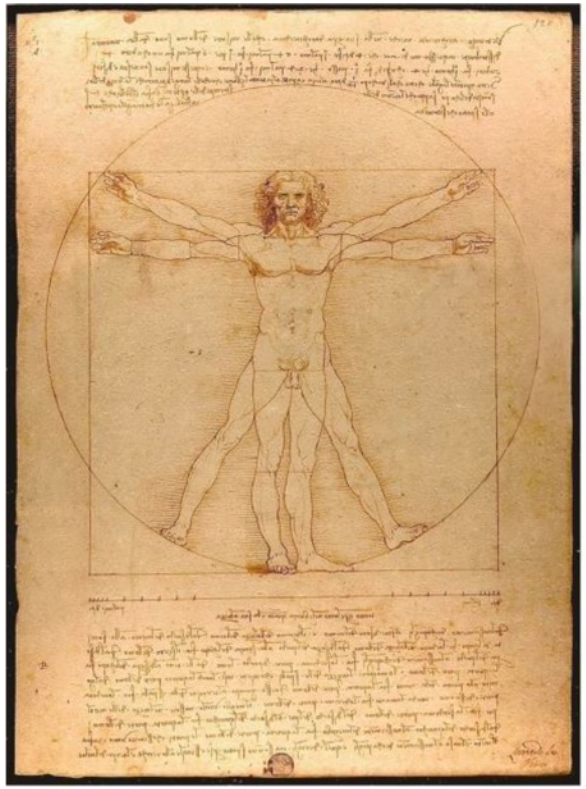
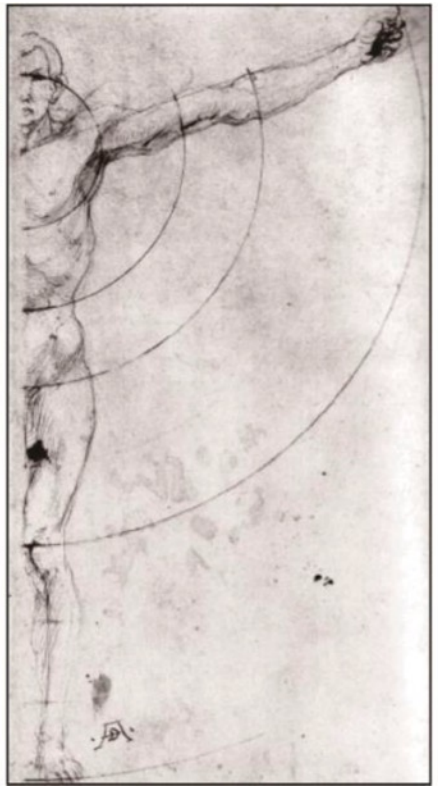
A Brief History of Geometric Morphometrics

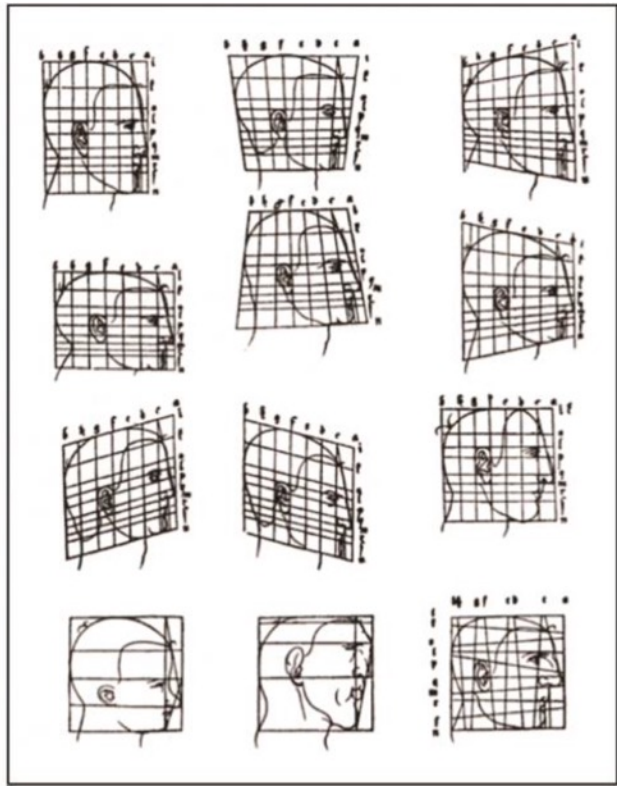
Ryan N. Felice

Analytical Paleobiology Workshop - Erlangen 2023

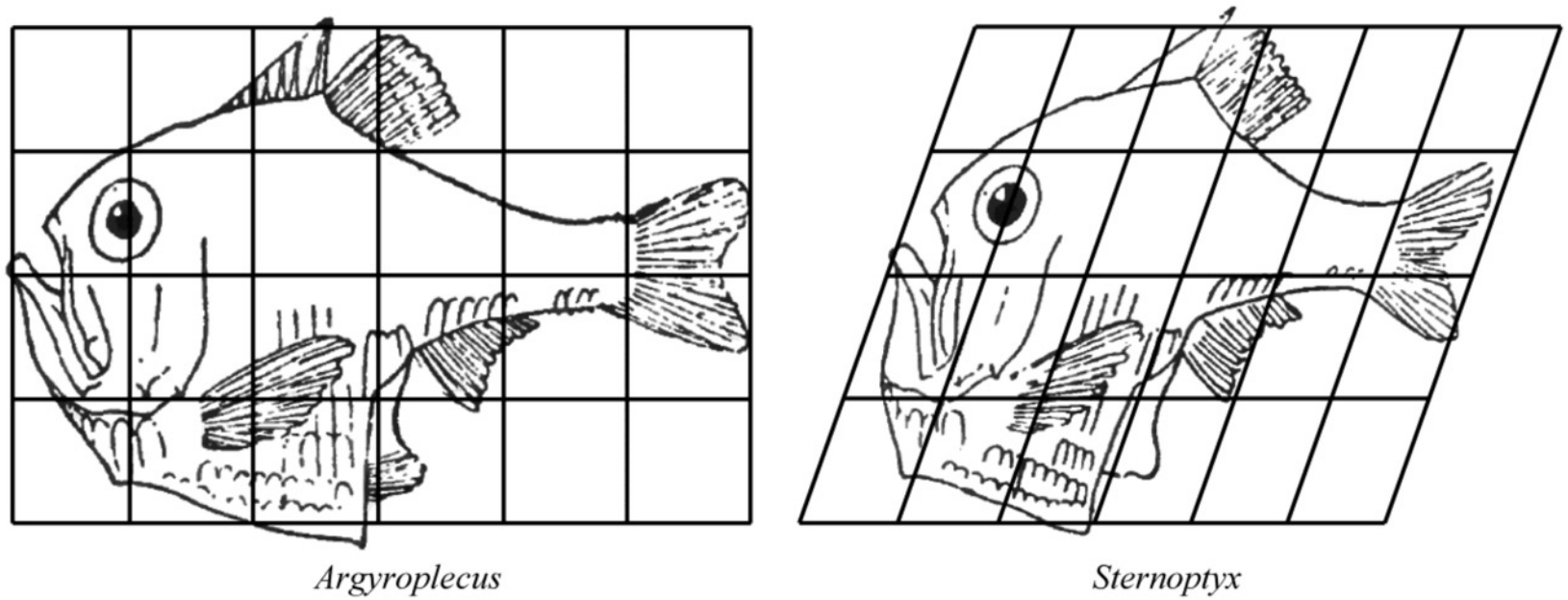


daVinci (c. 1480)

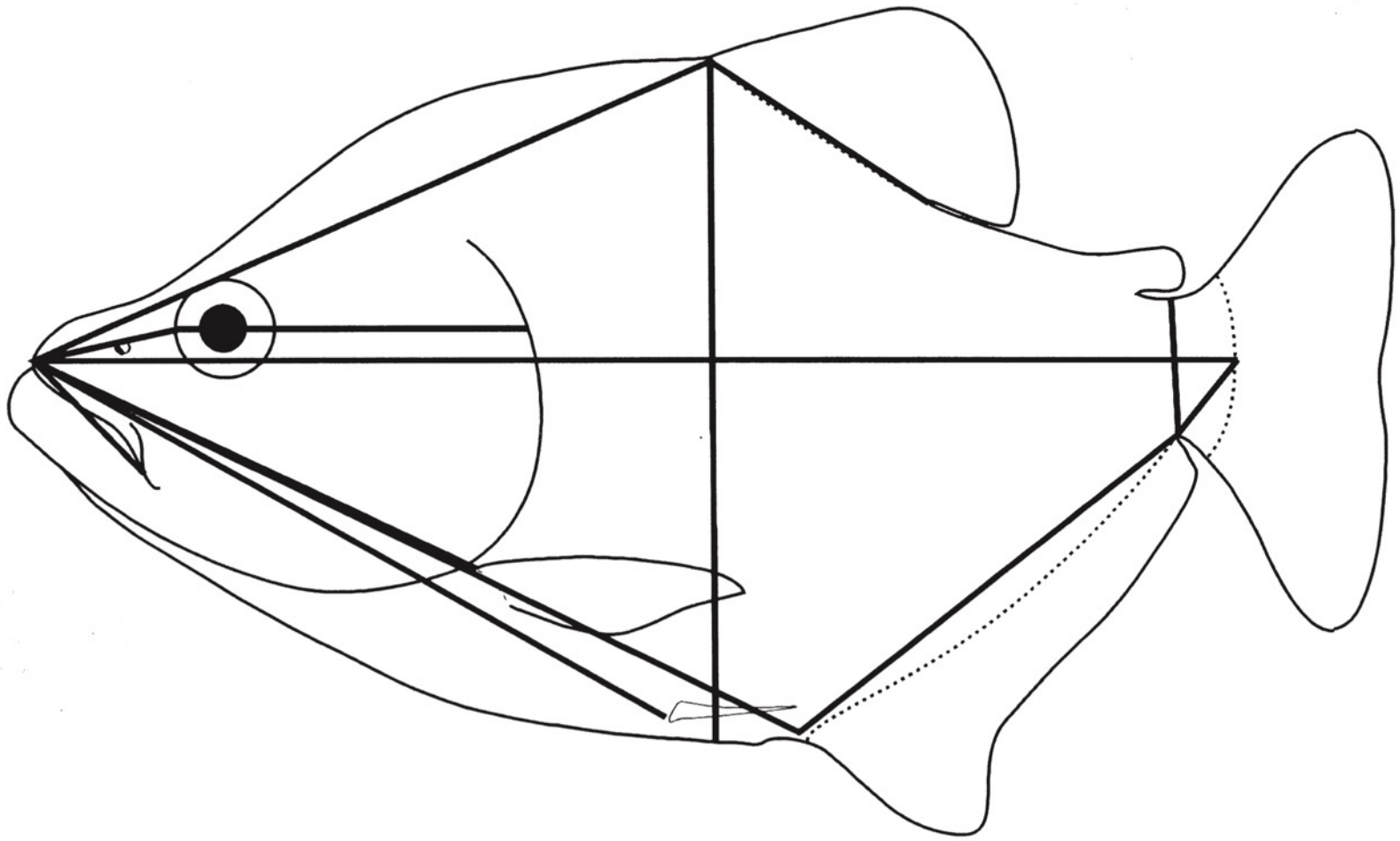




Düer (c. 1524)

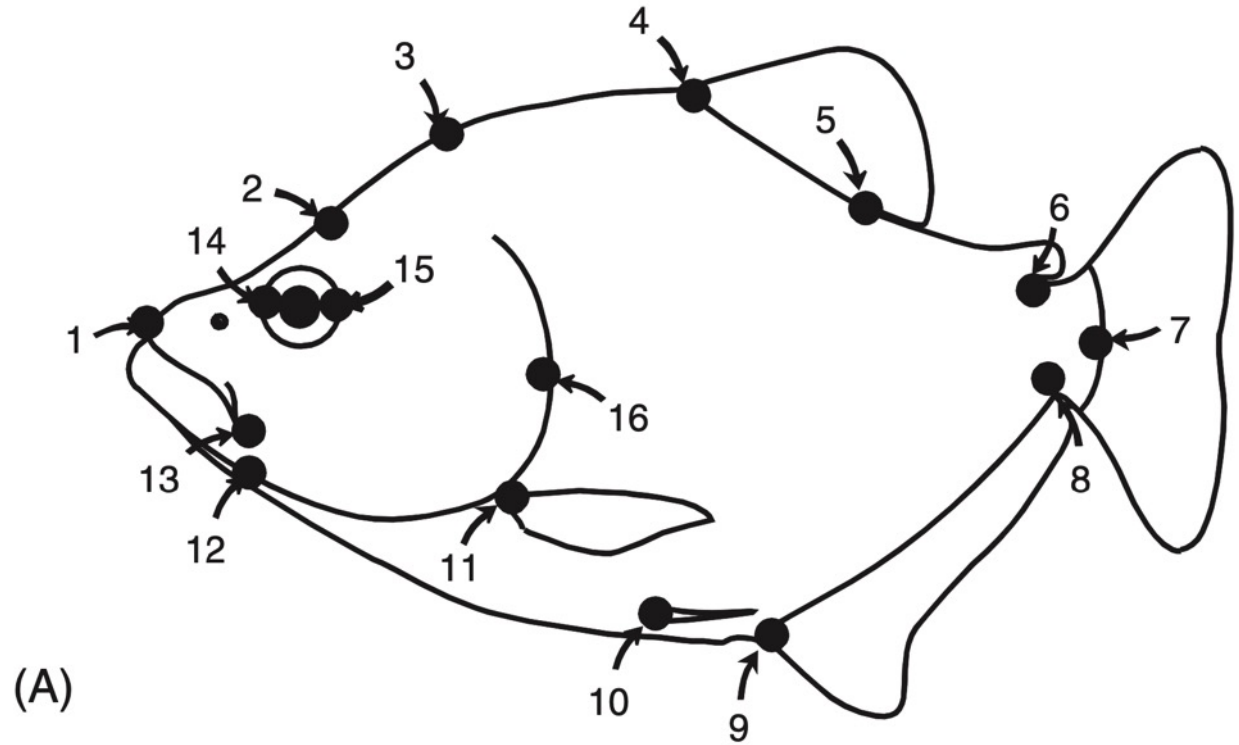


D’Arcy Thompson



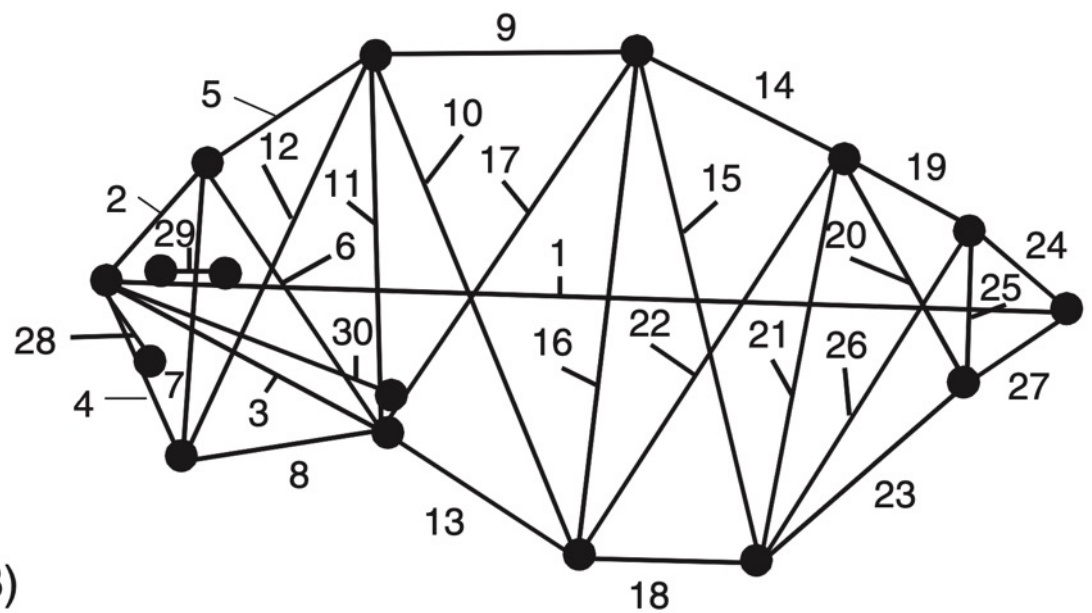
1930s/1940s

Truss measurements

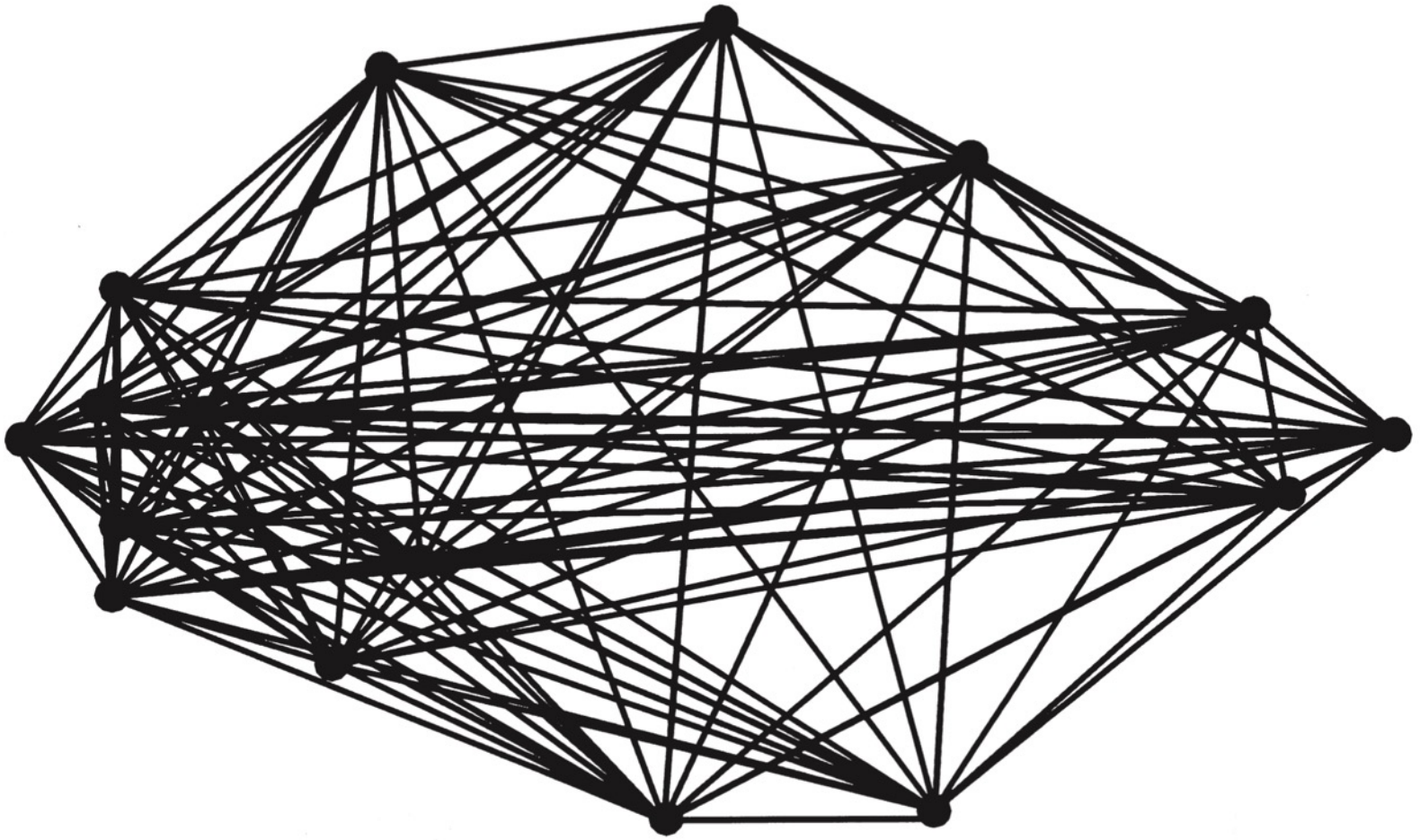


(A)

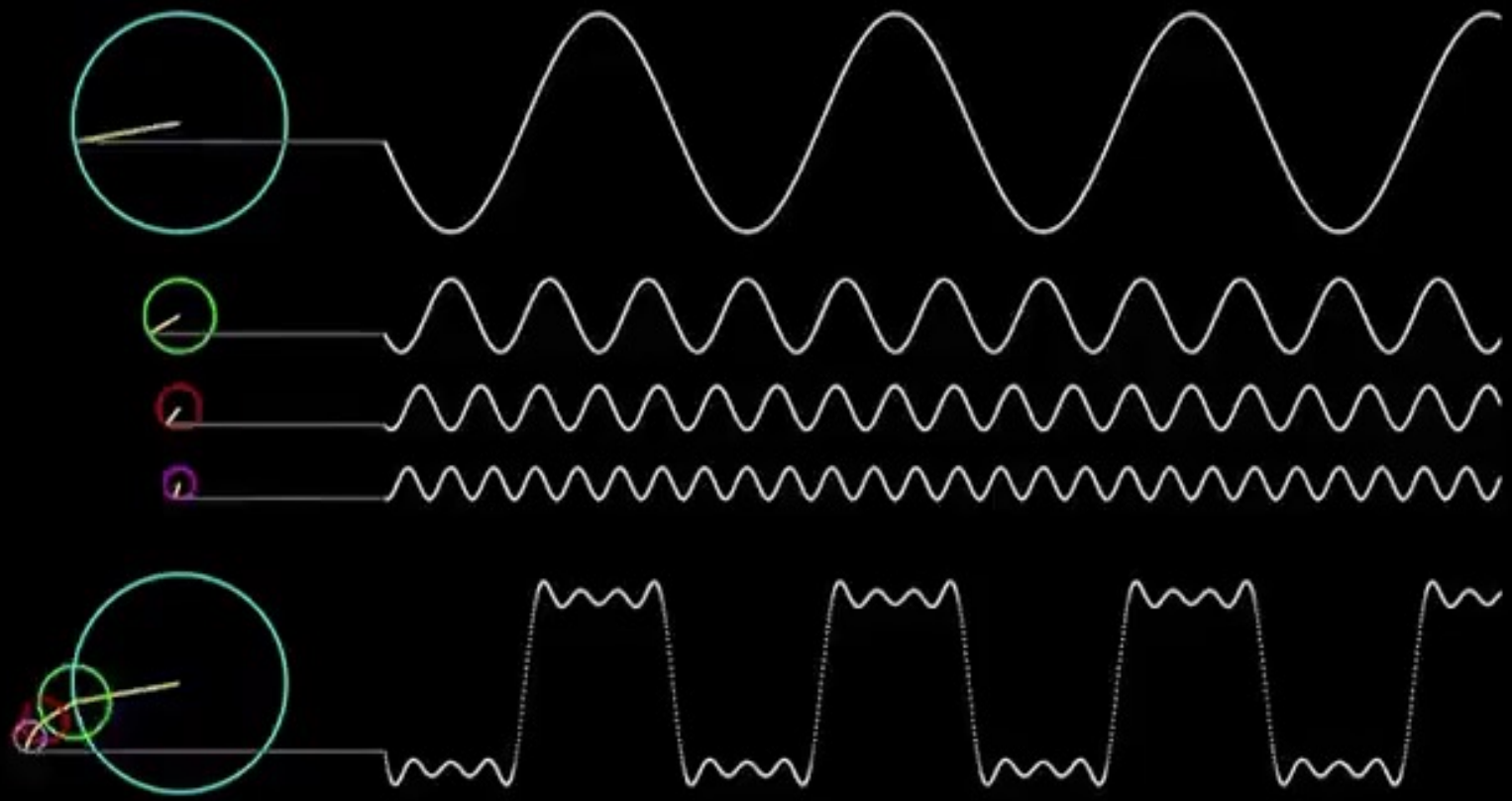
What is the drawback of this approach?

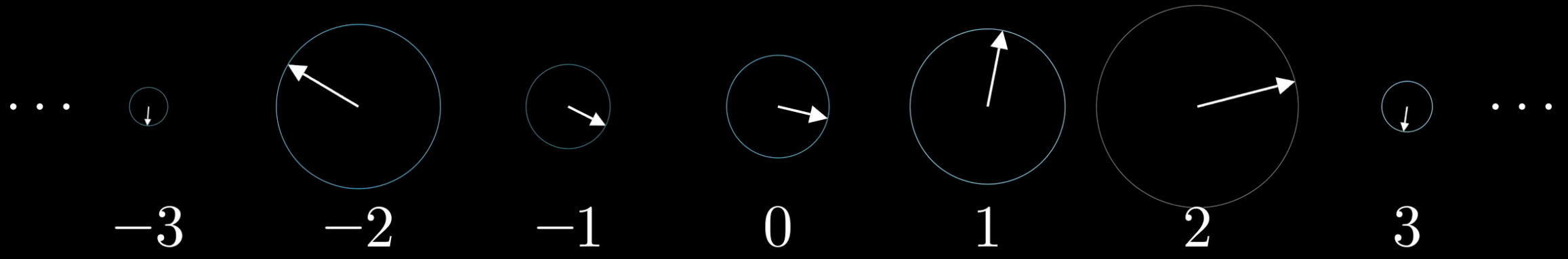


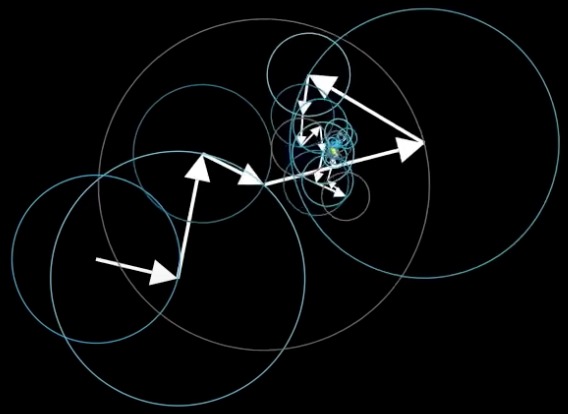
(B)



All pairwise truss measurements

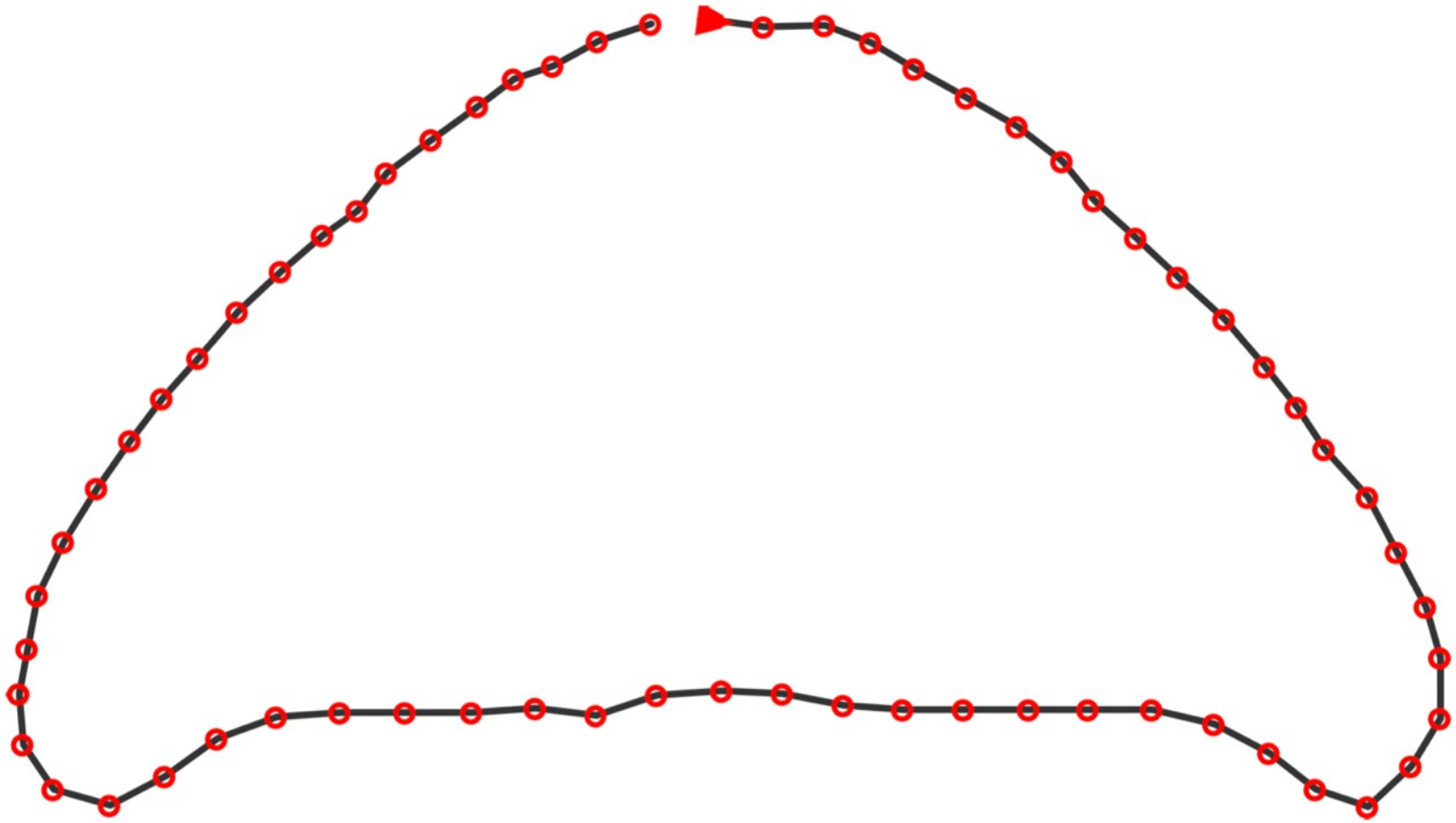


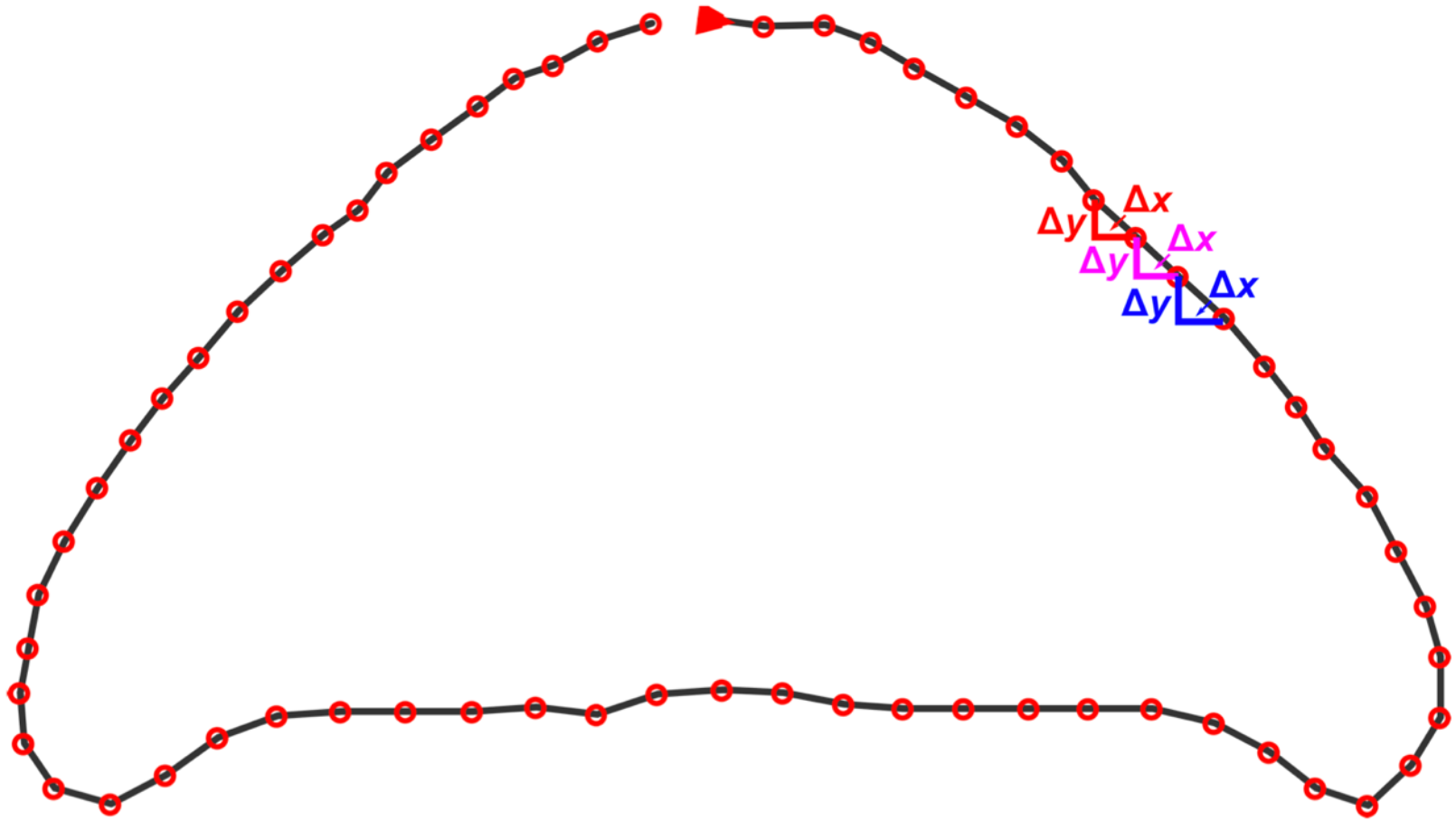




Play (k)

1960s/1970s- Outline analysis

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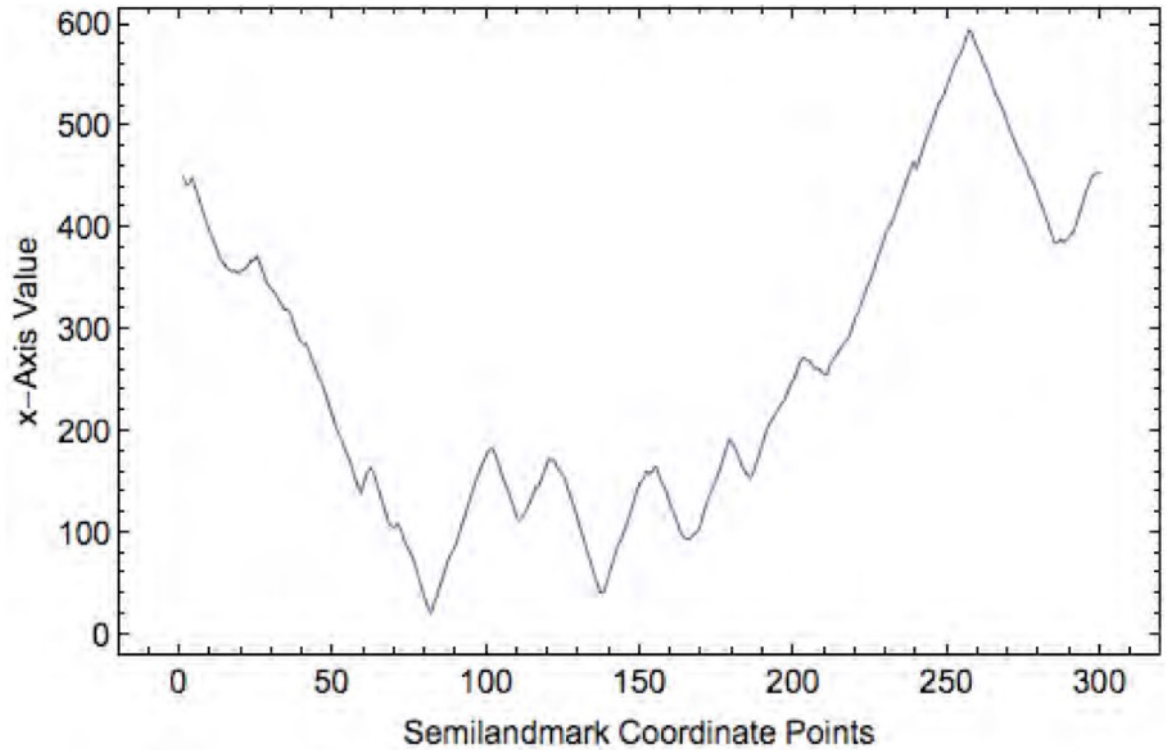
Δx
Δy
Δx
Δy

A.


600
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x–Axis Value
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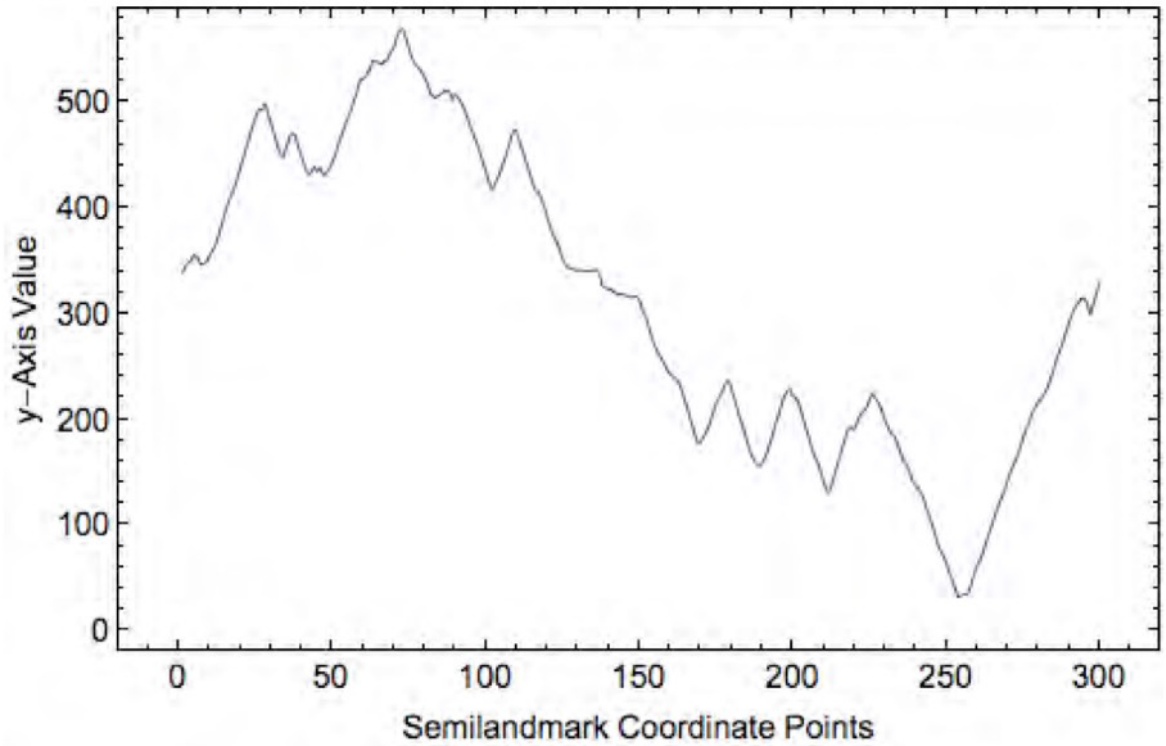
100
150
200
250
300
Semilandmark Coordinate Points

B.


500
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y–Axis Value
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Semilandmark Coordinate Points

$$A_n = \frac{T}{2n^2\pi^2}\sum_{p=1}^{k}\frac{\Delta x_p}{\Delta t_p}\left[\cos\left(\frac{2\pi n t_p}{T}\right)-\cos\left(\frac{2\pi n t_{p-1}}{T}\right)\right]$$

$$B_n = \frac{T}{2n^2\pi^2}\sum_{p=1}^{k}\frac{\Delta x_p}{\Delta t_p}\left[\sin\left(\frac{2\pi n t_p}{T}\right)-\sin\left(\frac{2\pi n t_{p-1}}{T}\right)\right]$$

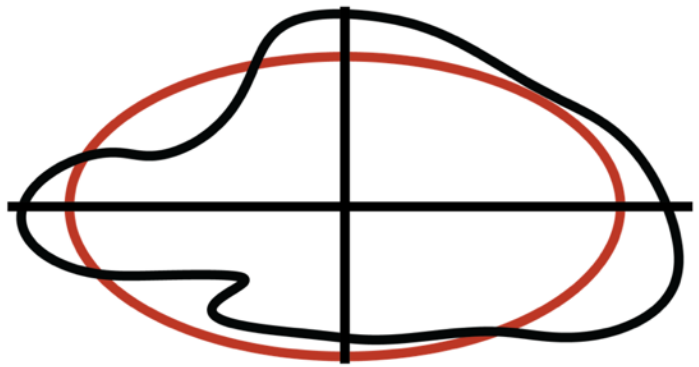
$$C_n = \frac{T}{2n^2\pi^2}\sum_{p=1}^{k}\frac{\Delta y_p}{\Delta t_p}\left[\cos\left(\frac{2\pi n t_p}{T}\right)-\cos\left(\frac{2\pi n t_{p-1}}{T}\right)\right]$$

$$D_n = \frac{T}{2n^2\pi^2}\sum_{p=1}^{k}\frac{\Delta y_p}{\Delta t_p}\left[\sin\left(\frac{2\pi n t_p}{T}\right)-\sin\left(\frac{2\pi n t_{p-1}}{T}\right)\right]$$

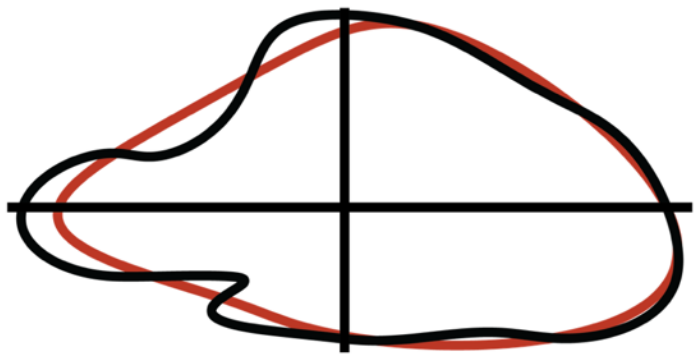
Where: k = the total number of steps around the outline
n = the harmonic number
Δx = the displacement along the x axis between point p and p+1
Δt = the length of the step between point p and p+1
t_p = accumulated length of step segments at point p
T = sum of lengths of all steps around outline

See This PDF from Norm MacLeod for more math

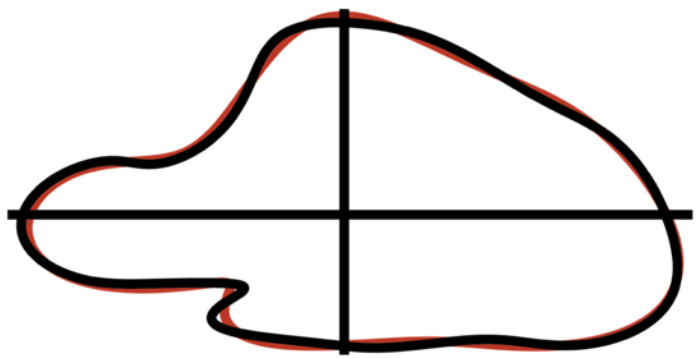
One Harmonic



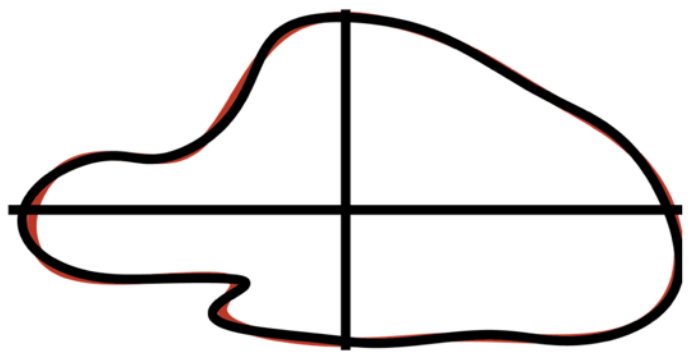
Three Harmonics



Six Harmonics



Eight Harmonics



Heart Chambers

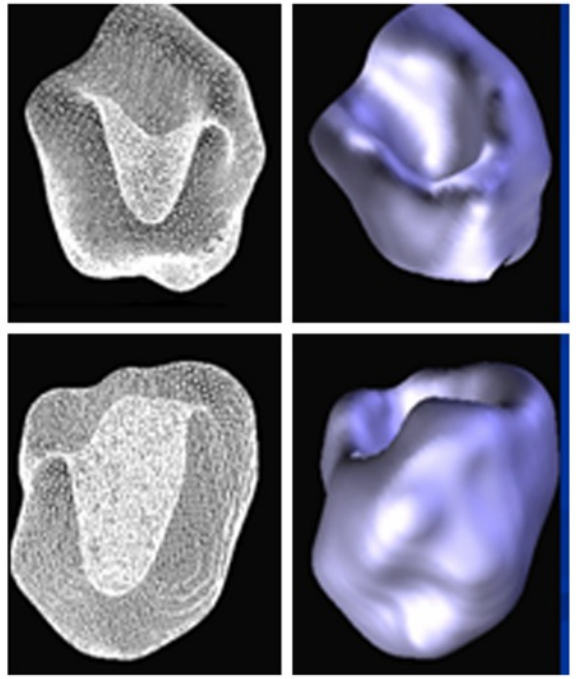


Fig. 19. Segmentation and separation of scintigraphic image sequences in one patient resulting in the structures of the epicardium wall and the endocardium wall.

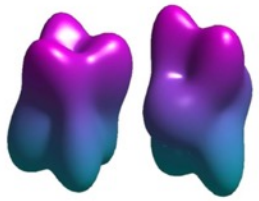
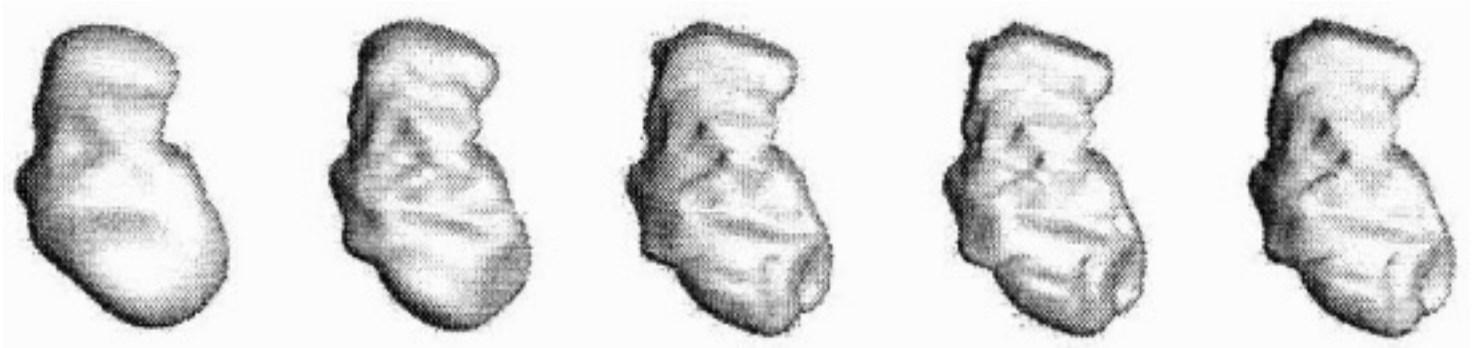


Fig. 20. Rendered surface of the left ventricle at stress and rest respectively after segmentation of scintigraphic image sequences in one patient.

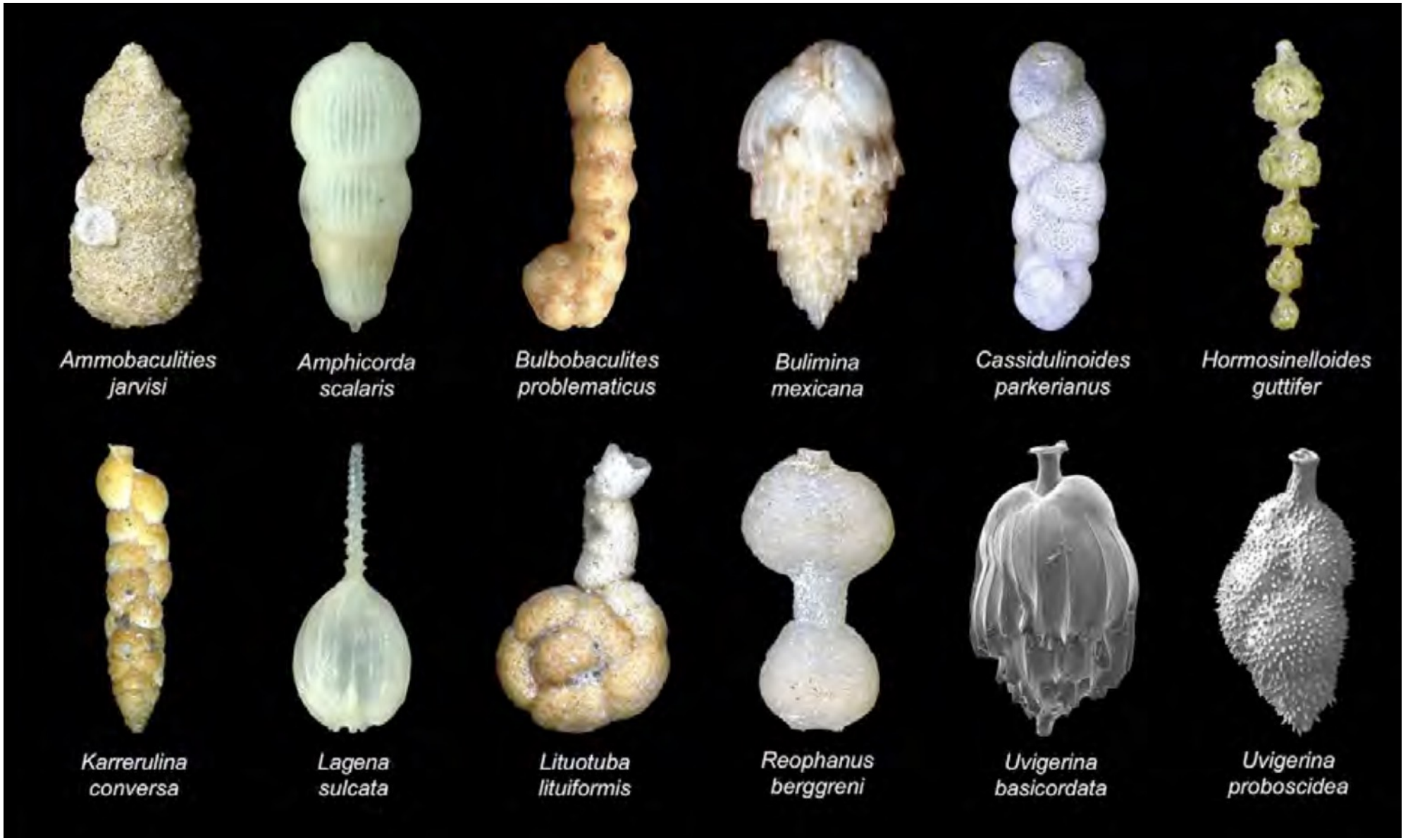
Ben Abdallah et al 2010

Hippocampus

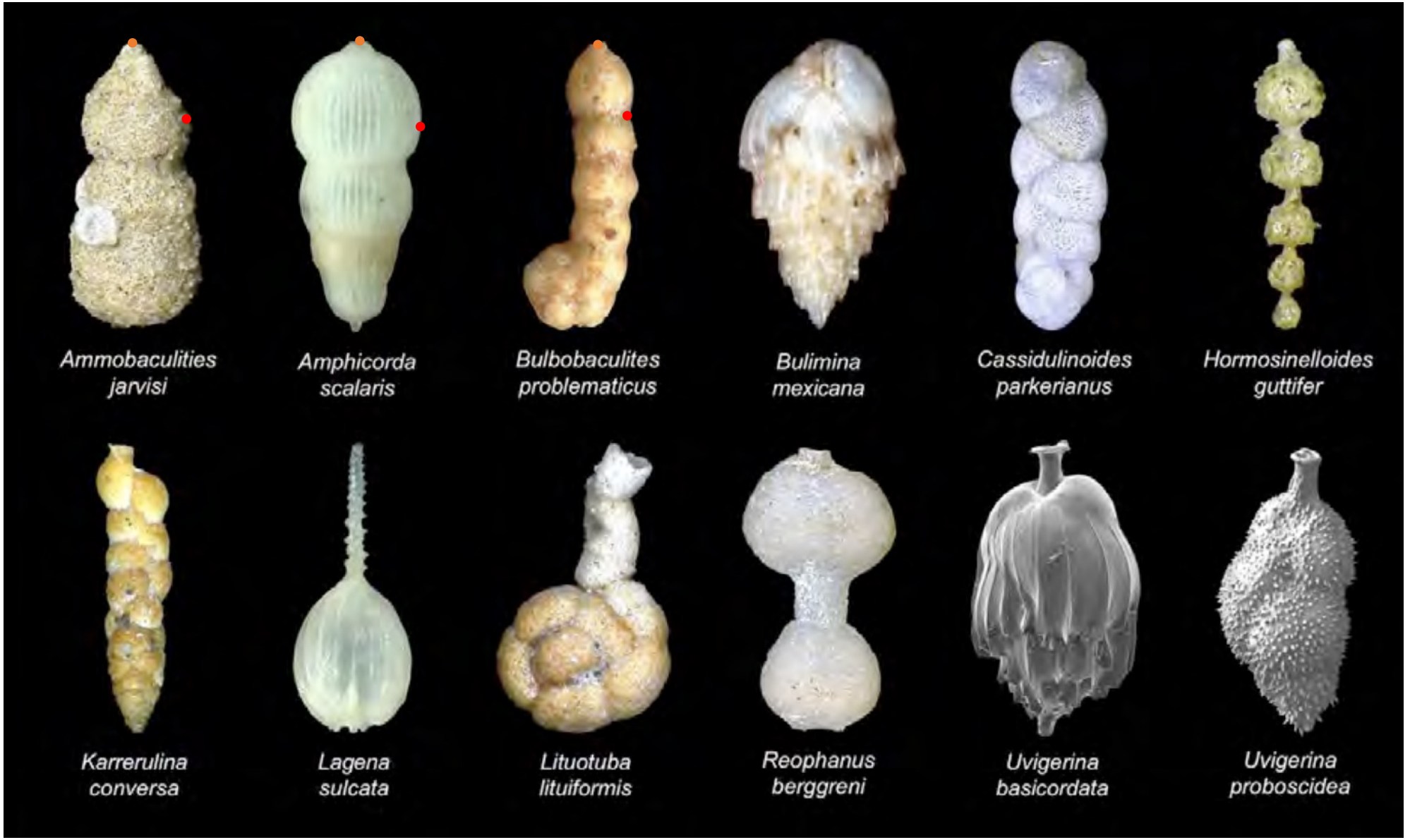


Gutman et al 2006

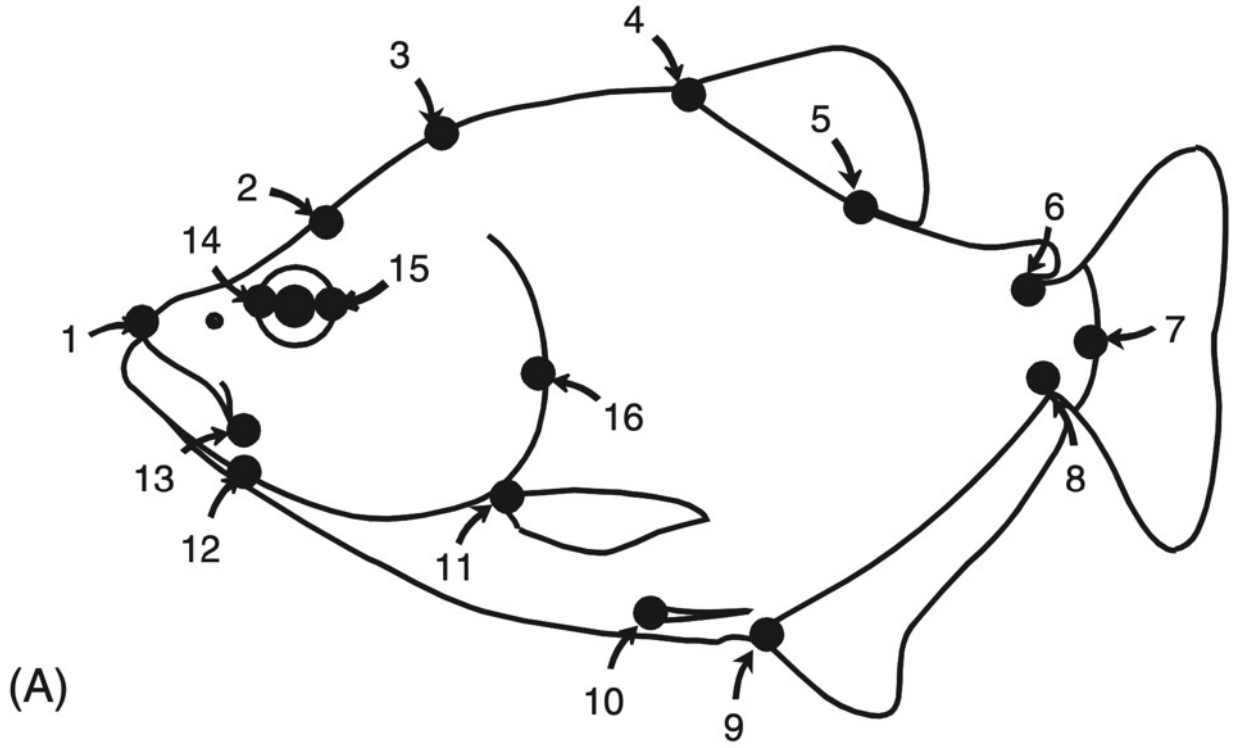
- What is the drawback??

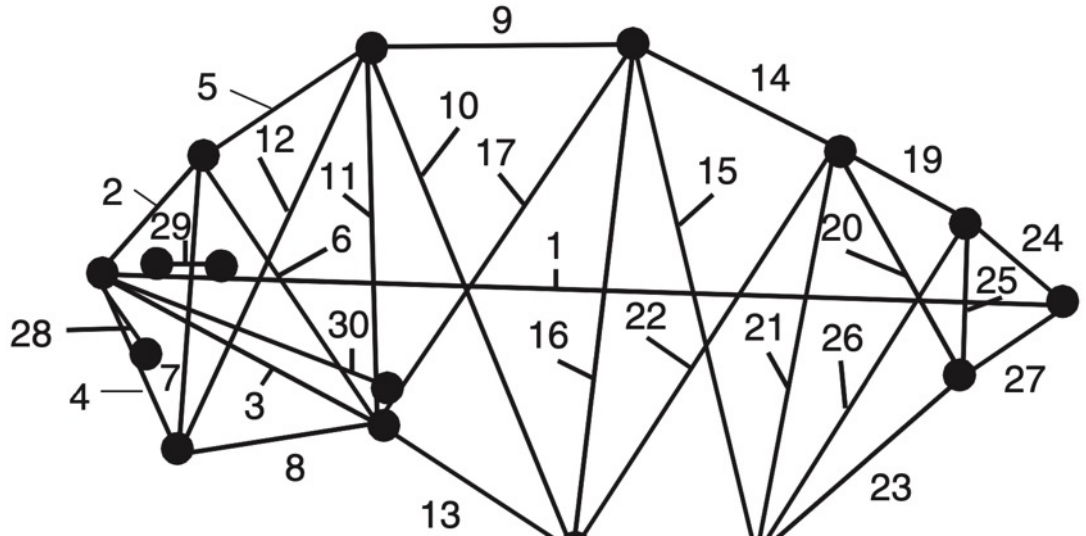


- What is the drawback??

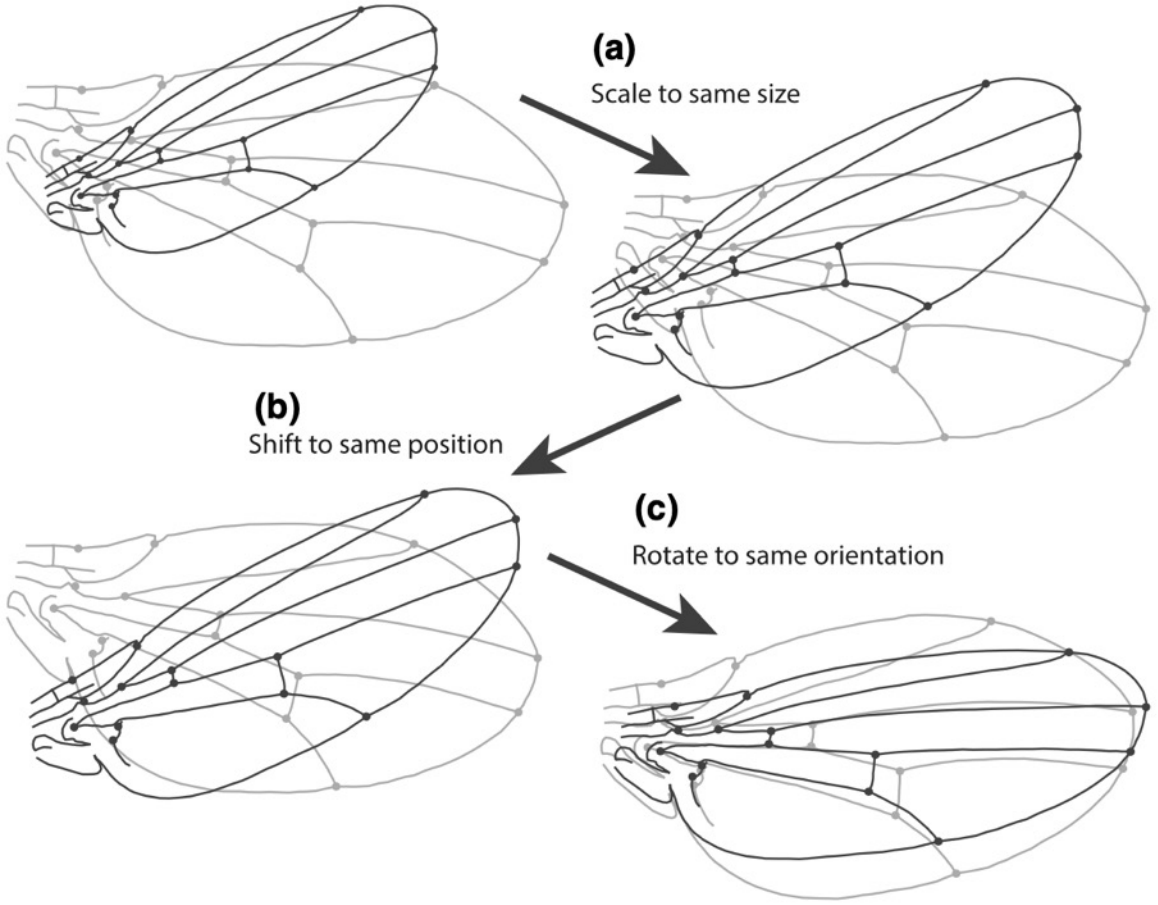


The “Revolution”: La
Morphometrics



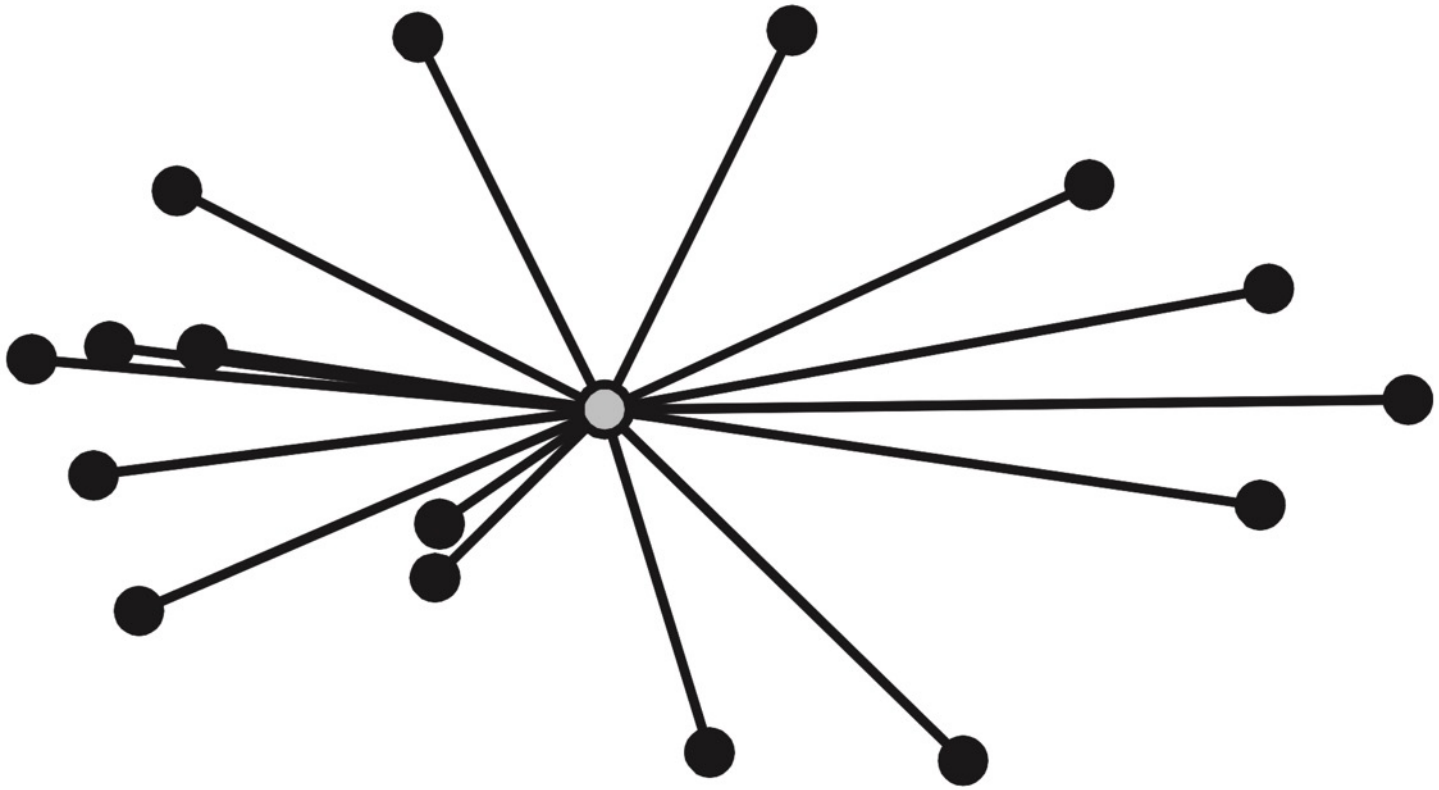


Procrustes superimposition

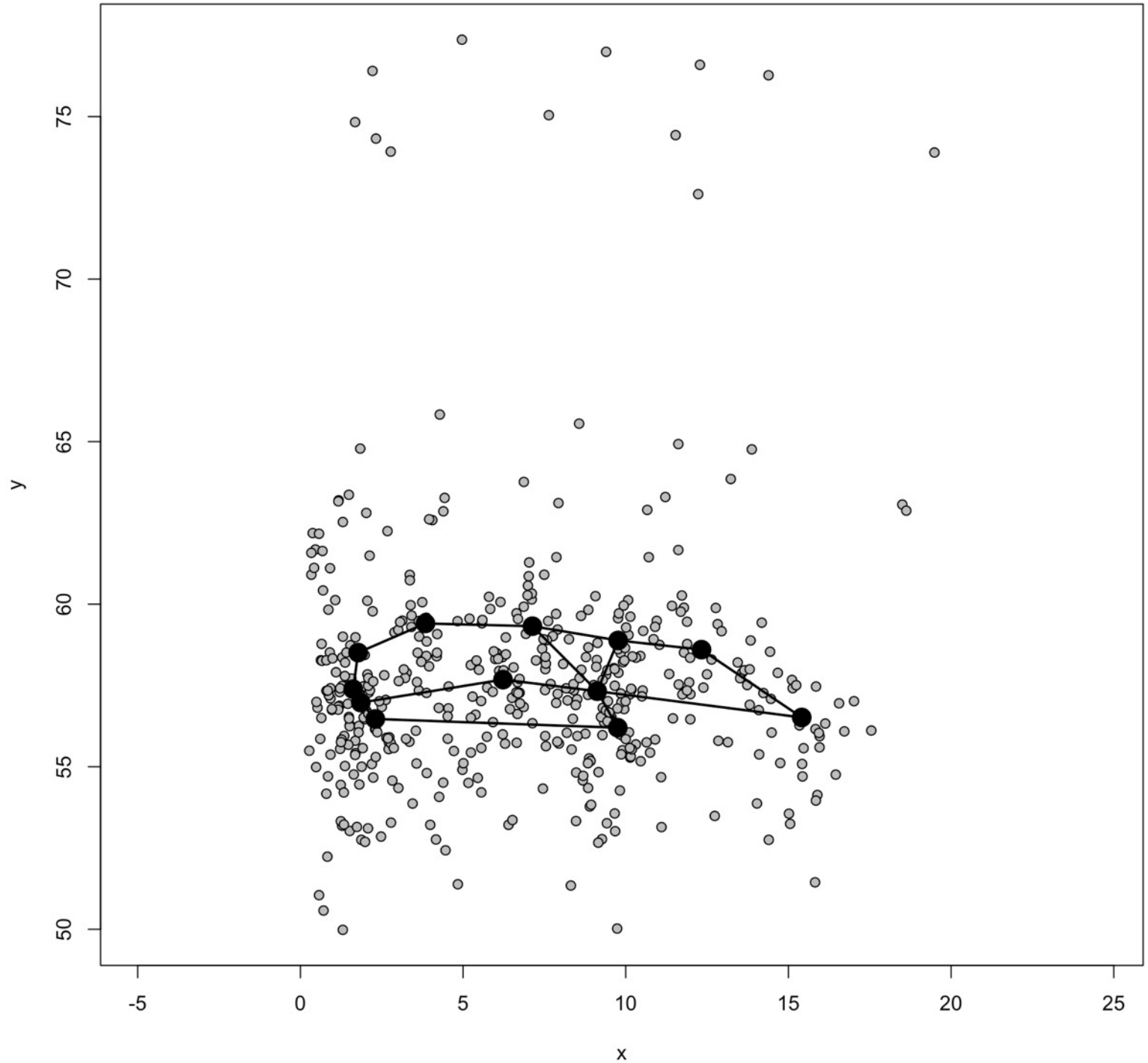


Procrustes had a stronghold on Mount Korydallos at Erineus, on the sacred way between Athens and Eleusis. There he had a bed, in which he invited every passer-by to spend the night, and where he set to work on them with his smith's hammer, to stretch them to fit. … If the guest proved too tall, Procrustes would amputate the excess length; if the guest was too short Procrustes would stretch them until they died; nobody ever fit the bed exactly.

What is centroid size?



Sum of the squared distances from each point to the geometric center of all of the points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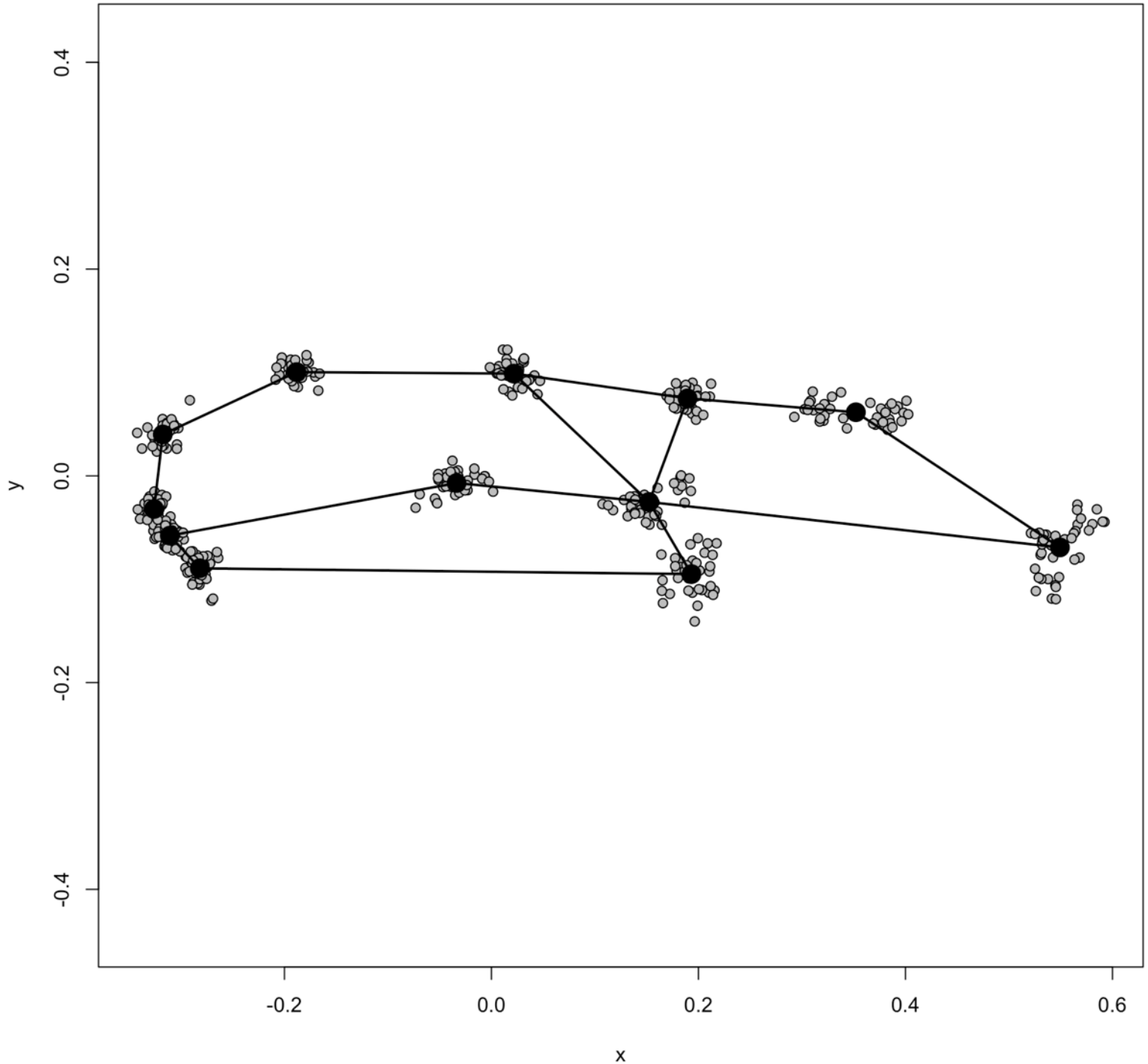
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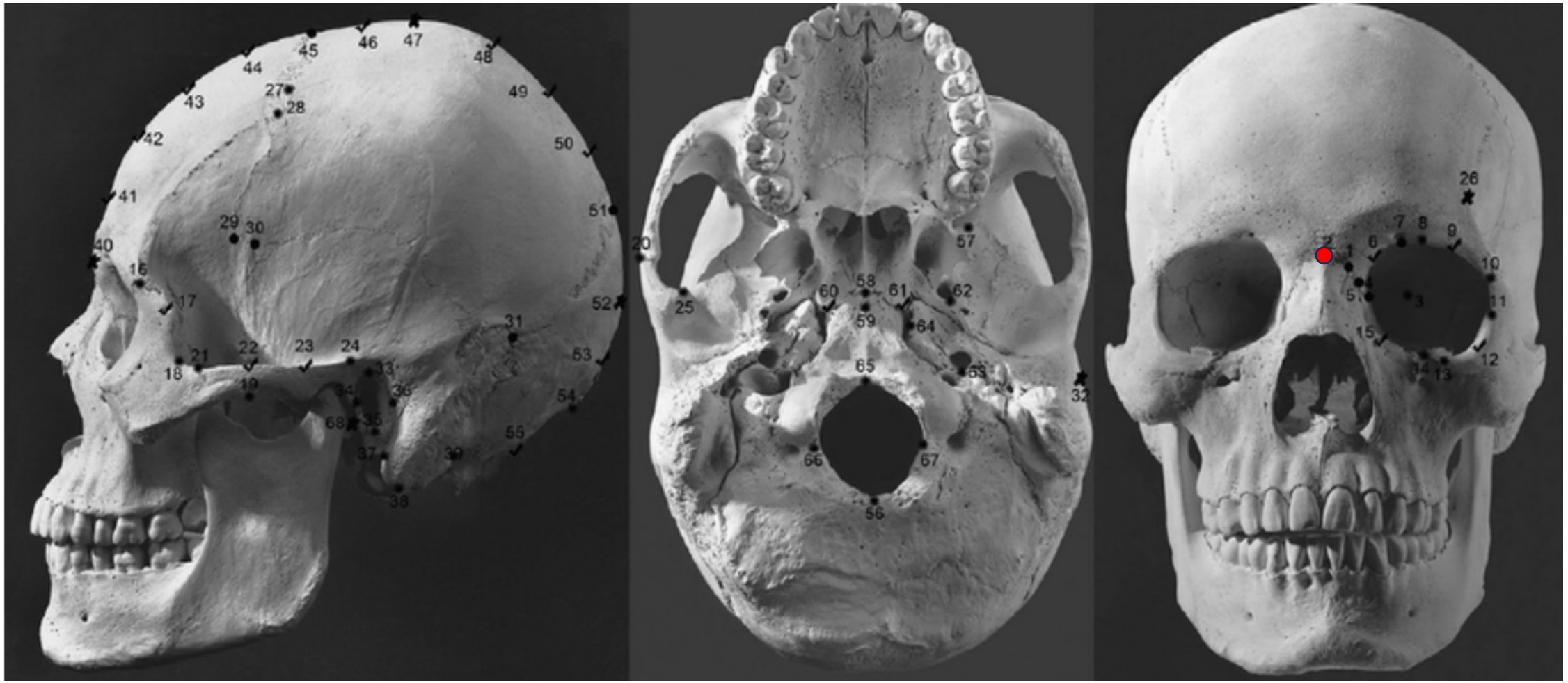
x

Landmark Types (Bookstein, 1991)

- **Type 1: Anatomical:** homology is provided by biologically unique patterns on the form (e.g. intersection of three bones)
- **Type 2: Processual:** homology is provided only by geometric, not biological or histological, criteria (*e.g.*, point of maximum curvature along a boundary)
- **Type 3 extremal:** “end-points of diameters, centroids, intersections of interlandmark segments, points farthest from such segments, constructions involving perpendiculars or evenly spaced radial intercepts, and the like”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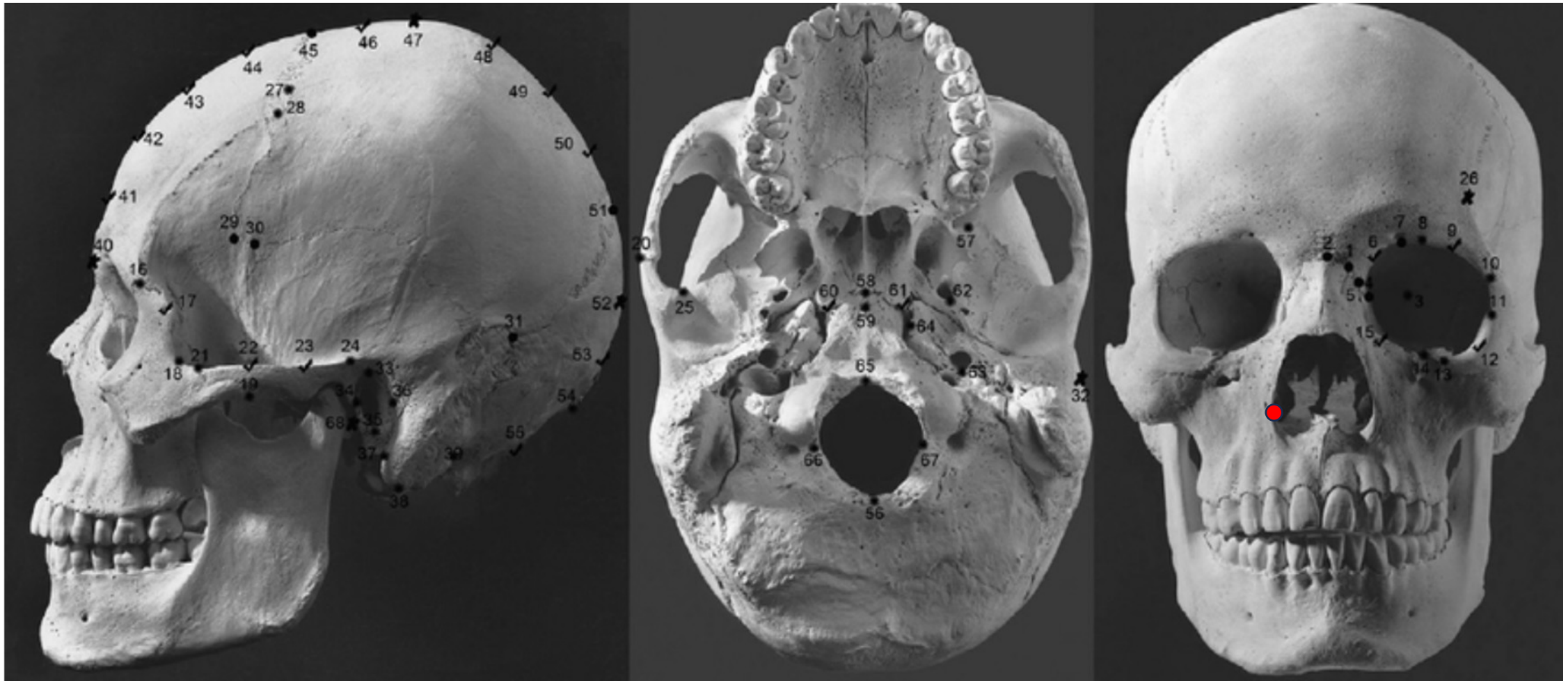
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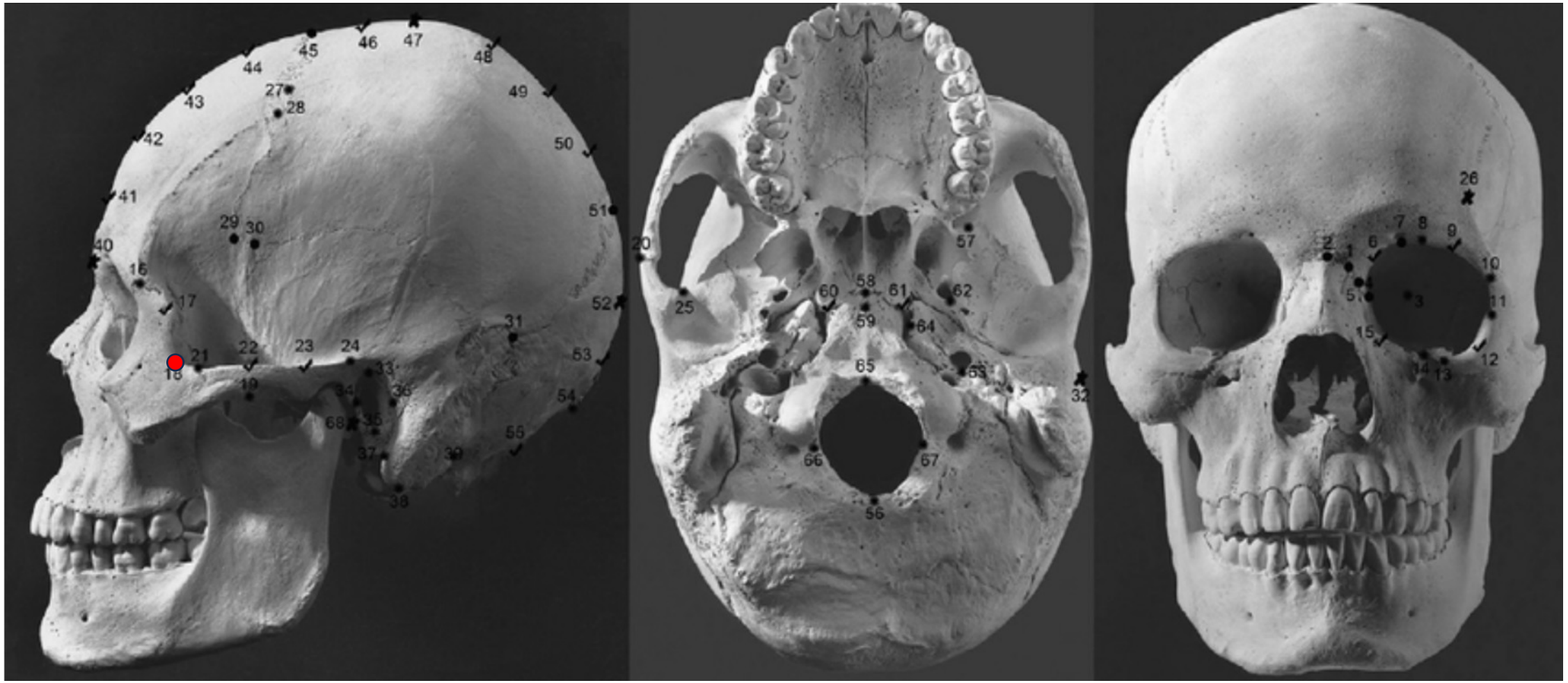
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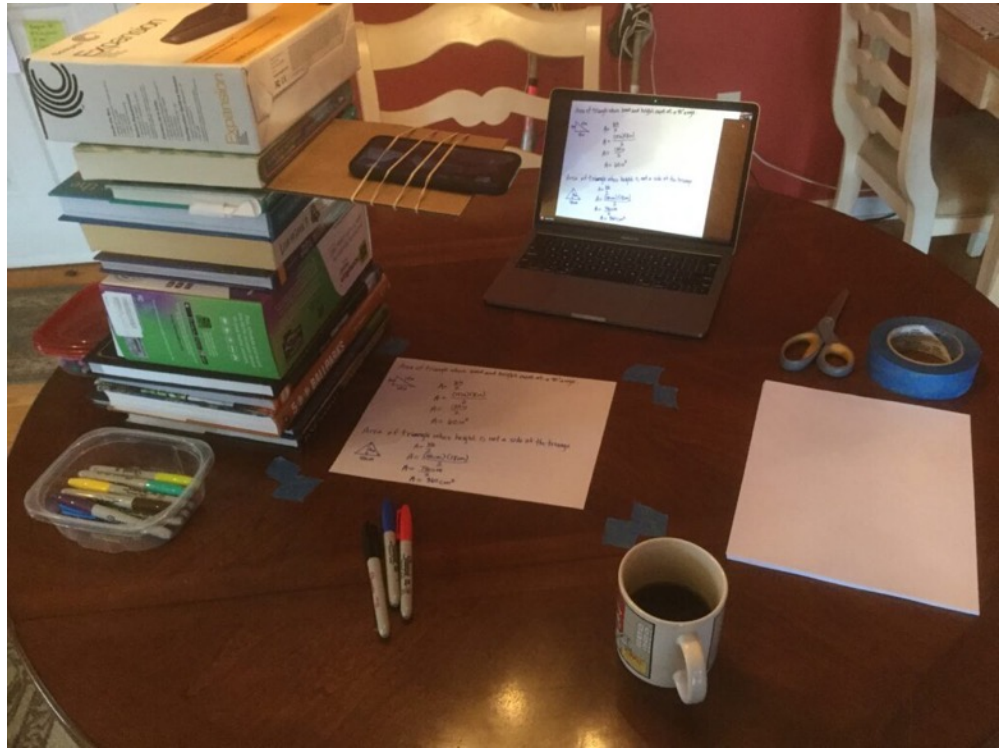
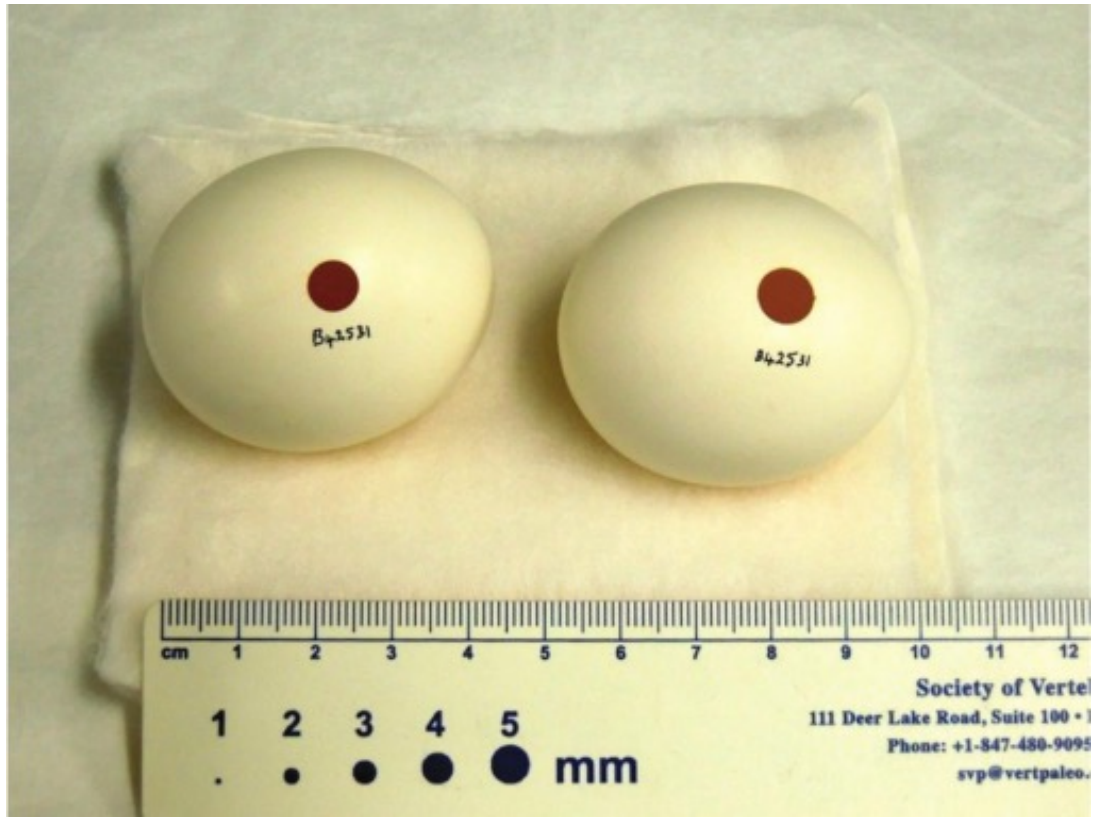
Collecting 2D Data

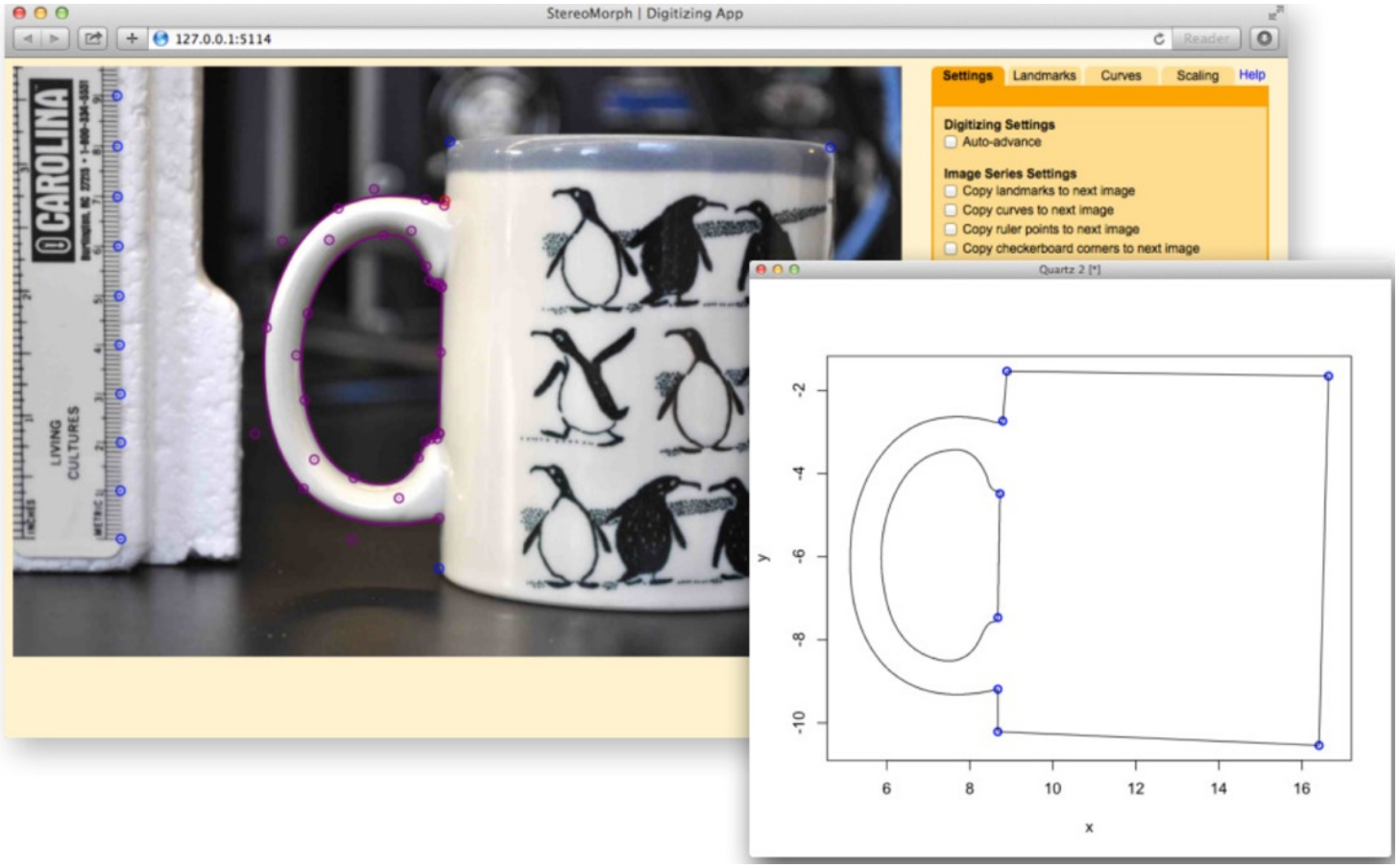
Collecting 2D Data



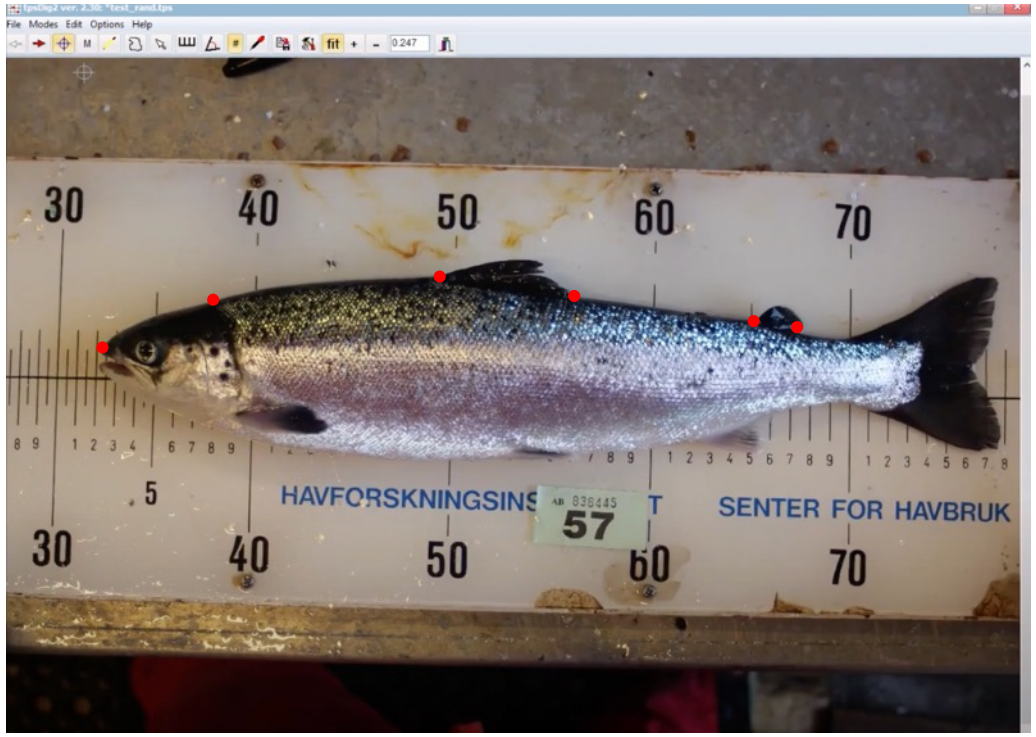
Collecting 2D Data



Collecting 2D Data



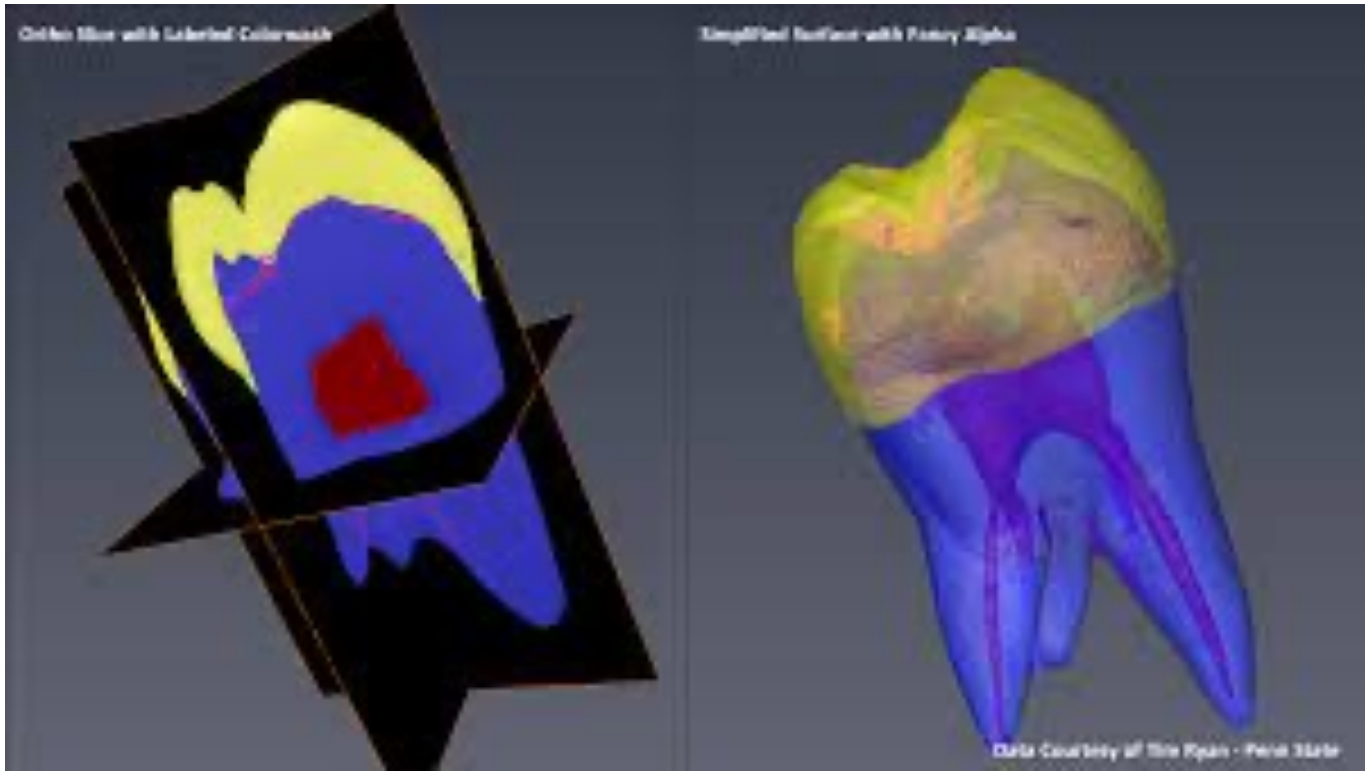
Stereomorph R package

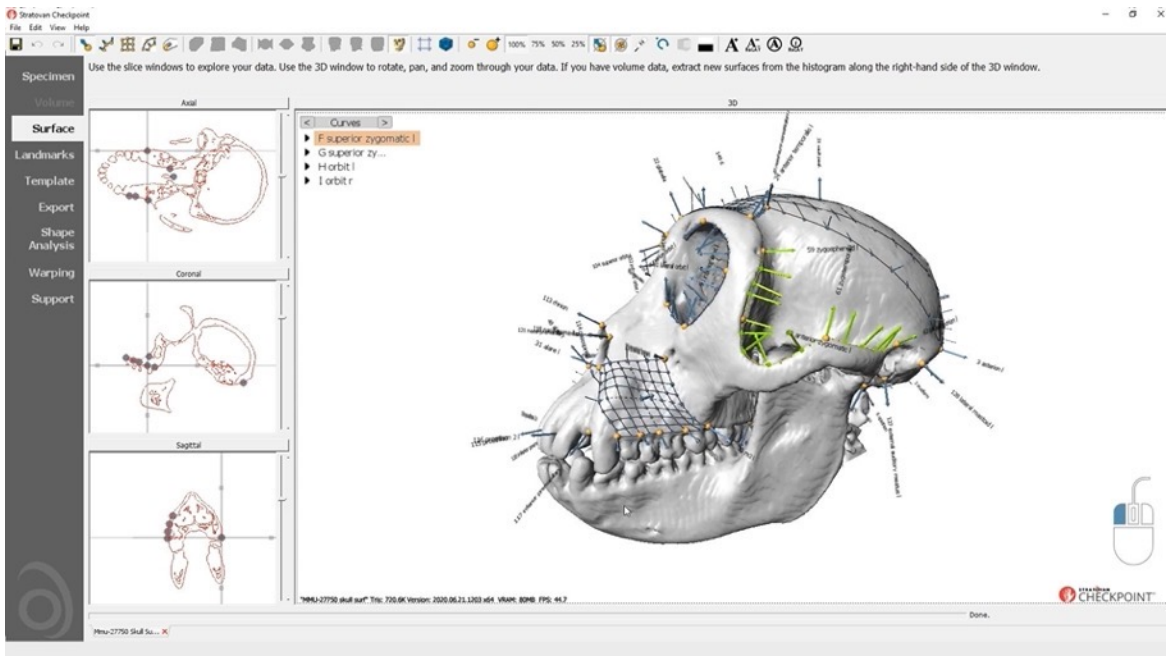


TPS-dig (Windows only)

Collecting 3D Data

Collecting 3D Data

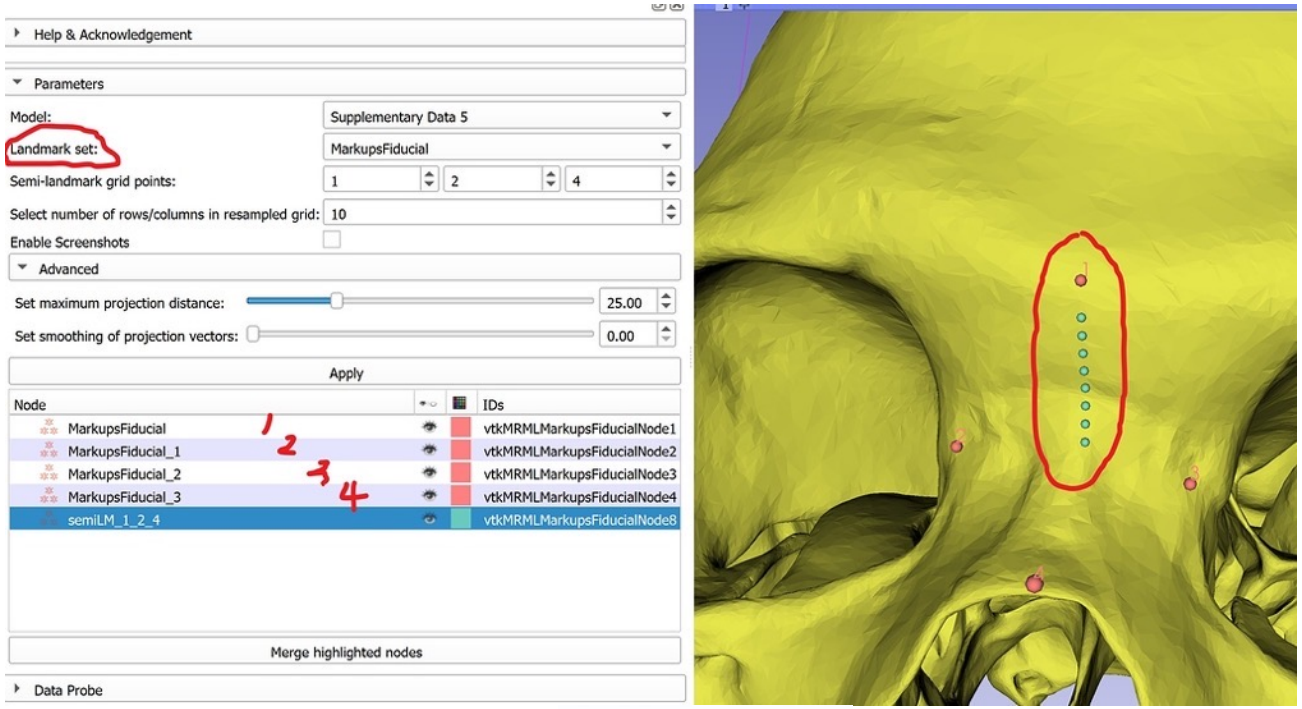


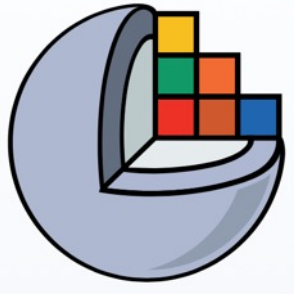


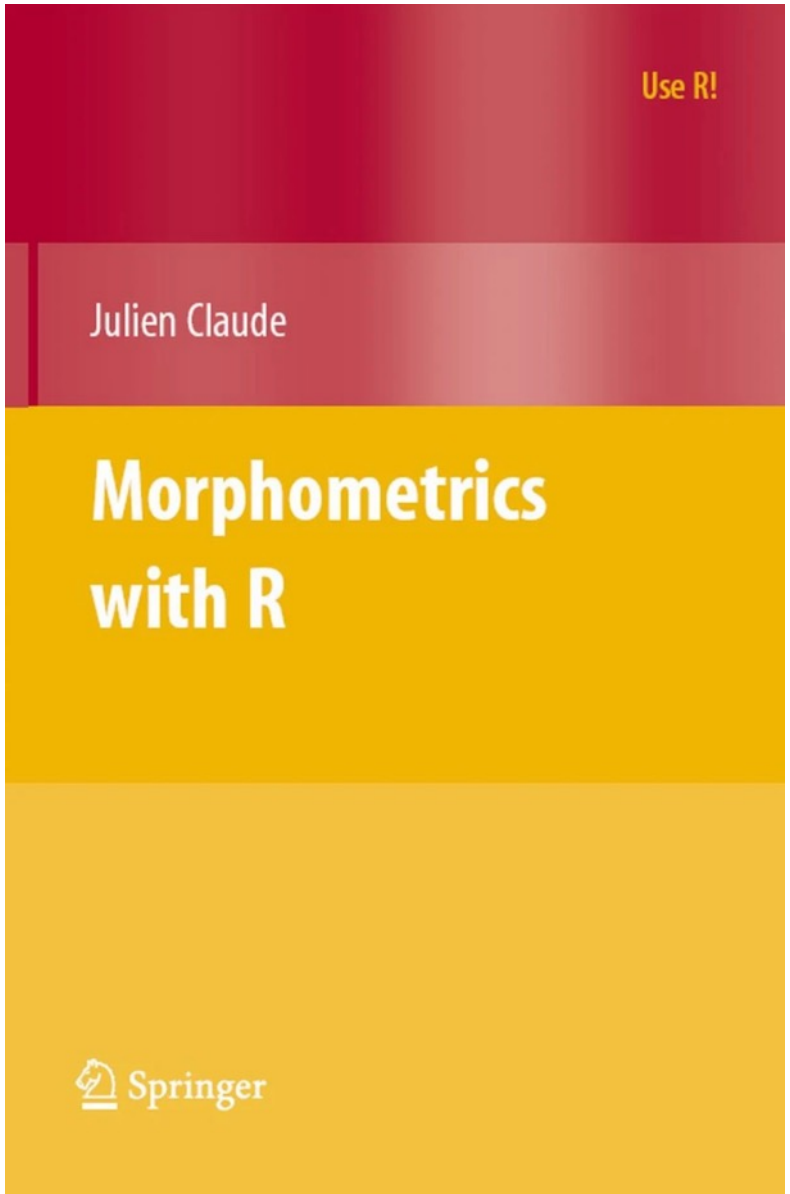
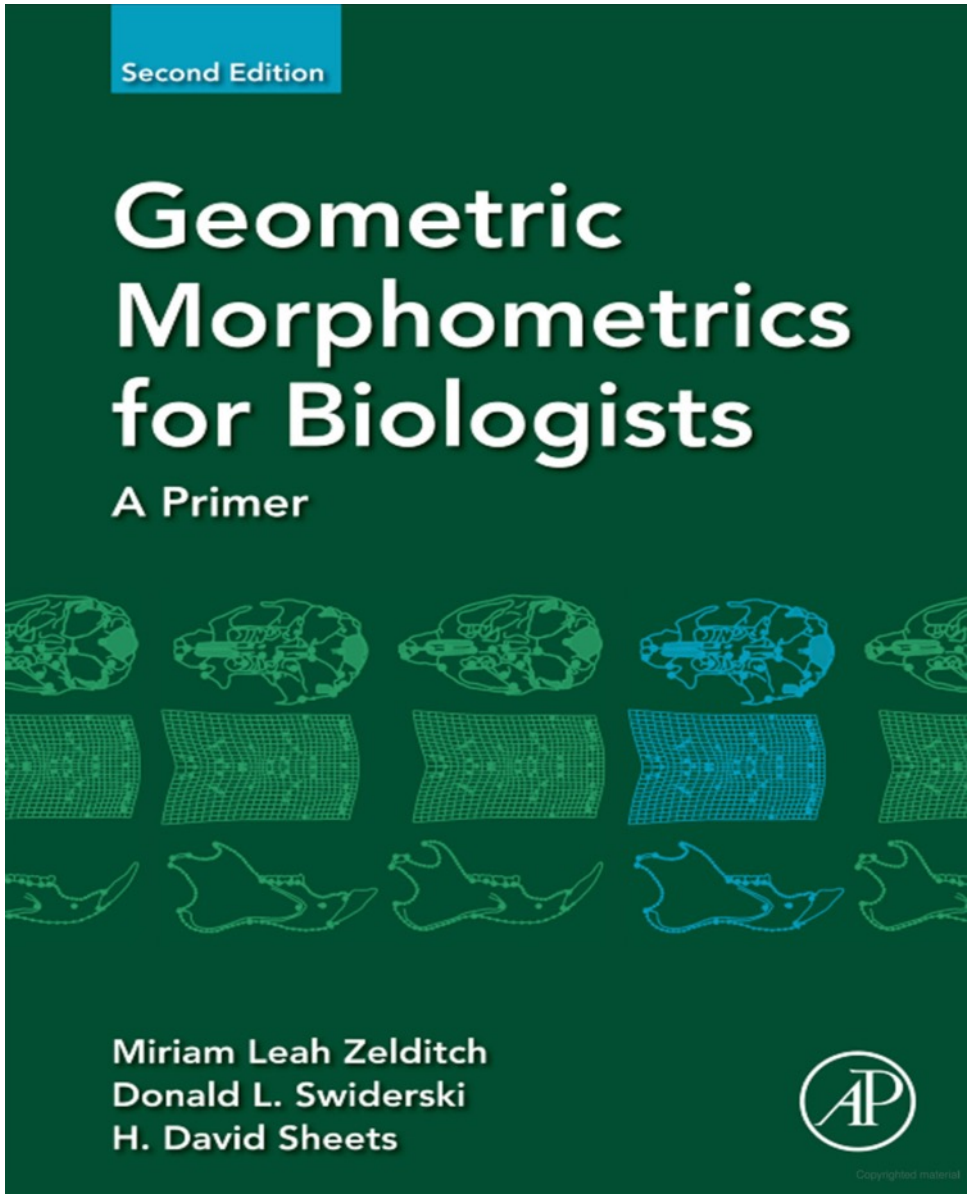
STRATOVAN™

CHECKPOINT™

Checkpoint (Windows Only, expensive)





SlicerMorph (multiplatform, free)

Recommended References