Entrega 2:

$$y'' = \underbrace{\boxed{\frac{1}{2EI} W_0}}_{=A} (Lx - x^2)\left(1+(y')^2\right)^{3/2}$$

$$\frac{\partial^2 y}{\partial x^2} - A(Lx - x^2)\left(1+\left(\frac{\partial y}{\partial x}\right)^2\right)^{3/2} = 0$$

para aproximar las derivadas usaré esquemas de diferencias centradas:

$$\boxed{\begin{aligned} \frac{\partial^2 y}{\partial x^2} &= (y_{i-1} - 2y_i + y_{i+1})/(\Delta x)^2 \\ \frac{\partial y}{\partial x} &= (y_{i-1} + y_{i+1})/2\Delta x \end{aligned}} \quad i = 2, \dots, n-1$$

(Discretizaré el eje x en n puntos)

∴ La ecuación a resolver la veo como

$$\boxed{\frac{y_{i-1} - 2y_i + y_{i+1}}{(\Delta x)^2} - A(Lx_i - x_i^2)\left(1+\left(\frac{y_{i-1}+y_{i+1}}{2\Delta x}\right)^2\right)^{3/2}}$$

Definición: $\vec{y} = (y_2, y_3, \dots, y_{n-1})$

Tomaré una función $F: \mathbb{R}^{n-1} \to \mathbb{R}^{n-1}$ dada por $F(\vec{y}) = \begin{bmatrix} f_2(\vec{y}) \\ f_3(\vec{y}) \\ \vdots \\ f_{n-1}(\vec{y}) \end{bmatrix}$

donde $\boxed{f_i(\vec{y}) = \frac{y_{i-1} - 2y_i + y_{i+1}}{(\Delta x)^2} - A(Lx_i - x_i^2)\left(1+\left(\frac{y_{i-1}+y_{i+1}}{2\Delta x}\right)^2\right)^{3/2}}$ $i = 2, \dots, n-1$

Ahora buscaré un $\vec{y}_{sol}$ tal que $F(\vec{y}_{sol}) = 0$, para esto usaré Newton:

$$y^{k+1} = y^k + \Delta y^k$$

con Δy^k solución de $J(y^k)\,\Delta y^k = F(y^k)$ $\quad k = 1, 2, 3, \dots$

donde $J(y^k)$ es el Jacobiano de y^k

$$J(y) = \begin{bmatrix} \frac{\partial f_2}{\partial y_2} & \frac{\partial f_2}{\partial y_3} & \cdots & \frac{\partial f_2}{\partial y_{n-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{n-1}}{\partial y_2} & \frac{\partial f_{n-1}}{\partial y_3} & \cdots & \frac{\partial f_{n-1}}{\partial y_{n-1}} \end{bmatrix}$$

$$\boxed{\frac{\partial f_i}{\partial y_k} = \begin{cases} -\frac{2}{(\Delta x)^2} & k = i \\ \frac{1}{(\Delta x)^2} - A(Lx_i - x_i^2) \cdot \frac{3}{2}\left(1+\left(\frac{y_{i-1}+y_{i+1}}{2\Delta x}\right)^2\right)^{1/2} \cdot 2\left(\frac{y_{i-1}+y_{i+1}}{2\Delta x}\right) \cdot \frac{1}{2\Delta x} & k = i \pm 1 \\ 0 & \text{c.c} \end{cases}}$$

$$\frac{\partial f_i}{\partial y_K} = \begin{cases} -2/(\Delta x)^2 & K = i \\ \dfrac{1}{(\Delta x)^2} - \dfrac{3W_0}{8EI(\Delta x)^2}\left(Lx_i - x_i^2\right)\left(y_{i-1} + y_{i+1}\right)\left(1 + \left(\dfrac{y_{i-1}+y_{i+1}}{2\Delta x}\right)^2\right)^{1/2} & K = \pm i \\ 0 & \text{c.c} \end{cases}$$