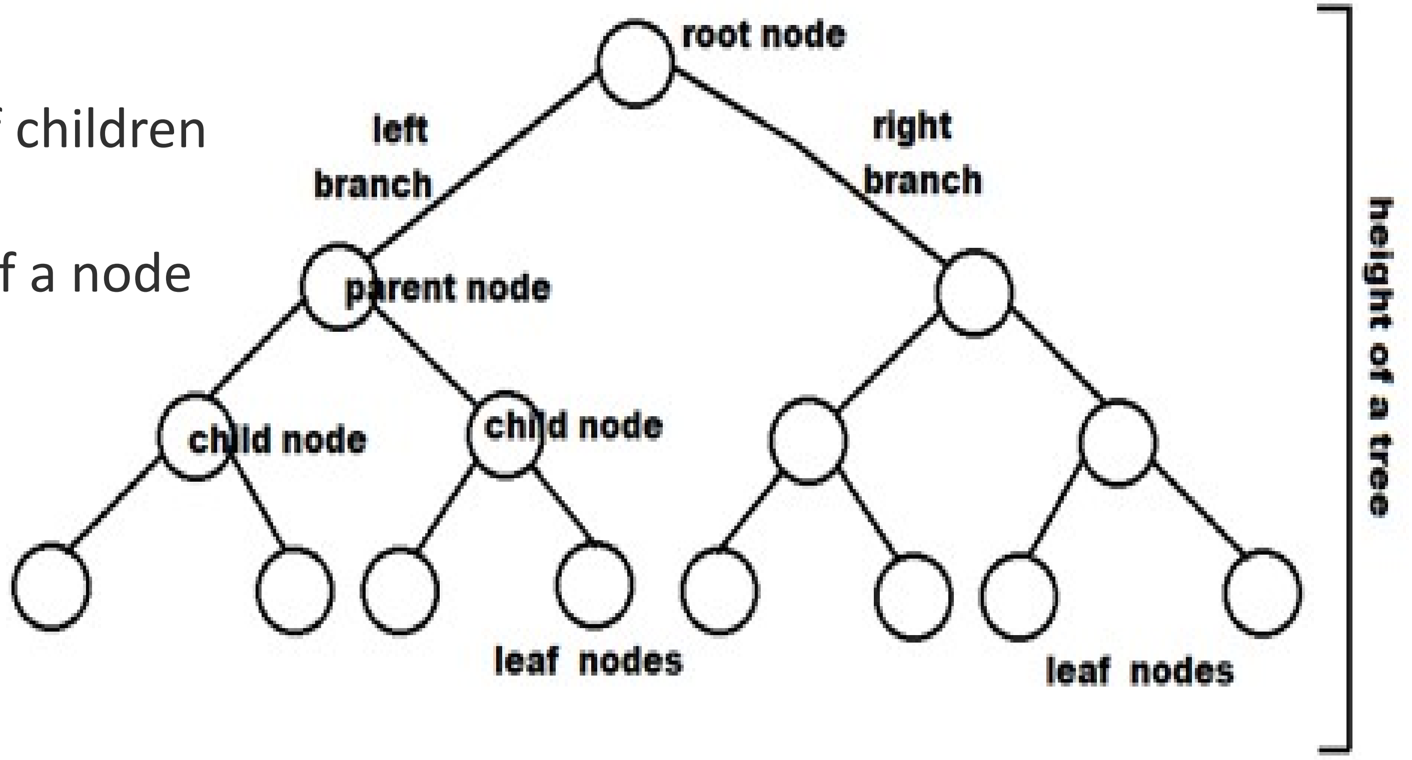
Binary Tree
and
Binary Search Tree

Tree

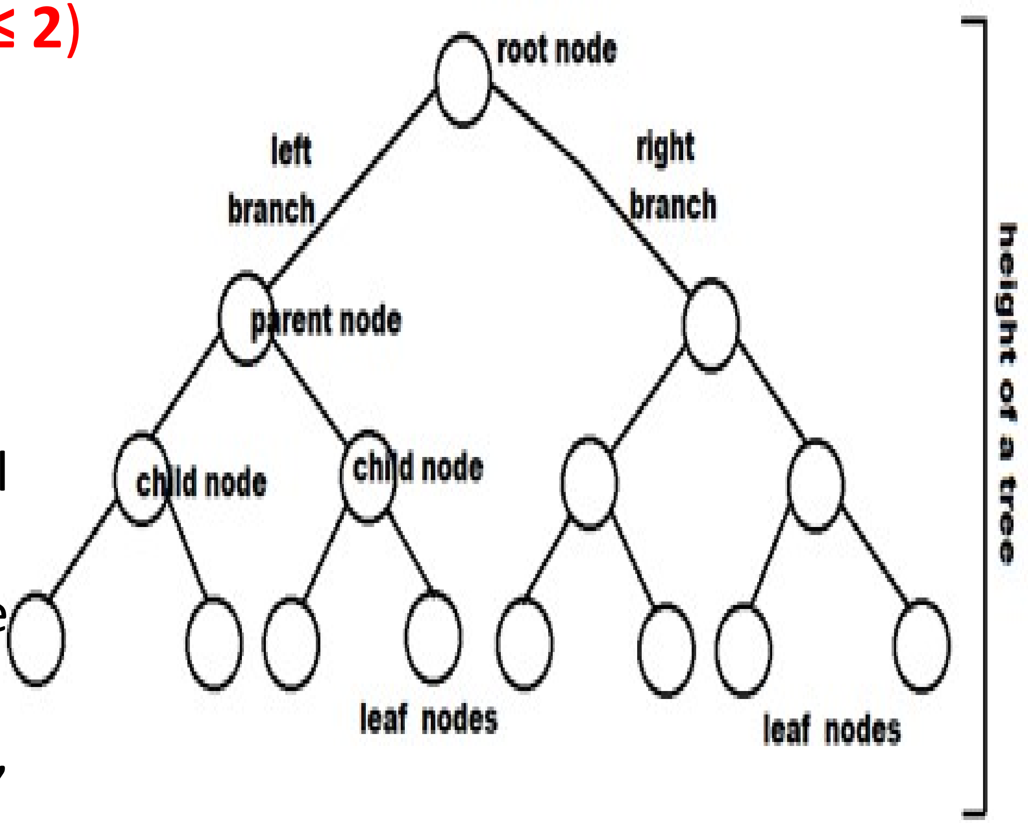
- Trees are Non-Linear and Abstract Data Structures that simulate a hierarchical structure.
- There is one and only one path between every pair of vertices in a tree.
- There is (n-1) edges in a tree with n vertices.
  - Any connected graph with n vertices and (n-1) edges is a tree.

- **Degree of a node** is the total number of children of that node.
- **Degree of a tree** is the highest degree of a node among all the nodes in the tree.
- Height of a node
- Height of the Tree
- Depth of a node
- Forest



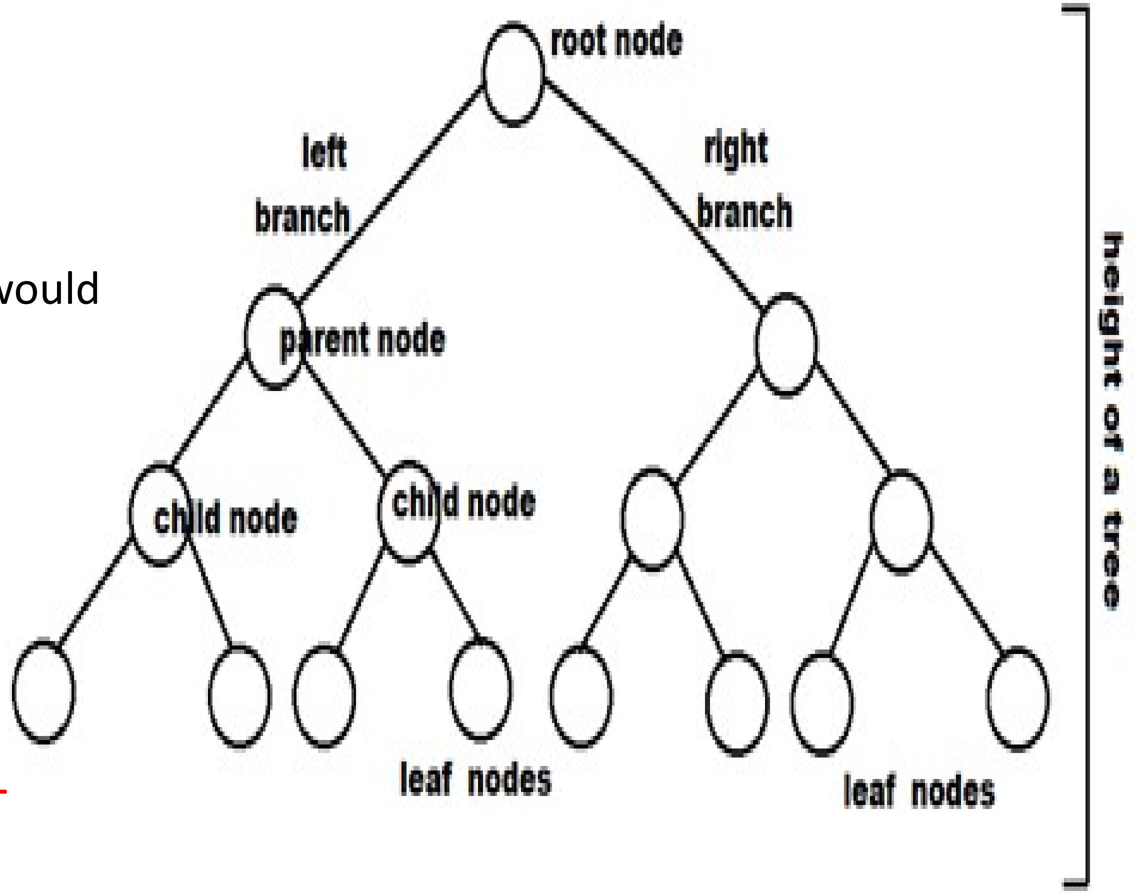
Binary Tree

- A binary tree is a data structure composed of nodes, where each node contains a value and two references to other nodes, called the left child and the right child.
- In a binary tree each node can have n children **($0 \le n \le 2$)**



- At each level of *i*, the maximum number of nodes is 2^i.
- If height = 4, the maximum number of nodes (1+2+4+8+16) = 31
  - at height h is $2^0 + 2^1 + 2^2 + \ldots 2^h = 2^{h+1} - 1$.
- The minimum number of nodes possible at height h is equal to **h+1**.
- If the number of nodes is minimum, then the height of the tree would be maximum.
- On the other hand, if the number of nodes is maximum, then the height of the tree would be minimum.

Binary Tree



- The minimum number of nodes possible at height h is equal to **h+1**.
- If the number of nodes is minimum, then the height of the tree would be maximum.
  - Let's n is number of nodes in the binary tree.
  - Then, n = h+1; meaning h= n-1

- At each level of *i*, the maximum number of nodes is 2^i.
- If height = 4, the maximum number of nodes (1+2+4+8+16) = 31
  - at height h is $2^0 + 2^1 + 2^2 + \ldots 2^h = 2^{h+1} - 1$.

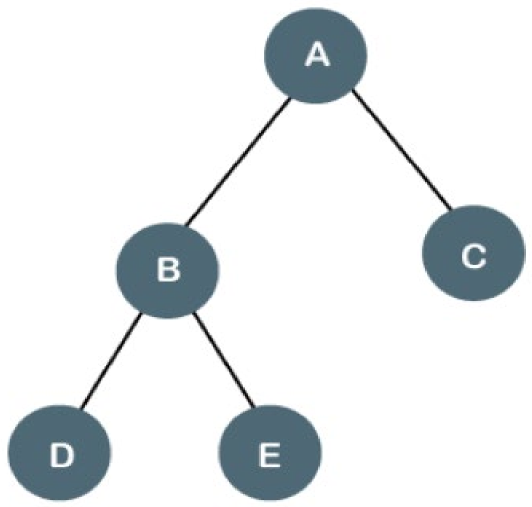
- if the number of nodes is maximum, then the height of the tree would be minimum.

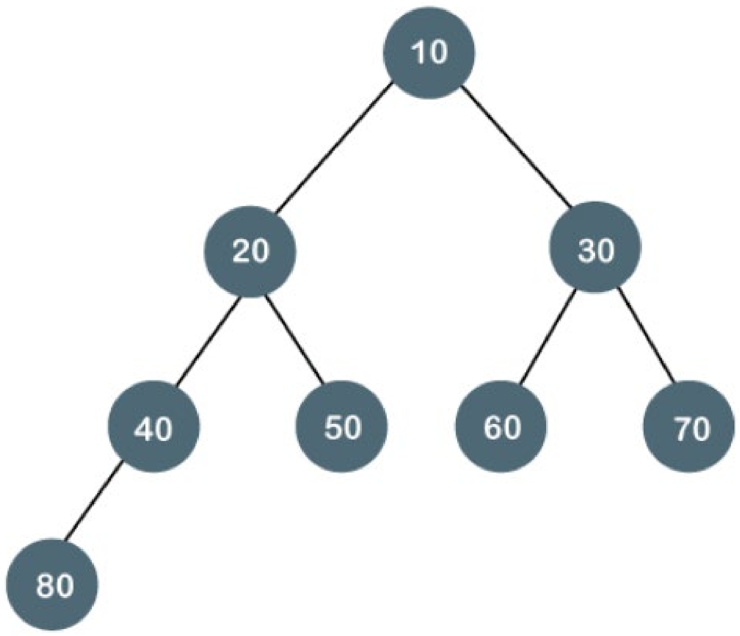
Let's n is number of nodes in the binary tree.

i.e. $n = 2^{h+1} - 1$;
hence, minimum height, $\mathbf{h = \log_2(n+1) - 1}$

Types of Binary tree

- ❑Full binary tree: A full binary tree is a binary tree in which every node has either two children or no children.
  - ❑ each node must contain 2 children except the leaf nodes.

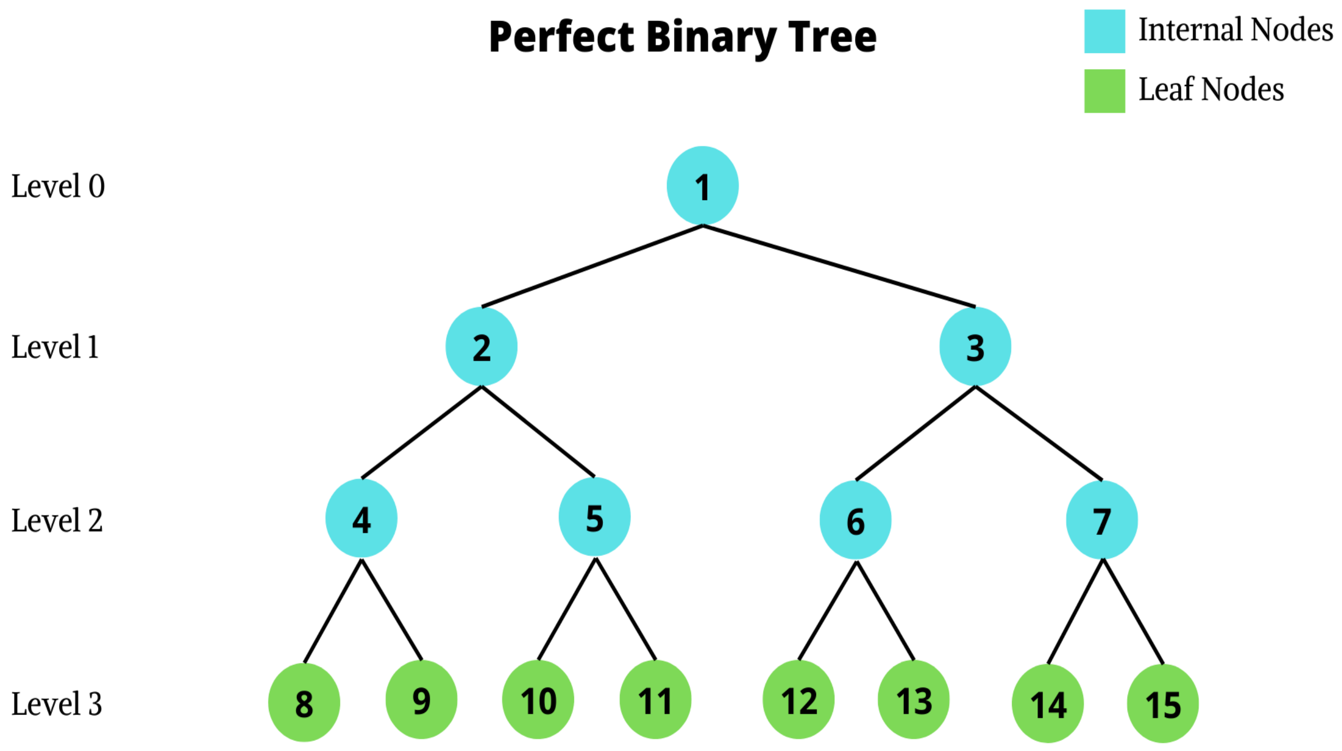




- ❑Complete binary tree: A complete binary tree is a binary tree in which all levels except the last are completely filled
  - ❑ All of the nodes must be as far to the left as feasible in the last level. The nodes should be added from the left

Types of Binary tree

Perfect binary tree: A perfect binary tree is a binary tree in which all internal nodes have two children and all leaf nodes are at the same level.

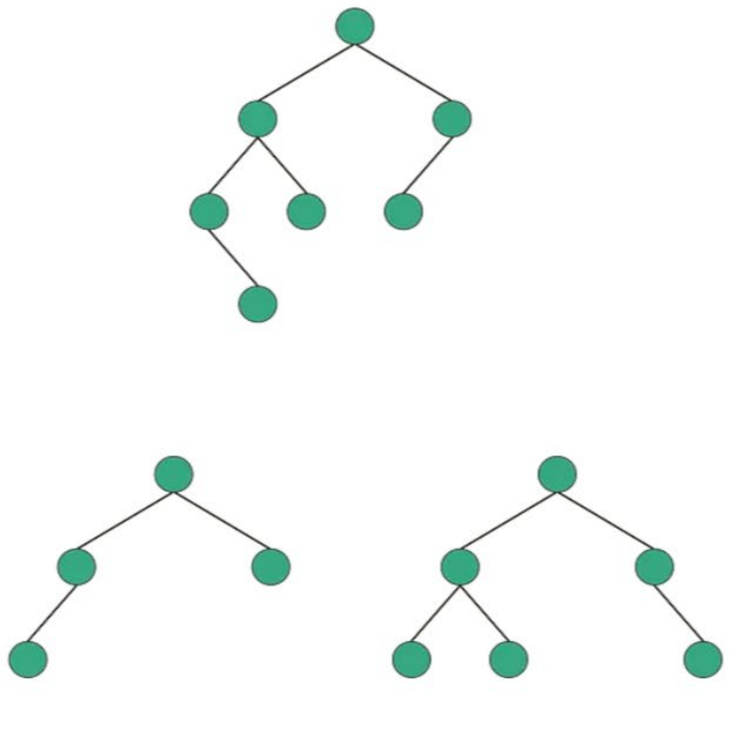


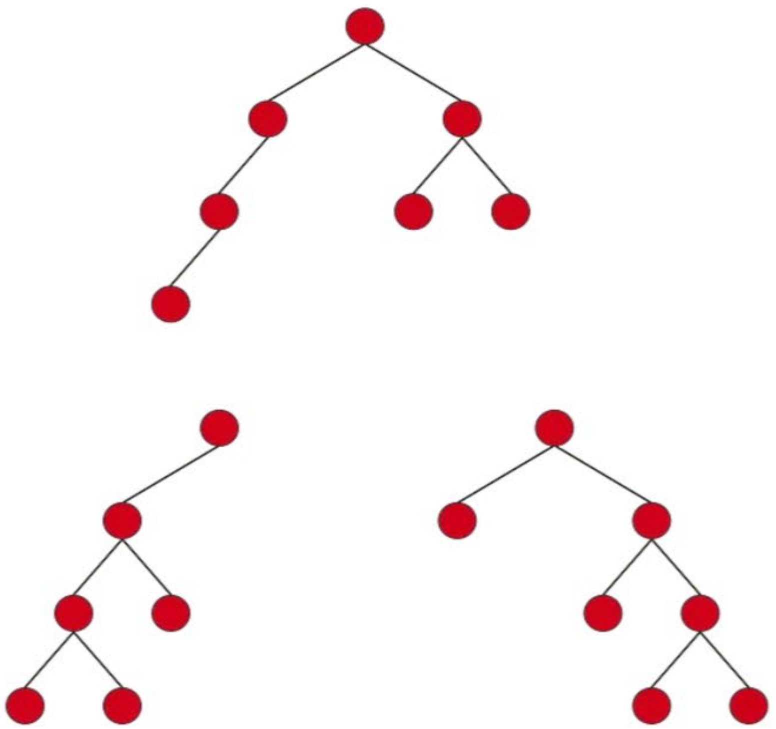
Types of Binary tree

Balanced binary tree: A balanced binary tree is a binary tree in which the heights of the left and right subtrees of every node differ by at most one.

Some operations in a an unbalanced tree may take longer than O(log n) time

AVL Tree and Red-Black Tree are well-known data structure to generate/maintain Balanced Binary Search Tree.

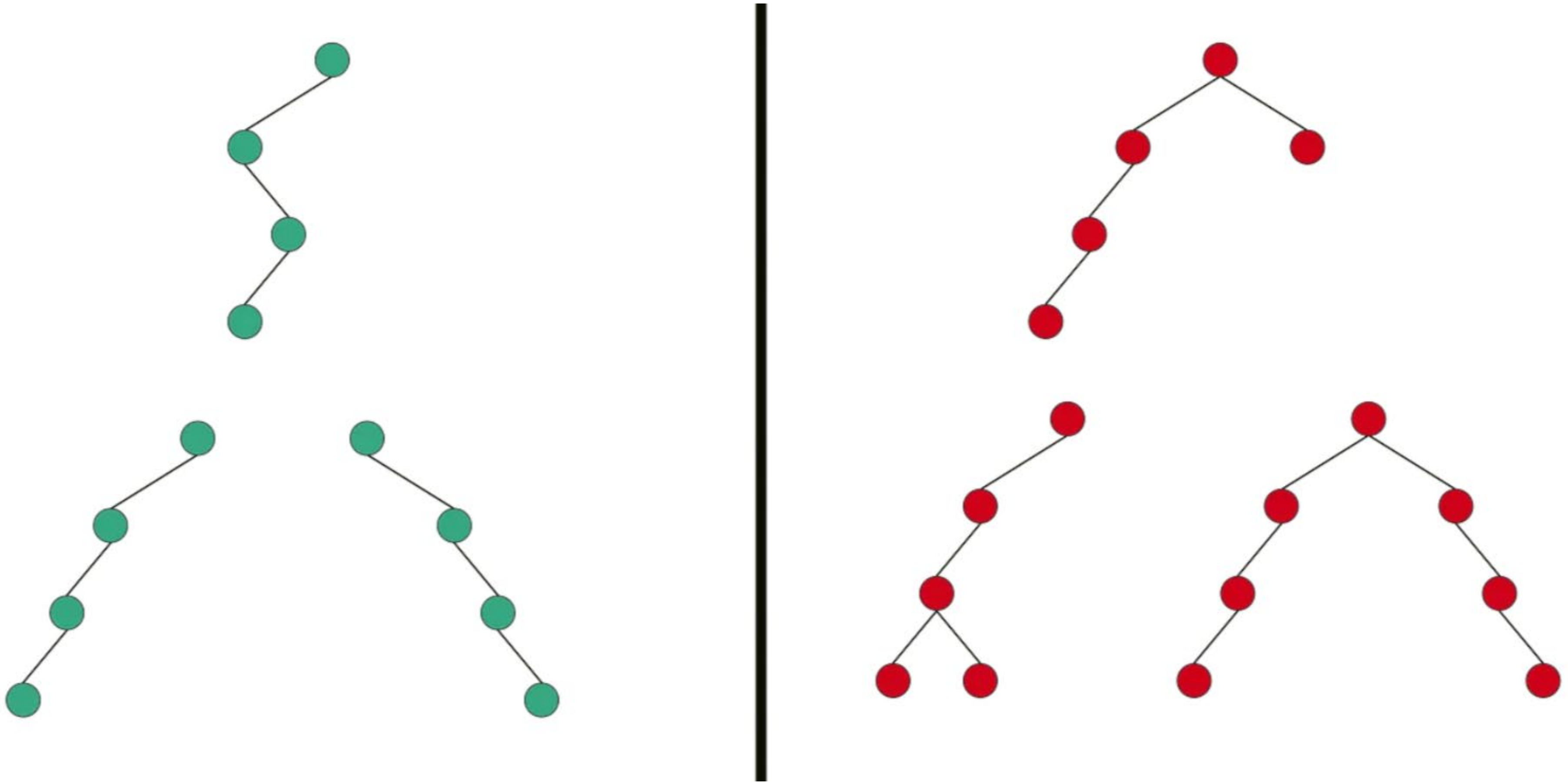




Types of Binary tree

Degenerate (or pathological) tree: A degenerate (or pathological) tree is a tree in which each parent node has only one associated child node.

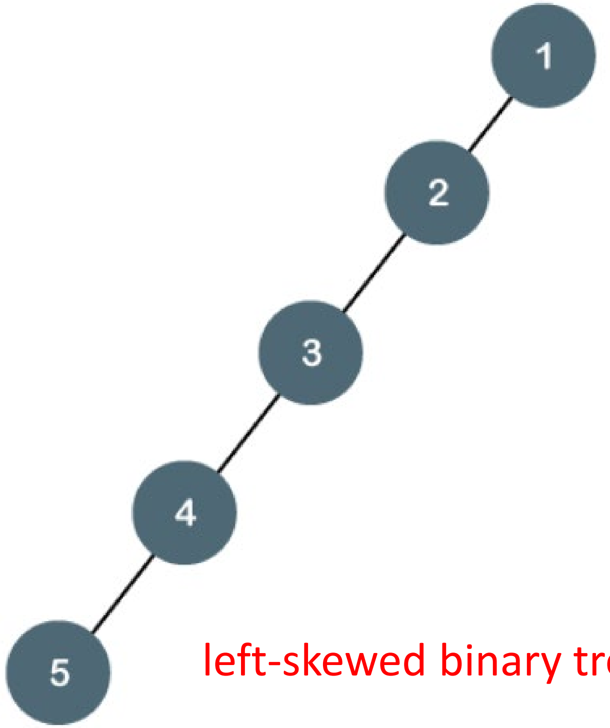
Height of a Degenerate Binary Tree is equal to Total number of nodes in that tree.



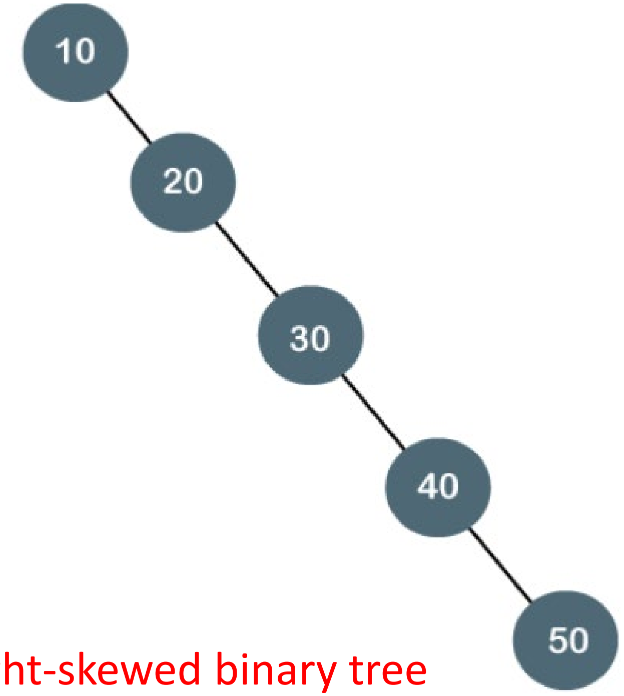
Types of Binary tree

Skewed binary tree

- In a skewed binary tree is a special type of Degenerate (or pathological) tree,
- Here, all the nodes are either left-skewed or right-skewed.
- A left-skewed binary tree is a tree in which each node has at most one right child.
- A right-skewed binary tree is a tree in which each node has at most one left child.



left-skewed binary tree



right-skewed binary tree

Binary Search Tree Traversal

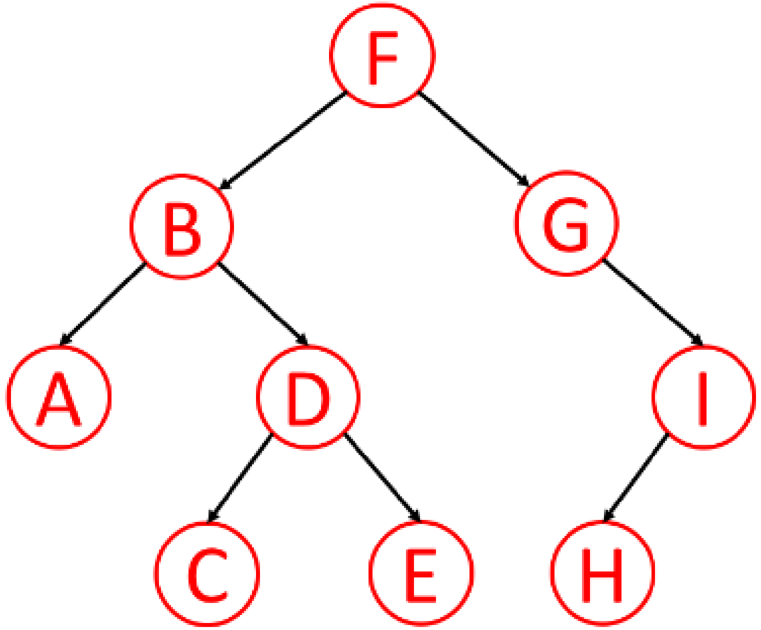
- Traverse the BST to visit all the nodes in the tree in a specific order. There are three main types of traversal: in-order, pre-order, and post-order.

- In-order traversal visits the nodes in ascending order of their keys
- pre-order traversal visits the current node before its children
- post-order traversal visits the current node after its children

- Traversal can be implemented using recursion or an iterative algorithm, and the time complexity is $O(n)$, where n is the number of nodes in the tree.

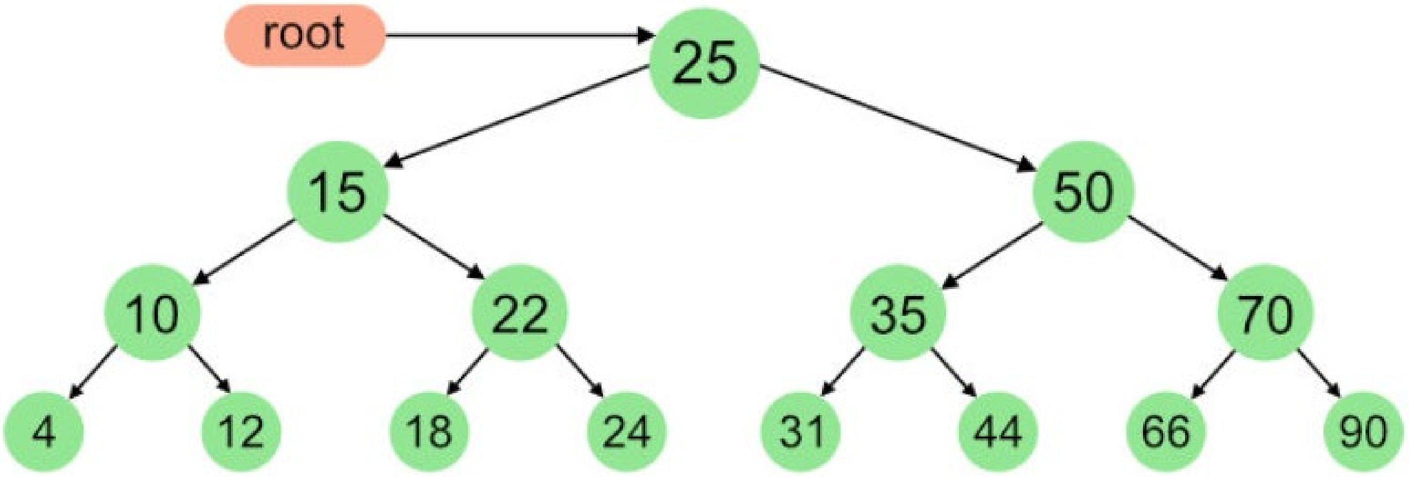
Binary Tree: Pre-order Traversal

- Pre-order traversal is to visit the root first. Then traverse the left subtree. Finally, traverse the right subtree.



Preorder:

F	B	A	D	C	E	G	I	H

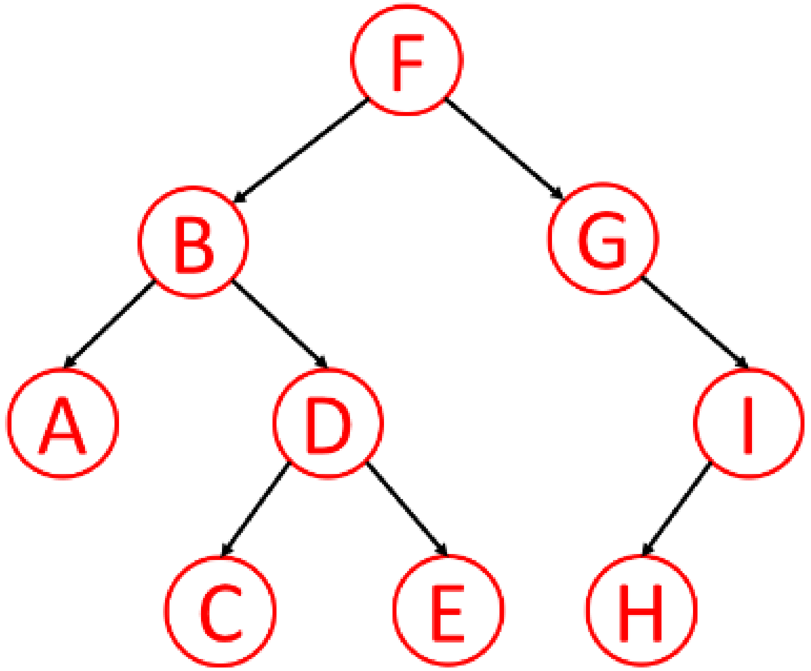


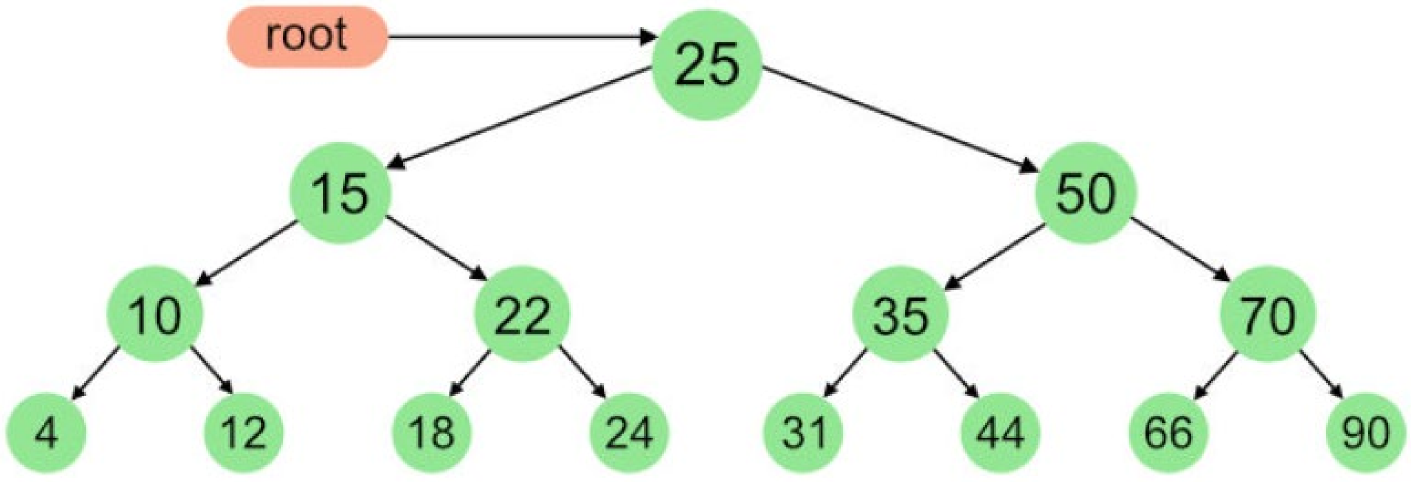
25, 15, 10, 4, 12, 22, 18, 24, 50, 35, 31, 44, 70, 66, 90

```python
def preorder(self):
    print(self.val)
    if self.left is not None:
        self.left.preorder()
    if self.right is not None:
        self.right.preorder()
```

Binary Tree: In-order Traversal

- In-order traversal is to traverse the left subtree first. Then visit the root. Finally traverse the right subtree.





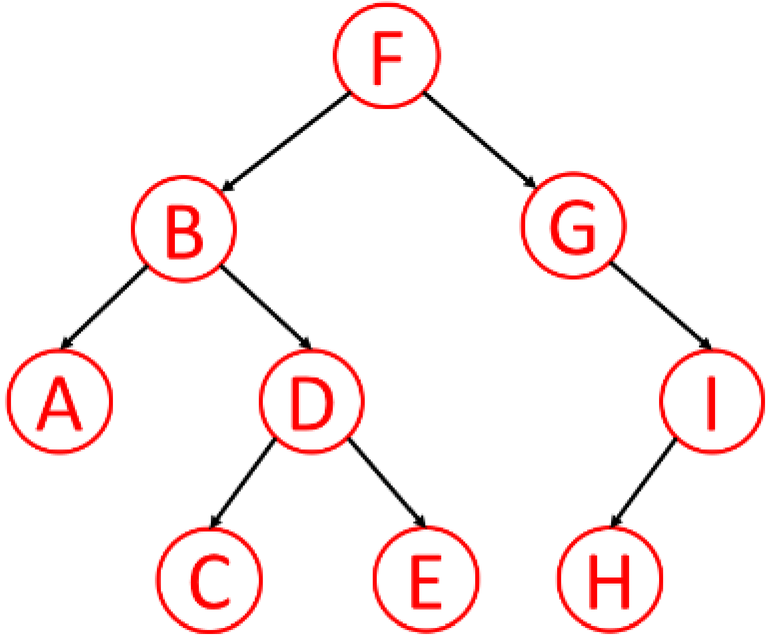
4, 10, 12, 15, 18, 22, 24, 25, 31, 35, 44, 50, 66, 70, 90

Inorder:	A	B	C	D	E	F	G	H	I

```python
def inorder(self):
    if self.left is not None:
        self.left.inorder()
    print(self.val)
    if self.right is not None:
        self.right.inorder()
```

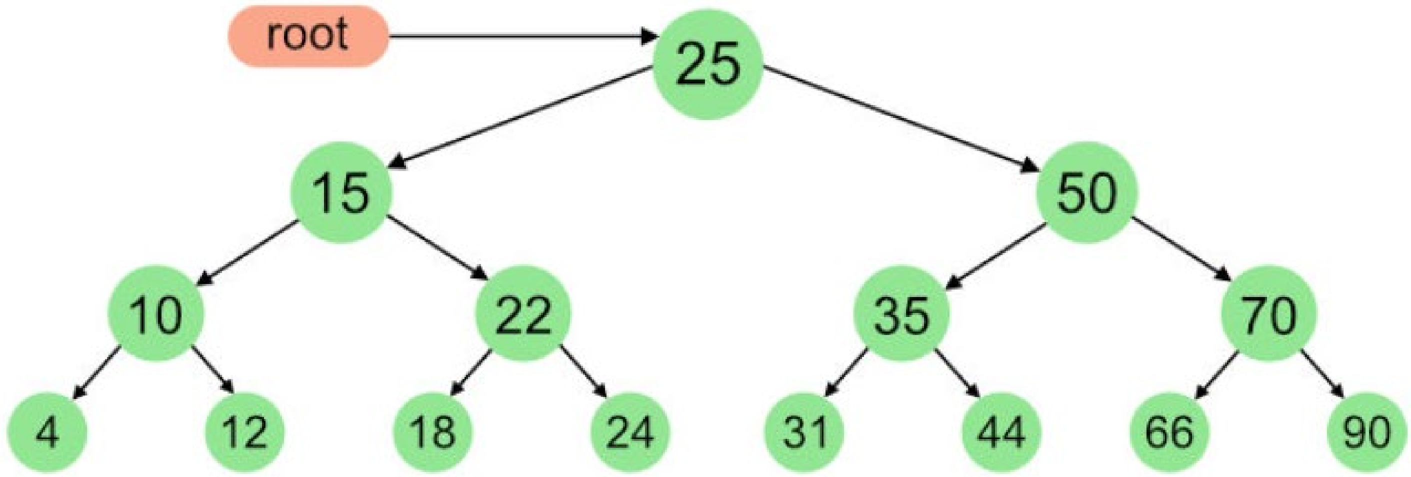
Binary Tree: Post-order Traversal

- Post-order traversal is to traverse the left subtree first. Then traverse the right subtree. Finally, visit the root.



Postorder:

A	C	E	D	B	H	I	G	F



4, 12, 10, 18, 24, 22, 15, 31, 44, 35, 66, 90, 70, 50, 25

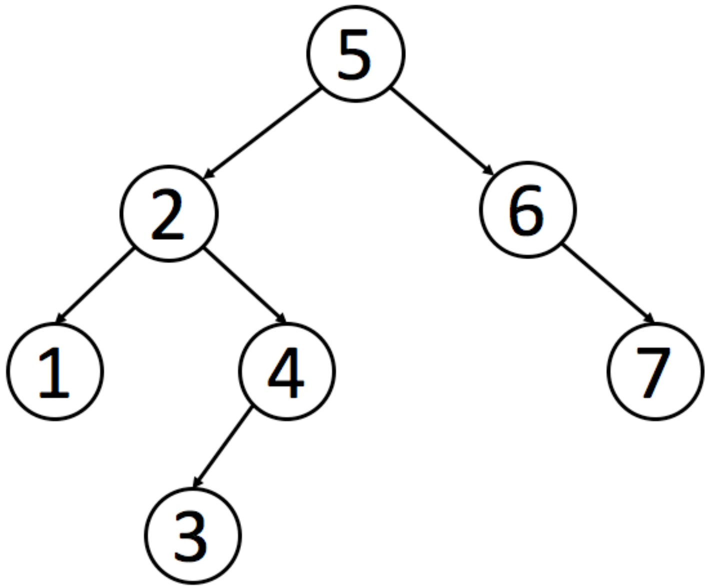
```python
def postorder(self):
    if self.left is not None:
        self.left.postorder()
    if self.right is not None:
        self.right.postorder()
    print(self.val)
```

Traversal code

```
def inorder(self):
    if self.left is not None:
        self.left.inorder()
    print(self.val)
    if self.right is not None:
        self.right.inorder()

def preorder(self):
    print(self.val)
    if self.left is not None:
        self.left.preorder()
    if self.right is not None:
        self.right.preorder()

def postorder(self):
    if self.left is not None:
        self.left.postorder()
    if self.right is not None:
        self.right.postorder()
    print(self.val)
```

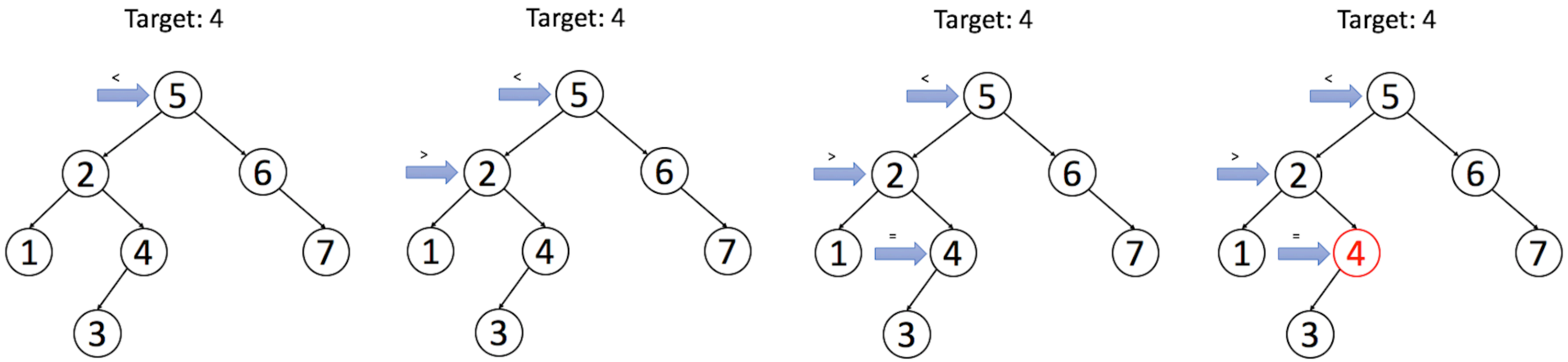
Binary Search Tree

- A Binary Search Tree is a special form of a binary tree.
- The value in each node must be greater than (or equal to) any values in its left subtree, but less than (or equal to) any values in its right subtree.



Binary Search Tree – Search Operation

- return the node if the target value is equal to the value of the node;
- continue searching in the *left subtree* if the target value is less than the value of the node;
- continue searching in the right subtree if the target value is larger than the value of the node.
- The time complexity of the search operation in a balanced BST is O(log n), where n is the number of nodes in the tree.



Binary Search Tree – Search Operation

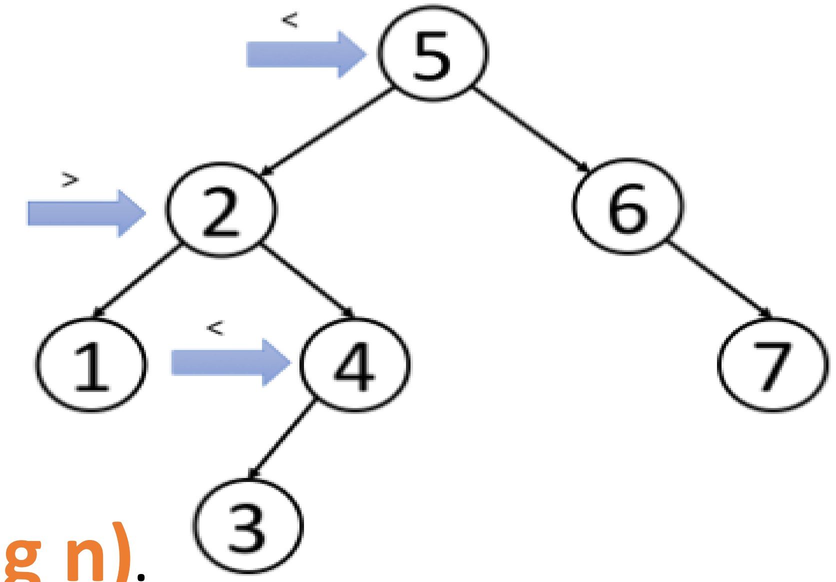
```python
def search(self, val):
    return self._search(val, self.root)

def _search(self, val, node):
    if not node:
        return False
    elif node.val == val:
        return True
    elif val < node.val:
        return self._search(val, node.left)
    else:
        return self._search(val, node.right)
```

Binary Search Tree – Insertion Operation

❑ Similar to the search strategy, for each node, we will:

1. search the left or right subtrees according to the relation of the value of the node and the value of our target node;
2. repeat STEP 1 until reaching an external node;
3. add the new node as its left or right child depending on the relation of the value of the node and the value of our target node.

Input Array: [5, 2, 6, 1, 7, 4, 3]



Time complexity of the insertion operation in a balanced BST is also **O(log n)**.

Binary Search Tree – Insertion Operation

```
class TreeNode:
    def __init__(self, val):
        self.val = val
        self.left = None
        self.right = None
```

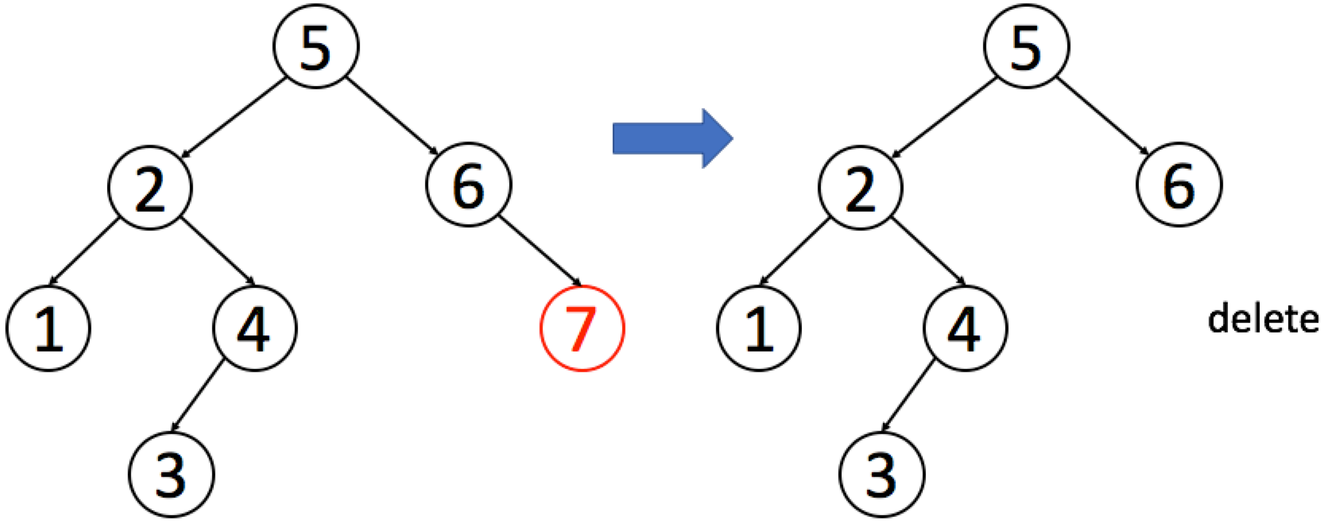
```
class BinarySearchTree:
    def __init__(self):
        self.root = None

    def insert(self, val):
        if not self.root:
            self.root = TreeNode(val)
        else:
            self._insert(val, self.root)

    def _insert(self, val, node):
        if val < node.val:
            if node.left:
                self._insert(val, node.left)
            else:
                node.left = TreeNode(val)
        else:
            if node.right:
                self._insert(val, node.right)
            else:
                node.right = TreeNode(val)
```

Binary Search Tree – Deletion Operation

- ❑ Deletion is more complicated than the two operations we mentioned before.
- ❑ There are also many different strategies for deletion.

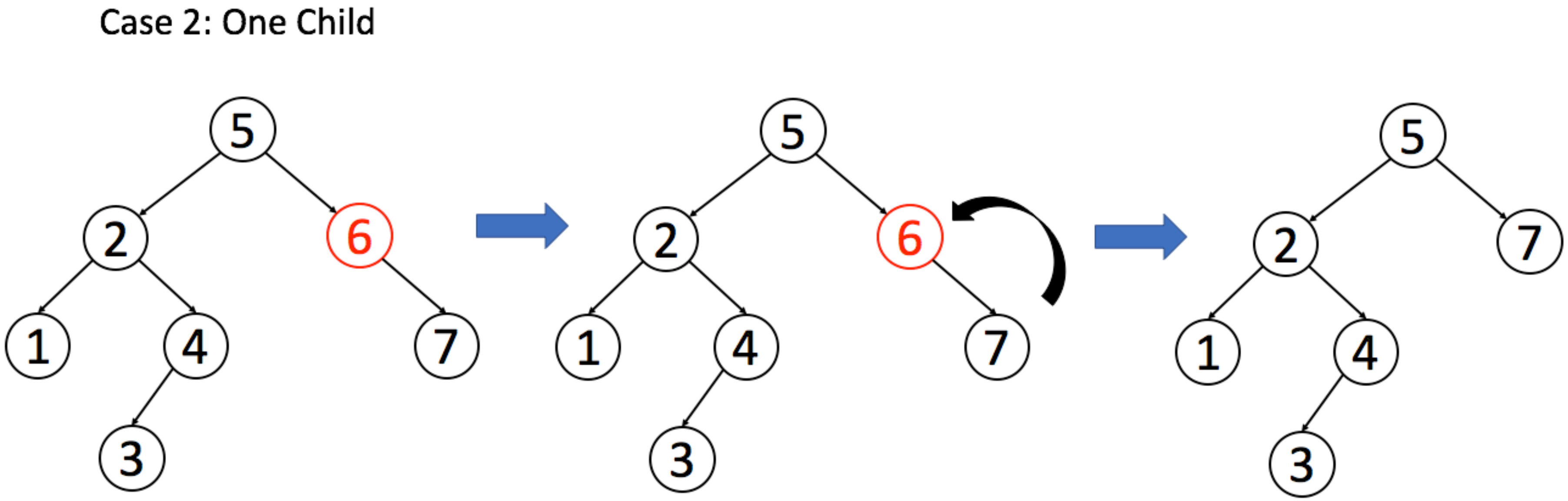
- If the target node has ***no child***, we can simply remove the node

Case 1: No Child



Binary Search Tree – Deletion Operation

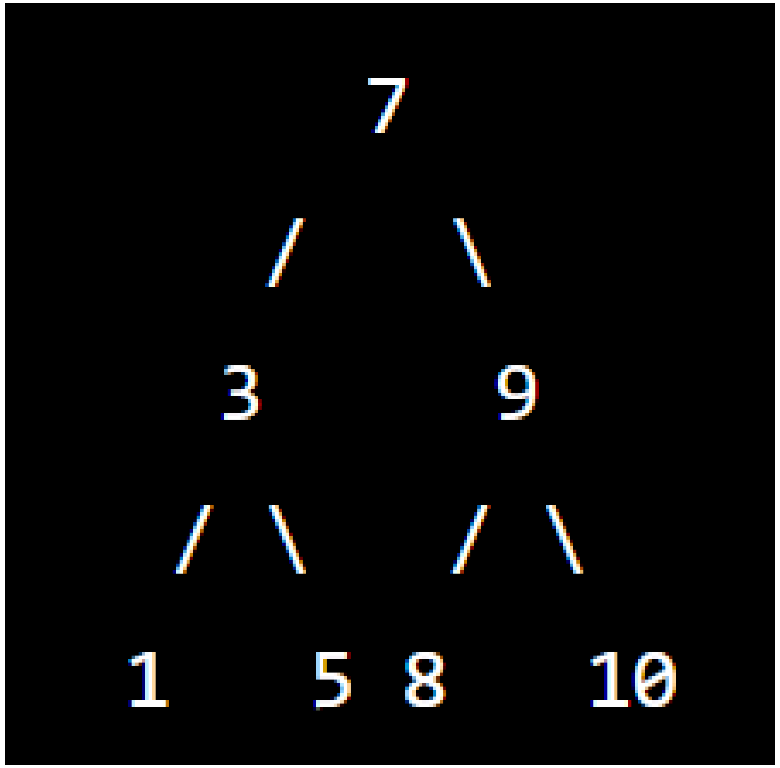
- If the target node has ***no child***, we can simply remove the node
- If the target node has ***one child***, we can use its child to replace itself



Binary Search Tree – Deletion Operation

- If the target node has ***two children***, replace the node with its in-order successor or predecessor node and delete that node.

Inorder: 1, 3, 5, 7, 8, 9, 10



Binary Search Tree – Deletion Operation

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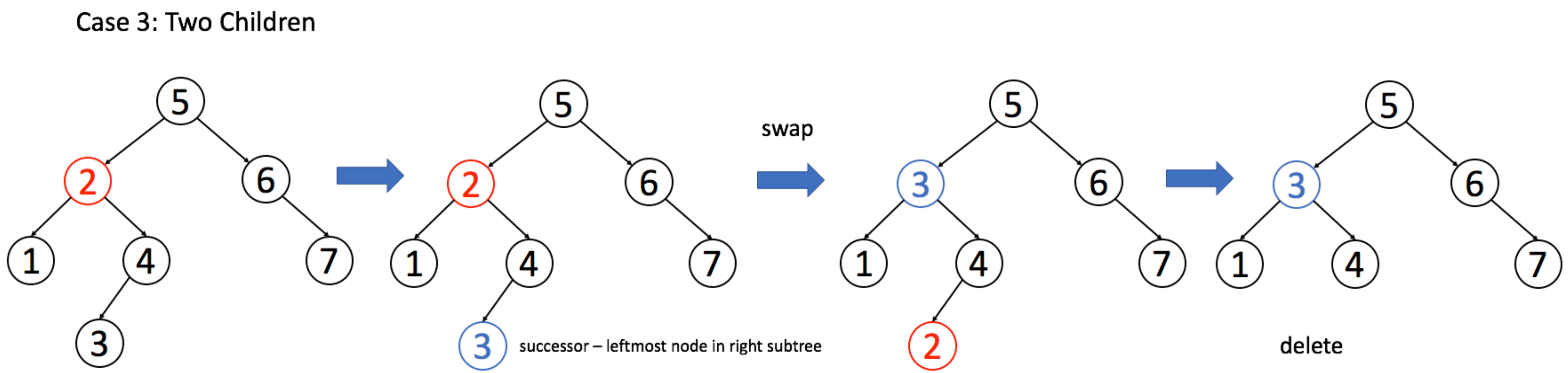

❑ Delete Operation

```python
def delete(self, val):
    self.root = self._delete(val, self.root)

def _delete(self, val, node):
    if not node:
        return node

    if val < node.val:
        node.left = self._delete(val, node.left)
    elif val > node.val:
        node.right = self._delete(val, node.right)
    else:
        # Case 1: Node has no children
        if not node.left and not node.right:
            node = None
        # Case 2: Node has one child
        elif not node.left:
            node = node.right
        elif not node.right:
            node = node.left
        # Case 3: Node has two children
        else:
            successor = self._find_min(node.right)
            node.val = successor.val
            node.right = self._delete(successor.val, node.right)

    return node
```

```python
def _find_min(self, node):
    while node.left:
        node = node.left
    return node
```