

Rewinding a bit..... To yesterday

We have detection histories – now what?

$\Pr(eh = 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0) =$

$$(1-\varepsilon_3)(1-p_4) (1-\varepsilon_4)p_5 (1-\varepsilon_5)p_6 [\varepsilon_6 + (1-\varepsilon_6)(1-p_7)\{\varepsilon_7+(1-\varepsilon_7)(1-p_8)\}] = H_1$$

Likelihood (parameters | data) = $H_1^{No.\ cases} H_2^{No.\ cases} H_x^{No.\ cases}$

0001010010	10
0010001101	3
0101010100	4
1001000000	18
0001010111	2
0011101000	1
	3
	No. cases

Maximize likelihood function so that, under the assumed statistical model, the observed data is most probable.

https://en.wikipedia.org/wiki/Maximum_likelihood_estimation

Likelihood (parameters | data) = $H_1^{\text{No. cases}} H_2^{\text{No. cases}} H_x^{\text{No. cases}}$

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Likelihood (parameters | data) = $H_1^{\text{No. cases}} H_2^{\text{No. cases}} H_x^{\text{No. cases}}$

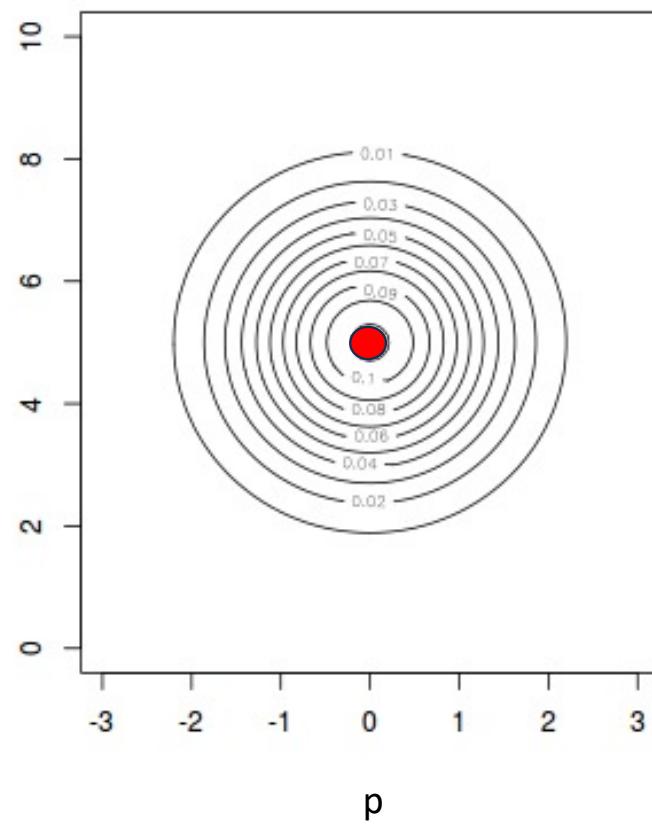
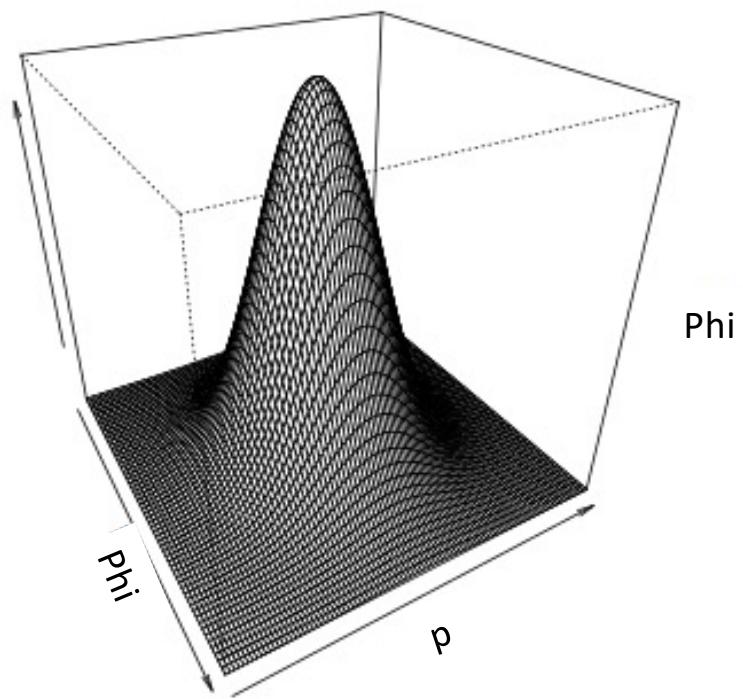
- $\varepsilon(\text{constant})p(\text{time-varying})$
- $\varepsilon(\text{time-varying})p(\text{sea-levels})$

Maximize likelihood function so that, under the assumed statistical model, the observed data is most probable.

https://en.wikipedia.org/wiki/Maximum_likelihood_estimation

Likelihood (parameters|data) = $H_1^{\text{No. cases}} H_2^{\text{No. cases}} H_x^{\text{No. cases}}$

- $\epsilon(\text{constant})p(\text{time-varying})$ 0001010010
- $\epsilon(\text{time-varying})p(\text{sea-levels})$ 0010001101
- $\phi(\text{constnat}))p(\text{sconstant})$ 0101010100
1001000000
0001010111
0011101000



<https://www.flutterbys.com.au/stats/tut/tut4.3.html>

```
> AIC(dipper.const,dipper.phi.t, dipper.p.t, dipper.phi.t.p.t)
```

	model	npar	rank	logLik	AIC	AICc	dAIC	AICwt
dipper.const	p~1 phi~1	2	2	-333.4188	670.838	671.068	0.000	0.7953
dipper.phi.t	p~1 phi~t	7	7	-329.8650	673.730	676.113	2.892	0.1873
dipper.p.t	p~t phi~1	7	7	-332.2401	678.480	680.863	7.642	0.0174
dipper.phi.t.p.t	p~t phi~t	12	12	-328.4751	680.950	688.379	10.112	0.0000

>

Higher log-likelihood value better fit to the data

Least negative (highest) is dipper.phi.t.p.t

Why is dipper.constant on “top” (the code rearranges the models with the best as the first row)

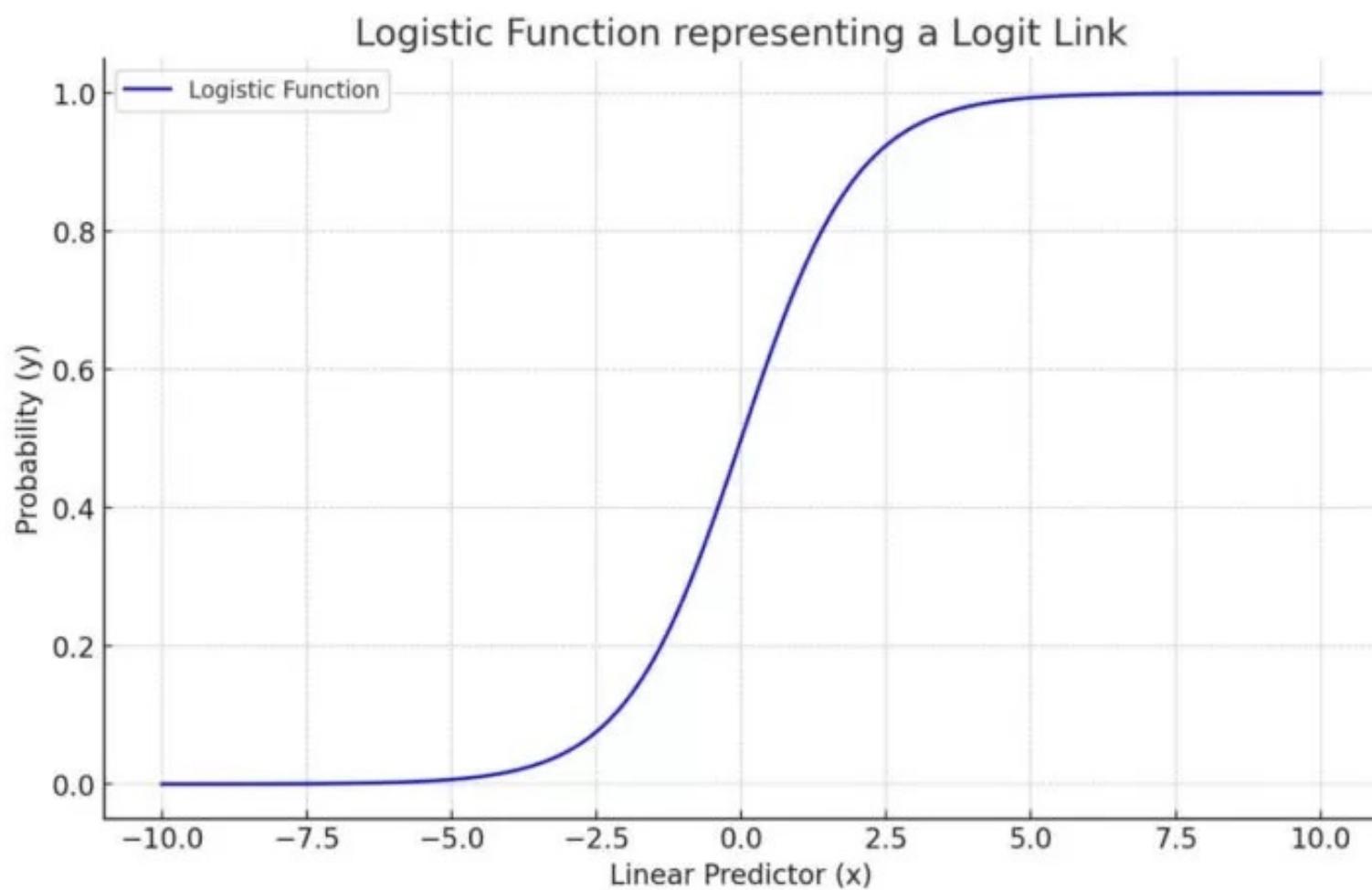
$$\mathrm{AIC} = 2k - 2 \ln \widehat{L}$$

$$AICc = AIC + \frac{2k(k+1)}{n-k-1}$$

Covariate modeling via link functions

- Link functions allows model to accommodate response variables that do not naturally fit a linear scale, such as probabilities between 0 and 1
- In logistic regression model (one example of a GLM Generalized Linear Model), the logit link function transforms the probability scale to an unbounded scale, where linear regression can be applied
- **logit link function is ideal for binary outcome modelling (survive or not survive, preserved or not preserved)**
- **correct link function improves model fit and accuracy**
- Identity, Probit, Log , Inverse, Sine are other link functions you might encounter.

<https://statisticseasily.com/link-functions-in-generalized-linear-models/>



Covariate modeling via link functions

$$\text{logit}(\varepsilon_{i,t}) = \log\left(\frac{\varepsilon_{i,t}}{1 - \varepsilon_{i,t}}\right) = \beta_0 + \beta_1 x_i + \beta_2 y_t$$

Time specific covariates

- Duration of bin
- Sea-level
- Temperature

$$\text{logit}(\varepsilon_{i,t}) = \log\left(\frac{\varepsilon_{i,t}}{1 - \varepsilon_{i,t}}\right) = \beta_0$$

```
invlogit(2.2262629)
[1] 0.9025833
```

```
invlogit(0.2421485)
[1] 0.560243
```

RStudio Console output:

```
R 4.4.1 · ~/→
p      logit
phi    logit

Beta parameters (coefficients)
  beta   SE.beta     lcl     ucl
p  2.2262629 0.3251461 1.58898831 2.863537
phi 0.2421485 0.1020139 0.04220503 0.442092

Eigenvalues : 1 0.09066
Numerical rank of Hessian : 2 ( svtol = 1e-05 )

Variance-covariance matrix of beta parameters
  p      phi
p  0.105719960 -0.008508673
phi -0.008508673  0.010406826

Fitted (real) parameters evaluated at base levels of covariates

p
session estimate SE.estimate     lcl     ucl
  1981     NA       NA       NA       NA
  1982 0.9025833 0.02858903 0.8304737 0.9460142
  1983 0.9025833 0.02858903 0.8304737 0.9460142
  1984 0.9025833 0.02858903 0.8304737 0.9460142
  1985 0.9025833 0.02858903 0.8304737 0.9460142
  1986 0.9025833 0.02858903 0.8304737 0.9460142
  1987 0.9025833 0.02858903 0.8304737 0.9460142

phi
session estimate SE.estimate     lcl     ucl
  1981 0.560243 0.02513323 0.5105497 0.6087574
  1982 0.560243 0.02513323 0.5105497 0.6087574
  1983 0.560243 0.02513323 0.5105497 0.6087574
  1984 0.560243 0.02513323 0.5105497 0.6087574
  1985 0.560243 0.02513323 0.5105497 0.6087574
  1986 0.560243 0.02513323 0.5105497 0.6087574
  1987     NA       NA       NA       NA
```

Covariate modeling via link functions

$$\text{logit}(\varepsilon_{i,t}) = \log\left(\frac{\varepsilon_{i,t}}{1 - \varepsilon_{i,t}}\right) = \boxed{\beta_0} + \boxed{\beta_1}x_i + \boxed{\beta_2}y_t$$

$$\varepsilon_{i,t} = \frac{e^{\boxed{\beta_0} + \boxed{\beta_1}x_i + \boxed{\beta_2}y_t}}{1 + e^{\boxed{\beta_0} + \boxed{\beta_1}x_i + \boxed{\beta_2}y_t}}$$

A bit of everything related to CMR in paleo

Lee Hsiang Liow

Natural History Museum and Centre for Planetary Habitability, University of Oslo, Norway

16.8.2024 (Friday)



Friedrich-Alexander-Universität
Erlangen-Nürnberg



Summary

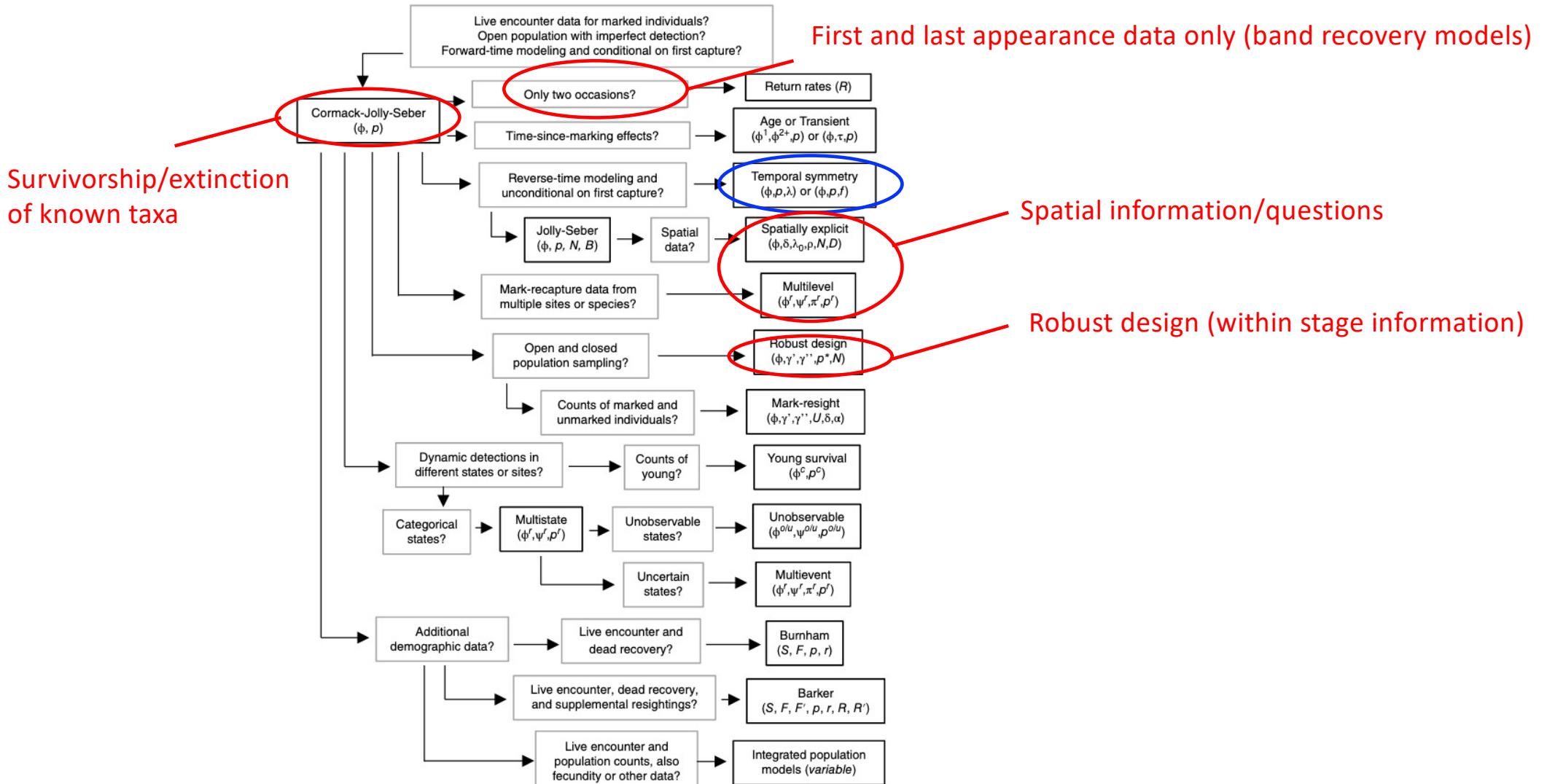
$p \neq 1$

Two types of zeros

Common tutorial questions

- Additive versus multiplicative models
- Model-averaging

Many models: few explored in paleo



First and last appearance data only (band recovery models)

Spatial information/questions

Robust design (within stage information)

MARK is the “main” software and easy to read reference (but you must “translate”)



The screenshot shows a web browser displaying the homepage of the program MARK. The URL in the address bar is www.phidot.org/software/mark/docs/book/. The page has a blue header with the text "program MARK" and "'A Gentle Introduction'". Below the header is a green navigation bar with links for Home, About, Download, Support Forum, and Training. The main content area features a large image of a computer monitor displaying the book cover of "Program MARK - a gentle introduction - (edition 2.2)". To the right of the image, there is an "Overview" section with text about the program's purpose and its relationship to previous applications. Below the overview is an "About the book..." section with information about the book's format and availability. On the left side of the main content area, there are two sidebar sections: "Book chapters & data files" (with a dropdown menu labeled "Select chapter") and "Thoughts for the day...". The "Thoughts for the day..." section contains a quote by Gary White: "Debugging is twice as hard as writing the code in the first place. Therefore, if you write the code as cleverly as possible, you are, by definition, not smart enough to debug it."

<http://www.phidot.org/software/mark/docs/book/>

Gary White

Self learning material and software

Gary White MARK

<http://www.phidot.org/software/mark/>

(great if you are windows user, a bit more involved if you us Mac) great to pair with Rmark (Jeff Laake)

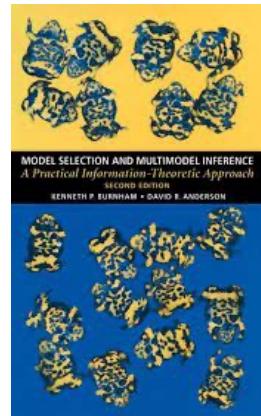
Gary White MARK book

<http://www.phidot.org/software/mark/docs/book/>

Michael Conroy lecture notes (bare bones description of models; useful for Mark/Rmark code and data)
<https://sites.google.com/site/cmrsoftware/>

Burnham and Anderson 2022:

Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach



R packages for CMR

- [RMark](#) (runs MARK from R, works best with Windows)
- [serc](#) and [openCR](#)
- [Marked](#) (some overlap with openCR and MARK but this doesn't have Pradel models)
- [R2ucare](#) (for goodness of fit tests and simulations for CMR)
- [mra](#), [Rcapture](#), [BaSTA](#)

Other languages for capture recapture

Python <https://www.python.org/>

- <https://austinrochford.com/posts/2018-01-31-capture-recapture.html>
- https://pyro.ai/examples/capture_recapture.html

Stan <https://mc-stan.org/>

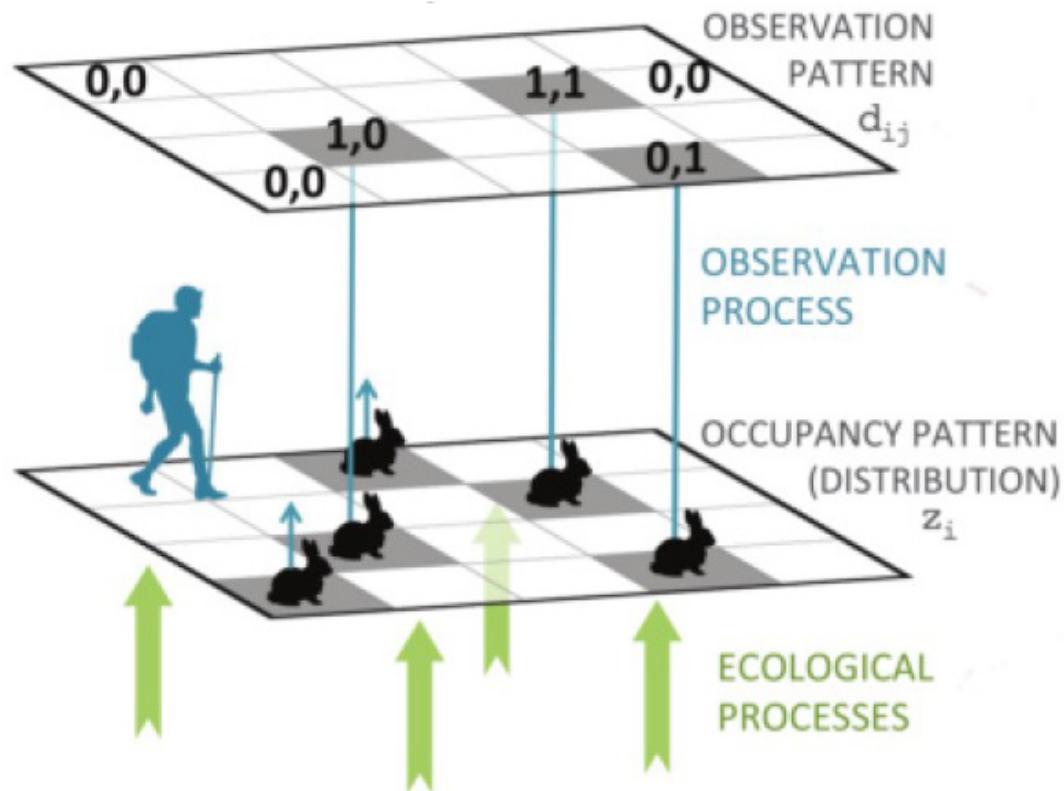
- <https://mc-stan.org/docs/stan-users-guide/latent-discrete.html#mark-recapture-models>

BUGS, JAGS

<https://bccs.org.my/tut/bayes-with-jags-a-tutorial-for-wildlife-researchers/abundance-from-capture-recapture-data/basic-spatial-capture-recapture-models/>

<https://esajournals.onlinelibrary.wiley.com/doi/full/10.1002/ecs2.3810>

Site-occupancy modeling in statistical ecology

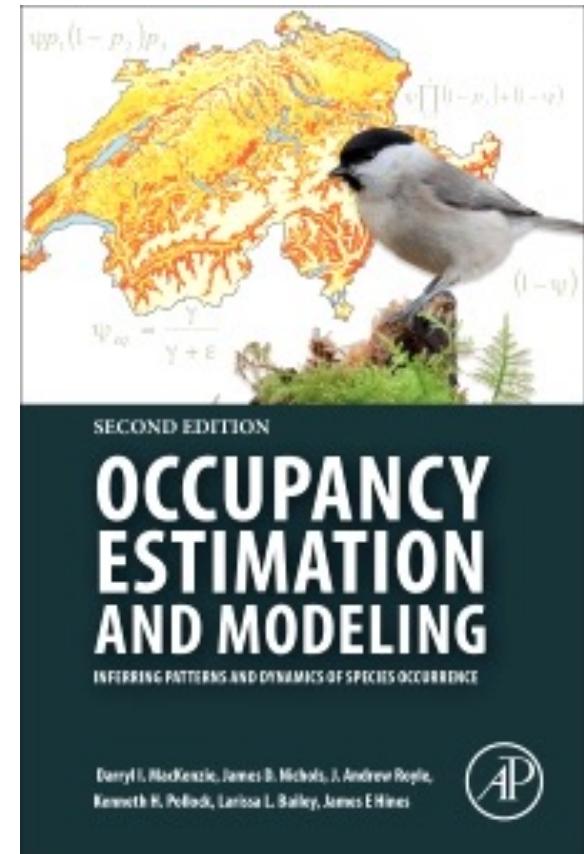


Not to be confused with

Foote, M. et al. (2007) Rise and fall of species occupancy in Cenozoic fossil molluscs. *Science* (80-.). 318, 1131–11341

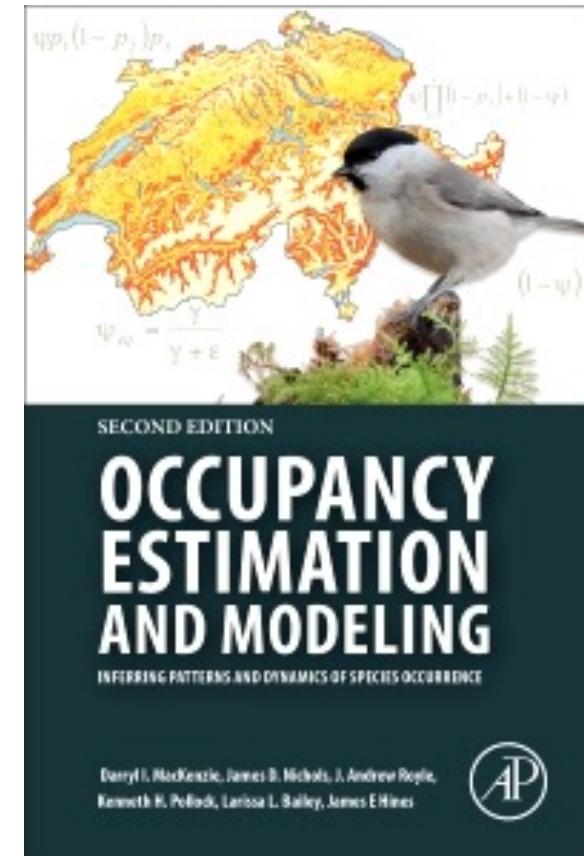
Foote, M. (2016) On the measurement of occupancy in ecology and paleontology. *Am. Nat.* 42, 707–729

(Site)-Occupancy modeling

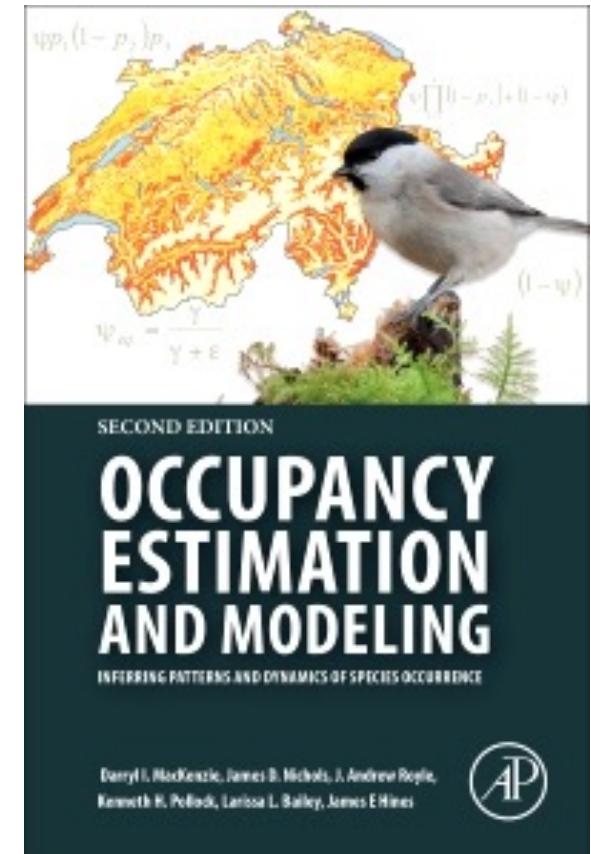
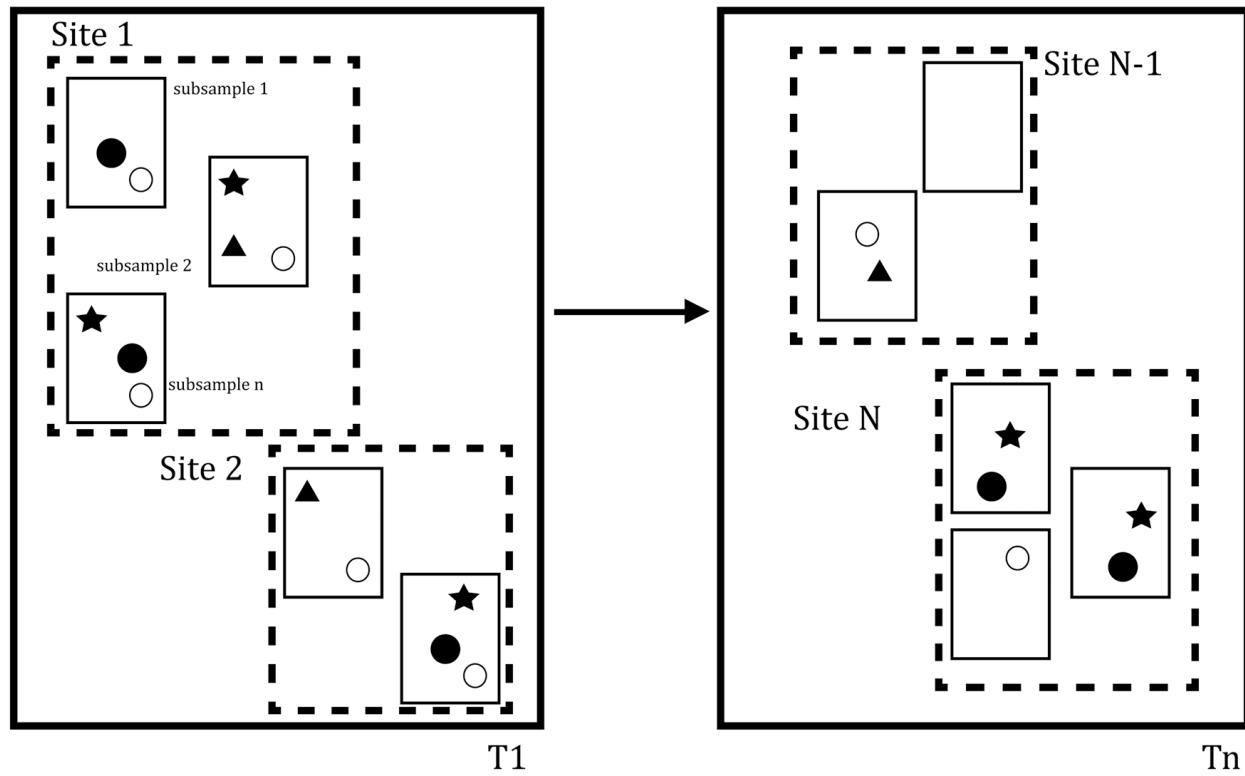


Occupancy modeling in paleo

- Liow, L.H. (2013) Simultaneous estimation of occupancy and detection probabilities: an illustration using Cincinnatian brachiopods. *Paleobiology* 39, 193–213
- Lawing, A. Michelle, et al. (2021). Occupancy models reveal regional differences in detectability and improve relative abundance estimations in fossil pollen assemblages. *Quaternary Science Reviews* 253: 106747.
<https://doi.org/10.1016/j.quascirev.2020.106747>
- Reitan, T., Ergon, T., & Liow, L. H. (2022). Relative species abundance and population densities of the past: Developing multispecies occupancy models for fossil data. *Paleobiology*, 1-16.
doi:10.1017/pab.2022.17
- Reitan, T., E. D. Martino, and L. H. Liow. (2024). Estimating relative species abundance using fossil data identified to different taxonomic levels. *Ecography* 2024:e06866.

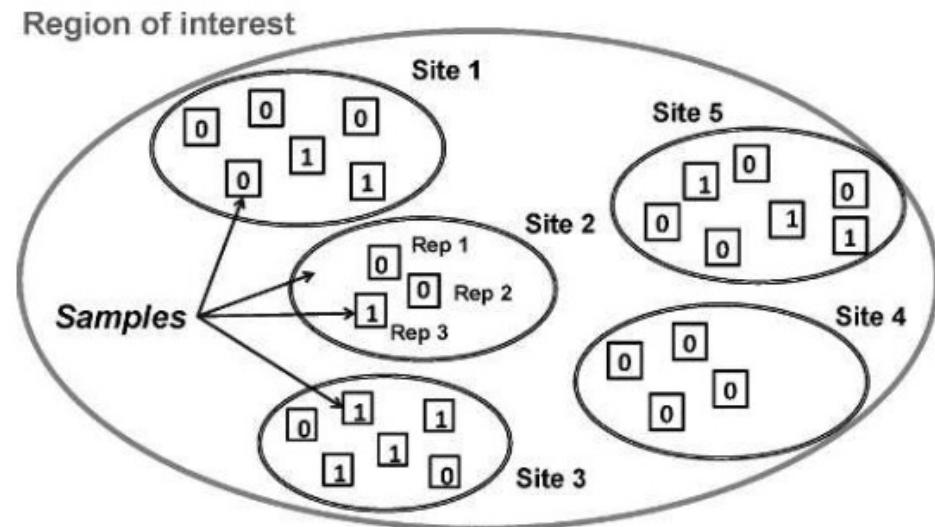


Occupancy modeling in paleo



$$Pr(h = 1010) = \varphi(1 - p)^2 p^2$$

$$Pr(h = 0000) =?????$$



$$Pr(h=1010)=\varphi(1-p)^2p^2$$

$$Pr(h=0000)=(1-\varphi)+\varphi(1-p)^4$$

$$L(\varphi,p|h_1,h_2,\dots~h_s,) = \prod_{i=1}^s \Pr(h_i) \qquad \qquad Pr(h=1010)=\varphi(1-p)^2p^2$$

$$Pr(h=0000)=(1-\varphi)+\varphi(1-p)^4$$

$$L(\varphi,p|h_1,h_2,\dots~h_s,) = \prod_{i=1}^s \Pr(h_i)$$

R package occupancy modeling

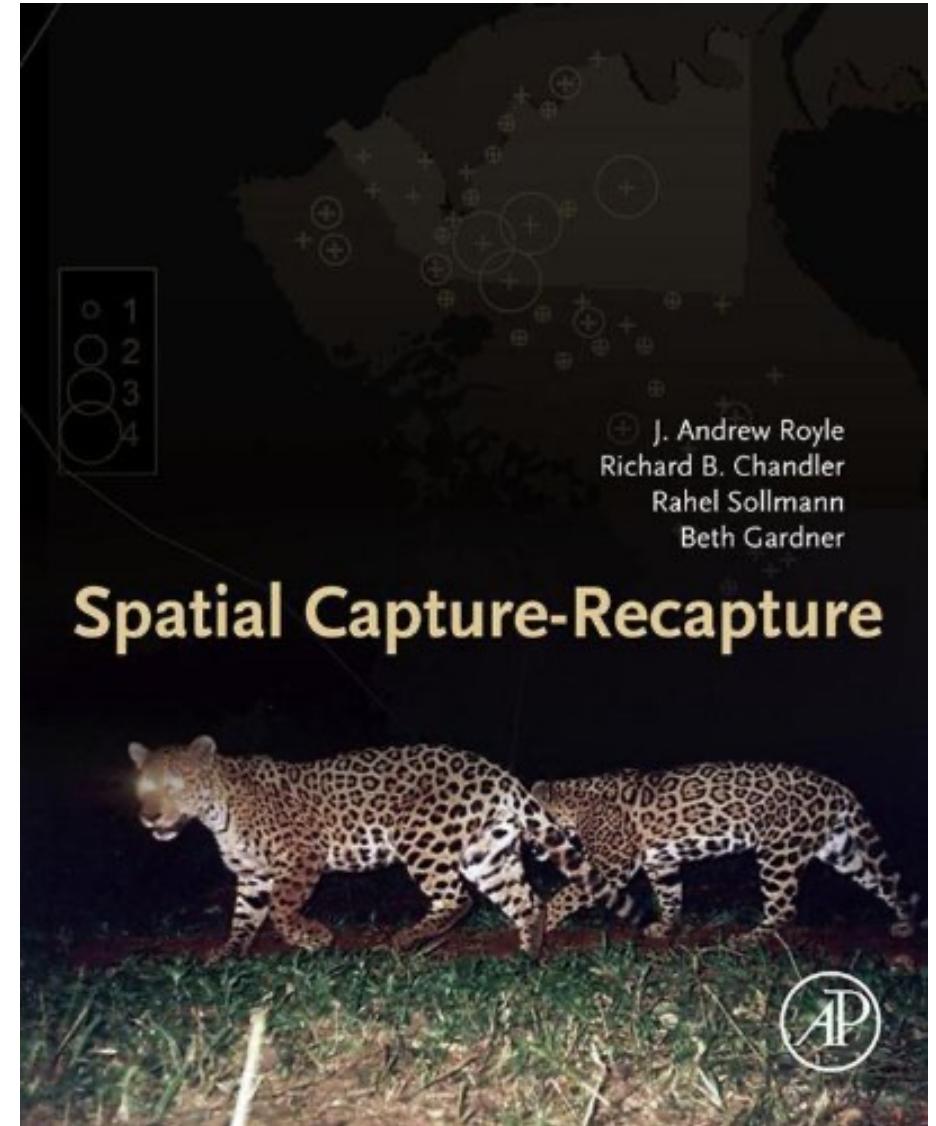
- openCR, serc
- unmarked

Some resources for occupancy modeling

- <https://kevintshoemaker.github.io/NRES-746/Occupancy.html>
- <https://science.uct.ac.za/seec/stats-toolbox-seminars-spatial-and-species-distribution-toolboxes/single-season-occupancy-models-using-bayesian-approach>

Spatial capture recapture

- Combining the best of capture recapture and occupancy
- Also implemented in serc, openCR, unmarked



Brief comparisons of CMR with other approaches (paleo context)

HISTORY OF ESTIMATION TAXONOMIC RICHNESS AND DIVERSIFICATION RATES

Nichols & Pollock 1983 Estimating taxonomic diversity, extinction rates, and speciation rates from fossil data using capture-recapture models. *Paleobiology* 9, 150–163

Foote & Raup 1996 Fossil preservation and the stratigraphic ranges of taxa. *Paleobiology*

Foote 1999/2001 (Boundary crossers method)

Alroy et al. 2001 (sampling standardization)

Connolly and Miller papers 2001-2 using CMR (Connolly is an ecologist)

Foote 2003 (few people use this) – CMR-like, but accounts for origination and extinction within time interval (but see robust design)

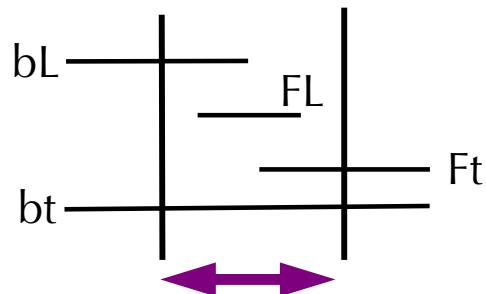
(2009) My own first capture recapture paleo-paper – I met Nichols in 2006; short course paper with Nichols

Silvestro, Schinitzler & Liow Syst bio 2014 Pyrate model paper (not the software)

Warnock et al. 2020 RevBayes (starting from birth death models but dropping the “relationships”)

Comparisons with other approaches

Foote's per capita origination and extinction rates (boundary crossers)



- Easy to use
- Lose information of “singletons” (FL class information not used)
- Hard to compare different models

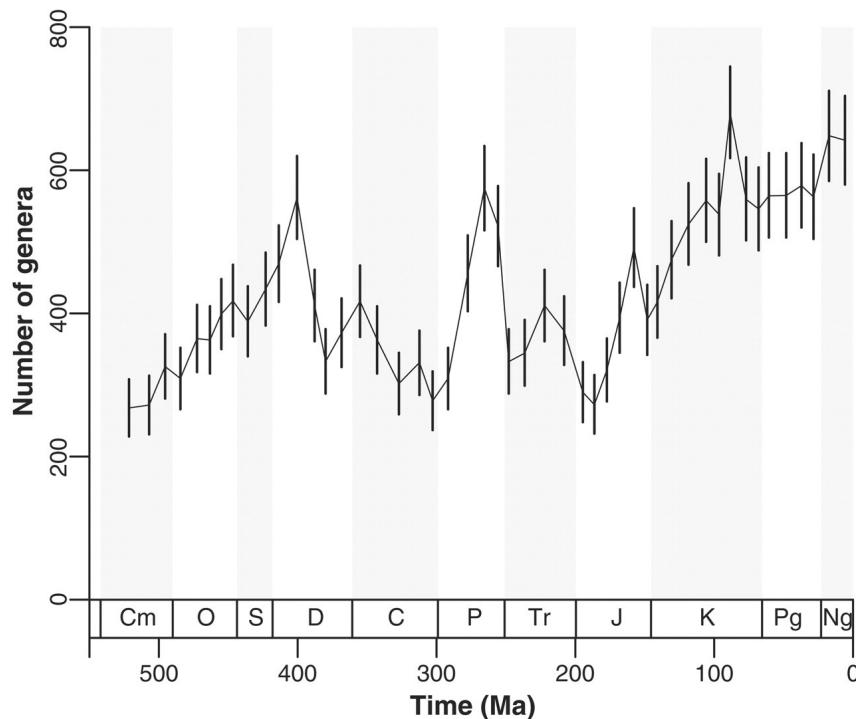
$$\hat{p} = \ln\left(\frac{N_t}{N_{bt}}\right) / \Delta t$$

$$\hat{q} = \ln\left(\frac{N_b}{N_{bt}}\right) / \Delta t$$

Foote, M. 2000. Origination and extinction components of taxonomic diversity: general problems. *Paleobiology* 26:74-102.

Comparisons with other approaches

Sampling standardization approaches (including SQS)



- Easy to use
- Assume that even sample or quorums will allow unbiased relative change to be estimated
- Ad hoc rather than modelling approach (preservation is not modelled although it is part of the process)
- Hard to compare different models

Comparisons with other approaches

CMR-like approaches in paleo independently developed

Foote 2003 Journal of Geology

- Some hard, some easy to use, not easy to understand
- Hard to compare different models
- (very) Special cases of CMR

Alroy “three-timer” and related methods

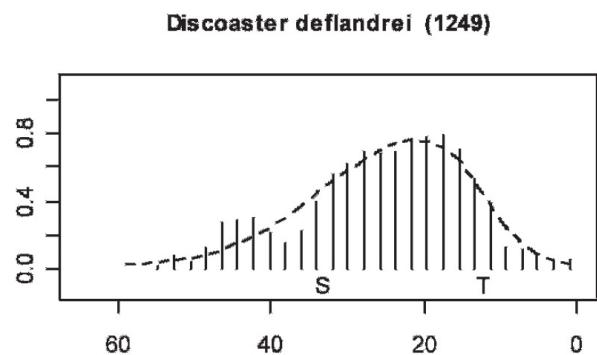
Comparisons with other approaches

PyRate (Silvestro et al. Sys Bio 2014)

Key preservation assumption (different) is the shape of species observations (beta distribution based on “hat”)

Smooths out temporal information

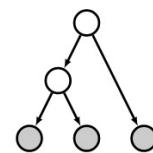
Conditioned on at least one observation per taxon (like the CMR models conditioned on first observation)



RevBayes

https://revbayes.github.io/tutorials/fbd_range/

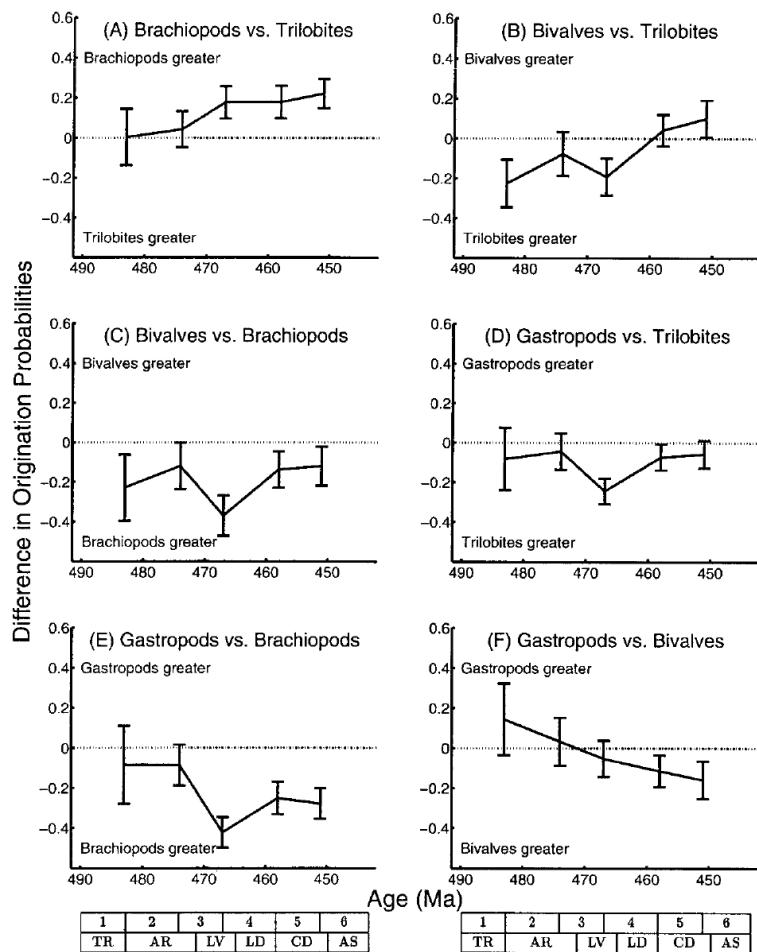
Rachel's lecture!



RevBayes

Bayesian phylogenetic inference using probabilistic graphical models and an interpreted language

Some paleo papers using CMR



E 4. Model-averaged estimates of between-class differences in genus origination probabilities over time ob-



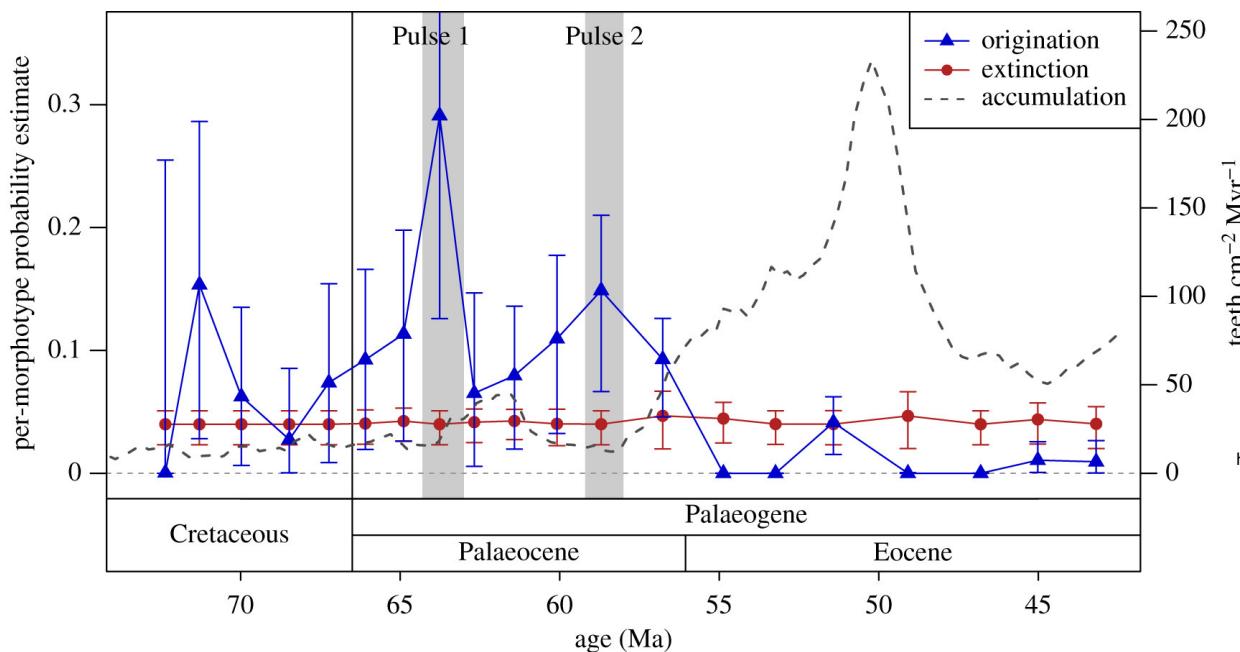
Nichols, J.D. and Pollock, K.H. (1983) Estimating taxonomic diversity, extinction rates, and speciation rates from fossil data using capture-recapture models. *Paleobiology* 9, 150–163

Connolly, S.R. and Miller, A.I. (2001) Joint estimation of sampling and turnover rates from fossil databases: Capture-Mark-Recapture methods revisited. *Paleobiology* 27, 751–767

Connolly, S.R. and Miller, A.I. (2001) Global Ordovician faunal transitions in the marine benthos: proximate causes. *Paleobiology* 27, 779–795

Connolly, S.R. and Miller, A.I. (2002) Global Ordovician faunal transitions in the marine benthos: ultimate causes. *Paleobiology* 28, 26–40

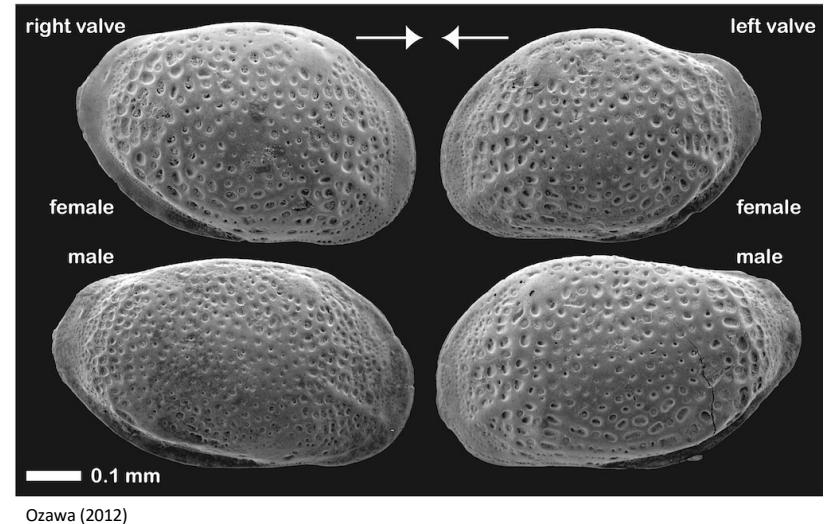
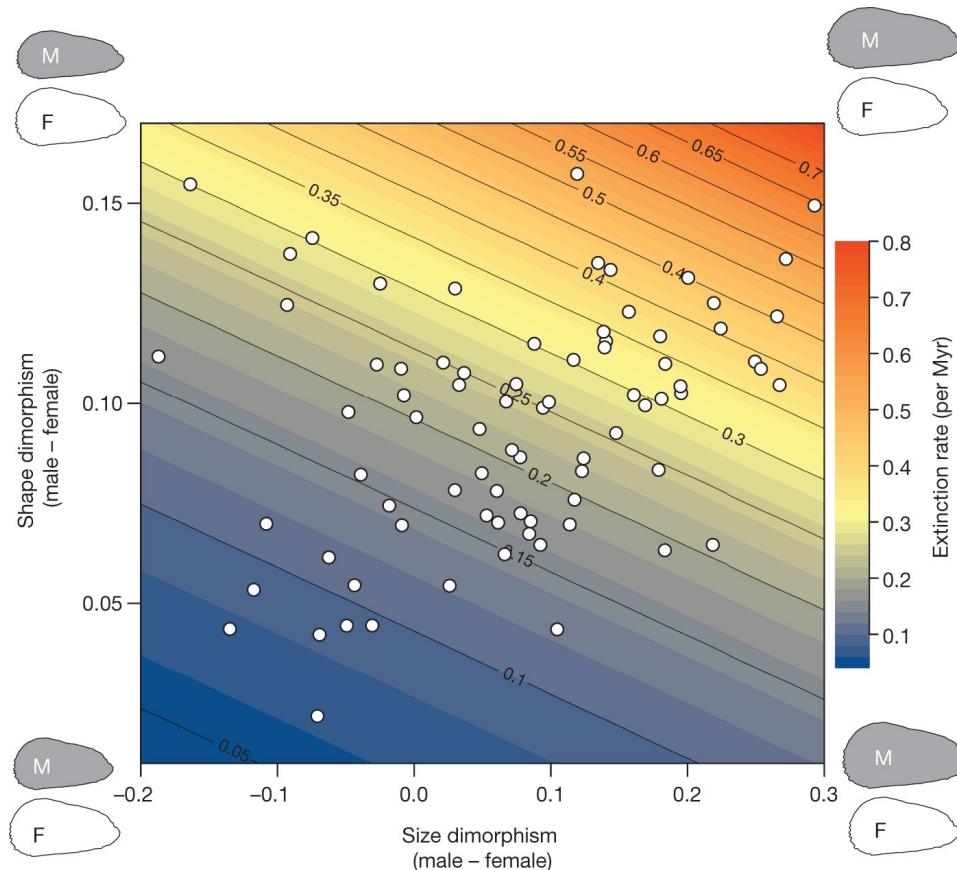
Some paleo papers using CMR



<https://news.harvard.edu/gazette/story/2018/11/tiny-teeth-tell-the-story-of-two-fish-species-rapid-evolution/>

Sibert, E. et al. (2018) Two pulses of morphological diversification in Pacific pelagic fishes following the Cretaceous - Palaeogene mass extinction. *Proc. R. Soc. B-BIOLOGICAL Sci.* 285,

Some paleo papers using CMR



Martins, M.J.F. et al. (2018) High male sexual investment as a driver of extinction in fossil ostracods. *Nature*

Some paleo papers using CMR

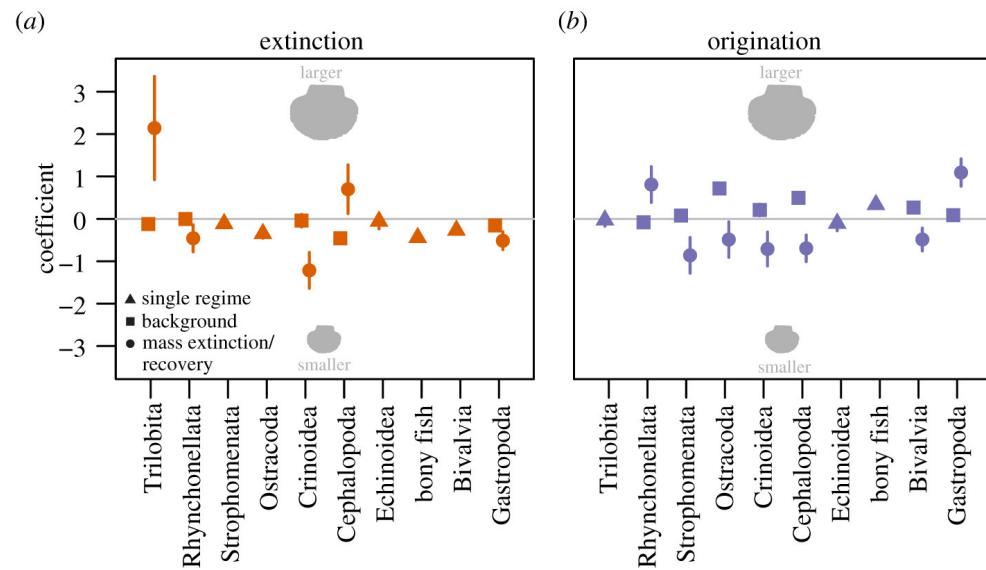
Payne, J., & Heim, N. (2020). Body size, sampling completeness, and extinction risk in the marine fossil record. *Paleobiology*, 46(1), 23-40. doi:10.1017/pab.2019.43

Pedro M. Monarrez, Noel A. Heim and Jonathan L. Payne 2021

[Mass extinctions alter extinction and origination dynamics with respect to body size](#)

Proc B

“CMR analysis of the fossil record reveals a
gradient of increasing extinction risk
against the sampling of smaller-bodied
genera within classes ”



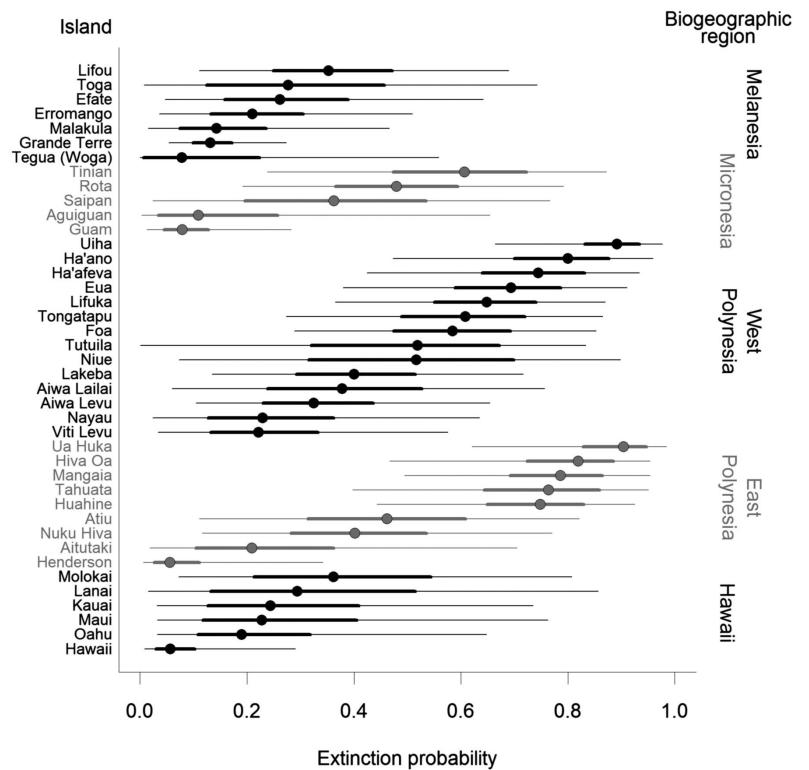
Some paleo papers using CMR

Magnitude and variation of prehistoric bird extinctions in the Pacific 2013

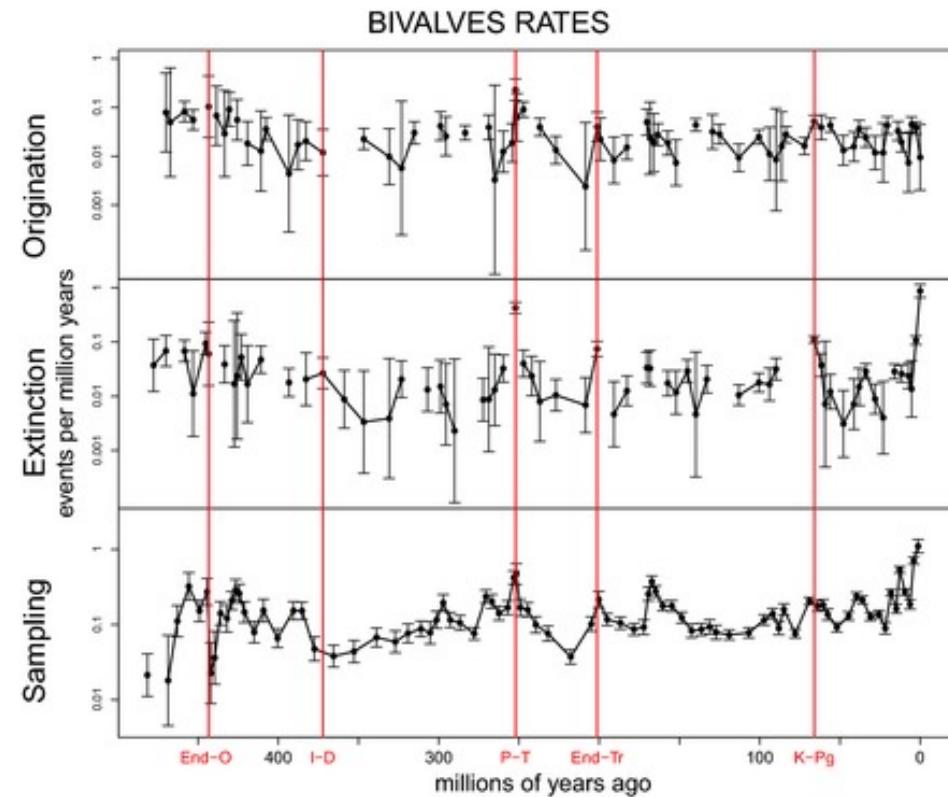
Richard P. Duncan richard.duncan@canberra.edu.au, Alison G. Boyer, and Tim M. Blackburn

PNAS

"We use a Bayesian mark-recapture approach to model gaps in the fossil record and to quantify losses of nonpasserine landbirds on 41 Pacific islands."



Some paleo papers using CMR



Liow, L.H. *et al.* (2015) Ecological interactions on macroevolutionary time scales: clams and brachiopods are more than ships that pass in the night. *Ecol. Lett.* 18, 1030–1039