

EE 510, Spring 2021, GPP
MIDTERM 2
Tuesday, April 6th 2021, 11:00-12:45

ALL PROBLEMS COUNT EQUALLY

Problem 1:

Given a 3x3 matrix A , with three distinct eigenvalues, the $\exp(At)$ may contain all of the following terms:

(i) $e^{-t}, e^{-2t}, t^2 e^{+t}$

(ii) t, t^2, t^3

(iii) $e^t, e^{2t}, e^{3t}, e^{-t}$

(iv) $e^t, e^{-t}, 1$

(v) e^t, e^{-t}, te^t

(vi) none of the above.

.ANSWER:

- (i)
- (ii)
- (iii)
- (iv)
- (v)
- (vi)

Problem 2:

A 3x3 matrix A is equal to $A=U \begin{bmatrix} 1 & \sqrt{2} & 0 \\ 3 & 3\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix} U'$, $UU' = I$. (U' means transpose of U). Its singular values are:

(i) $\sqrt{2}, \sqrt{2}, 2$

(ii) $1, \sqrt{2}, 2$

(iii) $2, \sqrt{2}, 2 + \sqrt{2}$

(iv) $1, 1, \sqrt{3}$

(v) $0, 2, \sqrt{30}$

(vi) *none of the above*

ANSWER:

(i)

(ii)

(iii)

(iv)

(v)

(vi)

Problem 3:

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

$A^4 = \alpha_1 A + \alpha_2 I$. The (α_1, α_2) pair is equal to:

(i) $(4, 3)$

(ii) $(16, 25)$

(iii) $(20, 21)$

(iv) $(4, -2)$

(v) $(16, 4)$

(vi) *none of the above*

ANSWER:

- (i)
- (ii)
- (iii)
- (iv)
- (v)
- (vi)

Problem 4:

A 3x3 real matrix A is symmetric ($A=A'$). Which of the following are possible eigenvalues of A ?

| <i>Case i</i> | <i>Case ii</i> | <i>Case iii</i> | <i>Case iv</i> | <i>Case v</i> |
|---------------|----------------|-----------------|----------------|---------------|
| $1, -1, 1+j$ | $1, 1, 1-j$ | $2, 1-j, 1+j$ | $1, 2, 3, 4$ | $1, 1, 0$ |

(vi) *none of the above*

ANSWER:

- (i)
- (ii)
- (iii)
- (iv)
- (v)
- (vi)

Problem 5:

Given a 2x2 real symmetric matrix A , with eigenvalues 1 and 2; which of the following can be eigenvectors of A corresponding to 1 and 2 respectively:

| Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|---|--|--|---|--|
| $\begin{bmatrix} 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 8 \\ -7 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ | $\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ |

(v) none of the above

ANSWER:

- (i)
- (ii)
- (iii)
- (iv)
- (v)
- (vi)

Problem 6:

$$A = \begin{bmatrix} 4 & 5 & 3 \\ 1 & 2 & 0 \\ 5 & 5 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Consider the column spaces of A and B , A_c and B_c in R^3 respectively, and the row spaces of A and B , A_r and B_r in R^3 respectively. Which of the following is true?

- (i) $A_c = B_c$
- (ii) $A_c = B_r$
- (iii) $A_r = B_r$
- (iv) $B_c = B_r$
- (v) $A_r = B_c$
- (vi) none of the above

ANSWER:

- (i)
- (ii)
- (iii)
- (iv)
- (v)
- (vi)

Problem 7:

$$A = T \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} T^{-1}, \quad T \text{ is } 2 \times 2, \text{ invertible. } e^{5A} = ?$$

(i) $e^{10}[-9I - 5A]$

(ii) $e^{10}[-9I + 5A]$

(iii) $e^2[+9I + 5A]$

(iv) $e^{10}[9I + 5A]$

(v) $2^5[-9I + 5A]$

(vi) none of the above

ANSWER:

- (i)
- (ii)
- (iii)
- (iv)
- (v)
- (vi)

Problem 8:

A is a 3x3 matrix with eigenvalues: a, 1 and 2. If:

$$\det[I - A^3(A - I)(A - 2I)^2 - A^2(I - A)(A - 2I)^3] = -35$$

Then

(i) $a=2$

(ii) $a=3$

(iii) $a=8$

(iv) $a=0$

(v) $a=-1$

(vi) none of the above

ANSWER:

(i)

(ii)

(iii)

(iv)

(v)

(vi)

Problem 9:

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & a & 2 \\ 0 & 1/3 & 1/a \end{bmatrix} \quad x'Ax \text{ is positive for all } x \neq 0 \text{ vector, if "a" equals:}$$

(x' is the transpose of x)

(i) -5, (ii) 40, (iii) 1, (iv) any a positive, (v) 6, (vi) none of the above

ANSWER:

(i)

(ii)

(iii)

(iv)

(v)

(vi)

Problem 10:

Calculate the pseudoinverse of

$$A = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$(i) \begin{bmatrix} \sqrt{2} & 0 \\ +\sqrt{2} & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(ii) \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{3} \\ 0 & \frac{1}{3} \\ \frac{-1}{\sqrt{2}} & \frac{-1}{3} \\ 0 & 0 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ \sqrt{2} & 0 \\ 1 & \sqrt{2} \end{bmatrix}$$

$$(iv) \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 \\ 0 & 1 \\ \frac{1}{\sqrt{2}} & -1 \\ 0 & 0 \end{bmatrix}$$

$$(v) \begin{bmatrix} \frac{1}{3} & \frac{1}{\sqrt{2}} \\ \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{bmatrix}$$

(vi) *none of the above*

ANSWER:

(i)

(ii)

(iii)

(iv)

(v)

(vi)