PRACTICO 2

1. $\lim_{n\to\infty} z_n = w \iff \forall \varepsilon>0, \exists n_0 \in \mathbb{N} / n>n_0 \Rightarrow |z_n - w| < \varepsilon$

1

Sea $w=0$ y $\varepsilon>0$:

$$|z_n - 0| < \varepsilon$$
$$\iff |z_n| < \varepsilon$$
$$\iff |z_n| - 0 < \varepsilon$$
$$\iff \big||z_n| - 0\big| < \varepsilon$$

2

Por hipótesis $\forall \varepsilon>0, \exists n_0 \in \mathbb{N} / n>n_0 \Rightarrow |z_n - w| < \varepsilon$

Dado que $|z_n - w| \geq \big||z_n| - |w|\big|$

$$\Rightarrow \big||z_n| - |w|\big| < \varepsilon$$

La recíproca solo vale cuando $|z_n - w| = \big||z_n| - |w|\big|$

2.

$$z_n = n(\sqrt[n]{w} - 1) = n(\sqrt[n]{\overset{\text{módulo 1 por hipo}}{e^{i\theta}}} - 1) = n\left((e^{i\theta})^{\frac{1}{n}} - 1\right)$$

$$= n\left(\left(\cos\left(\tfrac{\theta}{n}\right) + i.\sin\left(\tfrac{\theta}{n}\right)\right) - 1\right)$$

$$= \underbrace{n\left(\cos\left(\tfrac{\theta}{n}\right) - 1\right)}_{X_n} + i\,\underbrace{n\sin\left(\tfrac{\theta}{n}\right)}_{Y_n}$$

Por teorema, siendo $z_n = x_n + i \cdot y_n$:

$$\lim_{n\to\infty} z_n = x + i \cdot y \iff \underbrace{\lim_{n\to\infty} x_n = x}_{A} \wedge \underbrace{\lim_{n\to\infty} y_n = y}_{B}$$

A

$$\lim_{n\to\infty} n\cdot\left(\cos\left(\frac{\theta}{n}\right)-1\right) = \lim_{n\to\infty} \frac{\cos\left(\frac{\theta}{n}\right)-1}{\frac{1}{n}} \underset{\text{lhopital de sucesiones reales}}{=} \lim_{n\to\infty} \frac{\frac{\theta\sin\left(\frac{\theta}{n}\right)}{n^2}}{-\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} -\theta\cdot\sin\left(\frac{\theta}{n}\right) = -\theta\cdot 0 = 0$$

B

$$\lim_{n\to\infty} n\cdot\sin\left(\frac{\theta}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{\theta}{n}\right)}{\frac{1}{n}} \overset{\text{lhopital de sucesiones reales}}{=} \lim_{n\to\infty} \frac{-\frac{\theta\cos\left(\frac{\theta}{n}\right)}{n^2}}{-\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \theta\cdot\cos\left(\frac{\theta}{n}\right) = \lim_{n\to\infty} (Arg(w)+2k\pi)\cdot\cos\left(\frac{Arg(w)+2k\pi}{n}\right)$$

$$= Arg(w)$$

$$\therefore \lim_{n\to\infty} z_n = i\cdot Arg(w)$$

3 Método para encontrar la fórmula explícita.

Dado $a_{n+2} = C_1 a_{n+1} + C_0 a_n$, resuelvo $x^2 - C_1 x - C_2 = 0$

Luego:

si $X_1 \neq X_2$: $a_n = A.X_1^n + BX_2^n$

si $X_1 = X_2$: $a_n = (A + B.n).X^n$

Valuo en $n=1$, $n=2$ y despejo A, B.

Entonces:

En este caso $C_1 = C_2 = \frac{1}{2}$. Luego

$$X^2 - \frac{1}{2}X - \frac{1}{2} = 0 \implies X = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4}+2}}{2} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

Luego $Z_n = A.1^n + B(-\frac{1}{2})^n$

$Z_1 = A - \frac{B}{2} = 0 \implies A = \frac{B}{2}$

$Z_2 = A + \frac{B}{4} = i$

$$\frac{B}{2} + \frac{B}{4} = i$$

$$\frac{2B+B}{4} = i$$

$$\frac{3B}{4} = i$$

$$B = \frac{4i}{3} \implies A = \frac{2i}{3}$$

$$\implies Z_n = \frac{2i}{3} + \frac{4i}{3}\left(-\frac{1}{2}\right)^n = \frac{2i}{3} + \frac{4i}{3}(-1)^n \cdot 2^{-n}$$

Parece que $\lim_{n\to\infty} Z_n = \frac{2i}{3}$...

$$\left|\frac{2i}{3} + \frac{4i}{3}(-1)^n 2^{-n} - \frac{2i}{3}\right| = \frac{4}{3}|i|.|(-1)^n|.|2^{-n}| = \frac{4}{3}\cdot 1 \cdot 1 \cdot 2^{-n} = \frac{4}{3}\cdot 2^{-n} < \varepsilon$$

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$$\rightsquigarrow \frac{\ln\left(\frac{4}{3\varepsilon}\right)}{\ln(2)} < n$$

luego tomando $n_0 = \left\lceil \frac{\ln\left(\frac{4}{3\varepsilon}\right)}{\ln(2)} \right\rceil$:

$\forall \varepsilon > 0, \exists n_0 \in \mathbb{N} / n > n_0 \Rightarrow |z_n - \frac{2i}{3}| < \varepsilon$

5

1 $\quad Dom(f) = \mathbb{C}$

Siendo $z = x+yi \Rightarrow z^2 = x^2 - y^2 + i2xy$

$$\Rightarrow f(z) = x^2 + 2xyi - y^2 + x + yi - 1$$

$$= \underbrace{(x^2-y^2+x-1)}_{\mu(x,y)} + i\underbrace{(2xy+y)}_{v(x,y)}$$

2 $\quad z^2+1 = 0 \iff z = i \lor z = -i$

$Dom(f) = \{ z \in \mathbb{C} / z \neq i \land z \neq -i \}$

Luego:

$$(z^2+1)^{-1} = \frac{1}{x^2+2xyi-y^2+1} = \frac{1}{(x^2-y^2+1)+i(2xy)}$$

$$= \frac{(x^2-y^2+1)-i(2xy)}{(x^2-y^2+1)^2+(2xy)^2} = \underbrace{\frac{x^2-y^2+1}{(x^2-y^2+1)^2+(2xy)^2}}_{\mu(x,y)} - i \cdot \underbrace{\frac{2xy}{(x^2-y^2+1)^2+(2xy)^2}}_{v(x,y)}$$

$z^{-1} = \frac{\bar{z}}{|z|^2}$

3 $Dom(f) = \mathbb{C} - \{0\}$

Sea $z = x+iy$:

$$f(z) = x+iy + \frac{x-iy}{x^2+y^2} = \underbrace{\left(x + \frac{x}{x^2+y^2}\right)}_{u(x,y)} + i \cdot \underbrace{\left(y - \frac{y}{x^2+y^2}\right)}_{v(x,y)}$$

5

$Dom(f) = \{z = x+iy \in \mathbb{C} / x^2+y^2 \neq 1\}$

$$f(z) = \underbrace{\frac{1}{1-(x^2+y^2)}}_{u(x,y)} \qquad v(x,y) = 0$$

6

$Dom(f) = \{z = x+iy \in \mathbb{C} / x^2+y^2 \neq 1\}$

Siendo $z = x+iy$, $z^2 = (x^2-y^2) + 2xy\, i$

$|z-1| = \sqrt{(x-1)^2+y^2}$

Luego:

$$f(z) = \frac{2(x^2-y^2) + i.4xy + 3}{(x-1)^2+y^2} = \underbrace{\frac{2(x^2-y^2)+3}{(x-1)^2+y^2}}_{u(x,y)} + i . \underbrace{\frac{4xy}{(x-1)^2+y^2}}_{v(x,y)}$$

6) Sean $z, w \in A$:

$$\frac{z+\frac{1}{z}}{2} = \frac{w+\frac{1}{w}}{2} \Rightarrow z+\frac{1}{z} = w+\frac{1}{w} \Rightarrow \frac{z^2+1}{z} = \frac{w^2+1}{w}$$

$$\Rightarrow z^2w + w = w^2z + z \Rightarrow z^2w - z = w^2z - w$$

$$\Rightarrow z(zw-1) = w(wz-1) \Rightarrow z(wz-1) = w(wz-1) \underset{\circledast}{\Rightarrow} z = w$$

$\circledast$ veamos que $wz-1 \neq 0$:

$$wz - 1 = 0 \Rightarrow wz = 1 \Rightarrow |w| . |z| = 1 \Rightarrow |w| = \frac{1}{|z|}$$

dado que $z \in A \Rightarrow |z| > 1 \Rightarrow \frac{1}{|z|} < 1 \Rightarrow |w| < 1$ Absurdo, pues $w \in A$

$\therefore$ Es inyectiva

9

1

$$\lim_{z\to i}\left(2z^2 - iz^3 + z.\operatorname{Arg}(\bar{z})\right) = 2i^2 - i.i^3 + i.\frac{3\pi}{2} = -2-1+i.\frac{3\pi}{2} = -3+i\frac{3\pi}{2}$$

2

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{xy^2}{x^2+y^4} = ?$$

Prueba 1

$x = \frac{1}{n}$

$y = \frac{1}{n}$

$$\rightsquigarrow \lim_{n\to\infty} \frac{\frac{1}{n}\cdot\frac{1}{n^2}}{\frac{1}{n^2}+\frac{1}{n^4}} = \lim_{n\to\infty}\frac{\frac{1}{n^3}}{\frac{n^2+1}{n^4}} = \lim_{n\to\infty}\frac{n}{n^2+1} = 0$$

Prueba 2

$x = \frac{1}{n^2}$

$y = \frac{1}{n}$

$$\rightsquigarrow \lim_{n\to\infty} \frac{\frac{1}{n^2}\cdot\frac{1}{n}}{\frac{1}{n^4}+\frac{1}{n^4}} = \lim_{n\to\infty}\frac{\frac{1}{n^3}}{\frac{2}{n^4}} = \lim_{n\to\infty}\frac{n}{2} = \infty$$

$\therefore \nexists \lim$

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3

$$\lim_{z\to i} \frac{iz^3-1}{z+i} = \frac{i.i^3-1}{i+i} = \frac{1-1}{2i} = \frac{0}{2i} = 0$$

4

$$\lim_{z\to i} \frac{z^4+i}{z+i} = \frac{1+i}{2i} = (-i).\frac{1+i}{2} = \frac{-i-i^2}{2} = \frac{1}{2}-\frac{i}{2}$$

5

$$\lim_{z\to 0} \frac{\bar{z}}{z} = \lim_{\substack{x\to 0\\ y\to 0}} \frac{x-iy}{x+iy} = \lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2-y^2-i.2xy}{x^2+y^2}$$

$$= \lim_{\substack{x\to 0\\ y\to 0}} \underbrace{\frac{x^2-y^2}{x^2+y^2}}_{u(x,y)} - i.\underbrace{\frac{2xy}{x^2+y^2}}_{v(x,y)}$$

Luego

$$\lim_{\substack{x\to 0\\ y\to 0}} u(x,y) = ?$$

Prueba 1

$x=\frac{1}{n}$

$y=\frac{1}{n}$ $\rightsquigarrow \lim_{n\to\infty} \dfrac{\frac{1}{n}-\frac{1}{n}}{\frac{1}{n^2}+\frac{1}{n^2}} = \dfrac{n^2.0}{2} = 0$

Prueba 2

$x=\frac{1}{n}$

$y=0$ $\rightsquigarrow \lim_{n\to\infty} \dfrac{\frac{1}{n^2}}{\frac{1}{n^2}} = 1$

$$\therefore \nexists \lim_{\substack{x\to 0\\ y\to 0}} u(x,y) \implies \nexists \lim_{z\to 0} f(z)$$

6 Tiene pinta que $\lim_{z \to 0} \frac{\bar{z}^2}{z} = 0$. Veamos por definición:

Sea $\varepsilon > 0$:

$$\left|\frac{\bar{z}^2}{z} - 0\right| = \left|\frac{\bar{z}^2}{z}\right| = \frac{|\bar{z}^2|}{|z|} = \frac{|z|^2}{|z|} = |z - 0| < \delta$$

tomo $\boxed{\delta = \varepsilon}$

$$\therefore \lim_{z \to 0} \frac{\bar{z}^2}{z} = 0$$

7

$$\lim_{z \to 0} \left(\frac{z}{\bar{z}}\right)^2 = \lim_{z \to 0} \left(\frac{z^2}{|z|^2}\right)^2 = \lim_{\substack{x \to 0 \\ y \to 0}} \left(\frac{(x^2 - y^2) + i2xy}{x^2 + y^2}\right)^2$$

$$= \lim_{\substack{x \to 0 \\ y \to 0}} \left(\frac{x^2 - y^2}{x^2 + y^2}\right)^2 + \frac{i \cdot 4xy(x^2 - y^2)}{(x^2 + y^2)^2} + (i)^2 \cdot \frac{(2xy)^2}{(x^2 + y^2)^2}$$

$$= \lim_{\substack{x \to 0 \\ y \to 0}} \underbrace{\frac{(x^2 - y^2)^2 - 4x^2y^2}{(x^2 + y^2)^2}}_{u(x,y)} + i \cdot \underbrace{\frac{(x^2 - y^2) \cdot 4xy}{(x^2 + y^2)^2}}_{v(x,y)}$$

$$\lim_{\substack{x \to 0 \\ y \to 0}} u(x,y) = ?$$

Prueba 1

$x = 0$, $y = \frac{1}{n}$ $\rightsquigarrow$ $\lim_{n \to \infty} \frac{\frac{1}{n^4} - 0}{\frac{1}{n^4}} = 1$

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Prueba 2

$x = \frac{1}{n}$
$y = \frac{1}{n}$ $\rightsquigarrow \lim\limits_{n \to \infty} \frac{-4 \cdot \frac{1}{n^4}}{\left(\frac{2}{n^2}\right)^2} = -\frac{4 \cdot \frac{1}{n^4}}{4 \cdot \frac{1}{n^4}} = -1$

$\therefore \nexists \lim\limits_{\substack{x \to 0 \\ y \to 0}} f(x,y) \Rightarrow \nexists \lim\limits_{z \to 0} \left(\frac{z}{\bar{z}}\right)^2$

8 $\lim\limits_{z \to -i} \frac{z^4 - 1}{z+i} = \lim\limits_{z \to -i} \frac{(z^2-1)(z^2+1)}{z+i} = \lim\limits_{z \to -i} \frac{(z^2-1) \cdot (z^2 - (-1))}{z+i}$

$= \lim\limits_{z \to -i} \frac{(z^2-1)(z - \sqrt{-1})(z + \sqrt{-1})}{z+i} = \lim\limits_{z \to -i} \frac{(z^2-1)(z-i)(z+i)}{z+i}$

$= \lim\limits_{z \to -i} (z^2-1)(z-i) = (-1-1)(-i-i) = 4i$

10

$\lim\limits_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} = \lim\limits_{z \to z_0} \frac{3z^2 + 2z - 3z_0^2 + 2z_0}{z - z_0}$

$= \lim\limits_{z \to z_0} \frac{3(z^2 - z_0^2) + 2(z - z_0)}{z - z_0} = \lim\limits_{z \to z_0} \frac{(z - z_0) \cdot (3(z + z_0) + 2)}{z - z_0}$

$= \lim\limits_{z \to z_0} 3(z + z_0) + 2 = 3(2z_0) + 2 = 6z_0 + 2$

11 $\lim\limits_{z \to 2i} \frac{z^2 + 4}{z - 2i} = \lim\limits_{z \to 2i} \frac{z^2 - (-4)}{z - 2i} = \lim\limits_{z \to 2i} \frac{(z - \sqrt{-4})(z + \sqrt{-4})}{z - 2i}$

$= \lim\limits_{z \to 2i} \frac{(z - 2i)(z + 2i)}{z - 2i} = \lim\limits_{z \to 2i} z + 2i = 4i$

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Probemos por definición:

$$\left|\frac{z^2+4}{z-2i}-4i\right| = \left|\frac{z^2+4-4iz-8}{z-2i}\right| = \left|\frac{z^2-4iz-4}{z-2i}\right| = \left|\frac{(z-2i)^2}{z-2i}\right| = |z-2i| < \delta$$

Luego siendo $\delta = \varepsilon$:

$$\forall \varepsilon > 0, \exists \delta > 0 / 0 < |z-2i| < \delta \Rightarrow \left|\frac{z^2+4}{z-2i}-4i\right| < \varepsilon$$

Para que sea continua podríamos definirla como:

$$f(z) = \begin{cases} \dfrac{z^2+4}{z-2i} & z \neq 2i \\ 4i & z = 2i \end{cases}$$

12

1

$$K = \{ z = x+iy \in \mathbb{C} / |z| = 2 \}$$

Luego sea $z \in K$, $z = 2.e^{i\theta}$

$$f(2e^{i\theta}) = 2e^{i\theta} - \frac{1}{2}e^{i(-\theta)} = 2(\cos(\theta) + i.\sin(\theta)) - \frac{1}{2}(\cos(\theta) - i.\sin(\theta))$$

$$= \underbrace{\frac{3}{2}\cos(\theta)}_{Re(f)} + i.\underbrace{\frac{5}{2}\sin(\theta)}_{Im(f)}$$

es claro que

$\max Im(z)(\theta) = Im(z)(\frac{\pi}{2}) = \frac{5}{2}$ $\quad\quad$ $\max Re(z)(\theta) = Re(z)(0) = \frac{3}{2}$

$\min Im(z)(\theta) = Im(z)(0) = 0$ $\quad\quad$ $\min Re(z)(\theta) = Re(z)(\frac{\pi}{2}) = 0$

Como el coeficiente de la parte real es menor que el de la parte imaginaria $\theta = \frac{\pi}{2} + 2k\pi$ es el máx y $\theta = 2k\pi$ es el mín. Es decir $|f| = 2,5$ y $|f| = 1,5$ respectivamente

13

1

$$|f(z)^2-1| = |(f(z)-1)(f(z)+1)| = |f(z)-1|.|f(z)+1|$$

Si $f(z)=\pm 1$ $\begin{cases} + \text{ satisface que } |f(z)^2-1|<1 \text{ y que } |f(z)-1|<1 \text{ y } |f(z)+1|\geq 1 \\ - \text{ satisface que } |f(z)^2-1|<1 \text{ y que } |f(z)+1|<1 \text{ y } |f(z)-1|\geq 1 \end{cases}$

Sea $f(z)\neq\pm 1$, es decir que $|f(z)-1|\neq 0$:

Caso 1 $|f(z)+1|>1$:

$\Longleftrightarrow |f(z)-1| < \frac{1}{|f(z)+1|} < 1$

$\Downarrow$

$|f(z)-1|.|f(z)+1|<1$

Caso 2 Sea $|f(z)+1| < 1$

$\Longleftrightarrow |f(z)+1| < \frac{1}{|f(z)-1|} \Longleftrightarrow |f(z)-1| > 1$

$|f(z)-1| . |f(z)+1| < 1$

Es decir que siempre tenemos uno solo de los dos factores que es menor estricto que 1.

2

Sea $f(z) = f(x+iy) = u(x,y) + i.v(x,y)$.

$$e^{u}.e^{iv} = 1 = 1.e^{i0} \Longleftrightarrow \begin{cases} e^{u} = 1 \\ v = 0 + 2k\pi \end{cases} \Longleftrightarrow \begin{cases} u = 0 \\ v = 2k\pi \end{cases} \quad \otimes$$

$$\left.\begin{aligned} &\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = 0 \\ &\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = 0 \end{aligned}\right\} \Rightarrow u, v \text{ son constantes en } D \Rightarrow f \text{ es constante en } D$$

$\otimes \Rightarrow f(z) = u(x,y) + i.v(x,y) = i.2k\pi$

3

$\forall z \in \mathbb{C}$ tenemos que $f(z)^2 = z \Longleftrightarrow f(z) = \sqrt{z} \vee f(z) = -\sqrt{z}$

Ahora veamos que $\forall z \in D \Rightarrow f(z) = \sqrt{z} \veebar f(z) = -\sqrt{z}$

dado que $f(z) \neq c.i \ \forall c \in \mathbb{R} \Rightarrow f(z) \neq 0 \Rightarrow z \neq 0$

$\Rightarrow -\sqrt{z}, \sqrt{z}$ no forman un cjto conexo, pues no existen z_i tq $\sqrt{z_1} = -\sqrt{z_2}$

Dado que D es un abierto conexo y que f es continua, por teorema la imagen de f debe ser conexa

$\Rightarrow f(z) = \sqrt{z} \veebar f(z) = -\sqrt{z}$

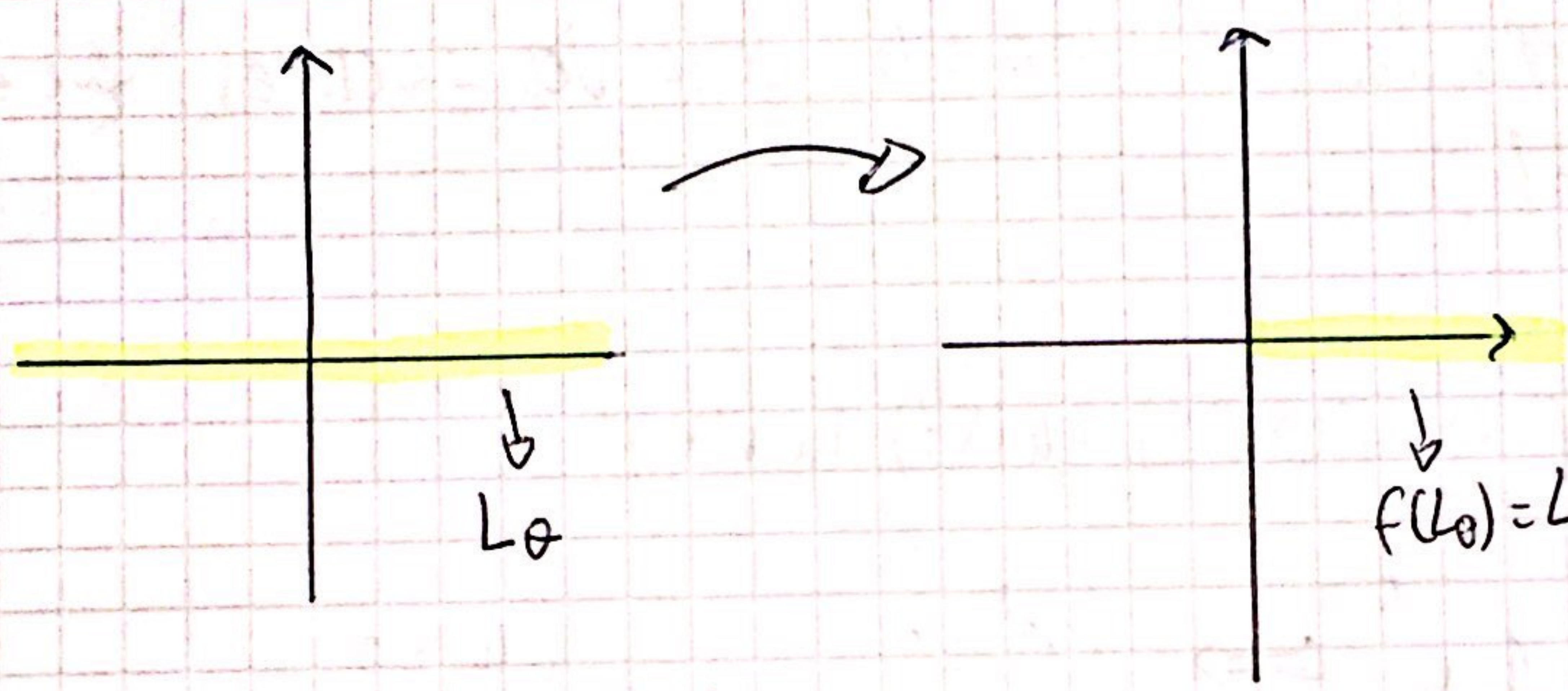
Pues $A = \{\sqrt{z} / z \in D\} \cup \{-\sqrt{z} / z \in D\}$ es no conexo

(14)

1 $z^2 = (x^2 - y^2) + i.2xy = f(z)$

Sea $L_\theta = \{ z = x + iy \in \mathbb{C} / y = 0 \}$ la recta horizontal.

Luego $f(L_\theta) = x^2$ → no Parte imaginaria nula, Parte real siempre positiva o nula



Dado $L_\alpha = \{ z \in \mathbb{C} / R_e(z) \geq 0 \wedge I_m(z) = 0 \}$ probemos que $f(L_\theta) = L_\alpha$.

$\subseteq$

Sea $z \in L_\theta \Rightarrow z = x$, luego:

$$f(z) = x^2 + i.0 = x^2 \Rightarrow R_e(f(z)) \geq 0 \wedge I_m(f(z)) = 0$$

$$\Rightarrow f(L_\theta) \in L_\alpha$$

$\supseteq$

Sea $t \geq 0 \in \mathbb{R}$ y $w = t$. Luego $w \in L_\alpha$

Sea $z = \pm\sqrt{t} \in L_\theta$:

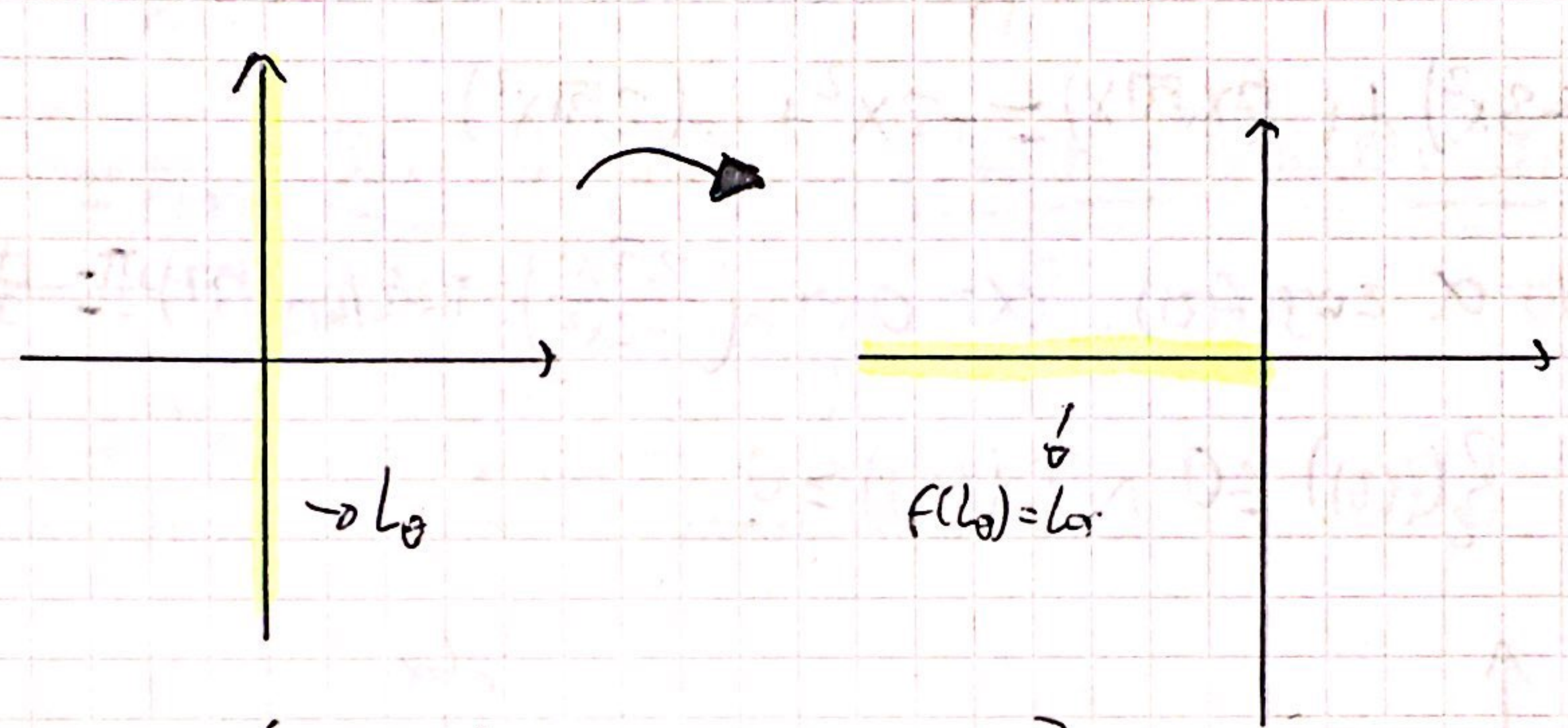
$$f(z) = (\pm\sqrt{t})^2 = t = w \Rightarrow L_\alpha \in f(L_\theta)$$

$$\therefore f(L_\theta) = L_\alpha$$

2

Sea $L_\theta = \{z = x+iy \in \mathbb{C} / x=0\}$ la recta vertical:

Luego $f(L_\theta) = -y^2$ } no parte imaginaria nula, parte real siempre negativa o nula.



Sea $L_\alpha = \{z \in \mathbb{C} / R_e(z) \leq 0 \wedge I_m(z) = 0\}$, probemos que $f(L_\theta) = L_\alpha$

$\subseteq$ Sea $z \in L_\theta \Rightarrow z = iy$

Luego:

$f(z) = -(y^2) \Rightarrow R_e(z) \leq 0 \wedge I_m(z) = 0$

$\Rightarrow f(L_\theta) \in L_\alpha$

$\supseteq$ Sea $T \geq 0$, $w = -T \in L_\alpha$:

Siendo $z = \pm\sqrt{T}.i \in L_\theta$:

$f(z) = (\pm\sqrt{T}i)^2 = T.i^2 = -T = w \Rightarrow L_\alpha \in f(L_\theta)$

$\therefore f(L_\theta) = L_\alpha$

3

$$f(z) = z^2 = (x^2 - y^2) + i \cdot 2xy$$

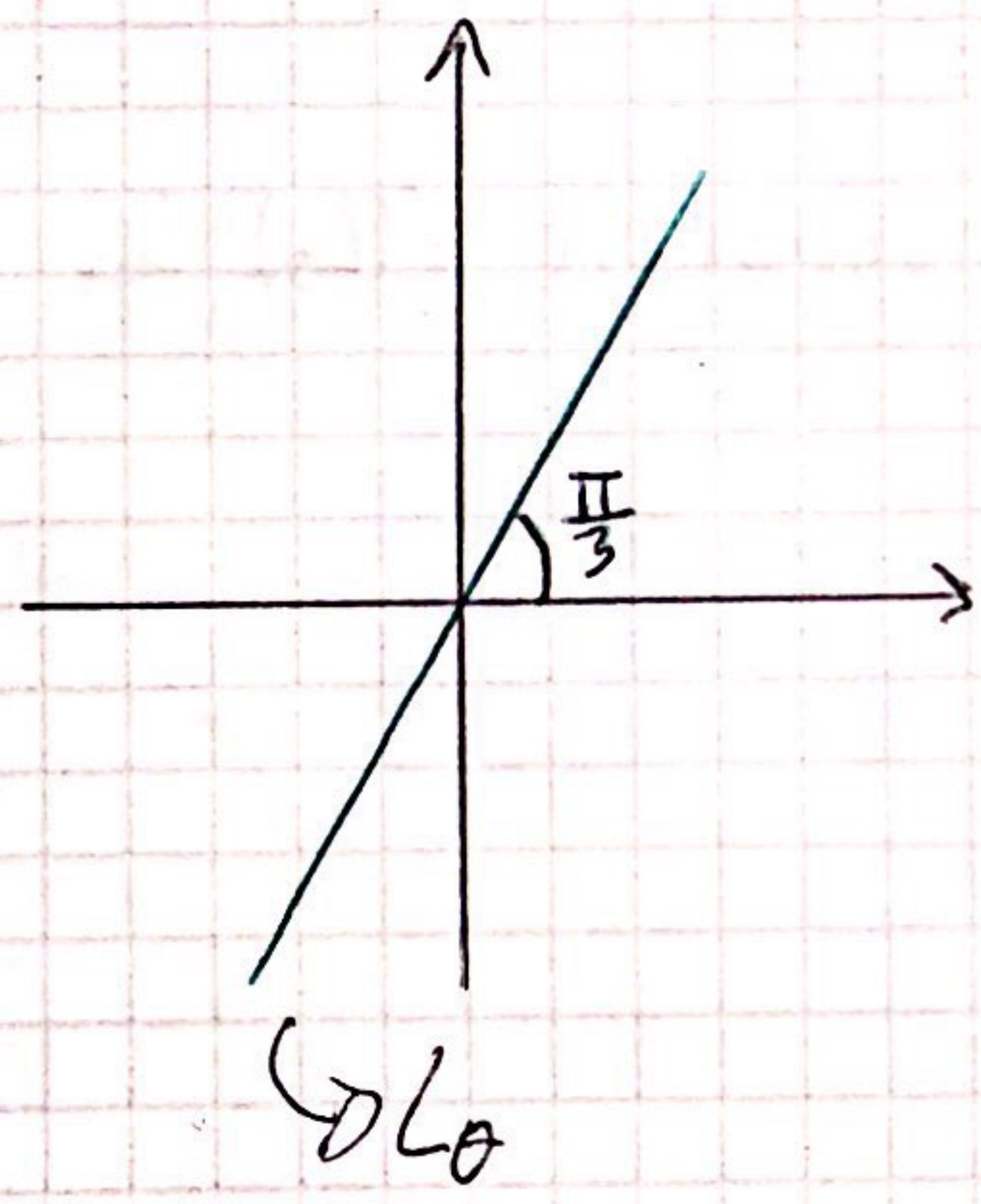
Siendo $y = \sqrt{3}\,x \Rightarrow z = x + i\sqrt{3}\,x \Rightarrow$ Siendo $\theta \in \arg(z)$, $\theta = \arctan\left(\frac{\sqrt{3}\,x}{x}\right)$
$= \arctan(\sqrt{3}) = \frac{\pi}{3}$

Por otro lado

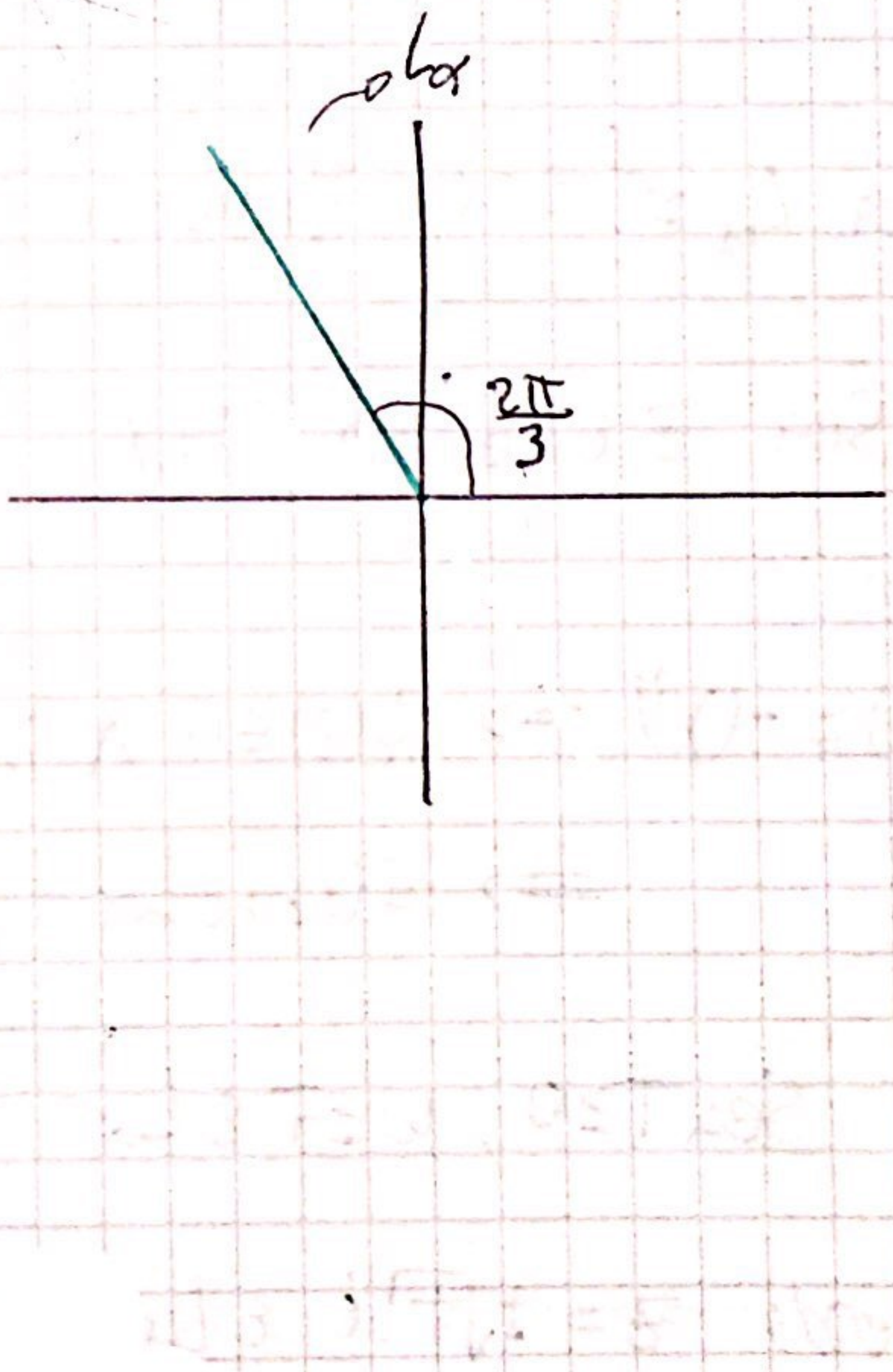
$$f(z) = (x^2 - 3x^2) + i \cdot (2x\sqrt{3}\,x) = -2x^2 + i \cdot (2\sqrt{3}\,x^2)$$

$\Rightarrow$ Siendo $\alpha \in \arg(f(z))$, $\alpha = \arctan\left(\frac{2\sqrt{3}x^2}{-2x^2}\right) + \pi = \arctan(-\sqrt{3}) + \pi = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$

y además $R_e(f(z)) \leq 0 \wedge I_m(f(z)) \geq 0$



$\longrightarrow$



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