Note:

Dear Sir/madam,

I have uploaded all of the code into github.

If you are interested in them, just check

https://github.com/Vin453095545/EE511/invitations

You can check the README.txt and to find who I am.

Thanks!

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EE511 homework 3 Feb. 6

Question 1: A components manufacturer delivers a batch of 125 microchips to a parts distributor. The distributor checks for lot conformance by counting the number of defective chips in a random sampling (without replacement) of the lot. If the distributor finds any defective chips in the sample it rejects the entire lot. Suppose that there are six defective units in the lot of 125 microchips. Simulate the lot sampling to estimate the probability that the distributor will reject the lot if it tests five microchips. What is the fewest number of microchips that the distributor should test to reject this lot 95% of the time?

Theoretical results:

- Five tries

$\frac{6}{125}+\frac{119}{125}\times\frac{6}{124}+\frac{119}{125}\times\frac{118}{124}\times\frac{6}{123}+\frac{119}{125}\times\frac{118}{124}\times\frac{117}{123}\times\frac{6}{122}+\frac{119}{125}\times\frac{118}{124}\times\frac{117}{123}\times\frac{116}{122}\times\frac{6}{121}$ =0.2212

- Fewest number of microchips that the distributor should test to reject this lot 95% of the time:

$$\left(\frac{6}{125}+\frac{119}{125}\times\frac{6}{124}+\frac{119}{125}\times\frac{118}{124}\times\frac{6}{123}+\frac{119}{125}\times\frac{119-\mathrm{i}}{125-\mathrm{i}}\right)>95\%$$

As the code result shows, the result is that we need to try 49 times.

```
%theoretically
clear
TOTAL=125;
DEFECTIVE=6;
FINE=TOTAL-DEFECTIVE;
TRIES=5;i=0;P=0;REJECT=0.95;
while(sum(P)<REJECT)
    i=i+1;
    TOTAL=125;
    DEFECTIVE=6;
    FINE=TOTAL-DEFECTIVE;
    P(i)=1;
```

```
    for j=1:i-1
    P(i)=P(i)*FINE/TOTAL;
    TOTAL=TOTAL-1;
    FINE=FINE-1;
    end
    P(i)=P(i)*DEFECTIVE/TOTAL;
End
sprintf(' fewest number of microchips that the distributor should test to
reject this lot %d is: \n %d', REJECT, i)
```

```
ans =

 fewest number of microchips that the distributor should test to reject this lot 9.500000e-01 is:
 49
```

Simulation results:

- Analysis: Just to for loop until 95% reject it.
- Code

```
%Q1(2
clear
B=ones(1,125);
B(1:6)=0;
DEFECT_N=0;
N=10000
EXPERIMENT=0;
while(DEFECT_N/N<0.95)
     if(EXPERIMENT==5)
        sprintf('the propability of 5 tires to rejecct: %2f ',DEFECT_N/N)
     end
    EXPERIMENT=EXPERIMENT+1;
    DEFECT_N=0;
for z=1:N%N experiments
      A=B;
      i=0;
    while(i<EXPERIMENT)%the number of trials
    i=i+1;
    j=ceil(125*rand);
        if(A(j)==1)
            A(j)=2;%without replacement
        elseif(A(j)==0)
            DEFECT_N=DEFECT_N+1;
            break;
        else
            i=i-1;
        end
    end
end
end
sprintf('Try %d times to get 95%% reject', EXPERIMENT)
```

```
>> Q1_2

ans =

the propability of 5 tires to rejecct: 0.226100


ans =

Try 47 times to get 95% reject

>> Q1_2

ans =

the propability of 5 tires to rejecct: 0.225400


ans =

Try 49 times to get 95% reject
```

- Comment:

As we can see, the simulation result is very close the to the theoretical results.

Question2:

Suppose that 120 cars arrive at a freeway onramp per hour on average. Simulate one hour of arrivals to the freeway onramp: (1) subdivide the hour into small time intervals (< 1 second) and then (2) perform a Bernoulli trial to indicate a car arrival within each small time interval. Generate a histogram for the number of arrivals per hour. Repeat the counting experiment by sampling directly from an equivalent Poisson distribution by using the inverse transform method (described in class). Generate a histogram for the number of arrivals per hour using this method. Overlay the theoretical p.m.f. on both histograms. Comment on the results.

Analysis for title of statement:

Three tasks should be finished in this question:

a. Generate a histogram for number of arrivals per hour with Bernoulli trail

b. Generate a histogram for number of arrivals per hour with inverse transform methods

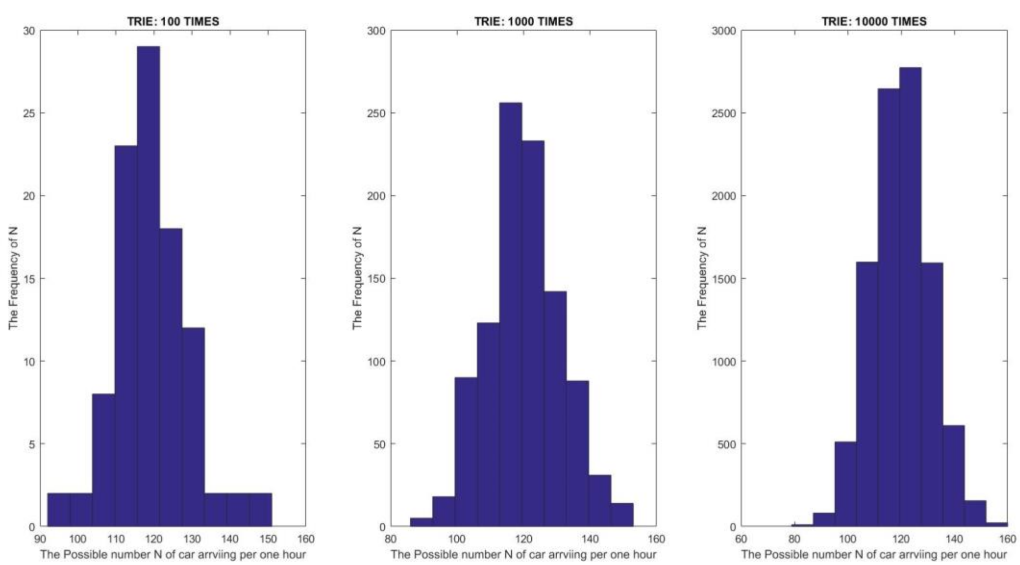
c. Overlay the theoretical on both histograms and make comment.

Solution for task a:

- Analysis:
  - Divide 1 hour into N(N=10000) parts.
  - Suppose only at most one car arrives in each interval.
  - The theoretical probability that one car comes into is 120/N.
  - Generate a random value, if it smaller than 10/N, than assume a car comes during this interval.
- Code

```
%Q2
clear
N=10000;
TRIES=[100,1000,10000];
E=120/N;
for z=1:3
    for y=1:TRIES(z)
        for i=1:N
        CarArriving(i)=rand<E;
        end
        SumCarArriving(y)=sum(CarArriving);
    end
        subplot(1,3,z);
        hist(SumCarArriving);
        title(['TRIE: ',num2str(TRIES(z)),' TIMES']);
        ylabel('The Frequency of N');
        xlabel('The Possible number N of car arrviing per one hour');
end
```

- Results：



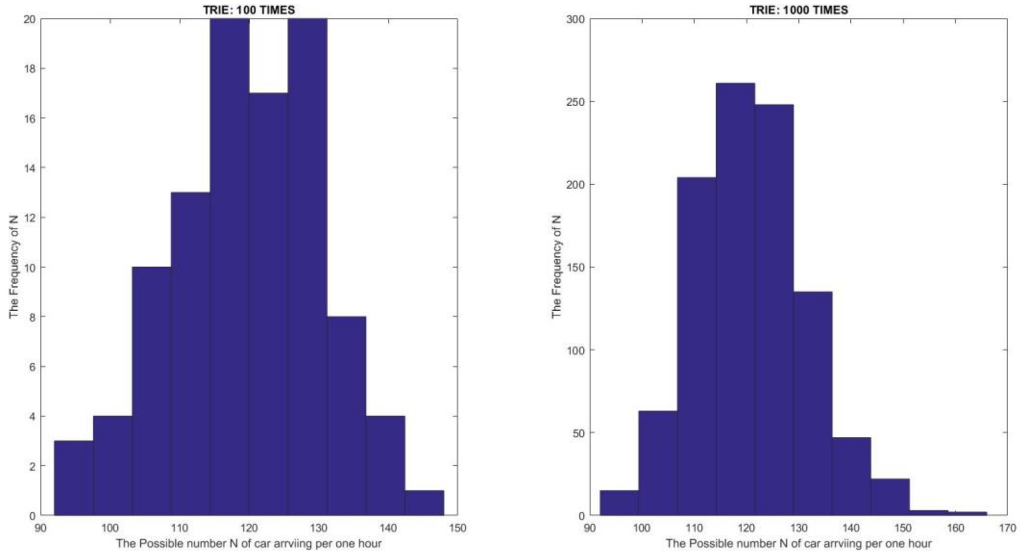
Solution for task b:

- Analysis: inverse transform methods algorithm:
    - Define a passion distribution: $P(X = k) = \frac{\lambda^k}{k!} e^{\lambda}, k = 0,1,2,3 \ldots$ In this question, $\lambda = 120.$
    - generate a rand to make it larger than $\sum_i^k P_i$, and record the k.
    - repeated above operation.
- Code

```
- %2_2
- clear
-
-
- TRIES=[100,1000];
- lambda=120;
- %generate a theoretical poisson p.d.f
- Theo_Poisson=poisspdf(lambda,90:0.1:159)
- %generate a simultaiton histgram with inverse
  transform method
  for z=1:2
      for i=1:TRIES(z)
  temp=rand;
  j(i)=1;
  while(temp>sum(poisspdf(lambda,1:j(i))))
      %inverse transform methodequal to exp(-
  lambda).*lambda.^k(i)/factorial(k(i))+p
  j(i)=j(i)+1;
  end
  end
          subplot(1,2,z);
           histogram(j,1)
           bins=  range(j)%rescale the bandwidth of p

  histogram(j,bins,'Normalization','probability')
          hold on
          plot(90:0.1:159,Theo_Poisson)
          title(['TRIE: ',num2str(TRIES(z)),' TIMES']);
         ylabel('The Frequency of N');
          xlabel('The Possible number N of car arrviing
  per one hour');
  end
```

- Results:



Solution for task b:

In this task, we need to rescale the histogram of the figure in task a and task b. And the unit width of simulation figure in x-axil to match the theoretical is necessary.

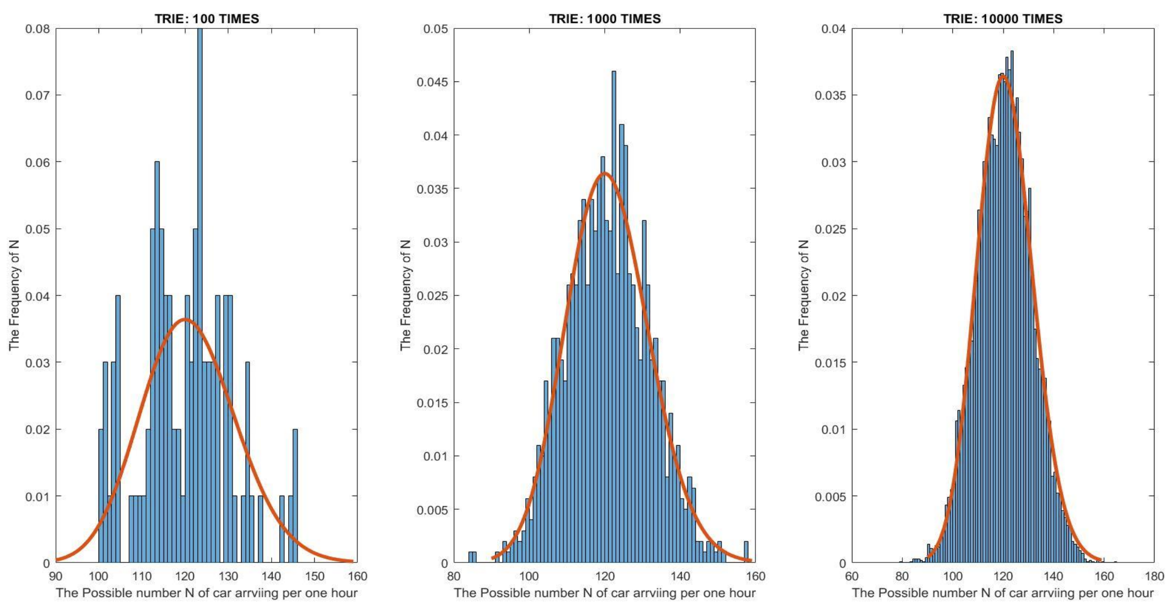


Figure: the overlay between theoretical and simulation with Bernoulli trail

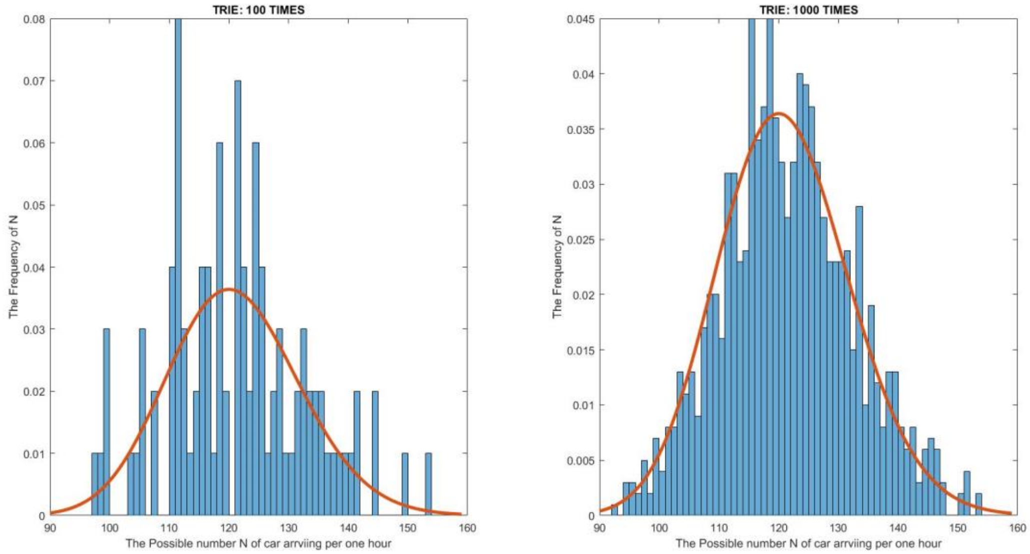


Figure: the overlay between theoretical and simulation with inverse transform method

- Comment:
  - As we can see from the above figure, both two methods fit the theoretical results well.
  - The more we try, the more the simulation will fit the theoretical results.

Question3 :

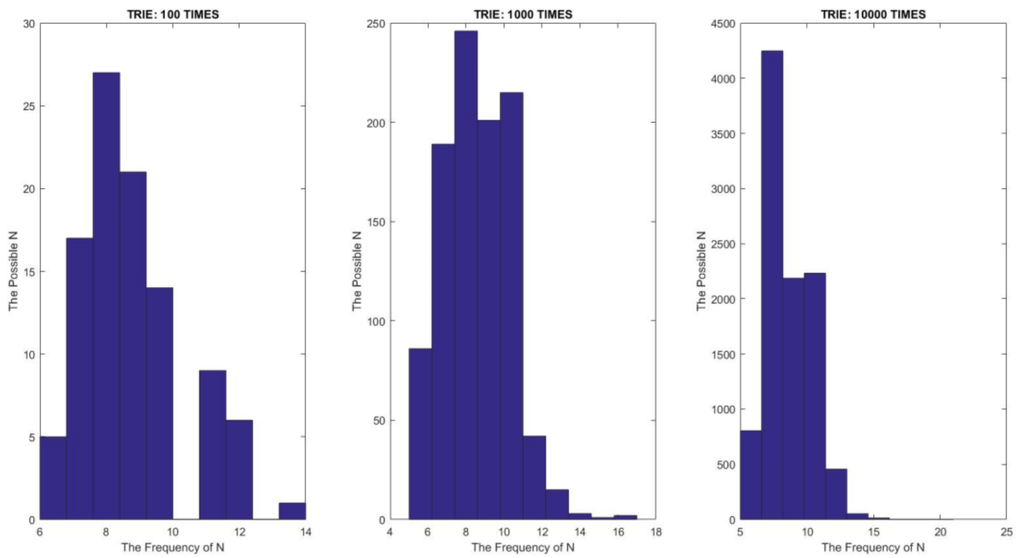
Define the random variable $N = \min\{n: \sum_{i=1}^{n} X_i > 4\}$ as the smallest number of uniform random samples whose sum is greater than four. Generate a histogram using 100, 1000, and 10000 samples for N. Comment on $E[N]$.

- **Codes:**

```
clear
TRIES=[100,1000,10000]
SUM=0;
i=0;
for z=1:3
    for j=1:TRIES(z)
     SUM=0;
     i=0;
        while(SUM<4)
            SUM=rand+SUM;
            i=i+1;
        end
    x(j)=i;
    end
    subplot(1,3,z)
    hist(x)
    title(['TRIE: ',num2str(TRIES(z)),' TIMES']);
    xlabel('The Frequency of N');
    ylabel('The Possible N');
```

```
end
```

- results:



The mean for three different simulation times are:

```
ans =

    8.5300


ans =

    8.6870


ans =

    8.6449
```

- Comment:
  - Since the theoretical N is 8.667, so when we do simulation, the frequency of 8.667 will be the highest.
  - The more we try, the higher of the frequency of 8.667.

Question4:

Produce a sequence $\{X_k\}$ where $p_j = \frac{1}{j}$ for $j = 1, 2, \cdots, 60$. [This is equivalent to spinning the minute hand on a clock and observing the stopping position if $P[\text{stop on minute } j] = \frac{1}{j}$]. Generate a histogram. Define the random variable $N_j = \min\{k: X_k = j\}$. Simulate sampling from N_{60}. Estimate $E[N_{60}]$ and $Var[N_{60}]$. Compare you estimates with the theoretical values.

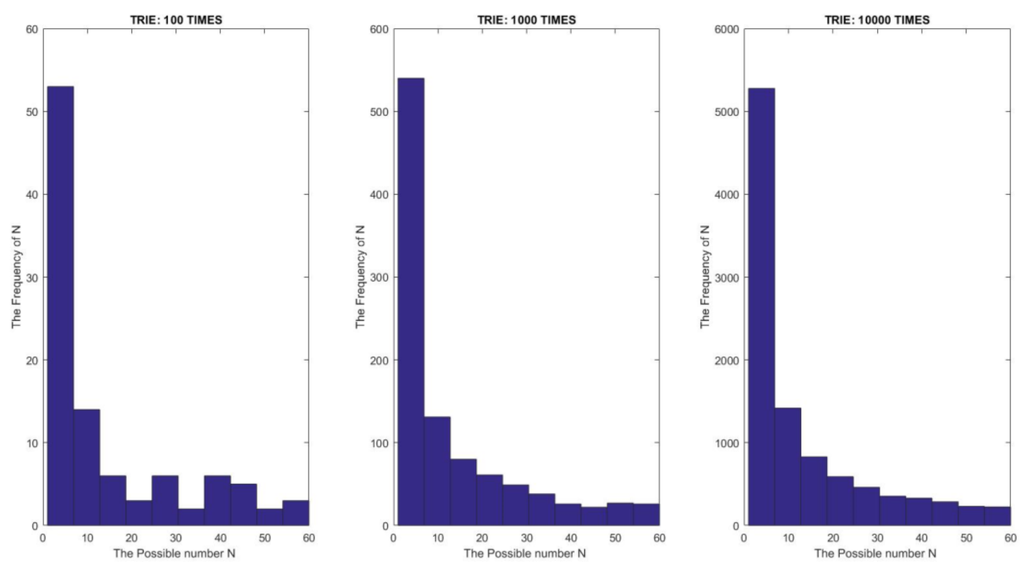
- Analysis:

- Generate a sequence $\{\frac{a}{1}, \frac{a}{2} \dots \frac{a}{60}\}$
- generate N rand, calculate the number that the rand fall into the target.
- Do loop until the generated rand number fall into X_{60}, and record the number of trials to obtain this rand number.

- Code for histogram

```
%Q4
p=1./(1:60)
p=p./sum(p)
TRIES=[100,1000,10000]
for z=1:3
y=randsrc(TRIES(z),1,[1:60; p])%to use randsrc, MATLAB shoud be updated to
2016b
        subplot(1,3,z);
           hist(y)
        title(['TRIE: ',num2str(TRIES(z)),' TIMES']);
        ylabel('The Frequency of N');
        xlabel('The Possible number N ');
end
```

- Results for histogram:



- Code for N_j

```
%4
clear
p=1./(1:60);
```

```
p=p./sum(p);
TRIES=[100,1000,10000];
for z=1:3
        for i=1:TRIES(z)
    N(i)=0;
    while(rand<(1-p(60)))
    N(i)=N(i)+1;
    end
        end
    subplot(1,3,z);
    hist(N)
    title(['TRIE: ',num2str(TRIES(z)),' TIMES']);
    ylabel('The Frequency of N');
        xlabel('The Possible number N of car arrviing per one hour');
    sprintf('Trials: %f, Mean: %f, Variance: %f\n', TRIES(z), mean(N),var(N))
end
     sprintf('Theoritically, we need to try %f times', 1/p(60))
```

- Results for N_j

```
ans =

Trials: 100.000000, Mean: 251.650000, Variance: 63096.775253


ans =

Trials: 1000.000000, Mean: 275.205000, Variance: 71053.406381


ans =

Trials: 10000.000000, Mean: 276.957800, Variance: 76182.416661


ans =

Theoritically, we need to try 280.792225 times
```

- Comment:
  - The more times we simulate, the closer to theoretical results the simulation results are.
  - From the simulation results, we can find that when the simulations times are not enough large, the results have great possibility to be smaller than the theoretical result. May be it is because the probability that the generated rand number fall into X_{60} is too low. So the results are either much larger than the theoretical results (when it falls in into X_{60}) or always smaller than the theoretical results.

Question 5:

Use the accept-reject method to sample from the following distribution p_j by sampling from the (non-optimal) uniform auxiliary distribution ($q_j = 0.05$ for $j = 1, \dots, 20$):

$$p_1 = p_2 = p_3 = p_4 = p_5 = 0.06, p_6 = p_9 = 0.15, p_7 = p_{10} = 0.13, p_8 = 0.14.$$

Generate a histogram and overlay the target distribution p_j. Compute the sample mean and sample variance and compare these values to the theoretical values. Estimate the efficiency of your sampler with the following ratio:

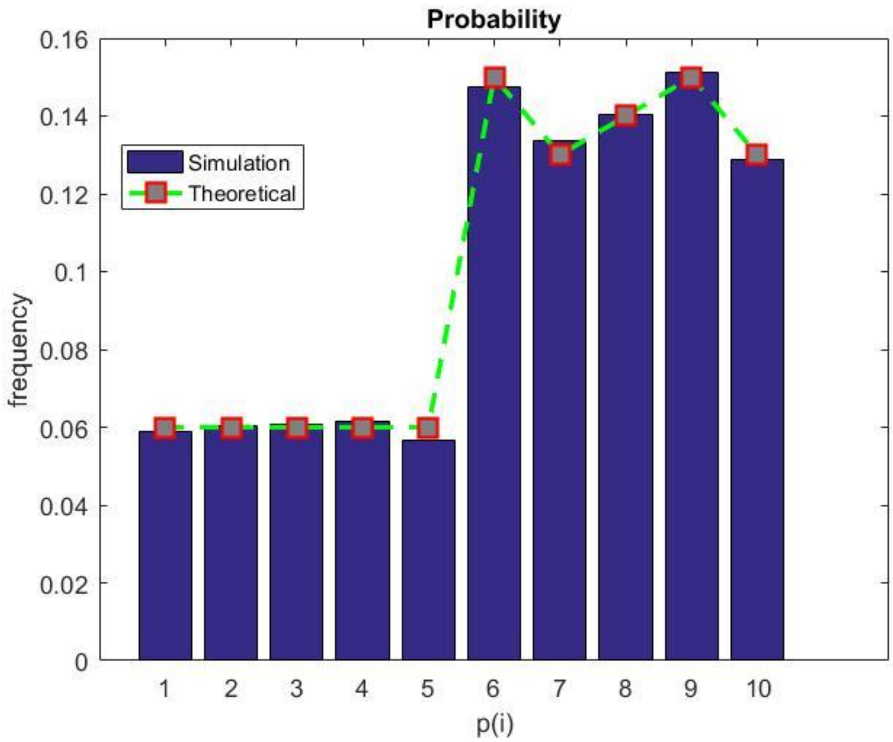
$$\text{Efficiency} = \frac{\#\, accepted}{\#\, accepted + \#\, rejected}$$

Compare your estimate of the efficiency to the theoretical efficiency given your choice for the constant c.

- **code for distribution of pj:**

```
%Q5
clear
N=10000
q=[6 6 6 6 6 15 13 14 15 13]/100
x(1:10)=0;
for j=1:N
temp=rand;
for i=1:10
    x(i)=x(i)+(temp<sum(q(1:i)));
end
end
X=[x(1),diff(x)];
sampleX=X/N;
bar(sampleX);
hold on
plot(q,'--gs',...
    'LineWidth',2,...
    'MarkerSize',10,...
    'MarkerEdgeColor','r',...
    'MarkerFaceColor',[0.5,0.5,0.5]);
title('Probability')
xlabel('p(i)');
ylabel('frequency')
legend('Simulation','Theoretical')
sprintf('Sample mean: %f Sample variance: %f\n Theoretical
mean:  %f ...Theoretical
variance: %f',mean(sampleX),var(sampleX),mean(q),var(q))
```

- **Results for distribution of pj:**




```
ans =

Sample mean: 0.100000 Sample variance: 0.001846
 Theoretical mean:  0.100000 ...Theoretical variance: 0.001822
```

- Analysis for the calculating efficiency；
  - Theoretically： the probability of the efficiency is the $\sum_{\mathrm{i}}^{10} p_i \times \frac{q}{\max\{p_i, q\}} = \frac{2}{3}$
- Code for the calculating efficiency：

```
clear
N=10000;
p=[6 6 6 6 6 15 13 14 15 13]/100
q=0.1;
c=max([p,q])/q
for i = 1:N,  k = 0;
    while(1)
        k = k + 1;
         j = 1 + floor(10*rand);    % Get Uniform j
     if (c*rand) < p(j)/0.1  % Accept p(j) if U<p(j)/c, q(j)= 0.1
     X(i) = p(j); C(i)  = k;
         break
         end
     end
end
```

```
sprintf('Simulation efficience: %f\n My estimate: %f',
1/mean(C),sum(p/c))
```

- Results for the calculating efficiency

```
ans =

Simulation efficience: 0.669344
 My estimate: 0.666667
```