EE510, Sp. 2020, GPP
MIDTERM II
Thursday, April 9th 2020

NAME:

USC ID:

SCORE

Problem / Answer	Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)	Case (vi)	Case (vii)
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							

NOTE: ***ALL PROBLEMS HAVE THE SAME VALUE AND ADD UP TO A TOTAL OF 20pts***

Problem 1:

Given a 3x3 matrix A, the exp(At) may contain all of the following terms:

(i) e^{-t}, e^{-2t}, $t^2 e^{-t}$

(ii) t , t^2, t^3

(iii) e^{t}, e^{2t}, e^{3t}, e^{-t}

(iv) e^{t}, e^{-t}, $t\,e^{t}$, *1*

(v) e^{t}, e^{-t}, te^{t}

(vi) e^{t}, e^{-t}, t

(vii) none of the above.

Problem 2:

A 3x3 matrix A is equal to $A=U\begin{bmatrix}1 & \sqrt{2} & 0\\ 2 & 2\sqrt{2} & 0\\ 0 & 0 & 2\end{bmatrix}U'$, $UU' = I.$ (U' means transpose of U). *Its singular values are:*

(i) $\sqrt{2}$, $\sqrt{2}$, *2*

(ii) *1* , $\sqrt{2}$, *2*

(iii) *1* , $\sqrt{2}$, $2+\sqrt{2}$

(iv) *1* , *1* , $\sqrt{2}$

(v) *1* , *2* , *2*

(vi) *1* , $\sqrt{2}$, $\sqrt{2}$

(vii) none of the above

Problem 3:

The singular values of $A=\begin{bmatrix} 5/2 & 3/2 \\ 3/2 & -3/2 \end{bmatrix}$ *are:*

(i) (3 , 2)

(ii) (5/2 , 3/2)

(iii) (1 , 3/2)

(iv) (5/2 , 3)

(v) (1 , 1.5)

(vi) (5/2 , -3/2)

(vii) none of the above.

Problem 4:

$A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 5 & 4 & 3 \\ 4 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, $C=\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

Which of the following is true ("p.d." stands for positive definite):

(i) A is p.d. , B is p.d. , C is p.d.

(ii) A is not p.d. , B is p.d. , C is p.d.

(iii) A is p.d. , B is not p.d. , C is p.d.

(iv) A is p.d. , B is p.d. , C is not p.d.

(v) A is not p.d. , B is not p.d. , C is not p.d.

(vi) A is p.d. , B is not p.d. , C is not p.d.

(vii) none of the above.

Problem 5:

A is an nxn matrix. $(A' \text{ means transpose of } A)$. *Which of the following is always correct:*

(i) $A\, e^{A.0} = 0$

(ii) $A'\, e^{At} = e^{At} A'$

(iii) $e^{At}\, e^{A't} = e^{(A+A')t}$

(iv) $e^{A}\, e^{A'} = e^{A\,.\,A'}$

(v) $e^{A}\, e^{A^2} = e^{A+A^2}$

(vi) $e^{A}\, e^{A^2} = e^{A^3}$

(vii) none of the above.

Problem 6:

A, B are symmetric positive definite matrices. Which of the following is not necessarily correct:

(i) $A+B$ is p.d.

(ii) $A+B^2$ is p.d.

(iii) ABA is p.d.

(iv) $(A+B)(A+B)$ is p.d.

(v) e^A is p.d.

(vi) $AB + BA$ is p.d.

(vii) none of the above.

Problem 7:

$\dot{x} = Ax$, *A is 3x3 ,* $x(o) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $A = U \begin{bmatrix} -2 & 3 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} U'$, (U' *means transpose U) and*

$$U = \begin{bmatrix} 1/\sqrt{3} & 2/\sqrt{8} & 1/\sqrt{6} \\ -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & -2/\sqrt{8} & 1/\sqrt{6} \end{bmatrix}.$$ *As* $t \to \infty$ *x(t) goes to:*

(i) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} (1/3)$

(ii) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$*(1/3)*

(iii) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$*(1/3*

(iv) $\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$*(1/3)*

(v) $\begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$*(1/3)*

(vi) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$*(1/3)*

(vii) none of the above.

<u>Problem 8:</u>

$$A = \begin{bmatrix} 1 & b + 1/2 \\ b - 1/2 & 2 \end{bmatrix}$$

The Jacobi method for solving Ax=c converges to the solution for "b" equal to:

(i) -3

(ii) -2

(iii) -1.5

(iv) 1.5

(v) 1.15

(vi) 2.3

(vii) none of the above.

<u>Problem 9:</u>

$$A = \begin{bmatrix} 1 & \sqrt{5} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

($\|A\| = max_{\|x\|\neq 0} \frac{\|A\|}{\|x\|}$ *, we use the norm:* $\left\|\begin{matrix} y_1 \\ y_2 \end{matrix}\right\| = \sqrt{{y_1}^2 + {y_2}^2}$*))*
The norm of A is:

(i) 1

(ii) 2

(iii) 5/2

(iv) 5

(v) 5+$\sqrt{5}$

(vi) $\sqrt{5}$

(vii) none of the above.

<u>Problem 10:</u>

A, B are nxn matrices, are diagonalizable and have the same eigenvectors. $(A'$ and B', are the transposes of A and B). *Which one is not true:*

(i) A (B+3 I) = (B + 3 I) A

(ii) A (B+B^2*) = (B+*B^2*) A*

(iii) $A\, e^B = e^B\, A$

(iv) $A'\, B' = B'\ A'$

(v) $A'\, B = B'\ A$

(vi) A (B - A) + (A – B) A = 0

(vii) none of the above.

Problem 11:

$A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^3 = \alpha_1 A + \alpha_2 I$ *. The* (α_1, α_2) *pair is equal to:*

(i) (4 , 3)

(ii) (11 , 27)

(iii) (27 , 10)

(iv) (4 , -2)

(v) (16 , 4)

(vi) (4 , 16)

(vii) none of the above.

Problem 12:

$$A = \begin{bmatrix} 3 & -1 & 3b \\ -1 & 3 & -1 \\ 3b & -1 & 3 \end{bmatrix}$$

For what numbers "b" is the A matrix positive definite?

(i) 3

(ii) -4/3

(iii) 6

(iv) 1/2

(v) -3

(vi) 9

(vii) none of the above

<u>Problem 13:</u>

The Gauss-Seidel method will converge to the solution of Ax=b, for A equal to:

$A1=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A2=\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, $A3=\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$,$A4=\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, $A5=\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$, $A6=\begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$

(i) A1

(ii) A2

(iii) A3

(iv) A4

(v) A5

(vi) A6

(vii) none of the above.

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