

21. Sea A la primera matriz del ejercicio anterior. Hallar matrices elementales E_1, E_2, \dots, E_k tales que $E_k E_{k-1} \cdots E_2 E_1 A = I$.

Para llevar $A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}$ a la matriz identidad realicé 9 operaciones elementales por fila,

entonces $\text{Id}_3 = E_9 E_8 E_7 E_6 E_5 E_4 E_3 E_2 E_1 A$, donde:

$$E_1 = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \frac{1}{3} f_1$$

$$E_2 = \begin{pmatrix} 1/3 & 0 & 0 \\ -2/3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f_2 - 2f_1$$

$$E_3 = \begin{pmatrix} 1/3 & 0 & 0 \\ -2/3 & 1 & 0 \\ -1/3 & 0 & 1 \end{pmatrix} \quad f_3 - f_1$$

$$E_4 = \begin{pmatrix} 1/3 & 0 & 0 \\ -2/5 & 3/5 & 0 \\ -1/3 & 0 & 1 \end{pmatrix} \quad \frac{3}{5} f_2$$

$$E_5 = \begin{pmatrix} 1/3 & 0 & 0 \\ -2/5 & 3/5 & 0 \\ -2/5 & 3/5 & 1 \end{pmatrix} \quad f_3 + \frac{3}{5} f_2$$

$$E_6 = \begin{pmatrix} 1/3 & 0 & 0 \\ -2/5 & 3/5 & 0 \\ 7/6 & -4/3 & 5/6 \end{pmatrix} \quad \frac{-5}{6} f_3$$

$$E_7 = \begin{pmatrix} 1/5 & 1/5 & 0 \\ -2/5 & 3/5 & 0 \\ 7/6 & -4/3 & 5/6 \end{pmatrix} \quad f_1 + \frac{1}{3} f_2$$

$$E_8 = \begin{pmatrix} -1/2 & 1 & -1/2 \\ -2/5 & 3/5 & 0 \\ 7/6 & -4/3 & 5/6 \end{pmatrix} \quad f_1 - \frac{3}{5} f_3$$

$$E_9 = \begin{pmatrix} -1/2 & 1 & -1/2 \\ -1/6 & 1/3 & -1/6 \\ 7/6 & -4/3 & 5/6 \end{pmatrix} \quad f_2 + \frac{1}{5} f_3$$