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Problem 1.

(a) $aabaab$ is not in $L(G)$. The reason is that there is no way to have a b at the end on its own. The only derivation that results in a b is $B \to bA$, and since there is no derivation $A \to \varepsilon$, there is no way to have b at the end.

(b) $aaaaba$ is in $L(G)$. The derivation is as follows

$$
\begin{aligned}
S &\to AB \\
&\to aAB \\
&\to aAbA \\
&\to aAba \\
&\to aaAba \\
&\to aaaAba \\
&\to aaaaba
\end{aligned}
$$

(c) $aabbaa$ is not in $L(G)$. The same reasoning for (a) applies. It is not possible to have the two b's together in the center, without any as, because the only derivation that results in a b is $B \to bA$. Since $A \to \varepsilon$ is not possible, there must be at least one a between b's

(d) $abaaba$ is in $L(G)$. The derivation is as follows

$$
\begin{aligned}
S &\to ABS \\
&\to ABAB \\
&\to aBaB \\
&\to abAabA \\
&\to abaaba
\end{aligned}
$$

Problem 2.

(a) Derivation:

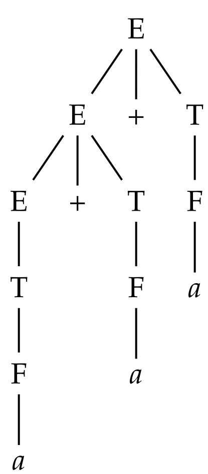
$$\begin{aligned} E &\to T \\ &\to F \\ &\to a \end{aligned}$$

Parse Tree:

(b) Derivation:

$$\begin{aligned} E &\to E + T \\ &\to E + T + T \\ &\to T + T + T \\ &\to F + F + F \\ &\to a + a + a \end{aligned}$$

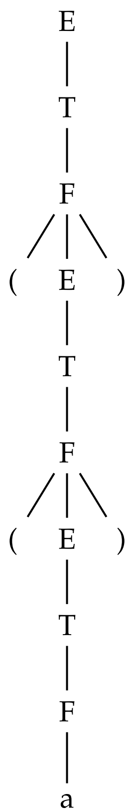
Parse Tree:



(c) Derivation:

$$
\begin{aligned}
E &\to T \\
&\to F \\
&\to (E) \\
&\to (T) \\
&\to (F) \\
&\to ((E)) \\
&\to ((T)) \\
&\to ((F)) \\
&\to ((a))
\end{aligned}
$$

Parse Tree:



Problem 3.

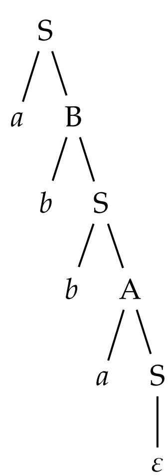
(a)

$$
\begin{aligned}
S &\to aB \\
&\to aaBB \\
&\to aabSB \\
&\to aabB \\
&\to aabbS \\
&\to aabb
\end{aligned}
$$

(b)

$$
\begin{aligned}
S &\to aB \\
&\to aaBB \\
&\to aaBaBB \\
&\to aaBaBbS \\
&\to aaBaBb \\
&\to aaBabSb \\
&\to aaBabb \\
&\to aabSabb \\
&\to aababb
\end{aligned}
$$

(c)



Problem 4.

(a) $V = \{S\}$
$\Sigma = \{a, b\}$
$R = \{\}$
$S = S$

(b) $V = \{S\}$
$\Sigma = \{a, b\}$
$R = \{S \to a \mid b \mid aaS \mid abS \mid baS \mid bbS\}$
$S = S$

(c) $V = \{S\}$
$\Sigma = \{a, b\}$
$R = \{S \to \varepsilon \mid Sb \mid aSb\}$
$S = S$

(d) $V = \{S, T\}$
$\Sigma = \{a, b\}$
$R = \{S \to TaTaTaTaT, \quad T \to \varepsilon \mid aT \mid bT\}$
$S = S$

(e) $V = \{S, T\}$
$\Sigma = \{a, b\}$
$R = \{S \to aTa \mid bTb, \quad T \to \varepsilon \mid aT \mid bT\}$
$S = S$

Problem 5.

(a) $V = \{S, T, F\}$
$\Sigma = \{a, b, c\}$
$R = \{S \to TF, \quad T \to \varepsilon \mid aTb, \quad F \to \varepsilon \mid bFc\}$
$S = S$
Reasoning: For every a or c, we need to have a b. We cannot mix the as on the left with the cs on the right, so we need intermediary variables to prevent mixing, but the bs can be joined in the middle.

(b) $V = \{S, T\}$
$\Sigma = \{a, b, c\}$
$R = \{S \to \varepsilon \mid aSc \mid T, \quad T \to \varepsilon \mid aTb\}$
$S = S$
Reasoning: For every b or c, we need to have a a. We can first fix the number of cs we want, and then squeeze the number of bs in between. Everything, is of course, prepended with an a, so the counts line up.

Problem 6.

$V = \{S, T, F\}$
$\Sigma = \{0, 1\}$
$R = \{S \to \varepsilon \mid 0S \mid 1T, \quad T \to 0F \mid 1S, \quad F \to 1F \mid 0T\}$
$S = S$
Reasoning: The substituions from HW2 10F translate nicely here, with each of S, T, F representing a state. S has the only terminal derviation, so all words generated need to end in S.

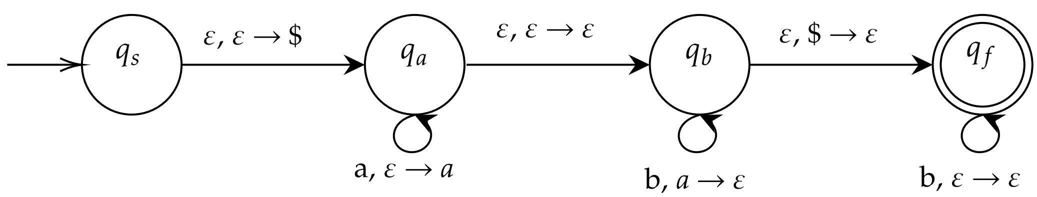
Problem 7.

(a) **Initial**
$S \to aSab \mid B$
$B \to bbC \mid bb$
$C \to \varepsilon \mid cC$
Pass 1
$S_{new} \to S$
$S \to aSab \mid B$
$B \to bbC \mid bb$
$C \to \varepsilon \mid cC$
Pass 2
$S_{new} \to S$
$S \to aSab \mid B$
$B \to bbC \mid bb$
$C \to cC$
Pass 3
$S_{new} \to aSab \mid bbC \mid bb$
$S \to aSab \mid bbC \mid bb$
$B \to bbC \mid bb$
$C \to cC$
Pass 4
$S_{new} \to aW_1 \mid bW_3 \mid bb$
$S \to aW_1 \mid bW_3 \mid bb$
$B \to bW_3 \mid bb$
$C \to cC$
$W_1 \to SW_2$
$W_2 \to ab$
$W_2 \to bC$
Pass 5
$S_{new} \to T_1 W_1 \mid T_b W_3 \mid T_b T_b$
$S \to T_a W_1 \mid T_b W_3 \mid T_b T_b$
$B \to T_b W_3 \mid T_b T_b$
$C \to T_c C$
$W_1 \to SW_2$
$W_2 \to T_a T_b$
$W_2 \to T_b C$

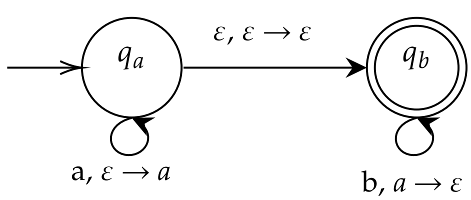
$T_a \to a$
$T_b \to b$
$T_c \to c$

(b) **Initial**
$S \to AB$
$A \to a \mid B$
$B \to b \mid A \mid \varepsilon$
Pass 1
$S_{new} \to S$
$S \to AB$
$A \to a \mid B$
$B \to b \mid A \mid \varepsilon$
Pass 2
$S_{new} \to S \mid \varepsilon$
$S \to AB \mid A \mid B$
$A \to a \mid B$
$B \to b \mid A$
Pass 3
$S_{new} \to S \mid \varepsilon$
$S \to AB \mid A \mid AA$
$A \to a \mid b$
$B \to b$

Problem 8.

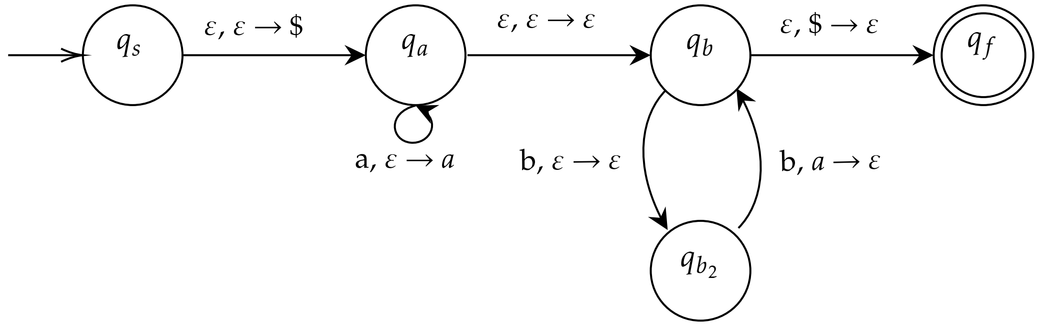


(a)



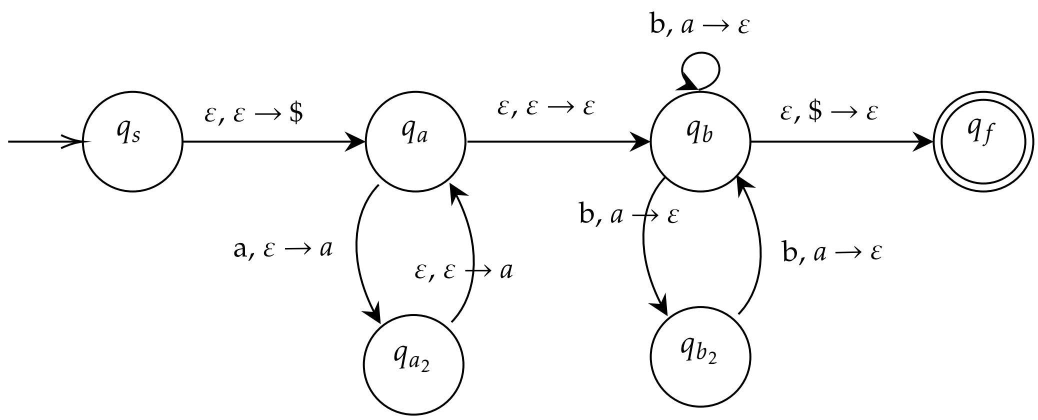
(b)

Problem 9.



Here, q_s is the start state. q_a is a state in which we are counting the number of as (by pushing onto the stack). q_b and q_{b_2} are states to ensure we read 2 bs for every a we are popping off the stack. q_f is a final accepting state, to ensure that we don't have anything more to read.

Problem 10.



Here, q_s is the start state. For every a, we are pushing two a's onto the stack (this is what q_a and q_{a_2} are for – conversion from extended pushdown). Then, in q_b, we have the option of either popping 2 as off the stack, or 1 a, until we reach the bottom of the stack, leaving us in q_f.