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Problem 1.

Tseitin's Rules:

1. $ac = ca$
2. $ad = da$
3. $bc = cb$
4. $bd = db$
5. $ce = eca$
6. $de = edb$
7. $cca = ccae$

(a) $bccdbc \neq cbabd$
The pair is not equivalent, because $bccdbc$ is a 6-letter word, and $cbabd$ is a 5-letter word, so we would need to remove a letter to go from left to right. Rules 1-4 are swap rules, and cannot remove a letter. Rules 5 and 6 require an e, which the left side does not have, nor can we use Rule 7 to create one (because there is no a on the left side).
Therefore, the two are not equivalent.

(b) $cadbcedb \neq caccaedb$
The pair is not equivalent, because $cadbcedb$ has two d's, and $caccaedb$ has only one. None of the rules can remove or create a d:
Rules 1-4 are swap rules, and does not create or remove letters
Rule 5 creates an a when going from left to right.
Rule 6 creates an b when going from left to right.
Rule 7 creates an e when going from left to right.
Therefore, the two are not equivalent.

(c)

$$\begin{aligned}
aecdab &\stackrel{?}{=} cade \\
&\stackrel{?}{=} caedb (\text{Rule 6: } de = edb) \\
&\stackrel{?}{=} acedb (\text{Rule 1: } ca = ac) \\
&\stackrel{?}{=} aecadb (\text{Rule 5: } ce = eca) \\
&\stackrel{?}{=} aecdab (\text{Rule 2: } ad = da) \\
aecdab &= aecdab \checkmark
\end{aligned}$$

Therefore, the two are equivalent.

Problem 2.

(a) If February has 30 days, then 7 is an odd number.
February has 30 days = False
7 is an odd number = True
False $\rightarrow$ True = **True**

(b) If January has 31 days, then 7 is an even number.
January has 31 days = True
7 is an even number = False
True $\rightarrow$ False = **False**

(c) If 7 is an odd number, then February does not have 30 days
7 is an odd number = True
February does not have 30 days = True
True $\rightarrow$ True = **True**

(d) If 7 is an even number, then January has exactly 28 days
7 is an even number = False
January has exactly 28 days = False
False $\rightarrow$ False = **True**

Problem 3.

Proof. Suppose $\sqrt{x}$ is rational. Then $\sqrt{x} \cdot \sqrt{x}$ is also rational. $x = \sqrt{x}^2$, so x must also be rational. Therefore, by the contrapositive, if x is irrational, then $\sqrt{x}$ must be irrational.

$\square$

Problem 4.

(a) $\forall x \ \ (x \cdot 2 \neq x \cdot 3)$ is False when $x = 0$

$$\begin{aligned}
&\neg(\forall x \ \ (x \cdot 2 \neq x \cdot 3)) \\
&\exists x \ \ \neg(x \cdot 2 \neq x \cdot 3) \\
&\exists x \ \ (x \cdot 2 = x \cdot 3)
\end{aligned}$$

(b) $\exists x \ \ (x + 2 = x + 3)$ is False.

$$\begin{aligned}
&\neg(\exists x \ \ (x + 2 = x + 3)) \\
&\forall x \ \ \neg(x + 2 = x + 3) \\
&\forall x \ \ (x + 2 \neq x + 3)
\end{aligned}$$

(c) $\forall x \ \ (x^2 \neq x)$ is False when $x = 0$ or $x = 1$

$$\begin{aligned}
&\neg(\forall x \ \ (x^2 \neq x)) \\
&\exists x \ \ \neg(x^2 \neq x) \\
&\exists x \ \ (x^2 = x)
\end{aligned}$$

(d) $\exists x \ \ (5 \leq x < 6)$ is True. $x = 5$ is possible.

$$\begin{aligned}
&\neg(\exists x \ \ (5 \leq x < 6)) \\
&\forall x \ \ \neg(5 \leq x < 6) \\
&\forall x \ \ \neg(5 \leq x \wedge x < 6) \\
&\forall x \ \ \neg(5 \leq x) \vee \neg(x < 6) \\
&\forall x \ \ (5 > x) \vee (x \geq 6)
\end{aligned}$$

Problem 5.

(a) $\exists x \;\; CS(x) \land \neg Charlie(x)$

(b) $\forall x \;\; Charlie(x) \rightarrow CS(x)$

(c) $\exists x_1 \exists x_2 \;\; x_1 \neq x_2 \land Charlie(x_1) \land Charlie(x_2)$

Problem 6.

(a) $\exists x \forall y \;\; Knows(x, y)$

(b) $\exists x \forall y \;\; CS(y) \land Knows(x, y)$

(c) $\forall x \exists y \;\; CS(x) \land \neg CS(y) \land Knows(x, y)$

(d) $\forall x \exists y \forall z \;\; Knows(x, y) \land CS(z) \land \neg Knows(y, z)$

Problem 7.

(a) $\{1, 2, 3, 4\}$

(b) $\{2\}$

(c) $\{(2,1), (2,2), (2,4), (3,1), (3,2), (3,4)\}$

(d) $\{3\}$

(e) $\varnothing$

Problem 8.

(a) As i goes to infinity, the interval shrinks to $[-0, 0]$, so the intersection would be $\{0\}$.

(b) As i goes to infinity, the interval is always shrinking to $[-0, 0]$, so the union would be largest interval: $[-1, 1]$ or S_1.

Problem 9.

(a) $\{(1, x, u), (1, x, v), (1, y, u), (1, y, v), (2, x, u), (2, x, v), (2, y, u), (2, y, v), (3, x, u), (3, x, v), (3, y, u), (3, y, v)\}$

(b) $|A \times B \times C \times D \times E| = |A| \times |B| \times |C| \times |D| \times |E| = 60$

Problem 10.

1. $(\varnothing, \varnothing)$
2. $(\varnothing, \{3\})$
3. $(\varnothing, \{4\})$
4. $(\varnothing, \{3, 4\})$
5. $(\{3\}, \{3\})$
6. $(\{3\}, \{3, 4\})$

Problem 11.

(Diagrams from Left to Right)

1. Is a function; every x produces a unique y.
2. Is not a function; there are some x with no result y.
3. Is a function; every x produces a unique y.
4. Is not a function; there are some x that produce multiple y.

Problem 12.

(Diagrams from Left to Right)

1. injection
2. none
3. bijection
4. surjection