14. Sean

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}.$$

(a) Realizar los productos AB, BA, AC, CA, BC, CB, ABC, ACB, BAC, BCA, CAB y CBA.

(b) Verificar que, en los productos de 3 matrices, da lo mismo asociar de una u otra forma.

Recordar que si $A = \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix}$ y $B = \begin{pmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{pmatrix}$, entonces:

$$A \cdot B = \begin{pmatrix} AJ+BM+CP & AK+BN+CQ & AL+BO+CR \\ DJ+EM+FP & DK+EN+FQ & DL+EO+FR \\ GJ+HK+IP & GK+HN+IQ & GL+HO+IR \end{pmatrix}$$

a) $AB = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 5 & 1 & 4 \\ 6 & 4 & 9 \\ 4 & -2 & -1 \end{pmatrix}$ $\qquad BA = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 4 & -8 & -1 \\ -3 & 6 & 1 \\ 9 & -18 & -2 \end{pmatrix}$

$AC = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & -1 & -1 \\ 0 & -1 & 0 \\ -6 & -1 & -2 \end{pmatrix}$ $\qquad CA = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -6 & -1 \\ 4 & -8 & -1 \end{pmatrix}$

$BC = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 9 & -1 & 4 \\ -5 & 2 & -3 \\ 22 & -1 & 9 \end{pmatrix}$ $\qquad CB = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 8 \\ 3 & 5 & 9 \\ 4 & 6 & 11 \end{pmatrix}$

$ABC = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 1 & 4 \\ 6 & 4 & 9 \\ 4 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 19 & -5 & 10 \\ 41 & -6 & 19 \\ -3 & -4 & 1 \end{pmatrix}$

$ACB = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} -3 & -1 & -1 \\ 0 & -1 & 0 \\ -6 & -1 & -2 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} -2 & -6 & -10 \\ 2 & 0 & 1 \\ -6 & -12 & -21 \end{pmatrix}$

$BAC = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -8 & -1 \\ -3 & 6 & 1 \\ 9 & -18 & -2 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -15 & -4 & -5 \\ 12 & 3 & 4 \\ -33 & -9 & -11 \end{pmatrix}$

$BCA = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 9 & -1 & 4 \\ -5 & 2 & -3 \\ 22 & -1 & 9 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 12 & -24 & -5 \\ -6 & 12 & 5 \\ 30 & -60 & -10 \end{pmatrix}$

$CAB = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -6 & -1 \\ 4 & -8 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 3 & -5 & -6 \\ 14 & 0 & 7 \\ 19 & 1 & 11 \end{pmatrix}$

$CBA = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 8 \\ 3 & 5 & 9 \\ 4 & 6 & 11 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 16 & -32 & -4 \\ 17 & -34 & -4 \\ 21 & -42 & -5 \end{pmatrix}$

b) Tengo que probar que $(AB)C = A(BC)$

$$\left(\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}\right)\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix}\left(\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}\right)$$

$$\begin{pmatrix} 5 & 1 & 4 \\ 6 & 4 & 9 \\ 4 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 9 & -1 & 4 \\ -5 & 2 & -3 \\ 22 & -1 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 19 & -5 & 10 \\ 41 & -6 & 19 \\ -3 & -4 & 1 \end{pmatrix} = \begin{pmatrix} 19 & -5 & 10 \\ 41 & -6 & 19 \\ -3 & -4 & 1 \end{pmatrix} \checkmark$$