EE 510, Spring 2021, GPP
MIDTERM 1
Tuesday, March 2nd 2021, 11:00-12:45

EACH PROBLEM COUNTS FOR 2,5 POINTS

NAME:

USC ID:

Problem 1:

Given the matrix $A=\begin{bmatrix}1 & 0 & 1 & 0\\ 3 & 2 & 3 & 1\\ 2 & 2 & 2 & 1\end{bmatrix}$ *the nullspace N (A) is spanned by:*

Case (i)	*Case(ii)*	*Case (iii)*	*Case (iv)*	*Case (v)*
$\begin{bmatrix}3\\-1\\-2\\1\end{bmatrix},\begin{bmatrix}0\\-1\\1\\1\end{bmatrix}$	$\begin{bmatrix}1\\0\\-1\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$	$\begin{bmatrix}0\\1\\2\\0\\1\end{bmatrix},\begin{bmatrix}0\\0\\1\\3\\1\end{bmatrix}$	$\begin{bmatrix}3\\0\\-1\end{bmatrix},\begin{bmatrix}-1\\0\\2\end{bmatrix}$	$\begin{bmatrix}0\\1\\0\\-3\end{bmatrix},\begin{bmatrix}-2\\0\\1\\0\end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

Problem 2:

Given a 4x5 matrix A ,the column space C(A) can have as basis:

Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)
$\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$	$\begin{bmatrix}1\\1\\-1\end{bmatrix}, \begin{bmatrix}0\\-1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$	$\begin{bmatrix}2\\3\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\3\end{bmatrix},$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

Problem 3:

What is the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{bmatrix}$ *?*

Case (i)	*Case (ii)*	*Case (iii)*	*Case (iv)*	*Case (v)*
3	*2*	*0*	*1*	*4*

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

Problem 4:

What is the rank of the matrix $A=\begin{bmatrix}0 & 0 & 1 & -1 & 0\\ -1 & 2 & 0 & 0 & -1\\ -1 & 0 & 3 & 0 & 2\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & -1\end{bmatrix}$ *?*

Case (i)	*Case (ii)*	*Case (iii)*	*Case (iv)*	*Case (v)*
2	*3*	*4*	*5*	*6*

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 5:</u>

Given are two nonzero four dimensional vectors a and b, where

$$b = \begin{bmatrix} 2 \\ 0 \\ -2 \\ 5 \end{bmatrix}$$

Which of the following vectors could be a projection of "b" on "a"?

Case (i)	*Case (ii)*	*Case (iii)*	*Case (iv)*	*Case (v)*
$\begin{bmatrix} 1 \\ -2 \\ -1 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 1 \\ -3 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -2 \\ -2 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \\ 1 \\ 5 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ -2 \\ 5 \end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 6:</u>

Given A=$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 2 & 1 \\ 0 & 3 & 2 \end{bmatrix}$ *, could any of the following matrices be an inverse of A?*

Case (i)	*Case (ii)*	*Case (iii)*	*Case (iv)*	*Case (v)*
$\begin{bmatrix} 1 & 1/3 & 1 \\ 0 & 1/3 & -1 \\ 0 & -3 & -2 \end{bmatrix}$	$\begin{bmatrix} 1 & 3 & 9 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & -3 \\ 0 & -2 \end{bmatrix}$	*It does not have an inverse*

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 7:</u>

For $n \times n$ *matrices A and B, which of the following is always true?*

(i) $AB = B \quad then \;\; A = I$

(ii) $(AB)^2 = A^2B^2$

(iii) $A^2 - 2A = -I \quad then \; A = I$

(iv) A is a matrix $, n \times n\, , a_{11} = a_{22} = \cdots = a_{nn} =$
$0 \;\; implies \; A \; is \; singular$

(v) I f the rows 1 and 3 of B are the same, so are the rows 1 and 3 of AB

(vi) none of the above

<u>ANSWER:</u>

<u>(i)</u>
<u>(ii)</u>
<u>(iii)</u>
<u>(iv)</u>
<u>(v)</u>
<u>(vi)</u>

Problem 8:

For which of the following vectors "b" ,both of the following systems are solvable for the x's and y's?

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 0 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = b \quad , \quad \begin{bmatrix} 0 & 2 \\ 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = b$$

(i) $b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(ii) $b = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

(iii) $b = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

(iv) $b = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$

(v) $b = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 9:</u>

What is the projection of $b = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ *onto the intersection of the planes*

$x - y - z = 0 \; and \; x + 3z = 0$?

(i)) $0.5 \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$

(ii) $0.5 \begin{bmatrix} 3 \\ -4 \\ -1 \end{bmatrix}$

(iii) $0.5 \begin{bmatrix} -3 \\ 4 \\ -1 \end{bmatrix}$

(iv) $0.5 \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$

(v) $0.5\begin{bmatrix}-3\\-4\\-1\end{bmatrix}$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

<u>Problem 10:</u>

Find the largest possible number of linearly independent vectors among the following vectors.

$$\begin{bmatrix}0\\0\\-2\\2\end{bmatrix},\begin{bmatrix}1\\-1\\0\\0\end{bmatrix},\begin{bmatrix}0\\3\\0\\-3\end{bmatrix},\begin{bmatrix}-1\\0\\1\\0\end{bmatrix},\begin{bmatrix}0\\3\\-3\\0\end{bmatrix},\begin{bmatrix}2\\0\\0\\-2\end{bmatrix}$$

(i) 5

(ii) 4

(iii) 3

(iv) 2

(v) 6

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)