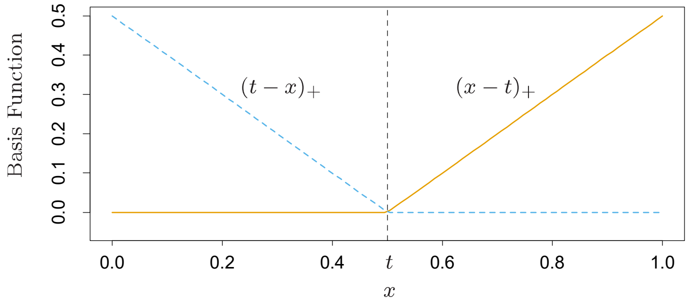

FIGURE 9.9. *The basis functions $(x-t)_+$ (solid orange) and $(t-x)_+$ (broken blue) used by MARS.*

As an example, the functions $(x-0.5)_+$ and $(0.5-x)_+$ are shown in Figure 9.9.

Each function is piecewise linear, with a *knot* at the value t. In the terminology of Chapter 5, these are linear splines. We call the two functions a *reflected pair* in the discussion below. The idea is to form reflected pairs for each input X_j with knots at each observed value x_{ij} of that input. Therefore, the collection of basis functions is

$$\mathcal{C} = \{(X_j - t)_+,\ (t - X_j)_+\}_{\substack{t \in \{x_{1j}, x_{2j}, \ldots, x_{Nj}\} \\ j = 1, 2, \ldots, p.}} \tag{9.18}$$

If all of the input values are distinct, there are $2Np$ basis functions altogether. Note that although each basis function depends only on a single X_j, for example, $h(X) = (X_j - t)_+$, it is considered as a function over the entire input space $\mathbb{R}^p$.

The model-building strategy is like a forward stepwise linear regression, but instead of using the original inputs, we are allowed to use functions from the set $\mathcal{C}$ and their products. Thus the model has the form

$$f(X) = \beta_0 + \sum_{m=1}^{M} \beta_m h_m(X), \tag{9.19}$$

where each $h_m(X)$ is a function in $\mathcal{C}$, or a product of two or more such functions.

Given a choice for the h_m, the coefficients β_m are estimated by minimizing the residual sum-of-squares, that is, by standard linear regression. The real art, however, is in the construction of the functions $h_m(x)$. We start with only the constant function $h_0(X) = 1$ in our model, and all functions in the set $\mathcal{C}$ are candidate functions. This is depicted in Figure 9.10.

At each stage we consider as a new basis function pair all products of a function h_m in the model set $\mathcal{M}$ with one of the reflected pairs in $\mathcal{C}$. We add to the model $\mathcal{M}$ the term of the form

$$\hat{\beta}_{M+1} h_\ell(X) \cdot (X_j - t)_+ + \hat{\beta}_{M+2} h_\ell(X) \cdot (t - X_j)_+,\ h_\ell \in \mathcal{M},$$