Solution Infinite Grid -1

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1 Solution

Before proceeding, let us reflect on what we know about infinite grids. From the previous problem, we learned that the equivalent resistance between adjacent vertices on the infinite grid is $R/2$. But how do we use that information here, where we have a hole where one of the resistors would be. Well, one way of looking at it is that a hole is simply an edge where no current is allowed to go through, which is effectively an infinite resistance resistor. We can build an infinite resistance resistor by placing two resistors in parallel, one with resistance R and the other one with $-R$ (bear with me, yes this is unphysical, but this is fine as long as the end result is physical). We can verify this by looking at the equivalent resistance

$$R_{\text{eq}} = \frac{R(-R)}{R - R}$$

which is divergent[1]. This then means, that we can think of the grid with an edge removed as simply the original infinite grid, but with an additional $-R$ resistance resistor in parallel. But the infinite grid contributes with $R/2$ equivalent resistance, such that the total equivalent resistance is simply what results from an $R/2$ resistor in parallel with a $-R$ resistor, that is

$$R_{\text{eq}} = \frac{R/2(-R)}{R - R/2} = R$$

Which is finite, positive and physical.

[1] You can always work with limits if the division by zero worries you.