COMP9334 Assignment1
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Question 1

Given conditions:

1 cpu. 1 disk $T_{monitor} = 60min$

# completed job	1267
# cpu accesses	2178
# disk accesses	2412
cpu busy time	2929 s
Disk busy time	2765 s

(a) Define CPU as device 1, Disk as device 2

Based on Service Demand Law $D(j) = \frac{U(j)}{X(0)}$, $U(j)$ is the utilization of device j

$X(0)$ is the throughput of the system

$U(j) = \frac{B(j)}{T}$, $B(j)$ is the busy time of device j

output rate $X(0) = \frac{C(0)}{T}$, $C(0)$ is the number of requests completed for the system

Therefore, the service demand of a job at device j $D(j) = \frac{B(j)}{C(0)}$

$\Rightarrow$ The service demand at cpu is $D(1) = \frac{2929}{1267} = 2.31s$

The service demand at Disk is $D(2) = \frac{2765}{1267} = 2.18s$

(b) Utilisation limit: $U(j) \leq 1 \Rightarrow X(0) \leq \frac{1}{D(j)}$

$\Rightarrow X(0) \leq \min \frac{1}{D_j}, X_0 \leq \frac{1}{\max D_j}$

$R \geq \sum_{j=1}^{K} D_j$, $R = \frac{M}{X(0)} - Z$. $X(0) = \frac{M}{Z+R} \leq \frac{M}{Z + \sum_{j=1}^{K} D_j}$

M: Interface Clients
Z: mean thinking time
R: mean response time of the computer system
$X(0)$ = throughput

$\Rightarrow X(0) \leq \min\left[\frac{1}{\max D_i}, \frac{N}{\sum_{i=1}^{K} D_i}\right]$

thinking time $= 14s$ Users $= 20$ $\Rightarrow Z = 14, N = 20$

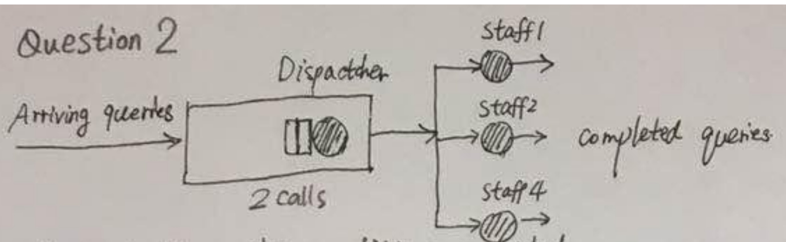
Because $D(1) = 2.31s$

$D(2) = 2.18s$

$D_{max} = 2.31s = D_{cpu} = D(1)$

$X(0) \leq \min\left[\frac{1}{2.31}, \frac{20}{2.31 + 2.18 + 14}\right] = \min(0.43, 1.082) = 0.43$

Question 2

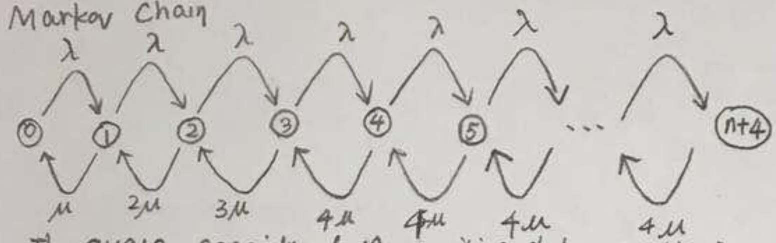


Solution: From the conditions provided

Voice-over-Ip records rate $\lambda=15$ queries $\mu=3$ queries

(a)

Markov Chain



The queue consists of n waiting slots, so the format is $M/M/4/4+n$ (kendall's notation)

state 0 : 0 calls in system (4 staffs are idle)

state 1 : 1 call in system (3 staffs are idle, 1 staff has jobs)

state 2 : 2 calls ------ (2 ------ idle, 2 ------)

state 3 : 3 calls ------ (1 ------, 3 ------)

state 4 : 4 calls ------ (All staffs has jobs)

state 5 : 5 calls ------ (------, There is 1 staff in waiting slot)

⋮

state n+4 : n+4 calls ------ (------ n ------)

The transition probabilities (with a small δ time interval)

$P[0\to1]=\lambda\delta$

$P[0\to2]=0$

$P[1\to0]=\mu\delta$

$P[2\to0]=0$

$P[2\to1]=2\mu\delta$

⋮

$P[k\to k+1]=\lambda\delta \ (0\le k\le n+3)$

$$P[k\to k-1]=\begin{cases} k\mu\delta & (1\le k\le 4)\\ 4\mu\delta & (5\le k\le 4+n)\end{cases}$$

(b) Balance equation for the continuous-time Markov Chain

state 0: $\lambda p_0=\mu p_1$

state 1: $\lambda p_0+2\mu p_2=(\lambda+\mu)p_1$

state 2: $\lambda p_1+3\mu p_3=(\lambda+2\mu)p_2$

state 3: $\lambda p_2+4\mu p_4=(\lambda+3\mu)p_3$

state 4: $\lambda p_3+4\mu p_5=(\lambda+4\mu)p_4$

....

State k: $\lambda p_{k-1}+4\mu p_{k+1}=(\lambda+4\mu)p_k \ (5\le k\le n+3)$

⋮

State n+3: $\lambda p_{n+2}+4\mu p_{n+4}=(\lambda+4\mu)p_{n+3}$

State n+4: $\lambda p_{n+3}=4\mu p_{n+4}$ $(\lambda=15, \mu=3)$

(c) We define $X_0 = \frac{\lambda}{\mu}$

When $1 \le n \le 4$ $P_n = (\frac{\lambda}{\mu})^n \cdot P_0 \cdot \frac{1}{n!} = X_0 \cdot P_0 \cdot \frac{1}{n!}$

$n > 4$ $P_n = \frac{\lambda^{n+4} \cdot P_0}{4^n \cdot 4! \cdot \mu^{n+4}} = (X_0)^{n+4} \cdot \frac{P_0}{4^n \cdot 4!}$

(d) (i) Based on 6 query, we use $n=6$ $\lambda=15$. $\mu=3$ to calculate P_n by matlab.

$P_6 = 0.2935$

(ii) $N_{avg} = X \cdot R_{avg}$ (Little's Law) $\begin{cases} X: \text{Throughput of the device} \\ R: \text{Average response time of the jobs} \\ N: \text{Avr \# of device} \end{cases}$

$n=6$ $P_6 = 0.2935$ $X = \text{rate} \times (1 - P_{reject})$

$\Rightarrow X = (1-P_6) \times 15 = 10.605$ transactions/h

$R_{avg} = \frac{N_{avg}}{X} = \frac{4.35}{10.605} \doteq 0.4$ hours

Mean waiting time = Mean response time − Mean service time $= 0.41 - \frac{1}{4 \cdot \mu} \approx 0.32$ hour

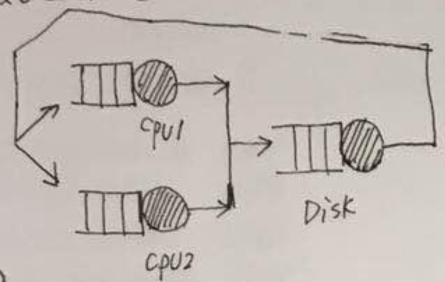
(e) We calculate these results from matlab programs

adding $\begin{cases} 5 \\ 10 \\ 15 \\ 20 \end{cases}$ slots $\Rightarrow n = \begin{cases} 5+2+4=11 \\ 10+2+4=16 \\ 15+2+4=21 \\ 20+2+4=26 \end{cases}$ requires.

We run the results as $\begin{cases} P_{11} = 0.2233 & P_{21} = 0.2071 \\ P_{16} = 0.2023 & P_{26} = 0.2007 \end{cases}$

(f) We can make a conclusion that the limit capability for system to process is adding 10 more waiting slots (overall 12). Because numbers of staffs are only 4. When adding more than 10 slots, it has little influence on the system. Under this limitation, Only when we enforce work efficiency of staff can help enhance the system efficiency.

Question 3



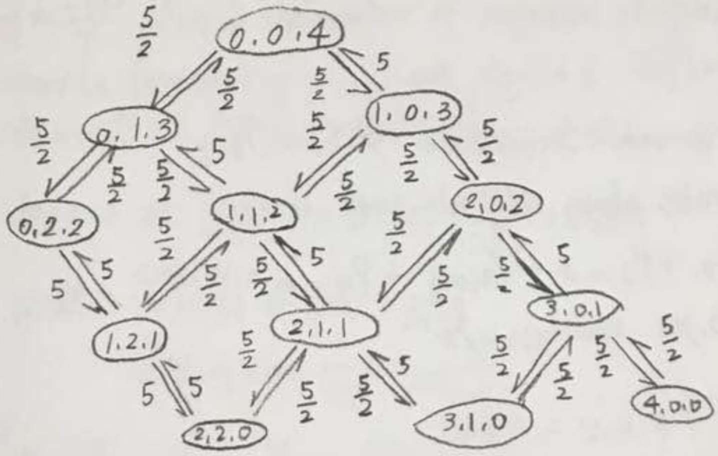
(a)

(1) Obviously we have 12 states. The list of states is (0.0.4), (1.0.3), (0.1,3), (2.0,2), (1.1.2), (0.2.2), (3.0.1), (2.1.1), (1.2.1), (4.0.0), (3,1.0), (2,2.0)

(2) From the condition we get that mean processing time $T_{disk} = 0.2s$, $T_{cpu1} = 0.2s$. $T_{cpu2} = 0.4s$

Thus, the transition rate of CPU1, Cpu2, Disk is

$R_{cpu1} = \frac{1}{0.2} = 5$ transactions/s, $R_{cpu2} = \frac{1}{0.4} = \frac{5}{2}$ transactions/s, $R_{Disk} = \frac{1}{0.2} = 5$ transactions/s



(b) The balance equation are based on the in & out of Nodes in Markov chain

$5P_{0,0,4} - 5P_{1,0,3} - 2.5P_{0,1,3} + 0P_{2,0,2} + 0P_{1,1,2} + 0P_{0,2,2} + 0P_{3,0,1} + 0P_{2,1,1} + 0P_{1,2,1} + 0P_{4,0,0} + 0P_{3,1,0} + 0P_{2,2,0} +$

$-\frac{5}{2} P_{0,0,4} + 10P_{1,0,3} + 0P_{0,1,3} - 5P_{2,0,2} - \frac{5}{2} P_{1,1,2} + 0P_{0,2,2} + 0P_{3,0,1} + 0P_{2,1,1} + 0P_{1,2,1} + 0P_{4,0,0} + 0P_{3,1,0} + 0P_{2,2,0} +$

$-\frac{5}{2} P_{0,0,4} + 0P_{1,0,3} + \frac{15}{2}P_{0,1,3} + 0P_{2,0,2} - 5P_{1,1,2} - \frac{5}{2} P_{0,2,2} + 0P_{3,0,1} + 0P_{2,1,1} + 0P_{1,2,1} + 0P_{4,0,0} + 0P_{3,1,0} + 0P_{2,2,0} +$

$0P_{0,0,4} - \frac{5}{2} P_{1,0,3} + 0P_{0,1,3} + 10P_{2,0,2} + 0P_{1,1,2} + 0P_{0,2,2} - 5P_{3,0,1} - \frac{5}{2} P_{2,1,1} + 0P_{1,2,1} + 0P_{4,0,0} + 0P_{3,1,0} + 0P_{2,2,0} +$

$0P_{0,0,4} - \frac{5}{2} P_{1,0,3} - \frac{5}{2} P_{0,1,3} + 0P_{2,0,2} + \frac{25}{2}P_{1,1,2} + 0P_{0,2,2} + 0P_{3,0,1} - 5P_{2,1,1} - \frac{5}{2} P_{1,2,1} + 0P_{4,0,0} + 0P_{3,1,0} + 0P_{2,2,0} +$

$0P_{0,0,4} + 0P_{1,0,3} - \frac{5}{2} P_{0,1,3} + 0P_{2,0,2} + 0P_{1,1,2} + \frac{15}{2}P_{0,2,2} + 0P_{3,0,1} + 0P_{2,1,1} - 5P_{1,2,1} + 0P_{4,0,0} + 0P_{3,1,0} + 0P_{2,2,0} +$

$0P_{0,0,4} + 0P_{1,0,3} + 0P_{0,1,3} - \frac{5}{2} P_{2,0,2} + 0P_{1,1,2} + 0P_{0,2,2} + 10P_{3,0,1} + 0P_{2,1,1} + 0P_{1,2,1} - 5P_{4,0,0} - \frac{5}{2} P_{3,1,0} + 0P_{2,2,0} +$

$0P_{0,0,4} + 0P_{1,0,3} + 0P_{0,1,3} - \frac{5}{2} P_{2,0,2} - \frac{5}{2} P_{1,1,2} + 0P_{0,2,2} + 0P_{3,0,1} + \frac{25}{2}P_{2,1,1} + 0P_{1,2,1} + 0P_{4,0,0} - 5P_{3,1,0} - \frac{5}{2} P_{2,2,0} +$

$0P_{0,0,4} + 0P_{1,0,3} + 0P_{0,1,3} + 0P_{2,0,2} + 0P_{1,1,2} - 5P_{0,2,2} + 0P_{3,0,1} + 0P_{2,1,1} + \frac{25}{2}P_{1,2,1} + 0P_{4,0,0} + 0P_{3,1,0} - 5P_{2,2,0} +$

$0P_{0,0,4} + 0P_{1,0,3} + 0P_{0,1,3} + 0P_{2,0,2} + 0P_{1,1,2} + 0P_{0,2,2} - \frac{5}{2} P_{3,0,1} + 0P_{2,1,1} + 0P_{1,2,1} + 5P_{4,0,0} + 0P_{3,1,0} + 0P_{2,2,0} +$

$0P_{0,0,4} + 0P_{1,0,3} + 0P_{0,1,3} + 0P_{2,0,2} + 0P_{1,1,2} + 0P_{0,2,2} - \frac{5}{2} P_{3,0,1} - \frac{5}{2} P_{2,1,1} + 0P_{1,2,1} + 0P_{4,0,0} + \frac{15}{2}P_{3,1,0} + 0P_{2,2,0} +$

$0P_{0,0,4} + 0P_{1,0,3} + 0P_{0,1,3} + 0P_{2,0,2} + 0P_{1,1,2} + 0P_{0,2,2} + 0P_{3,0,1} - \frac{5}{2} P_{2,1,1} - 5P_{1,2,1} + 0P_{4,0,0} + 0P_{3,1,0} + \frac{15}{2}P_{2,2,0} +$

$= 1$

(c). We use Matlab to solve the question. The solution is

$$\begin{cases} P_{0,0,4} = 0.1711 \\ P_{1,0,3} = 0.0912 \\ P_{0,1,3} = 0.1598 \\ P_{2,0,2} = 0.0501 \\ P_{1,1,2} = 0.0935 \\ P_{0,2,2} = 0.1213 \\ P_{3,0,1} = 0.0259 \\ P_{2,1,1} = 0.0572 \\ P_{1,2,1} = 0.1021 \\ P_{4,0,0} = 0.0130 \\ P_{3,1,0} = 0.0277 \\ P_{2,2,0} = 0.0871 \end{cases}$$

(d) Throughput = Utilisation × Service Rate $\Rightarrow Th = U \times r$

We compute the rate where only the part works.

Thus, $r_{cpu1} = P_{1,0,3} + P_{2,0,2} + P_{3,0,1} + P_{4,0,0} = 0.1802$

$Th_{cpu1(only)} = 5 \times 0.1802 = 0.901$ transactions/s

$r_{cpu2} = P_{0,1,3} + P_{0,2,2} = 0.2811$

$Th_{cpu2(only)} = \frac{5}{2} \times 0.2811 = 0.7028$ transactions/s

$Th_{cpu1+cpu2} = r_{cpu1+cpu2} \cdot U_{1+2} = U(P_{1,1,2} + P_{2,1,1} + P_{1,2,1} + P_{3,1,0} + P_{2,2,0}) = 0.368 \times 7.5 = 2.76$ transactions/s

$Th_{total} = 0.901 + 0.7028 + 2.76 = 4.364$ transactions/s

(e) Mean number of jobs.

$$N_{cpu1} = \bar{m} = \sum_{i=1}^{n} n_i p_i = (P_{1,2,1} + P_{1,0,3} + P_{1,1,2}) \times 1 + (P_{2,0,2} + P_{2,1,1} + P_{2,2,0}) \times 2 + (P_{3,0,1} + P_{3,1,0}) \times 3 + P_{4,0,0} \times 4$$
$$= 0.89$$

(f) Generally. $Th_{cpu1} = Th_{cpu1(only)} + Th_{total} U_1 = 0.901 + 0.368 \times 5 = 2.74$ transactions/s

Based on Little's Law

$$R = \frac{N}{X} = \frac{N_{cpu1}}{Th_{cpu1}} = \frac{0.89}{2.739} = 0.33 \text{ seconds}$$