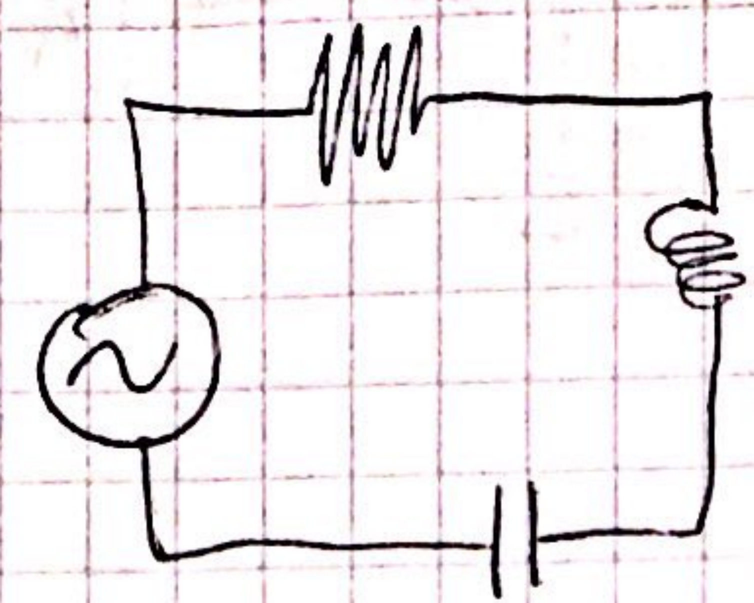
HOJA N°

FECHA

PRACTICO 9

1) a)



$$V_0 \cos(wt) = \frac{q}{C} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2}$$

Propongo $q(t) = A\cos(wt) + B\sin(wt)$

Luego:

$$V_0\cos(wt) = \frac{1}{C}\left[A\cos(wt) + B\sin(wt)\right] + R\left[-Aw\sin(wt) + Bw\cos(wt)\right] + L\left[-Aw^2\cos(wt) - Bw^2\sin(wt)\right]$$

$$= \sin(wt)\left[\frac{B}{C} - RAw - BLw^2\right] + \cos(wt)\left[\frac{A}{C} + RBw - LAw^2\right]$$

Solución

$$\begin{cases} (\alpha) & V_0 = \frac{A}{C} + RBw - LAw^2 \\ (\beta) & \frac{B}{C} - RAw - BLw^2 = 0 \end{cases}$$

De (β):

$$B\left(\frac{1}{C} - Lw^2\right) = RAw \implies B = \frac{RAw}{\frac{1}{C} - Lw^2}$$

Luego en (α)

$$A\left(\frac{1}{C} + \frac{(Rw)^2}{\frac{1}{C} - Lw^2} - Lw^2\right) = V_0 \implies A = V_0\left(\frac{1}{C} + \frac{(Rw)^2}{\frac{1}{C} - Lw^2} - Lw^2\right)^{-1}$$

NOTA

$$= V_0\left[\left(\frac{1}{C}-L\omega^2\right)+\left(\frac{(R\omega)^2}{\frac{1}{C}-L\omega^2}\right)\right]^{-1} = V_0\left[\frac{1}{\frac{1}{C}-L\omega^2}\cdot\left[\left(\frac{1}{C}-L\omega^2\right)^2+(R\omega)^2\right]\right]^{-1}$$

$$= V_0\left[\frac{\omega^2}{\frac{1}{C}-L\omega^2}\left[\left(\frac{1}{C\omega}-L\omega\right)^2+R^2\right]\right]^{-1} = V_0\left[\frac{\omega}{\frac{1}{C\omega}-L\omega}\left[R^2+\left(\frac{1}{C\omega}-L\omega\right)^2\right]\right]^{-1}$$

$$= V_0\left[\frac{\omega}{X_c-X_L}\left[Z^2\right]\right]^{-1} = \frac{V_0(X_c-X_L)}{\omega Z^2}$$

$$\leadsto A = \frac{V_0(X_c-X_L)}{\omega Z^2}$$

Y por lo tanto:

$$B = A\cdot\frac{R\omega}{\frac{1}{C}-L\omega^2} = A\cdot\frac{R}{\frac{1}{C\omega}-L\omega} = A\cdot\frac{R}{X_c-X_L} = \frac{V_0(X_c-X_L)}{\omega Z^2}\cdot\frac{R}{X_c-X_L} = \frac{V_0 R}{\omega Z^2}$$

Finalmente:

$$q(t) = \frac{V_0(X_c-X_L)}{\omega Z^2}\cos(\omega t) + \frac{VR}{\omega Z^2}\cdot\sin(\omega t)$$

$$\Longrightarrow i(t) = -\underbrace{\frac{V(X_c-X_L)}{Z^2}}_{\alpha}\sin(\omega t) + \underbrace{\frac{VR}{Z^2}}_{\beta}\cos(\omega t)$$

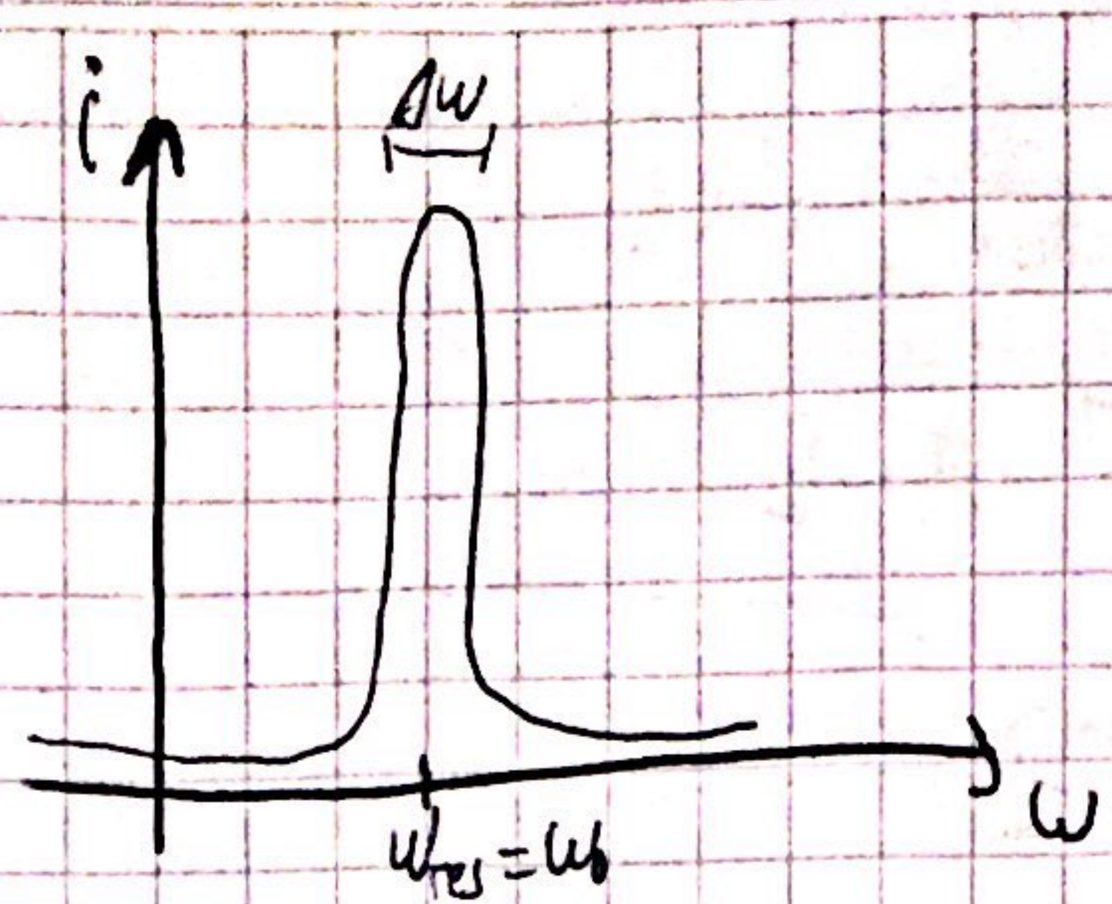
Se puede demostrar que el max de esta función es $\sqrt{\alpha^2+\beta^2}$. Luego:

$$I_{max} = \sqrt{\frac{V_0^2(X_c-X_L)^2}{Z^4}+\frac{V_0^2R^2}{Z^4}} = \frac{V}{Z^2}\cdot\sqrt{(X_c-X_L)^2+R^2} = \frac{V}{Z^2}\cdot Z = \frac{V}{Z}$$

Z depende de ω, por lo tanto I es máxima cuando Z es mínimo, para esto $\leadsto X_c = X_L \Longrightarrow \boxed{\omega = \frac{1}{\sqrt{LC}}}$

NOTA

b)



$$I_0(w) = \frac{I_{0max}}{\sqrt{2}}$$

$$I_0(w)^2 = \frac{I_{0max}^2}{2} = \frac{V_0^2}{2Z_{min}^2}$$

Por lo tanto buscamos w tal que $Z(w)^2 = Z_{min}^2 \cdot 2$

$Z(w) = \sqrt{R^2+(X_c-X_L)^2} \implies Z(w)^2 = R^2+(X_c-X_L)^2$

Z es mínimo cuando $X_c = X_L \implies Z_{min} = R^2$

Entonces:

$$R^2+(X_c-X_L) = 2R^2$$

$$\frac{1}{wC} - Lw = \pm R \quad (\alpha)$$

Desarrollo por taylor a $\frac{1}{w} = f(w)$ de orden 1 centrado en w_0.

$T_{1,w_0}(w) = f(w_0) + f'(w_0)(w-w_0) = \frac{1}{w_0} - \frac{w-w_0}{w_0^2}$

Luego;

$$\pm R \approx \left[\frac{1}{w_0} - \frac{w-w_0}{w_0^2}\right]\frac{1}{C} - wL$$

Uso $w = w_0 + \Delta w$:

$$\pm R \simeq \frac{1}{\omega_0 C} - \frac{\Delta\omega}{C\omega_0^2} - (\omega_0 - \Delta\omega)L$$

$$\simeq \frac{\sqrt{LC}}{C} - \frac{\Delta\omega \cdot LC}{C} - \frac{L}{\sqrt{LC}} - \Delta\omega L$$

$$\simeq \sqrt{\frac{L}{C}} - \Delta\omega \cdot L - \sqrt{\frac{L}{C}} - \Delta\omega \cdot L$$

$$\simeq -2L \cdot \Delta\omega \implies \boxed{\Delta\omega \simeq \pm \frac{R}{2L}}$$

c)

$$Q = \frac{\omega_0}{2\Delta\omega}$$

$$= \frac{\omega_0}{2} \cdot \pm \frac{2L}{R} = \pm \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \pm \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$$

2

a)

$$i = i_C + i_L + i_R = \frac{V_0}{\ } + \frac{V_0}{X_L} + \frac{V_0}{R}$$

b) $$i = \frac{V_0}{R} + \frac{V_0}{X_L} + \frac{V_0}{X_C} = \frac{V_0}{R} + \frac{V_0}{\omega L} + V_0 \omega C$$

c) Veamos cuando para que ω I es mínimo:

$$I = V_0 \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2} \implies I^2 = V_0^2\left[\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2\right]$$

$$= V_0^2\left[\frac{1}{R^2} + \frac{1}{(L\omega)^2} - \frac{2C}{L} + (\omega C)^2\right]$$

su mínimo

$$\frac{\partial (I^2)}{\partial \omega} = 0 \implies -\frac{2}{\omega^3 L^2} + 2\omega C^2 \implies 2\omega C^2 = \frac{2}{\omega^3 L^2}$$

$$\implies \omega^4 = \frac{1}{C^2 L^2} \implies \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

NOTA

HOJA N°

FECHA

3

a) $I_0 = V_0 \cdot \frac{1}{Z} = V_0 \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$

$= V_0 \sqrt{\frac{1}{R^2} + \left(\frac{1}{wL} - wC\right)^2}$

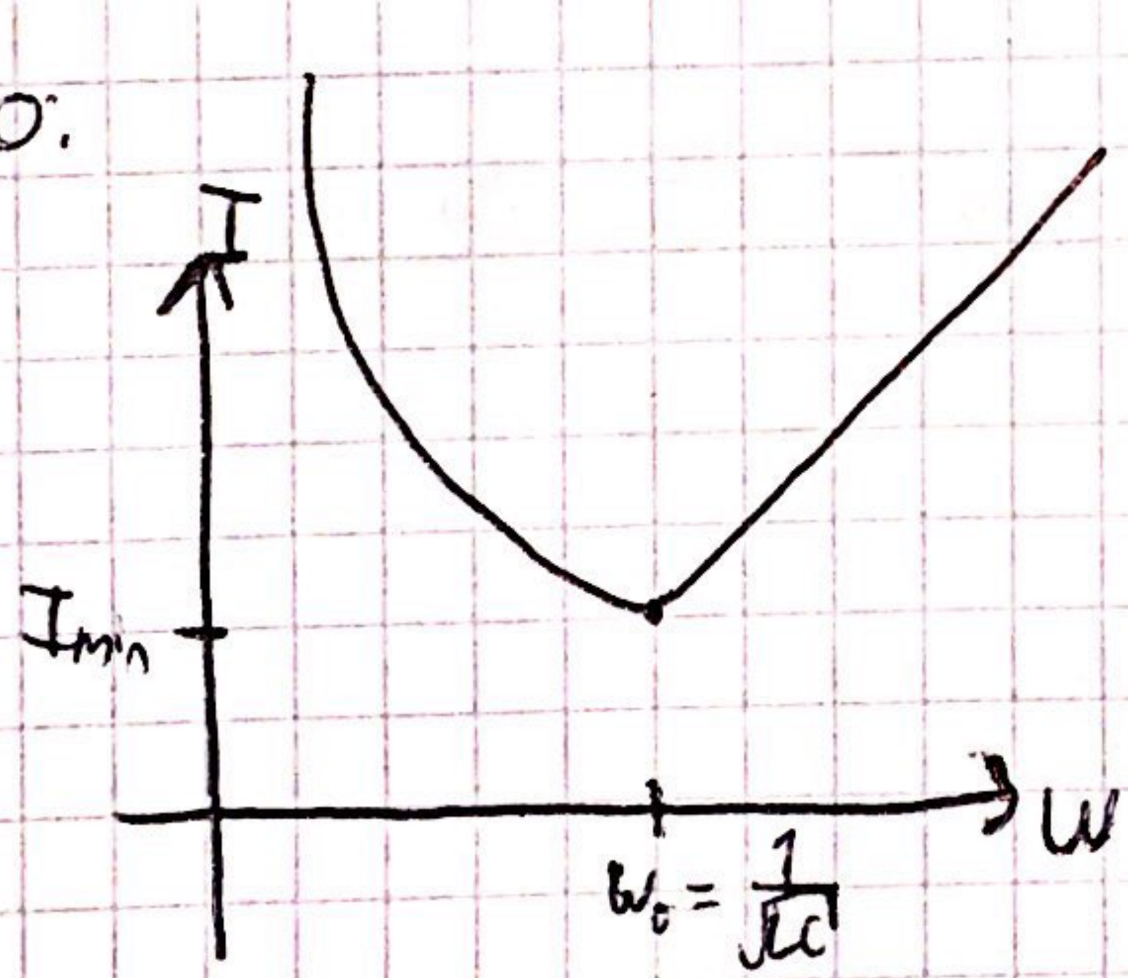
$\hookrightarrow$ cuando w es chico cae como $\frac{1}{X}$, cuando w es grande crece como X

Además:

$w \to \infty \implies \begin{cases} \frac{1}{wL} \to 0 \\ wC \to \infty \end{cases}$

$w \to 0 \implies \begin{cases} \frac{1}{wL} \to \infty \\ wC \to 0 \end{cases}$

Luego:



b) I es mínimo cuando $\overbrace{\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}^{\frac{1}{Z}(w)}$ es mínimo. es decir cuando $X_L = X_C$.

$\frac{1}{Z}(w) = \frac{1}{Z}_{min} \cdot \frac{1}{\sqrt{2}}$

$\left(\frac{1}{Z}(w)\right)^2 = \frac{1}{Z}_{min}^{\;2} \cdot \frac{1}{2}$

NOTA

Luego:

$$\frac{1}{R^2}+\left(\frac{1}{\omega L}-\omega C\right)^2=\frac{1}{R^2}\cdot\frac{1}{2}$$

$$\left(\frac{1}{\omega L}-\omega C\right)^2=\frac{1}{2R^2}$$

$$\left(\frac{1}{\omega L}-\omega C\right)=\frac{1}{\sqrt{2}R}$$

Uso taylor: $\frac{1}{\omega}\approx\frac{1}{\omega_0}-\frac{\omega-\omega_0}{\omega_0^2}$:

$$\frac{1}{\omega_0 L}-\frac{\omega-\omega_0}{\omega_0^2 L}-\omega c=\frac{1}{\sqrt{2}R}$$

Uso $\omega=\omega_0+\Delta\omega$, y que $\omega_0=\frac{1}{\sqrt{LC}}$

$$\frac{\sqrt{LC}}{L}-\frac{\Delta\omega . LC}{L}-\frac{C}{\sqrt{LC}}-\Delta\omega . C=\frac{1}{\sqrt{2}R}$$

$$\sqrt{\frac{C}{L}}-\Delta\omega . C-\sqrt{\frac{C}{L}}-\Delta\omega . C=\frac{1}{\sqrt{2}R}$$

$$-2\Delta\omega C=\frac{1}{\sqrt{2}R}\implies\Delta\omega=-\frac{1}{CR}\cdot\frac{1}{2\sqrt{2}}$$

Luego:

$$Q=\frac{\omega_0}{2\Delta\omega}=\frac{1}{2\sqrt{LC}}\cdot CR\,2\sqrt{2}=\sqrt{2}\sqrt{\frac{C}{L}}\cdot R$$

c) en serie $Q=\frac{1}{R}\sqrt{\frac{L}{C}}$

en paralelo $Q=\sqrt{2}R\sqrt{\frac{C}{L}}$

4

$$V(t)=V_0\cos(\omega t)$$

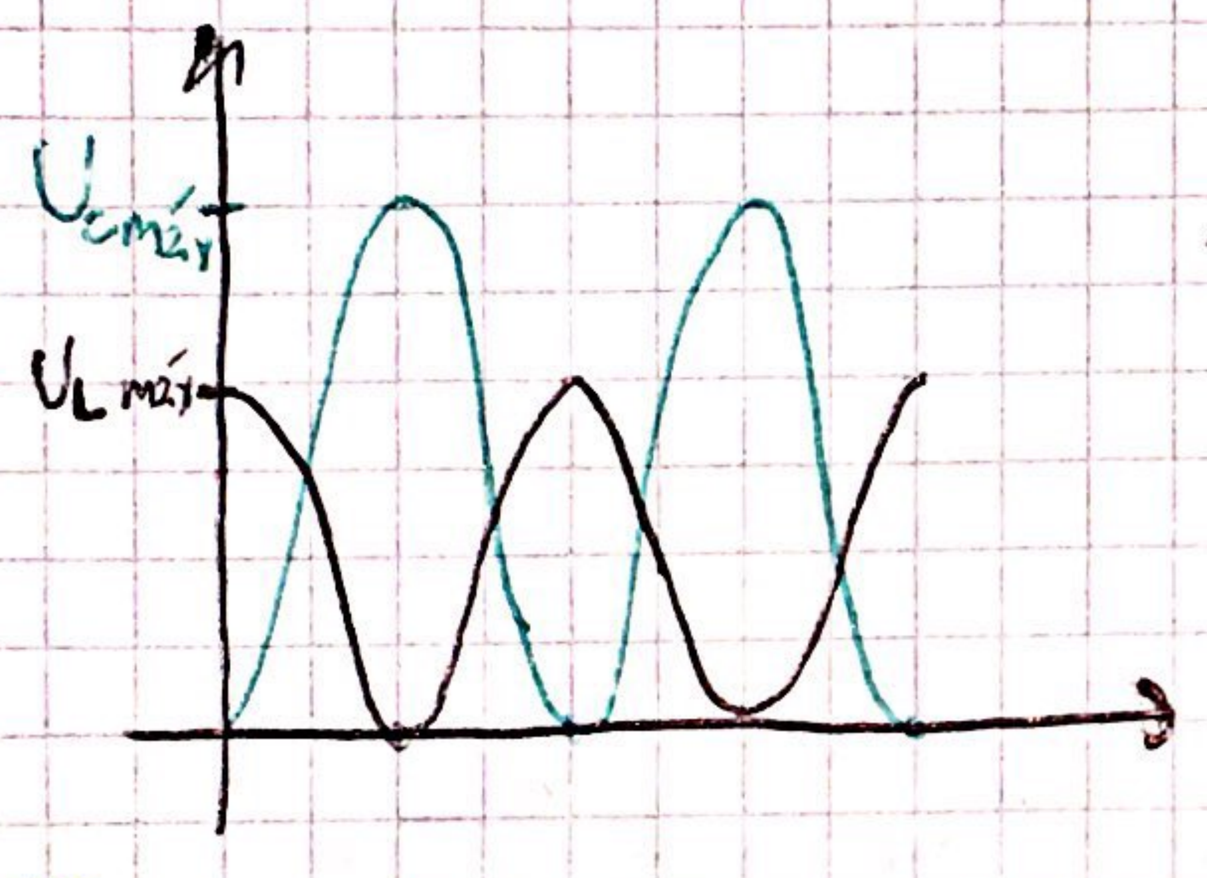
$$i(t)=I_0\cos(\omega t-\varphi)$$

$$q(t)=\int i(t)dt=\frac{I_0}{\omega}\sin(\omega t-\quad)$$

$$U_c = \frac{q^2(t)}{2C} = \frac{I_0^2}{w^2C} \cdot \sin^2(wT-\emptyset)$$

$$U_L = \frac{1}{2}L \cdot i(t)^2 = \frac{LI_0^2}{2} \cdot \cos^2(wT-\emptyset)$$

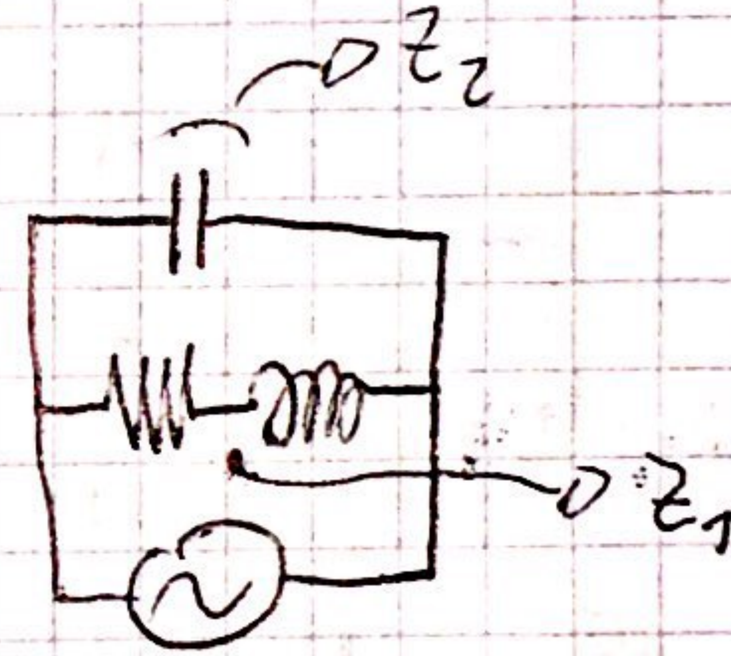
Por seno y coseno sabemos que si U_c es max, U_L es mínimo y viceversa. Es decir



suponiendo $U_c > U_L$

5

$$V(t) = V_0 \cdot e^{jwT} \cdot e^{j\varphi_1}$$
$$i(t) = I_0 \cdot e^{jwT} \cdot e^{j\varphi_2}$$



$$Z = \frac{V(t)}{i(t)} = \frac{V_0}{I_0} e^{j(\varphi_1-\varphi_2)}$$

Es de notar que $Z \in \mathbb{R} \iff e^{j(\varphi_1-\varphi_2)} \in \mathbb{R} \iff \varphi_1 = \varphi_2$

↳ en fase

$$Z_{eq} = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \left(\frac{1}{R+jwL} - \frac{wC}{j}\right)^{-1}$$

$$= \left(\frac{R-jwL}{R^2+jwL} + jwC\right)^{-1} = \left(\frac{R}{R^2+(wL)^2} + j\left(-\frac{wL}{R^2+w^2L^2} + wC\right)\right)^{-1}$$

es de notar que $Z_{eq} \in \mathbb{R} \iff -\frac{wL}{R^2+(wL)^2} + wC = 0$

Luego:

$$\omega C = \frac{\omega L}{R^2+\omega^2 L^2}$$

$$C = \frac{L}{R^2+\omega^2 L^2}$$

$$R^2+\omega^2 L^2 = \frac{L}{C}$$

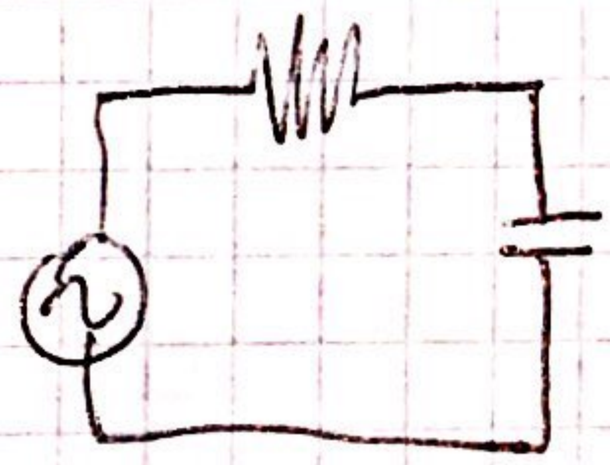
$$\omega^2 L^2 = \frac{L}{C} - R^2$$

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

6 a)

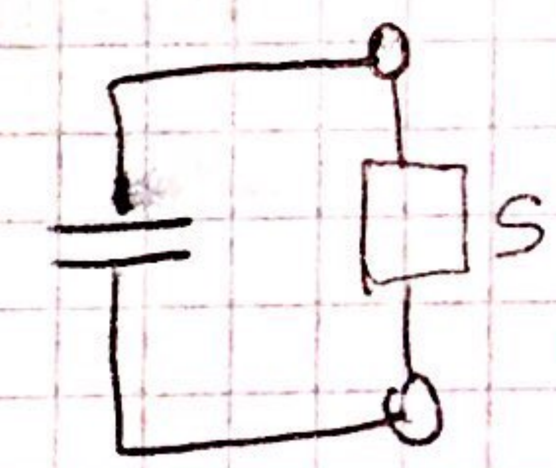
Veamos por separado:

(1°)



$$I_1 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2+X_C^2}} = \frac{V_0}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

(2°)



$$I_2 = \frac{\overset{V_C}{V_1}}{Z} = \frac{V_1}{\sqrt{X_C^2}} = \frac{V_1}{X_C} = V_1 \omega C$$

La corriente que pasa por (1°) es la misma que pasa por el C del (2°), pues por S no pasa corriente, hay un corte. Entonces:

$$I_1 = I_2$$

$$\frac{V_0}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} = V_1 \cdot \omega C$$

$$\frac{V_0}{V_1} = \omega C \sqrt{R^2+\left(\frac{1}{\omega C}\right)^2} \Longrightarrow \frac{V_0^2}{V_1^2} = (\omega C)^2\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]$$

NOTA

HOJA N°

FECHA

$$= (wCR)^2 + 1 \Rightarrow \frac{V_1^2}{V_0^2} = \frac{1}{(wCR)^2 + 1}$$

b) Cuando $w >> 1 \Rightarrow \frac{V_1^2}{V_0^2} = \frac{1}{(wCR)^2}$

Luego si $w = 2w'$

$$\frac{V_1^2}{V_0^2} = \frac{1}{(2w'CR)^2} = \frac{1}{4} \cdot \frac{1}{(w'CR)^2}$$

c) $f = 5000 Hz$, $f = \frac{w}{2\pi} \Rightarrow w = 10000\pi \, Hz$

Luego:

$$((wCR)^2 + 1)^{-1} = 0{,}01$$

$$w^2 c^2 R^2 + 1 = 100$$

$$w^2 c^2 R^2 = 99$$

$$c^2 R^2 = \frac{99}{w^2}$$

$$(CR)^2 = \frac{99}{10\pi^2 Hz^2}$$

$$C.R = \frac{\sqrt{99}}{10^4 \pi \, Hz} \longrightarrow \text{elegimos} \quad C = 1F, \quad R = \frac{\sqrt{99}}{10^4 \pi} \Omega$$

NOTA