Data Science for Energy System Modelling
Lecture 6: Optimisation in Electricity Markets

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Hybrid Format Information: Zoom and Recordings

The recordings will be **publicly available** in this ISIS course
and may also be published on YouTube or other platforms.

- The camera and microphone are directed **only at the lecturer**.
- Student contributions are **not recorded**.
  When interaction takes place, the recording will be **paused**.
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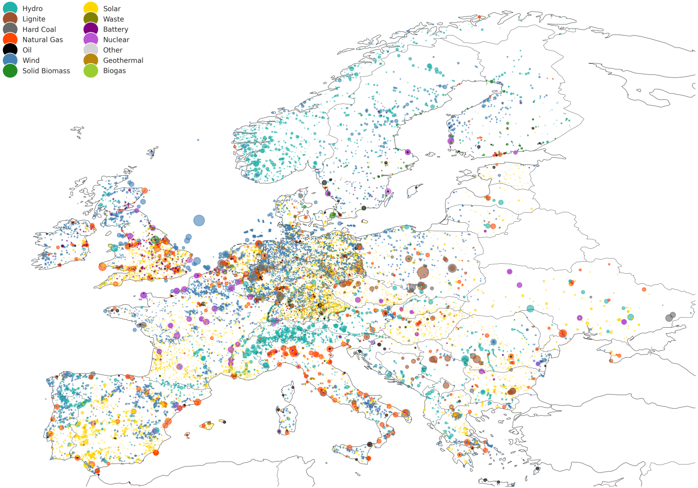
- Participants who wish to remain anonymous may join under a **pseudonym**
  and/or use the **chat or Q&A** function instead of speaking.

In the `pandas` tutorial, we looked at power plant data for Europe.



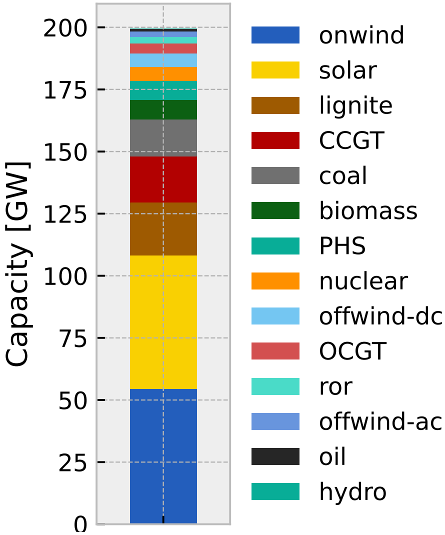
Many **power plants** in Europe.

Many **consumers** in Europe.

Interconnected through **power grid**.

How to decide which of these power plants produce at a given time?

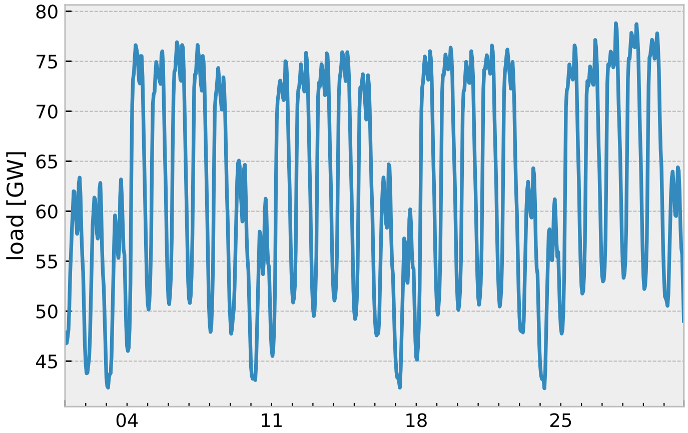
Generation capacities in Germany in 2022



Note that:

- available wind, solar, hydro generation capacity depends on **weather conditions**.
- available conventional capacity might vary due to **scheduled maintenance** or **unplanned outages**.
- nuclear phase-out in 2023
- 113 GW of solar PV in fall 2025
- 67 GW of onshore wind in fall 2025
- 9 GW of offshore wind in fall 2025

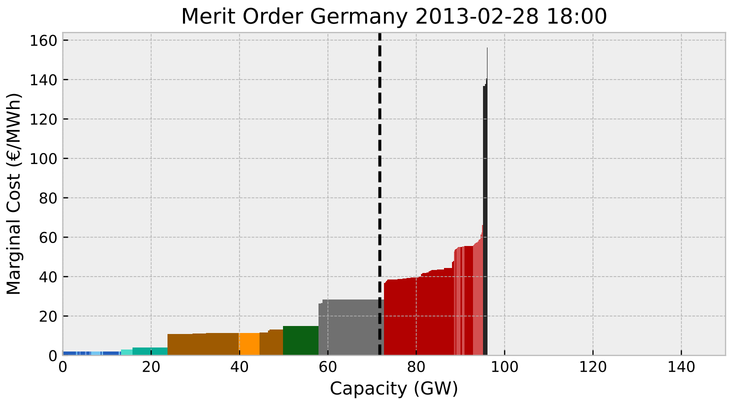
Demand must be matched from available generation capacity.



Note that this is neglecting all kinds of **real-world complications** (e.g. grid constraints, cross-border power flows, demand flexibilities, etc.)

We are focusing on the **conceptual aspects**!

For one market region and one hour we can start with the merit order



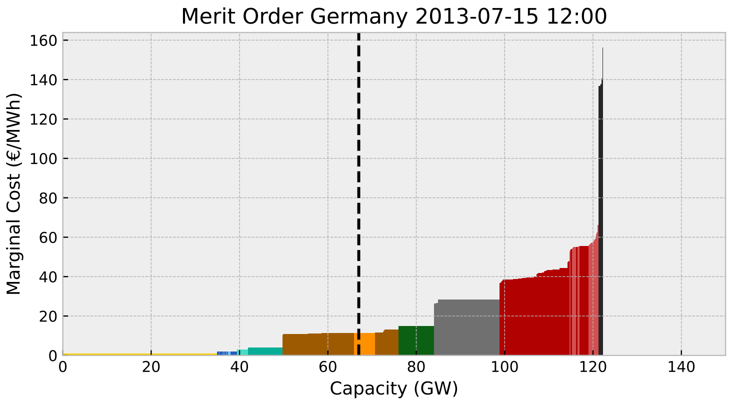
Sort generators by marginal cost and how much they could produce in a given hour.

Consider wind, solar, hydro availability & outages.

Here:
Assume fixed demand.

Price set by the generator intersecting with demand.

For one market region and one hour we can start with the merit order



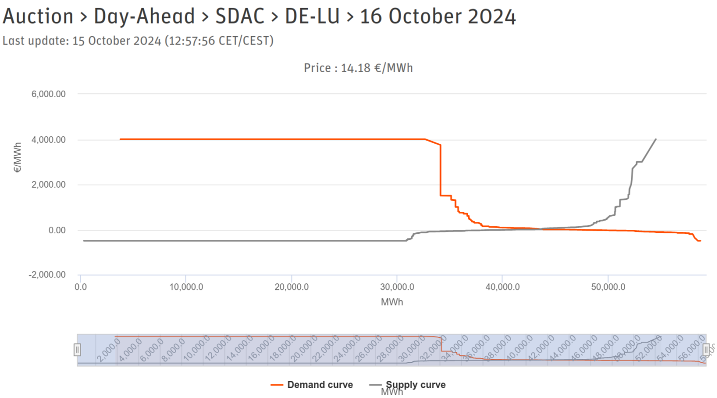
Sort generators by marginal cost and how much they could produce in a given hour.

Consider wind, solar, hydro availability & outages.

Here:
Assume fixed demand.

Price set by the generator intersecting with demand.

And in reality?



Displayed aggregated curves include all orders submitted to the operating coupled NEMOs for resp. SDAC or SIDC auction in the concerned bidding zone ; the displayed "Price" information reflects the EPEX SPOT market clearing price of the respective EPEX SPOT auction. For delivery date 26/06/2024, the displayed aggregated curves for SDAC auction in AT, BE, DE, FR and NL include all orders submitted to EPEX SPOT only.

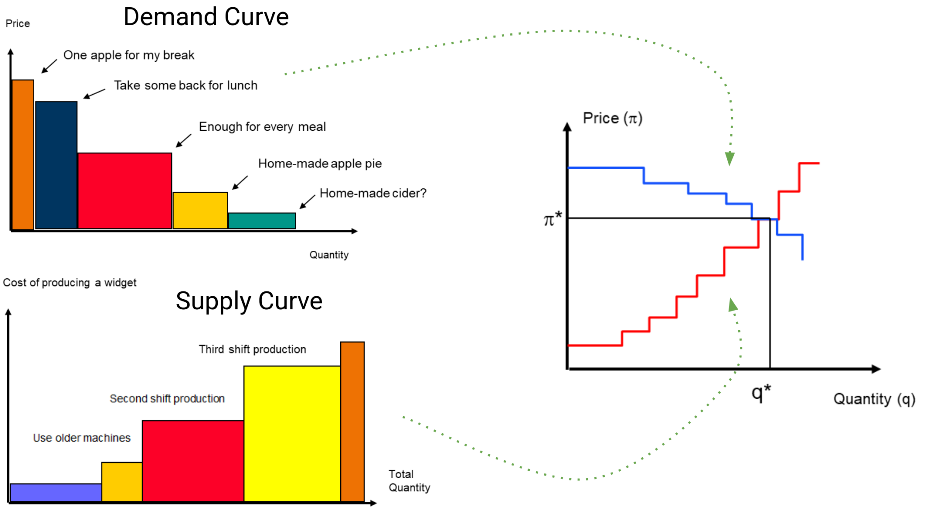
EPEXSPOT day-ahead electricity market

Shown for 1 hour in German market zone.

Demand side has varying willingness to pay, just like supply side (power plants) has varying marginal cost.

Source: https://www.epexspot.com/en/market-results

Economics 101: Price formation from supply & demand curves



How to determine the supply curve? (i.e. generator marginal cost)

The **short-term marginal generation cost** of a generator *s* can be calculated from the sum of **fuel cost** (fc_s), the operation-dependent **operation & maintenance cost** (VOM_s), and potentially the cost **CO_2 emission certificates** γ $\left(\frac{€}{t_{CO2}}\right)$.

$$STMGC_s = o_s = \frac{fc_s}{\eta_s} + VOM_s + \gamma \frac{\rho_s}{\eta_s},$$

where ρ_s is are the specific emissions of the fuel used $\left(\frac{t_{CO2}}{MWh_{th}}\right)$ and η_s is the conversion efficiency from thermal to electrical energy $\left(\frac{MWh_{el}}{MWh_{th}}\right)$.

Let's check that the units align:

$$\frac{€}{MWh_{el}} = \frac{\frac{€}{MWh_{th}}}{\frac{MWh_{el}}{MWh_{th}}} + \frac{€}{MWh_{el}} + \frac{€}{t_{CO2}} \frac{\frac{t_{CO2}}{MWh_{th}}}{\frac{MWh_{el}}{MWh_{th}}}$$

NB: STMGC neglects all kinds of fixed costs, mainly the cost of building the power plant.

Simplified merit order example

Power Plant	Capacity [GW]	Efficiency [MWh_{el}/MWh_{th}]	Fuel Price [€/MWh_{th}]	VOM [€/MWh_{el}]	Emissions [tCO2/MWh_{th}]
Lignite	10	0.4	4.5	1	0.36
Hard Coal	10	0.45	11	1	0.36
Gas (CCGT)	20	0.58	50	1.5	0.2
Uranium	5	0.33	3.3	2	≈ 0

Assume fixed load of 30 GW in a particular hour and no storage or trade of electricity.

Which power plants should supply electricity, with

- no CO_2 price?
- CO_2 price of 150 €/t_{CO_2}?
- 10 GW of wind / solar production with zero marginal cost?

All this can be formulated as an optimisation problem!

For a single region and hour, we have s generators with marginal costs o_s (€/MWh) and (available) capacities G_s (MW). We seek to minimise the operational costs of generator dispatch g_s to fulfil the electricity load of d MW. The **optimisation problem**:

$$\min_{g_s} \sum_s o_s g_s \tag{1}$$

such that

$$g_s \leq G_s \tag{2}$$

$$g_s \geq 0 \tag{3}$$

$$\sum_s g_s = d \tag{4}$$

(1) is the **objective function**, (2-4) are inequality and equality **constraints**. g_s are the **optimisation variables** or **decision variables**. G_s, o_s and d are **input parameters**.

General form of optimisation problems

Find the **best possible decision within given limits**.

Objective:
(the goal)

$$\min_x f(x)$$

- minimise costs, emissions, travel time
- maximise profits, speed

Decision variables:
(choices, levers)

$$x \in \mathbb{R}^n$$

- how many pizzas to bake
- how far to cycle each day

Constraints:
(rules to obey)

$$g(x) \leq a$$

- budget caps
- maximum load on a bridge
- emissions reduction targets

$$h(x) = b$$

- supply = demand
- inflow = outflow

Let’s consider a simple example

There is a power demand of 500 MW that must be met exactly by two power plants. Unit 1 is a cheaper coal-fired plant with a cost of 3 €/MWh, a minimum output of 50 MW and a maximum output of 300 MW. Unit 2 is a more expensive gas-fired plant with costs of 4 €/MWh, a minimum output of 100 MW and a maximum output of 400 MW. **What is the dispatch of each unit that minimises costs?**

Decision variables: x_1 and x_2 for the dispatch of units 1 and 2 [MW]

Objective function: minimize $3x_1 + 4x_2$

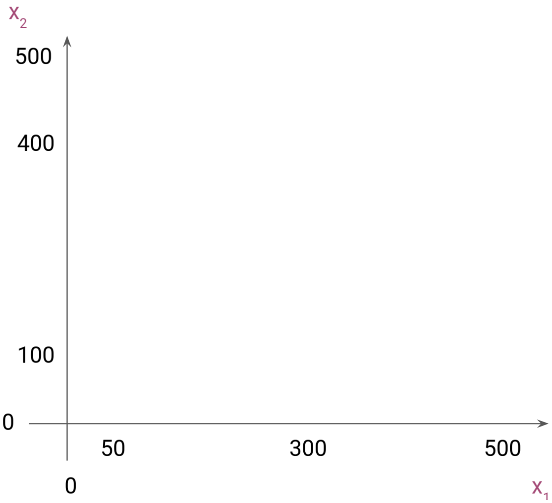
Constraints:

$x_1 + x_2 = 500$ (energy balance)

$50 \leq x_1 \leq 300$ (dispatch limits of unit 1)

$100 \leq x_2 \leq 400$ (dispatch limits of unit 2)

Let's solve it graphically



Objective function: minimize $3\,x_1 + 4\,x_2$

Constraints:

$x_1 + x_2 = 500$

$50 \leq x_1 \leq 300$

$100 \leq x_2 \leq 400$

Calculate optimal solution

Variables:

$x_1 = 300$

$x_2 = 500 - x_1 = 200$

Objective function: $3\,x_1 + 4\,x_2 = 1700$

Interpretation: Cheaper plant runs at full capacity of 300 MW, remaining 200 MW served by more expensive plant at 1700 € total cost.

Check some other solution: $x_1 = 100$, $x_2 = 400$

$3\,x_1 + 4\,x_2 = 1900$

You might wonder: Why optimisation?

We could find minimum-cost dispatch & market clearing prices by sorting by marginal cost.

Answer: We have many more **degrees of freedom / decision variables**:

1. Dispatch of power plants
2. Operation of storage units
3. Curtailment of renewables
4. Cross-border flows between market zones

but we also have to respect many **physical constraints**

1. Meet energy demand
2. Do not exceed generators or storage capacities
3. Do not overload transmission lines

which become **too complex and intertwined** to solve by hand.

Formulated as an optimisation problem (same as before)

$$\min_{g_s} \sum_s o_s g_s \tag{5}$$

such that

$$g_s \leq G_s \tag{6}$$

$$g_s \geq 0 \tag{7}$$

$$\sum_s g_s = d \tag{8}$$

Adding a spatial component i $\rightarrow$ e.g. multiple market zones

$$\min_{g_{i,s},f_\ell} \sum_s o_{i,s} g_{i,s}$$

such that

$$g_{i,s} \leq G_{i,s}$$

$$g_{i,s} \geq 0$$

$$\sum_s g_{i,s} - \sum_\ell K_{i\ell} f_\ell = d_i \qquad \text{Kirchhoff Current Law}$$

$$|f_\ell| \leq F_\ell \qquad \text{line limits}$$

$$\sum_\ell C_{\ell c} x_\ell f_\ell = 0 \qquad \text{Kirchhoff Voltage Law}$$

where f_ℓ is the power flow (e.g. into/from other market zones), $K_{i\ell}$ is the incidence matrix and F_ℓ is the transmission line capacity, x_ℓ is the line reactance, and $C_{\ell c}$ is the cycle matrix.

Adding a temporal component $t \rightarrow$ e.g. multiple points in time

$$\min_{g_{i,s,t}, f_{\ell,t}} \sum_s o_{i,s} g_{i,s,t}$$

such that

$$g_{i,s,t} \leq \hat{g}_{i,s,t} G_{i,s}$$

$$g_{i,s,t} \geq 0$$

$$\sum_s g_{i,s,t} - \sum_\ell K_{i\ell} f_{\ell,t} = d_{i,t} \qquad \text{Kirchhoff Current Law}$$

$$|f_{\ell,t}| \leq F_\ell \qquad \text{line limits}$$

$$\sum_\ell C_{\ell c} x_\ell f_{\ell,t} = 0 \qquad \text{Kirchhoff Voltage Law}$$

where $\hat{g}_{i,s,t}$ can, for instance, represent the capacity factor of a wind turbine.

NB: This problem is **separable** in t as there are no dependencies between time steps.

Adding a temporal component t → **now with storage**

But this changes with storage units r, which do **temporal arbitrage**, i.e. charge/buy when prices are low, discharge/sell when prices are high, subject to its physical constraints:

Add constraints where storage can dispatch power within its **discharging capacity**:

$$0 \leq g_{i,r,t,\text{discharge}} \leq G_{i,r,\text{discharge}}$$

and consume power to charge the storage within its **charging capacity**:

$$0 \leq g_{i,r,t,\text{charge}} \leq G_{i,r,\text{charge}}$$

Also add a constraint on the limit $E_{i,r}$ on the **state of charge** $e_{i,r,t}$:

$$0 \leq e_{i,r,t} \leq E_{i,r}$$

The dispatch decisions are related by the following constraint:

$$e_{i,r,t} = \eta^{\text{self-discharge}}_{i,r,t} e_{i,r,t-1} + \eta^{\text{charge}}_{i,r} g_{i,r,t,\text{charge}} - \frac{1}{\eta^{\text{discharge}}_{i,r}} g_{i,r,t,\text{discharge}}$$

One more thing: Shadow prices in optimisation problems

Shadow prices (aka dual variables or KKT multipliers) measure the marginal change in the objective value of the optimal solution by relaxing the constraint by a small amount.

Let's return to the **optimisation problem** for the merit order now including dual variables:

$$\min_{g_s} \sum_s o_s g_s \qquad s.t. \tag{9}$$

$$g_s \leq G_s \qquad \leftrightarrow \bar{\mu}_s \tag{10}$$

$$-g_s \leq 0 \qquad \leftrightarrow \underline{\mu}_s \tag{11}$$

$$\sum_s g_s = d \qquad \leftrightarrow \lambda \tag{12}$$

The **dual variable** λ corresponds to the **market clearing price**, i.e.:

- the marginal cost of the marginal generator
- the cost of consuming an additional unit of power

How to solve large optimisation problems and find shadow prices?

In general finding the solution to optimisation problems is hard, at worst *NP*-hard. Nonlinear, nonconvex and discrete problems are particularly troublesome.

There is specialised software for solving particular classes of optimisation problems.

- Some of them are **open-source**: HiGHS, GLPK, CBC
- Others are **commercial** (with free academic licenses): CPLEX, Gurobi, COPT, Xpress

For **linear problems (LP)** the main two algorithms are:

- The **simplex algorithm**
- The **interior-point algorithm**

We can access these **solvers** through Python with packages like `pyomo`, `PuLP` or `linopy`.

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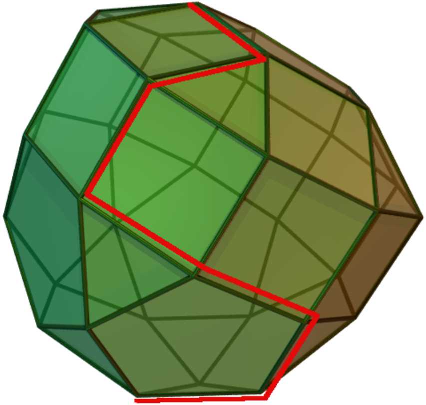
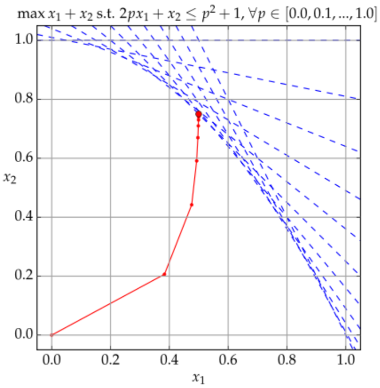
Sketch of the simplex algorithm



- Building the feasible space, which is a multi-dimensional polyhedron, and search its surface for the solution.
- If problem is feasible, optimum occurs at (at least) one of the vertices.
- Starts at feasible vertex. If not the optimum, follow edge that improves objective to the next vertex.
- Repeat until the optimum is found.
- **Complexity:** On *average* polynomial time, but worst cases with exponential time.

Source: Wikipedia

Sketch of interior point / barrier methods



- Can also be used on more general nonlinear problems.
- Search the interior of the feasible space rather than its surface.
- Extremise the objective function plus a **barrier term** penalising solutions coming close to constraints.
- As penality becomes less severe: convergence to optimum point at boundary.
- **Complexity:** For linear problems, polynomial time.

Source: Wikipedia

Up next

Next Thursday (1/2): Introduction to `linopy` for optimisation.

Next Thursday (2/2): Consultation hour for Assignment 2.

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