Report on " On the estimation of the number of components in multivariate functional principal component analysis" (LSSP-2024-0460)

In this paper, the authors conduct extensive simulations to explore how to best determine the number of principal components to retain based on Happ and Greven (2018) which introduced a method for performing principal components analysis on multivariate functional data observed over varying dimensional domains, using univariate functional principal components for each feature. The findings indicate that the conventional method of selecting components based on a percentage of variance explained for each univariate feature may be unreliable when trying to capture the overall variance in multivariate functional data. As a result, the authors caution practitioners to use this approach carefully.

Overall, the manuscript introduces an interesting statistical methodology and their findings are well-supported by numerical study. Nevertheless, there are a few concerns and comments that warrant attention:

1 Major Comments

1. Page 2: Although I understand how $s_1, \ldots, s_p$ are obtained, the authors need to provide a more detailed explanation of how these values are generated using a Bernoulli distribution, for example.

2. Page 2: It would be helpful to simulate a case with irregular time points, where each function has different time points. Moreover, the time points for each function do not need to be regular. I recommend that the authors consider these situations.

3. Pages 2,3: The paragraph in the end of Page 2 and Page 3 requires a complete rewrite as it lacks the necessary rigor in its explanations and methodology.

4. Pages 3,4: The simulation study and application sections of the paper need to be significantly revised. The current writing provides very limited explanation, which makes it difficult for readers to fully understand the intuition behind the results, the

significance of the findings, and how the method performs in practice. The authors should offer more detailed descriptions of the simulation settings including the rationale behind the chosen parameters and models. Moreover, the application section should clearly describe the dataset, the variables used, and how the results from the method apply to real-world scenarios. More emphasis should be placed on interpreting the application results and their implications which help readers understand the practical value of the proposed method.

5. Figure 2: The authors classify the number of simulations based on the size of the circles, which is very confusing. Please present the results using a clearer visualization method.

6. Page 4, Application: The authors use B-spline bases for the Canadian weather dataset. I recommend that they also use a Fourier basis and compare the results. This can demonstrate the robustness of the method, as Fourier bases typically provide a better fit for weather data.

7. Page 5, Figure 3: Figure 3 is missing a y-axis.