EE510, Fall 2022, GPP FINAL Monday, Dec. 12th 2022, 4:30pm-5:30pm

ALL PROBLEMS COUNT EQUALLY:3pts

Problem 1:

(x-1)(x+)(x+2)

A is a 3×3 matrix that has eigenvalues 1, -1,2, and corresponding eigenvectors:

$$\begin{bmatrix} 1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\sqrt{3}\\3\sqrt{3} \end{bmatrix}, \begin{bmatrix} 1\\1\\-1 \end{bmatrix}$$

$$a^3 + 1a^2 - \alpha - 2$$

Then
$$c_1 =$$

 $A^5 = c_0 I + c_1 A + C_2 A + C_3 A + C_4 A + C_4 A + C_5 A$

$$1 = (6 + 6 + 6) + (2)$$

$$-1 = -(6 - 6) + (2)$$

$$as = 2a^{6} + a^{3} - 2a^{6}$$

(iii) 1 C6+C1=(

(v) $\sqrt{3}$

m + 24;2

\[\langle \la

(vi) none of the above

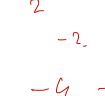
Problem 2:

Let A and B be two 3×3 matrices with eigenvalues: $\frac{1}{2}$, -1, -2 and $\frac{-1}{2}$, 2, -2 respectively. Then their Kronecker product has determinant equal to:

$$\frac{(iv)}{(v)} = -2$$

$$(vi)$$
 2

(vii) none of the above





Problem 3:

A is a 8×8 matrix. It has: eigenvalue 2 with multiplicity 3 and 3 eigenvectors eigenvalue 3 with multiplicity 4 and 2 eigenvectors eigenvalue 1 with multiplicity 1 and 1 eigenvector

(x-2) (x-3) (x-1)

Which is the following polynomials is possible as a minimal polynomial of A?

(i)
$$(\lambda - 2)^2 (\lambda - 3)^2 (\lambda - 1)^1$$

(ii) $(\lambda - 2)^3 (\lambda - 3)^2 (\lambda - 1)^1$
(iii) $(\lambda - 2)^2 (\lambda - 3)^4 (\lambda - 1)^1$

$$(iii)(\lambda - 2)^3(\lambda - 3)^4(\lambda - 1)^1$$

(iii)
$$(\lambda - 2)^3 (\lambda - 3)^4 (\lambda - 1)^1$$

(iv) $(\lambda - 2)^1 (\lambda - 3)^1 (\lambda - 1)^1$
(vi) none of the above

Problem 4:

A box in the three-dimensional Euclidean space is formed by the rows: (2,1,1), (1,2,1), (1,1,3). Starting at time 0 the box is transformed by:

$$\frac{d}{dt} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 6 & 2 & 2 \\ -2 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

The volume of the transformed box at time $t = \frac{2}{10} \ln 2$ is equal to:

(i) 21

(ii)16_

(iii)28

(iv)35

(v)14

(vi)none of the above

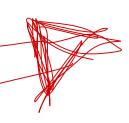
Problem 5:

$$A = \begin{bmatrix} -9 & -4 \\ 81 & 27 \end{bmatrix}$$
 , $A^{51} = ?$

$$(i)9^{50}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 9^{49} * 50\begin{bmatrix} 18 & 4 \\ -81 & -18 \end{bmatrix}$$

$$(ii)9^{50}\begin{bmatrix}9&4\\-81&7\end{bmatrix}-9^{49}\begin{bmatrix}25&4\\72&-18\end{bmatrix} \checkmark$$

$$(iii)9^{50}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + 9^{49} * 50\begin{bmatrix}18 & 4\\-81 & -18\end{bmatrix}$$



$$(iv)-9^{51}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + 51 * 9^{50}\begin{bmatrix}18 & 4\\ -81 & -18\end{bmatrix}$$

$$(v)9^{51}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + 9^{50} * 51\begin{bmatrix}-18 & 4\\ -81 & -18\end{bmatrix}$$

$$(vi)-9^{51}\begin{bmatrix}-9 & 4\\ -81 & 7\end{bmatrix} + 9^{50} * 51\begin{bmatrix}25 & 4\\ -72 & -18\end{bmatrix}$$

(vii) none of the above

Problem 6:

A is a 6×6 matrix with all the eigenvalues equal to $\lambda \neq 0.A^n$ contains terms $\lambda^n, n\lambda^{n-1}, n^2\lambda^{n-2}, n^3\lambda^{n-3}, n^4\lambda^{n-4}$. Which is the following Jordan forms is possible for A?

$$\begin{bmatrix}
\lambda & 1 & 0 & 0 & 0 \\
0 & \lambda & 1 & 0 & 0 & 0 \\
0 & 0 & \lambda & 1 & 0 & 0 \\
0 & 0 & 0 & \lambda & 1 & 0 \\
0 & 0 & 0 & 0 & \lambda & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda
\end{bmatrix}$$

(ii)
$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(iii)
$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(iv)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(v)

| гλ | 1 | 0 | 0 | 0 | 01 | |
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| ١ | 0 | 1 | 1 | | ۸I | |
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| 0 | 0 | 0 | 0 | λ | 1 | |
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(vi) none of the above

Problem 7:

Given a symmetric nonsingular matrix $A \times 3$, and a vector $b \in R^3$, consider that the linear system: Ax = b has condition number $A^2x = b$, is:

- *(i)* 2
- (ii) 3
- (iii) 6
- (iv) 4
- (v) 9
- (vi) none of the above

Problem 8:

In order to solve the linear system Ax = b, where $: A \ n \times n$, and $b \in R^n$, A symmetric and positive definite, we use the iteration: $x_{k+1} = x_k - \epsilon(A^T A x_k - A^T b)$. In order to have convergence to the solution of the linear system, for any initial condition, we need to have $: 0 < \epsilon < \frac{2}{8}$, where δ is:

- (i) $||A^{-1}||$, (where ||.|| is the usual sup norm)
- (ii) $\|A^2\|'$, (where $\|.\|$ is the usual sup norm)

(iii)

- (iv) $|\lambda_{min}(A)|$
- (v) $|\lambda_{max}(A)|$
- (vi) none of the above

Problem 9:

Let A and B be two 3×3 matrices with eigenvalues 1,2,3 and -1,-2,3, respectively. Then their Kronecker product has trace equal to:

- (i) ()
- (ii) 36
- (iii) -36
- (iv) -12
- (v) 12
- (vi) none of the above

<u>Problem 10:</u>

It holds for a real matrix $M, n \times n, M^3 \neq 1, Mv \neq v, v \in R^3, v \neq zero$ vector The minimal polynomial of M is:

ANSWER:

(i)
$$\lambda^3 - 1$$

(ii)
$$\lambda^2 + \lambda + 1$$

(iii)
$$\lambda^2 - \lambda + 1$$

- (iv) $\lambda + 1$
- (v) $\lambda 1$
- (vi) none of the above



$$(\lambda - 1) (\lambda^2 + \lambda + 1)$$

$$= 0$$