HOJA N°

FECHA

Profesora: Cecilia Cormick , cecilia.cormick@unc.edu.ar

Ayudante: Fabrizio Venturi , fabrizioventuri@mi.unc.edu.ar

PRACTICO 1

1

a) $e \simeq 1{,}602 \, . \, 10^{-19} \, C$ $\quad d = 5{,}292 \, . \, 10^{-11} \, m$

$m_e = 9{,}109 \, . \, 10^{-31} \, Kg$

$m_p = 1{,}673 \, . \, 10^{-27} \, Kg$

$K_c = 8{,}987 \, . \, 10^{9} \, Nm^2 \, C^{-2}$

$G \simeq 6{,}674 \, . \, 10^{-11} \, N \, m^2 . Kg^{-2}$

$$|F_g| = \frac{m_e . m_p . G}{d^2}$$

$$|F_g| = \frac{(9{,}109)(1{,}673)(6{,}674) . \, 10^{(-31)+(-27)+(-11)} \, N}{(5{,}292)^2 \, . \, 10^{-22}} = 3{,}63 \, . \, 10^{-69+22} \, N$$

$$\boxed{|F|_g = 3{,}63 \, . \, 10^{-47} \, N}$$

$$F_{el} = \frac{q_e . q_p . K_c}{d^2} = -\frac{(1{,}602)^2 (8{,}987)}{(5{,}292)^2} . \, 10^{-38+9+22} \, N$$

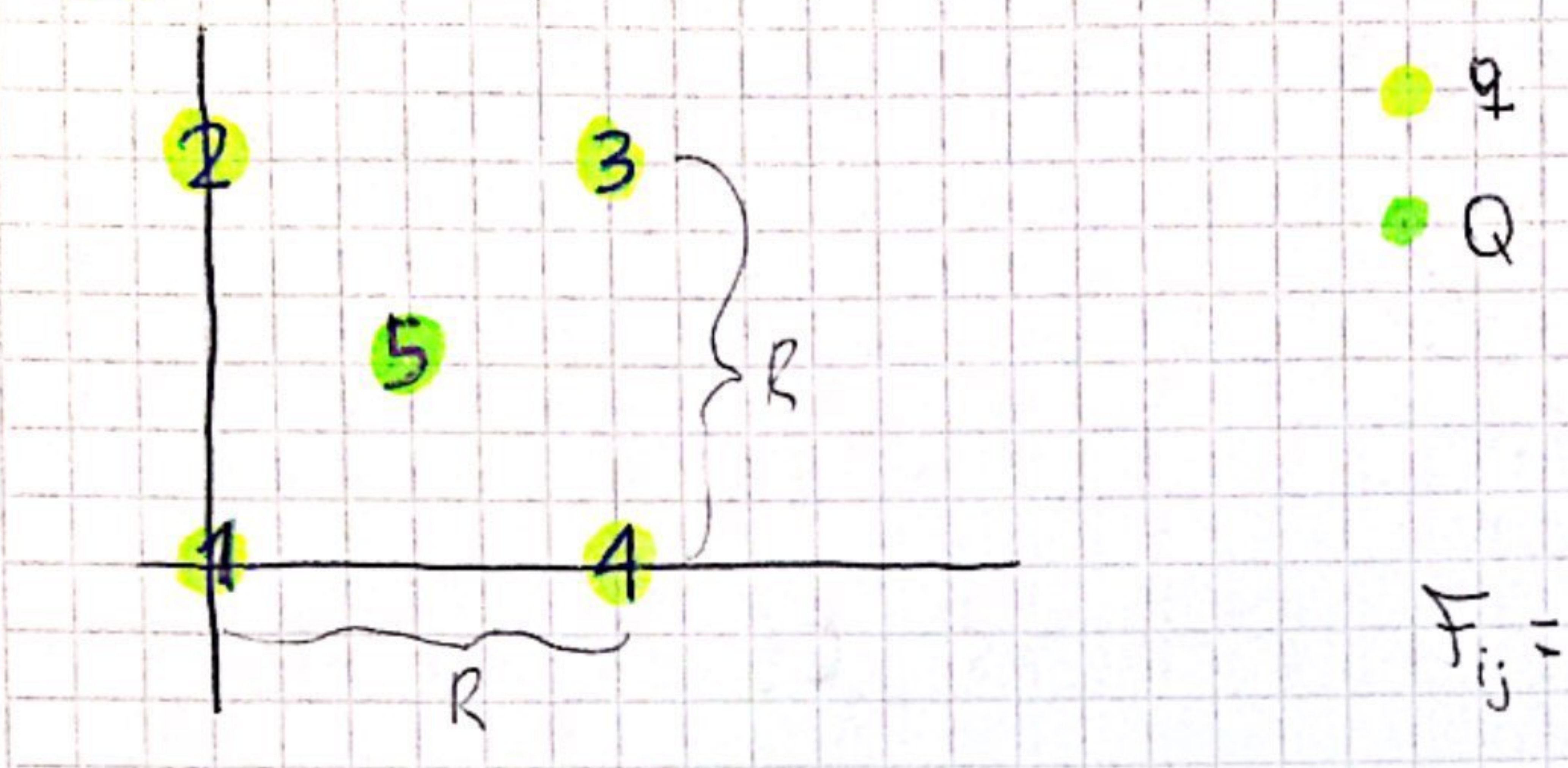
$$\boxed{F_{el} = -0{,}815 \, . \, 10^{-7} \, N}$$

b)

$$F_g = \frac{(6{,}67) . (1{,}673)^2 . 10^{-11} . 10^{-54}}{(5{,}292)^2 . 10^{-22}} . \frac{N m^2}{Kg^2} . Kg^2 . \frac{1}{m^2} . \hat{r} = 0{,}66661 . 10^{-43} \, N \, \hat{r}$$

NOTA

2



$$F_{ij} = K_c \frac{q_i \cdot q_j}{\|\vec{r}_j - \vec{r}_i\|^3} \cdot \|\vec{r}_j - \vec{r}_i\|$$

Tenemos que:

$$\vec{r}_1 = 0 \quad , \|\vec{r}_1\| = 0$$
$$\vec{r}_2 = R\hat{y} \quad , \|\vec{r}_2\| = R$$
$$\vec{r}_3 = R\hat{x} + R\hat{y} \quad , \|\vec{r}_3\| = \sqrt{2}R$$
$$\vec{r}_4 = R\hat{x} \quad , \|\vec{r}_4\| = R$$
$$\vec{r}_5 = \frac{R}{2}\hat{x} + \frac{R}{2}\hat{y} \quad , \|\vec{r}_5\| = \frac{R}{\sqrt{2}}$$

Luego:

$$\vec{F}_{12} = K_c \cdot \frac{q^2}{R^3} \cdot R\hat{y} = K_c \frac{q^2}{R^2}\hat{y}$$

$$\vec{F}_{13} = K_c \frac{q^2}{(\sqrt{2}R)^3} \cdot (R\hat{x} + R\hat{y}) = \frac{K_c q^2}{\sqrt{2}^3 R^2}\hat{x} + \frac{K_c q^2}{\sqrt{2}^3 R^2}\hat{y}$$

$$\vec{F}_{14} = K_c \frac{q^2}{R^3} \cdot R\hat{x} = K_c \cdot \frac{q^2}{R^2}\hat{x}$$

$$\vec{F}_{15} = K_c \cdot \frac{qQ}{\frac{R^3}{\sqrt{2}^3}} \cdot \left(\frac{R}{2}\hat{x} + \frac{R}{2}\hat{y}\right) = \frac{K_c qQ\sqrt{2}}{R^2}\hat{x} + \frac{K_c qQ\sqrt{2}}{R^2}\hat{y}$$

Luego:

$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} = \left(\frac{K_c q^2}{2\sqrt{2}R^2} + \frac{K_c q^2}{R^2} + \frac{K_c qQ\sqrt{2}}{R^2}\right)(\hat{x} + \hat{y})$$

NOTA

HOJA N°

FECHA

Para que sea nulo:

$$\frac{K_c q^2}{2\sqrt{2} R^2} + \frac{K_c q^2}{R^2} + \frac{\sqrt{2} K_c q Q}{R^2} = 0$$

$$\frac{q}{2\sqrt{2}} + q + \sqrt{2} Q = 0$$

$$q\left(\frac{1}{2\sqrt{2}} + 1\right) = -\sqrt{2} Q$$

$$-q\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = Q$$

$$\boxed{-q\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right) = Q}$$

NOTA

HOJA N°

FECHA

$$F_T=\left(\frac{q^2k_c}{2r^2}+\frac{q^2k_c}{2r^2}+\frac{qQk_c}{\frac{1}{2}r^2}\right)\hat{x}+\left(\frac{q^2k_c}{2r^2}+\frac{q^2k_c}{2r^2}+\frac{qQk_c}{\frac{1}{2}r^2}\right)\hat{y}$$

$$=0 \qquad\qquad =0$$

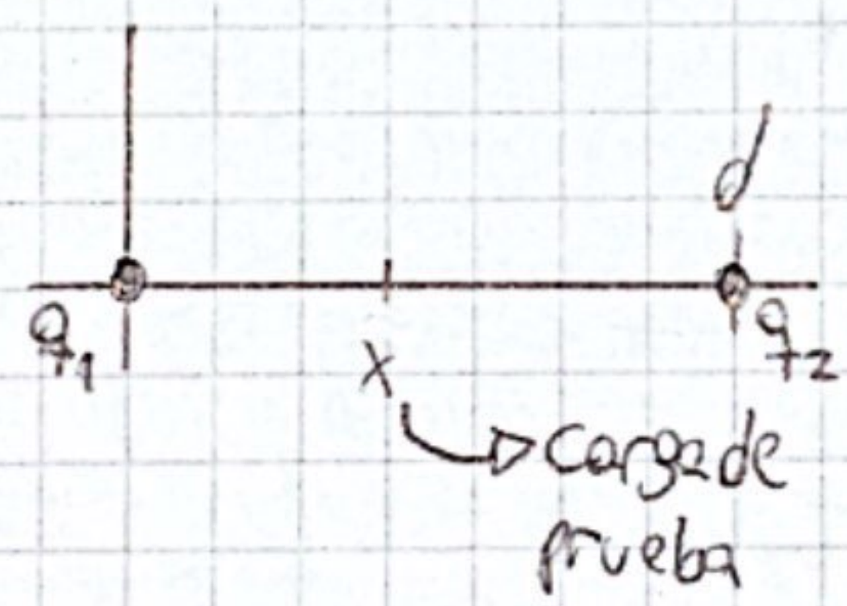
Para que esté en equilibrio

$$\frac{q^2k_c}{2r^2}+\frac{q^2k_c}{2r^2}+\frac{qQk_c}{\frac{1}{2}r^2}=0$$

$$\frac{q^2k_c}{r^2}=-2\frac{qQk_c}{r^2}$$

$$\boxed{-\frac{q}{2}=Q}$$

3



$\vec{E}=k_c\cdot q_1\cdot\frac{(\vec{r}-\vec{r}')}{\|\vec{r}-\vec{r}'\|^3}$

$\vec{r}'$: generador del campo

$\vec{r}$: punto de efecto

Siendo $E_y=E_z=0$

$E_x=E_{x_1}+E_{x_2}$, entonces:

$$E_{x_1}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{\|\vec{x}-0\|^3}\cdot(\vec{x}-0)=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{\|\vec{x}\|^3}\cdot\vec{x}$$

$$E_{x_2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{\|\vec{x}-\vec{d}\|^3}\cdot(\vec{x}-\vec{d})$$

NOTA

$$\vec{E}_T = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\|X\|^3}\cdot\vec{X} + \frac{q_2}{\|\vec{X}-d\|^3}\cdot(\vec{X}+\vec{d})\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\|X\|^3}X.\hat{X} + \frac{q_2}{\|\vec{X}-\vec{d}\|^3}\cdot\|X-d\|\cdot\widehat{x-d}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\|\vec{X}\|^2}\hat{X} + \frac{q_2}{\|\vec{X}-\vec{d}\|^2}\hat{X}\right)$$ → la dirección $\widehat{x-d}$ es $\hat{x}$

$$|\vec{E}_T| = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\|\vec{X}\|^2} + \frac{q_2}{\|\vec{X}-\vec{d}\|^2}\right)$$

debo buscar X tal que $|\vec{E}_1| = 0$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{X^2} + \frac{q_2}{(X-d)^2}\right) = 0$$

$$\frac{q_1}{X^2} = -\frac{q_2}{(X-d)^2}$$

$$q_1(X-d)^2 = -X^2 q_2$$

$$q_1X^2 - 2q_1Xd + q_1d^2 = -X^2q_2$$

$$(q_1+q_2)X^2 - 2q_1dX + q_1d^2 = 0$$

Si $q_1 = -q_2$

$$-2q_1dX + q_1d^2 = 0$$
$$2q_1dX = q_1d^2$$
$$2X = d$$
$$\boxed{X = \frac{d}{2}}$$

Si $q_1 \neq -q_2$

$$\frac{2q_1d \pm \sqrt{4q_1d^2 - 4(q_1+q_2)q_1d^2}}{2(q_1-q_2)} = \frac{2q_1 \pm d\sqrt{-4q_1q_2}}{2(q_1+q_2)}$$

$$= d\left[\frac{q_1 \pm \sqrt{-q_1q_2}}{(q_1+q_2)}\right]$$ → puntos en el que el campo es cero

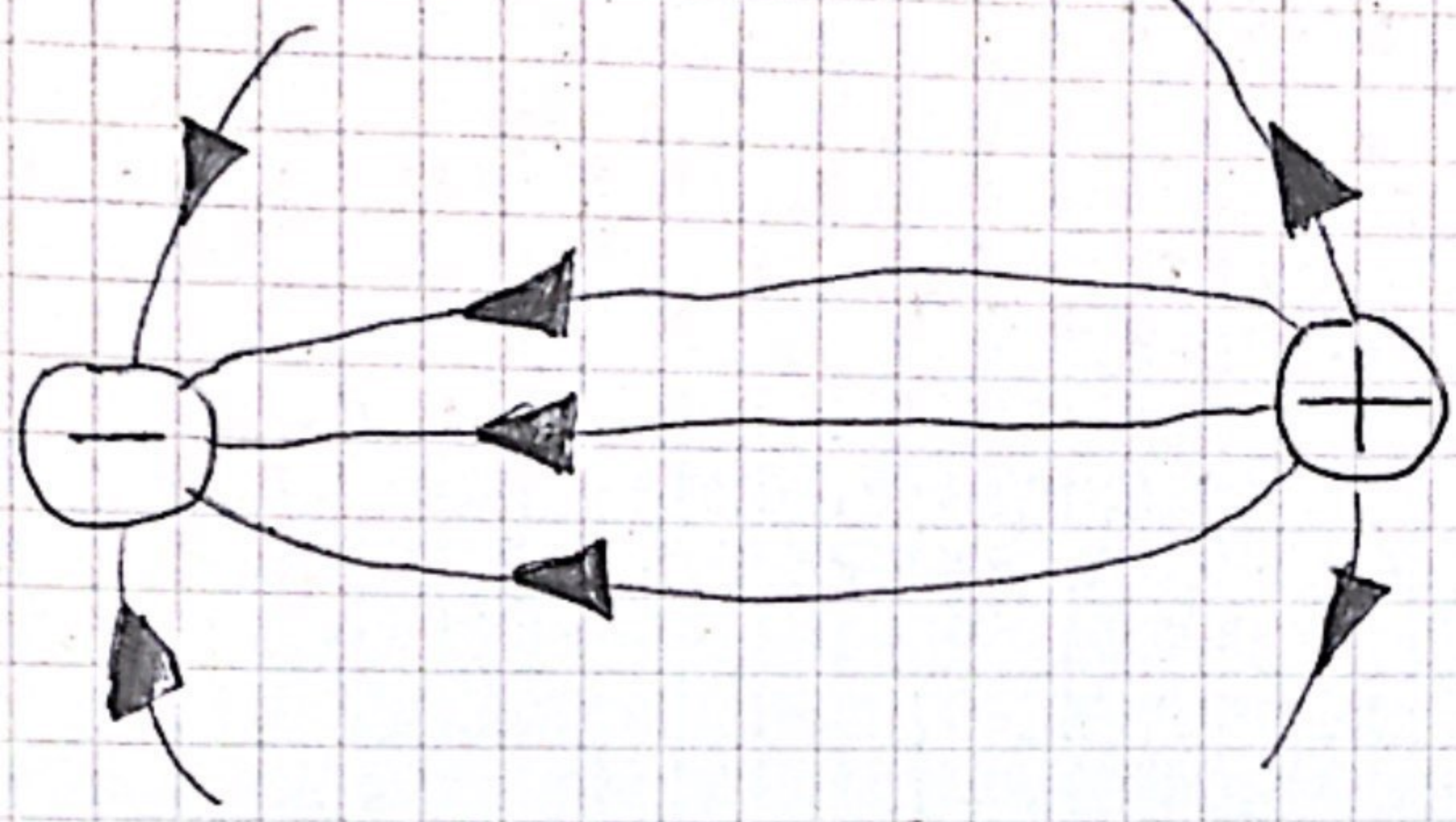
NOTA

Scanned with CamScanner

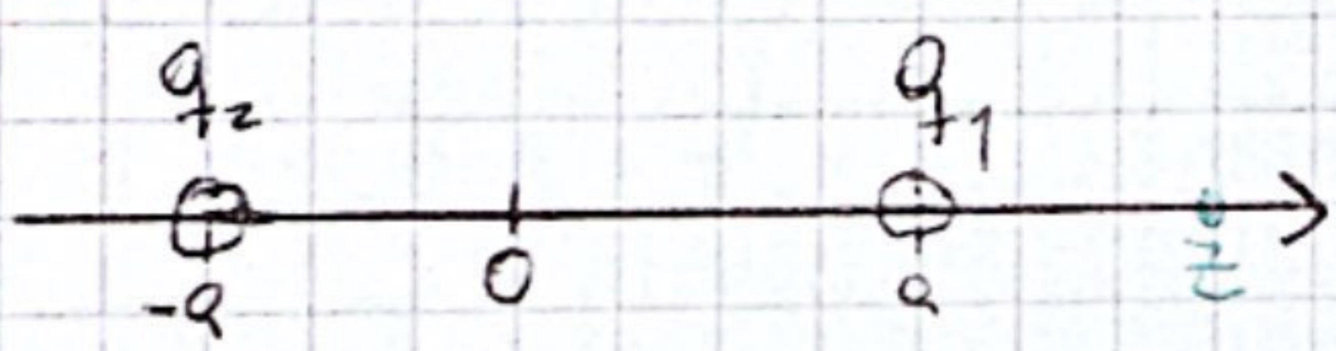
HOJA N°

FECHA

4 a)



b)



$q_1 = e$

$q_2 = -e$

$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{(z-a)^2}\hat{z} = \frac{1}{4\pi\varepsilon_0} \frac{e}{(z-a)^2} \hat{z}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{(z+a)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-e}{(z+a)^2} \hat{z}$$

$$E_T = \frac{1}{4\varepsilon_0\pi} \cdot e\left(\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right)\hat{z}$$

$$= \frac{e}{4\pi\varepsilon_0}\left(\frac{(z+a)^2-(z-a)^2}{(z-a)^2 \cdot (z+a)^2}\right)\hat{z} = \frac{e}{4\pi\varepsilon_0}\left(\frac{4az}{(z-a)^2(z+a)^2}\right)\hat{z}$$

$$= \frac{e}{\pi\varepsilon_0} \cdot \frac{az}{(z-a)^2(z+a)^2}\hat{z}$$

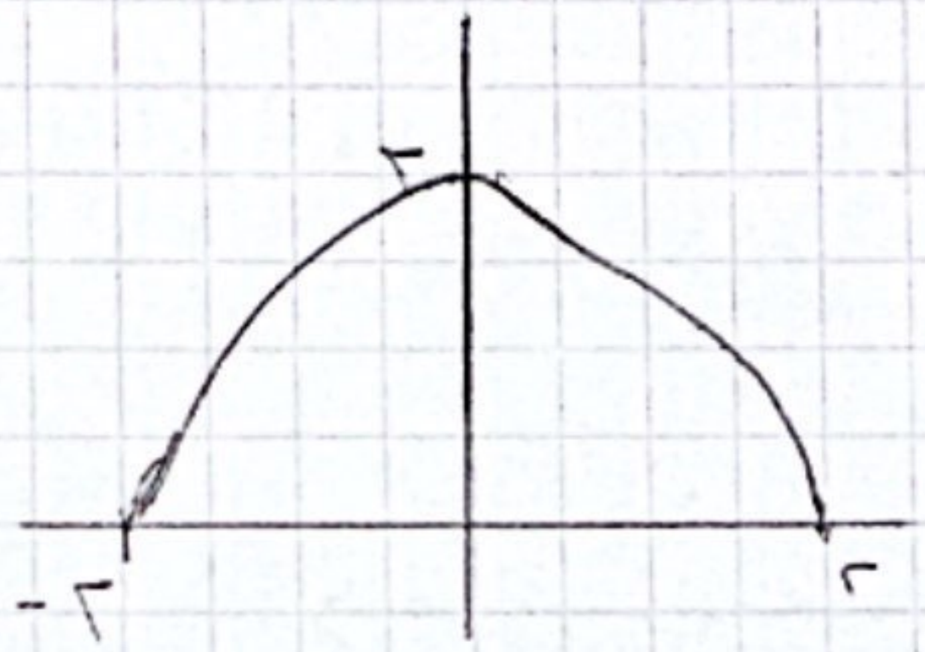
c) cuando $z \to \infty$:

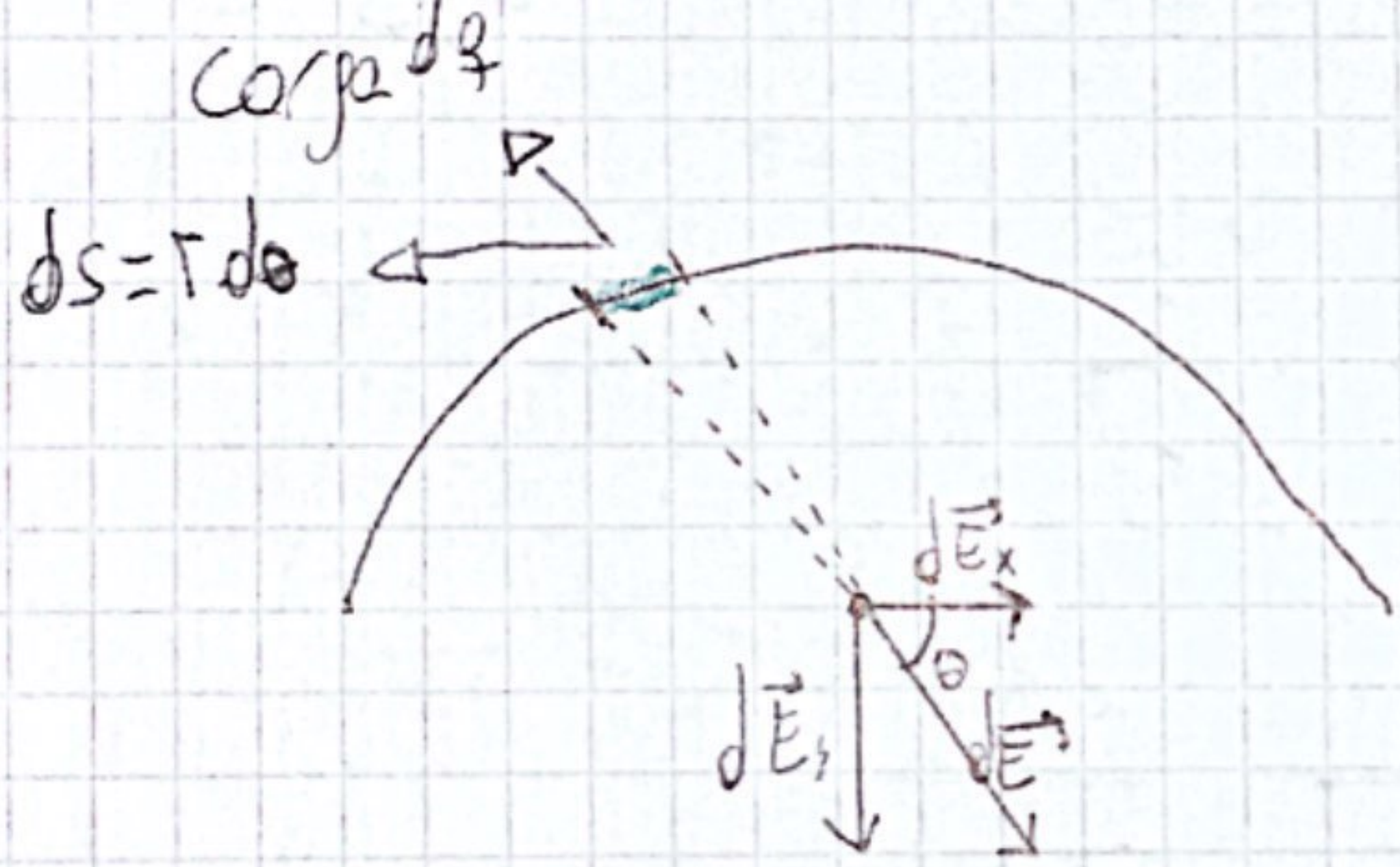
$$E = \frac{\rho}{\pi \varepsilon_0} \cdot \frac{a\, z}{z^2 . z^2} = \frac{\rho}{\pi \varepsilon_0} \cdot \frac{a}{z^3}$$

d)

$$E = 0$$

5





- $dE = K_c \cdot \frac{dq}{r^2}$
- $dE_x = dE . \cos(\theta)$
- $dE_y = dE . \sin(\theta)$

Siendo $\rho = \frac{Q}{A} = \frac{Q}{\pi . r}$:

$$dq = \rho . ds \Rightarrow dq = \frac{Q}{\pi . r} . r . d\theta = \frac{Q\, d\theta}{\pi}$$

Entonces:

$$dE_x = dE . \cos(\theta) = K_c . \frac{dq}{r^2} . \cos(\theta) = K_c . \frac{Q . d\theta}{\pi . r^2} . \cos(\theta)$$

$$dE_y = dE . \sin(\theta) = K_c \frac{dq}{r^2} . \sin(\theta) = K_c . \frac{Q . d\theta}{\pi r^2} . \sin(\theta)$$

HOJA N°

FECHA

Entonces:

$$E_x = \int_0^{\pi} dE_x = \int_0^{\pi} \frac{k_c Q}{\pi . r^2} . \cos(\theta)\, d\theta = \frac{k_c . Q}{\pi . r^2} \int_0^{\pi} \cos(\theta) . d\theta = \frac{k_c . Q}{\pi . r^2} . \left(\sin(\theta) \Big|_0^{\pi} \right) = 0$$

$$E_y = \int_0^{\pi} dE_y = \int_0^{\pi} \frac{k_c . Q}{\pi . r^2} . \sin(\theta)\, d\theta = \frac{k_c . Q}{\pi . r^2} \int_0^{\pi} \sin(\theta) . d\theta = \frac{k_c . Q}{\pi . r^2} . \left(-\cos(\theta) \Big|_0^{\pi} \right)$$

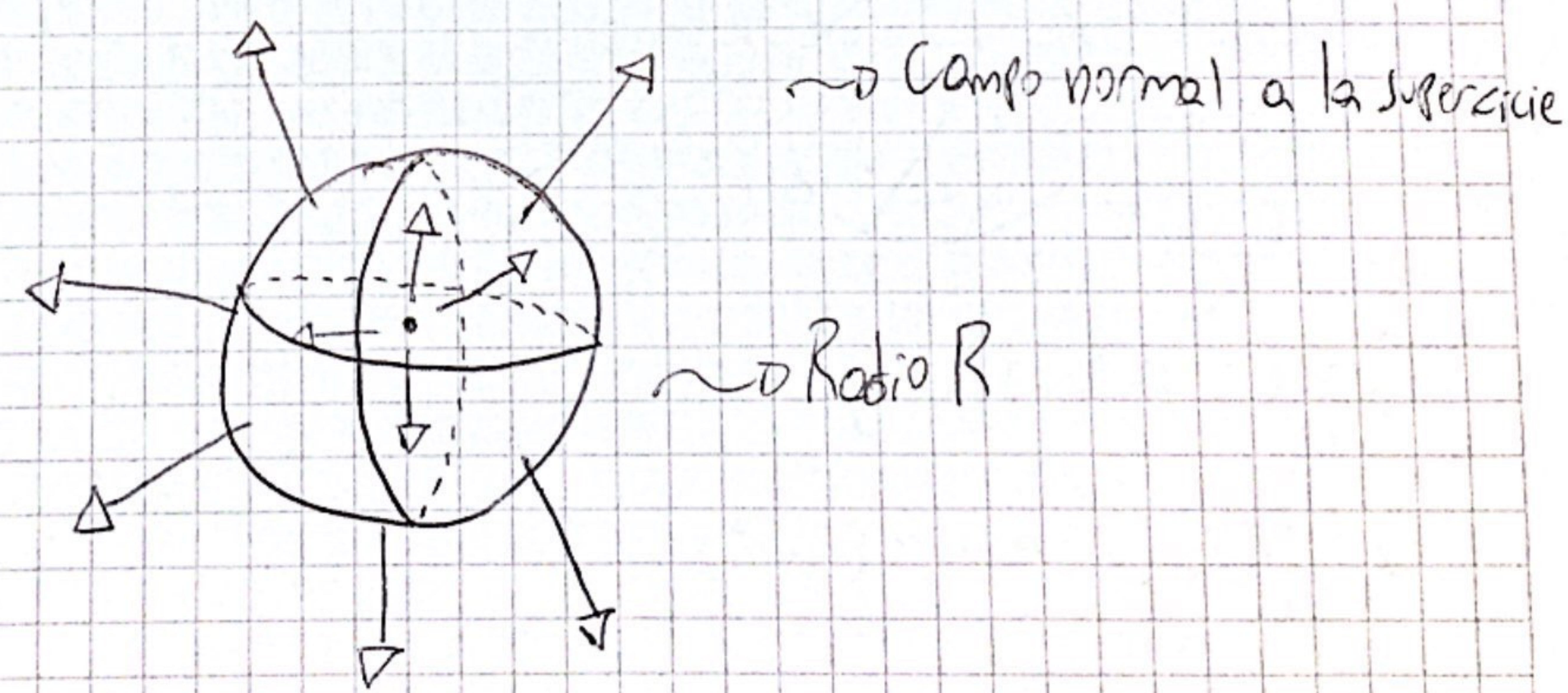
$$= \frac{k_c Q}{\pi r^2} . (1 - (-1)) = \frac{2 k_c Q}{\pi . r^2} \hat{y}$$

Finalmente:

$$\boxed{E_T = E_x + E_y = \frac{2 k_c Q}{\pi . r^2} \hat{y}}$$

8

a)



b) $E = A . r . \hat{r} = A . \vec{r}$

$$\Phi = \oint_{esf} A . \vec{r} . d\vec{s} = A \oint R . ds . \cos(0) = A . R \oint ds = A . R . 4\pi . R^2$$

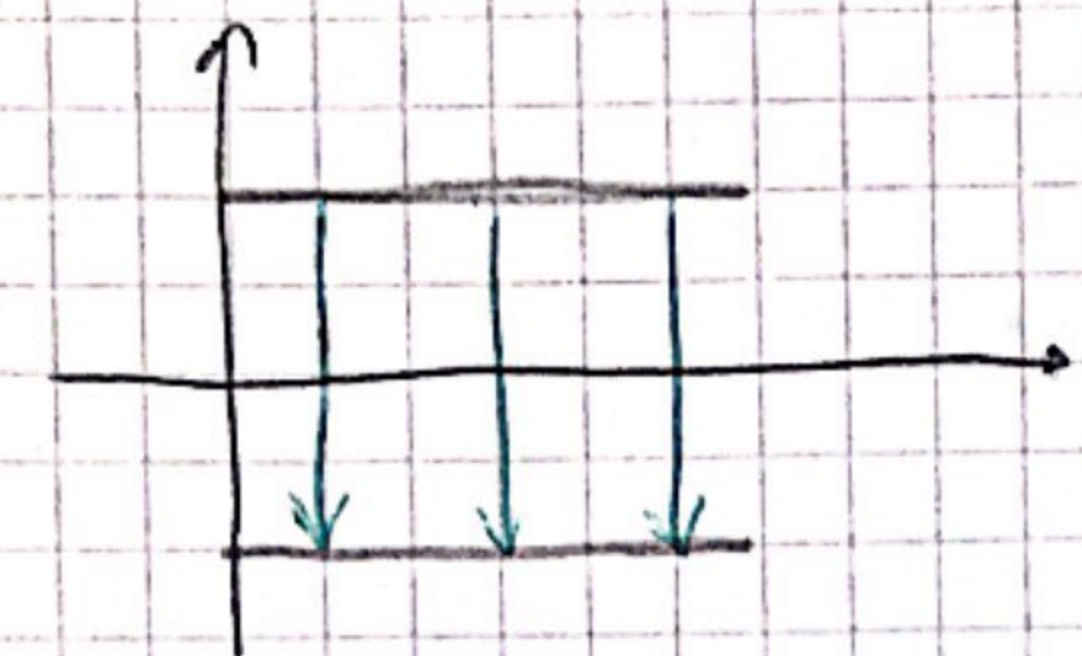
$$= 4\pi . A . R^3$$

NOTA

HOJA N°

FECHA

7



a) supongamos que el campo es uniforme entre las placas:

$$F = E.q = 3.10^{4}\,\frac{N}{C}.(1{,}602).10^{-19}C = 4{,}806.10^{-15}\,N$$

b) $F = m.a$

$$\Rightarrow m.a = E.q$$

$$\Rightarrow a = \frac{E.q}{m} = \frac{(4{,}806).10^{-15}N}{(9{,}109).10^{-31}Kg} = 0{,}5276.10^{16}\,\frac{m}{s^2}$$

c) Primero veamos cuanto tarda en salir de las placas:

$$X(t) = V_0.T \Rightarrow \frac{X(t)}{V_0} = T$$

$$\frac{4cm}{2{,}10^{7}\frac{m}{s}} = T$$

$$\boxed{0{,}02.10^{-7}s = T}$$

Luego:

$$Y(t) = \frac{1}{2}a\,T^2 + \overset{0}{V_{0y}}.T + \underbrace{Y_0}_{2cm}$$

$$Y(t) = \frac{1}{2}.0{,}5276.10^{16}\,\frac{m}{s}T^2 + 0{,}02m$$

$$Y(0{,}02.10^{-7}s) = \frac{1}{2}.0{,}5276.10^{16}\frac{m}{s}.(0{,}02)^2.10^{-14} + 0{,}02m$$

$$= 0{,}0105m + 0{,}02m = 0{,}0305\,m$$

NOTA

d)

$$V_{fx} = V_{0x} = 2 \cdot 10^7 \frac{m}{s}$$

Ahora veamos V_{fy}:

$$V_y(T) = a \cdot T + \underbrace{V_{y0}}_{=0} = a \cdot T$$

$$\Rightarrow V_{fy} = V_y(0{,}2 \cdot 10^{-7} s) = (0{,}5276) \cdot 10^{16} \cdot (0{,}2) \cdot 10^{-7} \frac{m}{s^2} \cdot s = 0{,}105 \cdot 10^{9} \frac{m}{s}$$

Luego: $\vec{V} = (2 \cdot 10^7, 0{,}105 \cdot 10^9)$

dirección original: $(1,0)$

Luego:

$$\theta = \arccos\left(\frac{(1,0) \cdot (2 \cdot 10^7, 0{,}105 \cdot 10^9)}{1 \cdot \sqrt{4 \cdot 10^{14} + (0{,}105)^2 \cdot 10^{18}}} \right) = 79{,}22°$$

e)

$$\frac{X(T)}{V_0} = T$$

$$\frac{16 cm}{2 \cdot 10^7 \frac{m}{s}} = T$$

$$\frac{0{,}16 m}{2 \cdot 10^7 \frac{m}{s}} = T$$

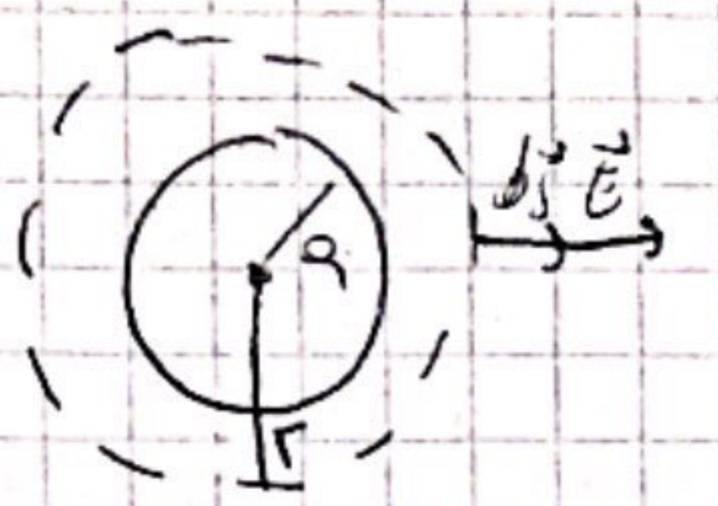
$$\boxed{0{,}08 \cdot 10^{-7} s = T}$$

Luego

$$y(T) = \frac{1}{2} (0{,}5276) \cdot 10^{16} \frac{m}{s^2} \cdot T^2 + 0{,}02 m$$

$$y(0{,}08 \cdot 10^{-7} s) = 0{,}168 m + 0{,}02 m = 0{,}188 m$$

(9)



$r > a$

b)

↳ Sup Gaussiana (Esfera)

Por Gauss

$$\varepsilon_0 \oint \vec{E}\, d\vec{s} = Q$$

$$\varepsilon_0 \oint E . ds . \cos(0) = Q$$

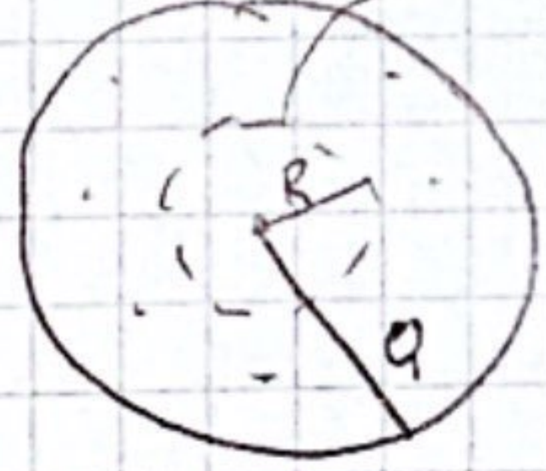
$$\varepsilon_0 E \int ds = Q$$

$$\varepsilon_0 . E . 4\pi r^2 = Q$$

$$\boxed{E = \frac{Q}{\varepsilon_0 4\pi r^2}}$$ ⇝ Campo fuera (y en el borde)

Por otro lado

→ Sup Gaussiana (Esfera)



$R \leq a$

Q : Carga esfera grande

Q' : Carga esfera pequeña

$$Q = \rho . v = \rho . \frac{4\pi a^3}{3} \Longrightarrow \rho = \frac{Q}{\frac{4}{3}\pi . a^3}$$

Entonces:

$$Q' = \rho . v = \frac{Q}{\frac{4}{3}\pi a^3} . \frac{4}{3} . \pi . R^3 = Q \left(\frac{R}{a}\right)^3$$

Por Gauss:

$$\varepsilon_0 \oint E\,ds = Q\left(\frac{R}{a}\right)^3$$

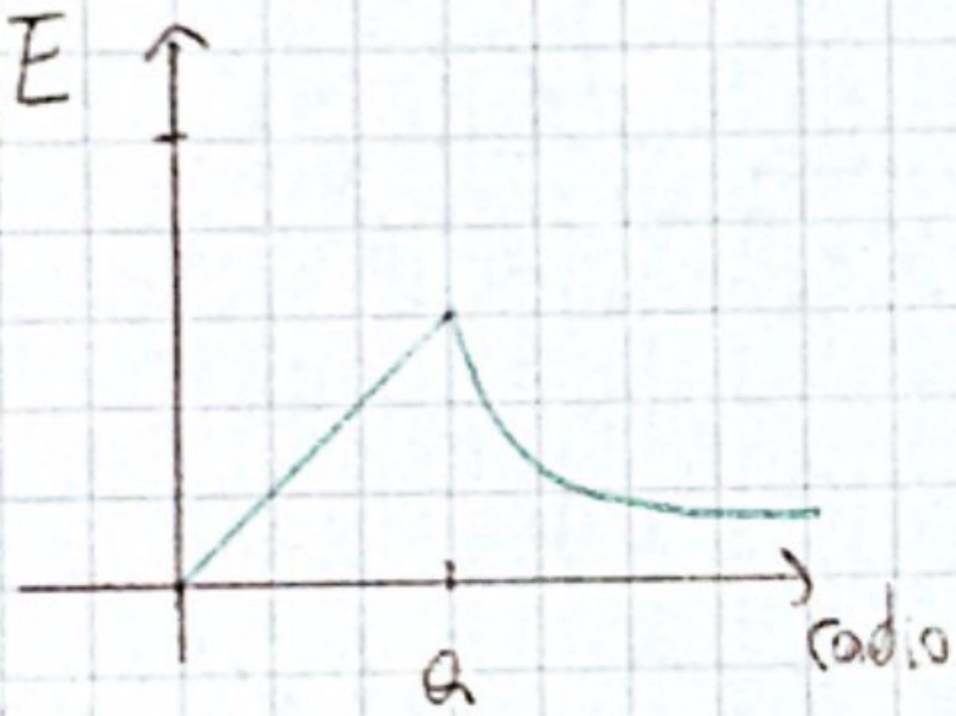
$$\varepsilon_0 E \int ds = Q\,\frac{R^3}{a^3}$$

$$\varepsilon_0 E \cdot 4\pi R^2 = Q\,\frac{R^3}{a^3}$$

$$\boxed{E = \frac{Q \cdot R}{\varepsilon_0 \cdot 4\pi \cdot a^3}}$$ ~ adentro de esfera "a"

~ R es una variable

c)

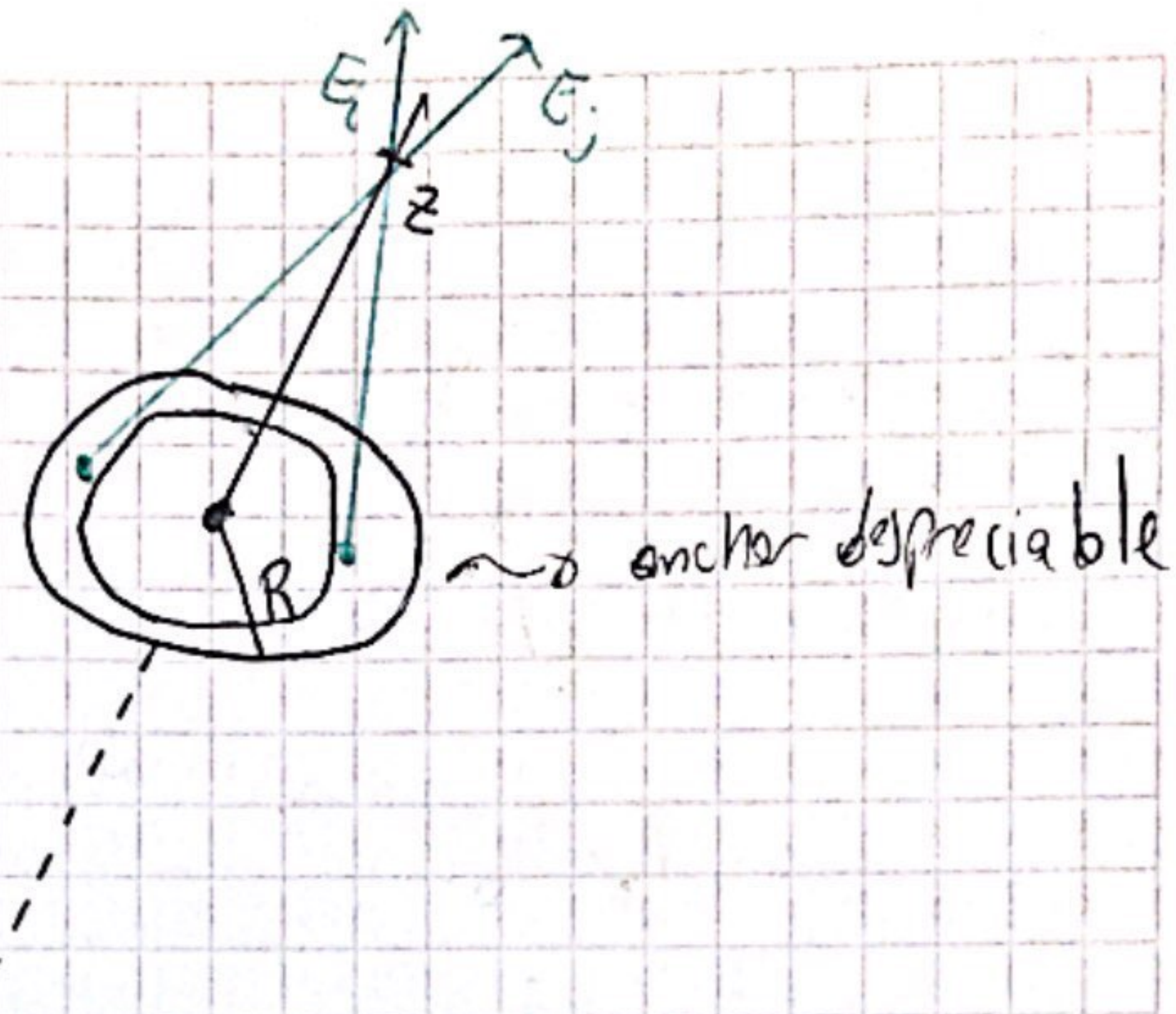


d) En la sup de la esfera $E = \frac{Q}{\varepsilon_0 4\pi a^2}$

En el centro de la esfera $R=0$:

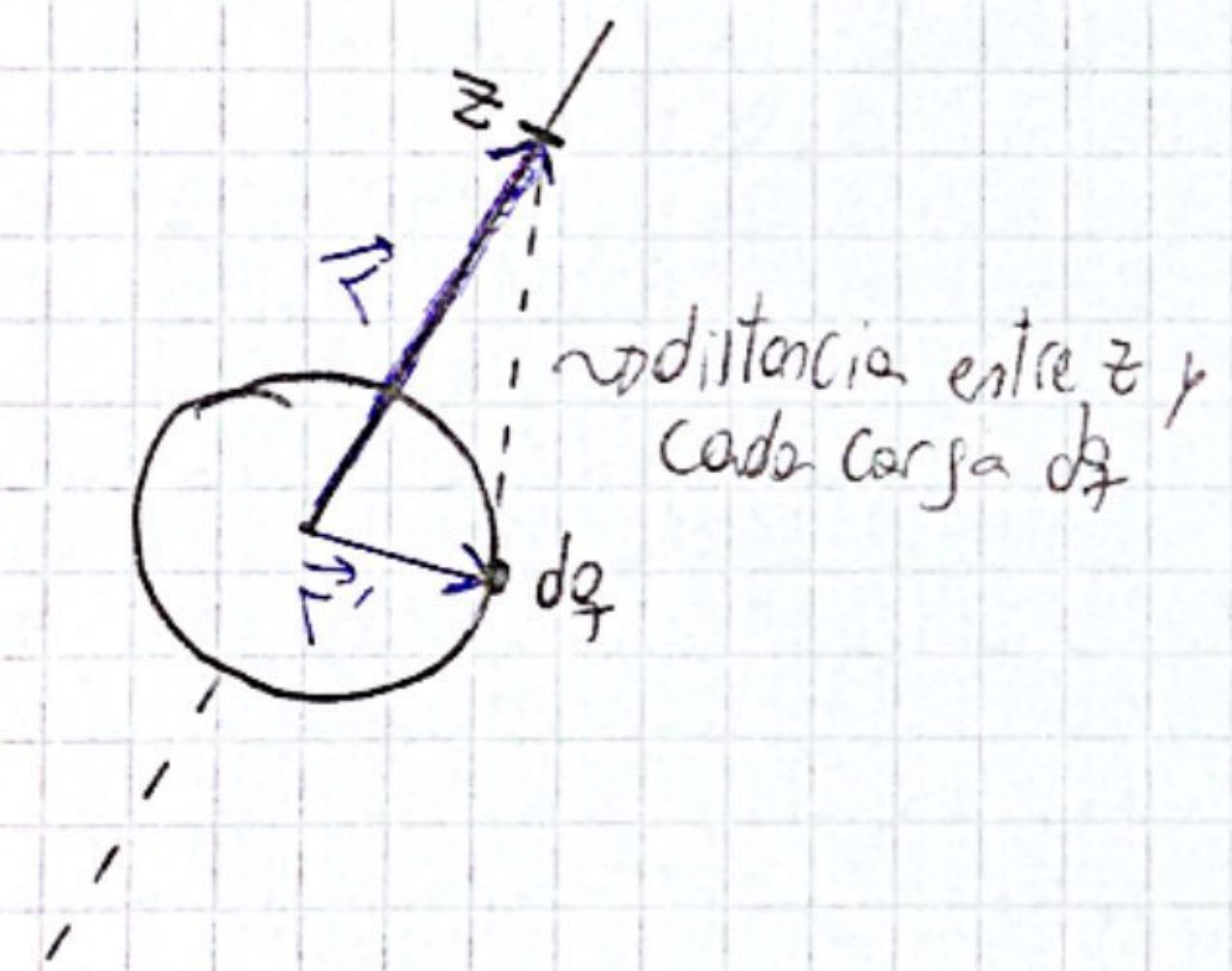
$$E = \frac{0}{\varepsilon_0 4\pi a^3} = 0$$

6



Notemos que por simetría, para cada carga q_i va a haber otra q_j tal que $E_i + E_j$ tiene dirección en z. Por lo tanto E_T depende de z. Esto no va a suceder para el punto del centro, pero de igual manera E_0 apunta hacia z.

Entonces:



$$E_{dq}(z) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{\|r - r'\|^3} \cdot (\vec{r} - \vec{r}')$$

donde : $\vec{r} = z.\hat{z}$

$\vec{r}' = R.\cos(\theta)\hat{x} + R.\sin(\theta)\hat{y}$

$$d = \|r - r'\| = \sqrt{r^2 + r'^2} = \sqrt{z^2 + R^2}$$

Es decir

$$E_T = \int_0^Q E_{dq} = \int_0^Q \frac{(z.\hat{z} - R\cos(\theta)\hat{x} - R\sin(\theta)\hat{y})}{4\pi\varepsilon_0 (\sqrt{z^2+R^2})^3} \cdot dq$$

$$dq = \rho \, dl = \rho . R \, d\theta$$

longitud de dicho arco

densidad de carga

$$\rho = \frac{Q}{A} = \frac{Q}{2\pi R}$$

$$dq = \frac{Q \, d\theta}{2\pi}$$

Reemplazando:

$$E_T = \int_0^{2\pi} \frac{(z\,\hat{z} - R\cos(\theta)\hat{x} - R\sin(\theta)\hat{y})}{4\pi\,\varepsilon_0 (\sqrt{z^2+R^2})^3} \cdot \frac{Q\,d\theta}{2\pi}$$

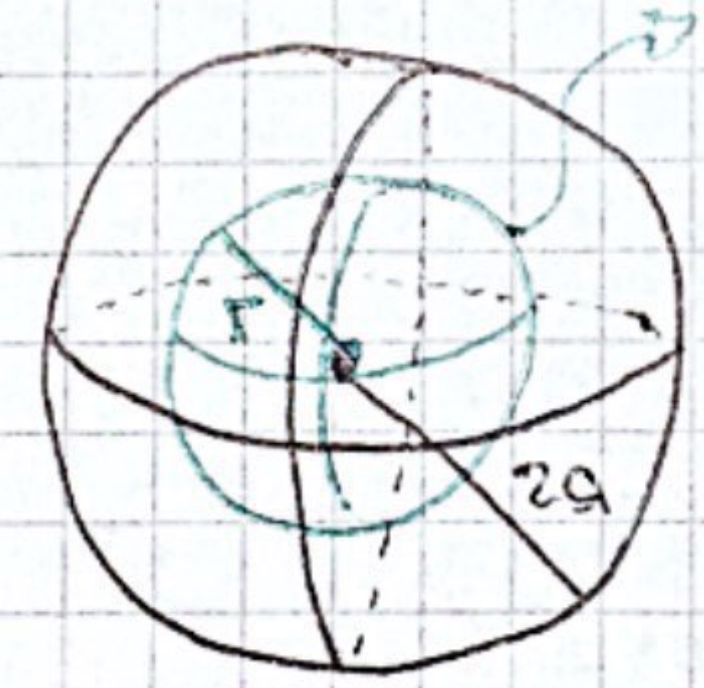
$$= \frac{Q}{8\pi^2\varepsilon_0(\sqrt{z^2+R^2})^3} \left[\hat{z}\int_0^{2\pi} z\,d\theta - R\hat{x}\int_0^{2\pi}\cos(\theta)\,d\theta - R\hat{y}\int_0^{2\pi}\sin(\theta) \right]$$

$$= \frac{Q}{8\pi^2\varepsilon_0(\sqrt{z^2+R^2})^3} \cdot 2\pi z\,\hat{z} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{z\,\hat{z}}{(\sqrt{z^2+R^2})^3} = \frac{k\,Q\,z}{(\sqrt{z^2+R^2})^3}\,\hat{z}$$

10

Primero calculo el campo dentro de la esfera de radio $2a$:

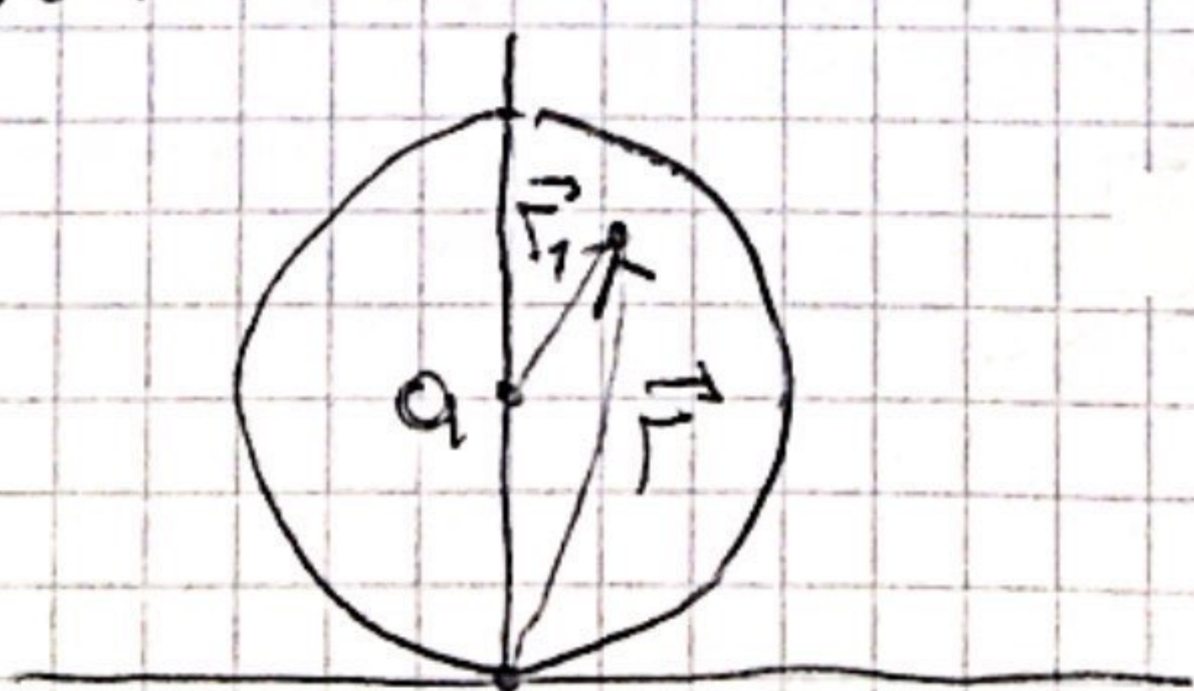
Sup Gaussiana con carga $Q' = \rho \cdot \frac{4}{3}\pi r^3$



$$\varepsilon_0 \oint \vec{E}\,d\vec{s} = Q'$$

$$\varepsilon_0 E\,4\pi r^2 = \frac{4}{3}\pi r^3 \cdot \rho \Rightarrow E = \frac{-\rho \cdot r}{3\varepsilon_0} \rightsquigarrow \vec{E}_1(\vec{r}) = \frac{-\rho}{3\varepsilon_0} \cdot \vec{r}$$

Por otro lado:



Del mismo $\vec{E}_2(\vec{r}_1) = \frac{-\rho}{3\varepsilon_0} \cdot \vec{r}_1$

donde $\vec{r}_1 = \vec{r} - a\,\hat{y}$

Campo medido desde el centro de la esfera

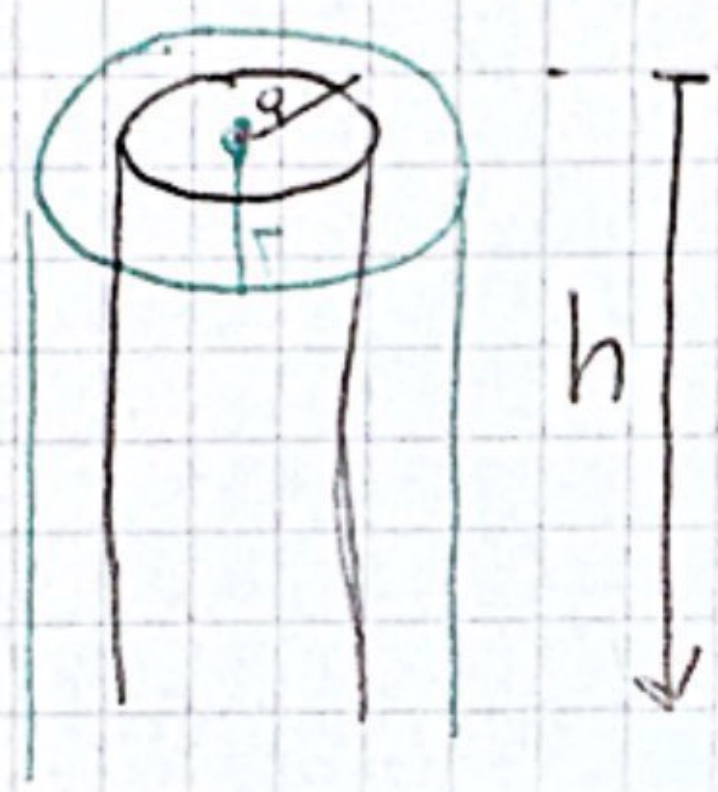
Entonces: $\vec{E}_2(\vec{r}_1) = \frac{-\rho}{3\varepsilon_0}\cdot(\vec{r} - a\hat{y})$

Por lo tanto:

$$\vec{E}_T = \frac{\rho}{3\varepsilon_0}\cdot\vec{r} - \frac{\rho}{3\varepsilon_0}(\vec{r} - a\hat{y}) = \frac{\rho\vec{r}}{3\varepsilon_0} - \frac{\rho}{3\varepsilon_0}\vec{r} + \frac{\rho a}{3\varepsilon_0}\hat{y} = \frac{\rho a}{3\varepsilon_0}\hat{y}$$

↳ Solo depende de $y \Rightarrow E_x = 0$, $E_z = 0$

12



Tenemos simetria rotacional por lo tanto podemos describir el campo con solo su componente radial

$$\varepsilon_0 \oint \vec{E}\, d\vec{s} = Q$$

$$\varepsilon_0 E \oint ds = \sigma 2\pi a h$$

$$\varepsilon_0 E\, 2\pi r h = \sigma 2\pi a h$$

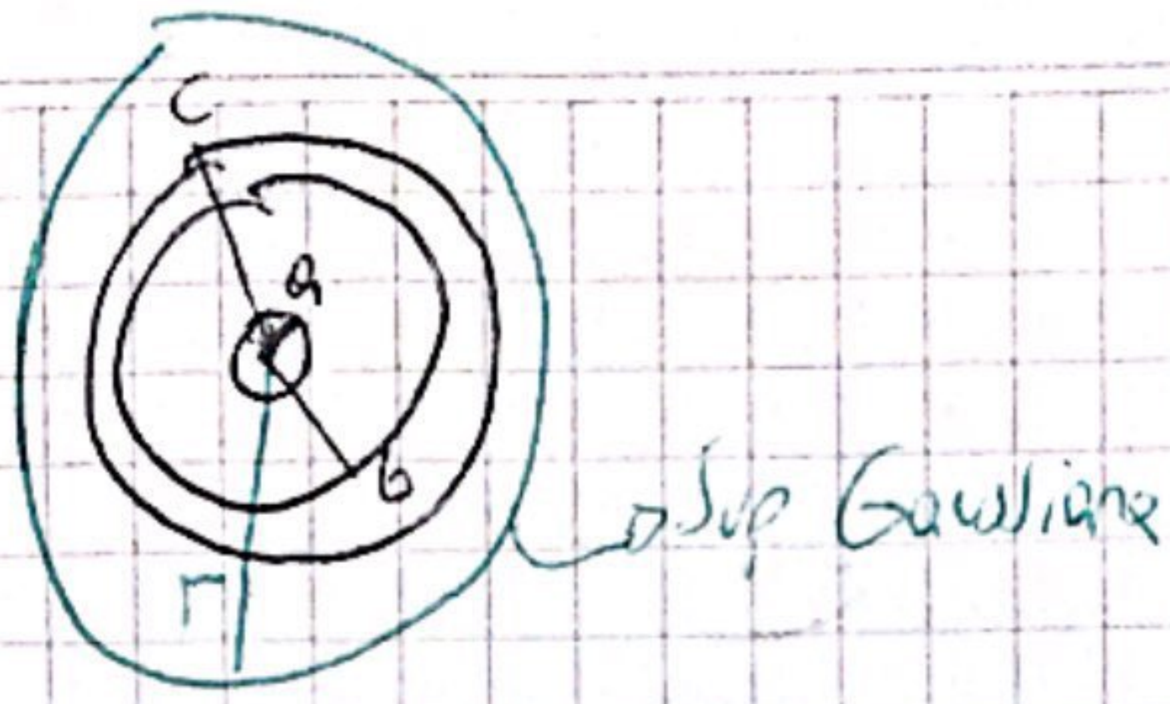
$$\boxed{E = \frac{\sigma a}{\varepsilon_0 r}} \rightsquigarrow \boxed{E(\vec{r}) = \frac{\sigma a}{\varepsilon_0 r}\cdot\hat{r}}$$

14

a) Entre a y b, es el campo de una esfera uniformemente cargada, entonces:

$$E = \frac{2Q}{4\pi\varepsilon_0 r^2} \quad , b \geq r \geq a$$

Por otra parte, para $r \geq c$:



$\varepsilon_0 \oint E\,ds = 2Q + (-Q)$

$\varepsilon_0 \oint E\,ds = Q$

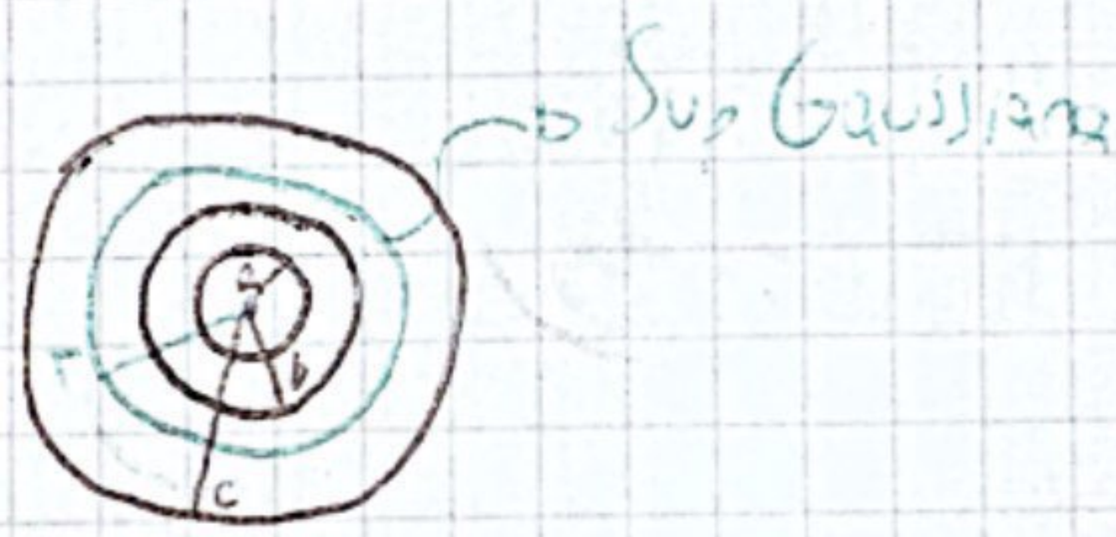
$\varepsilon_0 E\, 4\pi r^2 = Q$

$$\boxed{E = \frac{Q}{4\pi R^2 \varepsilon_0}}$$

Si $c \geq r \geq b$:

$\vec{E} = 0$ pues es un conductor

b)



Dado que la sup. gaussiana está dentro del conductor, en su interior el campo debe ser cero, para esto su carga encerrada debe ser cero. Dado que el cascarón es un conductor, su carga deben estar en "b" y "c". Dado que "a" tiene carga 2Q, "b" debe tener carga -2Q. Y finalmente c debe tener carga Q.