Name: Kevin Zhang

Problem 1.

(a) $A \leq_T B$ is true. A is already decidable, so we can ignore the oracle for B, no matter what B is.

(b) $A \leq_m B$ is false. If $B = \emptyset$, there is no $f : \Sigma^* \to \Sigma^*$ that can turn an element of A into nothing.

Problem 2.

(a) L_1 is mapping-reducible to A_{TM}. We can define a f as follows:

$$f(x) = \begin{cases} x & x \text{ is not an encoding} \langle M \rangle \\ \langle M, s \rangle & \text{if x is an encoding } \langle M \rangle \text{ and s is a string consisting only of 1's that is accepted by M} \end{cases}$$

(b) L_2 is not mapping-reducible to A_{TM}. $A_{TM} = \{\langle M, s \rangle \mid \text{ M is a TM that accepts s }\}$. Since the machines in L_2 does not accept anything, there is no way to turn all those machines into acceptors.

Problem 3.

Want to show $A \subseteq \Sigma^*$ is decidable $\leftrightarrow A \leq_m 0^*1^*$.

(a) Prove $A \subseteq \Sigma^*$ is decidable $\to A \leq_m 0^*1^*$.

Assume A is decidable. Then, we can definte a f as follows:

$$f(x) = \begin{cases} 1 & \text{if x is accepted by A} \\ 0 & \text{if x is rejected by A} \end{cases}$$

Thus, $A \leq_m 0^*1^*$.

(b) Prove $A \subseteq \Sigma^*$ is decidable $\leftarrow A \leq_m 0^*1^*$.

Assume $A \leq_m 0^*1^*$.

0^*1^* is a regular expression, which means it can be represented with some DFA D.

D is decidable, because all DFAs are decidable.

$A \leq_m B$ and B is decidable $\rightarrow$ A is decidable (Shown in class, Lec22ab, Slide 150).

Therefore, A must be decidable.

Problem 4.

Prove that $HALT$ is enumerably complete. From the definition of enumerably complete, have two show two things:

(1) Prove $HALT$ is enumerable / recognizable. We can do so by constructing a recognizer:

On given input x, if x is not encoding $\langle M, w\rangle$, reject. If it is, then run M on input w. If M accepts or rejects (halts), then $HALT$ will accept.

(2) Prove for all $L \subseteq Sigma^*$, $L \leq_m HALT$.

We know that A_{TM} is enumerably complete (hw10, pb 7). This means that for all $L \subseteq Sigma^*$, $L \leq_m A_{TM}$.

$A_{TM} \leq_m HALT$ was shown previously (Lec 22ab, Slide 148).

For any $L \subseteq \Sigma^*$, if $L \leq_m A_{TM}$ and $A_{TM} \leq_m HALT$, then $L \leq_m HALT$ (from class).

Thus, for all $L \subseteq \Sigma^*$, $L \leq_m HALT$.

Problem 5.

$L = \{w \mid$ either w = 0x for some $x \in A_{TM}$ or w = 1y for some $y \in \overline{A_{TM}}\}$.

(1) Prove L is not recognizable.

$\overline{A_{TM}} \leq_m L$. We can create a mapping function f that adds a 1 before every element of $\overline{A_{TM}}$.

$\overline{A_{TM}}$ is not recognizable. We can prove this because A_{TM} is recognizable (shown in class), A_{TM} is not decidable (shown in class), and $\overline{A_{TM}}$ cannot be recognizable because A recognizable and $\bar{A}$ recognizable A decidable (shown in class).

$A \leq_m B$ and A not recognizable $\rightarrow B$ is not recognizable (Lec22ab, slide 150).

Thus, L is not recognizable.

(2) Prove $\bar{L}$ is not recognizable.

We can use a similar proof. $\overline{A_{TM}} \leq_m \bar{L}$. We can create a mapping function f that adds a 0 before every element of $\overline{A_{TM}}$.

$\overline{A_{TM}}$ is not recognizable (See above).

$A \leq_m B$ and A not recognizable $\rightarrow B$ is not recognizable (Lec22ab, slide 150).

Thus, $\bar{L}$ is not recognizable.

Problem 6.

True. 0^*1^* is a regular language, which is in class P (Lec 22c, Slide 7). Then, we apply property 2 on Lec23, Slide 28 to show $L \in P$.

Problem 7.

$K \stackrel{?}{=}$ recognizable . From Lecture 18c, we know that A is recognizable iff there exists decidable language B such that for all $w \in \Sigma^*$: $w \in A \leftrightarrow \exists s(\langle w, s\rangle \in B)$. In this case, $B \in P$, which makes B a decidable language. Therefore, K is most similar to recognizable languages. Most likely, $K \subseteq$ recognizable.

K cannot be decidable, because there is no constraint on $|s|$, so searching for s could potentially take forever, so decidable $\subseteq K$.

Thus, the full-chain might look like so:

regular $\subseteq$ context-free $\subseteq$ P $\subseteq$ NP $\subseteq$ decidable $\subseteq$ K $\subseteq$ recognizable .

Problem 8.

Show that $ISO \in NP$.

(1) ISO can be solved by non-deterministically assigning numbers to nodes of each graph, sorting the adjacency matrixes of the graphs (by node-number), and then comparing the two results. Differently-sized adjacency matrixes will be rejected outright.

(2) ISO can be verified in polynomial time. Compare the resulting adjacency matricies of G and H, to see if both graphs have the same verticies + edges. This will take $O(n^2)$.