Frequent Itemset Mining

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(PART II)

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Condensed representation of Frequent Itemsets: Closed and Maximal Itemsets

Maximal Itemsets

- The set of Maximal (frequent) Itemsets:

$$M_\theta = \{P \subset I \,|\, freq(P) \geq \theta \wedge \forall P' \supset P : freq(P') < \theta\}$$

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- That is:

$$\forall \theta, \forall P \in F_\theta : (P \in M_\theta) \vee (\exists P' \supset P : freq(P') \geq \theta)$$

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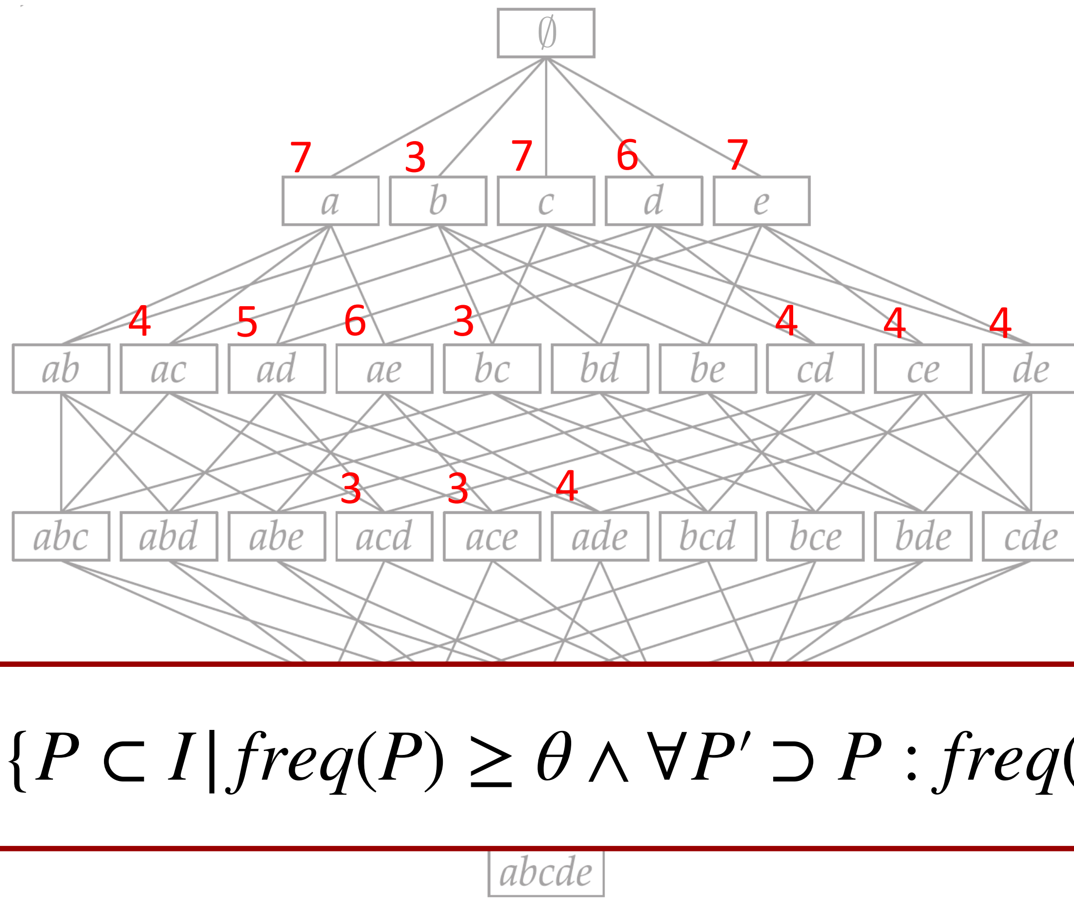
Example (5)

Here are the Frequent itemset with minsup θ=3

Q: What are the maximal itemsets minsup θ=3?



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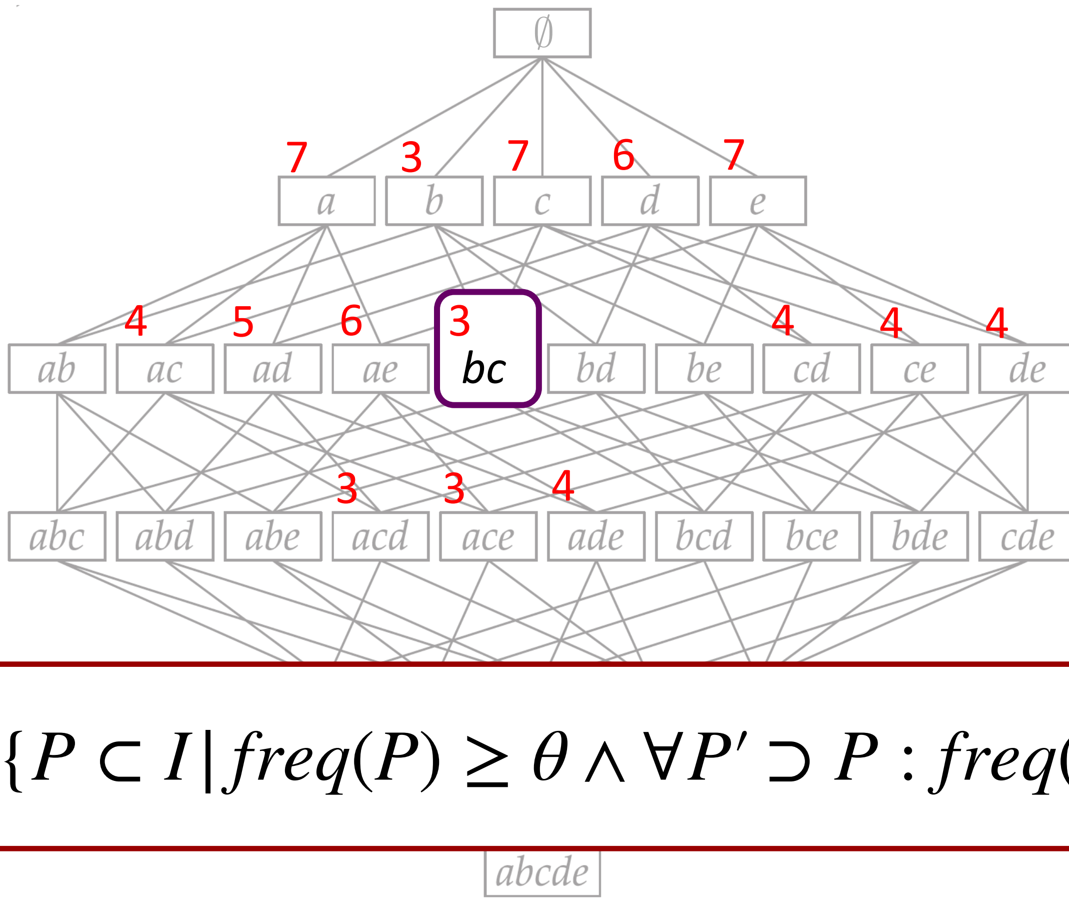

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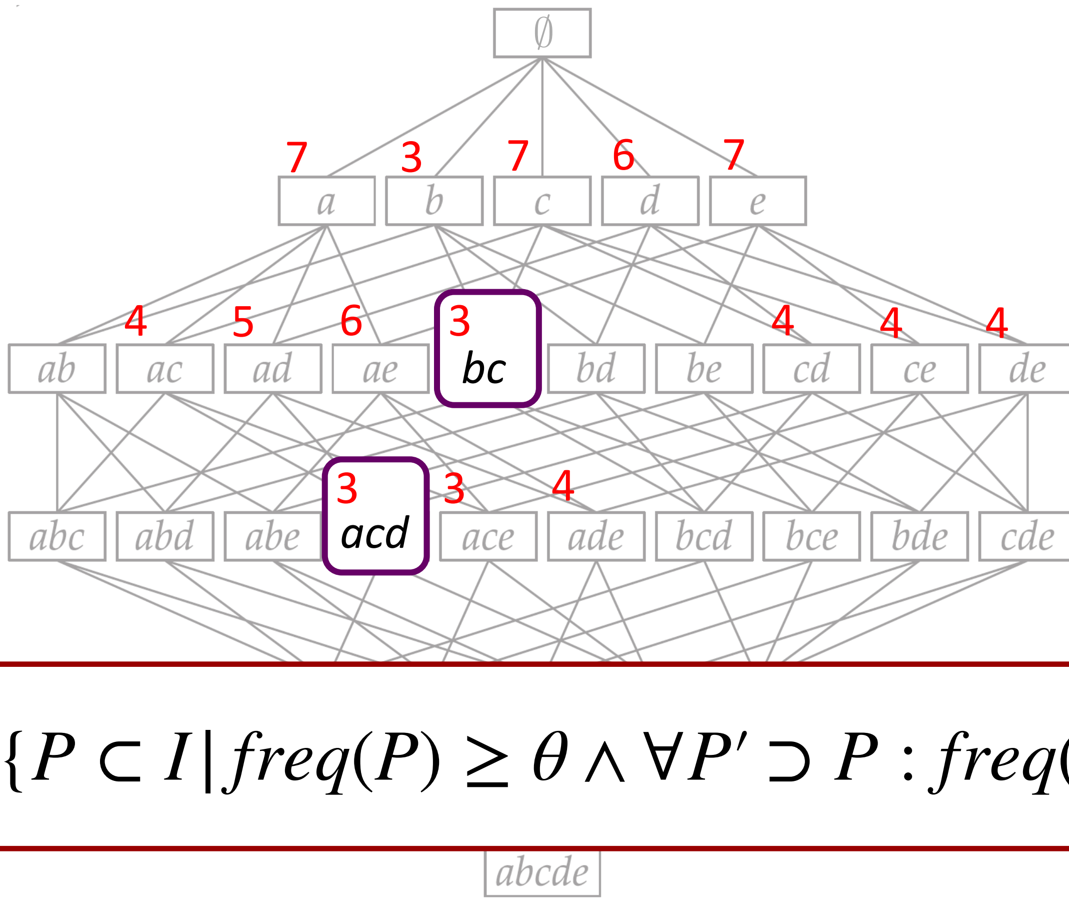

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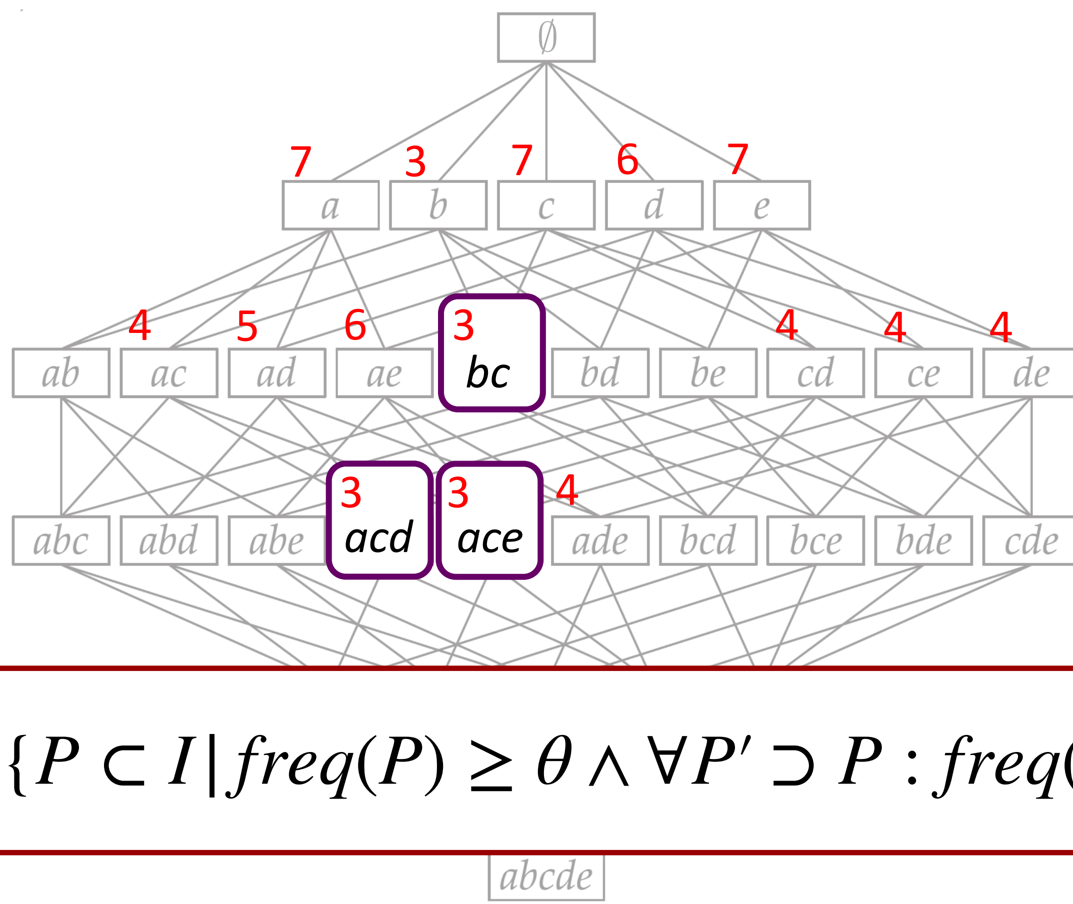

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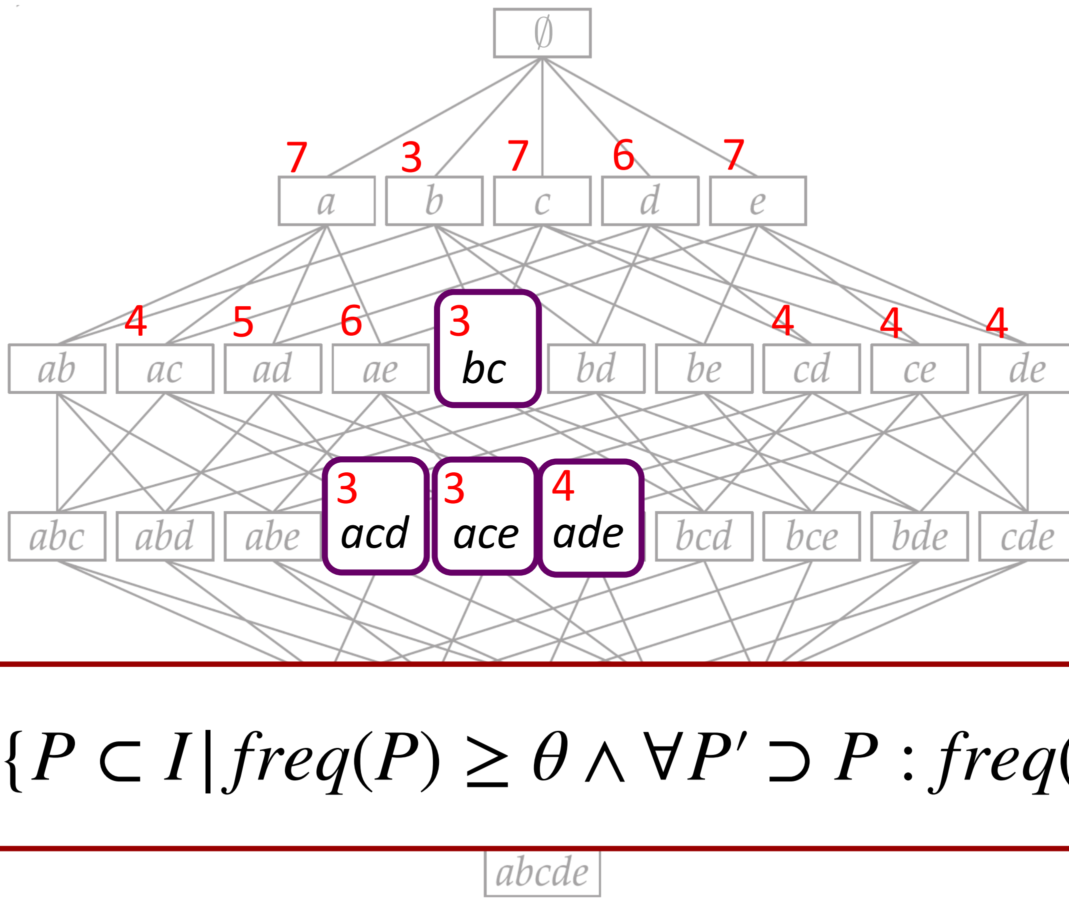

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THE NEED Can we have a condensed representation of the set of frequent itemsets, which preserves knowledge of all support values?

Closed Itemsets

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- Which is not the case with the maximal itemsets:

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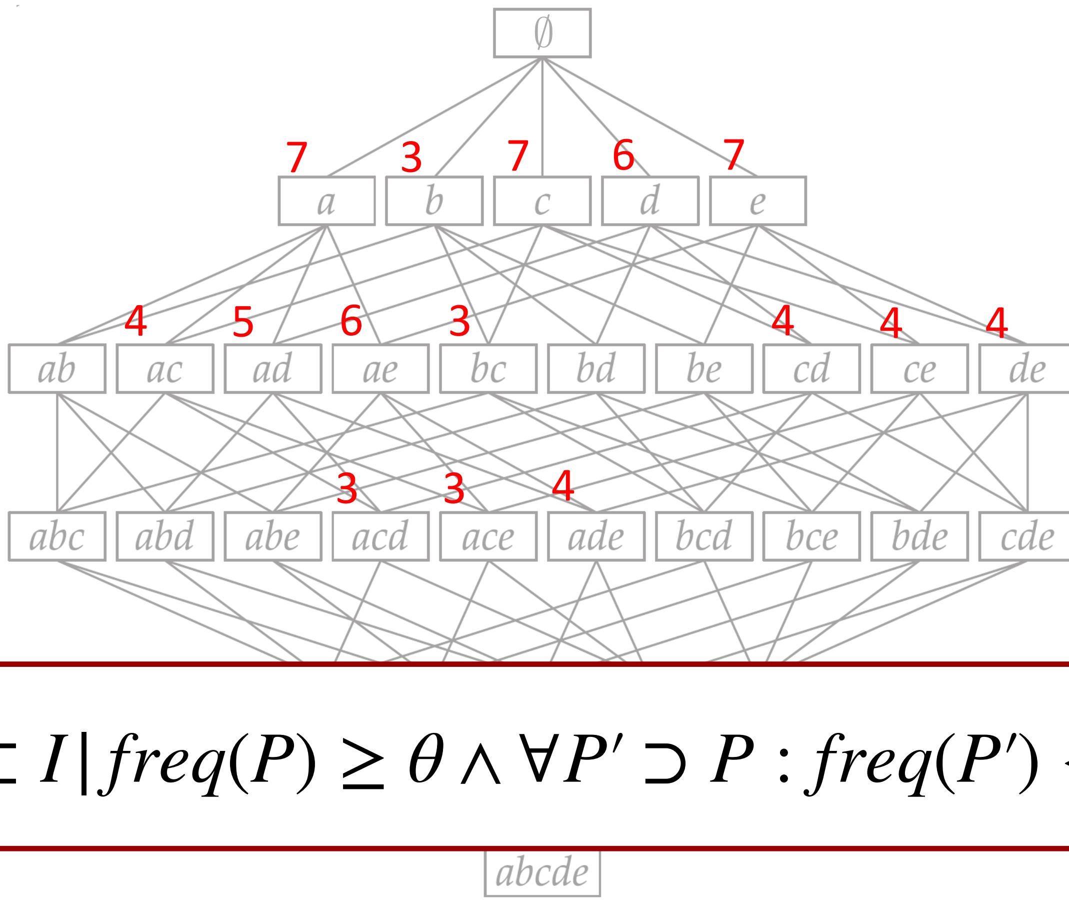
Example (6)

Here are the Frequent itemset with minsup θ=3

Q: are b and de Closed itemsets?



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matrix representation

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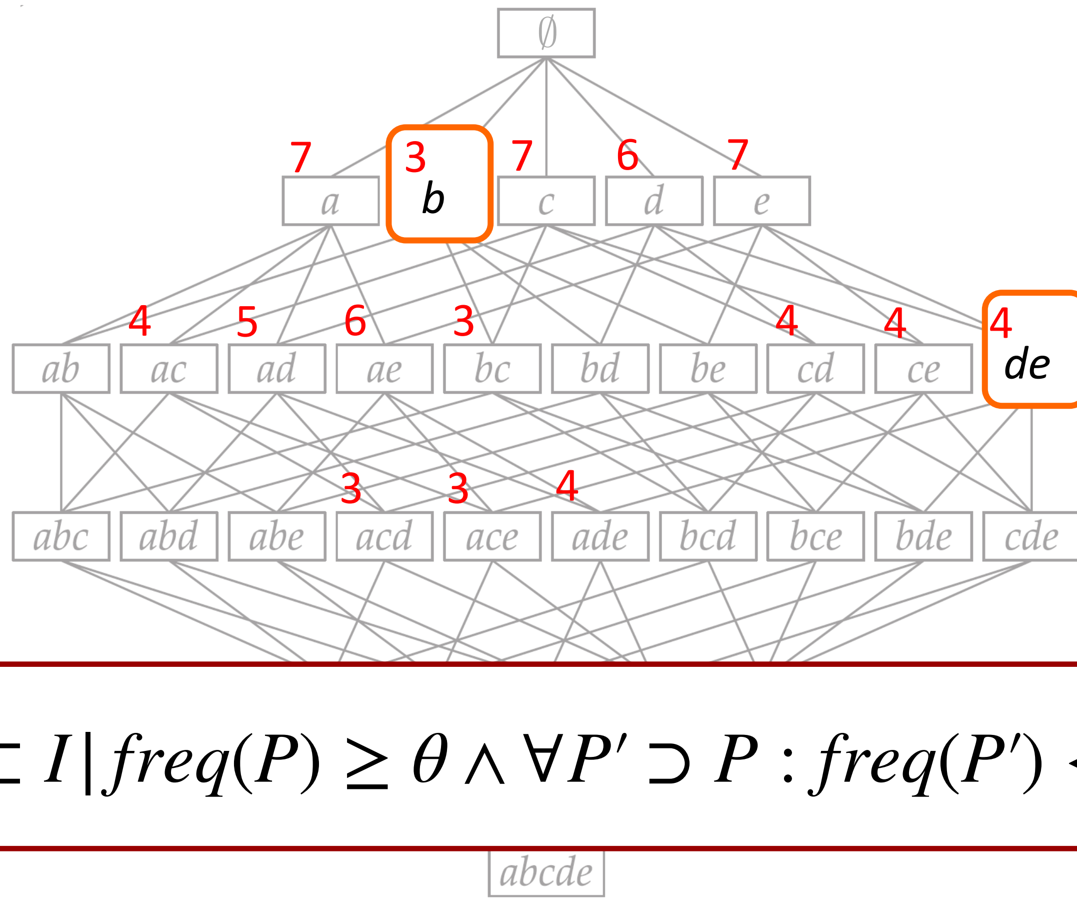

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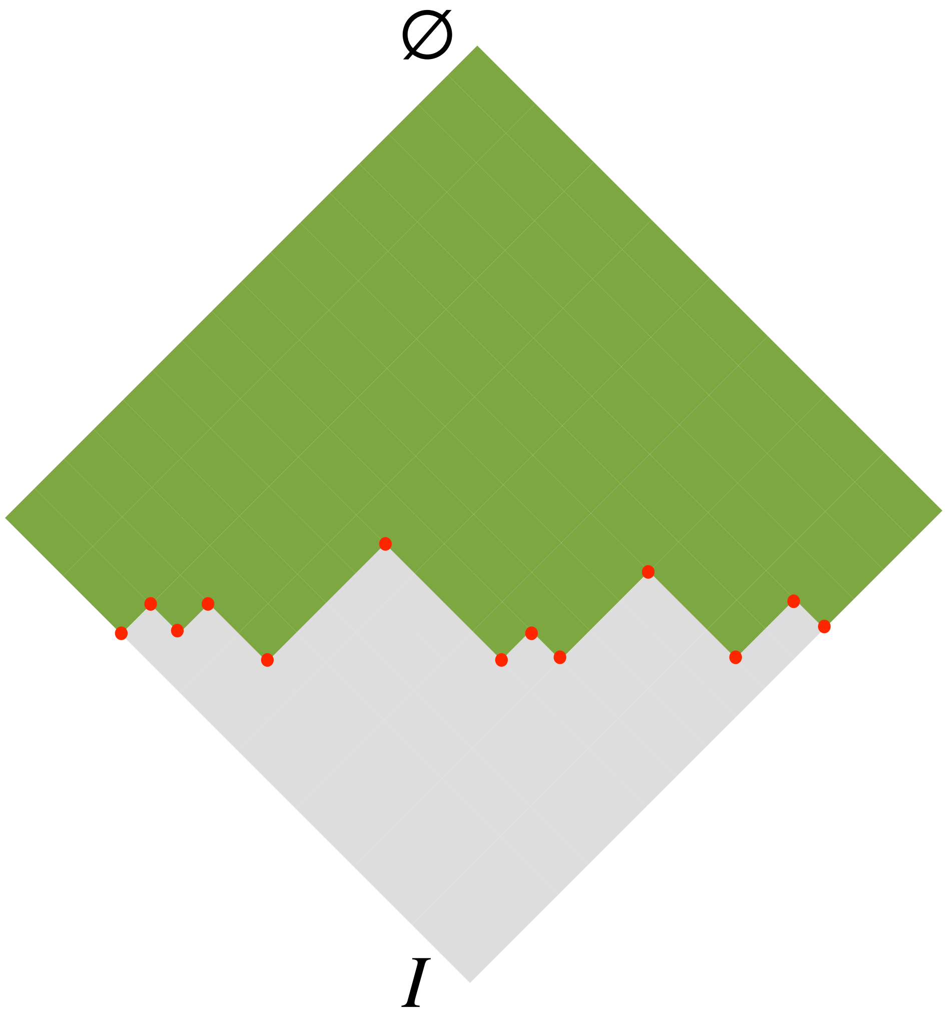
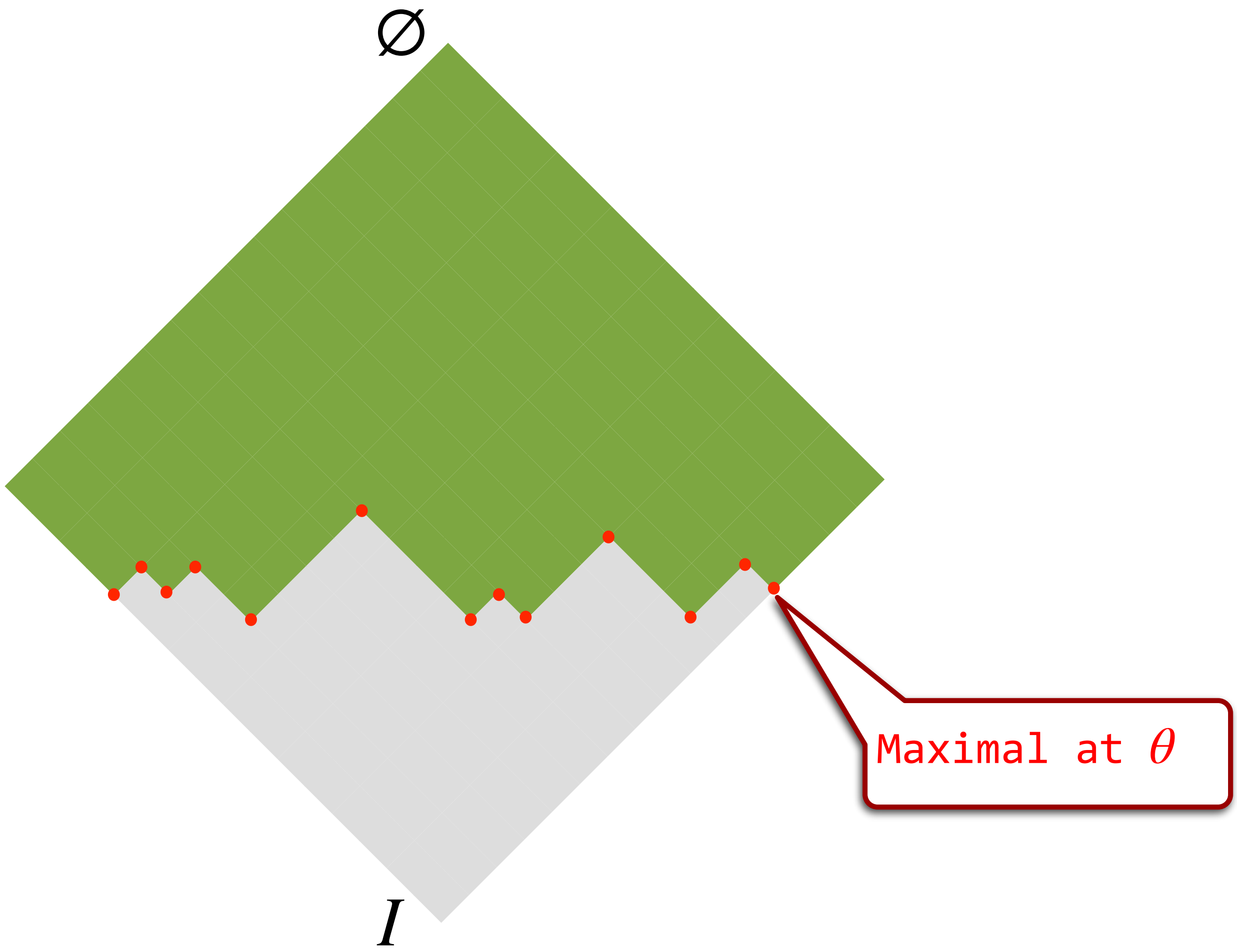
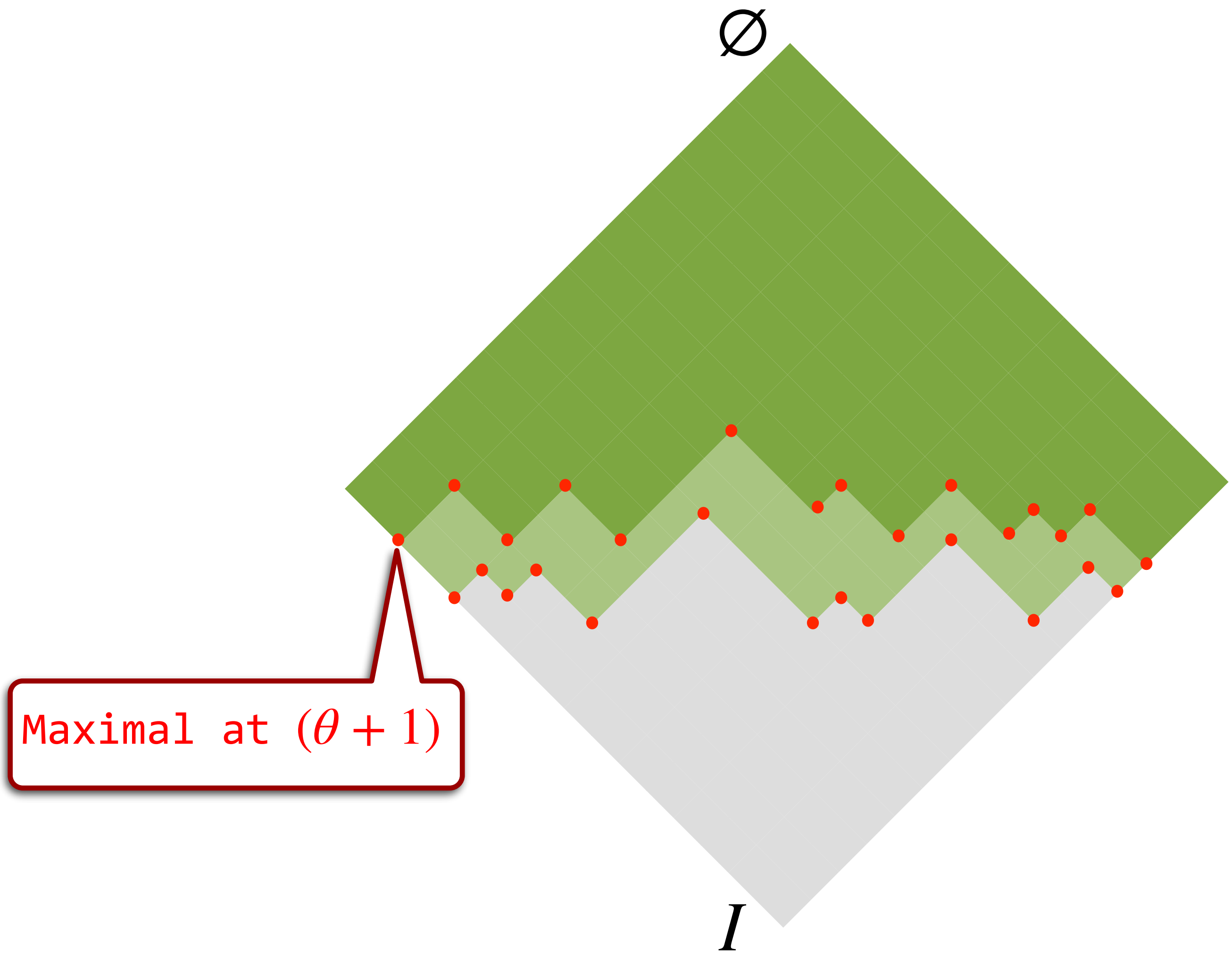
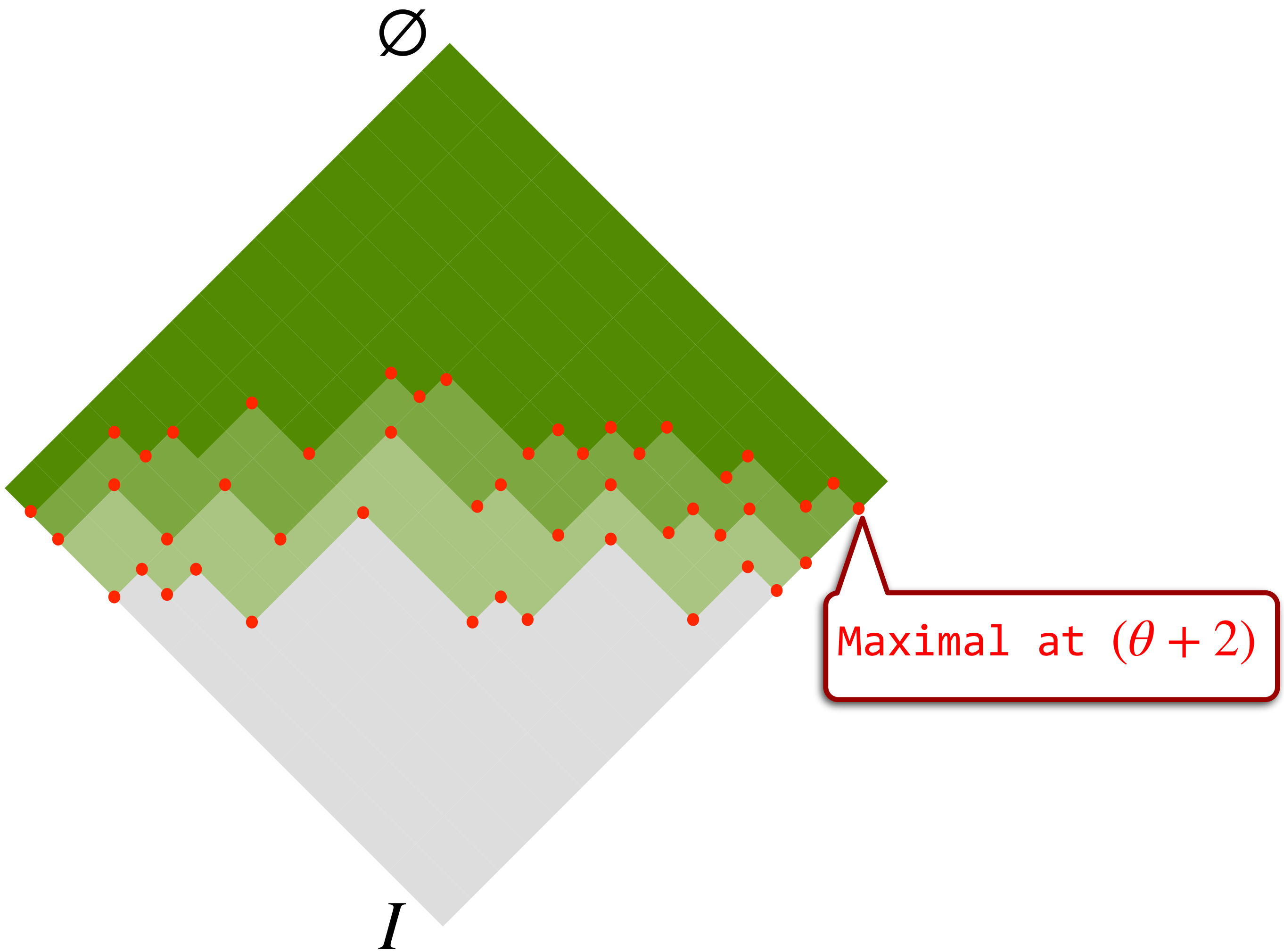

∅
I

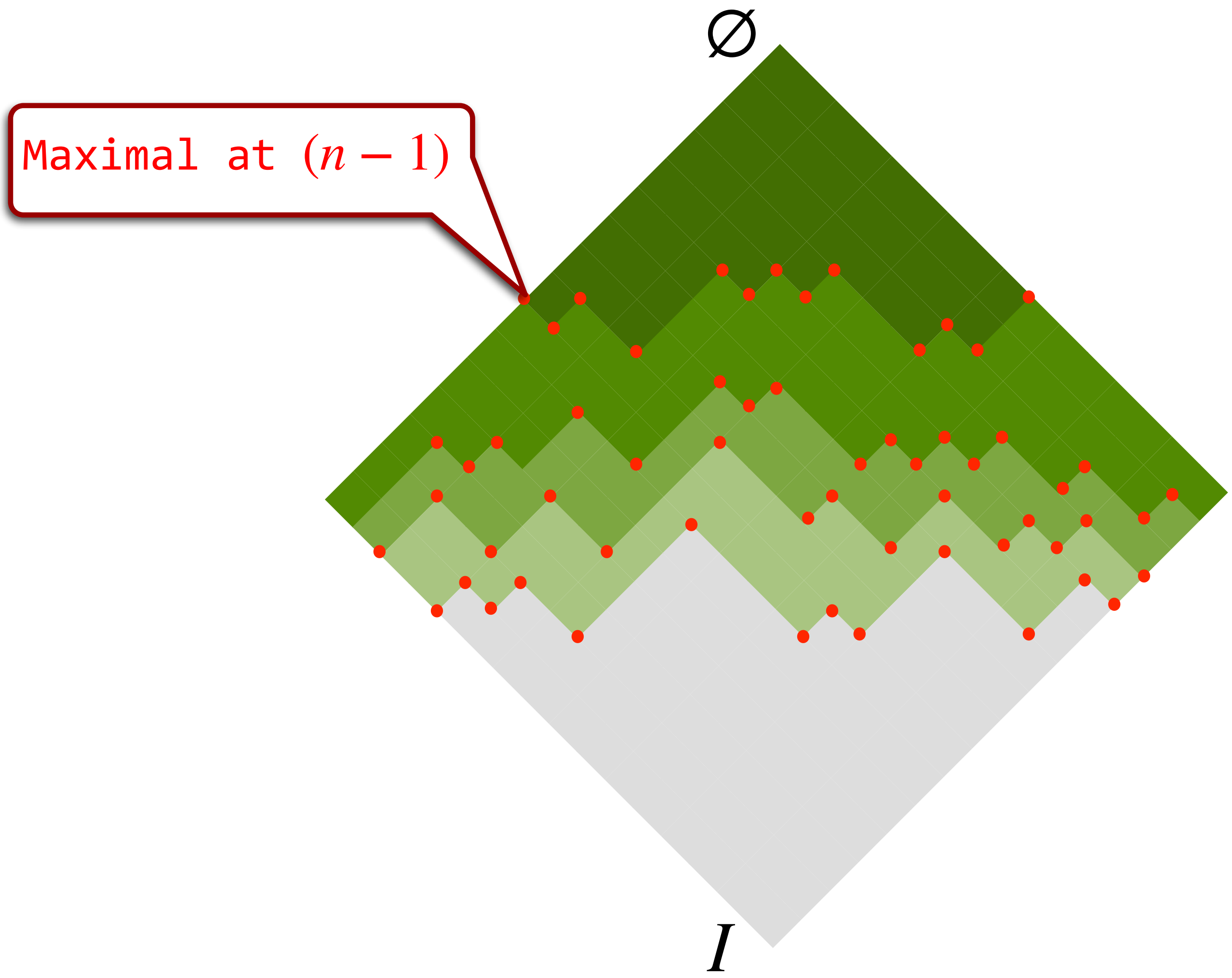


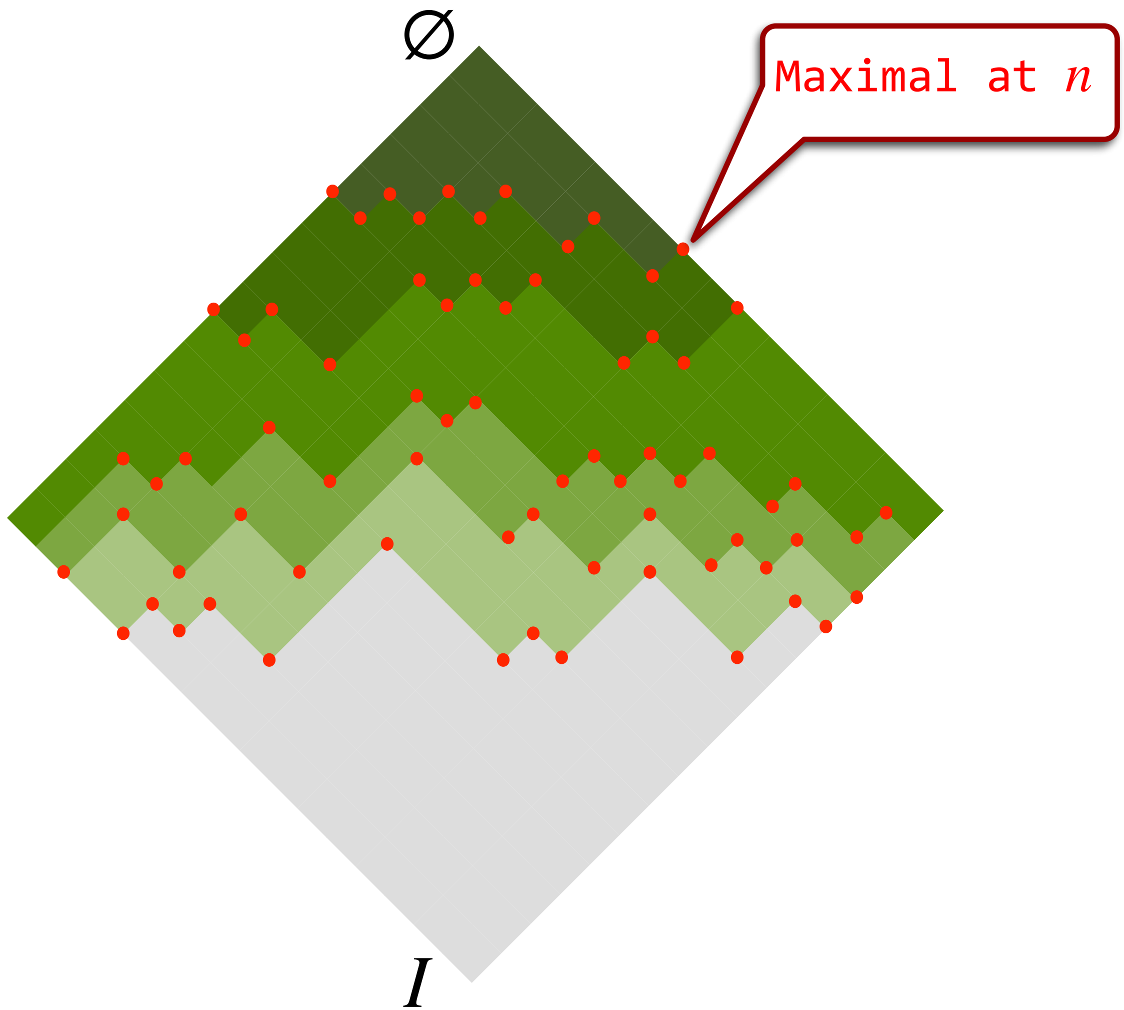
∅
Maximal at θ
I

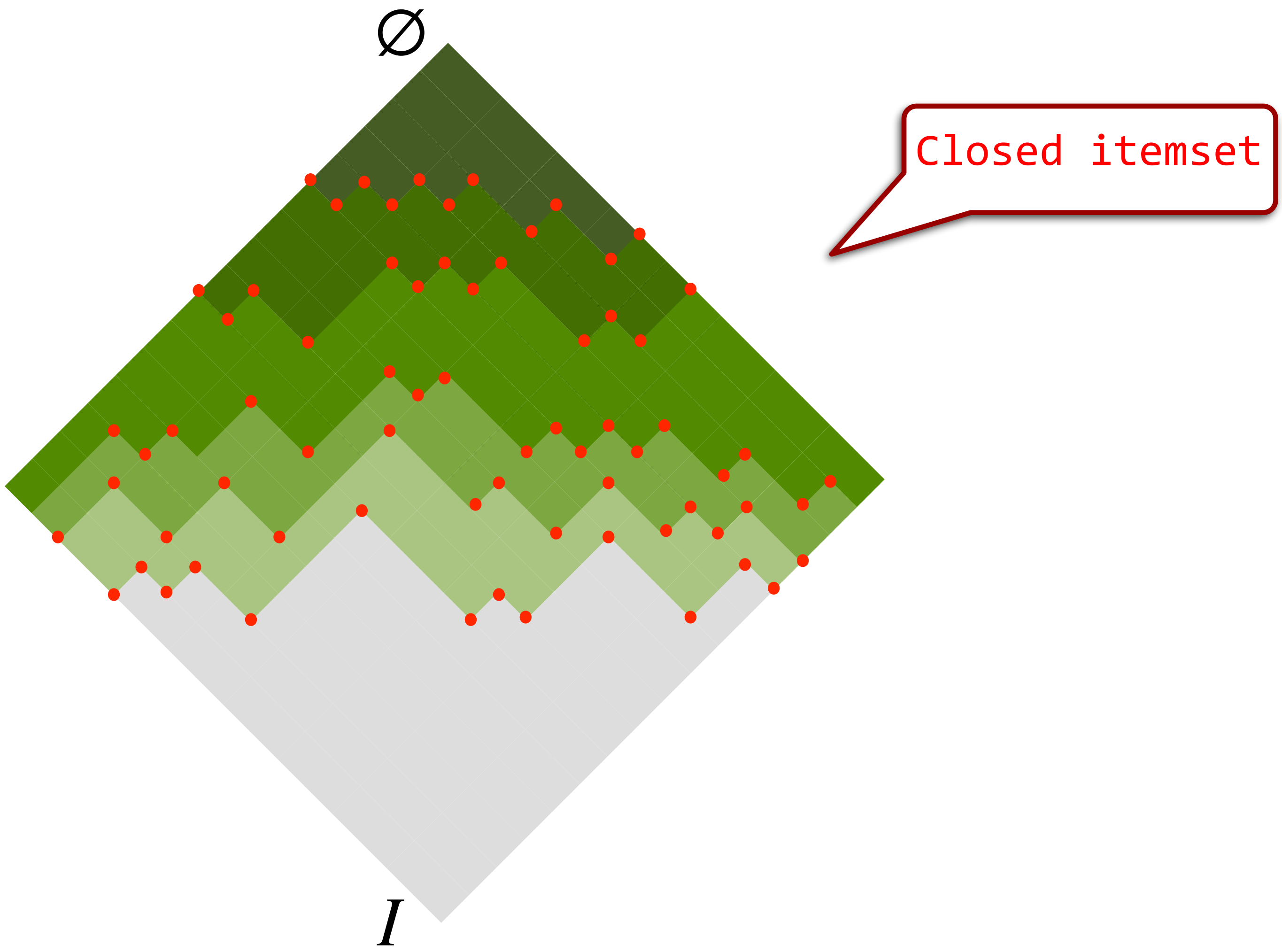

$\varnothing$
Maximal at $(\theta + 1)$
I

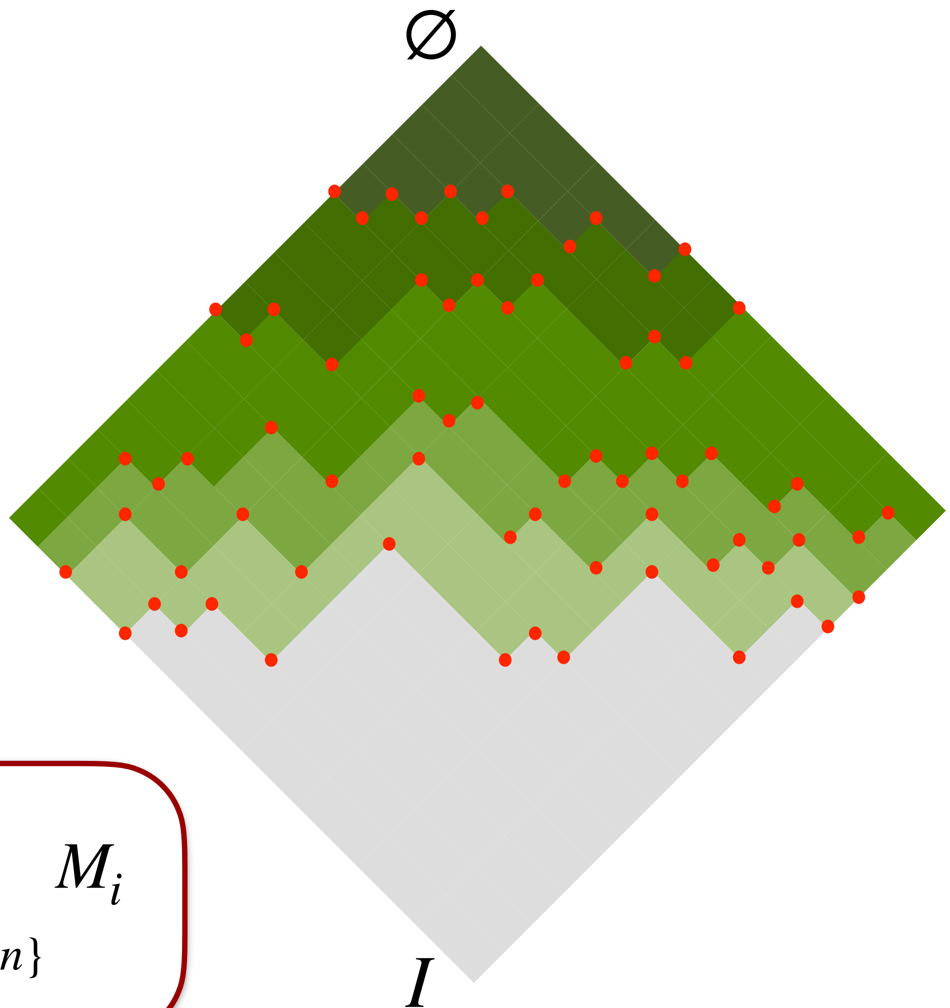


$\varnothing$
I
Maximal at $(\theta + 2)$


Maximal at $(n-1)$
$\varnothing$
I


$\varnothing$
Maximal at n
I


∅
I
Closed itemset



$$C_\theta = \bigcup_{i \in \{\theta, \ldots, n\}} M_i$$

Frequent/Closed/Maximal

http://fimi.ua.ac.be/data/

Frequent/Closed/Maximal

Dataset	#Frequent	#Closed	#Maximal
Zoo-1	151 807	3 292	230
Mushroom	155 734	3 287	453
Lymph	9 967 402	46 802	5 191
Hepatitis	$27 \cdot 10^7$+	1 827 264	189 205

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Tutorials

github.com/FDSInfoMontp-HMIN233/FIM2