Faculty of Computer & Information Sciences
Ain Shams University
Course Name: Discrete Math and Linear Algebra
Offering dept.: Scientific Computing
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Instructors: Dr. Mohammed Marey

Exam: (Midterm - C)22/11/2018
Year: (1st term) 2nd year
Duration: 1 hours
Total Grade: 10 points

Question 1 DM

A Translate the following statement into a sentence in English.
Is this statment is true? Explan your answer.
$\forall m,\ n \in \mathbb{N}\ ((m \bmod n) \in \mathbb{N} \vee (n \bmod m) \in \mathbb{N})$.

B For the following statement, find its negation and simplify your answer. Is this negated statement true? Explain your answer.

$$\forall x \in \mathbb{R}, \quad (x > 2) \longrightarrow (x^2 > 3)$$

C Show that the argument

$$u \rightarrow q,\ \ r \vee s,\ \ \sim s \rightarrow \sim p,\ \ ((\sim p) \wedge r) \rightarrow u,\ \ \sim s,\ \ \therefore\ q$$

is valid by deducing the conclusion from the premises step by step through the use of the basic rules of inference or laws of logic.

D Consider we define the following predicates
- $C(a,b)$: b is the child of a
- $W(c)$: c works at a grocery store

Translations $\forall y \exists x (C(x,y) \wedge W(y))$ to the corresponding English sentence.

Question 2 DM

A If $A = \{a, b, c\}$ and $B = \{d, e\}$ find

(i) $\mathcal{P}(A)$, the power set of A

(ii) $A \times B$.

B Use mathematical induction to show that

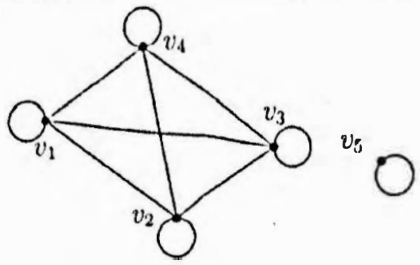
$$n! \geq 2^{n-1} \quad \text{for } n = 1, 2, \ldots$$

C Given n elements, count the worst case time complexity to perform the following sorting algorithm.

Procedure bubblesort
{sort n integers $a_1, a_2, \ldots, a_n$ in ascending order}
for i:= 1 to n-1
for j:= 1 to n-i
if $a_j > a_{j+1}$ then swap (a_j, a_{j+1})

Which of the following is/are the order of the algorithm? Why?
$O(n)$, $O(x^2)$, $O(x^3)$, $\Omega(n)$, $\Omega(x^2)$, $\Omega(x^3)$, $\Theta(n)$, $\Theta(x^2)$, $\Theta(x^3)$.

A Let V denote the set of vertices of the following graph G. ed by



Define a relation R on V by

$(v, w) \in R$ iff there exists at least one edge in G
which connects vertex v to vertex w directly.

Is R an equivalence relation? Justify your answer.

B Using prime factorization find the value of GCD (140,228).

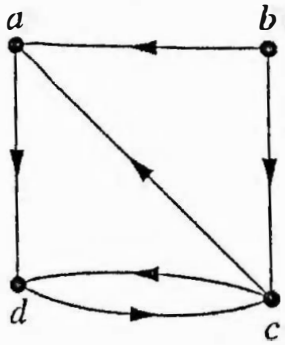
C Prove or disprove (if possible) the following proposition:
a) $(A - B = \phi) \rightarrow (A \cap B = B \cap A)$
b) $313(x^3 + y^3) = z^3$ has no solution when x, y, z $\in Z^+$.

f(n)

D For each of the following mappings indicate what type of function they are (if any). Use the following key:
i) Not a function
ii) A function which is neither onto nor one-to-one
iii) A function which is onto but not one-to-one
iv) A function which is one-to-one but not onto
v) A function which is both onto and one-to-one

a) The mapping f from $Q^+ to\ Q^+$ defined by $f(x) = 2x$.
b) The mapping f from $[0, \infty)$ to $[0, \infty)$ defined by $f(x) = |x - 0.5|$.

E Let R be the relation with digraph shown in Figure.
Write the matrix relation of R then use it to find the transitive closure matrix of R.



Good Luck