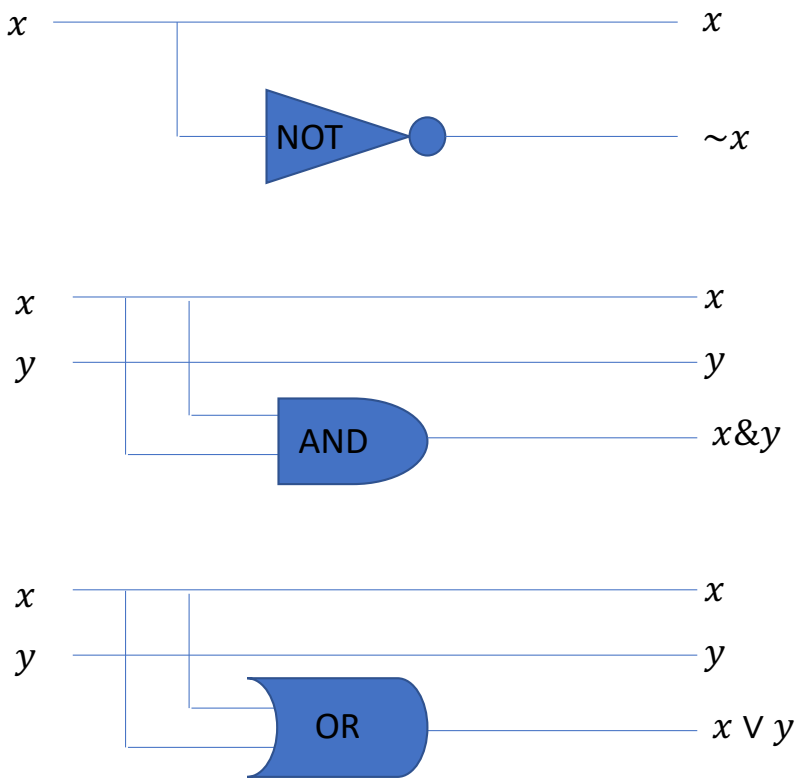
EE 520 Homework 3

In Nielsen and Chuang, do the following problems:

Sec 3.1: Exercises 3.7
Sec 3.2: Exercises 3.9-3.13, 3.31
Sec 4.5: Exercises 4.41-4.43
Sec 5.1: Exercises 5.4, 5.5

Problem 1 (Hard Problems Exist)

Argue that there exist Boolean functions that map n input bits to 1 output bit that require at least $2^n/n$ classical logic gates to compute. Assume the universal set of gates AND, OR, NOT and FANOUT (where FANOUT makes one copy of a bit). It is helpful to think of each step of the circuit as an *assignment*, where a new bit value is calculated from one or two existing bit values by applying a gate, and each new gate can draw on any of the current bit values:



After each assignment, the number of bits in the circuit has increased by 1.

Problem 2 (Classical Reversible Computation)

Show that the classical Fredkin gate can perform AND, OR, NOT, and NAND gates reversibly, provided that wires and ancilla bits (initialized in state 0 or 1) are available.

Problem 3 (Another Universal Set)

Show that the controlled-$\left(iR_X(\pi a)\right)$ and controlled-$\left(iR_Z(\pi a)\right)$ gates, with a an irrational number, together form a universal set of quantum gates, provided that ancilla qubits (initialized in states $|0\rangle$ or $|1\rangle$) are available.

Show all work.

Due Thursday 6 October 2022 before midnight. Please hand in your assignment by uploading it as a PDF file through the Blackboard site.