Quadratic Equations (For WAEC & NECO Success)

# Objectives

By the end of this lesson, students should be able to:

- Define and identify quadratic equations.  
- Solve quadratic equations by factorization, completing the square, and the quadratic formula.  
- Interpret the nature of roots using the discriminant.  
- Relate roots and coefficients.  
- Form quadratic equations given the roots.  
- Solve quadratic inequalities.  
- Apply quadratic equations to solve real-life problems.

# 1. Introduction to Quadratic Equations

A quadratic equation is an equation of the form:  
ax^2 + bx + c = 0  
where a, b, c are real numbers and a ≠ 0.

Identifying coefficients:

- a: Coefficient of x²  
- b: Coefficient of x  
- c: Constant term

## Example 1:

Identify a, b, and c in 3x² - 5x + 2 = 0.  
a = 3, b = -5, c = 2

## Try This:

Find a, b, and c in 2x² + 7x - 6 = 0.

# 2. Methods of Solving Quadratic Equations

## (i) Factorization

Express the quadratic as a product of two binomials.

## Example 2:

Solve x² + 5x + 6 = 0.  
Solution:  
x² + 2x + 3x + 6 = 0  
x(x + 2) + 3(x + 2) = 0  
(x + 2)(x + 3) = 0  
Thus, x = -2 or x = -3.

## Try This:

Solve x² - 7x + 12 = 0.

## (ii) Completing the Square

Make the left-hand side a perfect square trinomial.

## Example 3:

Solve x² + 4x - 5 = 0.  
Solution:  
x² + 4x = 5  
x² + 4x + 4 = 9 (Add 4 to both sides)  
(x + 2)² = 9  
x + 2 = ±3  
x = 1 or x = -5

## Try This:

Solve x² - 6x + 5 = 0 by completing the square.

## (iii) Using the Quadratic Formula

The formula is:  
x = (-b ± √(b² - 4ac)) / 2a

## Example 4:

Solve 2x² - 4x - 6 = 0.  
Solution:  
a = 2, b = -4, c = -6  
x = [-(−4) ± √((-4)² - 4(2)(−6))] / (2(2))  
x = [4 ± √(16 + 48)] / 4  
x = [4 ± √64] / 4  
x = [4 ± 8] / 4  
Thus, x = 3 or x = -1.

## Try This:

Solve x² + 3x - 10 = 0 using the quadratic formula.

# 3. Nature of Roots

Discriminant D = b² - 4ac  
Interpretation:  
- D > 0: Two distinct real roots.  
- D = 0: Equal real roots.  
- D < 0: Complex roots.

## Example 5:

Find the nature of roots of x² + 2x + 5 = 0.  
Solution:  
D = (2)² - 4(1)(5) = 4 - 20 = -16  
Thus, roots are complex.

## Try This:

Find the nature of roots of x² - 4x + 4 = 0.

# 4. Relationship Between Roots and Coefficients

Sum of roots = -b/a  
Product of roots = c/a

## Example 6:

Find the sum and product of roots of 2x² - 5x + 3 = 0.  
Solution:  
Sum = -(-5)/2 = 5/2  
Product = 3/2

## Try This:

Find the sum and product of roots of x² + 7x + 10 = 0.

# 5. Formation of Quadratic Equations from Given Roots

If α and β are roots:  
(x - α)(x - β) = 0  
Expand to get:  
x² - (α + β)x + αβ = 0

## Example 7:

Form the quadratic equation whose roots are 2 and -3.  
Solution:  
(x - 2)(x + 3) = 0  
x² + x - 6 = 0

## Try This:

Form a quadratic equation with roots 5 and -1.

# 6. Graphical Method

Plot the graph of y = ax² + bx + c.  
Steps:  
- Create a table of values.  
- Plot points.  
- Draw a smooth curve.

## Example 8:

Draw the graph of y = x² - 4.

## Try This:

Draw the graph of y = x² + 2x - 3.

# 7. Solving Quadratic Inequalities

Solve inequalities like ax² + bx + c > 0 or < 0.  
Steps:  
- Solve ax² + bx + c = 0.  
- Sketch the graph.  
- Identify the required region.

## Example 9:

Solve x² - 5x + 6 < 0.  
Solution:  
(x - 2)(x - 3) < 0  
Solution set: 2 < x < 3

## Try This:

Solve x² - x - 6 > 0.