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Decoding Quantum Error Correcting Codes with a Quantum Annealer

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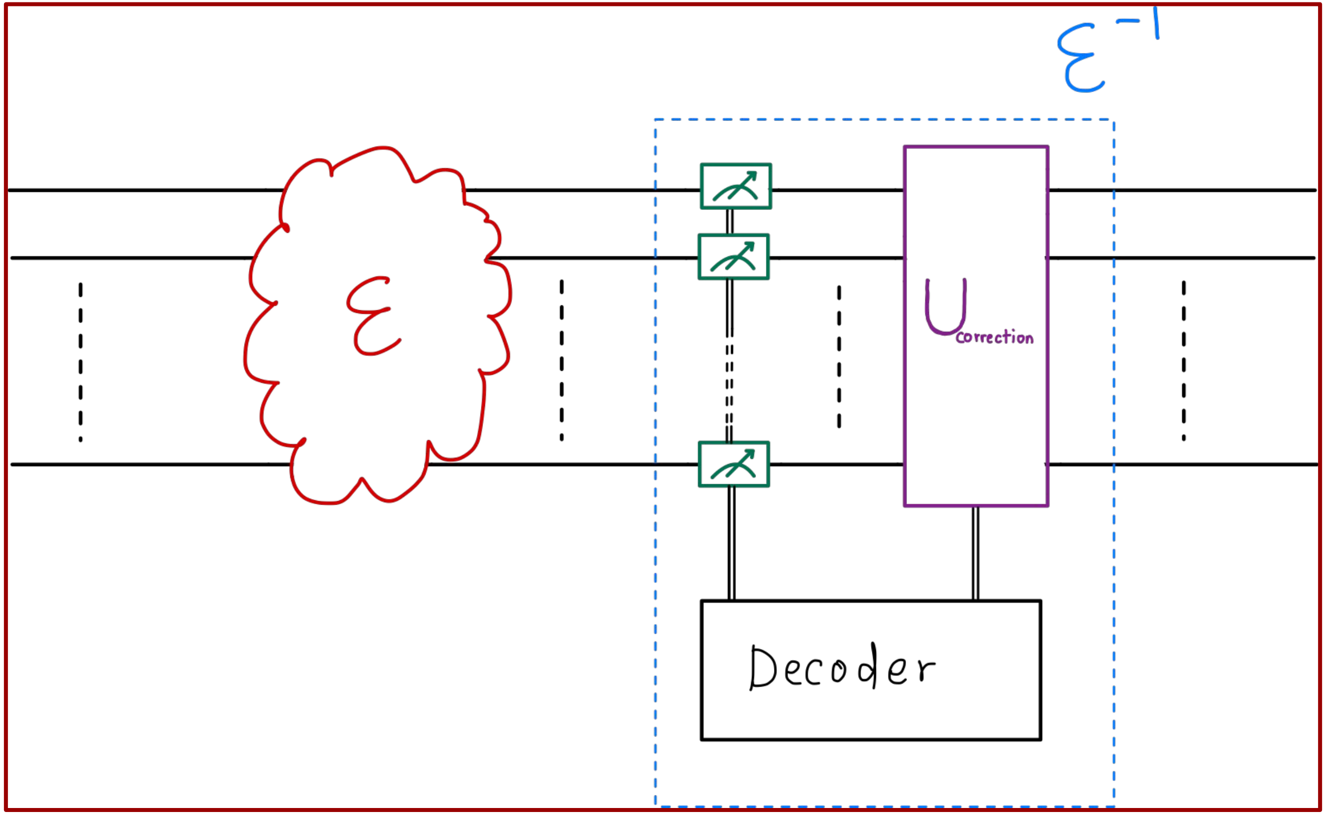
Overview

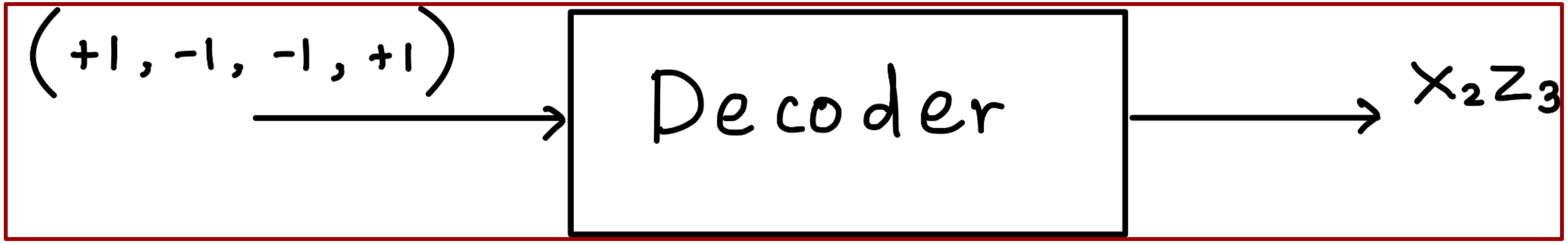
- Quantum Error Correction and decoders
- Prior work by Fujisaki et al.
- Our improvement
- Fujisaki et al.'s results
- Our results
- Conclusion

General QEC Protocol

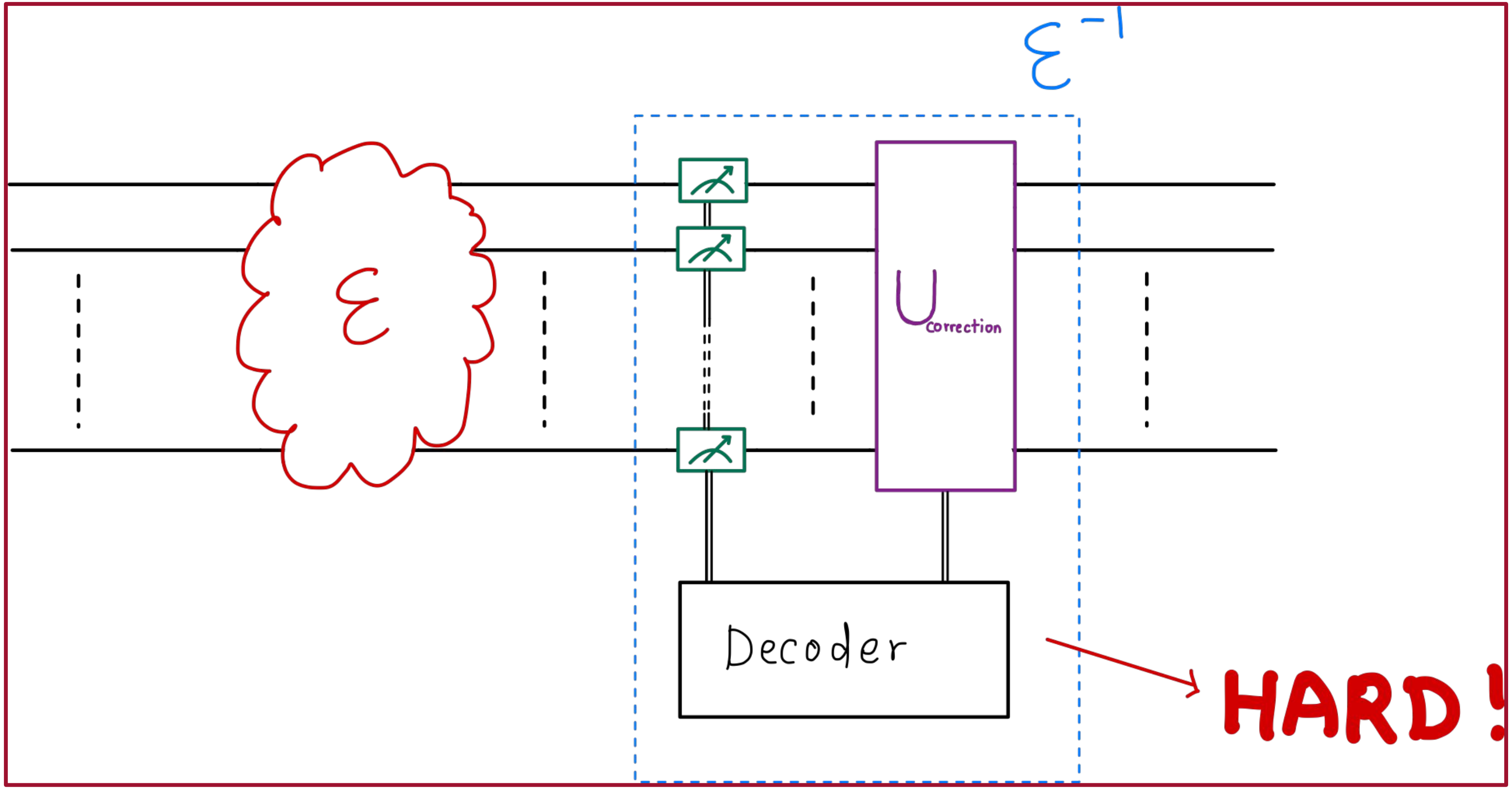


What does a decoder do?

- Given a syndrome (stabilizer measurement outcome), it determines which error has occurred.



General Quantum Error Correction (QEC) Protocol



Why is decoding an issue?

[[3,1,3]] bitflip repetition code

- Stabilizer Generators:

$$S_1 = Z_1 Z_2\,,$$

$$S_2 = Z_2 Z_3\,.$$

- Measuring the stabilizers gives the syndrome:

$$s_1, s_2 \in \{+1, -1\}\,.$$

[[3,1,3]] bitflip repetition code

s_1	s_2	Error
+1	+1	No Error
-1	+1	X_1
+1	-1	X_2
-1	-1	X_3

So, why is decoding an issue?

[[n,k,d]] code

- Number of stabilizers = $n - k$

- Number of different possible syndrome is = $2^{n - k}$

So what’s the problem exactly?

[[228,12,18]] qLDPC code

- Number of stabilizers = 228 - 12 = 216
- Number of different possible syndrome is = 2^{216}
- Assume that you magically generate an entry in the lookup table for one syndrome in 0.000000000000001 seconds.
- How much time would it take to generate the lookup table?

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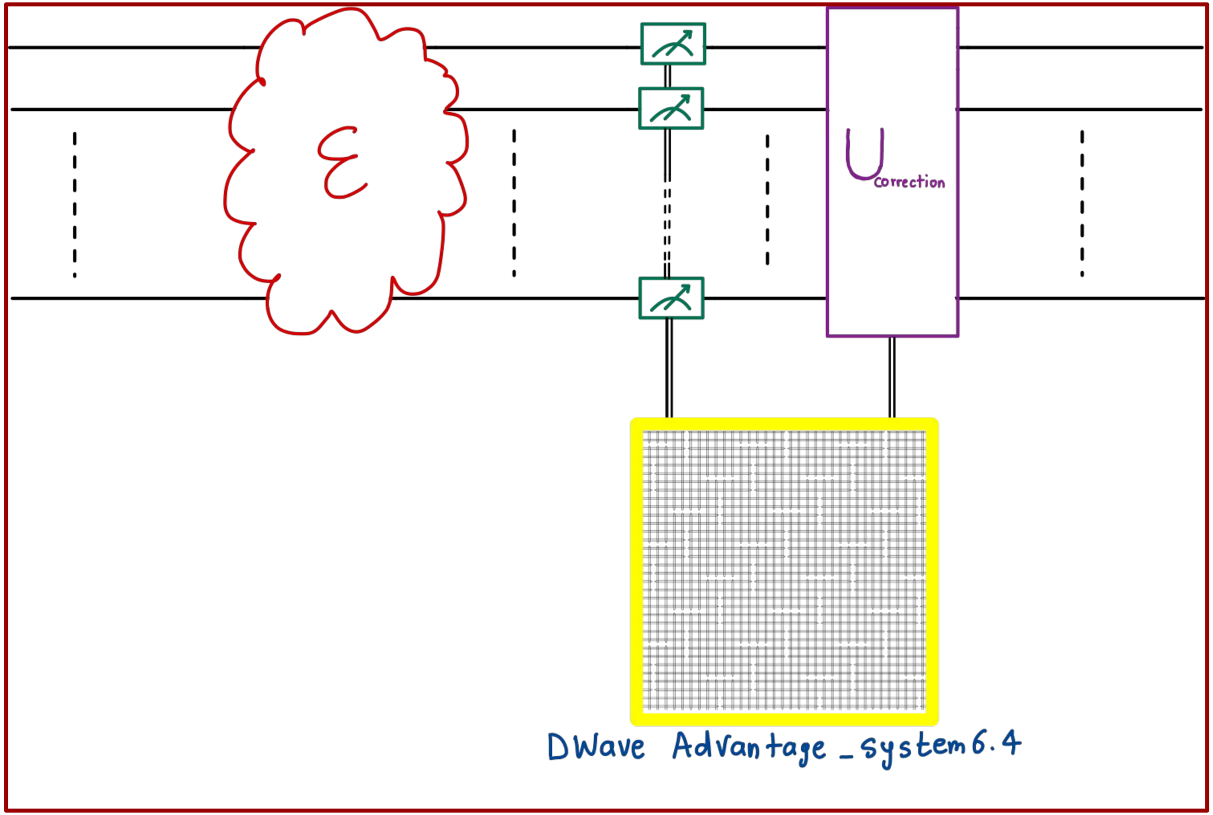
10000000000000000000000000000000000000000000 years

And it would also take you 1000000000000000000000000000000000000000000000000000 Terrabytes of memory

Our Protocol



*Our work

But why?

How can we decode using an annealer?

- Annealing devices can be used to find ground states of Hamiltonians.

- Can we define a Hamiltonian such that its ground state is the answer to the decoding problem?

  Yes!

- How can we construct such a Hamiltonian?

Constructing the *Decoder* Hamiltonian

Fujisaki et al.[1] devised a construction for the *decoder* Hamiltonian, which is given by:

$$H = -J \sum_{i=1}^{N_s} s_i \prod_{j \in \hat{S}_i}^{4} \sigma_j - h \sum_{i=1}^{N_d} \sigma_i \,,$$

where,

- N_s = # of X stabilizers
- N_d = # of data qubits in QEC code
- $\hat{S}_i$ = i^{th} X stabilizer

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[1] Fujisaki et al., Phys. Rev. Research 4, 043086 (2022)

Improving Fujisaki et al.'s Hamiltonian expression

$$H = -J \sum_{i=1}^{N_s} \left(s_i \prod_{j \in \hat{S}_{i_z}} \sigma_j \prod_{k \in \hat{S}_{i_x}} \sigma_{k+N_d} \right) - h \sum_{i=1}^{2N_d} \sigma_i \, .$$

Fujisaki et al.'s expression	**Our expression**
Works only for Surface codes	Works for all Stabilizer codes
Explicit only for Z error correction	Explicit for correcting all, X, Y and Z errors

Annealing Decoding Example using the [[3,1,3]] Bitflip code

- Stabilizer Generators:

$$S_1 = Z_1 Z_2\,,$$
$$S_2 = Z_2 Z_3\,.$$

- Measuring the stabilizers gives the syndrome:

$$s_1, s_2 \in \{+1, -1\}\,.$$

- Many-body Ising Hamiltonian (*decoder* Hamiltonian):

$$H = -J(s_1 Z_1 Z_2 + s_2 Z_2 Z_3) - h(Z_1 + Z_2 + Z_3)\,.$$

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Ground States for Various Syndromes

$$H = -J(s_1 Z_1 Z_2 + s_2 Z_2 Z_3) - h(Z_1 + Z_2 + Z_3) .$$

- $s_1 , s_2 = +1, +1 \Rightarrow |\psi_g\rangle = |000\rangle \Rightarrow$ **No error**
- $s_1 , s_2 = +1, -1 \Rightarrow |\psi_g\rangle = |001\rangle \Rightarrow$ **X error on qubit 3**
- $s_1 , s_2 = -1, +1 \Rightarrow |\psi_g\rangle = |100\rangle \Rightarrow$ **X error on qubit 1**
- $s_1 , s_2 = -1, -1 \Rightarrow |\psi_g\rangle = |010\rangle \Rightarrow$ **X error on qubit 2**

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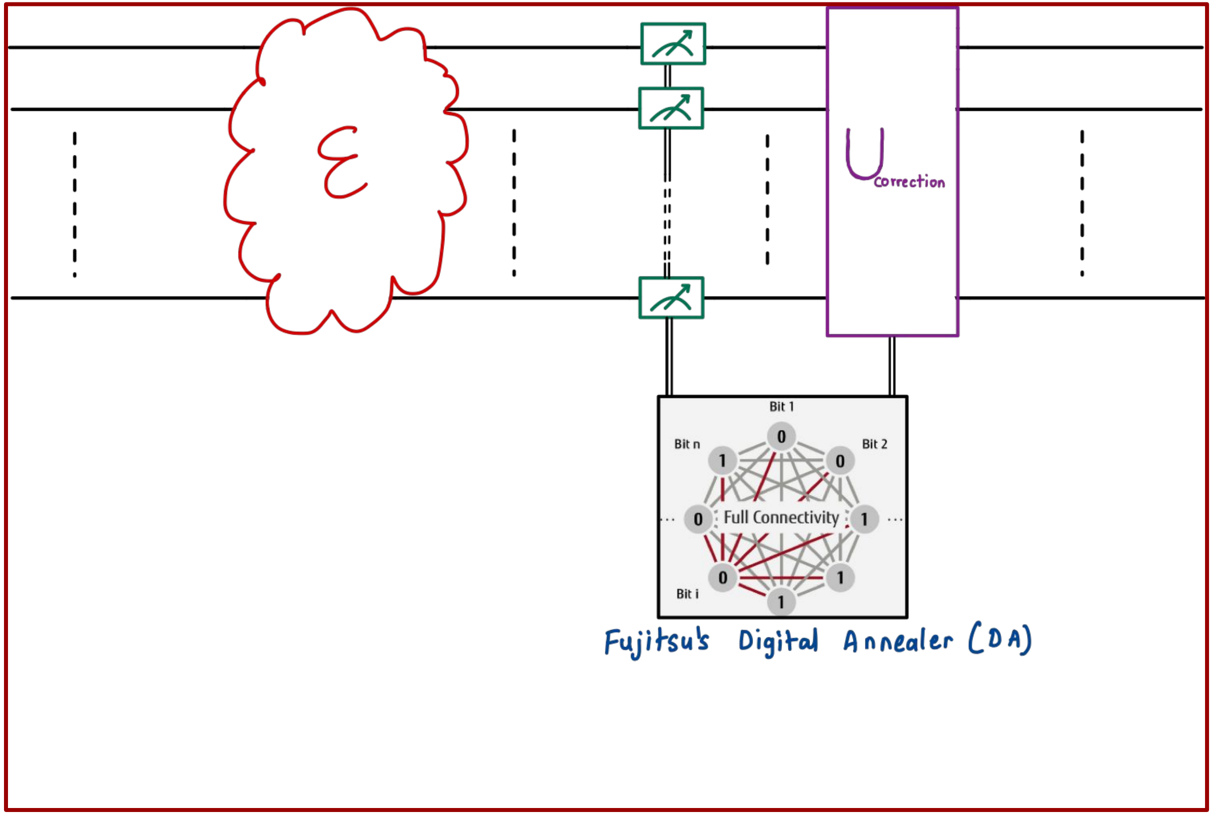
[[3,1,3]] Bitflip Code Takeaways

- We can use the error syndrome to define a unique *decoder* Hamiltonian.
- Each unique *decoder* Hamiltonian has a non-degenerate ground state, which is the answer to the decoding problem.
- Using an annealer, we can find this ground state.
- Assuming the error that created the syndrome is correctable, we will successfully recover the logical state.

Universality of the Annealing Decoder

- We have constructed the *decoder* Hamiltonian for the following codes, and verified that the ground states give the solution for all correctable errors to the decoding problem:
    - [[3,1,3]] Bitflip code
    - [[5,1,5]] Bitflip code
    - [[5,1,3]] Perfect code
    - [[7,1,3]] Color code
    - [[9,1,3]] Shor code
    - [[9,1,3]] Surface code

Fujisaki et al.'s Work



Performance of the Annealer Decoder

- Fujisaki et al.[1] used a Digital Annealer (DA) to solve the decoding problem for the surface code.

- There, the authors benchmark the decoder to understand its performance using two metrics:

  1. Accuracy: How good is the annealing decoder at finding the solution to the decoding problem as the difficulty increases?
  2. Scaling: How many iterations of the DA are needed to find the solution as a function of the size of the syndrome?

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[1] Fujisaki et al., Phys. Rev. Research 4, 043086 (2022)

Error Model

- Fujisaki et al.[1] assumes a single error rate noise model (X, Y, and Z errors occur on each qubit with equal probability, independently of one another), often referred to as depolarizing noise.

- Using this noise model, the performance of the decoder is analyzed as a function of increasing error rate.

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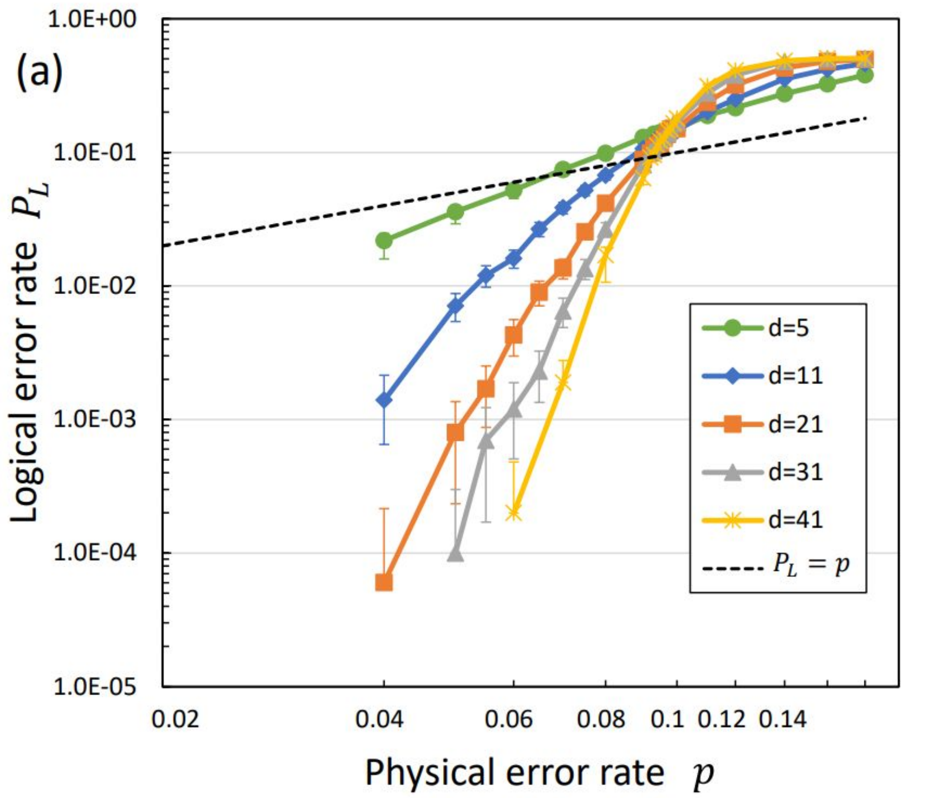
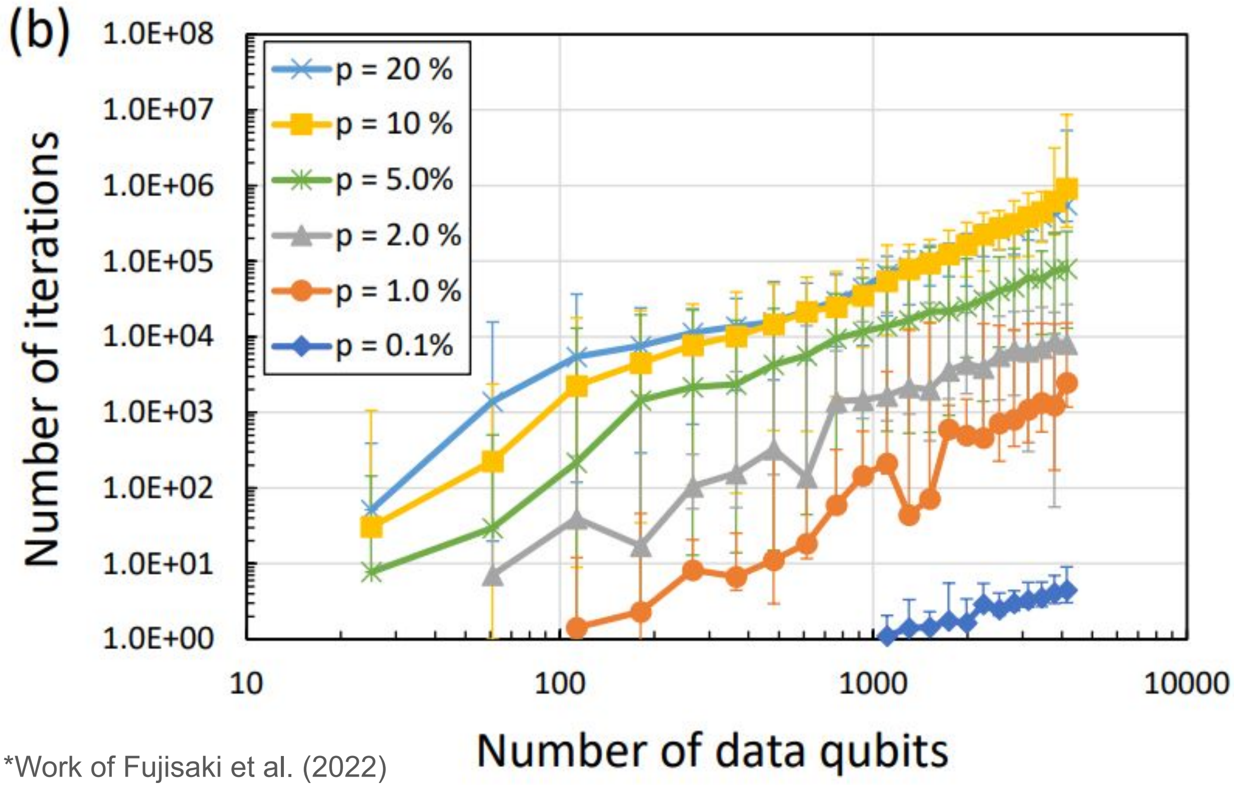
[1] Fujisaki et al., Phys. Rev. Research 4, 043086 (2022)

DA Decoder Logical Error Rate vs. Physical Error Rate



*Work of Fujisaki et al. (2022)

Scaling of the Annealing Decoder on DA



*Work of Fujisaki et al. (2022)

DA Decoder Scaling vs. SA vs. MWPM

TABLE II. Order of polynomial n for the fixed physical error rates of the DA, SA, and MWPM decoders.

Physical error rate p (%)	n for DA	n for SA	n for MWPM
0.1	1.01	2.77	2.06
1.0	1.79	2.38	2.10
2.0	1.74	2.25	2.24
5.0	1.84	2.11	2.50
10	1.81	2.05	2.72
20	1.54	2.04	2.83

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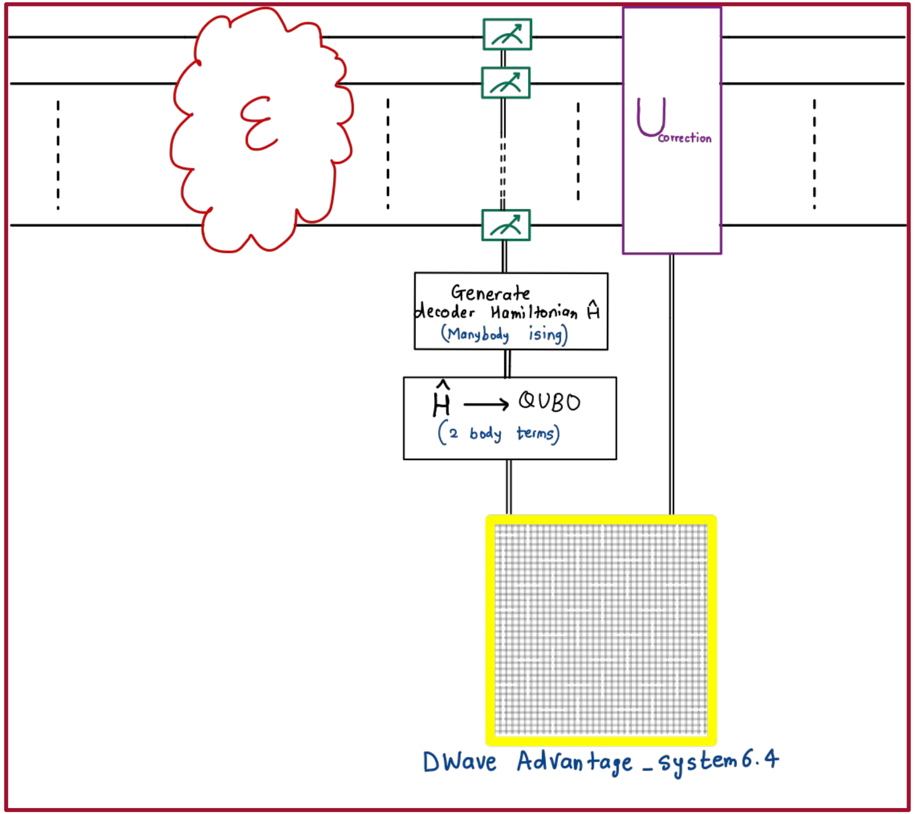
*Work of Fujisaki et al. (2022)

Our Work

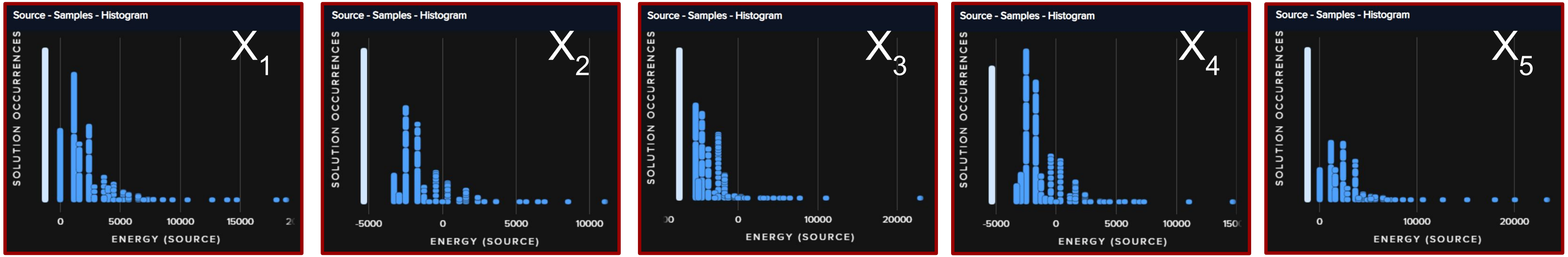


*Our work

Implementing Decoding on a Quantum Annealer

- In order to find the ground state of the many-body *decoder* Hamiltonian, we must construct a QUBO problem whose solution is the same as the ground state of the *decoder* Hamiltonian

- Fujisaki et al.[1] provides a mapping to the QUBO, which we used to decode the [[5,1,5]] bitflip code on `dwave_advantage_system_6.4`



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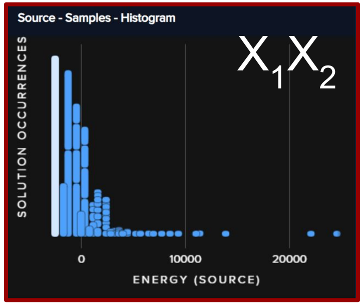
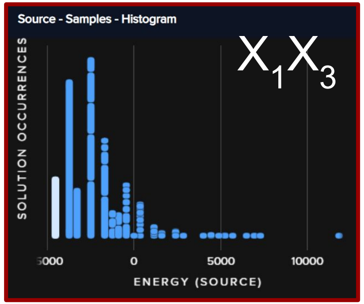
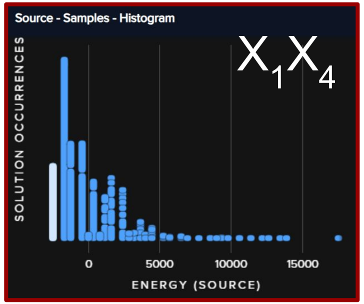
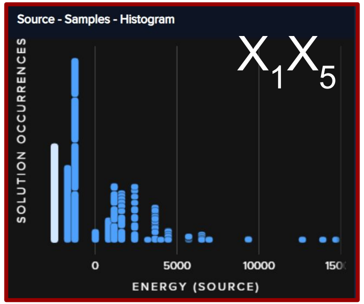
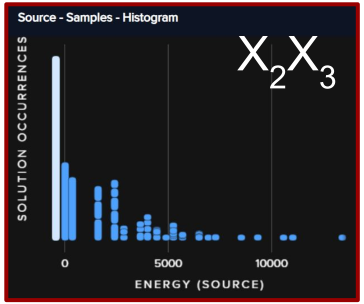
Implementing Decoding on a Quantum Annealer

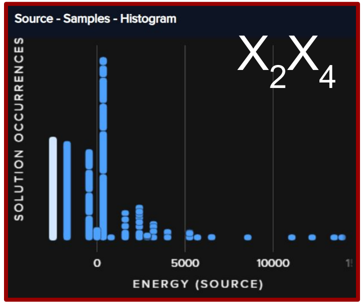


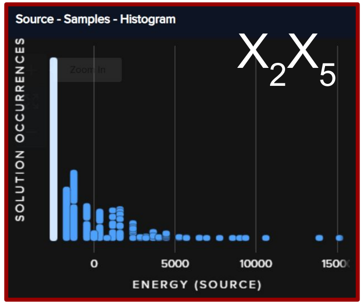


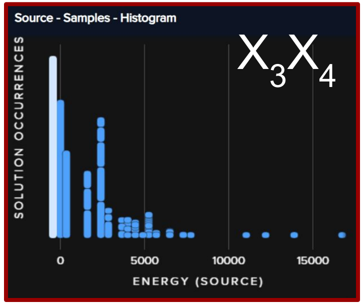


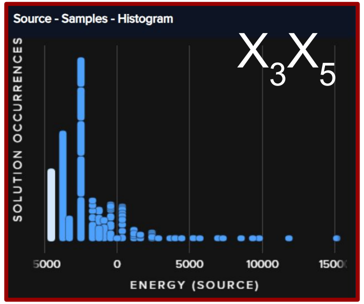


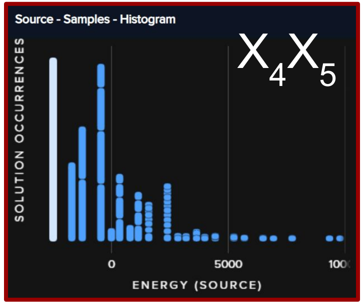












*Our work

Advantages

- Decodes *all* stabilizer codes

- Scales well as the code/system size increases; better than SA or MWPM by one polynomial order

- Annealers are *very good* at giving a *decent* answer *very fast*, which sounds like ideal case when we reach ~100k+ qubits

- Photonic interconnects would potentially eliminate hot classical buses and allow our apparatus to be integrated into the same housing (DR) as the QPU

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Disadvantages

- In case of Y_1 error and Z_1Z_4 error having the same syndrome, this decoder will prefer Z_1Z_4 error.

- Only viable in the cases of qubit modalities where decoherence time is much higher than the order of the annealing time.

  (Useless for superconducting qubits)

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Future Work

- Currently in the process of making our code robust to run large scale experiments for arbitrary stabilizer codes

- Would like to improve the formulation of our equation to take into account device specific noise models that are more representative of real world noise processes, and not just simplified, unrealistic noise such as depolarizing noise

- Potentially extend to non-additive codes

Thank you! ☺