EE 810. Home work #2

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DBO ID .

1.4 (42)

- (a) If A^2 is defined then A is necessarily square.
- (b) If AB and BA are defined then A and B are square.
- (c) If AB and BA are defined then AB and BA are square.
- (d) If AB = B then A = I.

TRUE Assume

with dimention mxn.

(m×n) x (m×n)

must

for this multiplication valid, to be equal

 $A \times A = A^2$

(mxm) x (mxn) = (mxm)

any thing.

Square

Matrix

FALSE

Assume

: A · B = AB(mxq) (mxn) (nxg)

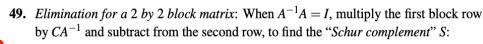
However, if BA also exist, then

- B A

 $(n \times q) \cdot (m \times n)$ $(n \times n)$ For BA to valid, q = m.

.: Possible dimension of A 4 Bor Amra & Brxm for both AB 4 BA to exist, However man can be

© TRUE	As proven in part (B) of this question,
	if AB&BA exist, then A: dim mxn
	B : dim nxm.
	: A·B = AB
	men num mem
	∴ B·A = BA -> square matrix
	NXM MXM OXN
(D) False	IF AB=B, does not have to I
	in all the cases.
	If B is a matrix with all zeros, then
	A can be anything & AB will Still
	equal B; i.e. matrix with zeros.
	4.0.



$$\begin{bmatrix} I & 0 \\ -CA^{-1} & I \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & S \end{bmatrix}.$$

given A"A = I

$$S = -CA'B+D$$

1.5 C1)

1.	When is an upper triangular matrix nonsingular (a full set of pivots)?
••	whom is an apper triangular matrix nonsingular (a run set of privots).

o d e	Consider	Upper	triongular	matrix	A =	a	Ь	_ c
0 D f		, ,	. ,					e
						0	0	f

l a

0 de > 0	
0 0 f	

402 - 0				
	_ Mear	15 (atica	57
	One	٥f	the	element
	a	0		



i.e. it has a full set of pivot, if a only if all the diagonal elements are non-zero

1.5(11)

11. Solve as two triangular systems, without multiplying LU to find A:

$$LUx = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}.$$

$$L \quad V \quad n \quad = b \quad ---- \text{ (given)}$$

• Say,
$$Lc = b$$
 where $C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 4 & 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence,

$$2U+4V+4W=2 : U=5$$

$$V+2W=-2 \longrightarrow V=-2 \longrightarrow$$

$$W=0 : W=0$$



	•						_
(I)	Representing	the given	three	egy ations	10	a	matrix form.
\sim	•	<i>U</i> .					

1	4	2	_ _2
-2	-8	3	32
0	ı	1	1

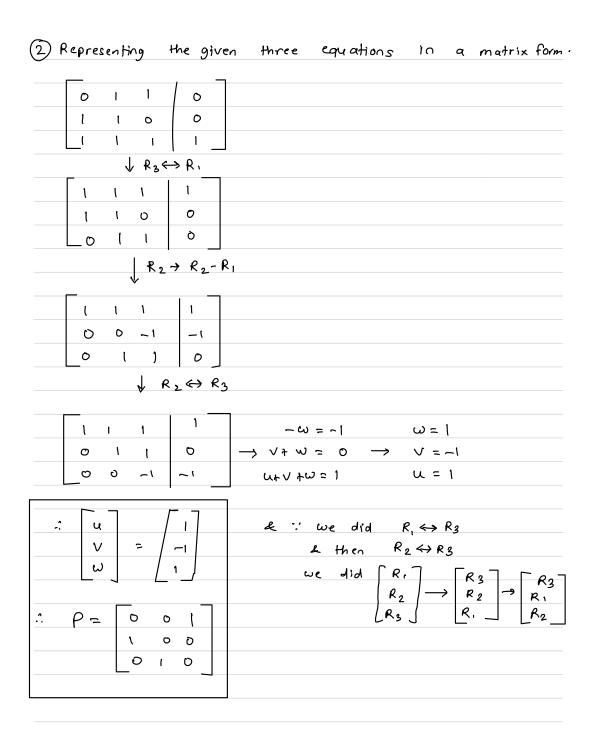
$$R_2 \rightarrow R_2 + 2R_1$$

$$R_2 \leftrightarrow R_3$$

1. U=2

→ *· V = -3

$$\begin{bmatrix} 0 \\ V \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} \qquad \begin{cases} 4 & \text{if we did} \\ R_2 \leftrightarrow R_s, \end{cases} \qquad \begin{cases} 0 & \text{if } 0 & \text{if } 0 \\ 0 & \text{if } 0 \end{cases}$$



1.6(2)

(a) Find the inverses of the permutation matrices

$P = [0 \ 1 \ 0]$	o and	$P = 1 \ 0 \ 0$	· -
1 0 (0	0 1 0	
L	_	L	_

(de+(A) =

> = -1 ... Pis non-singular - P-1 exists.

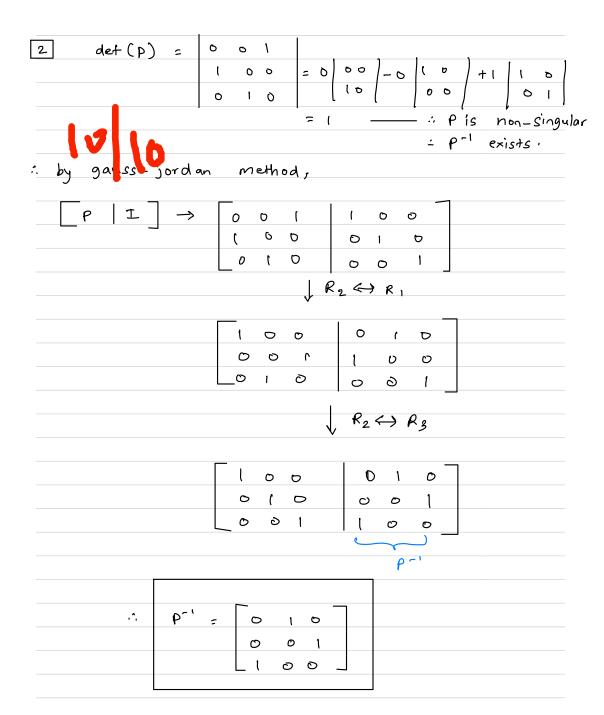
by gauss-jordan method,

١ 0 0

 $\downarrow R_3 \leftrightarrow R_1$

O O \circ 0 1 0 1 0 \circ

P-1



- (b) Explain for permutations why P^{-1} is always the same as P^{T} . Show that the 1s are in the right places to give $PP^{T} = I$.
 - · What permutation do is to exchanges rows of a matrix.
 - · Inverse of a matrix always undo the transformation. · for permutations, P-1 should un-exchange the rows exchanged by P.
 - · consider :

$$P_{1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P_{2} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P_1^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$
 $P_2^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

but
$$P_1^{T} = \begin{cases} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{cases}$$
 $P_2^{T} = \begin{cases} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{cases}$

give pp+= I

1.6(10)

10. Find the inverses (in any legal way) of

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}.$$

U using gauss - Jordan method,

A, II =	0001	1000	
	0020	0 0 0 0	
	0300	0 0 1 0	
	4000	0001	

 $\begin{array}{c|c}
R_{2} = R_{2}/2 \\
R_{3} = R_{3}/3 \\
R_{4} = R_{4}/4
\end{array}$

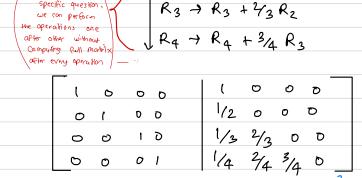
0001	(000)
0010	0 1/2 0 0
0 1 0 0	00130
0001	000/4

 $\begin{array}{c} R_1 \longleftrightarrow R_4 \\ R_2 \longleftrightarrow R_3 \end{array}$

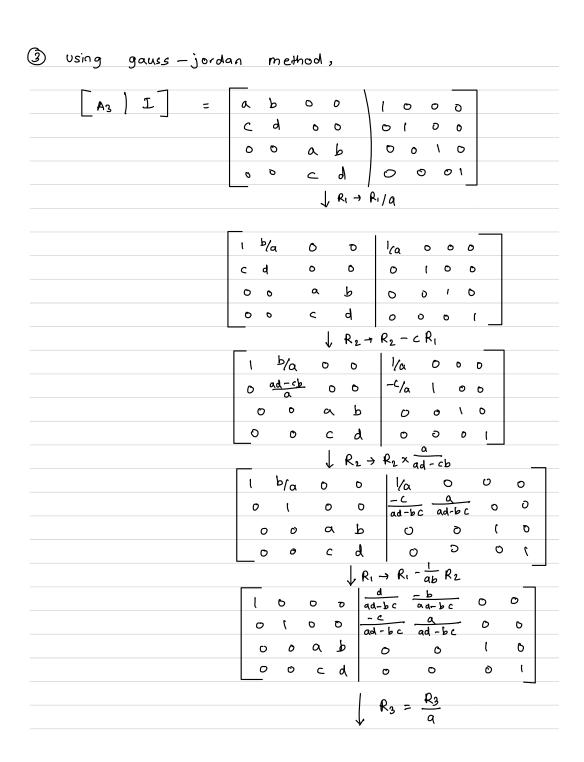
$$A_{1}^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1/4 \\ 0 & 0 & 1/3 & 0 \\ 0 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} A_2 & I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

only for this \ specific question,



 $R_2 \rightarrow R_2 + \frac{1}{2}R_1$



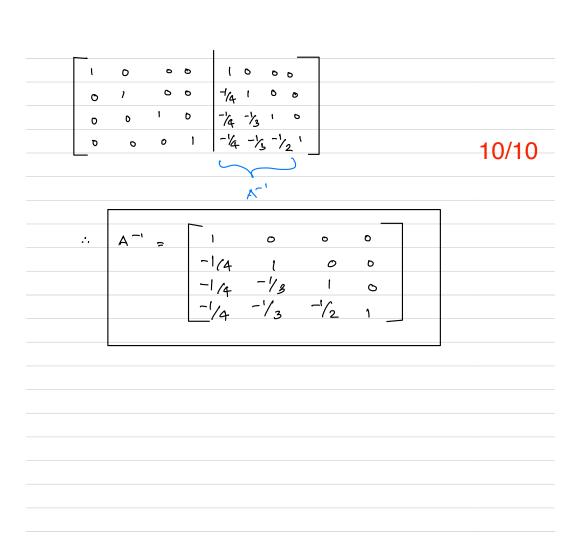
A3-1

		_	^	
١	1	6	(20)	

20. Find the inverse of

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 1 \end{bmatrix}$$

using gauss-jordan	method,
[A I] =	1/4 , 00 6100
	1/3 1/3 1 0 00 1 0
	$R_{2} \rightarrow R_{2} \sim \frac{R_{1}/4}{R_{5} \rightarrow R_{3} - R_{4}/3}$ $R_{4} \rightarrow R_{4} - R_{1}/2$
	10000
	0 1/3 1 0 -1/3 0 1 0
	$\begin{array}{c} R_4 \rightarrow R_4 - R_2/2 \\ R_5 \rightarrow R_3 - R_2/3 \end{array}$
	0 1 0 0 74 1 0 0
	0 0 1 0 -14-13 1 0
	/ R4 -> R4 - R3/2



		_	$\overline{}$
		/ b	- 1
Α.	6 1	4	COL
1,	_	$r \rightarrow$	\sim $_{\prime}$



- **40.** True or false (with a counterexample if false and a reason if true):
 - (a) A 4 by 4 matrix with a row of zeros is not invertible.
 - (b) A matrix with Is down the main diagonal is invertible.
 - (c) If A is invertible then A^{-1} is invertible.
 - (d) If A^{T} is invertible then A is invertible.

TRUE if 4x4 matrix has row of zeros,

its determinant will be zero.

it means it is a singular matrix.

Inverse of singular matrix is

not possible.

. It is not invertible.

B FALSE

Consider a simple modern

M = (1

 $de+(M) = 0 \longrightarrow singular.$

1. inverse NOT possible

C TRUE

· A is invertible, At exist.

A-1 A = I

but also $\rightarrow AA^{-1} = \pm$

This mean inverse

6-11-1

1.13 Solve Ax = b by solving the triangular systems Lc = b and Ux = c:

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

What part of A^{-1} have you found, with this particular b?

Let	<mark>አ</mark> ዶ	2,	L	د ے	[]	
		22			C ₂	
		×3 _			C3	

$$LC = b$$

$$R_2 \rightarrow R_2 - 4R_1$$

$$R_3 \rightarrow R_3 - R_1$$

<u> </u>	0	0	۲,		0
O	1	D	C2	=	0
0	ō	1	C_3		ŗ

$$C_{3} = 1$$
 $C_{2} = 0$ $C_{1} = 0$

$$\begin{array}{c|cccc} x_1 & & & & & & & \\ x_2 & = & & & & \\ x_3 & & & & & & \\ \end{array}$$

2.
$$2x_1 + 2x_2 + 4x_3 = 0$$

 $x_2 + 3x_3 = 0$

$$x_3 = 0 \longrightarrow x_2 = -3$$

$$\chi_3 = 1 \qquad \qquad \chi_1 = 1$$

Solution of given triangular system is
$$k = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix}$$

- **1.22** (a) If A is invertible what is the inverse of A^{T} ?
 - (b) If A is also symmetric what is the transpose of A^{-1} ?
 - (c) Illustrate both formulas when $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.
- (a) consider A = \[\bar{a} \ \bar{b} \]

 C d
 - $A^{-1} = 1 \qquad \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

can be shown easily using matrix identities

$$How$$
, $A^T = a c$

$$A^{T} \int_{ad-bc}^{a} \left[\begin{array}{cccc} d & -c \\ -b & a \end{array} \right] - C(ii)$$

from equation (i) & (ii), we can clearly see that inverse of AT is nothing but transpose of A-1

$$\therefore (A^T)^{-1} = (A^T)^T - (given, A is invertible)$$

$$A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}; \quad A^{-1} = \begin{bmatrix} 1 \\ ab - c^2 \end{bmatrix} \begin{bmatrix} b - c \\ -c & a \end{bmatrix} \begin{bmatrix} (A^{-1})^{-1} = (A^{-1})^{-1} \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} \alpha & C \\ C & b \end{bmatrix}; A^{-1} = \begin{bmatrix} 1 \\ ab - c^{2} \end{bmatrix} \begin{bmatrix} b & -C \\ -c & \alpha \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$
 $A^{-1} = \frac{1}{2-1} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

$$Now, A^{T} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} (A^{T})^{-1} = \underbrace{1}_{2-1} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$(A^{-1})^{T} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = A^{-1}$$

$$(A^{T})^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = A^{-1}$$

Hence, both formulas have been illustrated.

10/10