$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\pi_i\pi_j\, d_T(m_i, m_j)$$

$$= \frac{1}{2}\sum_i\sum_j \pi_i\pi_j \langle\langle x_i - x_j, x_i - x_j \rangle\rangle_T$$

$$= \frac{1}{2}\sum_i\sum_j\sum_p\pi_i\pi_j\int_{T_p}(x_i^{(p)}(t_p) - x_j^{(p)}(t_p))\,T(x_i - x_j)^{(p)}(t_p)\,dt_p$$

$$= \frac{1}{2}\sum_i\sum_j\sum_p\pi_i\pi_j\int_{T_p}(x_i^{(p)}(t_p) - x_j^{(p)}(t_p))\,(T(x_i)^{(p)}(t_p) - T(x_j)^{(p)}(t_p))\,dt_p$$

$$= \frac{1}{2}\sum_i\sum_j\sum_p\int_{T_p} x_i^{(p)}(t_p)\,T(x_i)^{(p)}(t_p) - x_j^{(p)}T(x_i)^{(p)}(t_p) - x_i^{(p)}T(x_j)^{(p)}(t_p) + x_j^{(p)}T(x_j)^{(p)}(t_p)\,dt_p$$

$$= \frac{1}{2}\sum_p\int_{T_p}\underbrace{\sum_j\pi_j}_{=1}\sum_i x_i^{(p)}T(x_i)^{(p)}(t_p)\,dt_p$$

$$-\frac{1}{2}\sum_p\int_{T_p}\sum_i\sum_j\pi_i\pi_j\, x_j^{(p)}(t_p)\,T(x_i)^{(p)}(t_p)\,dt_p$$

$$\sum_p \int_{T_p} - \frac{1}{2}\sum_p \int_{T_p} \sum_i \sum_j \pi_i \pi_j \, x_i^{(p)}(t_p)\, T(x_j)^{(p)}(t_p)\, dt_p$$

$$+ \frac{1}{2}\sum_p \int_{T_p} \sum_i \pi_i \sum_j \pi_j \, x_j^{(p)}(t_p)\, T(x_j)^{(p)}(t_p)\, dt_p$$

$$= \frac{1}{2}\sum_i \pi_i \underbrace{\sum_p \int_{T_p} x_i^{(p)}(t_p)\, T(x_i)^{(p)}(t_p)\, dt_p}_{= \|x_i\|_T^2}$$

$$- \frac{1}{2}\sum_i \sum_j \pi_i \pi_j \underbrace{\sum_p \int_{T_p} x_j^{(p)}(t_p)\, T(x_i)^{p}(t_p)}_{\langle\langle x_j , x_i \rangle\rangle_T}$$

$$- \frac{1}{2}\sum_i \underbrace{\sum_j \pi_i \pi_j \sum_p \int_{T_p} x_i^{(p)}(t_p)\, T(x_j)^{(p)}(t_p)\, dt_p}_{= \text{same}}$$

$$+ \frac{1}{2}\sum_j \pi_j \underbrace{\sum_p \int_{T_p} x_j^{(p)}\, T(x_j)^{(p)}(t_p)\, dt_p}_{\|x_j\|_T^2}$$

$$= \sum_i \pi_i \|x_i\|_T^2 - \sum_i \sum_j \pi_i \pi_j \langle\langle x_i , x_j \rangle\rangle_T$$

Now just need to show

$\sum_i \sum_j \pi_i \pi_j \langle\langle x_i , x_j \rangle\rangle$ and result follows from last rest.

$$\sum_j \pi_i \pi_j \sum_p \int_{T_p} x_i^{(p)}(t_p)\, T(x_j)^{(p)}(t_p)\, dt_p$$

$$= \sum_p \int_{T_p} \underbrace{\sum_i \pi_i x_i^{(p)}}_{\mu^{(p)}(t)} \sum_j \pi_j (T x_j)^{(p)}(t_p)\, dt_p$$

$$= \sum_p \int_{T_p} \mu^{(p)}(t_p)\, T\Big(\underbrace{\sum_j \pi_j x_j}_{\mu}\Big)^{(p)}(t_p)\, dt_p$$

$$= \sum_p \int_{T_p} \mu^{(p)}(t_p)\, T(\mu)^{(p)}\, dt_p$$

$\rightarrow$ Eq 5 so ok.