FAETERJ-Rio
Cálculo I
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Solução dos Exercícios 3.2a – 3.2w

Exercícios 3.2: Resolva as integrais a seguir.

a) $\int \cos(2x)\ dx$

$$u = 2x$$
$$u' = 2$$
$$du = u'dx$$
$$du = 2dx$$
$$\frac{du}{2} = dx$$

$$\int \cos(u)\frac{du}{2} = \frac{1}{2}\int \cos(u)\, du$$

$$= \frac{1}{2} sen\ u + C = \frac{1}{2} sen\ (2x) + C$$

b) $\int xe^{(x^2)}\ dx$

$$u = x^2$$
$$u' = 2x$$
$$du = u'dx$$
$$du = 2x\ dx$$
$$\frac{du}{2x} = dx$$

$$\int xe^{(u)}\frac{du}{2x} = \frac{1}{2}\int e^u\ du$$
$$= \frac{1}{2}e^u + C = \frac{1}{2}e^{(x^2)} + C$$

c) $\int x^2\sqrt{x^3+1}\ dx$

$$u = x^3 + 1$$
$$u' = 3x^2$$
$$du = u'dx$$
$$du = 3x^2\ dx$$
$$\frac{du}{3x^2} = dx$$

$$\int x^2\sqrt{u}\frac{du}{3x^2} = \frac{1}{3}\int \sqrt{u}\ du = \frac{1}{3}\int u^{\frac{1}{2}}\ du$$

$$= \frac{1}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{3}\frac{2}{3}u^{\frac{3}{2}} = \frac{2}{9}\sqrt{u^3} + C = \frac{2}{9}\sqrt{(x^3+1)^3} + C$$

d) $\int sin^2\theta \cos\theta\ \ d\theta$

$$u = \sin\theta$$
$$u' = \cos\theta$$
$$du = u'd\theta$$
$$du = \cos\theta\ \ d\theta$$
$$\frac{du}{\cos\theta} = d\theta$$
$$\int u^2 \cos\theta \frac{du}{\cos\theta} = \int u^2\, du$$
$$= \frac{u^3}{3} + C = \frac{1}{3}(\sin\theta)^3 + C$$

e) $\int \frac{x^3}{x^4-5}\ dx$

$$u = x^4 - 5$$
$$u' = 4x^3$$
$$du = u'dx$$
$$du = 4x^3dx$$
$$\frac{du}{4x^3} = dx$$

$$\int \frac{x^3}{u}\frac{du}{4x^3} = \frac{1}{4}\int \frac{1}{u}\ du$$

$$= \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln |x^4 - 5| + C$$

f) $\int \sqrt{2t+1}\ dt$

$$u = 2t + 1$$
$$u' = 2$$
$$du = u'dt$$
$$du = 2dt$$
$$\frac{du}{2} = dt$$

$$\int \sqrt{u} \frac{du}{2} = \frac{1}{2} \int \sqrt{u}\, du = \frac{1}{2} \int u^{\frac{1}{2}}\, du$$

$$= \frac{1}{2} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{2} \frac{2}{3} u^{\frac{3}{2}} = \frac{1}{3} \sqrt{u^3} + C = \frac{1}{3} \sqrt{(2t+1)^3} + C$$

g) $\int x\sqrt{1-x^2}\ dx$

$$u = 1 - x^2$$
$$u' = -2x$$
$$du = u'dx$$
$$du = -2x\ dx$$
$$\frac{du}{-2x} = dx$$

$$\int x\sqrt{u} \frac{du}{-2x} = -\frac{1}{2} \int \sqrt{u}\, du = -\frac{1}{2} \int \sqrt{u}\, du = -\frac{1}{2} \int u^{\frac{1}{2}}\, du$$

$$= -\frac{1}{2} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{1}{2} \frac{2}{3} u^{\frac{3}{2}} = -\frac{1}{3} \sqrt{u^3} + C = -\frac{1}{3} \sqrt{(1-x^2)^3} + C$$

h) $\int (3x+2)^{20}\ dx$

$$u = 3x + 2$$
$$u' = 3$$
$$du = u'dx$$

$$du = 3dx$$
$$\frac{du}{3} = dx$$

$$\int (u)^{20} \frac{du}{3} = \frac{1}{3} \int u^{20}\, du$$
$$= \frac{1}{3} \frac{u^{21}}{21} = \frac{1}{63} u^{21} + C = \frac{1}{63} (3x+2)^{21} + C$$

i) $\int \cos\left(\frac{\pi t}{2}\right)\, dt$

$$u = \frac{\pi t}{2}$$
$$u' = \frac{\pi}{2}$$
$$du = u' dt$$
$$du = \frac{\pi}{2}\, dt$$
$$\frac{du}{\frac{\pi}{2}} = dt$$
$$\frac{2}{\pi} du = dt$$

$$\int \cos(u)\, \frac{2}{\pi} du = \frac{2}{\pi} \int \cos(u)\, du$$

$$= \frac{2}{\pi} \sin u + C = \frac{2}{\pi} \sin\left(\frac{\pi t}{2}\right) + C$$

j) $\int \cos(\pi t)\, dt$

$$u = \pi t$$
$$u' = \pi$$
$$du = u' dt$$
$$du = \pi\, dt$$
$$\frac{du}{\pi} = dt$$

$$\int \cos(u) \frac{du}{\pi} = \frac{1}{\pi} \int \cos(u)\, du$$

$$= \frac{1}{\pi} \sin u + C = \frac{1}{\pi} \sin(\pi t) + C$$

k) $\int \cos^2(\theta) \sin(\theta)\ d\theta$

$$u = \cos\theta$$
$$u' = -\sin\theta$$
$$du = u'd\theta$$
$$du = -\sin\theta\, d\theta$$
$$\frac{du}{-\sin\theta} = d\theta$$

$$\int u^2 \sin(\theta) \frac{du}{-\sin\theta} = -\int u^2\, du$$
$$= -\frac{u^3}{3} + C = -\frac{1}{3} cos^3\theta + C$$

l) $\int \frac{e^u}{(1-e^u)^2}\ du$

$$z = 1 - e^u$$
$$z' = -e^u$$
$$dz = u'du$$
$$dz = -e^u\ du$$
$$\frac{dz}{-e^u} = du$$

$$\int \frac{e^u}{z^2} \frac{dz}{-e^u} = -\int \frac{1}{z^2} dz = -\int z^{-2}\, dz$$
$$= -\left(\frac{z^{-1}}{-1}\right) = \frac{1}{z} + C = \frac{1}{1-e^u} + C$$

m) $\int \frac{(\ln x)^2}{x}\ dx$

$$u = \ln x$$
$$u' = \frac{1}{x}$$
$$du = u'dx$$
$$du = \frac{1}{x} dx$$
$$xdu = dx$$

$$\int \frac{(u)^2}{x} \, xdu = \int u^2 \, du$$
$$= \frac{u^3}{3} + C = \frac{1}{3}(\ln|x|)^3 + C$$

n) $\int \cos^4(\theta)\sin(\theta) \; d\theta$

$$u = \cos\theta$$
$$u' = -\sin\theta$$
$$du = u'd\theta$$
$$du = -\sin\theta \, d\theta$$
$$\frac{du}{-\sin\theta} = d\theta$$

$$\int u^4 \sin(\theta) \frac{du}{-\sin\theta} = -\int u^4 \, du$$
$$= -\frac{u^5}{5} + C = \frac{1}{5} cos^5\theta + C$$

o) $\int x^2 e^{x^3} \; dx$

$$u = x^3$$
$$u' = 3x^2$$
$$du = u'dx$$
$$du = 3x^2 dx$$
$$\frac{du}{3x^2} = dx$$

$$\int x^2 e^u \frac{du}{3x^2} = \frac{1}{3}\int e^u \, du$$

$$= \frac{1}{3} e^u + C = \frac{1}{3} e^u + C = \frac{1}{3} e^{x^3} + C$$

p) $\int 3\sin(3t)\ dt$

$$u = 3t$$
$$u' = 3$$
$$du = u'dt$$
$$du = 3dt$$
$$\frac{du}{3} = dt$$

$$\int 3\sin(u)\frac{du}{3} = \int \sin u\ \ du$$
$$-\cos u + C = -\cos(3t) + C$$

q) $\int sec^2\,(2\theta)\ d\theta$

$$u = 2\theta$$
$$u' = 2$$
$$du = u'd\theta$$
$$du = 2d\theta$$
$$\frac{du}{2} = d\theta$$

$$\int sec^2\,(u)\frac{du}{2} = \frac{1}{2}\int sec^2\,(u)\ du$$

$$= \frac{1}{2}tg\ u + C = \frac{1}{2}tg\ (2\theta) + C$$

r) $\int y^2(4 - y^3)^{\left(\frac{2}{3}\right)}\ dy$

$$u = 4 - y^3$$
$$u' = -3y^2$$
$$du = u'dy$$
$$du = -3y^2dy$$
$$\frac{du}{-3y^2} = dy$$

$$\int y^2 (u)^{\left(\frac{2}{3}\right)} \frac{du}{-3y^2} = -\frac{1}{3}\int (u)^{\left(\frac{2}{3}\right)}\ du$$

$$= -\frac{1}{3}\frac{u^{\frac{5}{3}}}{\frac{5}{3}} = -\frac{1}{3}\frac{3}{5}u^{\frac{5}{3}} = -\frac{1}{5}\sqrt[3]{u^5} + C = -\frac{1}{5}\sqrt[3]{(4-y^3)^5} + C$$

s) $\int e^{-5r}\ dr$

$$u = -5r$$
$$u' = -5$$
$$du = u'dr$$
$$du = -5dr$$
$$\frac{du}{-5} = dr$$

$$\int e^u \frac{du}{-5} = -\frac{1}{5}\int e^u\ du$$

$$-\frac{1}{5}e^u + C = -\frac{1}{5}e^{(-5r)} + C$$

t) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx$

$$u = \sqrt{x}$$
$$u' = \frac{1}{2\sqrt{x}}$$
$$du = u'dx$$
$$du = \frac{1}{2\sqrt{x}}dx$$
$$2\sqrt{x}\ du = dx$$

$$\int \frac{\sin u}{\sqrt{x}} 2\sqrt{x}\ du = 2\int \sin u\, du$$

$$= 2(-\cos u) + C = -2\left(\cos\sqrt{x}\right) + C$$

u) $\int \frac{z^2}{z^3+1}\ dz$

$$u = z^3 + 1$$
$$u' = 3z^2$$
$$du = u'dz$$
$$du = 3z^2dz$$
$$\frac{du}{3z^2} = dz$$

$$\int \frac{z^2}{u}\frac{du}{3z^2} = \frac{1}{3}\int \frac{1}{u}du$$

$$= \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|z^3+1| + C$$

v) $\int \sin x \sin(\cos x)\ \ dx$

$$u = \cos x$$
$$u' = -\sin x$$
$$du = u'dx$$
$$du = -\sin x\, dx$$
$$\frac{du}{-\sin x} = dx$$

$$\int \sin x \sin(u)\frac{du}{-\sin x} = -\int \sin(u)\ \ du$$
$$= -(-\cos u\ ) + C = \cos(\cos x) + C$$

w) $\int x\sqrt{x+2}\ \ dx$

$$u = x + 2$$
$$u' = 1$$
$$du = u'dx$$
$$du = dx$$

$$\int (u-2)\sqrt{u}\ \ du = \int u\sqrt{u} - 2\sqrt{u}\ \ du = \int \sqrt{u^3}\,du\ - 2\int \sqrt{u}\ \ du$$

$$\int \sqrt{u^3}\, du = \int u^{\frac{3}{2}}\, du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{5}u^{\frac{5}{2}} + C$$

$$-2\int \sqrt{u}\; du = -2\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = -2\frac{2}{3}\sqrt{u^3} + C = -\frac{4}{3}\sqrt{u^3} + C =$$

$$\int (u-2)\sqrt{u}\; du = \frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}\sqrt{u^3} + C = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}\sqrt{(x+2)^3} + C$$

$$= \frac{2}{5}(x+2)\sqrt{(x+2)^3} - \frac{4}{3}\sqrt{(x+2)^3} + C$$

$$= \sqrt{(x+2)^3}\left(\frac{2}{5}(x+2) - \frac{4}{3}\right)$$

$$= \sqrt{(x+2)^3}\,\frac{6(x+2)-20}{15}$$

$$= \sqrt{(x+2)^3}\,\frac{2[3(x+2)-10]}{15}$$

$$= \sqrt{(x+2)^3}\,\frac{2(3x+6-10)}{15}$$

$$= \sqrt{(x+2)^3}\,\frac{2}{15}(3x-4)$$

$$= \frac{2}{15}\sqrt{(x+2)^3}(3x-4) + C$$

Gabarito:

3.2a) $\frac{1}{2}\sin(2x) + C$. 3.2b) $\frac{1}{2}e^{x^2} + C$. 3.2c) $\frac{2}{9}\sqrt{x^3+1} + C$.
3.2d) $\frac{1}{3}sin^3\theta + C$. 3.2e) $\frac{1}{4}\ln|x^4-5| + C$. 3.2f) $\frac{1}{3}\sqrt{(2t+1)^3} + C$.
3.2g) $-\frac{1}{3}\sqrt{(1+x^2)^3} + C$. 3.2h) $\frac{1}{63}(3x+2)^{21} + C$. 3.2i) $\frac{2}{\pi}\sin\left(\frac{\pi t}{2}\right) + C$.
3.2j) $\frac{1}{\pi}\sin(\pi t) + C$. 3.2k) $-\frac{1}{3}\cos^3\theta + C$. 3.2l) $-\frac{1}{1-e^u} + C$.
3.2m) $\frac{1}{3}ln^3|x| + C$. 3.2n) $-\frac{1}{5}(\cos\theta)^5 + C$. 3.2o) $\frac{1}{3}e^{x^3} + C$.
3.2p) $-\cos(3t) + C$. 3.2q) $\frac{1}{2}tg(2\theta) + C$.

3.2r) $\frac{1}{5}\sqrt[3]{(4-y^3)^5}+C$. 3.2s) $-\frac{1}{5}e^{-5r}+C$. 3.2t) $-2\cos(\sqrt{x})+C$.
3.2u) $\frac{1}{3}\ln|z^3+1|+C$. 3.2v) $\cos(\cos x)+C$.
3.2w) $\frac{2}{15}\sqrt{(x+2)^3}(3x-4)+C$.