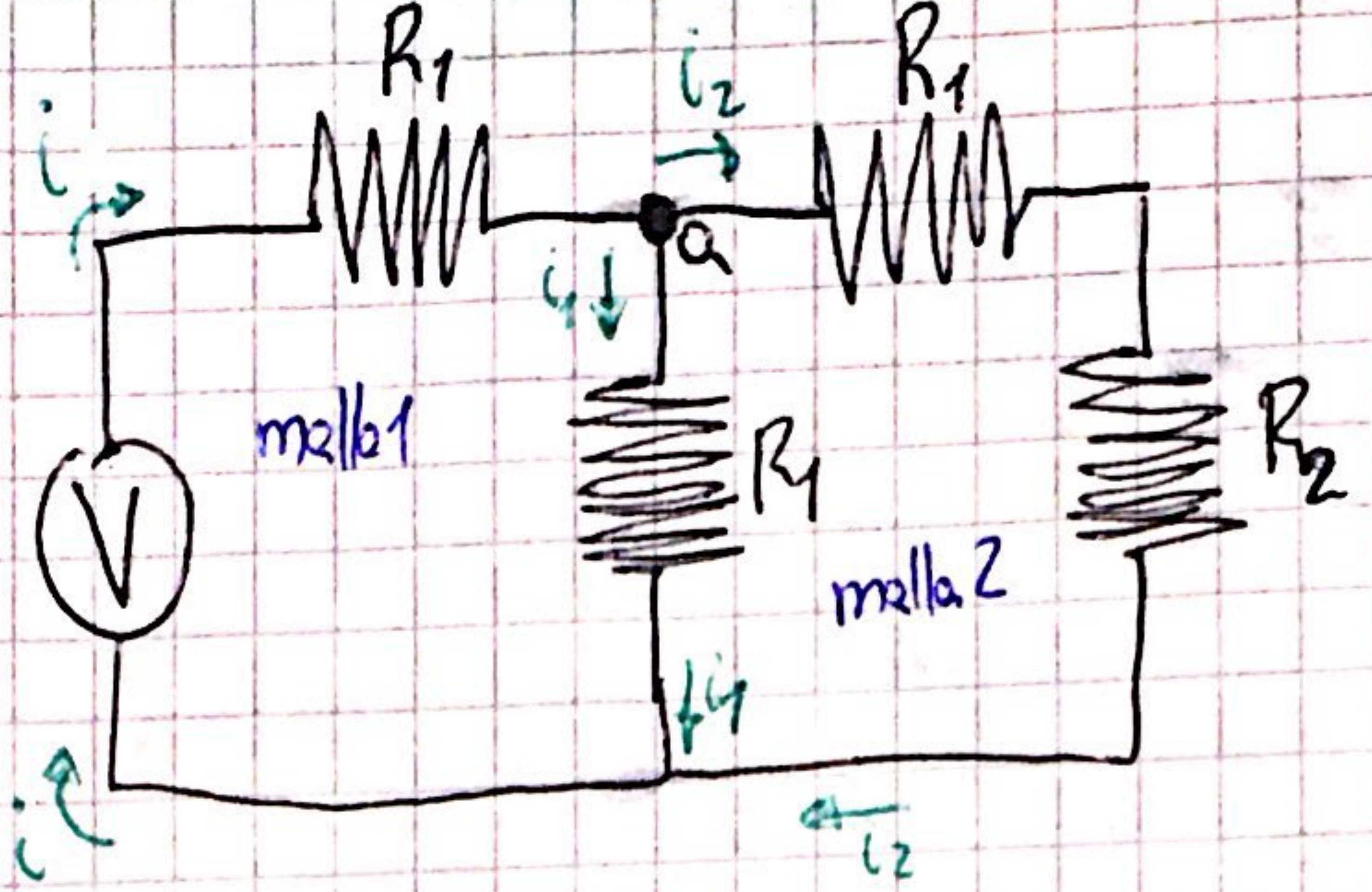
HOJA N°

FECHA

PRACTICO 8

1

a)



Por nodo a: $i = i_1 + i_2$ (1')

malla 1: $V_0 = i R_1 + i_1 R_1$ (1)

malla 2: $0 = i_2(R_1+R_2) - i_1 R_1$ (2)

↳ pues la corriente circula en diferente sentido

b) Reemplazo 1' en 1:

$$V_0=(i_1+i_2)R_1+i_1R_1 \Longrightarrow V_0 = i_1(R_1+R_1)+i_2R_2 \Longrightarrow \underbrace{V_0=i_1 2R_1+i_2R_2}_{(\alpha)}$$

Con (α) y (2):

$$\begin{array}{cc} i_1 & i_2 \\ \end{array}$$

$$\left(\begin{array}{cc|c} 2R_1 & R_1 & V_0 \\ -R_1 & R_1+R_2 & 0 \end{array}\right) \xrightarrow{2.F_2} \left(\begin{array}{cc|c} 2R_1 & R_1 & V_0 \\ -2R_1 & 2R_1+2R_2 & 0 \end{array}\right) \xrightarrow{F_2+F_1}$$

$$\left(\begin{array}{cc|c} 2R_1 & R_1 & V_0 \\ 0 & 3R_1+2R_2 & V_0 \end{array}\right)$$

NOTA

Entonces:

$$\begin{cases} 2R_1 \cdot i_1 + R_1 \cdot i_2 = V_0 \\ i_2(3R_1+2R_2) = V_0 \end{cases} \Longrightarrow \boxed{i_2 = \frac{V_0}{3R_1+2R_2}}$$

Remplazo arriba:

$$2R_1 i_1 + \frac{R_1 V_0}{3R_1+2R_2} = V_0 \iff 2R_1 i_1 = V_0 - \frac{R_1 V_0}{3R_1+2R_2}$$

$$\iff 2R_1 i_1 = \frac{3R_1V_0 + 2R_2V_0 - R_1V_0}{3R_1+2R_2} = \frac{2R_1V_0+2R_2V_0}{3R_1+2R_2}$$

$$\iff i_1 = \frac{1}{2R_1} \cdot \frac{2R_1V_0+2R_2V_0}{3R_1+2R_2} = \frac{R_1V_0+R_2V_0}{R_1(3R_1+2R_2)}$$

$$\iff \boxed{i_1 = V_0 \cdot \frac{R_1+R_2}{R_1(3R_1+2R_2)}}$$

Luego:

$$i = i_1 + i_2 = V_0\left[\frac{(R_1+R_2)}{R_1(3R_1+2R_2)} + \frac{1}{3R_1+2R_2}\right] = V_0\left[\frac{(2R_1+R_2)}{R_1(3R_1+2R_2)}\right]$$

c)

$$R_{eq} = \frac{V_0}{i} = \frac{R_1(3R_1+2R_2)}{2R_1+R_2}$$

d)

En la Guia pasada:

$$R_{eq} = R_1 + \left(\frac{1}{R_1} + \frac{1}{R_1+R_2}\right)^{-1} = R_1 + \left(\frac{2R_1+R_2}{R_1(R_1+R_2)}\right)^{-1}$$

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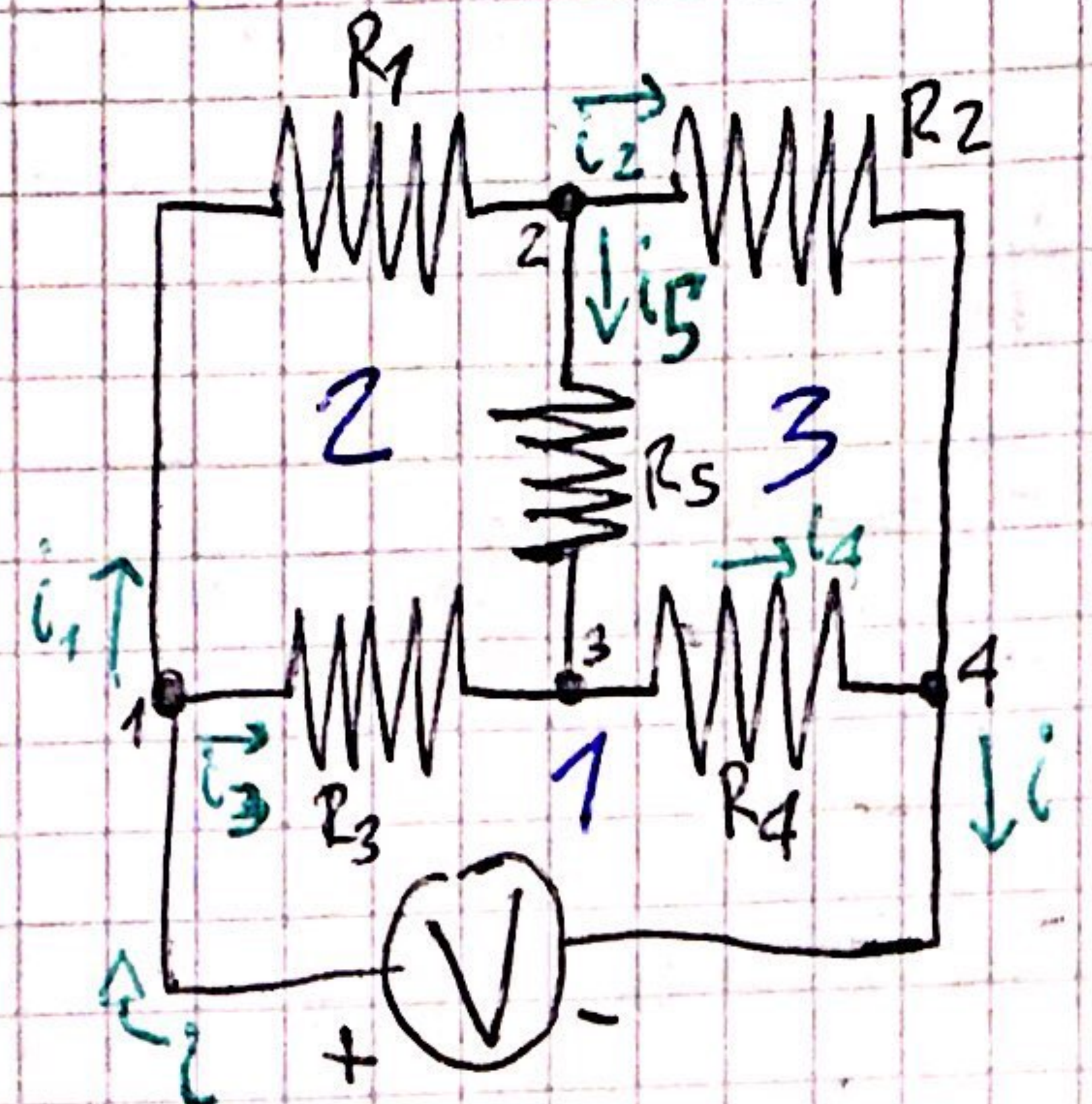
$$= R_1 + \frac{R_1(R_1+R_2)}{2R_1+R_2} = \frac{R_1(2R_1+R_2)+R_1(R_1+R_2)}{2R_1+R_2} = \frac{R_1(3R_1+2R_2)}{2R_1+R_2}$$

2 a)



nodo 1: $i = i_1 + i_3$

nodo 2: $i_1 = i_2 + i_5$

nodo 3: $i_4 = i_3 + i_5$

nodo 4: $i = i_4 + i_2$

igualando a cero $\rightsquigarrow$

$$\begin{array}{cccccc} i & i_1 & i_2 & i_3 & i_4 & i_5 \end{array}$$
$$\begin{pmatrix} 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 \\ 1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

No intercambio filas

Luego las ecuaciones LI:

$$\begin{cases} i = i_2 + i_4 & (a) \\ i_1 = i_2 + i_5 & (b) \\ i_4 = i_3 + i_5 & (c) \end{cases}$$

Además

malla 1: $V = i_3 R_3 + i_4 R_4$ (d)

malla 2: $0 = i_1 R_1 + i_5 R_5 - i_3 R_3$ (e)

malla 3: $0 = i_5 R_5 + i_4 R_4 - i_2 R_2$ (f)

NOTA

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b) Condiciones:

Por b: $i_s = 0 \iff \boxed{i_1 = i_2}$

Por c: $i_s = 0 \iff \boxed{i_4 = i_3}$

Por e: $i_s = 0 \iff \frac{-i_1 R_1 + i_3 R_3}{R_s} = 0 \iff \boxed{i_3 R_3 = i_1 R_1}$

Por f: $i_s = 0 \iff \frac{-i_4 R_4 + i_2 R_2}{R_s} = 0 \iff \boxed{i_2 R_2 = i_4 R_4}$

Entonces tenemos:

me saco de encima i_4, i_2

$$\begin{cases} i = i_2 + i_4 \\ i_1 = i_2 \\ i_3 = i_4 \\ i_3 R_3 = i_1 R_1 \\ i_4 R_4 = i_2 R_2 \\ V = i_3 R_3 + i_4 R_4 \end{cases} \approx \begin{cases} i = i_1 + i_3 & \alpha \\ i_3 R_3 = i_1 R_1 & \beta \\ i_3 R_4 = i_1 R_2 & \gamma \\ V = i_3 (R_3 + R_4) & \delta \end{cases}$$

Por δ: $\boxed{i_3 = \frac{V}{R_3 + R_4} = i_4}$

Reemplaz i_3 en γ:

$$\boxed{i_1 = \frac{V R_4}{R_2 (R_3 + R_4)} = i_2}$$

Luego: $\boxed{i = i_2 + i_4 = \frac{V}{R_3 + R_4} \left(\frac{R_4}{R_2} + 1 \right)}$

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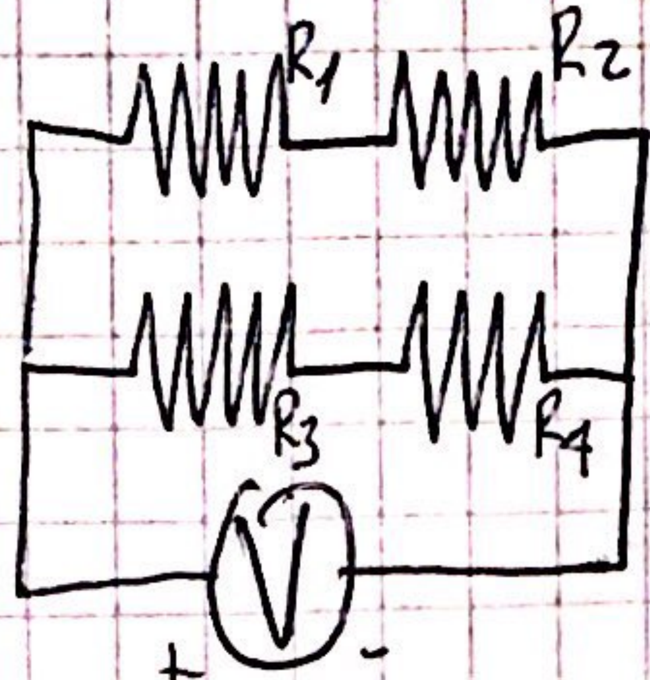
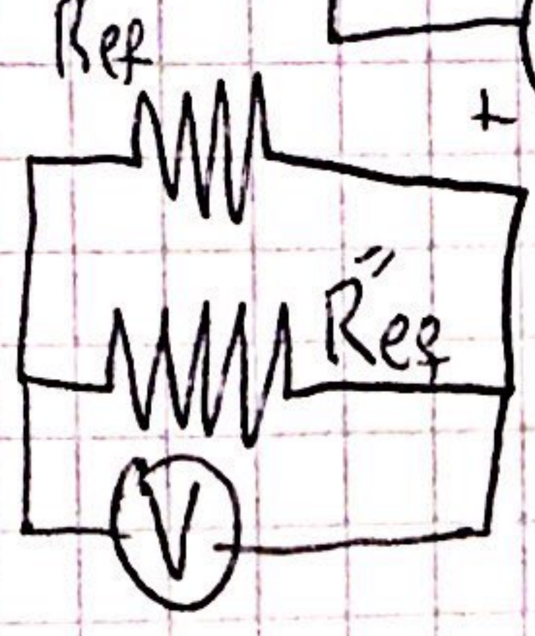
Dado que $i_s = 0$, el circuito nos queda:



$R'_{ef} = R_1 + R_2$

$R''_{ef} = R_3 + R_4$

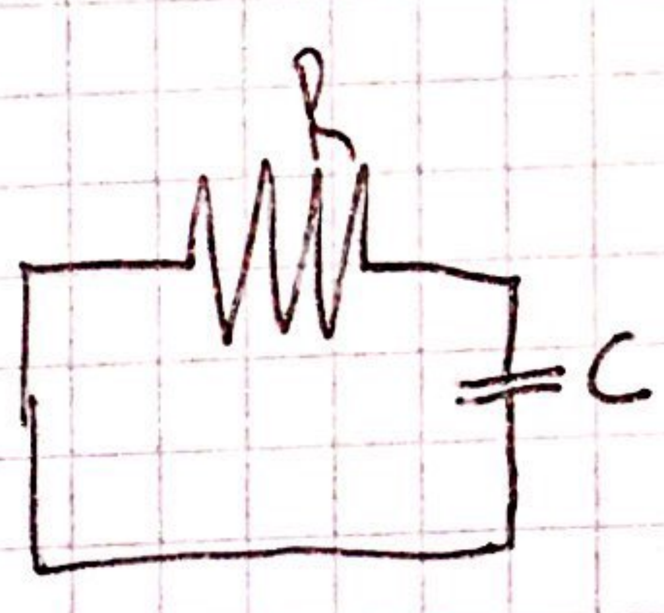
$\equiv$



$$R_{EQ} = \left(\frac{1}{R'_{ef}} + \frac{1}{R''_{ef}}\right)^{-1} = \left(\frac{1}{R_1+R_2} + \frac{1}{R_3+R_4}\right)^{-1} = \left(\frac{R_3+R_4+R_1+R_2}{(R_1+R_2)(R_3+R_4)}\right)^{-1}$$

$$= \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

3



$V_c = \frac{q}{C}$

$V_R = i.R$

a) Por malla: $0 = i.R + \frac{q}{C}$

(1) $0 = \frac{dq}{dt}.R + \frac{q}{C}$ $\rightsquigarrow$ ec diferencial.

Supongamos que $q(t) = A.e^{at} + B.e^{-bt} + D \Rightarrow \frac{dq(t)}{dt} = Aae^{at} - Bbe^{-bt}$

Entonces en (1):

$$0 = R(Aae^{at} - Bbe^{-bt}) + (e^{at}.A + Be^{-bt} + D)\cdot\frac{1}{C}$$

NOTA

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$$0 = A e^{at}\left(a R + \frac{1}{C}\right) + B e^{-bt}\left(\frac{1}{C} - R b\right) + \frac{D}{C}$$

Posible solución:

$$\begin{cases} A e^{at} = 0 & \Longrightarrow A = 0 \\ \frac{1}{C} = R b & \Longrightarrow b = \frac{1}{RC} \\ \frac{D}{C} = 0 & \Longrightarrow D = 0 \end{cases}$$

Entonces $\quad q(t) = B \cdot e^{-\frac{t}{RC}} + 0$

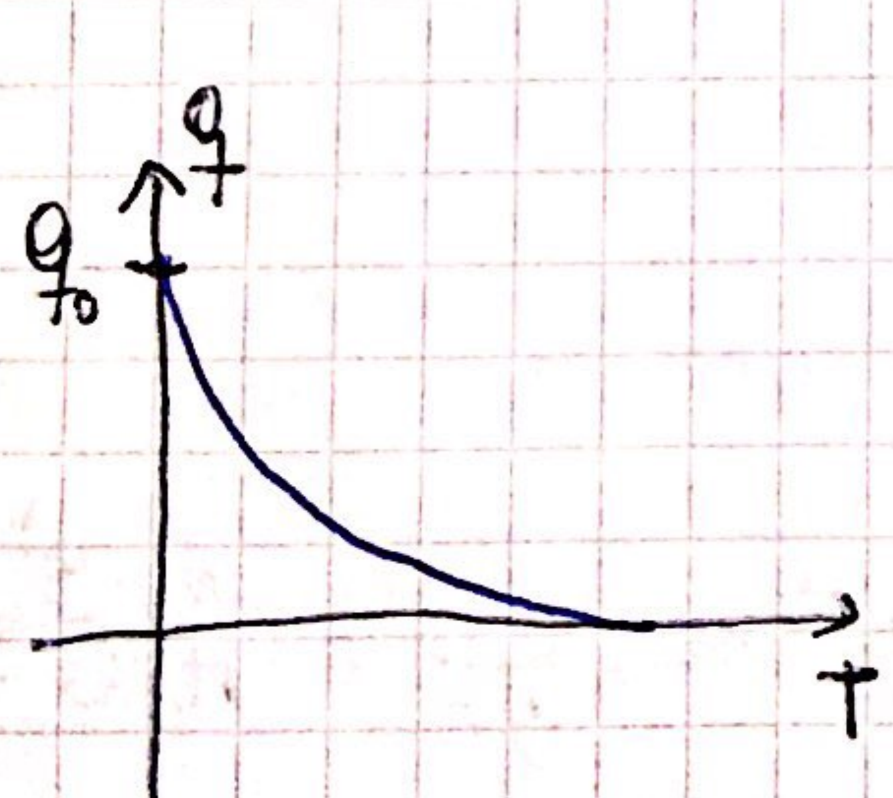
Dado que en $t=0$ el capacitor tiene una carga maxima q_0:

$$q(0) = B \cdot e^{0} = q_0 \Longrightarrow B = q_0$$

Por lo tanto

$$q(t) = q_0 \cdot e^{-\frac{t}{RC}}$$

$$i(t) = \frac{dq}{dt} = -\frac{q_0}{RC} \cdot e^{-\frac{t}{RC}} = -\frac{V_0}{R} e^{-\frac{t}{RC}} = -i_0 e^{-\frac{t}{RC}}$$



el capacitor se descarga

NOTA

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b)

Energía inicialmente almacenada: $E_p = \frac{q_0^2}{2C}$

Sabemos que $P_r = i^2 R$ es la energía disipada. Pero en este caso la energía varía con el tiempo, por lo tanto:

$$E_p = \int_0^\infty i(t)^2 . R . dt = \int_0^\infty R . i_0^2 e^{-\frac{2t}{RC}} dt = -\frac{R^2C}{2} i_0^2 e^{-\frac{2t}{RC}} \Big|_0^\infty$$

$$= -\lim_{t\to\infty} \frac{R^2C . i_0^2}{2e^{\frac{2t}{RC}}} + \frac{R^2Ci_0^2}{2} = \frac{R^2Ci_0^2}{2} = \frac{R^2C}{2} . \frac{q_0^2}{R^2C^2} = \frac{q_0^2}{2C}$$

$i_0 = \frac{V_0}{R} = \frac{q_0}{RC}$

$i_0^2 = \frac{q_0^2}{R^2C^2}$

c)

$$q_0 = C.V_0 = 100\mu F . 10V = 1000 \mu F . V = 1000 . 10^{-6} F . V = 10^{-3} \frac{C}{V} . V = 10^{-3} C$$

entonces $q(t) = 10^{-3} C . e^{-\frac{t}{100\Omega . 100\mu F}} = 10^{-3} C . e^{-\frac{t . 100}{\Omega F}}$

Tenemos que hallar t tal que $q(t) = 1{,}602 . 10^{-19} C$. Entonces:

$$1{,}602 . 10^{-19} C = 10^{-3} C . e^{-\frac{t 100}{\Omega F}}$$

$$1{,}602 . 10^{-16} = e^{-\frac{t 100}{\Omega F}}$$

$$\ln(1{,}602 . 10^{-16}) = -\frac{t . 100}{\Omega . F}$$

$[F] = \frac{s^4 A^2}{m^2 kg}$

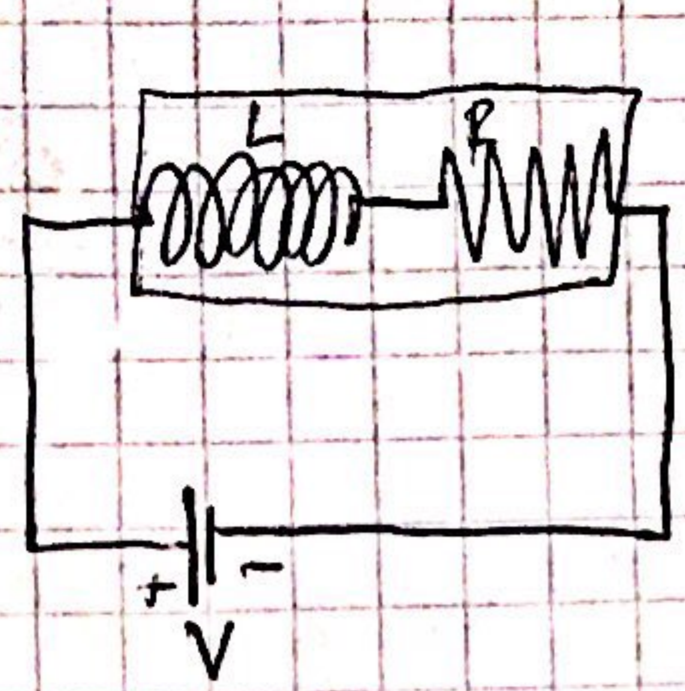
$[\Omega] = \frac{m^2 kg}{s^3 A^2}$

$$\Longrightarrow t = -10^{-2} \ln(1{,}602 . 10^{-16}) . \Omega . F = 0{,}3637 \ s$$

NOTA

4

a)



$V_0 = 6V$

$V_L = -L\frac{di}{dt}$

$V_R = i.R$

$$V = -V_L + V_R = L\cdot\frac{di}{dt} + i.R$$

b)

$$0 = V_0 - L\frac{di}{dt} - i.R = 0$$

Supongamos $i(t) = Ae^{at} + Be^{-bt} + D$:

$$V_0 - L[Aae^{at} - Bbe^{-bt}] - R[Ae^{at} + Be^{-bt} + D]$$

$$V_0 + Ae^{at}(-R - La) + Be^{at}(-R + Lb) - RD = 0$$

solución:

$$\begin{cases} V_0 = RD & \Rightarrow D = \frac{V_0}{R} \\ A = 0 & \\ R = Lb & \Rightarrow b = \frac{R}{L} \end{cases}$$

Luego:

$$i(t) = Be^{-\frac{Rt}{L}} + \frac{V_0}{R}$$

Sabemos que el inductor se opone a la variación de la corriente,

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Por lo tanto no aparecerá la corriente de forma instantánea, es decir $i(t)=0$.

$$i(0)=B\cdot 1+\frac{V_0}{R}=0 \Rightarrow B=-\frac{V_0}{R}$$

Finalmente:

$$\boxed{i(t)=\frac{V_0}{R}\left(1-e^{-\frac{Rt}{L}}\right)}$$

Por otro lado:

$$\lim_{t\to\infty}\frac{V_0}{R}\left(1-\frac{1}{e^{\frac{Rt}{L}}}\right)=\frac{V_0}{R}\left(1-\frac{1}{\infty}\right)=\frac{V_0}{R}=\frac{6V}{0,1\Omega}=\boxed{60A}$$

valor asintótico.

Dado que es creciente y tiende a $\frac{V_0}{R}$, entonces $i_{max}=\frac{V_0}{R}$. Por lo tanto

$\frac{9V_0}{10R}$ es el 90%.

Tenemos:

$$\frac{V_0}{R}\left(1-e^{-\frac{Rt}{L}}\right)=\frac{9}{10}\frac{V_0}{R}$$

$$-e^{-\frac{Rt}{L}}=-\frac{1}{10} \Rightarrow -\frac{Rt}{L}=\ln\left(\frac{1}{10}\right)$$

$\mu H=10^{-3}H$

$[H]=\frac{m^2 Kg}{s^2A^2}$

$$\Rightarrow T=-\ln\left(\frac{1}{10}\right)10^{-2}s=0,02\ s$$

$[\Omega]=\frac{m^2 Kg}{s^3A^2}$

$\Rightarrow \frac{1mH}{0,1\Omega}=10^{-2}s$

NOTA

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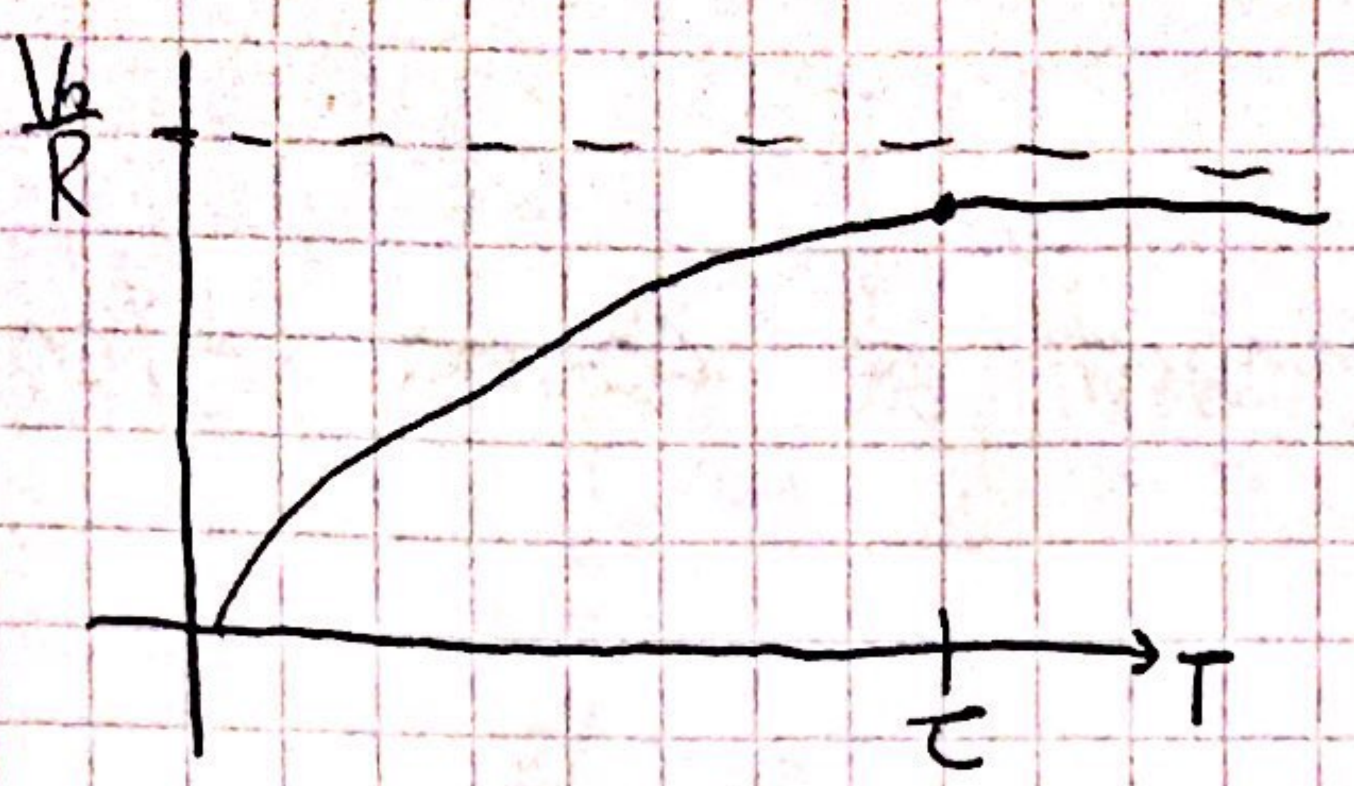
c)



$\tau = \frac{L}{R}$

d)

$$U_L = \frac{1}{2} L\, i(t)^2 = \frac{L V_0^2}{2R^2}\left(1 - e^{-\frac{R}{L}t}\right)^2 = \frac{L V_0^2}{2R^2}\left(1 - 2e^{-\frac{R}{L}t} + e^{-\frac{2R}{L}\cdot t}\right)$$

$$U_L \stackrel{?}{=} E_{P_B} - \int_0^t P_r\, dt$$

La energía disipada por una batería tiene que ver con el trabajo que hace para mover una carga a través de un potencial.

$$V = \frac{W}{q} \Rightarrow W = E_p = V_0 . q \text{. Entonces:}$$

$$E_{P_B} = \int_0^t dq\, V_0 \quad \text{, dado que } \frac{dq}{dt} = i \Rightarrow dq = i . dt \text{. Entonces:}$$

$$E_{P_B} = \int_0^t \underbrace{i . V_0}_{P_B} . dt .$$

Además:

$$P_R = i(t)^2 R = \frac{V_0^2}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right)^2$$

Entonces:

NOTA

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FECHA

$$E_{P_B} - \int_0^t P_R\,dt = \int_0^t (P_B - P_R)\,dt = \frac{V_0^2}{R}\int_0^t \left(1 - e^{-\frac{R t}{L}} - 1 + 2e^{-\frac{R}{L}\cdot t} - e^{-\frac{2R}{L}\cdot t}\right)dt$$

$$= \frac{V_0^2}{R}\int_0^t \left(e^{-\frac{R}{L}\cdot t} - e^{-\frac{2R}{L}\cdot t}\right)dt = \frac{V_0^2}{R}\left(\int_0^t e^{-\frac{R}{L}\cdot t}dt - \int_0^t e^{-\frac{2R}{L}\cdot t}dt\right)$$

$$= \frac{V_0^2}{R}\left[\left(-\frac{L}{R}\cdot e^{-\frac{R}{L}\cdot t}\Big|_0^t\right) - \left(-\frac{L}{2R}e^{-\frac{2R}{L}\cdot t}\Big|_0^t\right)\right]$$

$$= \frac{V_0^2}{R}\left[\left(-\frac{L}{R}e^{-\frac{R}{L}t} + \frac{L}{R}\right) - \left(-\frac{L}{2R}e^{-\frac{2R}{L}\cdot t} + \frac{L}{2R}\right)\right]$$

$$= \frac{V_0^2 L}{R^2}\left(\frac{1}{2} - e^{-\frac{R}{L}t} + \frac{e^{-\frac{2R}{L}t}}{2}\right) = \frac{V_0^2 L}{2R^2}\left(1 - 2e^{-\frac{R}{L}t} + e^{-\frac{2R}{L}t}\right)$$

5

NOTA

(6) $V = A\,e^{\beta_1 t} + B\,e^{\beta_2 t}$

a)

$V = V_c - V_L + V_R = \frac{q}{c} + L\frac{di}{dt} + i.R = \frac{q}{c} + L\cdot\frac{d^2q}{dt^2} + \frac{dq}{dt}\cdot R$

Supongamos: $q(t) = A\,e^{\beta_1 t} + B\,e^{\beta_2 t}$

$\Longrightarrow q'(t) = A\beta_1 e^{\beta_1 t} + B\beta_2 e^{\beta_2 t}$

$\Longrightarrow q''(t) = A\beta_1^2 e^{\beta_1 t} + B\beta_2^2 e^{\beta_2 t}$

Luego:

$V = \frac{1}{c}\left[Ae^{\beta_1 t} + B\,e^{\beta_2 t}\right] + R\left[A\beta_1 e^{\beta_1 t} + B\beta_2 e^{\beta_2 t}\right] + L\left[A\beta_1^2 e^{\beta_1 t} + B\beta_2^2\, e^{\beta_2 t}\right]$

derivo respecto a t:

$0 = A\beta_1 e^{\beta_1 t}\left[\frac{1}{c} + R\beta_1 + L\beta_1^2\right] + B\beta_2 e^{\beta_2 t}\left[\frac{1}{c} + R\beta_2 + L\beta_2^2\right]$

Pues si bien depende del tiempo, es constante.

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Solución:

$$\begin{cases} L\beta_1^2 + R\beta_1 + \frac{1}{C} = 0 & \Longrightarrow \beta_1 = \dfrac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L} \\ L\beta_2^2 + R\beta_2 + \frac{1}{C} = 0 & \Longrightarrow \beta_2 = \dfrac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L} \end{cases}$$

Elijo:

$$\beta_1 = \frac{-R - \sqrt{R^2 - \frac{4L}{C}}}{2L}, \quad \beta_2 = \frac{-R + \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

$$\beta_1 = \frac{-600\Omega - \sqrt{600^2\Omega^2 - \frac{4.100.10^{-6}}{10^{-4}}\frac{H}{F}}}{2.100.10^{-6}\,H} = -5999983{,}33\,\frac{1}{s}$$

$$\beta_2 = \frac{-600\Omega + \sqrt{600^2\Omega - \frac{4.100.10^{-6}}{10^{-4}}\frac{H}{F}}}{2\,100.10^{-6}\,H} = -16{,}66\cdot\frac{1}{s}$$

Dado que propuse q como exponencial:

$$\omega = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$, es sobreamortiguado si $\omega > 0$.

$$\Longrightarrow \frac{R^2}{4L^2} > \frac{1}{LC} \iff \frac{R^2}{4L} > \frac{1}{C}$$

$$\frac{R^2}{4L} = \frac{600^2\Omega^2}{100.10^{-6}.H} = \frac{3600000}{100.10^{-6}}\frac{\Omega^2}{H} = 36.10^8\,\frac{m^2 Kg}{s^4A^2}$$

$$\frac{1}{C} = \frac{1}{0{,}01\mu F} = \frac{1}{10^{-2}10^{-6}\,F} = 10^8\,F^{-1} = 10^8\,\frac{m^2Kg}{s^4A^2}$$

$\therefore$ Es sobreamortiguado.

NOTA

b)

$$q(t) = A.e^{\beta_1 t} + B e^{\beta_2 t} \Longrightarrow q(0) = Q_0 = A + B$$

$$i(t) = A\beta_1 e^{\beta_1 t} + B\beta_2 e^{\beta_2 t} \Longrightarrow i(0) = A\beta_1 + B\beta_2 = i_0$$

$$\begin{cases} A + B = Q_0 & \Longrightarrow \quad A = Q_0 - B \quad (\alpha) \\ A\beta_1 + B\beta_2 = i_0 \end{cases}$$

$\hookrightarrow$ Reemplazo (α):

$$(Q_0 - B)\beta_1 + B\beta_2 = i_0$$

$$B(\beta_2 - \beta_1) = i_0 - Q_0\beta_1$$

$$\boxed{B = \frac{i_0 - Q_0\beta_1}{\beta_2 - \beta_1}}$$

$$\Longrightarrow \boxed{A = Q_0 - \frac{i_0 - Q_0\beta_1}{\beta_2 - \beta_1}}$$

Estos son los A,B que yo planteé, no los que me piden

NOTA

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Entonces:

$$V = Ae^{\beta_1 T} + Be^{\beta_2 T} + LA\beta_1^2 \cdot e^{\beta_1 T} + LB\beta_2^2 e^{\beta_2 T} + RA\beta_1 e^{\beta_1 T} + RB\beta_2 e^{\beta_2 T}$$

$$= e^{\beta_1 T}\underbrace{[A + LA\beta_1^2 + RA\beta_1]}_{\text{Este es el "A" que se pide}} + e^{\beta_2 T}\underbrace{[B + LB\beta_2^2 + RB\beta_2]}_{\text{Este es el "B" que se pide}}$$

Luego:

$$\text{"}A\text{"} = A[1 + L\beta_1^2 + R\beta_1] = \left(Q_0 - \frac{i_0 - Q_0\beta_1}{\beta_2 - \beta_1}\right) \cdot [1 + L\beta_1^2 + R\beta_1]$$

$$\text{"}B\text{"} = B[1 + L\beta_2^2 + R\beta_2] = \left(\frac{i_0 - Q_0\beta_1}{\beta_2 - \beta_1}\right) \cdot [1 + L\beta_2^2 + R\beta_2]$$

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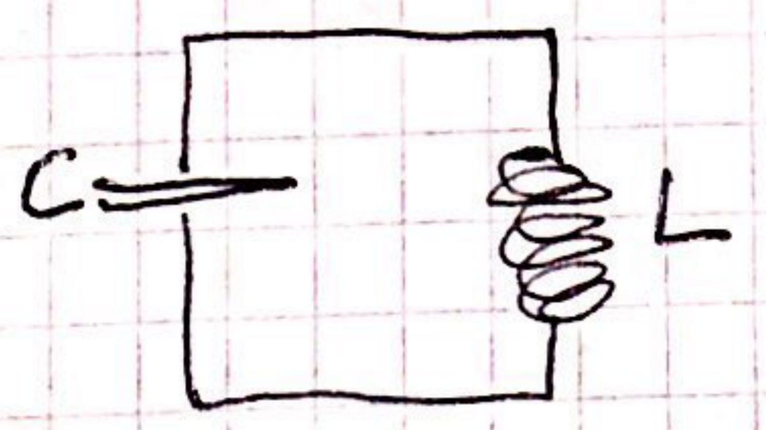
7 a)



No hay resistencia $\Rightarrow$ no hay termino disipativo $\Rightarrow$ $w = \sqrt{\frac{1}{LC}}$

$L.C = 2,81 . 9 \text{ mH} . \text{pF} = 25,29 . 10^{-3} H . 10^{-12} F = 25,29 . 10^{-15} s^2$

$\Rightarrow \quad w = \sqrt{\frac{10^{15}}{25,29} \cdot \frac{1}{s}} = 6288188 \frac{1}{s}$

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b) primero calculemos q, i en función del tiempo. Tenemos:

$$0 = V_c - V_L = \frac{q}{C} + L \cdot \frac{di}{dt} = \frac{q}{C} + L \cdot \frac{d^2 q}{dt^2}$$

Supongo que $q(t) = A \cdot \sin(\omega t + \phi) + B\cos(\omega t + \phi)$

$$\Rightarrow q''(t) = -A\omega^2 \sin(\omega t + \phi) - B\omega^2 \cos(\omega t + \phi)$$

Entonces:

$$0 = \frac{1}{C}\left[A\sin(\omega t + \phi) + B\cos(\omega t + \phi)\right] + L\left[-A\omega^2 \sin(\omega t + \phi) - B\omega^2 \cos(\omega t + \phi)\right]$$

$$0 = A\sin(\omega t + \phi)\left[\frac{1}{C} - L\omega^2\right] + B\cos(\omega t + \phi)\left[\frac{1}{C} - L\omega^2\right]$$

Solución

$$\begin{cases} A = 0 \\ \frac{1}{C} - L\omega^2 = 0 \end{cases} \Rightarrow \omega = \sqrt{\frac{1}{LC}}$$

Entonces

$$q(t) = B \cdot \cos(\omega t + \phi)$$

Dado que $q(0) = q_0 = B\cos(\phi) \Rightarrow B = q_0 \wedge \phi = 0$

Por lo tanto $q(t) = q_0 \cos(\omega t)$. Entonces

$$-1 \leq \cos(\omega t) \leq 1 \qquad \forall t$$

$$-q_0 \leq q_0 \cdot \cos(\omega t) \leq q_0 \Rightarrow q_0 \text{ es el máx}$$

$C = \frac{q_0}{V} \Rightarrow Q_0 = CV = 9_{pF} \cdot 12V = 9 \cdot 12 \cdot 10^{-12}\ F \cdot V = 108 \cdot 10^{-10}\ C$

Además:

$i(t) = -q_0 w \sin(wt + \phi) \Rightarrow -q_0 w = i_0$ es la corriente máx.

$i_0 = 6288188 \cdot 1{,}08 \cdot 10^{-10}\ \frac{C}{s} = 0{,}00067\ A$

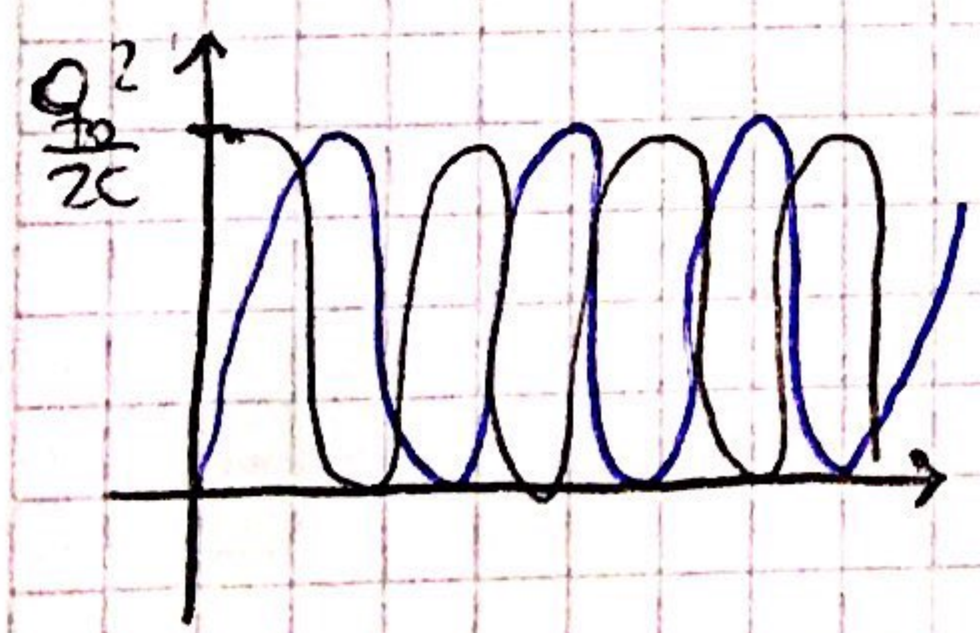
c)

$$U_{total} = U_c + U_L = \frac{1}{2}\frac{q^2(t)}{C} + \frac{1}{2}L \cdot i(t)^2 = \frac{q_0^2 \cos^2(wt)}{2C} + \frac{L q_0^2 w^2 \sin^2(wt)}{2}$$

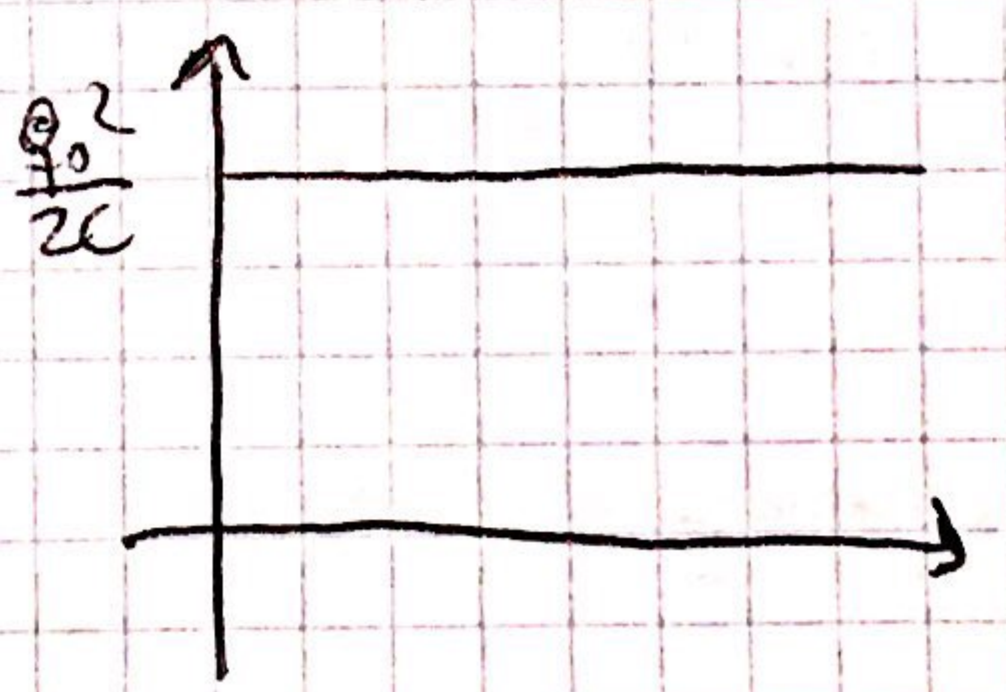
$w^2 = \frac{1}{LC}$

$$= \frac{q_0^2 \cos^2(wt)}{2C} + \frac{q_0^2 \sin^2(wt)}{2C} = \frac{q_0^2}{2C}$$

U_c U_L

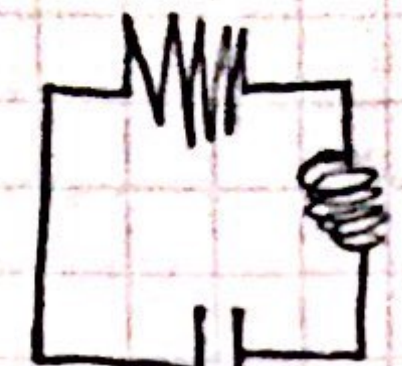


U_{total}



8

a)



$$0 = R.i + L.\frac{di}{dt} + \frac{q}{c}$$

$$0 = R\frac{dq}{dt} + L.\frac{d^2q}{dt^2} + \frac{q}{c} \qquad (1)$$

Propongo $q(t) = A.e^{at}.\cos(wt+\phi)$

$$q'(t) = A\left[ae^{at}\cos(wt+\phi) - we^{at}\sin(wt+\phi)\right]$$

$$= Aae^{at}\cos(wt+\phi) - Awe^{at}\sin(wt+\phi)$$

$$q''(t) = Aa\left[ae^{at}\cos(wt+\phi) - we^{at}\sin(wt+\phi)\right] - Aw\left[ae^{at}\sin(wt+\phi) + we^{at}\cos(wt+\phi)\right]$$

$$= \left[Aa^2 - Aw^2\right]\cos(wt+\phi) - 2Aaw\,e^{at}\sin(wt+\phi)$$

Luego reemplazo en (1)

$$0 = e^{at}\cos(wt+\phi).\left[RAa + LAa^2 - LAw^2 + \frac{A}{c}\right] + e^{at}\sin(wt+\phi)\left[-RAw - 2AawL\right]$$

Solución

$$\begin{cases} RAa + LAa^2 - LAw^2 + \frac{A}{c} = 0 & \alpha \\ -RAw - 2AawL = 0 & \beta \end{cases}$$

Por (α):

$$-RwA - 2AawL = 0 \Rightarrow Rw + 2awL = 0 \Rightarrow R + 2aL = 0 \Rightarrow a = -\frac{R}{2L}$$

Luego reemplazo en (β):

NOTA

HOJA N°

FECHA

$$Ra + La^2 - L\omega^2 + \frac{1}{C} = 0$$

$$L\omega^2 = Ra + La^2 + \frac{1}{C}$$

$$\omega^2 = \frac{Ra}{L} + a^2 + \frac{1}{LC}$$

Reemplazo $\rightarrow$

$$\omega^2 = -\frac{R^2}{2L^2} + \frac{R^2}{4L^2} + \frac{1}{LC}$$

$$\omega^2 = -\frac{R^2}{4L^2} + \frac{1}{LC}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Entonces:

$$\omega = \sqrt{\frac{1}{(2,2)(1,8)(10^{-12}).\ HF} - \frac{(7,6)\ \Omega}{4(2,2)10^{-6}H}} \approx \sqrt{0,25.\ 10^{12} - 0,86.\ 10^{6}} \cdot \frac{1}{s}$$

$$= 4,9.10^{5} \cdot \frac{1}{s}$$

b) $q(t) = A.e^{-\frac{R}{2L}\cdot T}\cos(\omega T + \phi)$

Siendo q_0 la carga inicial:

$$q(0) = q_0 = A.\cos(\phi) \rightsquigarrow \text{Tomo } A = q_0,\ \phi = 0$$

Luego:

$$q(t) = q_0.e^{-\frac{R}{2L}\cdot T}\cos(\omega T)$$

$$i(t) = \frac{dq}{dt} = q_0.\omega.\ e^{-\frac{R}{2L}\cdot T}\cos(\omega T) = i_0.e^{-\frac{R}{2L}}\cos(\omega T)$$

Lo que nos pide el ej se lo llama "la envolvente" y tiene que ver solo con el prefactor exponencial:

NOTA

$i(T) = \frac{i_0}{2}$ cuando $i_0 e^{-\frac{R}{2L}T} = \frac{i_0}{2}$

$$\iff e^{-\frac{R}{2L}T} = \frac{1}{2}$$

$$\iff -\frac{R}{2L}T = \ln\left(\frac{1}{2}\right)$$

$$\iff \boxed{T = \frac{-2L}{R}\ln\left(\frac{1}{2}\right)}$$

c)

$$\frac{1}{LC} = \frac{R^2}{4L^2}$$

$$R^2 = \frac{4L}{C}$$

$$R = 2\sqrt{\frac{L}{C}} = 2\sqrt{\frac{2,2}{1,8}}\ \Omega = 2,21\ \Omega$$

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