CS 3800 (2+3) Fall 2020
W. Schnyder 11/17/2020

Homework 9

(due Tuesday, November 24)

Instructions: This homework is to be submitted on GradeScope as a *single* pdf (not in parts) by 11:59 pm on the due date. You may either type your solutions in a word processor and print to a pdf, or write them by hand and submit a scanned copy. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc....).

Begin by reviewing your class notes, the slides, and the textbook. Then do the exercises below. Show your work. An unjustified answer may receive little or no credit.

Problems 6 and 7 require lecture 20 (Thursday-Friday).

Read: 5.1, 6.3 (for Thursday-Friday), 5.3, 7.1 (for Tuesday)

Note: Except in problems 1 and 2, assume that all Turing machines have the same input alphabet $\Sigma = \{0, 1\}$ and are themselves encoded in Σ.

1. [10 Points] (Past topic, may be used in future assignment.)
   Construct a pushdown automaton P such that
   $$L(P) = \{w \in \{a, b\}^* \mid w \text{ has more } b's \text{ than } a's\}.$$
   Specify the components of your automaton and draw a state-diagram.

2. [5 Points] **Tint.** Let $\Gamma = \{a, b, c\}$. You are to design an **iterator** for Γ^*, i.e., a one-way Turing machine with input alphabet Γ, that on input any string $w \in \Gamma^*$, outputs the string next(w) that follows w in shortlex order. In the end configuration:
   - the tape content consists of next(w) beginning in the first cell (all other cells blank)
   - the head is under the first cell
   - the state is q_{accept}

3. [6 Points] If Turing machines are encoded as described in Lecture 18b and M is the machine whose encoding $\langle M \rangle$ is the string

   011 000 110 000 011 000 110 111 010 011 000 110 111 011 111 110 000 100 011 000 110 111 000 011 111 000 110 111 010

   (a) What are the transitions of M?

   (b) What is the language accepted by M?

   (c) Is M a decider?

4. [10 Points] Let SUB_{DFA} consist of all (encoded) pairs $\langle A, B\rangle$ of DFAs such that every word accepted by A is also accepted by B.

$$SUB_{DFA} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) \subseteq L(B)\}$$

Show that to decide whether every word $w \in \Sigma^*$ that is accepted by A is also accepted by B, it is sufficient to test the two DFAs on all strings up to a certain length. Show how you would calculate a length that works.

5. [10 Points] Let Odd-Self be the language

$$\{\langle M\rangle \mid M \text{ is a Turing machine that halts on input } \langle M\rangle \text{ after an odd number of steps}\}$$

That is, on input $\langle M\rangle$, machine M enters state q_{accept} or q_{reject} after executing an odd number of transitions.

(a) We want to show that Odd-Self is undecidable by a proof from scratch (i.e., by diagonalization, not by reduction). Assume Odd-Self is decidable, that is, there is a TM T that on input $\langle M\rangle$

- accepts if M is a Turing machine that halts on input $\langle M\rangle$ after an odd number of steps
- rejects otherwise

Prove that this assumption is contradictory.

(b) Is Odd-Self a recognizable language? Prove your answer.

(c) Let Odd-Halt be the language

$\{\langle M, w\rangle \mid M$ is a Turing machine that halts on input w after an odd number of steps$\}$.

Is the language Odd-Halt decidable? Prove your answer.

6. [15 Points] Use Rice's theorem to prove the undecidability of each of the following languages. Do explain why you may use the Theorem and how you are using it.

(a) $\{\langle M\rangle \mid M \text{ is a TM and } 1011 \in L(M)\}$.

(b) $ALL_{\text{TM}} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \Sigma^*\}$.

(c) $E_{TM} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$.

7. [4 Points] Can Rice's Theorem be used directly to show that the language

$$\text{Self-}A_{TM} = \{< M >\mid M \text{ is a Turing machine and } M \text{ accepts } \langle M\rangle\}$$

is undecidable? Either do so, or explain why it isn't possible.

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8. [**0** Point] **Do not submit.** Problem 3.22 page 190. The solution is in the book page 191, this is for practice only.

9. [**0** Point] **Do not submit.** Problem 3.10 page 188. The solution is in the book page 191, this is for practice only.

10. [**0** Point] **Do not submit.** Problem 4.5 page 211. Solution page 213. Let

$$E_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}.$$

Show that $\overline{E_{TM}}$, the complement of E_{TM} is Turing-recognizable.

11. [**0** Point] **Do not submit.** Problem 4.10 page 211. The solution is in the book page 213, this is for practice only.

12. [**0** Point] **Do not submit.** Problem 4.12 page 211. The solution is in the book page 214, this is for practice only.

13. [**0** Point] **Do not submit.** Problem 4.14 page 211. The solution is in the book page 214, this is for practice only.