Esercizi sulla trasformata Fourier

fondamenti di telecomunicazioni

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1.
$$x(t) = \frac{A}{2} rect\left(\frac{t}{T}\right) - \frac{A}{T} t\ rect\left(\frac{t}{T}\right)$$
$$\frac{dx(t)}{dt} = x'(t) = A\delta\left(t + \frac{T}{2}\right) - \frac{A}{T} rect\left(\frac{t}{T}\right) \xrightarrow{\mathscr{F}} X'(f) = Ae^{2i\pi f \frac{T}{2}} - \frac{A}{\cancel{T}}\cancel{T} sinc(Tf) = 2i\pi f X(f)$$
$$\Rightarrow X(f) = \frac{X'(f)}{2i\pi f} = \frac{A}{2i\pi f}\left[e^{i\pi f T} - sinc(Tf)\right]$$

2.
$$Y(f) = \frac{1}{2}\left[X(f - f_0) + X(f + f_0)\right]\cos\left(\frac{2\pi f}{f_0}\right)$$
$$y(t) = \left[\frac{1}{2}x(t)e^{2i\pi f_0 t} + \frac{1}{2}x(t)e^{-2i\pi f_0 t}\right] * \left[\frac{1}{2}\delta\left(t - \frac{1}{f_0}\right) + \frac{1}{2}\left(t + \frac{1}{f_0}\right)\right] =$$
$$= x(t)\cos(2\pi f_0 t) * \left[\frac{1}{2}\delta\left(t - \frac{1}{f_0}\right) + \frac{1}{2}\left(t + \frac{1}{f_0}\right)\right] =$$
$$= \frac{1}{2}x\left(t - \frac{1}{f_0}\right)\cos\left[2\pi f_0\left(t - \frac{1}{f_0}\right)\right] + \frac{1}{2}x\left(t + \frac{1}{f_0}\right)\cos\left[2\pi f_0\left(t + \frac{1}{f_0}\right)\right] =$$
$$= \frac{1}{2}\left[x\left(t - \frac{1}{f_0}\right) + x\left(t + \frac{1}{f_0}\right)\right]\cos(2\pi f_0 t)$$

3.
$$x(t) = rect(2t) - rect\left[4\left(t + \frac{3}{8}\right)\right] - rect\left[4\left(t - \frac{3}{8}\right)\right] \xrightarrow{\mathscr{F}}$$
$$\xrightarrow{\mathscr{F}} X(f) = \frac{1}{2} sinc\left(\frac{f}{2}\right) - \frac{1}{4} sinc\left(\frac{f}{4}\right) e^{-2i\pi \frac{3}{8} f} - \frac{1}{4} sinc\left(\frac{f}{4}\right) e^{2i\pi \frac{3}{8} f} =$$
$$\underset{\text{formule di triplicazione del coseno}}{=\!=\!=} \frac{1}{2}\left[sinc\left(\frac{f}{2}\right) - sinc\left(\frac{f}{4}\right)\cos\left(\frac{3\pi f}{4}\right)\right]$$

4.
$$x(t) = \sum_{n \in \mathbb{Z}} rect\left(\frac{t - nT}{T}\right)$$
$$c_n = \frac{\tau}{T} sinc\left(\frac{\tau n}{T}\right)$$
$$\Rightarrow X(f) = \sum_{n \in \mathbb{Z}} \frac{\tau}{T} sinc\left(\frac{\tau n}{T}\right)\delta\left(f - \frac{n}{T}\right)$$

5.
$$\pi(t) = \sum_{n \in \mathbb{Z}} \delta(t - nT)$$
$$c_n = \frac{1}{T}$$
$$\Rightarrow \Pi(f) = \frac{1}{T} \sum_{n \in \mathbb{Z}} \delta \left(f - \frac{n}{T} \right)$$

Quindi il pettine (o treno campionatore) si autotrasforma

6. Treno di trapezi
$$\dot{x} = 2rect\left(\frac{2t}{3T}\right) * rect\left(\frac{2t}{T}\right)$$
$$\overline{T} = \frac{5}{2}T$$
$$c_n = 2\frac{1}{\overline{T}}\frac{3T}{2}sinc\left(\frac{3T}{2}\frac{n}{\overline{T}}\right)\frac{T}{2}sinc\left(\frac{T}{2}\frac{n}{\overline{T}}\right) = \frac{3T}{5}sinc\left(\frac{3n}{2}\right)sinc\left(\frac{n}{2}\right)$$
$$\Rightarrow X(f) = \frac{3T}{5}\sum_{n \in \mathbb{Z}} sinc\left(\frac{3n}{2}\right)sinc\left(\frac{n}{2}\right)\delta\left(f - \frac{n}{T}\right)$$

7.
$$x(t) = [\cos(2\pi f_0 t)\sin(2\pi f_0 t)] * \frac{\sin(3\pi f_0 t)}{\pi t} = \frac{1}{2}\sin(4\pi f_0 t) * 3f_0 sinc(3f_0 t)$$
$$X(f) = \frac{1}{4}\left[\frac{\delta(f - 2f_0)}{i} - \frac{\delta(f + 2f_0)}{i}\right]\frac{\cancel{3f_0}}{\cancel{3f_0}}rect\left(\frac{f}{3f_0}\right) = \frac{1}{4i}\left[\delta(f - 2f_0)rect\left(\frac{2\cancel{f_0}}{3\cancel{f_0}}\right) - \delta(f + 2f_0)rect\left(-\frac{2\cancel{f_0}}{3\cancel{f_0}}\right)\right] =$$
$$= \frac{1}{4i}rect\left(\frac{2}{3}\right)[\delta(f - 2f_0) - \delta(f + f_0)] \stackrel{rect(\frac{2}{3})=0}{=\!=\!=\!=} 0 \Rightarrow x(t) = 0$$