Homework 10
(due Wednesday, December 2)

Instructions: This homework is to be submitted on GradeScope as a *single* pdf (not in parts) by 11:59 pm on the due date. You may either type your solutions in a word processor and print to a pdf, or write them by hand and submit a scanned copy. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc....).

Begin by reviewing your class notes, the slides, and the textbook. Then do the exercises below. Show your work. An unjustified answer may receive little or no credit.

Read: 5.1 (pages 215-220), 5.3, 7.1 (for Tuesday)

Note: In all problems, $\Sigma = \{0, 1\}$. Assume that all Turing machines have this input alphabet and are themselves encoded in it.

1. [5 Points] Let
$$E_{TM} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \emptyset\}.$$
Show that E_{TM}, is not recognizable. You may use results from previous homework problems, provided you give explicit references to these problems.

2. [10 Points] Let $S = \{\langle M\rangle \mid M$ is a DFA that accepts w^R whenever it accepts $w\}$. Show that S is decidable.

3. [10 Points] Let $E = \{\langle M\rangle \mid M$ is a DFA that accepts some string with more 1s than 0s$\}$. Show that E is decidable. (Hint: Theorems about CFLs are helpful here).

4. [10 Points]
   (a) Show that for each Turing machine M there exists a Turing machine $\widehat{M}$ such that
   - $\widehat{M}$ never prints a blank
   - $L(\widehat{M}) = L(M)$

   (b) Consider the problem of determining whether, started on a blank tape, a Turing machine will ever print a blank. Formulate this problem as a language and show that it is undecidable. (Hint: use a reduction from ACCEPT-EMPTY.)

5. [15 Points] An *unreachable state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any unreachable states. Formulate this problem as a language and show that it is undecidable.

6. [10 Points] Let $\Sigma = \{0, 1\}$ and consider the following languages $A, B \subseteq \Sigma^*$:

   $A = \{w \in \Sigma^* \mid \ w \text{ begins with } 0\}$
   $B = \{w \in \Sigma^* \mid \ |w| \geq 4 \text{ and } w \text{ begins with } 1\}$

   Show that $A \leq_m B$ by defining an explicit reduction f from A to B.

7. [10 Points] In this problem we show that for each language $L \subseteq \Sigma^*$

$$L \text{ is enumerable } \Leftrightarrow \ L \leq_m A_{TM}$$

   The direction "$\Leftarrow$" will be proven in class. Here you are to prove the direction "$\Rightarrow$".