EE510, Fall 2022 GPP
MIDTERM 1
Wednesday, October 5th 2022
EACH PROBLEM COUNTS EQUALLY (2.083 pts)

Problem 1:

Given a matrix A:

$$\begin{bmatrix} a & -1 & 0 & 0 \\ 0 & a & -1 & 0 \\ 0 & 0 & a & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$$

For which of the following values of a the matrix has rank 3?

Case i	*Case ii*	*Case iii*	*Case iv*	*Case v*
-6	*-1*	*6*	*11*	*1*

rank 3

(vi) none of the above

Problem 2:

For which vector "b" both of the following systems are solvable

$$\begin{bmatrix} 3 & -1 & 2 \\ 3 & -1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = b \quad , \quad \begin{bmatrix} 5 & 2 \\ 5 & 2 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = b$$

x y

(i) $b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(ii) $b = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

(iii) $b = \begin{bmatrix} -5 \\ -5 \\ 1 \end{bmatrix}$?

(iv) $b = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$

(v) $b = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$

(vi) none of the above

Problem 3:

What is the matrix P that projects every point in R^3*, onto the intersection of the planes*

$3x + 2y - z = 0$ *and* $x + y = 0$ *?*

(i) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$-x = y$
$y = y$
$z = -y$

$\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

$$(ii)\ P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(iii)\ \ P = \frac{1}{\sqrt{6}} \begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

$$(iv)\ P = \frac{1}{3} \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$(v)\ P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(vi) none of the above

<u>Problem 4:</u>

Find the largest possible number of linearly independent vectors among

$$\begin{bmatrix} 4 \\ -3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -1 \\ 2 \end{bmatrix}$$

(i) 5

(ii) 4

(iii) 3

(iv) 2

(v) 6

(vi) none of the above

1 2 4 2
0 −6 −12 1
0 −2 −4

<u>Problem 5:</u>

What is the rank of the matrix $A=\begin{bmatrix}1 & 2 & 4\\ 5 & 4 & 8\\ 2 & 2 & 4\end{bmatrix}$ *?*

Case (i)	*Case (ii)*	*Case (iii)*	*Case (iv)*	*Case (v)*
3	*2*	*0*	*1*	*4*

rank = 2.

(vi) none of the above

<u>Problem 6:</u>

What is the rank of the matrix $A=\begin{bmatrix}9 & 6 & 7 & -1 & 0\\ 1 & 2 & 2 & 0 & -1\\ -1 & 0 & 3 & 0 & 2\\ 0 & 0 & 0 & 0 & 1\\ -1 & 0 & 0 & 0 & 1\end{bmatrix}$ *?*

Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)
2	3	4	5	6

(vi) none of the above

$$\begin{bmatrix} 1-p \\ -p \\ -1-p \\ 1-p \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = 0$$

Problem 7:

Given are two nonzero four dimensional vectors a and b, where

$$b = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Which of the following vectors could be a projection of "b" on "a"?

Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)
$\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

$-p=0$

$p=0$

(vi) none of the above

$2(1-p)\ldots$ $\quad -2(-1-p)=0$ $\quad 2(1-p)-p-1$ $\quad 3-2p=0$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & \frac{1}{2} & 0 \\ 0 & 5 & 2 & 0 & 0 & 1 \end{array}\right]$$

Problem 8:

Given $A=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 5 & 2 \end{bmatrix}$*, could any of the following matrices be an inverse of A?*

$1 \quad -3$

Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)

$\begin{bmatrix}1 & 1/3 & 1\\ 0 & 2 & -1\\ 0 & -5 & -2\end{bmatrix}$	$\begin{bmatrix}1 & 3 & 9\\ 0 & -3 & -9\\ 0 & 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 3 & -3\\ 0 & 2 & -1\\ 1 & -5 & 2\end{bmatrix}$	$\begin{bmatrix}1 & -3\\ 0 & -2\end{bmatrix}$	*It does not have an inverse*

(vi) none of the above

Problem 9:

Given the matrix $A=\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 2 & 0 & 1\\ 1 & 2 & 3 & -1\end{bmatrix}$ *the nullspace N(A) is spanned by:*

Case i	*Case ii*	*Case iii*	*Case iv*	*Case v*
$\begin{bmatrix}4\\-2\\-1\\6\end{bmatrix},\begin{bmatrix}4\\0\\-1\\0\end{bmatrix}$	$\begin{bmatrix}-1\\1\\1\\1\end{bmatrix},\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$	$\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\\1\end{bmatrix}$	$\begin{bmatrix}2\\0\\-1\end{bmatrix},\begin{bmatrix}-1\\0\\3\end{bmatrix}$	$\begin{bmatrix}2\\1\\-2\\-2\end{bmatrix},\begin{bmatrix}-4\\-2\\4\\4\end{bmatrix}$

(vi) none of the above

Problem 10:

Given a 3x5 matrix A the column space C(A) can have as basis:

Case i	*Case ii*	*Case iii*	*Case iv*	*Case v*
$\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}$	$\begin{bmatrix}1\\2\\-1\end{bmatrix},\begin{bmatrix}0\\-2\\2\end{bmatrix},\begin{bmatrix}2\\0\\0\end{bmatrix}$	$\begin{bmatrix}5\\2\\-2\end{bmatrix},\begin{bmatrix}0\\-2\\2\end{bmatrix},\begin{bmatrix}2\\0\\0\end{bmatrix}$ rank 2

(vi) none of the above

Problem 11:

What is the rank of the matrix $A=\begin{bmatrix}1 & -1 & 2\\ 2 & -2 & 4\\ 3 & -3 & 6\end{bmatrix}$?

Case i	*Case ii*	*Case iii*	*Case iv*	*Case v*
3	*2*	*0*	*1*	*4*

(vi) none of the above

Problem 12:

What is the rank of the matrix $A=\begin{bmatrix}3 & -1 & 1 & -2 & 6\\ 0 & 2 & -3 & 7 & 5\\ 0 & 0 & 1 & -7 & 2\\ 0 & 0 & 0 & 0 & -1\\ 0 & 0 & 0 & 0 & 2\end{bmatrix}$?

Case i	*Case ii*	*Case iii*	*Case iv*	*Case v*
2	*3*	*4*	*5*	*6*

(vi) none of the above