

# Reverse time models in CMR (Pradel seniority)

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1.9.2025 Monday



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# Different classes of CMR (MR) models

## CJS models

- Cormack-Jolly-Seber models
- Originally “Closed-population model”
- “conditional upon the animal being released alive at first encounter, and survival and catchability refer only to these marked animals”
- Developed initially for survival rates
- Translation: we can get extinction rates if we are only interested in the taxa that entered the fossil record and are sampled at least once.

## JS models

- Jolly-Seber models
- “Open-population model”
- “unmarked animals in the population have the same probability of capture as marked animals in the population, i.e., that newly captured unmarked animals are a random sample of all unmarked animals in the population”
- Developed for parameters like abundance, population growth, recruitment.
- Translation: we can get extinction rates, origination rates, taxonomic richness and we include those taxa never sampled or have never entered the fossil record.

Which class of model do you expect a greater uncertainty in estimates? Why?

# Different classes of models

## **CJS models**

- Cormack-Jolly-Seber (CJS) models
- “Closed-population model”
- <http://www.phidot.org/software/mark/docs/book/pdf/chap3.pdf>

## **JS models**

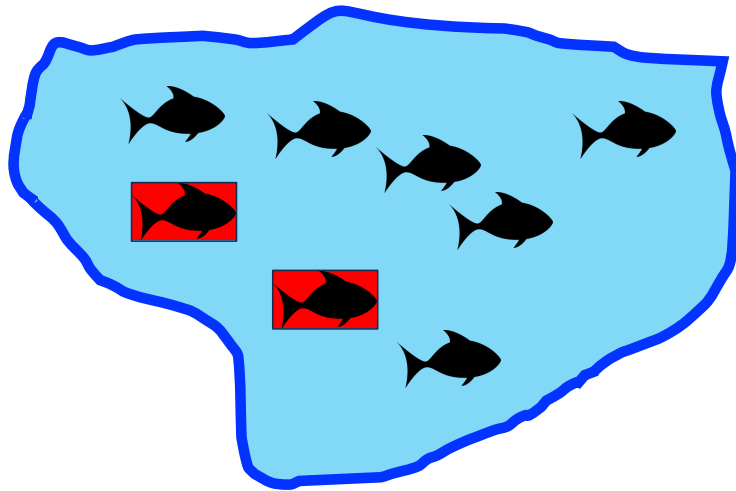
- Jolly-Seber (JS) models
- “Open-population model”
- <http://www.phidot.org/software/mark/docs/book/pdf/chap12.pdf>

## **RD models**

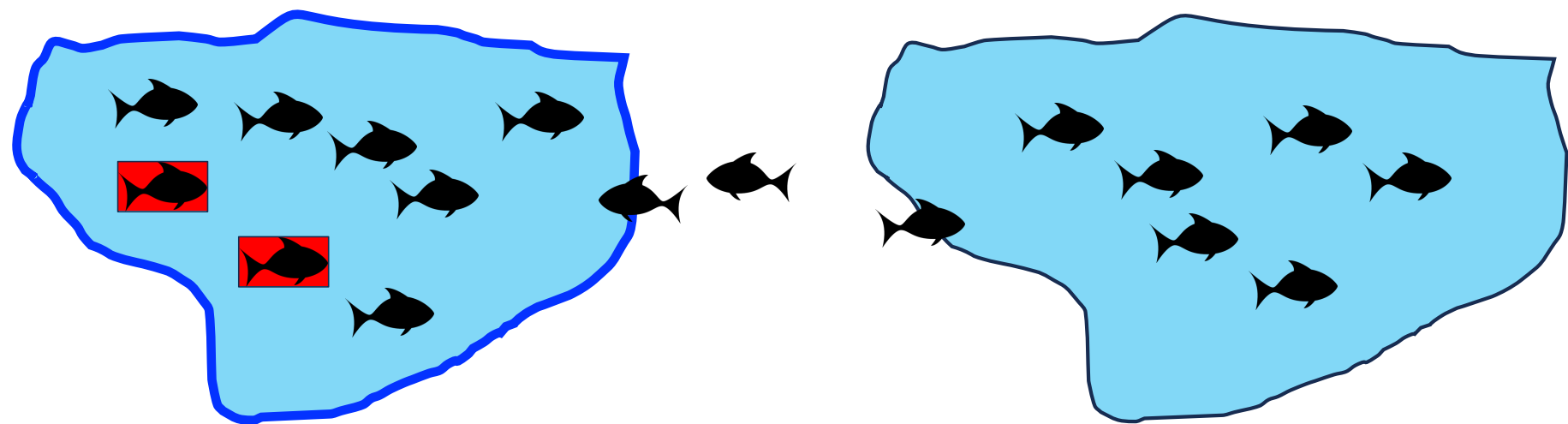
- Robust-design (RD) models
- Hybrid “open-closed population model”
- <http://www.phidot.org/software/mark/docs/book/pdf/chap16.pdf>

Within each class, many different model types,  
with each model type, you can formulate many different specific models (e.g. time-varying, time constant, covariates)

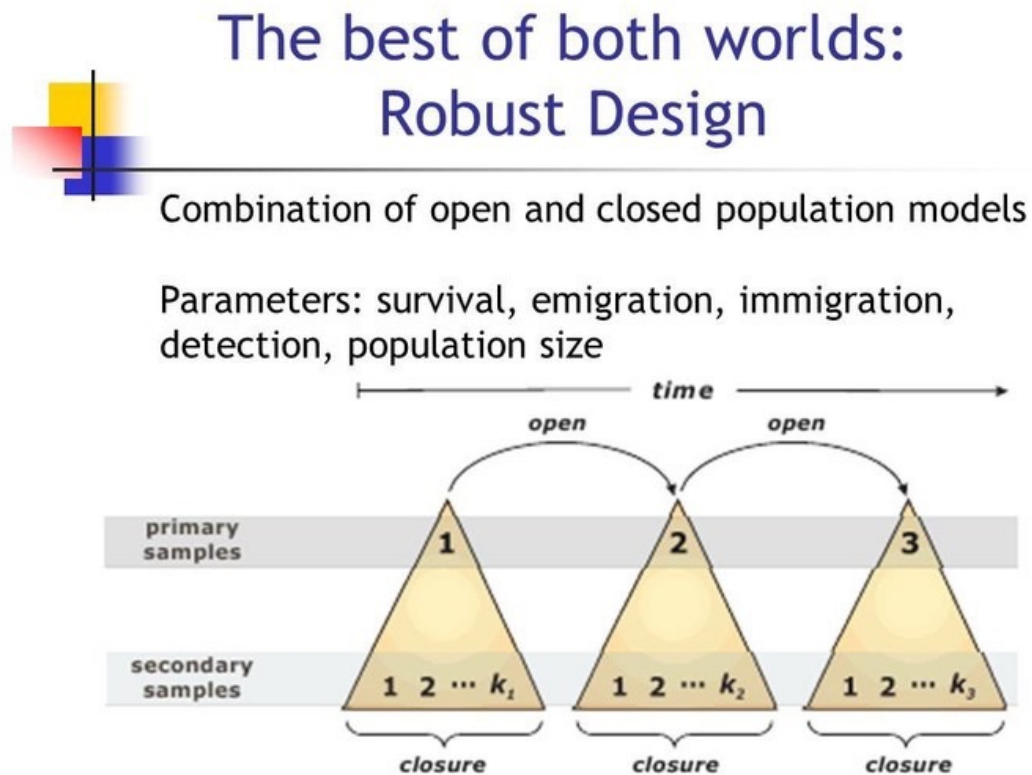
Closed-population: births and deaths happen in your sampling universe only



Open-population



## Robust design (closed and open)



## RT models

$$\text{Pr. } CJS(01101 | \text{release in } 2) = \phi_2 p_3 \phi_3 (1 - p_4) \phi_4 p_5$$

$\phi_i$  = probability that if alive in  $i$ , also alive in  $i + 1$

Turn 01101 around = 10110

$\gamma$  = seniority parameter (complement of origination probability in paleo speak)

$\gamma_i$  = probability that if alive in  $i$ , also alive in  $i - 1$

$$\text{Pr. } RT(01101 | \text{last capture in } 5) = \gamma_5 (1 - p_4) \gamma_4 p_3 \gamma_3 p_2 (1 - \gamma_2 p_1)$$

## RT models

$$\phi_t N_t = \gamma_{t+1} N_{t+1}$$

$$E(\lambda_t) = E \left[ \frac{N_{t+1}}{N_t} \right]$$

$$E(\lambda_t) = \frac{\phi_t}{\gamma_{t+1}}$$



# Pradel seniority model

The Pradel seniority (1996) model  $(\phi_t, \gamma_t, p_t)$  can be reparameterized in multiple ways, including

$(\phi_t, \lambda_t, p_t)$  population growth rate

$(\phi_t, f_t, p_t)$ , recruitment as functions of covariates, for example

Note that the POPAN; the Link-Barker and Pradel-recruitment; and the Burnham JS and Pradel-lambda formulations.

## Maximum likelihood estimation

- statistical approach for estimating the parameters of a model
- parameters are chosen to maximize the likelihood for the assumed model given the data

So do to MLE, we have to

- assume a model (e.g. data generating process) for the data in questions
- derive the likelihood function for our data, given our assumed model

# Pradel (1996): Likelihood expression to maximize

In terms of the  $u_i$ 's,  $n_i$ 's,  $v_i$ 's, and  $d_i$ 's,

$$\begin{aligned}
 L(\phi, p, \gamma, \mu) = & \prod_{i=1}^s \binom{\xi_i^{u_i}}{\gamma_i^{\sum_{j<i} u_j}} \binom{p_i^{n_i}}{\left[ (1-p_i)^{\sum_{j \leq i} u_j - \sum_{j<i} v_j - n_i} \right]} \binom{\phi_i^{\sum_{j>i} v_j}}{\mu_i^{n_i - d_i}} \\
 & \cdot \left[ (1-\mu_i)^{d_i} \right] \left[ (1-p_i(1-\mu_i))^{\sum_{j>i} u_j} \right] \left( \chi_i^{v_i - d_i} \right) \\
 & / \left( \sum_{i=1}^s \xi_i \left\{ \prod_{j=1}^{i-1} \phi_j (1-p_j(1-\mu_j)) \right\} \left\{ \prod_{j=i+1}^s \gamma_j \right\} p_i \right)^{\sum_{i=1}^s u_i}.
 \end{aligned} \tag{2}$$

Pradel 1996

openCR to the rescue

## 5.1 Non-spatial openCR models

### 5.1.1 Parameters and model types

Table 2. Parameter definitions and default link functions (nonspatial models)

Parameter	Symbol	Link	Description
p	$p$	logit	capture probability (recapture probability for CJS)
phi*	$\phi$	logit	apparent survival
b	$b$	mlogit	entry probability of PENT in MARK
f*	$f$	log	per capita recruitment rate
gamma*	$\gamma$	logit	seniority (Pradel 1996)
lambda*	$\lambda$	log	population growth rate (finite rate of increase)
superN	$N$	log	superpopulation size
BN	$B_N$	log	number of entrants
N	$N_j$	log	time-specific population size

\* parameters marked with an asterisk are scaled by the interval between primary sessions.

Table 3. Parameters of nonspatial **openCR** models

Type	Alias	p	phi	b	f	gamma	lambda	superN	BN	N
CJS		+	+							
JSSAbCL	PLBb	+	+	+						
JSSAfCL	PLBf	+	+		+					
JSSAgCL	PLBg	+	+			+				
JSSAI	PLBI	+	+				+			
JSSAb		+	+	+				+		
JSSAf		+	+		+			+		
JSSAg		+	+			+		+		
JSSAI		+	+				+	+		
JSSAB		+	+						+	
JSSAN		+	+							+

Models with type ending in CL are of the Pradel–Link–Barker type, with aliases as shown.

<https://cran.r-project.org/web/packages/openCR/vignettes/openCR-vignette.pdf>

### “Translations”

p	Sampling probability
phi	Survival probability (1-phi is extinction probability)
b	Similar to 1-gamma but not “scaled”
f	Per capita origination/speciation
gamma	Seniority probability (1-gamma is origination probability)
lamda	Net diversification rate
superN	Richness of the whole “data” (including those not seen)
BN	Number of new taxa appearing
N	Number of taxa in time interval

Conditional (closed populations)

Open populations, hence the estimates of some form of N

# Chapters in MARK book most relevant (if not using MARK)

- Chapter 1 (introduction)
- Chapter 4 (dipper example, but skip the MARK specific bits)
- Chapter 5 (goodness of fit –not covered in lectures but important)
- Chapter 6 (more on covariates and link functions)
- Chapter 11 (individual covariates)
- Chapter 12 (Pradel) and 13 (JS models in general)
- Liow, L.H. and Nichols, J.D. (2010) Estimating rates and probabilities of origination and extinction using taxonomic occurrence data: Capture-recapture approaches. In *Short Courses in Paleontology: Quantitative Paleobiology* (Hunt, G. and Alroy, J., eds), pp. 81–94, Paleontological Society (Supplementary has step by step for MARK if you are a windows user)