Arithmetic for Computers

Chapter Three

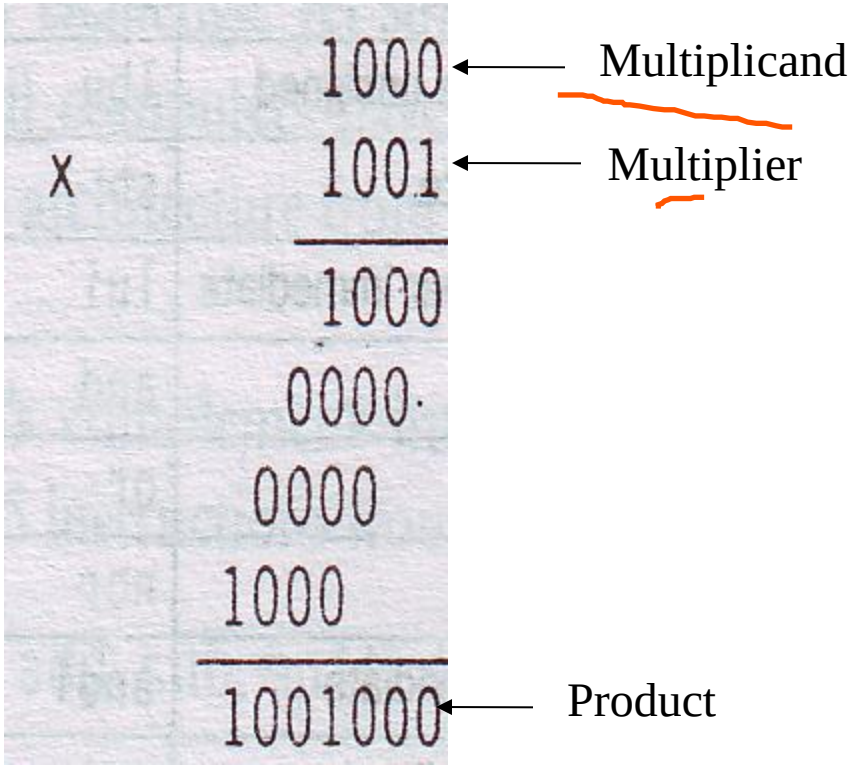
Book of David A. Patteron

P. Hayes

page-239-240

General Concept of Multiplication

$1000_2 \times 1001_2$

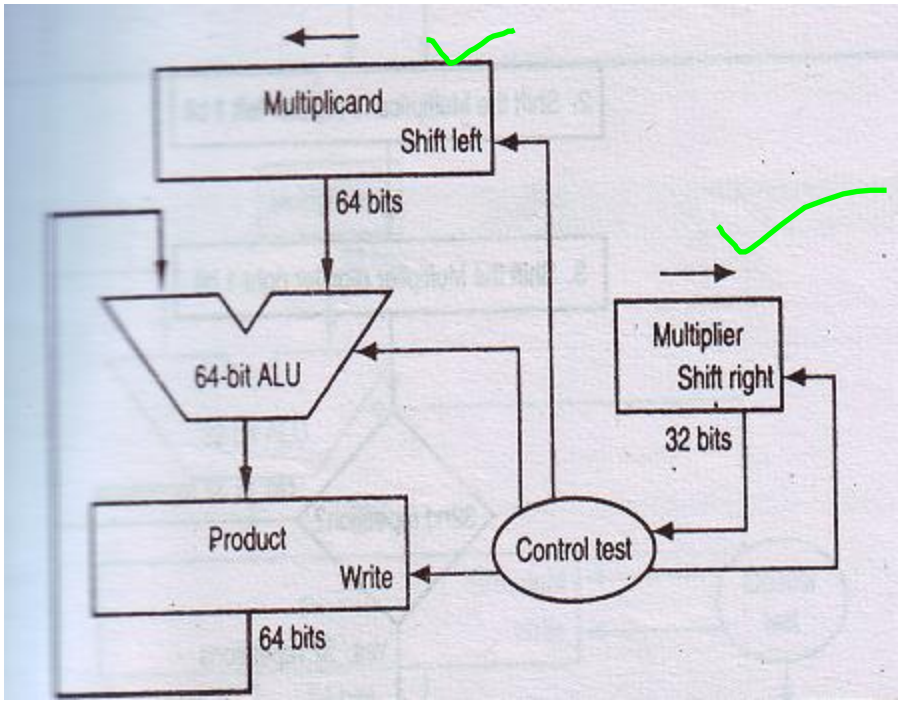


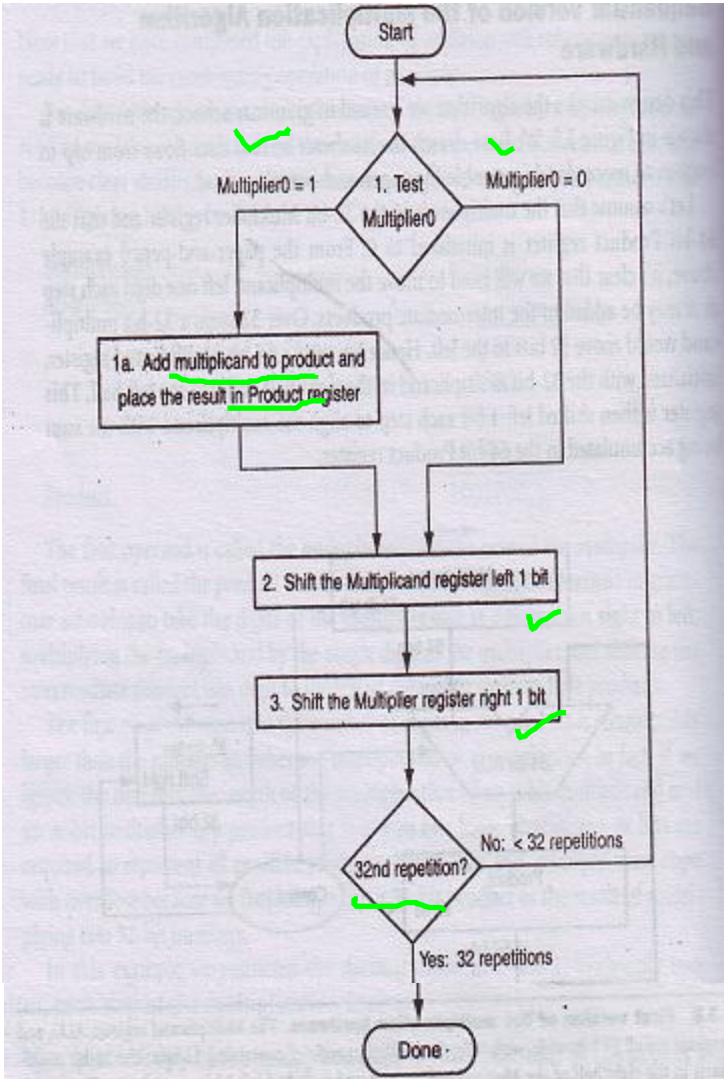
- ✔ If multiplicand = ***n*** bits and multiplier = ***m*** bits then product = ***n*** + ***m*** bits.
- ✔ Two rules:

1. Place a copy of multiplicand in the proper place if multiplier bit=1.
2. Place 0 in the proper place if multiplier bit = 0.

- ✔ Like addition, overflow can occur.

Sequential Version of the Multiplication Algorithm and Hardware (Unsigned Number)





✔ A moderate frequency for a slow operation can limit performance.

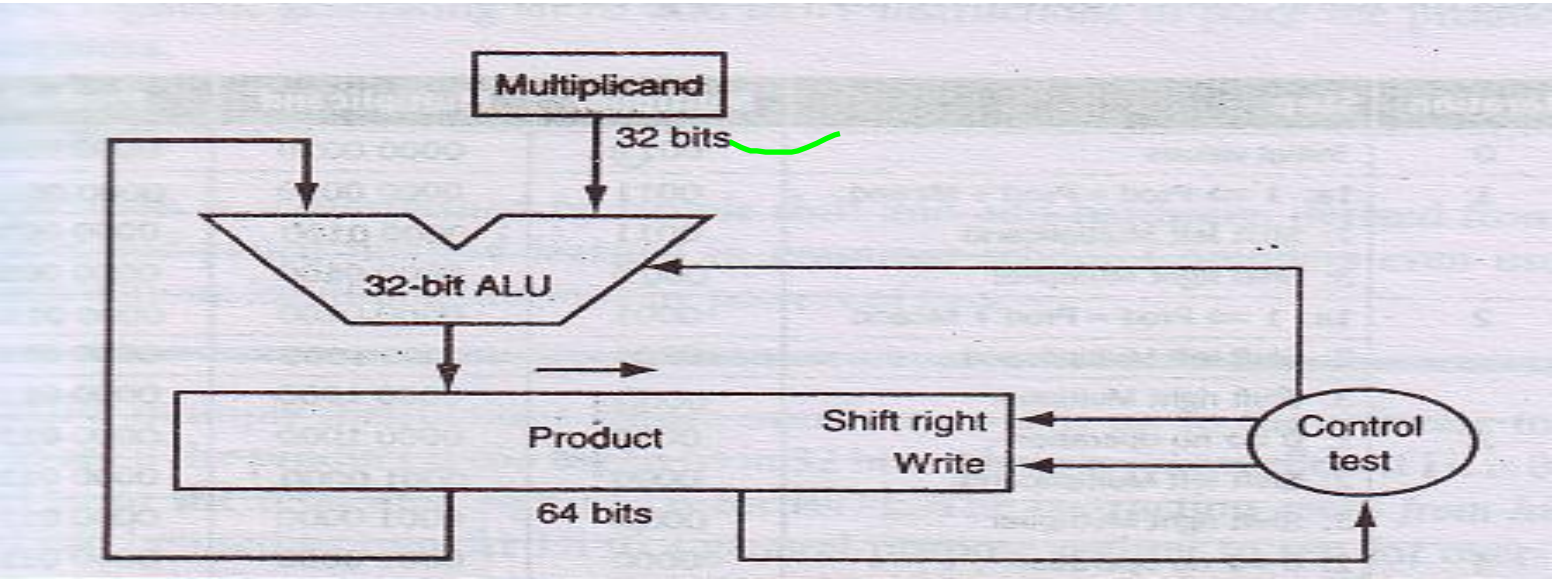
Example

Iteration	Step	Multiplier	Multiplicand	Product
0	Initial values	0011	0000 0010	0000 0000
1	1a: 1 ⇒ Prod = Prod + Mcand	0011	0000 0010	0000 0010
	2: Shift left Multiplicand	0011	0000 0100	0000 0010
	3: Shift right Multiplier	0011	0000 0100	0000 0010
2	1a: 1 ⇒ Prod = Prod + Mcand	0001	0000 0100	0000 0110
	2: Shift left Multiplicand	0001	0000 1000	0000 0110
	3: Shift right Multiplier	0000	0000 1000	0000 0110
3	1: 0 ⇒ no operation	0000	0000 1000	0000 0110
	2: Shift left Multiplicand	0000	0001 0000	0000 0110
	3: Shift right Multiplier	0000	0001 0000	0000 0110
4	1: 0 ⇒ no operation	0000	0001 0000	0000 0110
	2: Shift left Multiplicand	0000	0010 0000	0000 0110
	3: Shift right Multiplier	0000	0010 0000	0000 0110

Works for negative number. Consider sign extension during shift operation.

Refined Version of Multiplication Algorithm

- Execution time can be reduced through parallelism.
- Hardware cost can be reduced by using non-utilized portion of product register.



Example

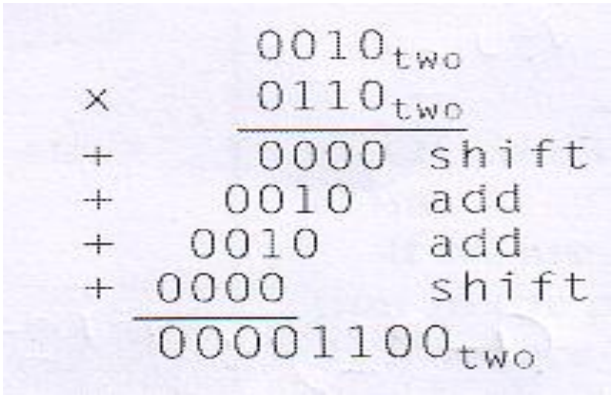
Iteration	Multiplicand	Original algorithm	
		Step	Product
0	0010	Initial values	0000 0110
1	0010	1: 0 ⟹ no operation	0000 0110
	0010	2: Shift right Product	0000 0011
2	0010	1a: 1 ⟹ Prod = Prod + Mcand	0010 0011
	0010	2: Shift right Product	0001 0001
3	0010	1a: 1 ⟹ Prod = Prod + Mcand	0011 0001
	0010	2: Shift right Product	0001 1000
4	0010	1: 0 ⟹ no operation	0001 1000
	0010	2: Shift right Product	0000 1100

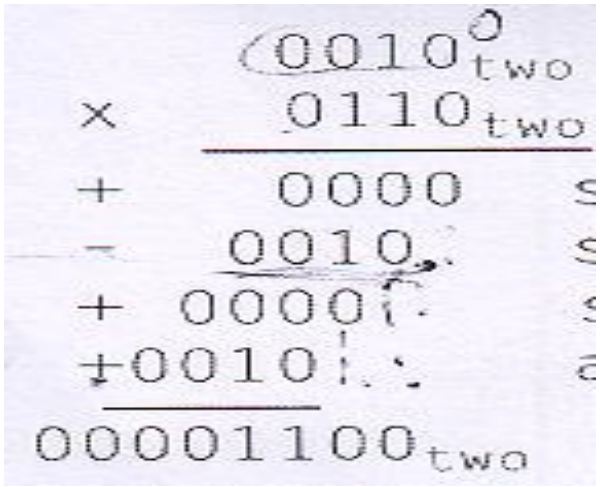
Booth's Multiplication

- ✔ Used for both positive and negative number (2's complement number).
- ✔ It uses both add and sub to compute the product.
- ✔ Instead of 1 bit of a multiplier it considers 2 bits.
- ✔ It provides faster execution.

Concepts of Booth's Algorithm

$2_{10} \times 6_{10} = 12_{10}$





- ✔ $6_{10} = -2_{10} + 8_{10}$
  - » $6_{10} \times 2_{10} = -2_{10} \times 2_{10} + 8_{10} \times 2_{10}$
  - » $12_{10} = -4_{10} + 16_{10}$
- ✔ Condition:
  1. subtract when first see a 1. (10).
  2. add when see the bit after the last 1. (01)

Validity of Booth's Algorithm

$$X^* = x_i x_{i-1} x_{i-2} \cdots x_{i-k+1} x_{i-k} x_{i-k-1}$$
$$= 0\ 1\ 1\ \ldots\ 1\ 1\ 0$$

- In normal multiplication the contribution of X^* to $P = X \times Y$ is $\sum_{j=i-k}^{i-1} 2^j Y$

- In booth's multiplication, $x_i x_{i-1} = 01$ which contributes $2^i Y$.
- When $x_{i-k} x_{i-k-1} = 10$ the contribution is $-2^{i-k} Y$.
- Net Contribution:

$$2^i Y - 2^{i-k} Y = 2^{i-k} Y(2^k - 1) Y$$
$$= 2^{i-k} \sum_{m=0}^{k-1} 2^m Y$$
$$= \sum_{m=0}^{k-1} 2^{m+i-k} Y$$

- If j=m+i-k then we get $\sum_{j=i-k}^{i-1} 2^j Y$

Booth's Algorithm

- ✔ Step 1:

  Depending on the current and previous bit, do one of the following:

  00: No arithmetic operation.

  01:Add multiplicand to the left half of the product.

  10: Sub the multiplicand from the left half of the product.

  11: No arithmetic operation.

- ✔ Step 2:

  Shift the product register right 1 bit.

Example

Itera-tion	Multi-plicand	Original algorithm		Booth's algorithm	
		Step	Product	Step	Product
0	0010	Initial values	0000 0110	Initial values	0000 0110 0
1	0010	1: 0 ⟹ no operation	0000 0110	1a: 00 ⟹ no operation	0000 0110 0
	0010	2: Shift right Product	0000 0011	2: Shift right Product	0000 0011 0
2	0010	1a: 1 ⟹ Prod = Prod + Mcand	0010 0011	1c: 10 ⟹ Prod = Prod − Mcand	1110 0011 0
	0010	2: Shift right Product	0001 0001	2: Shift right Product	1111 0001 1
3	0010	1a: 1 ⟹ Prod = Prod + Mcand	0011 0001	1d: 11 ⟹ no operation	1111 0001 1
	0010	2: Shift right Product	0001 1000	2: Shift right Product	1111 1000 1
4	0010	1: 0 ⟹ no operation	0001 1000	1b: 01 ⟹ Prod = Prod + Mcand	0001 1000 1
	0010	2: Shift right Product	0000 1100	2: Shift right Product	0000 1100 0