FCIS – Ain Shams University
Subject: Discrete Math and Linear Algebra
Exam: (Final) 6/1/2018
Year: (2^{nd}) undergraduate

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Offering Dept.: Scientific Computing
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Duration: 3 hours

Answer the following four questions: The total marks: 90 (+ 10 Bonus)

1^{st} Question	Discrete Math	marks: 25

A. Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are:
Box 1 "*The gold is not here*", Box 2 "*The gold is not here*", Box 3 "*The gold is in Box 2*"
Only one message is true; the other two are false. Which box has the gold?

B. Show that $p \leftrightarrow q$ and $(p \land q) \lor (\neg p \land \neg q)$ are logically equivalent by a truth table and by developing a series of logically equivalences.

C. Let P(m,n) ::="m divides n," where the domain for both variables consists of all positive integers. Determine the truth values of each of these statements.
a) P(4, 5) b) P(2, 4) c) $\forall m \, \forall n \, P(m,n)$
d) $\exists m \, \forall n \, P(m,n)$ e) $\exists n \, \forall m \, P(m,n)$ f) $\forall n \, P(1,n)$

D. Prove using direct proof that if m + n and n + p are even integers, then m + p is even, where m, n, and p are integers.

E. Let P (n) be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that P(n) is true for $n \geq 18$.

2^{nd} Question	Discrete Math	marks: 25

A. Find the integer a such that $a \equiv -12 \pmod{20}, where\ 20 \leq a \leq 80$.

B. Find each of these values.
a. GCD (140,228). b. LCM (140,228).
c. $(-133 \bmod 23 + 261 \bmod 23) \bmod 23$.

C. Let R_1 and R_2 be relations on a set A represented by $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_{R_2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find the matrices that represent $R1 \cup R2$, $R1 \cap R2$, $R2 \circ R1$, $R1 \oplus R2$, R_1^{-1}, symmetric, reflexive, and transitive closures of R_1.

D. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

E. What are the equivalent classes of the following relations:
$R_1 = \{(a, b) : a \equiv b \pmod 4, a, b \in \{-20, -19, -18, \dots, 18, 19, 20\}\}$.

3^{rd} Question	Linear Algebra	marks: 25

A. Given the matrix $A = \begin{bmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \mathbf{0} \end{bmatrix}$, find the following:

a. Base for the row space of A.
b. Base of the column space of A.
c. Spanning set for the null space of the matrix A.
d. What is the dimension of Row A, Col A, and Nul A.

B. Using row reduction find the determinant of A where $A = \begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & 1 & -2 & 3 \end{bmatrix}$.

C. Let $v_1 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$ and H=span$\{v_1, v_2, v_3\}$.

i- Write the linearly dependence relation of v_1, v_2, and v_3.
ii- Show that span $\{v_1, v_2, v_3\}$=span $\{v_2, v_3\}$.

D. Find the inverse of A where $A = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

4^{th} Question	Linear Algebra	marks: 25

A. Given the matrix $A = \begin{bmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \mathbf{2} \end{bmatrix}$, find the following:

a. Characteristic equation and Eigenvalues of A.
b. Bases of eigenspaces corresponding to each eigen value of A.
c. The matrices D and P such that $A = PDP^{-1}$.
d. A^{50}.

B. Find the parametric form of the general solution of the linear system whose augmented matrix $[A\ b] = \begin{bmatrix} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

C. Find the value(s) of h for which the vectors $\begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -6 \\ 4 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix}$ are linearly dependent.

D. Show that the subset of all symmetric matrices $S_{3\times3}$ is a subspace of the set of all matrices $M_{3\times3}$.

Good Luck

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