Problem: Stable equilibrium in Electrostatics

By: Francisco Dahab, inspired by Feynman Lectures

Prelude

It's often easy to hide our lack of physical understanding of a situation behind mathematical theorems, but it's important to, when possible, fight this urge. This is one such problem where understanding the physical implications of what we're asking makes the answer very evident.

Problem Statement

It's a known fact that there exists no stable equilibrium in three dimensional Electrostatics, but why is this the case? To answer this, people often think in terms of understanding the properties that solutions to the Laplace equation

$$\nabla^2 V = 0$$

must respect. But we're physicists, so let's also try a more palpable approach.

Thinking only in terms of Gauss' Law and what stable equilibrium implies about the shape of the electric field, devise an explanation for why stable equlibrium **cannot exist**.