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If we suppose that  $L = \{a^n b a^n b a^n b \mid n \geq 1\}$  is context-free, then we can assume some pumping length  $p$ . Suppose we have  $w = a^{p-1} b a^{p-1} b a^{p-1} b$ . The pumping lemma shows that we can construct  $w$  as  $uvxyz$ , such that  $|xyv| \leq p$  and  $v \neq \varepsilon$  or  $y \neq \varepsilon$ . No matter what we pick for  $v$  and  $y$ , pumping those  $v$  or  $y$  will result in a word that is not in  $L$ . If either  $v$  or  $y$  is  $\varepsilon$ , letting the other select a substring in  $w$ , the largest substring that the other can be is  $a^{p-1} b$  or  $b a^{p-1}$ , which if pumped, will result in an imbalance in the number of  $a$ s. If we have values for both  $v$  and  $y$ , we still run into an issue, because there will always be a third set of  $a$ s that is not being pumped. Therefore,  $L$  is not context-free.