FCIS – Ain Shams University
Subject: Discrete Mathematics & Linear Algebra
Exam date: 10/1/2015
Year: (1st term) 2st undergraduate

Instructor: Mohammed Marey
Offering Dept.: Bioinformatics Dept.
Academic year: 2014-2015
Duration: 3 hours
Total mark: 50

Part 1: Discrete Mathematics - Solve (**DM1** ∧ (**DM2** ∨ **DM3**)) (Part 1total marks:25)

1st Question : DM1 marks: 15

A. Show that the argument $u \to q, (\neg q) \vee t, r \vee s, \neg s \to \neg p, \big((\neg p) \wedge r\big) \to u, \neg s, \therefore\ t$ is valid by deducing the conclusion from the premises step by step through the use of the basic rules of inference or lows of logic.

B. Give a proof by contradiction of the theorem " If 3n+2 is odd, then n is odd".

C. Prove that the sum of the first n positive odd integers is n^2, i.e.

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

D. Find the value of $\sum_{k=99}^{200}(k-3)^2$, where $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

2nd Question : DM 2 marks: 10

A. Given a directed graph G=(V,E) in (Fig: D-Graph),

- i- State the two sets V and E.
- ii- Show that $\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$
- iii- Determine whether G is strongly connected and if not, whether it is weakly connected.
- iv- Using adjacency matrix of G, determine the number of paths of length 4 from any vertex to any other vertex in G.
- v- Represent the graph using the incidence matrix.

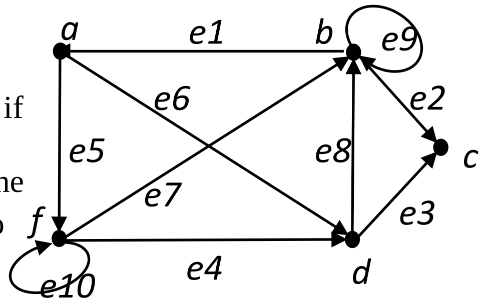

Fig: D-graph

B. Given $A = \{a_1, a_2, a_3, \ldots, a_n\}$, deduce the formulas representing the (number of all relation, number of all reflexive relations, number of all symmetric relations , number of all (symmetric ∧ reflexive) relations) which can be defined on set A.

3nd Question : DM 3 marks: 10

A. Find x, y, z, and w if you know that :

- i- x = gcd (92928, 123552).
- ii- y = lcm (92928, 123552).
- iii- $z \equiv -11 \pmod{21}$ where $(90 \le x \le 110)$.
- iv- $w = ((-133 \bmod 23 + 261 \bmod 23) \bmod 23)$.

B. Given A = {1,2,3,4}, if R and S are two relations on A such that $R = \{(x, y),\quad x \mid y\}$ and $S = \{(x, y), x \equiv y \bmod 2\}$

- i- Write the corresponding matrix representation for each of R and S.
- ii- Determine whether R is reflexive, symmetric, anti-symmetric or transitive.
- iii- Find the symmetric, reflexive and transitive closures of R.
- iv- Prove that S is an equivalence relation and find its equivalence classes.
- v- Find $\overline{R}$, S^{-1}, $R \cap S$, $R \circ S$.

Part 2: Linear Algebra - Solve (LA1 $\wedge$ LA2) (Part 2 total marks:25)

4st Question **:** LA1 marks: 15

A. Let $v_1 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$ and H=span$\{v_1, v_2, v_3\}$.

- i- Write the linearly dependence relation of $v_1, v_2,$ and v_3.
- ii- Show that span $\{v_1, v_2, v_3\}$=span $\{v_2, v_3\}$.
- iii- Find the base and dimension of the subspace H.

B. Using the augmented matrix [A I], find the inverse of $A = \begin{bmatrix} 0 & 5 & 1 \\ 1 & 4 & 0 \\ 3 & 6 & 2 \end{bmatrix}$, if it exists.

C. Let H = { $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$: $a, b, c \in R$} be the set of diagonal 3x3 matrices and let V be the vector space V of all 3×3 matrices.

- i- Show that H is a subspace of V.
- ii- Write the set of bases of the subspace H.
- iii- If G has a definition as H after setting (b=1), show that G is not a subspace of V

5st Question : LA 2 marks: 10

A. Given $A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$:

- i- Find base for the row space of A
- ii- Find base of the column space of A
- iii- Find a spanning set for the null space of the matrix A
- iv- What is the dimension of Row A, Col A, and Nul A.

B. Let $M_{2\times2}$ be the vector space of all 2×2 matrices, and define $T: M_{2\times2} \to M_{2\times2}$ by $T(A) = A + A^T$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

- i- Show that T is a linear transformation.
- ii- Show that the range of T is the set of B in $M_{2\times2}$ with the property that $B^T = B$.
- iii- Describe the kernel of T.

Good Luck …. (f)

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