FAETERJ-Rio
Cálculo I
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Solução dos Exercícios 2.24a – 2.24g

Exercício 2.24: Calcule os limites a seguir.

a) $\lim\limits_{x\to\infty} \frac{x^2+4}{8^x}$ (obs.: $Dx(a^x) = a^x.\ln a$)

$$\lim_{x\to\infty} \frac{x^2+4}{8^x} = \frac{\infty}{\infty}$$

$$Dx(x^2+4) = 2x$$

$$Dx(8^x) = 8^x \ln 8$$

$$\lim_{x\to\infty} \frac{x^2+4}{8^x} = \lim_{x\to\infty} \frac{2x}{8^x \ln 8} = \infty$$

$$Dx(2x) = 2$$

$$Dx(8^x \ln 8) = 8^x 8^x \ln 8 = 8^x (\ln 8)^2$$

$$\lim_{x\to\infty} \frac{x^2+4}{8^x} = \lim_{x\to\infty} \frac{2x}{8^x \ln 8} = \lim_{x\to\infty} \frac{2}{8^x (\ln 8)^2} = 0$$

b) $\lim\limits_{x\to\infty} \left(\frac{\ln x}{x^2}\right)$

$$\lim_{x\to\infty} \left(\frac{\ln x}{x^2}\right) = \frac{\infty}{\infty}$$

$$Dx(\ln x) = \frac{1}{x}$$

$$Dx(x^2) = 2x$$

$$\lim_{x\to\infty}\left(\frac{\ln x}{x^2}\right)=\lim_{x\to\infty}\left(\frac{\frac{1}{x}}{2x}\right)=\lim_{x\to\infty}\left(\frac{1}{2x^2}\right)=\frac{1}{\infty}=0$$

c) $\lim_{x\to\infty}\left(\frac{x^3}{e^{3x}}\right)$

$$\lim_{x\to\infty}\left(\frac{x^3}{e^{3x}}\right)=\frac{\infty}{\infty}$$

$$Dx(x^3)=3x^2$$

$$Dx(e^{3x})=e^{3x}.3=3e^{3x}$$

$$\lim_{x\to\infty}\left(\frac{x^3}{e^{3x}}\right)=\lim_{x\to\infty}\left(\frac{3x^2}{3e^{3x}}\right)=\frac{\infty}{\infty}$$

$$Dx(3x^2)=6x$$

$$Dx(3e^{3x})=3e^{3x}.3=9e^{3x}$$

$$\lim_{x\to\infty}\left(\frac{x^3}{e^{3x}}\right)=\lim_{x\to\infty}\left(\frac{3x^2}{3e^{3x}}\right)=\lim_{x\to\infty}\left(\frac{6x}{9e^{3x}}\right)=\frac{\infty}{\infty}$$

$$Dx(6x)=6$$

$$Dx(9e^{3x})=9e^{3x}.3=27e^{3x}$$

$$\lim_{x\to\infty}\left(\frac{x^3}{e^{3x}}\right)=\lim_{x\to\infty}\left(\frac{3x^2}{3e^{3x}}\right)=\lim_{x\to\infty}\left(\frac{6x}{9e^{3x}}\right)=\lim_{x\to\infty}\left(\frac{6}{27e^{3x}}\right)=\frac{6}{\infty}=0$$

d) $\lim_{x\to1}\left(\frac{sen\,(\pi x)}{x-1}\right)$ (obs.: $\cos\pi=-1$)

$$\lim_{x\to1}\left(\frac{sen\,(\pi x)}{x-1}\right)=\frac{0}{0}$$

$$Dx(sen\,(\pi x))=cos(\pi x).\pi=\pi cos(\pi x)$$

$$Dx(x-1)=1$$

$$\lim_{x\to 1}\left(\frac{sen\ (\pi x)}{x-1}\right)=\lim_{x\to 1}\left(\frac{\pi cos(\pi x)}{1}\right)=-\pi$$

e) $\lim_{x\to 0}\left(\frac{e^x-cos\ x}{4.sen\ x}\right)$

$$\lim_{x\to 0}\left(\frac{e^x-cos\ x}{4.\ sen\ x}\right)=\frac{1-1}{4.0}=\frac{0}{0}$$

$$Dx(e^x-cos\ x)=e^x+sen\ x$$

$$Dx(4.sen\ x)=4cos\ x$$

$$\lim_{x\to 0}\left(\frac{e^x-cos\ x}{4.\ sen\ x}\right)=\lim_{x\to 0}\left(\frac{e^x+sen\ x}{4cos\ x}\right)=\frac{1+0}{4}=\frac{1}{4}$$

f) $\lim_{x\to 0}\left(\frac{2e^{2x}-2}{x^2+x}\right)$

$$\lim_{x\to 0}\left(\frac{2e^{2x}-2}{x^2+x}\right)=\frac{2-2}{0-0}=\frac{0}{0}$$

$$Dx(2e^{2x}-2)=2.2.e^{2x}=4e^{2x}$$

$$Dx(x^2+x)=2x+1$$

$$\lim_{x\to 0}\left(\frac{2e^{2x}-2}{x^2+x}\right)=\lim_{x\to 0}\left(\frac{4e^{2x}}{2x+1}\right)=\frac{4}{1}=4$$

g) $\lim_{x\to 0}\left(\frac{x.cos\ x}{(x+1).sen\ x}\right)$

$$\lim_{x\to 0}\left(\frac{x.\ cos\ x}{(x+1).\ sen\ x}\right)=\frac{0.1}{1.0}=\frac{0}{0}$$

$$Dx(x.\ cos\ x)=x(-sen\ x)+\big(1(\cos x)\big)=-xsen\ x+\cos x$$

$$Dx\big((x+1).\ sen\ x\big)=(x+1)\cos x+sen\ x$$

$$\lim_{x\to 0}\left(\frac{x.cos\ x}{(x+1).sen\ x}\right)=\lim_{x\to 0}\left(\frac{-xsen\ x+\cos x}{(x+1)\cos x+sen\ x}\right)=\frac{1}{1}=1$$

Gabarito:

2.24a) 0. 2.24b) 0. 2.24c) 0. 2.24d) $-\pi$. 2.24e) $\frac{1}{4}$. 2.24f) 4. 2.24g) 1.