

1 Calculate the eigenvalues & eigenvectors of

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= [A - \lambda I] x = 0$$

$$\left[\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right] x = 0$$

$$\left[\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right] x = 0$$

$$\begin{bmatrix} 2-\lambda & 1-0 \\ 1-0 & 2-\lambda \end{bmatrix} x = 0$$

To find λ

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$[(2-\lambda)(2-\lambda) - 1 \times 1] = 0$$

$$= 4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$= (\lambda - 3)(\lambda - 1) = 0$$

$$\lambda = 3, \lambda = 1$$

Eigen values are

Find Eigen Vector

when $\lambda = 3$

$$\begin{bmatrix} 2-3 & 1 \\ 1 & 2-3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$-x + y = 0$$

$$-x + y = 0 \Rightarrow -x = -y$$

$$x = y$$

$$x = 1, y = 1$$

$$x + y = 0$$

$$x = -y$$

$$1 = -1$$

$$= -1$$

when $\lambda = 1$

$$\begin{bmatrix} 2-1 & 1 \\ 1 & 2-1 \end{bmatrix} x = 0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x = 0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x = 0$$

$$x + y = 0$$

$$x = -y$$

$$1 = -1$$

$$\text{eigen vector} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\text{Eigen vector} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2 Find the Singular value decomposition of

$$A = \begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix}$$

SVD =

$$A = U \Sigma V^T$$

$$U = A \cdot A^T$$

$$\begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} -4 & 1 \\ -7 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 65 & -32 \\ -32 & 17 \end{bmatrix}$$

$$\text{Unit form} = \begin{bmatrix} 0.89 \\ -0.45 \end{bmatrix}$$

$$\lambda = 1$$

Eigen vector in unit form.

$$= \frac{1}{\sqrt{1^2 + 2^2}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\rightarrow \frac{1}{\sqrt{1^2 + 2^2}} = \frac{1}{\sqrt{1+4}} = \frac{1}{\sqrt{5}} = \frac{1}{2.23}$$

$$= \underline{\underline{0.45}}$$

$$\rightarrow \frac{2}{\sqrt{1^2 + 2^2}} = \frac{2}{\sqrt{1+4}} = \frac{2}{\sqrt{5}} = \frac{2}{2.23}$$

$$= \underline{\underline{0.89}}$$

$$= \begin{bmatrix} 0.45 \\ 0.89 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.89 & 0.45 \\ -0.45 & 0.89 \end{bmatrix}$$

$$= \frac{1}{\sqrt{16+9}} \begin{bmatrix} 16 \\ 9 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.6 \end{bmatrix}$$

$$\text{Find } V = \frac{1}{\lambda} A$$

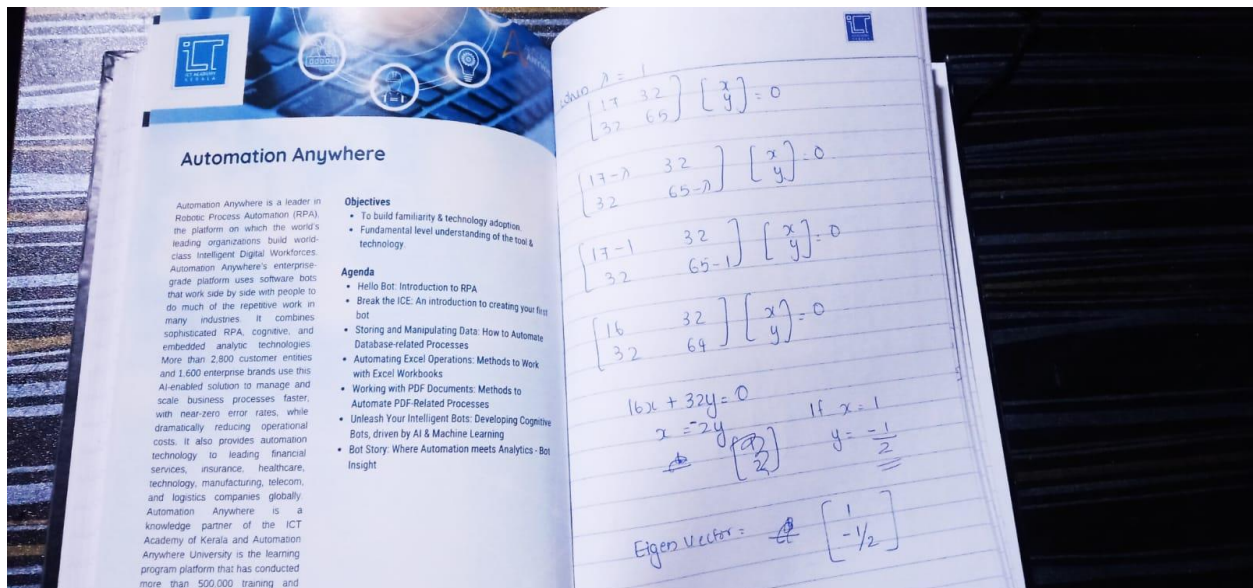
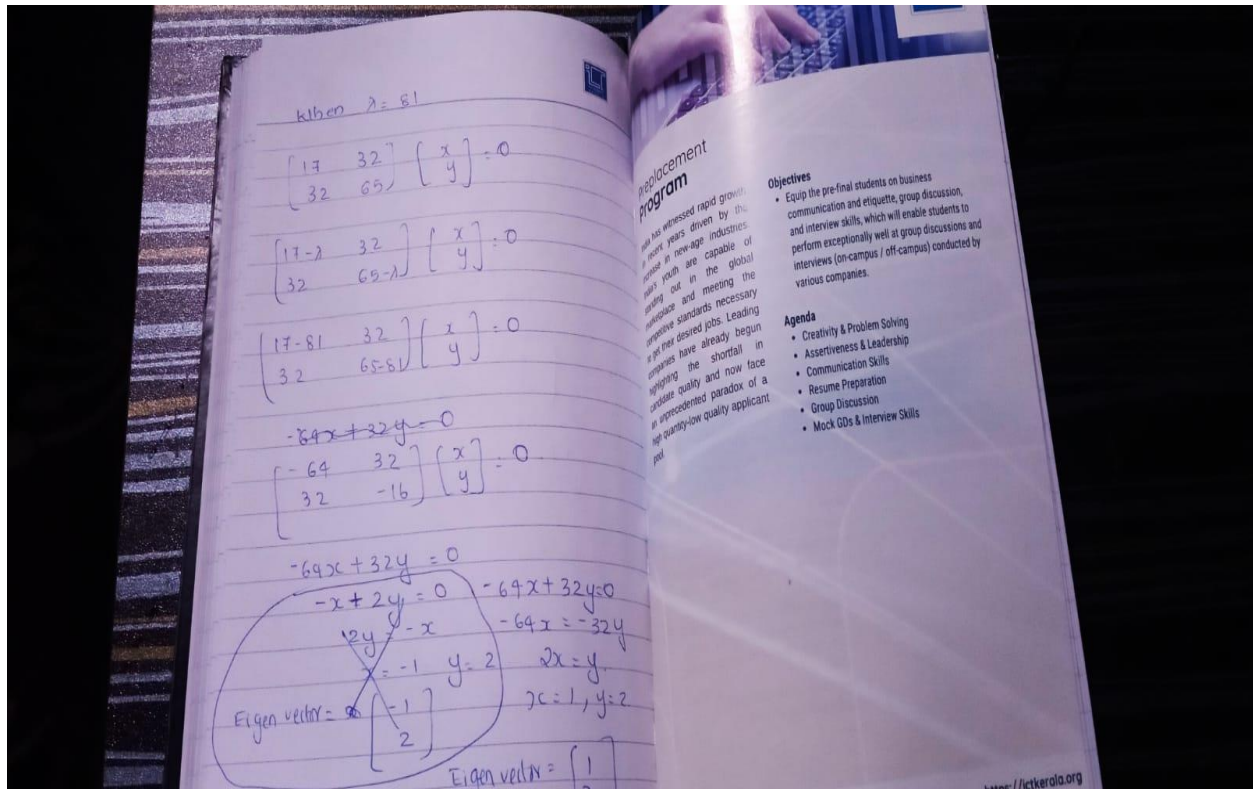
$$V = A^T \cdot A$$

$$\begin{bmatrix} -4 & 1 \\ -7 & 4 \end{bmatrix} \cdot \begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 17 & 32 \\ 32 & 65 \end{bmatrix}$$

$$\begin{bmatrix} 17-\lambda & 32 \\ 32 & 65-\lambda \end{bmatrix} = 0$$

$$\begin{aligned} & (17-\lambda)(65-\lambda) - 32^2 = 0 \\ & = 1105 - 17\lambda - 65\lambda + \lambda^2 - 1024 = 0 \\ & = 81 - 82\lambda + \lambda^2 = 0 \\ & \lambda^2 - 82\lambda + 81 = 0 \\ & \lambda = 81 \quad \lambda = 1 \end{aligned}$$



Unit Eigen Vector

$$\lambda = 81 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\rightarrow \frac{1}{\sqrt{1^2 + 2^2}} = \frac{1}{\sqrt{5}} = \frac{1}{2.23} = 0.44$$

$$\rightarrow \frac{2}{\sqrt{1^2 + 2^2}} = \frac{2}{\sqrt{5}} = \frac{2}{2.23} = 0.89$$

$$\begin{pmatrix} 0.44 \\ 0.89 \end{pmatrix}$$

Unit Eigen Vector in unit form

When $\lambda = 1$

$$\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$$

$$= \frac{1}{\sqrt{1^2 + (-1/2)^2}} = \frac{1}{\sqrt{1 + 0.25}} = \frac{1}{\sqrt{1.25}} = \frac{1}{1.11}$$

$$= 0.90$$

$$\frac{-1/2}{\sqrt{1^2 + (-1/2)^2}} = \frac{-1/2}{\sqrt{1.25}} =$$

$$= \frac{-0.5}{1.11} = -0.45$$

$$\begin{pmatrix} 0.90 \\ -0.45 \end{pmatrix}$$

Apply form of SVD

$$A = U \cdot \Sigma \cdot V^T$$

$$U = \begin{pmatrix} 0.89 & 0.45 \\ -0.45 & 0.89 \end{pmatrix} \cdot \Sigma = \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix} V^T \begin{pmatrix} 0.44 & 0.89 \\ 0.90 & -0.45 \end{pmatrix}$$