SNAIL Parametric Amplifier (SPA)

PHYS 559 - Homework 3
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1 What is a SNAIL-based Parametric Amplifier?

SNAIL Parametric Amplifier (SPA) is a quantum-limited Josephson-junction-based 3-wave-mixing parametric amplifier which uses an array of SNAILs (see Section 1.2) as a source of tunable nonlinearity. The SPA was developed at Yale [1][2][3].

1.1 3-wave-mixing

Parametric amplifiers convert energy from a pump to a signal field with an extra idler field when stimulated by a signal field. For this energy conversion, when third-order nonlinearity is used, that is called 3-wave-mixing. The pump is at a higher frequency than the signal and idler mode, and one photon from the pump is converted into a photon in the signal mode and a photon in the idler mode.

$$\omega_{\text{Pump}} = \omega_{\text{Signal}} + \omega_{\text{Idler}} \,. \tag{1}$$

1.2 SNAIL

SNAIL stands for **S**uperconducting **N**onlinear **A**symmetric **I**nductive e**L**ement[1]. To perform this 3-wave-mixing, we need an asymmetrical potential around the minima, and SNAIL provides this as shown in figure 2. SNAIL consists of three large Josephson junctions (tunnelling energy E_J each) and one smaller Josephson junction (tunnelling energy αE_J) in parallel, as shown in figure 1. The ratio of the inductance of the smaller Josephson junction to the larger Josephson junction is α.

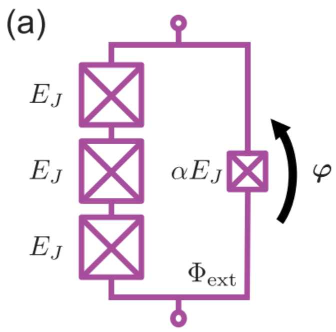

Figure 1: Circuit for a SNAIL [1]

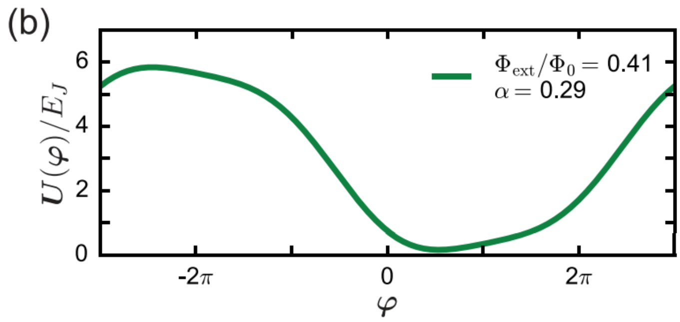

Figure 2: Potential of a SNAIL [1]

1.3 Physical realization of SPA

The SPA is created by embedding an array of M SNAILs at the centre of a $\frac{\lambda}{2}$ section of a microstrip transmission line. One end of the SPA is coupled to the signal transmission line (left-hand side of figure 3(b)) and the other end to the pump (right-hand side of figure 3(b)).
During the manufacturing, you can engineer the following parameters, i.e., knobs set during the fabrication:

[1] Fun fact: The reason why scientists went out of the way to name this circuit SNAIL is that it will then be in line with other circuits with acronyms named after Crustacean creatures like SLUG and SQUID.

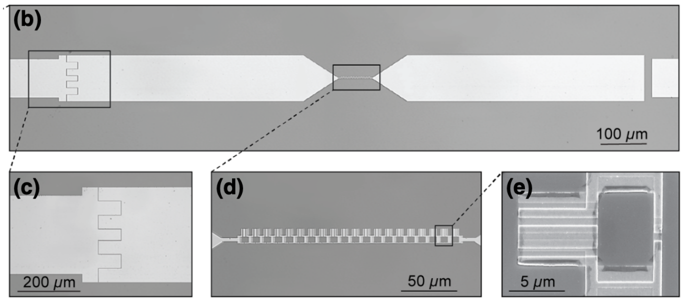

Figure 3: (b) Microstrip resonator, (c) The capacitor coupling the microstrip resonator to signal port (d) The SNAIL array with $M = 20$ (e) A single SNAIL [3]

- The Josephson inductance (E_J), Junction size ratio (α) seen in figure 1,
- Number of SNAILs in the array (M) seen in figure 3(d)
- Linear inductance, Coupling capacitance (κ) seen in figure 3(c).

Other than these, you can also control the flux (Φ) through the device *in situ*.

2 How SPA Works?

We have the Hamiltonian of the following form:

$$H/\hbar = \omega_a(\Phi)a^\dagger a + g_3(\Phi)(a + a^\dagger)^3 + g_4(\Phi)(a + a^\dagger)^4 + \cdots \tag{2}$$

The first term of Eq(2) is a linear harmonic oscillator term with resonant frequency ω_a. The second term is associated with the cubic nonlinearity of the potential energy; The third term is associated with the quartic nonlinearity of the potential energy, and so on, where g_n are the n^{th}-order nonlinearities. For a 3-wave-mixing amplifier, we use the first two terms; the rest of the terms are *parasitic terms*, which we try to suppress as much as possible. All the terms are functions of external flux (Φ) through each SNAIL loop, which we can tune *in situ*. It turns out that for specific values of Φ, this g_4 nonlinearity actually goes to zero, and you can suppress this g_4 nonlinearity very well [3], which is also known as Kerr-nonlinearity.

To operate the SPA as a 3-wave-mixing amplifier, we apply a strong microwave pump tone at pump frequency $\omega_p \approx 2\omega_a$, with amplitude α_p. Under the presence of this pump, the Hamiltonian transforms such that the effective Hamiltonian looks as follows:

$$H/\hbar = \Delta a^\dagger a + g(a^2 + a^{\dagger 2}), \quad \text{where, } g = 2g_3\alpha_p \ \& \ \Delta = \omega_a - \frac{\omega_p}{2} \tag{3}$$

The second term, due to the presence of a^2 and $a^{\dagger 2}$ essentially tells us that photons are being created or destroyed in pairs of two, where the prefactor g depends on the amplitude of the pump α_p and g_3 nonlinearity which itself depends on Φ. Therefore, in the presence of this type of interaction, if we apply a small signal to the input port, then due to the second term of Eq(3), the pump photon gets destroyed into a pair of photons, one at the signal frequency (ω_s) which comes out of the device and the second photon at the idler frequency ($\omega_i = \omega_p - \omega_s$) as described in section 1.1.

So, we can see that a weak signal simulates many more photons to come out of the device at the signal frequency. Hence, under the action of the pump, this device behaves like a parametric amplifier, and we obtain power gain G, given in [2]. We always get the maximum gain at $\omega_s = \frac{\omega_p}{2}$.

3 Conclusion

In conclusion, SNAIL parametric amplifiers can be optimized for the larger compression power necessary for the readout of medium-scale quantum processors. SPA achieves 1-dB compression powers (on par with other resonant quantum-limited para-amps) but over the larger tunable bandwidth of 1 GHz of the device for $M = 20$, and without sacrificing any other desirable characteristics such as quantum-limited noise performance, etc. [2]. $P_{1\text{ dB}} = -102$ dBm has been demonstrated in [3]. It also gives a methodology for suppressing spurious Kerr effects and improved dynamic range.

References

[1] N. E. Frattini et al. "3-wave mixing Josephson dipole element". In: *Applied Physics Letters* 110.22 (May 2017). DOI: 10.1063/1.4984142. URL: https://doi.org/10.1063%2F1.4984142.

[2] N. E. Frattini et al. "Optimizing the Nonlinearity and Dissipation of a SNAIL Parametric Amplifier for Dynamic Range". In: *Physical Review Applied* 10.5 (Nov. 2018). DOI: 10.1103/physrevapplied.10.054020. URL: https://doi.org/10.1103%2Fphysrevapplied.10.054020.

[3] V.V. Sivak et al. "Kerr-Free Three-Wave Mixing in Superconducting Quantum Circuits". In: *Physical Review Applied* 11.5 (May 2019). DOI: 10.1103/physrevapplied.11.054060. URL: https://doi.org/10.1103%2Fphysrevapplied.11.054060.