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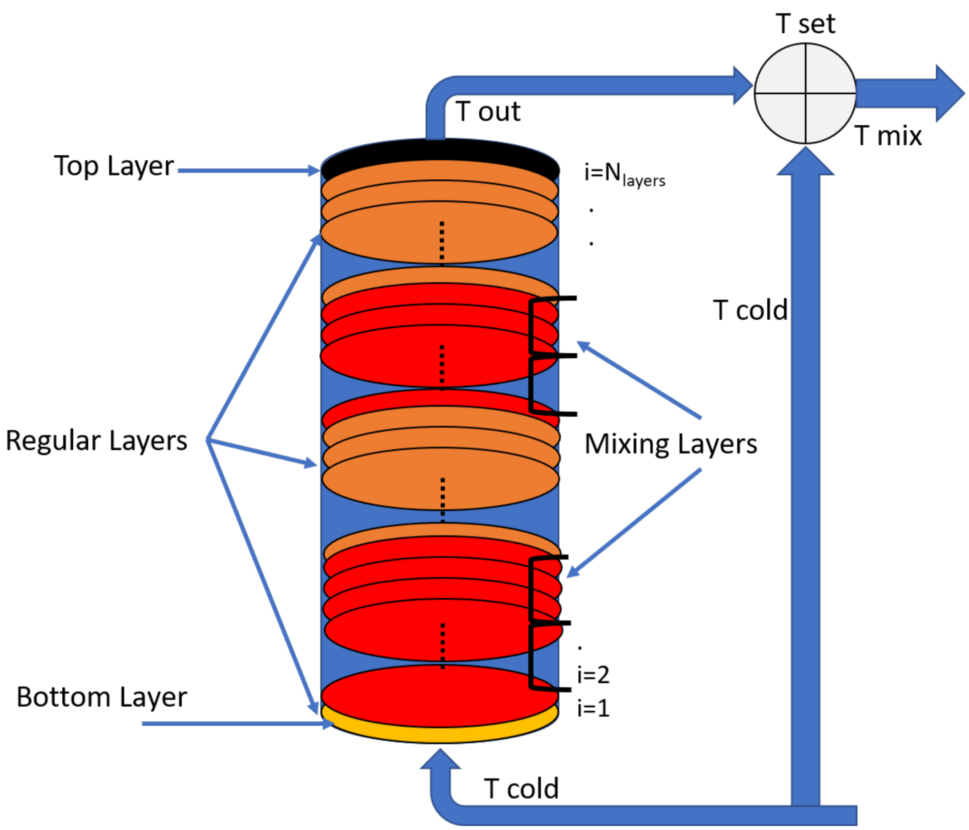
Fixed Thermally Stratified Layer Model

My inclination towards proposing a fixed thermally stratified layer model is the fact that it automates the procedure of defining varying number of layers on the go. The main idea of this model is that the water tank is assumed to be composed of 4 types of layers: 1) Mixing layers, 2) Regular layers, 3) Top layer and Bottom Layer. The following figure shows the arrangement of these layers.



The assumption for each layer remains that they are isothermal with equal volume. For more information regarding the volume draws, control capabilities and temperature settings, see the document by Rob (DSO+T Water Heater Agent).

<u>Regular Layer Modeling:</u>

Consider i^{th} regular layer, then heat balance around this layer can be considered as:

$$\rho\, c_p\, V_i\, dT_i/dt = \rho\, c_p\, \dot{V}\, (T_{i-1} - T_i) - UA_i\, (T_i - T_{amb}) + R_{cond,i} \qquad (1)$$

In (1), UA_n is the heat loss coefficient, ρ is the density c_p is the specific heat, $\dot{V}$ is the rate of volume of water draw from the tank, and V_i is the volume of the layer. While the rest of the terms have been earlier defined and explained quite clearly in a document by Rob (See DSO+T Water Heater Agent.docx), there is a new term popping here, denoted as $R_{cond,i}$. This $R_{cond,i}$ stands for the conductive heat transfer between i^{th} layer and its adjacent layers. In its full generality, the term R_{cond} is expressed as the onedimensional heat equation, which solves for temperature T as a function of space x, time t, and constant coefficient α.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, 0 \leq x \leq L, t \geq 0$$

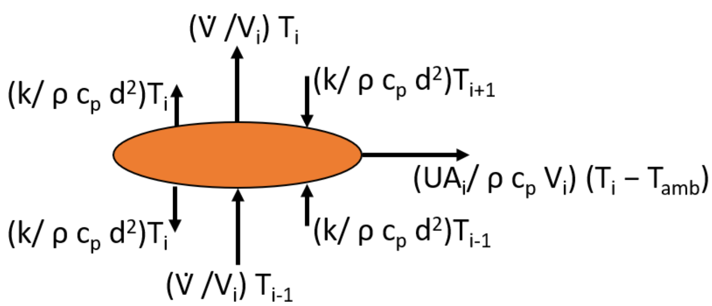
However, thanks to some works (Recktenwald 2011, Y. Han. 2009, L. Mongibello 2014), this partial differential equation has been shown to be well approximated by the following difference equation, defined for each i^{th} layer:

$$R_{cond,i} = (V_i\, k/d^2)(T_{i-1} - 2\, T_i + T_{i+1})$$

where d is the width of the layer i and k is the thermal conductivity. Including these expressions in (1) yields:

$$\rho\, c_p\, V_i\, dT_i/dt = \rho\, c_p\, \dot{V}_i\, (T_{i-1} - T_i) - UA_i\, (T_i - T_{amb}) + (V_i\, k/d^2)(T_{i-1} - 2\, T_i + T_{i+1})$$

$$dT_i/dt = (\dot{V}_i / V_i)\, (T_{i-1} - T_i) - (UA_i / \rho\, c_p\, V_i)\, (T_i - T_{amb}) + (k/ \rho\, c_p\, d^2)(T_{i-1} - 2\, T_i + T_{i+1}) - (2)$$



Regular Layer

Mixed Layer Modeling:

Now using the same procedure adopted for the regular layer, we model the mixing layer and its associated heat and energy balances. Consider i^{th} mixing layer, then heat balance around this layer can be considered as:

$$\rho\, c_p\, V_i\, dT_i/dt = \rho\, c_p\, \dot{V}_i\, (T_{i-1} - T_i) - UA_i\, (T_i - T_{amb}) + R_{cond,i} + P_i + R_{circ,i} - (3)$$

Here, P_i is the power received by the i^{th} mixing layer from the heating element, i.e., $P_i = P_{rated}/N_{mixlayer}$ with $N_{mixlayer}$ are the number of mixing layers. This distributes power to all mixing layer proportionally.

Another new term pops up here: $R_{circ,i}$. This term has been proposed by researchers [Stephen Koch, 2012] to make life easier when capturing the effect of turbulence and mixing when the heating element is turned on. Otherwise, we would need to model density differences and buoyancy effects which would have made life extremely difficult. However, another approach has been worked out right now by Rob, by dynamically creating and merging layers around heating elements. See the other document for that.

$R_{circ,i}$ is modeled similar to $R_{cond,i}$, albeit with a heuristic circular volume flow rate $\dot{V}_{circ}$

$$R_{circ,i} = \rho\, c_p\, \dot{V}_{circ}\, (T_{i-1} - 2\, T_i + T_{i+1}) u_i$$

$\dot{V}_{circ}$ is then a heuristic value, which can be adjusted and observed till plausible mixing phenomena is observed in the model. Note that u_i is the input element control switch. This models the circulation flow only when the heating element is turned on.

Now combining everything, we get for the i^{th} mixing layer,

$$dT_i/dt = (\dot{V} / V_i)\, (T_{i-1} - T_i) - (UA_i / \rho\, c_p\, V_i)\, (T_i - T_{amb}) + (k/ \rho\, c_p\, d^2)(T_{i-1} - 2\, T_i + T_{i+1}) + P_{rated}/(\rho\, c_p\, V_i\, N_{mixlayer})$$

$$+ (\dot{V}_{circ} / V_i)(T_{i-1} - 2\, T_i + T_{i+1}).u_i \qquad (4)$$

Bottom Layer Modeling:

The bottom layer is modeled similar to regular layer, however, with the two difference: 1) There is no layer beneath the bottom layer, so from R_{cond} the term T_{i-1} vanishes, 2) in the loss term the area exposed to the ambient temperature is modified, as it is more than the regular layer. Since, we have one bottom layer, and we have fixed it at position 1 (see figure), we can be a bit more specific here and write its temperature evolution final expression as:

$$dT_1/dt = (\dot{V} / V_1)\, (T_{inflow} - T_1) - (UA_{bottom} / \rho\, c_p\, V_1)\, (T_1 - T_{amb}) + (k/ \rho\, c_p\, d^2)(T_2 - 2\, T_1) - (5)$$

Top Layer Modelling:

Similar to the bottom layer, the top layer is fixed at position N_{Layers} (see figure) and its temperature evolution follows:

$$dT_{NLayers} /dt = (\dot{V} / V_{NLayers})\, (T_{NLayers-1} - T_{NLayers}) - (UA_{top} / \rho\, c_p\, V_{NLayers})\, (T_{NLayers} - T_{amb})$$

$$+ (k/ \rho\, c_p\, d^2)(T_{NLayers-1} - 2\, T_{NLayers}) - (6)$$

Compacting the Model:

Defining a state vector x of the form,

$$\mathbf{x} = [T_{inflow}, T_1, T_2, \ldots., T_{NLAYERS}, T_{amb}],$$

the control vector,

$$\mathbf{u} = [u_1, u_2],$$

with u_1 and u_2 being the lower and upper element switch controls and the individual the whole system of equations defined above can be written as:

$$\mathbf{dx/dt} = \mathbf{A}(\dot{V}, \mathbf{u})\, \mathbf{x} + \mathbf{Bu}$$

where A and B can be defined using the collecting the terms from (2), (4), (5) and (6).

Model Characteristics:

Note that the collected terms in $\mathbf{A}(\dot{V}, \mathbf{u})$ are the function of $\dot{V}$ and $\mathbf{u}$. This means that these differential equations have varying parameters. However, for the given parameters, they are of the first order. These types of models are also called as Linear Parameter Varying Model (LPV), which are well known for control engineers. I don't know how we are going to tackle this issue, but in general for famous control problems in the literature, we can also analyze their stability, controllability etc. However, I don't know much about them.

Degree of Freedom:

How many layers are enough? There is no direct answer to that except, when we could calibrate the model. We can start with a smaller number of layers (5, 10 etc) and then see if we need more or these are enough.

Solution:

Now question comes regarding how we can solve such a system! I would say, we try Rob's discretization method, which he proposed for the fixed 2-node method. The method essentially, for a given interval, takes smaller steps in a linear domain, i.e., assuming the temperatures remain constant within this small step, to accommodate for nonlinearity. With this assumption, we can represent our system of equation as:

$$\Delta\mathbf{x}/\Delta t = \mathbf{A}(\dot{V}, \mathbf{u})\ \mathbf{x} + \mathbf{B}\mathbf{u},$$

Which translates to computing x(k+1), while knowing ($\dot{V}$ (k), **u**(k),x(k)) at step k;

$$\mathbf{x}(k+1) = \mathbf{x}(k) + (\mathbf{A}(\dot{V}(k), \mathbf{u}(k))\ \mathbf{x}(k) + \mathbf{B}\mathbf{u}(k))\ \Delta t$$

In the above equation, the temperatures x(k) are initialized at the very beginning of the simulation and are then consequently computed for each next time step (k+1). The term $\dot{V}$ (k) is computed with the knowledge of water draw volume rate in combination with the mixing valve position (see DSO+T Water Heater Agent.docx for more information) and **u**(k) is calculated using hysteresis controller, i.e. turned on/off when the corresponding output temperature is outside the allowable temperature range with the upper element taking piority.

Simulations:

Preliminary results are available and they look proimising.

References:

Stephen Koch, "Demand Response Methods for Ancillary Services and Renewable Energy Integration in Electric Power Systems", 2012.

L. Mongibello et. al., "Numerical Simulation of a Solar Domestic Hot Water System", 2014.

Y Han, et. al., "Thermal stratification within the water tank", 2009.

Geral W. Recktenwald, "Finite-Difference Approximations to the Heat Equation", 2011.