CS 3800 (2+3) Fall 2020
W. Schnyder 11/10/2020

Homework 8
(due Tuesday, November 17)

Instructions: This homework is to be submitted on GradeScope as a *single* pdf (not in parts) by 11:59 pm on the due date. You may either type your solutions in a word processor and print to a pdf, or write them by hand and submit a scanned copy. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc....).

Begin by reviewing your class notes, the slides, and the textbook. Then do the exercises below. Show your work. An unjustified answer may receive little or no credit.

Read: 3.2 (pp 180-182), 3.3 (for Thursday-Friday), 4.1, 4.2 (for Tuesday)

1. [8 Points] **Recognize vs. Decide.** Let L be the set of nonempty bitstrings that begin with 1. That is,

$$L = \{w \in \{0,1\}^+ \mid w \text{ begins with } 1\}$$

Four Turing machines are given below. For each machine state whether it *decides* L, or *recognizes* L, or *both*, or *neither* and justify your answers.

Each machine has the form $(Q, \Sigma, \Gamma, \delta, s, q_{\text{accept}}, q_{\text{reject}})$ with $\Sigma = \{0,1\}$, $\Gamma = \{0, 1, \textvisiblespace\}$, and $Q = \{s, q, q_{\text{accept}}, q_{\text{reject}}\}$. Transitions δ are specified in each case.

(a) M_1 has transitions

s	1	1	R	q_{accept}
s	0	0	R	q
s	$\textvisiblespace$	$\textvisiblespace$	R	q
q	1	1	R	q
q	0	0	R	q
q	$\textvisiblespace$	$\textvisiblespace$	R	q_{reject}

(b) M_2 has transitions

s	1	1	R	q_{accept}
s	0	0	R	s
s	$\textvisiblespace$	$\textvisiblespace$	R	q_{reject}
q	1	1	R	q
q	0	0	R	q
q	$\textvisiblespace$	$\textvisiblespace$	R	q_{reject}

(c) M_3 has transitions

s	1	1	R	q_{accept}
s	0	0	R	q
s	$\textvisiblespace$	$\textvisiblespace$	R	s
q	1	1	R	q
q	0	0	R	q
q	$\textvisiblespace$	$\textvisiblespace$	R	q_{reject}

(d) M_4 has transitions

s	1	1	R	q_{accept}
s	0	0	R	q
s	␣	␣	R	q_{reject}
q	1	1	R	q
q	0	0	R	q
q	␣	␣	R	q_{reject}

2. [10 Points] **One-way vs. Two-way.** We wish to construct a Turing Machine

$$M = (Q, \Sigma, \Gamma, \delta, s, q_{\text{accept}}, q_{\text{reject}})$$

that decides the language

$$L = \{w \in \{0,1\}^* \mid |w| \geq 2 \text{ and } w \text{ ends in } \texttt{00} \text{ or in } \texttt{01}\}$$

s	0	s	0	R
s	1	s	1	R
s	␣	q_1	␣	L
q_1	0	q_2	0	L
q_1	1	q_2	1	L
q_1	␣	q_{reject}	␣	R
q_2	0	q_{accept}	0	R
q_2	1	q_{reject}	1	R
q_2	␣	q_{reject}	␣	R

For which, if any, of the following machine models does M decide L? Explain your answers.

(a) M is a standard machine with a one-way infinite tape.

(b) M has a two-way infinite tape.

3. [10 Points] **Two tapes.** We wish to construct a 2-tape Turing Machine

$$M = (Q, \Sigma, \Gamma, \delta, s, q_{\text{accept}}, q_{\text{reject}})$$

that decides the language $L = \{a^n b^n \mid n \geq 1\}$. For simplicity, our machine's tapes are 2-way infinite and it allows the stay-put motion S.

Initially, the input w is written on tape 1, with head 1 under its first symbol (or under a blank if the input is empty). At the end of its computation, the machine should be in state q_{accept} if $w \in L$ or in state q_{reject} if $w \notin L\}$. A **constraint** is that your machine should never print b on tape 2.

We choose $Q = \{s, q, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{a, b\}$, $\Gamma = \Sigma \cup \{␣\}$ and begin with the transitions:

s	$\binom{\sqcup}{\sqcup}$	q_{reject}	$\binom{\sqcup}{\sqcup}$	$\binom{S}{S}$
s	$\binom{a}{\sqcup}$	s	$\binom{a}{a}$	$\binom{R}{R}$
s	$\binom{b}{\sqcup}$	q	$\binom{b}{\sqcup}$	$\binom{S}{L}$

A transition is inessential if no input will ever cause M to execute this transition, otherwise it is *essential*. Of the missing 15 transitions of M only 6 (for q) are essential. Write these 6 transitions to complete δ.

4. [10 Points] **Trading states for symbols**

   Show that for each string $w \in \{0,1\}^*$ there exists a stay-put one-way Turing machine

   $$M_w = (Q, \{0,1\}, \Gamma, \delta, s, q_{accept}, q_{reject})$$

   with $|Q| \leq 5$ states that starting on a blank tape in state s ends in state q_{accept} with the tape content w beginning in the first cell and the head under that cell.

   That is, starting on a blank tape, the machine outputs w and stops in q_{accept}.

5. [10 Points] **Decidability and Enumerators.** Show that a language $L \subseteq \Sigma^*$ is *decidable* if and only if there exists an enumerator that enumerates L in shortlex string order (where strings are arranged by length and within the same length alphabetically). Note: this is an *iff* statement, you are thus expected to prove both directions.

6. [10 Points] **Large parts of recognizable languages are decidable.** Show that every infinite Turing-recognizable language has an infinite decidable subset.

7. [6 Points] **Tint-two-way**. Design a two-way Turing machine that on input any binary number bin(x) (the binary representation of $x \in \mathbb{N}$) outputs bin($x+1$). The machine starts with bin(x) written on an otherwise empty tape with the head under the first symbol and ends in state q_{accept} with bin($x+1$) written anywhere on this otherwise empty tape with the head under the first symbol.

8. [6 Points] **Tint-two-way**. Design a two-way Turing machine that on input the string bin(x)+bin(y) (where $x, y \in \mathbb{N}$) outputs bin($x+y$). The machine starts with bin(x)+bin(y) written on an otherwise empty tape with the head under the first symbol and ends in state q_{accept} with bin($x+y$) written anywhere on this otherwise empty tape with the head under the first symbol.