EE 510, Spring 2021, GPP
FINAL
Tuesday, May 11th 2021, 11:00am-1:00pm

ALL PROBLEMS COUNT EQUALLY

<u>Problem 1:</u>

Given A *,*2×2 $matrix$ and $b \in R^2$ *,assume that there is no vector* x $such$ $that$ $Ax = b$ $where$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq 0, A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.
.Then for whatever b this holds, there is a $[y_1, y_2]$ $such$ $that$ $[y_1, y_2]B \geq [0,0], [y_1, y_2]b < 0$, *where the matrix B equals*
.

(i) $\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 3 & -2 \\ -6 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & -2 \\ 4 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 2 \\ 4 & 1 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$

(vi) *none of the above*

ANSWER:

(i)
(ii)
(iii)
(iv) Y
(v)
(vi)

<u>Problem 2:</u>

Given a symmetric nonsingular matrix A $n \times n, and$ a $vector$ $b \in R^n$ *,consider that the linear system:*$Ax = b$ *has condition number 2.Then the condition number of* $A^3x = b$*,is :*

(i) 2
(ii) 3
(iii) 6
(iv) 5
(v) 8
(vi) *none of the above*

ANSWER:

(i)
(ii)
(iii)
(iv)
(v) Y
(vi)

Problem 3:

In order to solve the linear system $Ax = b$,*where :* $A \;\; n \times n, and \;\; b \in R^n$,*A symmetric and positive definite, we use the iteration:*$x_{k+1} = x_k - \epsilon(A^T A\, x_k - A^T b)$ *.In order to have convergence to the solution of the linear system, for any initial condition, we need to have :* $0 < \epsilon < \frac{2}{\delta}$,*where* δ *is:*

(i) $\|A^{-1}\|$ *, (where* $\|.\|$ *is the usual sup norm)*

(ii) $\|\mathrm{A}^2\|$ *, (where* $\|.\|$ *is the usual sup norm)*

(iii) 1

(iv) $|\lambda_{min}(A)|$

(v) $|\lambda_{max}(A)|$

(vi) *none of the above*

ANSWER:

(i)
(ii) Y

(iii)
(iv)
(v)
(vi)

<u>Problem 4:</u>

Let A and B be two 3×3 *matrices with eigenvalues 1,2,3 and -1, 2,-3, respectively. Then their Kronecker product has trace equal to:*

(i) 0
(ii) 36
(iii) -36
(iv) -12
(v) 12
(vi) *none of the above*

ANSWER:

(i)
(ii)
(iii)
(iv) Y
(v)
(vi)

<u>Problem 5:</u>

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 4 & 2 \\ 1 & 1 & 5 \end{bmatrix}$$

The eigenvalues of A are:

(i) 2,1,4
(ii) 3,3,7
(iii) 5,4,4
(iv) 4,4,2
(v) 8,-1,6
(vi) *none of the above*

ANSWER:

(i)
(ii) Y
(iii)
(iv)
(v)
(vi)

<u>Problem 6:</u>

A is a 6×6 *matrix with all the eigenvalues equal to* $\lambda \neq 0.A^n$ *contains terms* $\lambda^n, n\lambda^{n-1}, n^2\lambda^{n-2}, n^3\lambda^{n-3}, n^4\lambda^{n-4}, n^5\lambda^{n-5}$ *.Which is the following Jordan forms is possible for A?*

(i)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(ii)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(iii)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(iv)

$$\begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(v)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi) Y

<u>Problem 7:</u>

$$A = \begin{bmatrix} 1 & b + 1/2 \\ b - 1/2 & 2 \end{bmatrix}$$

The Gauss-Seidel method for solving Ax=c converges to the solution for any initial condition, for "b" equal to:

(i) -3

(ii) -2

(iii) -1.5

(iv) 1.5

(v) 1

(vi) none of the above

ANSWER:

(i)
(ii)
(iii)
(iv)
(v) Y
(vi)

Problem 8:

$\mathrm{A} = \begin{bmatrix} \sqrt{2} & 1 \\ 0 & \sqrt{2} \end{bmatrix}$, $\|\mathrm{A}\| = ?$,

(where $\|.\|$ *is the usual sup norm for matrices and for vectors we use the Euclidean norm)*

ANSWER:

(i) $\sqrt{2}$

(ii) 2
(iii) $2\sqrt{2}$
(iv) 1
(v) $2+\sqrt{2}$
(vi) *none of the above*

ANSWER:

(i)
(ii) Y
(iii)
(iv)
(v)
(vi)

<u>Problem 9:</u>

$$b = \begin{bmatrix}1\\2\\c\end{bmatrix}, A = c\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} + \begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}1 & 2 & c\end{bmatrix}$$

For which value of c is the pair (A,b) controllable?

ANSWER:

(i) 3
(ii) *2*
(iii) *1*
(iv) *4*
(v) *0*
(vi) *none of the above*

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi) Y

Problem 10:

$$V=\begin{bmatrix}1 & x_1^1 & x_1^2 & x_1^3\\ 1 & x_2^1 & x_2^2 & x_2^3\\ 1 & x_3^1 & x_3^2 & x_3^3\\ 1 & x_4^1 & x_4^2 & x_4^3\end{bmatrix}, x_1 = 5, x_2 = \frac{9}{2}, x_3 = 4, x_4 = 3$$

$$|\det(V)| = ?$$

ANSWER:

(i) $\frac{9}{2}$

(ii) $\frac{3}{4}$

(iii) $\mathbf{\frac{5}{3}}$

(iv) $\frac{4}{3}$

(v) *0*

(vi) *none of the above*

ANSWER:

(i)
(ii) Y
(iii)
(iv)
(v)
(vi)