EE510, Spring 2020, GPP
FINAL
Tuesday, May 12th 2020

NAME:

USC ID:

SCORE

Problem / Answer	Case i	Case ii	Case iii	Case iv	Case v
1					
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13					

Problem 1:

For, $n \times n$ *matrices A and B, which of the following is always true?*

(i) $AB = B \quad then \;\; A = I$

(ii) $(AB)^2 = A^2B^2$

(iii) $A^2 + A = I \quad then \; A^{-1} = I + A$

(iv)A is a matrix $, n \times n \, , a_{11} = a_{22} = \cdots = a_{nn} = 0 \;\; implies \; A \; is \; singular$

(v) $I f$ *the rows 1 and 3 of B are the same, so are the rows 1 and 3 of AB*

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

Problem 2:

For which vector "b" both of the following systems are solvable

$$\begin{bmatrix} 1 & 4 & 1 \\ 2 & 8 & 1 \\ -1 & -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = b \qquad , \qquad \begin{bmatrix} 0 & 2 \\ 0 & 4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = b$$

(i) $b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(ii) $b = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

(iii) $b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(iv) $b = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$

(v) $b = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

Problem 3:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Consider the column spaces of A and B, A_c and B_c *in* R^3 *respectively , and the raw spaces of A and B,* A_r and B_r *in* R^3 *respectively . Which of the following is true?*

(i) $A_c = B_c$

(ii) $A_c = A_r$

(iii) $A_c = B_r$

(iv) $B_c = B_r$

(v) $B_c = A_r$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)

(iv)
(v)
(vi)

Problem 4:

What is the matrix P that projects every point in R^n, *onto the intersection of the planes*

$x + y + z = 0$ *and* $x + 2z = 0$ *?*

(i)$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(ii)$P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(iii) $P = \frac{1}{\sqrt{6}} \begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$

(iv)$P = \frac{1}{6} \begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

(v)$P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

Problem 5:

$$A = \begin{bmatrix} 15 & 9 \\ -16 & -9 \end{bmatrix} \ , \ A^{100} = ?$$

(i)$3^{100}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 3^{99}100\begin{bmatrix} 12 & 9 \\ -16 & -12 \end{bmatrix}$

(ii) $3^{99}\begin{bmatrix} 6 & 3 \\ 10 & -5 \end{bmatrix} + 3^{100}\begin{bmatrix} 15 & 9 \\ -16 & 7 \end{bmatrix}$

(iii)$3^{100}\begin{bmatrix} -6 & -3 \\ 10 & -5 \end{bmatrix} + 3^{98}\begin{bmatrix} 5 & 3 \\ 10 & 6 \end{bmatrix}$

(iv)$3^{101}\begin{bmatrix} -4 & 3 \\ -10 & 7 \end{bmatrix} + 3^{100}\begin{bmatrix} -5 & 3 \\ 10 & -6 \end{bmatrix}$

(v)$\begin{bmatrix} -4 & 3 \\ -10 & 7 \end{bmatrix} + 3^{100}\begin{bmatrix} 7 & 3 \\ -10 & -4 \end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 6:</u>

$$A = \begin{bmatrix} 15 & 9 \\ -16 & -9 \end{bmatrix}, \ e^{2A} = ?$$

(i)$e^3\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$

(ii)$e^3\begin{bmatrix} 15 & 9 \\ -16 & -9 \end{bmatrix}$

(iii)$e^6\begin{bmatrix} -23 & -16 \\ 18 & 12 \end{bmatrix}$

(iv) $e^6\begin{bmatrix} 25 & 18 \\ -32 & -23 \end{bmatrix}$

(v) $3^2\begin{bmatrix} 13 & 18 \\ -16 & -23 \end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 7:</u>

Calculate the pseudoinverse of

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 1 & 1 & -1 & 0 \end{bmatrix}$$

(i) $\begin{bmatrix} \sqrt{2} & 0 \\ -\sqrt{2} & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{3} \\ 0 & \frac{1}{3} \\ \frac{1}{\sqrt{2}} & \frac{-1}{3} \\ 0 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \sqrt{2} & 0 \\ 1 & \sqrt{2} \end{bmatrix}$

(iv) $\begin{bmatrix} \frac{1}{\sqrt{2}} & 1 \\ 0 & 1 \\ \frac{1}{\sqrt{2}} & -1 \\ 0 & 0 \end{bmatrix}$

(v) $\begin{bmatrix} \frac{1}{3} & \frac{1}{\sqrt{2}} \\ \frac{1}{3} & 0 \\ \frac{-1}{3} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 8:</u>

A is a 3x3 matrix with eigenvalues 0, 1 and 2. Then

$$\det\left[A^2(A-I)(A-2I)^2 + A(I-A)(A-2I)^3\right] = ?$$

(i) 2

(ii)4

(iii)8

(iv)0

(v)3

(vi) none of the above

ANSWER:

(i)

(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 9:</u>

Find the largest possible number of linearly independent vectors among

$$\begin{bmatrix} 2 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

(i) 5

(ii) 4

(iii) 3

(iv) 2

(v) 6

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 10:</u>

$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 6 & a \\ 0 & a & -1 \end{bmatrix}$ *is positive definite for "a" equal to:*

(i) 5, (ii) 4, (iii) 3, (iv) 2, (v) 6, (vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 11:</u>

Given an mxn nonsquare matrix A, what are the dimensions of the matrices associated with the SVD decomposition of A ($A = U\Sigma V^*$)?

(i) U is m×n, Σ is n×n, V is n×n,

(ii) U is m×n, Σ is n×m, V is n×m,

(iii) U is m×n, Σ is n×n, V is n×n,

(iv) U is m×m, Σ is m×n, V is n×n,

(v) U is m×m, Σ is m×m, V is n×m,

(vi) none of the above

ANSWER:
(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 12:</u>

A box has edges from (0,0,0) to (2,1,1), (1,2,1), (1,1,3). The volume is equal to:

(i) 4
(ii)6
(iii)7
(iv)13
(v)12
(vi)none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

Problem 13:

A is a 6×6 *matrix with all the eigenvalues equal to* $\lambda \neq 0.A^n$ *contains terms* λ^n,$n\lambda^{n-1}$,$n^2\lambda^{n-2}$, $n^3\lambda^{n-3}$ *.Which is the following Jordan forms is possible for A?*

(i)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(ii)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(iii)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(iv)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(v)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)