Constraint Programming

the user states the problem, the computer solve it

Nadjib Lazaar
Ing - Phd - Assistant Professor - University of Montpellier - COCONUT Team
http://www.lirmm.fr/~lazaar/

27/09/2021

Artificial Intelligence

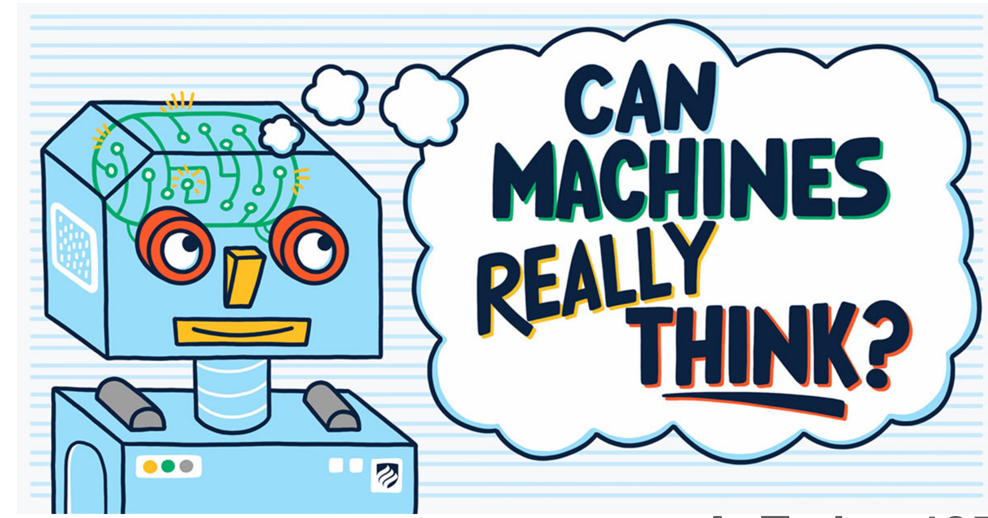
Received ideas

Blind men and an elephant "盲人摸象"

- ☑ AI is 100% artificial
  - AI needs our help
- ☑ AI would overall surpass human intelligence
  - Living being is far from a machine being
- ☑ AI is dangerous
  - Not about AI, but about data sovereignty
- ☑ An algorithm = an IA
  - 40 % of European AI start-ups are not using IA

Artificial Intelligence

Definition



A. Turing 1950

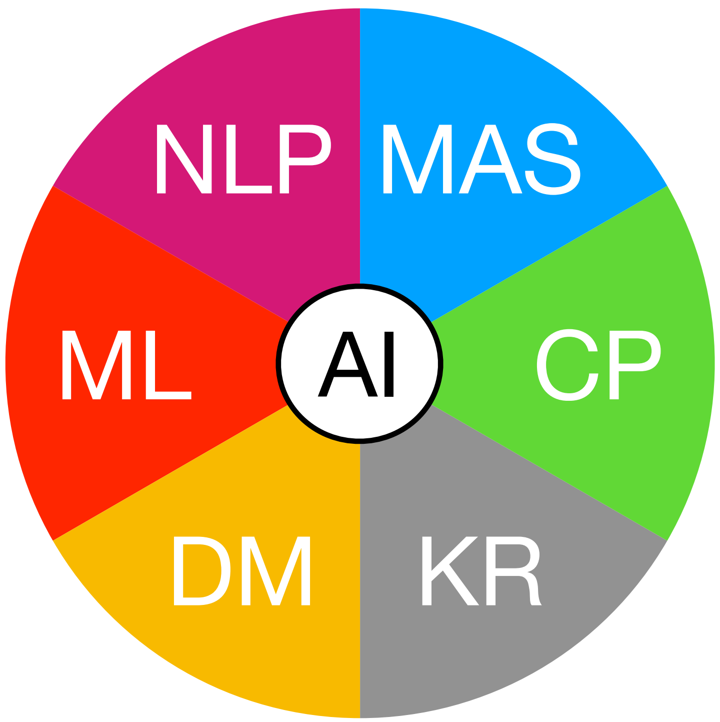
« The science of making machines do things that would require intelligence if done by men » Marvin Minsky

That is, be able to:

- use a language
- make abstractions and to express concepts
- solve different kind of human problems,
- improve themselves

Artificial Intelligence

AI Topics

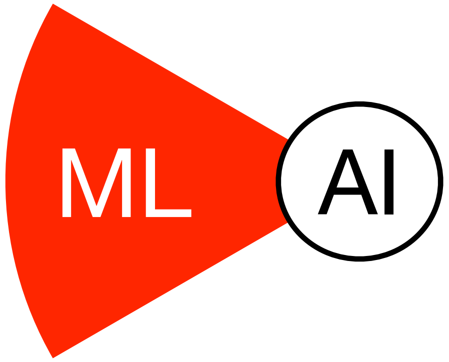


- NLP: Natural Language Processing
- MAS: Multi-Agent Systems
- KR: Knowledge Representation
- DM: Data Mining
- ML: Machine Learning
- CP: Constraint Programming

COCONUT

Artificial Intelligence

ML Topics



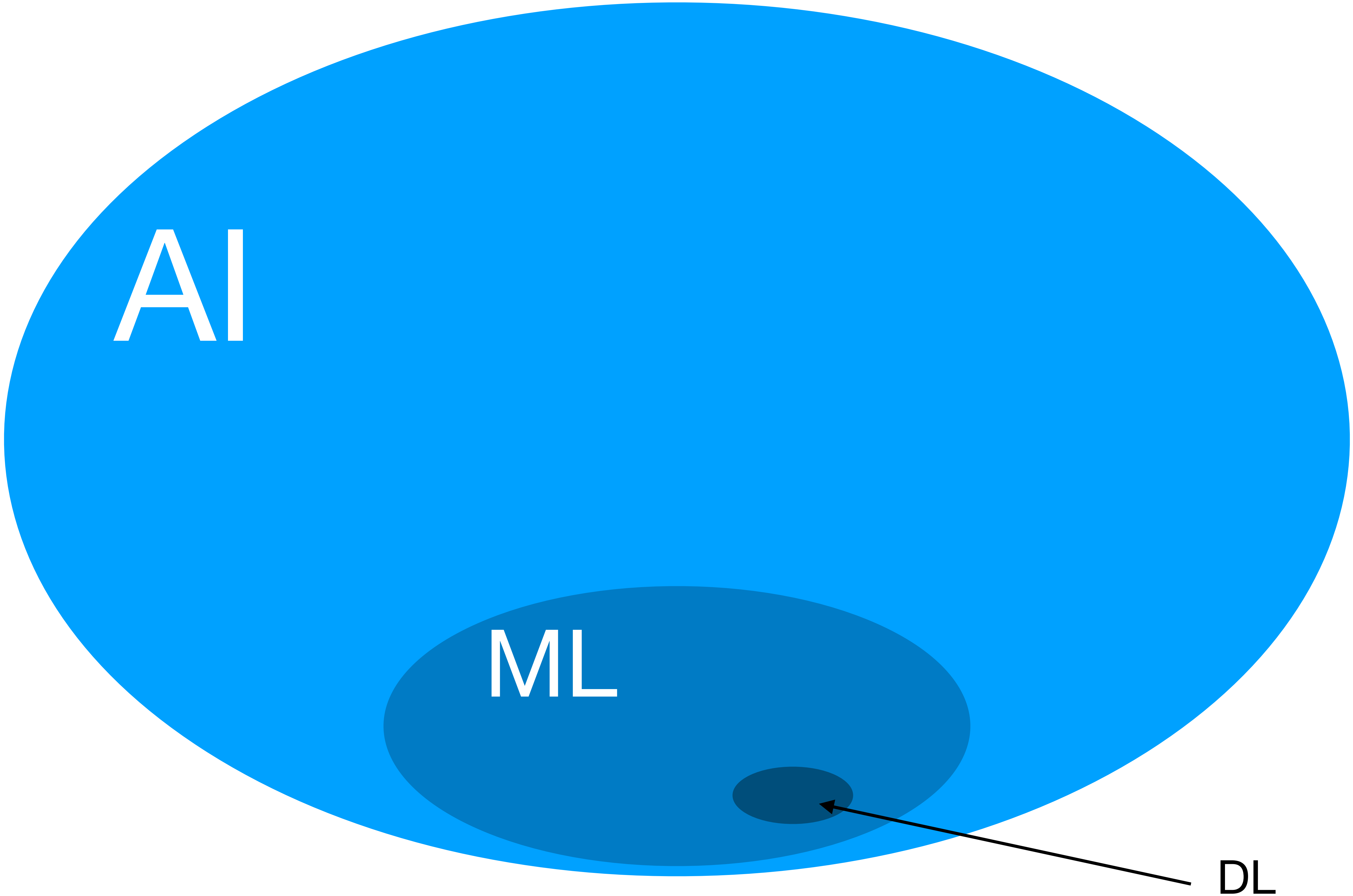
1. Active Learning
2. Adversarial Machine Learning
3. Bayesian Optimization
4. Classification
5. Clustering
6. Cost-Sensitive Learning
7. Deep Generative Models

8. Deep Learning

9. Developmental Learning
10. Dimensionality Reduction and Manifold Learning
11. Ensemble Methods
12. Explainable Machine Learning
13. Feature Selection
14. Learning Sparse Models
15. Federated Learning
16. Interpretability
17. Kernel Methods
18. Knowledge-based Learning
19. Learning Generative Models
20. Learning Graphical Models
21. Learning Preferences or Rankings
22. Learning Theory
23. Multi-instance;Multi-label;Multi-view learning
24. Neuro-Symbolic Methods
25. Online Learning
26. Probabilistic Machine Learning
27. Recommender Systems
28. Reinforcement Learning
29. Relational Learning
30. Semi-Supervised Learning
31. Structured Prediction
32. Tensor and Matrix Methods
33. Time-series;Data Streams
34. Transfer, Adaptation, Multi-task Learning
35. Trusted Machine Learning
36. Unsupervised Learning

Artificial Intelligence

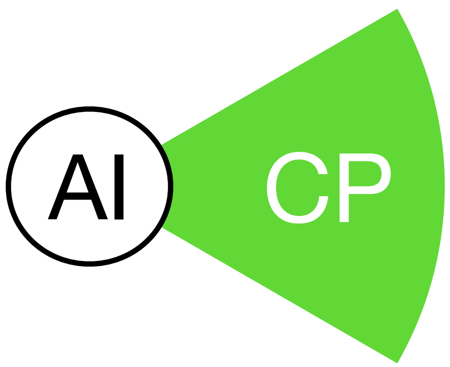
ML Topics



Constraint Programming

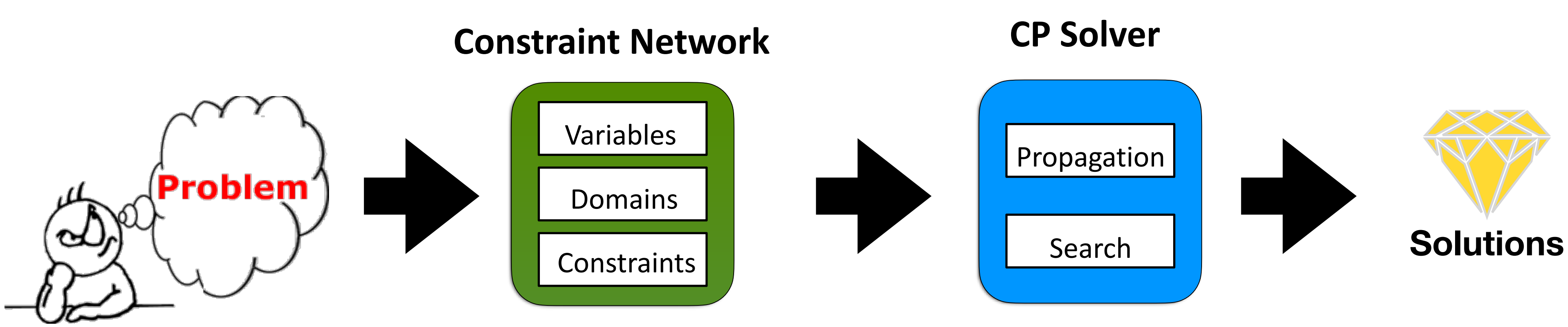
Definition

A formalism to model (**constraint network**) and to solve (**constraint solver**) combinatorial problems (scheduling, planning,…).



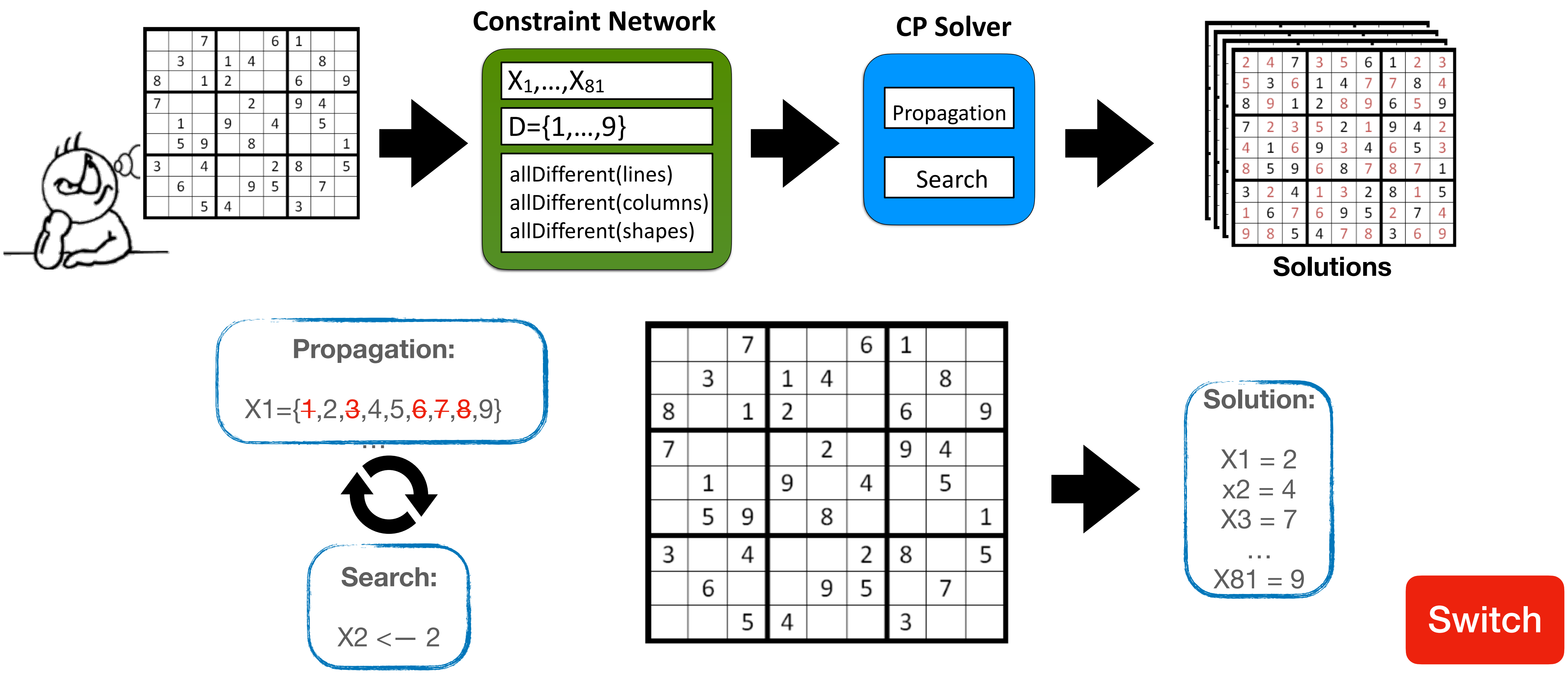
Examples of constraints :

- $X_1+X_2+X_3=5$
- $\text{allDifferent}(X_1,..X_n)$
- $c(X_i,X_j)= \{(1,1), (2,1),(2,3), (4,4)\}$



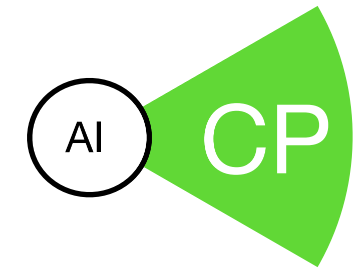
Constraint Programming

Example



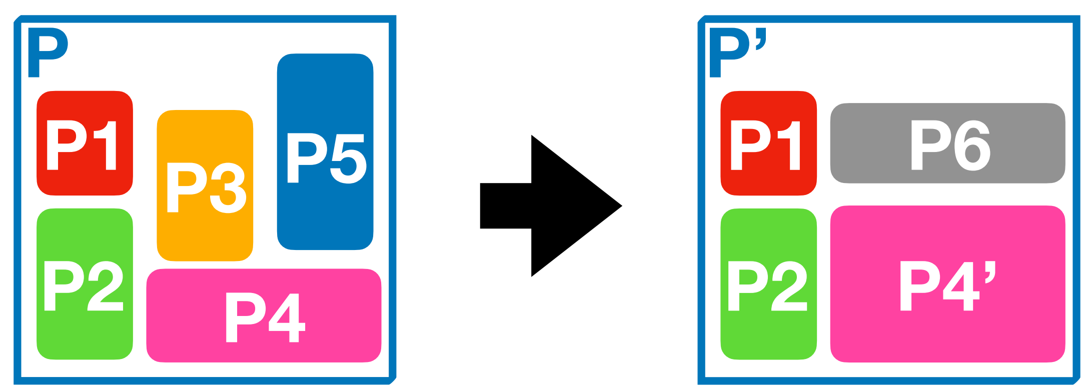
Constraint Programming

Why CP?



- Combinatorial problems can be solved by Integer Linear Programming (ILP) or by propositional satisfiability (SAT)

- Advantages of CP :
  - Compactness
  - Expressiveness
  - And (often) efficiency



Constraint Programming

Why CP?

AI CP

- Sudoku 9x9:
  - SAT = 6 500 clauses
  - CP: `sudoku.mod` with n=9

Variables $X = \{X_{1,1}, \ldots X_{n,n}\}$

Domaine des variables : $\{1,\ldots,n\}$

Contraintes :

allDifferent($X_{i,1},..,X_{i,n}$), $\forall i$ (rows)

allDifferent($X_{i,1},..,X_{i,n}$), $\forall i$ (columns)

allDifferent($X_{i,j}$), $\forall i,j$ (squares)

sudoku.mod

Sudoku 36x36

17		5				16		24												2		19				27		13									
	22			18						32		9	4		21	3		12		10			28						8		15	6		23	25		31
2		19				27		13												17		5				16		24									
						4		21				16	32	24		9								6		23				27	8	13		15	
	10			28						8		15	6		23	25		31		22			18						32		9	4		21	3		12
						6		23				27	8	13		15								4		21				16	32	24		9	
32		9								16		24					17		5	8		15								27		13					2		19
				17		5							3		12										2		19							25		31			
8		15								27		13					2		19	32		9								16		24					17		5
7		20				1		11										16		24	26		34				29		35									27		13
				2		19							25		31									17		5							3		12			
26		34				29		35									27		13	7		20				1		11									16		24
	17		22	5	18					36		14									2		10	19	28					30		33						
1		11	16		24	32		9						36			14			29		35	27		13	8		15						30			33	
	2		10	19	28					30		33									17		22	5	18					36		14						
										4		21					1		11											6		23					29		35
29		35	27		13	8		15					30			33			1		11	16		24	32		9							36			14	
										6		23					29		35											4		21					1		11
5		17				24		16												19		2				13		27									
	18			22						9		32	21		4	12		3		28			10						15		8	23		6	31		25
19		2				13		27												5		17				24		16									
						21		4				24	9	16		32								23		6				13	15	27		8	
	28			10						15		8	23		6	31		25		18			22						9		32	21		4	12		3
						23		6				13	15	27		8								21		4				24	9	16		32	
9		32								24		16					5		17	15		8								13		27					19		2
				5		17							12		3										19		2							31		25			
15		8								13		27					19		2	9		32								24		16					5		17
20		7				11		1										24		16	34		26				35		29									13		27
				19		2							31		25									5		17							12		3			
34		26				35		29									13		27	20		7				11		1									24		16
	5		18	17	22					14		36									19		28	2	10					33		30						
11		1	24		16	9		32						14			36			35		29	13		27	15		8						33			30	
	19		28	2	10					33		30									5		18	17	22					14		36						
										21		4					11		1											23		6					35		29
35		29	13		27	15		8					33			30			11		1	24		16	9		32							14			36	
										23		6					35		29												4					11		1

Constraint Programming

Why CP?

AI CP

- Sudoku 36x36:
  - SAT ~ 2 millions of clauses : cnf file of 160Gb
  - CP: `sudoku.mod` with n=36

```
Variables X= {X1,1, …Xn,n}

Domaine des variables : {1,…,n}

Contraintes :
allDifferent(Xi,1,..,Xi,n), ∀i (rows)
allDifferent(Xi,1,..,Xi,n), ∀i (columns)
allDifferent(Xi,j), ∀i,j (squares)
```

sudoku.mod

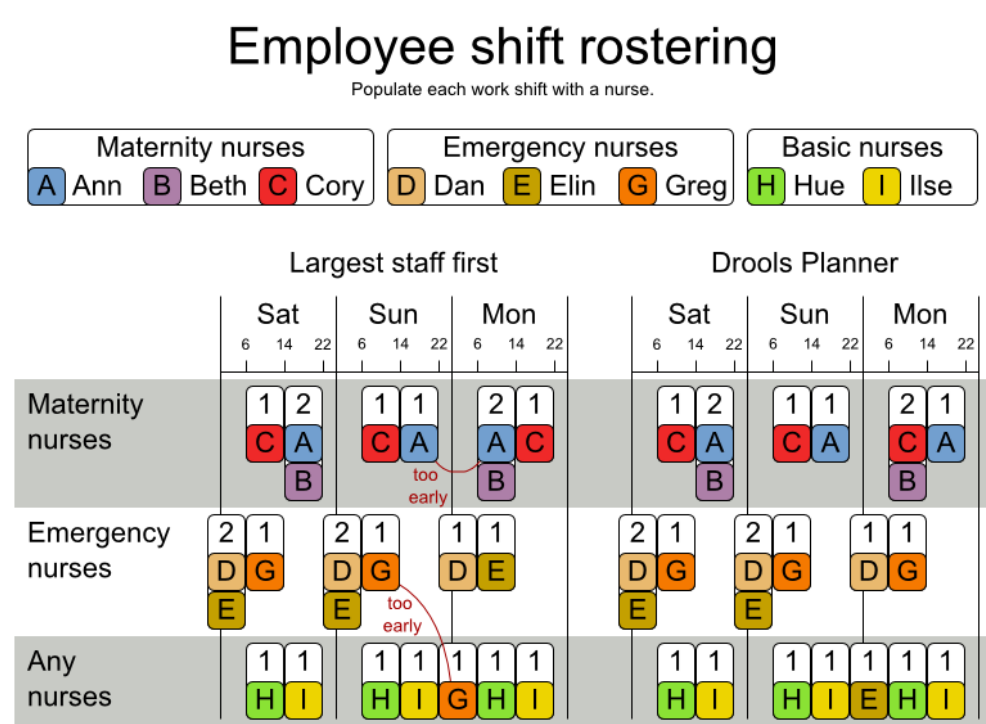
Switch

Constraint Programming

Why CP?

AI CP

- Nurse Rostering Problem
  - Best Linear Program: more than 10 000 lines



Switch

Constraint Programming

Solvers and CP platforms

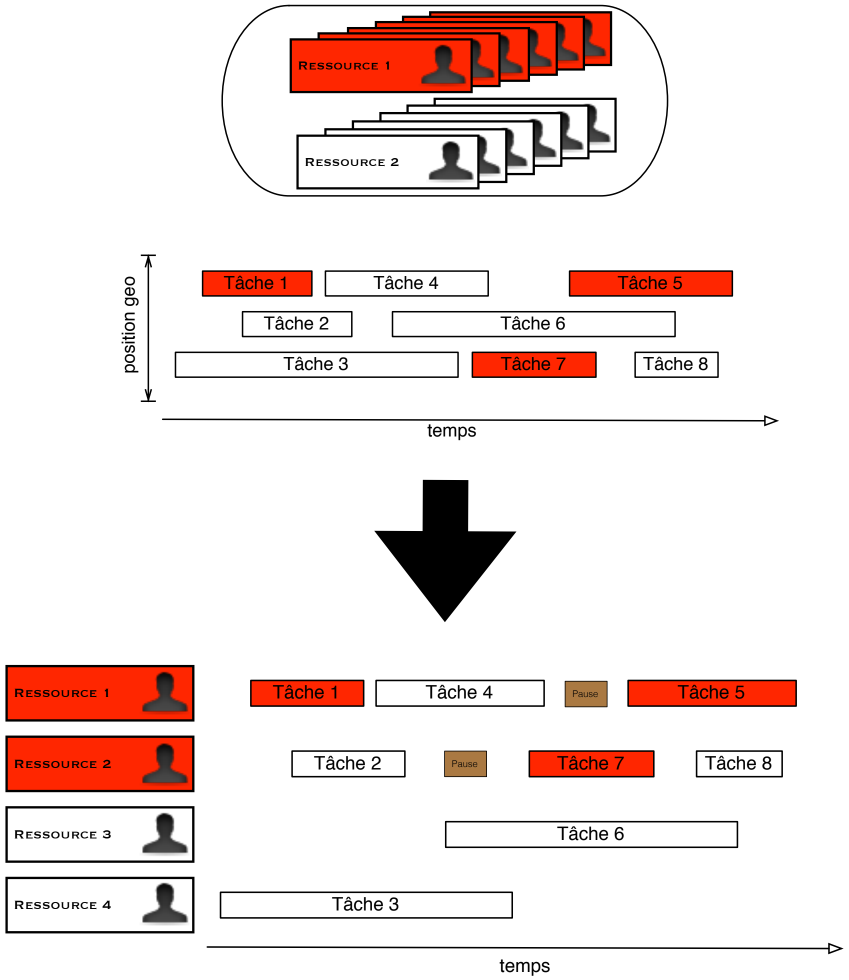
AI CP

IBM ILOG CP Optimizer (Java, C++,	IBM
Google OR-Tools (C++, Java, C#, Python)	Google OR-Tools
Artelys Kalis (Java, C++, Python)	ARTELYS KALIS™ Constraint Programming Library
SICStus Prolog (CLPFD bib in prolog)	SICStus 4
Gecode (C++)	G
Choco (Java)	CHOCO
Minizinc (high-level, solver-independent)	Mini Zn

Constraint Programming

SNCF train driver planning using CP

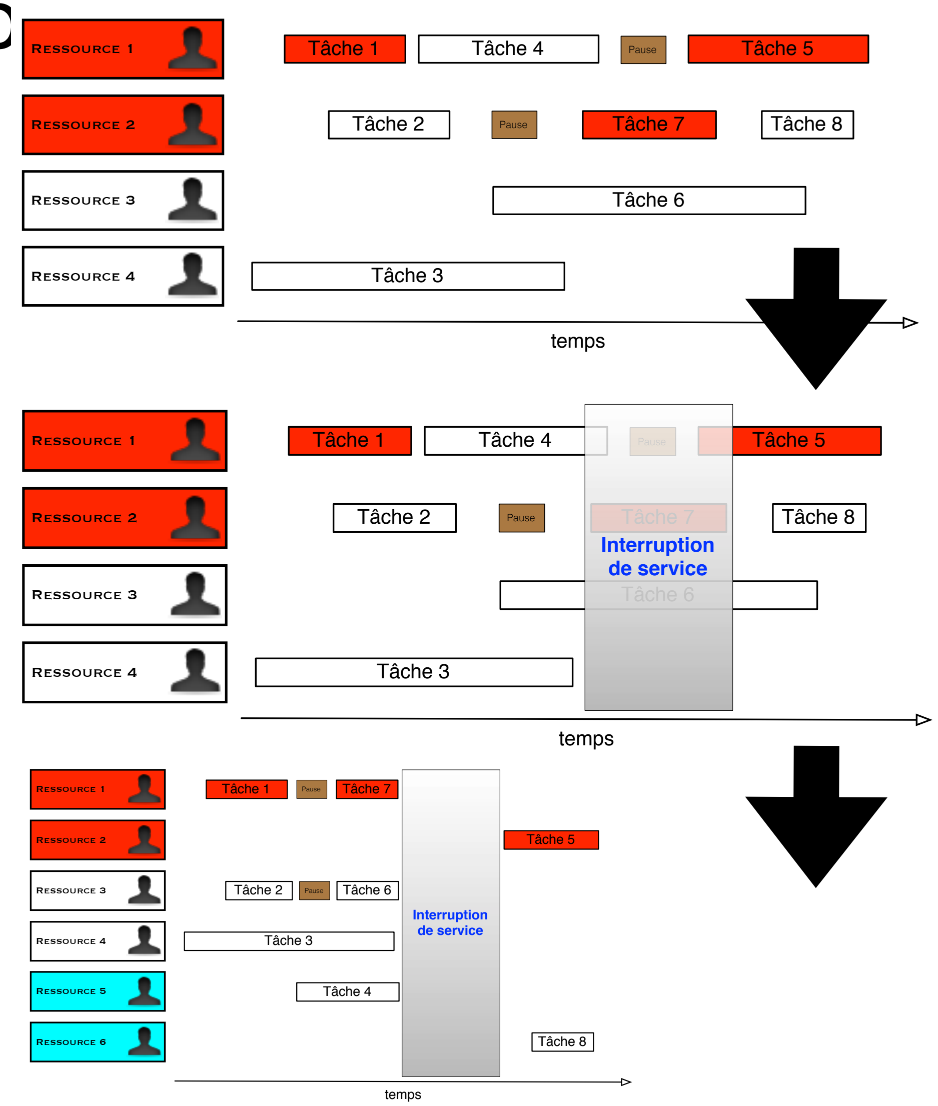
- Legal rules constraints:
  - daily working time
  - daily rest periods
  - …
- Preferences:
  - Type of lines
  - Day of rest
  - Place of rest
  - …
- Solution:
  - Best in terms of ressources
  - Robust one
  - Cheapest one
  - …



Constraint Programming

SNCF train driver planning using CP

- Maintaining an existing p



Constraint Programming

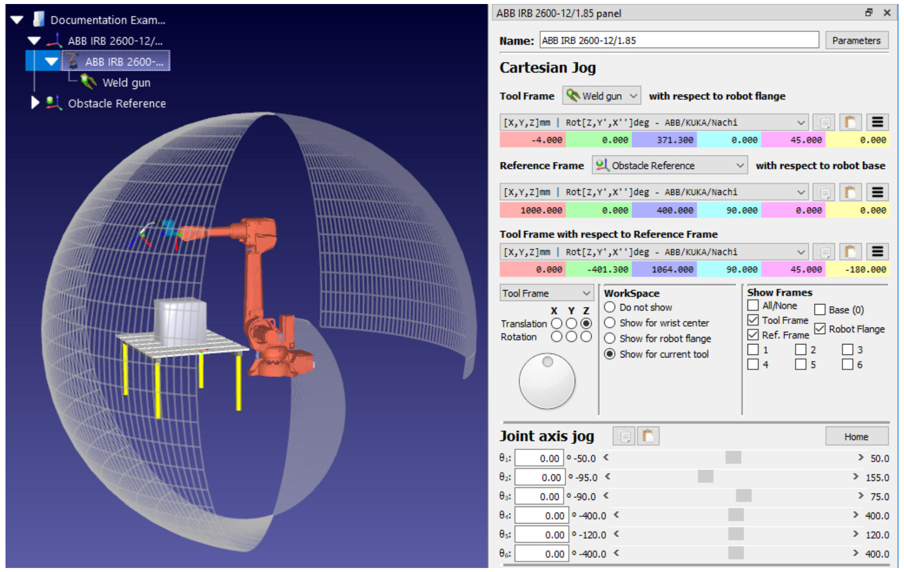
ABB Robotics partner projects

- SWMOD: Test Case Execution Scheduling with CP

"SWMOD deployed at ABB Robotics and used every day to schedule tests throughout several ABB centers in the world (Norway, Sweden, India, China)"

https://github.com/Makouno44/Robtest

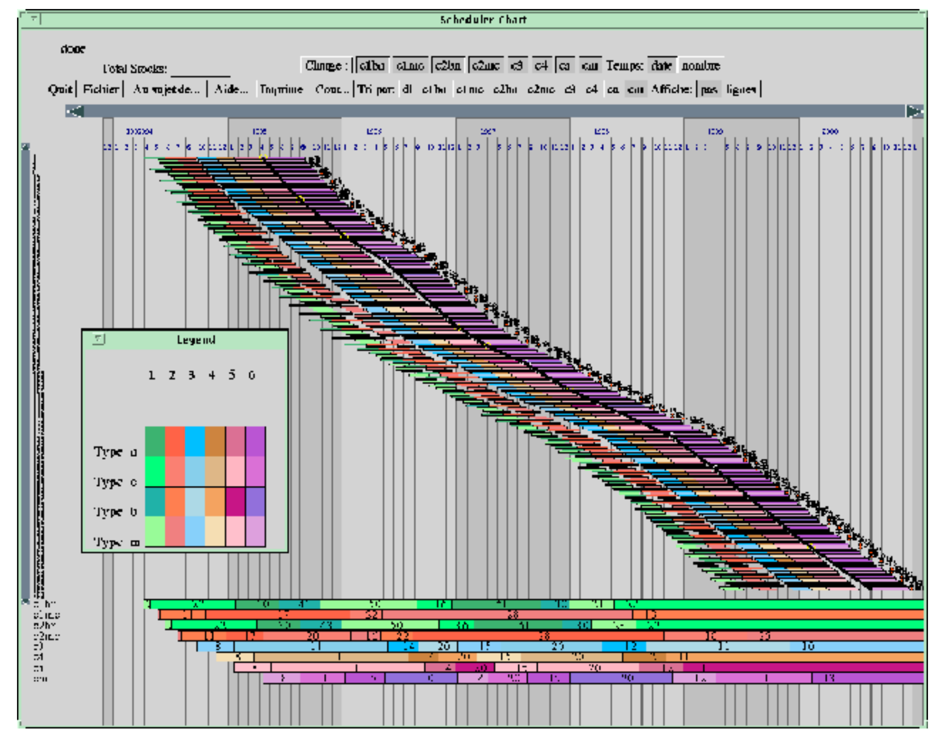
- Robtest: Optimal Stress Test Trajectories for Robots with CP



Constraint Programming

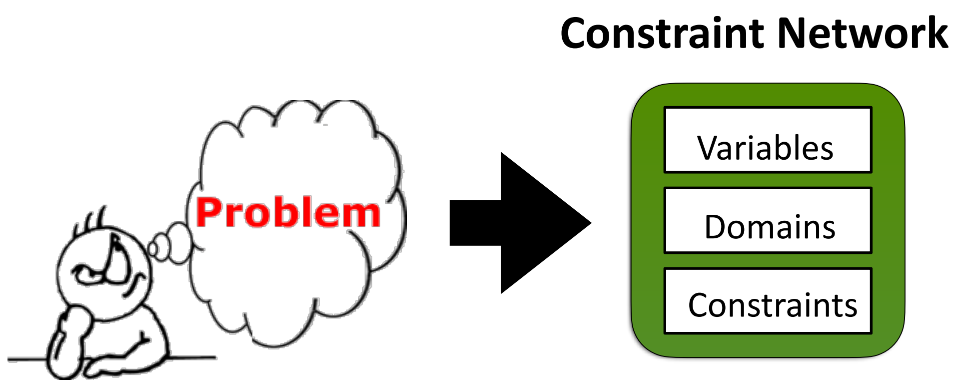
Aircraft Industry - Dassault Aviation

Assembly of Mirage aircraft



Constraint Programming

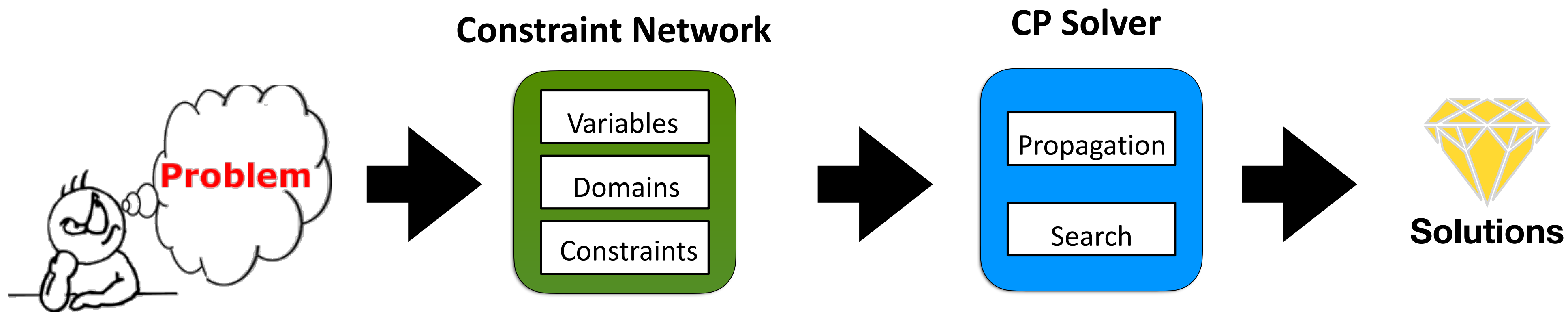
Modeling




```
Constraint Network N=(X,D,C)

X: finite set of variables
D: domain on X, where D(Xi) is a finite set of values for Xi
C: a set of constraints
```

Constraint Programming

Solving



Instantiation I

- Complete: assignment on X
- Partial: assignment on Y subset X
- Valid: values in D
- Violating c: assignment on var(c) is not in c
- Locally consistent: assignment not violating any constraint on Y
- Solution: locally consistent on X

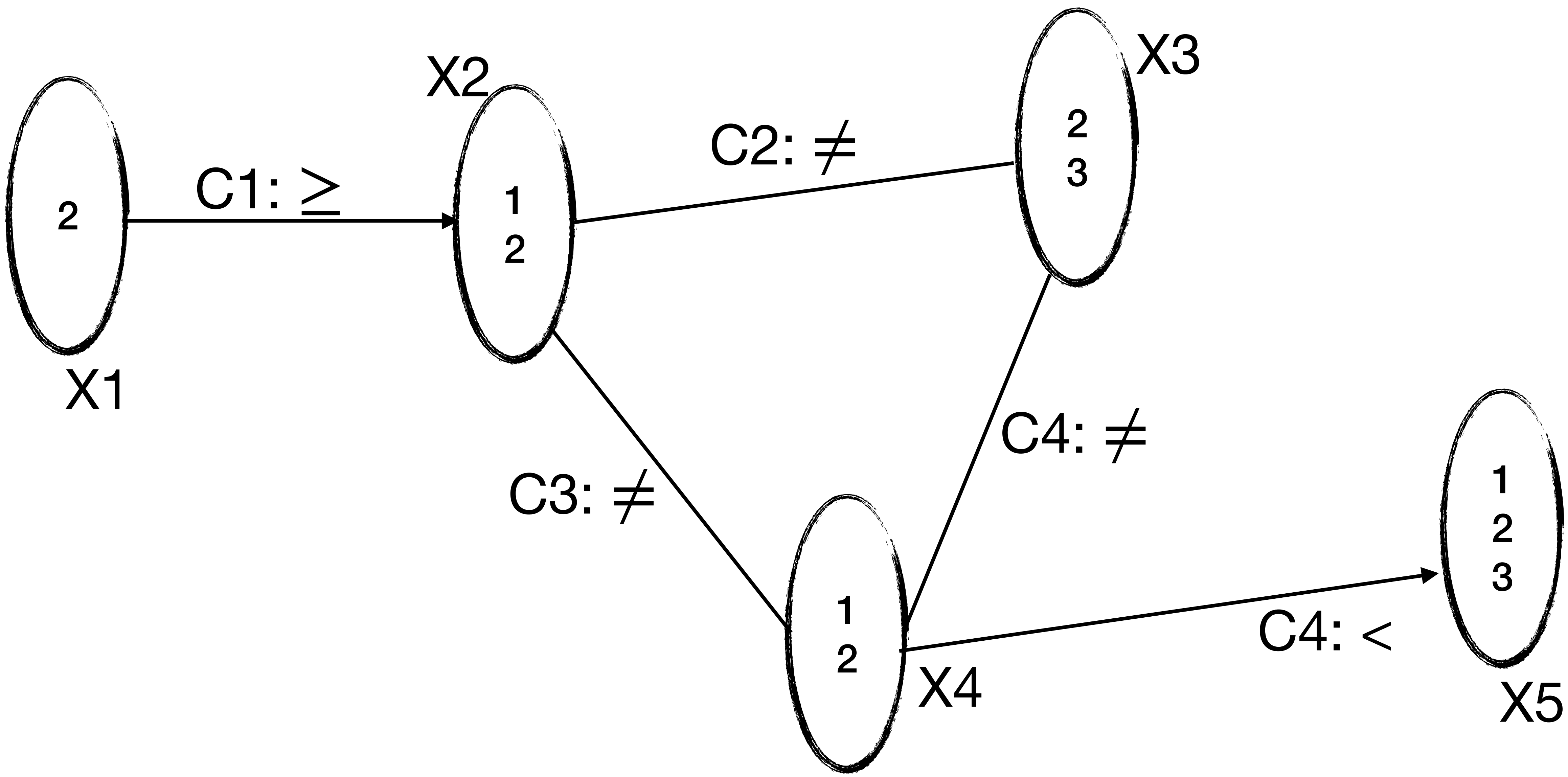
Constraint Programming

Solving - search

```
Backtracking(<X,D,C>, I):

If I is complete then return true

Select a variable Xi not in I

ForEach v in D(Xi) do

If I union <Xi, v> is locally consistent then

   If Backtracking(<X,D,C>, I union <Xi, v>) then
      return true

return false
```

Constraint Programming

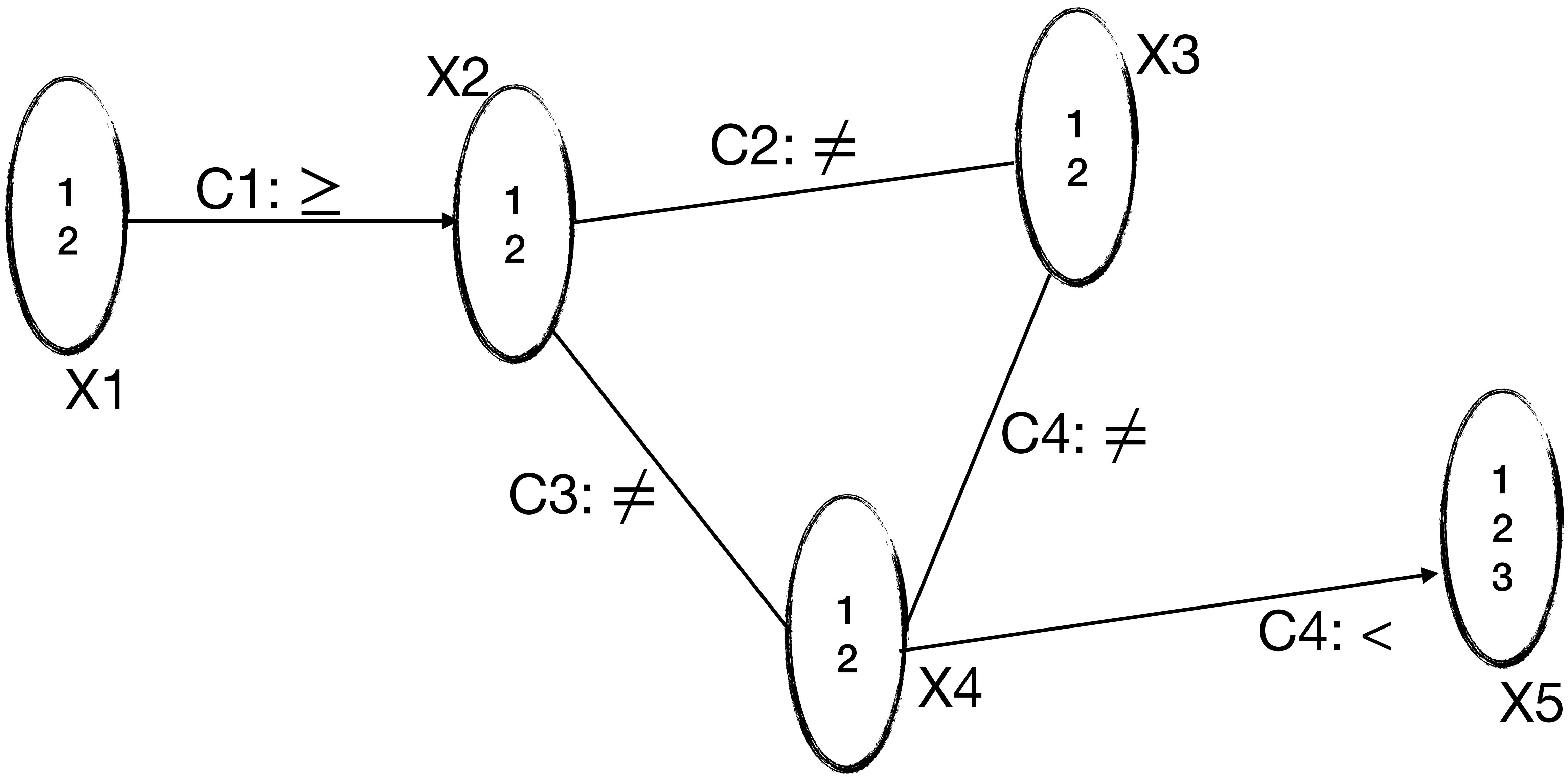
Solving - propagation (local consistency)



Situation 1

Constraint Programming

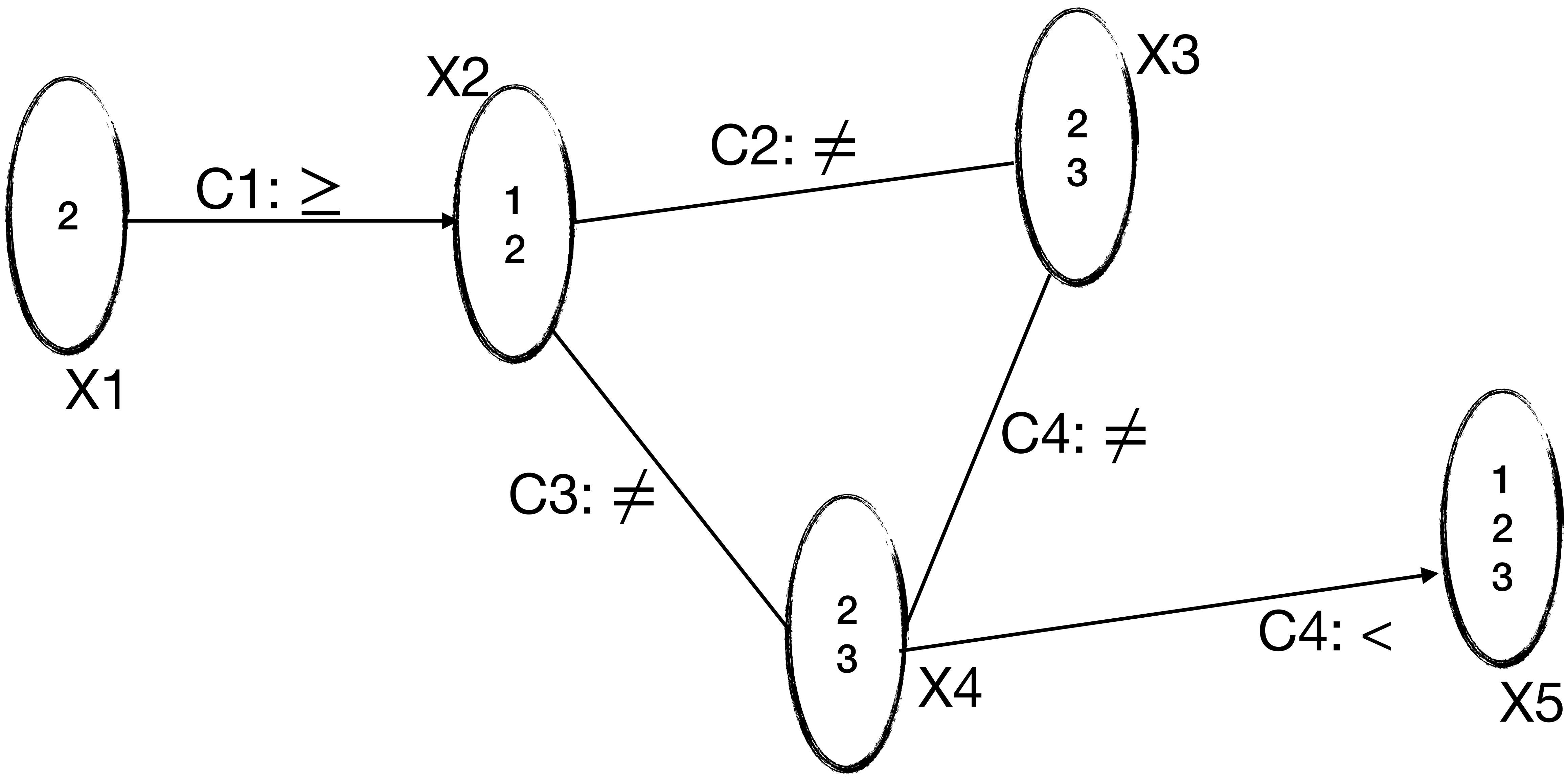
Solving - propagation (local consistency)



Situation 2

Constraint Programming

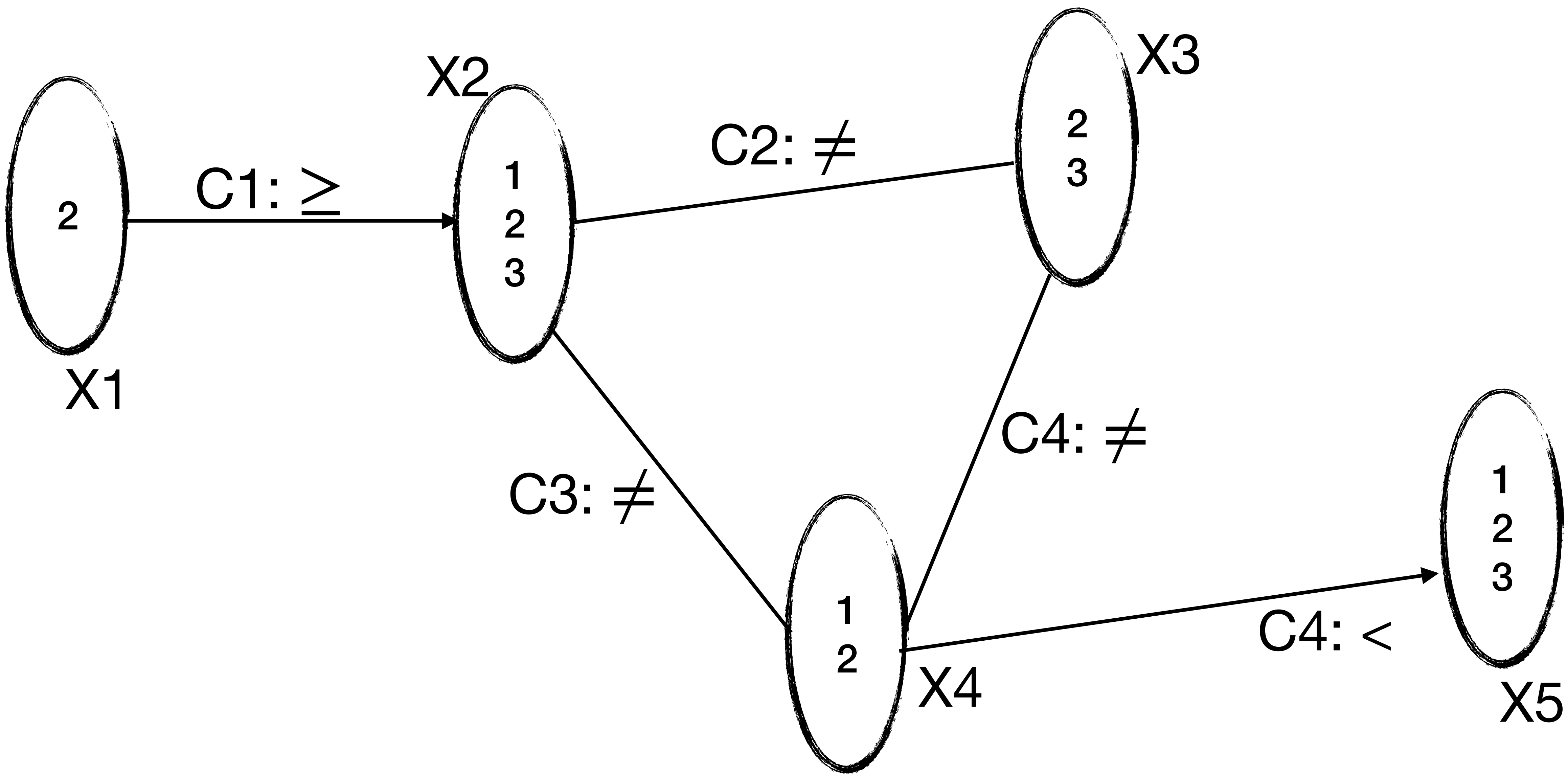
Solving - propagation (local consistency)



Situation 3

Constraint Programming

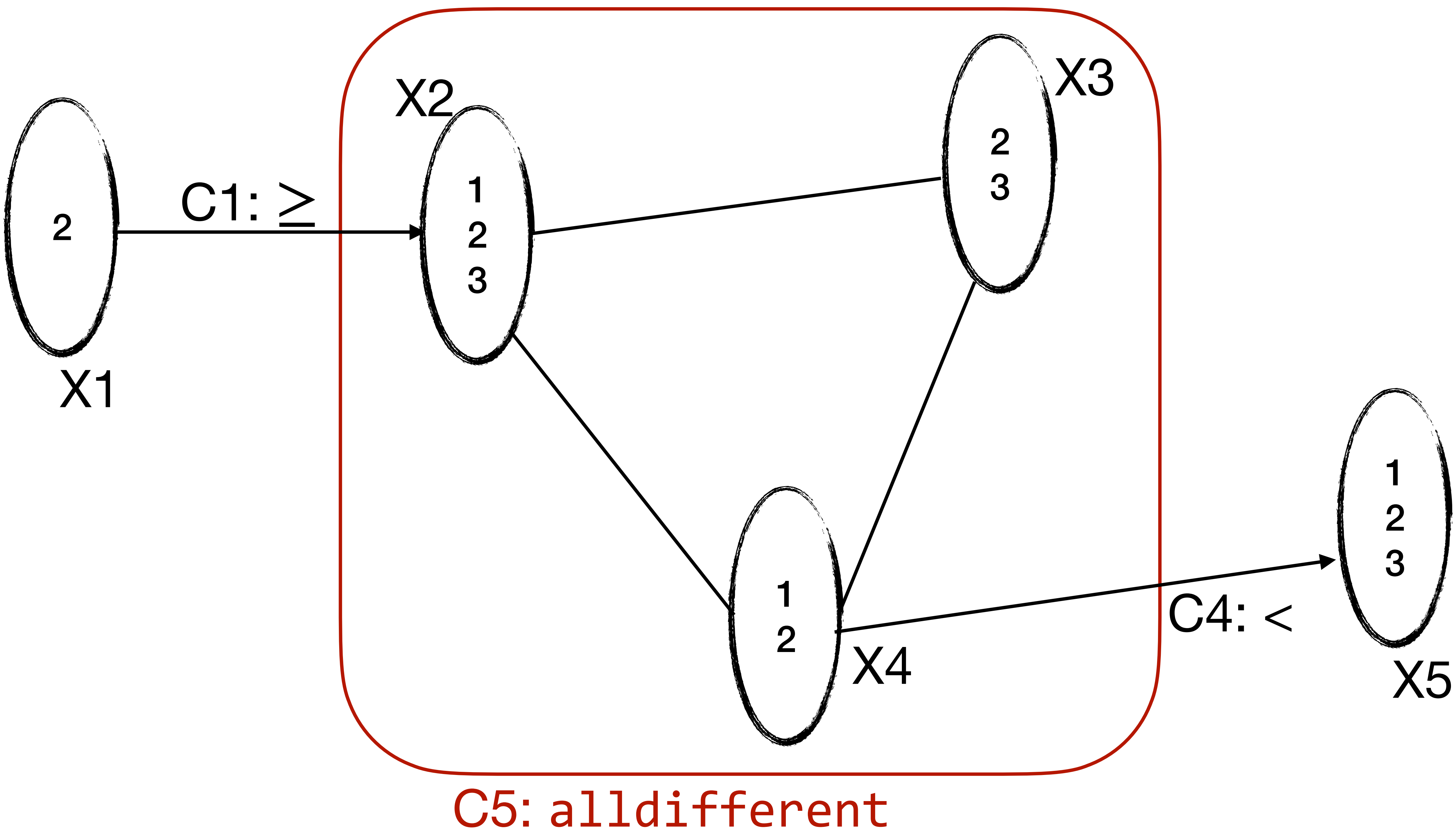
Solving - propagation (local consistency)



Situation 4

Constraint Programming

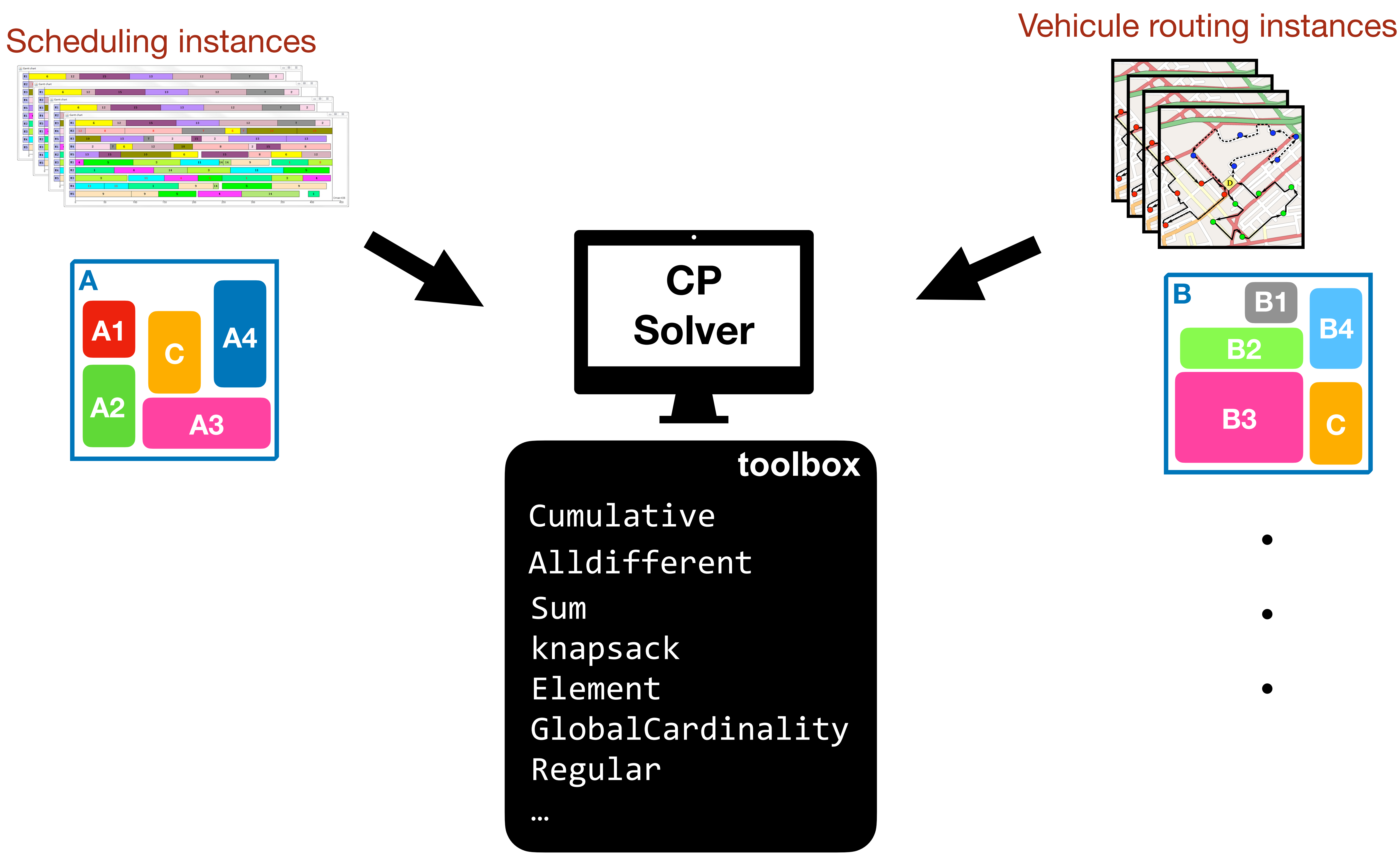
Solving - propagation (local consistency)



Situation 5

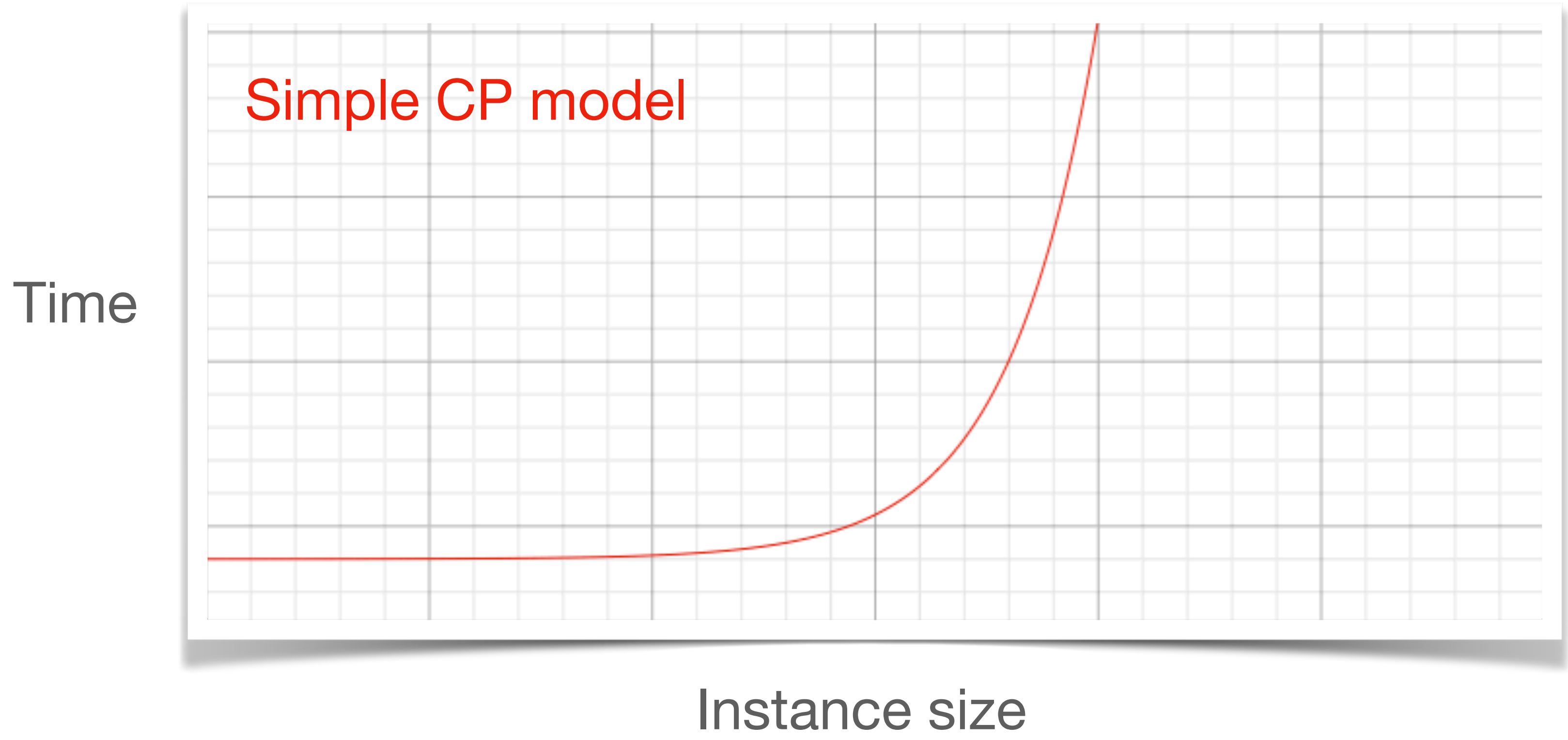
Constraint Programming

Global constraints



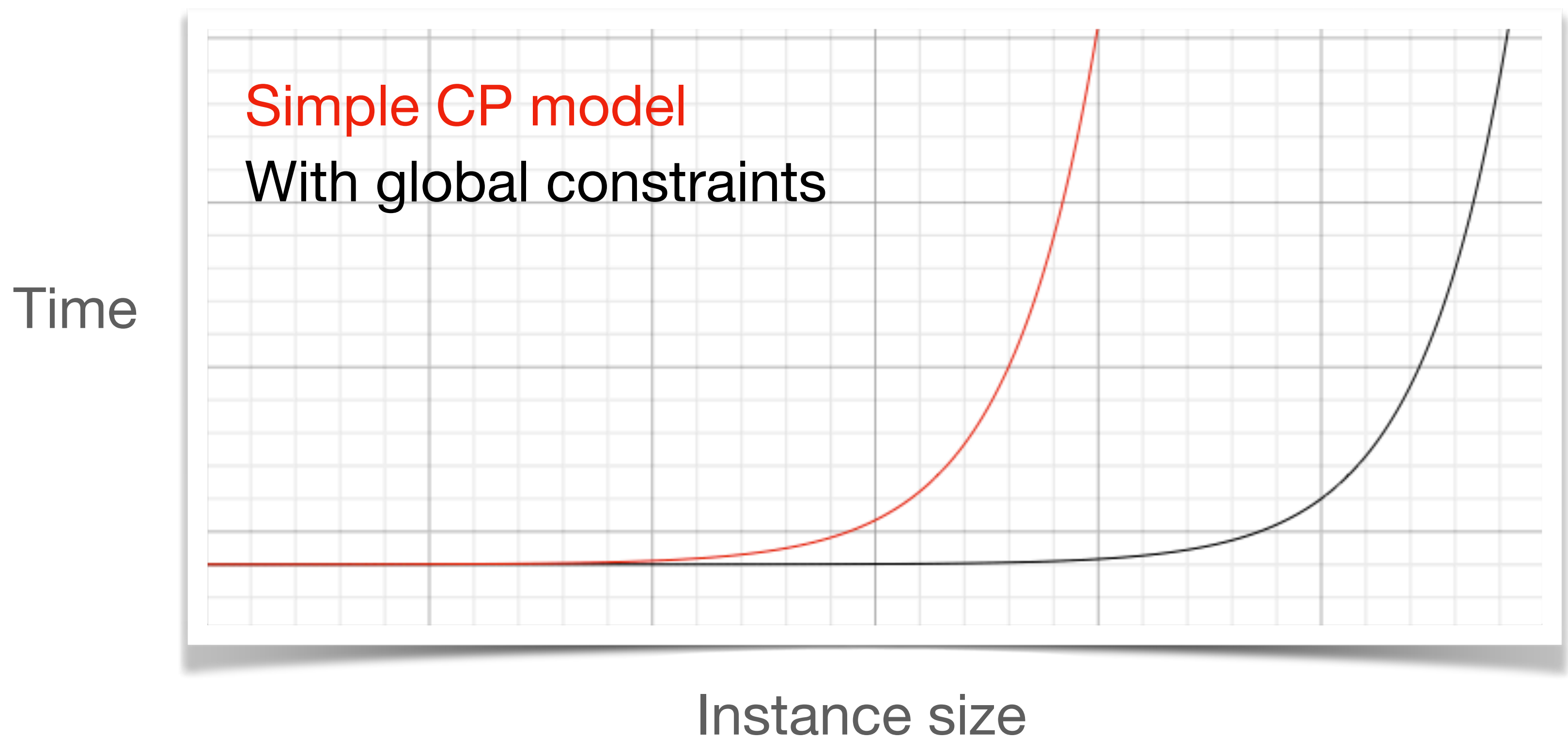
Constraint Programming

Global constraints



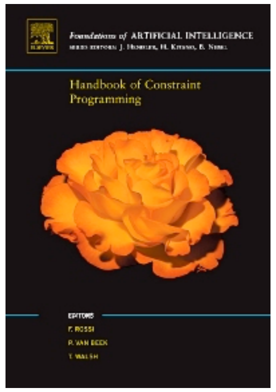
Constraint Programming

Global constraints



Constraint Programming

References & links



- **Handbook of Constraint Programming.** Francesca Rossi Peter van Beek Toby Wals. Elsevier Science 2006
- ACP: Association for Constraint Programming
- Guide to Constraint Programming
- CP conference Series
- Global Constraint Catalog
- COCONUT Team, LIRMM