$x \vee x = \sup \underbrace{\{x,x\}}_{\{x\}} = \sup\{x\} = x$ $\qquad$ y cota sup. de $\{x\}$

$\Updownarrow$

$y \geq x$

la mínima $\leftarrow$ x

$$\boxed{x \vee x = x}$$

$$x \vee (x \wedge y) = x$$

$$\sup\{x, \inf\{x,y\}\}$$

Fundamental: $x \wedge y \leq x, y$

$x \leq x \vee y \qquad y \leq x \vee y$

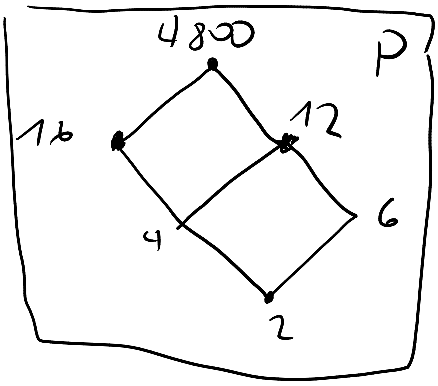
$x \vee y \leq u \iff x \leq u \ \& \ y \leq u$

Probar $x \vee (x \wedge y) = x$ vía dos desigualdades $\leq, \geq$

$\boxed{\geq}$ $x \vee (x \wedge y) \geq x$ ✓

$\boxed{\leq}$ $x \vee (x \wedge y) \leq x \Leftarrow x \leq x \ \& \ x \wedge y \leq x$

$x \leq x$ ✓ reflex, $\quad x \wedge y \leq x$ ✓

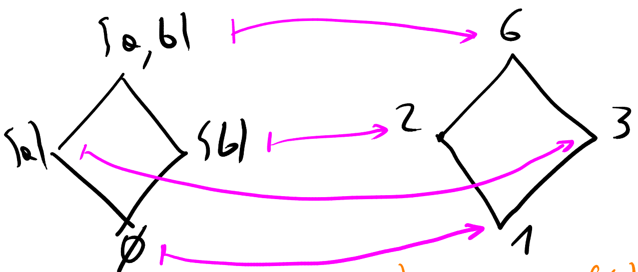


sub poset de $\langle \mathbb{N}, | \rangle$ es reticulado, pero no es subreticulado de $(\mathbb{N}, |)$

$\sup^P\{16, 12\} = 4800$

$\sup^{\mathbb{N}}\{16, 12\} = 48$

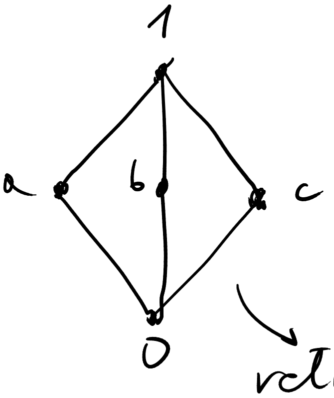
Ejemplo de isomorfismo de retículos. $f: \langle \overset{L}{\mathcal{P}(\{a,b\})}, \overset{\vee}{\cup}, \overset{\wedge}{\cap} \rangle \searrow \langle \overset{L'}{D_6}, \overset{\vee'}{\text{mcm}}, \overset{\wedge'}{\text{mcd}} \rangle$



$$\overset{f(\ x \quad \vee \quad y\ )}{f(\{a\} \cup \{b\})} = \overset{f(x) \quad \vee' \quad f(y)}{f(\{a\}) \text{ mcm } f(\{b\})}$$

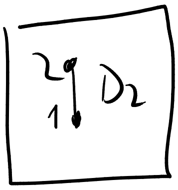
$$\underset{6}{\overset{\|}{f(\{a,b\})}} \qquad = 3 \text{ mcm } 2$$

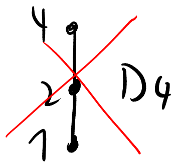
$$= 6 \text{ ✓}$$



retículo acotado complementado.

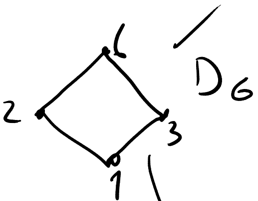
En conjuntos $A \longmapsto A^c$ está unívocamente determinado el complemento.





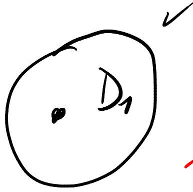
$2 \vee 4 = 4$

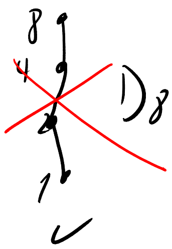
$2 \wedge 4 = 2 \neq 1$



$2 \vee 3 = 6$

$2 \wedge 3 = 1$



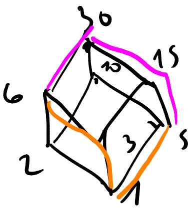


Nota: 0^c es siempre el (único) complemento de 1. (y viceversa)

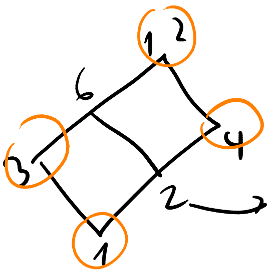
$0^c \vee 1^c = 1^c$

$0^c \wedge 1^c = 0^c$

$30 = 2 \cdot 3 \cdot 5$



es complementado



→ no tiene complemento.

$p^4 = 16$ $\leftarrow p = 2$