Strength Of Materials

Contents

1 Introduction

The office hour is Tuesday 14:00 to 15:00 in C217 Engineering Central.

This module is the study of how materials can sustain external actions without failure by using simplified mathematical models. Types of actions are:

- Forces
- Temperatures changes
- Settlements

While failure types include:

- Rupture
- Excessive deformation

1.1 Assessment

80% is a closed book exam at the end of the term. It is a core module, so must acieve at least 40% in the exam. // 20% is based off of three blackboard tests; basic concepts (7%), Basic beam theory (7%), Stresses and strains and advanced beam theory (6%).
You will have one week exactly for each test, they will go live at 16:30 on a friday.

1.2 Recommended Reading

- Hibbeler, RC, Mechanics of Materials, Prentice Hall, SI Eighth Edition
- Case, Chilver & Ross, Strength of Materials & Structures, 4th Edition
- D. Gross, W. Hauger, J. Shroder, W. Wall, J. Bonet, Engineering Mechanics 2: Mechanics of Materials, Springer

2 Basic Concepts

2.1 Governing Principles

How a load is transmitted through a material is goverened by two basic principles.

2.1.1 Equilibrium

The sum of all forces and moments on a body, or any part of a body, must sum to zero. If a problem can be solved using only equilibrium conditions then it is statically determinate.

2.1.2 Compatibiblity

The movements resulting from the external loads must be internally compatible (the material must not break from movement caused by external loads) when also considering the external supports.

2.2 St. Venant's Principle

Regardless of the complexity of the distribution of external forces at a small region on the surface of a body, the resulting effect a small distance away depends only on the statically equivalent force.
Results in stress concentrations in a material, rather than uniform stress distribution.

2.3 Stress

Stress is the amount of internal force per unit area:

$$\sigma = \frac{F}{A}$$

and can be either tensile or compressive.

The force acting on an area can be normal or tangentail to the area. The direct stress (σ") is the normal force per unit area, while shear stress (τ) is the tangential force per unit area.

2.4 Temperature Stresses

$$\varepsilon_T = \alpha \Delta T \tag{1}$$
$$\Delta L_T = \alpha \Delta T L \tag{2}$$

Where:

$$\begin{aligned} \varepsilon_T &= \text{Thermal strain} \\ \alpha &= \text{Thermal coefficient} \\ T &= \text{Temperature} \\ \Delta L_T &= \text{Change in length due to thermal expansion} \\ L &= \text{Original length} \end{aligned}$$

Thermal stress doesn't cause any strain, unless the expansion is restricted by a support or constraint. The general procedure for solving problems of thermal stress is:

1. Remove one of the constrainst and assume the body can extend or contract freely
2. Calculate ΔL_T
3. Calculate the force required to return the body to its original length

2.5 Force Stress

$$\Delta L_F = \frac{FL}{AE} \tag{3}$$

Or

$$\Delta L_F = \sigma \frac{L}{E} \tag{4}$$

Where:

$$\begin{aligned} \Delta L_F &= \text{Change in length due to force applied} \\ \sigma &= \text{Stress} \\ F &= \text{Force} \\ L &= \text{Original length} \\ A &= \text{Cross sectional area} \\ E &= \text{Young's modulus} \end{aligned}$$

2.6 Poisson's Ration

When there is a direct stress along a beam in one direction this results in a direct strain in the same direction as well as a lateral strain, so a strain acting perpendicular to the direction of stress. The relationship between direct and lateral strain is given by Poisson's coefficient (v), and is usually 0.3.

$$v = -\frac{\varepsilon_2}{\varepsilon_1} \tag{5}$$

Where:

$$\begin{aligned} v &= \text{Poisson's Ration} \\ \varepsilon_2 &= \frac{\Delta d}{d} \\ \varepsilon_1 &= \frac{\Delta l}{l} \end{aligned}$$

2.7 Strain Energy

When a material is deformed the work done by external forces is stored as elastic strain energy within the material.

$$W = \int F \delta l = \int \sigma A l \delta \varepsilon = V \int \sigma \delta \varepsilon$$

The strain energy per unit volume (w) is the area under the stress-strain graph

$$w = \int \sigma \varepsilon \delta \varepsilon$$