EE 510, Fall 2022, GPP
MIDTERM 2
Wednesday, November 9th 2022, 7:30pm-8:30pm

ALL PROBLEMS COUNT EQUALLY

Problem 1:

Given a 3x3 matrix A, with three distinct eigenvalues, the exp(At) may contain all of the following terms:

(i) e^{-t}, e^{t}, $t^2 e^{-t}$

~~*(ii)*~~ $\cancel{t, t^2, t^3}$

(iii) e^{t}, e^{2t}, e^{3t}, *1*

(iv) e^{t}, e^{-t}, t^2

(v) e^{t}, e^{-t}, te^{t}

(vi) e^{t}, e^{-t}, *t*

(vii) none of the above

Problem 2:

A 3x3 matrix A is equal to $A = U\begin{bmatrix} 1 & \sqrt{2} & 0 \\ \sqrt{8} & \sqrt{8}\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}U'$, $UU' = I$. *Its singular values are:*

(i) $\sqrt{2}$, $\sqrt{2}$, *2*

(ii) *1*, $\sqrt{2}$, *0*

(iii) *2*, $\sqrt{2}$, $2+\sqrt{2}$

(iv) *1*, *1*, $3\sqrt{3}$

$\sqrt{4}$ $\downarrow$ 2 $\sqrt{27}$ $\downarrow$ $\sqrt{9\times 3}$ $3\sqrt{3}$ $\sqrt{0}$ 0

(v) 0 , 2 , $3\sqrt{3}$

(vi) 0 , 2 , - $3\sqrt{3}$

(vii) none of the above

Problem 3:

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} \sqrt{2} & 1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

$A^{31} = \alpha_1 A + \alpha_2 I$. The (α_1, α_2) pair is equal to:

(i) (2 , 3)

(ii) (1 , 0)

(iii) ($\sqrt{2}$,$\sqrt{3}$)

(iv) ($-\sqrt{2}$,$\sqrt{3}$)

(v) (-2 , 3)

(vi) (0 , 1)

(vii) none of the above

$A^2 - I = 0$

$A^2 = I$.

$A^{31} = A^{30} A$

$= (A^2)^{15} A$

$= I(A)$

$= A$.

Problem 4:

A 3x3 real matrix A is symmetric (A=A') positive definite. Which of the following are possible eigenvalues of A?

(i) ~~1, -1, +j~~ ✗

(ii) ~~$1, 3+j, 3-j$~~ ✗

(iii) ~~2, -1, 1~~ ✗

(iv) 1,2,3,4 ✗

(v) 1,1,0 α

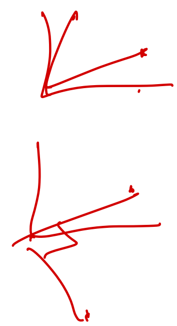
(vi) 0, 1, -1 α

$\boxed{\textit{(vii) none of the above}}$

Problem 5:

Given a 2x2 real symmetric matrix A, with eigenvalues 1 and 2; which of the following can be eigenvectors of A corresponding to 1 and 2 respectively:

(i) $\begin{bmatrix}7\\3\end{bmatrix}, \begin{bmatrix}3\\7\end{bmatrix}$ α 42



(ii) $\begin{bmatrix}1\\-1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$ α -1 +1 0

(iii) $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}-2\\-1\end{bmatrix}$ −4

(iv) $\begin{bmatrix}3\\2\end{bmatrix}, \begin{bmatrix}-6\\-4\end{bmatrix}$ −18 −26

$\boxed{\textit{(v)} \begin{bmatrix}7\\3\end{bmatrix}, \begin{bmatrix}3\\-7\end{bmatrix}}$ 0

(vi) both cases (i) and (v)

(vii) none of the above

Problem 6:

$$A = T\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}T^{-1}, \qquad T \text{ is } 2 \times 2\text{, invertible. } e^{4A} = ?$$

(i)$e^8[-7I - 4A]$

$\boxed{\textit{(ii)}\, e^8[-7I + 4A]}$

$e^{2(4)}\,[I + (A - 2I)\,4]$

(iii)$e^4[+7I + 4A]$

$e^8\,[I + 4A - 8I]$

(iv) $e^{10}[7I + 4A]$

$e^8\,[4A - 7I]$

(v)$2^4[-7I + 4A]$

(vi) $e^{10}[-7I + 4A]$

$e^{2(4)}\,[1 - 2(4)I + \cancel{t}A]$

(vii) none of the above

$e^8\,[-7I + 4A]$

Problem 7:

$$A = \begin{bmatrix} 2 & a & 0 \\ 0 & 1 & 0 \\ 0 & a & 2 \end{bmatrix}$$ *and* $x'Ax$ *is positive for all* $x \neq 0$ *vectors in* R^3*, if "a" equals:*

(x' is the transpose of x)

(i) 3

A → non symmetric ,

(ii) -3

(iii) 1

(iv) any a positive

(v) 6

(vi)4

(vii) *none of the above*

Problem 8:

$\dot{x} = Ax$, *A is 3x3* , $x(0) = \begin{bmatrix}1\\1\\1\end{bmatrix}$, $A = U\begin{bmatrix}-4 & 3 & 0\\0 & -3 & 0\\0 & 0 & 0\end{bmatrix}U'$, *(U′ means transpose U) and*

$U = \begin{bmatrix}1/\sqrt{3} & 2/\sqrt{8} & 1/\sqrt{6}\\-1/\sqrt{3} & 0 & 2/\sqrt{6}\\1/\sqrt{3} & -2/\sqrt{8} & 1/\sqrt{6}\end{bmatrix}$. *As* $t \to \infty$ *x(t) goes to:*

$\lambda = -4, -3, 0$

(i) $\begin{bmatrix}1\\1\\1\end{bmatrix}(1/3)$

$e^{Bt} = \begin{bmatrix}e^{-4t} & & \\ & e^{-2t} & \\ & & e^{0}\end{bmatrix}$

(ii) $\begin{bmatrix}1\\-1\\1\end{bmatrix}$*(1/3)*

$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 1\end{bmatrix}$

(iii) $\begin{bmatrix}1\\2\\1\end{bmatrix}$*(1/3*

(iv) $\begin{bmatrix}2\\0\\-2\end{bmatrix}$*(1/3)*

(v) $\begin{bmatrix}2\\4\\2\end{bmatrix}$*(1/3)*

(vi) $\begin{bmatrix}1\\-2\\1\end{bmatrix}$*(1/3)*

(vii) *none of the above.*

Problem9:

$$A = \begin{bmatrix} 2 & b+1 \\ b-1 & 5 \end{bmatrix}$$

The Jacobi method for solving Ax=c converges, for any initial condition, to the solution for "b" equal to:

~~*(i) 5*~~ 1.5

~~*(ii) -5*~~ 1.7

~~*(iii) -3.5*~~ 1.066

~~*(iv) 3.5*~~

$\boxed{\textit{(v) 3}}$

~~*(vi) 6*~~

(vii) none of the above.

$$J = \begin{bmatrix} 2 & \\ & 5 \end{bmatrix}^{-1} \begin{bmatrix} & -b-1 \\ -b+1 & \end{bmatrix}$$

Problem10:

$$A = \frac{1}{2}\begin{bmatrix} -1 & c \\ c & -1 \end{bmatrix}$$

The iteration $x_{k+1} = \frac{1}{2}Ax_k + \frac{1}{2}Ax_{k-1} + b$ *for solving* $x = Ax + b$ *,uses the average of the last two values of* x_k*. We use the equivalent formulation:*

$$z_{k+1} = \begin{bmatrix} \frac{1}{2}A & \frac{1}{2}A \\ I & 0 \end{bmatrix} z_k + \begin{bmatrix} b \\ 0 \end{bmatrix},\ z_k = \begin{bmatrix} x_k \\ x_{k-1} \end{bmatrix}$$

in order to study the convergence. The iteration converges to the solution for any initial conditions if c equals:

(i) 5

(ii) -5

(iii) -6

(iv) 6

$$\begin{bmatrix} -\frac{1}{2} & \\ & -\frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} & -c \\ -c & \end{bmatrix}$$

(v) -2

(vi) 3

(vii) none of the above.