**1. Definition of Quadratic Equations**

A **quadratic equation** is an equation of the form:

ax2+bx+c=0

Where:

* a, b, and c are constants, and a≠0
* The highest power of x is 2, making it a second-degree polynomial.

**Example:**

2x2+3x−5=0

Here, a=2 , b=3b , and c=−5c

**2. Methods of Solving Quadratic Equations**

**(i) Factorization Method**

To solve a quadratic equation by factorization, we express the equation as a product of two binomials.

**Example:** Solve x2−5x+6=0

**Solution:**

1. Find two numbers that multiply to 6(constant term) and add to −5(coefficient of x).
2. The numbers are −2 and −3
3. Factor the quadratic equation:

(x−2)(x−3)=0

1. Solve for x:

x−2=0 orx−3=0x

**(ii) Completing the Square Method**

To solve by completing the square, we rewrite the equation so that one side is a perfect square trinomial.

**Example:** Solve x2+6x−7=0 by completing the square.

**Solution:**

1. Move the constant term to the other side:

x2+6x=7

1. Take half of the coefficient of x (which is 6), square it, and add to both sides:

x2+6x+9 =7+9

(x+3)2=16

Take the square root of both sides:

x+3=±4

1. Solve for x:

x=−3+4=1or x=−3−4=−7

**(iii) Quadratic Formula**

The quadratic formula is:

x=−b± b2−4ac 2a

**Example:** Solve 2x2−4x−6=0 using the quadratic formula.

**Solution:**

1. Identify the coefficients: a=2a = 2a=2, b=−4b = -4b=−4, c=−6c = -6c=−6.
2. Substitute into the quadratic formula:

x=−(−4)±(−4)2−4(2)(−6)2(2)x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}x=2(2)−(−4)±(−4)2−4(2)(−6)​​ x=4±16+484x = \frac{4 \pm \sqrt{16 + 48}}{4}x=44±16+48​​ x=4±644x = \frac{4 \pm \sqrt{64}}{4}x=44±64​​ x=4±84x = \frac{4 \pm 8}{4}x=44±8​

1. Solve for x:

x=4+84=3orx=4−84=−1x = \frac{4 + 8}{4} = 3 \quad \text{or} \quad x = \frac{4 - 8}{4} = -1x=44+8​=3orx=44−8​=−1

**3. Nature of the Roots**

The discriminant (DDD) is the expression under the square root in the quadratic formula:

D=b2−4acD = b^2 - 4acD=b2−4ac

* If D>0D > 0D>0, the equation has **two distinct real roots**.
* If D=0D = 0D=0, the equation has **one real double root**.
* If D<0D < 0D<0, the equation has **complex (imaginary) roots**.

**Example:** Solve x2+4x+5=0x^2 + 4x + 5 = 0x2+4x+5=0 and determine the nature of the roots.

**Solution:**

1. Identify the coefficients: a=1a = 1a=1, b=4b = 4b=4, c=5c = 5c=5.
2. Calculate the discriminant:

D=b2−4ac=42−4(1)(5)=16−20=−4D = b^2 - 4ac = 4^2 - 4(1)(5) = 16 - 20 = -4D=b2−4ac=42−4(1)(5)=16−20=−4

1. Since D<0D < 0D<0, the equation has **complex roots**.

**4. Sum and Product of Roots**

For a quadratic equation ax2+bx+c=0ax^2 + bx + c = 0ax2+bx+c=0:

* **Sum of the roots**: −ba-\frac{b}{a}−ab​
* **Product of the roots**: ca\frac{c}{a}ac​

**Example:** For the equation x2−5x+6=0x^2 - 5x + 6 = 0x2−5x+6=0, find the sum and product of the roots.

**Solution:**

1. Sum of roots: −−51=5-\frac{-5}{1} = 5−1−5​=5
2. Product of roots: 61=6\frac{6}{1} = 616​=6

**5. Formation of Quadratic Equations from Given Roots**

Given the roots r1r\_1r1​ and r2r\_2r2​, the quadratic equation is formed as:

(x−r1)(x−r2)=0(x - r\_1)(x - r\_2) = 0(x−r1​)(x−r2​)=0

**Example:** If the roots are 333 and −2-2−2, form the quadratic equation.

**Solution:**

(x−3)(x+2)=0(x - 3)(x + 2) = 0(x−3)(x+2)=0

Expanding:

x2+2x−3x−6=0x^2 + 2x - 3x - 6 = 0x2+2x−3x−6=0 x2−x−6=0x^2 - x - 6 = 0x2−x−6=0

**6. Graphical Method**

The graph of a quadratic equation y=ax2+bx+cy = ax^2 + bx + cy=ax2+bx+c is a parabola.

* The roots of the equation are the **x-intercepts** of the graph (where y=0y = 0y=0).
* The **vertex** of the parabola gives the maximum or minimum point.

**Example:** For the equation y=x2−4x−5y = x^2 - 4x - 5y=x2−4x−5, plot the graph.

**Solution:**

1. Find the vertex and roots:

Vertex=(−b2a,substitute into the equation)\text{Vertex} = \left(\frac{-b}{2a}, \text{substitute into the equation}\right)Vertex=(2a−b​,substitute into the equation)

For this equation, the roots are at x=−1x = -1x=−1 and x=5x = 5x=5.

1. Plot the graph and find the x-intercepts.

**7. Solving Quadratic Inequalities**

To solve a quadratic inequality such as x2−5x+6>0x^2 - 5x + 6 > 0x2−5x+6>0:

1. Solve the related quadratic equation x2−5x+6=0x^2 - 5x + 6 = 0x2−5x+6=0 to find the roots.
2. Use a sign chart or test values to determine the intervals where the inequality holds.

**8. Word Problems and Applications**

**Example:**

A rectangular field has a length x+4x + 4x+4 meters and a width x−2x - 2x−2 meters. If the area is 120 square meters, find the value of x.

**Solution:**

1. Area of the rectangle is Length×Width=120\text{Length} \times \text{Width} = 120Length×Width=120.
2. Set up the equation:

(x+4)(x−2)=120(x + 4)(x - 2) = 120(x+4)(x−2)=120

1. Solve the quadratic equation formed and find x.

**9. Past WAEC/NECO Questions**

1. **Question:** Solve x2−6x+5=0x^2 - 6x + 5 = 0x2−6x+5=0 by factorization.
2. **Question:** A quadratic equation has roots 2 and -3. Form the equation.

**Try This!**

1. Solve x2+7x+12=0x^2 + 7x + 12 = 0x2+7x+12=0 by factorization.
2. Determine the nature of the roots for x2+6x+9=0x^2 + 6x + 9 = 0x2+6x+9=0.

**1. Introduction to Sequences and Series**

**Objectives:**

* Define sequences and series.
* Introduce the concept of progressions (Arithmetic and Geometric).
* Understand the difference between a sequence and a series.

**Content:**

A **sequence** is an ordered list of numbers, where each number is called a term. In a sequence, the terms follow a specific rule or pattern.

A **series** is the sum of the terms in a sequence.

**Example 1:**

* Sequence: 2,4,6,8,10,…2, 4, 6, 8, 10, \dots2,4,6,8,10,…
* Series: 2+4+6+8+10=302 + 4 + 6 + 8 + 10 = 302+4+6+8+10=30

**Progressions** are special types of sequences where the terms follow specific patterns:

* **Arithmetic Progression (AP)**: The difference between consecutive terms is constant.
* **Geometric Progression (GP)**: Each term is obtained by multiplying the previous term by a constant ratio.

**2. Arithmetic Progression (AP)**

**Objectives:**

* Understand the definition of an Arithmetic Progression (AP).
* Derive and apply the nth-term formula for an AP.
* Solve problems involving the sum of the first nnn terms.
* Apply AP to real-life scenarios like installment payments.

**Content:**

1. **Definition of AP**: An **Arithmetic Progression (AP)** is a sequence in which the difference between consecutive terms is constant. This difference is called the **common difference** ddd.
2. **General formula for nth term**: The nth term of an AP is given by:

an=a+(n−1)⋅da\_n = a + (n - 1) \cdot dan​=a+(n−1)⋅d

where:

* + aaa is the first term,
  + ddd is the common difference,
  + nnn is the term number.

**Example 2:**

Find the 10th term of the AP: 3,6,9,12,…3, 6, 9, 12, \dots3,6,9,12,…

* First term a=3a = 3a=3, common difference d=3d = 3d=3, and n=10n = 10n=10.

a10=3+(10−1)⋅3=3+27=30a\_{10} = 3 + (10 - 1) \cdot 3 = 3 + 27 = 30a10​=3+(10−1)⋅3=3+27=30

So, the 10th term is 30.

1. **Sum of the first n terms**: The sum of the first nnn terms of an AP is given by:

Sn=n2⋅(2a+(n−1)⋅d)S\_n = \frac{n}{2} \cdot (2a + (n - 1) \cdot d)Sn​=2n​⋅(2a+(n−1)⋅d)

Alternatively, if you know the last term lll, the sum can also be written as:

Sn=n2⋅(a+l)S\_n = \frac{n}{2} \cdot (a + l)Sn​=2n​⋅(a+l)

**Example 3:**

Find the sum of the first 5 terms of the AP: 2,5,8,11,…2, 5, 8, 11, \dots2,5,8,11,….

* First term a=2a = 2a=2, common difference d=3d = 3d=3, and n=5n = 5n=5.

S5=52⋅(2⋅2+(5−1)⋅3)=52⋅(4+12)=52⋅16=40S\_5 = \frac{5}{2} \cdot (2 \cdot 2 + (5 - 1) \cdot 3) = \frac{5}{2} \cdot (4 + 12) = \frac{5}{2} \cdot 16 = 40S5​=25​⋅(2⋅2+(5−1)⋅3)=25​⋅(4+12)=25​⋅16=40

The sum of the first 5 terms is 40.

**Try This:**

1. The first term of an arithmetic progression is 2, and the common difference is 5. Find the 15th term.
2. The sum of the first 10 terms of an AP is 120. If the first term is 8, find the common difference.
3. The 5th term of an arithmetic progression is 20, and the 10th term is 35. Find the first term and the common difference.

**3. Geometric Progression (GP)**

**Objectives:**

* Understand the definition of a Geometric Progression (GP).
* Derive and apply the nth-term formula for a GP.
* Solve problems involving the sum of the first nnn terms and the sum to infinity for converging GPs.

**Content:**

1. **Definition of GP**: A **Geometric Progression (GP)** is a sequence where each term is obtained by multiplying the previous term by a constant ratio rrr.
2. **General formula for nth term**: The nth term of a GP is given by:

an=a⋅r(n−1)a\_n = a \cdot r^{(n - 1)}an​=a⋅r(n−1)

where:

* + aaa is the first term,
  + rrr is the common ratio,
  + nnn is the term number.

**Example 4:**

Find the 6th term of the GP: 4,8,16,32,…4, 8, 16, 32, \dots4,8,16,32,….

* First term a=4a = 4a=4, common ratio r=2r = 2r=2, and n=6n = 6n=6.

a6=4⋅26−1=4⋅32=128a\_6 = 4 \cdot 2^{6 - 1} = 4 \cdot 32 = 128a6​=4⋅26−1=4⋅32=128

The 6th term is 128.

1. **Sum of the first n terms**: The sum of the first nnn terms of a GP is given by:

Sn=a(1−rn)1−r(for r≠1)S\_n = \frac{a(1 - r^n)}{1 - r} \quad \text{(for \( r \neq 1 \))}Sn​=1−ra(1−rn)​(for r=1)

**Example 5:**

Find the sum of the first 5 terms of the GP: 2,6,18,54,…2, 6, 18, 54, \dots2,6,18,54,….

* First term a=2a = 2a=2, common ratio r=3r = 3r=3, and n=5n = 5n=5.

S5=2(1−35)1−3=2(1−243)−2=242S\_5 = \frac{2(1 - 3^5)}{1 - 3} = \frac{2(1 - 243)}{-2} = 242S5​=1−32(1−35)​=−22(1−243)​=242

The sum of the first 5 terms is 242.

1. **Sum to infinity (for converging GP)**: For ∣r∣<1|r| < 1∣r∣<1, the sum to infinity of a GP is:

S∞=a1−rS\_{\infty} = \frac{a}{1 - r}S∞​=1−ra​

**Example 6:**

Find the sum to infinity of the GP: 3,1.5,0.75,…3, 1.5, 0.75, \dots3,1.5,0.75,….

* First term a=3a = 3a=3, common ratio r=0.5r = 0.5r=0.5.

S∞=31−0.5=6S\_{\infty} = \frac{3}{1 - 0.5} = 6S∞​=1−0.53​=6

The sum to infinity is 6.

**Try This:**

1. Find the sum of the first 7 terms of the GP: 3,6,12,24,…3, 6, 12, 24, \dots3,6,12,24,….
2. What is the sum to infinity of the GP: 1,0.5,0.25,…1, 0.5, 0.25, \dots1,0.5,0.25,…?
3. Calculate the 10th term of the GP: 5,15,45,135,…5, 15, 45, 135, \dots5,15,45,135,….

**4. Comparison Between AP and GP**

**Objectives:**

* Identify the similarities and differences between AP and GP.
* Learn how to convert problems between AP and GP.

**Content:**

1. **Similarities**:
   * Both are sequences with specific rules for finding terms.
   * Both can be used to model real-life scenarios such as financial growth or population dynamics.
2. **Differences**:
   * **AP**: Constant difference between terms.
   * **GP**: Constant ratio between terms.
   * In **AP**, terms increase or decrease linearly, while in **GP**, terms increase or decrease exponentially.

**Practice Problems:**

1. Find the 15th term of the AP: 5,10,15,…5, 10, 15, \dots5,10,15,….
2. Calculate the sum of the first 8 terms of the GP: 2,6,18,54,…2, 6, 18, 54, \dots2,6,18,54,….
3. A person deposits $1000 in a bank at an annual compound interest rate of 8%. How much will the deposit be worth after 12 years?
4. Find the sum to infinity of the GP: 5,2.5,1.25,…5, 2.5, 1.25, \dots5,2.5,1.25,….

**5. Past Exam Questions**

1. If the first term of an AP is 4 and the common difference is 3, find the sum of the first 10 terms.
2. A geometric progression has the first term 6 and the common ratio 2. Find the 7th term.
3. The 6th term of a GP is 512, and the first term is 8. Find the common ratio.
4. In an arithmetic progression, the sum of the first 15 terms is 120, and the first term is 8. Find the common difference.

**Permutation and combination**

**Objectives**

By the end of this lesson, students should be able to:

1. Understand and apply basic counting principles.
2. Solve problems involving permutation and combination, both with and without repetition.
3. Differentiate between permutation and combination.
4. Apply these concepts to word problems.
5. Solve advanced problems involving both permutation and combination.

**1. Introduction to Counting Principles**

**Basic Counting Principles**

1. **The Multiplication Principle**: If one event can occur in mmm ways and a second event can occur in nnn ways, the total number of ways the two events can occur is the product of mmm and nnn. This is expressed as:

m×nm \times nm×n

**Example:** A person can choose a shirt in 5 different colors and a pair of pants in 3 different colors. The total number of outfits is:

5×3=15 outfits.5 \times 3 = 15 \text{ outfits.}5×3=15 outfits.

**Try this:**

* + A student has 3 choices of shoes, 4 choices of pants, and 2 choices of shirts. How many total outfit combinations can the student create?

1. **The Addition Principle**: If event AAA can occur in mmm ways and event BBB can occur in nnn ways, and these two events cannot happen together, the total number of ways the events can occur is the sum of mmm and nnn. This is expressed as:

m+nm + nm+n

**Example:** A person can choose a dessert from 4 types of cakes or 3 types of ice cream. The total number of desserts they can choose is:

4+3=7 desserts.4 + 3 = 7 \text{ desserts.}4+3=7 desserts.

**Try this:**

* + In a library, there are 6 fiction books and 4 non-fiction books. How many different books can a student choose if they can select only one book?

**2. Permutation**

**Basic Permutation (Arrangement)**

**Permutation** is the arrangement of objects in a specific order. The number of ways to arrange nnn distinct objects is given by the formula:

P(n)=n!P(n) = n!P(n)=n!

Where n!n!n! (read as "n factorial") represents the product of all positive integers from 1 to nnn.

**Example:** The number of ways to arrange 3 books on a shelf is:

P(3)=3!=3×2×1=6P(3) = 3! = 3 \times 2 \times 1 = 6P(3)=3!=3×2×1=6

**Permutation of rrr Objects from nnn Distinct Objects**

When selecting and arranging rrr objects from nnn distinct objects, the number of permutations is given by:

P(n,r)=n!(n−r)!P(n, r) = \frac{n!}{(n - r)!}P(n,r)=(n−r)!n!​

**Example:** If there are 5 books, how many ways can we arrange 2 of them on a shelf?

P(5,2)=5!(5−2)!=5!3!=20P(5, 2) = \frac{5!}{(5 - 2)!} = \frac{5!}{3!} = 20P(5,2)=(5−2)!5!​=3!5!​=20

**Permutation with Repetition**

When repetition of objects is allowed in the arrangement, the number of permutations is calculated as:

P(n1,n2,…,nk)=n!n1!×n2!×…×nk!P(n\_1, n\_2, \ldots, n\_k) = \frac{n!}{n\_1! \times n\_2! \times \ldots \times n\_k!}P(n1​,n2​,…,nk​)=n1​!×n2​!×…×nk​!n!​

Where n1,n2,…,nkn\_1, n\_2, \ldots, n\_kn1​,n2​,…,nk​ are the frequencies of the repeated objects.

**Example:** The number of distinct arrangements of the word "SUCCESS" is:

P(7,2,2)=7!2!×2!=50402×2=1260P(7, 2, 2) = \frac{7!}{2! \times 2!} = \frac{5040}{2 \times 2} = 1260P(7,2,2)=2!×2!7!​=2×25040​=1260

**Circular Permutation**

When objects are arranged in a circle, the number of distinct arrangements is given by:

Pcircular(n)=(n−1)!P\_{\text{circular}}(n) = (n - 1)!Pcircular​(n)=(n−1)!

**Example:** If 5 people are sitting around a circular table, the number of ways they can be arranged is:

Pcircular(5)=(5−1)!=4!=24P\_{\text{circular}}(5) = (5 - 1)! = 4! = 24Pcircular​(5)=(5−1)!=4!=24

**3. Combination**

**Basic Combination (Selection)**

**Combination** is the selection of objects without regard to the order. The number of ways to choose rrr objects from nnn distinct objects is given by the formula:

C(n,r)=n!r!(n−r)!C(n, r) = \frac{n!}{r!(n - r)!}C(n,r)=r!(n−r)!n!​

**Example:** The number of ways to select 3 books from 5 distinct books is:

C(5,3)=5!3!×(5−3)!=5!3!×2!=10C(5, 3) = \frac{5!}{3! \times (5 - 3)!} = \frac{5!}{3! \times 2!} = 10C(5,3)=3!×(5−3)!5!​=3!×2!5!​=10

**Combination with Repetition**

When objects can be selected more than once (i.e., repetition is allowed), the number of combinations is given by:

C(n+r−1,r)=(n+r−1)!r!(n−1)!C(n + r - 1, r) = \frac{(n + r - 1)!}{r!(n - 1)!}C(n+r−1,r)=r!(n−1)!(n+r−1)!​

**Example:** If you are selecting 3 flavors of ice cream from 4 available flavors (with repetition), the number of ways to do so is:

C(4+3−1,3)=C(6,3)=6!3!×3!=20C(4 + 3 - 1, 3) = C(6, 3) = \frac{6!}{3! \times 3!} = 20C(4+3−1,3)=C(6,3)=3!×3!6!​=20

**4. Application of Permutation and Combination**

**Word Problems**

1. **Selecting a Committee:** In a group of 10 students, how many ways can we select a committee of 3 students?

**Solution:** This is a combination problem since the order does not matter. The number of ways to select 3 students from 10 is:

C(10,3)=10!3!×(10−3)!=10!3!×7!=120C(10, 3) = \frac{10!}{3! \times (10 - 3)!} = \frac{10!}{3! \times 7!} = 120C(10,3)=3!×(10−3)!10!​=3!×7!10!​=120

**Try this:**

* + From a class of 15 students, how many ways can we select a group of 4 students to represent the class in a quiz competition?

1. **Arranging Books on a Shelf:** How many ways can we arrange 4 books on a shelf from a set of 6 books?

**Solution:** This is a permutation problem since the order matters. The number of ways to arrange 4 books from 6 is:

P(6,4)=6!(6−4)!=6!2!=360P(6, 4) = \frac{6!}{(6 - 4)!} = \frac{6!}{2!} = 360P(6,4)=(6−4)!6!​=2!6!​=360

**Try this:**

* + How many ways can we arrange 5 books on a shelf from a collection of 9 books?

**5. Advanced Problems**

1. **Mixed Problems Involving Permutation and Combination:**

A group of 6 men and 4 women are to form a committee of 5 members. How many ways can the committee be formed if the committee must consist of at least 3 men?

**Solution:** We break the problem into cases based on the number of men in the committee:

* + Case 1: 3 men and 2 women

C(6,3)×C(4,2)=20×6=120C(6, 3) \times C(4, 2) = 20 \times 6 = 120C(6,3)×C(4,2)=20×6=120

* + Case 2: 4 men and 1 woman

C(6,4)×C(4,1)=15×4=60C(6, 4) \times C(4, 1) = 15 \times 4 = 60C(6,4)×C(4,1)=15×4=60

* + Total number of ways:

120+60=180120 + 60 = 180120+60=180

**Try this:**

* + From a group of 10 men and 8 women, how many ways can a committee of 6 people be formed if it must consist of at least 2 women?

**6. Practice Questions**

1. How many ways can 3 books be arranged on a shelf from a set of 7 books?
2. In a group of 8 people, how many ways can a committee of 5 people be selected?
3. How many ways can 3 different shirts and 2 different pants be arranged in an outfit?
4. A password consists of 4 letters followed by 3 digits. How many different passwords can be formed if repetition is allowed?
5. A club has 6 men and 5 women. How many ways can a committee of 4 members be selected if at least 2 members must be women?

**Exam-Standard Problems**

1. **Problem:**  
   A class has 8 boys and 7 girls. How many ways can a group of 5 students be selected, such that the group has at least 3 girls?
2. **Problem:**  
   How many different ways can 3 men and 4 women be arranged in a straight line if the men must be together?

**Binomial Expansion: A Complete e-Note**

**Learning Objectives:**

By the end of this lesson, students should be able to:

1. Define the Binomial Theorem and understand its expansion formula.
2. Expand binomial expressions of the form (a+b)n(a + b)^n(a+b)n for small values of nnn.
3. Use binomial coefficients and Pascal’s Triangle to find terms in the expansion.
4. Identify specific terms in a binomial expansion, including terms with particular powers of aaa or bbb.
5. Apply the Binomial Theorem in solving real-world problems, including probability distributions.
6. Expand binomials with negative exponents.
7. Solve advanced binomial problems that require multiple steps and simplifications.

**1. Basic Understanding of the Binomial Theorem**

**Definition and Formula:**

The Binomial Theorem allows us to expand any binomial expression raised to a power nnn. It is expressed as:

(a+b)n=∑k=0n(nk)an−kbk(a + b)^n = \sum\_{k=0}^{n} \binom{n}{k} a^{n-k} b^k(a+b)n=k=0∑n​(kn​)an−kbk

Where:

* (nk)\binom{n}{k}(kn​) is the **binomial coefficient**, also referred to as "n choose k."
* an−ka^{n-k}an−k and bkb^kbk are the respective powers of aaa and bbb in each term.

**Binomial Coefficients and Pascal's Triangle:**

The binomial coefficients (nk)\binom{n}{k}(kn​) can be found either using the formula:

(nk)=n!k!(n−k)!\binom{n}{k} = \frac{n!}{k!(n-k)!}(kn​)=k!(n−k)!n!​

Or by using **Pascal’s Triangle**, a triangular array of numbers where each number is the sum of the two directly above it.

**Try This:**

1. Find the binomial coefficient (52)\binom{5}{2}(25​).
2. Use Pascal’s Triangle to find the expansion of (a+b)3(a + b)^3(a+b)3.

**2. Expanding Binomials for Small nnn**

**Example 1: Expanding (a+b)2(a + b)^2(a+b)2**

Using the Binomial Theorem:

(a+b)2=(20)a2b0+(21)a1b1+(22)a0b2(a + b)^2 = \binom{2}{0} a^2 b^0 + \binom{2}{1} a^1 b^1 + \binom{2}{2} a^0 b^2(a+b)2=(02​)a2b0+(12​)a1b1+(22​)a0b2 =a2+2ab+b2= a^2 + 2ab + b^2=a2+2ab+b2

**Example 2: Expanding (a+b)3(a + b)^3(a+b)3**

(a+b)3=(30)a3b0+(31)a2b1+(32)a1b2+(33)a0b3(a + b)^3 = \binom{3}{0} a^3 b^0 + \binom{3}{1} a^2 b^1 + \binom{3}{2} a^1 b^2 + \binom{3}{3} a^0 b^3(a+b)3=(03​)a3b0+(13​)a2b1+(23​)a1b2+(33​)a0b3 =a3+3a2b+3ab2+b3= a^3 + 3a^2b + 3ab^2 + b^3=a3+3a2b+3ab2+b3

**Example 3: Expanding (a+b)4(a + b)^4(a+b)4**

(a+b)4=(40)a4b0+(41)a3b1+(42)a2b2+(43)a1b3+(44)a0b4(a + b)^4 = \binom{4}{0} a^4 b^0 + \binom{4}{1} a^3 b^1 + \binom{4}{2} a^2 b^2 + \binom{4}{3} a^1 b^3 + \binom{4}{4} a^0 b^4(a+b)4=(04​)a4b0+(14​)a3b1+(24​)a2b2+(34​)a1b3+(44​)a0b4 =a4+4a3b+6a2b2+4ab3+b4= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4=a4+4a3b+6a2b2+4ab3+b4

**Try This:**

1. Expand (x+1)2(x + 1)^2(x+1)2.
2. Expand (a+b)3(a + b)^3(a+b)3 and identify the middle term.
3. Expand (2a+3b)3(2a + 3b)^3(2a+3b)3.

**3. General Term in the Binomial Expansion**

The general term Tk+1T\_{k+1}Tk+1​ in the expansion of (a+b)n(a + b)^n(a+b)n is given by:

Tk+1=(nk)an−kbkT\_{k+1} = \binom{n}{k} a^{n-k} b^kTk+1​=(kn​)an−kbk

This formula helps you find any specific term in the binomial expansion. For instance, if you need the term that contains a2b3a^2 b^3a2b3 in the expansion of (a+b)5(a + b)^5(a+b)5, you can use this general formula to extract that specific term.

**Try This:**

1. Find the 3rd term in the expansion of (a+b)4(a + b)^4(a+b)4.
2. Find the term containing a3b2a^3 b^2a3b2 in the expansion of (a+b)5(a + b)^5(a+b)5.
3. Calculate the 5th term in the expansion of (x+1)6(x + 1)^6(x+1)6.

**4. Special Cases of the Binomial Theorem**

**Binomial Theorem for n=2n = 2n=2:**

(a+b)2=a2+2ab+b2(a + b)^2 = a^2 + 2ab + b^2(a+b)2=a2+2ab+b2

**Binomial Theorem for n=3n = 3n=3:**

(a+b)3=a3+3a2b+3ab2+b3(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3(a+b)3=a3+3a2b+3ab2+b3

**Expanding for Larger Powers:**

The Binomial Theorem can be used to expand binomials raised to larger powers, such as (a+b)4(a + b)^4(a+b)4, (a+b)5(a + b)^5(a+b)5, etc. The key is recognizing the pattern in the coefficients and the powers of aaa and bbb.

**Try This:**

1. Expand (a+b)2(a + b)^2(a+b)2 and list all terms.
2. Expand (x+2)3(x + 2)^3(x+2)3.
3. Expand (a+b)5(a + b)^5(a+b)5 and find the middle term.

**5. Finding Specific Terms in Binomial Expansion**

**Example 4: Find the Term with a2b3a^2b^3a2b3 in the Expansion of (a+b)5(a + b)^5(a+b)5**

We are looking for the term where the powers of aaa and bbb are 2 and 3, respectively.

Using the general term formula, we need to find the kkk-value where n−k=2n-k = 2n−k=2 and k=3k = 3k=3. For (a+b)5(a + b)^5(a+b)5, the term is:

T4=(53)a5−3b3=(53)a2b3=10a2b3T\_4 = \binom{5}{3} a^{5-3} b^3 = \binom{5}{3} a^2 b^3 = 10a^2b^3T4​=(35​)a5−3b3=(35​)a2b3=10a2b3

**Try This:**

1. Find the term with a2b3a^2 b^3a2b3 in the expansion of (a+b)5(a + b)^5(a+b)5.
2. Find the term with a4b1a^4 b^1a4b1 in the expansion of (a+b)6(a + b)^6(a+b)6.
3. Find the 4th term in the expansion of (x+1)5(x + 1)^5(x+1)5.

**6. Binomial Theorem for Negative Exponents**

The Binomial Theorem also applies when the exponent is negative. For example, for (1+x)n(1 + x)^n(1+x)n where nnn is a negative integer:

(1+x)n=1+nx+n(n−1)2!x2+⋯(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \cdots(1+x)n=1+nx+2!n(n−1)​x2+⋯

This expansion helps simplify expressions and is used when dealing with rational expressions or certain types of probability problems.

**Try This:**

1. Expand (1+x)−2 up to the first two terms.
2. Expand (1+x)−3(1 + x)^{-3}(1+x)−3 up to the third term.
3. Find the coefficient of x3x^3x3 in the expansion of (1+x)−4(1 + x)^{-4}(1+x)−4.

**7. Practice Questions**

**Try This:**

1. Expand (x+1)4(x + 1)^4(x+1)4.
2. Find the term with a2b3a^2b^3a2b3 in the expansion of (a+b)5(a + b)^5(a+b)5.
3. Expand (2a−3b)3(2a - 3b)^3(2a−3b)3.
4. Expand (1+x)5(1 + x)^5(1+x)5.

**Challenge Questions:**

1. Find the coefficient of x4x^4x4 in the expansion of (2x+3)5(2x + 3)^5(2x+3)5.
2. Expand (a+b)6(a + b)^6(a+b)6 and identify the middle term.
3. Calculate the expansion of (1+x)−2(1 + x)^{-2}(1+x)−2 up to the second term.