3. Demostrar que en $(\mathbb{C}, +, \cdot)$ se cumple:

(a) $\overline{\overline{z}} = z$ Si $z = a+bi \Rightarrow \overline{\overline{z}} = \overline{\overline{a+bi}} = \overline{a-bi} = a+bi = z$

(b) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ Si $z_1 = a+bi$ y $z_2 = c+di$, $\overline{z_1+z_2} = \overline{(a+bi)+(c+di)} = \overline{(a+c)+i(b+d)} = (a+c)-i(b+d)$

$\overline{z_1}+\overline{z_2} = \overline{a+bi} + \overline{c+di} = a-bi+c-di = (a+c)-i(b+c)$

(c) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ Si $z_1 = a+bi$ y $z_2 = c+di$, $\overline{z_1 z_2} = \overline{(a+bi)(c+di)} = \overline{(ac+bd)+i(ad-bc)} = (ac-bd)-i(ad+bc)$

$\overline{z_1}\,\overline{z_2} = \overline{a+bi}\;\overline{c+di} = (a-bi)(c-di) = (ac-bd)-i(ad+bc)$

(d) $|\bar{z}| = |z|$ Si $z = a+bi$, $|\overline{z}| = |\overline{a+bi}| = |a-bi| = \sqrt{a^2+(-b)^2} = \sqrt{a^2+b^2} = |z|$

(e) $z\bar{z} = |z|^2$ Si $z = a+bi$, $z\overline{z} = (a+bi)\overline{a+bi} = (a+bi)(a-bi) = a^2-bi^2 = a^2+b^2 = |z|^2$

(f) $z^{-1} = \frac{1}{|z|^2}\bar{z}$, $\forall z \neq 0$

Si $z = a+bi$, $z^{-1} = (a+bi)^{-1} = \dfrac{1}{a+bi} = \dfrac{1}{a+bi}\cdot\dfrac{a-bi}{a-bi} = \dfrac{a-bi}{(a+bi)(a-bi)} = \dfrac{a-bi}{a^2+b^2} = \dfrac{\overline{z}}{|z|} = \dfrac{1}{|z|}\overline{z}$

(g) $|z_1 z_2| = |z_1|\,|z_2|$.

Si $z_1 = a+bi$ y $z_2 = c+di$,

$|z_1 z_2| = |(a+bi)(c+di)| = |(ac-bd)+i(ad+bc)| = \sqrt{(ac-bd)^2+(ad+bc)^2} = \sqrt{(ac)^2+(bd)^2-2acbd+(ad)^2+(bc)^2+2acbd}$

$= \sqrt{(ac)^2+(bd)^2+(ad)^2+(bc)^2} = \sqrt{(a^2+b^2)(c^2+d^2)} = \sqrt{a^2+b^2}+\sqrt{c^2+d^2} = |a+bi|+|c+di|$

$= |z_1|+|z_2|$

(h) $|z| \geq |\mathfrak{Re}(z)|$ y $|z| \geq |\mathfrak{Im}(z)|$ Si $z = a+bi$, $|z| = \sqrt{a^2+b^2} \geqslant \sqrt{a^2} = a = Re(z)$

$|z| = \sqrt{a^2+b^2} \geqslant \sqrt{b^2} = b = Im(z)$

(i) $z + \overline{z} = 2\mathfrak{Re}(z)$. Si $z = a+bi$, $z+\overline{z} = a+\cancel{bi}+a-\cancel{bi} = 2a = 2Re(z)$