A Study on the L_∞/L_2 Performance of a Computed Torque Controller

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***Abstract*—This paper introduces a new performance analysis method for a computed torque controller against disturbances with the framework of the induced norm from L_2 to L_∞ (which is also denoted by the L_∞/L_2-induced norm). To this end, we first derive a linear time-invariant (LTI) representation of the system obtained by connecting a robot manipulator and a computed torque controller. We then formulate the problem definition associated with the L_∞/L_2 performance of the computed torque controller in which the maximum magnitude of the output for disturbances of finite energy is adopted as the performance measure. We next show that the L_∞/L_2-induced norm of the derived LTI system can be explicitly computed by solving the continuous-time Lyapunov equation. Finally, some experiment results are provided to demonstrate the effectiveness of the introduced performance analysis method.**

I. Introduction

Computed torque method [1]–[3] involves computing accurate system dynamics of a given robot manipulator, and it has been regarded as one of the most effective methods for achieving a high tracking performance of robot systems. This is because an exact system model of a robot manipulator allows us to design an adequate controller that leads to a good tracking performance. More precisely, a linear time-invariant (LTI) and decoupled dynamics of robot manipulators is derived through a sort of feedback linearization in the computed torque method, and a number of novel control strategies with the method have been introduced in [4]–[10] by using the derived LTI and decoupled nature of robot manipulators. Furthermore, the effectiveness of the computed torque method is also experimentally demonstrated in [11],[12] through some real implementations.

On the other hand, because constructing accurate mathematical models of robot manipulators is often very difficult due to their model uncertainties such as friction, environmental changes and so on, there have been a number of studies on robust or adaptive control schemes [13]–[16]. These schemes are basically based on the Lyapunov methods [17],[18] by which the closed-loop systems obtained by connecting robot manipulators and the controllers [13]–[16] are guaranteed to robustly stable for the unmodeled elements; the Lyapunov stability of the closed-loop systems is proved by introducing adequate Lyapunov functions under the assumption of some constraints on the unmodeled elements. Even though the control schemes in [13]–[16] are quite meaningful in the sense of the stability proof, it is difficult to derive a quantitative performance measure for the unmodeled elements through the control schemes [13]–[16] .

In connection with this, this paper introduces a new quantitative performance measure for the computed torque method based on the fact that model uncertainties have been assumed to have finite energy and the associated performance specifications could be naturally represented in terms of time-domain bounds. In other words, this paper takes the induced norm from L_2 to L_∞ as a performance measure to evaluate computed torque controllers against the model uncertainties since the L_∞ norm of a signal corresponds to the maximum amplitude of the signal while the L_2 norm of a signal is used to represent the energy of the signal. Even though the induced norm from L_2 to L_∞ has been well studied in the filed of control theory, e.g., [19],[20] for continuous-time systems and [21],[22] for sampled-data systems, application of the induced norm to the performance analysis of the computed torque method is considered in this paper for the first time.

To this end, this paper first derives a linear time-invariant (LTI) representation of the system obtained by connecting a robot manipulator and a computed torque controller, in which a continuous-time LTI generalized plant and a continuous-time full-state feedback controller are concerned with. This paper next formulates the problem definition in the sense of the induced norm from L_2 to L_∞, by which a performance measure for the computed torque method against the model uncertainties could be constructed. It is then shown that the induced norm from L_2 to L_∞ can be explicitly computed by solving the continuous-time Lyapunov equation. Finally, we demonstrate the effectiveness of the new performance measure through some experiment results.

The organization of this paper is as follows. Section II states the dynamics of robot manipulators through the Lagrangian equation and reviews the computed torque method. Section III formulates the problem definition by defining the induced norm from L_2 to L_∞ for the computed torque treatment and provides its computation method. The effectiveness of the induced norm is demonstrated through some experiments in Section IV. Finally, concluding remarks are stated in Section V.

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In the following, the notation $\mathbb{R}^\nu$, $d_{\max}(\cdot)$ and $\mathrm{tr}(\cdot)$ are used to denote the set of ν-dimensional real numbers, the maximum diagonal entry and the trace of a real symmetric matrix, respectively. The notations $|\cdot|_2$ and $|\cdot|_\infty$ are used to denote the 2-norm and ∞-norm of a finite-dimensional vector, i.e.,

$$|x|_2 := (x^T x)^{1/2}, \quad |x|_\infty := \max_i |x_i| \tag{1}$$

We also use the notations $\|\cdot\|_2$ and $\|\cdot\|_\infty$ to imply the L_2 and L_∞ norms of a vector function, respectively, i.e.,

$$\|f(\cdot)\|_2 := \left(\int_0^\infty |w(t)|_2^2 dt\right)^{1/2} \tag{2}$$

$$\|f(\cdot)\|_\infty := \operatorname*{ess\,sup}_{0\le t<\infty} |f(t)|_\infty \tag{3}$$

Finally, the induced norm from L_2 to L_∞ of a system or an operator is denoted by $\|\cdot\|_{\infty/2}$ and we call such induced norm (i.e., the induced norm from L_2 to L_∞) the L_∞/L_2-induced norm, for simplicity.

II. Dynamics of Robot Manipulators and Computed Torque Method

Let us consider the robot manipulator described by the Lagrangian equation

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \tau_d = \tau \tag{4}$$

where $q(t) \in \mathbb{R}^n$ is the joint angle vector, $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ is the Coriolis and centrifugal torque vector, $G(q) \in \mathbb{R}^n$ is the gravitational torque vector, $\tau_d(t) \in \mathbb{R}^n$ is the disturbance torque vector and $\tau(t) \in \mathbb{R}^n$ is the control input torque vector. For the robot manipulator described by (4), we also consider the trajectory tracking problem

$$\begin{bmatrix} q^T(t) & \dot{q}^T(t) \end{bmatrix}^T \to \begin{bmatrix} q_d^T(t) & \dot{q}_d^T(t) \end{bmatrix}^T \ (t\to\infty) \tag{5}$$

where $[q_d^T(t)\ \dot{q}_d^T(t)] \in \mathbb{R}^{2n}$ is the desired trajectory. To deal with the trajectory tracking problem more tractably, the computed torque method has been considered to transform the nonlinear dynamics of (4 into a linear and decoupled one. More precisely, the computed torque controller is generally described by

$$\tau = \hat{M}(q)(\ddot{q}_d - u) + \hat{C}(q,\dot{q})\dot{q} + \hat{G}(q) \tag{6}$$

where the notation $\hat{(\cdot)}$ means the nominal or computed value of $(\cdot)$. Substituting (6) into (4) derives

$$\begin{aligned} \ddot{q} &= M^{-1}(q)\hat{M}(q)(\ddot{q}_d - u) \\ &\qquad + M^{-1}(q)(\tilde{C}(q,\dot{q})\dot{q} + \tilde{G}(q) - \tau_d) \\ &= \ddot{q}_d - u + M^{-1}(q)\tilde{M}(q)(\ddot{q}_d - u) \\ &\qquad + M^{-1}(q)(\tilde{C}(q,\dot{q})\dot{q} + \tilde{G}(q) - \tau_d) \end{aligned} \tag{7}$$

where $\tilde{M}(q) := \hat{M}(q) - M(q)$, $\tilde{C}(q,\dot{q}) := \hat{C}(q,\dot{q}) - C(q,\dot{q})$ and $\tilde{G}(q) := \hat{G}(q) - G(q)$. Here, if we define the disturbance w and the trajectory tracking error e respectively as

$$\begin{aligned} w &:= -M^{-1}(q)\tilde{M}(q)(\ddot{q}_d - u) \\ &\qquad - M^{-1}(q)(\tilde{C}(q,\dot{q})\dot{q} + \tilde{G}(q) - \tau_d) \end{aligned} \tag{8}$$

$$e := q_d - q \tag{9}$$

we then readily have the error dynamics described by

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w + \begin{bmatrix} 0 \\ I \end{bmatrix} u \tag{10}$$

On the other hand, we can define the regulated output depending on the desired performance specifications for the trajectory tracking problem, and it is generally regarded as a function of the trajectory tracking error such that

$$z := C_p e + C_d \dot{e} \tag{11}$$

where C_p and C_d are weighting constant matrices to be suitably selected by the user. Regarding to a controller synthesis, we further assume that both e and $\dot{e}$ can be directly measured throughout the paper. This immediately leads us to define the continuous-time LTI generalized plant P given by

$$P : \begin{cases} \dot{x} &= Ax + Bw + Bu \\ z &= Cx \\ y &= x \end{cases} \tag{12}$$

where $x(t) := [e^T(t)\ \dot{e}^T(t)]^T \in \mathbb{R}^{2n}$ is the state, $w(t) \in \mathbb{R}^n$ is the disturbance, $u(t) \in \mathbb{R}^n$ is the control input, $y(t) \in \mathbb{R}^n$ is the measured output and $z(t) \in \mathbb{R}^{n_z}$ is the regulated output, respectively, with

$$A := \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \ B := \begin{bmatrix} 0 \\ I \end{bmatrix}, \ C := \begin{bmatrix} C_p & C_d \end{bmatrix} \tag{13}$$

Based on the LTI nature of (12), a number of studies on the computed torque method have introduced a sort of the full-state feedback controller described by

$$u = Ky = Kx \tag{14}$$

by which the resulting closed-loop system P_K given by

$$P_K : \begin{cases} \dot{x} &= A_K x + Bw \\ z &= Cx \end{cases} \tag{15}$$

becomes stable (i.e., every eigenvalues of A_K has strictly negative real part), where $A_K := A + BK$. The values of K can be determined in connection with the desired natural frequencies and damping rations dependent on the performance specifications of the system. However, such a controller design scheme cannot consider the effect of the disturbance w because natural frequencies and damping rations do not fit into performance analysis for disturbances. In this sense, it is required to introduce a quantitative performance measure for the computed torque method relevant to the effect of disturbances, and thus the following section formulates such a performance measure in the sense of the L_∞/L_2-induced norm.

III. Performance Measure with the L_∞/L_2-Induced Norm

This section is concerned with introducing a new performance measure for the computed torque method with the L_∞/L_2-induced norm. The rationale behind the choice of such a specific induced norm is that it is practically useful to represent performance specifications in the terms of time-domain bounds rather than frequency-domain bounds and disturbances with model uncertainties in the treatment of the computed torque method have been generally assumed to have finite energy. In other words, because the maximum magnitude of a signal coincides with the L_∞ norm of the signal while the energy of a signal can be represented by using the L_2 norm of the signal, computing the L_∞/L_2-induced norm of the closed-loop system P_K is quite meaningful in the practical sense for the computed torque method.

w is assumed to have a finite energy, and the supremum of the L_2 norm of w is also assumed to be bounded, i.e.,

$$\|w\|_2 = \left(\int_0^\infty |w(t)|_2^2 dt\right)^{1/2} \le \gamma_2 < \infty \tag{16}$$

We further assume that A_K has all its eigenvalues in the left-half-plane (LHP), i.e., the stability of the closed-loop system P_K. Then, the input-output behavior of P_K can be described by the convolution integral

$$\begin{aligned} z(t) &= \int_0^t C\exp(A_K(t-\tau))Bw(\tau)d\tau \\ &=: (\mathbf{T}w)(t) \quad (0 \le t < \infty) \end{aligned} \tag{17}$$

where $\mathbf{T}$ is defined as an operator from L_2 to L_∞. If we note that $(\mathbf{T}w)(t)$ is a continuous function and $\mathbf{T}$ is a linear operator, then it readily follows that

$$\begin{aligned} \|\mathbf{T}\|_{\infty/2} &:= \sup_{\|w\|_2\le 1} \|\mathbf{T}w\|_\infty = \sup_{\|w\|_2\le 1} \sup_t |\mathbf{T}w(t)|_\infty \\ &= \sup_t \sup_{\|w\|_2\le 1} \left|\int_0^t C\exp(A_K(t-\tau))Bw(\tau)d\tau\right|_\infty \\ &= \lim_{t\to\infty} \sup_{\|w\|_2\le 1} \left|\int_0^t C\exp(A_K(t-\tau))Bw(\tau)d\tau\right|_\infty \\ &= \sup_{\|w\|_2\le 1} \left|\int_0^\infty C\exp(A_K\tau)Bw(\tau)d\tau\right|_\infty \end{aligned} \tag{18}$$

Regrading to the computation of (18), we are in a position to introduce the continuous-time Cauchy-Schwarz inequality, with the vector-valued functions f_1 and f_2, described by

$$\left(\int_0^t f_1^T(\tau)f_2(\tau)d\tau\right)^2 \le \int_0^t |f_1(\tau)|_2^2 d\tau \cdot \int_0^t |f_2(\tau)|_2^2 d\tau \tag{19}$$

where the equality holds if and only if $f_1(\tau) = \lambda f_2(\tau)$ $(\forall \tau \in [0, t])$ for a constant λ. By applying this inequality to (18), we readily see that

$$\begin{aligned} &\|\mathbf{T}\|_{\infty/2} \\ &= \max_{1\le i\le n} \left(\int_0^\infty C_i \exp(A_K\tau)BB^T\exp(A_K^T\tau)C_i^T d\tau\right)^{1/2} \end{aligned} \tag{20}$$

because

$$\begin{aligned} &\left(\int_0^\infty C_i\exp(A_K\tau)Bw(\tau)d\tau\right)^2 \\ \le &\|B^T\exp(A_K^T\tau)C_i^T\|_2^2 \cdot \|w(\tau)\|_2^2 \end{aligned} \tag{21}$$

where C_i $(1 \le i \le n)$ denotes the ith row of C. It is well known that the integral in the right-hand-side of (20) can be computed by solving the continuous-time Lyapunov equation

$$A_K P + PA_K^T + BB^T = 0 \tag{22}$$

by which we have

$$\|\mathbf{T}\|_{\infty/2} = \max_{1\le i\le n}(C_i P C_i^T) = d_{\max}^{1/2}(CPC^T) \tag{23}$$

By (23), we have the following results.

Theorem 1: The L_∞/L_2-induced norm $\|\mathbf{T}\|_{\infty/2}$ relevant to (15) coincides with $d_{\max}^{1/2}(CPC^T)$.

Corollary 2: It readily follows that

$$\|z\|_\infty \le \|\mathbf{T}\|_{\infty/2}\cdot\gamma_2 = d_{\max}^{1/2}(CPC^T)\cdot\gamma_2 \tag{24}$$

Theorem 1 undoubtedly means that the L_∞/L_2-induced norm can be readily obtained by solving the continuous-time Lyapunov equation while Corollary 2 clearly implies that the L_∞ norm of the regulated output z decreases by taking $d_{\max}^{1/2}$ smaller. On the other hand, even though Theorem 1 is essentially equivalent to the results in [19],[20] or [21], [22], which are associated with the L_∞/L_2-induced norm of continuous-time LTI systems or sampled-data systems, respectively, Theorem 1 is meaningful in the sense that a quantitative performance of computed torque controllers against disturbances is dealt with in the sense of the L_∞/L_2-induced norm for the first time.

IV. Experiment Results

This section demonstrates the effectiveness of the L_∞/L_2-induced norm as a performance measure for computed torque controllers through some experiments with the arguments of Theorem 1 and Corollary 2. To this end, we employ the 3-degrees of freedom (3-DoF) robot manipulator as shown in Fig. 1, which consists of three brush-less direct current (BLDC) motors with three linkages whose lengths are 0.3, 0.3 and 0.1 m, respectively. The desired trajectories of three joints considered in this section are shown in Fig. 2, which are cosine functions such that the angles and velocities are set to be zero at $t = 0$ [s] and 4 [s], and the total execution time is 4 [s].

The controller used in this section is given by

$$u = Ky = Kx = K\begin{bmatrix} e \\ \dot{e} \end{bmatrix} =: -\begin{bmatrix} K_p & K_d \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} \tag{25}$$

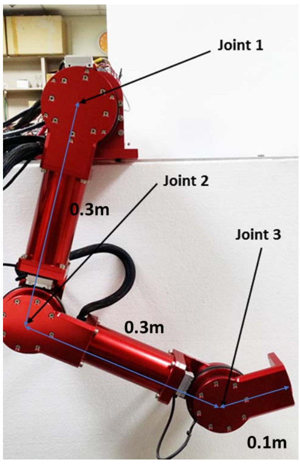


Fig. 1. 3-DoF robot manipulator

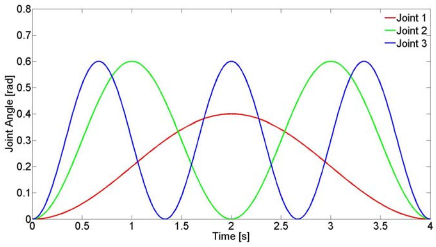


Fig. 2. Desired trajectories of three joints.

with

$$K_p = \begin{bmatrix} k_p & 0 & 0 \\ 0 & k_p & 0 \\ 0 & 0 & k_p \end{bmatrix}, \quad K_d = \begin{bmatrix} k_d & 0 & 0 \\ 0 & k_d & 0 \\ 0 & 0 & k_d \end{bmatrix} \tag{26}$$

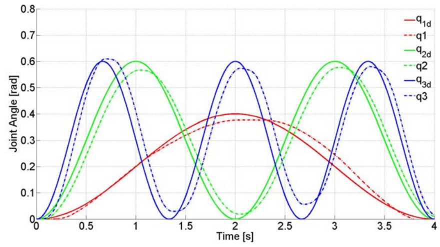
To examine the effectiveness of the L_∞/L_2-induced norm as a performance measure for computed torque controllers, we take various values of the parameter (K_p, K_d) by which the associated L_∞/L_2-induced norm is bounded (i.e., A_K has its all eigenvalues in the LHP.) We also consider the regulated output $z = Cx$ with the parameter

$$C = \begin{bmatrix} C_p & C_d \end{bmatrix} = \begin{bmatrix} 15 & 0 & 0 & 1 & 0 & 0 \\ 0 & 15 & 0 & 0 & 1 & 0 \\ 0 & 0 & 15 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

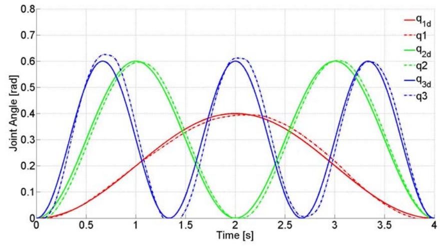
and observe its L_∞ norm to evaluate the effectiveness of the L_∞/L_2-induced norm as a performance measure for computed torque controllers.

The associated experiment results for the joint angles are shown in Fig. 3 while the experiment results for the L_∞ norms of the regulated output z together with the computation results for the L_∞/L_2-induced norm of the closed-loop system of (15) are given in Table I.

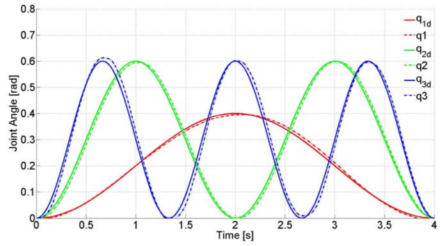
We can see from Fig. 3 and Table I that taking the L_∞/L_2-induced norm smaller leads to better performance of the trajectory tracking control. We can also observe from Table I that the L_∞ norm of the regulated output z is decreasing as the L_∞/L_2-induced norm $\|\mathbf{T}\|_{\infty/2}$ becomes smaller. These observations undoubtedly validate the effectiveness of the L_∞/L_2-induced norm as a performance measure for computed torque controllers against model uncertainties and disturbances.



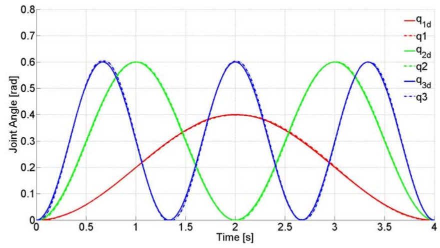
(a) $(\Psi_p, \Psi_d) = (30, 10)$



(b) $(\Psi_p, \Psi_d) = (90, 10)$



(c) $(\Psi_p, \Psi_d) = (120, 20)$



(d) $(\Psi_p, \Psi_d) = (400, 30)$

Fig. 3. Comparison between the experiment results for the joint angles $q(t)$ and the reference joint angles $q_d(t)$.

TABLE I
EXPERIMENT RESULTS FOR THE L_∞ NORM OF THE REGULATED OUTPUT z AND THE ASSOCIATED L_∞/L_2-INDUCED NORM.

(k_p, k_d)	$(30, 10)$	$(90, 10)$	$(120, 20)$	$(400, 30)$
$\|z\|_\infty$	2.0155	1.0628	0.7328	0.3570
$\|\mathbf{T}\|_{\infty/2}$	0.6519	0.3748	0.2681	0.1614

V. Conclusion

This paper dealt with a new framework for computed torque controllers by introducing a new performance measure with the L_∞/L_2-induced norm. To this end, we first derived a linear time-invariant (LTI) representation of the treatment of the computed torque method. We then formulated the problem definition by adopting the L_∞/L_2-induced norm as a performance measure against model uncertainties and disturbances. This was based on the fact that model uncertainties and disturbances are assumed to have finite energy and the maximum magnitude of tracking errors is adopted as a performance index to be minimized. We also provided the method for explicitly computing the L_∞/L_2-induced norm by solving the continuous-time Lyapunov equation. We further demonstrated the validity and effectiveness of the L_∞/L_2-induced norm as a performance measure for computed torque controllers through some simulation results. Finally, the success in this paper in formulating the new performance measure for computed torque controllers with the L_∞/L_2-induced norm is believed to contribute to wider applications of computed torque control approach to a number of practical problems.

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