

# Data Structures and Algorithms

## CSE 2101

Class time:

Wednesday and Thursday – 11.10am

Google classroom code: **mpsx2hl**

**Course Teacher:**

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Lectures	Topics
1	Complexity analysis
2	<b>Searching</b> <ul style="list-style-type: none"> <li>Linear search, binary Search, application of Binary Search- finding element in a sorted array</li> <li>finding nth root of a real number, solving equations.</li> </ul>
3-6	<b>Recursion</b> <ul style="list-style-type: none"> <li>Basic idea of recursion (3 laws-base case, call itself, move towards base case by state change)</li> <li>tracing output of a recursive function</li> <li>applications- merge sort, permutation, combination.</li> </ul> Memoization
	<b>Sorting</b> <ul style="list-style-type: none"> <li>Insertion sort, selection sort, bubble sort,</li> <li>merge sort, quick sort (randomized quick sort)</li> <li>distribution sort (counting sort, radix sort, bucket sort)</li> </ul> lower bounds for sorting, external sort
7	<b>Linked List</b> <ul style="list-style-type: none"> <li>Singly/doubly/circular linked lists,</li> <li>basic operations on linked list (insertion, deletion, and traverse),</li> <li>dynamic array and its application.</li> </ul>
8-9	<b>Stack Basic</b> <ul style="list-style-type: none"> <li>stack operations (push/pop/peek),</li> <li>stack-class implementation using Array and linked list,</li> <li>in-fix to post-fix expressions conversion and evaluation,</li> <li>balancing parentheses using stack,</li> </ul>
10-11	<b>Queue</b> <ul style="list-style-type: none"> <li>basic queue operations</li> <li>(Enqueue, dequeue), circular queue/ dequeue,</li> <li>queue-class implementation using array and linked list</li> <li>application- Josephus problem, palindrome checker using stack and queue.</li> </ul>

12	<b>Binary tree</b> <ul style="list-style-type: none"> <li>Binary tree representation using array and Pointer</li> <li>traversal of Binary Tree (in-order, pre-order and post-order).</li> </ul>
13-14	<b>Binary Search Tree</b> <ul style="list-style-type: none"> <li>BST representation</li> <li>basic operations on BST (creation, insertion, deletion, querying and traversing),</li> <li>application- searching, sets.</li> </ul>
15-16	<b>Heap</b> <ul style="list-style-type: none"> <li>Min-heap, max-heap,</li> <li>Fibonacci-heap</li> <li>applications-priority queue</li> <li>heap sort.</li> </ul>
17	<b>General Tree</b> Implementation, application of general tree- file system
18	<b>Disjoint Set</b> Union finds, path compression.
20	<b>Huffman Coding: application- Compression.</b>
21 -22	<b>Graph representation (adjacency matrix/adjacency list), basic operations on graph (node/edge insertion and deletion), traversing a graph: breadth-first search (BFS), depth-first search (DFS), graph-bi-colouring.</b>
23	<b>Self-balancing Binary Search Tree: AVL tree</b> (Rotation, insertion).
24-25	<b>Set Operations: Set representation using bitmask, set/clear bit, querying the status of a bit, toggling bit values, LSB, application of set operations.</b>
26-27	<b>String ADT: The concatenation of two strings, the extraction of substrings, searching a string for a matching substring, parsing.</b>

# Data Structures ?

- A data structure is a way of **organizing** and **storing** data in a **computer** so that it can be **accessed** and **modified efficiently**.
- Different types of data structures are suited to different kinds of applications:
- One of the first recorded uses of a data structure was the Jacquard loom in 1801, which used a punched card to control the pattern of a woven textile.
- some are highly specialized to specific tasks
- Some common examples of data structures include
  - Arrays
  - Linked lists
  - Stacks
  - Queues
  - Trees
  - graphs.

# Algorithms?

- Algorithms are a **set of instructions** for carrying out a specific task or solving a specific problem
- A key part of an algorithm is the **use** of one or more data structures in order to store and organize the data that **the algorithm operates on**.

## Relationship –

- data structures provide a way to store and organize data, and
- algorithms use the data structures to accomplish a specific task or solve a specific problem.

# Searching Algorithms: Linear and Binary Search

## Linear Search

- Sequential search

- It traverses the array sequentially to locate the required element.
- It searches for an element by comparing it with each element of the array one by one.

- Applicability

- No information is given about the array.
- The given array is unsorted or the elements are unordered.
- The list of data items is smaller.

# Linear Search

## Best Case

- The element being searched may be found at the first position.
- In this case, the search terminates in success with just one comparison.
- Thus in best case, linear search algorithm takes  $O(1)$  operations.

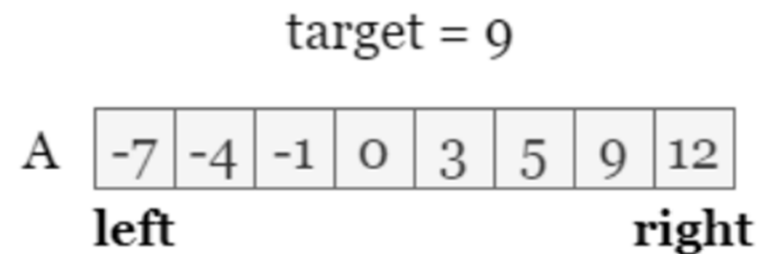
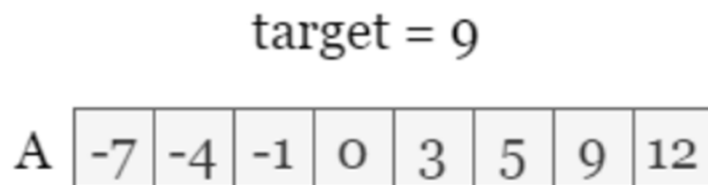
## Worst Case

- The element being searched may be present at the last position or not present in the array at all.
- In the former case, the search terminates in success with  $n$  comparisons.
- In the later case, the search terminates in failure with  $n$  comparisons.
- Thus in worst case, linear search algorithm takes  $O(n)$  operations.

# Binary Search

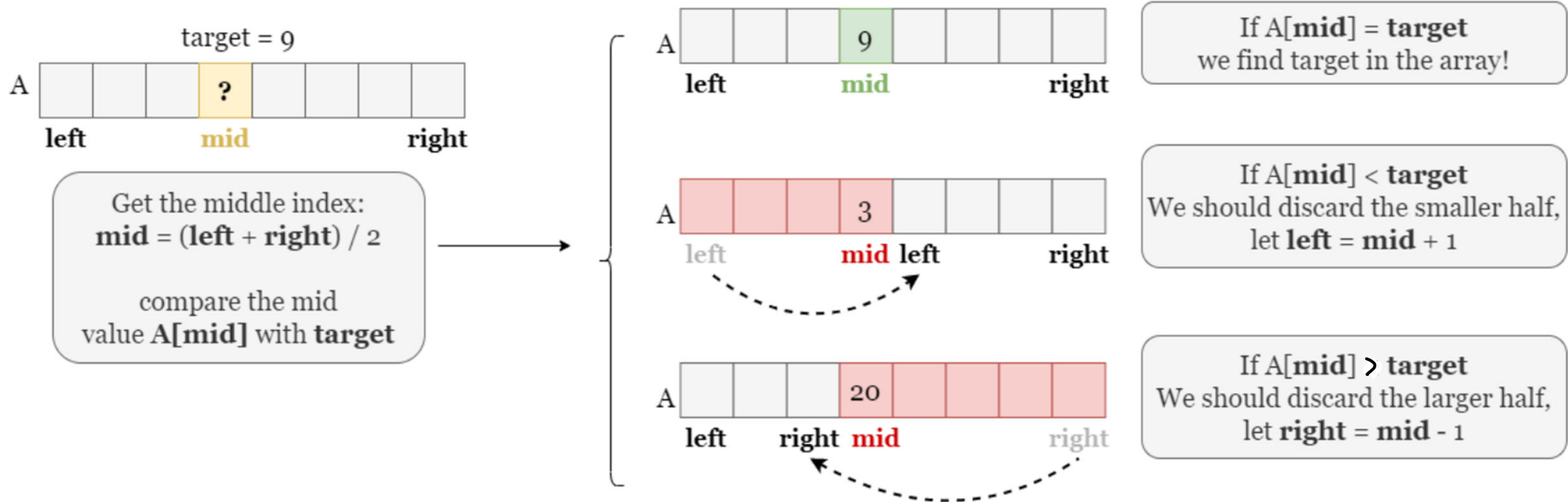
First, we define the search space using two boundary indexes, `left` and `right`

- We shall continue searching over the search space as long as it is not empty.
- A while loop with a condition: `left <= right`



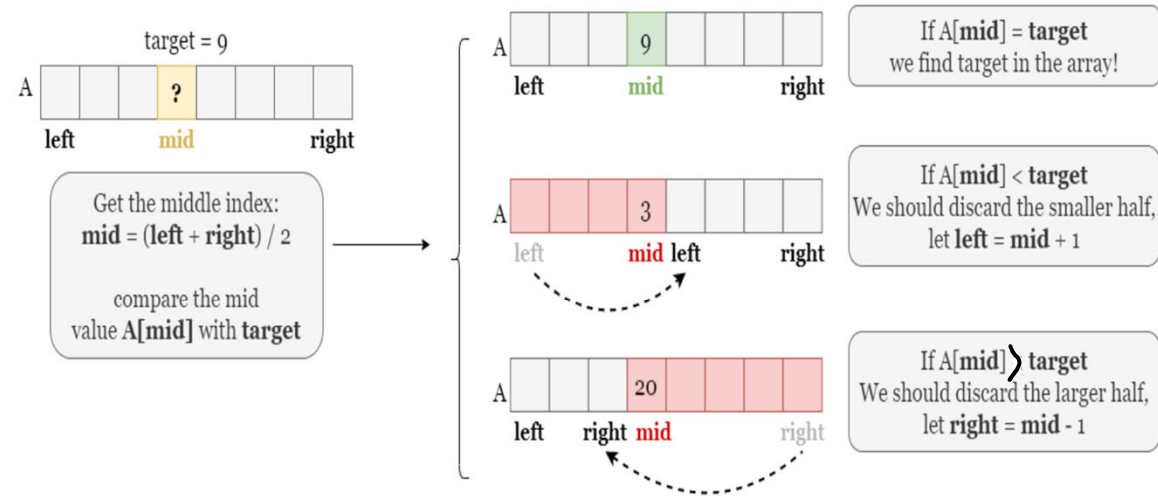
Determine the left and right boundaries

# Binary Search





# Binary Search



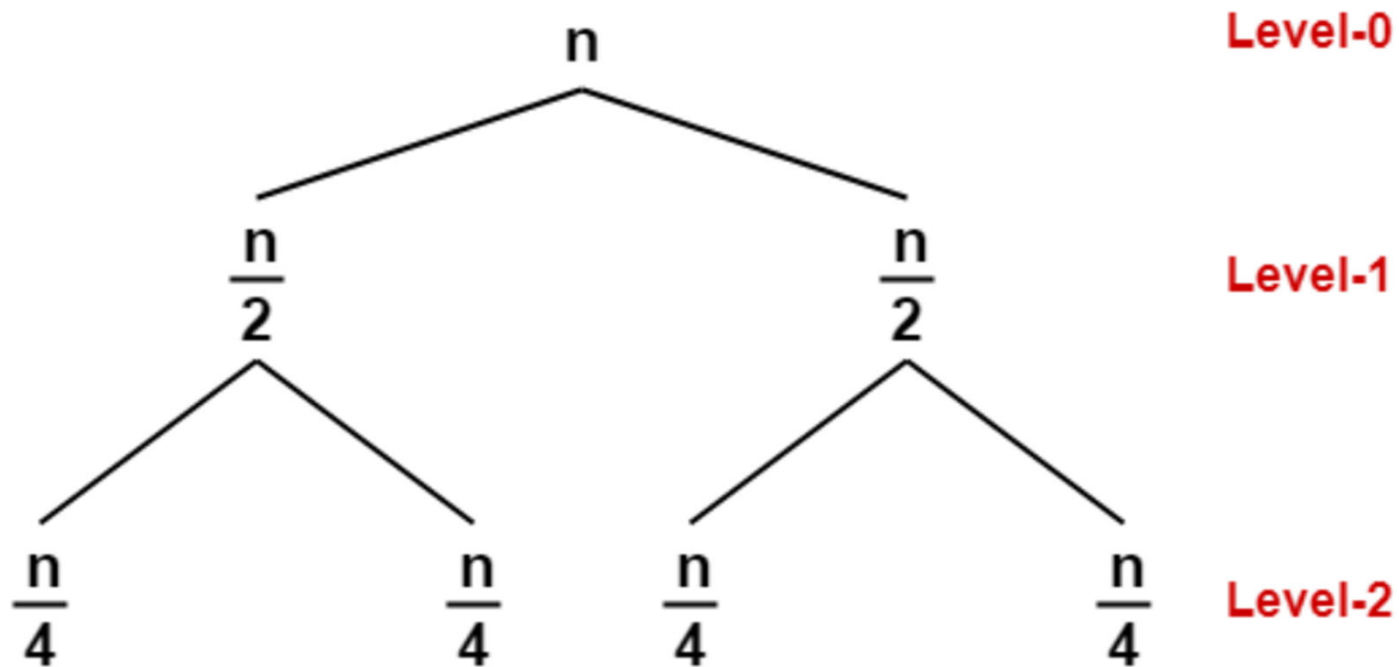
## Algorithm

1. Initialize the boundaries of the search space as  $left = 0$  and  $right = \text{nums.size} - 1$ .
2. If there are elements in the range  $[left, right]$ , we find the middle index  $mid = (left + right) / 2$  and compare the middle value  $\text{nums}[mid]$  with  $target$ :
  - If  $\text{nums}[mid] = target$ , return  $mid$ .
  - If  $\text{nums}[mid] < target$ , let  $left = mid + 1$  and repeat step 2.
  - If  $\text{nums}[mid] > target$ , let  $right = mid - 1$  and repeat step 2.
3. We finish the loop without finding  $target$ , return  $-1$ .

# Binary Search

```
1 class Solution:
2     def search(self, nums: List[int], target: int) -> int:
3         # Set the left and right boundaries
4         left = 0
5         right = len(nums) - 1
6
7         # Under this condition
8         while left <= right:
9             # Get the middle index and the middle value.
10            mid = (left + right) // 2
11
12            # Case 1, return the middle index.
13            if nums[mid] == target:
14                return mid
15            # Case 2, discard the smaller half.
16            elif nums[mid] < target:
17                left = mid + 1
18            # Case 3, discard the larger half.
19            else:
20                right = mid - 1
21
22            # If we finish the search without finding target, return -1.
23            return -1
```

# Binary Search



$n \rightarrow n/2 \rightarrow n/4 \rightarrow \dots \rightarrow 1$

$$n/2^k = 1$$

$$n = 2^k$$

$$k = \log_2 n$$

Assuming our search space is exhausted after k level

# Binary Search

## Complexity Analysis

Let  $n$  be the size of the input array `nums` .

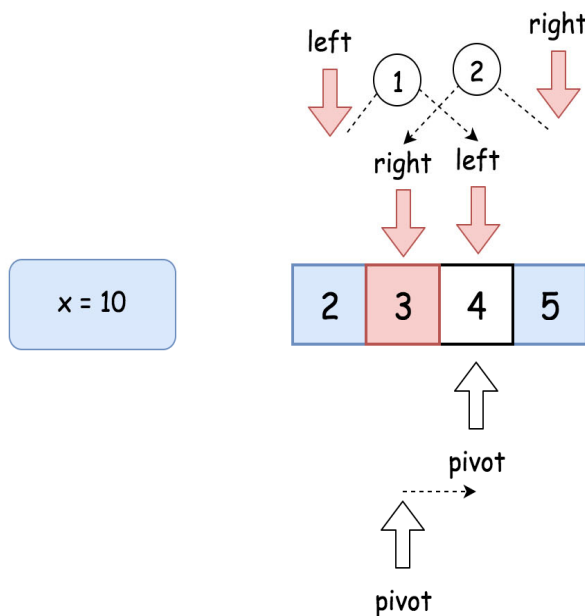
- Time complexity:  $O(\log n)$ 
  - `nums` is divided into half each time. In the worst-case scenario, we need to cut `nums` until the range has no element, and it takes logarithmic time to reach this break condition.
- Space complexity:  $O(1)$ 
  - During the loop, we only need to record three indexes, `left` , `right` , and `mid` , they take constant space.

# Binary Search – finding the square root of a real number $x$

Let's go back to the interview context. For  $x \geq 2$  the square root is always smaller than  $x/2$  and larger than 0 :  $0 < a < x/2$ .

Since  $a$  is an integer, the problem goes down to the iteration over the sorted set of integer numbers. Here the binary search enters the scene.

- If  $x < 2$ , return  $x$ .
- Set the left boundary to 2, and the right boundary to  $x / 2$ .
- While  $\text{left} \leq \text{right}$ :
  - Take  $\text{num} = (\text{left} + \text{right}) / 2$  as a guess. Compute  $\text{num} * \text{num}$  and compare it with  $x$ :
    - If  $\text{num} * \text{num} > x$ , move the right boundary  $\text{right} = \text{pivot} - 1$
    - Else, if  $\text{num} * \text{num} < x$ , move the left boundary  $\text{left} = \text{pivot} + 1$
    - Otherwise  $\text{num} * \text{num} == x$ , the integer square root is here, let's return it
- Return  $\text{right}$



# Binary Search – finding the square root of a real number x

```
1 class Solution:
2     def search(self, nums: List[int], target: int) -> int:
3         # Set the left and right boundaries
4         left = 0
5         right = len(nums) - 1
6
7         # Under this condition
8         while left <= right:
9             # Get the middle index and the middle value.
10            mid = (left + right) // 2
11
12            # Case 1, return the middle index.
13            if nums[mid] == target:
14                return mid
15
16            # Case 2, discard the smaller half.
17            elif nums[mid] < target:
18                left = mid + 1
19
20            # Case 3, discard the larger half.
21            else:
22                right = mid - 1
23
24            # If we finish the search without finding target, return -1.
25            return -1
```

```
1 class Solution:
2     def mySqrt(self, x):
3         if x < 2:
4             return x
5
6         left, right = 2, x // 2
7
8         while left <= right:
9             pivot = left + (right - left) // 2
10            print("This is the pivot value:", pivot)
11            num = pivot * pivot
12            if num > x:
13                right = pivot - 1
14            elif num < x:
15                left = pivot + 1
16            else:
17                return pivot
18
19        return right
20
21 sol = Solution()
22 print(sol.mySqrt(23))
```

```
This is the pivot value: 6
This is the pivot value: 3
This is the pivot value: 4
This is the pivot value: 5
```



# Binary Search – Recursive Implementation

```
if __name__ == '__main__':  
  
    nums = [2, 5, 6, 8, 9, 10]  
    target = 5  
  
    (left, right) = (0, len(nums) - 1)  
    index = binarySearch(nums, left, right, target)  
  
    if index != -1:  
        print('Element found at index', index)  
    else:  
        print('Element found not in the list')
```

```
# Recursive implementation of the binary search algorithm to return  
# the position of `target` in subarray nums[left...right]  
def binarySearch(nums, left, right, target):  
  
    # Base condition (search space is exhausted)  
    if left > right:  
        return -1  
  
    # find the mid-value in the search space and  
    # compares it with the target  
  
    mid = (left + right) // 2  
  
    # overflow can happen. Use below  
    # mid = left + (right - left) / 2  
  
    # Base condition (a target is found)  
    if target == nums[mid]:  
        return mid  
  
    # discard all elements in the right search space,  
    # including the middle element  
    elif target < nums[mid]:  
        return binarySearch(nums, left, mid - 1, target)  
  
    # discard all elements in the left search space,  
    # including the middle element  
    else:  
        return binarySearch(nums, mid + 1, right, target)
```