Report for "On the use of Gram matrix for multivariate functional principal components analysis"

The authors established the relationship between the eigen-elements of the covariance operator and those of the inner-product matrix, and proposed to use the Gram matrix to estimate eigen-components when the number of sampling points M is much higher than the number of sample size N. The derivations and results before Section 4.2 seem to be correct. The results are not surprising as they are direct extensions from the duality properties in the multivariate analysis.

The computational complexity in Section 4.3 does not seem to be accurate. The role of the dimension P was not correctly reflected in the results. The authors concluded that the number of features P did not have much impact on the computational complexity. This sounds counter intuitive. More discussion and careful examinations are needed for this section.

The simulation results are somewhat confusing. For the computational time, the authors reported the ratio of the computation time between the covariance method and the Gram matrix method. Figure 5 seems to indicate that when P is small, the ratio has a large variance. Please provide explanations for this.

The covariance decomposition method is faster for most of the cases in Figure 5. The authors need to explain the empirical results and check if they match the results in Section 4.3.

Figure 6 shows that the the Gram matrix method is faster in scenario 2. The authors then concluded that the Gram matrix method is faster for two-dimensional functional data without a clear explanation. It seems to me that this is primarily due to the fact that the number of sampling points is M^2, which is much larger than the sample size N.

In Figure 7 and Figure 9, the performance of the two methods are pretty much the same when the functions are defined on a one-dimensional domain. However, Figure 8 and Figure 10 indicate that when the domain is two-dimensional, the covariance decomposition method is more accurate for the estimation of eigenvalues; while the Gram matrix method is more accurate for the estimation of eigen-functions. This result needs further examination and explanation. The authors simply concluded that the Gram matrix method should be used for multi-dimensional functional data (image), which is pre-mature.