$$= \sum_{n=1}^{N} \pi_n \underbrace{\sum_p \int_{T_p} X_n^{(p)}(t_p)\, T(x)^{(p)}(t_p)\, dt_p}_{= \langle\langle X_n, X_n \rangle\rangle_T = \|X_n\|_T^2}$$

$$- \overbrace{\sum_p \int_{T_p} \underbrace{\sum_n \pi_n X_n^{(p)}(t_p)}_{= \mu^{(p)}(t_p)}\, T(\mu)^{(p)}(t_p)\, dt_p}^{= \|\mu\|_T}$$

$$- \underbrace{\sum_p \int_{T_p} \underbrace{\sum_{n=1}^{n} \pi_n T(X_n)^{(p)}(t_p)}_{\substack{= T\left(\sum_n \pi_n X_n\right)^{(p)}(t_p) \\ = T(\mu)^{(p)}(t_p)}}\, \mu^{(p)}(t_p)\, dt_p}_{= \|\mu\|_T}$$

$$+ \underbrace{\sum_n \pi_n}_{=1} \underbrace{\sum_p \int_{T_p} \mu^{(p)}(t_p)\, T(\mu)^{(p)}(t_p)\, dt_p}$$

$$= \sum_{n=1}^{N} \pi_n \|X_n\|_T^2 + \|\mu\|_T$$

$$= \sum_{n=1}^{N} \pi_n \|X_n\|_T^2 - \|\mu\|_T^2$$

$$= \sum_p \int_{T_p} \sum_n \pi_n X_n^{(p)}(t_p)\, T(X_n)^{(p)}(t_p)$$

$$- \mu^{(p)}(t_p)\, T(\mu)^{(p)}(t_p)\, dt_p$$

$$\sum_{n=1}^{N} \pi_n \, d_T^2(M_n, G_m) = \sum_{n=1}^{N} \pi_n \langle\langle x_n - \mu, x_n - \mu \rangle\rangle_T$$

$$= \sum_{n=1}^{N} \pi_n \langle\langle x_n - \mu, T(x_n - \mu) \rangle\rangle$$

$$= \sum_{n=1}^{N} \pi_n \sum_p \int_{T_p} (x_n(t_p) - \mu(t_p)) \, T(x_n - \mu)^{(p)}(t_p) \, dt_p$$

$$\boxed{\begin{aligned} T^{(p)}(x_n - \mu)(t_p) &= \langle\langle C_{p\cdot}(t_p, \cdot), x(\cdot) - \mu(\cdot) \rangle\rangle \\ &= \langle\langle C_{p\cdot}(t_p, \cdot), x(\cdot) \rangle\rangle + \\ &\quad \langle\langle C_{p\cdot}(t_p, \cdot), x(\cdot) \rangle\rangle \end{aligned}}$$

$$= \sum_n \pi_n \sum_p \int_{T_p} (x_n^{(p)}(t_p) - \mu^{(p)}(t_p)) \, (T(x)^{(p)}(t_p) - T\mu^{(p)}(t_p)) \, dt_p$$

$$= \sum_n \pi_n \sum_p \int_{T_p} x_n^{(p)}(t_p) \, T(x)^{(p)}(t_p) - x_n^{(p)}(t_p) \, T\mu^{(p)}(t_p)$$

$$- \mu^{(p)}(t_p) \, T(x)^{(p)}(t_p) + \mu^{(p)}(t_p) \, T(x)^{(p)}(t_p) \, dt_p$$

$$\int_{T_p}\left(\sum_n \pi_n X_n^{(p)}(t_p) \sum_q \int_{T_q} C_{pq}(t_p, s_q) X_n^{(q)}(s_q)\, ds_q\right.$$

$$\left. - \mu^{(p)}(t) \sum_q \int_{T_q} C_{pq}(t_p, s_q)\, \mu(s_q)\, ds_q \right) dt_p$$

$$= \sum_p \sum_q \int_{T_p} \int_{T_q} \left(\sum_n \pi_n X_n^{(p)}(t_p) X_n^{(q)}(s_q) - \mu^{(p)}(t_p)\mu^{(q)}(t_p)\right) \cdot$$

$$\cdot \left(C_{pq}(t_p, s_q)\right) ds_q\, dt_p.$$

$$= \sum_p \sum_q \int_{T_p} \underbrace{\int_{T_q} C_{pq}(t_p, s_q)^2\, ds_q}_{= \| C_{p\cdot}(t_p, \cdot) \|^2}\, dt_p$$

$$= \sum_p \int_{T_p} \| C_{p\cdot}(t_p, \cdot) \|^2\, dt_p$$

LHS of Eq 5 is ok!