FENIL DESAI

RBE 500 - Homework 4

4.6 $R = R_{x,\theta} R_{y,\phi}$ compute $\frac{\partial R}{\partial \phi}$. $\frac{\partial R}{\partial \phi}$ at $\theta = \frac{\pi}{2}$ & $\phi = \pi/2$

$\rightarrow R = \frac{\partial R}{\partial \phi} = \cancel{\frac{\partial R_{x,\theta}}{\partial \phi}}^{\to 0} . R_{y,\phi} + R_{x,\theta} . \frac{\partial R_{y,\phi}}{\partial \phi}$

$\therefore \frac{\partial R}{\partial \phi} = R_{x,\theta} \frac{\partial R_{y,\phi}}{\partial \phi}$

$\therefore \frac{\partial R}{\partial \phi} = R_{x,\theta} \cdot S(\hat{j}) R_{y,\phi}$

$$\frac{\partial R}{\partial \phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\frac{\partial R}{\partial \phi} = \begin{bmatrix} 0 & 0 & 1 \\ \sin\theta & 0 & 0 \\ -\cos\theta & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\frac{\partial R}{\partial \phi} = \begin{bmatrix} -\sin\phi & 0 & \cos\phi \\ \sin\theta\cos\phi & 0 & \sin\theta\sin\phi \\ -\cos\theta\cos\phi & 0 & -\sin\phi\cos\theta \end{bmatrix}$$

$$\left.\frac{\partial R}{\partial \phi}\right|_{\substack{\theta=\pi/2 \\ \phi=\pi/2}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer

$\boxed{4.10}$ $O_0 x_0 y_0 z_0$ & $O_1 x_1 y_1 z_1$

$$H_1^0 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad V_1^1(t) = [3, 7, 0]^T$$

→ Considering $V_1^1(t)$ to be the linear velocity of Particle 1 in frame 1.

& $w_1^1(t)$ be the angular velocity of particle 1 w.r.t frame 1.

$$w_1^1(t) = [0\ 0\ 0]^T$$

$$H_1^0 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \& \quad d_1^0 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

V_1^0 → linear velocity of Particle 1 in frame 0.
w_1^0 → angular velocity of Particle 1 in fram 0.

$$V_1^0 = RV_1^1 - RS(d)w_1^1$$

$$w_1^0 = Rw_1^1$$

$$\therefore \boxed{w_1^0 = [0\ 0\ 0]^T} \qquad (\because w_1^1 = [0\ 0\ 0]^T)$$

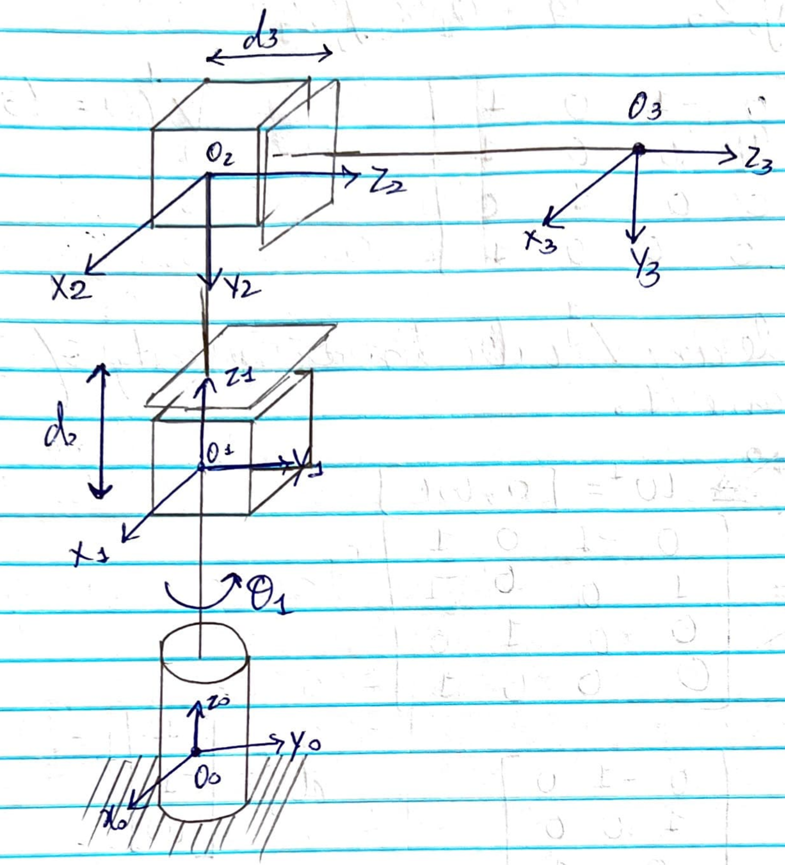
$$r_1^0 = R\, r_1^1$$

$$\therefore\ r_1^0 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$$\therefore\ r_1^0 = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$$\therefore\ \boxed{r_1^0 = [-1, 3, 0]^T}$$

$\boxed{4.15}$



$$J = \begin{bmatrix} Z_0^0 \times (O_3^0 - O_0^0) & Z_1^0 & Z_2^0 \\ Z_0^0 & 0 & 0 \end{bmatrix}_{6\times 3}$$

→ As this is an inverse kinematics problem we will know the position of end effector & H_3^0 will be known to use.

Since $H_3^0 = \begin{bmatrix} R_3^0 & t_3^0 \\ 0 & 1 \end{bmatrix}$

& $H_3^0 = A_1 A_2 A_3 = H_1^0 H_1^2 H_2^3$

$\therefore O_3^0 = t_3^0$

$O_0^0 = [0 \ 0 \ 0]$

$Z_0 = [0 \ 0 \ 1]$ z-axis of base joint.

$$H_1^0 = \begin{bmatrix} R_1^0 & t_1^0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore Z_1^0 = R_1^0 \cdot [0 \ 0 \ 1]^T$$

$$H_2^0 = \begin{bmatrix} R_2^0 & t_2^0 \\ 0 & 1 \end{bmatrix}$$

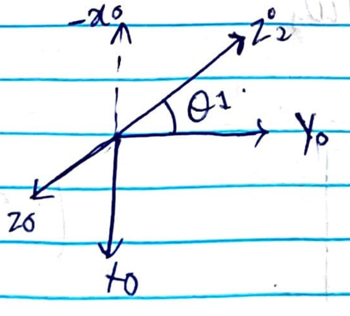
$$Z_2^0 = R_2^0 \cdot [0 \ 0 \ 1]^T$$

→ Let us ~~suppose~~ try to visualize the geometry of this robot

Here, we can clearly see $Z_0^0 = Z_1^0$ (both in same direction)

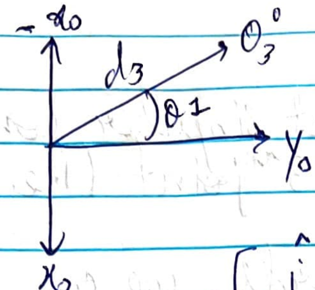
→ as we can see → for Z_2^0 if the Joint 1 is rotated by θ_1^0 then.

Z_2^0 will become



$$\therefore Z_2^0 = [-\sin\theta_1 \ \cos\theta_1 \ 0]$$

→ Similarly O_3^0 will become



$$O_3^0 = [-d_3 \sin\theta_1 \quad d_3 \cos\theta_1 \quad 0]$$

now $z_0^0 \times O_3^0 = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ +d_3 \sin\theta_1 & d_3 \cos\theta_1 & 0 \end{bmatrix}$

$$= -d_3 \cos\theta_1 \,\hat{i} - d_3 \sin\theta_1 \,\hat{j} + 0\hat{k}$$

$\therefore \; z_0^0 \times O_3^0 = [-d_3 \cos\theta_1 \quad -d_3 \sin\theta_1 \quad 0]$

$$\therefore \quad J = \begin{bmatrix} -d_3 \cos\theta_1 & 0 & -\sin\theta_1 \\ +d_3 \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

→ To simply find a singularity, we can do -

$$\det \begin{pmatrix} -d_3 \cos\theta_1 & 0 & -\sin\theta_1 \\ +d_3 \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & 1 & 0 \end{pmatrix} = 0$$

$\therefore \quad -d_3 \cos^2\theta_1 - d_3 \sin^2\theta_1 = 0$

$$= d_3 (\cos^2\theta_1 + \sin^2\theta_1) = 0$$

→ P.T.O

$\therefore \boxed{d_3 = 0}$

Therefore, we can get singular configuration if the joint value of the 3rd joint (i.e. d_3) is zero.

OR, in other words, we can say that if $d_3 = 0$, we can get infinite number of solutions for θ_1 to reach same end-effector position.

Therefore, $d_3 = 0$ will result in a singular configuration.

→ This observation can also simply be made by visualizing that when the 3rd joint will not be operated ($d_3 = 0$) the end effector will be exactly above joint 1 & joint 2 & hence there will exist infinite values of θ_1 that can yield same end effector position.