# A - derivation of clouds. Part 1

$$\sum_{n=1}^{N} \pi_n d^2(x_n, u) = \sum_{n=1}^{N} \pi_n \sum_{p=1}^{P} \| x_n^{(p)} - u^{(p)} \|^2$$

$$= \sum_{p=1}^{P} \sum_{n=1}^{N} \pi_n \left( \int_{T_p} x_n^{(p)}(t_p)^2 - 2 x_n^{(p)}(t_p) u^{(p)}(t_p) + u^{(p)}(t_p)^2 dt_p \right)$$

$$= \sum_{p=1}^{P} \Bigg( \underbrace{\sum_{n=1}^{N} \pi_n \int_{T_p} x_n^{(p)}(t_p) dt_p}_{= \| x_n^{(p)} \|^2} \underbrace{- 2 \int \Big( \sum_{n=1}^{N} \pi_n \underbrace{x_n^{(p)}(t_p)}_{= u^{(p)}(t_p)} \Big) dt_p}_{= \| u^{(p)} \|^2} + \int_{T_p} \underbrace{(u^{(p)}(t_p))^2}_{= \| u^{(p)} \|^2} dt_p \Bigg)$$

$$= \sum_{p=1}^{P} \left( \sum_{n=1}^{N} \pi_n \| x_n^{(p)} \|^2 - 2 \| u^{(p)} \|^2 + \| u^{(p)} \|^2 \right)$$

$$= \sum_{p=1}^{P} \left( \sum_{n=1}^{N} \pi_n \| x_n^{(p)} \|^2 - \| u^{(p)} \|^2 \right)$$

$$= \sum_{p=1}^{P} \int_{T_p} \operatorname{var}(x_n^{(p)}(t_p)) \, dt_p$$

# A Derivation of the Cluds part 2.

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i \pi_j d^2(x_i, x_j) = \sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i \pi_j \sum_{p=1}^{P} \| x_i^{(p)} - x_j^{(p)} \|^2$$

$$= \sum_{p=1}^{P} \left( \sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i \pi_j \| x_i^{(p)} - x_j^{(p)} \|^2 \right) \cdot$$

$$= \sum_{p=1}^{P} \left( \sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i \pi_j \left( \int_{T_p} (x_i^{(p)}(t_p))^2 - 2\, x_i^{(p)}(t_p)\, x_j^{(p)}(t_p) + x_j(t_p)^2 \, dt_p \right) \right)$$

$$= \sum_{p=1}^{P} \left( \underbrace{\sum_{j=1}^{N} \pi_j}_{=1} \sum_{i=1}^{N} \pi_i \underbrace{\int_{T_p} x_i^{(p)}(t_p)^2 dt_p}_{= \| x_i^{(p)} \|^2} - 2 \sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i \pi_j \int_{T_p} x_i^{(p)}(t_p)\, x_j^{(p)}(t_p)\, dt_p \right.$$

$$\left. + \underbrace{\sum_{i=1}^{N} \pi_i}_{=1} \underbrace{\sum_{i=1}^{N} \pi_j \int_{T_p} x_j^{(p)}(t_p)^2 dt_p}_{\text{same as in first term, use index } i \text{ from now on ...}} \right)$$

$$= \sum_{p=1}^{P} \left( 2 \sum_{i=1}^{N} \pi_i \| x_i^{(p)} \|^2 - 2 \sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i \pi_j \langle x_i^{(p)}, x_j^{(p)} \rangle \right)$$

$$= 2 \sum_{p=1}^{P} \left( \sum_{i=1}^{N} \pi_i \| x_i^{(p)} \|^2 \right.$$

$$= 2 \sum_{p=1}^{P} \left( \int_{T_p} \sum_{i=1}^{N} \pi_i (x_i^{(p)}(t_p))^2 - \sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i \pi_j x_i(t_p)\, x_j(t_p)\, dt_p \right)$$

$$= 2 \sum_{p=1}^{P} \int_{T_p} \sum_{i=1}^{N} \pi_i (x_i^{(p)}(t_p))^2 - \left( \sum_{i=1}^{N} \pi_i x_i(t_p) \right)^2 dt_p$$

$$= 2 \sum_{p=1}^{P} \int_{T_p} \operatorname{Var}(x_i^{(p)}(t_p)) \, dt_p$$