EE 810. Home work # F

Name. Onkar Vively Apte 1

5.60	(1)
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1.	If B is similar to A and C is similar to B, show that C is similar to A. (Let $B = M^{-1}AM$
	and $C = N^{-1}BN$.) Which matrices are similar to I?

•:	В	is	Similar	tо	A	:	В	=	Μſ	1 AM
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let Y be any non singular matrix.

5,	6(3)
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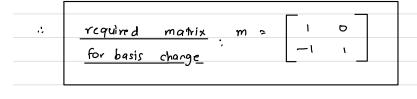
3/0(3)
3. Explain why A is never similar to $A + I$.
Consider B is similar to A.
: B= M ⁻¹ AM such that A&B have same λ_s .
: As of A will be given by $A-\lambda I = 0$
$\lambda = \underline{\lambda_1, \lambda_2 \dots \lambda_n} \qquad \bigcirc$
Noω Consider A+I.
· (A+1)~λI = 0
from () 4 (2), we see that eigenvalues of A 4 A+I Con never be the same.
: A 4 A+I ore never similar

5.6(8)

8. What matrix M changes the basis $V_1 = (1,1)$, $V_2 = (1,4)$ to the basis $v_1 = (2,5)$, $v_2 = (1,4)$? The columns of M come from expressing V_1 and V_2 as combinations $\sum m_{ij}v_i$ of the v's.

$$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
 $V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ $v_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

: here,
$$V_2 = V_2$$





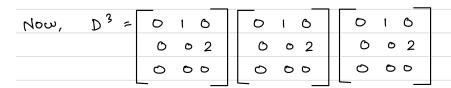
5.6(13)

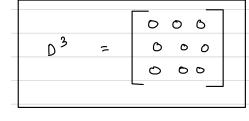
- 13. The derivative of $a + bx + cx^2$ is $b + 2cx + 0x^2$.
 - (a) Write the 3 by 3 matrix D such that

$$D \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} b \\ 2c \\ 0 \end{bmatrix}.$$

- (b) Compute D^3 and interpret the results in terms of derivatives.
- (c) What are the eigenvalues and eigenvectors of D?

Writing b, 2c 40 as linear combinations of a, b, c
will give us matrix D.





• D is a differentiation matrix.
• Multiplying D³ by a 3×1 vector will give zero vector since it is equivalent to differentiating a polynomial of degree 2 three times.

D-IA	=	- A	1	0	
,		0	٨~	2	
		0	D	-1	

$$= (-\lambda)^3$$

equating this with O gives

$$\lambda_1 = 0$$

$$\lambda_2 = 0$$

$$\lambda_3 = 0$$

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5-6(15)

15. On the space of 2 by 2 matrices, let T be the transformation that *transposes every matrix*. Find the eigenvalues and "eigenmatrices" for $A^{T} = \lambda A$.

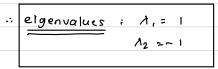
.. T is the transformation matrix.



 $TA = A^T = \lambda A$.

- .: a= da b= dc d= dd c= db
- $\lambda^2 b b = 0 \qquad \qquad b(\lambda^2 1) = 0$
 - c = 16

- : a=0 → A=1
 - b=0 → 1=+1,-1
 - d=0 → A=1
 - c = Ab



for $\lambda = 1$ eigen matrices:

Span { [] 0], [] 0]

for $\lambda = -1$ Span { [] 0]

5.6	(23)
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23. If A has eigenvalues 0, 1, 2, what are the eigenvalues of $A(A-I)(A-2I)$?								
Charactustic polynomial for A(A-I)(A-2I):								
P(d) = A(A-1)(A-2)								
However,								
using cayley-hamilton theorm,								
P(A) = A (A-I) (A-21)								
= 0								
\therefore eigenvalues of $A(A-1)(A-21)$ are 0,0,0								

5,6 (29)

29. Compute A^{10} and e^{A} if $A = MJM^{-1}$:

$$A = \begin{bmatrix} 14 & 9 \\ -16 & -10 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}.$$

$$A^{10} = (MJM')^{10} \qquad J^{10} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= M J^{10} M^{-1}$$

$$= \begin{bmatrix} 3 - 2 \\ -4 & 3 \end{bmatrix} \cdot 2^{10} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2^{10} & 2 \cdot 5 \cdot 2^{9} \\ 2^{10} & 2 \cdot 5 \cdot 2^{9} \end{bmatrix}$$

$$= 2^{10} \begin{bmatrix} 3 & 13 \\ -4 & -17 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

: Now,
$$e^{\Lambda} = M e^{5} M^{-1} = \begin{bmatrix} 3 & -2 \\ -43 \end{bmatrix} \begin{bmatrix} e^{2} & i \cdot e^{2} \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= e^{2} \begin{bmatrix} 3 & 1 & 3 & 2 \\ & -4 & -1 & 4 & 3 \end{bmatrix}$$

$$\therefore \frac{A^{10}}{} = 2^{10} \begin{bmatrix} 61 & 45 \\ -80 & -59 \end{bmatrix} & e \frac{e^{A}}{} = e^{2} \begin{bmatrix} 13 & 9 \\ -16 & -11 \end{bmatrix}$$

38. These Jordan matrices have eigenvalues 0, 0, 0, 0. They have two eigenvectors (find them). But the block sizes don't match and *J* is not similar to *K*:

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{bmatrix}.$$

For any matrix M, compare JM with MK. If they are equal, show that M is not invertible. Then $M^{-1}JM = K$ is impossible.

consider	M =	a,,	912	913	914
		azı	a_{ii}	a 23	924
		Q 31	az	a 33	934
		aqı	a 42	a 43	944

Now, to compare JM & Mk,

	_				 _		
JW=	б	1	0	6	a,,	912	913
	0	D	Ö	6	azı	a_{ii}	a 23
	٥	0	O	1	Q 51	a ₁₂ a ₂₁ a ₃₂	a 33
	0	0	0	٥	aqı	a 4 2	a 43

MK	ľı	a,,	912	913	914	
		α_{z_1}	a_{11}	a 23	924	
		Q 31	_	a 33	934	
		a ₄₁	a 42	a 43	944	

0	1	0	ь	
0	D	1	6	
۵	0	O	0	
0	0	0	٥	

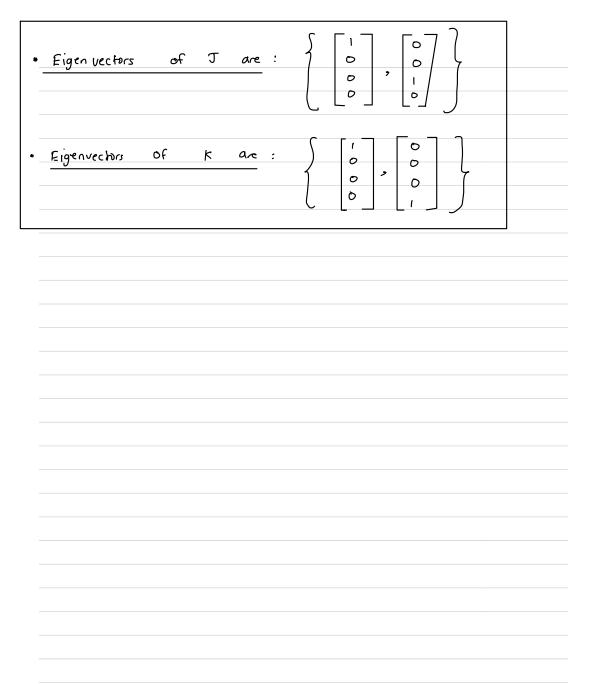
	_			-
=	0	912	913	0
	0	a_{11}	a 23	0
	0	a ,,	a 33	Б
	0	a 4 2	a 43	0

Now, assume JM = MK. $\therefore a_2 = a_{24}$ $a_{21} = a_{44}$ $a_{21} = a_{22}$ $a_{41} = a_{41}$ $a_{11} = a_{21}$ $a_{23} = a_{12}$ $a_{31} = a_{42}$ $a_{42} = a_{32}$

∴ M =	0	912	a 13	914	: 1st colomn = zero,
	0	0	a 23	0	M =0, matrix is singular.
	0	az	a 33	934	. M is not invertible.
	Q	0	a 43	0	J & K are not simlar.
			~ 43		J & K are not simlar.

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MJM-1= k is not possible.



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- **42.** Prove that AB has the same eigenvalues as BA.
 - tet AB = X
 - 4 BA = Y
 - we con with X = IX
 - = B-1B X
 - = B-1 B A B
 - = B (BA) B
 - X = B -1 (Y)B

. AB 4 BA are similar.

: , x & x are similar

i AB & BA have same eigenvalues.

Review 5.13

- **5.13** (a) Show that the matrix differential equation dX/dt = AX + XB has the solution $X(t) = e^{At}X(0)e^{Bt}$.
 - (b) Prove that the solutions of dX/dt = AX XA keep the same eigenvalues for all time.
- a To prove: dx = Ax + xB
 - $\therefore \chi(t) = 6 \times \chi(0) e^{8t}$
 - $\frac{1}{2} \frac{dx}{dt} = A \underbrace{e^{At} \times (0) e^{Bt}}_{x} + \underbrace{e^{At} \times (0) e^{Bt}}_{x} \cdot B$
 - ·· dx = Ax +xB
- (b) from the above result, we can conclude,
 - if dx = Ax xA,
 - x(4) = eAt x(0)eAt
 - : $X(+) = e^{A+} \times (0)(e^{A+})^{-1} (in the form A = B C B^{-1})$
 - ., X(t) 4 X(0) are similar matrix.
 - for all value, of to

5.15 Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 0 & -i & 0 \\ i & 1 & i \\ 0 & -i & 0 \end{bmatrix}.$$

What property do you expect for the eigenvectors, and is it true?

Now,
$$A - \lambda I = 0$$

$$-\lambda^3 + \lambda^2 + 2\lambda = 0 \qquad \Rightarrow \quad \lambda_1 = 0$$

for
$$\lambda = 0$$
,
$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

for
$$d = -1$$
; $\begin{bmatrix} 1 & -i & 0 & x \\ i & 2 & i & y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solving this gives:
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$
, say x_z

For
$$A=2$$
: $\begin{bmatrix} -2 & -1 & 0 \\ i & -1 & i \\ 0 & -i & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Solving this gives. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2i \\ 1 \end{bmatrix}$ say x_3

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2i \\ 1 \end{bmatrix}$$

Solving this gives. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2i \\ 1 \end{bmatrix}$

Say x_3

$$\begin{bmatrix} x \\ 1 & i \end{bmatrix} = \begin{bmatrix} 0 & -i & 0 \\ 0 & i & 1 \end{bmatrix}$$

Each other.

A $\begin{bmatrix} x_1 & x_2 & = & -1 \\ 1 & i & -i \\ 1 & i \end{bmatrix} = \begin{bmatrix} -1/(i) + O(i) + (i)(i) \\ -i & -i & -i \\ 1 & i \end{bmatrix} = \begin{bmatrix} -1/(i) + (i)(2i) + (i)(i) \\ -i & -i & -i \\ 1 & -i \end{bmatrix} = \begin{bmatrix} -1/(i) + (i)(2i) + (i)(i) \\ -i & -i & -i \\ 1 & -i & -i \end{bmatrix} = \begin{bmatrix} -1/(i) + (i)(2i) + (i)(i) \\ -i & -i & -i \\ 1 & -i & -i \end{bmatrix} = \begin{bmatrix} -1/(i) + (i)(2i) + (i)(i) \\ -i & -i & -i \\ 1 & -i & -i \end{bmatrix} = \begin{bmatrix} -1/(i) + (i)(2i) + (i)(i) \\ -i & -i & -i \\ 1 & -i & -i \\ 1 & -i & -i \end{bmatrix} = \begin{bmatrix} -1/(i) + (i)(2i) + (i)(i) \\ -i & -i & -i \\ 1 & -i & -i & -i \\ 1 & -i & -i \\ 1 & -i & -i \\ 1 & -i & -i & -i \\ 2i & -i & -i \\ 2i & -i & -i$

& Yes, a show above, it is True for the given matrix A.

5.30 What is the limit as
$$k \to \infty$$
 (the Markov steady state) of $\begin{bmatrix} .4 & .3 \\ .6 & .7 \end{bmatrix}^k \begin{bmatrix} a \\ b \end{bmatrix}$?

$$A = \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} \qquad (A) = 0.1 \neq D$$

$$Sum of Colourns = 1$$

.: A is a regular stochastic Mx,
.. steady state vector X is unique.

Let x be eigenvector corresponding to eigenvalue 1.

(A - I) x = 0

$$\begin{bmatrix} -0.6 & 0.3 & \chi_1 \\ 0.6 & -0.3 & \chi_2 & 0 \end{bmatrix}$$

$$\begin{array}{cccc}
 & \chi_1 + \chi_2 & = 0 \\
 & \chi_2 & = 2\chi_1
\end{array} \qquad \Rightarrow \qquad \begin{array}{c}
 & \chi & = & \frac{1}{3} \\
 & \frac{2}{3}
\end{array}$$

$$\lim_{k \to \infty} \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix}^{k} \begin{bmatrix} q \\ b \end{bmatrix} = (a + b) \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$$
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