PRACTICO 1

1

i $(-1+i)(3-2i)=-3+2i+3i-2i^2=-3+5i+2=-1+5i$

ii $\frac{3+i}{3-4i}-\frac{2-i}{8i}=(3+i).\frac{3+4i}{3^2+4^2}-(2-i).\frac{-8i}{8^2}$

$z^{-1}=\frac{\bar{z}}{|z|^2}$

$=\frac{9+12i+3i+4i^2}{25}-\frac{-16i+8i^2}{64}=\frac{9-4+15i}{25}-\frac{-8-16i}{64}$

$=\frac{5+15i}{25}+\frac{8+16i}{64}=\frac{1}{5}+\frac{3i}{5}+\frac{1}{8}+\frac{1}{4}i=\frac{13}{40}+\frac{17}{40}i$

iii

$\frac{1}{(1-i)(2-i)}=\frac{1}{2-i-2i+i^2}=\frac{1}{2-3i-1}=\frac{1}{1-3i}=\frac{1+3i}{1^2+3^2}$

$=\frac{1}{10}+\frac{3}{10}i$

iv

$i^{13}-i^9+1=i^{4.3+1}-i^{4.2+1}+1=(i^4)^3.i-(i^4)^2.i+1=1^3i-1^2i+1$

$=i-i+1=1$

v $(1-i)^4=((1-i)^2)^2=(1-2i+i^2)^2=(1-2i-1)^2=(-2i)^2=4i^2=-4$

vi $1-\frac{1}{1+\frac{1}{i}}=1-\frac{1}{1-i}=1-\frac{1+i}{1^2+1^2}=1-\frac{1}{2}-\frac{i}{2}=\frac{1}{2}-\frac{1}{2}i$

VII

• $(2i-1)^2 = 4i^2-4i+1 = -4-4i+1 = -3-4i$

• $\frac{4}{1-i} = 4\cdot\frac{1+i}{1^2+1^2} = 4\cdot\frac{1+i}{2} = 2+2i$

• $\frac{2-i}{1+i} = (2-i)\cdot\frac{1-i}{1^2+1^2} = \frac{(2-i)(1-i)}{2} = \frac{2-2i-i+i^2}{2} = \frac{2-3i-1}{2} = \frac{1}{2}-\frac{3}{2}i$

• $\frac{4}{1-i}+\frac{2-i}{1+i} = 2+2i+\frac{1}{2}-\frac{3}{2}i = \frac{5}{2}+\frac{1}{2}i$

Luego:

$(-3-4i)\cdot\left(\frac{5}{2}+\frac{1}{2}i\right) = -\frac{15}{2}-\frac{3}{2}i-10i-2i^2 = -\frac{15}{2}+2-\frac{3+20}{2}i$

$= -\frac{11}{2}-\frac{23}{2}i$

VIII

$\frac{(2+i)^2}{(6i-(1-2i))^2} = \frac{(2+i)^2}{(4i-1)^2} = \frac{4+2i+i^2}{16i^2-8i+1} = \frac{3+2i}{-15-8i} = (3+2i)\cdot\frac{-15+8i}{15^2+64}$

$= \frac{-45+24i-30i+16i^2}{289} = \frac{-61-6i}{289} = -\frac{61}{289}-\frac{6}{289}i$

IX

• $i^{11} = i^{4\cdot2+3} = (i^4)^2\cdot i^3 = i^3 = -i$

• $i^{20} = i^{4\cdot5} = (i^4)^5 = 1$

Luego:

$3i^{11}+6i^3+\frac{8}{i^{20}}+i^{-7} = -3i-6i+\frac{8}{1}-i = 8-10i$

②

i) Sea $z=(a,b)$, luego $\bar{z}=(a,-b) \Rightarrow \overline{(\bar{z})}=(a,b)$

$\Rightarrow \bar{\bar{z}}=z$

ii) Sea $z_1=(a,b)$, $z_2=(c,d) \Rightarrow \bar{z_1}=(a,-b)$, $\bar{z_2}=(c,-d)$

Luego $\bar{z_1}+\bar{z_2}=(a+c,-b-d)=(a+c,-(b+d))$

Por otro lado:

$z_1+z_2=(a+c,b+d) \Rightarrow \overline{z_1+z_2}=(a+c,-(b+d))=\bar{z_1}+\bar{z_2}$

iii) $\boxed{\begin{matrix} z_1=a+bi \\ z_2=c+di \end{matrix} \Rightarrow \begin{matrix} \bar{z_1}=a-bi \\ \bar{z_2}=c-di \end{matrix}}$

Luego

$\bar{z_1}.\bar{z_2}=(a-bi).(c-di)$

$=\underbrace{(ac-bd)-i(da+bc)}_{(\alpha)}$

Además:

$z_1.z_2=(a+bi).(c+di)=(ac-bd)+i(ad+bc)$

$\Rightarrow \overline{z_1.z_2}=(ac-bd)-i(ad+bc)=\bar{z_1}.\bar{z_2}$ por (α)

iv) $z=a+bi \Rightarrow |z|=\sqrt{a^2+b^2}$

$\bar{z}=a-bi \Rightarrow |\bar{z}|=\sqrt{a^2+(-b)^2}=\sqrt{a^2+b^2}$

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V

$z = a+bi$, $\overline{z} = a-bi$

$$z.\overline{z} = (a+bi)(a-bi) = a^2 - abi + abi + b^2 = a^2+b^2 = \sqrt{a^2+b^2}^2 = |z|^2$$

VI

$$z^{-1} \stackrel{?}{=} \frac{\overline{z}}{|z|^2}$$

Sea $z = a+bi$, $z^{-1} = \frac{1}{a+bi} = \frac{1}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{a-bi}{(a+bi)(a-bi)} = \frac{\overline{z}}{z.\overline{z}}$

$$= \frac{\overline{z}}{|z|^2}$$

ej anterior

VII

sea $z_1 = a+bi$, $z_2 = c+d.i$. Luego:

$|z_1| = \sqrt{a^2+b^2}$

$|z_2| = \sqrt{c^2+d^2}$

$|z_1|.|z_2| = \sqrt{(a^2+b^2)(c^2+d^2)}$

$z_1.z_2 = (ac-bd) + i(ad+bc)$

$|z_1.z_2| = \sqrt{(ac-bd)^2 + (ad+bc)^2}$

Luego:

$$|z_1||z_2| = |z_1.z_2|$$

$$\Longleftrightarrow \sqrt{(a^2+b^2)(c^2+d^2)} = \sqrt{(ac-bd)^2 + (ad+bc)^2}$$

$$\Longleftrightarrow (ac)^2 + (ad)^2 + (bc)^2 + (bd)^2 = (ac)^2 - 2acbd + (bd)^2 + (ad)^2 + 2abcd + (bc)^2$$

$$\Longleftrightarrow (ac)^2 + (ad)^2 + (bc)^2 + (bd)^2 = (ac)^2 + (ad)^2 + (bc)^2 + (bd)^2$$

viii Sea $z = a + b.i$

$$a^2 \geq a^2$$
$$a^2 + b^2 \geq a^2$$
$$\sqrt{a^2+b^2} \geq \sqrt{a^2}$$
$$\sqrt{a^2+b^2} \geq |a| \Rightarrow |z| \geq |R_e(z)|$$

$$b^2 \geq b^2$$
$$a^2+b^2 \geq b^2$$
$$\sqrt{a^2+b^2} \geq \sqrt{b^2}$$
$$\sqrt{a^2+b^2} \geq |b| \Rightarrow |z| \geq |I_m(z)|$$

ix

Sea $z = R_e(z) + i.I_m(z)$, $\bar{z} = R_e(z) - i I_m(z)$, luego:

$$z + \bar{z} = R_e(z) + I_m(z)i + R_e(z) - I_m(z)i = 2R_e(z)$$

X

Sea $z = R_e(z) + i.I_m(z)$, $\bar{z} = R_e(z) - I_m(z)$, luego:

$$z - \bar{z} = R_e(z) + i.I_m(z) - R_e(z) + i.I_m(z) = 2i.I_m(z)$$

xi

$$z^n = 1 \iff z.\cdots.z = 1$$
$$\iff \overline{z.\cdots.z} = 1$$
$$\iff \bar{z}.\cdots.\bar{z} = 1$$
$$\iff \bar{z}^n = 1$$

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③ i

Sea $z=a+bi$, $w=c+di$. Luego:

$|z|=\sqrt{a^2+b^2}$, $|w|=\sqrt{c^2+d^2}$

$|z|+|w|=\sqrt{a^2+b^2}+\sqrt{c^2+d^2}$

$|z+w|=\sqrt{(a+c)^2+(b+d)^2}$

Luego:

$$|z|+|w|\geq|z+w| \iff (|z|+|w|)^2\geq|z+w|^2$$
$$\iff a^2+b^2+2\sqrt{(a^2+b^2)(c^2+d^2)}+c^2+d^2\geq(a+c)^2+(b+d)^2$$
$$\iff 2\sqrt{(a^2+b^2)(c^2+d^2)}\geq 2ac+2bd$$
$$\iff (a^2+b^2)(c^2+d^2)\geq(ac)^2+2acbd+(bd)^2$$
$$\iff (ac)^2+(ad)^2+(bc)^2+(bd)^2\geq(ac)^2+(bd)^2+2abcd$$
$$\iff (ad)^2+(bc)^2\geq 2abcd$$
$$\iff (ad)^2-2abcd+(bc)^2\geq 0$$
$$\iff (ad-bc)^2\geq 0$$

$\hookrightarrow$ Verdadero $\forall a,b,c,d\in\mathbb{R}$

ii

Si $r=0$

$w=0 \Rightarrow |z|\leq|z|$ Verdadero

Si $r\neq 0$

de la demostración anterior

$a=r_1\cos(\theta_1)$ $c=r_2\cos(\theta_2)$

$b=r_1\sin(\theta_1)$ $d=r_2\sin(\theta_2)$

queremos que $(ad-bc)^2=0$:

$$\Longleftrightarrow r_1 r_2 \cos(\theta_1)\sin(\theta_2) - r_1 r_2 \sin(\theta_1)\cos(\theta_2) = 0$$

$$\Longleftrightarrow \cos(\theta_1)\sin(\theta_2) - \sin(\theta_1)\cos(\theta_2) = 0$$

$$\Longleftrightarrow \sin(\theta_2-\theta_1) = 0$$

$$\Longleftrightarrow \theta_2-\theta_1 = 0+2k\pi \ \vee \ \theta_2-\theta_1 = \pi + 2k\pi$$

$$\Longleftrightarrow \theta_2 = \theta_1 + 2k\pi \ \vee \ \theta_2 = (\theta_1+\pi) + 2k\pi$$

Es decir que z, w pertenecen a la misma recta.

iii

Prueba por inducción

Caso base: $n=1$

$|z_1| \geq |z_1|$ ✓

Paso inductivo

$$\sum_{i=1}^{k}|z_i| \geq \left|\sum_{i=1}^{k} z_i\right| \overset{?}{\Longrightarrow} \sum_{i=1}^{k+1}|z_i| \geq \left|\sum_{i=1}^{k+1} z_i\right|$$

$$\sum_{i=1}^{k+1}|z_i| = \sum_{i=1}^{k}|z_i| + |z_{k+1}| \underset{\text{Por H.I.}}{\geq} \left|\sum_{i=1}^{k} z_i\right| + |z_{k+1}|$$

Por otro lado:

$$\left|\sum_{i=1}^{k+1} z_i\right| = \left|\sum_{i=1}^{k} z_i + z_{k+1}\right| \overset{\text{Por H.I.}}{\leq} \left|\sum_{i=1}^{k} z_i\right| + |z_{k+1}|$$

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$$\Longrightarrow \sum_{i=1}^{K+1}|z_i| \geq \left|\sum_{i=1}^{K} z_i + z_{K+1}\right| = \left|\sum_{i=1}^{K+1} z_i\right|$$

4

Sea $z = a+bi$, $w = c+di$.

$$|z| = \sqrt{a^2+b^2},\ |w| = \sqrt{c^2+d^2} \Longrightarrow \big||w|-|z|\big| = \left|\sqrt{c^2+d^2} - \sqrt{a^2+b^2}\right|$$

$$w - z = (c-a) + i(d-b) \Longrightarrow |w-z| = \sqrt{(c-a)^2 + (d-b)^2}$$

Luego:

$$\big||w|-|z|\big| \leq |w-z|$$

$$\Longleftrightarrow \big||w|-|z|\big|^2 \leq |w-z|^2$$

$$\Longleftrightarrow (c^2+d^2) - 2\sqrt{(c^2+d^2)(a^2+b^2)} + (a^2+b^2) \leq (c-a)^2 + (d-b)^2$$

$$\Longleftrightarrow -2\sqrt{(c^2+d^2)(a^2+b^2)} \leq -2ca - 2bd$$

$$\Longleftrightarrow \sqrt{(c^2+d^2)(a^2+b^2)} \geq ac+bd$$

$$\Longleftrightarrow (a^2+b^2)(c^2+d^2) \geq (ac)^2 + 2acbd + (bd)^2$$

$\Downarrow$

Verdadero, lo probamos en el 3.

5

i

$z_1 = z_2 = 0$

$$\Longleftrightarrow z_1^2 = 0 \wedge z_2^2 = 0 \Longleftrightarrow z_1^2 + z_2^2 = 0$$

II Falso. Contraejemplo:

Sea $z_1 = i$, $z_2 = i^2 \Rightarrow z_1^2 + z_2^2 = i^2 + i^4 = -1+1 = 0$

7

$$\operatorname{Arg}(z) = \begin{cases} 0 & a>0,\ b=0 \\ \pi & a<0,\ b=0 \\ \frac{\pi}{2} & a=0,\ b>0 \\ -\frac{\pi}{2} & a=0,\ b<0 \\ \arctan\left(\frac{b}{a}\right) & a>0 \\ \arctan\left(\frac{b}{a}\right)+\pi & a<0,\ b\geq 0 \\ \arctan\left(\frac{b}{a}\right)-\pi & a<0,\ b<0 \end{cases}$$

están en los ejes

IV ó I

II

III

a)

$\operatorname{Arg}(z) = \arctan\left(\frac{72}{65}\right) = 0{,}83$

$|z| = \sqrt{65^2+72^2} = 97$

b)

$\operatorname{Arg}(z) = \arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

$|z| = \sqrt{1^2+\sqrt{3}^2} = 2$

c)

$\operatorname{Arg}(z) = \arctan\left(\frac{1}{-1}\right)+\pi = -\frac{\pi}{4}+\pi = \frac{3\pi}{4}$

$|z| = \sqrt{1^2+1^2} = \sqrt{2}$

d)

$\operatorname{Arg}(z) = \arctan\left(\frac{\sqrt{3}}{1}\right) - \pi = \frac{\pi}{3} - \pi = -\frac{2\pi}{3}$

$|z| = \sqrt{1+3} = 2$

e) $\cos\left(\frac{7\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$Arg(z) = \arctan(1) = \frac{\pi}{4}$

$|z| = \sqrt{\cos^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{4}\right)} = 1$

f)

$z = \frac{-2}{1+\sqrt{3}i} = \frac{-2}{1+\sqrt{3}i} \cdot \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{-2+2\sqrt{3}i}{1+3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$Arg(z) = \arctan\left(\frac{\sqrt{3}}{2} : -\frac{1}{2}\right) + \pi = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$

g)

$z = \frac{i}{-2-2i} = i \cdot \frac{-2+2i}{4+4} = \frac{-2i+2i^2}{8} = -\frac{1}{4} - \frac{1}{4}i$

$Arg(z) = \arctan(1) - \pi = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$

$|z| = \sqrt{\frac{1}{16} + \frac{1}{16}} = \frac{\sqrt{2}}{4}$

h)

Sea $w = \sqrt{3} - i \implies$ $|z| = 2$, $\theta = -\frac{\pi}{6}$

Luego $w = 2 . \left(\cos\left(-\frac{\pi}{6}\right) + i . \sin\left(-\frac{\pi}{6}\right)\right)$

$z = w^6 = 2^6 . \left(\cos\left(6 \cdot -\frac{\pi}{6}\right) + i . \sin\left(6 \cdot -\frac{\pi}{6}\right)\right)$

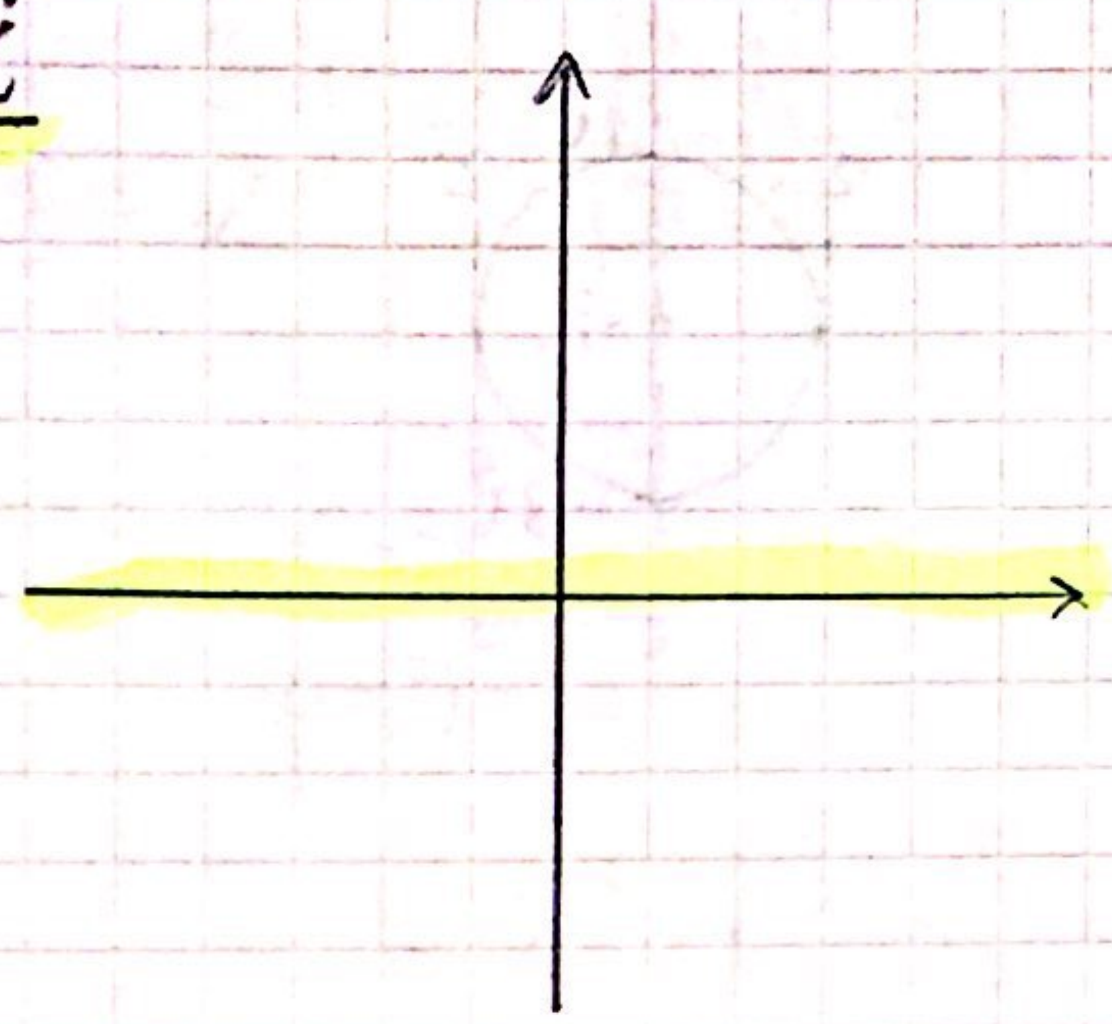
$= 2^6\left(\cos(-\pi) + i . \sin(-\pi)\right)$

$= 2^6(-1) = -2^6$

$\implies |z| = 2^6, \quad Arg(z) = \pi$

8

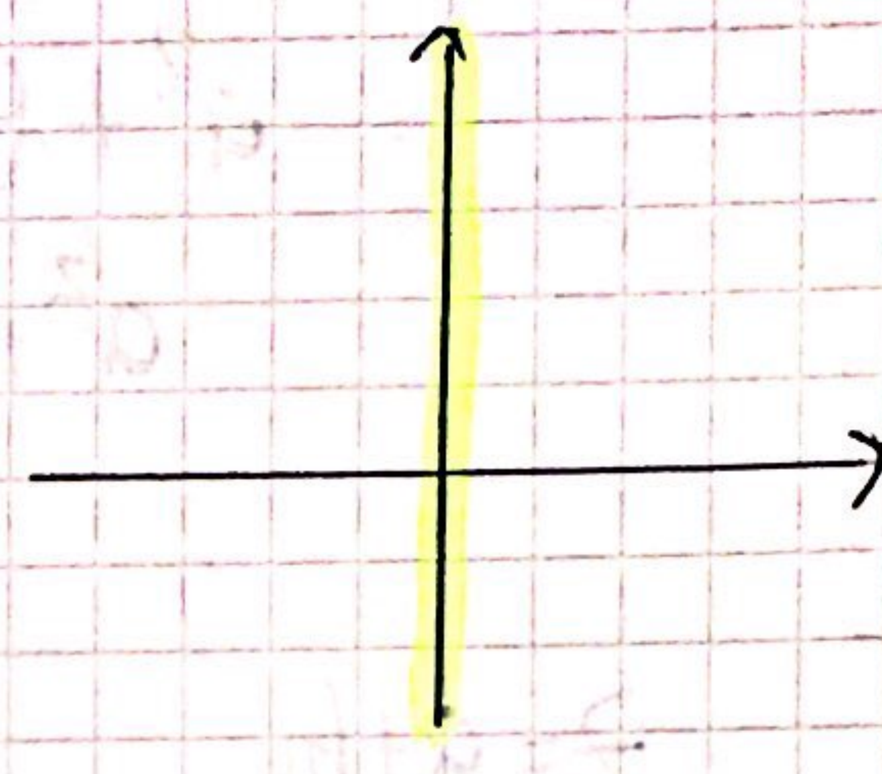
i



$a+bi = a-bi$

$\iff b=-b \iff b=0$

ii

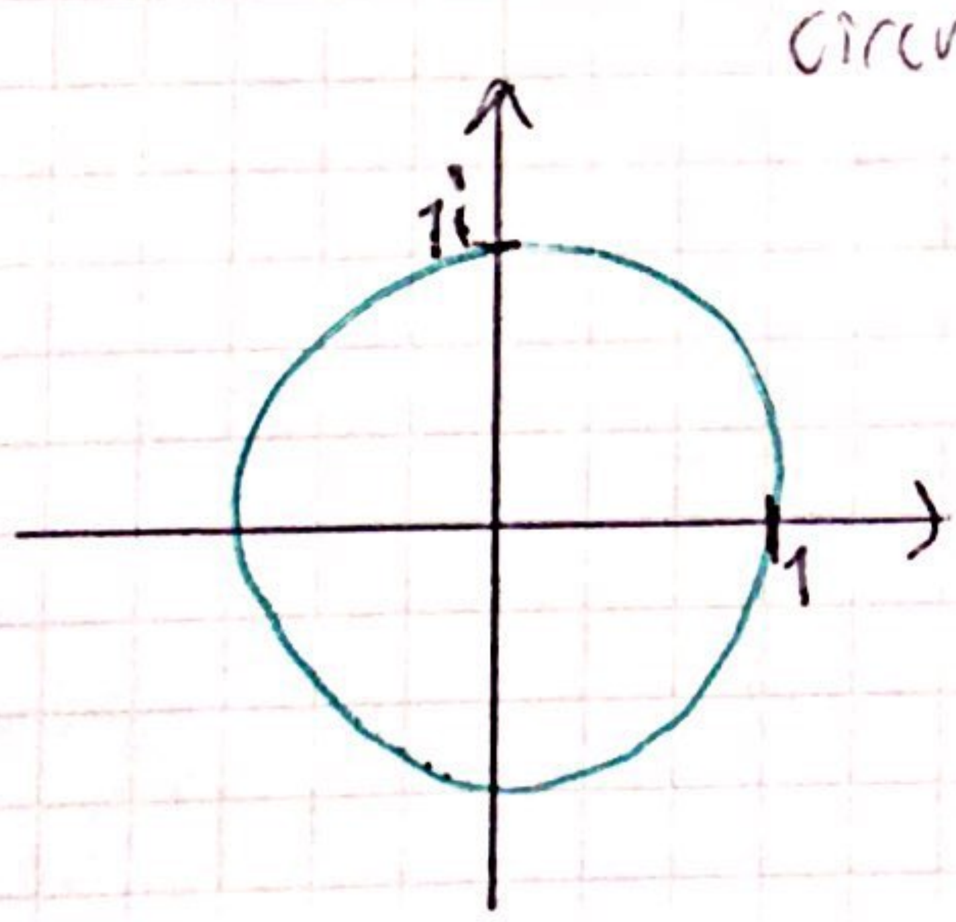


$\overline{a-bi} = -a-bi$

$a=-a \iff a=0$

iii

$z\cdot\bar{z} = a^2-(bi)^2 = \underbrace{a^2+b^2=1}_{\text{circunferencia}}$

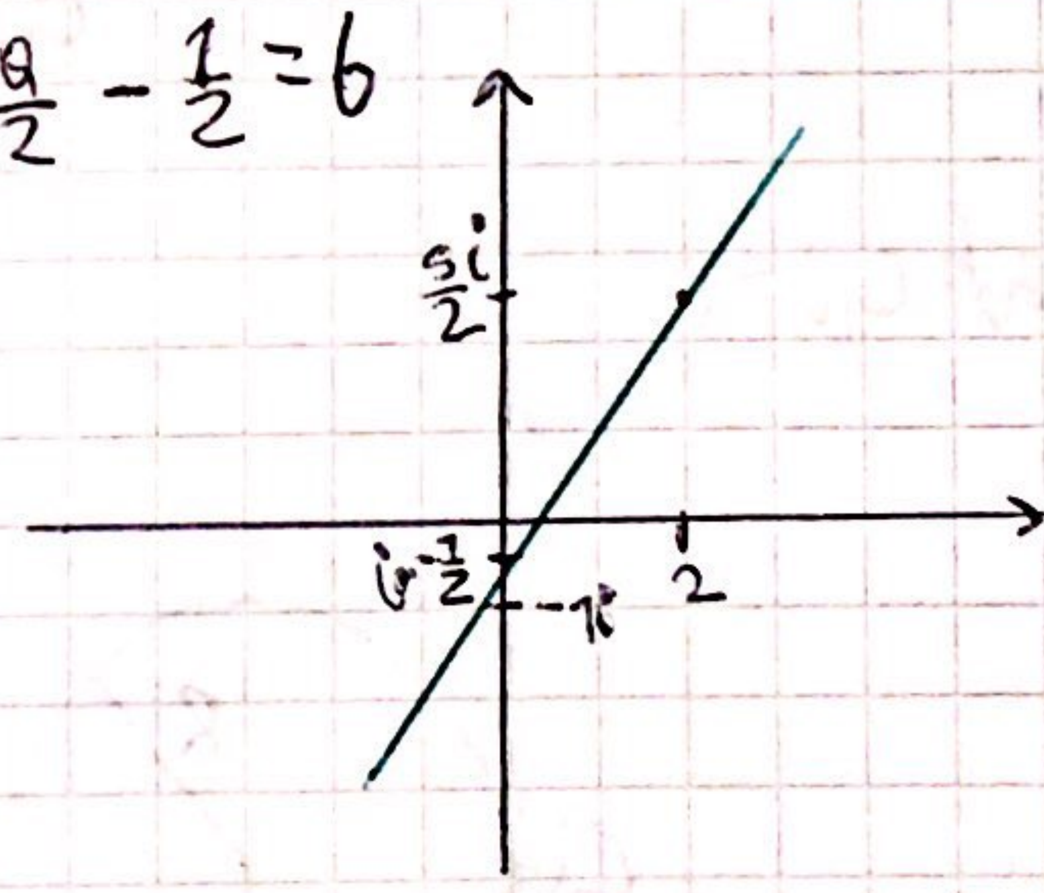


iv

$Re(z)=a, \quad Im(z)=b$

$3a-1=2b$

$\frac{3a}{2}-\frac{1}{2}=b$



v

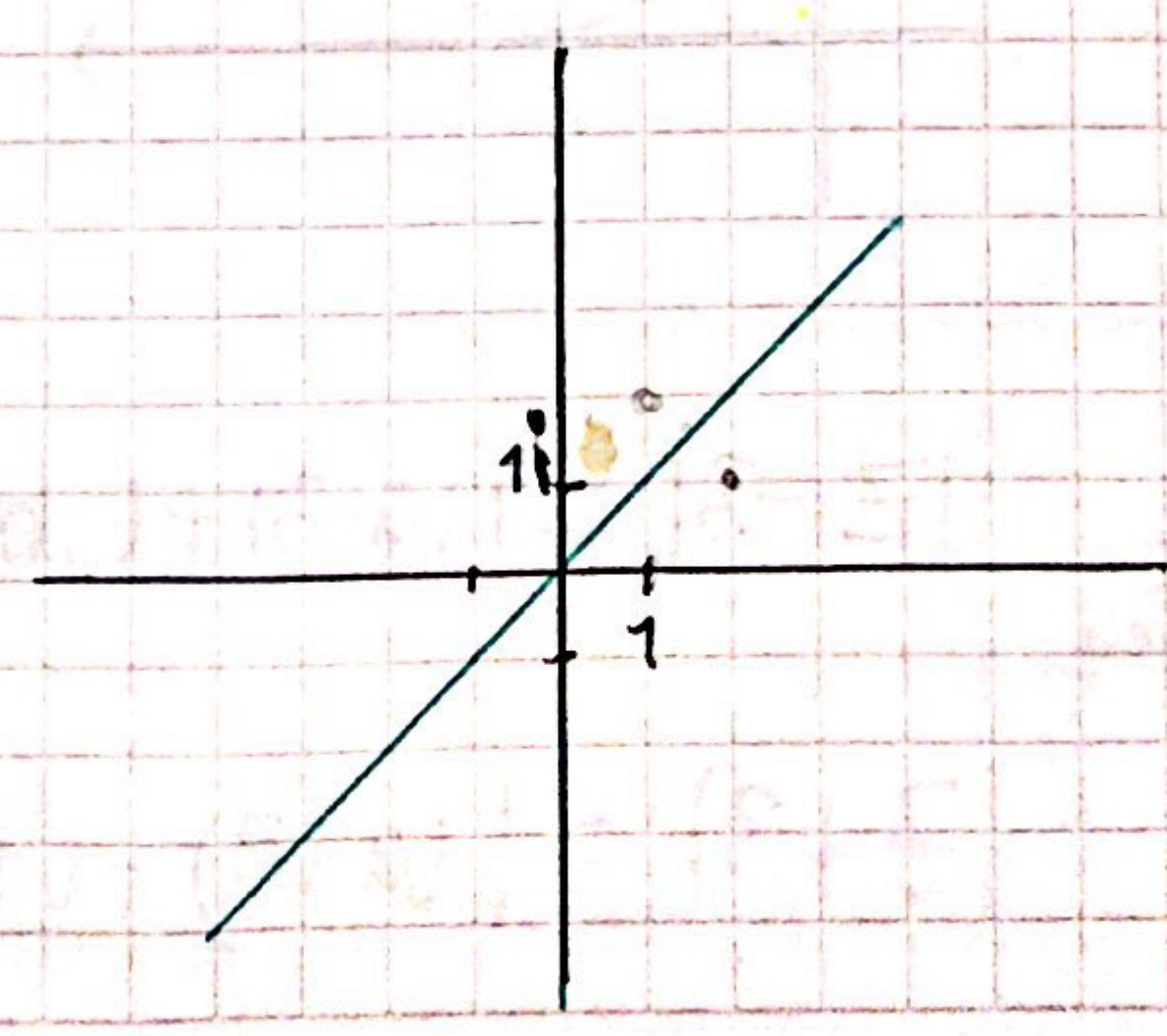
$|(a-1)+b.i| = |a+i.(b-1)|$

$\sqrt{(a-1)^2+b^2} = \sqrt{a^2+(b-1)^2}$

$(a-1)^2+b^2 = a^2+(b-1)^2$

$-2a+1=-2b+1$

$a=b$



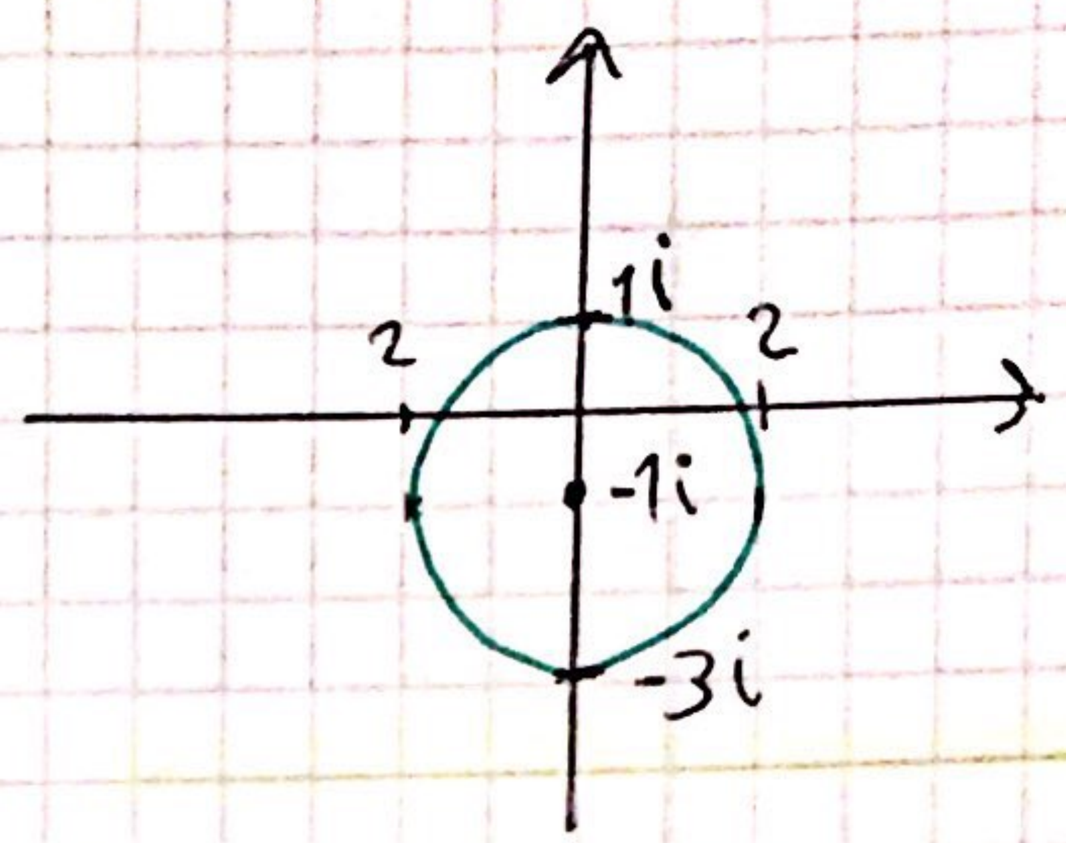
vi

$$\bar{z} - i = a + i(-1-b)$$

$$\Rightarrow |\bar{z} - i| = \sqrt{a^2 + (-1-b)^2} = 2$$

$$a^2 + (-1-b)^2 = 4$$

$$a^2 + (b+1)^2 = 4$$



vii

$$z = a + bi$$

$$w_0 = c + di$$

$$R_e(z.\bar{w}_0) = ac + bd$$

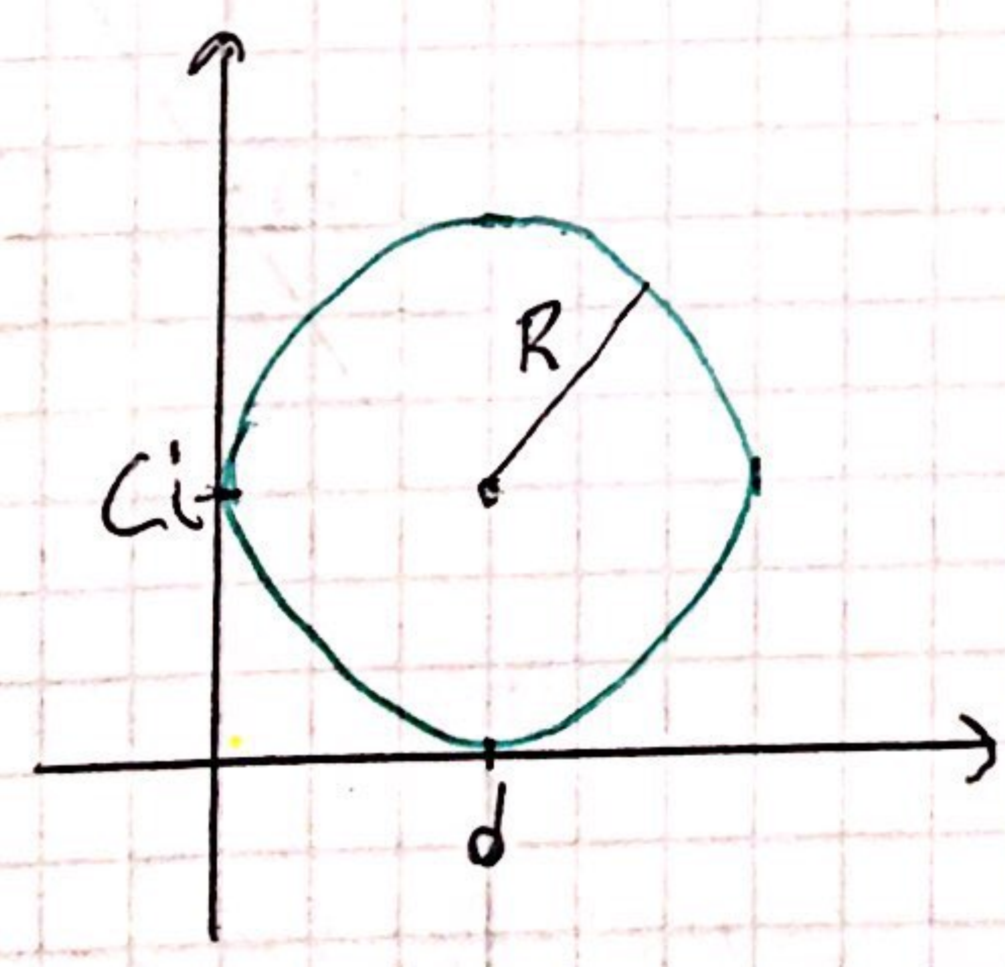
Luego:

$$|z|^2 - 2.R_e(z.\bar{w}_0) + |w_0|^2 = (a^2+b^2) - 2(ac+bd) + (c^2+d^2) = R^2$$

$$a^2 - 2ac + c^2 + b^2 - 2bd + d^2 = R^2$$

$$(a-c)^2 + (b-d)^2 = R^2$$

Suponiendo $c, d > 0$



viii

$$|z-3| = |(a-3) + i.b| = \sqrt{(a-3)^2 + b^2}$$

$$|z+3| = |(a+3) + i.b| = \sqrt{(a+3)^2 + b^2}$$

Luego:

$$\left|\frac{z-3}{z+3}\right| = \frac{|z-3|}{|z+3|} = 2 \iff \frac{|z-3|^2}{|z+3|^2} = 4$$

$$\iff \frac{(a-3)^2+b^2}{(a+3)^2+b^2} = 4$$

$$(a-3)^2+b^2 = 4[(a+3)^2+b^2]$$

$$a^2-6a+9+b^2 = 4[a^2+6a+9+b^2]$$

$$a^2-6a+9+b^2 = 4a^2+24a+36+4b^2$$

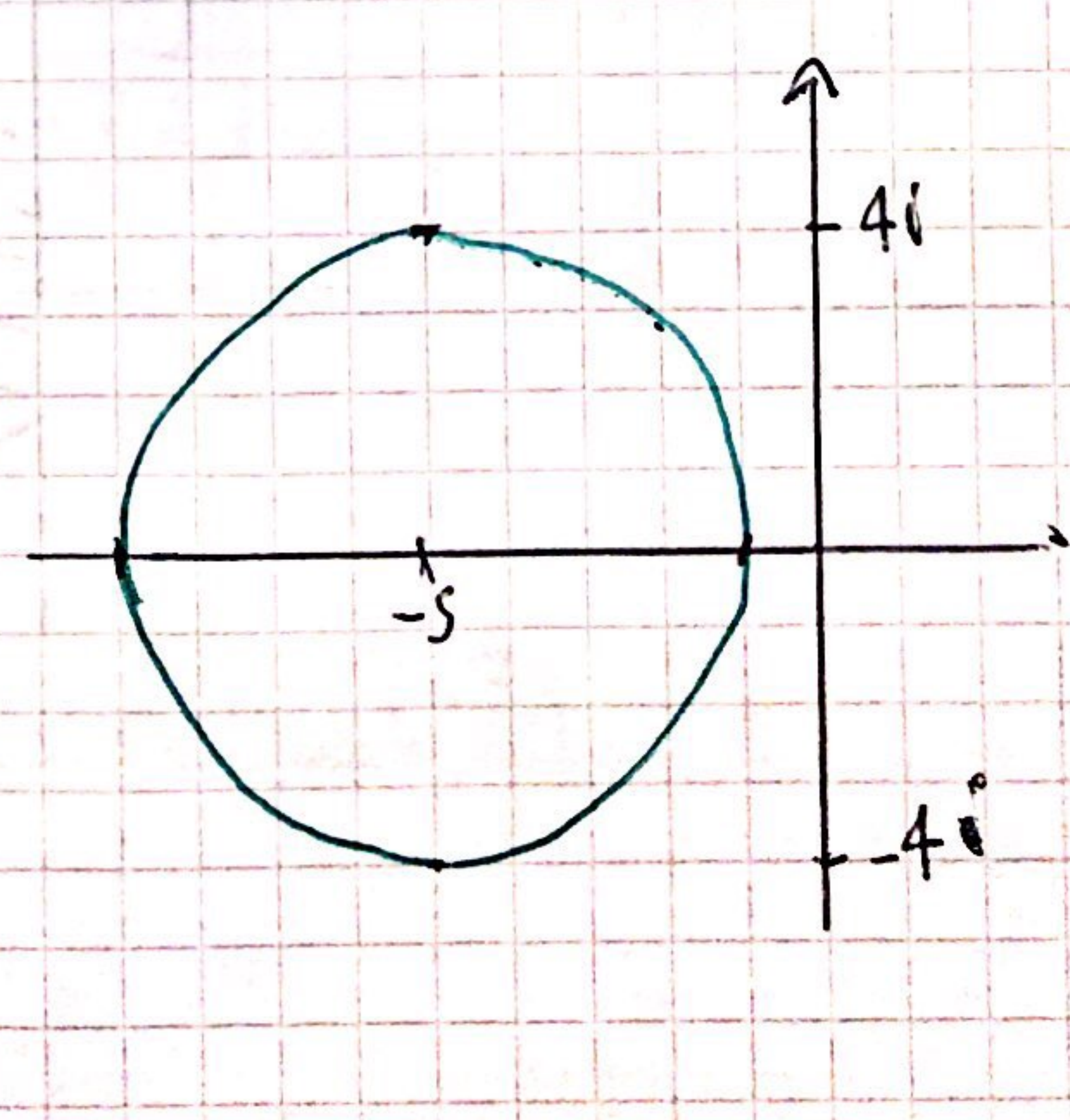
$$0 = 3a^2+30a+27+3b^2$$

$$-27 = 3a^2+30a+3b^2$$

$$-9 = a^2+10a+b^2$$

$$-9+25 = a^2+10a+25+b^2$$

$$16 = (a+5)^2+b^2$$



X

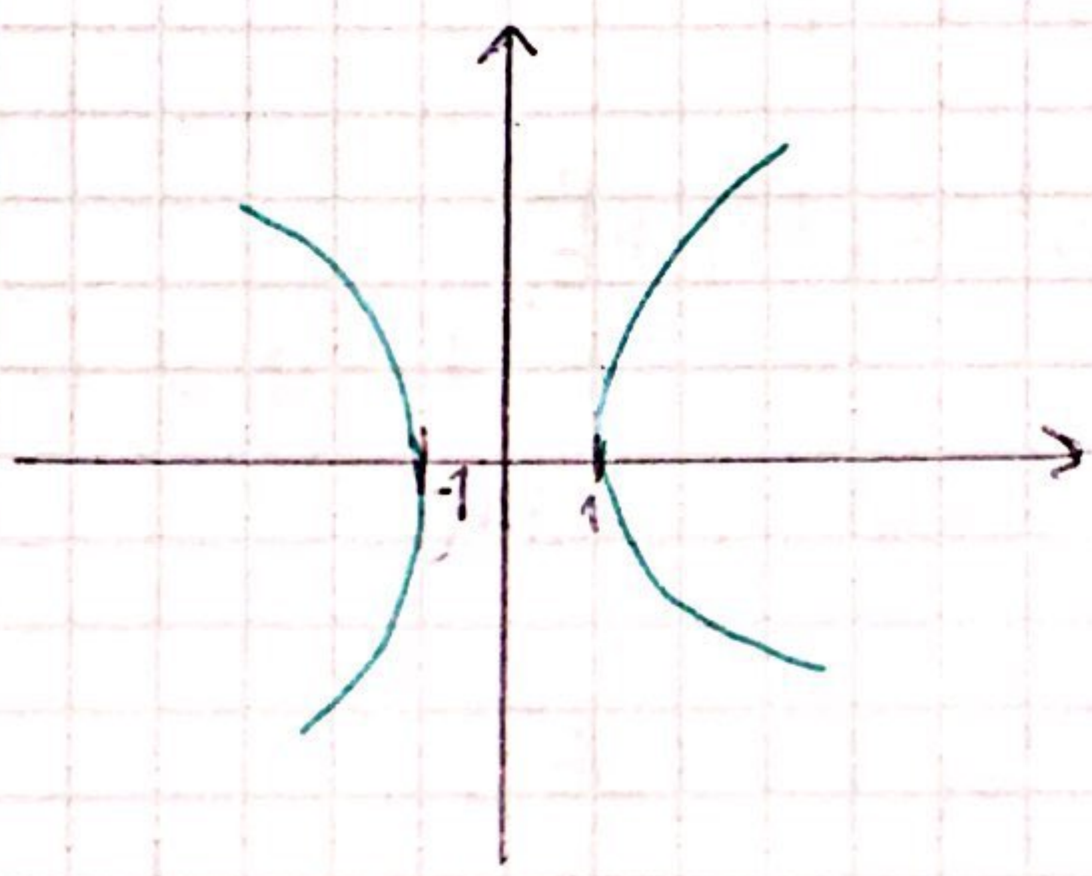
$z = a+bi \Rightarrow \bar{z} = a-bi$

Luego:

$z^2 = (a^2-b^2) + i.(2ab)$

$\bar{z}^2 = (a^2-b^2) - i.2ab$

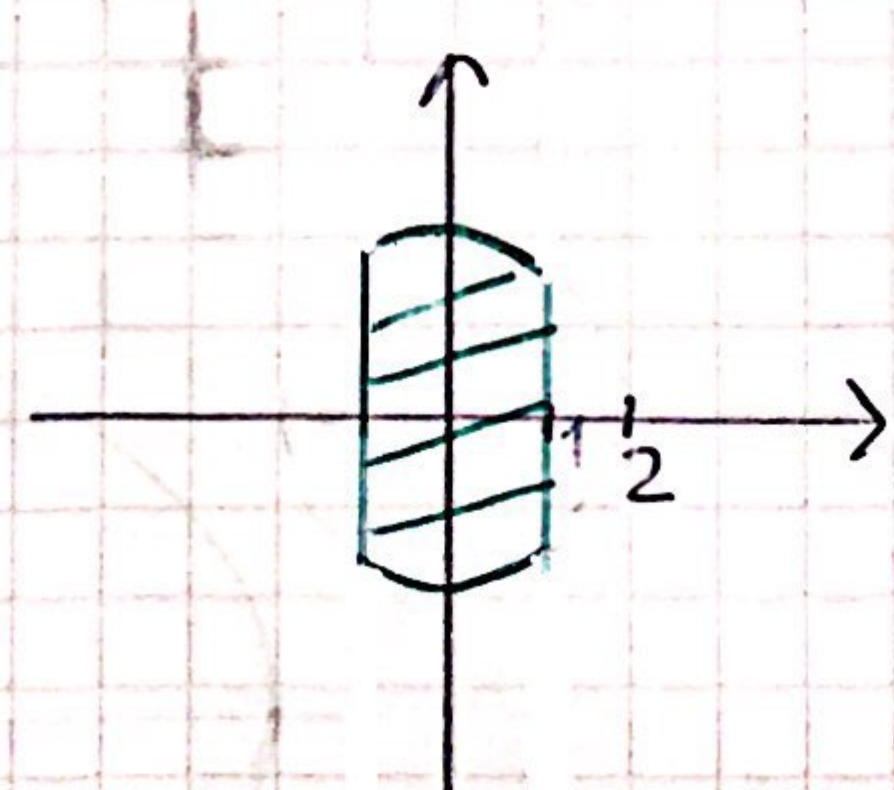
$z^2 + \bar{z}^2 = 2(a^2-b^2) = 2 \Rightarrow a^2-b^2=1$



XI $z=a+bi$

$-1 \leq a \leq 1$

$|z| = \sqrt{a^2+b^2} \leq 2 \Rightarrow a^2+b^2 \leq 4$



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10

i Sea $z = re^{i\theta_1}$, $w = 1-i = \sqrt{2}\, e^{i(\frac{\pi}{4})}$

Luego:

$$z^2 = 1-i$$

$$r^2 e^{i2\theta_1} = \sqrt{2}\, e^{i(-\frac{\pi}{4})}$$

$$\Longleftrightarrow \begin{cases} r^2 = \sqrt{2} \\ 2\theta_1 = -\frac{\pi}{4} + 2k\pi \end{cases} \Longleftrightarrow \begin{cases} r = \sqrt[4]{2} \\ \theta_1 = -\frac{\pi}{8} + k\pi \end{cases}$$

Tomo $k = 1, 2$:

$$\theta_1 = -\frac{\pi}{8} + \pi = \frac{7\pi}{8} \quad , \quad \theta_1 = -\frac{\pi}{8} + 2\pi = \frac{15\pi}{8}$$

$$\therefore\ z_1 = \sqrt[4]{2}\left(\cos\left(\frac{7\pi}{8}\right) + i.\sin\left(\frac{7\pi}{8}\right)\right)$$

$$z_2 = \sqrt[4]{2}\left(\cos\left(\frac{15\pi}{8}\right) + i.\sin\left(\frac{15\pi}{8}\right)\right)$$

ii

$$z = \frac{-2 \pm \sqrt{4-104}}{4} = \frac{-2 \pm \sqrt{-100}}{4} = \frac{-2 \pm 10i}{4} \left\langle \begin{array}{l} -\frac{1}{2} + \frac{5}{2}i \\ -\frac{1}{2} - \frac{5}{2}i \end{array} \right.$$

v

$$z = re^{i\theta_1}$$

$$4-3i = 5e^{i\theta_2} \quad \text{donde} \quad \theta_2 = \arctan\left(-\frac{3}{4}\right)$$

Luego:

$$r^2 e^{i2\theta_1} = 5e^{i\theta_2} \Longleftrightarrow \begin{cases} r^2 = 5 \\ 2\theta_1 = \theta_2 + 2k\pi \end{cases}$$

$$\iff \begin{cases} r = \sqrt{5} \\ \theta_1 = \dfrac{\arctan\left(-\frac{3}{4}\right)}{2} + k\pi \end{cases}$$

Tomo $k = 1, 2$:

$\theta_1 = -0{,}32 + \pi \approx 2{,}81$

$\theta_1 = -0{,}32 + 2\pi \approx 5{,}96$

$\therefore z = \sqrt{5}\left(\cos(2{,}81) + i.\sin(2{,}81)\right) \vee z = \sqrt{5}\left(\cos(5{,}96) + i.\sin(5{,}96)\right)$

vi $\quad z = r e^{i\theta}, \quad 8i = \sqrt{8}\, e^{i\frac{\pi}{2}}$

Luego

$$z^3 = r^3 e^{i3\theta} = \sqrt{8}\, e^{i\frac{\pi}{2}} \iff \begin{cases} r^3 = \sqrt{8} \\ 3\theta = \frac{\pi}{2} + 2k\pi \end{cases} \iff \begin{cases} r = \sqrt[6]{8} = 2 \\ \theta = \frac{\pi}{6} + \frac{2}{3}k\pi \end{cases}$$

Tomando $k = 0, 1, 2$:

$\theta = \frac{\pi}{6} \vee \theta = \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6} \vee \theta = \frac{\pi}{6} + \frac{4\pi}{3} = \frac{9\pi}{6} = \frac{3\pi}{2}$

$$\therefore \begin{aligned} z &= \overset{2}{\sqrt[6]{8}}\left(\cos\left(\tfrac{\pi}{6}\right) + i.\sin\left(\tfrac{\pi}{6}\right)\right) \\ z &= \overset{2}{\sqrt[6]{8}}\left(\cos\left(\tfrac{5\pi}{6}\right) + i.\sin\left(\tfrac{5\pi}{6}\right)\right) \\ z &= \underset{2}{\sqrt[6]{8}}\left(\cos\left(\tfrac{3\pi}{2}\right) + i.\sin\left(\tfrac{5\pi}{6}\right)\right) \end{aligned}$$

Vii $\quad z = r.e^{i\theta}, \qquad -i+1 = \sqrt{2}.e^{i(-\frac{\pi}{4})}$

Luego:

$$z^3 = r^3 e^{i3\theta} = \sqrt{2}\, e^{i(-\frac{\pi}{4})} \iff \begin{cases} r^3 = \sqrt{2} \\ 3\theta = -\frac{\pi}{4} + 2k\pi \end{cases} \iff \begin{cases} r = \sqrt[6]{2} \\ \theta = -\frac{\pi}{12} + \frac{2}{3}k\pi \end{cases}$$

Tomando $k = 1, 2, 3$:

$\theta = -\frac{\pi}{12} + \frac{2\pi}{3} = \frac{7\pi}{12}$

$\theta = -\frac{\pi}{12} + \frac{4\pi}{3} = \frac{15\pi}{12}$

$\theta = -\frac{\pi}{12} + 2\pi = \frac{23\pi}{12}$

$\therefore$

$z = \sqrt[6]{2}\left(\cos\left(\frac{7\pi}{12}\right) + i.\sin\left(\frac{7\pi}{12}\right)\right)$

$z = \sqrt[6]{2}\left(\cos\left(\frac{15\pi}{12}\right) + i.\sin\left(\frac{15\pi}{12}\right)\right)$

$z = \sqrt[6]{2}\left(\cos\left(\frac{23\pi}{12}\right) + i.\sin\left(\frac{23\pi}{12}\right)\right)$

Viii $\quad z = re^{i\theta}, \qquad 6+6i = \sqrt{2}\,6\,e^{i\frac{\pi}{4}}$

$$z^4 - 6 - 6i = 0 \iff z^4 = 6+6i \iff r^4 e^{i4\theta} = \sqrt{2}\,6.e^{i\frac{\pi}{4}}$$

$$\iff \begin{cases} r^4 = 6\sqrt{2} \\ 4\theta = \frac{\pi}{4} + 2k\pi \end{cases} \iff \begin{cases} r = \sqrt[4]{6}\sqrt[8]{2} = \sqrt[8]{36}\sqrt[8]{2} = \sqrt[8]{72} \\ \theta = \frac{\pi}{16} + \frac{k\pi}{2} \end{cases}$$

Tomando $k = 0, 1, 2, 3$:

$\theta = \frac{\pi}{16}$, $\quad \theta = \frac{\pi}{16} + \pi = \frac{17\pi}{16}$

$\theta = \frac{\pi}{16} + \frac{\pi}{2} = \frac{9\pi}{16}$, $\quad \theta = \frac{\pi}{16} + \frac{3\pi}{2} = \frac{25\pi}{16}$

$\therefore\; z = \sqrt[8]{72}\left(\cos\left(\frac{\pi}{16}\right) + i.\sin\left(\frac{\pi}{16}\right)\right)$

$z = \sqrt[8]{72}\left(\cos\left(\frac{9\pi}{16}\right) + i.\sin\left(\frac{9\pi}{16}\right)\right)$

$z = \sqrt[8]{72}\left(\cos\left(\frac{17\pi}{16}\right) + i.\sin\left(\frac{17\pi}{16}\right)\right)$

$z = \sqrt[8]{72}\left(\cos\left(\frac{25\pi}{16}\right) + i.\sin\left(\frac{25\pi}{16}\right)\right)$

ix

Sea $z^2 = U$, luego:

$$U^2 + 3U + 9 = 0 \implies U = \frac{-3 \pm \sqrt{9-49}}{2} = \frac{-3 \pm 27i}{2} \begin{cases} -\frac{3}{2} - \frac{27}{2}i \\ -\frac{3}{2} + \frac{27}{2}i \end{cases}$$

$$\implies \underbrace{z^2 = -\frac{3}{2} - \frac{27}{2}i}_{A} \quad \vee \quad \underbrace{z^2 = -\frac{3}{2} + \frac{27}{2}i}_{B}$$

A

$z = r_1 e^{i\theta}$, $\quad -\frac{3}{2} - \frac{27}{2}i = r_2 . e^{i\theta_2}$ donde $r_2 = \sqrt{\frac{9+27^2}{4}}$

$\theta_2 = \arctan(9) + \pi$

Luego

$$z^2 = r_1^2 e^{i2\theta} = r_2 e^{i\theta_2} \iff \begin{cases} r_1 = \sqrt{r_2} = \sqrt[4]{\frac{9+27^2}{3}} \\ \theta = \frac{\theta_2}{2} + k\pi \end{cases}$$

Tomando $k = 0,1$:

$\theta_a = \frac{\theta_2}{2} \approx 2{,}3$

$\theta_b = \frac{\theta_2}{2} + \pi \approx 5{,}44$

B $z=r_1e^{i\varrho}$, $-\frac{3}{2}+\frac{27}{2}i=r_2e^{i\theta_2}$ donde $r_2=\sqrt{\frac{9+27^2}{4}}$

$\theta_2=\arctan(-9)+\pi$

Luego:

$$z^2=r_1^2e^{i2\varrho}=r_2e^{i\theta_2}\iff\begin{cases}r_1=\sqrt{r_2}=\sqrt[4]{\frac{9+27^2}{4}}\\ \varrho=\frac{\theta_2}{2}+k\pi\end{cases}$$

Tomando $k=1,2$

$$\varrho_1=\frac{\theta_2}{2}+\pi\approx 0{,}84+\pi\approx 3{,}98$$

$$\varrho_2=\frac{\theta_2}{2}+2\pi\approx 0{,}84+2\pi\approx 7{,}12$$

Finalmente:

$$z=\sqrt[4]{\frac{9+27^2}{4}}\left(\cos(\theta_a)+i\sin(\theta_a)\right)$$

$$z=\sqrt[4]{\frac{9+27^2}{4}}\left(\cos(\theta_b)+i\sin(\theta_b)\right)$$

$$z=\sqrt[4]{\frac{9+27^2}{4}}\left(\cos(\varrho_1)+i\sin(\varrho_1)\right)$$

$$z=\sqrt[4]{\frac{9+27^2}{4}}\left(\cos(\varrho_2)+i\sin(\varrho_2)\right)$$

12

$$z=\frac{2c\pm\sqrt{4c^2-4}}{2}=\frac{2c\pm2\sqrt{c^2-1}}{2}=c\pm\sqrt{c^2-1}$$

Caso 1 Sea $c=1 \vee c=-1$:

$$\Rightarrow z=1\pm\sqrt{1-1}\quad\vee\quad z=-1\pm\sqrt{1-1}$$

$$=1\qquad\qquad =-1$$

$$\Rightarrow |z|=1\qquad\qquad \Rightarrow |z|=1$$

Caso 2 $c \in (-1,1)$:

$$z = c \pm \sqrt{c^2-1} = c \pm \sqrt{(-1)(1-c^2)} = c \pm i.\sqrt{1-c^2}$$

$$\Rightarrow |z| = \sqrt{c^2 + (1-c^2)} = \sqrt{1} = 1$$

13) i. Sea $z = a+bi$, $\bar{z} = a-bi$, luego:

$$i.z + 2\bar{z} = -b + ai + 2a - 2bi = (2a-b) + i.(a-2b)$$

Luego

$$i.z + 2\bar{z} = 1 + 2i \iff \begin{cases} 2a-b=1 \\ a-2b=2 \end{cases} \Rightarrow b = 2a-1$$

$$a - 4a + 2 = 2$$

$$-3a = 0$$

$$\boxed{a=0}$$

$$b = -1$$

$$\therefore z = -i$$

ii. Sea $z \neq 0$

$$z = re^{i\theta}, \quad \bar{z} = re^{i(-\theta)}$$

$$\Rightarrow z = \bar{z}^3 \iff re^{i\theta} = r^3 e^{i(-3\theta)}$$

pues $r>0$

$$\iff \begin{cases} r = r^3 \\ \theta = -3\theta + 2k\pi \end{cases} \iff \begin{cases} r = 1 \\ \theta = \frac{k\pi}{2} \end{cases}$$

Luego tomo $k=0,1,2,3$:

$\theta=0$, $\theta=\frac{\pi}{2}$, $\theta=\pi$, $\theta=\frac{3\pi}{2}$

Sea $z=0$

$\bar{z}=0 \Rightarrow z^3=0=z$

Soluciones: $z=0$, $z=e^{i0}=1$, $z=e^{i\frac{\pi}{2}}=i$, $z=e^{i\pi}=-1$, $z=e^{i\frac{3\pi}{2}}=-i$

iii)

Dado que $z\neq 0$:

$i.z^2+\bar{z}.z^{-1}=0 \Longleftrightarrow i.z^3+\bar{z}=0 \Longleftrightarrow i.z^3=-\bar{z}$

Luego sea $z=re^{i(\theta)}$:

$z^3=r^3e^{i(3\theta)}$

$\bar{z}=re^{i(\theta)}$

Dado que $-1=e^{i\pi} \Rightarrow -\bar{z}=(-1).\bar{z}=r.e^{i(\theta+\pi)}$

$i=e^{i\frac{\pi}{2}} \Rightarrow i.z^3=r^3e^{i(\theta+\frac{\pi}{2})}$

Luego:

$iz^3=-\bar{z} \Longleftrightarrow r^3e^{i(3\theta+\frac{\pi}{2})}=re^{i(-\theta+\pi)}$

$$\Longleftrightarrow \begin{cases} r^3=r \\ 3\theta+\frac{\pi}{2}=-\theta+\pi+2k\pi \end{cases} \Longleftrightarrow \begin{cases} r=1 \\ \theta=\frac{\pi}{8}+\frac{k\pi}{2} \end{cases}$$

Luego:

$k=0$, $\theta = \frac{\pi}{8}$

$k=1$, $\theta = \frac{5\pi}{8}$

$k=2$, $\theta = \frac{9\pi}{8}$

$k=3$, $\theta = \frac{13\pi}{8}$

$k=4$, $\theta = \frac{\pi}{8} + 2\pi$ $\rightsquigarrow$ se empiezan a repetir

Soluciones: $z = e^{i\frac{\pi}{8}}$, $z = e^{i\frac{5\pi}{8}}$, $z = e^{i\frac{9\pi}{8}}$, $z = e^{i\frac{13\pi}{8}}$

iv Si $z=0$, es claro que se cumple.

Sea $z \neq 0$:

$$\bar{z}^4 + |z| . z^2 (1-i) = 0 \iff \bar{z}^4 = (i-1)|z| . z^2$$

Sea $z = re^{i\theta}$, luego:

$\bar{z}^4 = r^4 e^{i(-4\theta)}$

$|z| = r$

$z^2 = r^2 e^{i(2\theta)}$

$(i-1) = \sqrt{2}\, e^{i(\frac{3\pi}{4})}$

Por lo tanto

$$\bar{z}^4 = (i-1)|z| . z^2 \iff r^4 e^{i(-4\theta)} = \sqrt{2}\, e^{i(\frac{3\pi}{4})} . r . r^2 . e^{i(2\theta)}$$

$$\iff r^4 e^{i(-4\theta)} = \sqrt{2}\, r^3 . e^{i(2\theta + \frac{3\pi}{4})}$$

$$\Longleftrightarrow \begin{cases} r^4 = \sqrt{2}\, r^3 \\ -4\theta = 2\theta + \frac{3\pi}{4} + 2k\pi \end{cases}$$

$$\Longleftrightarrow \begin{cases} r = \sqrt{2} \\ \theta = -\frac{\pi}{8} - \frac{k\pi}{3} \end{cases}$$

Luego:

$k=-1$, $\theta_1 = -\frac{\pi}{8} + \frac{\pi}{3} = \frac{5\pi}{24}$

$k=-2$, $\theta_1 = -\frac{\pi}{8} + \frac{2\pi}{3} = \frac{13\pi}{24}$

$k=-3$, $\theta_1 = -\frac{\pi}{8} + \pi = \frac{21\pi}{24}$

$k=-4$, $\theta_1 = -\frac{\pi}{8} + \frac{4\pi}{3} = \frac{29\pi}{24}$

$k=-5$, $\theta_1 = -\frac{\pi}{8} + \frac{5\pi}{3} = \frac{37\pi}{24}$

$k=-6$, $\theta_1 = -\frac{\pi}{8} + 2\pi = \frac{45\pi}{24}$

$k=-7$, $\theta_1 = -\frac{\pi}{8} + \frac{7\pi}{3} = \frac{53\pi}{24} = \frac{5\pi}{24} + 2\pi$ no se empiezan a repetir

$\therefore$ Soluciones:

$z = \sqrt{2}\, e^{i\frac{5\pi}{24}}$ $\qquad$ $z = \sqrt{2}\, e^{i\frac{29\pi}{24}}$

$z = \sqrt{2}\, e^{i\frac{13\pi}{24}}$ $\qquad$ $z = \sqrt{2}\, e^{i\frac{37\pi}{24}}$

$z = \sqrt{2}\, e^{i\frac{21\pi}{24}}$ $\qquad$ $z = \sqrt{2}\, e^{i\frac{45\pi}{24}}$

V si $z=0$ es claro que se cumple

Sea $z \neq 0$

Resolvamos $w^2 = z^2$ y veamos cuando alguna de sus soluciones es $-z$.

Tomemos $w = r_2 e^{i\theta_2}$, $z = r_2 e^{i\theta_2}$, luego:

$$w^2 = z^2 \iff r_2^2 e^{i(2\theta_2)} = r_1^2 e^{i(\theta_2)}$$

$$\iff \begin{cases} r_2^2 = r_1^2 \\ 2\theta_2 = 2\theta_1 + 2k\pi \end{cases} \iff \begin{cases} r_2 = r_1 \\ \theta_2 = \theta_1 + k\pi \end{cases}$$

Tomando $k = 0,1$:

$\theta_2 = \theta_1$

$\theta_2 = \theta_1 + \pi$

$$\implies w = \begin{cases} w_1 = r_1 e^{i\theta_1} \\ w_2 = r_1 e^{i(\theta_1 + \pi)} \end{cases}$$

Notar que $-1 = e^{i\pi} \implies -z = (-1).z = e^{i\pi}. r_1 e^{i\theta_1} = r_1 e^{i(\theta_1 + \pi)} = w_2$

$\therefore$ $-z$ es solución de la ecuación $\forall z \neq 0$.

$\therefore$ La ecuación se cumple $\forall z \in \mathbb{C}$

Vi Claramente $z \neq 0$, luego:

resolvamos $w^2 = \frac{z}{\bar{z}}$.

Tomemos $\begin{cases} z = r_1 e^{i(\theta_1)} \\ w = r_2 e^{i(\theta_2)} \end{cases} \implies \bar{z} = r_1 e^{i(-\theta_1)}$

Luego:

$$w^2 = \frac{z}{\bar{z}} \iff r_2^2 e^{i(\theta_2)} = \frac{r_1}{r_1} e^{i(\theta_1 - (-\theta_1))}$$

$$\iff r_2^2 e^{i(\theta_2)} = 1 . e^{i(2\theta_1)}$$

$$\Longleftrightarrow \begin{cases} r_2^2 = 1 \\ 2\theta_2 = 2\theta_1 + 2k\pi \end{cases} \Longleftrightarrow \begin{cases} r_2 = 1 \\ \theta_2 = \theta_1 + k\pi \end{cases}$$

$$\Longrightarrow w = \begin{cases} w_1 = e^{i\theta_1} \\ w_2 = e^{i(\theta_1+\pi)} \end{cases}$$

Por otro lado

$$\frac{z}{|z|} = \frac{r_1}{r_1} e^{i(\theta_1)} = e^{i\theta_1} = w_1$$

$\therefore$ Tiene solución $\forall z \in \mathbb{C} - \{0\}$

14 siendo $x = r\cos(\alpha)$, $y = r\sin(\alpha)$:

$$R(x,y) = (r\cos(\alpha+\theta), r\sin(\alpha+\theta))$$ $\rightarrow$ transformación lineal que rota a un punto en un ángulo θ alrededor del origen

$$= (r[\cos(\alpha).\cos(\theta) - \sin(\alpha)\sin(\theta)], r[\sin(\alpha)\cos(\theta) + \cos(\alpha)\sin(\theta)])$$

$$= (r\cos(\alpha)\cos(\theta) - r\sin(\alpha)\sin(\theta), r\sin(\alpha)\cos(\theta) + r\cos(\alpha)\sin(\theta))$$

$$= (x\cos(\theta) - y\sin(\theta), y\cos(\theta) + x\sin(\theta))$$

Luego:

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} . \begin{pmatrix} x \\ y \end{pmatrix} = (x\cos(\theta) - y\sin(\theta), x\sin(\theta) + y\cos(\theta)) = R(x,y)$$

Veamos que es transformación lineal:

dado $x' = r'\cos(\alpha)$, $y' = r'\sin(\alpha)$,

$R((x,y)+(x',y')) = R(x+x', y+y')$

$= ((x+x')\cos(\theta) - (y+y')\sin(\theta), (y+y')\cos(\theta) + (x+x')\sin(\theta))$

$= (x\cos(\theta) - y\sin(\theta), y\cos(\theta) + x\sin(\theta)) + (x'\cos(\theta) - y'\sin(\theta), y'\cos(\theta) + x'\sin(\theta))$

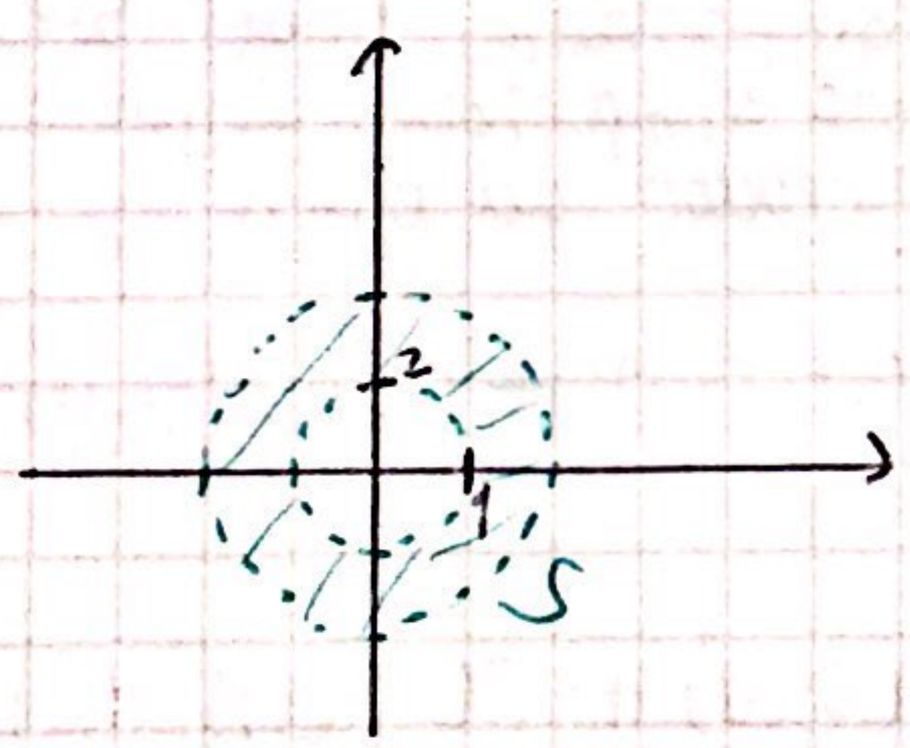
$= R(x,y) + R(x',y')$

$R(\lambda(x,y)) = R(\lambda x, \lambda y) = (\lambda x\cos(\theta) - \lambda y\sin(\theta), \lambda y\cos(\theta) + \lambda x\sin(\theta))$

$= \lambda(x\cos(\theta) - y\sin(\theta), y\cos(\theta) + x\sin(\theta)) = \lambda . R(x,y)$

16

1 $S = \{z \in \mathbb{C} / 1 < |z| < 2\}$



Note que $S = B_2(0) \cap (B_1(0) \cup F_r(B_1(0)))^c$

Por decinición $B_2(0)$ es abierto y $B_1(0) \cup F_r(B_1(0))$ es cerado, por lo tanto su complemento es abierto.

La intersección* de abiertos es abierto

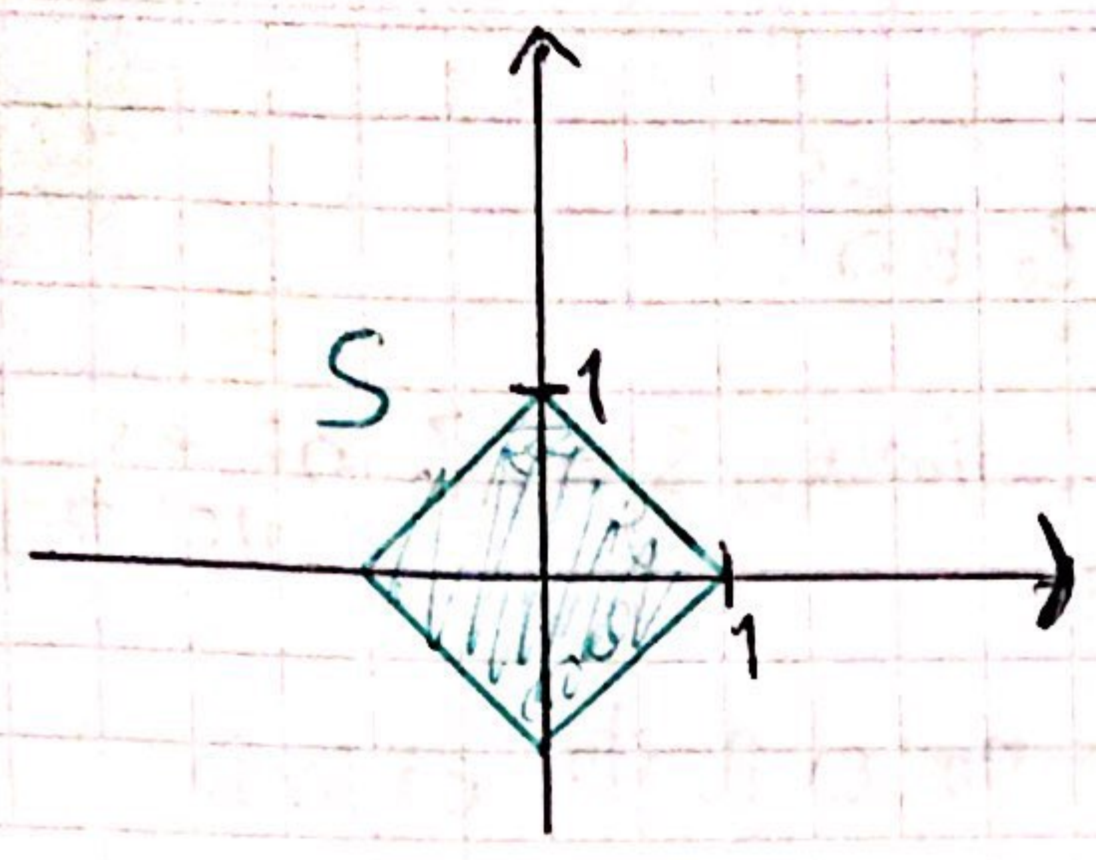
$\Longrightarrow$ S es abierto.

* finita

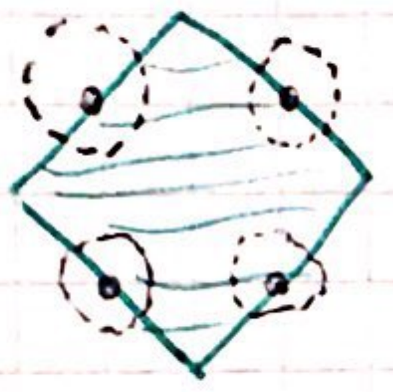
2 $S = \{z \in \mathbb{C} / |x| + |y| \le 1\}$

$|x| + |y| \le 1$

$$|y| \le 1 - |x| \begin{cases} y \le 1 - |x| & \begin{cases} y \le 1 - x & \text{si } x \ge 0 \wedge y \ge 0 \\ y \le 1 + x & \text{si } x < 0 \wedge y \ge 0 \end{cases} \\ y \ge |x| - 1 & \begin{cases} y \ge x - 1 & \text{si } x \ge 0 \wedge y < 0 \\ y \le -x - 1 & \text{si } x < 0 \wedge y < 0 \end{cases} \end{cases}$$



Notemos que dichos 4 semirectas son su frontera:



Pues para cualquier $r>0$ tenemos que:

Para cualquier z_0 contenido en las semirectas

$B_r(z_0)$ No está totalmente contenida en S $\Rightarrow$ No es punto interior

$B_r(z_0) \cap S \neq \emptyset$ $\Rightarrow$ No es punto exterior

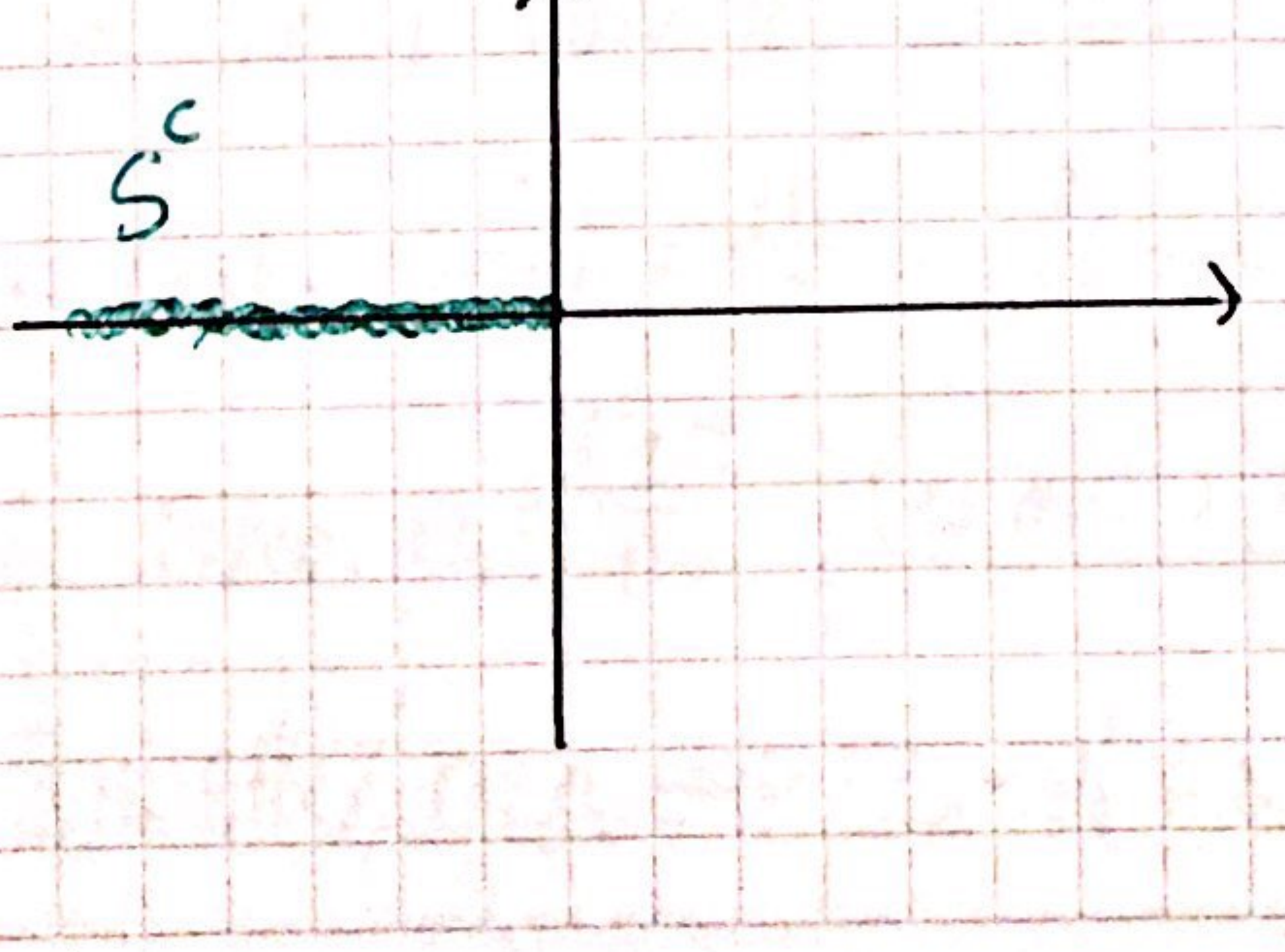
$\Rightarrow$ z_0 es un punto de frontera

$\Rightarrow$ S contiene a todos sus puntos frontera $\Rightarrow$ S es cerrado

3

$S = \{ z \in \mathbb{C} - \{0\} / -\pi < Arg(z) < \pi \}$

Luego $S^c = \{ z \in \mathbb{C} / Arg(z) = \pi \} \cup \{0\}$



Scanned with CamScanner

Notemos que:

S^c

Para todo $r>0$, siendo $z_0 \in S^c$:

$B_r(z_0)$ No está totalmente contenida en $S \Rightarrow z_0$ no es punto interior

$(B_r(z_0) \cap S) \neq \emptyset \Rightarrow z_0$ no es punto exterior

$\therefore z_0$ es punto frontera

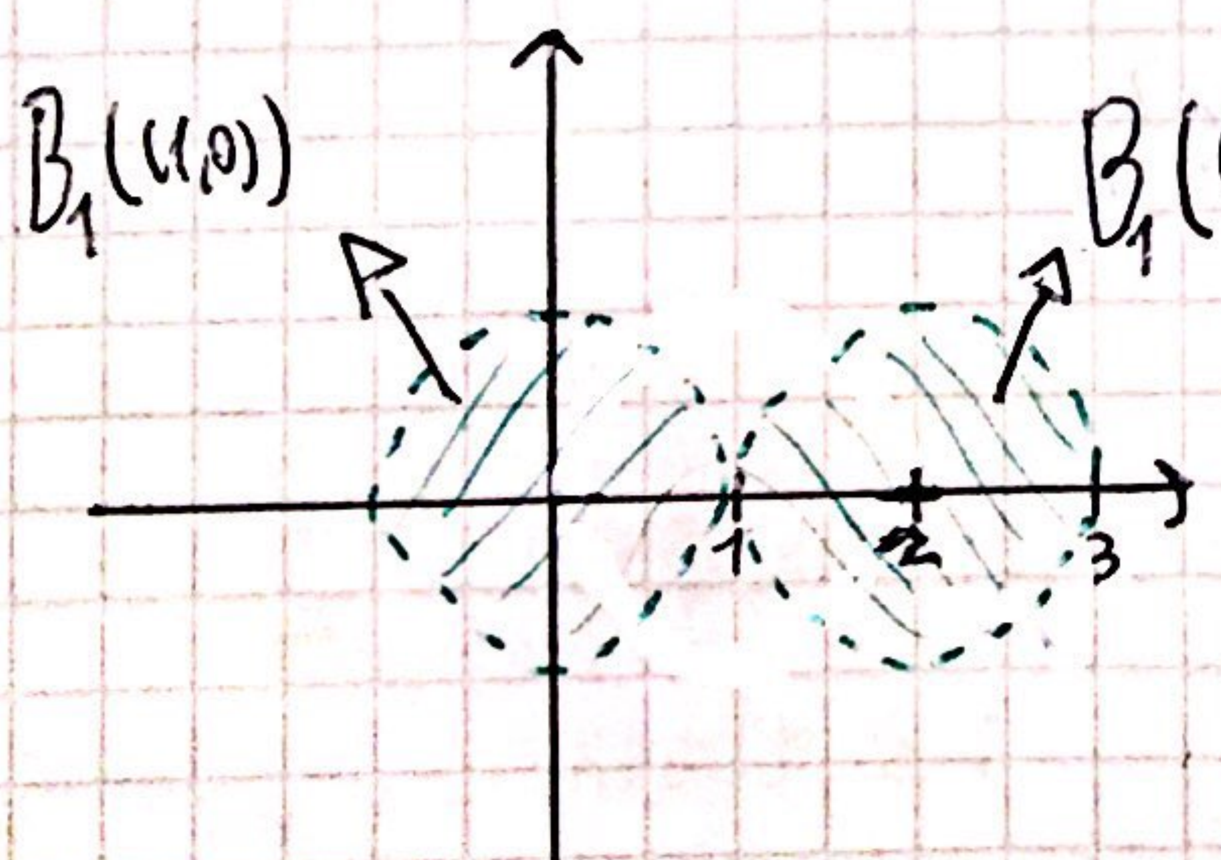
$\Rightarrow S^c = Fr(S^c) \Rightarrow S^c = Cl(S^c) \Rightarrow S^c$ es cerrado

$\Rightarrow S$ es abierto

4 abierto
5 cerrado
6 cerrado
7 abierto
8 ninguno

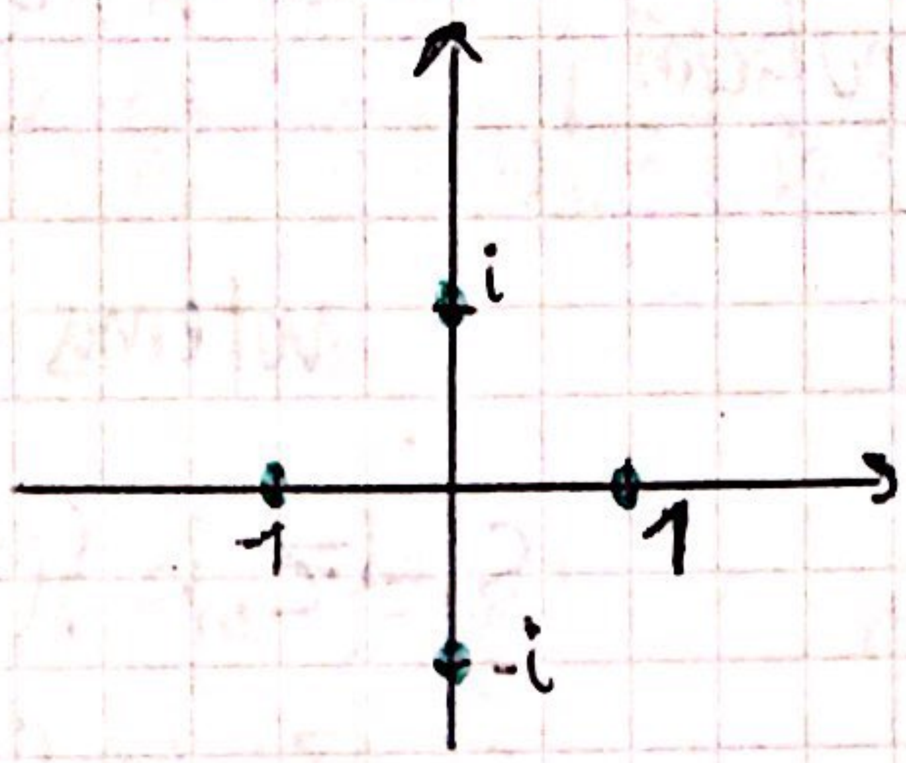
17

$A = \{z \in \mathbb{C} / |z|<1 \vee |z-2|<1\} = B_1((0,0)) \cup B_1((2,0))$, donde dichos conjuntos son bolas abiertas. Note que dichas bolas son disjuntas. Dado $z_1 \in B_1((0,0))$, $z_2 \in B_1((2,0))$ tenemos que no existe una curva que conecta a z_1, z_2 que pertenezca totalmente a $B_1((0,0)) \cup B_1((2,0))$. Dado que $z_1, z_2 \in A$, entonces A es no conexo.



18

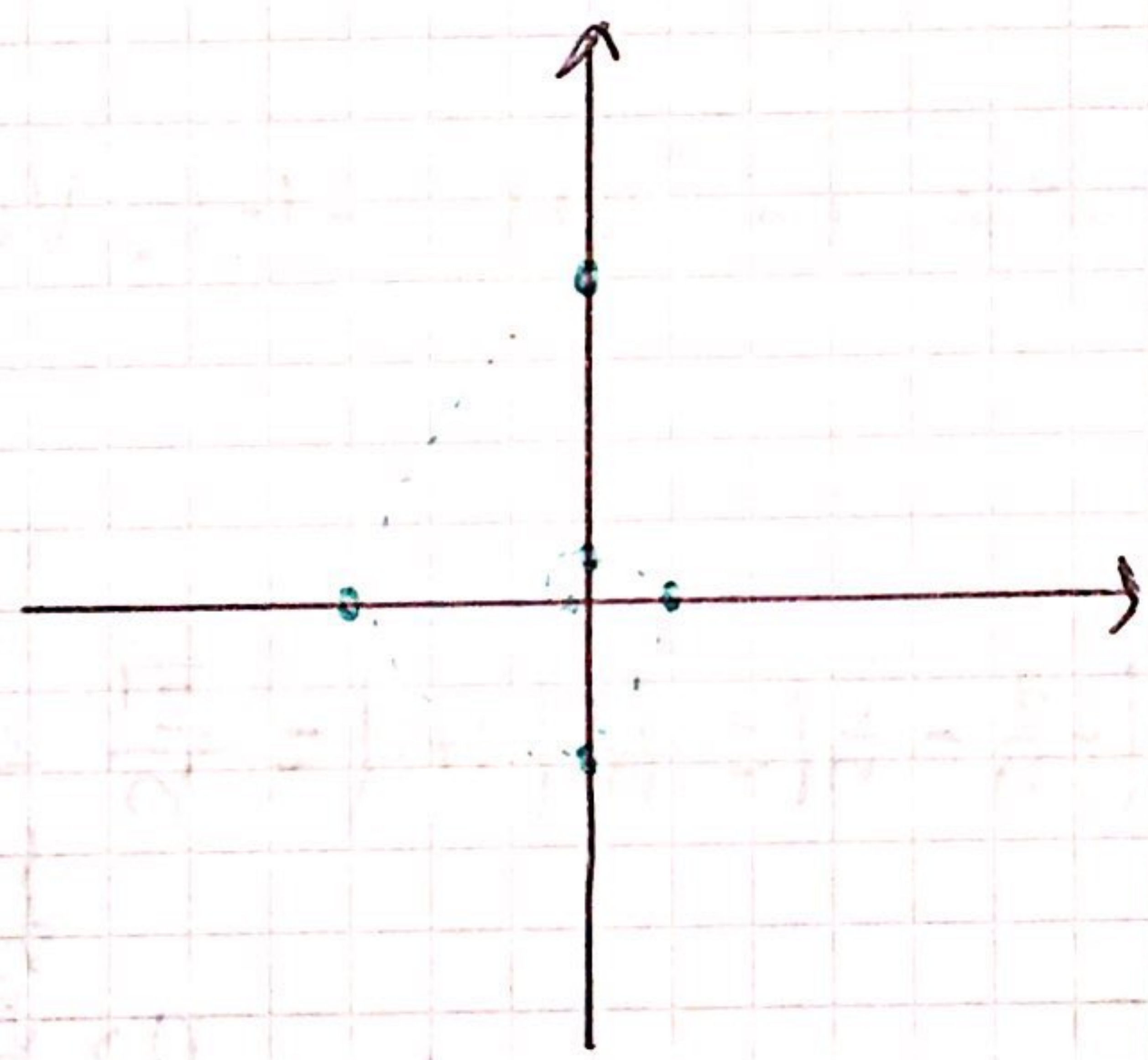
1



$z_n = \{1, -1, i, -i\}$

Claramente para cada $z \in \{z_n\}$ $\exists r > 0 \,/\, (B_r(z) - \{z\}) \cap S = \emptyset$, por lo tanto no tiene puntos de acumulación

2



Claramente para n pequeños $\exists r > 0 \,/\, (B_r(z) - \{z\}) \cap \{z_n\} = \emptyset$, es decir que no son puntos de acumulación.

Veamos si $\{z_n\}$ converge a 0:

$|z_n - 0| = |z_n| = \left|\frac{i^n}{n}\right| = \frac{|i|^n}{n} = \frac{1}{n} < \varepsilon \rightsquigarrow n > \frac{1}{\varepsilon}$

Luego si tomo $n_0 = \left\lceil \frac{1}{\varepsilon} \right\rceil$ (es decir su redondeo para arriba) tenemos que:

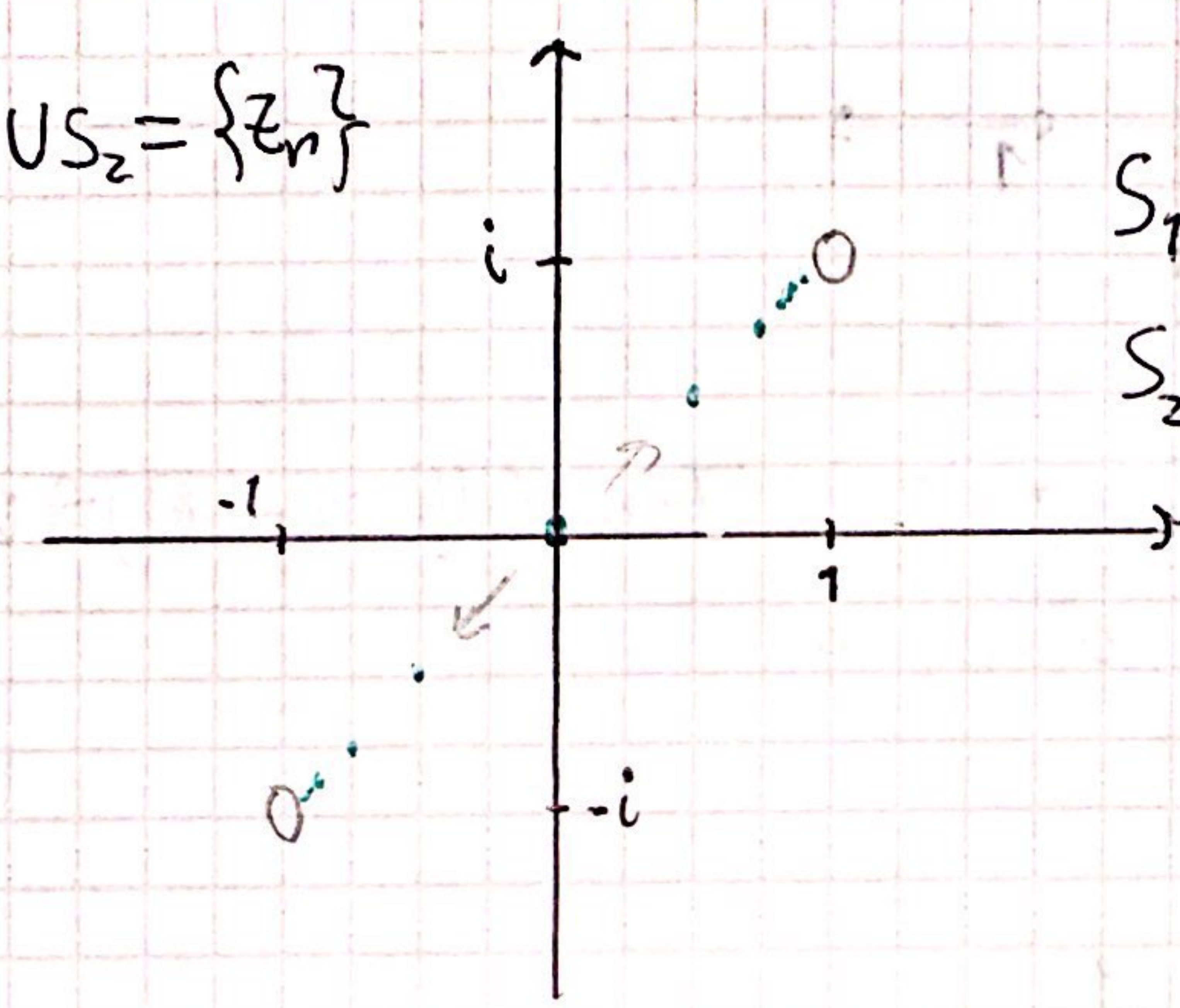
$\forall \varepsilon > 0, \; \exists n_0 \in \mathbb{N} \,/\, n > n_0 \Longrightarrow |z_n - 0| < \varepsilon$

Es decir que para $m > n_0$, tenemos que todo $z_m \in \{z_n\}$ pertenece a $B_\varepsilon(0)$. Es decir que $(B_\varepsilon(0) - \underbrace{\{0\}}_{\text{Pues } \frac{i^n}{n} \neq 0}) \cap \{z_n\} \neq \emptyset$

$\Longrightarrow$ $z = 0$ es punto de acumulación

3

Sea $S = S_1 \cup S_2 = \{z_n\}$



Notemos que:

$S_1 = \{z_{2n}\} = \{(1-\frac{1}{n}) + i(1-\frac{1}{n})\}$

$S_2 = \{z_{2n+1}\} = \{-(1-\frac{1}{n}) - i(1-\frac{1}{n})\}$

Parece que los puntos se acumulen en $1+i$, $-1-i$. Veamos:

Sea n Par

Sea $\varepsilon > 0$:

$$\left|(1-\tfrac{1}{n}) + i(1-\tfrac{1}{n}) - (1+i)\right| = \left|-\tfrac{1}{n} - \tfrac{i}{n}\right| = \tfrac{1}{n}\cdot|1+i| = \frac{\sqrt{2}}{n} < \varepsilon$$

$$\Downarrow$$

$$n > \frac{\sqrt{2}}{\varepsilon}$$

Luego si tomo $n_0 = \left\lceil \frac{\sqrt{2}}{\varepsilon} \right\rceil$ entonces:

$\forall \varepsilon > 0, \exists n_0 \in \mathbb{N} / n > n_0 \Longrightarrow |z - (1+i)| < \varepsilon$ para $z \in \{z_{2n}\}$

$\Longrightarrow (B_\varepsilon(1+i) - \{1+i\}) \cap S_1 \neq \emptyset \Longrightarrow (B_\varepsilon(1+i) - \{1+i\}) \cap \{z_n\} \neq \emptyset$

$\Longrightarrow w_1 = 1+i$ es un punto de acumulación

Sea n impar

Sea $\varepsilon > 0$:

$$\left| -(1-\tfrac{1}{n}) - i(1-\tfrac{1}{n}) - (-1-i) \right| = \left| \tfrac{1}{n} + \tfrac{i}{n} \right| = \tfrac{1}{n} |1+i| = \tfrac{\sqrt{2}}{n} < \varepsilon$$

$$\Downarrow$$

$$n > \frac{\sqrt{2}}{\varepsilon}$$

Luego si tomo $n_0 = \left\lceil \frac{\sqrt{2}}{\varepsilon} \right\rceil$:

$\forall \varepsilon > 0, \exists n_0 \in \mathbb{N} / n > n_0 \Rightarrow |z - (-1-i)| < \varepsilon$ para $z \in \{z_{2n+1}\}$

$\Rightarrow \left(B_\varepsilon(-1-i) - \{-1-i\}\right) \cap S_2 \neq \emptyset \Rightarrow \left(B_\varepsilon(-1-i) - \{-1-i\}\right) \cap \{z_n\} \neq \emptyset$

$\Rightarrow w_2 = -1-i$ es un punto de acumulación.

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Basta tomar $z = w = -1$

Por un lado:

$$h_1 = \sqrt[n]{z \cdot w} = \sqrt[n]{1} \iff h_1 = e^{i\frac{2k\pi}{n}}$$

$$\sqrt[n]{-1} = e^{i\left(\frac{-\pi + 2k\pi}{n}\right)}$$

Por otro lado:

$$h_2 = \sqrt[n]{z}\sqrt[n]{w} = \sqrt[n]{-1}\sqrt[n]{-1} \iff h_2 = e^{i\left(\frac{4k\pi}{n} - \frac{2\pi}{n}\right)}$$

Pero $h_1 \neq h_2 \Rightarrow \sqrt[n]{zw} \neq \sqrt[n]{z}\sqrt[n]{w}$