Homework 4
(due Tuesday, October 13)

Instructions: This homework is to be submitted on GradeScope as a *single* pdf (not in parts) by 11:59 pm on the due date. You may either type your solutions in a word processor and print to a pdf, or write them by hand and submit a scanned copy. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc....).

Begin by reviewing your class notes, the slides, and the textbook. Then do the exercises below. Show your work. An unjustified answer may receive little or no credit.

Read: 2.1 to p 107 (for Thursday-Friday), 2.1, 2.2 to p 116 (for Tuesday)

1. [14 Points] Let

$$\Sigma = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \ldots, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}.$$

Σ contains all size 3 columns of 0s and 1s. A string of symbols in Σ gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$A = \{w \in \Sigma^* \mid \text{ the bottom row of } w \text{ is the sum of the top two rows}\}.$$

(By convention, the empty string is, vacuously, in A.)
For example,

$$\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} \in A, \qquad \text{but} \qquad \begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} \notin A$$

(a) Show that the language A^R is regular and construct a DFA for this language (draw its diagram or table).

(b) Use general principles (not an automaton) to explain why A is a regular language.

(c) **tint.** Draw the state diagram of a DFA recognizing A, and submit it here. Then submit your DFA in the separate GradeScope assignment "hw4_tint". Due to the linearity of text in **tint**, represent each column $\begin{bmatrix}a\\b\\c\end{bmatrix}$ with a string *abc*. For example, your automaton should accept input `001 100 110` but reject input `111 110`.

2. [19 Points] For each of the following statements, state whether it is true or false. Explain.

(a) $baa \in a^*b^*a^*b^*$

(b) $a^* \cup b^* = (a \cup b)^*$

(c) $(a^*b^*)^* = (a \cup b)^*$

(d) $b^*a^* \cap a^*b^* = a^* \cup b^*$

(e) $(ab)^*a = a(ba)^*$

(f) $a^*b^* \cap c^*d^* = \emptyset$

(g) $abcd \in (a\,(cd)^*\,b)^*$

3. [12 Points] Let $\Sigma = \{a, b\}$. Write regular expressions (as simple as you can) for the sets

   (a) All strings in Σ^* with no more than three a's.

   (b) All strings in Σ^* that do not end with ab.

   (c) All strings in Σ^* with a number of a's divisible by 3.

4. [10 Points] Rewrite each of the following regular expressions as a simpler regular expression describing the same language. (For full credit simplify as much as possible).

   (a) $(a \cup b)^*(a \cup \varepsilon)b^*$

   (b) $\emptyset^* \cup a^* \cup b^* \cup (a \cup b)^*$

   (c) $((a^*a)\,b) \cup b$

   (d) $(a^*\,b)^* \cup (b^*\,a)^*$

   (e) $(a \cup b)^*\,a\,(a \cup b)^*$

5. [5 Points] Draw the state diagram of a three-state NFA recognizing the language described by $((a \cup c)^*\ (b \cup c)^*\ c\ b^*)$.

6. [4 Points] *Use the procedure from lecture 7c* to convert the expression $a^* \cup (ab)^*$ to a NFA.

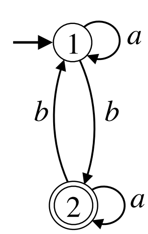
7. [6 Points] In a few words explain

   (a) The three main differences between a NFA and a DFA.

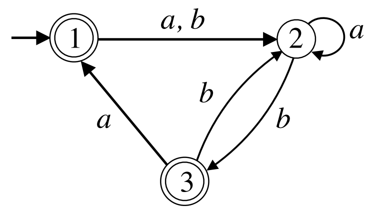
   (b) The main difference between a GNFA and a NFA.

8. [10 Points] *Use the method from lecture 7d* to convert the following finite automata to regular expressions. In each case first construct the corresponding GNFA (without $\emptyset$ labeled edges), then strip states one by one in numerical order (strip state 1 first, then state 2, ...). Show your work, including the diagrams after each stripping.

   (a)



(b)



9. [14 Points] **Pumping in regular languages.** For each of the following languages (over $\Sigma = \{0, 1\}$), give its minimum pumping length and justify your answer. Hint: See problem 1.55 page 91 and its solution page 99.

   (a) $001 \cup 0^*1^*$

   (b) $(01)^*$

   (c) ε

   (d) $1^*01^*01^*$

   (e) $10(11^*0)^*0$

   (f) 1011

   (g) Σ^*

10. [4 Points] Find the minimum pumping length for the language
    $\{w \in \{0, 1\}^* \mid w \text{ contains neither 0000 nor 1111 as substrings}\}$

11. [8 Points] Use the Pumping Lemma to show that the following languages are not regular. In each case, carefully describe the string that will be pumped and explain why pumping it leads to a contradiction.

    (a) $\{w\, w^R \mid w \in \{a, b\}^*\}$

    (b) $\{w\, \overline{w} \mid w \in \{a, b\}^*\}$
    ($\overline{w}$ stands for w with each occurrence of a replaced by b, and vice versa.)

12. [4 Points] Let $\Sigma = \{a\}$ and consider the language

    $$L = \{a^t \mid t \text{ is a prime number }\}$$

    Use the pumping Lemma to prove that L is not regular. Carefully describe the string that will be pumped and explain why pumping it leads to a contradiction.