## Basic overview/features of CMR for paleobiology

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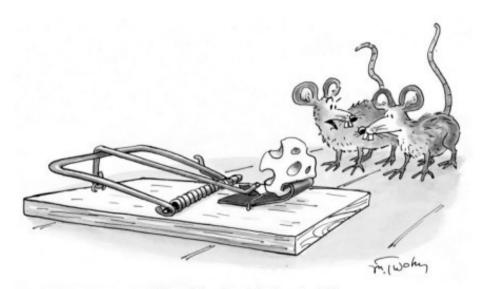
1.9.2025 Monday







#### THE ESSENCE OF CAPTURE RECAPTURE APPROACHES

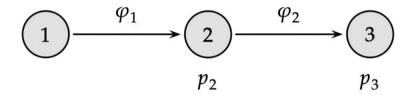


"Careful—it might be a trap!"

 $\frac{marked\ Day\ 2}{total\ for\ Day2} = \frac{marked\ Day1}{Estimated\ Total}$ 

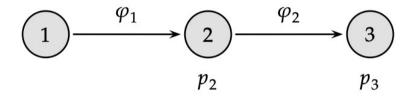
The Lincoln–Petersen method (Petersen–Lincoln index)



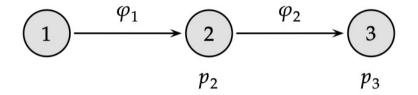


encounter history probability of encounter history

111  $\varphi_1 p_2 \varphi_2 p_3$ 



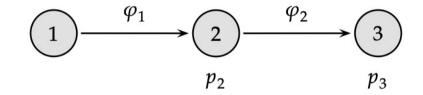
| encounter history | probability of encounter history                              |
|-------------------|---------------------------------------------------------------|
| 111               | $\varphi_1 p_2 \varphi_2 p_3$                                 |
| 110               | $\varphi_1 p_2 \big[ \varphi_2 (1-p_3) + (1-\varphi_2) \big]$ |
|                   | $= \varphi_1 p_2 (1 - \varphi_2 p_3)$                         |



| encounter history | probability of encounter history                                     |
|-------------------|----------------------------------------------------------------------|
| 111               | $\varphi_1p_2\varphi_2p_3$                                           |
| 110               | $\varphi_1 p_2 \left[ \varphi_2 (1 - p_3) + (1 - \varphi_2) \right]$ |
|                   | $=\varphi_1 p_2 (1 - \varphi_2 p_3)$                                 |
| 101               | $\varphi_1 \left( 1 - p_2 \right) \varphi_2 p_3$                     |

#### Trilobite taxa



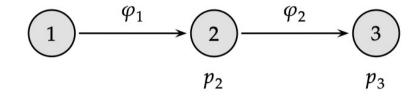


| encounter history | probability of encounter history                                                          |
|-------------------|-------------------------------------------------------------------------------------------|
| 111               | $\varphi_1 p_2 \varphi_2 p_3$                                                             |
| 110               | $\varphi_1 p_2 \left[ \varphi_2 (1 - p_3) + (1 - \varphi_2) \right]$                      |
|                   | $= \varphi_1 p_2 (1 - \varphi_2 p_3)$                                                     |
| 101               | $\varphi_1 \left(1 - p_2\right) \varphi_2 p_3$                                            |
| 100               | $\left(1-\varphi_1\right)+\varphi_1(1-p_2)(1-\varphi_2)+\varphi_1(1-p_2)\varphi_2(1-p_3)$ |
|                   | $= 1 - \varphi_1 p_2 - \varphi_1 (1 - p_2) \varphi_2 p_3$                                 |

#### Trilobite taxa



## **Encounter history**

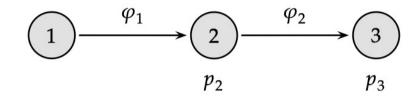


#### encounter history probability of encounter history

| Trilobite taxon A | 111 | $\varphi_1p_2\varphi_2p_3$                                                         |
|-------------------|-----|------------------------------------------------------------------------------------|
| Trilobite taxon B | 110 | $\varphi_1 p_2 [\varphi_2 (1 - p_3) + (1 - \varphi_2)]$                            |
|                   |     | $=\varphi_1 p_2 (1 - \varphi_2 p_3)$                                               |
| Trilobite taxon C | 101 | $\varphi_1 \left(1 - p_2\right) \varphi_2 p_3$                                     |
| Trilobite taxon D | 100 | $(1-\varphi_1) + \varphi_1(1-p_2)(1-\varphi_2) + \varphi_1(1-p_2)\varphi_2(1-p_3)$ |
|                   |     | $= 1 - \varphi_1 p_2 - \varphi_1 (1 - p_2) \varphi_2 p_3$                          |

#### Trilobite taxa





|              | encounter history | probability of encounter history                                                                 |
|--------------|-------------------|--------------------------------------------------------------------------------------------------|
| A, X, P      | 111               | $\varphi_1 p_2 \varphi_2 p_3$                                                                    |
| В, Е         | 110               | $\varphi_1 p_2 [\varphi_2 (1 - p_3) + (1 - \varphi_2)]$<br>= $\varphi_1 p_2 (1 - \varphi_2 p_3)$ |
| C, Q R, T, S | S, X, Z 101       | $\varphi_1 \left(1 - p_2\right) \varphi_2 p_3$                                                   |
| D, G H, J, I | K, Y 100          | $\left(1-\varphi_1\right)+\varphi_1(1-p_2)(1-\varphi_2)+\varphi_1(1-p_2)\varphi_2(1-p_3)$        |
|              |                   | $= 1 - \varphi_1 p_2 - \varphi_1 (1 - p_2) \varphi_2 p_3$                                        |

#### THE DATA

| Taxon | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| Α     | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| В     | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| С     | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| D     | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| Е     | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| F     | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| G     | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| Н     | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| I     | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| J     | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| K     | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| L     | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| M     | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| N     | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0     | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| Р     | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Q     | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| R     | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| S     | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| Т     | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

**Liow L.H.** & Nichols, J.D. **2010** Estimating rates and probabilities of origination and extinction using taxonomic occurrence data: Capture-recapture approaches. In Quantitative Paleobiology Short Course. Eds. Alroy J. & Hunt G. Paleontological Society pp. 81-94

#### **DECTECTION HISTORIES**

| Time interval | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------|---|---|---|---|---|---|---|---|
| L             | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| M             | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |

#### **Encounter/detection histories**

- Series of ones and zeros
- Ones are taken as true presences
- Two types of zeros
  - Not sampled
  - Not sampled or truly absent

|                | Taxon            | 1                          | 2                          | 3                     | 4                               | 5                     | 6                               | 7                               | 8                               |
|----------------|------------------|----------------------------|----------------------------|-----------------------|---------------------------------|-----------------------|---------------------------------|---------------------------------|---------------------------------|
|                | A                | 0                          | 0                          | 1                     | 1                               | 0                     | 0                               | 1                               | 0                               |
|                | В                | 0                          | 1                          | 0                     | 0                               | 0                     | 0                               | 0                               | 0                               |
|                | С                | 0                          | 0                          | 0                     | 0                               | 0                     | 1                               | 1                               | 1                               |
|                | D                | 1                          | 0                          | 1                     | 1                               | 0                     | 1                               | 0                               | 0                               |
| Detection      | Е                | 0                          | 0                          | 0                     | 1                               | 0                     | 0                               | 1                               | 1                               |
| probability    | F                | 0                          | 1                          | 0                     | 0                               | 1                     | 1                               | 0                               | 1                               |
| ·              |                  |                            |                            |                       |                                 |                       | 1                               | 1                               | 0                               |
| ^              |                  | - /                        | 10                         |                       | $\mathbf{\Omega}$               | _                     | 1                               | 1                               | 0                               |
| $D_{\epsilon}$ |                  | <b>)</b> //                |                            |                       | U.                              |                       | 1                               | 0                               | 1                               |
| $P_6$          |                  | •                          |                            |                       |                                 |                       | 0                               | 0                               | 0                               |
| <b>1</b> 0     |                  |                            |                            |                       |                                 |                       | U                               | U                               |                                 |
| 1 0            | K                | U                          | U                          | U                     | U                               |                       | 0                               | 1                               | 1                               |
| 1 0            | K<br>L           | 0                          | 0                          | 1                     | 1                               | 0                     | _                               |                                 |                                 |
| 1 0            | L<br>M           |                            |                            |                       |                                 | 0 1                   | 0                               | 1                               | 1                               |
| 1 0            |                  | 0                          | 0                          | 1                     | 1                               |                       | 0                               | 0                               | 0                               |
| 1 0            | M                | 0                          | 0                          | 1                     | 1                               | 1                     | 0 0                             | 0 0                             | 0 0                             |
| 1 0            | M                | 0 0 1                      | 0 1 1                      | 1<br>0                | 1 1 0                           | 1                     | 0 0 0                           | 1<br>0<br>0                     | 1<br>0<br>0                     |
| 1 0            | M<br>N<br>O      | 0 0 1 0                    | 0<br>1<br>1<br>0           | 1 0 0 0               | 1<br>1<br>0                     | 1<br>1<br>0           | 0 0 0 0 1                       | 1<br>0<br>0<br>1                | 1<br>0<br>0<br>0                |
| 1 0            | M<br>N<br>O      | 0<br>0<br>1<br>0           | 0<br>1<br>1<br>0<br>0      | 1<br>0<br>0<br>0      | 1<br>1<br>0<br>1                | 1<br>1<br>0<br>0      | 0<br>0<br>0<br>0<br>1           | 1<br>0<br>0<br>1<br>1           | 1<br>0<br>0<br>0<br>1           |
| 1 0            | M<br>N<br>O<br>P | 0<br>0<br>1<br>0<br>0      | 0<br>1<br>1<br>0<br>0      | 1<br>0<br>0<br>0<br>1 | 1<br>1<br>0<br>1<br>0<br>0      | 1<br>1<br>0<br>0<br>0 | 0<br>0<br>0<br>0<br>1           | 1<br>0<br>0<br>1<br>1<br>0      | 1<br>0<br>0<br>0<br>1<br>0<br>0 |
|                | M N O P Q R      | 0<br>0<br>1<br>0<br>0<br>0 | 0<br>1<br>1<br>0<br>0<br>0 | 1<br>0<br>0<br>0<br>1 | 1<br>1<br>0<br>1<br>0<br>0<br>0 | 1<br>0<br>0<br>0<br>0 | 0<br>0<br>0<br>0<br>1<br>0<br>0 | 1<br>0<br>0<br>1<br>1<br>0<br>1 | 1<br>0<br>0<br>0<br>1<br>0<br>0 |

|           | Taxon     | 1 | 2          | 3             | 4       | 5          | 6 | 7 | 8 |
|-----------|-----------|---|------------|---------------|---------|------------|---|---|---|
|           | Α         | 0 | 0          | 1             | 1       | 0          | 0 | 1 | 0 |
|           | В         | 0 | 1          | 0             | 0       | 0          | 0 | 0 | 0 |
|           | С         | 0 | 0          | 0             | 0       | 0          | 1 | 1 | 1 |
| Estimated | D         | 1 | 0          | 1             | 1       | 0          | 1 | 0 | 0 |
| no. taxa  | Е         | 0 | 0          | 0             | 1       | 0          | 0 | 1 | 1 |
| ^         |           | _ |            |               |         |            | 1 | 0 | 1 |
| = 2.      | $S_6 / j$ | ĥ | = $(8)$    | / <b>()</b>   | 5 =     | 16 <b></b> | 1 | 1 | 0 |
| $D_6$     | $^{3}6'I$ | 6 | <b>— O</b> | / <b>U.</b> . | <i></i> |            | 1 | 1 | 0 |
|           |           | 0 | 0          | 1             | 0       | 1          | 1 | 0 | 1 |
|           | J         | 1 | 0          | 0             | 0       | 0          | 0 | 0 | 0 |
|           | K         | 0 | 0          | 0             | 0       | 1          | 0 | 1 | 1 |
|           | L         | 0 | 0          | 1             | 1       | 0          | 0 | 0 | 0 |
|           | M         | 0 | 1          | 0             | 1       | 1          | 0 | 0 | 0 |
|           | N         | 1 | 1          | 0             | 0       | 1          | 0 | 1 | 0 |
|           | 0         | 0 | 0          | 0             | 1       | 0          | 1 | 1 | 1 |
|           | Р         | 0 | 0          | 1             | 0       | 0          | 0 | 0 | 0 |
|           | Q         | 0 | 0          | 0             | 0       | 0          | 0 | 1 | 0 |
|           | R         | 0 | 0          | 1             | 0       | 1          | 1 | 0 | 1 |
|           | S         | 0 | 0          | 0             | 0       | 1          | 0 | 0 | 1 |
|           | Т         | 0 | 0          | 0             | 1       | 0          | 0 | 0 | 0 |
|           |           |   |            |               |         |            |   |   |   |

|        | • -        |   |
|--------|------------|---|
| Time   | interva    | C |
| 111116 | iiitei vai | 3 |

|                   | Taxon         | 1       | 2     | 3 | 4 | 5  | 6 | 7 | 8 |  |
|-------------------|---------------|---------|-------|---|---|----|---|---|---|--|
|                   | Α             | 0       | 0     | 1 | 1 | 0  | 0 | 1 | 0 |  |
|                   | В             | 0       | 1     | 0 | 0 | 0  | 0 | 0 | 0 |  |
| Extinction        | С             | 0       | 0     | 0 | 0 | 0  | 1 | 1 | 1 |  |
| probability       | D             | 1       | 0     | 1 | 1 | 0  | 1 | 0 | 0 |  |
| ^                 | 1 (1          | ^r'     |       | 0 | 1 | 0  | 0 | 1 | 1 |  |
| $\mathcal{E}_5$ = | $=1-(\Lambda$ | $M_6$ / | $S_5$ | 0 | 0 | 1  | 1 | 0 | 1 |  |
|                   |               |         |       | 0 | 1 | 0  | 1 | 1 | 0 |  |
|                   |               |         |       | 0 | 0 | 0  | 1 | 1 | 0 |  |
|                   |               |         |       | 1 | 0 | 1  | 1 | 0 | 1 |  |
|                   |               |         |       | 0 | 0 | 0  | 0 | 0 | 0 |  |
|                   |               |         |       | 0 | 0 | 1) | 0 | 1 | 1 |  |
|                   |               |         |       | 1 | 1 | 0  | 0 | 0 | 0 |  |
|                   |               |         |       | 0 | 1 | 1  | 0 | 0 | 0 |  |
|                   |               |         |       | 0 | 0 | 1  | 0 | 1 | 0 |  |
|                   | _             |         |       | 0 | 1 | 0  | 1 | 1 | 1 |  |
|                   | Р             | 0       | 0     | 1 | 0 | 0  | 0 | 0 | 0 |  |
|                   | Q             | 0       | 0     | 0 | 0 | 0  | 0 | 1 | 0 |  |
|                   | R             | 0       | 0     | 1 | 0 | 1  | 1 | 0 | 1 |  |
|                   | S             | 0       | 0     | 0 | 0 | 1  | 0 | 0 | 1 |  |
|                   | Т             | 0       | 0     | 0 | 1 | 0  | 0 | 0 | 0 |  |

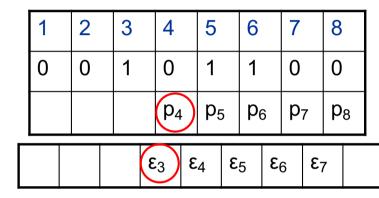
|                   | Taxon                       | 1                | 2     | 3 | 4 | 5 | 6 | 7 | 8 |  |
|-------------------|-----------------------------|------------------|-------|---|---|---|---|---|---|--|
|                   | Α                           | 0                | 0     | 1 | 1 | 0 | 0 | 1 | 0 |  |
|                   | В                           | 0                | 1     | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Extinction        | С                           | 0                | 0     | 0 | 0 | 0 | 1 | 1 | 1 |  |
| probability       | D                           | 1                | 0     | 1 | 1 | 0 | 1 | 0 | 0 |  |
| ^                 | 1 (1                        | ^ <u>-</u> ' /   |       | 0 | 1 | 0 | 0 | 1 | 1 |  |
| $\mathcal{E}_5$ = | $=1-(\Lambda$               | $I_6$ /          | $S_5$ | 0 | 0 | 1 | 1 | 0 | 1 |  |
|                   |                             |                  |       | 0 | 1 | 0 | 1 | 1 | 0 |  |
|                   | 100                         | / <b>n</b>       |       | 0 | 0 | 0 | 1 | 1 | 0 |  |
| =1-               | $-(\frac{m_6}{2})^{\prime}$ | $p_6$            | )     | 1 | 0 | 1 | 1 | 0 | 1 |  |
| 1                 | S                           | _                | ,     | 0 | 0 | 0 | 0 | 0 | 0 |  |
|                   | ۵                           | 5                |       | 0 | 0 | 1 | 0 | 1 | 1 |  |
|                   | 2/6                         | \ _              |       | 1 | 1 | 0 | 0 | 0 | 0 |  |
| _ 1               | 3/(                         | ).5 <sub>\</sub> |       | 0 | 1 | 1 | 0 | 0 | 0 |  |
| <u> </u>          | 16                          | <b>-</b>         |       | 0 | 0 | 1 | 0 | 1 | 0 |  |
|                   |                             |                  |       | 0 | 1 | 0 | 1 | 1 | 1 |  |
|                   | Р                           | 0                | 0     | 1 | 0 | 0 | 0 | 0 | 0 |  |
|                   | Q                           | 0                | 0     | 0 | 0 | 0 | 0 | 1 | 0 |  |
|                   | R                           | 0                | 0     | 1 | 0 | 1 | 1 | 0 | 1 |  |
|                   | S                           | 0                | 0     | 0 | 0 | 1 | 0 | 0 | 1 |  |
|                   | T                           | 0                | 0     | 0 | 1 | 0 | 0 | 0 | 0 |  |

|                    | Taxon                              | 1                 | 2               | 3 | 4 | 5 | 6 | 7 | 8 |  |
|--------------------|------------------------------------|-------------------|-----------------|---|---|---|---|---|---|--|
|                    | Α                                  | 0                 | 0               | 1 | 1 | 0 | 0 | 1 | 0 |  |
|                    | В                                  | 0                 | 1               | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Extinction         | C                                  | 0                 | 0               | 0 | 0 | 0 | 1 | 1 | 1 |  |
| probabi            | lity                               | 1                 | 0               | 1 | 1 | 0 | 1 | 0 | 0 |  |
|                    | ^ 1 7                              | <u>``</u>         | / \             | 0 | 1 | 0 | 0 | 1 | 1 |  |
|                    | $\hat{\varepsilon}_5 = 1 - (N_5)$  | $\hat{M}_{6}^{'}$ | $(S_5)$         | 0 | 0 | 1 | 1 | 0 | 1 |  |
|                    |                                    |                   |                 | 0 | 1 | 0 | 1 | 1 | 0 |  |
| Alive in T 5 and 6 | m                                  | / ĥ               |                 | 0 | 0 | 0 | 1 | 1 | 0 |  |
| A11 1 75           | $=1-(\frac{m_6}{2})^{\frac{1}{2}}$ | $/\hat{p}_{6}$    | <del>-}</del> - | 1 | 0 | 1 | 1 | 0 | 1 |  |
| Alive in T5        | S                                  | $S_5$             |                 |   | 0 | 0 | 0 | 0 | 0 |  |
|                    | ~                                  | 5                 |                 | 0 | 0 | 1 | 0 | 1 | 1 |  |
| Survival           | 2/6                                | ) <i>[</i>        |                 | 1 | 1 | 0 | 0 | 0 | 0 |  |
| Probability        | $-1$ $(\frac{3}{3})$               | 0.5               |                 | 0 | 1 | 1 | 0 | 0 | 0 |  |
| From T5 to T6      | - I <del> </del> 16                | <del></del>       |                 | 0 | 0 | 1 | 0 | 1 | 0 |  |
| 11011113 to 10     | 10                                 |                   |                 | 0 | 1 | 0 | 1 | 1 | 1 |  |
|                    | Р                                  | 0                 | 0               | 1 | 0 | 0 | 0 | 0 | 0 |  |
|                    | Q                                  | 0                 | 0               | 0 | 0 | 0 | 0 | 1 | 0 |  |
|                    | R                                  | 0                 | 0               | 1 | 0 | 1 | 1 | 0 | 1 |  |
|                    | S                                  | 0                 | 0               | 0 | 0 | 1 | 0 | 0 | 1 |  |
|                    | Т                                  | 0                 | 0               | 0 | 1 | 0 | 0 | 0 | 0 |  |
|                    |                                    |                   |                 |   |   |   |   |   |   |  |

| ſ | 1 | 2 | 3 |   | 4              |   | 5              |   | 6 |    | 7 |    | 8     |   |
|---|---|---|---|---|----------------|---|----------------|---|---|----|---|----|-------|---|
|   | 0 | 0 | 1 |   | 0              |   | 1              |   | 1 |    | 0 |    | 0     |   |
|   |   |   |   |   | p <sub>4</sub> | - | p <sub>5</sub> | 5 | p | 3  | p | 7  | р     | 8 |
|   |   |   |   | 3 | 3              | 3 | 4              | 3 | 5 | ε, |   | ε- | <br>7 |   |

probabilities
Extinction
probabilities

**Detection** 

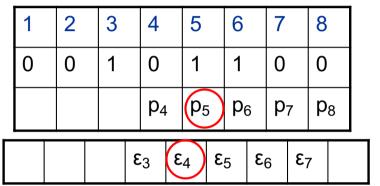


Detection probabilities Extinction probabilities

eh = 0 0 1 0 1 1 0 0

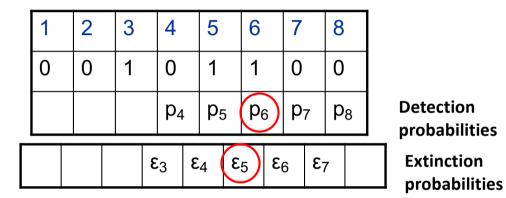
 $Pr(eh = 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ | initial encounter in interval 3) =$ 

$$(1-\epsilon_3)(1-p_4)(1-\epsilon_4)p_5(1-\epsilon_5)p_6[\epsilon_6+(1-\epsilon_6)(1-p_7)\{\epsilon_7+(1-\epsilon_7)(1-p_8)\}]$$

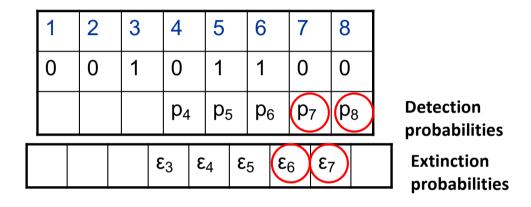


Detection probabilities Extinction probabilities

eh = 0 0 1 0 1 1 0 0 Pr(eh = 0 0 1 0 1 1 0 0 | initial encounter in interval 3) =  $(1-\epsilon_3)(1-p_4) \underbrace{ (1-\epsilon_4)p_5 (1-\epsilon_5)p_6 \left[\epsilon_6 + (1-\epsilon_6)(1-p_7)\{\epsilon_7 + (1-\epsilon_7)(1-p_8)\}\right] }$ 



eh = 0 0 1 0 1 1 0 0  $\text{Pr}(\text{eh} = 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ | \ \text{initial encounter in interval 3}) = \\ (1-\epsilon_3)(1-p_4) \ (1-\epsilon_4)p_5 \ (1-\epsilon_5)p_6 \ [\epsilon_5] + (1-\epsilon_6)(1-p_7)\{\epsilon_7+(1-\epsilon_7)(1-p_8)\}]$ 



eh = 0 0 1 0 1 1 0 0

Pr(eh = 0 0 1 0 1 1 0 0 | initial encounter in interval 3) =

+ 
$$(1-\epsilon_6)(1-p_7)\{\epsilon_7+(1-\epsilon_7)(1-p_8)\}$$

| 1 | 2 | 3 |   | 4  |   | 5              |    | 6 |                | 7 |                | 8 |   |                          |
|---|---|---|---|----|---|----------------|----|---|----------------|---|----------------|---|---|--------------------------|
| 0 | 0 | 1 |   | 0  |   | 1              |    | 1 |                | 0 |                | 0 |   |                          |
|   |   |   |   | p₄ |   | p <sub>5</sub> | 5  | p | 6              | p | 7              | p | 3 | Detection probabilities  |
|   |   |   | 8 | 3  | ٤ | 4              | ,3 | 5 | ε <sub>θ</sub> | 6 | ε <sub>7</sub> | 7 |   | Extinction probabilities |

eh = 0 0 1 0 1 1 0 0 Pr(eh = 0 0 1 0 1 1 0 0 | initial encounter in interval 3) =  $(1-\epsilon_3)(1-p_4) \; (1-\epsilon_4)p_5 \; (1-\epsilon_5)p_6 \; [\epsilon_6 + (1-\epsilon_6)(1-p_7)\{\epsilon_7 + (1-\epsilon_7)(1-p_8)\}]$ 

"sampling" and vital parameters are both explicit! (closer to generating process of the data)

#### We have detection histories – now what?

```
 Pr(eh = 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0) = \\  (1-\epsilon 3)(1-p4)\ (1-\epsilon 4)p5\ (1-\epsilon 5)p6\ [\epsilon 6 + (1-\epsilon 6)(1-p7)\{\epsilon 7 + (1-\epsilon 7)(1-p8)\}] = H1
```

Likelihood (parameters | data) =  $H_1^{No. cases} H_2^{No. cases} H_x^{No. cases}$ 

| 0001010010 | 10        |
|------------|-----------|
| 0010001101 | 3         |
| 0101010100 | 4         |
|            | 18        |
| 1001000000 | 2<br>1    |
| 0001010111 | 3         |
| 0011101000 | No. cases |

#### Maximum likelihood estimation

- statistical approach for estimating the parameters of a model
- parameters are chosen to maximize the likelihood for the assumed model given the data

#### So do to MLE, we have to

- assume a model (e.g. data generating process) for the data in questions
- derive the likelihood function for our data, given our assumed model

- •The event of falling right
- •The event of falling left
- •The falling : call it y
- •There are only two discrete values for y, 1 or 0 (right and left)
- •What's the probability of fall left? Or right?



## **The Data**

| T T | у | what |
|-----|---|------|
| 1   | 0 | L    |
| 2   | 1 | R    |
| 3   | 1 | R    |
| 4   | 0 | L    |
| 5   | 0 | L    |
| 6   | 1 | R    |
| 7   | 1 | R    |
| 8   | 1 | R    |
| 9   | 0 | L    |
| 10  | 0 | L    |



Probability mass function (PMF) for Bernoulli:  $p^{y_i}(1-p)^{1-y_i}$ 

$$y_i = 0$$

Negative outcome - falling left

$$y_i = 1$$

Positive outcome - falling right

PMF Example

if 
$$y_i = 1$$
, and  $p = 0.5$ 

$$p^{y_i}(1-p)^{1-y_i}$$

$$0.5^{1}(1 - 0.5)^{1-1} = 0.5$$

The likelihood function is the joint probability distribution (multiplication) of the PMFs for each observation. We are simply multiplying all the PMFs together

$$p^{y_1}(1-p)^{1-y_1} \cdot p^{y_2}(1-p)^{1-y_2} \dots \cdot p^{y_n}(1-p)^{1-y_n}$$

joint probability distribution



$$L(\theta) = \prod_{i=1}^{n} p^{y_i} (1-p)^{1-y_i}$$

likelihood function

We want to maximize this function. Rather than doing calculus to figure this out, let's play around with different values of p to see how it affects the likelihood. You can try it out using this workbook (tab: fair\_coin).

| p=0.3     |   |      |        |  |  |  |  |  |  |  |
|-----------|---|------|--------|--|--|--|--|--|--|--|
| i         | y | p(y) | PMF(y) |  |  |  |  |  |  |  |
| 1         | 0 | 0.3  | 0.7    |  |  |  |  |  |  |  |
| 2         | 0 | 0.3  | 0.7    |  |  |  |  |  |  |  |
| 3         | 0 | 0.3  | 0.7    |  |  |  |  |  |  |  |
| 4         | 1 | 0.3  | 0.3    |  |  |  |  |  |  |  |
| 5         | 1 | 0.3  | 0.3    |  |  |  |  |  |  |  |
| 6         | 0 | 0.3  | 0.7    |  |  |  |  |  |  |  |
| 7         | 1 | 0.3  | 0.3    |  |  |  |  |  |  |  |
| 8         | 1 | 0.3  | 0.3    |  |  |  |  |  |  |  |
| 9         | 1 | 0.3  | 0.3    |  |  |  |  |  |  |  |
| 10        | 0 | 0.3  | 0.7    |  |  |  |  |  |  |  |
| L= 0.0004 |   |      |        |  |  |  |  |  |  |  |

|    |   | p=0.5 |        |
|----|---|-------|--------|
| i  | y | p(y)  | PMF(y) |
| 1  | 0 | 0.5   | 0.5    |
| 2  | 0 | 0.5   | 0.5    |
| 3  | 0 | 0.5   | 0.5    |
| 4  | 1 | 0.5   | 0.5    |
| 5  | 1 | 0.5   | 0.5    |
| 6  | 0 | 0.5   | 0.5    |
| 7  | 1 | 0.5   | 0.5    |
| 8  | 1 | 0.5   | 0.5    |
| 9  | 1 | 0.5   | 0.5    |
| 10 | 0 | 0.5   | 0.5    |
|    |   | L=    | 0.0010 |
|    |   |       |        |

|    |   | p=0.7 |        |
|----|---|-------|--------|
| i  | y | p(y)  | PMF(y) |
| 1  | 0 | 0.7   | 0.3    |
| 2  | 0 | 0.7   | 0.3    |
| 3  | 0 | 0.7   | 0.3    |
| 4  | 1 | 0.7   | 0.7    |
| 5  | 1 | 0.7   | 0.7    |
| 6  | 0 | 0.7   | 0.3    |
| 7  | 1 | 0.7   | 0.7    |
| 8  | 1 | 0.7   | 0.7    |
| 9  | 1 | 0.7   | 0.7    |
| 10 | 0 | 0.7   | 0.3    |
|    |   | L=    | 0.0004 |

#### Likelihood of Detection histories

- 1. Estimate parameters (by maximizing the likelihood)
- 2. Estimate uncertainty in parameters
- 3. Compare models
  - e.g. same or different p's or ε's
  - e.g. with or without covariates (important factors that you think might influence p and  $\epsilon$ )
    - i. Akaike Information Criteria, AIC
    - ii. classical hypothesis testing
    - iii. extendable to Bayesian approaches
- 4. Good statistical properties

- 1. After initial encounters, detection/encounter probabilities are equal for all taxa in the data/group of interest
- 2. After initial encounters, extinction probabilities for all taxa are equal
- 3. Sampling intervals are short relative to the time over which extinction is to be estimated
- 4. The fate of each taxon (with respect to extinction and encounter) is independent of the fate of every other taxon

- 1. After initial encounters, detection/encounter probabilities are equal for all taxa in the data/group of interest
  - Taxon specific covariates
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- 2. After initial encounters, extinction probabilities for all taxa are equal
- 3. Sampling intervals are short relative to the time over which extinction is to be estimated
  - Simulations show that this is not a big problem; other models (e.g. robust design models, never applied in paleo) tackle this head-on
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- 1. After initial encounters, detection/encounter probabilities are equal for all taxa in the data/group of interest
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- 3. Sampling intervals are short relative to the time over which extinction is to be estimated
- 4. The fate of each taxon (with respect to extinction and encounter) is independent of the fate of every other taxon
  - Corrections for over-dispersion
  - Co-occurrence analyses

#### Covariate modeling

- A way to include factors or variables that may be important in explaining variation in the parameters (e.g. extinction, sampling) you are interested in
- Allows us to compare models with different [or no] covariates (Model Comparison and Selection)
- models to compare
  - ε(constant)p(time-varying)
  - ε(time-varying)p(sea-levels)

## Covariate modeling via link functions

$$\operatorname{logit}(\mathcal{E}_{i,t}) = \operatorname{log}(\frac{\mathcal{E}_{i,t}}{1 - \mathcal{E}_{i,t}}) = \beta_0 + \beta_1 x_i + \beta_2 y_t$$
Taxon specific covariates

- size
- minerology
- taxonomic group

#### Covariate modeling via link functions

$$\operatorname{logit}(\varepsilon_{i,t}) = \operatorname{log}(\frac{\varepsilon_{i,t}}{1 - \varepsilon_{i,t}}) = \beta_0 + \beta_1 x_i + \beta_2 y_t$$
Time specific covariates

Time specific covariates

- Duration of bin
- •Sea-level
- Temperature

## Covariate modeling via link functions

$$\operatorname{logit}(\varepsilon_{i,t}) = \operatorname{log}(\frac{\varepsilon_{i,t}}{1 - \varepsilon_{i,t}}) = \beta_0 + \beta_1 x_i + \beta_2 y_t$$

$$\varepsilon_{i,t} = \frac{e^{\beta_0 + \beta_1 x_i + \beta_2 y_t}}{1 + e^{\beta_0 + \beta_1 x_i + \beta_2 y_t}}$$

"Some simple rules of thumb are often useful in assessing the relative merits of models in the set: Models having  $\Delta i \leq 2$  have substantial support (evidence), those in which  $4 \leq \Delta i \leq 7$  have considerably less support, and models having  $\Delta i > 10$  have essentially no support." Burnham & Anderson 2004 – see dAIC

$$\Delta_i(AIC) = AIC_i - min(AIC)$$

$$L \propto exp\left(-rac{1}{2}\Delta_i(AIC)
ight)$$

$$AIC = -2\ln(L) + 2k$$

$$w_i(AIC) = rac{exp\left(-rac{1}{2}\Delta_i(AIC)
ight)}{\sum_{k=1}^{K}exp\left(-rac{1}{2}\Delta_k(AIC)
ight)}$$

|         | Model        | Npar (k) | Rank | logLik (L) | AIC | AICc | dAIC | AICwt |
|---------|--------------|----------|------|------------|-----|------|------|-------|
| Model A | p~1<br>phi~1 | 2        | 2    | -333       | 670 | 671  | 0    | 0.79  |
| Model B | p~1<br>phi~t | 7        | 7    | -330       | 673 | 676  | 2.9  | 0.19  |
| Model C | p~t<br>phi~1 | 7        | 7    | -332       | 678 | 678  | 7.6  | 0.02  |
| Model D | p~t phi~t    | 12       | 12   | -328       | 681 | 688  | 10.1 | 0.00  |

Model selection is cool, but don't stop there

Look at your parameter estimates and check if they make sense!

#### Why Capture-Mark-Recapture (CMR)?

- Detection probability
- Separating between
  - probability of detection (given presence)
  - probability of the parameters in question

(e.g. survivorship, origination, occupancy, immigration) and derived parameters such as species richness/diversity

The probability of detection or sampling is sometimes only a nuisance but sometimes interesting in itself.

- Covariates can be EASILY included in models for both vital parameters and sampling/detection estimates.
- Covariates can be modeled at a variety of levels (e.g. group factors, individual traits, temporal characteristics)

# Chapters in MARK book most relevant (if not using MARK)

- Chapter 1 (introduction)
- Chapter 4 (dipper example, but skip the MARK specific bits)
- Chapter 5 (goodness of fit –not covered in lectures but important)
- Chapter 6 (more on covariates and link functions)
- Chapter 11 (individual covariates)
- Chapter 12 (Pradel) and 13 (JS models in general)

http://www.phidot.org/software/mark/docs/book/

## Some newer fossil papers using CMR (but oldies are goodies too)

- Schachat et al. **2021.** A Cretaceous peak in family-level insect diversity estimated with mark-recapture methodology. Proceedings of the Royal Society, B 286:20192054.
- Monarrez et al. **2021** Mass extinctions alter extinction and origination dynamics with respect to body size Proceedings of the Royal Society, B <a href="https://doi.org/10.1098/rspb.2021.1681">https://doi.org/10.1098/rspb.2021.1681</a>
- Cohen et al. **2022** Plio-Pleistocene environmental variability in Africa and its implications for mammalian evolution, PNAS 119 (16) e2107393119
- Wilson et al. **2024**. Unveiling the underlying drivers of Phanerozoic marine diversification. Proceedings of the Royal Society, B 291: 20240165
- Leventhal S, Samuels-Fair M. **2025** Larval brooding correlated with high early origination rates in cheilostome Bryozoa. *Paleobiology*. 51(2):261-267