EJERCICIO 3 Verdadero o Falso ?

a) $W = \{(x,y) \in \mathbb{R}^2 \mid x^2 = y\}$ es Subespacio de $\mathbb{R}^2$

$0 \in W$; Sean $(1,1) \wedge (-1,1)$ ambos $\in W$

pero $(1,1) + (-1,1) = (0,2) \notin W \Rightarrow W$ NO ES SUBESPACIO DE $\mathbb{R}^2$

FALSO

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b) Si $W_1 \wedge W_2$ subespacios de $\mathbb{R}^8$ de $\dim = 5$,

ENTONCES $(W_1 \cap W_2) = 0$

Sabemos que (Teo. 2.3.10)

$$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$

$$\dim(W_1 + W_2) = 5 + 5 - \dim(W_1 \cap W_2)$$

Pero $\dim(W_1 + W_2) \leq \dim \mathbb{R}^8 = 8$:

$8 \geq 10 - \dim(W_1 \cap W_2) \Rightarrow \dim(W_1 \cap W_2) \neq 0$

$\therefore (W_1 \cap W_2) \neq 0$

Falso

4) Sea $T: R^{2\times 2} \rightarrow R_{<4}$

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a-c+2d)x^3+(b+2c-d)x^2 + (-a+2b+5c-4d)x + (2a-b-4c+5d)$$

a) Dadas $A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$ $C = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$

CUALES $\in N(T)$?

USANDO LA BASE CANONICA DE $R^{2\times 2}$ $A = \begin{pmatrix} 2 \\ 0 \\ 0 \\ -1 \end{pmatrix}$ $B = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ $C = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$

$$T = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 \\ -1 & 2 & 5 & -4 \\ 2 & -1 & -4 & 5 \end{bmatrix}$$

$[T](A) \neq 0$; $[T](B) = 0$; $[T](C) \neq 0 \Rightarrow B \in N(T)$

b) DADOS LOS POLINOMIOS $p = (x-1)(x-) = x^2 - 2x + 1$

$q = x^3 - x^2 - 3x + 3$

$r = x^3$

$p = \begin{pmatrix} 0 \\ 1 \\ -2 \\ 1 \end{pmatrix}$ $q = \begin{pmatrix} 1 \\ -1 \\ -3 \\ 3 \end{pmatrix}$ $r = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

RESUELVO PARA $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$: $T(M) = p, q, r$

$\exists M$? : satisfagan

$$T = \left[\begin{array}{cccc|c|c|c} 1 & 0 & -1 & 2 & & & \\ 0 & 1 & 2 & -1 & p & q & r \\ -1 & 2 & 5 & -4 & & & \\ 2 & -1 & -4 & 5 & & & \end{array}\right] \xrightarrow{\text{GAUSS}} \sim$$

Ec. identicas al ejercicio 2b

$p \wedge r$ no admiten solución, solo $q \in Im(T)$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 1 \\ 0 & 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 1 \\ 0 & 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right]$$

$p \notin Im(T)$

$$A = \begin{bmatrix} 1+c-2d & d-1-2c \\ c & d \end{bmatrix}$$

$r \notin Im\,T$

$$T(A) = q$$