EE 520 Homework 5

In Nielsen and Chuang, do the following problems:

Sec 6.1: Exercises 6.2, 6.3
Sec 2.4: Exercises 2.72
Sec 2.5: Exercise 2.77, 2.80, 2.82
Sec 4.4: Exercise 4.32 (To clarify this problem: the two projectors are $P_0 = I \otimes |0\rangle\langle 0|$ and $P_1 = I \otimes |1\rangle\langle 1|$.)
Sec 4.7: Exercise 4.49

Problem 1: Suppose that ρ is a density matrix of a quantum system. The von Neumann entropy of this state is $S(\rho) = -\mathrm{Tr}\{\rho \log_2 \rho\}$. For pure states, $S(\rho) = 0$; so having nonzero entropy is proof that a state is mixed.

For general states, it is sometimes hard to calculate the entropy; so instead, people often use a quantity called the *purity*: $P(\rho) = \mathrm{Tr}\{\rho^2\}$. Prove that $P(\rho) = 1$ for pure states, and $P(\rho) < 1$ for all mixed states.

Problem 2. Section 4.7.2 gives a proof of the Trotter formula, Eq. (4.98) in the textbook:

$$\lim_{n\to\infty} \left(e^{iA/n} e^{iB/n}\right)^n = e^{i(A+B)},$$

for any two Hermitian operators *A* and *B*. Using a similar argument, prove this formula:

$$\lim_{n\to\infty} \left(e^{iA/n} e^{iB/n} e^{-iA/n} e^{-iB/n}\right)^{n^2} = e^{-[A,B]}.$$

(This is a consequence of a theorem called the Baker-Campbell-Hausdorf formula.) Together with the usual Trotter formula, this formula allows one to generate new Hamiltonian terms from the commutators of other terms.

Show all work.

Due **Thursday 10 November 2022** before midnight. Please hand in your assignment by uploading it as a PDF file through the Blackboard site.