$

$$= \cos(2x^2 - 1) \,.\, 4x$$

$$= 4x.\cos(2x^2 - 1)$$

g) $f(x) = \sin\sqrt{x}$

$$u = \sqrt{x} = x^{\frac{1}{2}};$$

$$u' = \frac{1}{2}x^{\left(\frac{1}{2}-1\right)} = \frac{1}{2}x^{\left(-\frac{1}{2}\right)} = \frac{1}{2}\frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \sin(u)'.u'$$

$$= \cos(u) \,.\, \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}\cos\left(\sqrt{x}\right)$$

$$= \frac{\cos\left(\sqrt{x}\right)}{2\sqrt{x}}$$

h) $f(x) = e^{2x}.\cos(3x)$

$$f'(x) = e^{2x}\, cos(3x)' + e^{2x\prime}\cos(3x)$$

$$cos(3x)' = -3\sin(3x)$$

$$e^{2x\prime} = 2e^{2x}$$

$$f'(x) = e^{2x}.(-3\sin(3x)) + 2e^{2x}\cos(3x)$$

$$= e^{2x}(2\cos(3x) - 3\sin(3x))$$

i) $f(x) = \sec x$

$$\sec x = \frac{1}{\cos x}$$

$$f(x) = \frac{1}{\cos x}$$

$$f'(x) = \frac{1}{\cos x} = \frac{cosx.1' - (-sinx).1}{cos^2x}$$

$$= \frac{sinx}{cos^2x} = \frac{sinx}{cosx}.\frac{1}{cosx}$$

$$tgx = \frac{sinx}{cosx}$$

$$= \frac{sinx}{cosx}.\frac{1}{cosx} = tgx.secx$$

j) $f(x) = \text{cossec}\, x$

$$\text{cossec}\, x = \frac{1}{\sin x}$$

$$f(x) = \frac{1}{\sin x}$$

$$f'(x) = \frac{sinx.1' - cosx.1}{sin^2\, x} = -\frac{cosx}{sin^2\, x} = -\frac{cosx}{\sin x}\frac{1}{\sin x}$$

$$\frac{cosx}{\sin x} = \frac{1}{tgx} = cotgx$$

$$= -\frac{cosx}{\sin x}\frac{1}{\sin x} = -cotg\; x.cossec\; x$$

k) $f(x) = 7tg\ (2x^2 - 1)$

$$tg\ x' = sec^2 x$$

$$u = 2x^2 - 1;\ \ u' = 4x$$

$$f'(x) = 7(tg\ (u)'.u')$$

$$= 7(sec^2(2x^2 - 1).4x)$$

$$= 28x.sec^2(2x^2 - 1)$$

l) $f(x) = \sin(x^4)$

$$u = x^4;\ \ u' = 4x^3$$

$$f'(x) = \sin(u)'.u'$$

$$= \cos(x^4)\ 4x^3$$

$$= 4x^3.\cos(x^4)$$

m) $f(x) = sin^4\ x$

$$f(x) = sin^4\ x = sin^2 x.sin^2 x$$

$$f'(x) = sin^2 x.sin^2 x' + sin^2 x'.sin^2 x$$

$$sin^2 x' = sinx.sinx' + sinx'.sinx$$

$$= sinx.cosx + cosx.sinx$$

$$= 2.cosx.sinx$$

$$f'(x) = sin^2 x.2.cosx.sinx + 2.cosx.sinx.sin^2 x$$

$$= 2(2sin^3 x.\cos x)$$

$$= 4.sin^3 x.\cos x$$

n) $f(x) = \ln(\cos x)$

$$u = \cos x\,;\;\; u' = -\sin x$$

$$f'(x) = \ln(u).u'$$

$$= \frac{1}{\cos x}(-\sin x)$$

$$= -\frac{\sin x}{\cos x}$$

o) $f(x) = 5\sec(2x^2 - 1)$

$$u = 2x^2 - 1;\;\; u' = 4x$$

$$f'(x) = 5(\sec u'\,.u')$$

$$= 5(tg\ u.\sec u.4x)$$

$$= 20x(tg(2x^2 - 1)\sec(2x^2 - 1))$$

Gabarito:

2.25a) $-sin^2x + cos^2x$. 2.25b) $\cos x\, e^{(\sin x)}$. 2.25c) $\frac{\cos x}{x} - \frac{\sin x}{x^2}$.
2.25d) $e^x(\sin x + \cos x)$. 2.25e) $e^x\left(\frac{1}{x} + \ln x\right)$.
2.25f) $4x(\sin(2x^2 - 1))$. 2.25g) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$.
2.25h) $e^{2x}(2\cos(3x) - 3\sin(3x))$. 2.25i) $\tan x \sec x$.
2.25j) – $\cot x\ cossec\ x$. 2.25k) $28x.sec^2(2x^2 - 1)$. 2.25l) $4x^3\cos(x^4)$.
2.25m) $4sin^3x\cos x$. 2.25n) – $\tan x$.
2.25o) $20x\tan(2x^2 - 1)\sec(2x^2 - 1)$.