

Problem a: Equations of Motion

$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$
 $U = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$x_1 = l_1 \sin \theta_1$
 $x_2 = l_1 \sin \theta_1 + l_2 \sin (\theta_1 + \theta_2)$
 $y_1 = l_1 \cos \theta_1$
 $y_2 = l_1 \cos \theta_1 + l_2 \cos (\theta_1 + \theta_2)$

\rightarrow Velocity \rightarrow
 $\dot{x}_1 = \dot{\theta}_1 l_1 \cos \theta_1$
 $\dot{y}_1 = -\dot{\theta}_1 l_1 \sin \theta_1$
 $\dot{x}_2 = \dot{\theta}_1 l_1 \cos \theta_1 + (\dot{\theta}_1 + \dot{\theta}_2) l_2 \cos (\theta_1 + \theta_2)$
 $\dot{y}_2 = -\dot{\theta}_1 l_1 \sin \theta_1 - (\dot{\theta}_1 + \dot{\theta}_2) l_2 \sin (\theta_1 + \theta_2)$

\rightarrow Energy calculations \rightarrow
 $KE_1 = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} M_1 V_1^2$

$\left\{ \begin{array}{l} \omega_1 = \dot{\theta}_1 \\ V_1 = \sqrt{\dot{x}_1^2 + \dot{y}_1^2} \end{array} \right.$

$KE_2 = \frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} M_2 V_2^2$

$\left\{ \begin{array}{l} \omega_2 = \dot{\theta}_2 \\ V_2 = \sqrt{\dot{x}_2^2 + \dot{y}_2^2} \end{array} \right.$

$$PE_1 = M_1 g y_1 \cos(\theta_1) = M_1 g y_1$$

$$PE_2 = M_2 g y_2 = M_2 g (l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2))$$

$$L = KE_1 + KE_2 - PE_1 - PE_2$$

$$EOM \Rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \mu_i \quad i=1, 2$$

Equations of Motion –

$$\begin{aligned} \tau_{11} = & I_1 * \theta_{1ddot} + L_1^2 * M_2 * \theta_{1ddot} + M_1 * r_1^2 * \theta_{1ddot} + \\ & M_2 * r_2^2 * \theta_{1ddot} + M_2 * r_2^2 * \theta_{2ddot} - M_2 * g * r_2 * \sin(\theta_1 + \theta_2) \\ & - L_1 * M_2 * g * \sin(\theta_1) - M_1 * g * r_1 * \sin(\theta_1) - \\ & L_1 * M_2 * r_2 * \theta_{2dot}^2 * \sin(\theta_2) + 2 * L_1 * M_2 * r_2 * \theta_{1ddot} * \cos(\theta_2) \\ & + L_1 * M_2 * r_2 * \theta_{2ddot} * \cos(\theta_2) - \\ & 2 * L_1 * M_2 * r_2 * \theta_{1dot} * \theta_{2dot} * \sin(\theta_2) \end{aligned}$$

$$\begin{aligned} \tau_{22} = & I_2 * \theta_{2ddot} + M_2 * r_2^2 * \theta_{1ddot} + M_2 * r_2^2 * \theta_{2ddot} - \\ & M_2 * g * r_2 * \sin(\theta_1 + \theta_2) + L_1 * M_2 * r_2 * \theta_{1dot}^2 * \sin(\theta_2) + \\ & L_1 * M_2 * r_2 * \theta_{1ddot} * \cos(\theta_2) \end{aligned}$$

Problem b: State Space Representation of E.O.M

Here states $X(1) = \theta_1$, $X(2) = \theta_2$, $X(3) = \dot{\theta}_1$, $X(4) = \dot{\theta}_2$

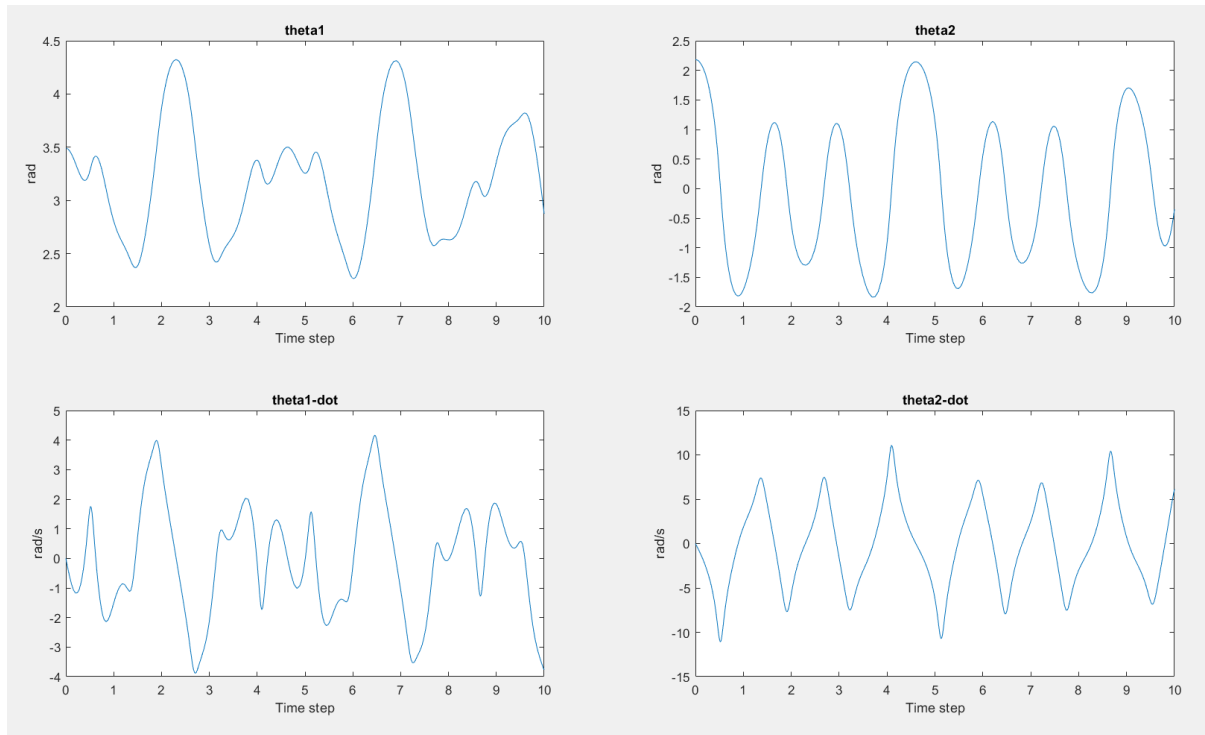
$$X(1)_{dot} = X(3) = \dot{\theta}_1$$

$$X(2)_{dot} = X(4) = \dot{\theta}_2$$

$$\begin{aligned} & I_2 * \tau_1 + M_2 * r_2^2 * \tau_1 - M_2 * r_2^2 * \tau_2 + \\ & L_1 * M_2^2 * g * r_2^2 * \sin(\theta_1) + \\ & I_2 * M_2 * g * r_2 * \sin(\theta_1 + \theta_2) + \\ & I_2 * L_1 * M_2 * g * \sin(\theta_1) \\ & + \frac{L_1^2 * M_2^2 * r_2^2 * \dot{\theta}_1^2 * \sin(2 * \theta_2)}{2} + \\ & I_2 * M_1 * g * r_1 * \sin(\theta_1) - \\ & L_1 * M_2 * r_2 * \tau_2 * \cos(\theta_2) + \\ & L_1 * M_2^2 * r_2^3 * \dot{\theta}_1^2 * \sin(\theta_2) + \\ & L_1 * M_2^2 * r_2^3 * \dot{\theta}_2^2 * \sin(\theta_2) - \\ & L_1 * M_2^2 * g * r_2^2 * \sin(\theta_1 + \theta_2) * \cos(\theta_2) + \\ & I_2 * L_1 * M_2 * r_2 * \dot{\theta}_2^2 * \sin(\theta_2) + \\ & M_1 * M_2 * g * r_1 * r_2^2 * \sin(\theta_1) + \\ & 2 * L_1 * M_2^2 * r_2^3 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_2) + \\ & 2 * I_2 * L_1 * M_2 * r_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_2) \\ X(3)_{dot} = & \frac{I_1 * I_2 + L_1^2 * M_2^2 * r_2^2 + I_2 * L_1^2 * M_2 + I_2 * M_1 * r_1^2 + \\ & I_1 * M_2 * r_2^2 + I_2 * M_2 * r_2^2 + \\ & M_1 * M_2 * r_1^2 * r_2^2 - L_1^2 * M_2^2 * r_2^2 * \cos^2(\theta_2) \\ & + 2 * I_2 * L_1 * M_2 * r_2 * \cos(\theta_2)}{I_1 * I_2 + L_1^2 * M_2^2 * r_2^2 + I_2 * L_1^2 * M_2 + I_2 * M_1 * r_1^2 + \\ & I_1 * M_2 * r_2^2 + I_2 * M_2 * r_2^2 + \\ & M_1 * M_2 * r_1^2 * r_2^2 - L_1^2 * M_2^2 * r_2^2 * \cos^2(\theta_2) \\ & + 2 * I_2 * L_1 * M_2 * r_2 * \cos(\theta_2)} \end{aligned}$$

$$\begin{aligned}
& M2 * r2^2 * \tau1 - L1^2 * M2 * \tau2 - \\
& M1 * r1^2 * \tau2 - I1 * \tau2 - \\
& M2 * r2^2 * \tau2 - L1^2 * M2^2 * g * r2 * \sin(\theta1 + \theta2) \\
& + L1 * M2^2 * g * r2^2 * \sin(\theta1) - \\
& I1 * M2 * g * r2 * \sin(\theta1 + \theta2) + \\
& L1^2 * M2^2 * r2^2 * \theta1_{dot}^2 * \sin(2 * \theta2) \\
& + \frac{L1^2 * M2^2 * r2^2 * \theta2_{dot}^2 * \sin(2 * \theta2)}{2} + \\
& L1 * M2 * r2 * \tau1 * \cos(\theta2) - \\
& 2 * L1 * M2 * r2 * \tau2 * \cos(\theta2) + \\
& L1 * M2^2 * r2^3 * \theta1_{dot}^2 * \sin(\theta2) + \\
& L1^3 * M2^2 * r2 * \theta1_{dot}^2 * \sin(\theta2) + \\
& L1 * M2^2 * r2^3 * \theta2_{dot}^2 * \sin(\theta2) - \\
& L1 * M2^2 * g * r2^2 * \sin(\theta1 + \theta2) * \cos(\theta2) + \\
& L1^2 * M2^2 * g * r2 * \cos(\theta2) * \sin(\theta1) - \\
& M1 * M2 * g * r1^2 * r2 * \sin(\theta1 + \theta2) + \\
& I1 * L1 * M2 * r2 * \theta1_{dot}^2 * \sin(\theta2) + \\
& M1 * M2 * g * r1 * r2^2 * \sin(\theta1) + \\
& L1^2 * M2^2 * r2^2 * \theta1_{dot} * \theta2_{dot} * \sin(2 * \theta2) + \\
& 2 * L1 * M2^2 * r2^3 * \theta1_{dot} * \theta2_{dot} * \sin(\theta2) + \\
& L1 * M1 * M2 * r1^2 * r2 * \theta1_{dot}^2 * \sin(\theta2) + \\
& L1 * M1 * M2 * g * r1 * r2 * \cos(\theta2) * \sin(\theta1) \\
X(4)_{dot} = & - \frac{I1 * I2 + L1^2 * M2^2 * r2^2 + I2 * L1^2 * M2 + \\
& I2 * M1 * r1^2 + I1 * M2 * r2^2 \\
& + I2 * M2 * r2^2 + M1 * M2 * r1^2 * r2^2 \\
& - L1^2 * M2^2 * r2^2 * \cos^2(\theta2) + \\
& 2 * I2 * L1 * M2 * r2 * \cos(\theta2)}{
\end{aligned}$$

Problem b: Simulation



Please toggle the visualization flag on line 3 of main.m file in the submission to “true” see the RRbot (the two-link robot) in action. !!!