15. Probar que si A y B son matrices $r \times n$ y C es una matriz $n \times q$, entonces

$$(A+B)C = AC + BC.$$

Tenemos $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{r1} & \cdots & a_{rn} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{r1} & \cdots & b_{rn} \end{pmatrix}$ y $C = \begin{pmatrix} c_{11} & \cdots & c_{1q} \\ \vdots & & \\ c_{n1} & \cdots & c_{nq} \end{pmatrix}$

Luego:

$$(A+B)C = \left(\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{r1} & \cdots & a_{rn} \end{pmatrix} + \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{r1} & \cdots & b_{rn} \end{pmatrix} \right) \begin{pmatrix} c_{11} & \cdots & c_{1q} \\ \vdots & & \\ c_{n1} & \cdots & c_{nq} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}+b_{11} & \cdots & a_{1n}+b_{1n} \\ \vdots & & \vdots \\ a_{r1}+b_{r1} & \cdots & a_{rn}+b_{rn} \end{pmatrix} \begin{pmatrix} c_{11} & \cdots & c_{1q} \\ \vdots & & \vdots \\ c_{n1} & \cdots & c_{nq} \end{pmatrix}$$

$$= \begin{pmatrix} (a_{11}+b_{11})c_{11}+\cdots+(a_{1n}+b_{1n})c_{n1} & \cdots & (a_{11}+b_{11})c_{1q}+\cdots+(a_{1n}+b_{1n})c_{nq} \\ \vdots & & \vdots \\ (a_{r1}+b_{r1})c_{11}+\cdots+(a_{rn}+b_{rn})c_{n1} & \cdots & (a_{r1}+b_{r1})c_{1q}+\cdots+(a_{rn}+b_{rn})c_{nq} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}c_{11}+b_{11}c_{11}+\cdots+a_{1n}c_{n1}+b_{1n}c_{n1} & \cdots & a_{11}c_{1q}+b_{11}c_{1q}+\cdots+a_{1n}c_{nq}+b_{1n}c_{nq} \\ \vdots & & \vdots \\ a_{r1}c_{11}+b_{r1}c_{11}+\cdots+a_{rn}c_{n1}+b_{rn}c_{n1} & \cdots & a_{r1}c_{1q}+b_{r1}c_{1q}+\cdots+a_{rn}c_{nq}+b_{rn}c_{nq} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}c_{11}+\cdots+a_{1n}c_{n1}+b_{11}c_{11}+\cdots+b_{1n}c_{n1} & \cdots & a_{11}c_{1q}+\cdots+a_{1n}c_{nq}+b_{11}c_{1q}+\cdots+b_{1n}c_{nq} \\ \vdots & & \vdots \\ a_{r1}c_{11}+\cdots+a_{rn}c_{n1}+b_{r1}c_{11}+\cdots+b_{rn}c_{n1} & \cdots & a_{r1}c_{1q}\cdots+a_{rn}c_{nq}+b_{r1}c_{1q}+\cdots+b_{rn}c_{nq} \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} a_{11}c_{11}+\cdots+a_{1n}c_{n1} & \cdots & a_{11}c_{1q}+\cdots+a_{1n}c_{nq} \\ \vdots & & \vdots \\ a_{r1}c_{11}+\cdots+a_{rn}c_{n1} & \cdots & a_{r1}c_{1q}\cdots+a_{rn}c_{nq} \end{pmatrix}}_{AC} + \underbrace{\begin{pmatrix} b_{11}c_{11}+\cdots+b_{1n}c_{n1} & \cdots & b_{11}c_{1q}+\cdots+b_{1n}c_{nq} \\ \vdots & & \vdots \\ b_{r1}c_{11}+\cdots+b_{rn}c_{n1} & \cdots & b_{r1}c_{1q}+\cdots+b_{rn}c_{nq} \end{pmatrix}}_{BC}$$

Por lo tanto, $(AB)+C = AC + BC$