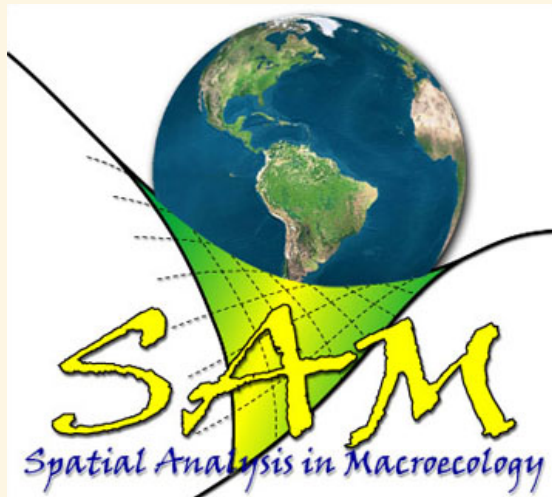


# SPATIAL MODELLING

DAVID ORME

# SPATIAL MODELLING TOOLS

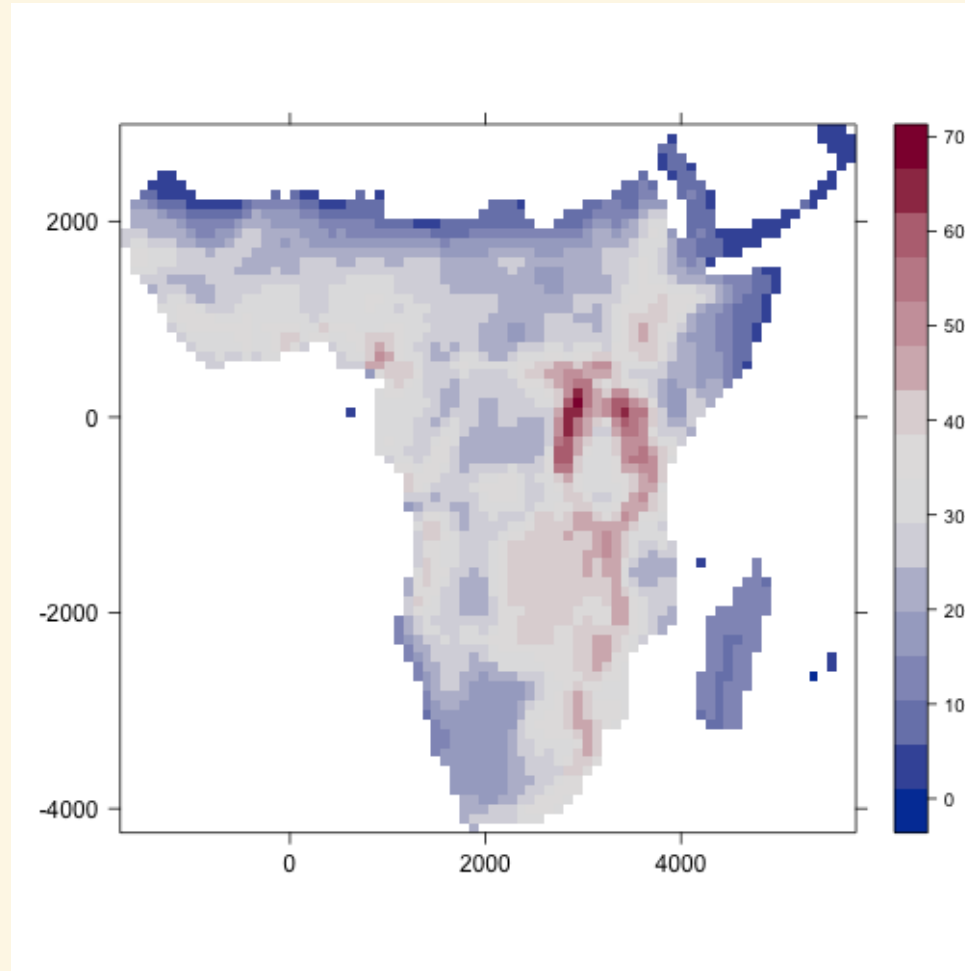
- The examples presented here use R
- Another excellent program with a nice GUI interface:
  - Spatial Analysis in Macroecology
  - <http://www.ecoevol.ufg.br/sam/>



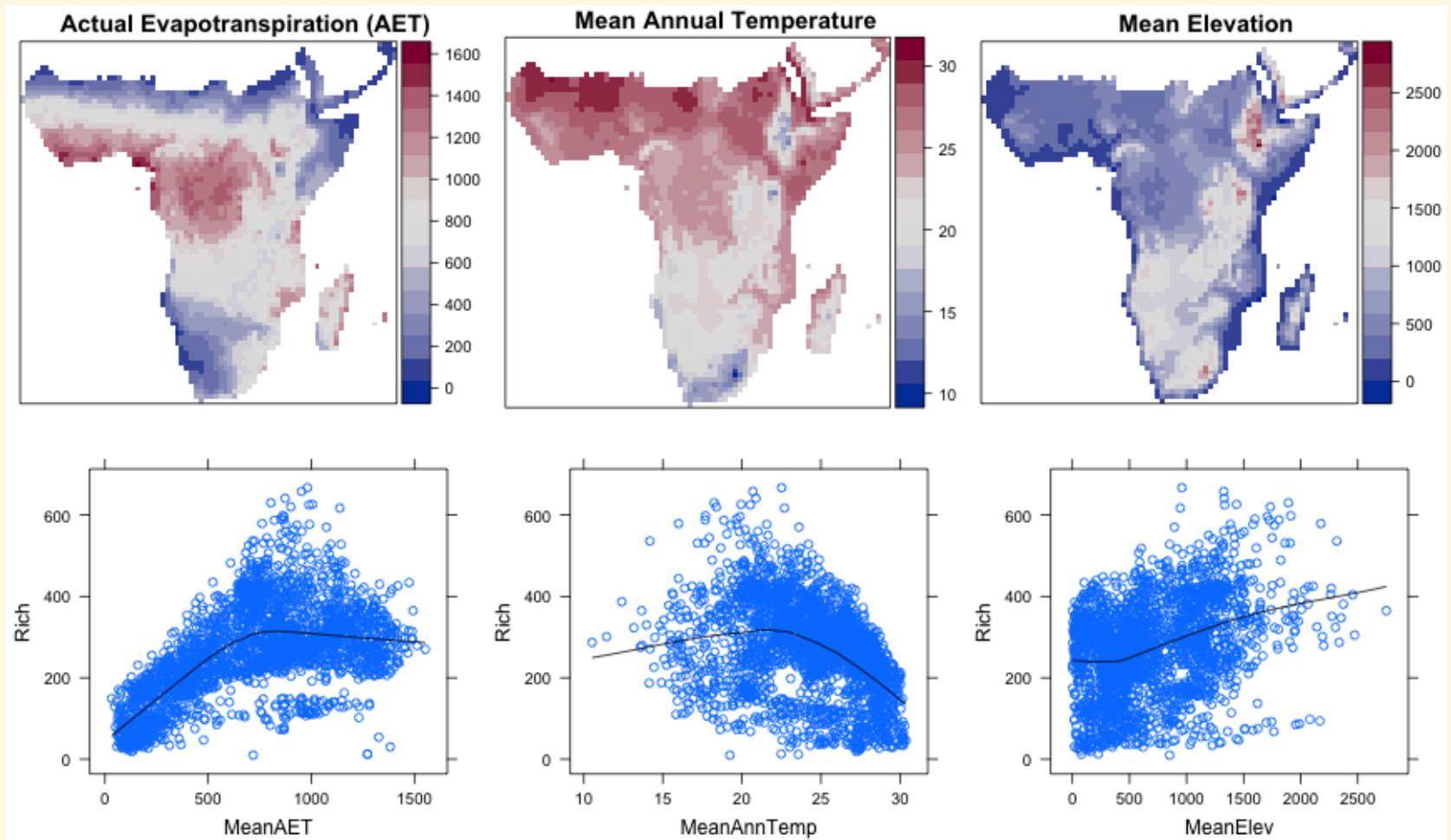
# OVERVIEW

- Example data: Afrotropical bird diversity
- Naive models
- Describing spatial autocorrelation
- Accounting for spatial autocorrelation

# AFROTROPICAL BIRD SPECIES RICHNESS



# EXPLANATORY VARIABLES



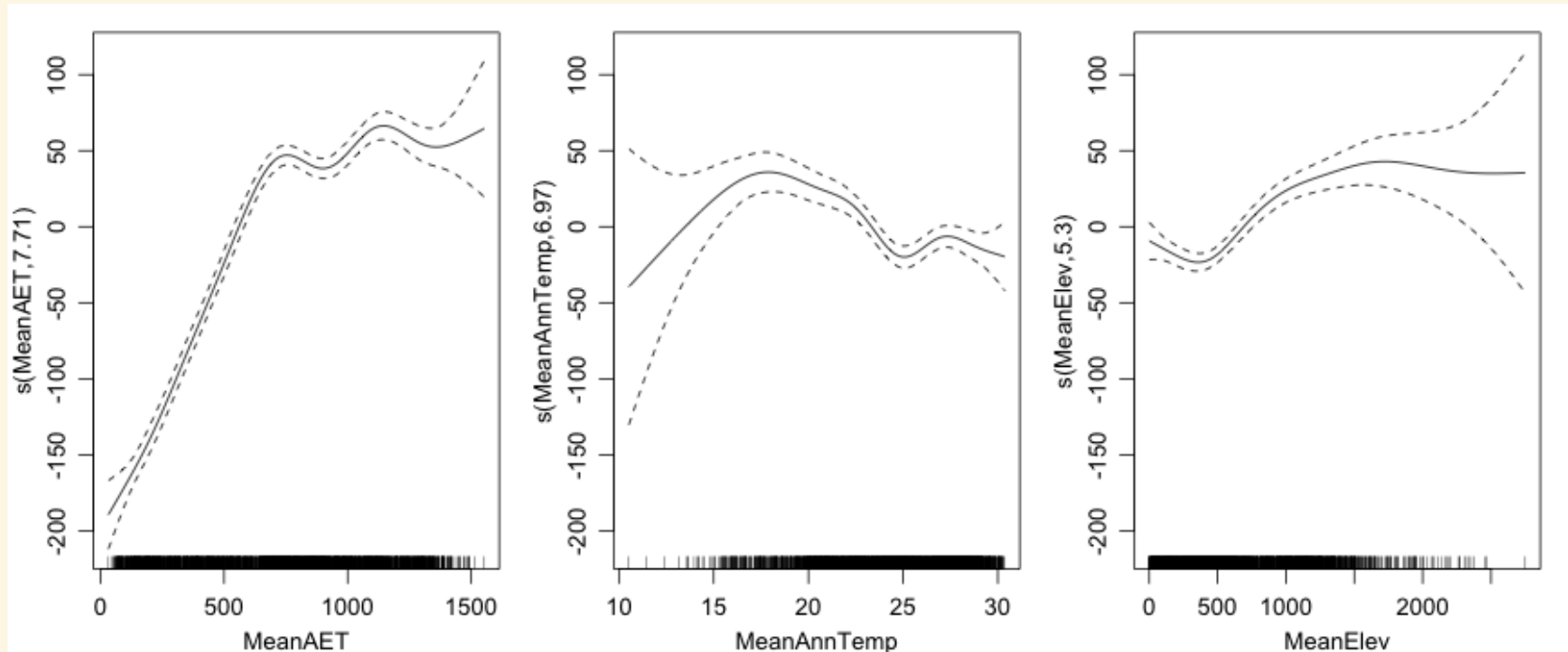
# A SIMPLE LINEAR MODEL

Richness ~ AET + Temperature + Elevation

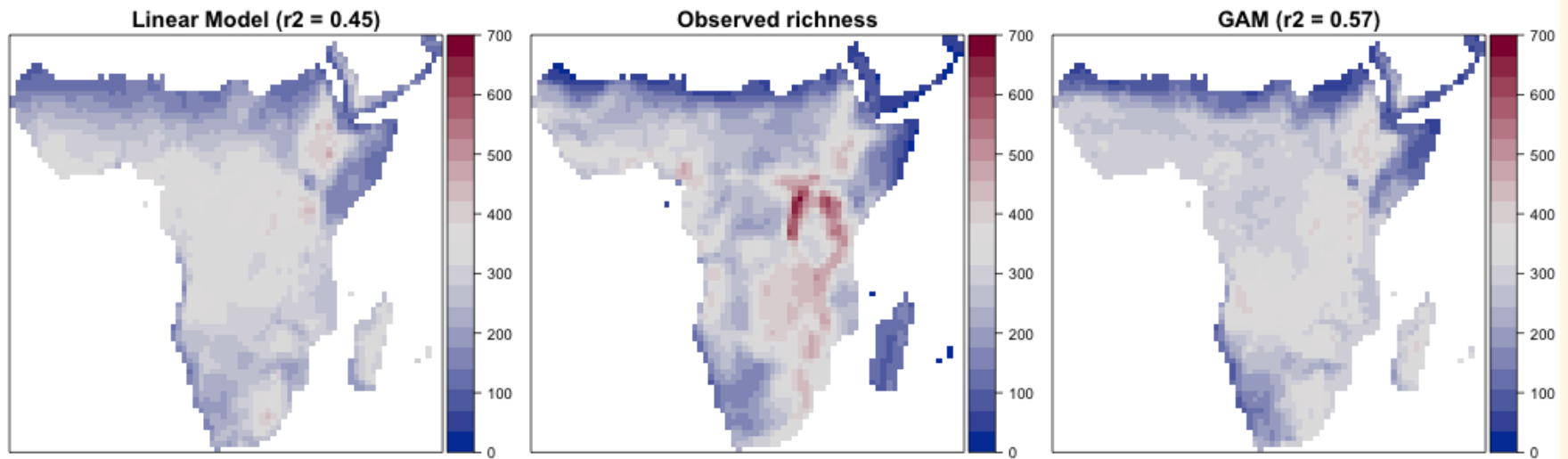
	Est	SE	t	p
(Intercept)	189.45	21.33	8.88	< 0.001
MeanAET	0.18	0.00	37.34	< 0.001
MeanAnnTemp	-4.18	0.72	-5.79	< 0.001
MeanElev	0.08	0.01	13.85	< 0.001

# A SIMPLE GAM

Richness  $\sim s(\text{AET}) + s(\text{Temperature}) + s(\text{Elevation})$

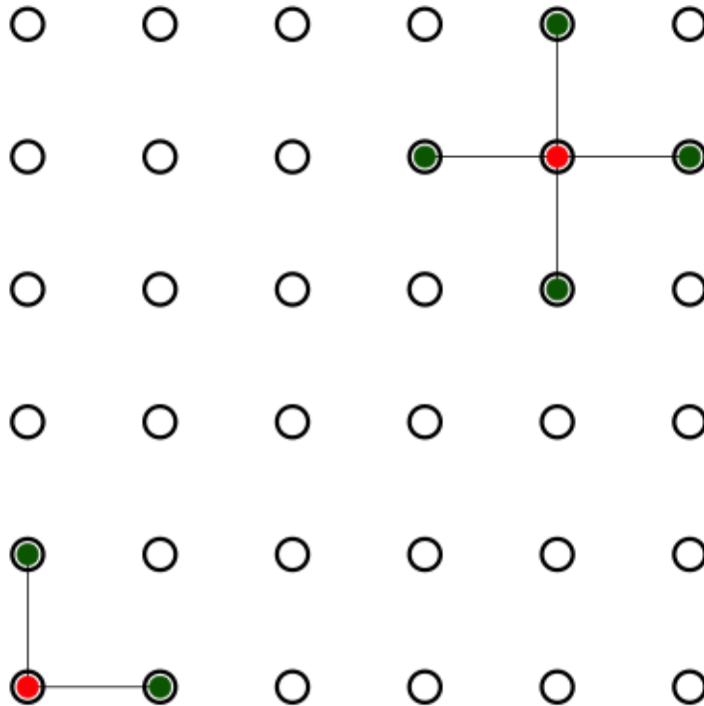


# MODEL PREDICTIONS





# NEIGHBOURHOODS

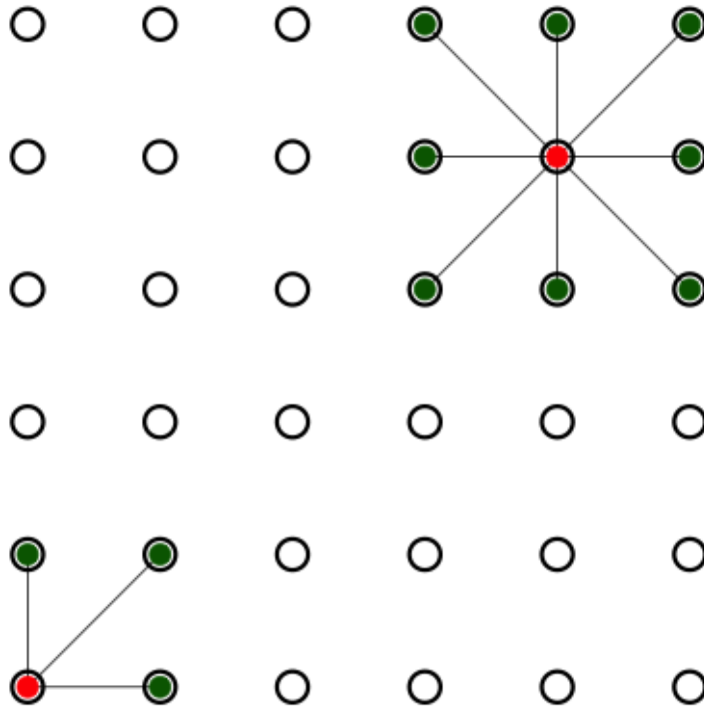


Rooks move

All cells within  
one step:

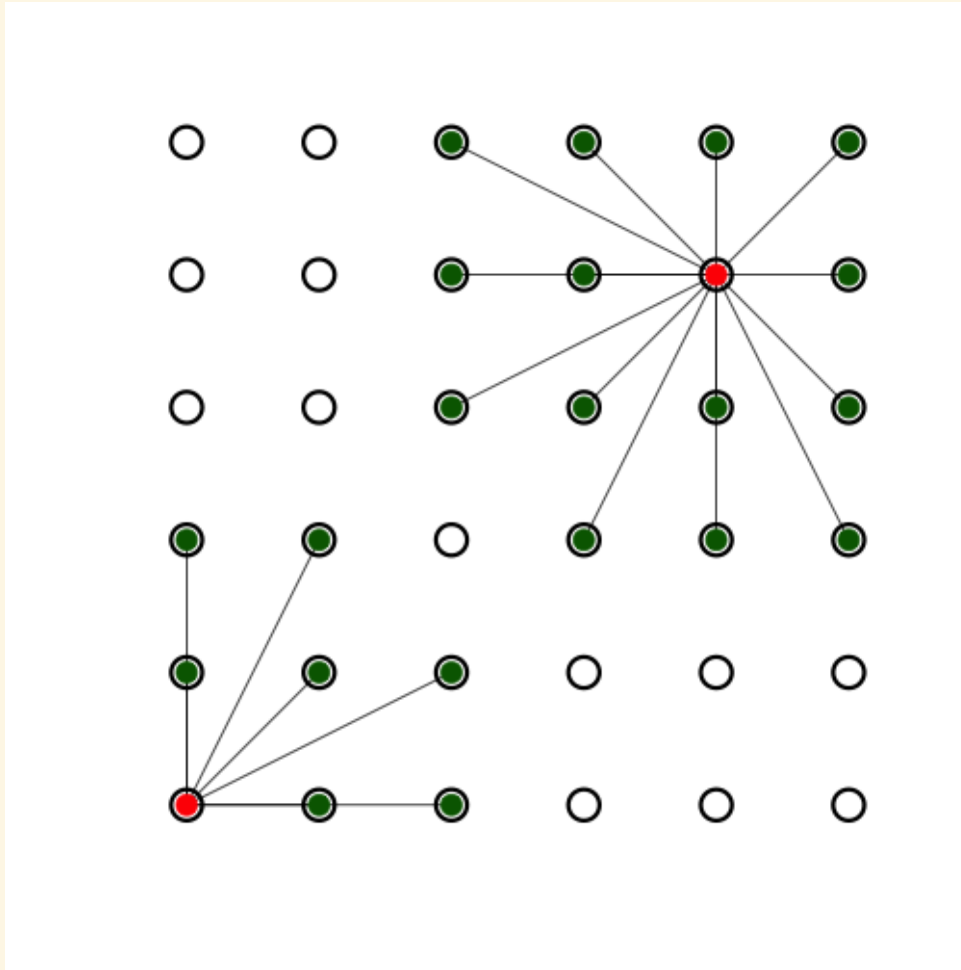
- vertically or
- horizontally

# NEIGHBOURHOODS



- Queens move
- All cells within one step:
- vertically,
  - horizontally
  - or
  - diagonally

# NEIGHBOURHOODS

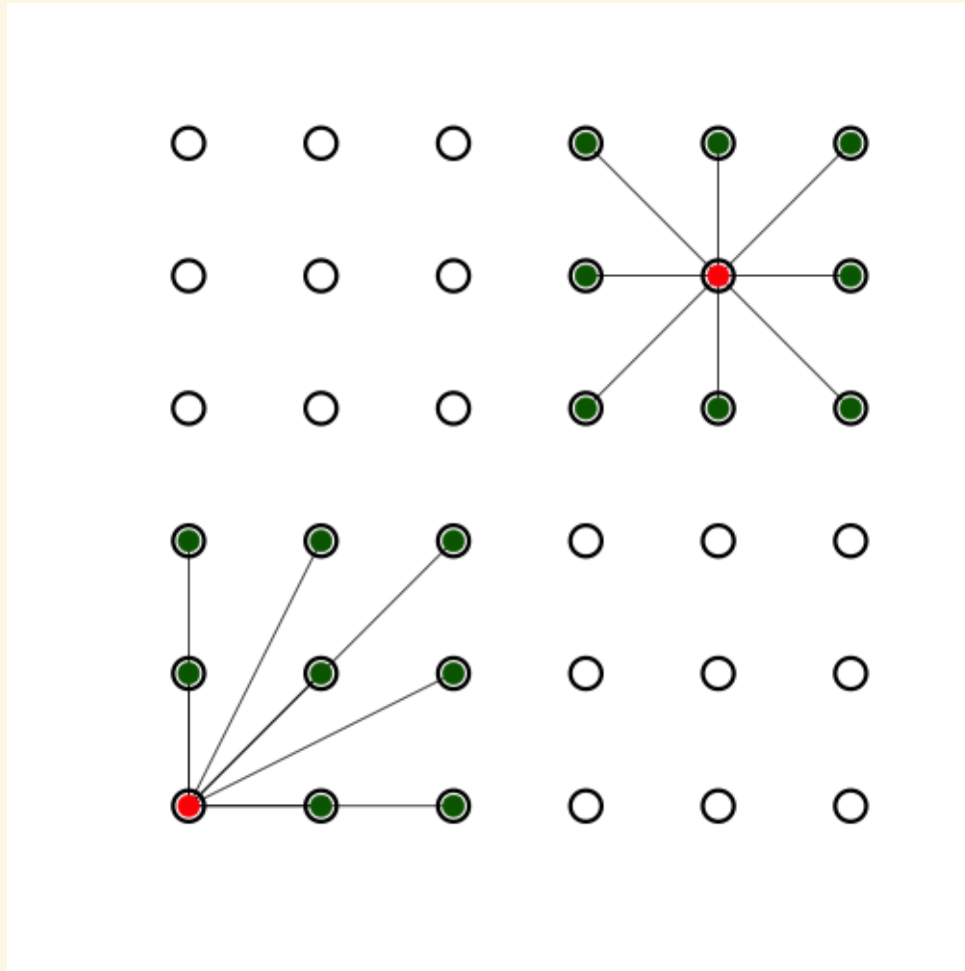


Distance based

All cells within:

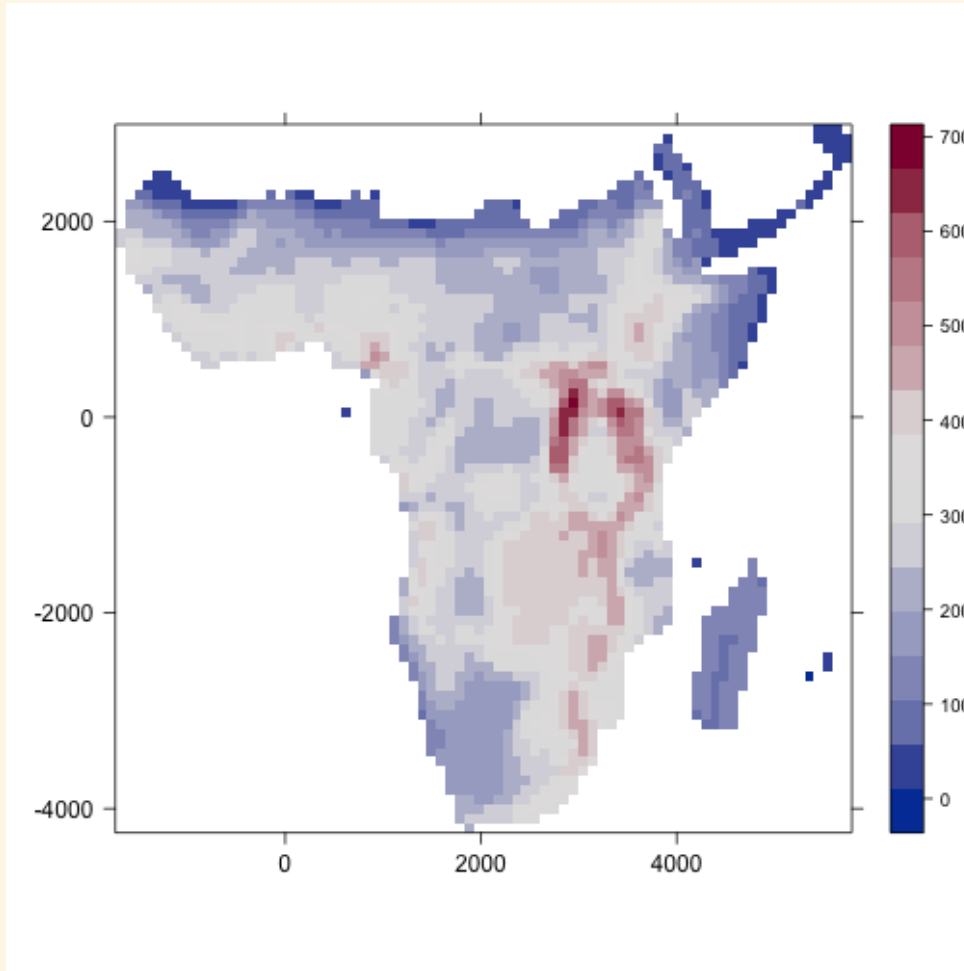
- 2.4 units

# NEIGHBOURHOODS



$k$  nearest  
The closest  $k$  cells

# SPATIAL AUTOCORRELATION



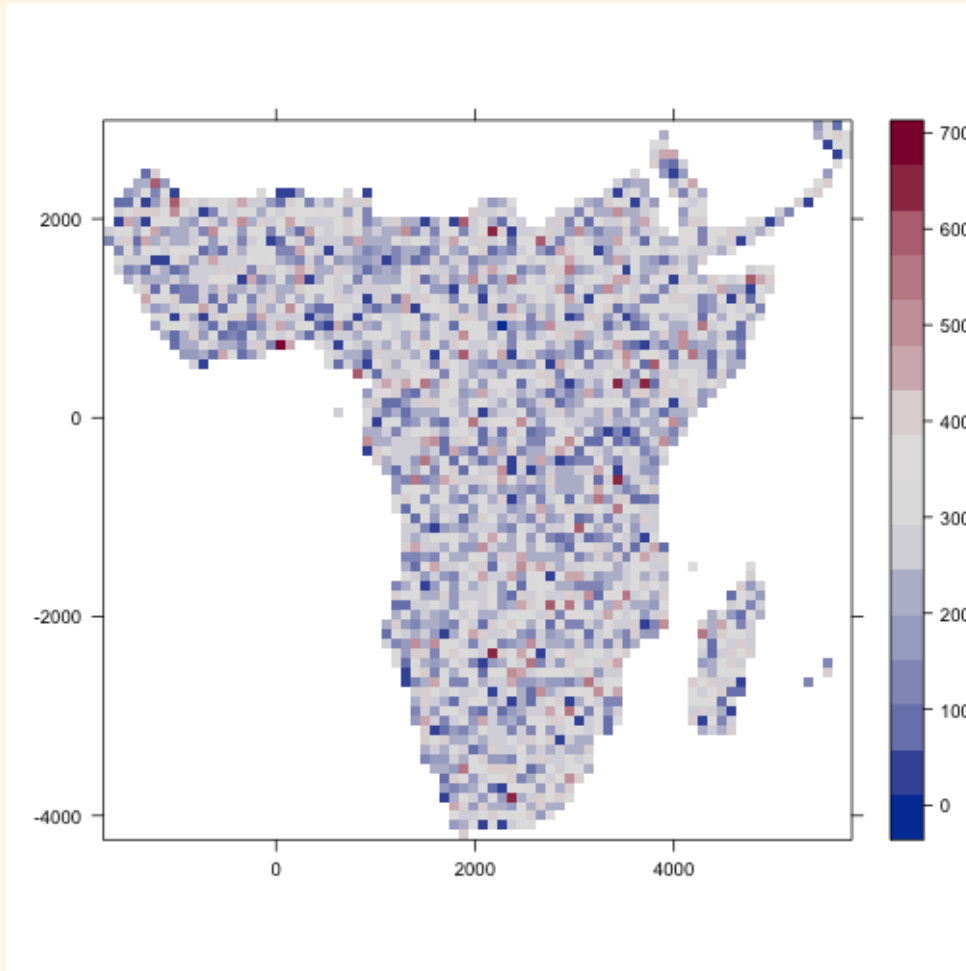
Global Moran's I

- $I = 0.922$
- $p < < 0.001$

Global Geary's C

- $C = 0.070$
- $p < < 0.001$

# SPATIAL AUTOCORRELATION



Global Moran's I

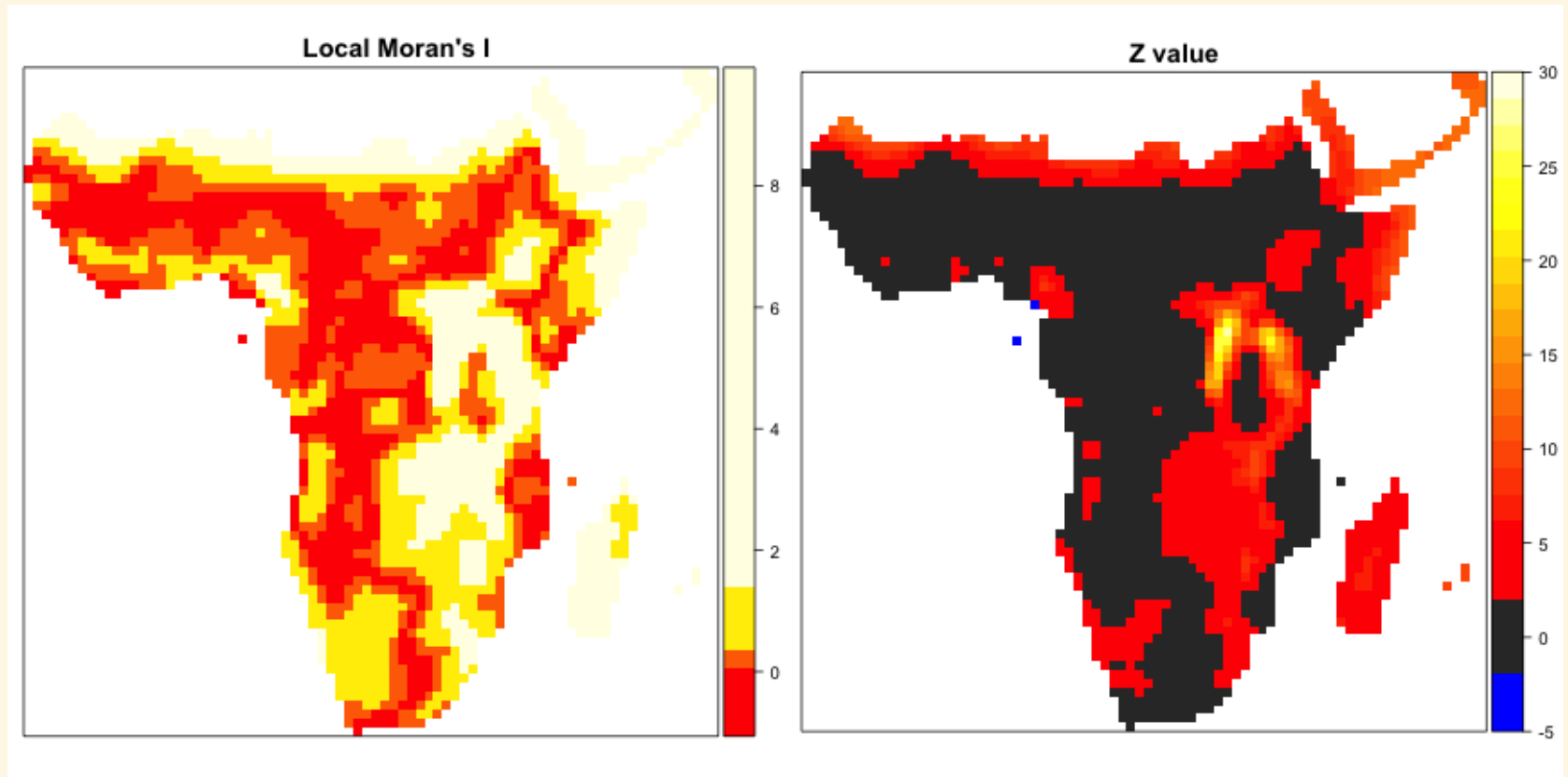
- $I = 0.015$
- $p = 0.059$

Global Geary's C

- $C = 0.984$
- $p = 0.059$

# LOCAL AUTOCORRELATION

Local indicators of spatial autocorrelation (LISA)



# EFFECTS OF SPATIAL AUTOCORRELATION

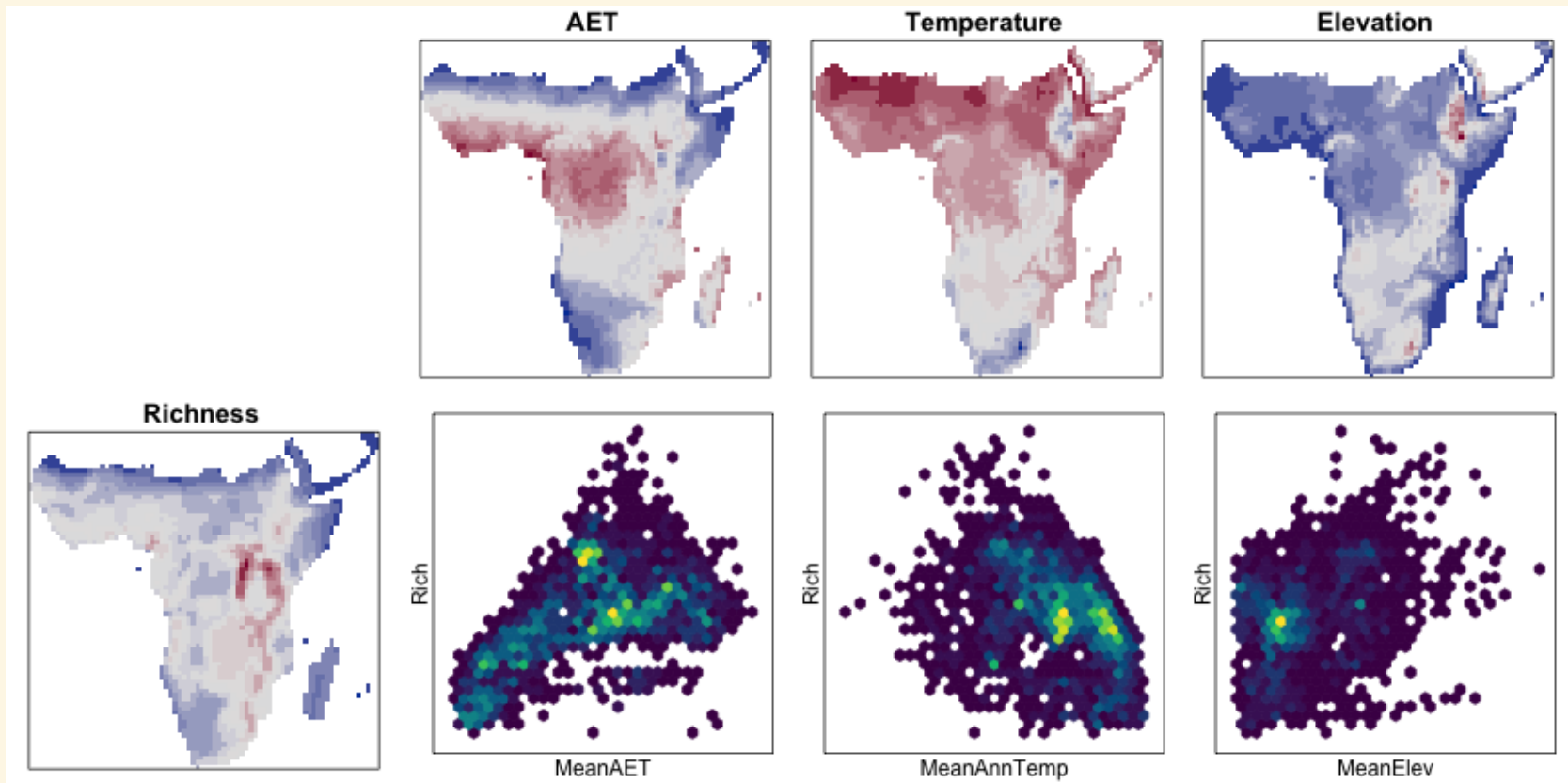
- Data points not independent
- Degrees of freedom reduced:
  - standard errors and significance testing affected
- Not equally weighted :
  - parameter estimation affected



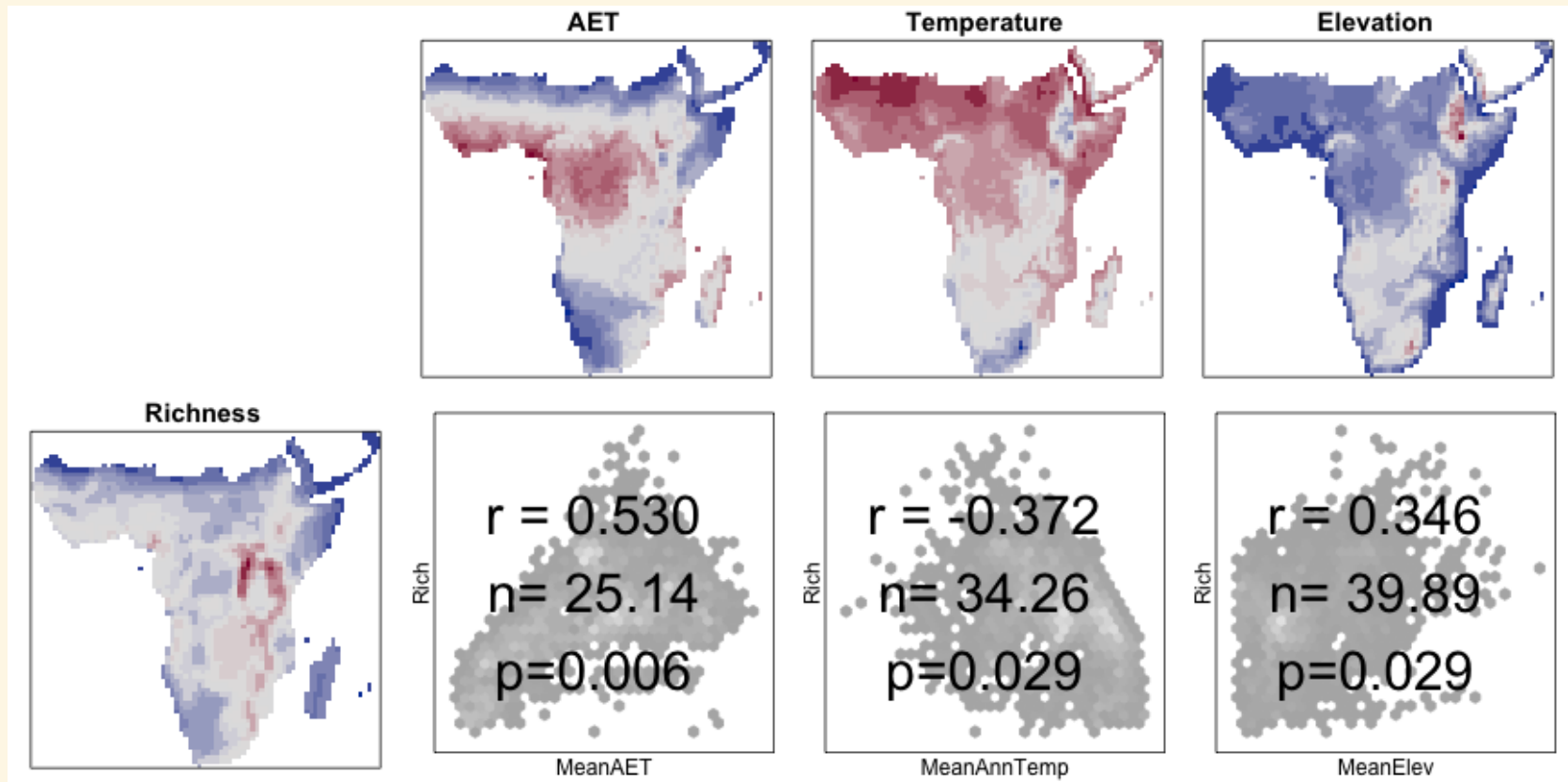
# DEALING WITH SPATIAL AUTOCORRELATION

- Modify the degrees of freedom in significance testing
- Account for autocorrelation in models:
  - Simultaneous autoregressive models
  - Generalised least squares
  - Eigenvector filtering
  - Geographically weighted regression

# DEGREES OF FREEDOM CORRECTION



# DEGREES OF FREEDOM CORRECTION



# SPATIAL AUTOREGRESSION

Solve for  $b$ :



$$bx_1 + \frac{1}{2}bx_2$$

$$\frac{1}{2}bx_1 + bx_2 + \frac{1}{2}bx_3$$

$$\frac{1}{2}bx_2 + bx_3 + \frac{1}{2}bx_4$$



$$\frac{1}{2}bx_3 + \frac{1}{2}bx_4$$

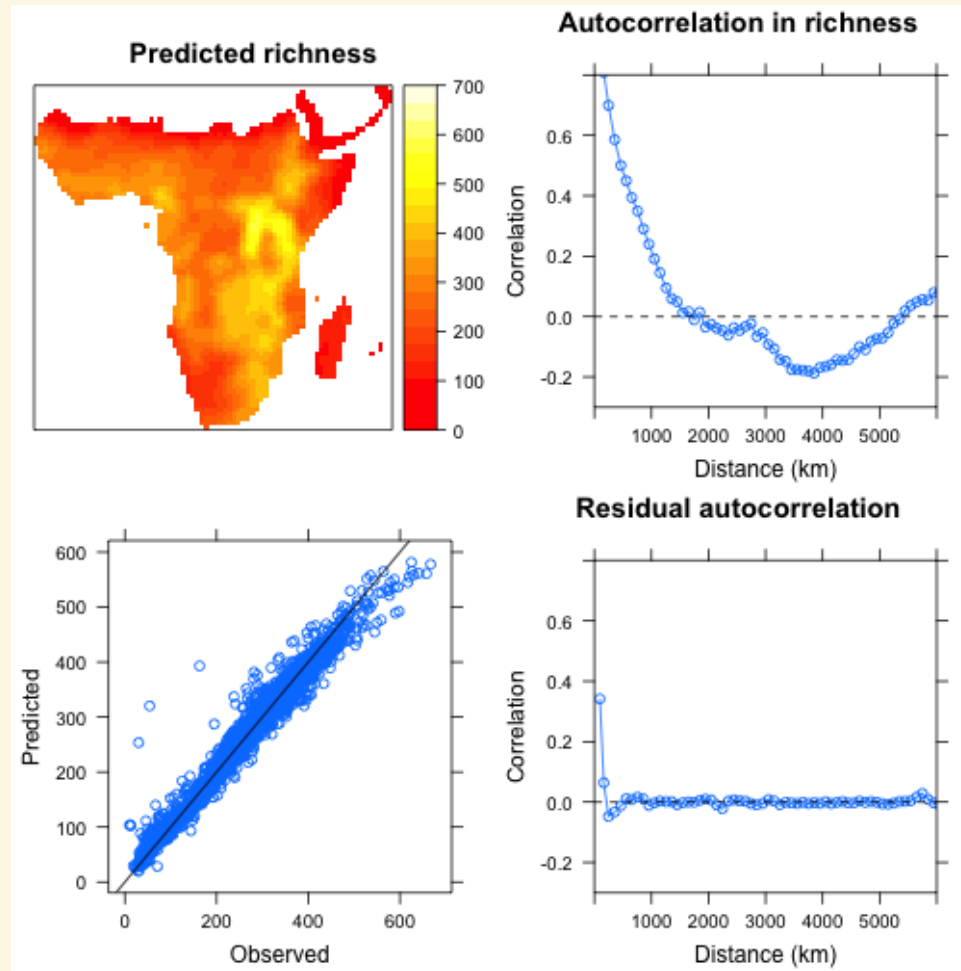
$x_1$

$x_1$

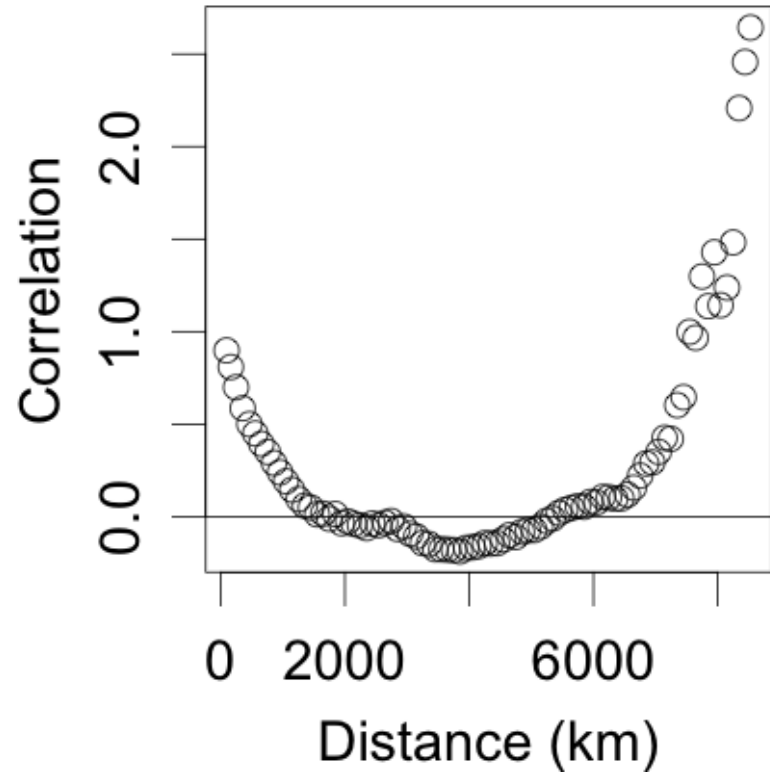
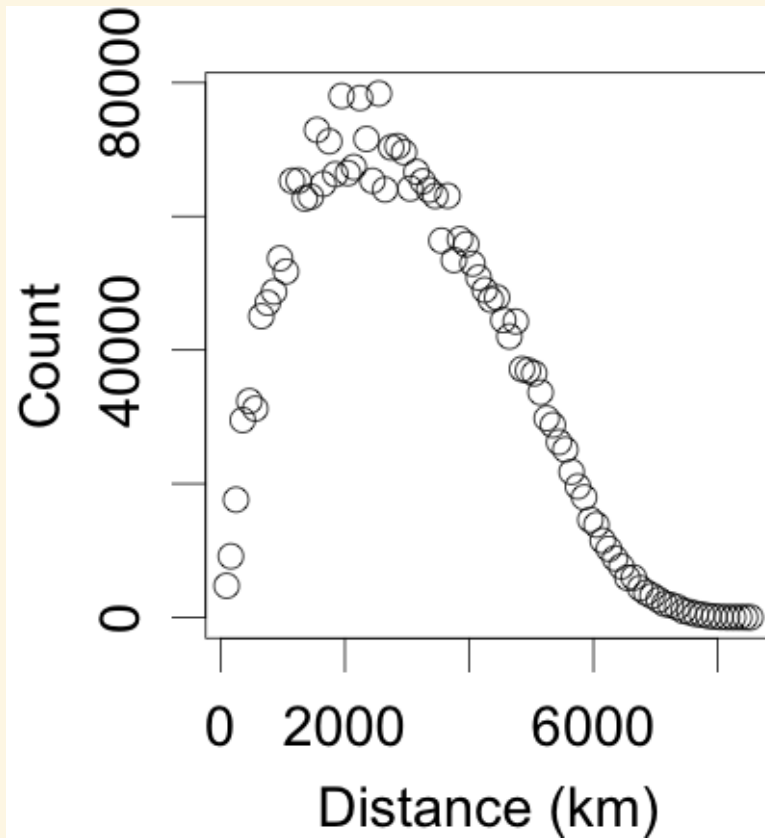
$x_3$

$x_4$

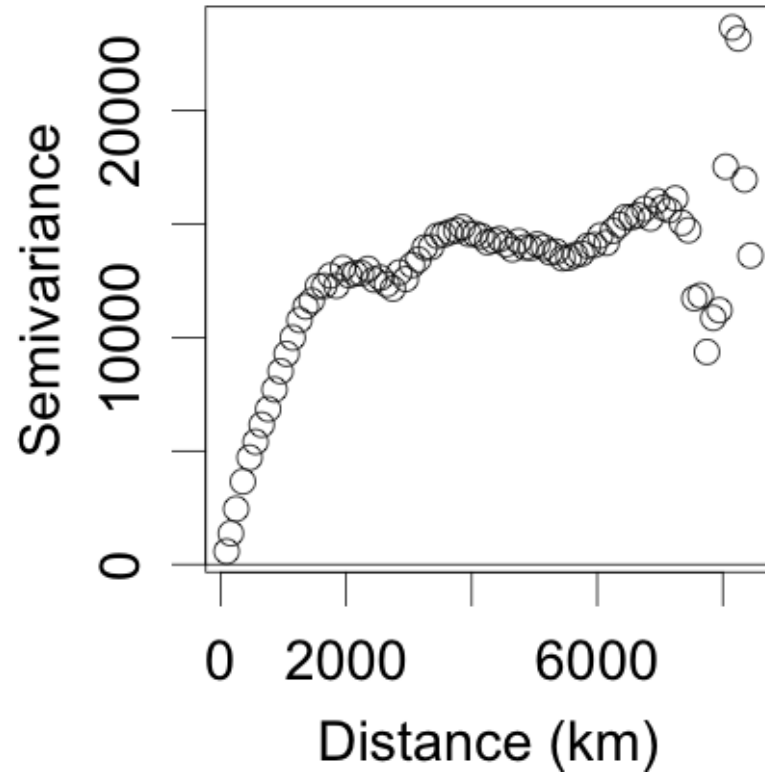
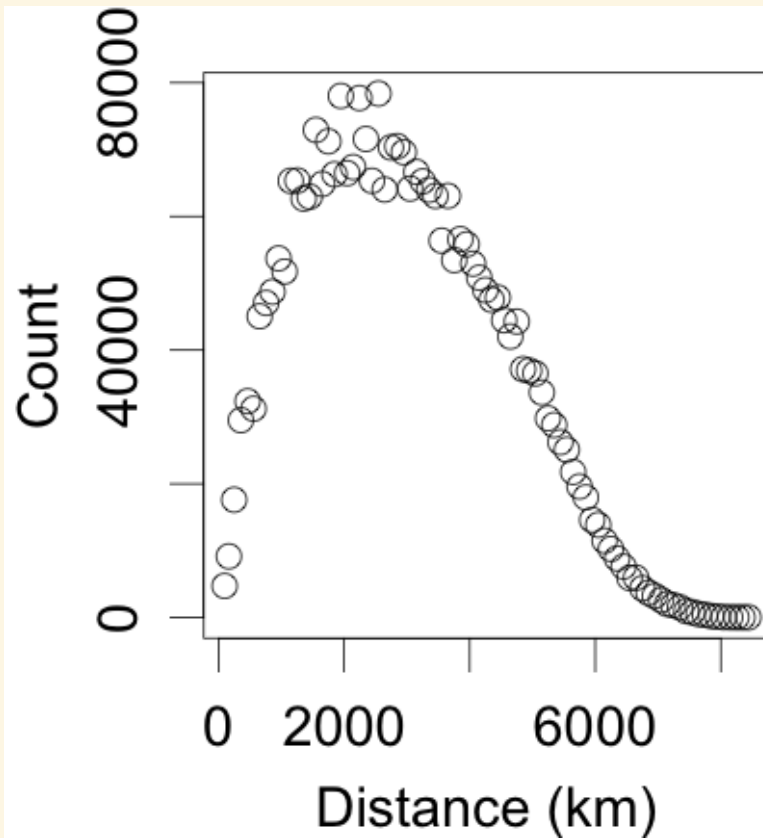
# SPATIAL AUTOREGRESSION



# CORRELOGRAM

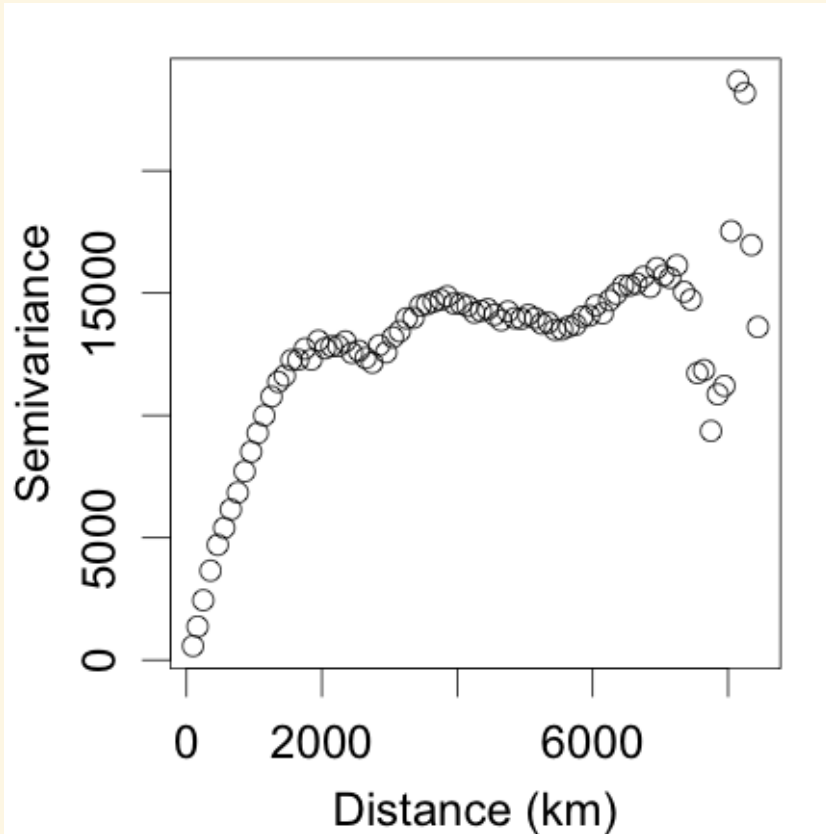


# VARIOGRAM



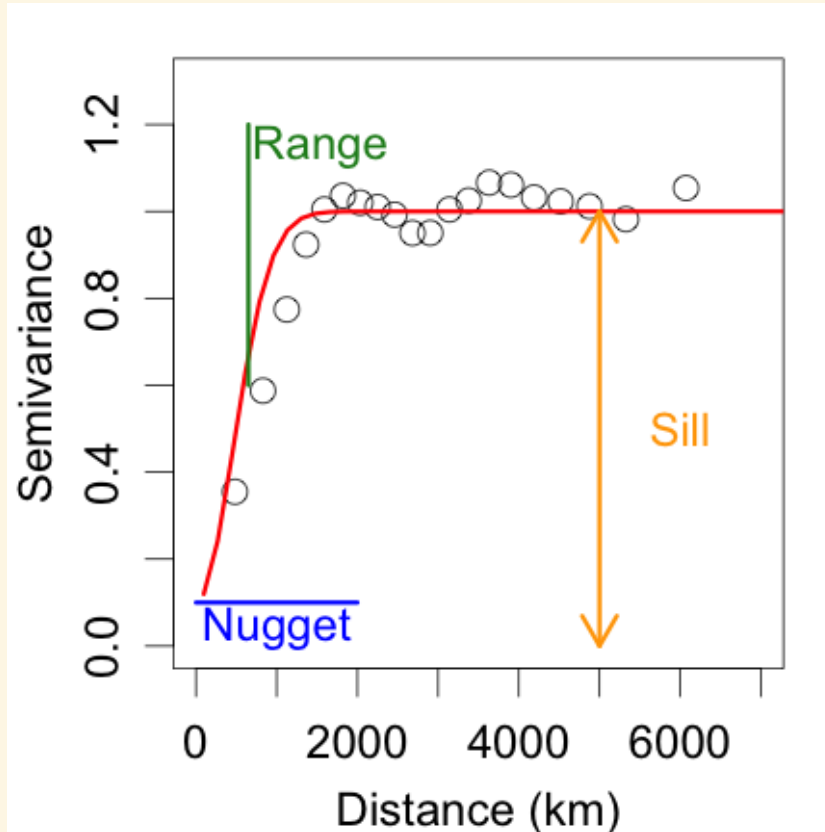


# GENERALISED LEAST SQUARES



- Model correlation as a function of **distance**
- Generate a correlation **matrix**

# GENERALISED LEAST SQUARES



- Different shapes:
  - Exponential
  - Spherical
  - Linear
- Parameters

# GENERALISED LEAST SQUARES

```
Generalized least squares fit by REML
Model: Rich ~ MeanAETScaled + MeanAnnTempScaled + MeanElevScaled
Data: figDat
      AIC      BIC    logLik
24676.89 24705.97 -12333.44

Correlation Structure: Gaussian spatial correlation
Formula: ~e_centre_behr + n_centre_behr
Parameter estimate(s):
range nugget
650.0    0.1

Coefficients:
              Value Std.Error   t-value p-value
(Intercept)  199.67323 16.755430 11.916927 0.0000
MeanAETScaled  17.65050  3.038337  5.809265 0.0000
MeanAnnTempScaled -27.53775  5.663121 -4.862645 0.0000
MeanElevScaled   3.59893  4.321932  0.832712 0.4051

Correlation:
              (Intr) MnAETS MnAnTS
MeanAETScaled  0.048
MeanAnnTempScaled 0.141  0.047
MeanElevScaled  0.156  0.079  0.936

Standardized residuals:
      Min      Q1      Med      Q3      Max
-2.35080108  0.09376345  0.73813223  1.30659392  4.48588628

Residual standard error: 97.85917
Degrees of freedom: 2484 total; 2480 residual
```

# STATIONARITY AND ISOTROPY

Is the same process happening in:

- different locations (stationarity)?
- different directions (isotropy)?

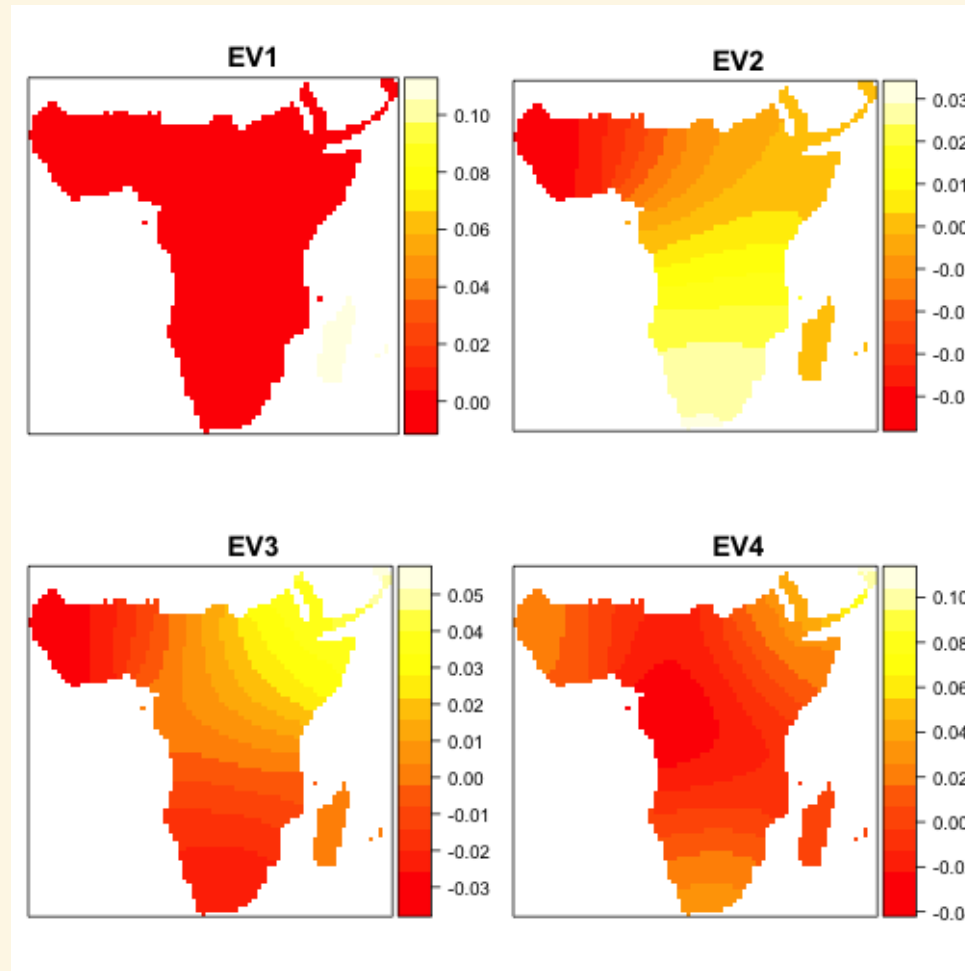
Is the problem in:

- the spatial structure of autocorrelation?
- differences in the actual relationship?

# EIGENVECTOR FILTERING

- Take the **eigendecomposition** of a spatial weights model
- Use the **eigenvectors** as variables in the model
- Use a selection process to identify and include only important eigenvectors

# EIGENVECTOR FILTERING



# EIGENVECTOR FILTERING

lm(Rich ~ MeanAET + MeanAnnTemp + MeanElev

	Est	SE	t	p
(Intercept)	189.45	21.33	8.88	< 0.001
MeanAET	0.18	0.00	37.34	< 0.001
MeanAnnTemp	-4.18	0.72	-5.79	< 0.001
MeanElev	0.08	0.01	13.85	< 0.001

# EIGENVECTOR FILTERING

lm(Rich ~ MeanAET + MeanAnnTemp +  
MeanElev + Re(spEV1) + Re(spEV2) + Re(spEV3)  
+ Re(spEV4))

	Est	SE	t	p
(Intercept)	80.23	33.00	2.43	1.5e-02
MeanAET	0.18	0.01	31.43	< 0.001
MeanAnnTemp	0.10	1.14	0.09	9.3e-01
MeanElev	0.08	0.01	12.70	< 0.001
Re(spEV1)	-1617.63	77.64	-20.83	< 0.001
Re(spEV2)	963.97	129.21	7.46	< 0.001
Re(spEV3)	-813.56	95.87	-8.49	< 0.001
Re(spEV4)	-150.38	100.28	-1.50	1.3e-01



# EIGENVECTOR FILTERING

lm(Rich ~ MeanAET + MeanAnnTemp +  
MeanElev + Re(spEV1) + Re(spEV2) + Re(spEV3))

	Est	SE	t	p
(Intercept)	58.39	29.62	1.97	4.9e-02
MeanAET	0.19	0.00	43.68	< 0.001
MeanAnnTemp	0.75	1.06	0.71	4.8e-01
MeanElev	0.08	0.01	13.78	< 0.001
Re(spEV1)	-1610.75	77.53	-20.78	< 0.001
Re(spEV2)	1031.60	121.11	8.52	< 0.001
Re(spEV3)	-848.19	93.07	-9.11	< 0.001

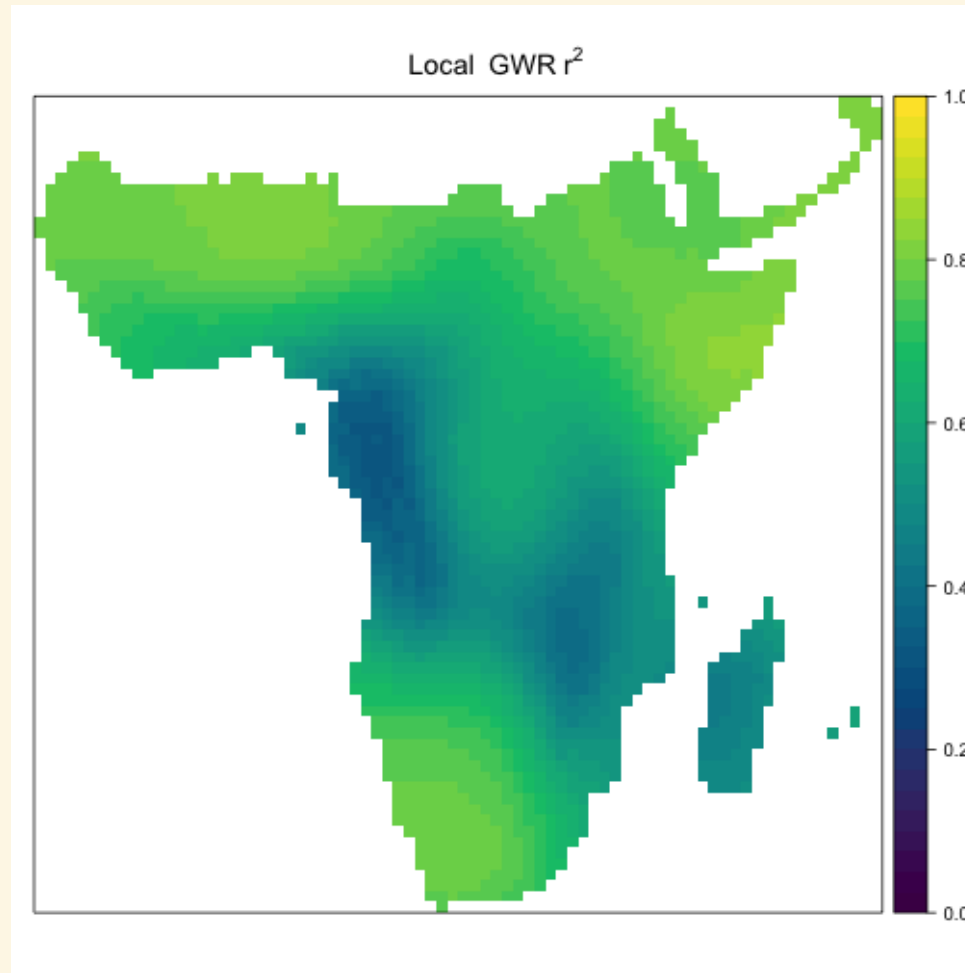
# GEOGRAPHICALLY WEIGHTED REGRESSION

Fit a model for **every cell**:

- Define a local **region size** and a **weighting function**
- Fit a weighted regression for each cell using the weights
- Look at how coefficients **vary in space**
- Possibly serious statistical issues!

# GEOGRAPHICALLY WEIGHTED REGRESSION

# GEOGRAPHICALLY WEIGHTED REGRESSION



# PROBLEMS

- Profusion of packages: sf, sp, spdep, mgcv, ncf, gstat, nlme, spgwr
- Different data structures
- Sometimes poor documentation
- Speed of calculation (= size of dataset)
- Memory hungry
- Too many options