EE 810. Home work #B

Name. Onhar Vively Apte/

2.1(8)

8. Which of the following descriptions are correct? The solutions x of

1 1		
_ <i> </i>	- 1	
Y	1	
	fo	

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- (a) a plane.
- (b) a line.
- (c) a point.
- (d) a subspace.
- (e) the nullspace of A.
- (f) the column space of A.

1 1	'	$\lceil x, \rceil$		6	
10	2	22	= /	0	
		713			

- $\therefore \quad ^{\mathcal{H}_{\mathfrak{l}}} + \mathcal{H}_{2} + \mathcal{H}_{3} = 0 \quad ---- \mathbb{C}$
- $\therefore \quad \chi_1 \quad +2\chi_3 \quad = 0 \quad ---- \boxed{2}$
 - $x_1 = -2x_3$

substituting this in equi

 $-2x_3+x_2+x_3=0.$

 $x_2 - x_3 = 0$ \longrightarrow This is a equal

of o line in R3

Also, by desination,

L {x | Ax = 0 } is a null space,

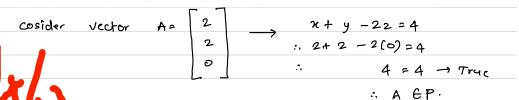
which is also a spacespace

in R3

- (d) Subspace V
 - (e) Nullspace V (f) Coloumnspace X

2.1 (15)/

15. Let **P** be the plane in \mathbb{R}^3 with equation x + y - 2z = 4. The origin (0,0,0) is not in **P**! Find two vectors in **P** and check that their sum is not in **P**.



osider vector $B = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$ $\Rightarrow x+y-2z=4$ $\Rightarrow x+y-2z=4$ $\Rightarrow x+y-2z=4$

· BEP

: Now consider Vector (A+B) =
$$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
 $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$ = $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 2+6-2 \end{bmatrix}$ $\begin{bmatrix} 3-4 \\ 4 \end{bmatrix}$ false

. (A+B) € P.

.:	Vector	Ą۶	2	&	vecto	r B=	6	are	fwo	vectors,
			2				4			•
			0				ן ס			
	which	li e	in	the	gi∨er	n pla	ne	P &	e th	eir addition,
	Vector	A٠	B =-	2	does	not li	e in	, the	plane	P ·
				6						
				0						

2.1(22)

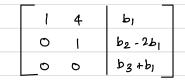
22. For which right-hand sides (find a condition on b_1, b_2, b_3) are these systems solvable?

(a)	$\begin{bmatrix} 1 & 4 \\ 2 & 8 \\ -1 & -4 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} x \\ x \\ x \end{bmatrix}$	$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$	(b)	$\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix}$	$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$	$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.	
	A	·	= B			κc		

(a) consider:
$$\begin{bmatrix} 1 & 4 & 2 & b_1 \\ 2 & 8 & 4 & b_2 \\ -1 & -4 & -2 & b_3 \end{bmatrix}$$

$$\begin{array}{ccc} R_2 \rightarrow & R_2 - 2R_1 \\ R_3 \rightarrow & R_3 + R_1 \end{array}$$

$$\therefore R_2 \rightarrow R_2 - 2R_1$$



2. 1	(27)								
27.	If A is any 8 by 8	invertible mat	trix, then its c	olumn spa	ce is	Why?			
	IF 8×8	matrix	is in	vertib	le ,	mean	s }	צ' {	
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	(5.6.7.0.7)	.,5 20							
	4 ∵ the	determin	nant	is 2e/	ro,	there	a~	ИО	
	Linearly	depend	ant ve	ctors.					
						_			
	Coloumn	space	w i jj	be	R ⁸			4	/4

10. Find a 2 by 3 system Ax = b whose complete solution is



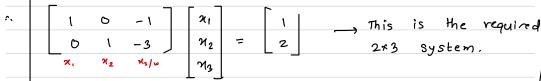
$$x = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + w \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

Find a 3 by 3 system with these solutions exactly when $b_1 + b_2 = b_3$.

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				N3			
.:	[x		۶	[1		_ t _	
	24.	ı		2	+ W	3	
	γ	3		٥		١	

.:	Тх,	1+W 2+3w	$\therefore \ \chi_{1} = 1 + \omega \rightarrow \chi_{1} - \omega = 1 \ \gamma \ \lceil b \rceil$
	74.2	2+3w	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
	N3 _	Lω	M3=w: X3 is a free vorsable.





. 3x3 matrix would be,

1	0	-1	$\lceil \varkappa, \rceil$		[1]
٥	1	- 3	×2	ŗ	2
Ð	O	٥	ηL		6

: now for case bitb2 = b3,

Perforn R3 + R3 + R2 +R1

		~ _				
<i>:.</i>	1 0 -1	፠ .		_ (17-	b1+b2=b3 1+2=3
	0 1 -3	74 2	11	2)	
	1 1-4	/ ×3 _		_ 3 _	-	

1. This is the required 3x3 system when b, +b2 = b3

2,2(45)

- **45.** Write all known relations between r and m and n if Ax = b has
 - (a) no solution for some b.
 - (b) infinitely many solutions for every b.
 - (c) exactly one solution for some b, no solution for other b.
 - (d) exactly one solution for every b.



(a) An = b has no solutions for some b.

. In case of no solutions for any b,

rank must be less than rows.

but no solution for some b means there is a ber

r < m

(b) An = b have infinitly many solutions.

.: ronk = rous, 4 rows < coloums.

" m = r

(C) Exactly one solution for some b (No sol for others.

.: No solution for b is when ronk is greater than rows,

" for some b we get a valid unique solution,

rank must be equal to colourna

∴ r = n
∠ m > n

(a)	Exa	ctly	00	e solu	ı tıon		for a	every	Ь		
	.:	rank	is	Same	as	ro ws	Ļ	Makir	Must	· be	squ or.
											rank.
		ء٣									
									•		

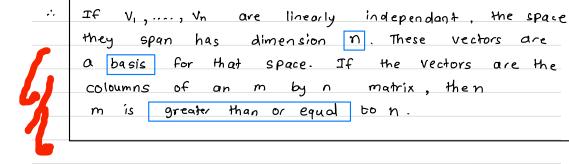
2.0 (0)	2.	3	(8)
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8. If w_1 , w_2 , w_3 are independent vectors, show that the sums $v_1 = w_2 + w_3$, $v_2 = w_1 + w_3$, and $v_3 = w_1 + w_2$ are *independent*. (Write $c_1v_1 + c_2v_2 + c_3v_3 = 0$ in terms of the w's. Find and solve equations for the c's.)

$$C_1 + C_3 = 0$$
 $-(2)$

2.3 (19)

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	-	V _s as a		•		
۶.	This mo	uhix will	have	n colo	ums	
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	•	as b	-		17 29 241	<i>5</i> C
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2.3 (22)
22. Suppose v_1, v_2, \ldots, v_6 are six vectors in \mathbb{R}^4 .
(a) Those vectors (do)(do not)(might not) span \mathbb{R}^4 .
(b) Those vectors (are)(are not)(might be) linearly independent.
(c) Any four of those vectors (are)(are not)(might be) a basis for R ⁴ .
(d) If those vectors are the columns of A , then $Ax = b$ (has) (does not have) (might not have) a solution.
(a) It is a possibility that all us lie in a single
l'ine.
" Those vectors might not span R4
(1)
(b) They are in R4, there can only be possible
combination of 4 linearly independent vector.
best case scnorlo, 4 of the given vectors can be
·
linearly independent but the remaining 2 must be
linearly dependant.
1. Those vectors are not linearly independent
G/G
6/6 (c) It is possible that any 4 out of 6 given vector
• · · · · · · · · · · · · · · · · · · ·
are linearly independent & hence can form basis in
- Any four of those vectors might be
a basis for R ⁴
cd) It is possible that in some case, Ax = b
might have a solution 4 in some cases it might
not
:. An = b might not have a solution.

2.4(2)

-2. Find the dimension and construct a basis for the four subspaces associated with each of the matrices

$$A = \begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 2 & 8 & 0 \end{bmatrix} \qquad \text{and} \qquad U = \begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Row space of A: basis = non zero rows of L

How,	0140	χ,		0	
] ×2	1	0	\rightarrow $\chi_2 + 4 \chi_3 = 0$
		213		0	:. x2 = -4 x3
		χ4		D	

										_ /	1
-:	χι		v,			1		ь		6	
	x_2	13	-4×3	=	ፖ ነ	0	+ X3	-4	+214	0	
	χş		u ₃			ь		t		0	
	χ4		24			0		0)	

Simillarly,
$$C(U^{T}) = \begin{cases} \begin{bmatrix} D \\ 1 \\ 4 \\ 0 \end{bmatrix} \end{cases}$$
 $\dim = 1$

Now, $U^{T} = \begin{bmatrix} D & O \\ 1 & O \\ 1 & O \\ 0 & O \end{bmatrix}$

$$\begin{bmatrix} A_{1} & O & O \\ 1 & O \\ 0 & O \\ 0 & O \end{bmatrix}$$

$$\vdots \quad X_{1} = 0.$$

$$X_{2} \longrightarrow \text{free variable}$$

$$\vdots \quad X_{n} \subseteq \begin{bmatrix} O \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore N(U^{T}) = \begin{bmatrix} A_{2} \begin{bmatrix} O \\ 1 \\ 1 \end{bmatrix} \mid A_{2} \in R \end{cases} \quad \dim = 1$$

		\wedge	. ^
2	4	(4)
_	•	_	• /

4. Describe the four subspaces in three-dimensional space associated with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

•:	A ۽	0	1	0										
		٥	٥	1	,	The	basis	for	C (A)	- f		7.	0	Ι Ί,
		0	٥	O					C (A)		0	,	0	

_:,	Th e	coloumn	Space	of A							
	İs	defined a	י :	C(A)	٤.) n	1 0	+ 9	0	x,y & TR }	_
									ر ہٰ	<u> </u>	

Now,	for	Null	Space,		A :	X	=	0			
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				o	ь	ь		N3		0	
				_ %.		-					

$$\lambda \qquad N(A) = \left\{ \chi \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mid \chi \in \mathbb{R} \right\}$$

Now,
$$A^{T} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and
$$C(A^T) = \left\{x\begin{bmatrix} 0\\ 0\\ 0\end{bmatrix} + y\begin{bmatrix} 0\\ 0\\ 1\end{bmatrix} \middle| x,y \in \mathbb{R}\right\}$$

$$\begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\chi_1 \\
\chi_2 \\
\chi_3
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}$$

$$A$$
 x_3 is a free variable, say $x_3 = \xi$.

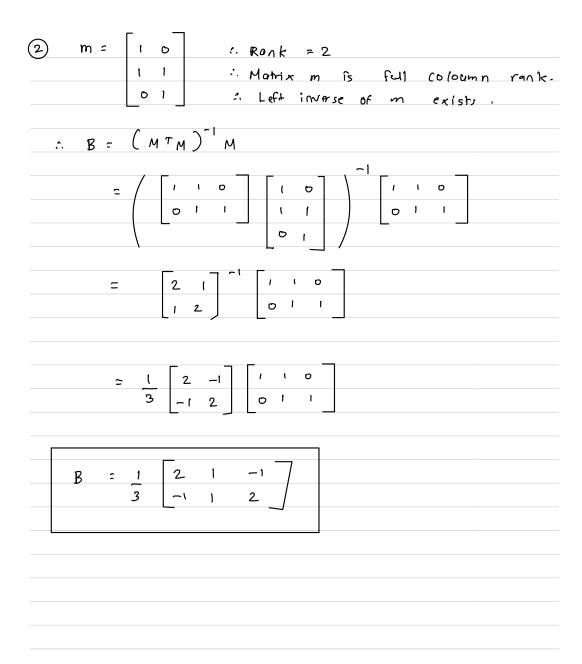
$$k \qquad N(A^{T}) = \begin{cases} x \begin{pmatrix} 0 \\ 0 \\ 1 \end{cases} & x \in \mathbb{R} \end{cases}$$

14. Find a left-inverse and/or a right-inverse (when they exist) for



$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$
 and $M = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $T = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$.

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$



(T	e a	b	— i <u>f a = b</u> ,
		0	۵	T does not have inverse.
			Į	if α ≠ δ,
				rank = 2
				T is full row rank or well
				as full coloumn rant.
٤٠	Left	invene		Right invase = 1 a -b
				q² [o a]

- **4.** Every straight line remains straight after a linear transformation. If z is halfway between x and y, show that Az is halfway between Ax and Ay.
 - Z is halfway between x 4 y,

$$Z = \frac{\chi_{+y}}{2} = \frac{1}{2} (\chi_{+y})$$

Every line remains a line after linear transformation, apply A on both sides,

$$Az = A \cdot \left(\frac{1}{2}(n+y)\right)$$

-: Az is also halfway between An a As

	a general	vector	in R3	,
(a, x, +	a ₂ N ₂ + a ₃ N ₃	,) , ,	where,	a, ,a, ,a3 are scal
				$a_1, a_2, a_3 \in R$
			4 2	M, , M2, M3 are a sex
				basis, x, x2, x3 6 R
Applying Lic	rear transform	nation T,		
	T. C.			•
	T (α, κ,)			
	= T(a, Mi) 1	-		
	= a, T(x,).	+ 92 T(N2)	+ a3 T	(n ₃)
you, Apply	a Iransfo	mation	T^2 ,	
	2 1			
		1 +a2 112	-	
	= T(T(a	lini tazuz	+ 93 43))
	= T(a,T(n			
	= 0, 72(ni)) + a ₂ T ² (12) + az	$\mathcal{T}^2(\mathcal{A}_3)$
/ , _				-
		T ² al	50	∟ where, α,, α, α, α, ∈ ₹ , ~ ~ ~ , ~ , ~ , ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~
		preserves a	ddilion T	$a_1, a_2, a_3 \in \mathbb{R}$
		•		

2.6 (22)

22. Which of these transformations is not linear? The input is
$$v = (v_1, v_2)$$
.

(a)
$$T(v) = (v_2, v_1)$$
.
(c) $T(v) = (0, v_1)$.

$$(v_1)$$
. (b) $T(v) = (v_1, v_1)$.
 (1) . (d) $T(v) = (0, 1)$.

Vector

$$T(x+y) = \begin{cases} x_2+y_2 \\ x_1+y_1 \end{cases}$$

$$T(x) = \begin{cases} x_2 \\ y_1 \end{cases}$$

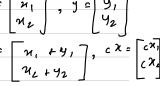
$$T(y) = \begin{bmatrix} u_2 \\ x_1 \end{bmatrix}, T(y) = \begin{bmatrix} y_2 \\ y_1 \end{bmatrix}, T(y) + T(y) = \begin{bmatrix} u_2 + y_2 \\ x_1 + y_1 \end{bmatrix}$$

 $T(x+y) = T(y) + T(y), given transformation preserves$

$$T(cx) = \begin{bmatrix} cx_1 \\ cx_1 \end{bmatrix}$$

$$T(X) = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, T(y) = \begin{bmatrix} y_1 \\ y_1 \end{bmatrix}, T(y) + T(y) = \begin{bmatrix} x_1 + y_1 \\ x_1 + y_1 \end{bmatrix}$$

$$T(n+y) = \begin{cases} n_1 + y_1 \\ x_2 + y_3 \end{cases}$$



$$\begin{bmatrix} x_1 + y_2 \end{bmatrix}$$

$$(y) = \begin{bmatrix} x_2 + y_2 \end{bmatrix}$$

$$\begin{bmatrix} c \mathcal{H}_1 \end{bmatrix} = \begin{bmatrix} c \mathcal{H}_2 \end{bmatrix} = \begin{bmatrix} c$$

$$T(v) = (V_2, v_1) \text{ is a linear transformation}$$

$$= [x_1] T(y) = [y_1] T(y) + T(y) = [x_1 + y_1]$$

$$\left[\begin{array}{c} \mathcal{A}_{1} \\ \mathcal{A}_{1} \end{array} \right] = \left[\begin{array}{c} \mathcal{A}_{1} \\ \mathcal{C}_{1} \end{array} \right]$$

$$C + (n) = C \cdot \begin{bmatrix} 0 \\ n_1 \end{bmatrix} = \begin{bmatrix} 0 \\ cn_1 \end{bmatrix}$$

$$T (cn) = \begin{bmatrix} 0 \\ cn_1 \end{bmatrix}$$

$$T(N+y) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \therefore T(N) + T(Y) \neq T(N+y)$$

$$CT(N) = C \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ C \end{bmatrix}$$

$$T(ch) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
 :. $c.T(h) \neq T(ch)$

2.6 (28)

- **28.** Find the *range* and *kernel* (those are new words for the column space and nullspace) of T.
 - (a) $T(v_1, v_2) = (v_2, v_1)$.
- (b) $T(v_1, v_2, v_3) = (v_1, v_2).$
- (c) $T(v_1, v_2) = (0, 0)$.
- (d) $T(v_1, v_2) = (v_1, v_1)$.

$$\textcircled{a}$$
 $\tau(v_1, v_2) = (v_2, v_1)$

$$\therefore C(T) = range(T) = \left\{ T(V_1, V_2) \mid (V_1, V_2) \in \mathbb{R}^2 \right\}$$

=
$$kemal(T) = \{ (v_1, v_2) \in \mathbb{R}^2 \mid T(v_1, v_2) = 0 \}$$

$$= \{ (v_1, v_2) \in \mathbb{R}^2 \mid (v_2, v_1) = 0 \}$$

$$= \{ (v_1, v_2) \in \mathbb{R}^2 \mid v_1 = 0, v_2 = 0 \}$$

$$= \{ (V_1, V_2) \in \mathbb{R}^2 \mid (V_1, V_2) = 0 \}$$

$$\begin{array}{lll} \text{(i)} & T(V_1, V_2) = (0, 0) \\ & : & (T) = range(T) = \left\{T(V_1, V_2) \mid (V_1, V_2) \in \mathbb{R}^2\right\} \\ & = \left\{(O_1 O) \mid V_1, V_2 \in \mathbb{R}\right\} \\ & & = \left\{(O_1 O)\right\} \\ & : & \text{rang}(T) = \left\{(O_1 O)\right\} \\ & : & \text{rang}(T) = \left\{(V_1, V_2) \in \mathbb{R}^2 \mid (V_1, V_2) = 0\right\} \\ & = \left\{(V_1, V_2) \in \mathbb{R}^2 \mid (V_1, V_2) \in \mathbb{R}\right\} \\ & = \left\{(V_1, V_2) \in \mathbb{R}^2 \mid (V_1, V_2) \in \mathbb{R}\right\} \\ & & \text{kernal}(T) = \mathbb{R}^2 \end{array}$$