Solutions to *Bruce Hansen's Econometrics*

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May 2, 2015

Contents

Revision History

Revision	Date	Author(s)	Description
1.0	01-05-2015	TC	created

Chapter 1

Introduction

No exercises in this chapter.

Chapter 2

Conditional Expectations and Projections

Problem 1: Find $\mathbb{E}\left(\mathbb{E}\left(\mathbb{E}\left(Y \mid X_1, X_2, X_3\right) \mid X_1, X_2\right) \mid X_1\right)$.
Solution: By the law of iterated expectations:

$$\mathbb{E}\left(\mathbb{E}\left(Y \mid X_1, X_2, X_3\right) \mid X_1, X_2\right) = \mathbb{E}\left(Y \mid X_1, X_2\right)$$

and

$$\mathbb{E}\left(\mathbb{E}\left(Y \mid X_1, X_2\right) \mid X_1\right) = \mathbb{E}\left(Y \mid X_1\right)$$

Problem 2: If $\mathbb{E}\left(Y \mid X\right) = a + bX$, find $\mathbb{E}\left(YX\right)$ in terms of the moments of X.
Solution: We know that:

$$\begin{aligned} \mathbb{E}\left(YX\right) &= \mathbb{E}\left(\mathbb{E}\left(Y \mid X\right) X\right) && \text{by LIE} \\ &= \mathbb{E}\left(aX + bX^2\right) \\ &= a\mu_X + b(\sigma_X^2 - \mu_X^2) \end{aligned}$$

Problem 3: Prove Theorem 2.8.1.4 (Properties of the CEF error). If $\mathbb{E}\left(|Y|\right) < \infty$, then for any function $h(\boldsymbol{X})$ such that $\mathbb{E}\left(|h(\boldsymbol{X})\varepsilon|\right) < \infty$, $\mathbb{E}\left(h(\boldsymbol{X})\varepsilon\right) = 0$.
Solution: Since $\mathbb{E}\left(|h(\boldsymbol{X})\varepsilon|\right) < \infty$, $h(\boldsymbol{X})\varepsilon$ is integrable, and by the conditioning theorem,

$$\mathbb{E}\left(h(\boldsymbol{X})\varepsilon\right) = \mathbb{E}\left(h(\boldsymbol{X})\mathbb{E}\left(\varepsilon \mid \boldsymbol{X}\right)\right)$$

but by the mean independence of the errors, $\mathbb{E}\left(\varepsilon \mid \boldsymbol{X}\right) = 0$, so

$$= 0$$

Problem 4**:** Suppose that $X, Y \in \{0, 1\}$ and the joint PDF is

	$X = 0$	$X = 1$
$Y = 0$	0.1	0.2
$Y = 1$	0.4	0.3

Find $\mathbb{E}(Y \mid X)$, $\mathbb{E}\left(Y^2 \mid X\right)$, $\mathbb{V}(Y \mid X)$ for $X = 0$ and $X = 1$.
Solution:

$$\begin{aligned}
\mathbb{E}(Y \mid X = 0) &= 0 \times 0.1 + 1 \times 0.4 \\
&= 0.4 \\
&= \mathbb{E}\left(Y^2 \mid X = 0\right) \\
\mathbb{E}(Y \mid X = 1) &= 0 \times 0.2 + 1 \times 0.3 \\
&= 0.3 \\
&= \mathbb{E}\left(Y^2 \mid X = 1\right) \\
\mathbb{V}(Y \mid X = 0) &= \mathbb{E}\left(Y^2 \mid X = 0\right) - \left(\mathbb{E}(Y \mid X = 0)\right)^2 \\
&= 0.24 \\
\mathbb{V}(Y \mid X = 1) &= \mathbb{E}\left(Y^2 \mid X = 1\right) - \left(\mathbb{E}(Y \mid X = 1)\right)^2 \\
&= 0.21
\end{aligned}$$

Problem 5**:** Show that $\sigma^2(\boldsymbol{X})$ is the best predictor of ε^2 given $\boldsymbol{X}$:

- Write down the MSE of a predictor $h(\boldsymbol{X})$ for ε^2.
- What does it mean to be predicting ε^2?
- Show that $\sigma^2(\boldsymbol{X})$ minimizes the MSE and is thus the best prediction.

Solution: The MSE of any function h for ε^2 is

$$\mathbb{E}\left(\varepsilon^2 - h(\boldsymbol{X})\right)^2$$

We say we are predicting ε^2 if we are attempting to estimate its value given the information in the observables $\boldsymbol{X}$, using the function estimate of h.

Chapter 3

The Algebra of Least Squares