EE510, Fall 2022, GPP
FINAL
Monday, Dec. 12th 2022, 4:30pm-5:30pm

ALL PROBLEMS COUNT EQUALLY:3pts

$(\lambda-1)(\lambda+1)(\lambda+2)$

Problem 1:

A is a 3×3 *matrix that has eigenvalues 1, -1,2, and corresponding eigenvectors:*

$$\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\sqrt{3}\\3\sqrt{3}\end{bmatrix}, \begin{bmatrix}1\\1\\-1\end{bmatrix},$$

$S\Lambda S^{-1}$ $\quad a^3+2a^2-a-2$ $\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} - \begin{bmatrix}1&2\sqrt{3}&3\sqrt{3}\\2\sqrt{3}&12&18\\3\sqrt{3}&18&27\end{bmatrix}$

$$A^5 = c_0 I + c_1 A + \cancel{c_2 A^2}$$

Then $c_1 =$

$1 = \cancel{c_0} + \cancel{c_1} + c_2$

$-1 = -\cancel{c_0} - \cancel{c_1} + c_2$

$a^5 = 2a^4 + a^3 - 2a^2$

(i) *10*

(ii) *-10*

(iii) *1*

$c_0 + c_1 = 1$

$2x + 2y = 2$

$\begin{bmatrix}1\\1\\-1\end{bmatrix}\begin{bmatrix}1 & 1 & -1\end{bmatrix}$ $\quad 2\begin{bmatrix}1&1&-1\\1&1&-1\\-1&-1&1\end{bmatrix}$

(iv) *-1*

(v) $\sqrt{3}$

(vi) *none of the above*

Problem 2:

Let A and B be two 3×3 *matrices with eigenvalues:* $\frac{1}{2}, -1, -2$ *and* $\frac{-1}{2}$*,2, -2 respectively. Then their Kronecker product has determinant equal to:*

0.5 $\quad \frac{1}{2} \times -1 \cdot 2$ $\quad$ -0.5

-1 $\quad$ 2

-2 $\quad$ -2

(i) -4
(ii) -8
(iii) *4*
(iv) -2
(v) 8
(vi) 2
(vii) *none of the above*

-4

$0.5 \times -2 \times -1$

1

2^3

8.

Problem 3:

A is a 8×8 *matrix. It has:*
eigenvalue 2 with multiplicity 3 and 3 eigenvectors
eigenvalue 3 with multiplicity 4 and 2 eigenvectors
eigenvalue 1 with multiplicity 1 and 1 eigenvector

$(\lambda-2)(\lambda-3)(\lambda-1)$

Which is the following polynomials is possible as a minimal polynomial of A?

(i) $(\lambda-2)^2(\lambda-3)^2(\lambda-1)^1$
(ii) $(\lambda-2)^3(\lambda-3)^2(\lambda-1)^1$
(iii) $(\lambda-2)^2(\lambda-3)^4(\lambda-1)^1$
(iv) $(\lambda-2)^3(\lambda-3)^4(\lambda-1)^1$
(v) $(\lambda-2)^1(\lambda-3)^1(\lambda-1)^1$
(vi) none of the above

Problem 4:

A box in the three-dimensional Euclidean space is formed by the rows: (2,1,1), (1,2,1), (1,1,3). Starting at time 0 the box is transformed by:

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 6 & 2 & 2 \\ -2 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

The volume of the transformed box at time $t = \frac{2}{10} \ln 2$ *is equal to:*

(i) 21
(ii)16
(iii)28
(iv)35
(v)14
(vi)none of the above

Problem 5:

$$A = \begin{bmatrix} -9 & -4 \\ 81 & 27 \end{bmatrix}, \quad A^{51} = ?$$

(i) $9^{50}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 9^{49} * 50 \begin{bmatrix} 18 & 4 \\ -81 & -18 \end{bmatrix}$

(ii) $9^{50}\begin{bmatrix} 9 & 4 \\ -81 & 7 \end{bmatrix} - 9^{49}\begin{bmatrix} 25 & 4 \\ 72 & -18 \end{bmatrix}$

(iii) $9^{50}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 9^{49} * 50 \begin{bmatrix} 18 & 4 \\ -81 & -18 \end{bmatrix}$

(iv) $-9^{51}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+51*9^{50}\begin{bmatrix}18 & 4\\-81 & -18\end{bmatrix}$ 2

(v) $9^{51}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+9^{50}*51\begin{bmatrix}-18 & 4\\-81 & -18\end{bmatrix}$

(vi) $-9^{51}\begin{bmatrix}-9 & 4\\-81 & 7\end{bmatrix}+9^{50}*51\begin{bmatrix}25 & 4\\-72 & -18\end{bmatrix}$

(vii) none of the above ?

Problem 6:

A is a 6×6 *matrix with all the eigenvalues equal to* $\lambda\neq 0.A^n$ *contains terms* $\lambda^n, n\lambda^{n-1}, n^2\lambda^{n-2}, n^3\lambda^{n-3}, n^4\lambda^{n-4}$ *.Which is the following Jordan forms is possible for A?*

5

(i)

$$\begin{bmatrix}\lambda & 1 & 0 & 0 & 0 & 0\\0 & \lambda & 1 & 0 & 0 & 0\\0 & 0 & \lambda & 1 & 0 & 0\\0 & 0 & 0 & \lambda & 1 & 0\\0 & 0 & 0 & 0 & \lambda & 0\\0 & 0 & 0 & 0 & 0 & \lambda\end{bmatrix}$$

(ii)

$$\begin{bmatrix}\lambda & 1 & 0 & 0 & 0 & 0\\0 & \lambda & 1 & 0 & 0 & 0\\0 & 0 & \lambda & 0 & 0 & 0\\0 & 0 & 0 & \lambda & 1 & 0\\0 & 0 & 0 & 0 & \lambda & 1\\0 & 0 & 0 & 0 & 0 & \lambda\end{bmatrix}$$

(iii)

$$\begin{bmatrix}\lambda & 1 & 0 & 0 & 0 & 0\\0 & \lambda & 1 & 0 & 0 & 0\\0 & 0 & \lambda & 1 & 0 & 0\\0 & 0 & 0 & \lambda & 0 & 0\\0 & 0 & 0 & 0 & \lambda & 0\\0 & 0 & 0 & 0 & 0 & \lambda\end{bmatrix}$$

(iv)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(v)

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

(vi) none of the above

Problem 7:

Given a symmetric nonsingular matrix A $3 \times 3, and\ a\ vector\ b \in R^3$ *,consider that the linear system:*$Ax = b$ *has condition number 2.Then the condition number of* $A^2 x = b$*,is :*

(i) 2
(ii) 3
(iii) 6
(iv) 4
(v) 9
(vi) *none of the above*

2^2

Problem 8:

In order to solve the linear system $Ax = b$*,where : A* $n \times n, and\ b \in R^n$*,A symmetric and positive definite, we use the iteration:*$x_{k+1} = x_k - \epsilon(A^T A\, x_k - A^T b)$ *.In order to have convergence to the solution of the linear system, for any initial condition, we need to have :* $0 < \epsilon < \frac{2}{\delta}$*,where* δ *is:*

(i) $\|A^{-1}\|$ *, (where* $\|.\|$ *is the usual sup norm)*

(ii) $\|A^2\|$ *, (where* $\|.\|$ *is the usual sup norm)*

(iii) 1

(iv) $|\lambda_{min}(A)|$

(v) $|\lambda_{max}(A)|$

(vi) *none of the above*

Problem 9:

Let A and B be two 3×3 *matrices with eigenvalues 1,2,3 and -1,-2,3, respectively. Then their Kronecker product has trace equal to:*

(i) 0
(ii) 36
(iii) -36
(iv) -12
(v) 12
(vi) *none of the above*

Problem 10:

It holds for a real matrix $M, n \times n, M^3 = I, Mv \neq v, v \in R^3, v \neq zero\ vector$
The minimal polynomial of M is:

(λ^2+1)

$(\lambda-1)(\lambda^2+\lambda+1)$
$\neq 0 \quad \neq 0.$

$Mv \neq v.$

ANSWER:

(i) $\lambda^3 - 1$

(ii) $\lambda^2 + \lambda + 1$

$A - I = 0.$

(iii) $\lambda^2 - \lambda + 1$

(iv) $\lambda + 1$

$A^3 - I = 0$

$A = I$

(v) $\lambda - 1$

$A \neq I.$

(vi) *none of the above*