CS 3800 (2+3) Fall 2020
W. Schnyder 12/3/2020

Homework 11

(due Tuesday, December 8)

Instructions: This homework is to be submitted on GradeScope as a *single* pdf (not in parts) by 11:59 pm on the due date. You may either type your solutions in a word processor and print to a pdf, or write them by hand and submit a scanned copy. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc....).

Begin by reviewing your class notes, the slides, and the textbook. Then do the exercises below. Show your work. An unjustified answer may receive little or no credit.

Read: 7.2 through page 289(for Thursday-Friday), 7.3, 7.4 (for Tuesday)

(Note that some definitions given in class are different from those in the text.)

Note: This assignment is due on Tuesday 12/8. Most problems are easy, just testing your understanding of definitions. Try to submit by the deadline. Late submissions will be accepted but not graded before the final exam.

1. [6 Points] Let $A, B \subseteq \Sigma^*$ and assume that A is decidable. For each of the following statements give either a proof or a counterexample.

   (a) $A \leq_T B$

   (b) $A \leq_m B$

2. [10 Points] Let $\Sigma = \{0, 1\}$. Which, if any, of the following languages are mapping-reducible to A_{TM} ? You may refer to problems from earlier assignments.

   (a) $L_1 = \{< M >|\ M$ accepts at least one string consisting only of 1's$\}$

   (b) $L_2 = \{< M >|\ M$ accepts no input$\}$

3. [7 Points] Show that language $A \subseteq \Sigma^*$ is decidable iff $A \leq_m 0^*\,1^*$. (Prove both directions.)

4. [10 Points] Prove that HALT is enumerably complete.

   Hint: in class we showed $A \leq_m B$ and $B \leq_m C \Rightarrow A \leq_m C$

5. [10 Points] Define the language L by

   $$L = \{w \mid \text{either } \ w = 0x \text{ for some } x \in A_{TM} \ \text{ or } \ w = 1y \text{ for some } y \in \overline{A_{TM}}$$

   Prove that neither L nor $\overline{L}$ is recognizable.

6. [3 Points] True or False? If $L \subseteq \{0, 1\}^*$ is such that $L \leq_p 0^*1^*$, then $L \in P$. Explain your answer.

7. [8 Points] Consider the class **K** of languages that is defined as follows. A language $A \subseteq \Sigma^*$ belongs to **K** if and only if there exists a language $B \in \mathbf{P}$ such that for all $w \in \Sigma^*$

$$w \in A \;\Leftrightarrow\; \exists s(\ \langle w, s \rangle \in B\ )$$

Compare **K** with the classes of languages that were discussed in class (regular, context-free, decidable, recognizable, **P**, **NP**). Justify your conclusion.

8. [6 Points] Call graphs G and H *isomorphic* if the nodes of G may be reordered so that it is identical to H. Let

$$ISO = \{\langle G, H \rangle \mid\ G \text{ and } H \text{ are isomorphic graphs}\}.$$

Show that $ISO \in NP$.

9. [**0** Point] **Do not submit.** Let $L = \{\langle M \rangle \mid L(M) = \Sigma^*\}$. Show that neither L nor $\overline{L}$ is recognizable. (This is a challenging problem.)