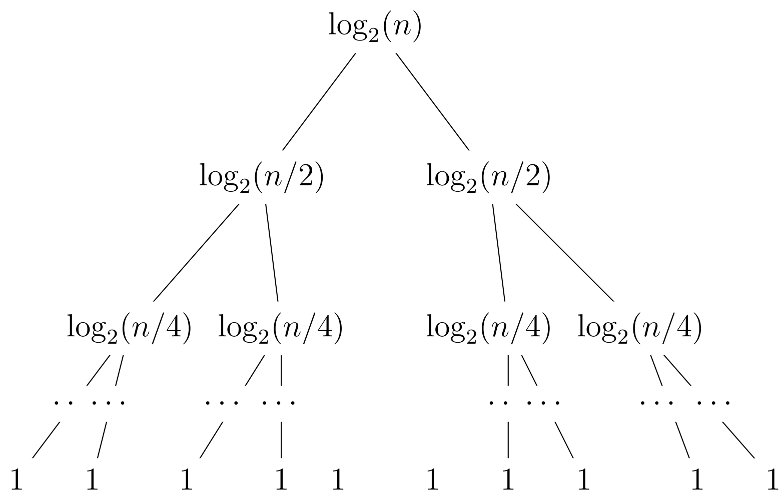
**CSE 317 Data Structures** **Issue Date:**
**Graphs and Recurrences** **Deadline:**

All graph questions (except those answered in this sheet or those labeled with "TA") are adapted from *https://chalmers.instructure.com/courses/7567/assignments/8084* and the answers are found at *https://chalmers.instructure.com/courses/7567/pages/exercises-week-6-graphs-solutions*.

**Graphs: Answer of P10** We find which bridges to build so that the total construction cost is minimum using Kruskal's algorithm. Kruskal's algorithm works by starting with each vertex in a separate cluster, and merging two clusters at each step. In this case, the clusters are merged in this order: 3 to 5 (115), 1 to 8 (120), 2 to 8 (155), 4 to 5 (160), 5 to 8 (170), 6 to 7 (175) and 2 to 6 (180).

**Recursive Formulas: Answer of P2**




The height $k$ of the recursion tree is at $T(1)$ for $T\left(\frac{n}{2^k}=1\right)$, for $k=\log_2 n$. To compute the recurrence equation exactly, first we observe that the base case $T(1)=1$ occurs at the leaf nodes of the tree at the height $k=\log_2(n)$ for number of leaf nodes $=2^{\log_2(n)}=n$. Second, as inferred from the recursion tree, the job at any height $i$ is $2^i\log_2\left(\frac{n}{2^i}\right)$. Accordingly, the solution of the recurrence equation exactly is

$$n+\sum_{i=1}^{\log_2(n)} 2^i\log_2\left(\frac{n}{2^i}\right)+\log_2 n$$

To solve the summation, we can first split it, using logarithms properties, into two summations

$$\sum_{i=1}^{\log_2(n)} 2^i\log_2\left(\frac{n}{2^i}\right)=\log_2(n)\left(\sum_{i=1}^{\log_2(n)} 2^i\right)-\sum_{i=1}^{\log_2(n)} i2^i$$

Note that $\sum_{i=1}^{z} 2^i = 2^{z+1}-2$, for any integer $z>1$, and to solve $I=\sum_{i=1}^{\log_2 n} i2^i$, introduce $i=m-1$, so

$$\sum_{i=1}^{\log_2 n} i2^i=\sum_{i=2}^{\log_2 n+1}(m-1)2^{m-1}=\frac{1}{2}\sum_{m=2}^{\log_2 n+1} m2^m-\frac{1}{2}\sum_{m=2}^{\log_2 n+1} 2^m$$

One can say that $I=0.5\left(I-2+2\left(\log_2 n+1\right)(n)\right)-0.5\sum_{m=2}^{\log_2 n+1} 2^m$

we can apply some algebraic manipulations to set $I$ into one side and evaluate its value. Finally,

$$T(n)=3n-\log_2 n-2$$