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Q1:Simulate tossing a fair coin(a Bernoili trial) 50 times. Count the number of heads. Record the longest run of heads. Generate a histogram for the Bernouli outcomes.

Methods:

- Record the run length:
    - the goal of the title of statement is to find the consecutive “1”. For example: the runs of lengths of 100111001111 is (1,3,4).
    - In stead of doing loop, my thought to find out and mark the change of digits(1->0 or o->1) of the array. For example, the mark of change of 100111001111 is 1,4,9(0->1) and 2,7,13(1->0). And calculate the length of the “1”.
    - Xor is used to realize the marking. For example, left shift one bit of the original data: 100111001111->[100111001111] and do XOR([100111001111,0],[0, 100111001111]) we will get the mark of (1->0): (1000010000010), then use find command in MATLAB to find the index of 1(that means the location where 1->0). The same as to obtain index of(0->1).
    - After that, the index array of (1->0) minus the index array of (0->1) and then we will get the runs length.
- Use max command to get the longest run of heads
- Use histogram to get the figure.

Discussions of the Algorithm:

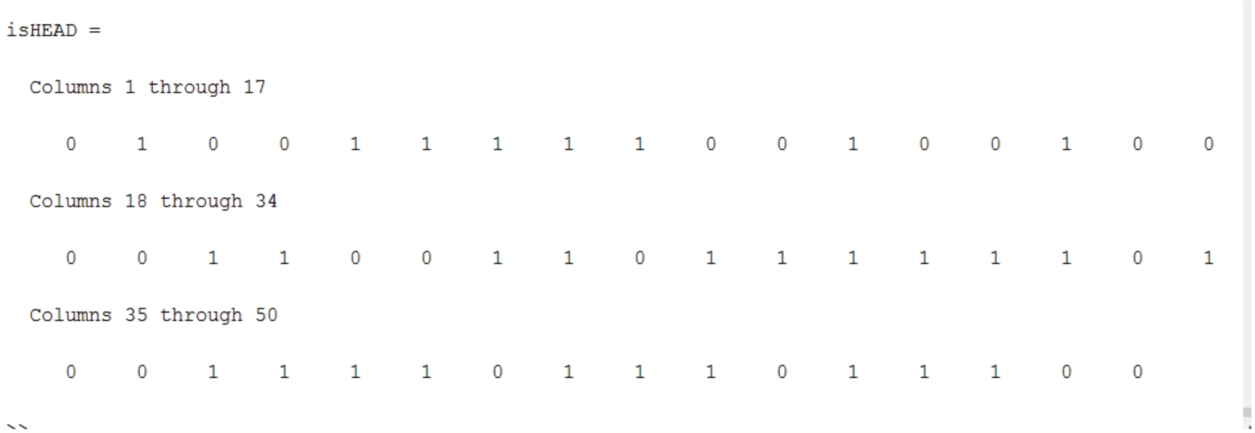
Compared with the for loop, this Algorithm will save a lot of time since it just do shifting, which especially fit for the architecture of our computer(CPU).

Codes:

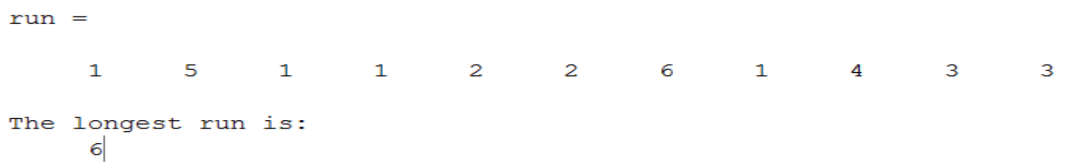
```
%Q1
clear
N=100;
TRIES=1;
THREDSHOD=0.5;
for i=1:TRIES
isHEAD=rand(1,N)>THREDSHOD;
runs=reshape(find(xor([0,isHEAD],[isHEAD,0])==1),2,[]);
run=runs(2,:)-runs(1,:)
longest=max(run);
T(i)=sum(runs(2,:)-runs(1,:));
end
histogram(isHEAD);
%title('The Histogram for the Bernoulli Outcomes')
%xlabel('Outcomes:HEADS(1)/TAILS(0)');
%ylabel('Frequency of the Outcome');
```

```
%disp('The longest run is:')
%disp(longest)

%histogram(T)
%title('The Histogram of Towssing(TRIES:1)')
%xlabel('The Number of Head');
%ylabel('Frequency of the Same Head Number');


histogram(run)
title('The Histogram of Runs Lengths of TOSSING 100 Times')
xlabel('The Frequency of Possible Runs Lengths');
ylabel('The Possible Runs Lengths');
```

Results:



```
isHEAD =

  Columns 1 through 17

     0     1     0     0     1     1     1     1     1     0     0     1     0     0     1     0     0

  Columns 18 through 34

     0     0     1     1     0     0     1     1     0     1     1     1     1     1     1     0     1

  Columns 35 through 50

     0     0     1     1     1     1     0     1     1     1     0     1     1     1     0     0

``
```



```
run =

     1     5     1     1     2     2     6     1     4     3     3

The longest run is:
     6
```

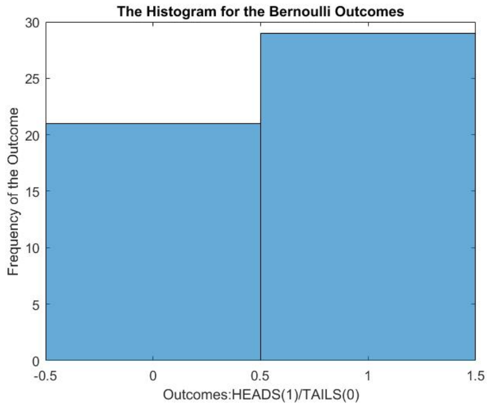


Figure1：The Histogram for the Bernouli Outcomes

Q1a: Reapeart the above experiment 20,100,200 amd 1000 times. Generate a histogram for each showing the number of heads in 50 flips. Comment on the limit of the histogram.

Methodes and codes:

The methods and codes are similar with the Q1's. Just revise the parameter: TRIES and the xlable and ylable to shows the different figure.

Results:

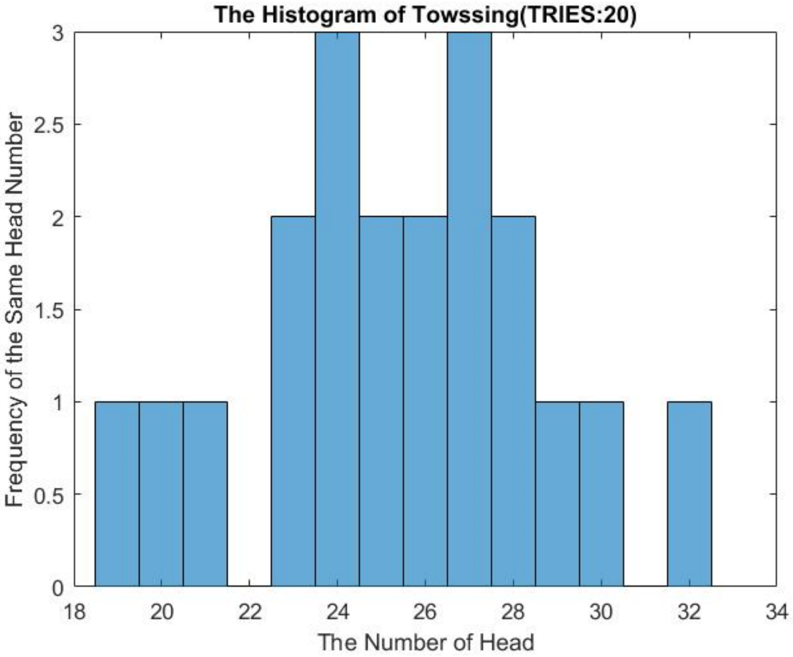


Figure 2: The historam of Towssing(TRIES:20)

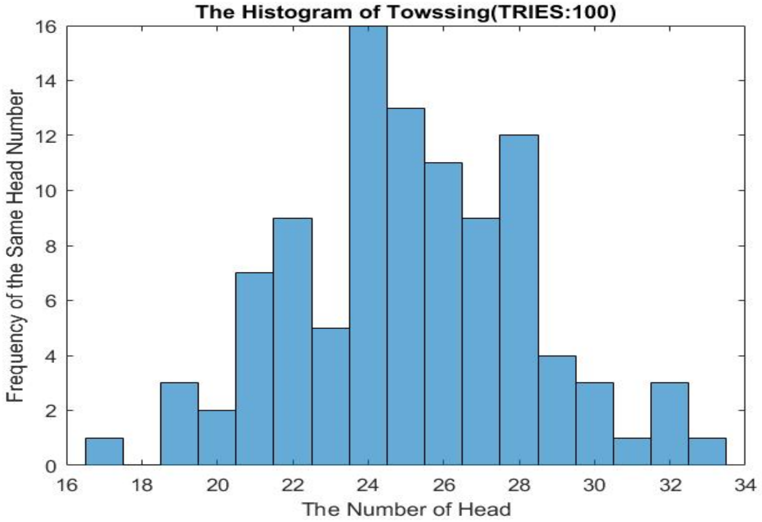


Figure 4: The historam of Towssing(TRIES:200)

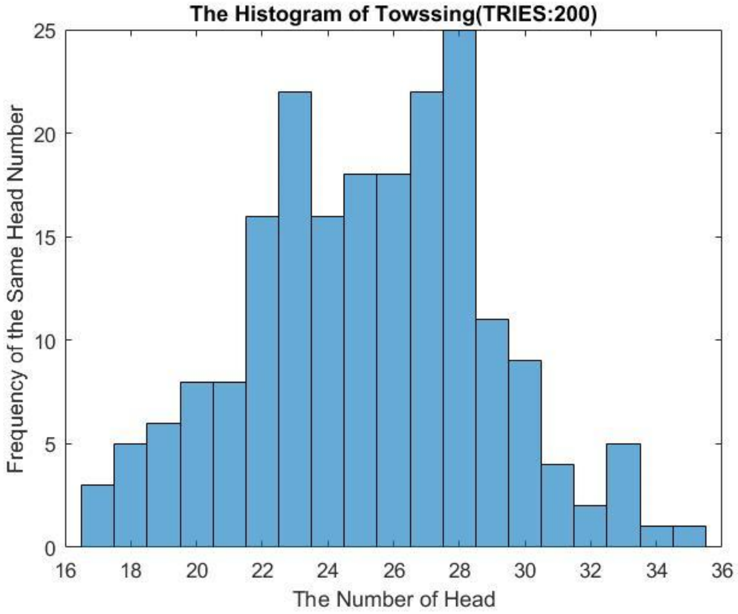


Figure 4: The historam of Towssing(TRIES:200)

Discussions:

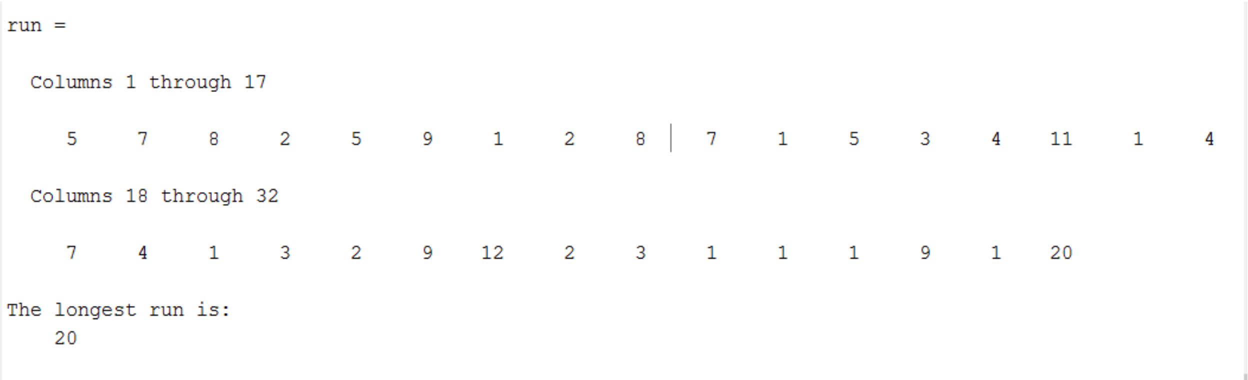
- The disadvantages of histogram is that we cannot get a smooth curve from the figure, which will result in some inaccuracy especially when the **quantity of the data is small.**

Q2： Simulate tossing a biased coin 200 times where P[HEAD]=0.8. Count the number of heads. Record the longest run of heads. Generate a histogram for the Bernoulli outcomes.

Methods:

The method is the same as the Q1’s. Set TRRIES=1, THRESHOD=0.2 and N=200.

Results:



```
run =

  Columns 1 through 17

     5     7     8     2     5     9     1     2     8     7     1     5     3     4    11     1     4

  Columns 18 through 32

     7     4     1     3     2     9    12     2     3     1     1     1     9     1    20

The longest run is:
    20
```

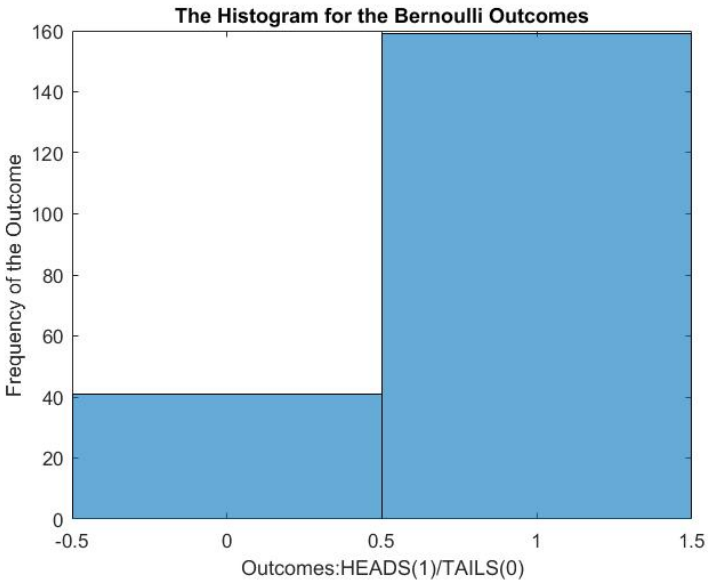


Figure 5: The histogram for the Bernoulli outcomes for P[HEAD]=0.8

Q3: Simulate tossing a fair coin 100 times. Generate a histogram showing the heads run lengths.

Method: The methods used here is the same as the Q1. Based on Q3, set THRESHOD=0.5 and N=100.

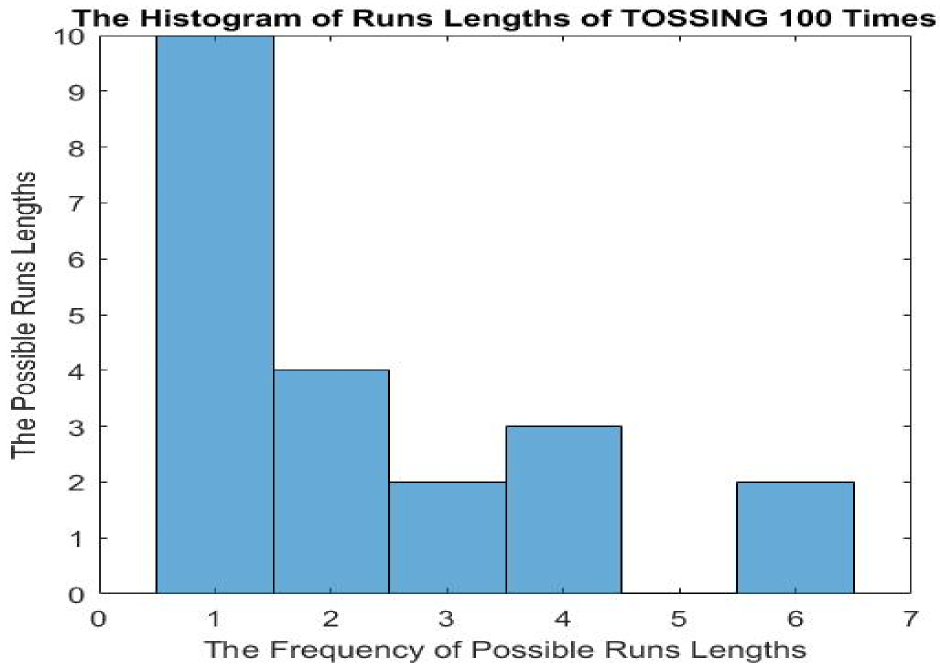


Figure 6: The Histogram of Runs lengths of tossing 100 times.

Discussion:

we can find in the figure that the frequency of the run of 1 is the largest since the propability to gain head is ½. So it is more likely to get one TAIL and one HEAD.

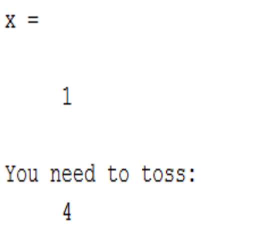
Q4：Simulate tossing a fair coin and count the number of tosses until reaching a user-specified positive number heads.

Methods: Our goal is to obtain the times you need to toss when the number of heads reach to the set number. So we just do a loop and until the HEAD number reach the to the one we set: x. The code is as the following shows.

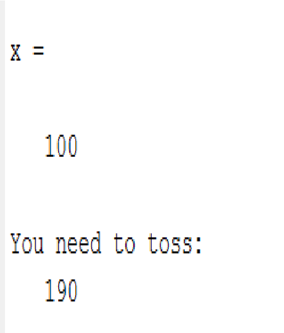
```
%4
clear
x=input('The number of heads:')
TRIES=1;
```

```
THREDSHOD=0.5;
Count=0;
HEAD=0;
    while(HEAD<x)
    HEAD=HEAD+(rand(1)<THREDSHOD);
    Count=Count+1;
    end
disp('You need to toss:')
disp(Count)
disp('times')
```

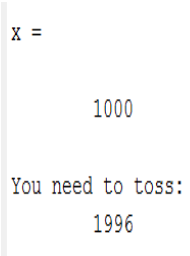
Results:



```
x =

     1

You need to toss:
     4
```



```
x =

   100

You need to toss:
   190
```



```
x =

        1000

You need to toss:
        1996
```

Discussion:

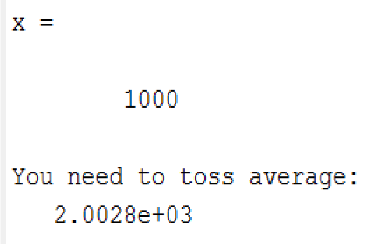
After several experiments we find that：

- If you want to get 1 head, the propability will always greater than 2.
- The more times you toss, the more closer that the propability reach to ½.

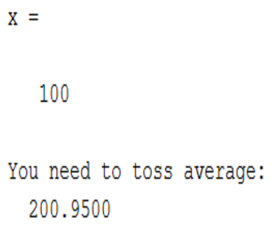
In order to get the more stable results, we improve the trials time and take the average times.

```
%4
clear
x=input('The number of heads:')
TRIES=1;
THREDSHOD=0.5;
Count=0;
for i=1:100
    HEAD=0;
    while(HEAD<x)
    HEAD=HEAD+(rand(1)<THREDSHOD);
    Count=Count+1;
    end
end
disp('You need to toss average:')
disp(Count/100)
disp('times')
```

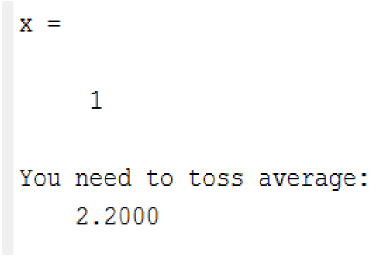
Results(Average times):



```
x =

        1000

You need to toss average:
   2.0028e+03
```



```
x =

   100

You need to toss average:
  200.9500
```



```
x =

     1

You need to toss average:
    2.2000
```

Discussion:

This will be more perfect to show the discussions above.