| الجامعة المصرية اليابانية للعلوم والتضنولوبية اليابانية للعلوم والتضنولية العلوم والتصنولية التصنولية العلوم والتصنولية العلوم والتصنولية العلوم والتصنولية العلوم والتصنولية العلوم والتصنولية العلوم والتصنولية التصنولية العلى التوليد العلوم والتلام والتولية التلام والتلام وال | Assignment 2: Spring 2021                  |
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| Department   | Computer Science and Engineering           |
| Submission due   | Tue, Apr 6th, 2021 by 9:30am               |
| Date   | 31/03/2021                                 |
| Course Title   | CSE 326: Analysis and Design of Algorithms |
| Instructor   | Prof. Walid Gomaa                          |
| Allowed Equipment  |  |

## Answer all of the following questions

1. What is the asymptotic complexity of an algorithm with runtime T(n,n) defined by the following recurrence equation?

$$T(x,y) = \begin{cases} T(x,c) = \Theta(x) & for \ c \le 2\\ T(c,y) = \Theta(y) & for \ c \le 2\\ T(x,y) = T(\frac{x}{2}, \frac{y}{2}) + \Theta(x+y) & otherwise \end{cases} \tag{1}$$

2. For each of the following recurrences, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply.

(a) 
$$T(n) = 3T(n/2) + n^2$$

(b) 
$$T(n) = 4T(n/2) + n^2$$

(c) 
$$T(n) = T(n/2) + 2^n$$

(d) 
$$T(n) = 2^n T(n/2) + n^n$$

(e) 
$$T(n) = 16T(n/4) + n$$

(f) 
$$T(n) = 2T(n/2) + n/\log n$$

(g) 
$$T(n) = 2T(n/4) + n^{0.51}$$

(h) 
$$T(n) = 0.5T(n/2) + 1/n$$

(i) 
$$T(n) = 16T(n/4) + n!$$

(j) 
$$T(n) = \sqrt{2}T(n/2) + \log n$$

(k) 
$$T(n) = 6T(n/3) + n^2 \log n$$

(1) 
$$T(n) = 4T(n/2) + n/\log n$$

(m) 
$$T(n) = 64T(n/8) - n^2 \log n$$

(n) 
$$T(n) = 7T(n/3) + n^2$$

(o) 
$$T(n) = 4T(n/2) + \log n$$

(p) 
$$T(n) = T(n/2) + n(2 - \cos n)$$

3. What is the asymptotic complexity of an algorithm with runtime T(n, n) defined by the following recurrence equation?

$$\begin{cases} T(x,c) = \Theta(x) & for \ c \leq 2 \\ T(x,y) = S(x,y/2) + \Theta(x) \\ S(c,y) = \Theta(y) & for \ c \leq 2 \\ S(x,y) = T(x/2,y) + \Theta(y) \end{cases}$$

$$(2)$$

4. Solve the following recurrences:

(a) 
$$T(n) = 2T(\lfloor \sqrt{n} \rfloor) + \log n \qquad with \ T(1) = \Theta(1)$$
 (3)

(b) 
$$T(n) = T(n/3) + T(2n/3) + \mathcal{O}(n)$$
 (4)