Homework 2

(due Tuesday, September 29)

Instructions: This homework is to be submitted on GradeScope as a *single* pdf (not in parts) by 11:59 pm on the due date. You may either type your solutions in a word processor and print to a pdf, or write them by hand and submit a scanned copy. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc....).

Begin by reviewing your class notes, the slides, and the textbook. Then do the exercises below. Show your work. An unjustified answer may receive little or no credit.

Read: 1.1 p 44-47, 1.2 p 47-54 (for Thursday-Friday), 1.2 p 55-63, 1.3 p 63-66 (for Tuesday)

1. [10 Points] Let X and Y be finite sets with $|X| = |Y|$ and consider a function $f : X \to Y$. Prove the following statements (only true for finite sets, see Problem 2).

   (a) f is injective $\Rightarrow$ f is surjective.

   (b) f is surjective $\Rightarrow$ f is injective.

   Hint: count arrows as we did in class.

2. [6 Points] Give examples of

   (a) A function $f : \mathbb{N} \to \mathbb{N}$ that is injective but not surjective.

   (b) A function $f : \mathbb{N} \to \mathbb{N}$ that is surjective but not injective.

3. [10 Points] In the standard numbering (shortlex ordering) of strings over the alphabet $\{a, b, c\}$ what number corresponds to string $b\,c\,a\,c\,b\,a\,b$?

4. [12 Points] In the standard numbering (shortlex ordering) of strings over the alphabet $\{a, b, c\}$, what string has number 909 ?

5. [5 Points] Cantor's Theorem states that for every function $f : X \to \mathcal{P}(X)$ from a set X to its power set, there exists a set $S \subseteq X$ such that $S \neq f(x)$ for all $x \in X$. This set S is constructed using Cantor's *diagonalization* method.

   Suppose that $X = \{a, b, c, d, e\}$ and that f is given by

   $f(a) = \{a, b\}$
   $f(b) = \{c, d, e\}$
   $f(c) = \{b, c\}$
   $f(d) = \varnothing$
   $f(e) = \{a, c, d\}$

   What is the set S constructed by Cantor's diagonalization method?

6. [9 Points] Use (a modified) diagonalization to show that for each sequence $S_0, S_1, S_2, \ldots$ of subsets of $\mathbb{N}$, there is a set $S \subseteq \mathbb{N}$ consisting only of *even numbers* such that

$$S \neq S_0, S_1, S_2, \ldots$$

7. [5 Points] Write down the formal (5-tuple) description of the DFA pictured in Exercise 1.21(b) on page 86 of the textbook. (Specify each of the 5 components.)

8. [5 Points] Draw the state transition diagram for the DFA whose formal description is

$$(\{q_1, q_2, q_3\}, \{a, b\}, \delta, q_1, \{q_1, q_2\})$$

where δ is the function given in the following table:

	a	b
q_1	q_2	q_3
q_2	q_2	q_1
q_3	q_3	q_3

9. [4 Points] Describe the language recognized by the DFA whose formal description was given in Problem 8.

10. [16 Points] For each of the following languages over the alphabet $\{0, 1\}$, draw the state transition diagram of a DFA with alphabet $\{0, 1\}$ that recognizes the language. For full credit, try to keep your solution simple (limit the number of states).

    (a) $\emptyset$

    (b) $\{\varepsilon\}$

    (c) $\{01, 10\}$

    (d) $\{w \mid w \text{ starts with 1 and ends with 1}\}$

    (e) $\{w \mid w \text{ has neither 00 nor 11 as substring}\}$

    (f) $\{w \mid w \text{ is a binary numeral divisible by 3}\}$.
    For example, 0, 10101 and 000011 are in the language but ε and 111 are not.

    (g) $\{w \mid \text{ there exist strings } x \text{ and } y \text{ such that } w = x111y\}$

11. [8 Points]. **Tint.** This problem requires knowledge of our **tint** machine simulator which will be covered Thursday-Friday. You are, however, encouraged to first solve it "on paper" using a state diagram (you already can do this now).

    Let language L consist of all binary strings in which any two successive ones are separated by an odd number of zeros. For example, $\varepsilon \in L$ and $10001 \in L$ and $010100010 \in L$ but $1001 \notin L$ and $0101001000 \notin L$. Write a tint program for a DFA over the alphabet $\{0, 1\}$ such that L is the language accepted by your DFA.