Lecture 6: Modeling Discrete Dynamics

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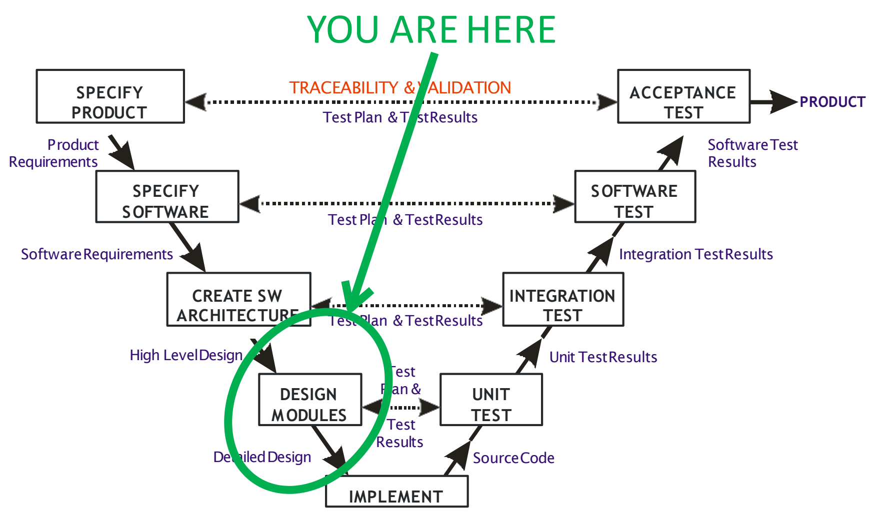
Based on the slides by Edward Lee and Peter Marwedel

Review

- High Level Design (HLD)
  - Architecture
    - Architecture description languages
  - Refined requirements
    - Sequence Diagrams

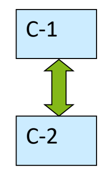


Discrete Systems

- **Discrete** = “individually separate / distinct”
- A **discrete system** is one that operates in a sequence of discrete *steps* or has signals taking discrete *values*.
- It is said to have **discrete dynamics**.

Recap: Models of Computation

- **What does it mean, "to compute"?**
- **Models of computation define**:
  - Components and an execution model for computations for each component
  - Communication model for exchange of information between components.



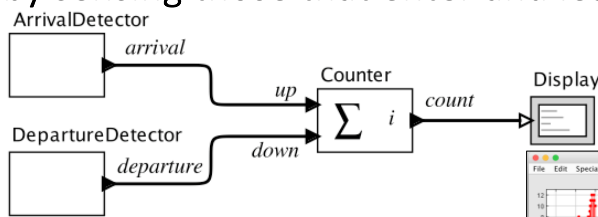
Models of Computation

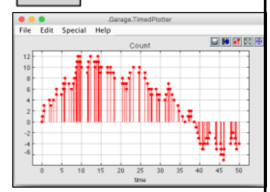
(Considered by Marwedel)

Communication/ local computations	**Shared memory**	**Message passing Synchronous**	**Asynchronous**
Undefined components	Plain text, use cases	(Message) sequence charts	
Communicating finite state machines	StateCharts		SDL
Data flow	Scoreboarding + Tomasulo Algorithm (☞ Comp.Archict.)		Kahn networks, SDF
Petri nets		C/E nets, P/T nets, …	
Discrete event (DE) model	VHDL*, Verilog*, SystemC*, …	Only experimental systems, e.g. distributed DE in Ptolemy	
Von Neumann model	C, C++, Java	C, C++, Java with libraries CSP, ADA	

Discrete Systems: Example Design Problem

Example: count the number of cars in a parking garage by sensing those that enter and leave:





Pure signal: $up\colon \mathbb{R} \to \{absent, present\}$

Discrete actor as a function:

$$Counter\colon (\mathbb{R} \to \{absent, present\})^P \to (\mathbb{R} \to \{absent\} \cup \mathbb{N})$$

$$P = \{up, down\}$$

Reflection

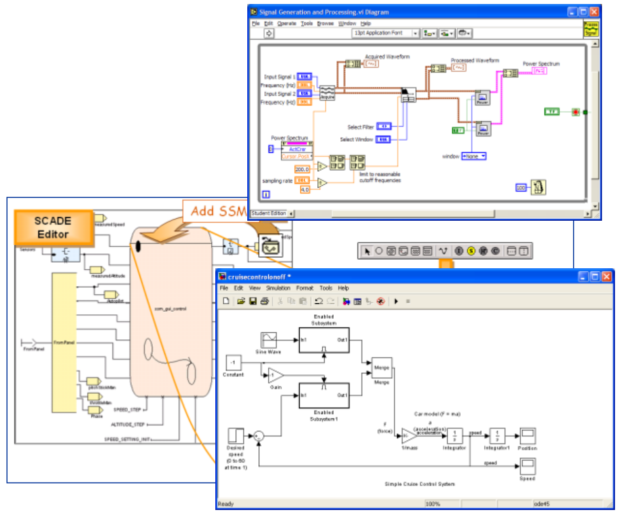
- What (mathematical) properties should a discrete signal have?

$$e\colon \mathbb{R} \to \{absent\} \cup X$$

Actor Modeling Languages / Frameworks

- LabVIEW
- Simulink
- Scade
- …

- Reactors
- StreamIT
- …

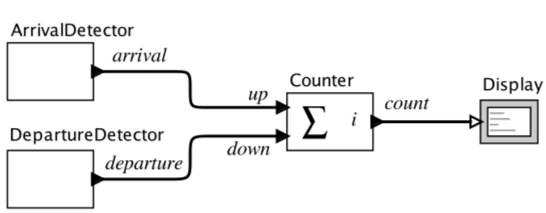


Reaction / Transition

For any $t \in \mathbb{R}$ where $up(t) \neq absent$ or $down(t) \neq absent$ the Counter **reacts**. It produces an output value in $\mathbb{N}$ and changes its internal **state**.

State: condition of the system at a particular point in time

- Encodes everything about the past that influences the system's reaction to current input



Reflection

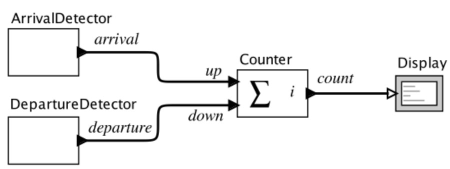
What are some scenarios that the given parking garage (interface) design does not handle well?

For $t \in \mathbb{R}$ the inputs are in a set

$$Inputs = (\{\text{up}, \text{down}\} \to \{\,absent, present\})$$

and the outputs are in a set

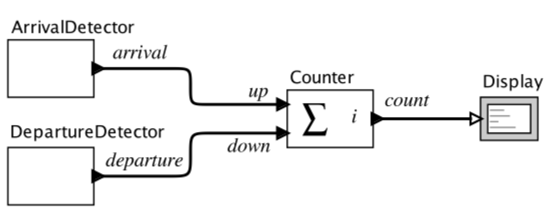
$$Outputs = (\{count\} \to \{\,absent\} \cup \mathbb{N})\ ,$$



State Space

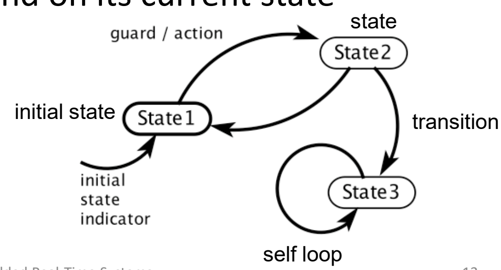
A practical parking garage has a finite number M of spaces, so the state space for the counter is

$$States = \{0, 1, 2, \cdots, M\}\ .$$

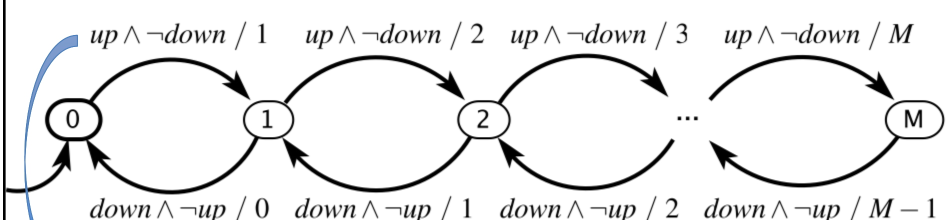


Finite State Machine (FSM)

- FSM
  - A model of a system with discrete dynamics
  - At each reaction maps valuations of the inputs to valuations of the outputs
  - The map may depend on its current state
- Transitions
  - Guards
  - Actions



Garage Counter Finite State Machine (FSM) in Pictures



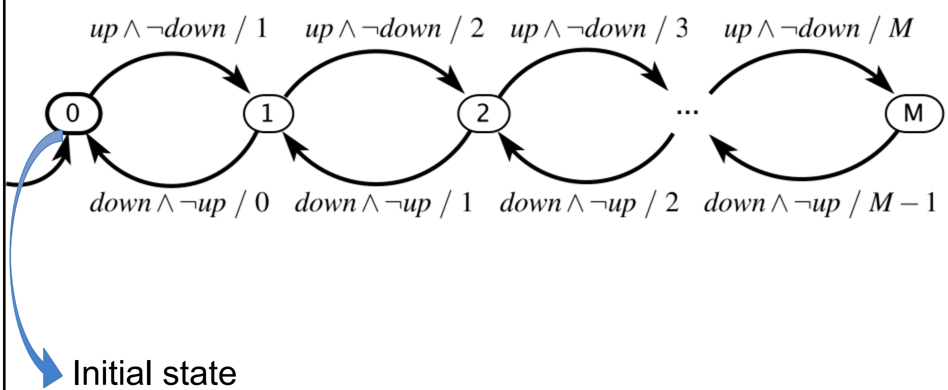
Guard $g \subseteq Inputs$ is specified using the shorthand

$$up \wedge \neg down$$

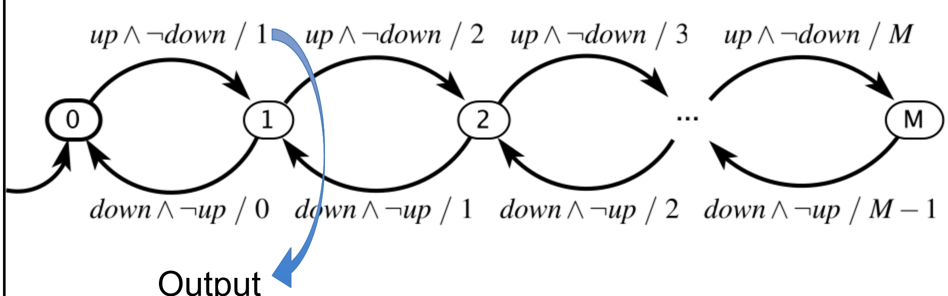
which means

$$g = \{\{up\}\} .$$

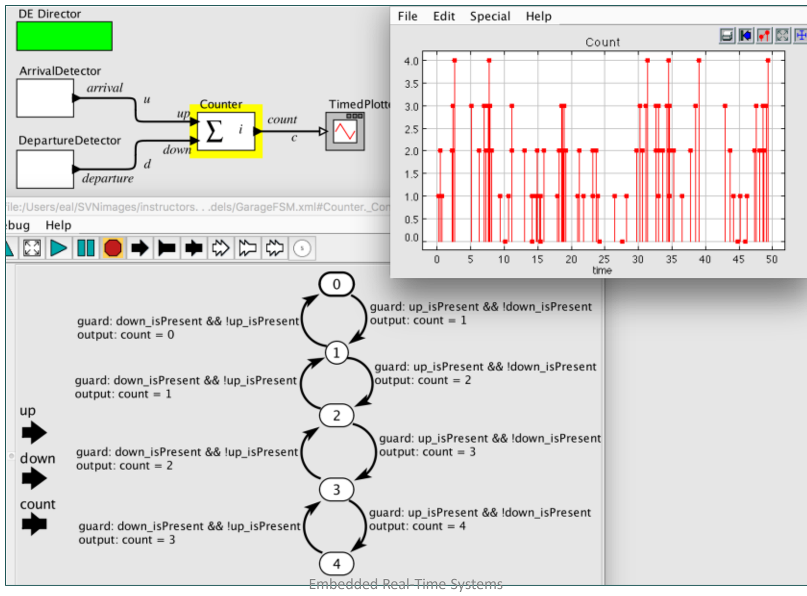
Garage Counter Finite State Machine (FSM) in Pictures



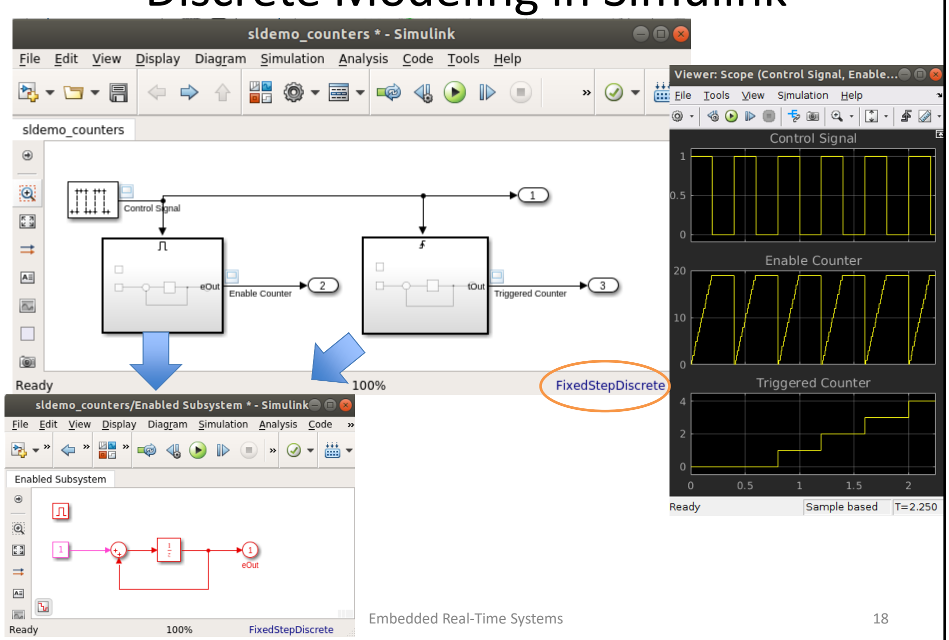
Garage Counter Finite State Machine (FSM) in Pictures



Ptolemy II Model

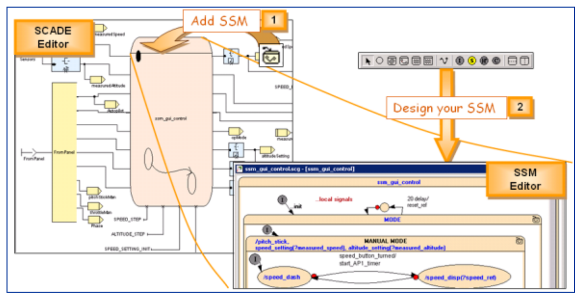


Discrete Modeling in Simulink

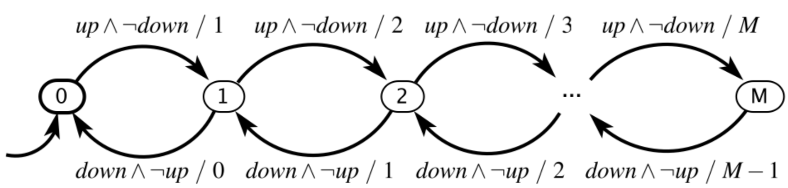


FSM Modeling Languages / Frameworks

- LabVIEW Statecharts
- Simulink Stateflow
- Scade
- …



Garage Counter Mathematical Model



Formally: $(States, Inputs, Outputs, update, initialState)$, where

- $States = \{0, 1, \cdots, M\}$
- $Inputs = (\{\text{up}, \text{down}\} \to \{absent, present\}$
- $Outputs = (\{count\} \to \{absent\} \cup \mathbb{N})$
- $update : States \times Inputs \to States \times Outputs$
- $initialState = 0$

The picture above defines the update function.

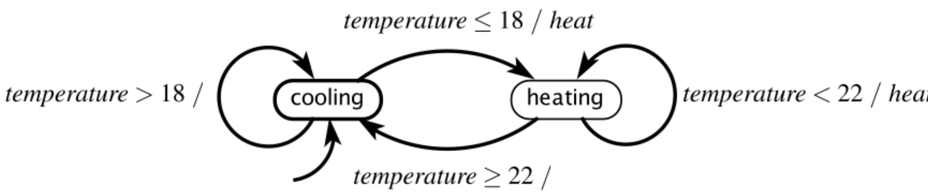
Examples of Guards for Pure Signals

true	Transition is always enabled.
p_1	Transition is enabled if p_1 is *present*.
$\neg p_1$	Transition is enabled if p_1 is *absent*.
$p_1 \wedge p_2$	Transition is enabled if both p_1 and p_2 are *present*.
$p_1 \vee p_2$	Transition is enabled if either p_1 or p_2 is *present*.
$p_1 \wedge \neg p_2$	Transition is enabled if p_1 is *present* and p_2 is *absent*.

Examples of Guards for Signals with Numerical Values

p_3	Transition is enabled if p_3 is *present* (not *absent*).
$p_3 = 1$	Transition is enabled if p_3 is *present* and has value 1.
$p_3 = 1 \wedge p_1$	Transition is enabled if p_3 has value 1 and p_1 is *present*.
$p_3 > 5$	Transition is enabled if p_3 is *present* with value greater than 5.

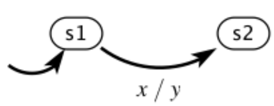
Example of *Modal* Model: Thermostat



When does a reaction occur?

- The FSM definition does NOT specify when it reacts.

input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$

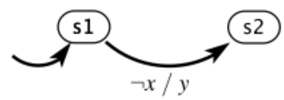


Suppose all inputs are discrete and a reaction occurs *when any input is present*. Then the above transition will be taken whenever the current state is s1 and *x* is present.

This is an *event-triggered model*.

When does a reaction occur?

input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$

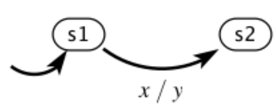


Suppose *x* and *y* are discrete and pure signals. When does the transition occur?

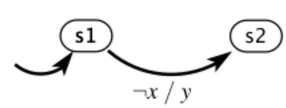
Answer: when the *environment* triggers a reaction and x is absent. If this is a (complete) event-triggered model, then the transition will never be taken because the reaction will only occur when x is present!

When does a reaction occur?

input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$



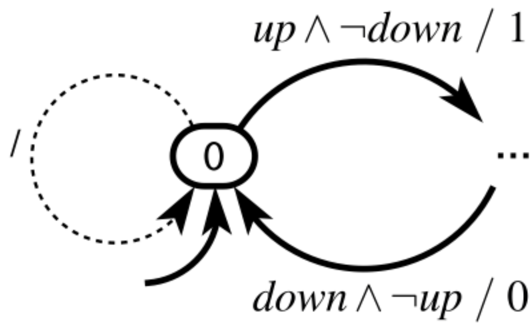
input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$



Suppose all inputs are discrete and a reaction occurs *on the tick of an external clock*.

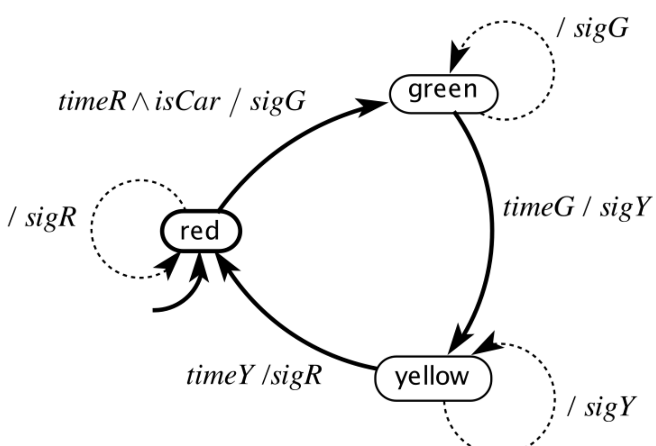
This is a *time-triggered model*.

More Notation: Default Transitions



- What happens if state=0 and a car departs?
- A default transition is enabled if no non-default transition is enabled and it either has no guard or the guard evaluates to true. When is the above default transition enabled?

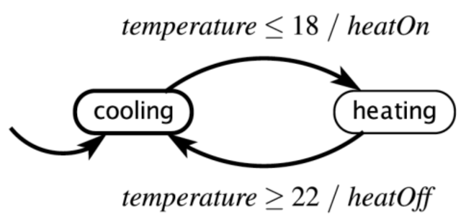
Example: Traffic Light Controller



Only show default transitions if they are guarded or produce outputs (or go to other states).

Where Default Transitions Need Not Be shown

input: *temperature* : $\mathbb{R}$
outputs: *heatOn*, *heatOff* : pure

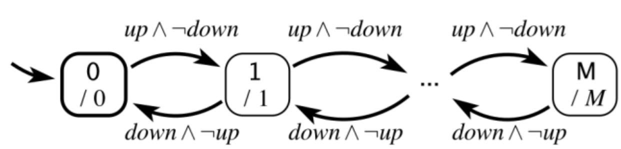


Exercise: From this picture, construct the formal mathematical model.

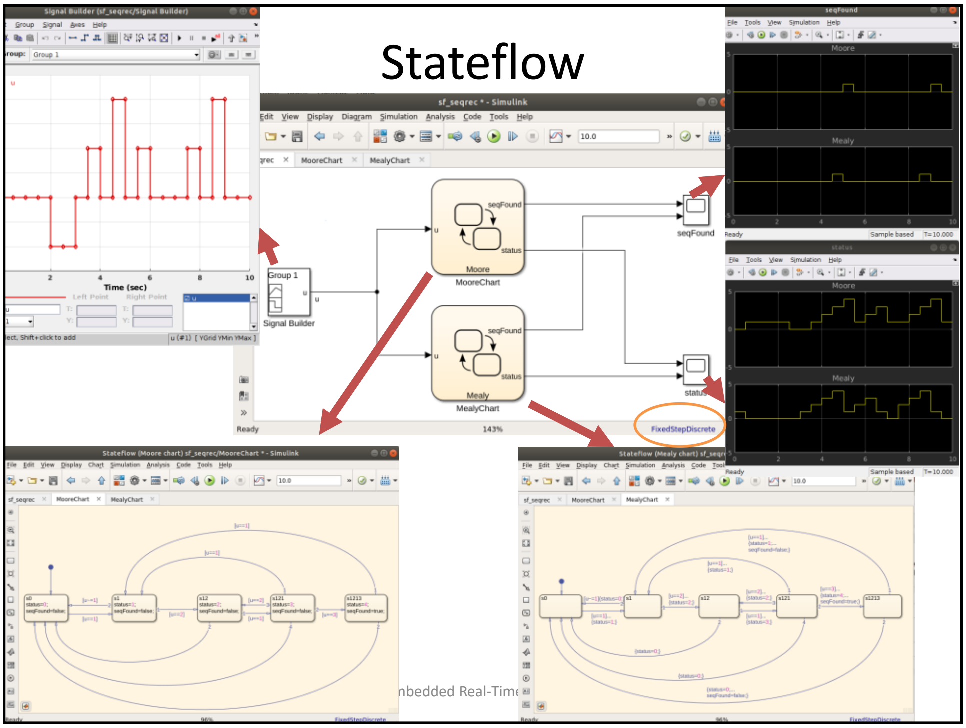
Mealy and Moore Machines

- The state machines we describe in this lecture are known as Mealy machines.
- Moore machine produces outputs when the machine is in a state, rather than when a transition is taken.

inputs: *up*, *down*: pure
output: *count*: $\{0, \cdots, M\}$



Stateflow



Some Definitions

- **Stuttering transition**: (possibly implicit) default transition that is enabled when inputs are absent, that does not change state, and that produces absent outputs.
- **Receptiveness**: For any input values, some transition is enabled. Our structure together with the implicit default transition ensures that our FSMs are receptive.
- **Determinism**: In every state, for all input values, exactly one (possibly implicit) transition is enabled.

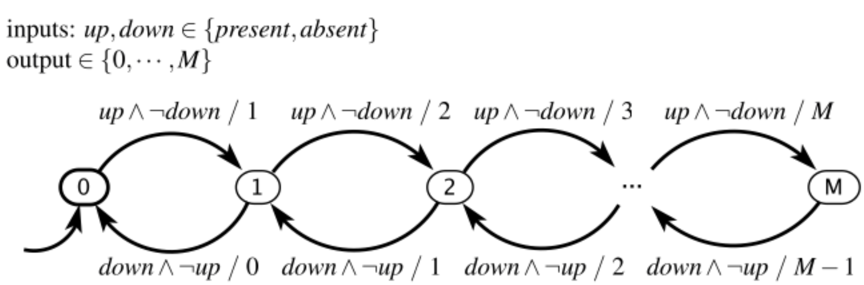
Reflection: Three Kinds of Transitions

- Self-Loop
- Default Transition
- Stuttering Transition

1. Is a default transition always a self-loop?
2. Is a stuttering transition always a self-loop?
3. Is a self-loop always stuttering?

Garage Counter Example

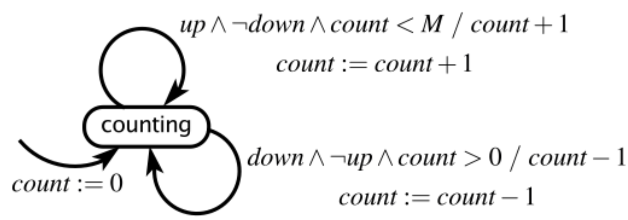
Is the following model manageable with large Ms?



Extended State Machines

Extended state machines augment the FSM model with *variables* that may be read or written. E.g.:

variable: $count \in \{0, \cdots, M\}$
inputs: $up, down \in \{present, absent\}$
output $\in \{0, \cdots, M\}$

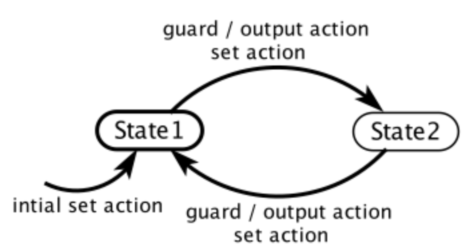


Question: What is the size of the state space?

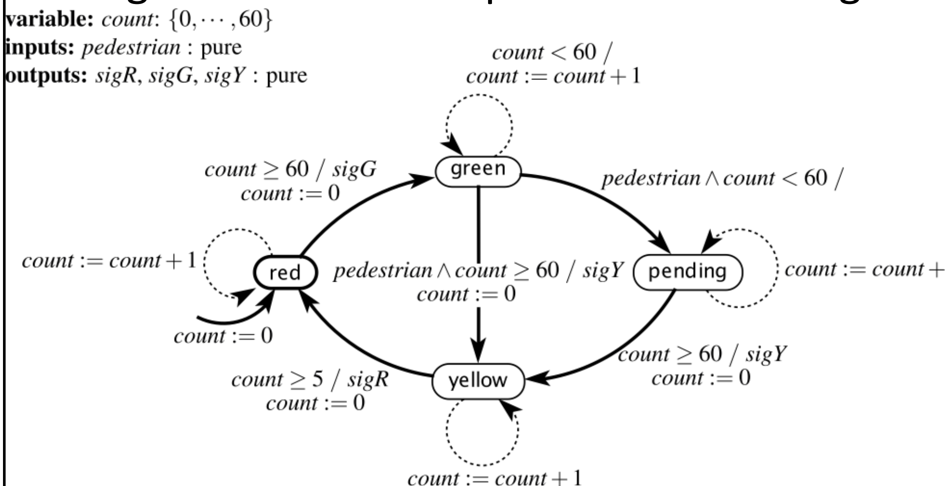
General Notation for Extended State Machines

We make explicit declarations of variables, inputs, and outputs to help distinguish the three.

variable declaration(s)
input declaration(s)
output declaration(s)

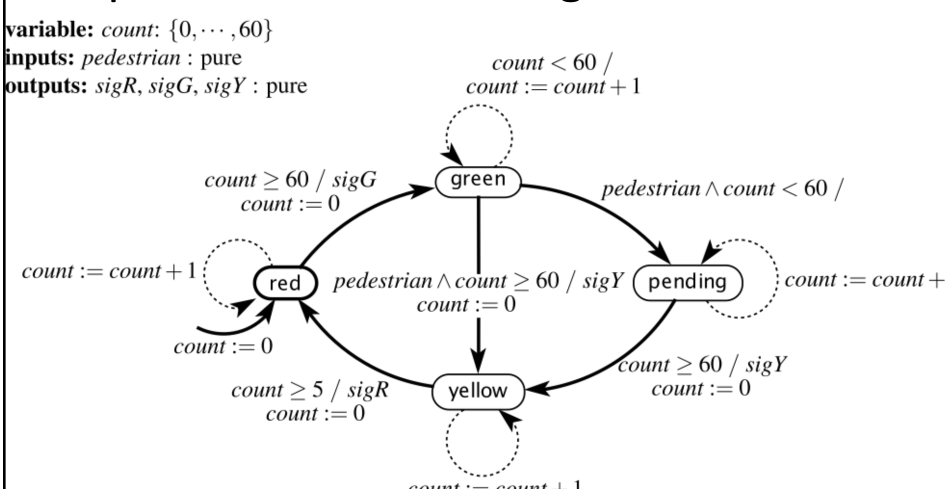


Extended state machine model of a traffic light controller at a pedestrian crossing



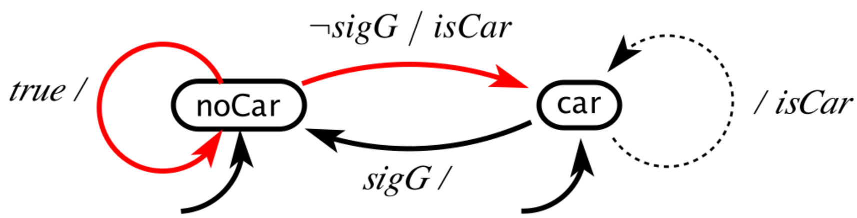
This model assumes one reaction per second
(a *time-triggered* model)

Quiz: What is the Size of the State Space for the Traffic Light Controller?



Example: Nondeterministic FSM

Model of the environment for a traffic light, abstracted using nondeterminism:



Formally, the update function is replaced by such a function:

$$possibleUpdates : States \times Inputs \rightarrow 2^{States \times Outputs}$$

Uses of Nondeterminism

- Modeling unknown aspects of the environment or system
  - Such as: how the environment changes a robot's orientation
- Hiding detail in a *specification* of the system
  - See the text

Any other reasons why nondeterministic FSMs might be preferred over deterministic FSMs?

Size Matters

Non-deterministic FSMs are more compact than deterministic FSMs

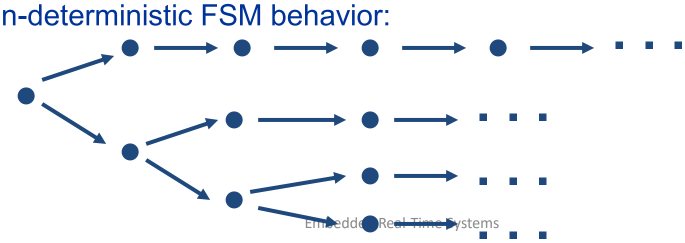
- A classic result in automata theory shows that a nondeterministic FSM has a related deterministic FSM that is equivalent in a technical sense.
- But the deterministic machine has, in the worst case, many more states (exponential in the number of states of the nondeterministic machine, Appendix B).

Non-deterministic Behavior: Tree of Computations

- For a fixed input sequence:
  - A deterministic system exhibits a single behavior
  - A non-deterministic system exhibits a **set of behaviors**
    - visualized as a *computation tree*

Deterministic FSM behavior:

Non-deterministic FSM behavior:



Non-deterministic ≠ Probabilistic/Stochastic

In a probabilistic FSM, each transition has an associated probability with which it is taken.

In a non-deterministic FSM, no such probability is known. We just know that any of the enabled transitions from a state can be taken.

Next Lecture

- Timed automata
- Hybrid systems
- Read chapter 4 of LeeSeshia