2.3 Sensitivity Analysis

As discussed in Section 3.2, the number M_j of univariate eigenfunctions used for MFPCA clearly has an impact on the results, as they control how much of the information in the univariate elements is used for calculating the multivariate FPCA. A standard approach in functional data analysis for quantifying the amount of information contributed by single eigenfunctions $\phi_m^{(j)}$ is the percentage of variance explained (pve), which is the ratio of the associated eigenvalue $\lambda_m^{(j)}$ and the sum of all eigenvalues. The following simulation systematically examines the sensitivity of the MFPCA result based on the pve of the univariate eigenfunctions.

Simulation Setup: The simulation is based on 100 replications with $N = 250$ observations of bivariate data on the unit interval (cf. setting 1 in Section 4.1), with $M = 8$ Fourier basis functions and exponentially decreasing eigenvalues for simulating the data. The number of univariate eigenfunctions M_1, M_2 for MFPCA is chosen based on pve $\in \{0.75, 0.90, 0.95, 0.99\}$ for both elements and $M_1 = M_2 = M = 8$ for comparison. The number of multivariate principal component functions is then set to $\min\{M_1+M_2, M\}$.

Results: The results of the sensitivity analysis are shown in Fig. 7 and Table 2. The number of estimated multivariate eigenvalues/eigenfunctions is for all 100 datasets $\hat{M} = 4$ for pve $= 0.75$, $\hat{M} = 6$ for pve $= 0.90$ and $\hat{M} = 8$ in all other cases. The results are as expected: Increasing the pve, and hence the information in the univariate FPCA, improves the estimation accuracy for both, multivariate eigenvalues and eigenfunctions. As a consequence, the reconstruction error reduces with increasing pve. Moreover, for a fixed m, the results show that there is a critical amount of information in univariate FPCA that is needed to describe the multivariate eigenvalues and eigenfunctions well. If this is reached (e.g. pve $= 0.95$ for $m = 5$, cf. Fig. 7), the additional benefit of using more univariate eigenfunctions (pve > 0.95) becomes negligible. If, in contrast, the univariate

FPCA does not contain enough information (pve < 0.95), the error rates for the MFPCA estimates are considerably increased. For fixed pve, the error rates rise abruptly for the last pair of eigenfunctions ($m \in \{\hat{M}_+ - 1, \hat{M}_+\}$). This is due to the fact that in this simulation, the multivariate functional principal components are derived from a Fourier basis. The last two eigenfunctions are hence sine and cosine functions with highest frequency and cannot be represented well by the univariate functions used, as they contain only functions with lower frequency, in other words, they do not contain enough information.

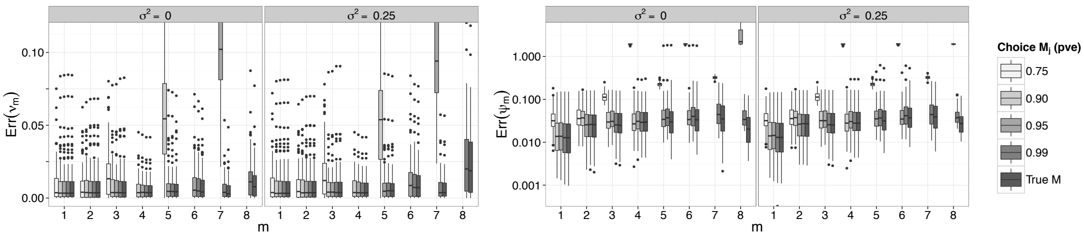

Figure 7: Relative errors for estimated eigenvalues (left) and eigenfunctions (right, log-scale) for the sensitivity analysis. Extreme values cut off for better comparability.

Table 2: Average MRSE (in %) in the sensitivity analysis.

	Choice of M_j (pve)				True M
	0.75	0.90	0.95	0.99	
$\sigma^2 = 0$	23.756	9.075	2.924	0.165	0.006
$\sigma^2 = 0.25$	24.099	9.583	3.593	0.842	0.740