Problem: Flux out of the face of a cube

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Prelude

A bit of backstory on this problem and why I find it important. It's no exaggeration to say that modern theoretical physics is all about symmetry, we often use symmetries to guide us when building Quantum Field Theories; we use them to dramatically simplify the quest for solutions to Einstein's equations; we use them, and their consequences, being conserved quantities, to reduce the dimensionality of the phase space when studying dynamical systems; *et cetera*. But symmetry is in many more places than where their use is canonical. Building the habit of looking for them whenever you're solving **any** physics problem early on, is invaluable. This is an exercise in practicing this skill.

Problem Statement

Suppose we have a cube with a positive charge placed on one of its vertices, the goal is to compute the electric field flux out of one of the cube's faces opposite to it, refer to the following diagram:

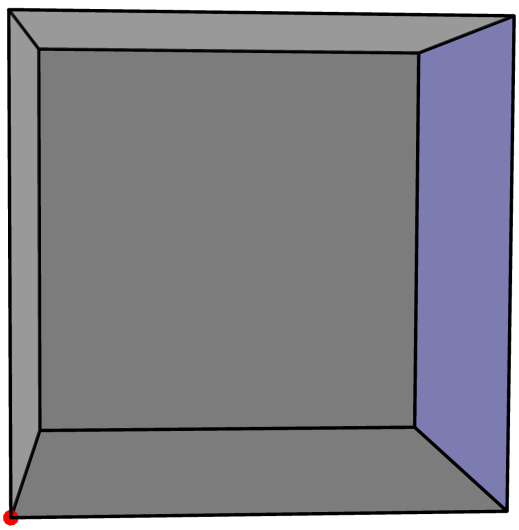

Figure 1: Red point indicates where the charge is, and blue hue indicates the face through which flux should be computed

Do this in two ways:

1. First, compute it by pure brute force, find the electric field and compute its flux through the face by direct integration.
2. Then, use symmetry and scaling arguments. Can you build another situation where Gauss' Law gives you the total flux directly and then by symmetry conclude what the flux in this problem is?