CHAPTER 2

Number Systems

(Solutions to Review Questions and Problems)

Review Questions

Q2-1. A number system shows how a number can be represented using distinct symbols.

Q2-2. In a positional number system, the position of a symbol determines the value it represents. In a non-positional number system each symbol has a value but the position of a symbol normally has no relation to its value; the value of each symbol is fixed.

Q2-3. The base (or radix) is the total number of symbols used in a positional number system.

Q2-4. The decimal system is a positional number system that uses ten symbols to represent a number. The word decimal is derived from the Latin root *decem* (ten) or *decimalis* (related to ten). In the decimal system, the base is 10.

Q2-5. The binary system is a positional number system that uses two symbols (0 and 1) to represent a number. The word binary is derived from the Latin root *bini* (two by two) or *binarius* (related to two). In the binary system, the base is 2.

Q2-6. The octal system is a positional number system that uses eight symbols to represent a number. The word octal is derived from the Latin root *octo* (eight) or *octalis* (related to eight). In the octal system, the base is 8.

Q2-7. The hexadecimal system is a positional number system with sixteen symbols. The word hexadecimal is derived from the Greek root *hex* (six) and the Latin root *decem* (ten). To be consistent with decimal and binary, it should have been called *sexadecimal*, from Latin roots *sex* and *decem*. In the hexadecimal system, the base is 16.

Q2-8. Conversion is easy because there is a direct relationship between the two systems (see the answer to question 9).

Q2-9. Four bits in binary is one hexadecimal digit.

Q2-10. Three bits in binary is one octal digit.

Problems

P2-1.

a.

Place values	64	32	16	8	4	2	1	1/2	1/4	1/8	
$(01101)_2$ =	0 +	0 +	0 +	8 +	4 +	0 +	1 +	0 +	0 +	0 =	13

b.

Place values	64	32	16	8	4	2	1	1/2	1/4	1/8	
$(1011000)_2$ =	64 +	0 +	16 +	8 +	0 +	0 +	0 +	0 +	0 +	0 =	88

c.

Place values	64	32	16	8	4	2	1	1/2	1/4	1/8	
$(011110.01)_2$ =	0 +	0 +	16 +	8 +	4 +	2 +	0 +	0 +	1/4 +	0 =	30.25

d.

Place values	64	32	16	8	4	2	1	1/2	1/4	1/8	
$(111111.111)_2$ =	0 +	32 +	16 +	8 +	4 +	2 +	1 +	1/2 +	1/4 +	1/8 =	63.875

P2-2.

a.

Place values	256	16	1	1/16	1/256	
$(AB2)_{16}$ =	10×256 +	11×16 +	2×1 +	0 +	0 =	2738

b.

Place values	256	16	1	1/16	1/256	
$(123)_{16}$ =	1×256 +	2×16 +	3×1 +	0 +	0 =	291

c.

Place values	256	16	1	1/16	1/256	
$(ABB)_{16}$ =	10×256 +	11×16 +	11×1 +	0 +	0 =	2747

d.

Place values	256	16	1	1/16	1/256	
$(35E.E1)_{16}$ =	3×256 +	5×16 +	14×1 +	$14 \times 1/16$ +	$1 \times 1/256$ =	862.879

P2-3.

a.

Place values	512	64	8	1	1/8	1/64	
$(237)_8$ =		+ 2×64 +	3×8 +	7×1 +	0 +	0 =	159

b.

Place values	512	64	8	1	1/8	1/64	
$(2731)_8$ =	2×512 +	7×64 +	3×8 +	1×1 +	0 +	0 =	1497

c.

Place values	512	64	8	1	1/8	1/64	
$(617.7)_8$ =	0	+ 6 × 64	+ 1 × 8	+ 7 × 1	+ 7 × 1 / 8	+ 0	= 399.875

d.

Place values	512	64	8	1	1/8	1/64	
$(21.11)_8$ =	0	+ 0	+ 2 × 8	+ 1 × 1	+ 1 × 1 / 8	+ 1 × 1 / 64	» 17.141

P2-4.

a. $1234 = (100\ 1101\ 0011)_2$ as shown below:

0	←	1	←	2	←	4	←	9	←	19	←	38	←	77	←	154	←	308	←	617	←	1234
		↓		↓		↓		↓		↓		↓		↓		↓		↓		↓		↓
		1		**0**		**0**		**1**		**1**		**0**		**1**		**0**		**0**		**1**		**0**

b. $88 = (1011000)_2$ as shown below:

0	←	1	←	2	←	5	←	11	←	22	←	44	←	88
		↓		↓		↓		↓		↓		↓		↓
		1		**0**		**1**		**1**		**0**		**0**		**0**

c. $124.02 = (111\ 1110.00000101)_2$ as shown below. Note that we start the integral part from the left, but the fraction part from the right

0	←	1	←	3	←	7	←	15	←	31	←	62	←	124
		↓		↓		↓		↓		↓		↓		↓
		1		**1**		**1**		**1**		**1**		**0**		**0**

	.02	→	.04	→	.08	→	.16	→	.32	→	.64	→	.28	→	.56	→	.12
	↓		↓		↓		↓		↓		↓		↓		↓		
•	**0**		**0**		**0**		**0**		**0**		**1**		**0**		**1**		

d. $14.56 = (1110.100011)_2$ as shown below:

0	←	1	←	3	←	7	←	14		.56	→	.12	→	.24	→	.48	→	.96	→	.92	→	.84
		↓		↓		↓		↓		↓		↓		↓		↓		↓		↓		
		1		**1**		**1**		**0**	•	**1**		**0**		**0**		**0**		**1**		**1**		

P2-5.

a. $1156 = (2204)_8$ as shown below:

0	←	2	←	18	←	144	←	1156
		↓		↓		↓		↓
		2		2		0		4

b. $99 = (134)_8$ as shown below:

0	←	1	←	12	←	99
		↓		↓		↓
		1		4		3

c. $11.4 = (13.3146)_8$ as shown below:

0	←	1	←	11		.4	→	.2	→	.6	→	.8	→	4
		↓		↓		↓		↓		↓		↓		
		1		3	•	3		1		4		6		

d. $72.8 = (110.6314)_8$ as shown below:

0	←	1	←	9	←	72		.8	→	.4	→	.2	→	.6	→	.8
		↓		↓		↓		↓		↓		↓		↓		
		1		1		0	•	6		3		1		4		

P2-6.

a. $576 = (237)_{16}$ as shown below:

0	←	2	←	35	←	567
		↓		↓		↓
		2		3		7

b. $1411 = (583)_{16}$ as shown below:

0	←	5	←	88	←	1411
		↓		↓		↓
		5		8		3

c. $12.13 = (C.2147AE)_{16}$ as shown below:

0	←	12		.13	→	.08	→	.48	→	.68	→	.88	→	.08
		↓		↓		↓		↓		↓		↓		
		C	•	2		1		7		A		E		

d. $16.5 = (10.8)_{16}$ as shown below:

0	←	1	←	16		.5	→	0
		↓		↓		↓		
		1		0	•	8		

P2-7.

a.

Change octal to binary						*Change binary to hexadecimal*					
$(514)_8$	=	101	001	100	•	=	1	0100	1100	•	= $(14C)_{16}$

b.

Change octal to binary						*Change binary to hexadecimal*					
$(411)_8$	=	100	001	001	•	=	1	0000	1001	•	= $(109)_{16}$

c.

Change octal to binary						*Change binary to hexadecimal*					
$(13.7)_8$	=	001	111	•	111	=	00	1011	•	1110	= $(B.E)_{16}$

d.

Change octal to binary							Change binary to hexadecimal					
$(1256)_8$	=	001	010	101	110	•	=	0010	0101	1110	•	= **$(25E)_{16}$**

P2-8.

a.

Change Hexadecimal to Binary						Change Binary to Octal					
$(51A)_{16}$	=	0101	0001	1010	•	=	010	100	011	010	• = **$(2432)_8$**

b.

Change Hexadecimal to Binary						Change Binary to Octal					
$(4E1)_{16}$	=	0100	1110	0001	•	=	010	011	100	001	• = **$(2341)_8$**

c.

Change Hexadecimal to Binary						Change Binary to Octal					
$(BB.C)_{16}$	=		1011	1011	• 1100	=		010	111	011	• 110 = **$(273.6)_8$**

d.

Change Hexadecimal to Binary						Change Binary to Octal						
$(ABC.D)_{16}$	=	1010	1011	1100	• 1101	=	101	010	111	100	• 110	100 = **$(5274.64)_8$**

P2-9.

a.

$(01101)_2$	=		001	101	•	= **$(15)_8$**

b.

$(1011000)_2$	=	001	011	000	•	= **$(130)_8$**

c.

$(011110.01)_2$	=		011	110	• 010	= **$(36.2)_8$**

d.

$(111111.111)_2$	=		111	111	• 111	= **$(77.7)_8$**

P2-10.

a.

$(01101)_2$	=		1101	•	= **$(0D)_{16}$**

b.

$(1011000)_2$	=	0101	1000	•	= **$(58)_{16}$**

c.

$(011110.01)_2$	=	0001	1110	• 0100	= **$(1E.4)_{16}$**

d.

$(111111.111)_2$	=	0011	1111	• 1110	= **$(3F.E)_{16}$**

P2-11.

a.

121	=	0	+	64	+	32	+	16	+	8	+	0	+	0	+	1	=	$(01111001)_2$

b.

78	=	0	+	64	+	0	+	0	+	8	+	4	+	2	+	0	=	$(01001110)_2$

c.

255	=	128	+	64	+	32	+	16	+	8	+	4	+	2	+	1	=	$(11111111)_2$

d.

214	=	128	+	64	+	0	+	16	+	0	+	4	+	2	+	0	=	$(11010110)_2$

P2-12.

a.

3 5/8	=	2	+	1	+	1/2	+	0	+	1/8	=	$(11.101)_2$

b.

12 3/32	=	8	+	4	+	0	+	0	+	0	+	1/16	+	1/32	=	$(1100.0001100)_2$

c.

4 13/64	=	4	+	0	+	0	+	1/8	+	1/16	+	0	+	1/64	=	$(100.0011010)_2$

d.

12 5/8	=	8	+	4	+	0	+	0	+	0	+	1/32	+	0	+	1/128	=	$(1100.0000101)_2$

P2-13.

a. binary: $2^6 - 1 = 63$

b. decimal: $10^6 - 1 = 999{,}999$

c. hexadecimal: $16^6 - 1 = 16{,}777{,}215$

d. octal: $8^6 - 1 = 262{,}143$

P2-14.

a. $\lceil 5 \times (\log 10) / (\log 2) \rceil = \lceil 16.6 \rceil = 17$

b. $\lceil 4 \times (\log 10) / (\log 8) \rceil = \lceil 4.4 \rceil = 5$

c. $\lceil 7 \times (\log 10) / (\log 16) \rceil = \lceil 5.8 \rceil = 6$

P2-15.

a. $\lceil 5 \times (\log 2) / (\log 10) \rceil = \lceil 16.6 \rceil = 2$

b. $\lceil 3 \times (\log 8) / (\log 10) \rceil = \lceil 16.6 \rceil = 3$

c. $\lceil 3 \times (\log 16) / (\log 10) \rceil = \lceil 16.6 \rceil = 4$

P2-16.

a. $0.1875 = 0.125 + 0.0625 = (1/8) + (1/16) = (3/16)$

b. $0.640625 = 0.5 + 0.125 + 0.015625 = (1/2) + (1/8) + (1/64) = (41/64)$

c. $0.40625 = 0.25 + 0.125 + 0.03125 = (1/4) + (1/8) + (1/32) = (13/32)$

d. $0.375 = 0.25 + 0.125 = (1/4) + 1/8) = 3/8$

P2-17. Using the result of previous exercise, we can find the equivalent as:

a. $7.1875 = (111)_2 + (0.001)_2 + (0.0001)_2 = (111.0011)_2$

b. $12.540625 = (1100)_2 + (0.1)_2 + (0.001)_2 + (0.000001)_2 = (1100.101001)_2$

c. $11.40625 = (1011)_2 + (0.01)_2 + (0.001)_2 + (0.00001)_2 = (1011.01101)_2$

d. $0.375 = (0.01)_2 + (0.001)_2 = (0.011)_2$

P2-18.

a. $10^{10} - 1 = 999$

b. $22^1 - 1 = 4095$

c. $8^8 - 1 = 16{,}777{,}215$

d. $16^7 - 1 = 268{,}435{,}455$

P2-19.

a. $\lceil \log_2 1000 \rceil = \lceil \log 1000 / \log 2 \rceil = \lceil 9.97 \rceil = 10$

b. $\lceil \log_2 100{,}000 \rceil = \lceil \log 100{,}000 / \log 2 \rceil = \lceil 16.6 \rceil = 17$

c. $\lceil \log_2 64 \rceil = \lceil \log_2 2^6 \rceil = \lceil 6 \times \log_2 2 \rceil = \lceil 6 \rceil = 6$

d. $\lceil \log_2 256 \rceil = \lceil \log_2 2^8 \rceil = \lceil 8 \times \log_2 2 \rceil = \lceil 8 \rceil = 8$

P2-20.

a. 14

b. 8

c. 13

d. 4

P2-21.

a.	17×256^3	+	234×256^2	+	34×256^1	+	14×256^0	=	**300,556,814**
b.	14×256^3	+	56×256^2	+	234×256^1	+	56×256^0	=	**238,611,000**
c.	110×256^3	+	14×256^2	+	56×256^1	+	78×256^0	=	**1,864,425,678**
d.	24×256^3	+	56×256^2	+	13×256^1	+	11×256^0	=	**406,326,539**

P2-22.

a.	00010001	11101010	00100010	00001110
b.	00001110	00111000	11101010	00111000
c.	01101110	00001110	00111000	01001110
d.	00011000	00111000	00001101	00001011

P2-23.

a. 15

b. 27

c. This is not a valid Roman Numeral (V cannot come before L)

d. 1157

P2-24.

a. XVII

b. XXXVIII

c. LXXXII

d. CMXCIX

P2-25.

a. Not valid because I cannot come before M

b. Not valid because I cannot come before C

c. Not valid because V cannot come before C

d. Not valid because 5 is written as V not VX

P2-26. First, we convert the four numbers to base 20 as shown below:

0	←	12
		↓
		12

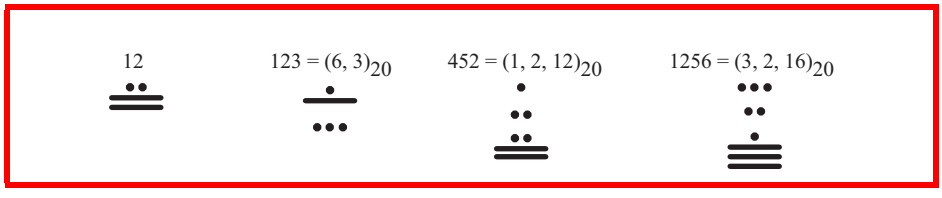
0	←	6	←	123
		↓		↓
		6		**3**

0	←	1	←	22	←	452
		↓		↓		↓
		1		**2**		**12**

0	←	3	←	62	←	1256
		↓		↓		↓
		3		**2**		**16**

The equivalent Myan numerals are in Figure 2.1.

Figure 2.1 *Solution to P2-26*



P2-27.

a. First, we convert the three numbers to base 60 as shown below:

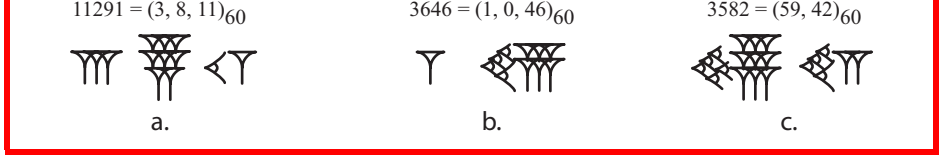
0	←	3	←	188	←	11291
		↓		↓		↓
		3		**8**		**11**

0	←	1	←	60	←	3646
		↓		↓		↓
		1		**0**		**46**

0	←	59	←	3582
		↓		↓
		59		**42**

The equivalent Babylonian numerals are shown in Figure 2.2.

Figure 2.2 *Solution to P2-27*



b. In Babylonian numerals, they used extra space when a zero was needed in the middle of the number. When a zero was need at left, they did not use anything; They probably recognized it from the context.