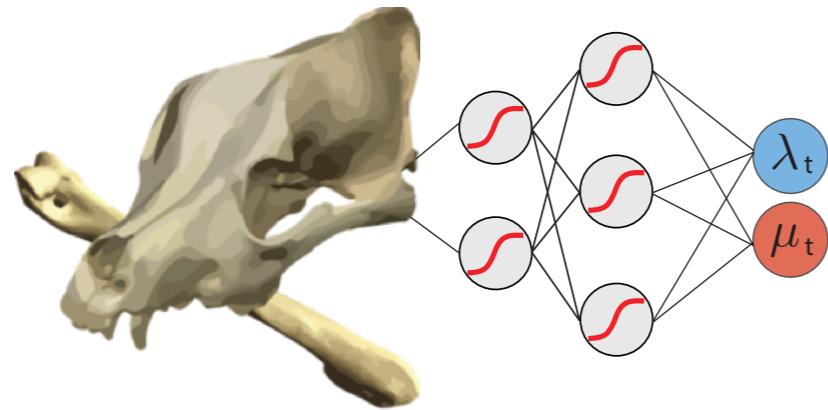


# Bayesian and deep learning for macroevolutionary analyses



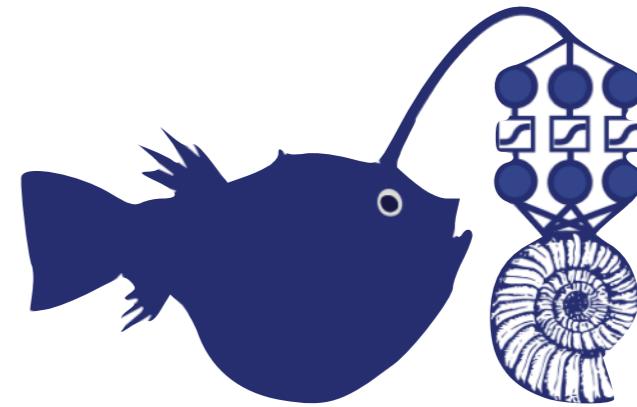
Gemini's take on our workshop

# Learning objectives



## PyRate and the BDNN model

Estimating trait and time dependent speciation and extinction rates form fossil occurrence data



## DeepDive

Estimating diversity trajectories through time using deep learning and mechanistic simulations

(And more!)



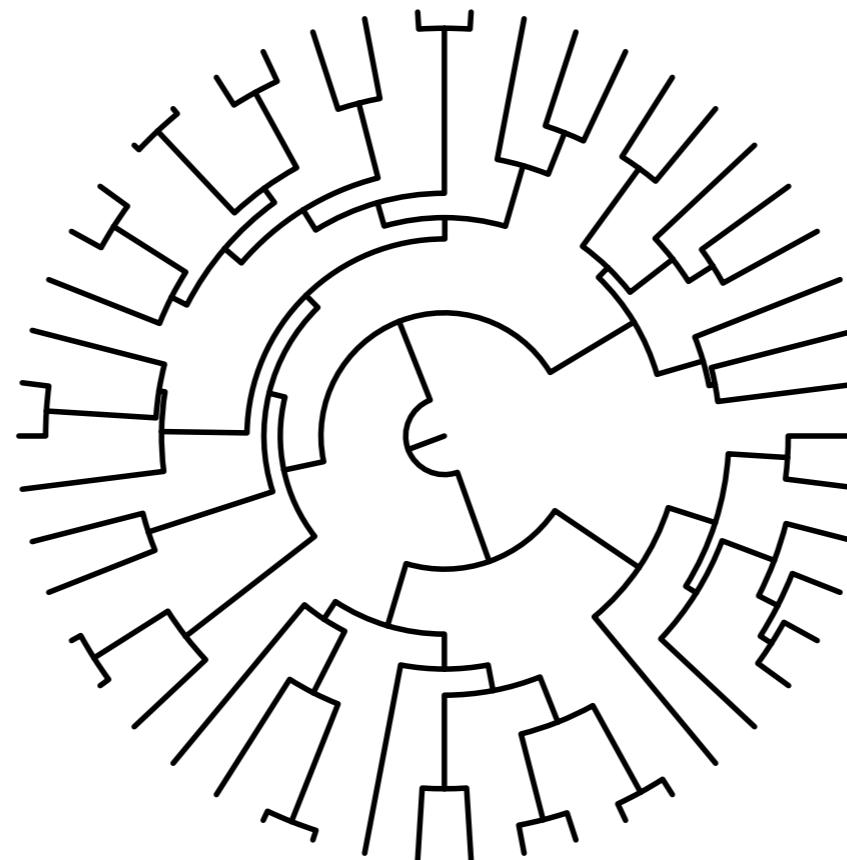
**PyRate**  
Bayesian estimation of  
macroevolutionary rates from  
fossil occurrence data

# Estimation of macroevolutionary rates from the fossil record





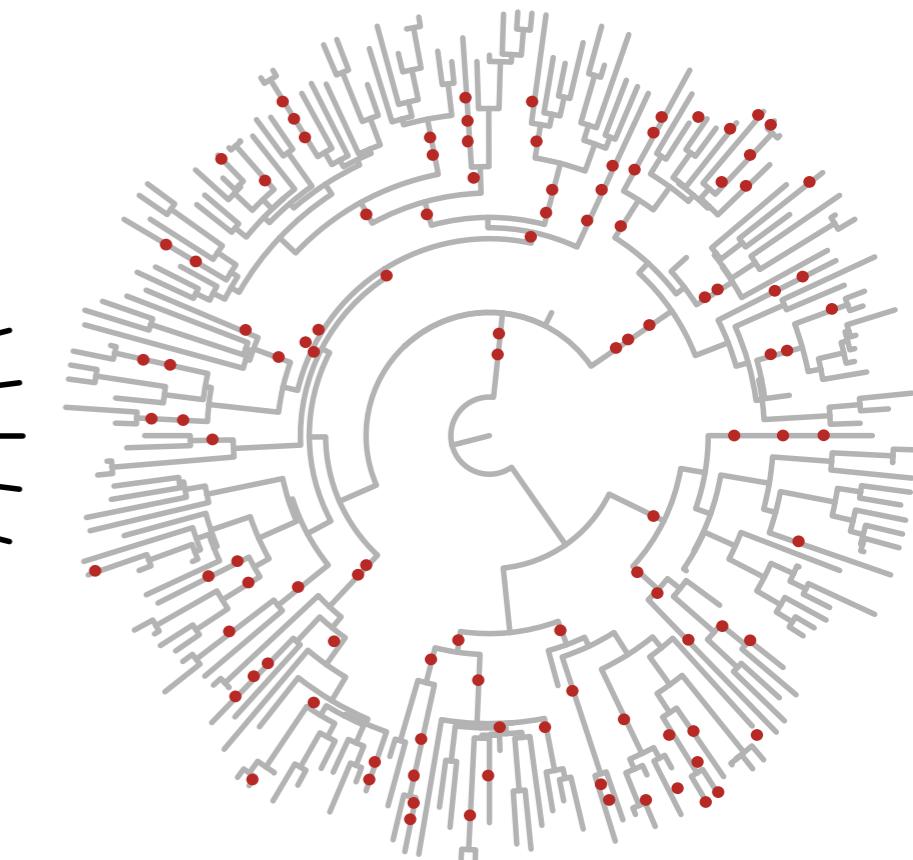
Complete evolutionary history of a clade



Reconstructed phylogeny of the extant taxa



Phylogenetic comparative methods



Fossil record

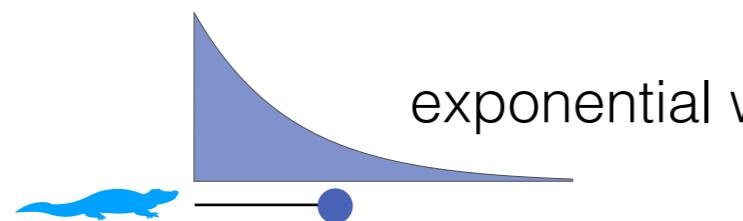


“Stratigraphic approaches”

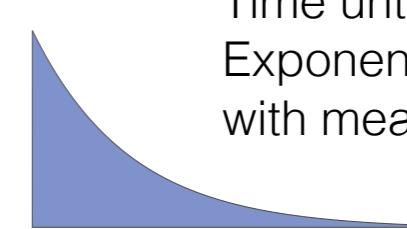
# Species diversification as a birth-death stochastic process

$\lambda$ : expected number of speciation events in 1 My per-lineage

$\mu$ : expected number of extinction events in 1 My per-lineage



exponential waiting time until speciation



Time until speciation:  
Exponential distribution  
with mean  $1/\lambda$

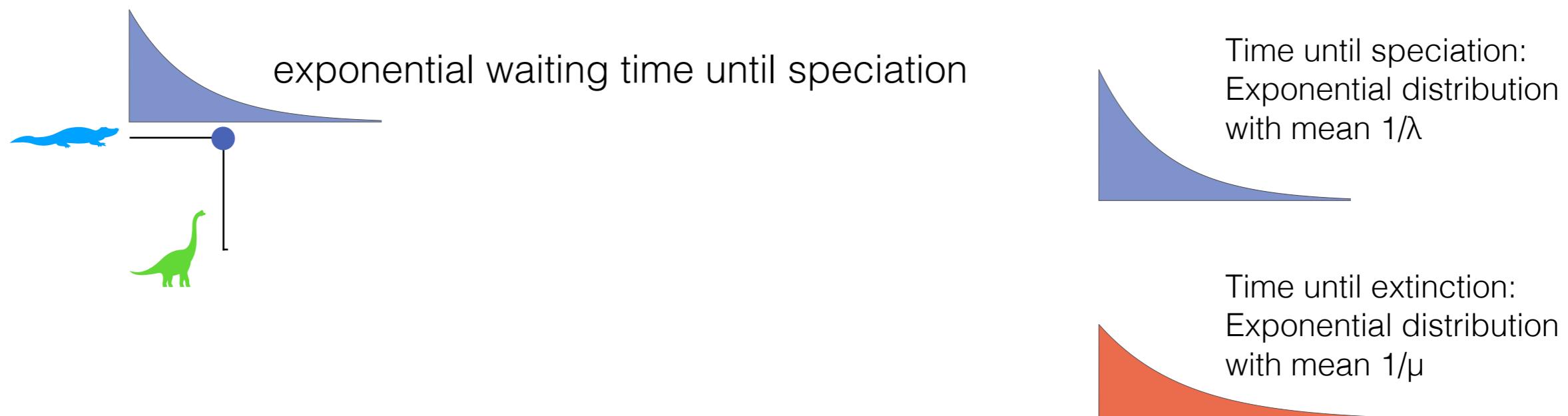


Time until extinction:  
Exponential distribution  
with mean  $1/\mu$

# Species diversification as a birth-death stochastic process

$\lambda$ : expected number of speciation events in 1 My per-lineage

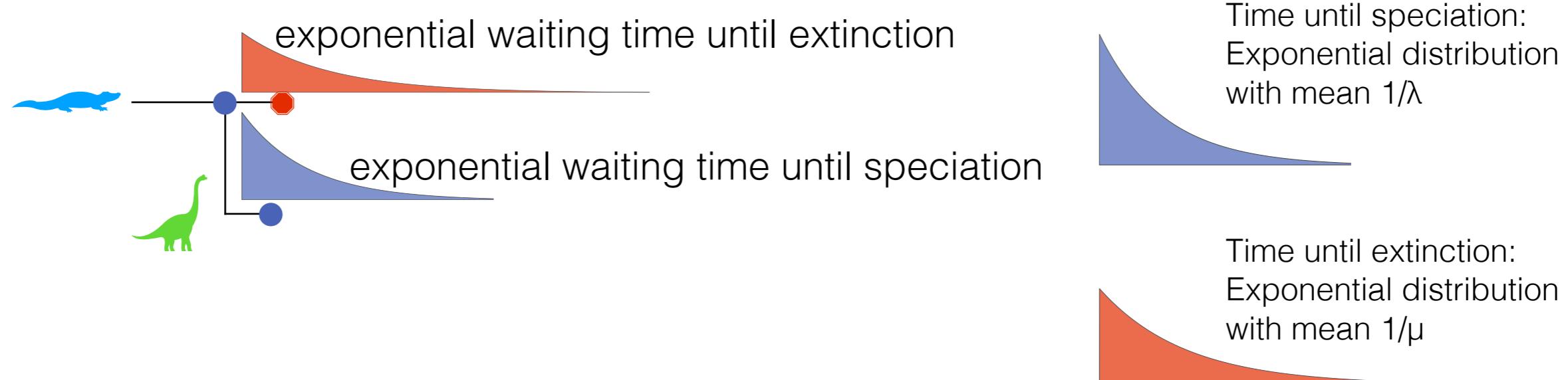
$\mu$ : expected number of extinction events in 1 My per-lineage



# Species diversification as a birth-death stochastic process

$\lambda$ : expected number of speciation events in 1 My per-lineage

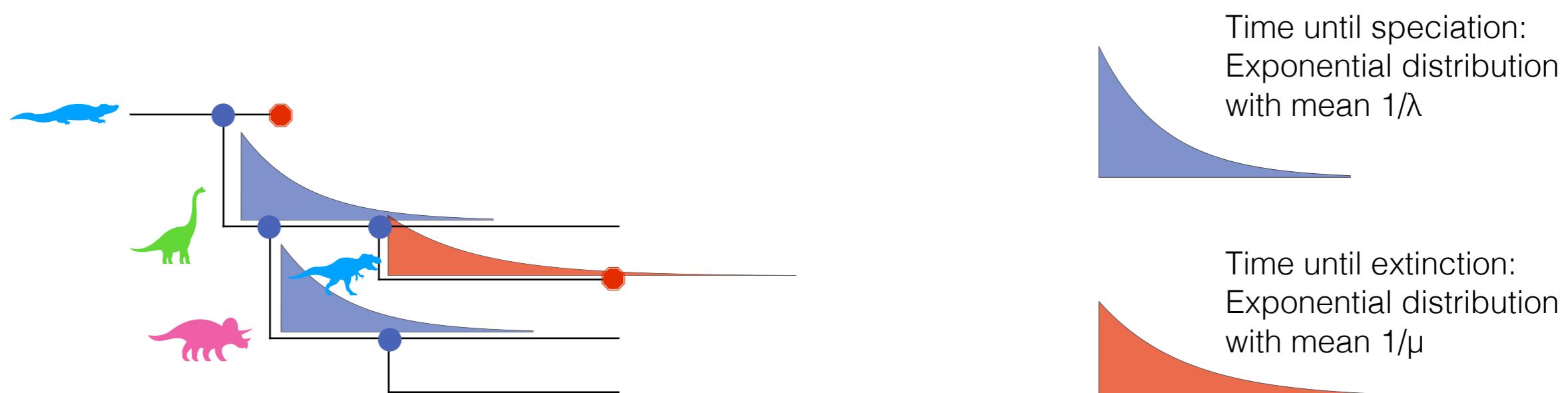
$\mu$ : expected number of extinction events in 1 My per-lineage



# Species diversification as a birth-death stochastic process

$\lambda$ : expected number of speciation events in 1 My per-lineage

$\mu$ : expected number of extinction events in 1 My per-lineage



# Species diversification as a birth-death stochastic process

## Why an exponential waiting time?

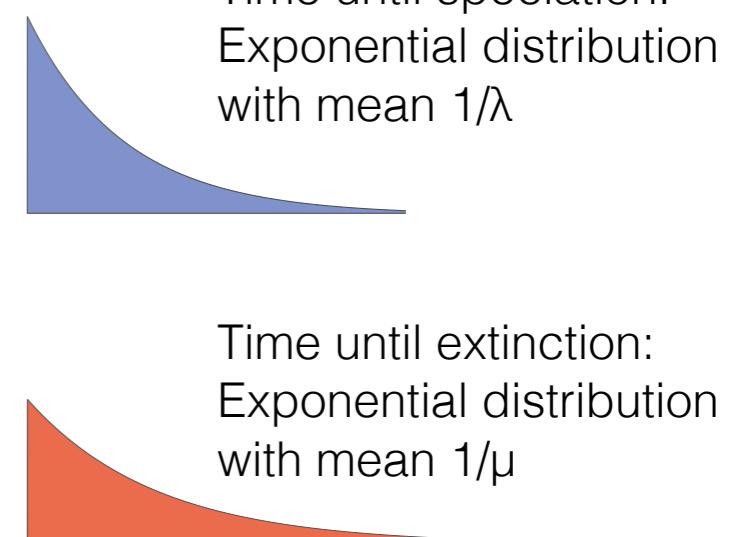
Expected pattern from a constant rate random process



$$P_{\text{event}}(t = 0) = 0.5$$



Probability of the event



# Species diversification as a birth-death stochastic process

## Why an exponential waiting time?

Expected pattern from a constant rate random process



$$P_{\text{event}}(t = 2) = 0.5 \times (1 - 0.5)^{t-1} = 0.25$$

↑              ↑  
Probability of the event  
|  
Probability that it didn't  
occur before time  $t$

# Species diversification as a birth-death stochastic process

## Why an exponential waiting time?

Expected pattern from a constant rate random process



$$P_{\text{event}}(t = 3) = 0.5 \times (1 - 0.5)^{t-1} = 0.125$$

↑                   ↑  
Probability of the event  
|  
Probability that it didn't  
occur before time  $t$

# Species diversification as a birth-death stochastic process

## Why an exponential waiting time?

Expected pattern from a constant rate random process



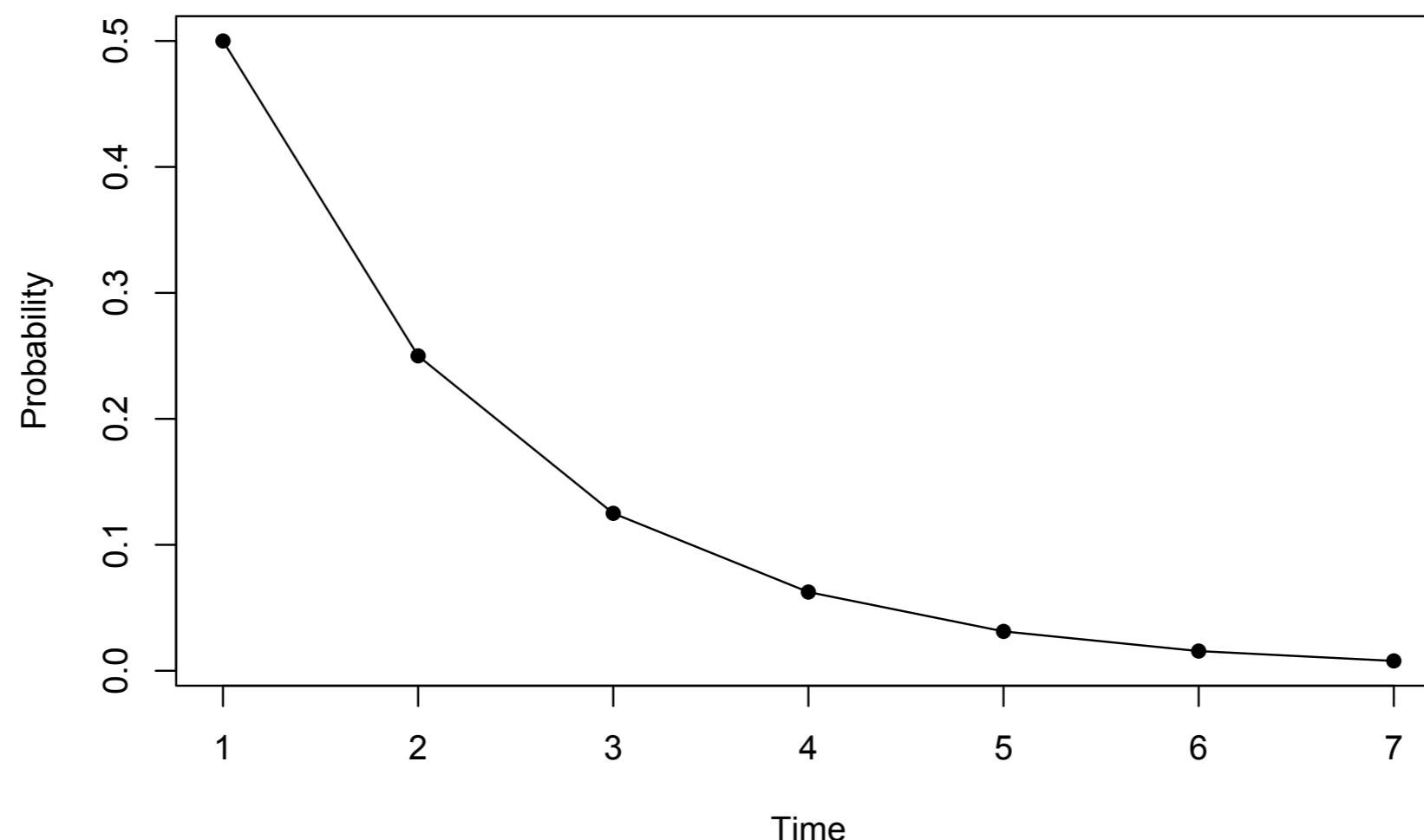
$$P_{\text{event}}(t = 4) = 0.5 \times (1 - 0.5)^{t-1} = 0.063$$

↑                      ↑  
Probability of the event  
|  
Probability that it didn't  
occur before time  $t$

# Species diversification as a birth-death stochastic process

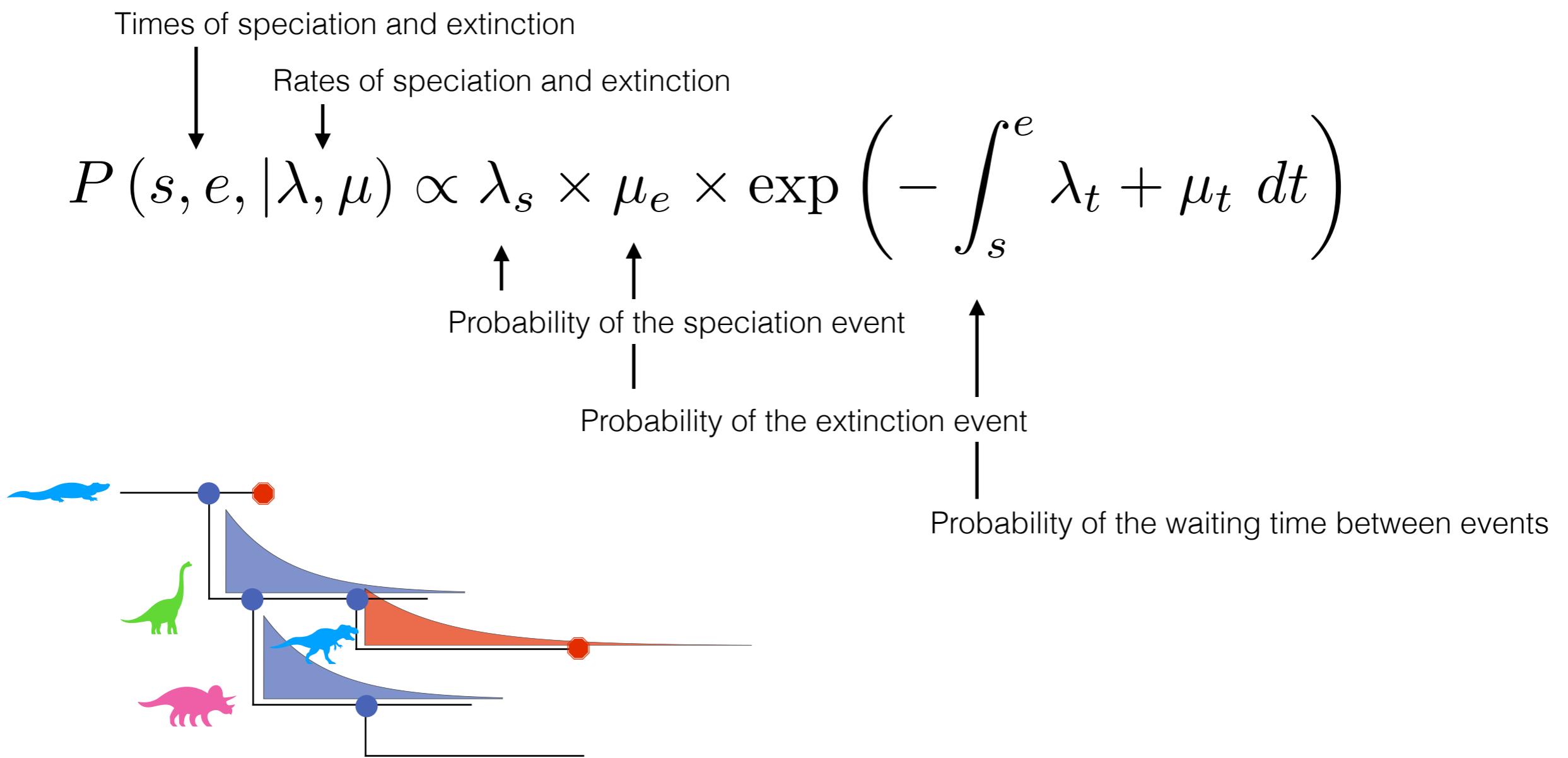
## Why an exponential waiting time?

Expected pattern from a constant rate random process



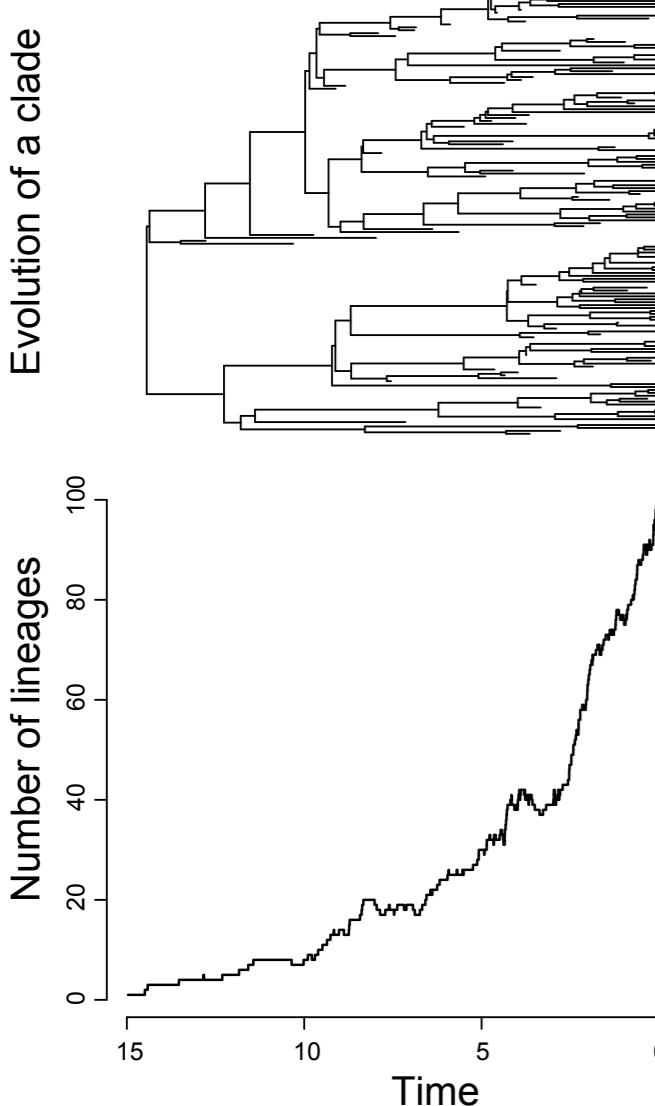
# Species diversification as a birth-death stochastic process

## Likelihood of a birth-death process

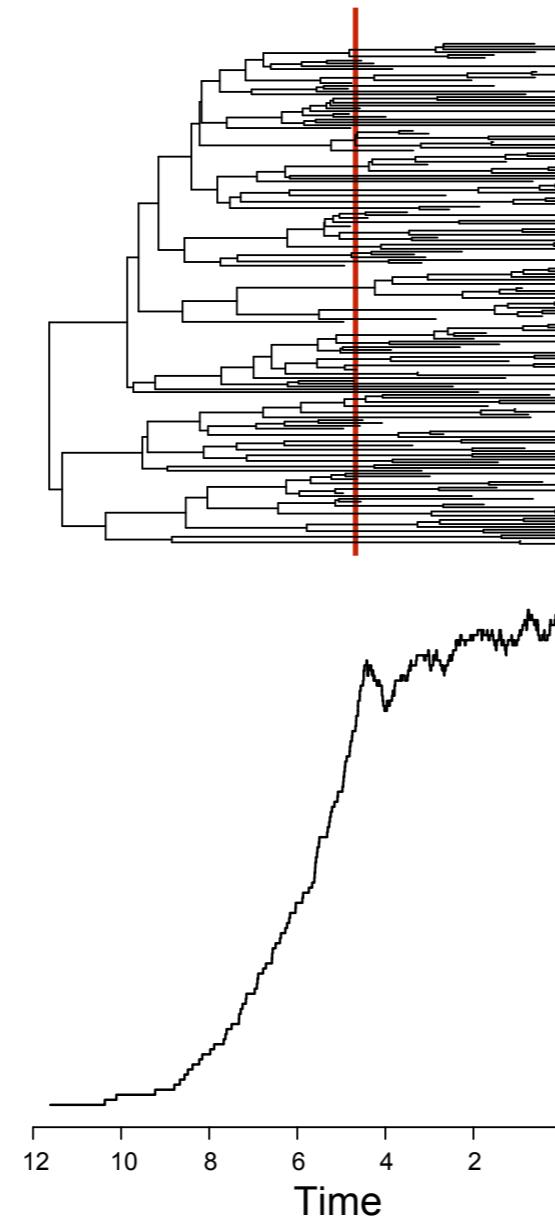


# Understanding the process of diversification

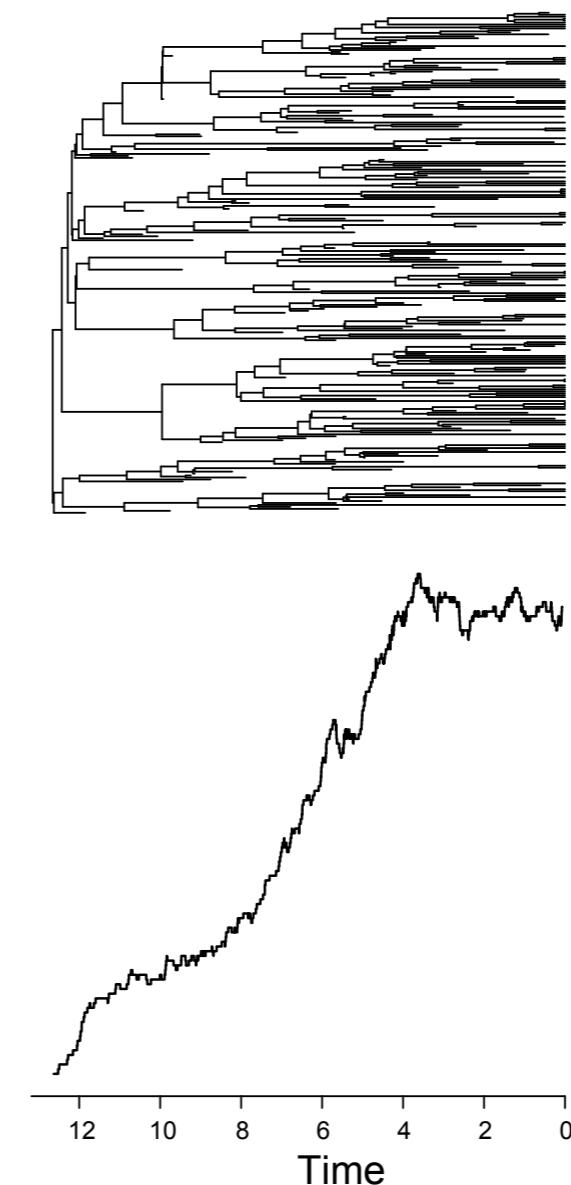
Constant speciation  
and extinction rate



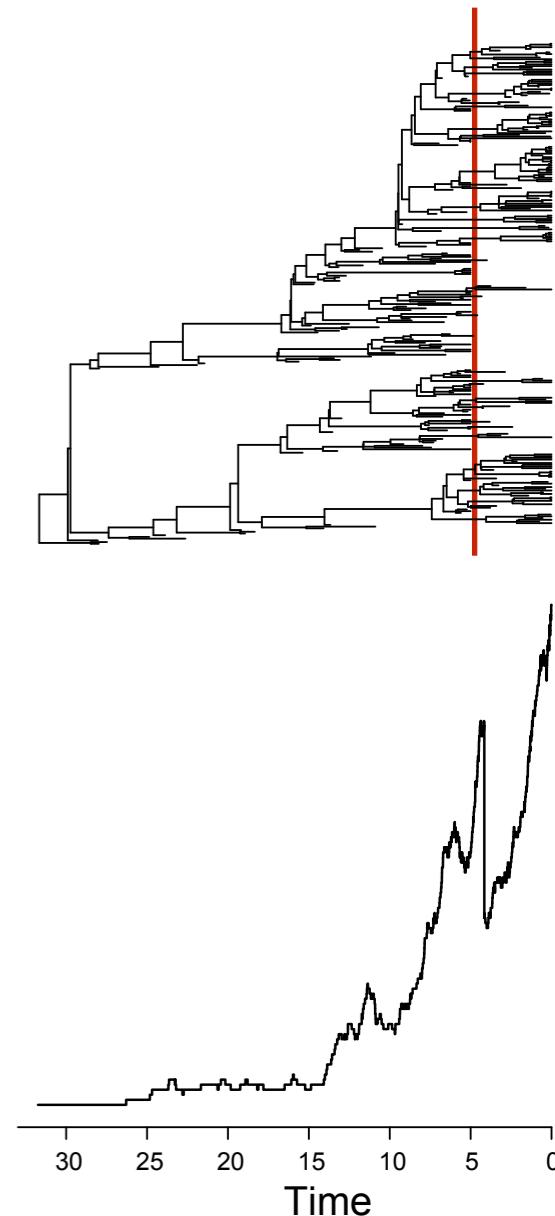
Rate shifts



Diversity dependent  
speciation rate

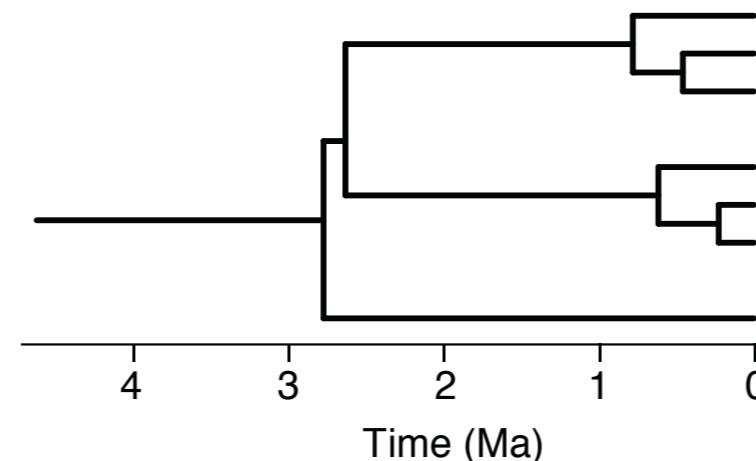


Mass extinction

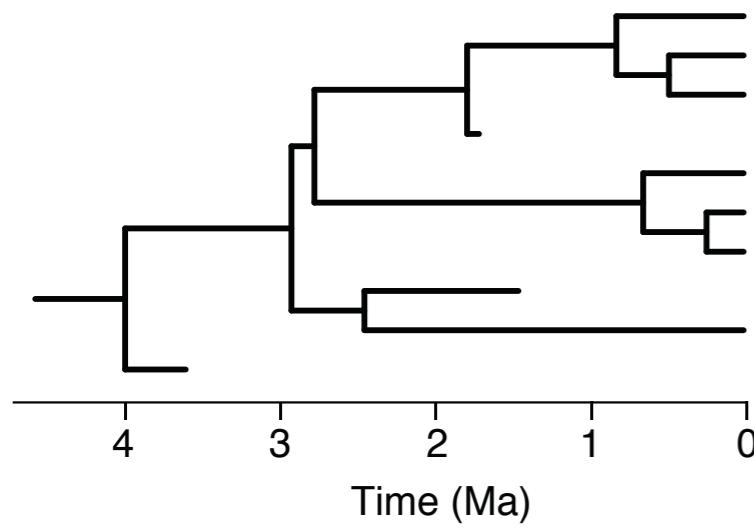


# Methods to infer speciation and extinction rates

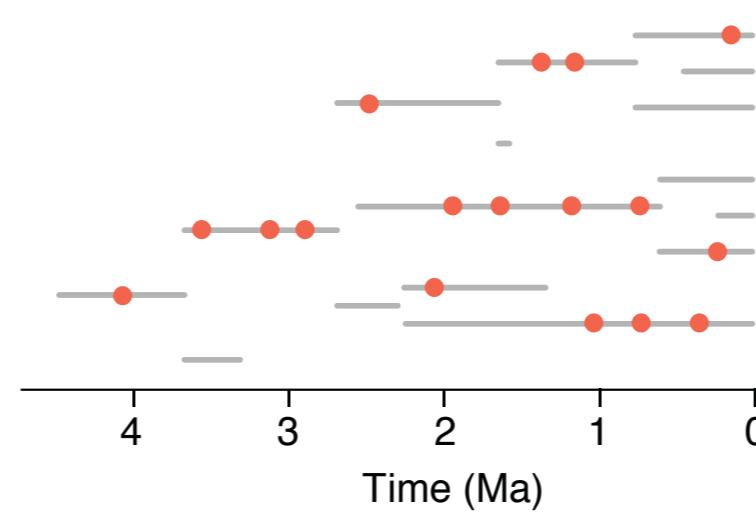
Phylogeny of extant species



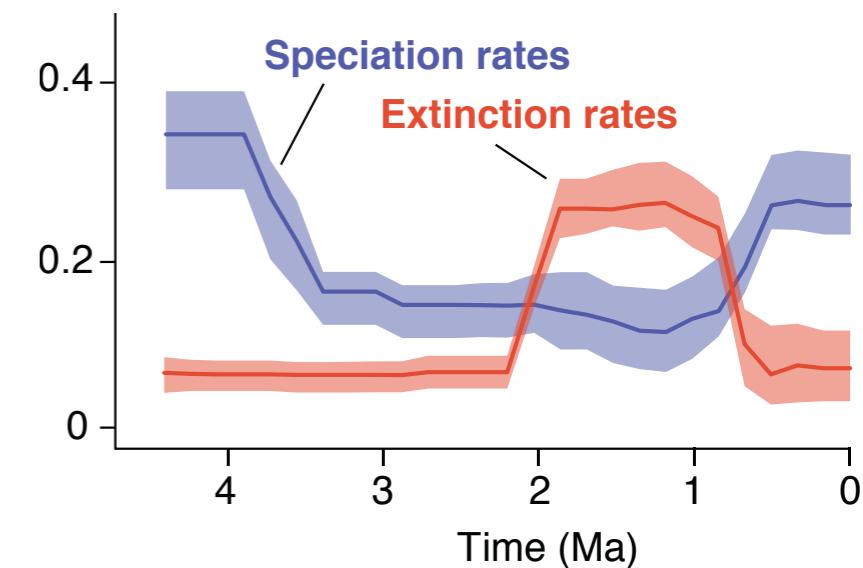
Complete evolutionary history



Fossil record



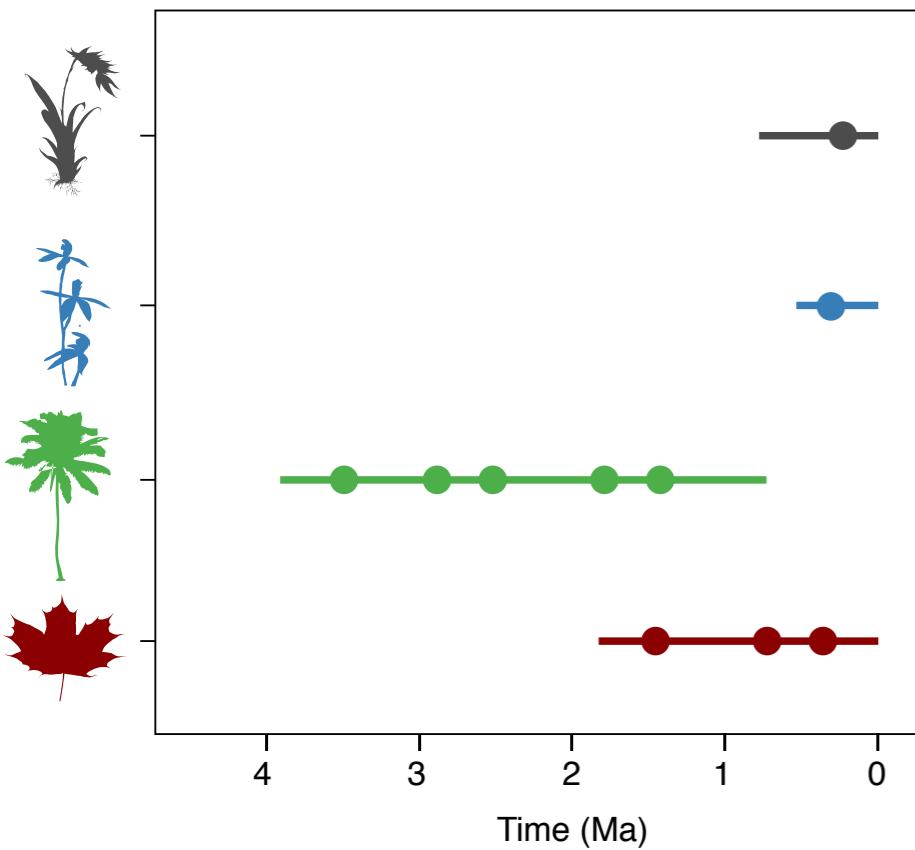
Diversification dynamics



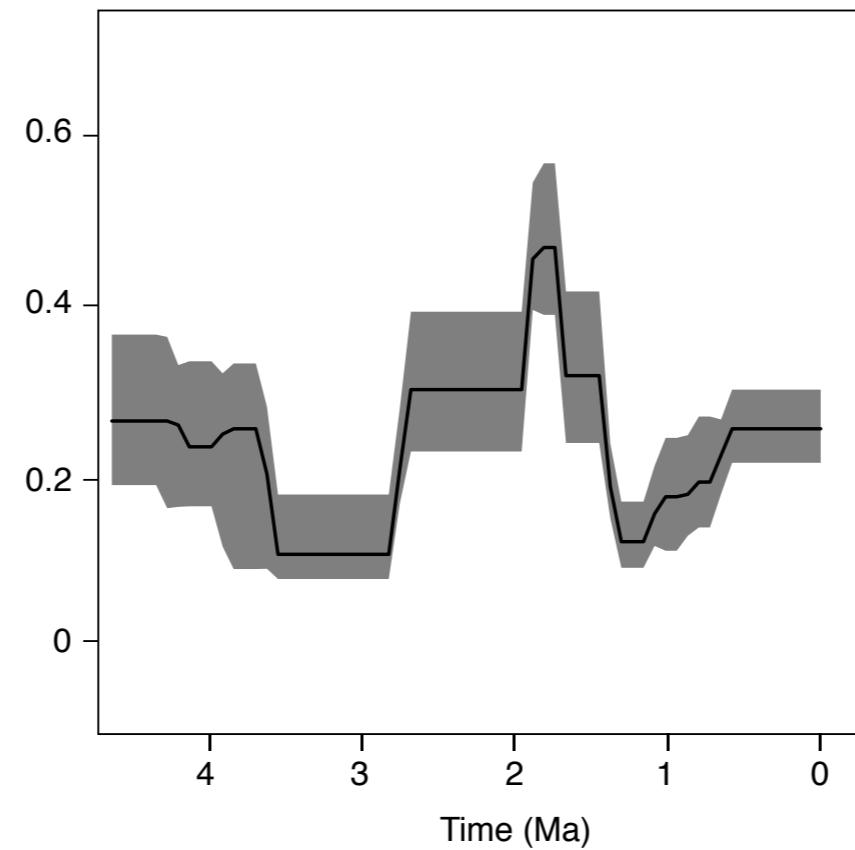
# Bayesian inference of macroevolutionary rates using fossil occurrences

$$\underbrace{P(q, s, e, \lambda, \mu | X)}_{\text{posterior}} \propto \underbrace{P(X | q, s, e)}_{\text{likelihood}} \times \underbrace{P(s, e | \lambda, \mu)}_{\text{BD prior}} \times \underbrace{P(q) P(\lambda, \mu)}_{\text{other (hyper-)priors}}$$

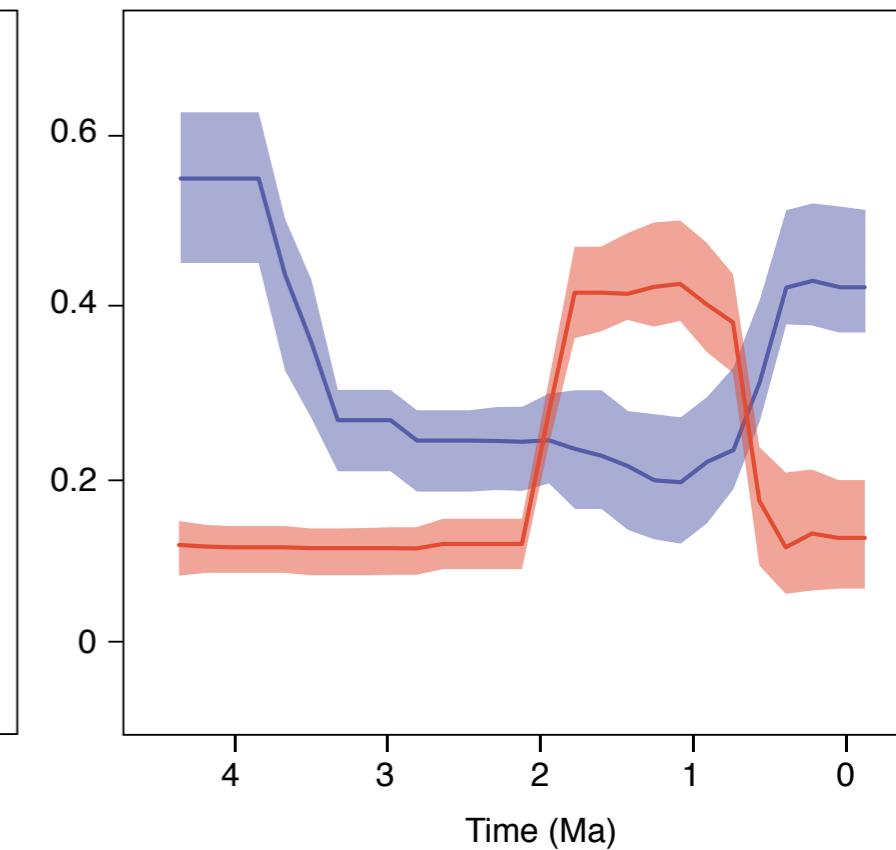
Taxa duration



**Preservation** rates



**Speciation & extinction** rates



**PyRATE**

<https://github.com/dsilvestro/PyRate>

Silvestro et al. 2014 Syst Biol, MEE

# A hierarchical probabilistic framework

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

POSTERIOR

LIKELIHOOD

PRIOR

HYPERTPRIORS



Probability of:

1. times of speciation/extinction ( $s, e$ )
2. preservation rate ( $q$ )
3. speciation/extinction rates ( $\lambda, \mu$ )

# A hierarchical probabilistic framework

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

Probability of the data given speciation and extinction times, preservation rate

POSTERIOR



Probability of:

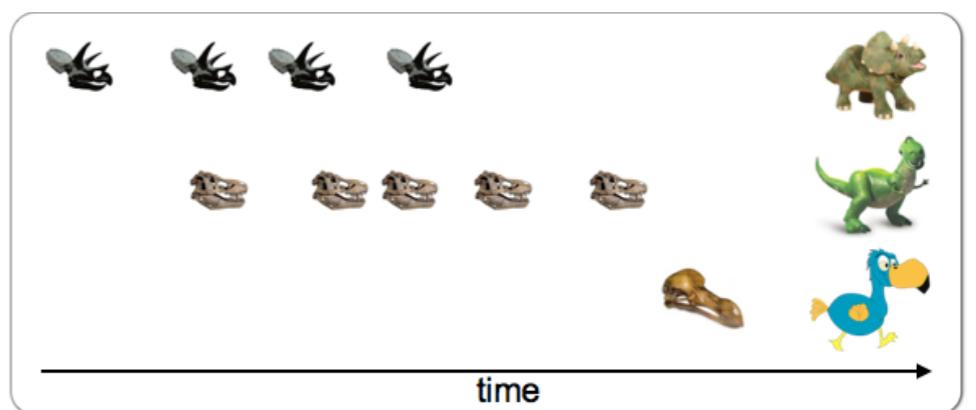
1. times of speciation/extinction ( $s, e$ )
2. preservation rate ( $q$ )
3. speciation/extinction rates ( $\lambda, \mu$ )

LIKELIHOOD

PRIOR

HYPERTPRIORS

$x =$



# A hierarchical probabilistic framework

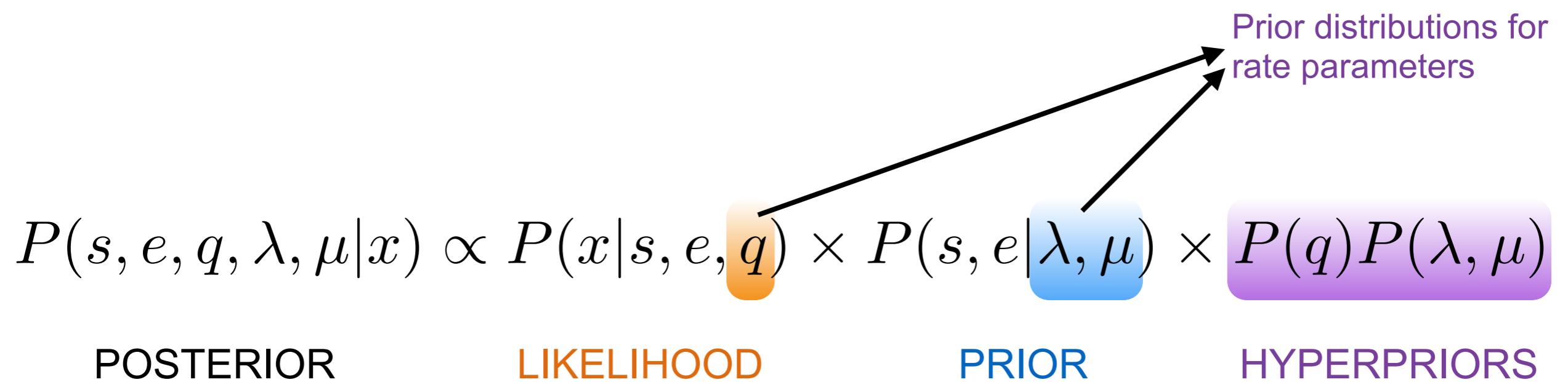
$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

POSTERIOR      LIKELIHOOD      PRIOR      HYPERPRIORS

Prior probability of the speciation and extinction times (Birth-Death process)

The diagram illustrates a hierarchical probabilistic framework. The posterior distribution is given by the equation  $P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$ . The terms are categorized as follows: POSTERIOR (the final result), LIKELIHOOD (the first term  $P(x | s, e, q)$ ), PRIOR (the second term  $P(s, e | \lambda, \mu)$ ), and HYPERPRIORS (the third term  $P(q)P(\lambda, \mu)$ ). The term  $P(s, e | \lambda, \mu)$  is highlighted in blue, and an arrow points from it to the text "Prior probability of the speciation and extinction times (Birth-Death process)".

# A hierarchical probabilistic framework

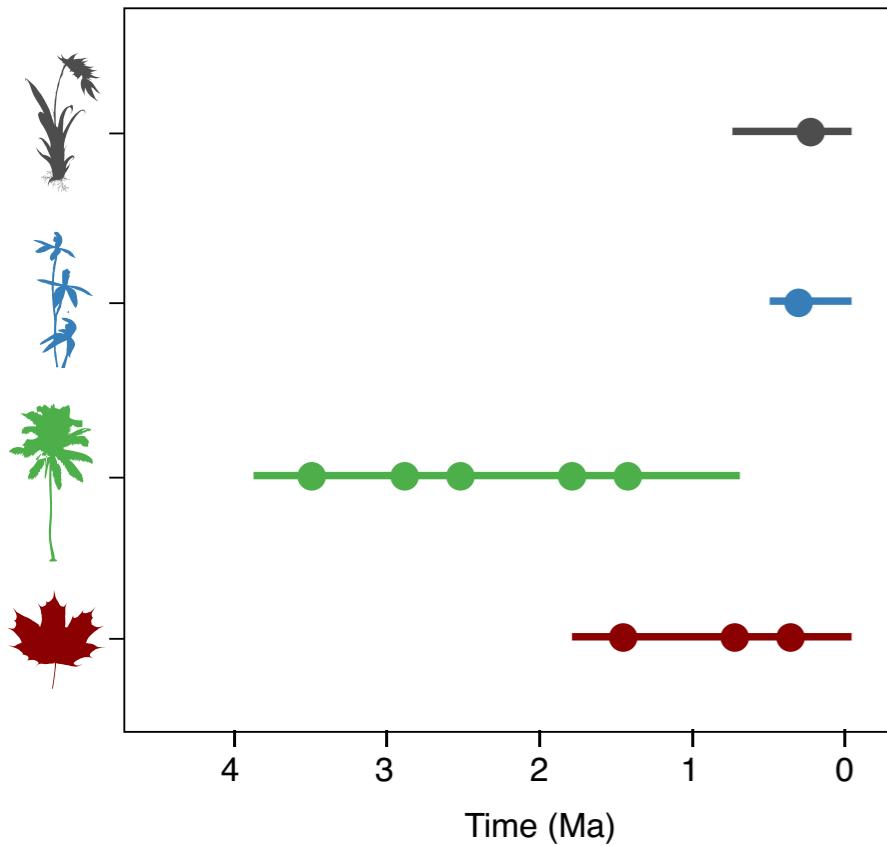


# Bayesian inference of macroevolutionary rates using fossil occurrences

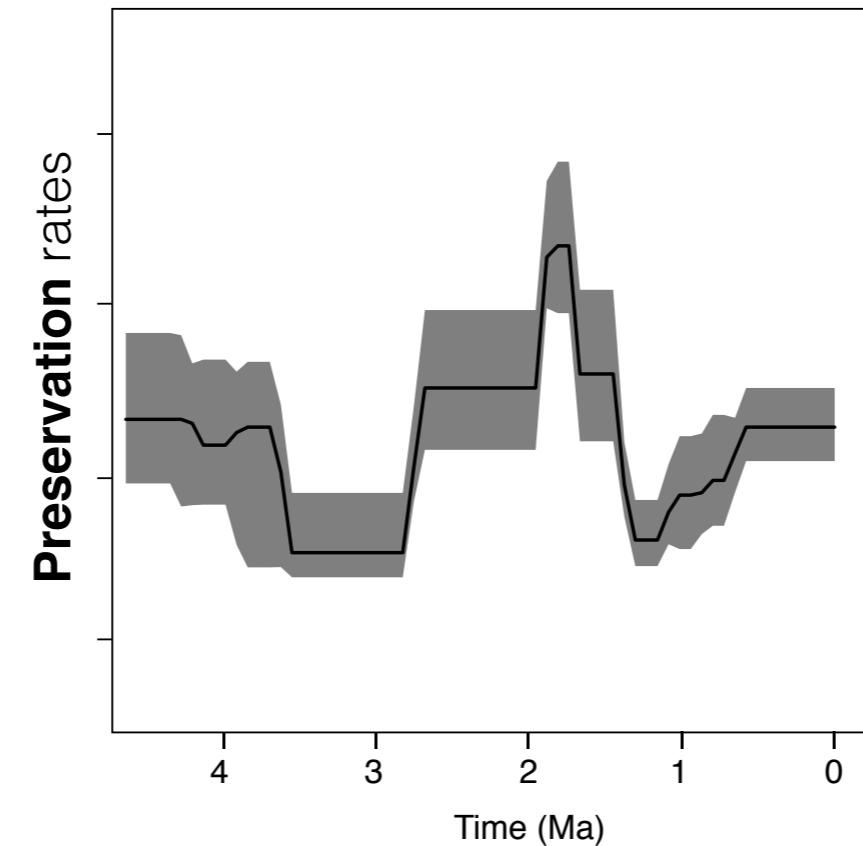
$$\underbrace{P(q, s, e, \lambda, \mu | X)}_{\text{posterior}} \propto \underbrace{P(X|q, s, e)}_{\text{likelihood}} \times \underbrace{P(s, e|\lambda, \mu)}_{\text{birth-death prior}} \times \underbrace{P(q)P(\lambda, \mu)}_{\text{other (hyper)priors}}$$

Metropolis-Hastings MCMC      Reversible-jump MCMC      Sampled directly from conjugate posterior

Species duration



Poisson process

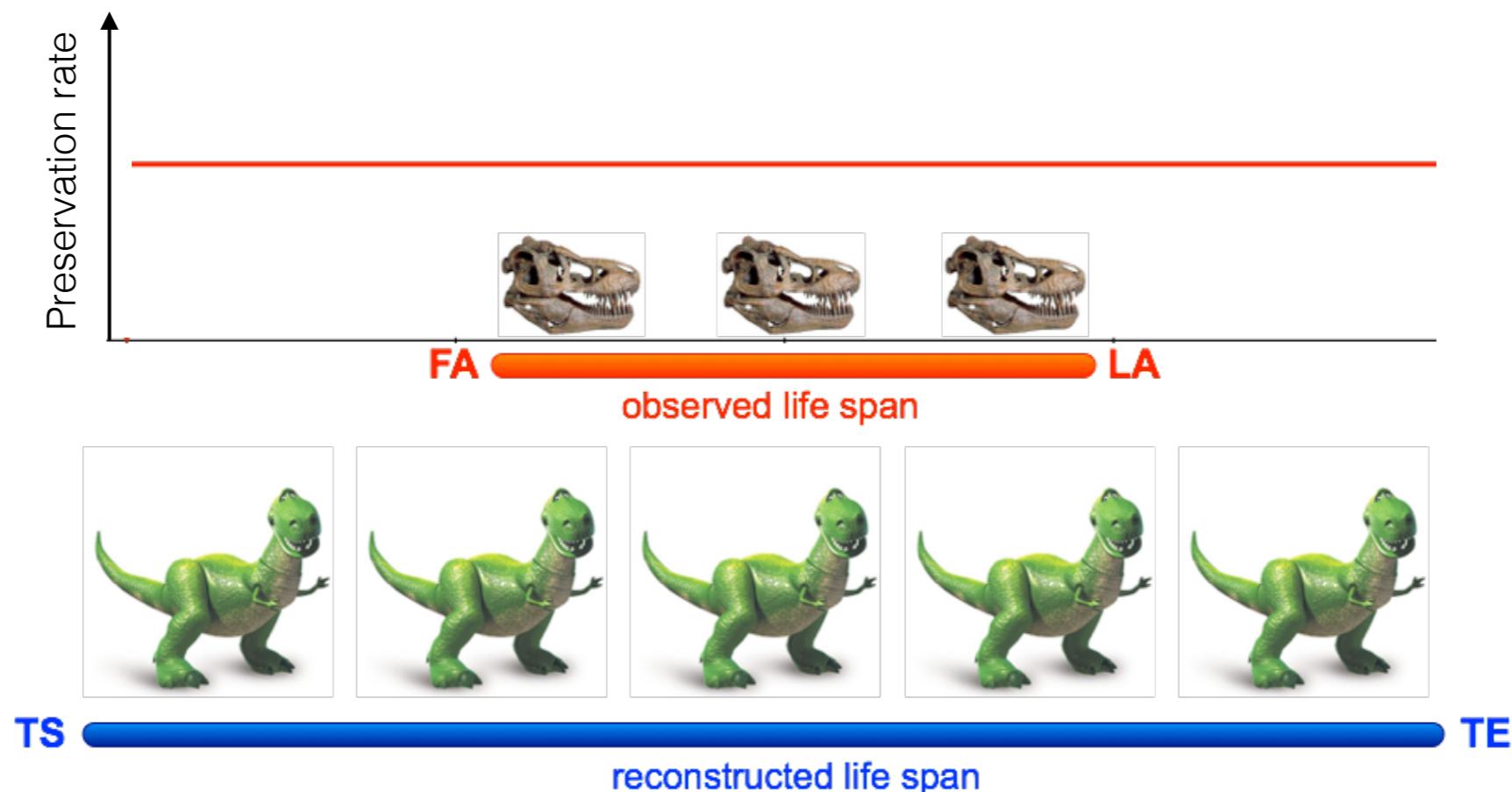


# Estimated preservation rates and times of speciation and extinction

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

LIKELIHOOD

Homogeneous Poisson process (HPP)



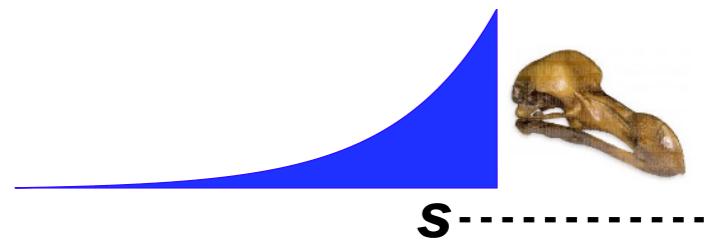
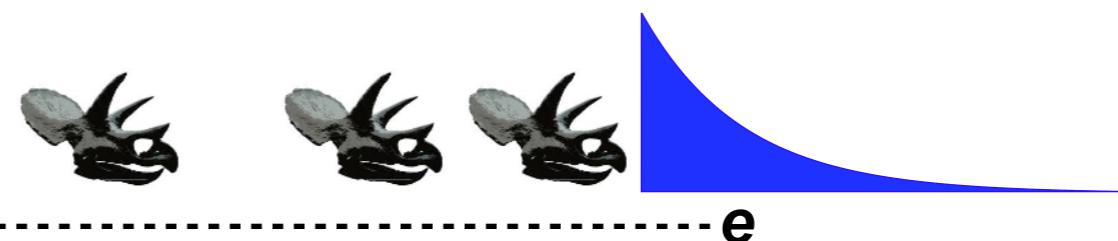
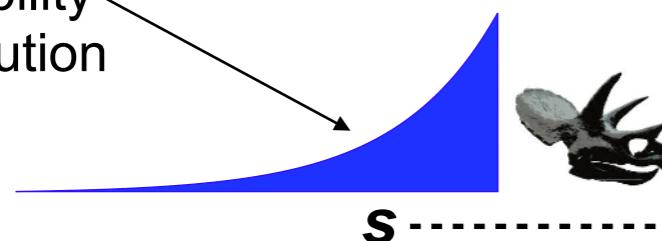
# Estimated preservation rates and times of speciation and extinction

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

LIKELIHOOD

Homogeneous Poisson process (HPP)

Posterior probability distribution



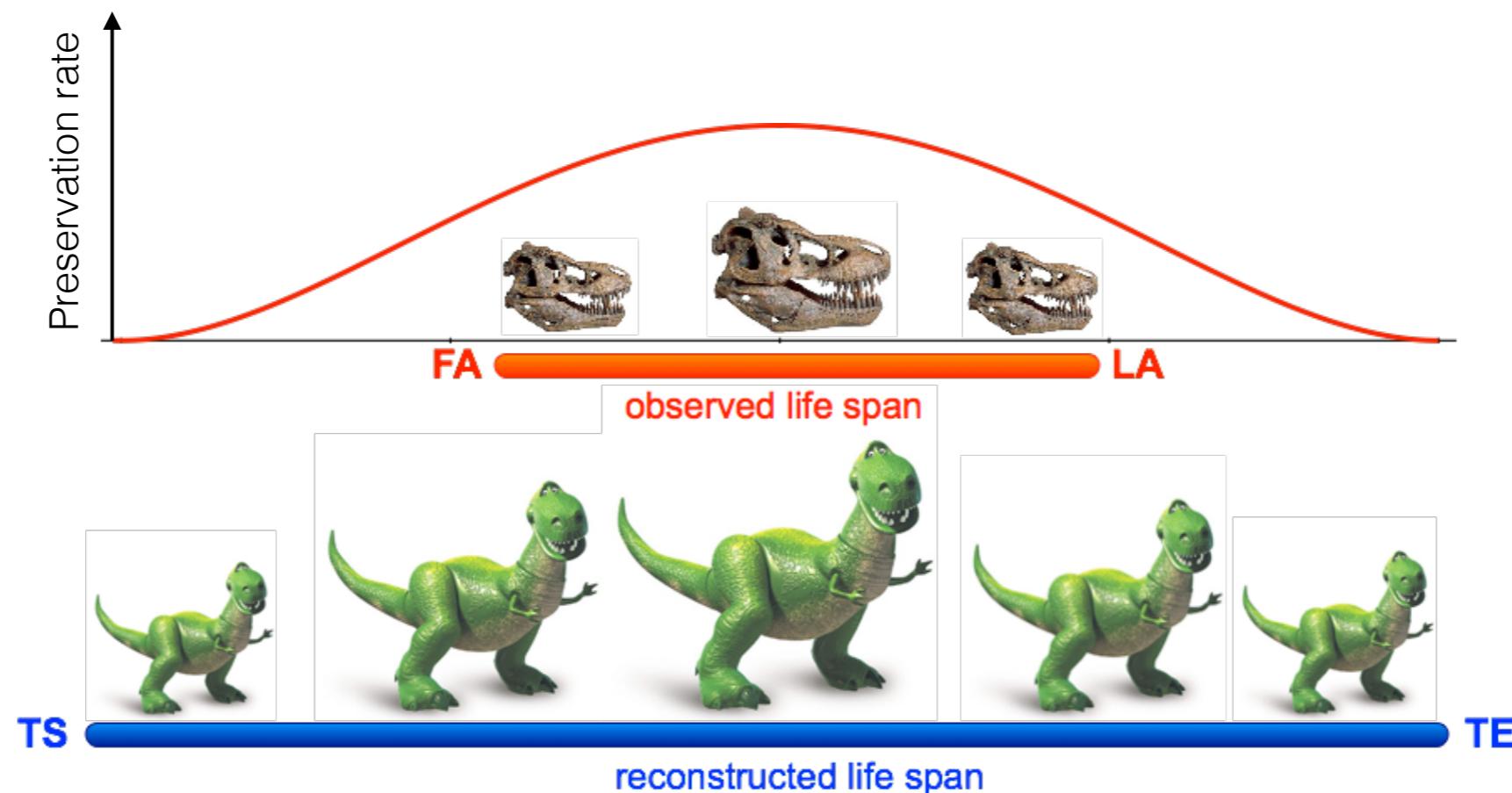
Constant preservation rate

# Estimated preservation rates and times of speciation and extinction

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

LIKELIHOOD

Non-homogeneous Poisson process (NHPP)

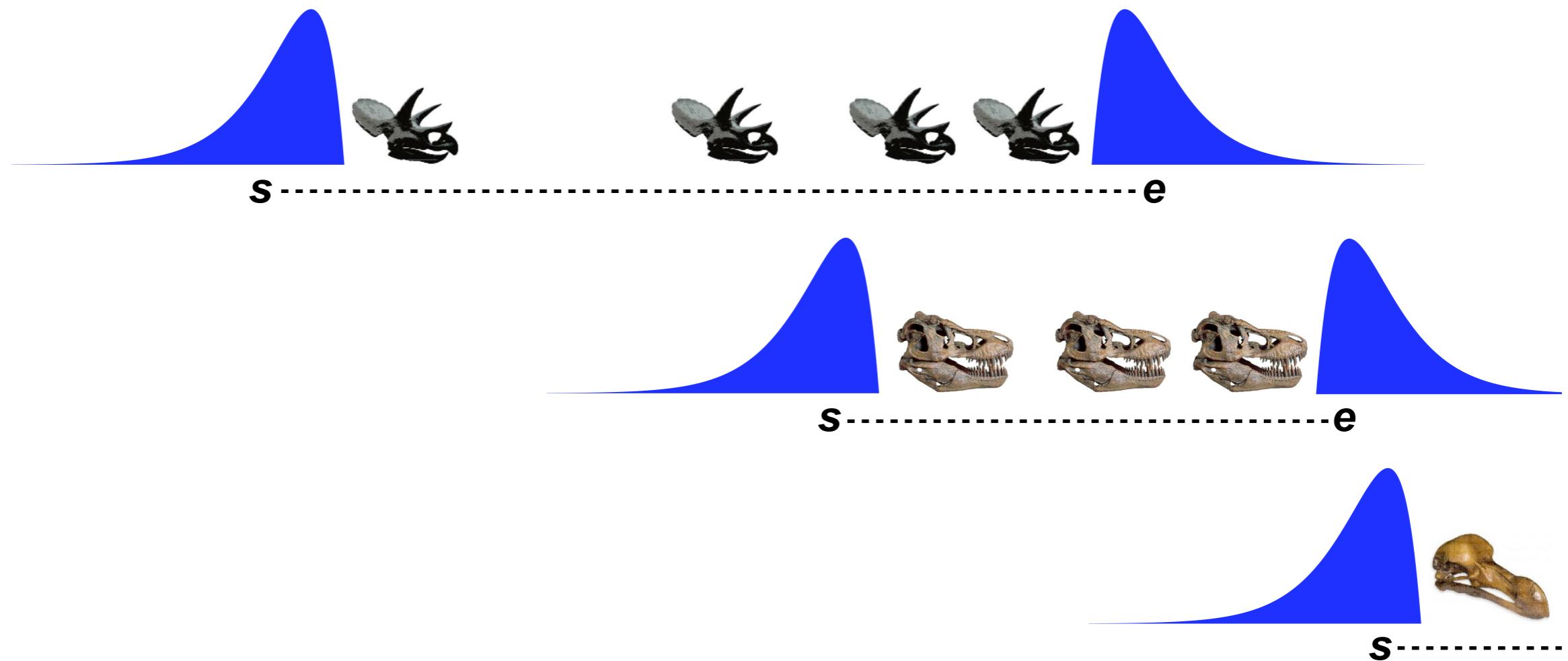


# Estimated preservation rates and times of speciation and extinction

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

LIKELIHOOD

Non-homogeneous Poisson process (NHPP)



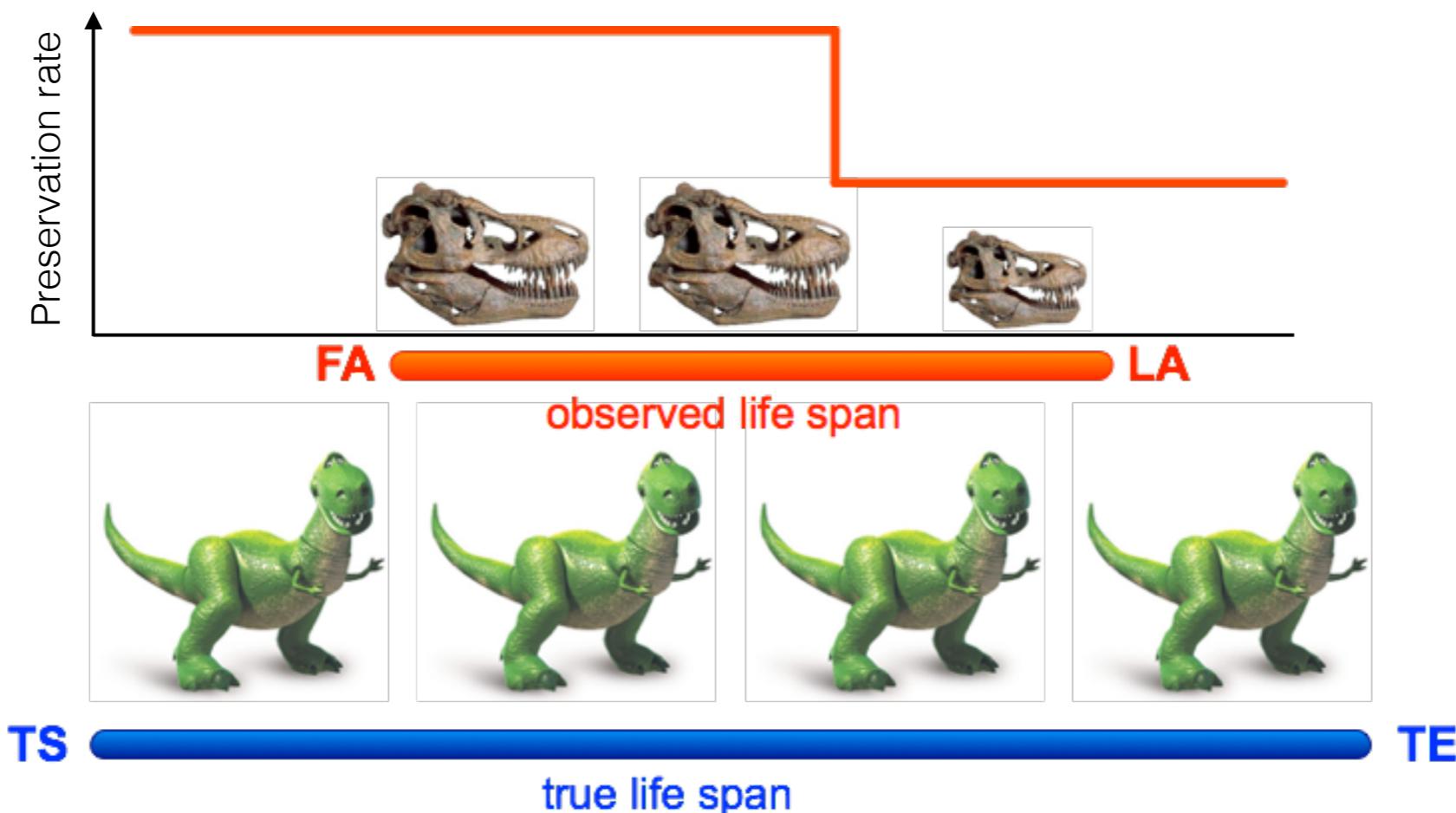
Constant mean preservation rate

# Estimated preservation rates and times of speciation and extinction

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

LIKELIHOOD

Time-variable Poisson process (TPP)

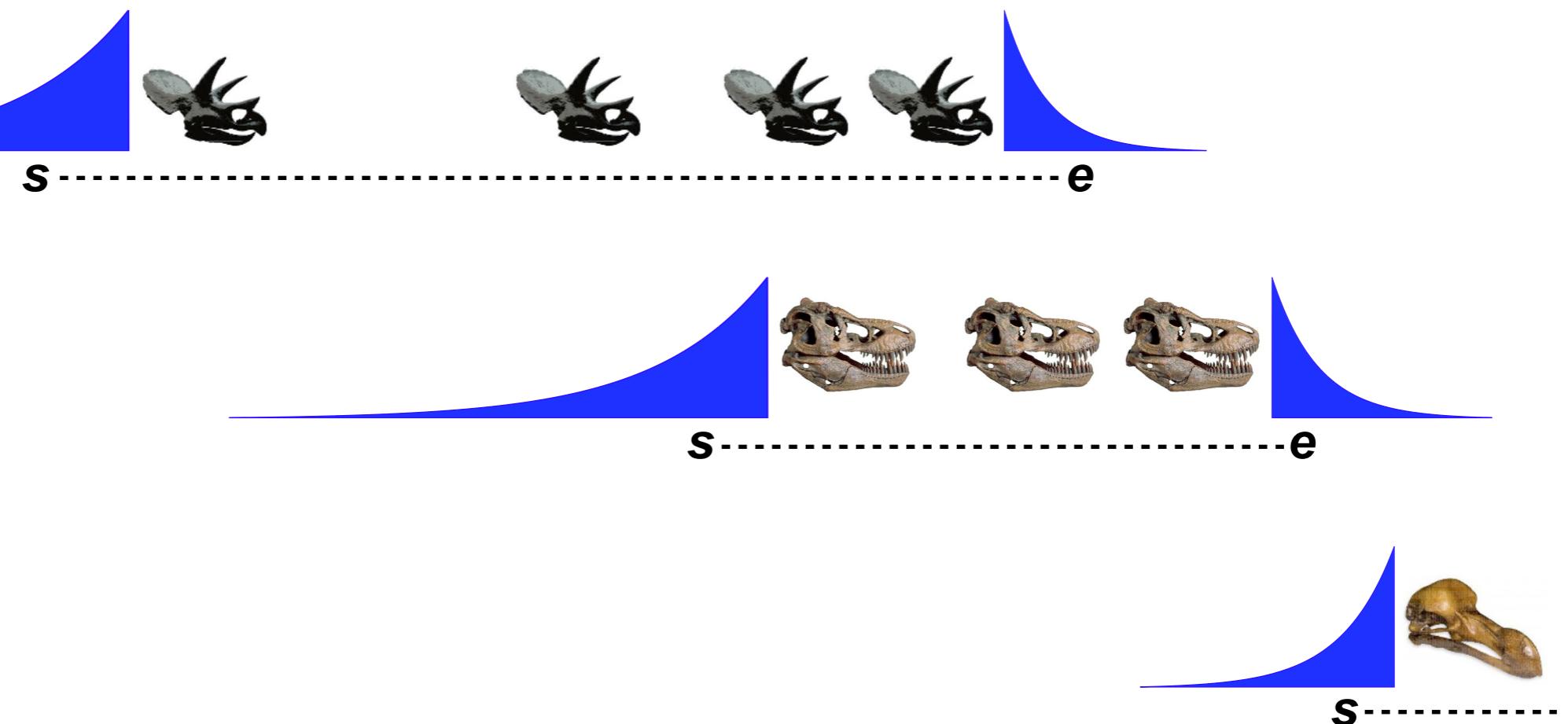


# Estimated preservation rates and times of speciation and extinction

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

LIKELIHOOD

Time-variable Poisson process (TPP)



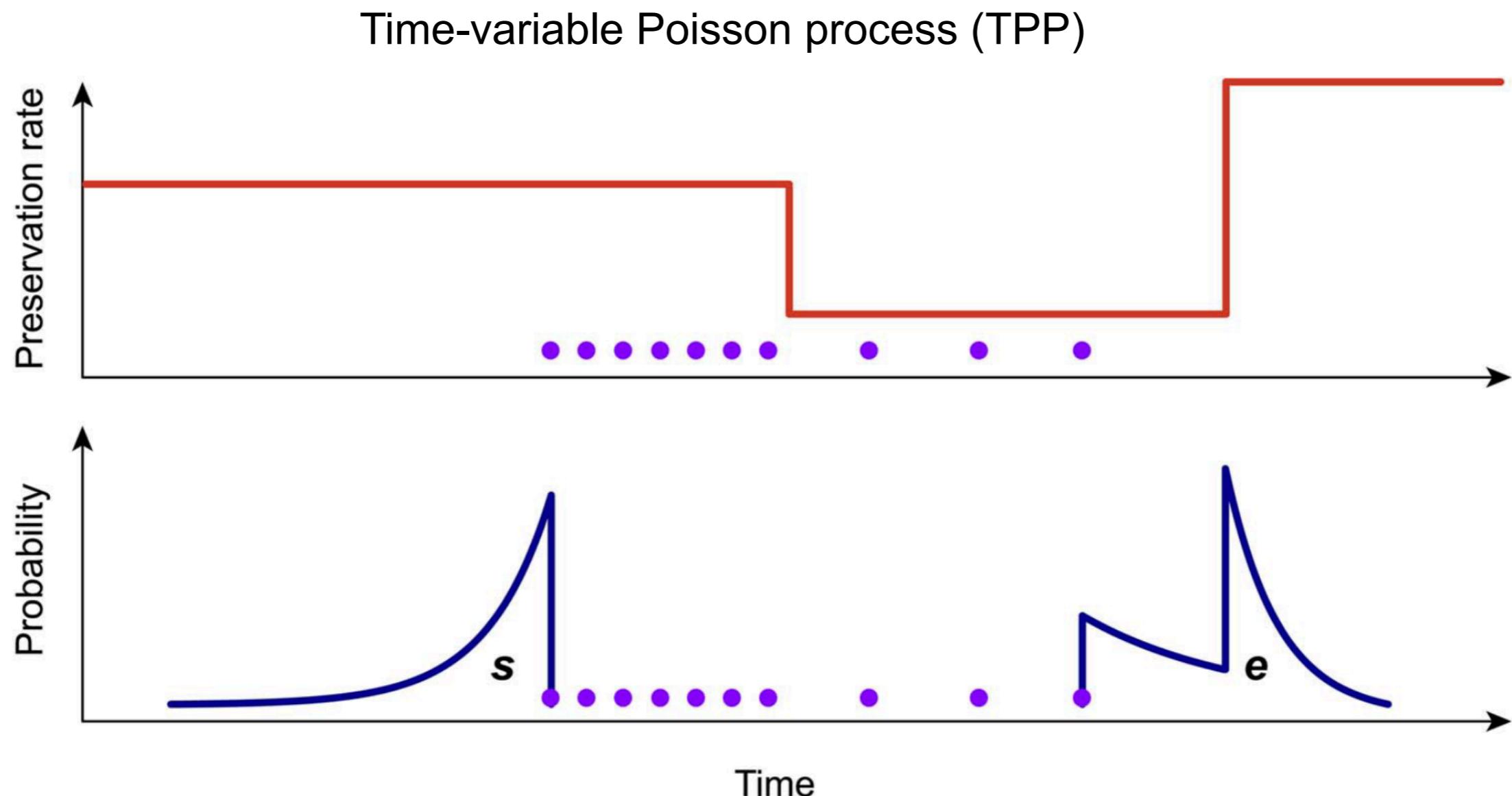
Low preservation rate

High preservation rate

# Estimated preservation rates and times of speciation and extinction

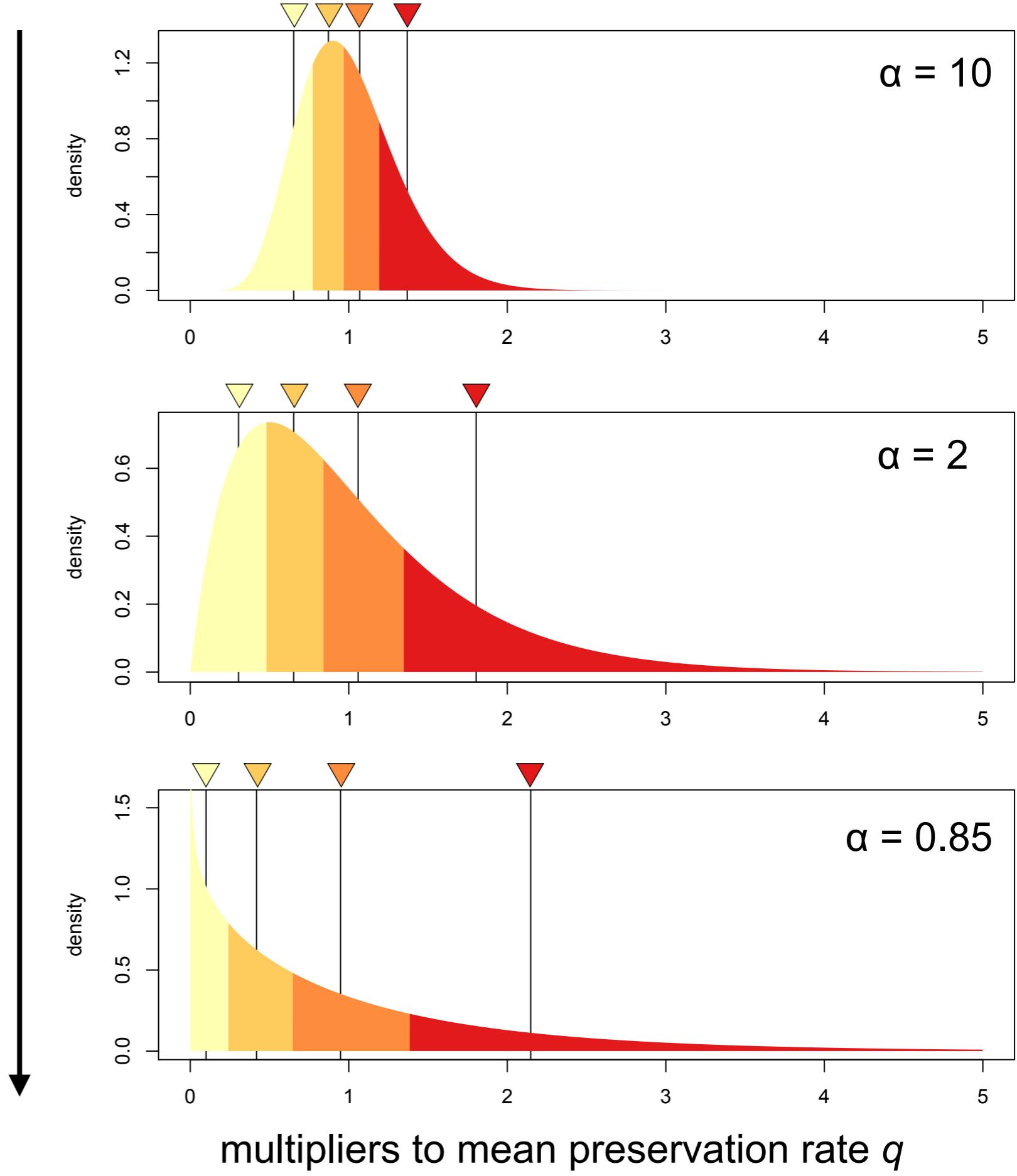
$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

LIKELIHOOD

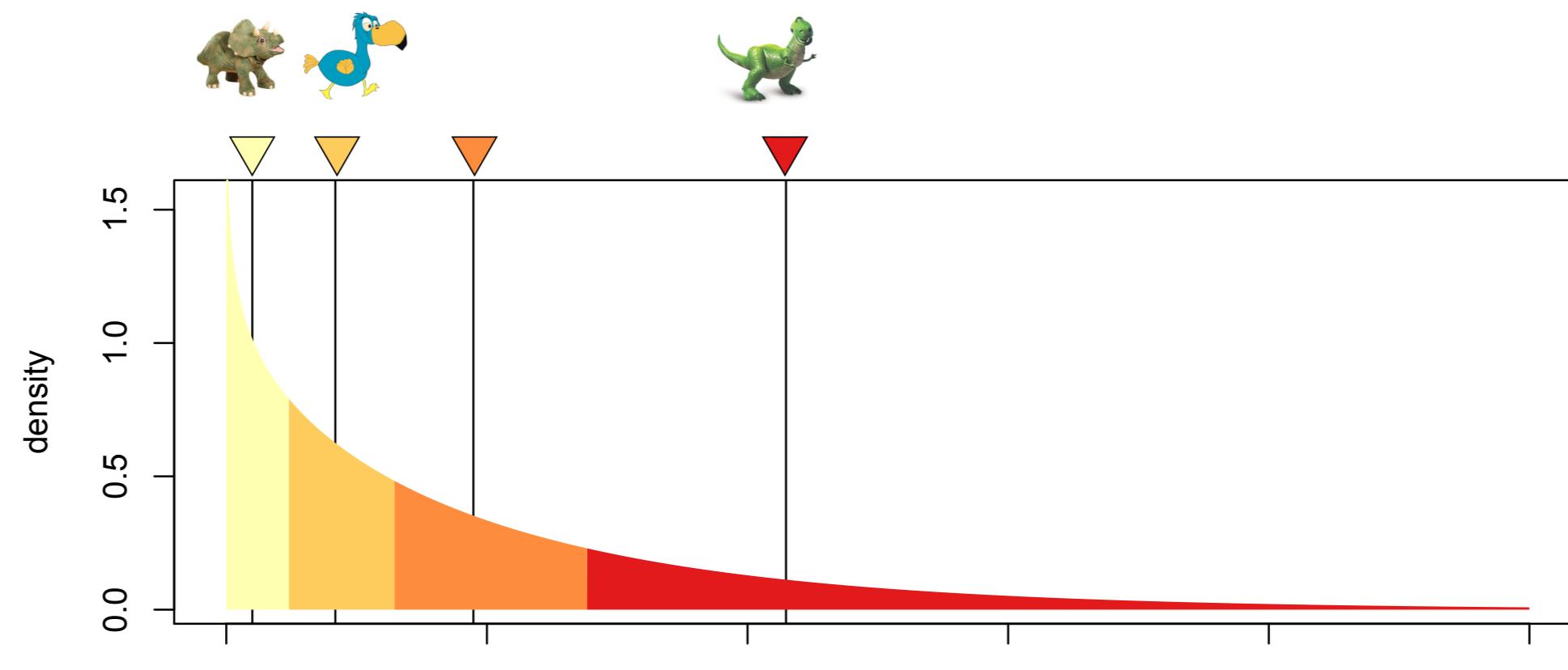
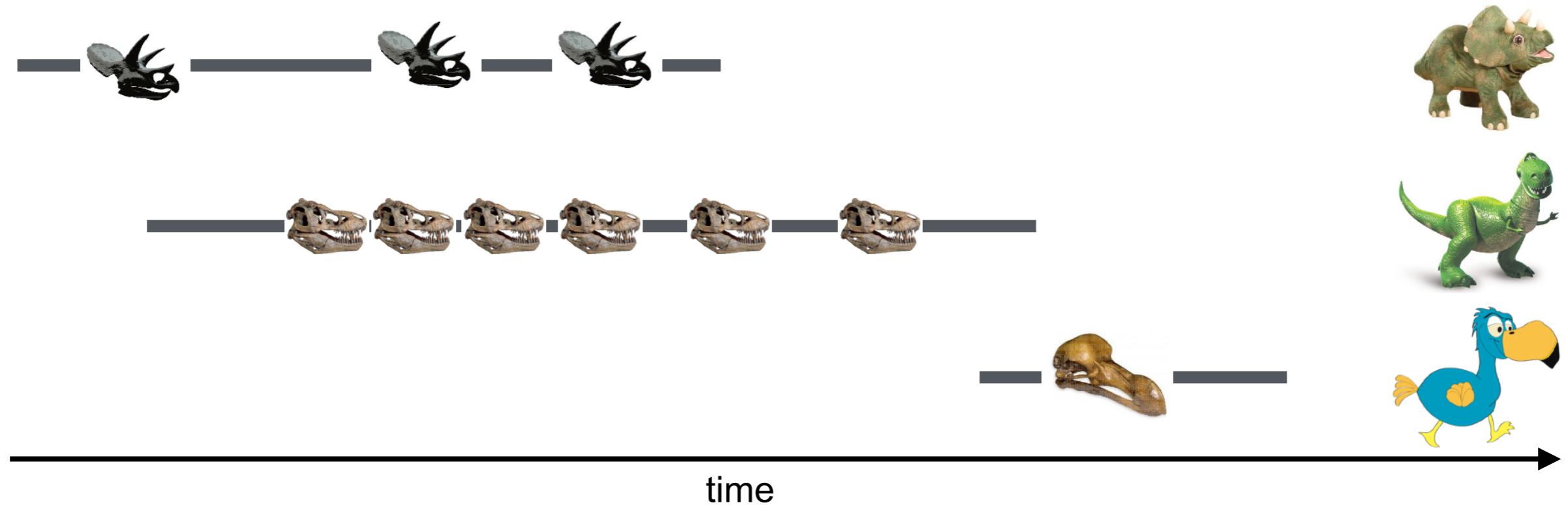


## Gamma model of rate heterogeneity

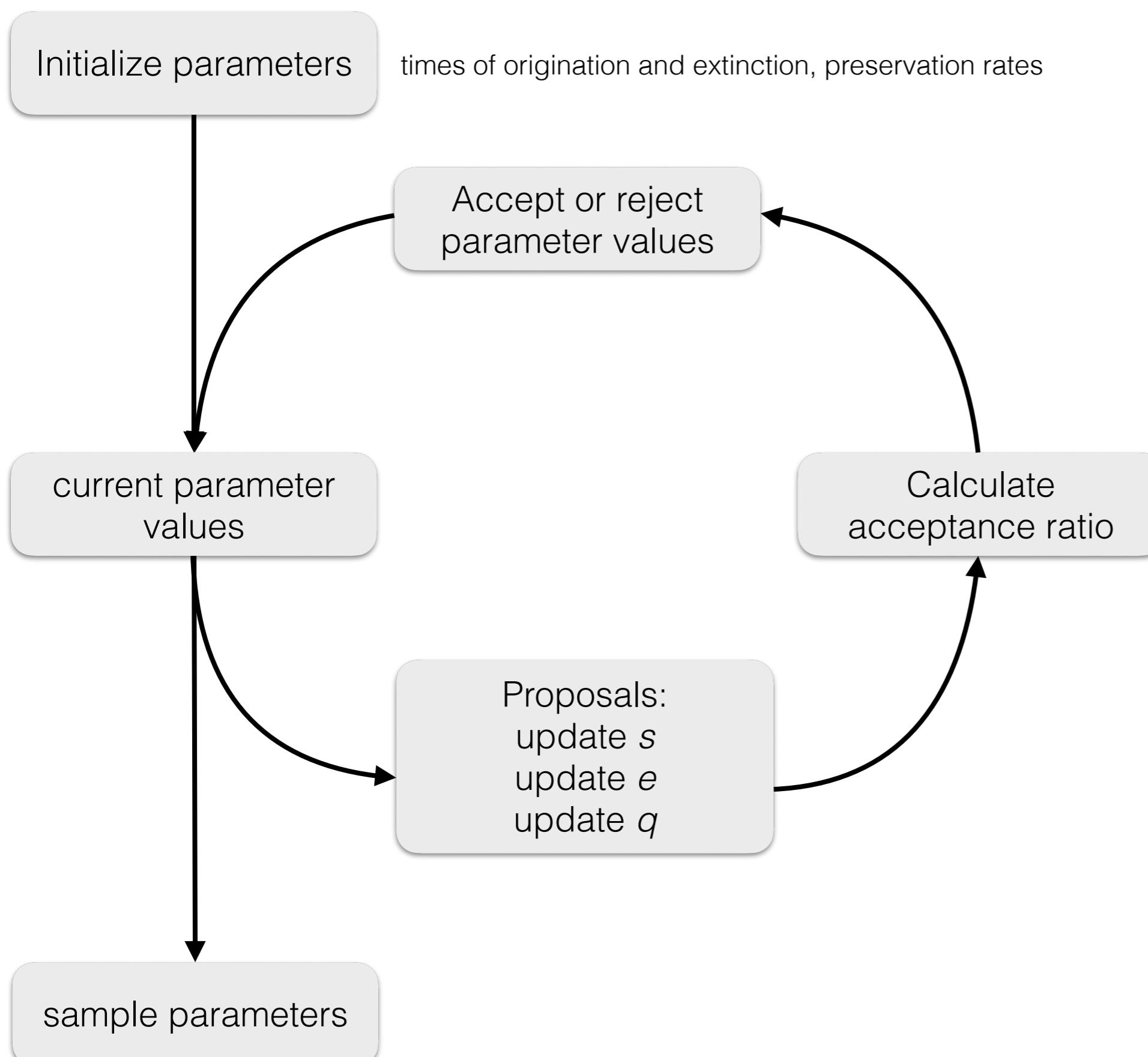
Increasing heterogeneity



# Poisson processes of preservation

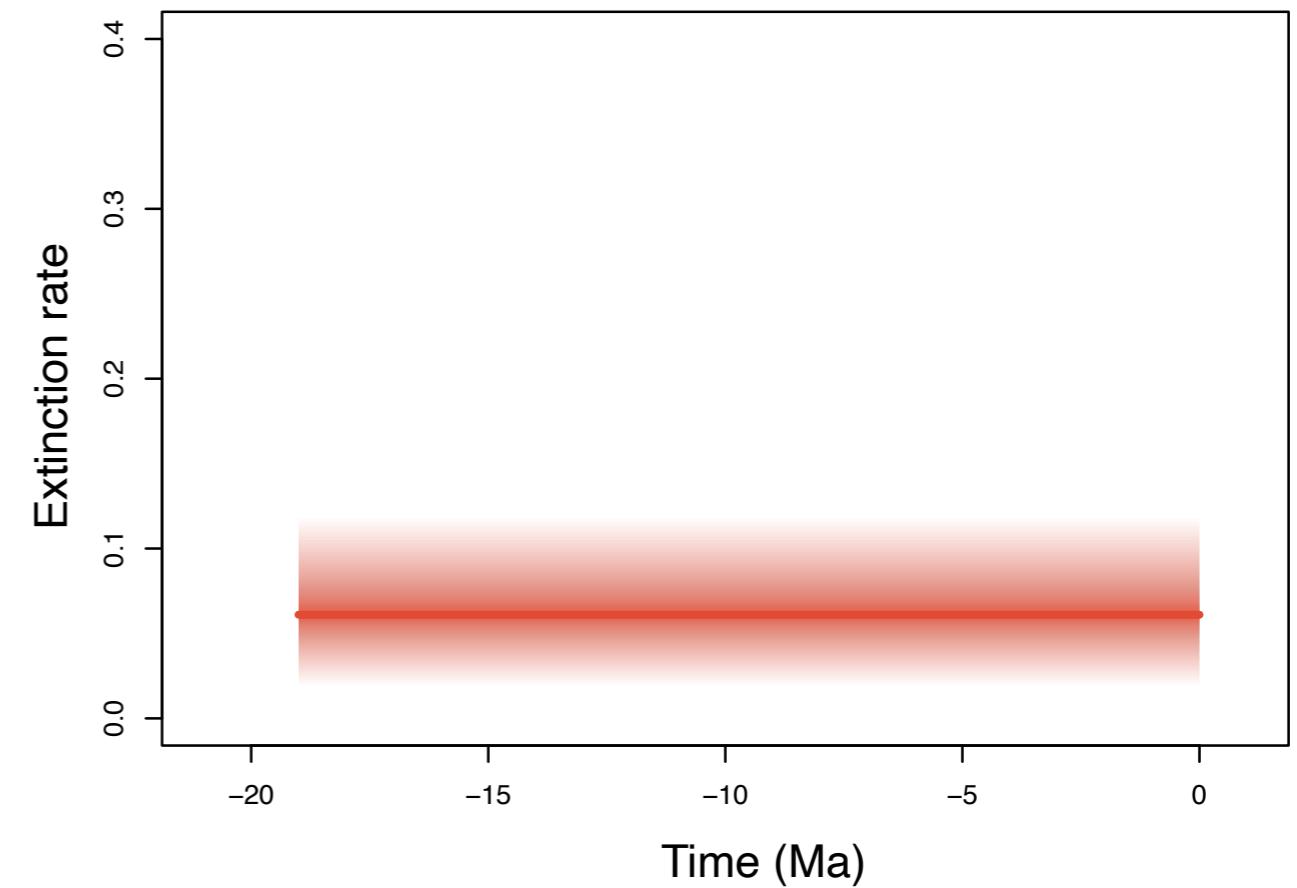
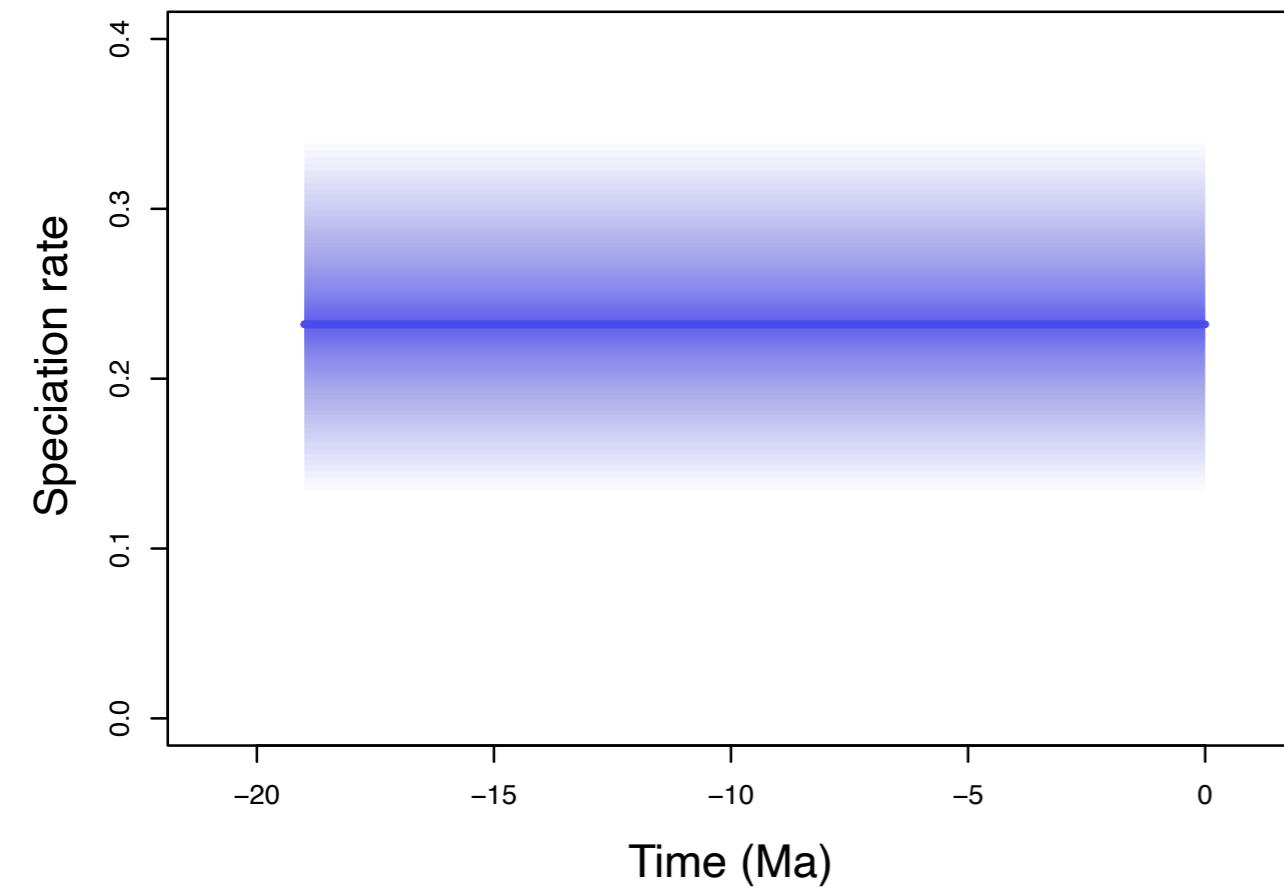


# Estimating times of origination, extinction and preservation – MCMC



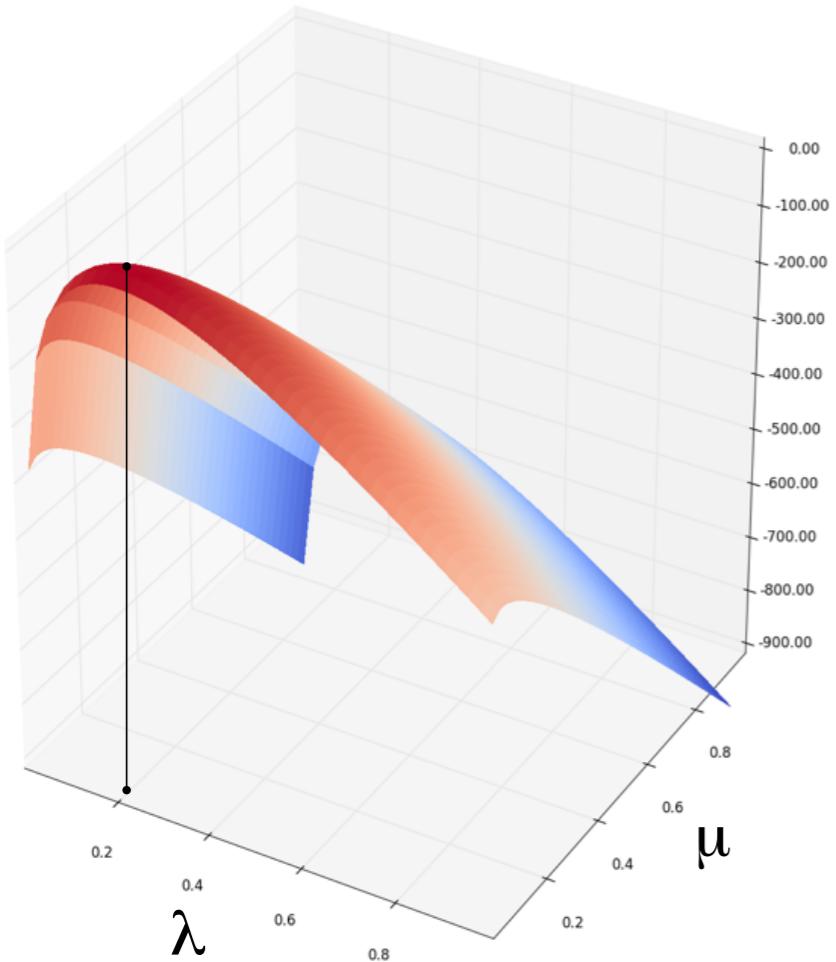
# Birth-death models: exploratory (“hypothesis-free”) algorithms

$$\underbrace{P(q, \mathbf{s}, \mathbf{e}, \lambda, \mu | X)}_{\text{posterior}} \propto \underbrace{P(X | q, \mathbf{s}, \mathbf{e})}_{\text{likelihood}} \times \underbrace{P(\mathbf{s}, \mathbf{e} | \lambda, \mu)}_{\text{birth-death prior}} \times \underbrace{P(q) P(\lambda, \mu)}_{\text{other (hyper)priors}}$$

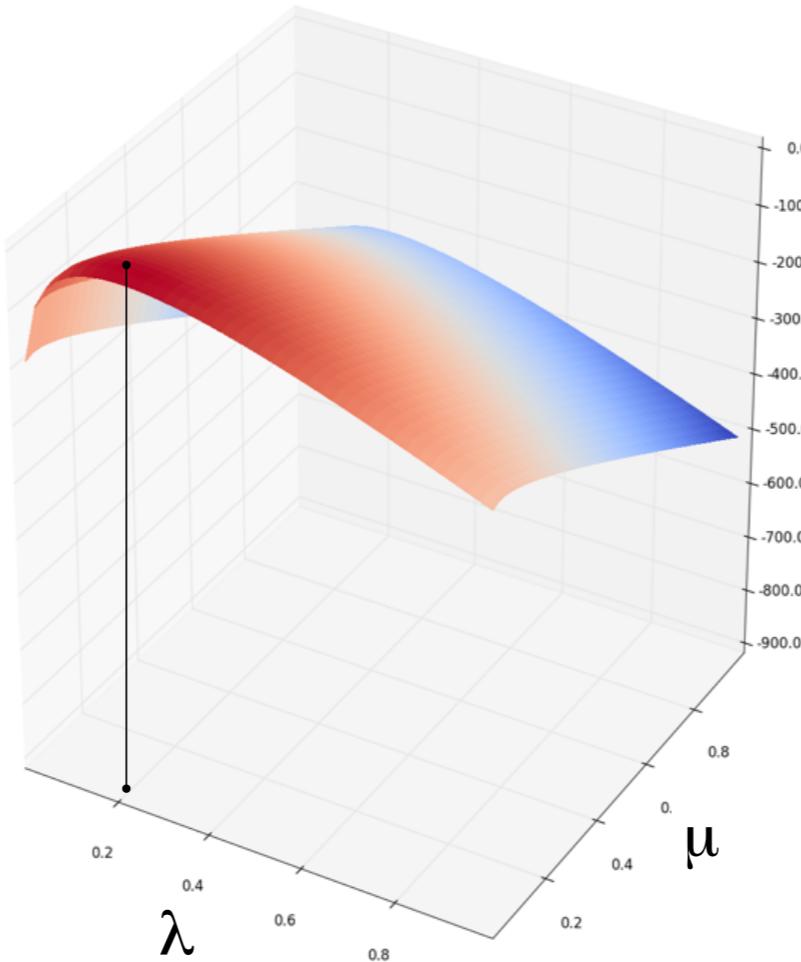


# Robustness to incomplete taxon sampling

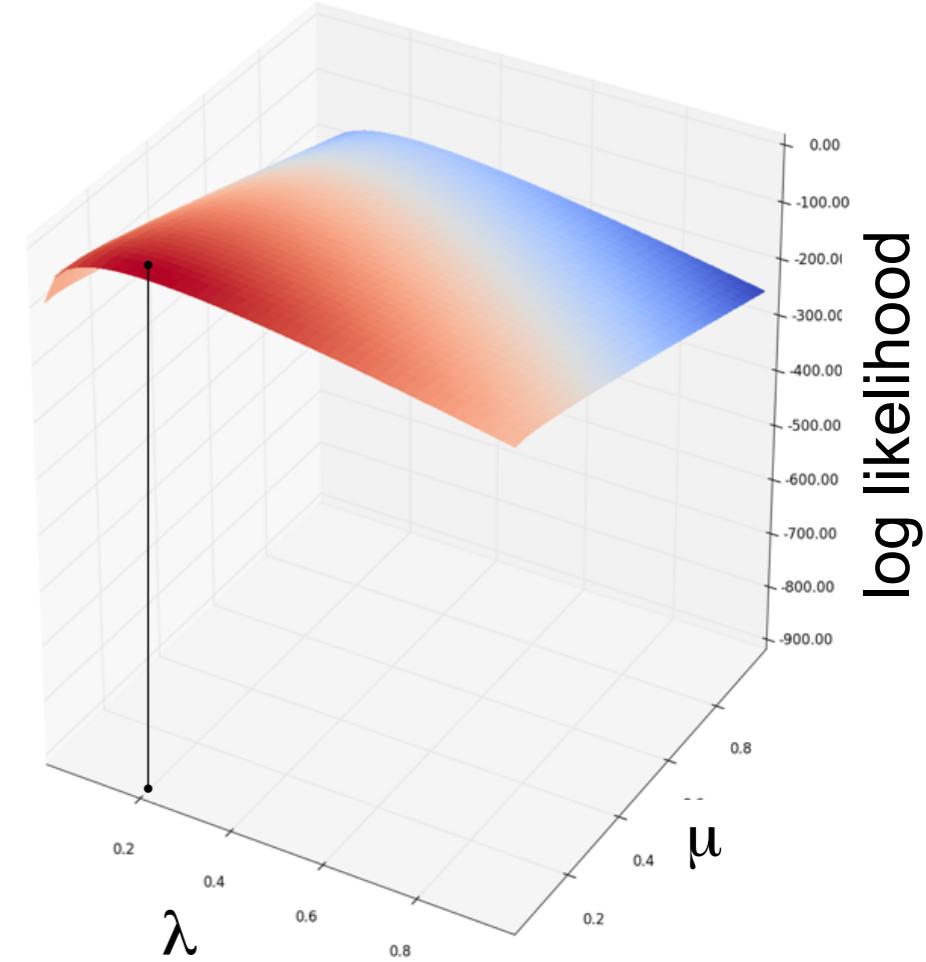
sampling: 100%



sampling: 50%



sampling: 25%

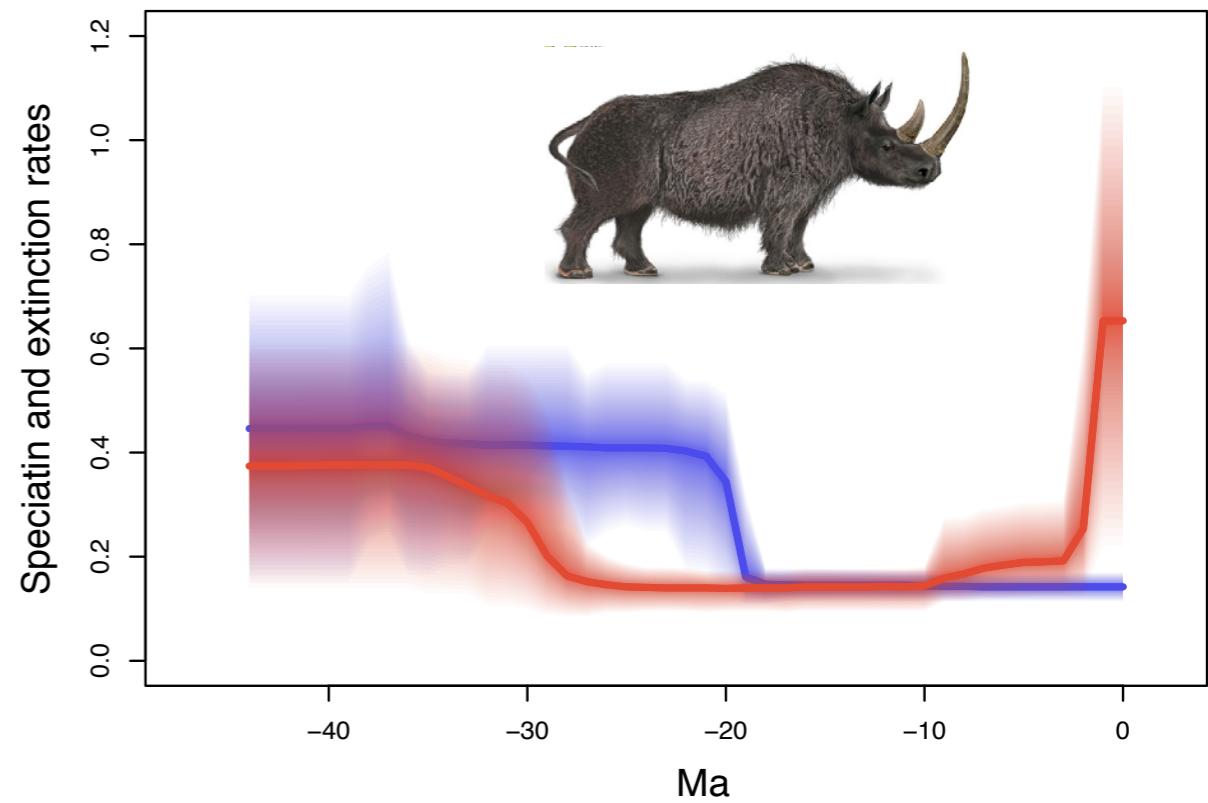
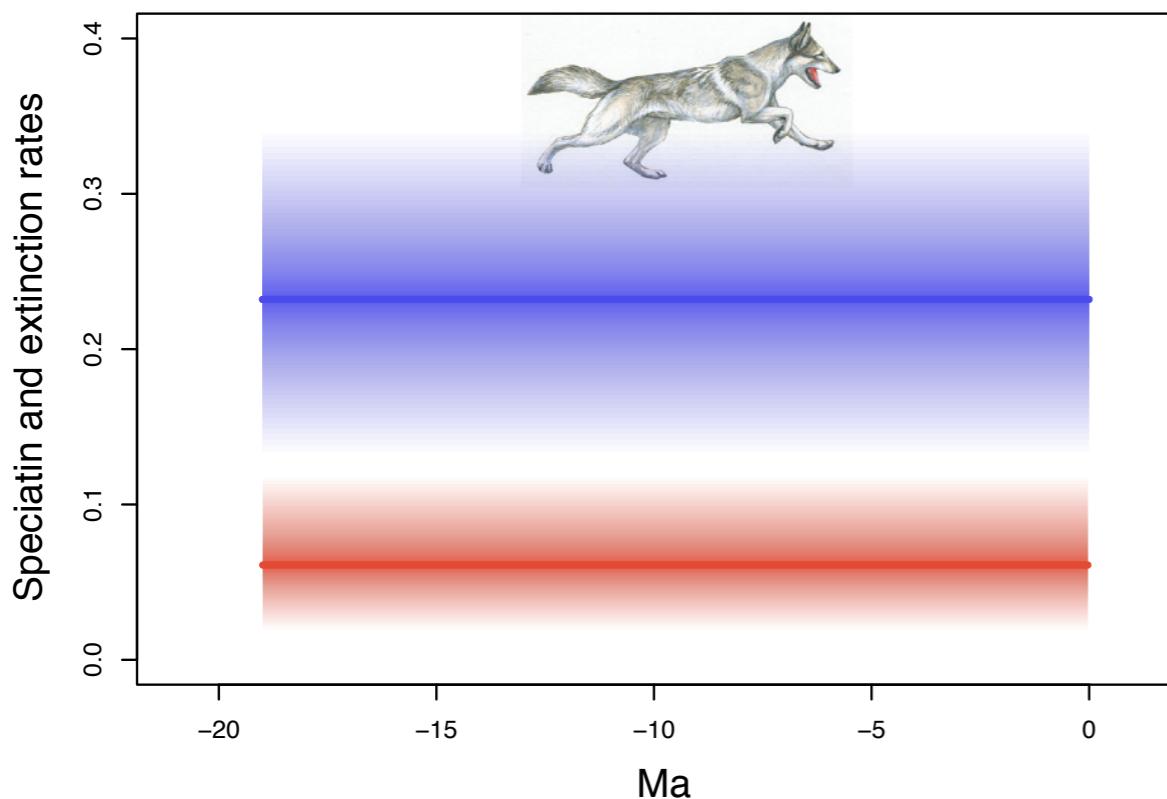


Random taxon sampling does not bias the estimation of speciation and extinction rates

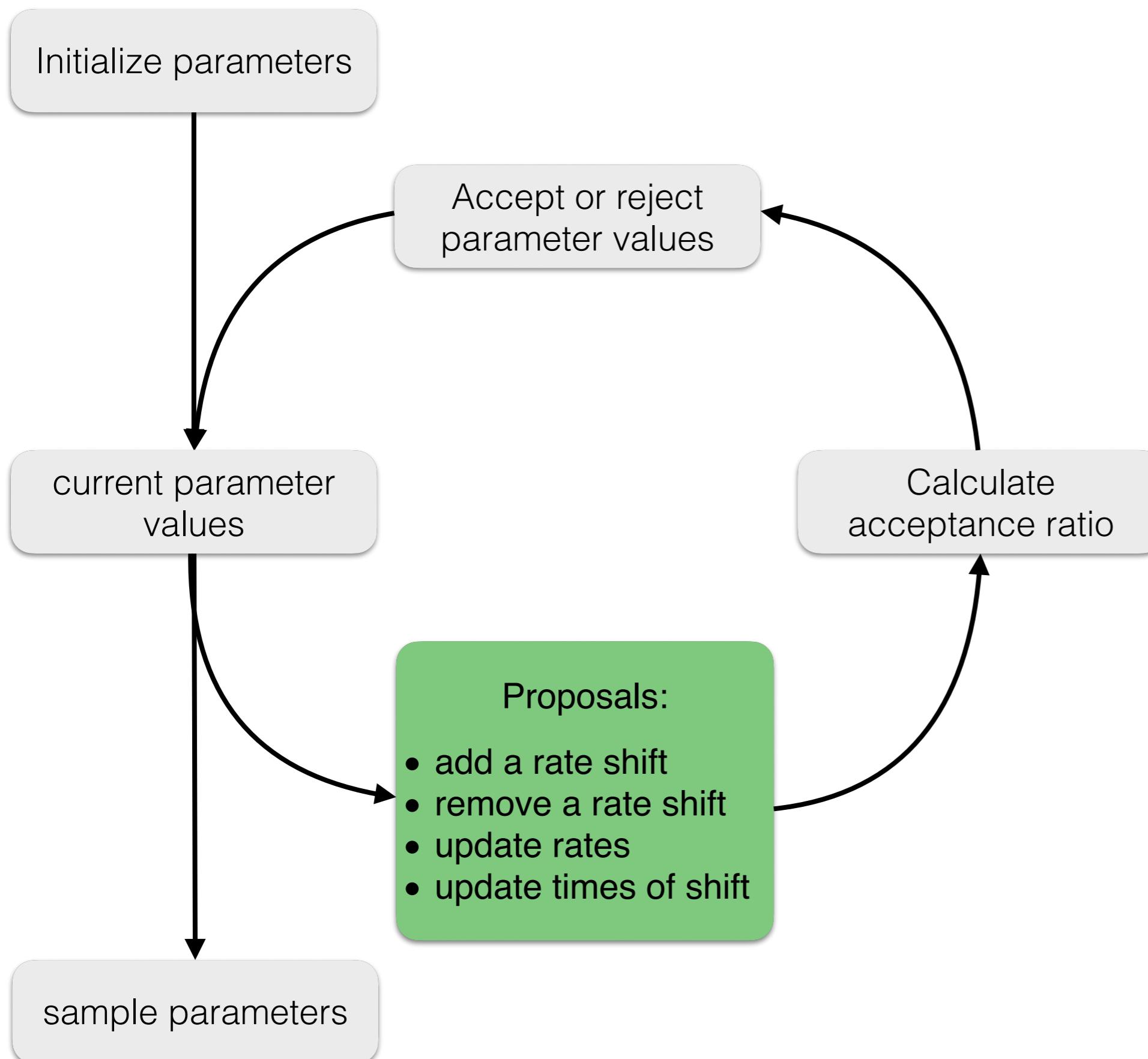
# Birth-death models: exploratory (“hypothesis-free”) algorithms

$$\underbrace{P(q, \mathbf{s}, \mathbf{e}, \lambda, \mu | X)}_{\text{posterior}} \propto \underbrace{P(X | q, \mathbf{s}, \mathbf{e})}_{\text{likelihood}} \times \underbrace{P(\mathbf{s}, \mathbf{e} | \lambda, \mu)}_{\text{birth-death prior}} \times \underbrace{P(q) P(\lambda, \mu)}_{\text{other (hyper)priors}}$$

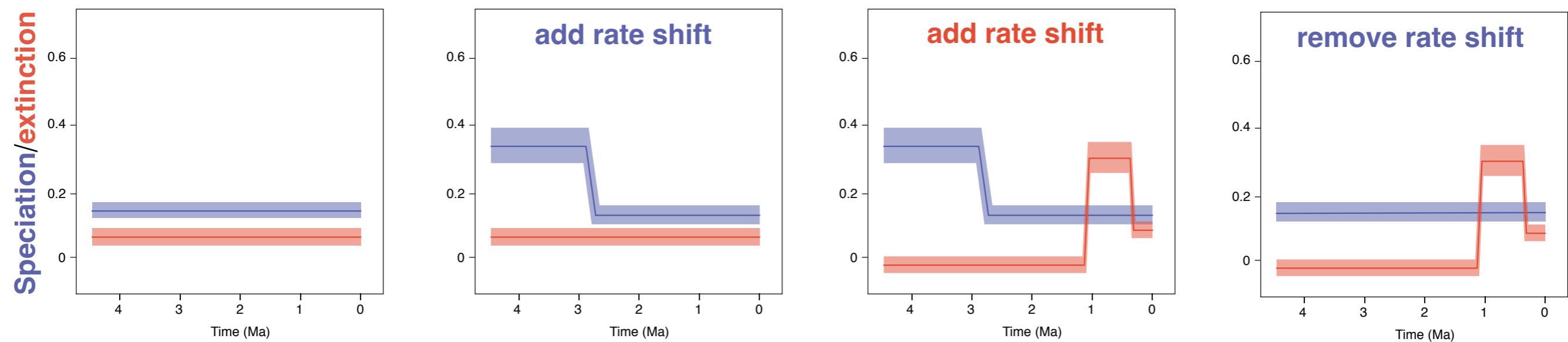
Reversible-jump  
MCMC



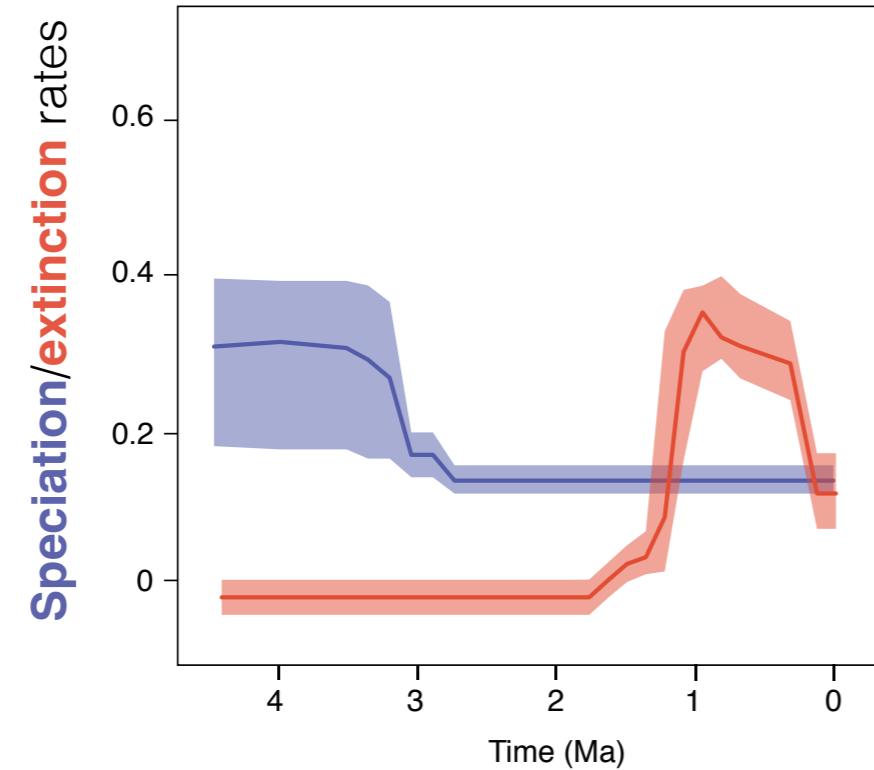
# Estimating rate heterogeneity through time – RJMCMC



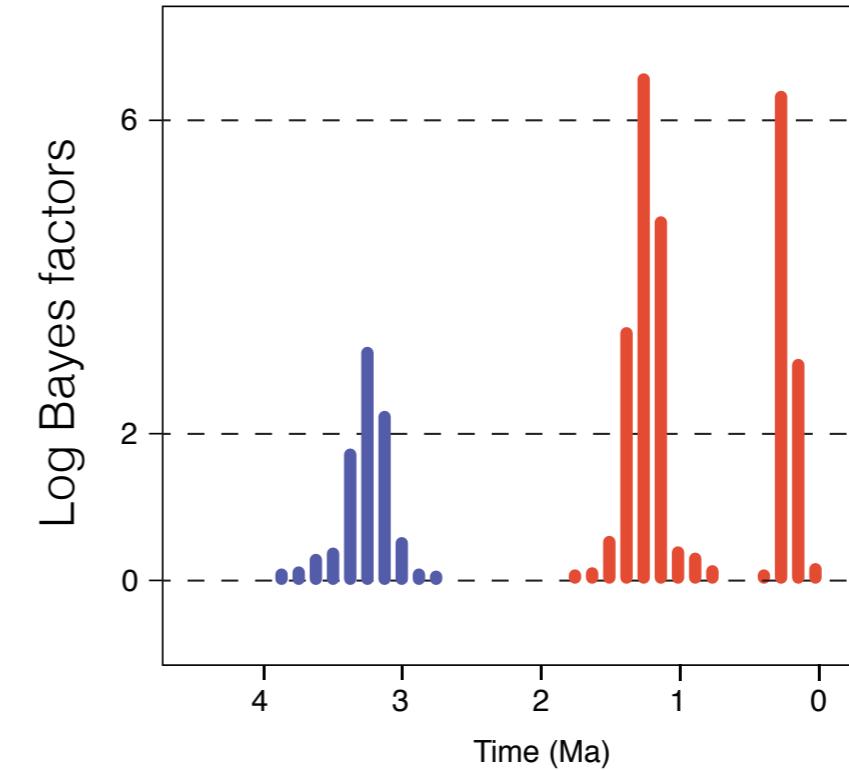
# Estimating rate heterogeneity through time – RJMCMC



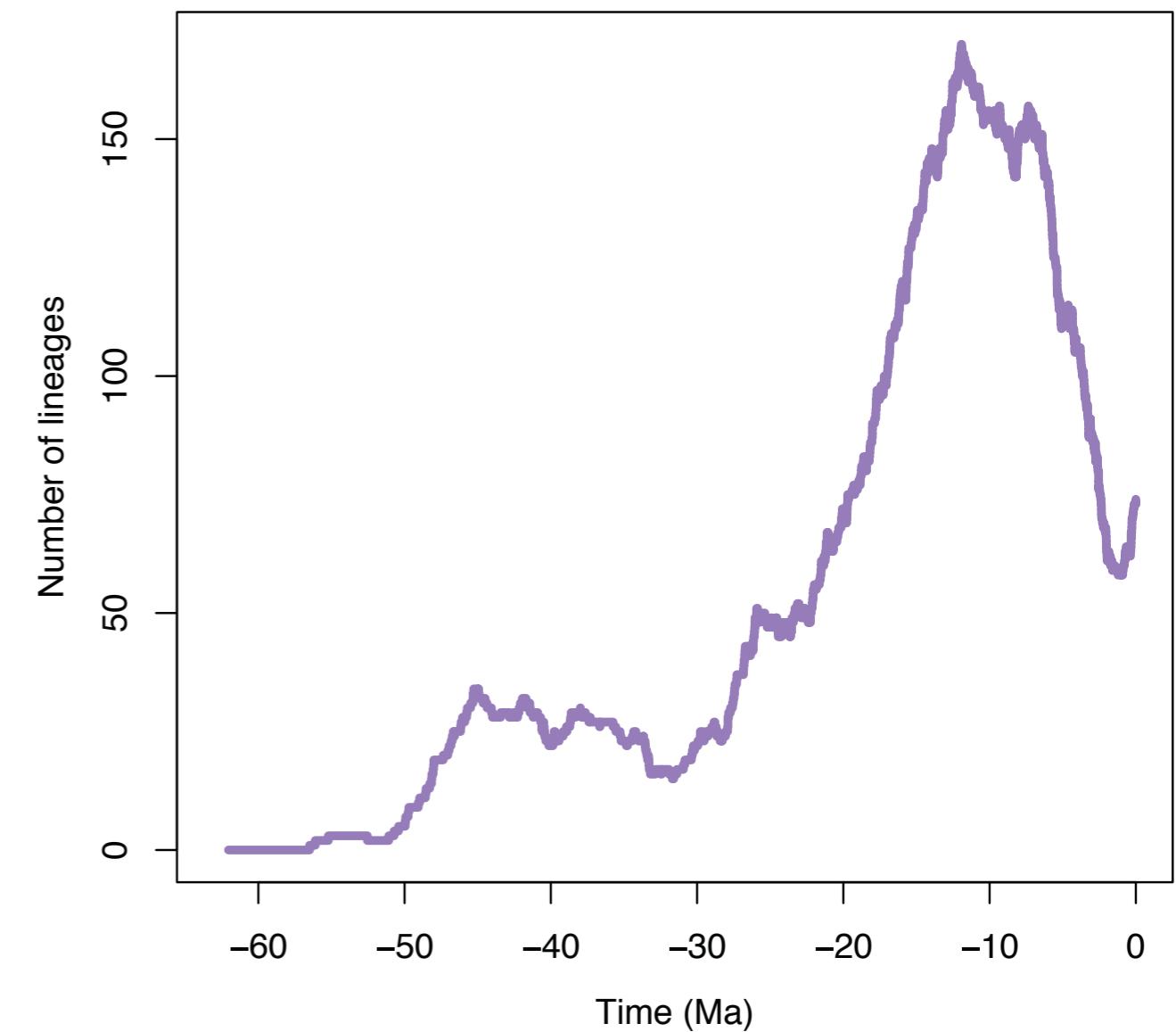
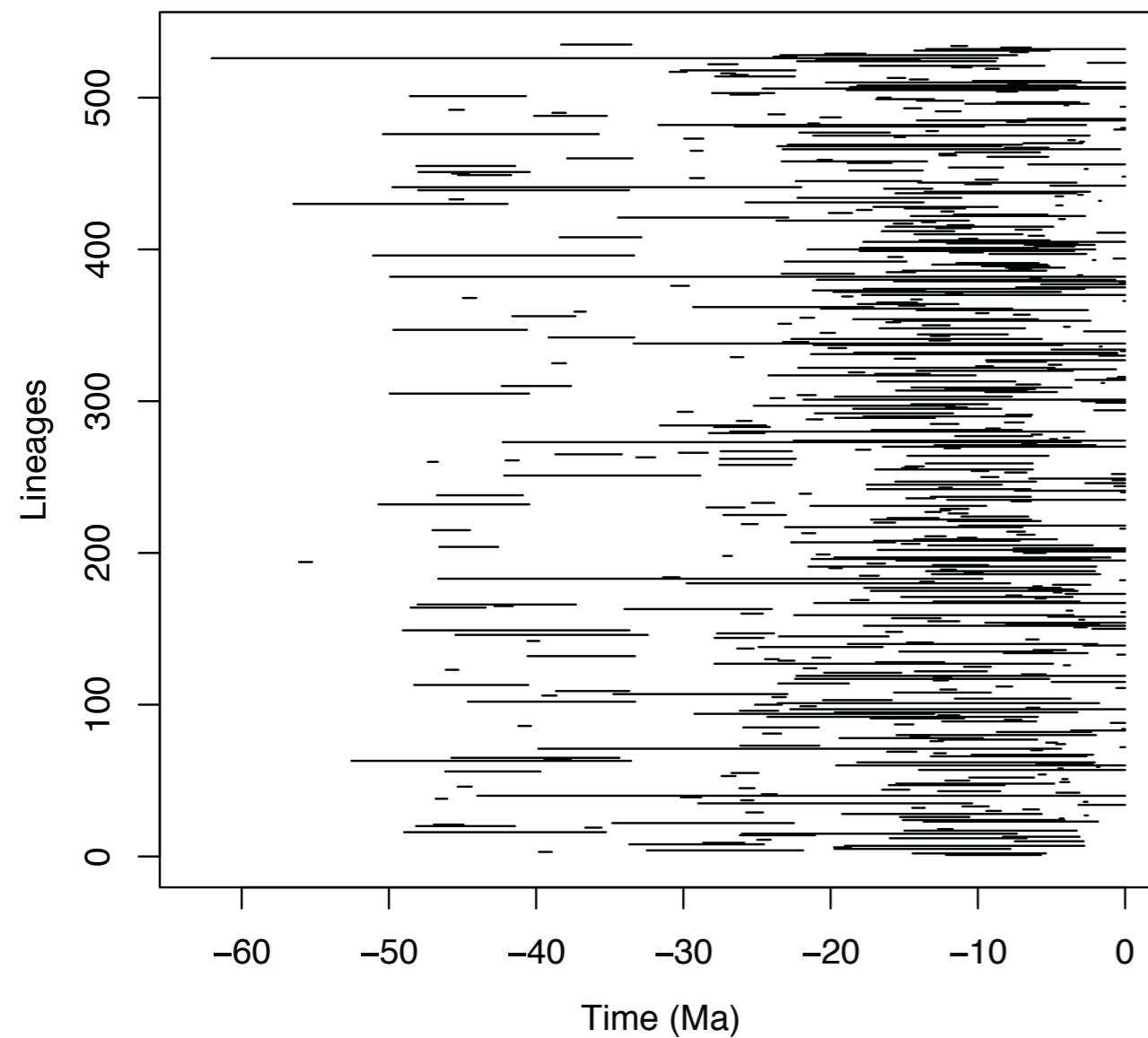
**Marginal rates**



**Times of rate shifts**



# Diversification and extinction of marine mammals



C Pimiento

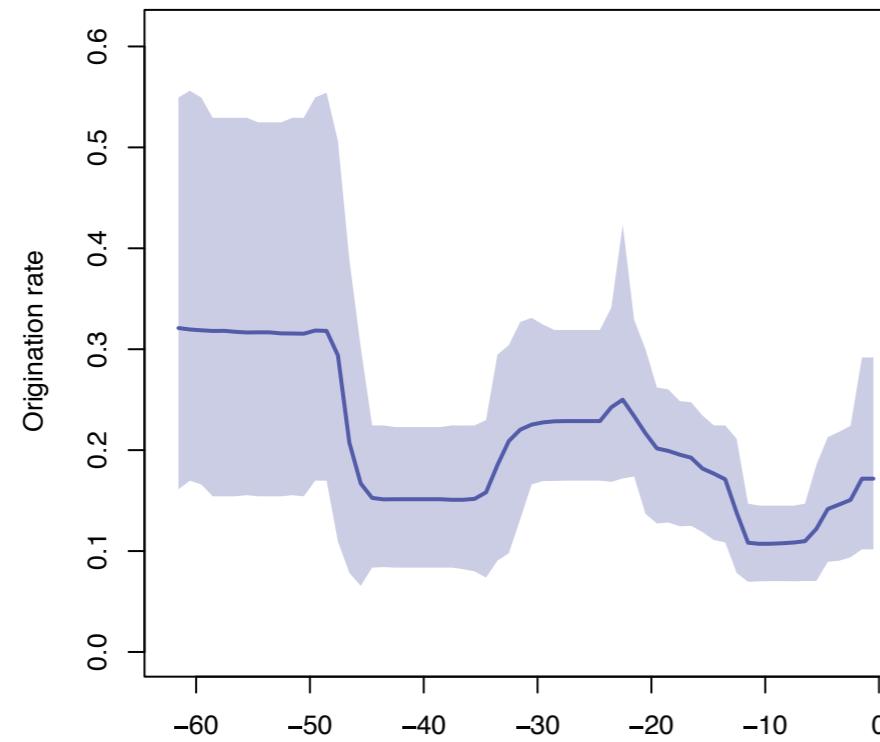


Pimiento et al. 2017 Nat Ecol Evol

# Diversification and extinction of marine mammals

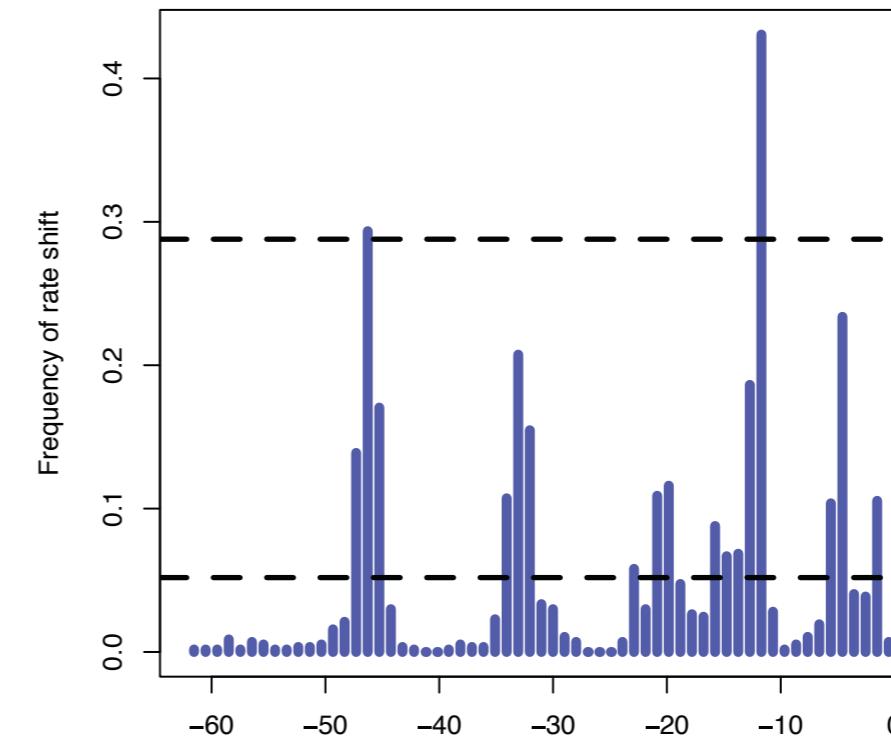
Marginal origination  
and extinction rates

Origination

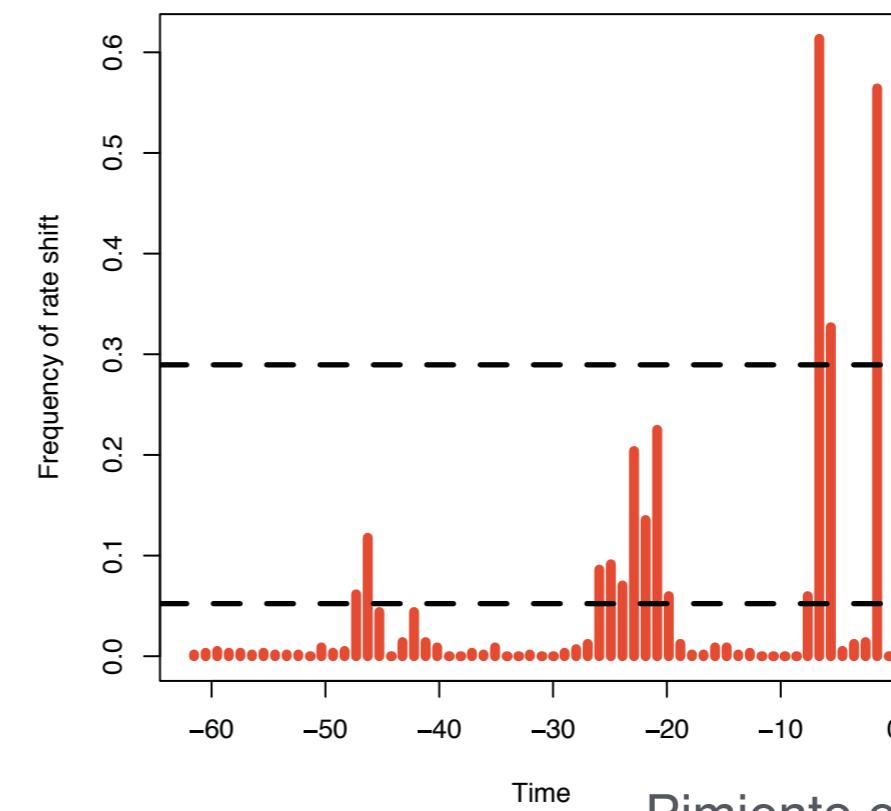
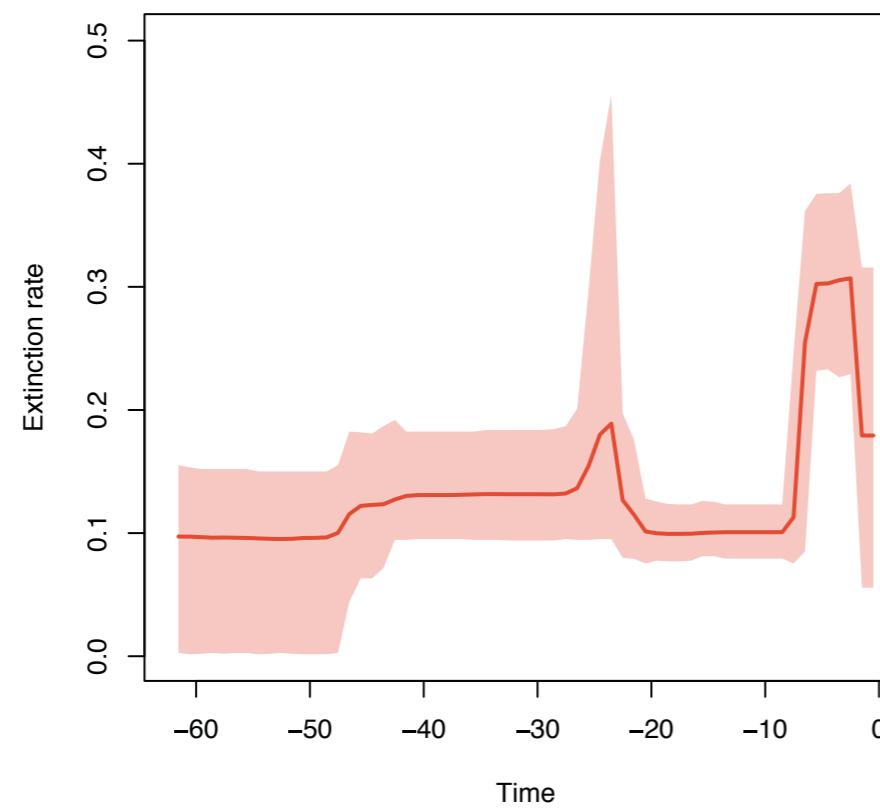


Estimated times  
of rate shift

Strong support for  
a rate shift



Extinction



## Hyper-prior distributions on the rate parameters

$$P(s, e, q, \lambda, \mu | x) \propto P(x | s, e, q) \times P(s, e | \lambda, \mu) \times P(q)P(\lambda, \mu)$$

POSTERIOR

LIKELIHOOD

PRIOR

HYPERPRIORS



Gamma prior on preservation rate



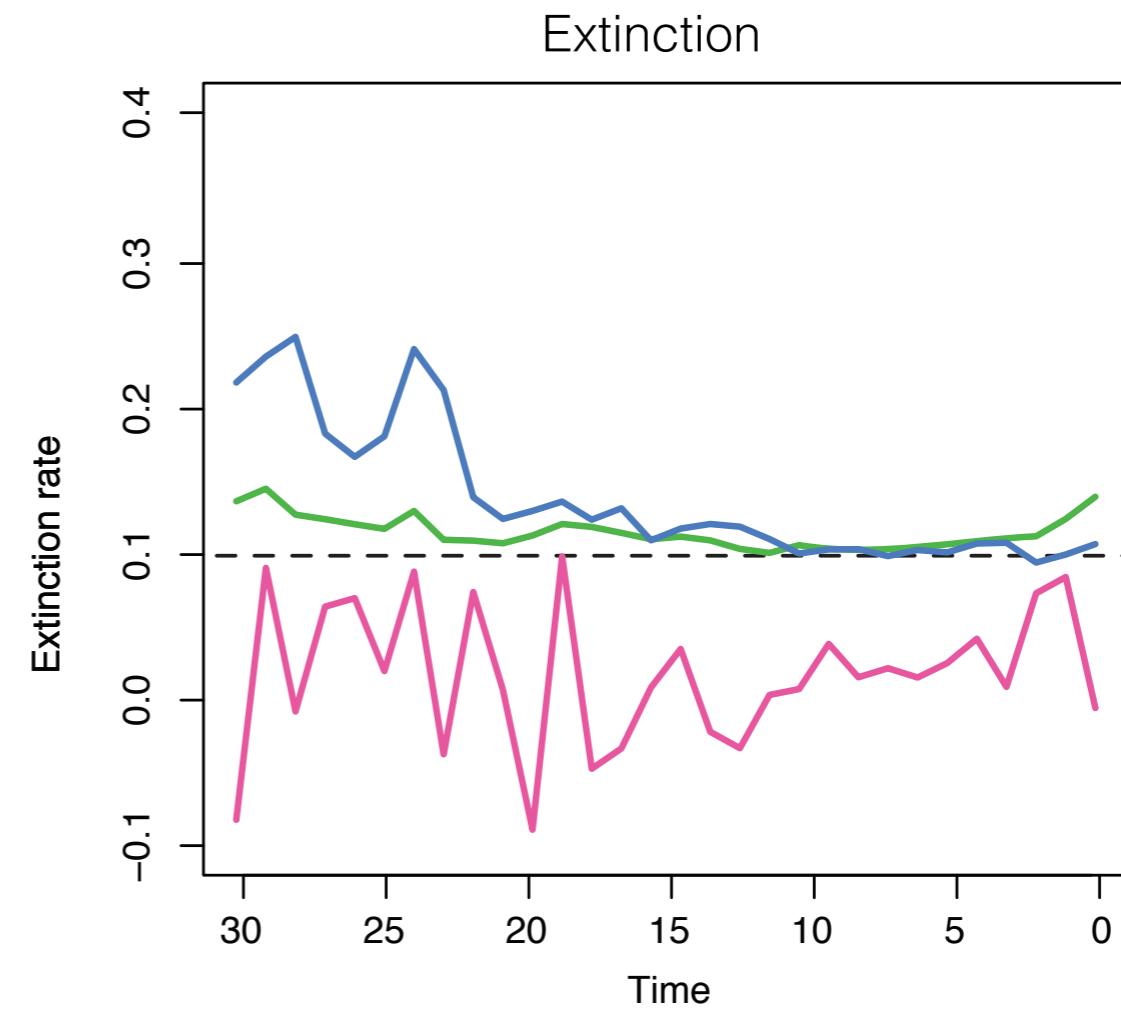
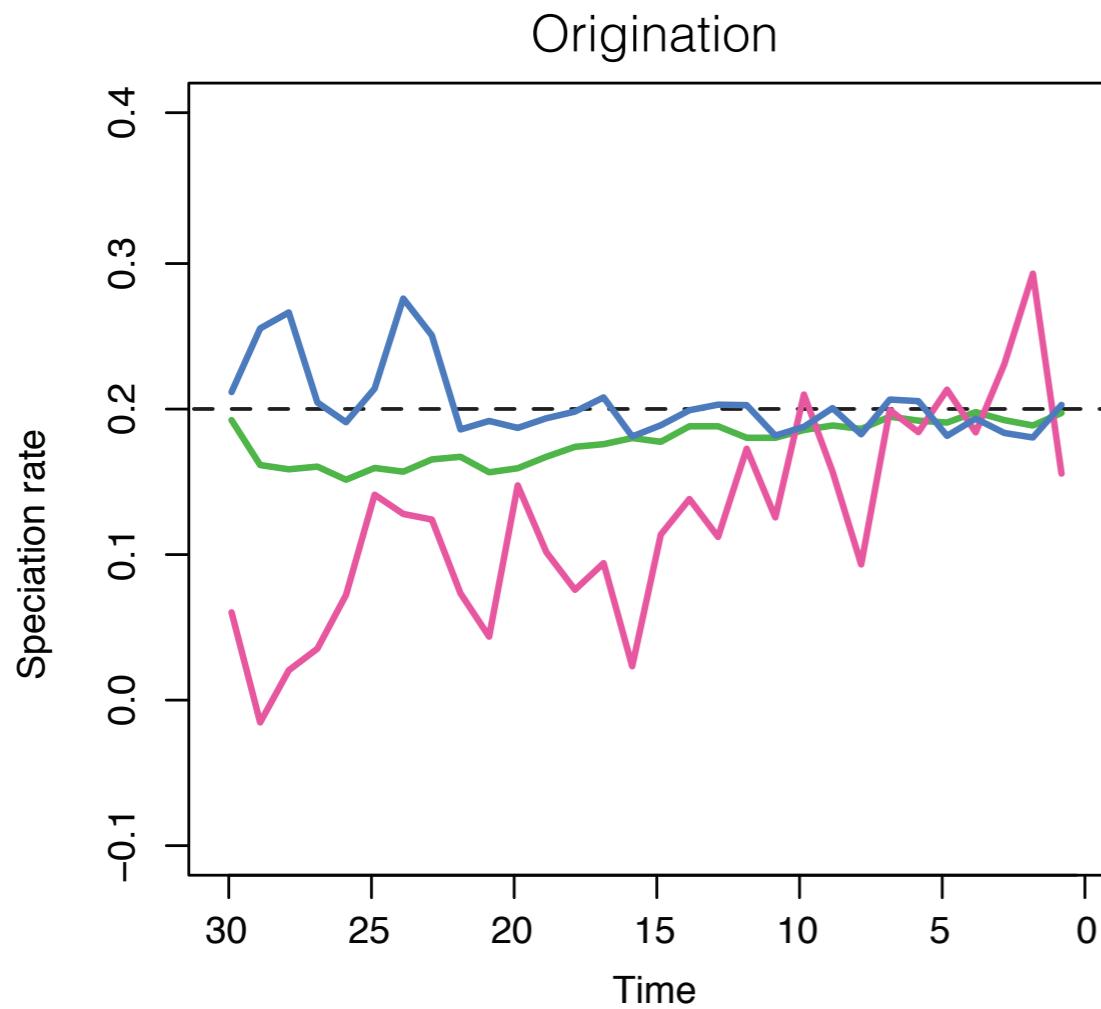
Uniform prior on alpha (rate heterogeneity)



Gamma or half-Cauchy priors on BD rates

# Effects of PyRate's regularization through RJMCMC

Constant birth-death simulation with constant, low preservation



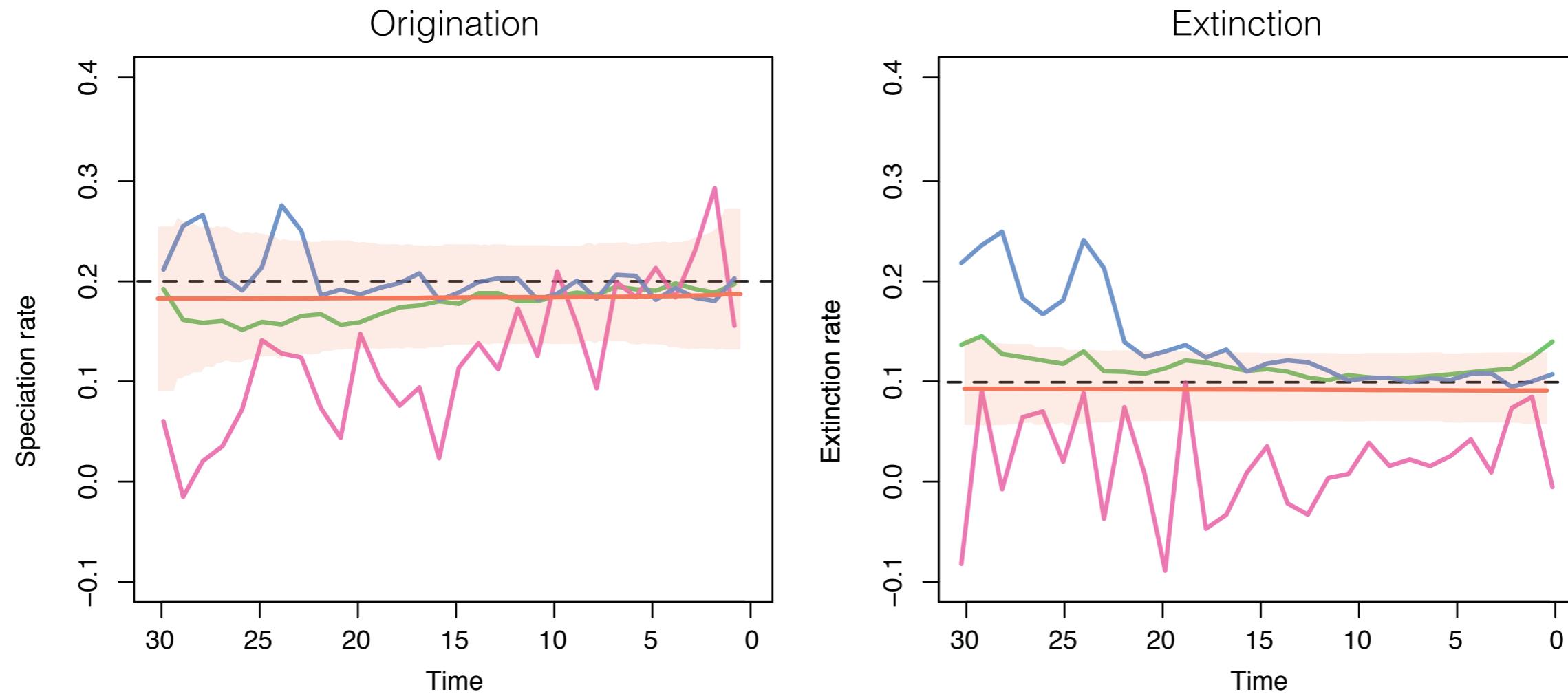
Mean per capita (Foote 2000)  
Three-timer (Alroy 2008)  
CMR (Liow & Finarelli 2014)



T M Smiley

# Effects of PyRate's regularization through RJMCMC

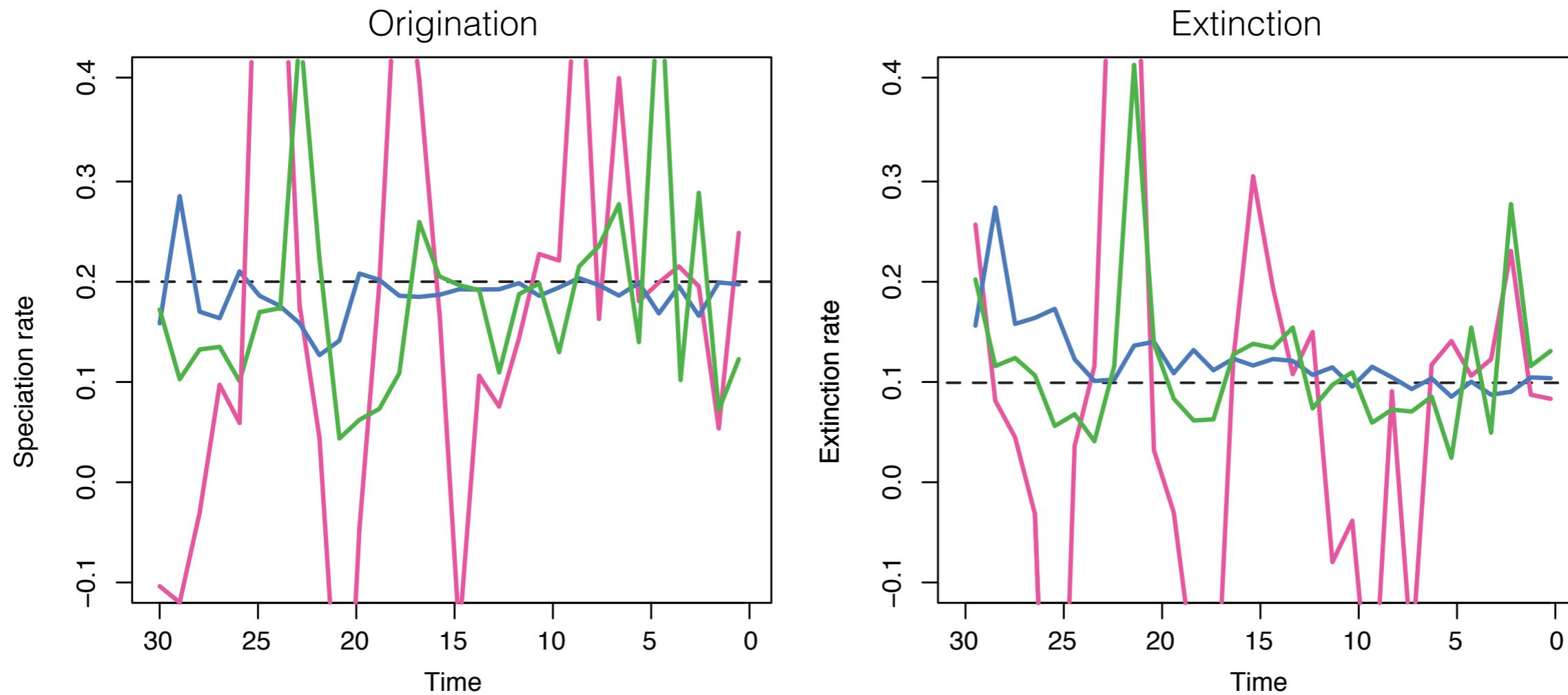
Constant birth-death simulation with constant, low preservation



Mean per capita (Foote 2000)  
Three-timer (Alroy 2008)  
CMR (Liow & Finarelli 2014)  
PyRate

# Effects of PyRate's regularization through RJMCMC

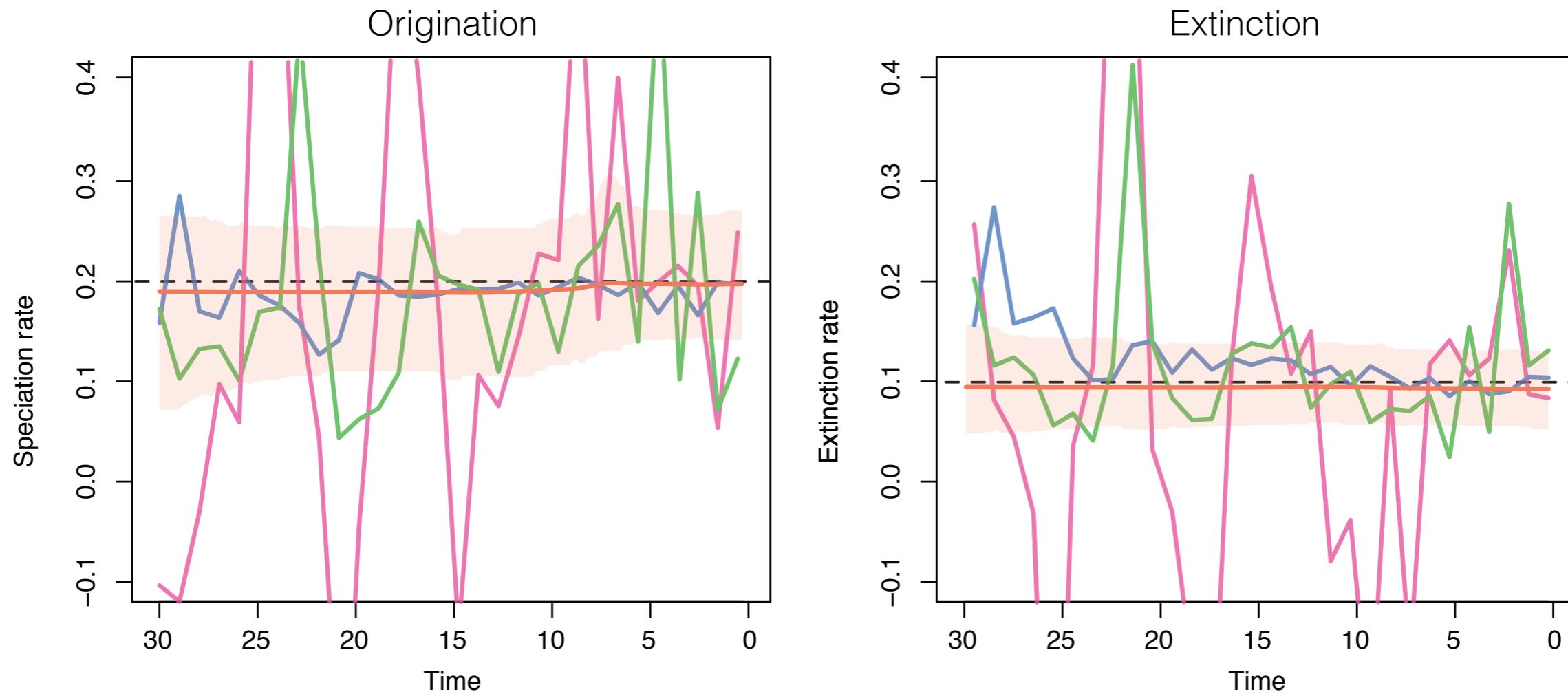
Constant birth-death simulation with time-variable preservation



Mean per capita (Foote 2000)  
Three-timer (Alroy 2008)  
CMR (Liow & Finarelli 2014)

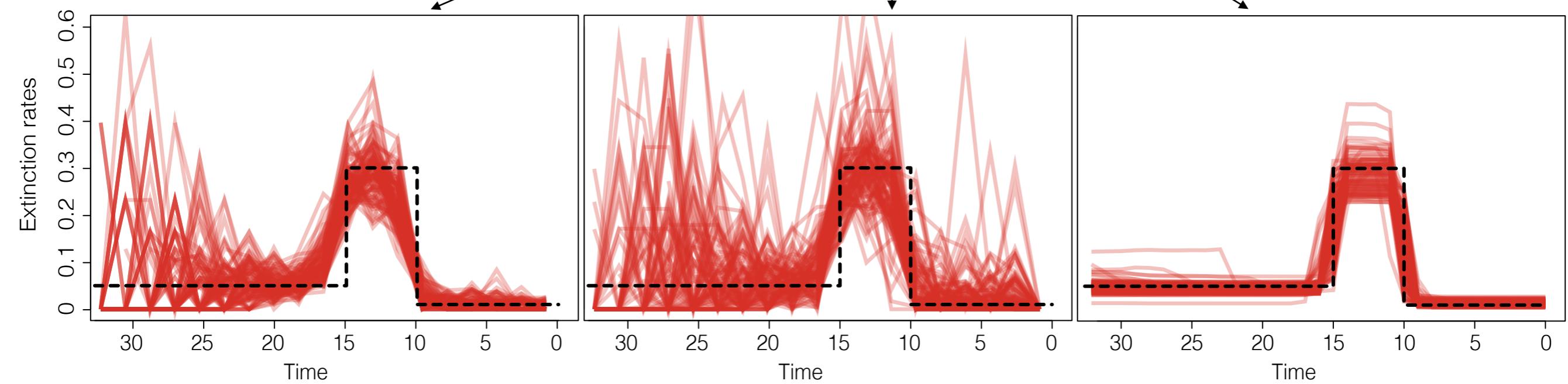
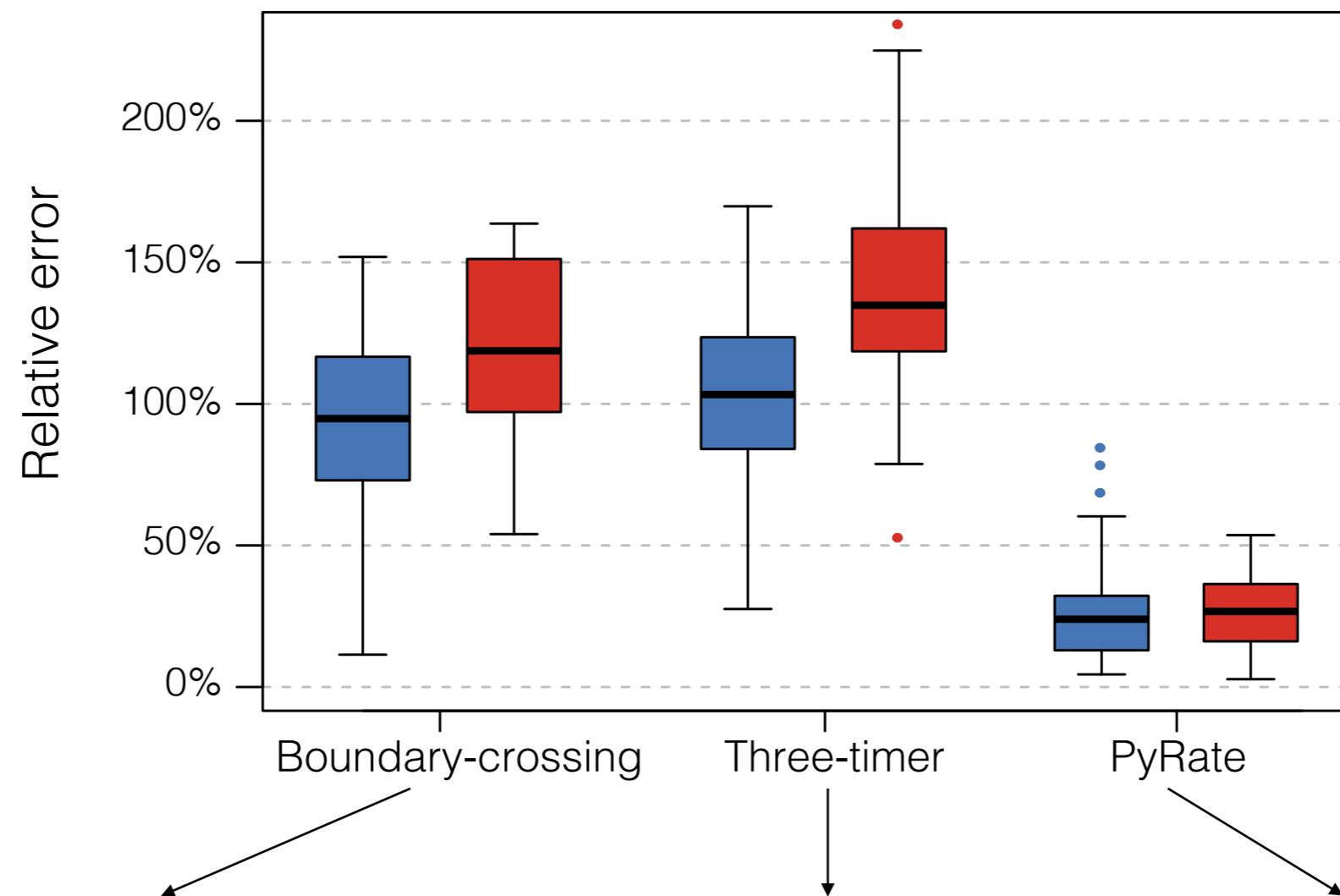
# Effects of PyRate's regularization through RJMCMC

Constant birth-death simulation with time-variable preservation

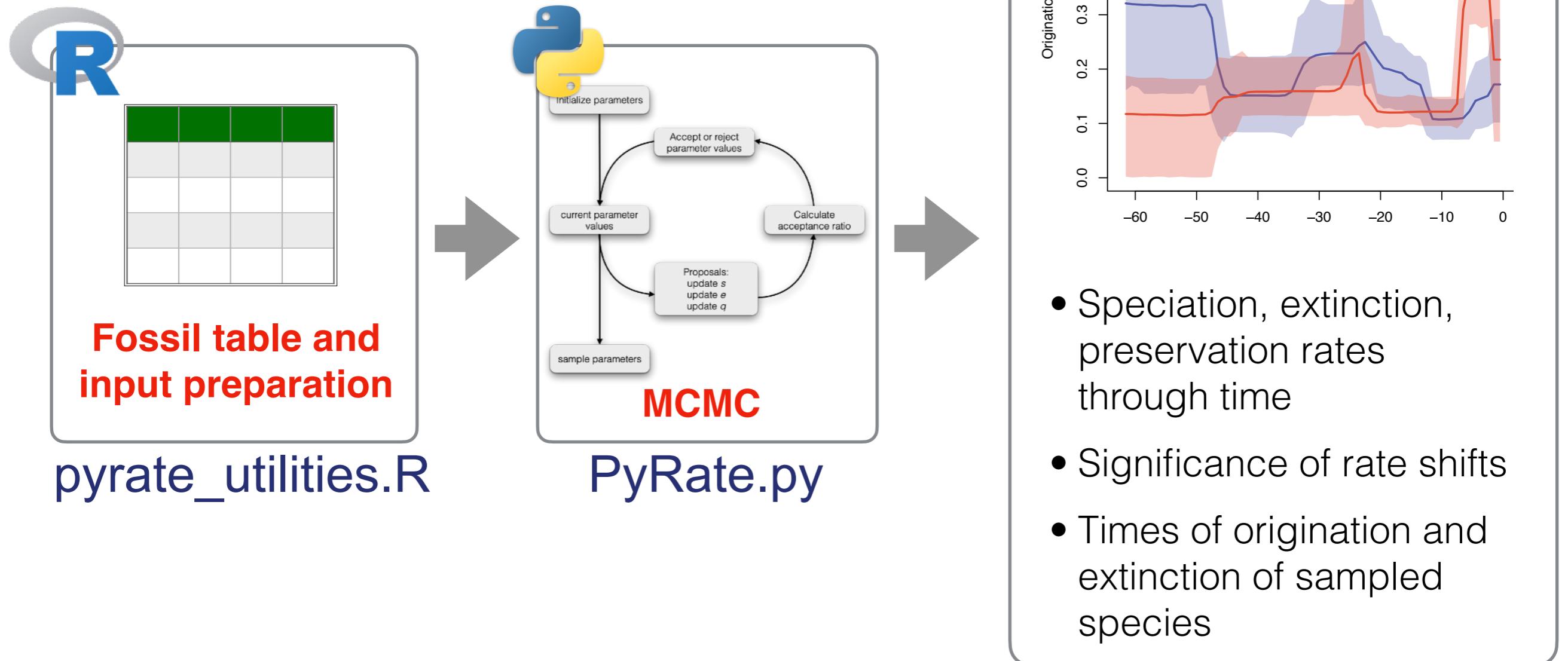


Mean per capita (Foote 2000)  
Three-timer (Alroy 2008)  
CMR (Liow & Finarelli 2014)  
PyRate

# Error in estimated **Speciation** & **extinction** rates



# How PyRate works



# How PyRate works

Taxon	Status	MinAge	MaxAge
<i>Ailurus</i>	Extant	0.6	1.3
<i>Alopecocyon</i>	Extinct	9.5	10.35
<i>Alopecocyon</i>	Extinct	9.7	11.11
...	...	...	...

Fossil occurrence data with age range and info on whether it's extant or extinct



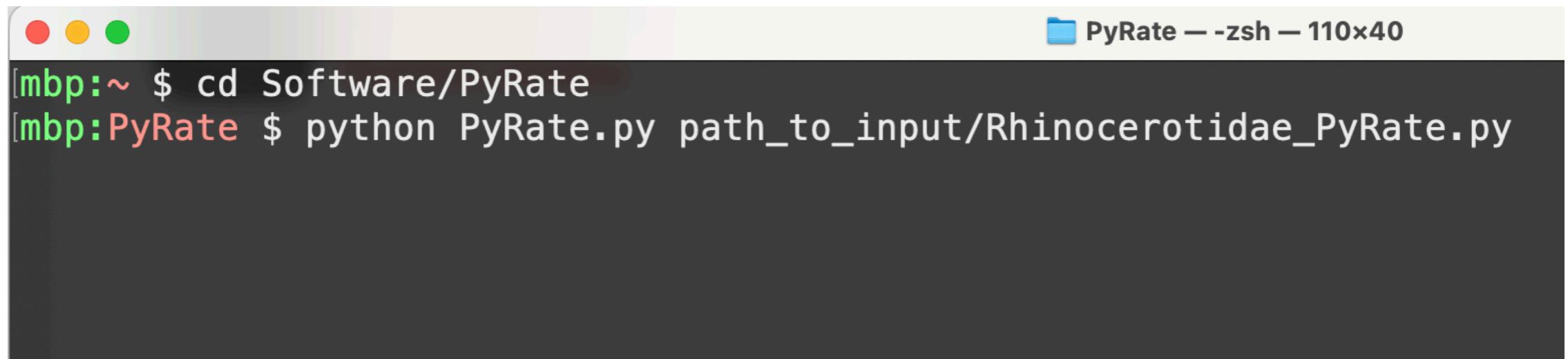
**Parse fossil table and create PyRate input file**

```
1 # load PyRate utilities
2
3 source("../PyRate-master/pyrate_utilities.r")
4 ▼ extract.ages(file = "path_to_your_fossil_table/Ursidae.txt",
5   replicates = 10, # specify number of age randomizations
6   cutoff = 10 # drop occurrences with wide age range (e.g. 10 myr)
7 ▲
8
9 )
```

# How PyRate works



**Run PyRate in a  
Terminal window**

A screenshot of a macOS Terminal window titled "PyRate — -zsh — 110x40". The window shows the command line interface with the following text:

```
[mbp:~ $ cd Software/PyRate
[mbp:PyRate $ python PyRate.py path_to_input/Rhinocerotidae_PyRate.py
```

The window has the standard OS X title bar with red, yellow, and green buttons, and the terminal interface is in light mode.

This will run a default analysis with RJMCMC

# How PyRate works



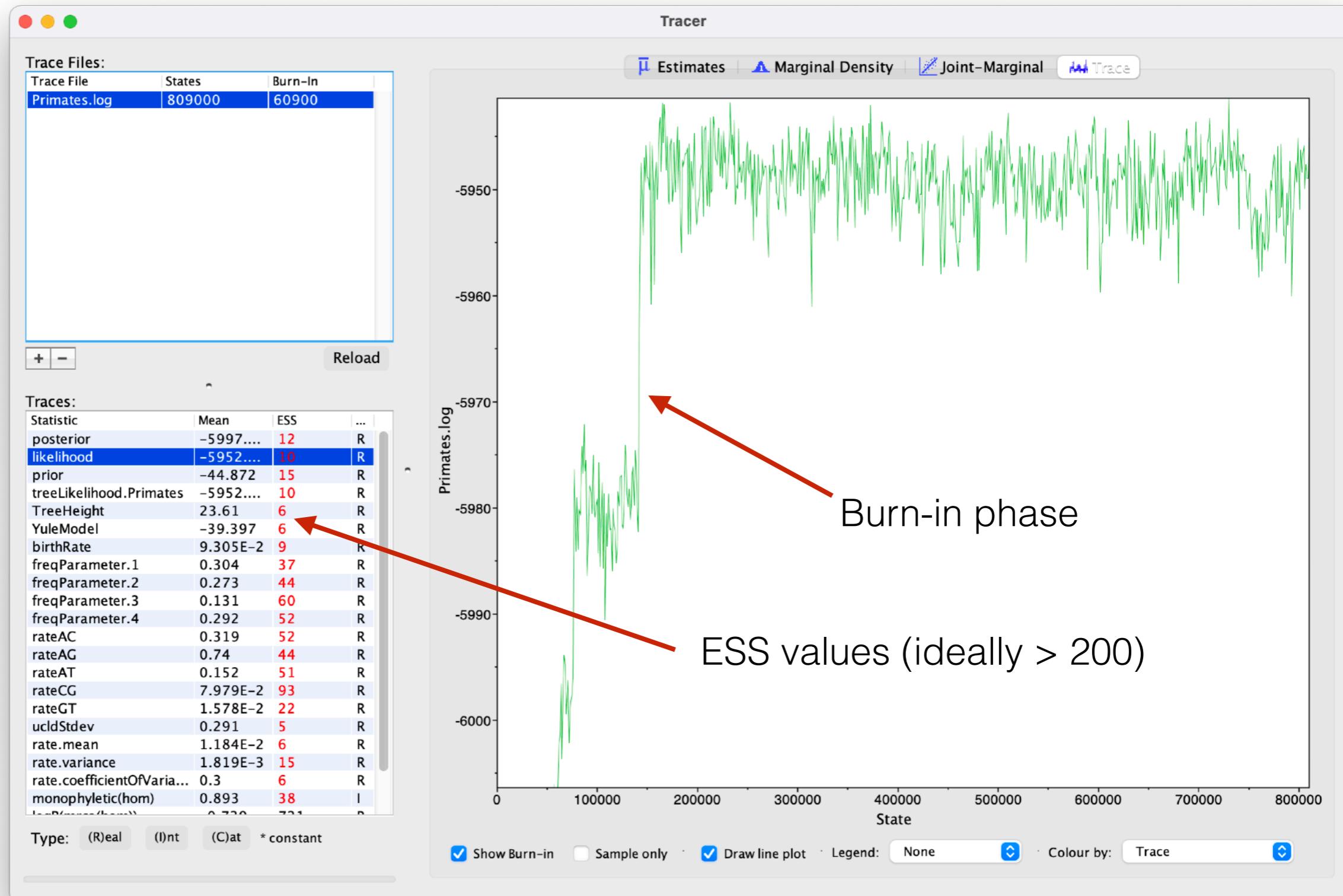
**Add options to the analysis: time variable preservation rates**

```
mbp:~ $ cd Software/PyRate
mbp:PyRate $ python PyRate.py path_to_input/Rhinocerotidae_PyRate.py -qShift path_to_file/epochs.txt
```

Simple text file defining the time bins for piece-wise constant preservation rates

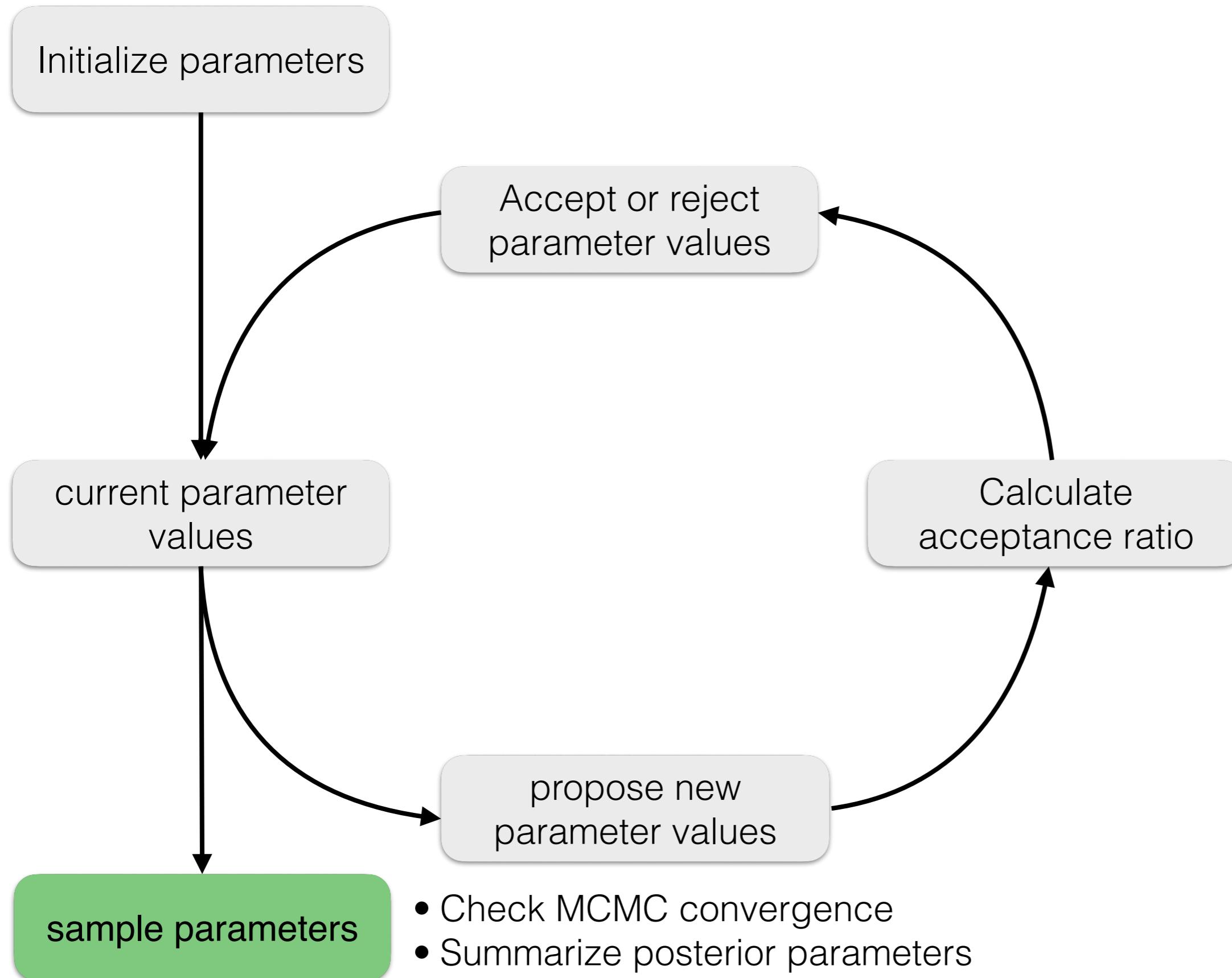
2.58
5.333
23.03
33.9
56.0
66.0

# Check MCMC convergence

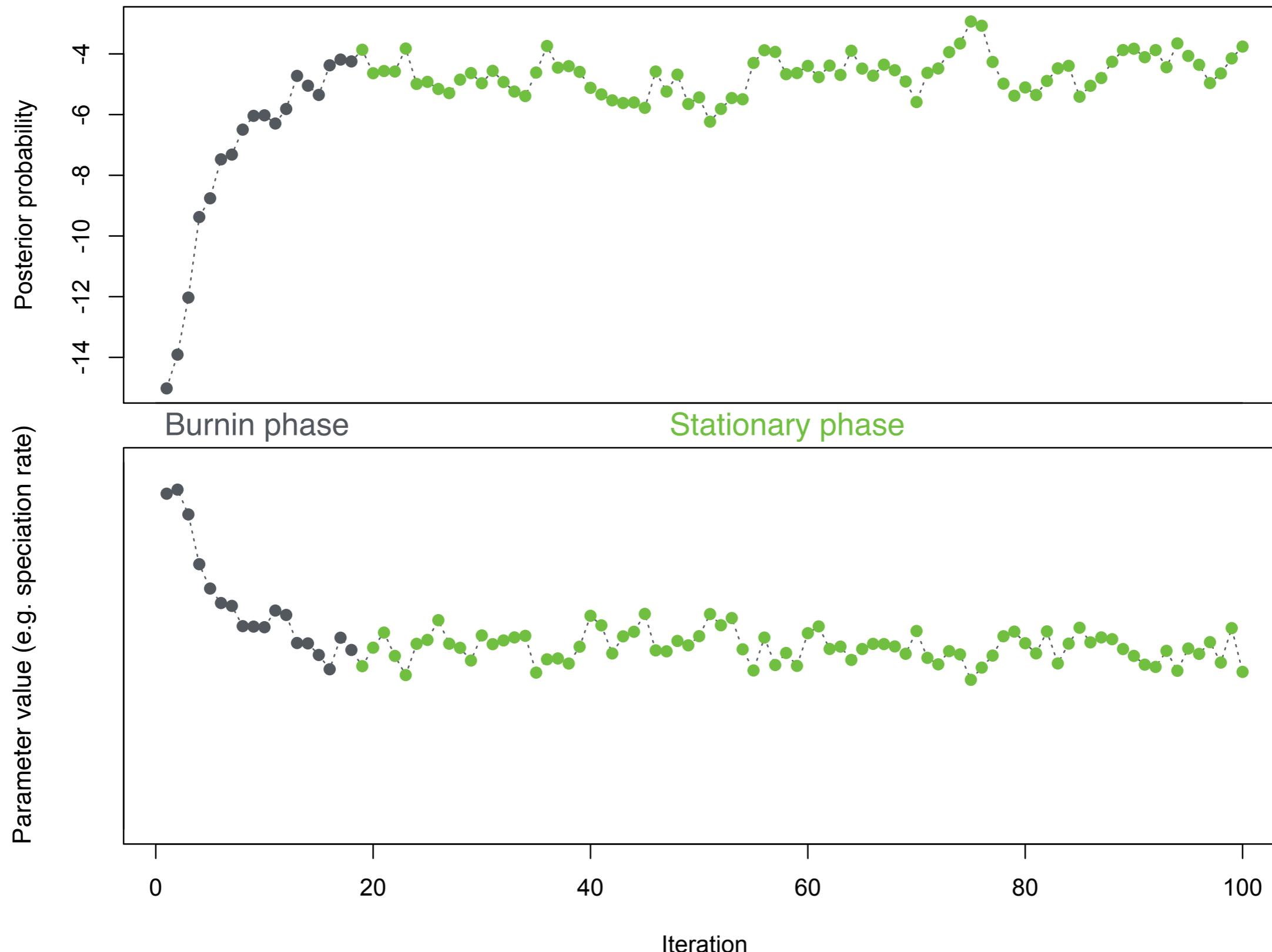


Tracer

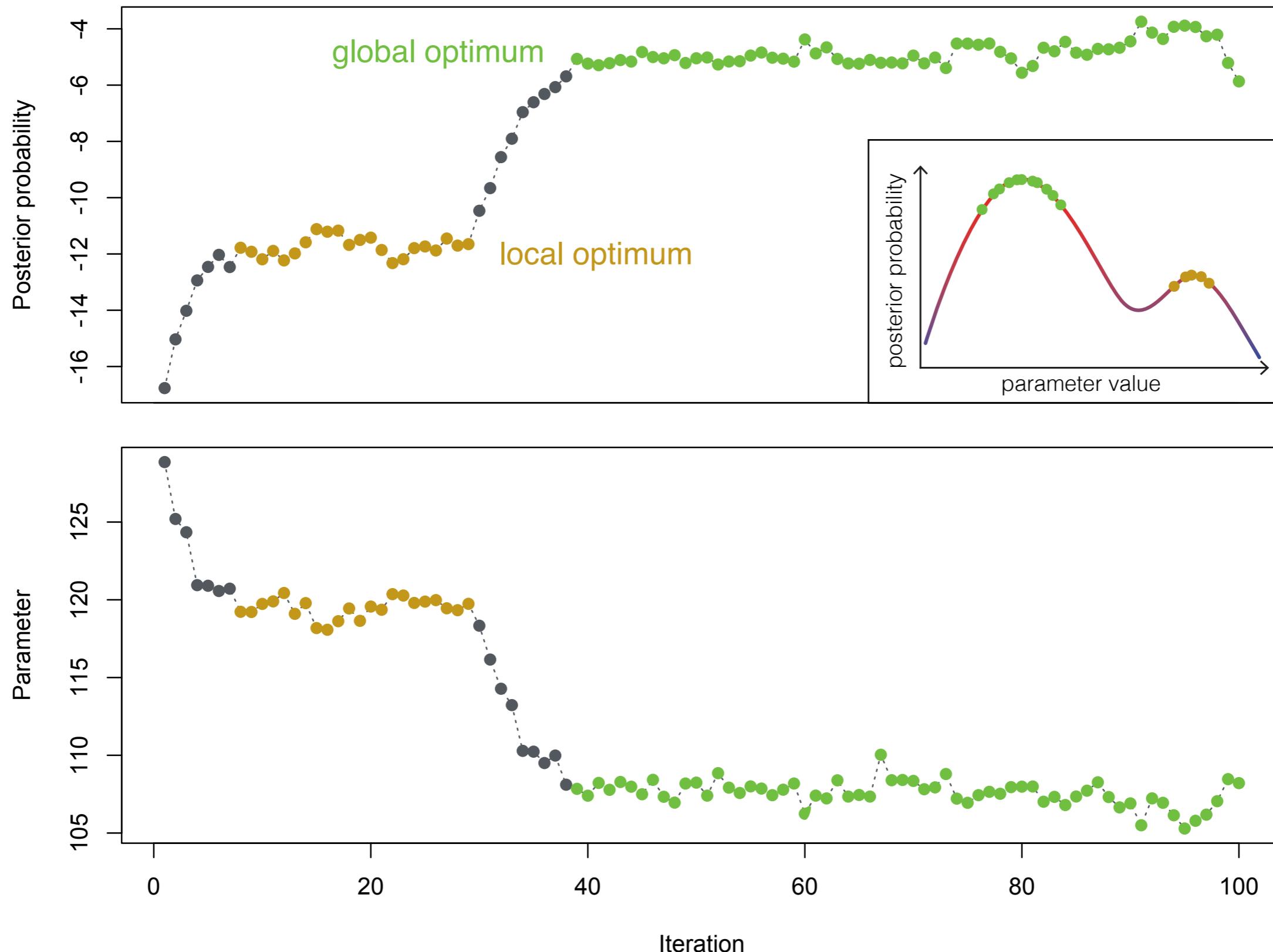
# Metropolis-Hastings MCMC



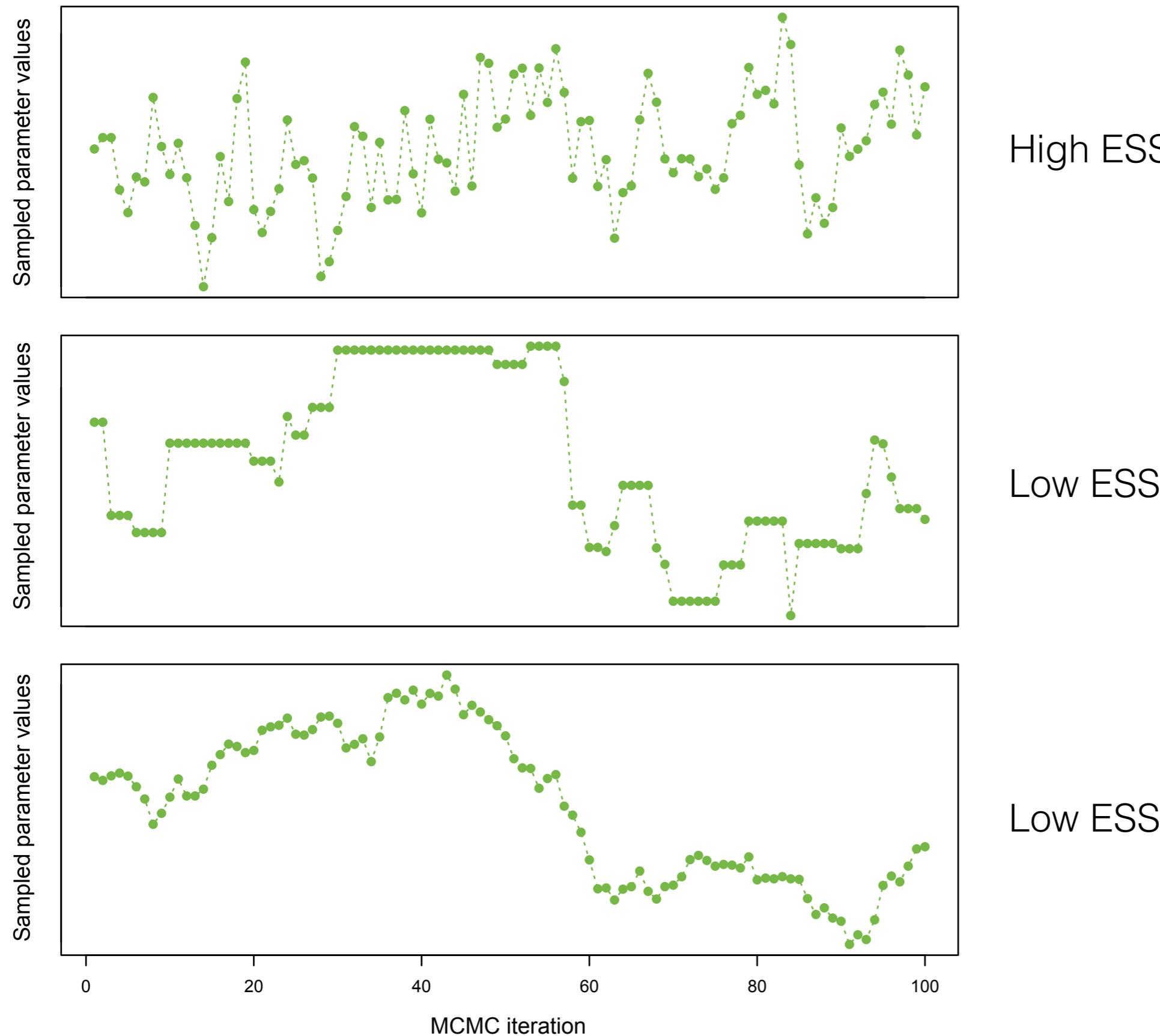
# Burn-in and MCMC convergence



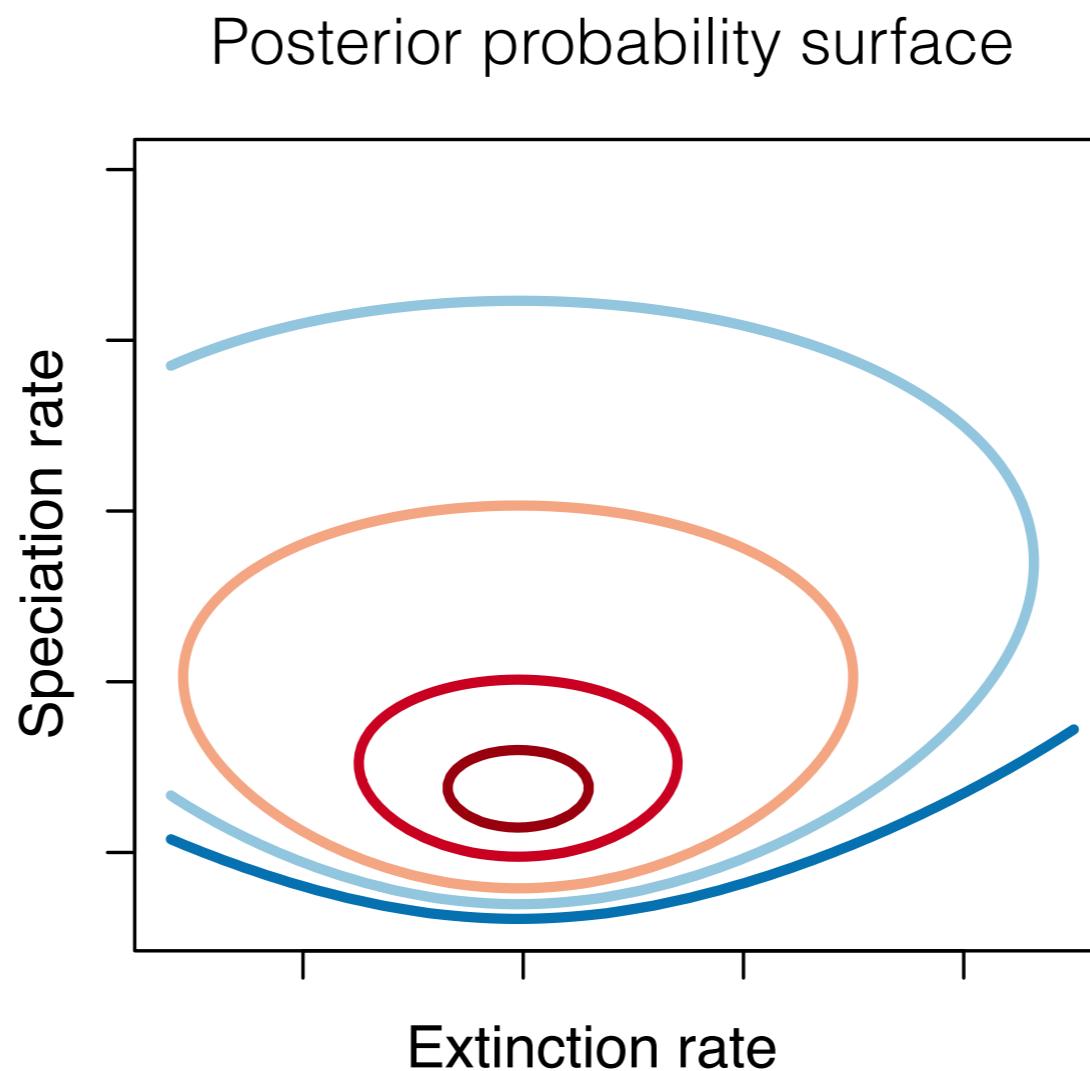
# Burn-in and MCMC convergence



# Posterior samples: Effective Sample Size (ESS)

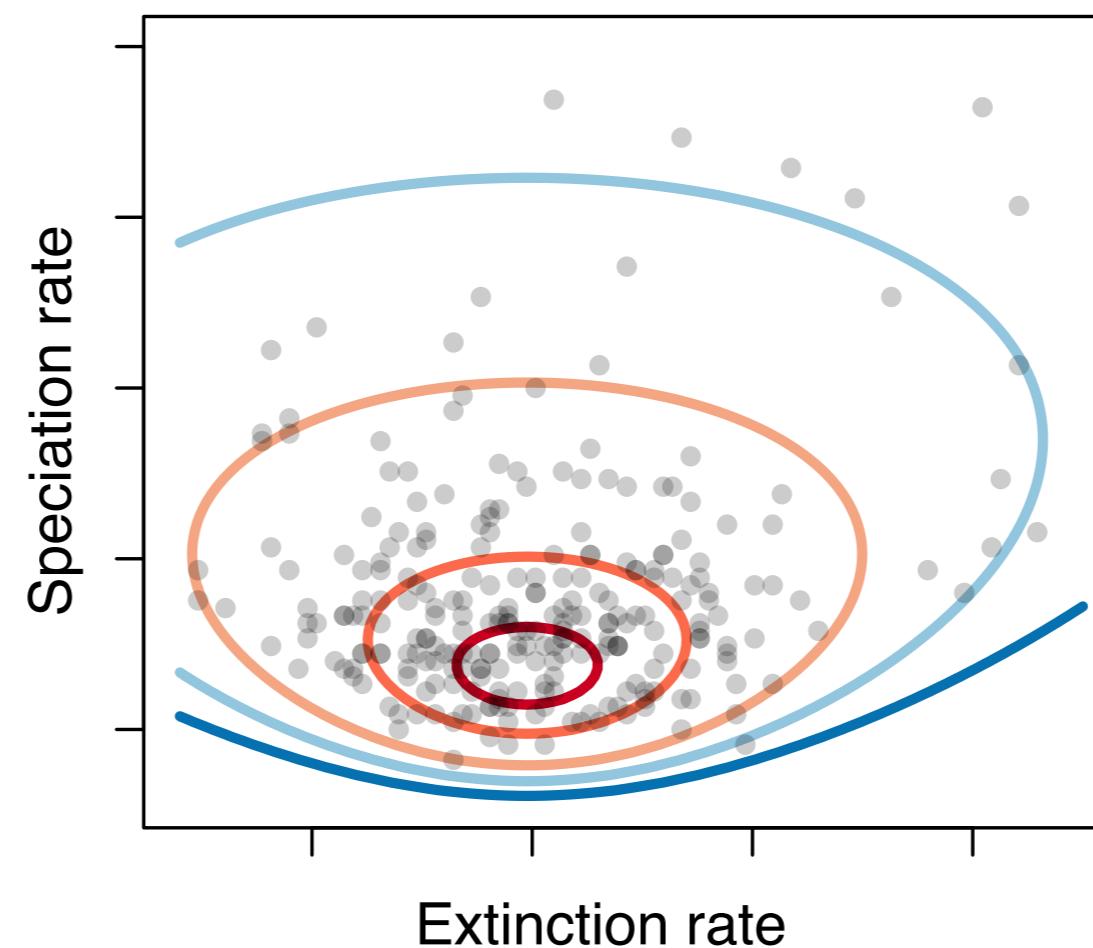


# Summarizing MCMC samples



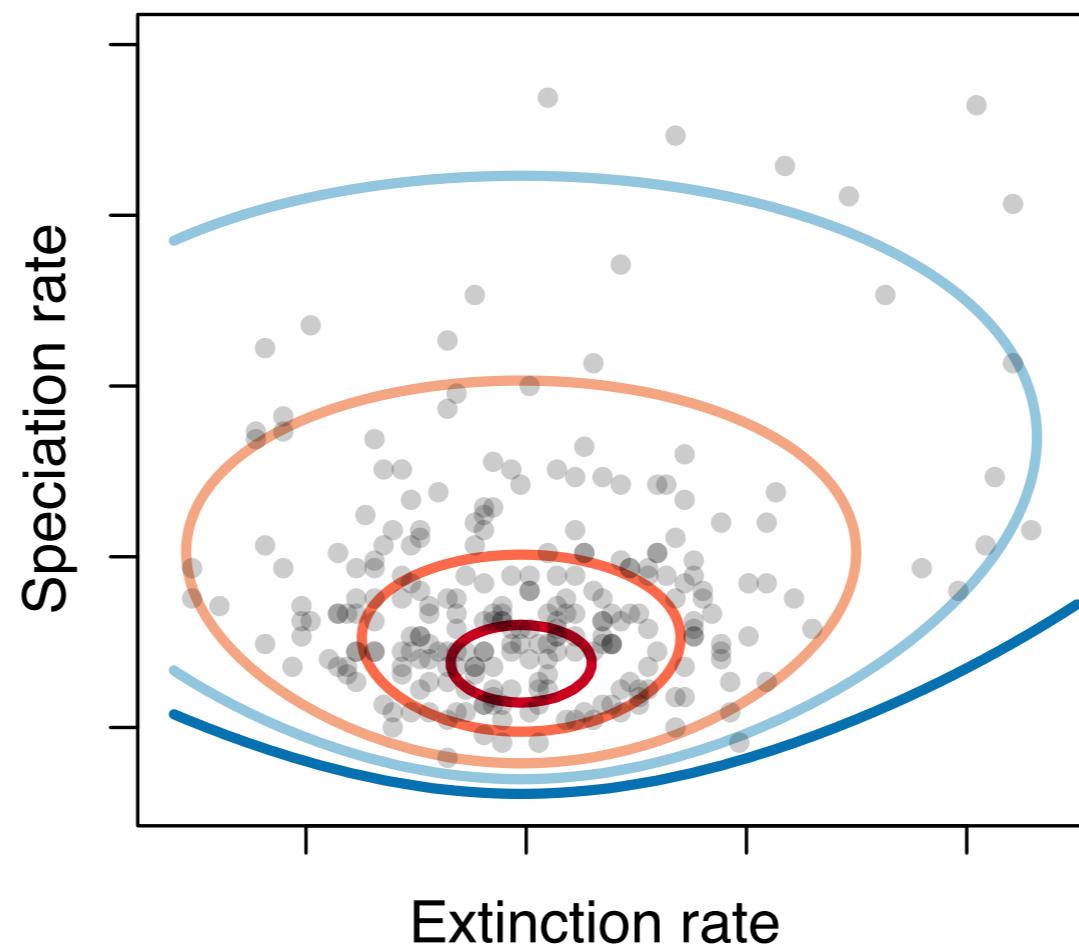
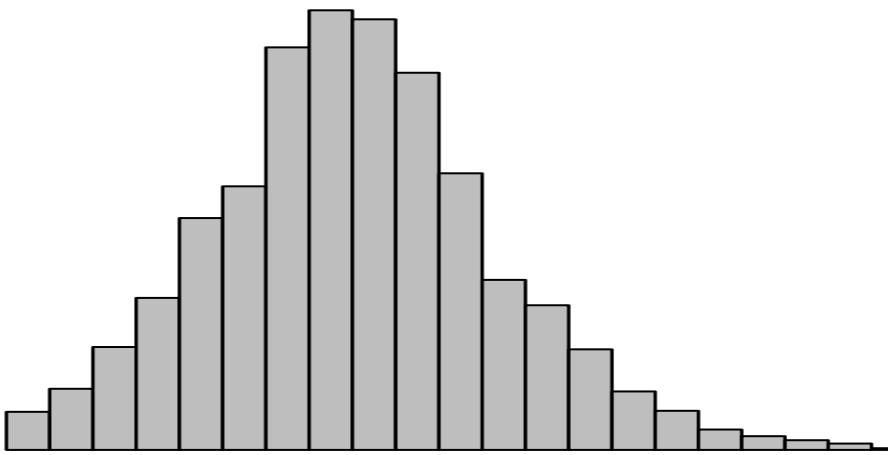
# Summarizing MCMC samples

Posterior samples from MCMC

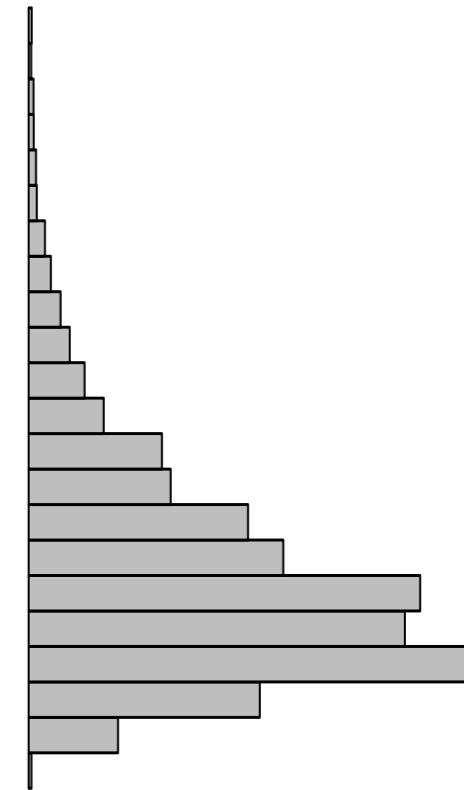


# Summarizing MCMC samples

Marginal distribution of  $\mu$



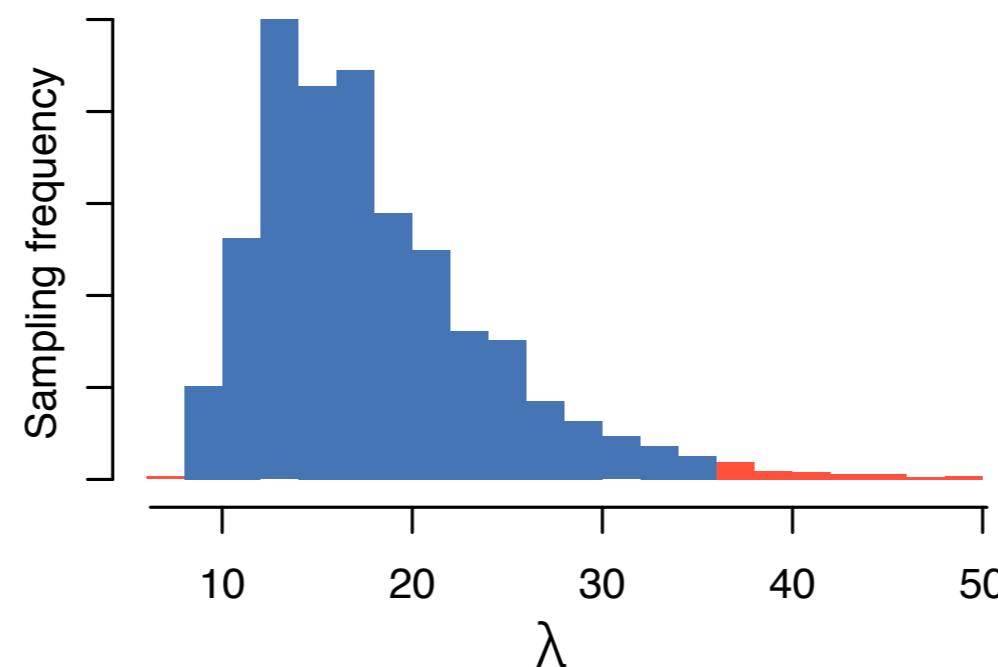
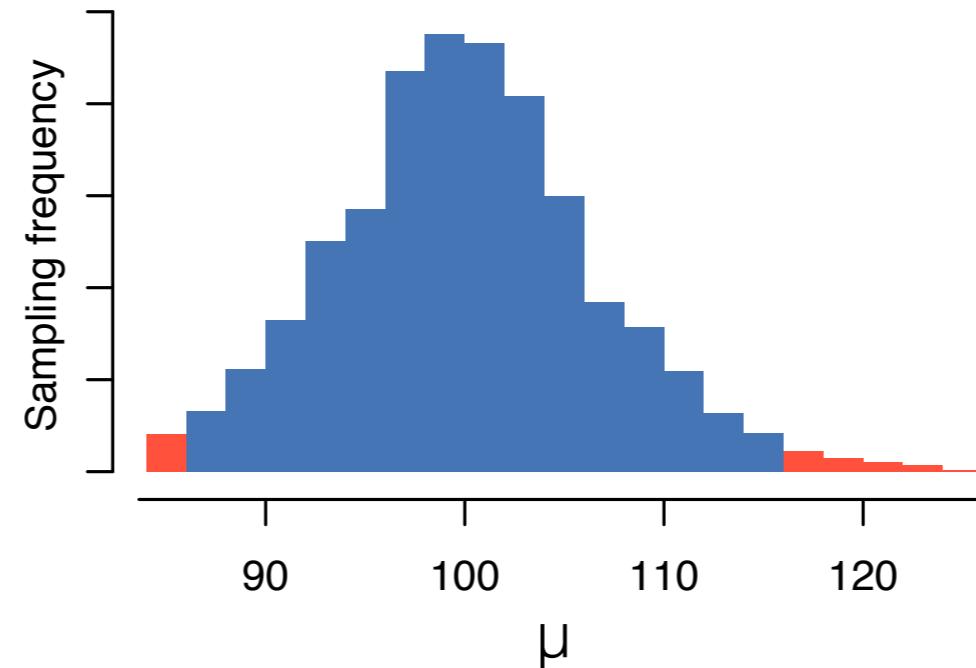
Marginal distribution of  $\lambda$



# Summarizing MCMC samples

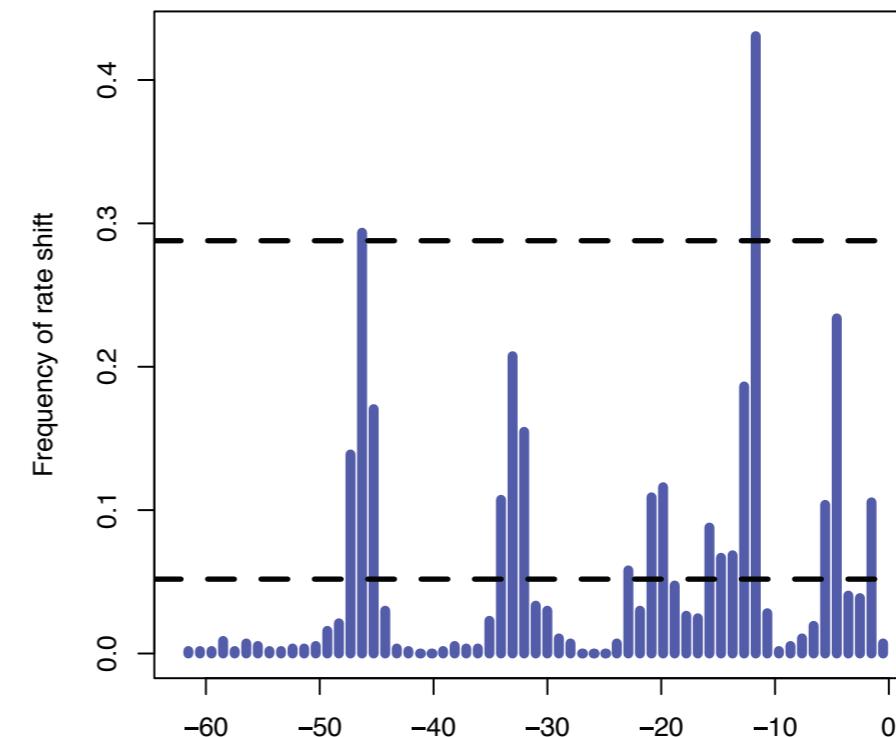
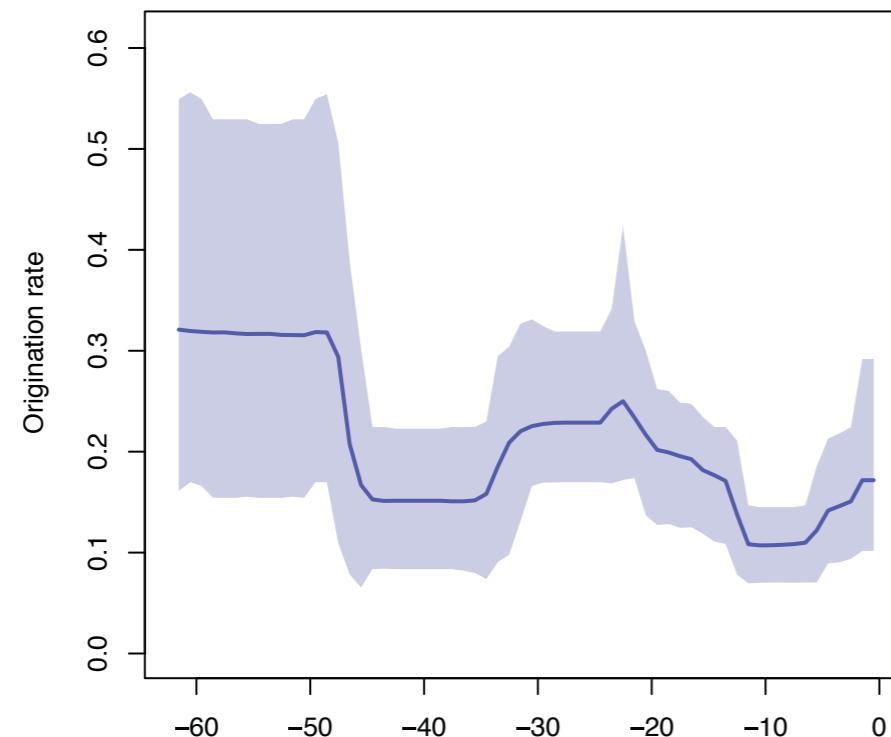
Posterior estimate: mean of  
MCMC samples

95% Credible intervals



# Plot results

```
[mbp:~ $ cd Software/PyRate  
[mbp:PyRate $ python PyRate.py -plotRJ your_path/pyrate_mcmc_logs -b 0.2
```



Strong support for  
a rate shift  
( $\text{logBF} > 6$ )

Significant rate shift  
( $\text{logBF} > 2$ )