

1.

No, assume three rotation angles are $[\theta_1, \theta_2, \theta_3]$,

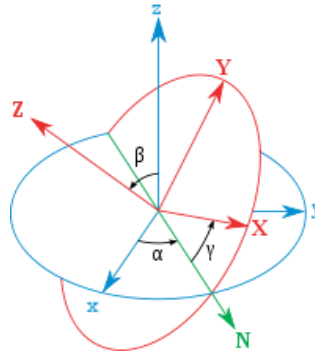
for the sequence of proper Euler angle (e.g. xyz), $[\pi + \theta_1, \pi - \theta_2, \pi + \theta_3]$ has same rotation matrix,

$$\begin{aligned}
 & \left(R_x(\theta_1) R_y(\theta_2) R_z(\theta_3) \right) \cdot \left(R_x(\pi + \theta_1) R_y(\pi - \theta_2) R_z(\pi + \theta_3) \right)^{-1} \\
 &= R_x(\theta_1) \cdot \underbrace{R_y(\theta_2) \cdot R_z(\theta_3) \cdot \left(R_z(\pi + \theta_3) \right)^{-1} \cdot \left(R_y(\pi - \theta_2) \right)^{-1} \cdot \left(R_x(\pi + \theta_1) \right)^{-1}}_{\substack{R_z(\pi) \\ R_y(\theta_2 + (\pi - \theta_2)) R_z(\pi) = R_y(\pi) R_z(\pi)}} \\
 &= R_x(\pi) R_z(\pi) R_y(\pi) = 1
 \end{aligned}$$

for the sequence of Tait–Bryan angle (e.g. xyx), $[\pi + \theta_1, 2\pi - \theta_2, \pi + \theta_3]$ has same rotation matrix.

$$\begin{aligned}
 & \left(R_x(\theta_1) R_y(\theta_2) R_x(\theta_3) \right) \cdot \left(R_x(\pi + \theta_1) R_y(2\pi - \theta_2) R_x(\pi + \theta_3) \right)^{-1} \\
 &= R_x(\theta_1) \cdot \underbrace{R_y(\theta_2) \cdot R_x(\theta_3) \cdot \left(R_x(\pi + \theta_3) \right)^{-1} \cdot \left(R_y(2\pi - \theta_2) \right)^{-1} \cdot \left(R_x(\pi + \theta_1) \right)^{-1}}_{\substack{R_x(\pi) \\ R_y(\theta_2 + (2\pi - \theta_2)) R_x(\pi) = R_x(\pi)}} \\
 &= R_x(\theta_1) R_x(\pi) \cdot \left(R_x(\pi + \theta_1) \right)^{-1} = 1
 \end{aligned}$$

2. The rotation axes of Euler angle change along with the Euler angles.



3. The two method have different physical meanings (different positions and rotation matrices), but they have same rotation angle about the same principal axis.

