Name: Kevin Zhang

Problem 1.

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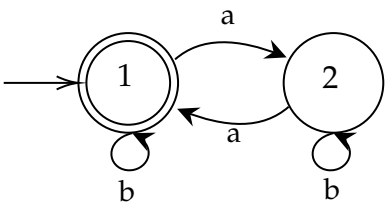
Problem 2.

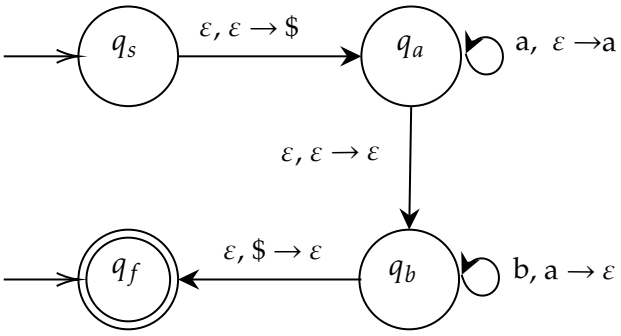
Intentionally Left Blank

Problem 3.

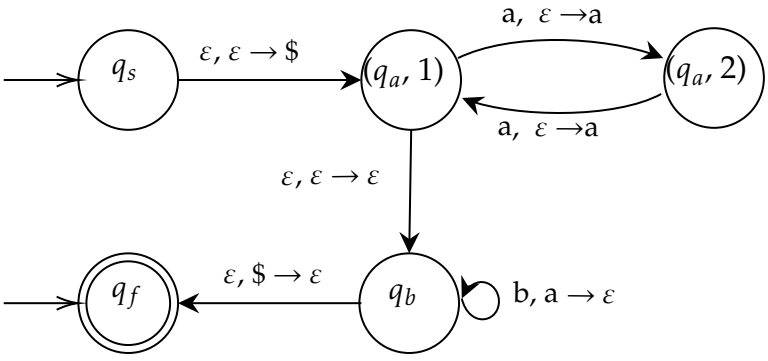
(a)



(b)



(c)



Problem 4.

(a) $L_1 = \{a^n b^m \mid n \leq m\}$ can be concatenated with $L_2 = \{b\}$ to form $L_3 = \{a^n b^m \mid n < m\}$. By closure properties, L_1 and L_2 are context-free, so L_3 must be as well. A similar argument can be made for $\{a^n b^m \mid n > m\}$, except by concatenating $\{a\}$ instead.

(b) If we let $L_1 = \{a^n b^m \mid n < m\}$, and $L_2 = \{a^n b^m \mid n > m\}$ $L = \{a, b\}^* - \{a^n b^n \mid n \in \mathbb{N}\}$ can be expressed as $L_1^* \cup L_2^* \cup (L_1^R)^* \cup (L_2^R)^*$. Union, Reveseral, and Star are closed under context-free languages, so L must also be context-free.

Problem 5.

(a) If we suppose that $L = \{a^n b a^n b a^n b \mid n \geq 1\}$ is context-free, then we can assume some pumping length p. Suppose we have $w = a^{p-1} b a^{p-1} b a^{p-1} b$. The pumping lemma shows that we can construct w as $uvxyz$, such that $|xyv| \leq p$ and $v \neq \varepsilon$ or $y \neq varepsilon$. No matter what we pick for v and y, pumping those v or y will result in a word that is not in L. If either v or y is ε, letting the other select a substring in w, the largest substring that the other can be is $a^{p-1} b$ or $b a^{p-1}$, which if pumped, will result in an imbalance in the number of as. If we have values for both v and y, we still run into an issue, because there will always be a third set of as that is not being pumped. Therefore, L is not context-free.

(b) We can use a regular language such as $aa*baa*baa*b$. $L \cap aa*baa*baa*b = \{a^n b a^n b a^n b \mid n \geq 1\}$. We already know that the right hand side is not context-free, which means L cannot be context-free.