Classification Methods: Supervised Machine Learning

- *Input:*
  - a document d
  - a fixed set of classes $C = \{c_1, c_2,\dots, c_J\}$
  - A training set of m hand-labeled documents $(d_1,c_1),\dots,(d_m,c_m)$
- *Output:*
  - a learned classifier $\gamma{:}d \rightarrow c$

Text Classification and Naïve Bayes

Naïve Bayes (I)

Naïve Bayes Intuition

- Simple ("naïve") classification method based on Bayes rule
- Relies on very simple representation of document
  - Bag of words

The bag of words representation

$\gamma($

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun… It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.

$)=c$

Bayes' Rule Applied to Documents and Classes

- For a document *d* and a class *c*

$$P(c \mid d) = \frac{P(d \mid c)P(c)}{P(d)}$$

Naïve Bayes Classifier (I)

$$c_{MAP} = \operatorname*{argmax}_{c \in C} P(c \mid d)$$

MAP is "maximum a posteriori" = most likely class

$$= \operatorname*{argmax}_{c \in C} \frac{P(d \mid c)P(c)}{P(d)}$$

Bayes Rule

$$= \operatorname*{argmax}_{c \in C} P(d \mid c)P(c)$$

Dropping the denominator

Naïve Bayes Classifier (II)

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}}\, P(d \mid c) P(c)$$

$$= \underset{c \in C}{\operatorname{argmax}}\, P(x_1, x_2, \ldots, x_n \mid c) P(c)$$

Document d represented as features x1..xn

Multinomial Naïve Bayes Independence Assumptions

$$P(x_1, x_2, \ldots, x_n \mid c)$$

- **Bag of Words assumption**: Assume position doesn't matter
- **Conditional Independence**: Assume the feature probabilities $P(x_i|c_j)$ are independent given the class c.

$$P(x_1, \ldots, x_n \mid c) = P(x_1 \mid c) \bullet P(x_2 \mid c) \bullet P(x_3 \mid c) \bullet \ldots \bullet P(x_n \mid c)$$

Applying Multinomial Naive Bayes Classifiers to Text Classification

positions $\leftarrow$ all word positions in test document

$$c_{NB} = \underset{c_j \in C}{\operatorname{argmax}}\, P(c_j) \prod_{i \in positions} P(x_i \mid c_j)$$

Learning the Multinomial Naïve Bayes Model

- First attempt: maximum likelihood estimates
  - simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{doccount(C = c_j)}{N_{doc}}$$

$$\hat{P}(w_i \mid c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

Parameter estimation

$$\hat{P}(w_i \mid c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

fraction of times word w_i appears among all words in documents of topic c_j

- Create mega-document for topic *j* by concatenating all docs in this topic
  - Use frequency of *w* in mega-document

Problem with Maximum Likelihood

- What if we have seen no training documents with the word ***fantastic*** and classified in the topic **positive** (***thumbs-up)***?

$$\hat{P}(\text{"fantastic"} \mid \text{positive}) = \frac{count(\text{"fantastic"}, \text{positive})}{\sum_{w \in V} count(w, \text{positive})} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$c_{MAP} = \operatorname{argmax}_c \hat{P}(c) \prod_i \hat{P}(x_i \mid c)$$

Laplace (add-1) smoothing for Naïve Bayes

$$\hat{P}(w_i \mid c) = \frac{count(w_i, c) + 1}{\sum_{w \in V} \left(count(w, c) + 1 \right)}$$

$$= \frac{count(w_i, c) + 1}{\left(\sum_{w \in V} count(w, c) \right) + |V|}$$

Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*

- Calculate $P(c_j)$ terms
  - For each c_j in C do

    $docs_j \leftarrow$ all docs with class $= c_j$

$$P(c_j) \leftarrow \frac{|docs_j|}{|\text{total \# documents}|}$$

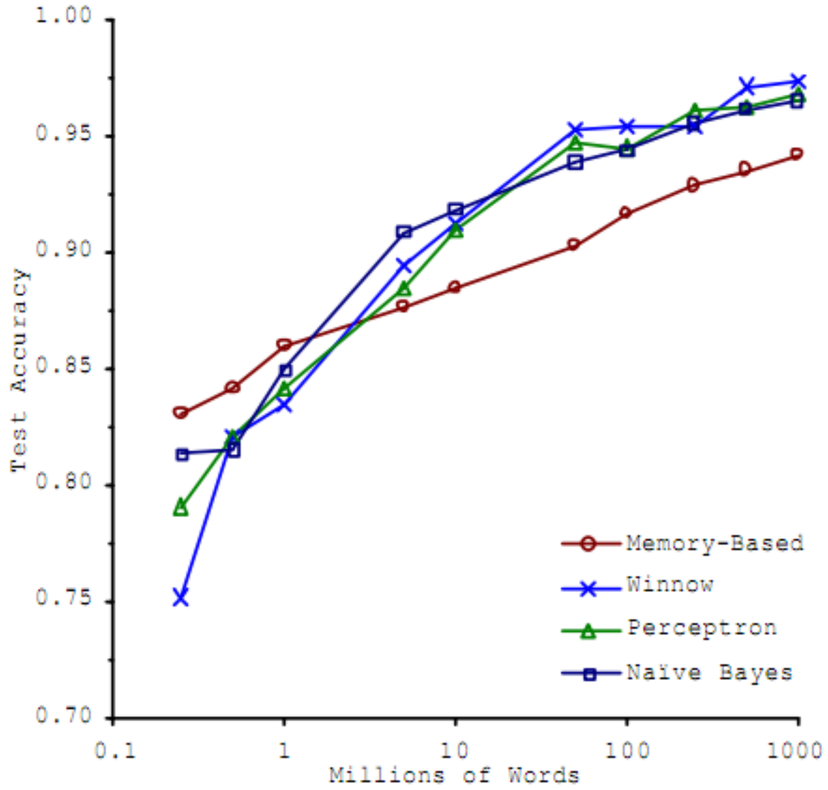
- Calculate $P(w_k \mid c_j)$ terms
  - $Text_j \leftarrow$ single doc containing all $docs_j$
  - For each word w_k in *Vocabulary*

    $n_k \leftarrow$ # of occurrences of w_k in $Text_j$

$$P(w_k \mid c_j) \leftarrow \frac{n_k + \alpha}{n + \alpha |Vocabulary|}$$

Accuracy as a function of data size

- With enough data
  - Classifier may not matter



Brill and Banko on spelling correction