الجامعة المصرية اليابانية للعلوم والتكنولوجيا E-JUST Egypt - Japan University of Science and Technology エジプト日本科学技術大学	**Assignment 1: Spring 2021**
Department	Computer Science and Engineering
Submission due	Tue, Mar 30th, 2021 by 9:30am
Date	23/03/2021
Course Title	CSE 326: Analysis and Design of Algorithms
Instructor	Prof. Walid Gomaa
Allowed Equipment	None

Answer all of the following questions

1. Compute the asymptotic order of growth of the following pairs of functions. In each case determine if:
   - $f(n) \in \mathcal{O}(g(n))$,
   - $f(n) \in \Omega(g(n))$,
   - $f(n) \in \Theta(g(n))$.

$f(n)$	$g(n)$
$100n + \log n$	$n + (\log n)^2$
$\log n$	$\log n^2$
$\frac{n^2}{\log n}$	$n(\log n)^2$
$(\log n)^{\log n}$	$\frac{n}{\log n}$
$\sqrt{n}$	$(\log n)^5$
$n2^n$	3^n
$2^{\sqrt{\log n}}$	$\sqrt{n}$

2. The following algorithm computes a power of 2 with the exponent which is itself a power of 2. Analyze the algorithm in terms of both the **unit cost** and **bit cost**.

Algorithm 1 Calculate $y = 2^m$

Require: $m = 2^n$
Ensure: $y = 2^m$
  $y \leftarrow 2$
  for $i \leftarrow 1$ to $\log m$ **do**
    $y \leftarrow y \cdot y$
  end for
  return y

3. Describe and analyze an algorithm that determines whether a given graph is bipartite. The time complexity should be linear in both the number of nodes and edges of the graph.

4. Assume that each of the expressions below gives the processing time $T(n)$ spent by an algorithm for solving a problem of size n. Select the dominant term(s) having the steepest increase in n and specify the lowest $\mathcal{O}$ complexity of each algorithm.

Expression	Dominant term(s)	$\mathcal{O}(\cdots)$
$5 + 0.001n^3 + 0.025n$		
$500n + 100n^{1.5} + 50n \log_{10} n$		
$0.3n + 5n^{1.5} + 2.5n^{1.75}$		
$n^2 \log_2 n + n(\log_2 n)^2$		
$n \log_3 n + n \log_2 n$		
$3 \log_8 n + \log_2 \log_2 log_2 n$		
$100n + 0.01n^2$		
$0.01n + 100n^2$		
$2n + n^{0.5} + 0.5n^{1.25}$		
$0.01n \log_2 n + n(\log_2 n)^2$		
$100n \log_3 n + n^3 + 100n$		
$0.003 \log_4 n + \log_2 \log_2 n$		

5. The statements below show some features of the $\mathcal{O}$ notation for the functions $f(n)$ and $g(n)$. Determine whether each statement is TRUE or FALSE and correct the formula in the latter case.

Statement	true/false	in case of false, write the correct formula
Rule of sums: $\mathcal{O}(f + g) = \mathcal{O}(f) + \mathcal{O}(g)$		
Rule of products: $\mathcal{O}(f \cdot g) = \mathcal{O}(f) \cdot \mathcal{O}(g)$		
Transitivity: if $g = \mathcal{O}(f)$ and $h = \mathcal{O}(f)$ then $g = \mathcal{O}(h)$		
$5n + 8n^2 + 100n^3 = \mathcal{O}(n^4)$		
$5n + 8n^2 + 100n^3 = \mathcal{O}(n^2 \log n)$		

6. Sort each of the following sets of functions in increasing order of asymptotic $\mathcal{O}$ complexity:

a)

$$\begin{aligned} f_1(n) &= n^{0.999999} \log n \\ f_2(n) &= 10000000n \\ f_3(n) &= 1.000001n \\ f_4(n) &= n^2 \end{aligned} \tag{1}$$

b)

$$\begin{aligned} f_1(n) &= 2^{2^{1000000}} \\ f_2(n) &= 2^{100000n} \\ f_3(n) &= \binom{n}{2} \\ f_4(n) &= n\sqrt{n} \end{aligned} \tag{2}$$

c)

$$\begin{aligned} f_1(n) &= n^{\sqrt{n}} \\ f_2(n) &= 2^n \\ f_3(n) &= n^{10} \cdot 2^{n/2} \\ f_4(n) &= \sum_{i=1}^{n} (i+1) \end{aligned} \tag{3}$$