Homework 3

(due Tuesday, October 6)

Instructions: This homework is to be submitted on GradeScope as a *single* pdf (not in parts) by 11:59 pm on the due date. You may either type your solutions in a word processor and print to a pdf, or write them by hand and submit a scanned copy. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc....).

Begin by reviewing your class notes, the slides, and the textbook. Then do the exercises below. Show your work. An unjustified answer may receive little or no credit.

Read: 1.3 (for Thursday-Friday), 1.4 (for Tuesday)

1. [5 Points]. **Tint**. Let

$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

   A string of symbols from Σ defines two rows of 0s and 1s. Consider each row to be a binary number and let

$$D = \{w \in \Sigma^* \mid \text{the top row of } w \text{ is a smaller number than the bottom row}\}$$

   For example, $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in D$ because, as binary numbers, $0011 < 0100$.

   Construct a finite automaton that accepts D. Note that, due to the linearity of symbols in tint, the columns above have to be represented by strings of length 2. That is, $\Sigma = \{00, 01, 10, 11\}$. And "00 01 10 10" $\in D$ because, as binary numbers, $0011 < 0100$.

2. [10 Points] Consider the language

$$L = \{w \in \{a, b\}^* \mid w \text{ has odd length and an even number of } a\text{'s}\}$$

   This language is the intersection of the simpler languages $L_1 = \{w \in \{a, b\}^* \mid w \text{ has odd length}\}$ and $L_2 = \{w \in \{a, b\}^* \mid w \text{ has an even number of } a\text{'s}\}$.

   (a) Give the diagram of a DFA accepting L_1.

   (b) Give the diagram of a DFA accepting L_2.

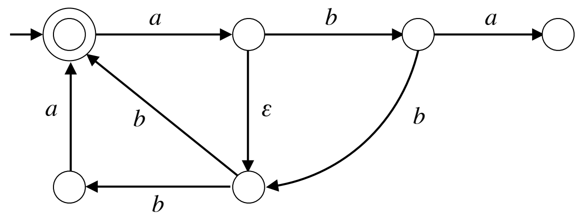
   (c) Use the construction of Lecture 5b to give the state diagram of a DFA for L.

3. [8 Points] Let $L = \{w \in \{a, b, c\}^* \mid w \neq a \text{ and } w \neq b \text{ and } w \neq c\}$.

   (a) Give the state diagram of a DFA for the complement $\overline{L}$ of L.

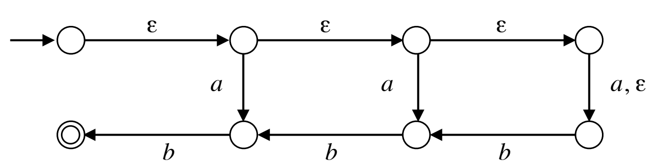
   (b) Apply the methods of Lecture 5b to give the state diagram of a DFA for L.

4. [5 Points] Which, if any, of the strings below are accepted by the following nondeterministic finite automaton?



   (a) aa
   (b) aba
   (c) abb
   (d) ab
   (e) $abab$

5. [8 Points] Describe the language $L \subseteq \{a, b\}^*$ that is accepted by the nondeterministic automaton



6. [20 Points] Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{a, b\}$.

   (a) $\{a\}$ with 2 states.
   (b) $\{\varepsilon\}$ with 1 state.
   (c) $\{a\}^*$ with 1 state.
   (d) $\{w \mid w$ contains the substring $abab\}$ with 5 states.
   (e) $\{w \mid w$ contains an even number of a's, or contains exactly two b's$\}$ with 6 states.

7. [15 Points] Let $\Sigma = \{0, 1\}$ and consider the languages $L_1 = \{w \in \Sigma^* \mid w \text{ has length at most } 5\}$ and $L_2 = \{w \in \Sigma^* \mid \text{ every odd position[1] of } w \text{ is a } 1\}$.

   (a) Draw the state diagram of a DFA for L_1.
   (b) Draw the state diagram of a DFA for L_2.
   (c) Use the method from Lecture 6 to draw the state diagram of a NFA for $L_1 \circ L_2$.

[1] Assume that the string is indexed from 1, that is position 1 is its first symbol.

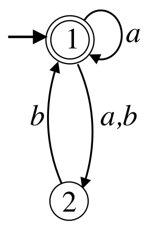
8. [8 Points] Let $L = \{w \in \{0,1\}^* \mid\ w \text{ contains at least two 0's and at most one 1}\}$.

   (a) Give the state diagram of a DFA for L.

   (b) Use the method from Lecture 6 to construct the state diagram of a NFA for L^*.

9. [11 Points] Use the subset method from Lecture 6 to convert the following nondeterministic finite automata to equivalent deterministic finite automata.

(a)



(b)