Homework I Due January 21, 2016

This homework addresses fitting data with linear models. We can do this for time series and we call the method *adaptive filtering*, or we can apply it to static data, assuming a stochastic generation model and we will call this *regression*. Here we are using THE SAME learning algorithm to solve both problems. The only thing that changes is the mapper.

Problem I

Our goal is to do system identification in the following nonlinear system (assumed unknown).

$$y(n) = \frac{y(n-1)}{0.1 + y^2(n-4)} + \sin^2 x(n-3)$$

We just observe the input x(n) (a white noise Gaussian sequence with unit variance) and the output y(n). Create 2,000 samples of the input x(n) and of the output y(n), starting from a zero initial condition. Select a FIR filter of order M=5 and M=15.IS the filter linear?

Design a Wiener filter (w=R⁻¹P) that will approximate the input-output map. Test the quality of the solution with MSE and also by verifying the degree of whiteness of the error signal. Implement also the LMS algorithm, i.e. the iterative solution that uses gradient information.

Estimate the largest stepsize.

Estimate the convergence time.

Show the difference between batch of 100 samples and on-line learning for the same step-size.

Show the weight tracks.

Compare the performance of the LMS for the best set of parameters in terms of misadjustment and speed of convergence by varying the stepsize. Experiment with different filter orders to see if the performance improves.

Problem II

For the obesity dataset in the website, fit the best hyperplane through the data using the optimal solution (least squares) and the iterative solution (LMS). Compare the performance of both and estimate the correlation coefficient. Then test your solution in the unused data (test set) and compare the performance with that of the training set.