20. Para cada una de las siguientes matrices, usar operaciones elementales por fila para decidir si son inversibles y hallar la inversa cuando lo sean.

$$\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 3 & -8 \\ -2 & 1 & 2 & -2 \\ 1 & 2 & 1 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & i \end{bmatrix}.$$

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[f_1 \cdot 1/3]{} \left(\begin{array}{ccc|ccc} 1 & -1/3 & 2/3 & 1/3 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{f_2 - 2f_1 \\ f_3 - f_1}]{} \left(\begin{array}{ccc|ccc} 1 & -1/3 & 2/3 & 1/3 & 0 & 0 \\ 0 & 5/3 & -1/3 & -2/3 & 1 & 0 \\ 0 & -8/3 & -2/3 & -1/3 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[f_2 \cdot \frac{3}{5}]{} \left(\begin{array}{ccc|ccc} 1 & -1/3 & 2/3 & 1/3 & 0 & 0 \\ 0 & 1 & -1/5 & -2/5 & 3/5 & 0 \\ 0 & -8/3 & -2/3 & -1/3 & 0 & 1 \end{array}\right) \xrightarrow[f_3 + \frac{8}{3} f_2]{} \left(\begin{array}{ccc|ccc} 1 & -1/3 & 2/3 & 1/3 & 0 & 0 \\ 0 & 1 & -1/5 & -2/5 & 3/5 & 0 \\ 0 & 0 & -6/5 & -7/5 & 8/5 & 1 \end{array}\right)$$

$$\xrightarrow[-\frac{5}{6} \cdot f_3]{} \left(\begin{array}{ccc|ccc} 1 & -1/3 & 2/3 & 1/3 & 0 & 0 \\ 0 & 1 & -1/5 & -2/5 & 3/5 & 0 \\ 0 & 0 & 1 & 7/6 & -4/3 & -5/6 \end{array}\right) \xrightarrow[f_1 + \frac{1}{3} f_2]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 3/5 & 1/5 & 1/5 & 0 \\ 0 & 1 & -1/5 & -2/5 & 3/5 & 0 \\ 0 & 0 & 1 & 7/6 & -4/3 & -5/6 \end{array}\right)$$

$$\xrightarrow[\substack{f_1 - \frac{3}{5} f_3 \\ f_2 + \frac{1}{5} f_3}]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & 1 & -1/2 \\ 0 & 1 & 0 & -1/6 & 1/3 & -1/6 \\ 0 & 0 & 1 & 7/6 & -4/3 & -5/6 \end{array}\right)$$

Entonces, $\begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} -1/2 & 1 & -1/2 \\ -1/6 & 1/3 & -1/6 \\ 7/6 & -4/3 & -5/6 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 2 & 5 & -1 & 1 & 0 & 0 \\ 4 & -1 & 2 & 0 & 1 & 0 \\ 6 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[f_1 \cdot \frac{1}{2}]{} \left(\begin{array}{ccc|ccc} 1 & 5/2 & -1/2 & 1/2 & 0 & 0 \\ 4 & -1 & 2 & 0 & 1 & 0 \\ 6 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{f_2 - 4f_1 \\ f_3 - 6f_1}]{} \left(\begin{array}{ccc|ccc} 1 & 5/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & -11 & 4 & -2 & 1 & 0 \\ 0 & -11 & 4 & -3 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[f_3 - f_2]{} \left(\begin{array}{ccc|ccc} 1 & 5/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & -11 & 4 & -2 & 1 & 0 \\ \underbrace{0 \quad 0 \quad 0}_{\text{fila nula}} & & & -1 & -1 & 1 \end{array}\right)$$

$\begin{pmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{pmatrix}$ es no inversible.

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & -3 & 3 & -8 & 0 & 1 & 0 & 0 \\ -2 & 1 & 2 & -2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 4 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{f_2 - f_1 \\ f_3 + 2f_1 \\ f_4 - f_1}]{} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & -4 & 2 & -10 & -1 & 1 & 0 & 0 \\ 0 & 3 & 4 & 2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[f_2 \leftrightarrow f_4]{} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 3 & 4 & 2 & 2 & 0 & 1 & 0 \\ 0 & -4 & 2 & -10 & -1 & 1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow[\substack{f_1 - f_2 \\ f_3 - 3f_2 \\ f_4 + 4f_2}]{} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 0 & 4 & -4 & 5 & 0 & 1 & -3 \\ 0 & 0 & 2 & -2 & -5 & 1 & 0 & 4 \end{array}\right) \xrightarrow[f_3 - 2f_4]{} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 15 & -2 & 1 & -11 \\ 0 & 0 & 2 & -2 & -5 & 1 & 0 & 4 \end{array}\right)$$

$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 3 & -8 \\ -2 & 1 & 2 & -2 \\ 1 & 2 & 1 & 4 \end{pmatrix}$ es no inversible.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 3 & 0 & i & 0 & 0 & 1 \end{array}\right) \xrightarrow[¿?]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-1+5i}{78} & \frac{1-5i}{39} & \frac{25+5i}{78} \\ 0 & 1 & 0 & \frac{47-i}{78} & \frac{-8-i}{39} & \frac{-5-i}{78} \\ 0 & 0 & 1 & \frac{-5-i}{26} & \frac{5+i}{13} & \frac{-5-i}{26} \end{array}\right)$$

Entonces, $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & i \end{pmatrix}^{-1} = \begin{pmatrix} \frac{-1+5i}{78} & \frac{1-5i}{39} & \frac{25+5i}{78} \\ \frac{47-i}{78} & \frac{-8-i}{39} & \frac{-5-i}{78} \\ \frac{-5-i}{26} & \frac{5+i}{13} & \frac{-5-i}{26} \end{pmatrix}$