Let $G(x, y)$ be the Green's function for the Laplace equation in a bounded domain with smooth boundary $\partial\Omega$, and let $\nu(y)$ be the outward normal vector to $\partial\Omega$ at $y \in \partial\Omega$. Then we want to show that the normal derivative of $G(x, y)$ with respect to $\partial\Omega$ is positive for $x \in \Omega$ and $y \in \partial\Omega$, i.e.,

To prove this, we first note that the Green's function $G(x, y)$ satisfies the following properties:

$G(x, y) = G(y, x)$ $\Delta_x G(x, y) = \delta(x - y)$ in the sense of distributions $G(x, y) = 0$ for $x \in \partial\Omega$ or $y \in \partial\Omega$ The boundary values of $G(x, y)$ satisfy the boundary condition $G(x, y)|_{\partial\Omega} = 0$ for $x, y \in \partial\Omega$.

Using these properties, we can compute the normal derivative of $G(x, y)$ as follows:

$\partial G \overline{\partial\nu_y(x,y) = \lim_{\epsilon\to 0+} \frac{G(x,y+\epsilon\nu(y)) - G(x,y)}{\epsilon} \; = \lim_{\epsilon\to 0+} \frac{1}{\epsilon}\int_0^\epsilon \frac{d}{dt} G(x,y+t\nu(y))dt \; = \lim_{\epsilon\to 0+} \int_0^\epsilon \frac{d}{dt}\left(\frac{1}{\epsilon} G(x,y+t\nu(y))\right)dt \; = \lim_{\epsilon\to 0+}}$

where $H_\epsilon(z, y)$ is the function

Note that $H_\epsilon(z, y)$ is a non-negative function, since $G(z, y + t\nu(y))$ is positive for $t > 0$ and $G(z, y)$ is zero. Therefore, to show that the normal derivative of $G(x, y)$ is positive, it suffices to show that the integral