Homework 1
(due Tuesday, September 22)

Instructions: This homework is to be submitted on GradeScope as a *single* pdf (not in parts) by 11:59 pm on the due date. You may either type your solutions in a word processor and print to a pdf, or write them by hand and submit a scanned copy. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc....).

Begin by reviewing your class notes, the slides, and the textbook. Then do the exercises below. Show your work. An unjustified answer may receive little or no credit.

Look ahead: Problems 11 and 12 require knowledge of function properties that will be covered in Thursday-Friday classes.

Read: 0.1, 0.2 (for Thursday-Friday), 1.1 (for Tuesday)

1. [6 Points] For the Tseitin word problem discussed in class, are the words in the following pairs equivalent? Prove your answers.
   - **(a)** $bccdbc$ and $cbabd$
   - **(b)** $cadbcedb$ and $caccaebd$
   - **(c)** $aecdab$ and $cade$

2. [8 Points] Which of the following conditional statements are true and why?
   - **(a)** If February has 30 days, then 7 is an odd number.
   - **(b)** If January has 31 days, then 7 is an even number.
   - **(c)** If 7 is an odd number, then February does not have 30 days.
   - **(d)** If 7 is an even number, then January has exactly 28 days.

3. [4 Points] Let x be a positive real number. Prove that if x is irrational (i.e., not a rational[1] number), then $\sqrt{x}$ is also irrational. For full credit your proof should be simple. Think about an optimal method to complete such a proof (in class we learned of two methods to prove implications, choose the most appropriate for this case).

4. [8 Points] In this problem, the domain is the set $\mathbb{Z}$ of integers. Write the negation of each of the following statements, simplifying to the point that no '$\neg$' symbol occurs in any of the statements (you may, however, use binary symbols such as '$\neq$' and '$<$'). For each of the statements, indicate whether the original statement is true.
   - **(a)** $\forall x\ (x \cdot 2 \neq x \cdot 3)$

[1] A rational number is a number x that can be written in the form $x = p/q$ where p and q are integers and $q \neq 0$. The set of rational numbers is written $\mathbb{Q}$.

(b) $\exists x\ (x\ + 2 = x + 3)$

(c) $\forall x\ (x^2 \neq x)$

(d) $\exists x\ (5 \leq x < 6)$

5. [7 Points] For this problem, the domain is the set of Northeastern students. Consider the following two predicates

Charlie(x)	meaning " Charlie knows x "
CS(x)	meaning " x is a CS student "

Using only variables, logic symbols (selected from $\neg, \vee, \wedge, \rightarrow, \leftrightarrow, \exists, \forall$), and the predicates Charlie() and CS(), formulate the statements:

(a) Charlie doesn't know every CS student.

(b) The only students known to Charlie are CS students.

(c) Charlie knows at least two students. Here you may also use symbols "=" and "≠", in addition to those listed above.

6. [8 Points] For this problem, the domain is the set of all students at Northeastern. Consider the following two predicates

CS(x)	meaning	"x is a CS student"
Knows(x, y)	meaning	"x knows y"

Using *only* variables, logic symbols (selected from $\neg, \vee, \wedge, \rightarrow, \leftrightarrow, \exists, \forall$), and the predicates CS() and Knows(), formulate the statements:

(a) Some student knows every student

(b) Some student knows every CS student

(c) Each CS student knows at least one student who isn't a CS student.

(d) Each student knows at least one student who doesn't know all CS students.

7. [5 Points] For each of the following set operations, specify the result by listing its elements inside curly braces. Be sure to use proper notation.

(a) $\{2, 3\} \cup \{1, 2, 4\}$

(b) $\{2, 3\} \cap \{1, 2, 4\}$

(c) $\{2, 3\} \times \{1, 2, 4\}$

(d) $\{2, 3\} - \{1, 2, 4\}$

(e) $\{2, 4\} - \{1, 2, 4\}$

8. [6 Points] For each positive integer i define set S_i to be the interval

$$S_i = \left[-\frac{1}{i}\ ,\ \frac{1}{i}\right] \ = \ \left\{x \in \mathbb{R} \ \middle|\ -\frac{1}{i} \le x \le \frac{1}{i}\right\}$$

Determine each of the following sets (giving as simple a description as possible).

(a) $\bigcap_{i=1}^{\infty} S_i$

(b) $\bigcup_{i=1}^{\infty} S_i$

9. [4 Points] Let $A = \{1, 2, 3\}$, $B = \{x, y\}$, $C = \{u, v\}$, $D = \{x\}$, $E = \{1, 2, x, u, v\}$.

(a) Use the listing method to write the set $A \times B \times C$ (be sure to use proper notation).

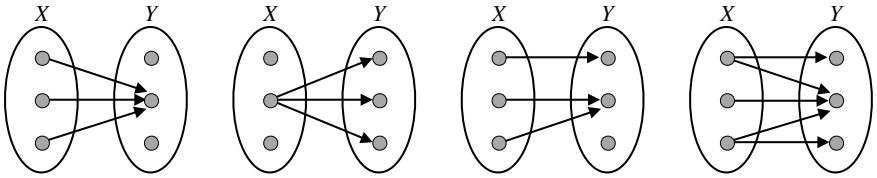
(b) What is the cardinality of set $A \times B \times C \times D \times E$?

10. [6 Points] List six different elements of the set

$$S = \left\{(X, Y) \in \ \mathcal{P}\big(\{1, 2, 3\}\big) \times \mathcal{P}\big(\{3, 4\}\big) \ \middle|\ X \subseteq Y\right\}$$

Make sure to use proper notation.

11. [4 Points] Under each of the following arrow diagrams, state whether it represents a function from X to Y.



12. [4 Points] Under each of the following arrow diagrams, state whether it represents an *injection*, a *surjection*, a *bijection*, or *none* of these types of functions. For bijections, just write 'bijection', without listing other types.