DIJKSTRA'S ALGORITHM

Dijkstra's algorithm is based on a cost/value based system where each node is assigned a minimum by a set of rules (known as relaxation).

It is represented using graphs and tables.

It is applicable to both direct and indirect paths.

Algorithm is as follows-

$$D (u) + c (u, v) < d (v)$$

Where u, v are nodes.

D (u) represents the distance from the initial node to node 'u'

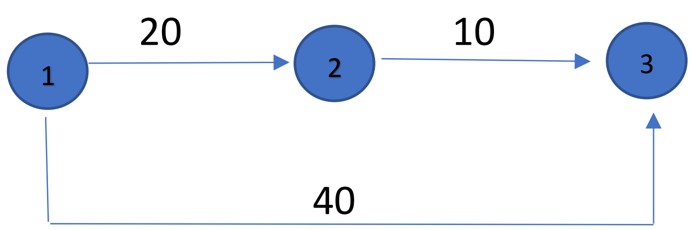
C (u, v) represents the cost to travel to final node v from initial node u.

<u>RULES OF THE ALGORITHM</u>

- Each node's value is initially initialized to infinity.
- Later when paths between the nodes is established the value (infinity) is reduced to a minimum value.
- Each set of nodes has an initial start point is initialised to zero.

Let's understand this concept using two examples

1.



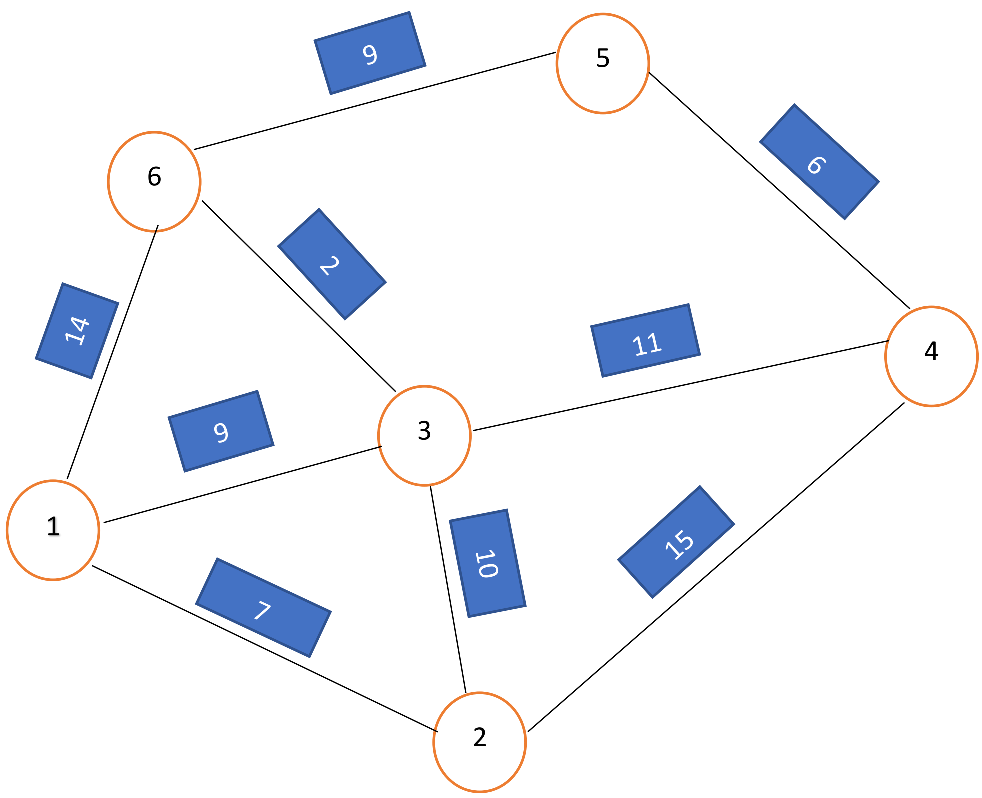
Here, $D (u) + c (u, v) < d (v)$ – eqn 1

$D (v) = d (u) + c (u, v)$ – eqn 2

Node 1 is the initial node so we set its value to zero, node 2, 3 are set to infinity.
Putting values in eqn 1 for moving from 1 to 2
0+20<infinity
As this is true we can set the value of 2 as 20 instead of infinity.
Putting values in eqn 1 for moving from 2 to 3
0+40<infinity
As this is true we can set the value of 3 as 40 instead of infinity.

Now taking distance between 1 and 3 which is 40, and substituting it in eqn 1, we get
=20+10<40
=30<40 which is true
So we can once again we can reduce (relaxation) the value of node 3 as 30.

2.



This is an example of indirect path approach which can be solved using the help of a table

Source	Destinations				
1	2	3	4	5	6
	infinity	infinity	infinity	infinity	infinity
1,2	7	9	infinity	infinity	14
1,2,3	7	9	22	infinity	14
1,2,3,6	7	9	20	infinity	11
1,2,3,6,4	7	9	20	20	11
1,2,3,6,4,5	7	9	20	20	11

Here, all nodes except 1 are set to infinity
We see that distance of 2 from 1 is 7,

Distance of 3 from 1 is 9,

Distance of 6 from 1 is 14.

Direct path

Distance of 4 from 1 via 2 is 7+15=22,
Distance of 6 from 1 via 3 is 9+2=11<14,
So we now update the value of node 6 as 11.

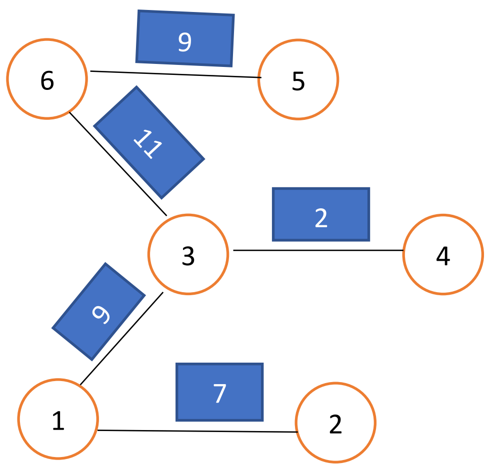
Distance of 4 from 1 via 3 is 9+11=20.

Note there is no direct or indirect path from 1 to 5 as of now.

Indirect path

Distance of 5 from 1 via updated value of 6(ie, 11) is 11+9=20.

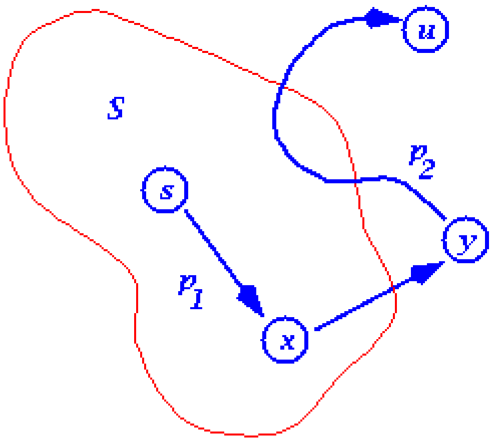
Here is the updated schematic diagram after applying the dijkstra's algorithm.



OTHER APPLICATIONS OF THE ALGORITHM

- Used for navigation purpose in google maps.
- Used in telephone networking.

PROOF OF DIJSTRA'S ALGORITHM



Proof using contradiction method:-

Let 's' be the source.

P1 is the path from s to x;

P2 is the path from y to u.

Let us denote the shortest distance between any points as del( ), and the distance from the initial node to a particular node say 'a', is given by d(a).

Let's assume 'u' to be the initial node then,

1. 'U' can't be's', as d[s]=0.

2. There also should be a path from s to u, otherwise d[u]=infinity(by the property of the algorithm).

3. If there is a path then there must also be a shortest path.

When 'x' is inserted into's' then d[x]=del(x,s) {since we assume that u is the first node for which this is not true}

Now (x,y) is relaxed so,

D[y]=del(s,y)<=del(s,u)<=d[u]

But since u is the initial node so d[u]<=d[y]

Thus the two inequalities must be equalities

D[y] =del(s, y) =del(s, u) =d[u]

So d[u]=del(s,u) is contradicting our assumption

Therefore when ant ‘u’ is added we get, d[u]=del(S,u)