EE 810: Homework # 0

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Problem set A 11

11. What is the 4 by 4 Fourier matrix $F_{2D} = F \otimes F$ for $F = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$?

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$$F_{2D} = F \otimes F$$

$$= \begin{bmatrix} F & F \\ F & -F \end{bmatrix}$$

$$\boxed{F_{2D} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}}$$

Problem set A (12)

12. Suppose $Ax = \lambda(A)x$ and $By = \lambda(B)y$. Form a long column vector z with n^2 components, $x_1 y$, then $x_2 y$, and eventually $x_n y$. Show that z is an eigenvector for $(A \otimes I)z = \lambda(A)z$ and $(A \otimes B)z = \lambda(A)\lambda(B)z$.

$$(A \otimes I)_z = \begin{bmatrix} a_{11}I & a_{12}I & \cdots & a_{1n}I \\ \vdots & \ddots & & \\ a_{n1}I & \cdots & & a_{nn}I \end{bmatrix} \begin{bmatrix} x_1 y_1 \\ \vdots \\ x_n y_n \end{bmatrix}$$

This vector will have n^2 components.

$$[(A \otimes I)_z]_i = a_{i1}x_1y_1 + 0\, x_1 y_2 + \cdots 0\, x_1 y_n$$
$$+ a_{i2}x_2y_1 + 0\, x_2y_2 + \cdots + 0\, x_2 y_n$$
$$\cdots$$
$$+ a_{in}x_ny_1 + \cdots + (0)x_n y_n$$

$\because$ $Ax = \lambda(A)x$ & $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = \lambda x_i$

$\therefore$ our expression becomes : $[(A \otimes I)z]_a = \lambda x_i y_1$

$[(A \otimes I)_z]_{n2} = \lambda x_i y_2$

$$\therefore \left[(A \otimes I)z\right] = \begin{bmatrix} \lambda x_1 y_1 \\ \lambda x_1 y_2 \\ \vdots \\ \lambda x_1 y_n \\ \lambda x_2 y_1 \\ \lambda x_2 y_2 \\ , \\ \vdots \\ \lambda x_n y_n \end{bmatrix} = \lambda \begin{bmatrix} x_1 y_1 \\ x_1 y_2 \\ \vdots \\ x_1 y_n \\ x_2 y_1 \\ x_2 y_2 \\ , \\ \vdots \\ x_n y_n \end{bmatrix} = \lambda (A) z$$

$$\boxed{\text{Hence, } (A \otimes I)_z = \lambda (A)_z}$$

Now, considering $(A \otimes B)z = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \\ a_{n1}B & \cdots & a_{nn}B \end{bmatrix} \begin{bmatrix} x_1 y_1 \\ \vdots \\ x_n y_n \end{bmatrix}$

I^{th} components containing n components will be as follows:

$$\left[(A \otimes B) z\right]_i = (a_{i1}B, \; a_{i2}B \, \ldots, a_{in}B) \begin{bmatrix} x_1 y_1 \\ \vdots \\ x_n y_n \end{bmatrix}$$

$$= a_{i1}B\,(x_1 y_1 + \cdots x_1 y_n) + a_{i2}B\,(x_2 y_1 \cdots x_2 y_n) + a_{in}B\,(x_n y_1 \ldots x_n y_n)$$

$$= a_{i1} x_1 B y_1 + \cdots + a_{n1} x_1 B y_n + \cdots + a_{in} x_n B y_1 + a_{in} x_n B y_n$$

$$\boxed{\text{Hence, } (A \otimes B) z = \lambda(A) \lambda(B) z}$$

Laub's handout 2

2. Prove that for all matrices A and B, $(A \otimes B)^+ = A^+ \otimes B^+$.

we can write A & B as:

$$\underset{m_A \times n_A}{A} = \underset{m_A \times m_A}{U_A} \underset{m_A \times n_A}{\begin{bmatrix} C_A & 0 \\ 0 & 0 \end{bmatrix}} \underset{n_A \times n_A}{V_A^T}$$

$$\underset{m_B \times n_B}{B} = \underset{m_B \times m_B}{U_B} \underset{m_B \times n_B}{\begin{bmatrix} C_B & 0 \\ 0 & 0 \end{bmatrix}} \underset{n_B \times n_B}{V_B^T}$$

$\therefore$ By the defination of psudo inverse,

$$A^+ = V_A \begin{bmatrix} C_A^{-1} & 0 \\ 0 & 0 \end{bmatrix} U_A^T$$

$$\& \quad B^+ = V_B \begin{bmatrix} C_B^{-1} & 0 \\ 0 & 0 \end{bmatrix} U_B^T$$

$$\therefore \quad A^+ \otimes B^+ = \left(V_A \begin{bmatrix} C_A^{-1} & 0 \\ 0 & 0 \end{bmatrix} U_A^T \right) \otimes \left(V_B \begin{bmatrix} C_B^{-1} & 0 \\ 0 & 0 \end{bmatrix} U_B^T \right)$$

$$= (V_A \otimes V_B) \begin{bmatrix} C_A^{-1} \otimes C_B^{-1} & 0 \\ 0 & 0 \end{bmatrix} (U_A^T \otimes U_B^T)$$

L (i)

Now, consider:

$$(A \otimes B)^+ = \left\{ \left(U_A \begin{bmatrix} C_A & 0 \\ 0 & 0 \end{bmatrix} V_A^T \right) \otimes \left(U_B \begin{bmatrix} C_B & 0 \\ 0 & 0 \end{bmatrix} V_A^T \right) \right\}^+$$

$$= \left\{ (U_A \otimes U_B) \left[\begin{pmatrix} C_A & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} C_B & 0 \\ 0 & 0 \end{pmatrix} \right] (V_A^T \otimes V_B^T) \right\}^+$$

$$= \left\{ (U_A \otimes U_B) \begin{bmatrix} C_A \otimes C_B & 0 \\ 0 & 0 \end{bmatrix} (V_A \otimes V_B)^T \right\}^+$$

$$= (V_A \otimes V_B) \begin{bmatrix} C_A \otimes C_B^{-1} & 0 \\ 0 & 0 \end{bmatrix} (U_A \otimes U_B) \quad \text{②}$$

comparing this w/ eq ①, eq ① = eq ②

$$\boxed{\text{Hence, showed that} \quad (A \otimes B)^+ = A^+ \otimes B^+}$$

laub's handout 4

4. Show that the general linear equation

$$\sum_{i=1}^{k} A_i X B_i = C$$

can be written in the form

$$[B_1^T \otimes A_1 + \cdots + B_k^T \otimes A_k]\mathrm{vec}(X) = \mathrm{vec}(C).$$

$\because \ \mathrm{vec}\left(A_1 x B_1 + \dots + A_k x B_k\right) = \mathrm{Vec}\,(C)$

$\therefore \ \mathrm{vec}\left(\sum_{i=1}^{K} A_i\, x\, B_i\right) = \sum_{j=1}^{K} \mathrm{vec}\,(A_i\, x\, B_i)$

also, $\mathrm{vec}\,(A_i\, x\, B_i)$ can be written as $(B_i' \otimes A_i)\ \mathrm{vec}\,(x)$

$\therefore \ \sum_{i=1}^{K} \mathrm{vec}\,(A_i\, x\, B_i) = \sum_{j=1}^{K} (B_j^T \otimes A_j)\ \mathrm{vec}\,(x)$

$$\boxed{\therefore \ \left[B_1^T \otimes A_1 + B_2^T \otimes A_2 + \dots + B_k^T \otimes A_k\right] \mathrm{vec}\,(x) = \mathrm{vec}\,(C)}$$

Problem set 5.1 32

32. Construct any 3 by 3 Markov matrix M: positive entries down each column add to 1. If $e = (1,1,1)$, verify that $M^T e = e$. By Problem 11, $\lambda = 1$ is also an eigenvalue of M. Challenge: A 3 by 3 singular Markov matrix with trace $\frac{1}{2}$ has eigenvalues $\lambda =$ ____.

1/3

Let $M^T = \begin{bmatrix} 1/2 & 1/6 & 1/3 \\ 1/3 & 1/6 & 1/2 \\ 1/3 & 1/2 & 1/6 \end{bmatrix}$ $\therefore M^T e = \begin{bmatrix} 1/2 & 1/6 & 1/3 \\ 1/3 & 1/6 & 1/2 \\ 1/3 & 1/2 & 1/6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = e$

So $M^T e = e$. 1 is an eigenvalue of M^T.

but M^T & M have same eigenvalues.

$\therefore$ M has one eigenvalue 1

Hence 1 is an eigenvalue of any markov matrix M.

Now, a 3×3 Markov matrix has trace 1.

$\because$ trace $= \lambda_1 + \lambda_2 + \lambda_3$, $\longleftarrow$ $\begin{cases} \lambda_1 = 0 & \text{— (given matrix is singular)} \\ \lambda_2 = 1 & \text{— (every markove matrix has } \lambda = 1) \end{cases}$

$\frac{1}{2} = 0 + 1 + \lambda_3$

$\therefore \lambda_3 = \frac{-1}{2}$

$\therefore$ Challenge: A 3 by 3 singular Markov matrix with trace $\frac{1}{2}$ has eigenvalues $\lambda = 0, 1, \frac{-1}{2}$

PS 5.3 10

10. Find the limiting values of y_k and $_k$ $(k \to \infty)$ if

$$y_{k+1} = .8y_k + .3z_k \qquad y_0 = 0$$
$$z_{k+1} = .2y_k + .7z_k \qquad z_0 = 5.$$

Also find formulas for y_k and z_k from $A^k = S\Lambda^k S^{-1}$.

say :

$$\begin{bmatrix} y_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} y_k \\ z_k \end{bmatrix}, \quad u_0 = \begin{bmatrix} y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

Trace 1.5,
one $\lambda = 1$,
$\therefore$ second $\lambda = 0.5$

$$u_{k+1} = A \, u_k$$

Now, finding the eigenvalues of A : $\lambda_1 = 0.5$, $\lambda_2 = 1$

$\therefore$ Finding eigenvectors,

for $\lambda = 0.5$, $x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

for $\lambda = 1$, $x_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\therefore$ writing A as $S\Lambda S^{-1}$, $\rightarrow A^n = S\Lambda^n S^{-1}$

$$\therefore \quad A^k = \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix}$$

$\therefore$ Using u_0, we can write:

$$U_k = A^k U_0$$

$$\therefore \begin{bmatrix} y_k \\ z_k \end{bmatrix} = S \Lambda^k S^{-1} \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3(0.5)^k \\ 1^k \end{bmatrix}$$

$$= \begin{bmatrix} (-3)\cdot(0.5^k) + (3)\cdot(1^k) \\ 3 \times 0.5^k + 2 \times (1^k) \end{bmatrix}$$

$\therefore$ as $k \to \infty$,

$$\begin{bmatrix} y_\infty \\ z_\infty \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$\therefore$ $\boxed{\text{limiting values : } y_k = 3, \; z_k = 2}$

PS 5.3 11

11. (a) From the fact that column 1 + column 2 = 2(column 3), so the columns are linearly dependent find one eigenvalue and one eigenvector of A:

$$A = \begin{bmatrix} .2 & .4 & .3 \\ .4 & .2 & .3 \\ .4 & .4 & .4 \end{bmatrix}.$$

(b) Find the other eigenvalues of A (it is Markov).

(c) If $u_0 = (0, 10, 0)$, find the limit of $A^k u_0$ as $k \to \infty$.

a $\because$ its a markov mx, $\lambda_1 = 1$

$\because$ its has linear dependent coloum, $\lambda_2 = 0$

$\because$ trace $= 0.8$, $0.8 = \lambda_1 + \lambda_2 + \lambda_3$

$0.8 = 1 + 0 + \lambda_3$

$\therefore \lambda_3 = -0.2$ ———— (1)

for $\lambda = 0$, $\left[\begin{array}{ccc|c} 2 & 4 & 3 & 0 \\ 4 & 2 & 3 & 0 \\ 4 & 4 & 4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 4 & 3 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore 2x_1 + 4x_2 + 3x_3 = 0$

$2x_2 + x_3 = 0.$

$\therefore 2x_1 = -4x_2 - 3x_3$

$x_3 = -2x_2$

$\therefore x_1 = x_2$

$\therefore x_3 = -2x_2$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

$\therefore$ one eigenvalue of given matrix is 0 & corrosponding eigenvector is $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

(b) as shown in eqⁿ (1), $\boxed{\lambda = 0, 1, -0.2}$

(c) Finding eigenvector for $\lambda = 1$,

$$\left[\begin{array}{ccc|c} -8 & 4 & 3 & 0 \\ 4 & -8 & 3 & 0 \\ 4 & 4 & -6 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} -8 & 4 & 3 & 0 \\ 0 & -6 & 9/2 & 0 \\ 0 & 6 & -9/2 & 0 \end{array}\right]$$

$$\downarrow$$

$$\therefore -8x_1 + 4x_2 + 3x_3 = 0 \qquad \longleftarrow \left[\begin{array}{ccc|c} -8 & 4 & 3 & 0 \\ 0 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$-4x_2 + 3x_3 = 0$$

$\therefore -8x_1 = -4x_2 - 3x_3$

$\therefore x_3 = \frac{4}{3} x_2$

$\therefore x_1 = x_2$ $\quad\therefore$ for $x_2 = 3$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$

$\because u_0 = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$, we know sum of entries of u_k will always be 10

Thus $u_k = A^k u_0$ will have sum 10.

as $k \to \infty$, is a multiple of eigenvector corresponding to $\lambda = 1$.

& $\because$ sum needs to be 10, we can say $\boxed{u_\infty = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}}$

PS 5.3 (12)

12. Suppose there are three major centers for Move-It-Yourself trucks. Every month half of those in Boston and in Los Angeles go to Chicago, the other half stay here they are, and the trucks in Chicago are split equally between Boston and Los Angeles Set up the 3 by 3 transition matrix A, and find the steady state u_∞ corresponding to the eigenvalue $\lambda = 1$.

Let b_k = trucks after k months in boston

l_k = ——"—— in LA

c_k = ——"—— chicago

$= \quad b_{k+1} = b_k - \frac{1}{2} b_k + \frac{1}{2} c_k \Rightarrow \frac{1}{2} b_k + 0 l_k + \frac{1}{2} c_k$

$l_{k+1} = l_k - \frac{1}{2} l_k + \frac{1}{2} c_k \Rightarrow 0 b_k + \frac{1}{2} l_k + \frac{1}{2} c_k$

$c_{k+1} = c_k - c_k + \frac{1}{2} b_k + \frac{1}{2} l_k \Rightarrow \frac{1}{2} b_k + \frac{1}{2} l_k + 0 c_k$

$$\therefore \begin{bmatrix} b_{k+1} \\ l_{k+1} \\ c_{k+1} \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} b_k \\ l_k \\ c_k \end{bmatrix}$$

Steady state vector u_∞ is one corresponding to $\lambda = 1$.

$= $ eigenvector for $\lambda = 1$: $\left[\begin{array}{ccc|c} -1/2 & 0 & 1/2 & 0 \\ 0 & -1/2 & 1/2 & 0 \\ 1/2 & 1/2 & -1 & 0 \end{array}\right]$

$\downarrow$

$$\therefore \left.\begin{aligned} -\frac{1}{2} x_1 + \frac{1}{2} x_3 &= 0 \\ -\frac{1}{2} x_2 + \frac{1}{2} x_2 &= 0 \\ \frac{1}{2} x_1 + \frac{1}{2} x_2 - x_3 &= 0. \end{aligned}\right\} \quad \begin{aligned} x_1 &= x_3 \\ x_2 &= x_3 \end{aligned}$$

$\therefore$ for $x_1 = 1$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$\boxed{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is the required steady state.}}$$

Meyer's 8.2.1

8.2.1. Verify Perron's theorem by by computing the eigenvalues and eigenvectors for

$$\mathbf{A} = \begin{pmatrix} 7 & 2 & 3 \\ 1 & 8 & 3 \\ 1 & 2 & 9 \end{pmatrix}.$$

Find the right-hand Perron vector $\mathbf{p}$ as well as the left-hand Perron vector $\mathbf{q}^T$.

$$A = \begin{bmatrix} 7 & 2 & 3 \\ 1 & 8 & 3 \\ 1 & 2 & 9 \end{bmatrix}$$

$\therefore$ to find characterstic polynomial of A,

$$|A - I\lambda| = 0 \Rightarrow \begin{vmatrix} 7-\lambda & 2 & 3 \\ 1 & 8-\lambda & 3 \\ 1 & 2 & 9-\lambda \end{vmatrix} = 0$$

$$\therefore \quad p(\lambda) = (7-\lambda)\left[(8-\lambda)(9-\lambda) - 6\right] - 2\left[9-\lambda-3\right] + 3\left[2-8-\lambda\right]$$

$$= (7-\lambda)(8-\lambda)(9-\lambda) - 6(7-\lambda) - 2(6-\lambda) + 3(\lambda-6)$$

$$= (\lambda-6)^2(12-\lambda)$$

$\therefore$ Perron root is $\lambda = 12$ & other λs are $6, 6$.

Let $p = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be eigenvector for $\lambda = 12$

$\therefore \quad Ap = \lambda p$

$$\therefore \quad \begin{bmatrix} 7 & 2 & 3 \\ 1 & 8 & 3 \\ 1 & 2 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 12 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore \quad \left.\begin{aligned} 7x + 2y + 3z &= 12x \\ x + 8y + 3z &= 12y \\ x + 2y + 9z &= 12z \end{aligned}\right\} \quad \begin{aligned} x &= y \\ x &= z \end{aligned} \qquad \therefore \; x = y = z$$

$$\therefore \quad p = \begin{bmatrix} x \\ x \\ x \end{bmatrix} .$$

$$\boxed{\begin{aligned} &\text{for } x > 0, \; p \text{ is perron vector.} \\ &\therefore \text{ for } x = 1, \quad p = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \end{aligned}}$$

w/ normalisation

similarly, $q^T = \begin{bmatrix} x & y & z \end{bmatrix}$, $q^T A = \lambda q^T$

$$\therefore \quad \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 7 & 2 & 3 \\ 1 & 8 & 3 \\ 1 & 2 & 9 \end{bmatrix} = \begin{bmatrix} 12x & 12y & 12z \end{bmatrix}$$

$$\therefore \quad \left.\begin{aligned} 7x + y + z &= 12x \\ 2x + 8y + 2z &= 12y \\ 3x + 3y + 9z &= 12z \end{aligned}\right\} \quad \left.\begin{aligned} x + z &= 2y \\ x + y &= z \end{aligned}\right\} \quad \begin{aligned} 2x &= y \\ 3x &= z \end{aligned}$$

$$\therefore \quad q^T = \begin{bmatrix} x & 2x & 3x \end{bmatrix}$$

$\therefore$ for $x = 1$, left perron vector = $\boxed{q^T = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}}$

w/ normalisation

Meyer's 8.2.4

10/10

8.2.4. Find the Perron root and the Perron vector for

$$\mathbf{A} = \begin{pmatrix} 1-\alpha & \beta \\ \alpha & 1-\beta \end{pmatrix},$$

where $\alpha + \beta = 1$ with $\alpha, \beta > 0$.

$$|A - I\lambda| = \begin{vmatrix} 1-\alpha-\lambda & \beta \\ \alpha & 1-\beta-\lambda \end{vmatrix} = 0$$

$$\therefore (1-\alpha-\lambda)(1-\beta-\lambda) - (\alpha\beta) = 0$$

$$\therefore 1 - \beta - \lambda - \alpha + \alpha\beta + \alpha\lambda - \lambda + \beta\lambda + \lambda^2 - \alpha\beta = 0$$

$$\therefore \lambda^2 + \lambda \underbrace{(\alpha+\beta)}_{=1} - 2\lambda - \underbrace{(\alpha+\beta)}_{=1} + 1 = 0$$

$$\therefore \lambda^2 + \lambda - 2\lambda - 1 + 1 = 0$$

$$\therefore \lambda(\lambda - 1) = 0$$

$$\therefore \lambda = 0.$$

$$\lambda = 1$$

for $\lambda = 1$,

$$\begin{bmatrix} 1-\alpha-1 & \beta \\ \alpha & 1-\beta-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore -\alpha x_1 + \beta x_2 = 0 \quad \Rightarrow \quad x_1 = \frac{\beta}{\alpha} x_2$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix} \quad \text{for } x_1 = \beta.$$

$\therefore$ Perron eigen vector is $\dfrac{1}{\sqrt{\alpha^2+\beta^2}}\begin{bmatrix}\beta \\ \alpha\end{bmatrix}$

For $\lambda=0$, $\begin{bmatrix}1-\alpha & \beta \\ \alpha & 1-\beta\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} = 0$

$\therefore x_1 = \dfrac{-\beta}{1-\alpha} x_2$ $\therefore \begin{bmatrix}x_1 \\ x_2\end{bmatrix} = \begin{bmatrix}\beta \\ \alpha-1\end{bmatrix}$

for $x_1 = \beta$.

$\therefore$ perron eigen vector $= \dfrac{1}{\sqrt{\beta^2+(1-\alpha)^2}}\begin{bmatrix}\beta \\ 1-\alpha\end{bmatrix}$