### EE510, Fall 2022 GPP MIDTERM 1

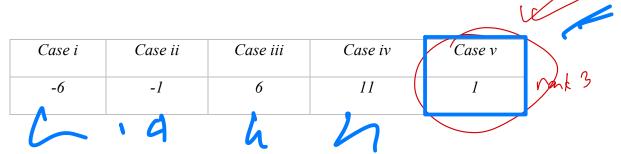
## Wednesday, October 5<sup>th</sup> 2022 EACH PROBLEM COUNTS EQUALLY (2.083 pts)

#### Problem 1:

Given a matrix A:

$$\begin{bmatrix} a & -1 & 0 & 0 \\ 0 & a & -1 & 0 \\ 0 & 0 & a & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$$

For which of the following values of a the matrix has rank 3?



(vi) none of the above

# Problem 2:

For which vector "b" both of the following systems are solvable

$$\begin{bmatrix} 3 & -1 & 2 \\ 3 & -1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = b , \begin{bmatrix} 5 & 2 \\ 5 & 2 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = b$$

$$(i) b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$(ii) \ b = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

$$(iii) \ b = \begin{bmatrix} -5 \\ -5 \\ 1 \end{bmatrix} \quad ?$$

$$(iv) b = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

$$(v) b = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$$

### Problem 3:

What is the matrix P that projects every point in  $\mathbb{R}^3$ , onto the intersection of the planes

$$3x + 2y - z = 0$$
 and  $x + y = 0$ ?

$$(i)P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad -\mathcal{N} = \mathcal{Y}$$

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$$(ii)P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

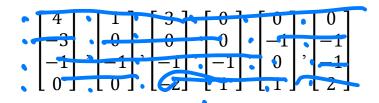
(iii) 
$$P = \frac{1}{\sqrt{6}} \begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

$$(iv)P = \frac{1}{3} \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

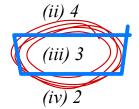
$$(v)P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

# Problem 4:

Find the largest possible number of linearly independent vectors among



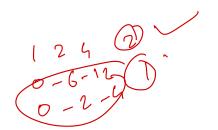
(i) 5



- (v) 6
- (vi) none of the above

# Problem 5:

What is the rank of the matrix  $A = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 4 & 8 \\ 2 & 2 & 4 \end{bmatrix}$ 



Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)
	/			
3	2	0	1	4
	(4)			

(vi) none of the above

## Problem 6:

What is the rank of the matrix  $A = \begin{bmatrix} 8 & 6 & 7 & 1 & 9 \\ 1 & 2 & 2 & 0 & -1 \\ -1 & 0 & 3 & 0 & 2 & ? \\ 8 & 0 & 0 & 0 & 1 & \\ -1 & 8 & 8 & 9 & 1 & ? \end{bmatrix}$ 

Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)
2	3	4	5	6

### Problem 7:

Given are two nonzero four dimensional vectors a and b, where

$$b = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

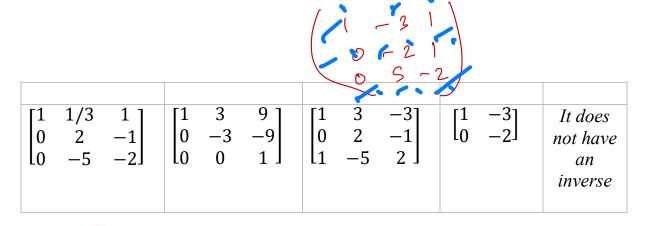
\[ \begin{aligned} & \cdot & \

Which of the following vectors could be a projection of "b" on "a"?

	Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)	
_0 = 0	$\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -2\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}_{0.050}$	
	ري) none of the	above of F	-2(-40-3	\(\rangle \langle \lan		0 0
	<u>Problem 8:</u>			0 5 1		120
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Given  $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ , could any of the following matrices be an inverse of A?

Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)
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## Problem 9:

Given the matrix  $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 2 & 3 & -1 \end{bmatrix}$ 

the nullspace N(A) is spanned by:

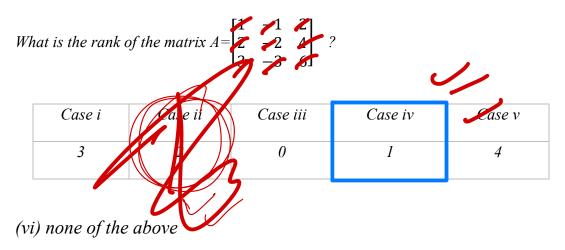
Case i	Case ii	Case iii	Case iv	Case v	
$\begin{bmatrix} 4 \\ -2 \\ -1 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -1\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 0\\1\\0\\1 \end{bmatrix}$	$\begin{bmatrix} 0\\1\\0\\0\\1\end{bmatrix},\begin{bmatrix} 0\\0\\1\\0\\1\end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2\\1\\-2\\-2 \end{bmatrix}, \begin{bmatrix} -4\\-2\\4\\4 \end{bmatrix}$	
(vi) none of the d	ıbove	[ -l <sub>1</sub>	1 ] — )	-2 -1 -1 -1 -2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	

#### Problem 10:

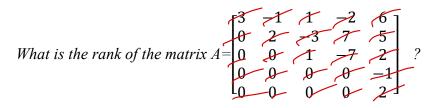
Given a 3x5 matrix A the column space C(A) can have as basis:

Case i	Case ii	Case iii	Case iv	Case v
$\begin{bmatrix} 1\\0\\0\\0\\0\end{bmatrix},\begin{bmatrix} 0\\1\\0\\0\\0\end{bmatrix},\begin{bmatrix} 0\\0\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$		$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}$	[8], [8], [8] 2], [8], [8] 2], [8] 2], [8]

#### Problem 11:



#### Problem 12:



Case i	Case ii	Case iii '	Case iv	Case v
2	3	4	5	6
		( )		

(vi) none of the above