Lecture 7: Modeling Hybrid Systems

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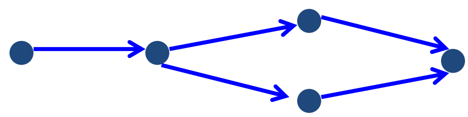
Based on the slides by Edward Lee

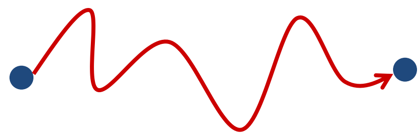
Review

- Discrete systems
- Finite-state machines (FSMs)
  - Graphical notation
  - Formal model
- Event-triggered vs. time-triggered models
- Deterministic vs. Non-deterministic FSMs
- Stuttering, Receptiveness, …
- Extended FSMs

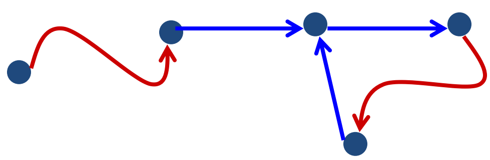
Discrete System (FSM)

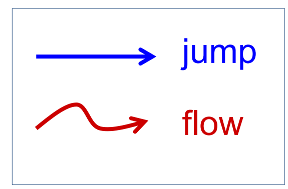


Continuous System



Hybrid System





Where Do Hybrid Systems Arise?

Digital controller of physical "plant"

- thermostat
- intelligent cruise/powertrain control in cars
- aircraft auto pilot

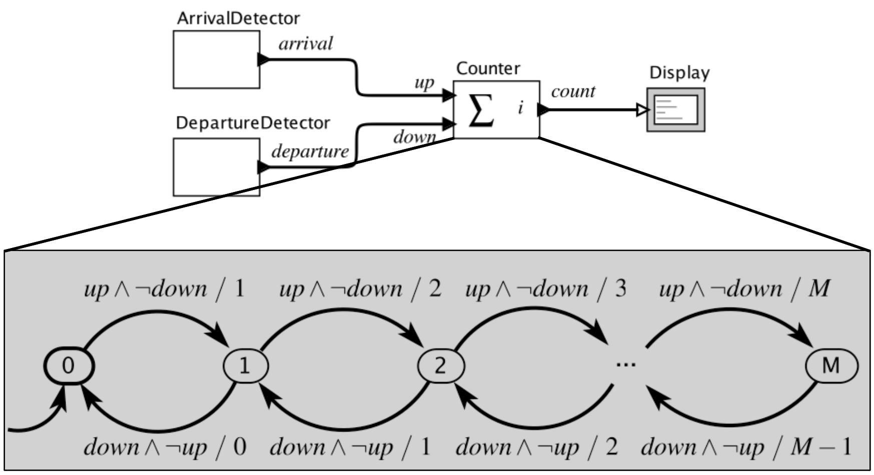
Phased operation of natural phenomena

- bouncing ball
- biological cell growth

Multi-agent systems

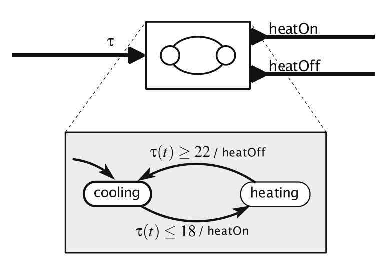
- ground and air transportation systems
- interacting robots

FSMs as Actors



FSMs with Continuous-Time Signals Modal Models

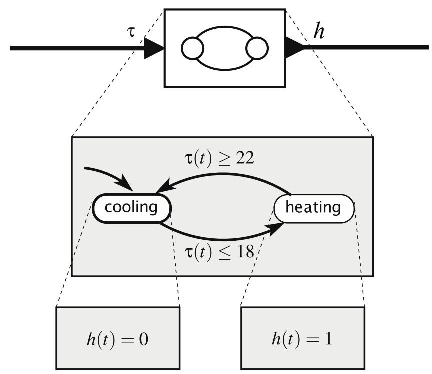
- Continuous-time input and/or output signals
- Inputs/outputs does not require to be absent between reactions
- A transition occurs when a guard on an outgoing transition from the current state becomes enabled



Discrete outputs in this example.

State Refinement

- A hybrid system associates with each state of an FSM a dynamic behavior.
- Example: Thermostat with continuous output.



Three Classes of Hybrid Systems

Timed automata

- Finite automata extended with a finite set of real-valued clocks

Higher-order dynamics

- System variables change in the state refinements

Supervisory control

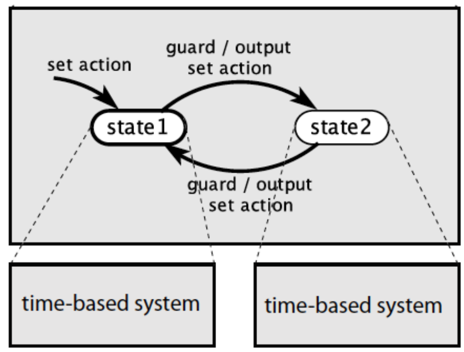
- Two-level control strategy

Timed Automata: A special case of hybrid systems

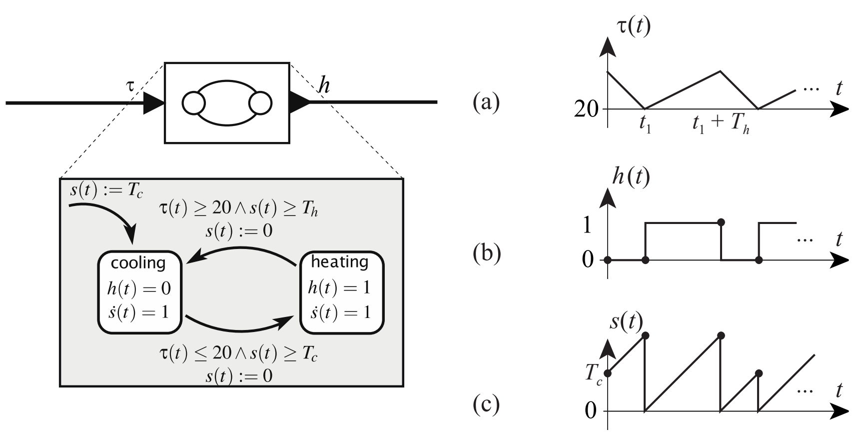
An alternative to FSMs that is explicit about the passage of time

A device that measures the passage of time, a **clock**, has a particularly simple *dynamics*: its state progresses linearly in time.

$\forall\, t \in T_m,\ \dot{s}(t) = a$

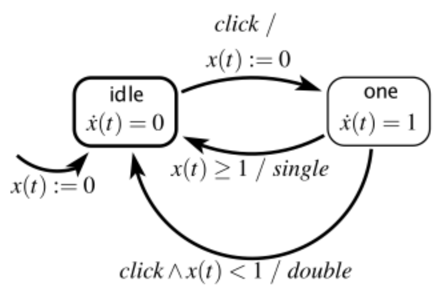


Example: Thermostat with Minimum Stay Time in Each State



Example: Mouse Double Click Detector

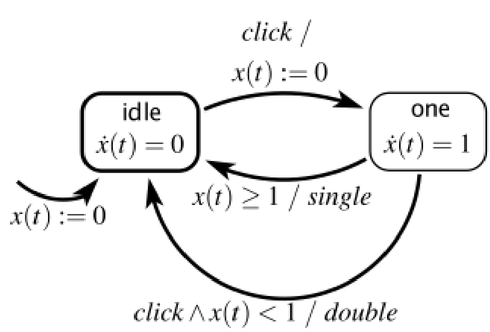
continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$



This simple form of hybrid system is called a *timed automaton*, where the dynamics is just passage of time.

Reflection

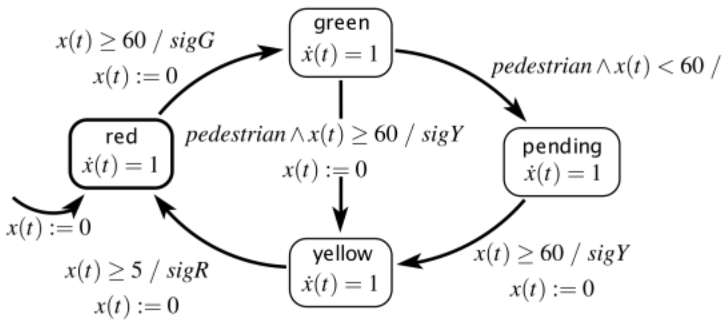
continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$



How many states does this automaton have?

Timed automaton model of a traffic light controller

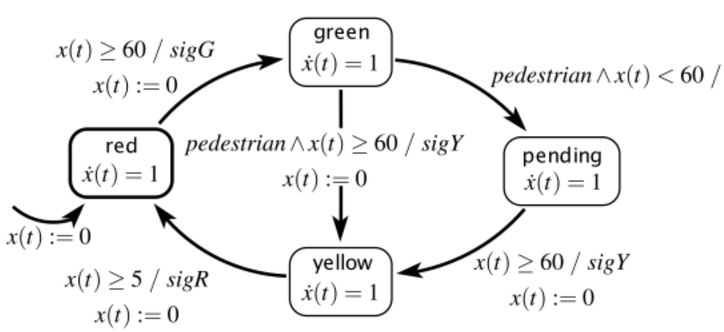
continuous variable: $x(t) \colon \mathbb{R}$
inputs: *pedestrian*: pure
outputs: *sigR*, *sigG*, *sigY*: pure



This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

When do reactions occur in a hybrid automaton?

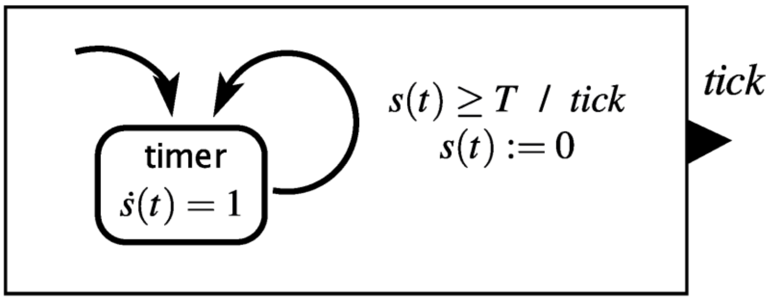
continuous variable: $x(t) \colon \mathbb{R}$
inputs: *pedestrian*: pure
outputs: *sigR*, *sigG*, *sigY*: pure



Reactions are occurring continually, with the continuous state variable *x* being continually updated.

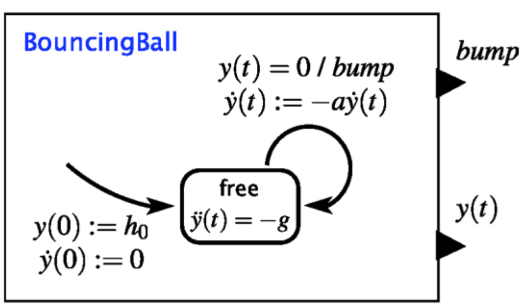
Example: "Tick" Generator (Timer)

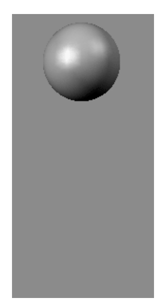
How would you model a timer that generates a 'tick' each time T time units elapses?



A similar timed automaton can model a generator of a timer interrupt.

Higher-Order Dynamics: Hybrid Automaton for Bouncing Ball

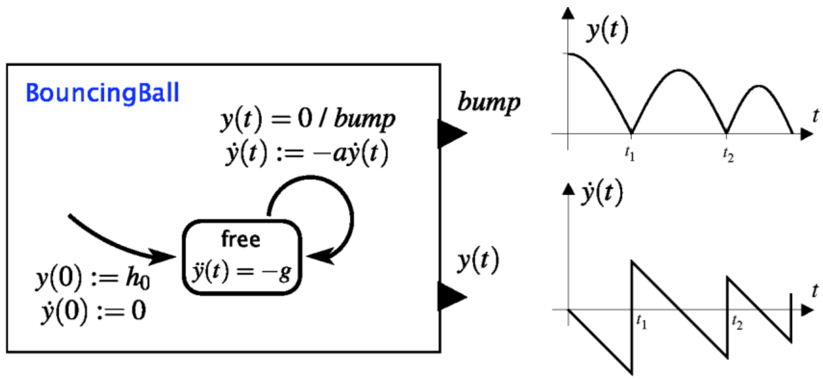




y – vertical distance from ground (position)
a – coefficient of restitution, $0 \leq a \leq 1$

If you plotted y(t), what would it look like?

Hybrid Automaton for Bouncing Ball



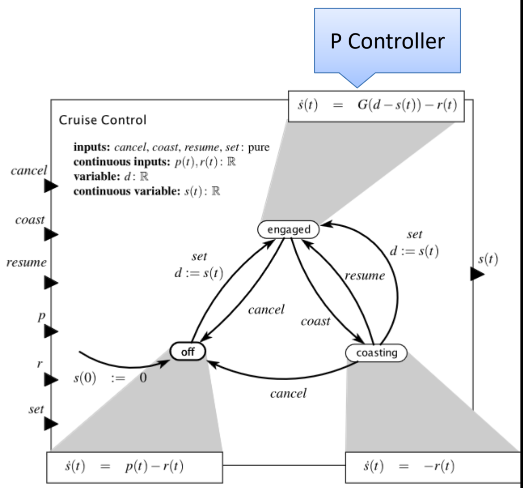
y – vertical distance from ground (position)
a – coefficient of restitution, $0 \leq a \leq 1$

Supervisory Control

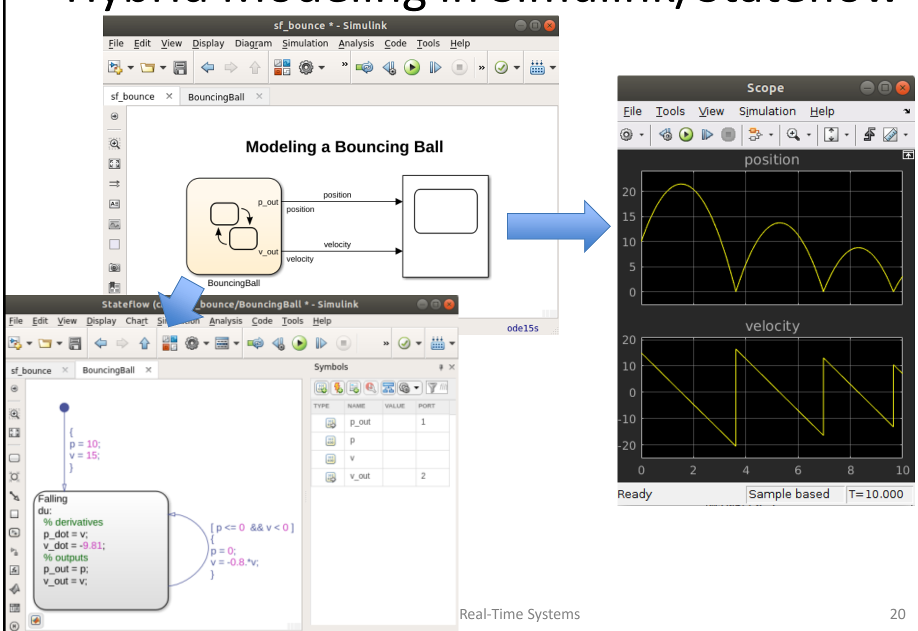
- Control system involves four components
  - Plant
  - Environment
  - Sensors
  - Controller
- Controller with two levels
  - *supervisory control* that determines the mode transition structure
  - *low-level control* that determines the time-based inputs to the plant

An Automotive **Cruise Control** System

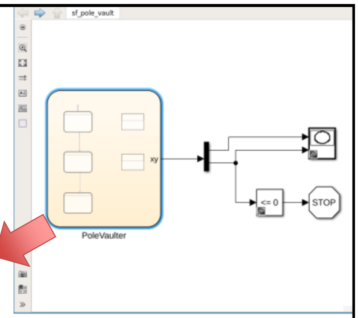
- Pure inputs (driver)
  - *set*: turn on + set speed
  - *cancel*: manual control
  - *coast*: temporary disable
  - *resume*: resume with last speed
- Real variables (at time *t*)
  - $s(t)$: speed
  - $p(t)$: accelerator pedal
  - $r(t)$: resistance (drag, friction, slope)
  - *d*: desired speed
  - *G*: gain of p controller

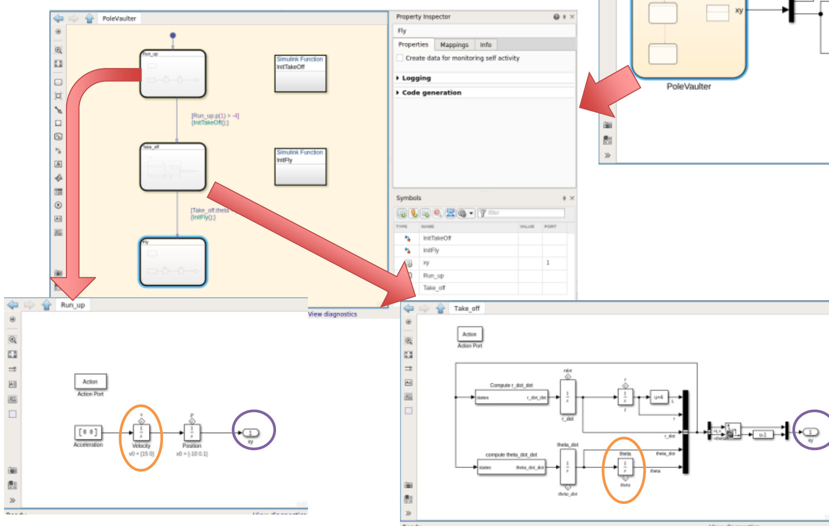


Hybrid Modeling in Simulink/Stateflow



Simulink Subsystems as Stateflow States





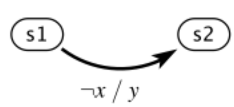
Next Lecture

- Composition of state machines
  - Synchronous composition
  - Asynchronous composition
- Statecharts
- Read chapter 5 of LeeSeshia

SPARE SLIDES

When do reactions occur in a hybrid automaton?

input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$



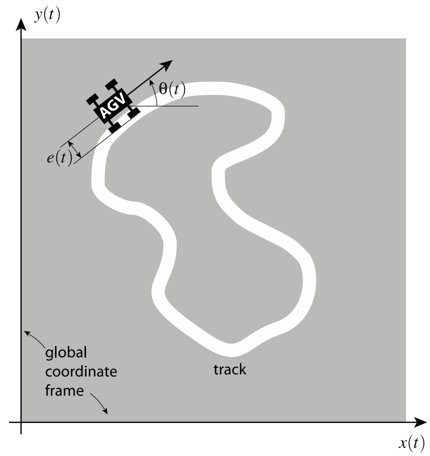
Suppose *x* and *y* are discrete and pure signals. When does the transition occur?

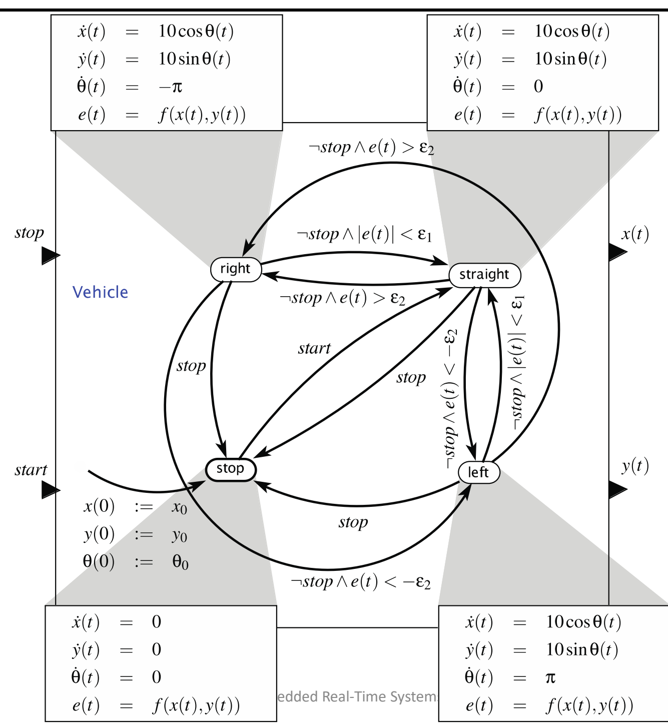
Answer: at the earliest time t when x is absent after entering s1. This will always be the same time when s1 is entered. Why?

If x is absent when s1 is entered, then the transition is taken then. If x is present when s1 is entered, then it will be absent at a time infinitesimally larger. How to model this rigorously?

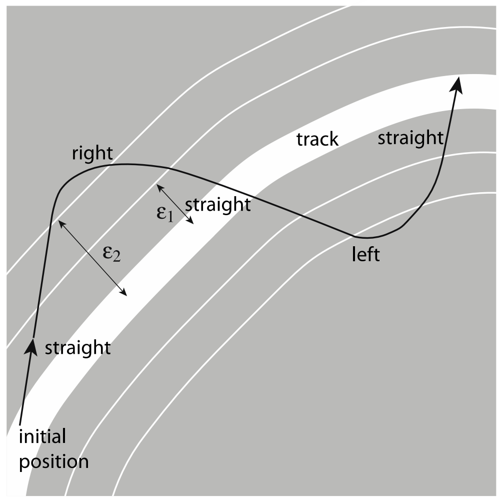
An Automated Guided Vehicle (AGV)

- The vehicle
  - Can move forward along its body axis with speed $0 \le u(t) \le 10$ km/h
  - Rotate about its center of gravity with an angular speed $-\pi \le \omega(t) \le \pi$ rad/sec
- The controller guides the vehicle in four modes left, right, straight, and stop.
- Position given by
  - $\dot{x}(t) = u(t) \cos \theta(t)$
  - $\dot{y}(t) = u(t) \sin \theta(t)$
  - $\theta(t) = \omega(t)$

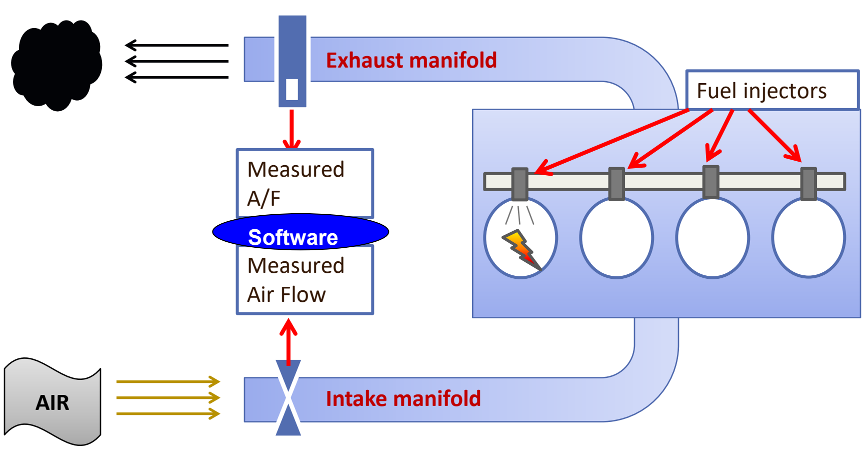




A trajectory of the AGV in different modes

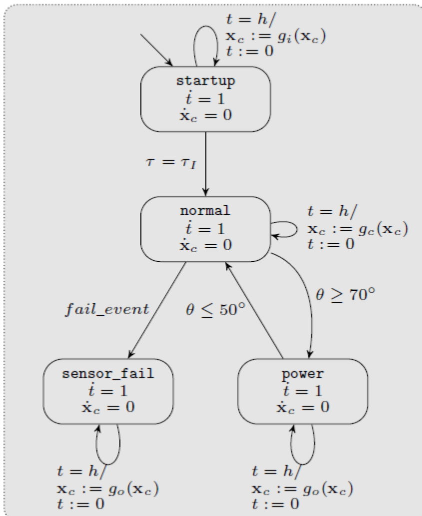


Hybrid Automaton Model of Toyota Powertrain Control Example



[Slide due to J. Deshmukh, Toyota]

Hybrid Automaton Model of Toyota Powertrain Control Example



Four Operating Modes:
1. Startup: Wait for O2 sensors to start giving accurate readings (temp dependent), employ open-loop control
2. Normal: Use combination of feedback PI control and feedforward control to regulate A/F ratio
3. Power: Driver depresses gas pedal more (higher throttle angle) – switch to feedforward
4. Sensor Failure: switch to feedforward control

"Powertrain Control Verification Benchmark", Jin et al., HSCC 2014