Ejercicio 9 - Práctico 3

Enunciado: Sea $A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}$. Hallar matrices elementales $E_1, E_2, \cdots, E_k$ tales que $E_k \cdot \cdots \cdot E_2 \cdot E_1 \cdot A =$Id.

Solución: Empezamos reduciendo la matriz ampliada, haciendo operaciones elementales por fila, para hallar la matriz inversa de A.

$$\begin{pmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow_{F_1 \leftrightarrow F_3} \begin{pmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 3 & -1 & 2 & 1 & 0 & 0 \end{pmatrix} \longrightarrow_{F_2 - 2F_1} \begin{pmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 7 & 1 & 0 & 1 & -2 \\ 3 & -1 & 2 & 1 & 0 & 0 \end{pmatrix}$$

$$\longrightarrow_{F_3 - 3F_1} \begin{pmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 7 & 1 & 0 & 1 & -2 \\ 0 & 8 & 2 & 1 & 0 & -3 \end{pmatrix} \longrightarrow_{\frac{1}{7}F_2} \begin{pmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & \frac{1}{7} & 0 & \frac{1}{7} & -\frac{2}{7} \\ 0 & 8 & 2 & 1 & 0 & -3 \end{pmatrix}$$

$$\longrightarrow_{F_3 - 8F_2} \begin{pmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & \frac{1}{7} & 0 & \frac{1}{7} & -\frac{2}{7} \\ 0 & 0 & \frac{6}{7} & 1 & -\frac{8}{7} & -\frac{5}{7} \end{pmatrix} \longrightarrow_{\frac{7}{6}F_3} \begin{pmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & \frac{1}{7} & 0 & \frac{1}{7} & -\frac{2}{7} \\ 0 & 0 & 1 & \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{pmatrix}$$

$$\longrightarrow_{F_2 - \frac{1}{7}F_3} \begin{pmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\ 0 & 0 & 1 & \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{pmatrix} \longrightarrow_{F_1 + 3F_2} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\ 0 & 0 & 1 & \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{pmatrix}.$$

Por lo tanto $A^{-1} = \begin{pmatrix} -\frac{1}{2} & 1 & \frac{1}{2} \\ -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\ \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{pmatrix}$. Escribamos las matrices elementales:

$$E_1 = e_{F_1 \leftrightarrow F_3}(\mathrm{Id}) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ E_2 = e_{F_2 - 2F_1}(\mathrm{Id}) = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_3 = e_{F_3 - 3F_1}(\mathrm{Id}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix},\ E_4 = e_{\frac{1}{7}F_2}(\mathrm{Id}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{7} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_5 = e_{F_3 - 8F_2}(\mathrm{Id}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -8 & 1 \end{pmatrix},\ E_6 = e_{\frac{7}{6}F_3}(\mathrm{Id}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{7}{6} \end{pmatrix}$$

$$E_7 = e_{F_2 - \frac{1}{7}F_3}(\mathrm{Id}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{7} \\ 0 & 0 & 1 \end{pmatrix},\ E_8 = e_{F_1 + 3F_2}(\mathrm{Id}) = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Chequear que

$$E_8 E_7 E_6 E_5 E_4 E_3 E_2 E_1 A = \mathrm{Id}.$$

De hecho $E_8 E_7 E_6 E_5 E_4 E_3 E_2 E_1 = A^{-1}$.