

Llamo $\alpha = EI$ por comodidad.

Grilla: $X_i = ih$, $i = 0, \dots, n$, $X_i \in [0, L]$

Utilizo las diferencias finitas: $Y'(x) \approx \frac{Y_{i+1} - Y_{i-1}}{2h}$
 $Y''(x) \approx \frac{Y_{i+1} - 2Y_i + Y_{i-1}}{h^2}$

$$\text{Luego: } \alpha Y'' = \frac{1}{2} w_0 (Lx - x^2) (1 + (Y')^2)^{\frac{3}{2}} \iff \alpha Y'' + \frac{1}{2} w_0 (x^2 - Lx) (1 + (Y')^2)^{\frac{3}{2}} = 0$$

Con las diferencias finitas:

$$\frac{\alpha}{h^2} (Y_{i+1} - 2Y_i + Y_{i-1}) + \frac{1}{2} w_0 (X_i^2 - LX_i) \left(1 + \frac{(Y_{i+1} - Y_{i-1})^2}{4h^2} \right)^{\frac{3}{2}} = 0$$

Dado que $Y_0 = Y(0) = 0$, $Y_n = Y(L) = 0$:

$i = 1$

$$F_1(Y_1, \dots, Y_{n-1}) = \frac{\alpha}{h^2} (Y_2 - 2Y_1) + \frac{1}{2} w_0 (X_1^2 - LX_1) \left(1 + \frac{Y_2^2}{4h^2} \right)^{\frac{3}{2}} = 0$$

$i \in \{2, \dots, n-2\}$

$$F_i(Y_1, \dots, Y_{n-1}) = \frac{\alpha}{h^2} (Y_{i+1} - 2Y_i + Y_{i-1}) + \frac{1}{2} w_0 (X_i^2 - LX_i) \left(1 + \frac{(Y_{i+1} - Y_{i-1})^2}{4h^2} \right)^{\frac{3}{2}} = 0$$

$i = n-1$

$$F_{n-1}(Y_1, \dots, Y_{n-1}) = \frac{\alpha}{h^2} (-2Y_{n-1} + Y_{n-2}) + \frac{1}{2} w_0 (X_{n-1}^2 - LX_{n-1}) \left(1 + \frac{Y_{n-2}^2}{4h^2} \right)^{\frac{3}{2}} = 0$$

Luego tendremos resolver $F(Y_1, \dots, Y_{n-1}) = 0$ donde

$$F(Y_1, \dots, Y_{n-1}) = (F_1, \dots, F_{n-1}).$$

Para esto utilizamos Newton. Sabiendo que:

$$\frac{\partial F_i}{\partial Y_n} = \begin{cases} -\frac{2\alpha}{h^2} & K=i \\ \frac{\alpha}{h^2} + \frac{3}{2}(4h^2)^{\frac{2}{3}}(X_i^2 - LX_i)(Y_{i+1} - Y_{i-1})(4h^2 + (Y_{i+1} - Y_{i-1})^2)^{\frac{1}{2}} & \begin{matrix} K=i+1 \\ 0 \\ K=i-1 \end{matrix} \\ 0 & \text{C.L.} \end{cases}$$