Time-Resolved Shadow Tomography of Open Quantum Systems

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Motivation for Open Quantum Systems (OQS)

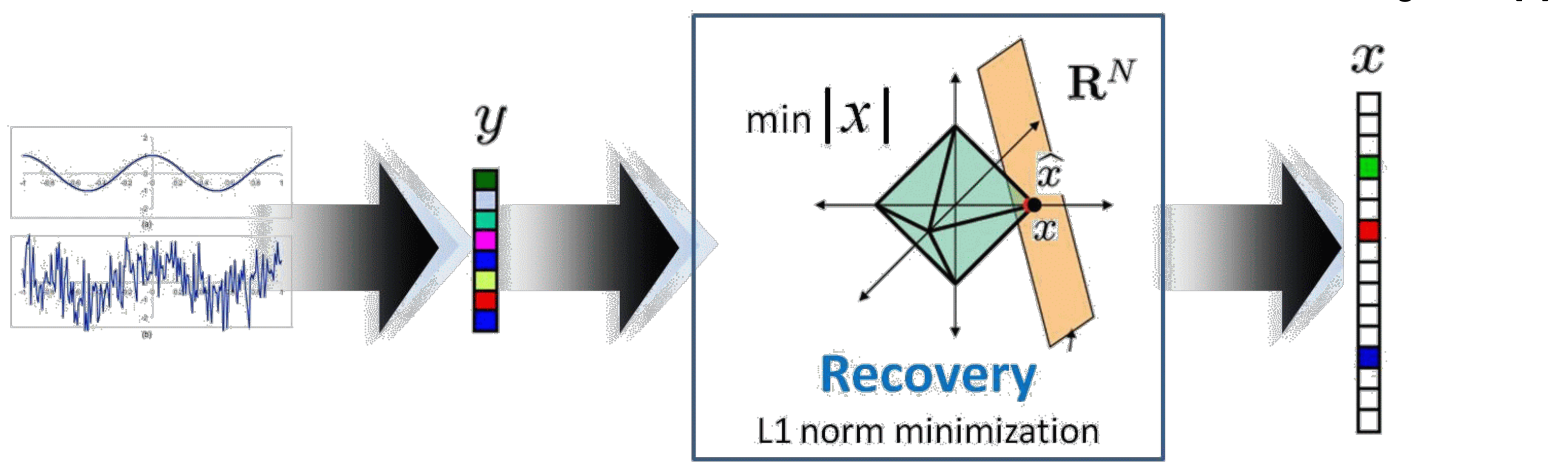
- The time evolution of an expectation value in an OQS is

$$S(t) = \langle O, \rho(t)\rangle = Tr(O\rho(t)) = Tr(Oe^{\mathcal{L}t}(\rho_0))$$
$$= \langle\langle O|e^{Lt}|\rho_0\rangle\rangle = \sum_k e^{\lambda_k t}\langle\langle O|v_k\rangle\rangle\langle\langle v_k|\rho\rangle\rangle$$

- How efficiently can we estimate $S(t)$ and extract some of the λ_ks?
- Applications include spectroscopy, device characterization, algorithm verification, etc.

Prior Work – Compressed Sensing (CS)

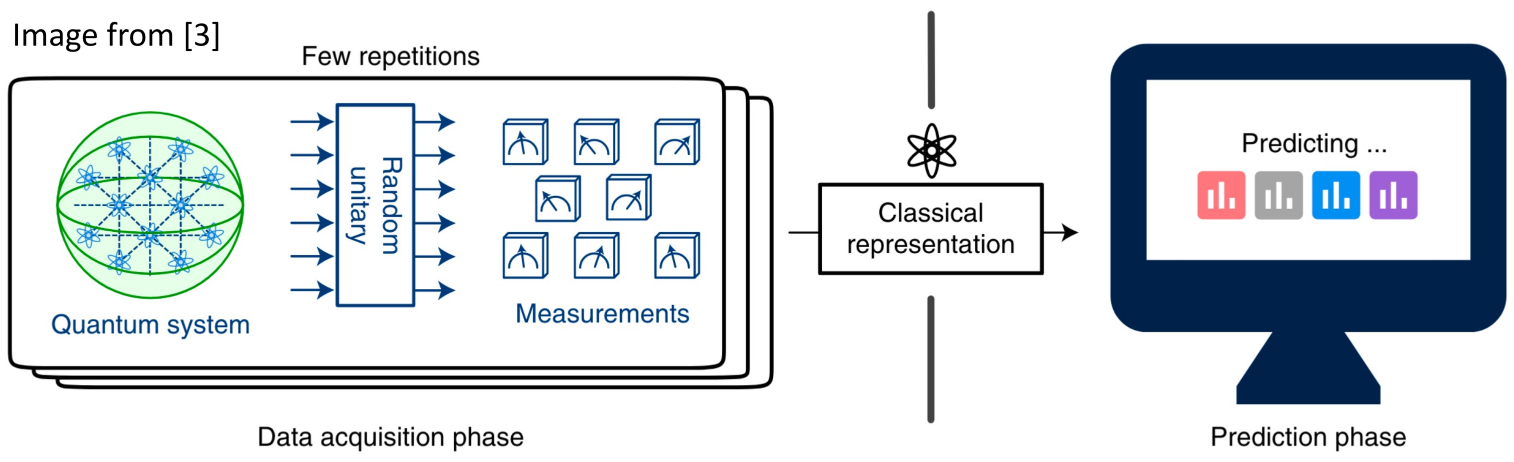
Image from [1]



- CS poses that a signal $x \in \mathcal{R}^n$, the measurement $y \in \mathcal{R}^m$ of x, and the sparse representation $\alpha \in \mathcal{R}^d$ of x, are related via
$$y = \Phi x = \Phi\mathcal{D}\alpha \equiv \Psi\alpha$$
where $\mathcal{D} \in \mathcal{R}^{n\times D}$ is the dictionary matrix, $\Phi \in \mathcal{R}^{m\times n}$ is the measurement matrix, and we are given that α is k-sparse
- Recovery of α (or x) from y is possible if $\Psi \in \mathcal{C}^{m\times D}$ obeys the restricted isometry property (RIP) of order k and some constant δ_K, which means that for every k-sparse vector $\alpha \in \mathcal{C}^D$, we have
$$(1-\delta_k)||\alpha||_2^2 \le ||\Psi\alpha||_2^2 \le (1+\delta_k)||\alpha||_2^2$$
- If x_k is the restriction of x to its k largest values (in magnitude) and $y = Ax + e$ is a noisy measurement of x obeying $||e||_2 \le \epsilon$, then[2] the solution x^* to the l_1-minimization
$$\min ||x||_1 \;\; s.t. \;\; ||Ax - y||_2 \le \epsilon$$
- obeys $||x^* - x||_2 \le C_{1k}\epsilon + C_{2k}\cdot||x^* - x||_1/\sqrt{k}$
If the signal is exactly k-sparse and noiseless, then this works perfectly

Prior Work – Shadow Tomography (ST)

Image from [3]



- ST[3] is an efficient method for estimating an arbitrary set of expectation values from a classical dataset of shadows which approximate a given quantum state
- ST promises that given $N \cdot K$ shadows of ρ, we can estimate a set of M expectation values $Tr(\rho O_i) \equiv o_i$ such that
$$|\hat{o}_i - o_i| \le \epsilon \;\text{ if } K = 2\log(2M/\delta)),\, N = \frac{34}{\epsilon^2}\cdot 4^w$$
where O_i is a Pauli string of max weight w, δ is the probability that at least one error exceeds ϵ, and each estimate is produced via median-of-means (MoM) over K sets of N shadows each.
- Other than its size and max weight, the shadow dataset is independent of the the observable set $\{O_i\}$

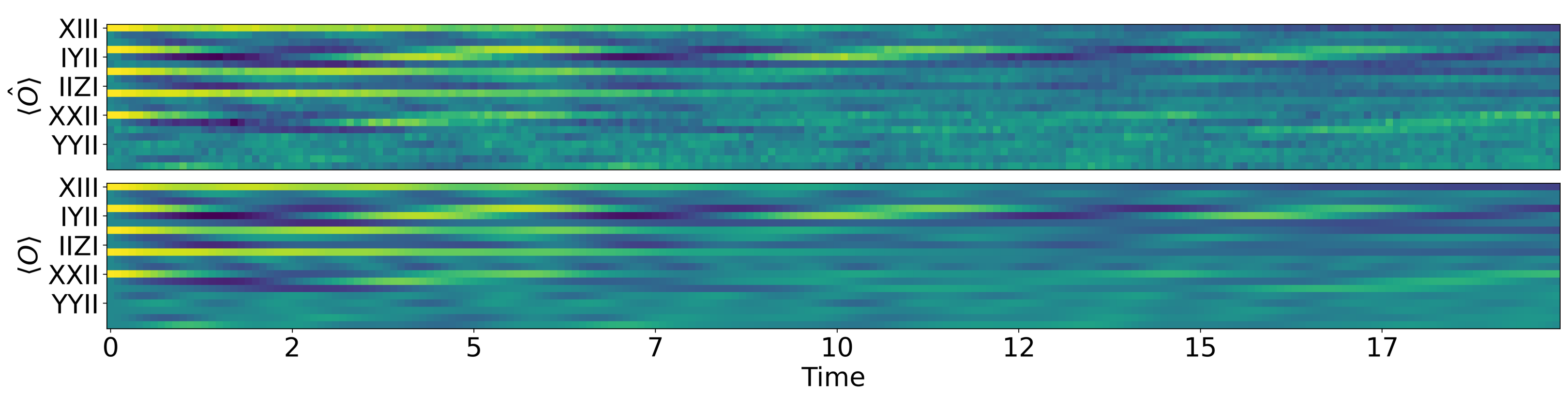
Data processing pipeline

- For time-domain signals of length n which are k-sparse in frequency space, it is known that for a unitary matrix $\mathrm{U} \in \mathcal{C}^{n\times n}$ and a sufficiently small $\delta_k > 0$, the following holds[4]:

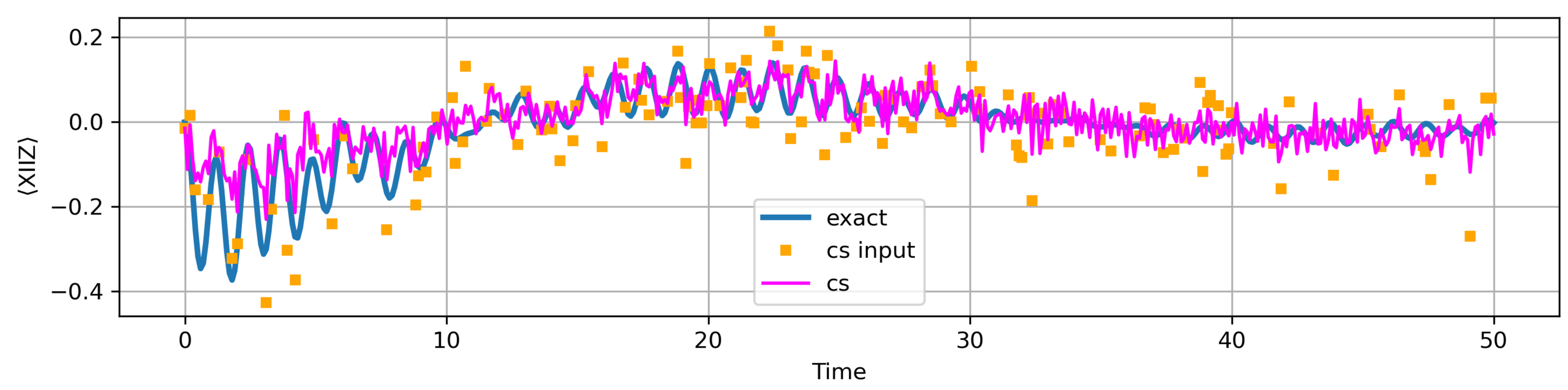
For some $q = O(\delta_k^{-4}\cdot k \cdot \log^2(k/\delta_k)\cdot\log n)$, let $\Psi \in \mathcal{C}^{q\times n}$ be a matrix whose q rows are chosen uniformly and independently from the rows of U. Then, with probability $1 - 2\exp[-\Omega(\delta_k^{-2}\cdot\log n\cdot\log(k/\delta_k))]$, the matrix Ψ satisfies the restricted isometry property of order k with constant δ_k

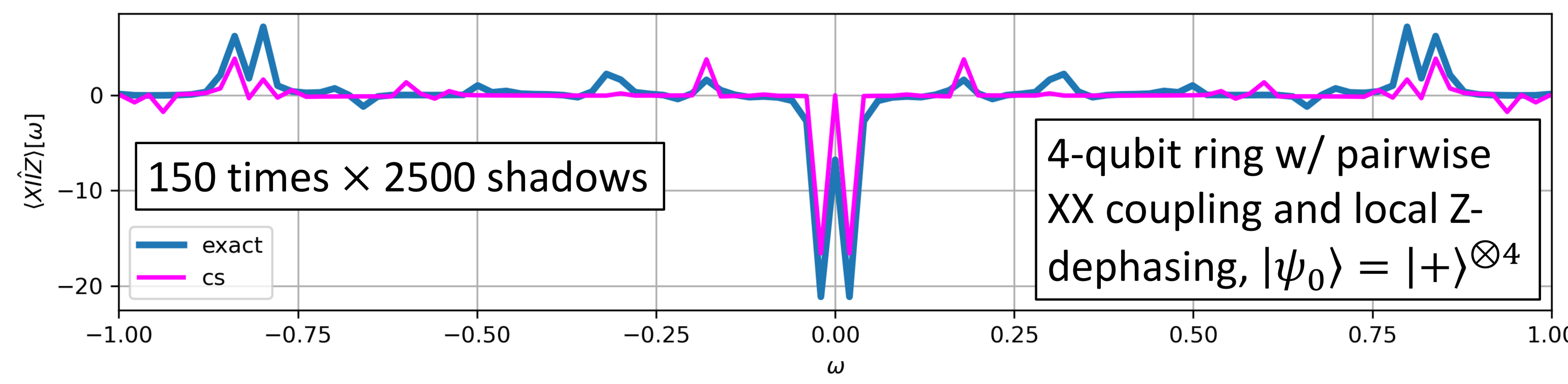
- Thus, with $\mathcal{D} = U$ (the $n\times n$ DFT transform), and Φ a subset of the rows of the identity matrix, we can compose them to obtain a subsampled Fourier matrix $\Psi = \Phi U$ which obeys the RIP

1) Perform ST at random times



2) Solve the CS problem with a subsampled Fourier sensing matrix





- Components beyond the Nyquist limit are recoverable
- Shot noise from ST is mitigated by enforcing sparsity
- Some weak components are missed

Summary & next steps

- Total number of shots needed to reconstruct all M signals of length n which are at most k-sparse
$$N_{tot} \sim O(k\log M\log^2 k\log n\,/\epsilon^2)$$
where the l_2-reconstruction error is bounded by $O(\epsilon\sqrt{n})$ for decay-less signals of weight-w Pauli strings
- Use a harmonic inversion algorithm to fit the reconstructed signal to a sum of decaying sinusoids to extract the evolution parameters, and quantify the effect of shot noise from ST on these estimates[5]
- Combine the various observable signals in a clever way to process more components in parallel[6]
- Can we upper bound the number of components λ_k given ρ_0? Are there better sensing matrices for weakly-decaying signals?

References

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[6] Chen et al. "Algorithmic Shadow Spectroscopy" (2023)

SQuInT, Albuquerque, NM (October 2023)