Data Science for Energy System Modelling
Lecture 5: Transmission Networks

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Hybrid Format Information: Zoom and Recordings

The recordings will be **publicly available** in this ISIS course
and may also be published on YouTube or other platforms.

- The camera and microphone are directed **only at the lecturer**.
- Student contributions are **not recorded**.
  When interaction takes place, the recording will be **paused**.
- If you **voluntarily speak** while the recording is running, you **consent** to the recording and publication of your contribution.
- You may **withdraw** this consent at any time for the future; that part will then be deleted or edited.
- Participants who wish to remain anonymous may join under a **pseudonym**
  and/or use the **chat or Q&A** function instead of speaking.

Balancing renewables in space versus time – Let's explore!

Build your own zero-emission electricity supply

Introduction

This tool calculates the cost of meeting a constant electricity demand from a combination of wind power, solar power and storage for different regions of the world.

First choose your location to determine the weather data for the wind and solar generation. Then choose your cost and technology assumptions to find the solution with least cost. Storage options are batteries and hydrogen from electrolysis of water.

Fun things to try out:

- remove technologies with the checkboxes, e.g. hydrogen gas storage or wind, and see system costs rise
- set solar or battery costs very low, to simulate breakthroughs in manufacturing

See also this Twitter thread for an overview of the model's features and capabilities.

This is a **toy model** with a **strongly simplified** setup. Please read the warnings below.

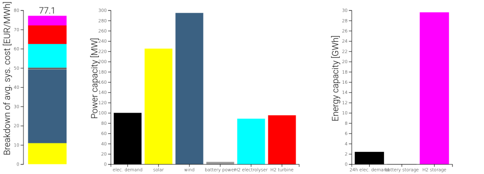
Step 1: Select location and weather year

- ◉ Select country
- ○ Select state or province (for Australia, China, Germany, India, Russia and United States)
- ○ Select point, rectangle or polygon, using the toolbox that appears at top-right

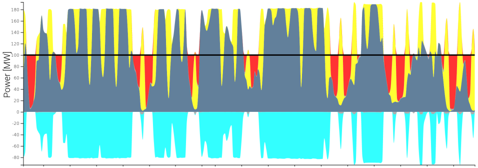
Average system cost [EUR/MWh]: 77.1

Cost is per unit of energy delivered in 2015 euros. For comparison, household electricity rates (including taxes and grid charges) averaged 211 EUR/MWh in the European Union in 2018 & 132 USD/MWh in the United States in 2019

Average marginal price of hydrogen [EUR/MWh LHV]: 78.8, [EUR/kg]: 2.60



Dispatch over a year to meet the constant demand (electricity demand is the black line; you can zoom and pan to see the details; negative values correspond to storage consuming electricity):



Source: `https://model.energy/`

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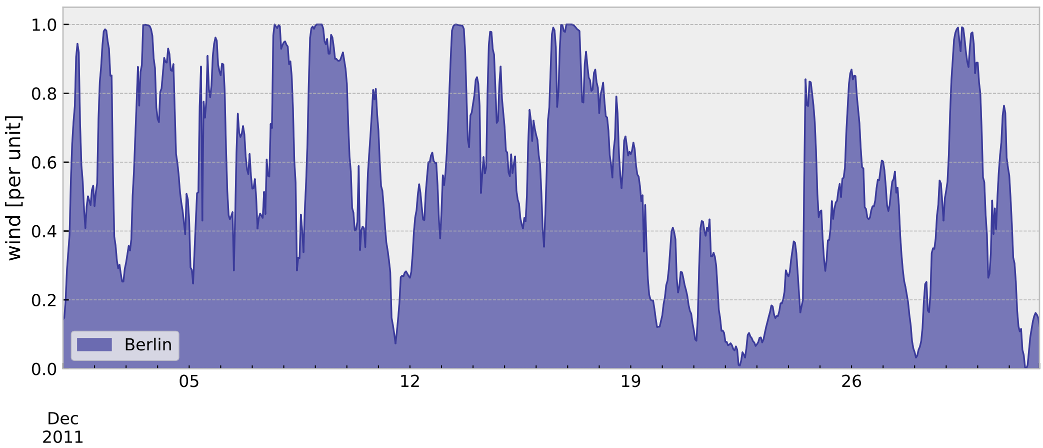
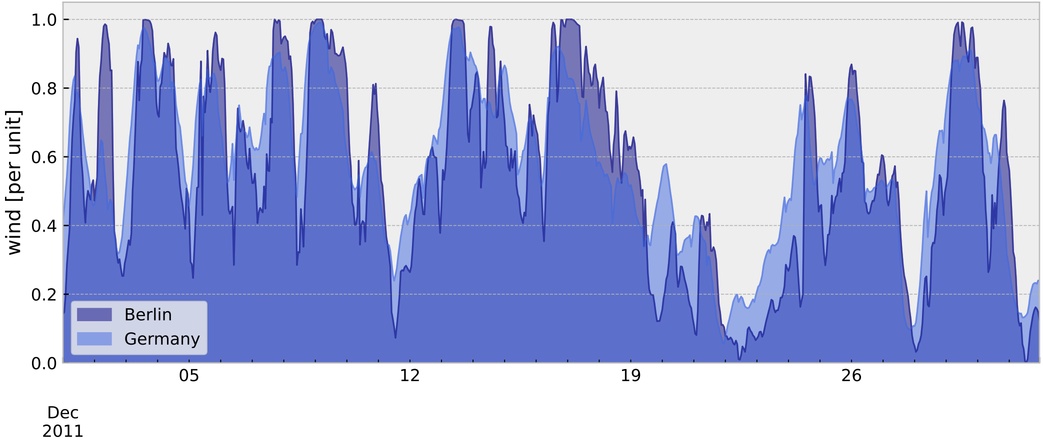
Wind production at a single site is highly variable. . .



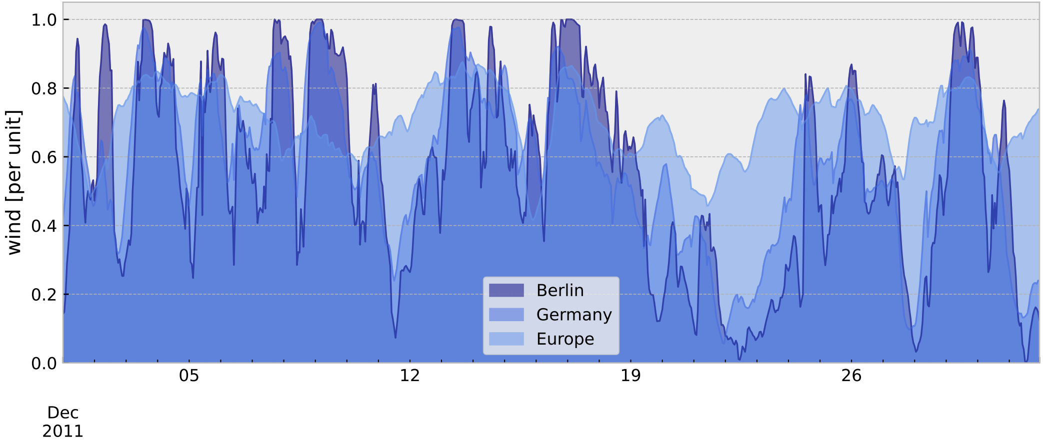
Source: https://model.energy/

. . . but aggregating over a whole country reduces variability . . .



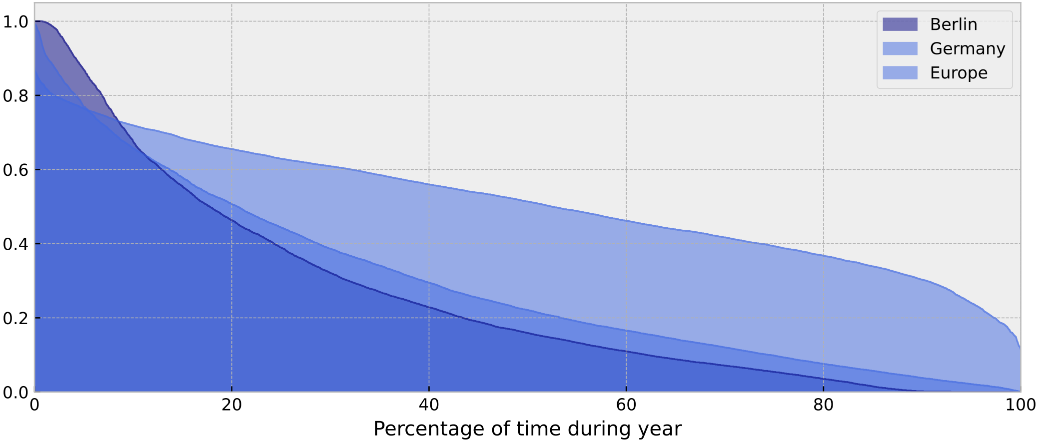
Source: https://model.energy/

. . . & integration across continent results in much smoother curve



Source: https://model.energy/

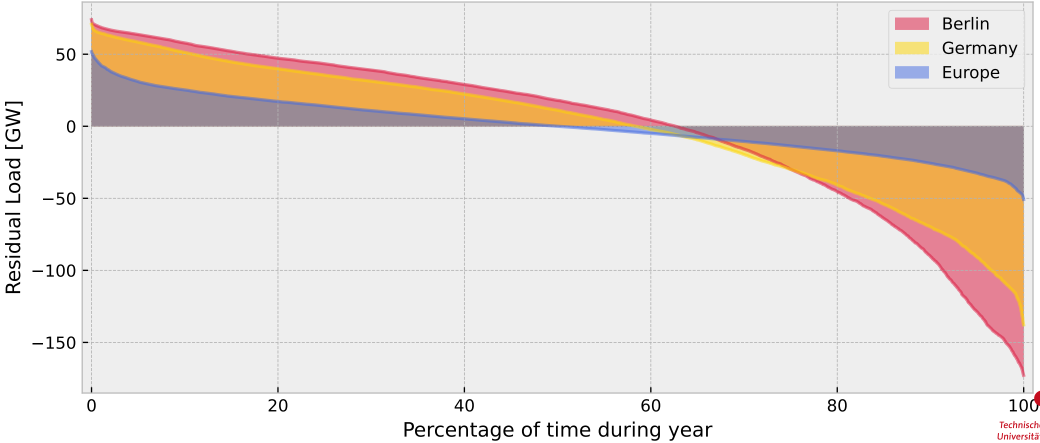
We can see this also in the corresponding duration curves...



. . . and the duration curve of the mismatch time series!

Lower peak mismatch and, thus, lower backup capacity or energy storage required.

Balancing renewables **in space** (networks) competes with balancing **in time** (storage)!



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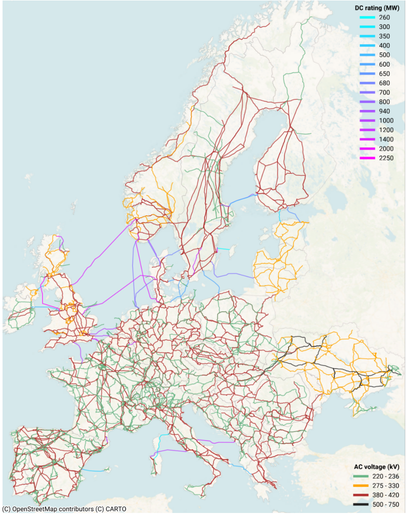
European power transmission network → potential bottlenecks



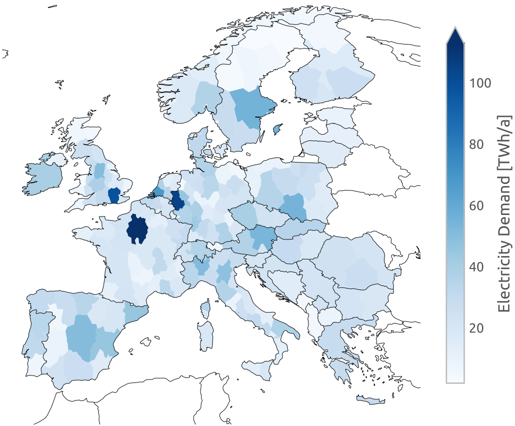
- In reality, we can only transport electricity within the **restrictions** of the power transmission network.
- Power flows in the network must respect **Kirchhoff's laws** for physical flow and the **thermal limits** of the transmission lines at any time.
- Different power imbalances $p_{i,t}$ at each location i and instead of $p_t = 0$ we will have

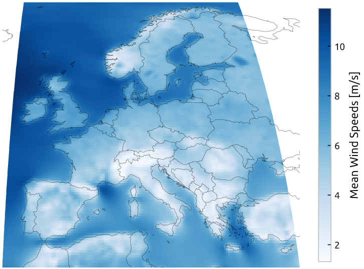
$$\sum_i p_{i,t} = 0 \quad \forall t$$

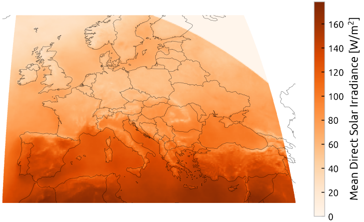
(neglecting transmission losses).

Spatial mismatch of demand and best renewable sites...

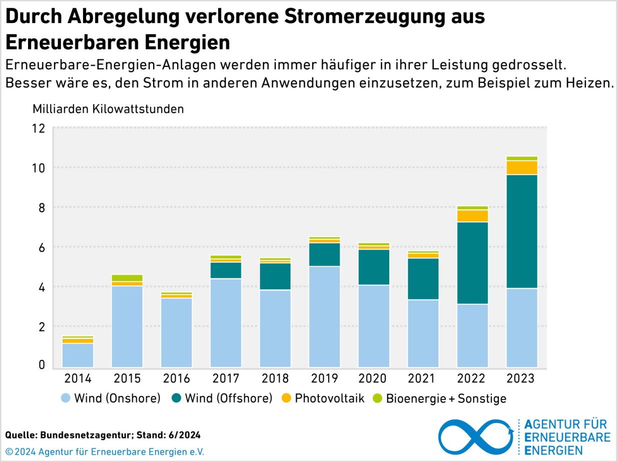
...may cause line **overloading** and **curtailment** of renewables without grid expansion!







Recent curtailment statistics for Germany



- mostly offshore and onshore wind curtailed ($\geq$ 90%)
- most curtailment caused at transmission level ($\approx$ 75%)
- approximately 2-3% of German electricity consumption
- increasing trend over last years

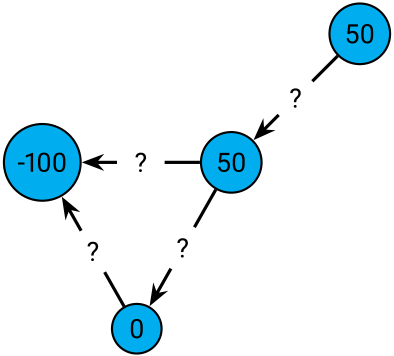
Source: https://www.unendlich-viel-energie.de/mediathek/grafiken/durch-abregelung-verlorene-stromerzeugung-aus-erneuerbaren-energien

How does power flow distribute in the transmission network?

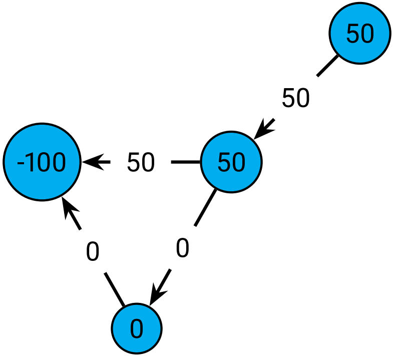
The goal of a power/load flow analysis is to find the flows in the lines of a network given the power injections/withdrawals at the nodes.
I.e. given power injection at the nodes

$$\mathbf{P}_i = \begin{pmatrix} 50 \\ 50 \\ 0 \\ -100 \end{pmatrix}$$

what are the flows in lines?



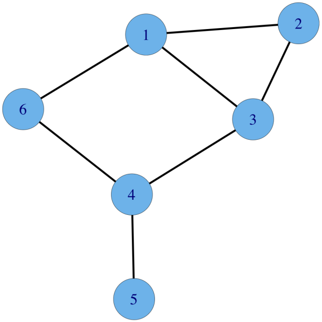
Is this correct?



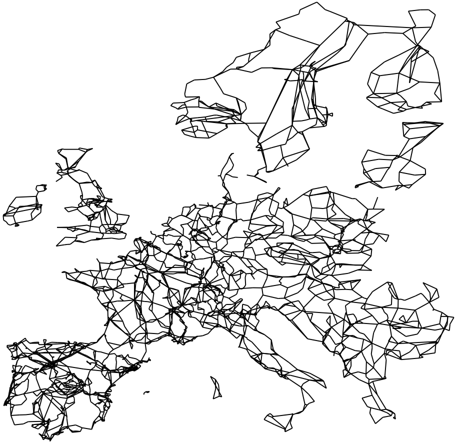
Definition of a network graph

A **network** (graph) is a collection of **vertices** (nodes) joined by **edges** (links).

- **vertices**: 1,2,3,4,5,6
- **edges**: (1,2), (1,3), (1,6), (2,3), (3,4), (4,5), (4,6)
- **order**: $N = 6$ vertices
- **size**: $L = 7$ edges



Example: European electricity transmission system as a graph



This dataset includes:

- 5380 **vertices**
- 6492 **edges**
- additional **attributes** for each edge (e.g. capacity, resistance, length) and node (e.g. voltage, coordinates)

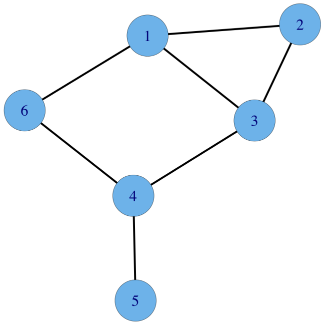
Question: How can we describe relations between vertices and edges of a graph.

Source: https://www.entsoe.eu/data/map/

Adjacency matrix A

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge between vertices i and j} \\ 0 & \text{otherwise.} \end{cases}$$

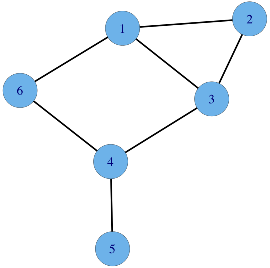
$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



- Diagonal elements are (usually) zero.
- Symmetric matrix for an undirected graph.
- If there are N vertices, it is an $N \times N$ matrix.

Degree

- The **degree** k_i of a vertex i is defined as the number of edges connected to i.
- Average degree of the network: $\langle k \rangle = \frac{1}{N} \sum_{i \in \{1,\ldots,N\}} k_i$.

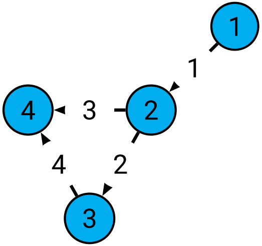


$$k_5 = 1$$
$$k_2 = k_6 = 2$$
$$k_1 = k_3 = k_4 = 3$$
$$\langle k \rangle = 2.33$$

The incidence matrix

For a directed graph (every edge has an orientation) $G = (V, E)$ with N nodes and L edges, the node-edge **incidence matrix** $K \in \mathbb{R}^{N \times L}$ has components

$$K_{i\ell} = \begin{cases} 1 & \text{if edge } \ell \text{ starts at node } i \\ -1 & \text{if edge } \ell \text{ ends at node } i \\ 0 & \text{otherwise} \end{cases}$$



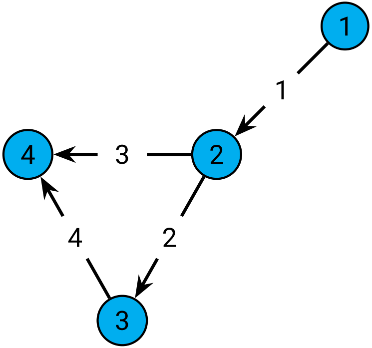
$$\mathbf{K} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

- columns (ℓ) sum to zero, since every edge starts (+1) and ends (-1) at some node
- rows (i) tell you which edges start (+1) and end (-1) there

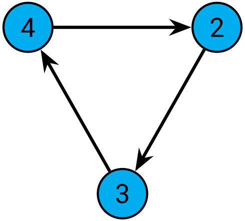
A cycle

A cycle is a path through the network that returns to its starting node.

For our graph:



The combination of edges $(0, 1, -1, 1)^\top$ corresponds to the cycle:



This can be expressed as a cycle matrix $C_{\ell c}$ which says which edges ℓ are part of cycle c.

Setting for power flow calculation

Let's take N nodes labelled by i, and L edges labelled by ℓ forming a directed graph.

Suppose at each node we have a **power imbalance** p_i.

($p_i > 0$ means more generation than consumption and $p_i < 0$ means more consumption)

Since we cannot create or destroy energy (and we're ignoring losses):

$$\sum_i p_i = 0$$

Question: How do the flows f_ℓ in the network relate to the nodal power imbalances?

$\rightarrow$ **Kirchhoff Laws!**

For **Kirchhoff's Current Law (KCL)** we use the incidence matrix

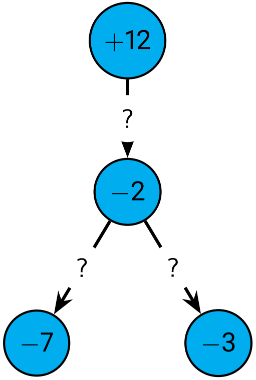
KCL enforces **energy conservation** at each vertex.

The power imbalance equals what goes out minus what comes in.

This can be expressed compactly with the incidence matrix

$$p_i = \sum_\ell K_{i\ell} f_\ell \quad \forall i$$

But KCL is **not enough** to determine the flow as soon as there are **cycles** in the network. For this, we need KVL.



For **Kirchhoff's Voltage Law (KVL)** need to linearise AC power flow

Active power flow f_ℓ and reactive power flow q_ℓ of line ℓ in voltage-polar coordinates:

$$f_\ell = \frac{r_\ell}{x_\ell^2}|V_i|^2 - |V_i|\,|V_j|\left[\frac{r_\ell}{x_\ell^2}\cos(\theta_i - \theta_j) - \frac{1}{x_\ell}\sin(\theta_i - \theta_j)\right]$$
$$q_\ell = \frac{1}{x_\ell}|V_i|^2 - |V_i|\,|V_j|\left[\frac{r_\ell}{x_\ell^2}\sin(\theta_i - \theta_j) - \frac{1}{x_\ell}\cos(\theta_i - \theta_j)\right]$$

with $|V_i|$ voltage magnitude, θ_i voltage angle, r_ℓ resistance, x_ℓ reactance. **Assuming:**

1. reactive power flows q_ℓ are negligible compared to real power flows f_ℓ ($q_\ell \approx 0$),
2. all per-unit voltage magnitudes are close to one ($|V_i| \approx 1$),
3. resistances r_ℓ are negligible relative to reactances x_ℓ ($x_\ell \gg r_\ell$),
4. voltage angle differences are small ($\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$)

Result: $$f_\ell = \frac{\theta_i - \theta_j}{x_\ell}$$

Source: AC = Alternating Current

Let's do it step by step

Active power flow f_ℓ and reactive power flow q_ℓ of line ℓ in voltage-polar coordinates:

$$f_\ell = \frac{r_\ell}{x_\ell^2}|V_i|^2 - |V_i||V_j|\left[\frac{r_\ell}{x_\ell^2}\cos(\theta_i - \theta_j) - \frac{1}{x_\ell}\sin(\theta_i - \theta_j)\right]$$

$$q_\ell = \frac{1}{x_\ell}|V_i|^2 - |V_i||V_j|\left[\frac{r_\ell}{x_\ell^2}\sin(\theta_i - \theta_j) - \frac{1}{x_\ell}\cos(\theta_i - \theta_j)\right]$$

with $|V_i|$ voltage magnitude, θ_i voltage angle, r_ℓ resistance, x_ℓ reactance.

1. $q_\ell \approx 0$

Active power flow f_ℓ and reactive power flow q_ℓ of line ℓ in voltage-polar coordinates:

$$f_\ell = \frac{r_\ell}{x_\ell^2}\left|V_i\right|^2 - \left|V_i\right|\left|V_j\right|\left[\frac{r_\ell}{x_\ell^2}\cos(\theta_i - \theta_j) - \frac{1}{x_\ell}\sin(\theta_i - \theta_j)\right]$$

with $|V_i|$ voltage magnitude, θ_i voltage angle, r_ℓ resistance, x_ℓ reactance.

2. $|V_i| \approx 1$

Active power flow f_ℓ and reactive power flow q_ℓ of line ℓ in voltage-polar coordinates:

$$f_\ell = \frac{r_\ell}{x_\ell^2} - \left[\frac{r_\ell}{x_\ell^2}\cos(\theta_i - \theta_j) - \frac{1}{x_\ell}\sin(\theta_i - \theta_j)\right]$$

with $|V_i|$ voltage magnitude, θ_i voltage angle, r_ℓ resistance, x_ℓ reactance.

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3. $x_\ell \gg r_\ell$

Active power flow f_ℓ and reactive power flow q_ℓ of line ℓ in voltage-polar coordinates:

$$f_\ell = \frac{1}{x_\ell} \sin(\theta_i - \theta_j)$$

with $|V_i|$ voltage magnitude, θ_i voltage angle, r_ℓ resistance, x_ℓ reactance.

4. $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$

Active power flow f_ℓ and reactive power flow q_ℓ of line ℓ in voltage-polar coordinates:

$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell}$$

with $|V_i|$ voltage magnitude, θ_i voltage angle, r_ℓ resistance, x_ℓ reactance.

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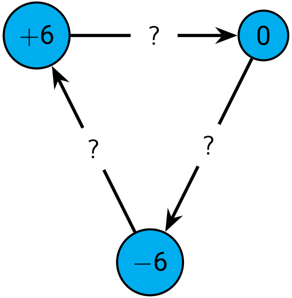
Kirchhoff's Voltage Law (KVL)

KVL: sum of voltage differences around each cycle add up to zero.



If the voltage angle at any node is given by θ_i, then the voltage difference across edge ℓ is

$$\sum_i K_{i\ell}\theta_i$$

KVL can be expressed using cycle matrix

$$\sum_\ell C_{\ell c} \sum_i K_{i\ell}\theta_i = 0 \qquad \forall c$$

Kirchhoff's Voltage Law (KVL)

Earlier we derived flow f_ℓ on line ℓ with reactance x_ℓ in terms of nodal voltage angles θ_i

$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell} = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

KVL now becomes $L - N + 1$ binding constraints on the line flows f_ℓ

$$\sum_\ell C_{\ell c} x_\ell f_\ell = 0 \qquad \forall c$$

With KVL and KCL together, we can solve for the line flows!

Now we have $N-1$ equations (one is linearly dependent!) for the flows f_ℓ from KCL:

$$p_i = \sum_\ell K_{i\ell} f_\ell \qquad \forall i \in \{1, \ldots, N-1\}$$

and $L-N+1$ equations from KVL:

$$\sum_\ell C_{\ell c} x_\ell f_\ell = 0 \qquad \forall c \in \{1, \ldots, L-N+1\}$$

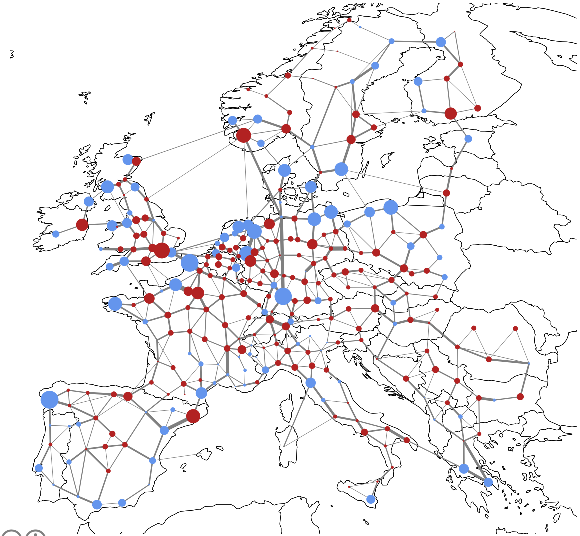
So L independent linear equations for L variables f_ℓ. We can solve this!

Let's calculate the power flows for two examples!

Example: Inputs are network graph and line parameters

- simulation for 100% renewable electricity, March 8, 2013, 16:00
- input: network graph G
- impedances of lines ℓ

Example: Further inputs are net generation and consumption $p_{i,t}$



- simulation for 100% renewable electricity, March 8, 2013, 16:00
- input: network graph G
- input: impedances of lines ℓ
- input: net generation at i (blue)
- input: net consumption at i (red)

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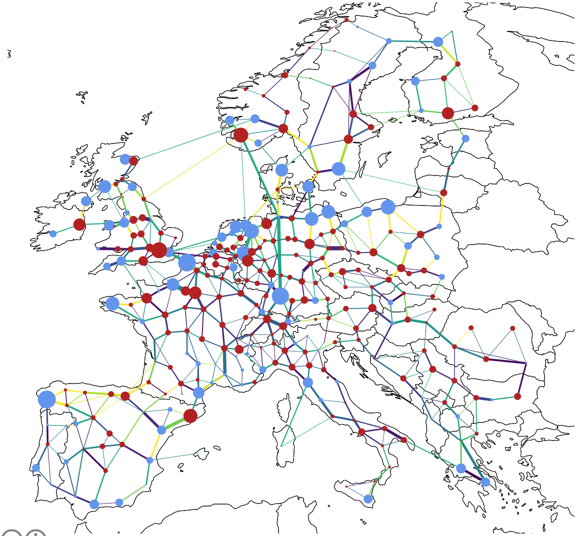
Example: Power flows calculated by applying one of alternatives 1-3



- simulation for 100% renewable electricity, March 8, 2013, 16:00
- input: network graph G
- input: impedances of lines ℓ
- input: net generation at i
- input: net consumption at i
- solve equation system
- output: power flows in ℓ
- yellow means high loading
- blue means low loading

Source: Another example: Copper Sushi

Next: Where does actually the power imbalance come from p_i?

So far, we assumed that the power imbalance (i.e. net generation & consumption) is given.

For today's system, we mostly know demand (relatively inflexible) and the unknown variables are the dispatch of power plants (and hydro storage).

For future systems, it gets more complicated because of the stronger interplay of generation, storage, transmission and flexible demands.

In the next lectures, we want to study how to operate electricity generation and consumption assets efficiently to achieve the most cost-effective supply of electricity.

For that, we need to learn about **how electricity markets work** and how we can model them as **optimisation problems**.

Up next

Next Thursday (1/2): Introduction to `pysheds` and `networkx`

Next Thursday (2/2): Lecture on Optimisation in Electricity Markets

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