Faculty of Computer & Information Sciences
Ain Shams University
Course Name: Discrete Math and Linear Algebra
Offering Dept: Scientific Computing
Academic year: 2017-2018
Instructors: Dr. Mohammed Marey

Exam: (Midterm-G2-A)11/11/2017
Year: (1^{st} term) 2^{nd} year
Duration: 1 hour
Total Grade: 15 points

Question 1 DM

I- *Aladdin finds two trunks A and B in a cave. He knows that each of them either contains a treasure or a fatal trap.*
- *On trunk A is written:* "At least one of these two trunks contains a treasure."
- *On trunk B is written:* "In *A* there's a fatal trap."

Aladdin knows that either both the inscriptions are true, or they are both false.
Can Aladdin choose a trunk being sure that he will find a treasure?
If this is the case, which trunk should he open?

II- B- Is $\forall x\ \exists y\ 2xy = x^2 + y^2$, $x, y \in R$ true? Why?

Question 2 DM

III- Use a truth table to show that the implication $\neg q \rightarrow \neg[\, p \wedge (\neg p \vee q)]$ is a tautology.

IV- Prove by induction that $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\ldots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$ for all $n \geq 2$.

Question 3 LA

I- Given the system of linear equations

$$\begin{aligned} 3x + 5y - 4z + w &= 7 \\ -3x - 2y + 4z &= -1 \\ 6x + y - 8z - 2w &= -4 \end{aligned}$$

i- Write the matrix of coefficients.
ii- Write the corresponding matrix equation.
iii- Write the corresponding vector equation.
iv- Using row operation on the corresponding augmented matrix, find the following:
- Set of free variables.
- Set of basis variables.
- The general solution as a parametric vector equation.

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Exam: (Midterm - B)16/11/2017
Year: (1st term) 2nd year
Duration: 3 hours
Total Grade: 15 Points

Question 1 DM

I- Three boxes are presented to you. One contains gold, the other two are empty.
Each box has imprinted on it a clue as to its contents; the clues are:
Box 1 "The gold is not here"
Box 2 "The gold is not here"
Box 3 "The gold is in Box 2"
Only one message is true; the other two are false. Which box has the gold?

II- Is $\exists m\ \exists n\ ((m + n = 14) \land (2m - n = 4))$, $m, n \in Z$ true? Why?

Question 2 DM

I- Determine whether $\forall x(P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

II- Let $n \geq 1$ be an integer. Recall from the lectures that the n-th harmonic number H_n is defined as

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n-1} + \frac{1}{n}$$

Using mathematical induction, prove that $H_{2^m} \leq 1 + m$ for all $m \geq 0$.

(In H_{2^m}, the subscript 2^m is 2 raised to the power m.)

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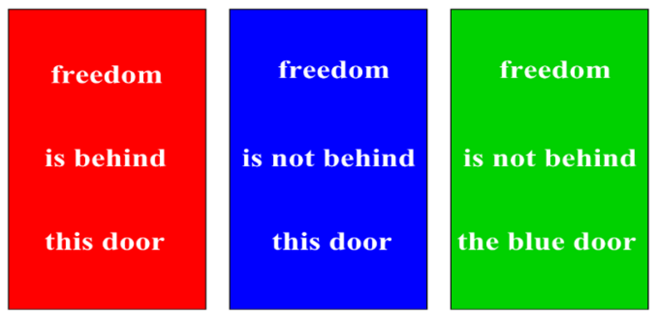
Exam: (Midterm - C)16/11/2017
Year: (1st term) 2nd year
Duration: 1 hours
Total Grade: 15 points

Question 1 DM

I- Kyle, Neal, and Grant find themselves trapped in a dark and cold dungeon (HOW they arrived there is another story). After a quick search the boys find three doors, the first one red, the second one blue, and the third one green. Behind one of the doors is a path to freedom. Behind the other two doors, however, is an evil fire-breathing dragon. Opening a door to the dragon means almost certain death. On each door there is an inscription:



Red door Blue door Green door

Given the fact that at LEAST ONE of the three statements on the three doors is true and at LEAST ONE of them is false, which door would lead the boys to safety?

II- Suppose that the domain of the propositional function P(x) consists of the integers $\{-2, -1, 0, 1, 2\}$. Write out each of these propositions using disjunctions, conjunctions, and negations.

a) $\exists x P(x)$ b) $\forall x P(x)$ c) $\exists x\ \neg P(x)$

d) $\forall x\ \neg P(x)$ e) $\neg\exists x P(x)$ f) $\neg\forall x P(x)$

Question 2 DM

I- Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

II- For any integer n, using **contradiction** give a proof of the theorem:

“if $(n^2 + 2)$ is odd, then n is odd”

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Question 1 DM

I- You are walking in a labyrinth and all of a sudden you find yourself in front of three possible roads: the road on your left is paved with gold, the one in front of you is paved with marble, while the one on your right is made of small stones. Each street is protected by a guardian.

You talk to the guardians and this is what they tell you:
The guardian of the gold street: "This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center".
The guardian of the marble street: "Neither the gold nor the stones will take you to the center".
The guardian of the stone street: "Follow the gold and you'll reach the center, follow the marble and you will be lost".

Given that you know that all the guardians are liars, can you choose a road being sure that it will lead you to the center of the labyrinth? If this is the case, which road you choose?

II- Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x \forall y(x2 = y2 \rightarrow x = y)$ b) $\forall x \exists y(y2 = x)$
c) $\forall x \forall y(xy \geq x)$ d $\forall x \exists y(xy > x)$

Question 2 DM

I- For any integer n, using **contraposition** give a proof of the theorem :

"if $((n+1)^2 + 1)$ is odd, then n is odd"

II- Prove by induction that $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + (n)(n+1) = \frac{(n)(n+1)(n+2)}{3}$ for all $n \geq 2$.