UNIVERSITY OF GHANA

BSc. ENGINEERING/FIRST SEMESTER EXAMINATIONS: 2020/2021

DEPARTMENT OF COMPUTER ENGINEERING

CPEN 205: DISCRETE MATHEMATICAL STRUCTURES (2 CREDITS)

INSTRUCTIONS:

ANSWER ALL QUESTIONS

EACH QUESTION CARRIES 25 MARKS

TIME ALLOWED: TWO (2) HOURS

Q1.

a)

i. Define a *recurrence relation*. [1 MARK]

ii. What is the solution of the linear homogeneous recurrence relation?
$a_n = -2a_{n-1} + 15a_{n-2}$
with $a_0 = 0$, and $a_1 = 1$ [3 MARKS]

iii. What is the solution of the linear homogeneous recurrence relation?
$a_n = 4a_{n-1} - 4a_{n-2}$
with $a_0 = 1$ and $a_1 = 2$ [3 MARKS]

iv. What is the solution of the same equation in (iii) with
$a_0 = 1$ and $a_1 = 8$? [2 MARKS]

b) The Fibonacci numbers satisfy the linear homogeneous recurrence relation

$f_n = f_{n-1} + f_{n-2}$ with initial conditions $f_0 = 0$ and $f_1 = 1$.

i. List the *first fifteen* Fibonacci numbers using the recurrence relation above. [3 MARKS]

ii. Prove that the solution (explicit formula) to the Fibonacci recurrence is

$$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

[5 MARKS]

c) A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.

i. Find a recurrence relation for $\{Ln\}$, where Ln is the number of lobsters caught in year n, under the assumption for this model. [2 MARKS]

ii. Find Ln if 100,000 lobsters were caught in year 1 and 300,000 were caught in year 2. [5 MARKS]

Q2.

a) How many license plates can be made using either *two* or *three letters* followed by either *two* or *three digits* and contain no letter or digit twice? [4 MARKS]

b) The *sixth* permutation of the lexicographic permutations of the 24 elements of the set {1, 2, 3, 4} is *1432*. Find the next **ten** permutations in *lexicographic order* after *1432*.

[5MARKS]

c) Suppose that a department contains 10 men and 15 women. How many ways are there to select a committee with *six* members if it must have

i. At most three women? [3 MARKS]
ii. At least 1 woman and at least 1 man? [4 MARKS]

d) Prove that

$\binom{2n}{2} - 2\binom{n}{2} = n^2$, where n is a positive integer. [4 MARKS]

e) Each user on a computer system has a password, which is *six to eight characters long*, where each character is an *uppercase letter* or *digit*. Each password must contain *at least one digit*. How many possible passwords are there? [5 MARKS]

Q3.

a) Show by means of **truth tables** that each of these conditional statements is a **tautology**.

i. $(p \wedge q) \rightarrow (p \rightarrow q)$ [4 MARKS]

ii. $\neg(p \rightarrow q) \rightarrow p$ [4 MARKS]

b) Use De Morgan's laws to find the *negation* of each of the following statements.

i. Jan is rich and happy. [1 MARK]

ii. Carlos will bicycle or run tomorrow. [1 MARK]

iii. Mei walks or takes the bus to class. [1 MARK]

iv. Ibrahim is smart and hard working. [1 MARK]

c) Draw Venn diagrams to discover whether or not the following are true?

i. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [4 MARKS]

ii. $(A \cap B)' = A' \cup B'$ [3 MARKS]

d) Let $T(x, y)$ be the statement "*x trusts y*," where the domain consists of all people in the
world. Use quantifiers to express each of these statements.

i. Everybody trusts Bob. [1 MARK]

ii. Bob trusts somebody. [1 MARK]

iii. Alice trusts herself. [1 MARK]

iv. Everyone trusts somebody. [1 MARK]

v. Someone trusts everybody. [1 MARK]

vi. Somebody is trusted by everybody. [1 MARK]

Q4.

a) A new car is worth only about 82 % of its value from the previous year during the first *three* years. Approximately how much will a $20,000 car be worth in 3 years? [4 MARKS]

b) Let *p* be "Bonsu is rich" and let *q* be "Bonsu is happy". Write each of the following in symbolic form using logical *operators* or *connectives*.

i. Bonsu is poor but happy; [1 MARK]

ii. Bonsu is neither rich nor happy; [1 MARK]

iii. Bonsu is either rich or unhappy; and [1 MARK]

iv. Bonsu is poor or else he is both rich and unhappy. [2 MARKS]

c) Let $Q(x, y)$ be the statement "*x has sent an e-mail message to y*," where the domain for
both x and y consists of all students in your class. Express each of these quantifications
in English.

i. $\exists x \exists y Q(x, y)$ [1 MARK]

ii. $\exists x \forall y Q(x, y)$ [1 MARK]

iii. $\forall x \exists y Q(x, y)$ [1 MARK]

iv. $\exists y \forall x Q(x, y)$ [1 MARK]

v. $\forall y \exists x Q(x, y)$ [1 MARK]

vi. $\forall x \forall y Q(x, y)$ [1 MARK]

d) Let the universe be the set of *all animals*, and define the following predicates:

- B(x) ≡ *x is a bird*
- D(x) ≡ *x is a dove*
- C(x) ≡ *x is a chicken*
- P(x) ≡ *x is a pig*
- F(x) ≡ *x can fly*
- W(x) ≡ *x has wings*
- M(x, y) ≡ *x has more feathers than y does*

Based on the predicates above, translate the following sentences into logic.

i. Chickens are birds [1 MARK]

ii. Some doves can fly [1 MARK]

iii. Pigs are not birds [1 MARK]

iv. Some birds can fly and some can't [1 MARK]

v. If a chicken can fly, then pigs have wings [1 MARK]

vi. Chickens have more feathers than pigs do. [1 MARK]

e) Let $A(x)$, $C(x)$, $S(x)$ and $GP(x)$ be the statements "*x is an animal*," "*x is a cat*," "*x is small*," and "*x is a good pet*," respectively. Express each of these quantifications in English where the universe consists of *all animals*.

i. $\forall x\ (C(x) \rightarrow A(x))$ [1 MARK]

ii. $\neg \exists x (C(x) \wedge \neg S(x))$ [1 MARK]

iii. $\forall x\ (C(x) \rightarrow S(x) \wedge A(x))$ [1 MARK]

iv. $\forall x\ (S(x) \wedge A(x) \rightarrow GP(x))$ [1 MARK]