Time-resolved Shadow Tomography of Open System Dynamics

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I. Introduction

• We show that shadow state tomography is applicable to open quantum systems by using shadows of an initial state and an assumed model of the dynamics to estimate arbitrary observables over time without the need to actually evolve the state in time.

II. Background

• Using shadow tomography, we can estimate the probabilities of exponentially many measurements only using polynomial copies of ρ. We follow the procedure for given by [2].

• Their main theorem asserts that for a chosen ensemble $\mathcal{U}$, a set of observables $O_1, \ldots, O_M$ and parameters $\epsilon, \delta \in [0,1]$, pick two sampling parameters N & K such that

$$N = 2\log\left(\frac{2M}{\delta}\right),$$

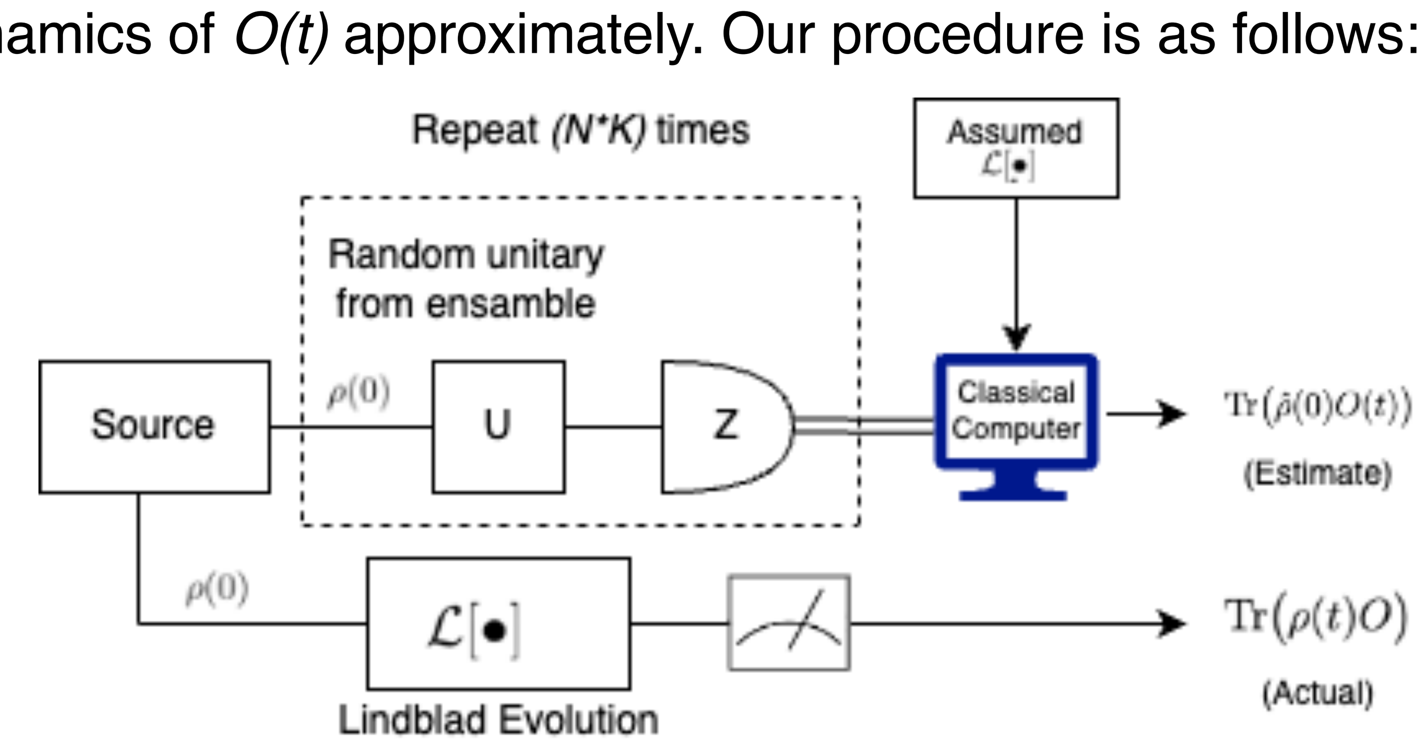
$$K = \frac{34}{\epsilon^2}\max_{1\le i\le M} \left\| O_i - \frac{Tr(O_i)}{2^n} I \right\|^2_{shadow}.$$

• A collection of $N{\cdot}K$ classical shadows is sufficient to estimate all $\mathrm{Tr}(O_i\,\rho)$ up to error ϵ and success probability of at least $1-\delta$.

• However, this shadow norm can grow exponentially depending on the locality of the observable and the choice of $\mathcal{U}$.

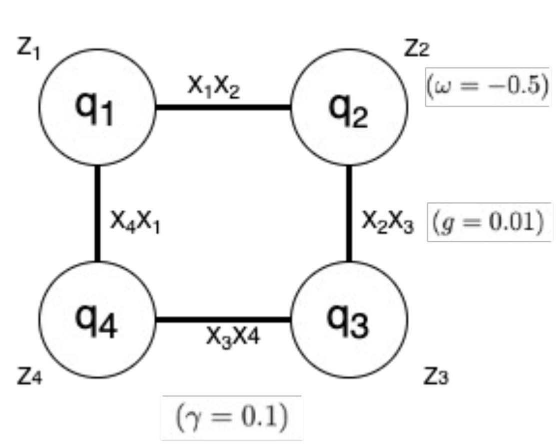
III. Analysis

• Given an assumed model for the open system dynamics, the shadow estimate of $\rho(0)$ alone is sufficient to predict the dynamics of $O(t)$ approximately. Our procedure is as follows:



• To run our simulations, we assumed a simple dephasing model governed by the Lindblad master equation with a random initial state.

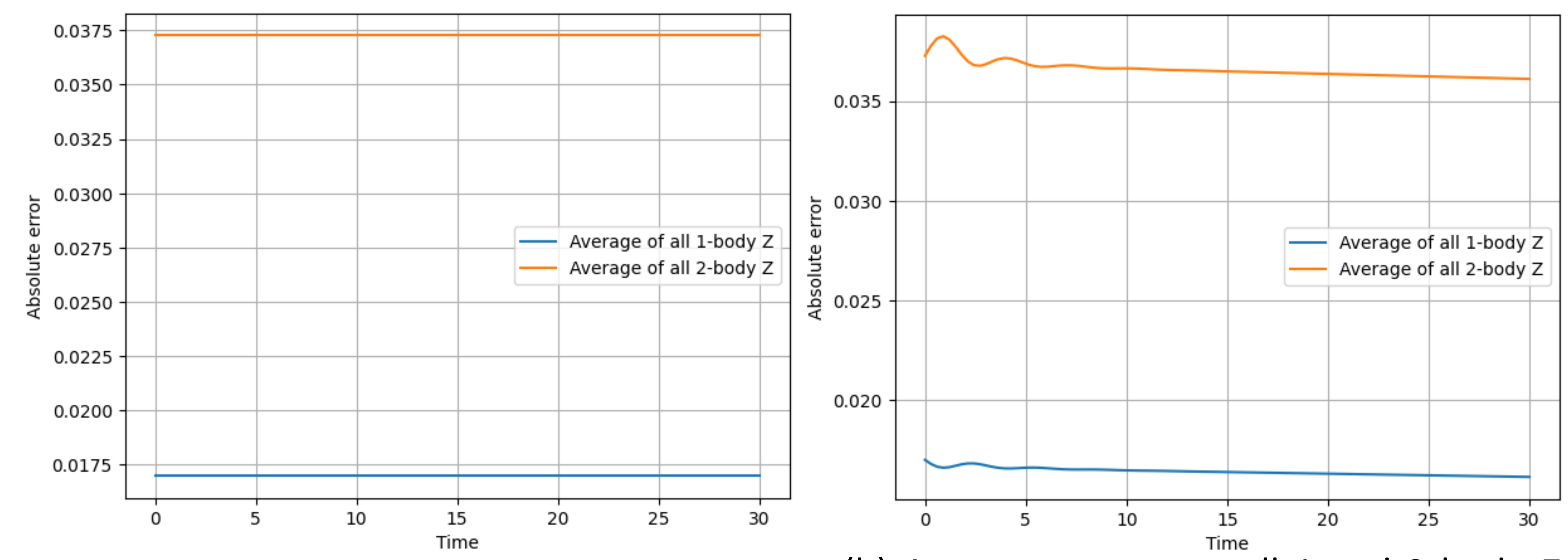
• We consider two cases:
(i) Without the XX coupling.
(ii) With the XX coupling.



• For case (i), we have a guaranteed bound on shadow norm independent of the size of the system, so the number of copies required to get a good estimate of the measurement outcome is logarithmic in the system dimension.

• For case (ii), the locality of local observables changes. This does not immediately guarantee that one could estimate the outcomes of local observables for $\rho(t)$ efficiently from copies of $\rho(0)$.

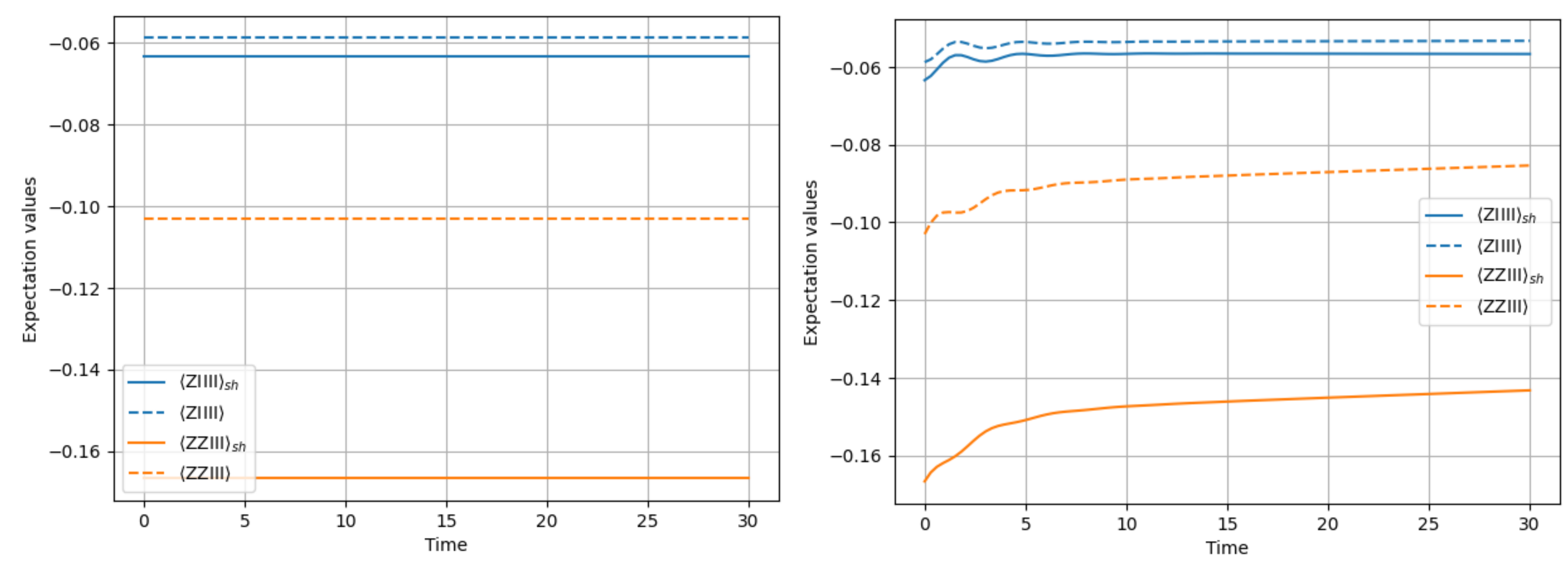
V. Results



(a) Average error over all 1 and 2 body Z observables, non-interacting case

(b) Average error over all 1 and 2 body Z observables, interacting case

• Figure 1: Plots of absolute errors of 1 and 2 body operators. Absolute error is defined as the absolute difference between the expectation value of the exact $\rho(t)$ and the shadow.



(a) Selected 1 and 2 body Z observables over time, non-interacting case

(b) Selected 1 and 2 body Z observables over time, interacting case

• Figure 2: The label $\langle \ldots \rangle_{sh}$ refers to the expectation value of the operator using shadow state tomography

VI. Discussion

• Even though we expected that the estimation errors of measurement outcomes of local operators would worsen as the locality of the operators grows as a function of time, the plots illustrate that the expectation value of operators can be approximated efficiently without loss of accuracy for seemingly long times.

• This could have multiple explanations.
- The system sizes we used are too small to resolve the operator weight increase over time
- We have some preliminary evidence which suggests that the theoretical bound is several orders-of-magnitude larger than the actual bound in practice

• One possible way to further investigate the role of locality and errors obtained from shadow tomography would be to perform a similar analysis for larger system sizes. It might be that the time-scale over which the operator weight increases is short enough that we need larger systems to resolve this effect. Additionally, we need to identify which initial states and evolution types are better suited to saturating the bound, which is fairly loose.

VII. References

[1] Aaronson. *Shadow Tomography of Quantum States*, 2018. doi:10.48550/arXiv.1711.01053

[2] Huang et al. *Predicting many properties of a quantum system from very few measurements*, 2020. doi:10.1038/s41567-020-0932-7