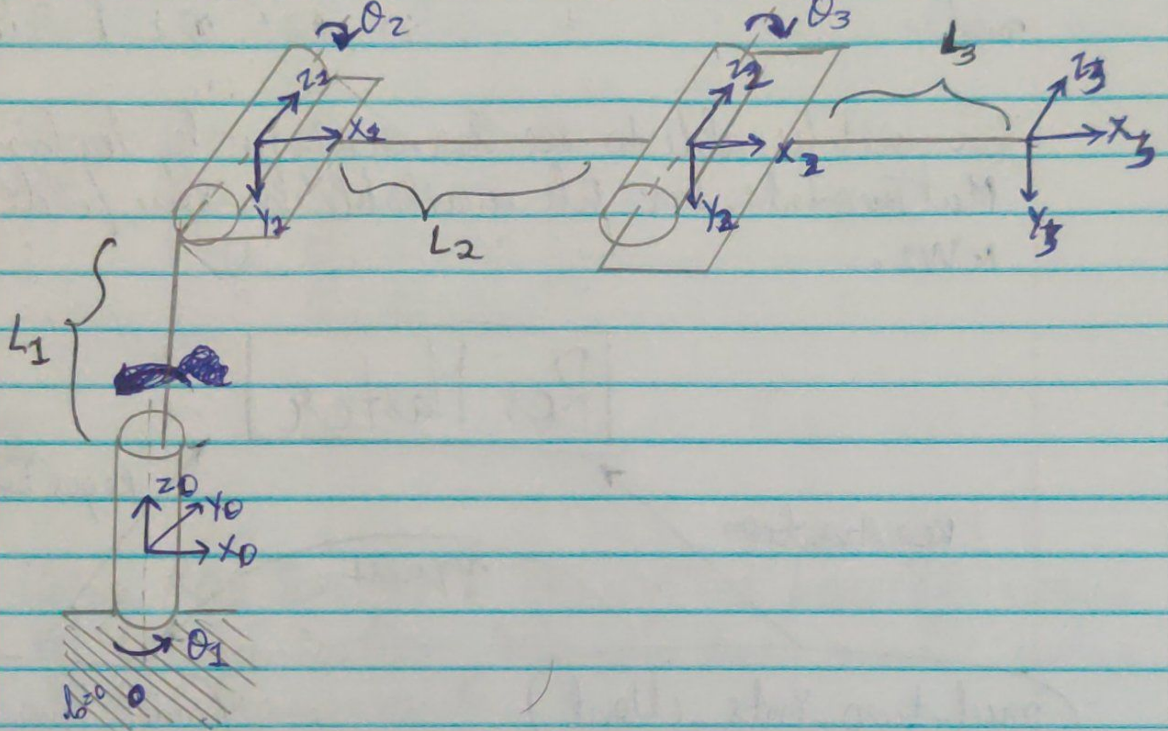
FENIL DESAI

RBE-500 [HW2]

3.5



→ For the problem under consideration, the reference frames are assigned to each of the joints according to D-H convention.

→ Now, let us calculate the D-H parameter table.

a	θ	d	α
0	θ_1	L1	-90°
L2	θ_2	0	0°
L3	θ_3	0	0°

we know that

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & S\theta_i S\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \cdot \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

— in the following representations $S \rightarrow \sin$ & $C \rightarrow \cos$.

$$A_1 = \begin{bmatrix} C\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & L_2 \cdot C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & L_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & L_3 \cos\theta_3 \\ S\theta_3 & C\theta_3 & 0 & L_3 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 \cdot A_2 \cdot A_3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & dx \\ r_{21} & r_{22} & r_{23} & dy \\ r_{31} & r_{32} & r_{33} & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

now Here $C_1 = \cos\theta_1$ & $S_1 = \sin\theta_1$, similarly for all θ_1, θ_2 & θ_3 we can write $C_1, S_1, C_2, S_2, C_3, S_3$

$$A_1 \circ A_2 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} C_2 & -S_2 & 0 & L_2 C_2 \\ S_2 & C_2 & 0 & L_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 \cdot A_2 = \begin{bmatrix} C_1C_2 & -C_1S_2 & -S_1 & L_2C_1C_2 \\ S_1C_2 & -S_1S_2 & C_1 & L_2S_1C_2 \\ -S_2 & -C_2 & 0 & -L_2S_2+L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 \cdot A_2 \cdot A_3 = \begin{bmatrix} C_1C_2 & -C_1S_2 & -S_1 & L_2C_1C_2 \\ S_1C_2 & -S_1S_2 & C_1 & L_2S_1C_2 \\ -S_2 & -C_2 & 0 & -L_2S_2+L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore\ r_{11} = C_1C_2C_3 - C_1S_2S_3$

$r_{12} = -C_1C_2S_3 - C_1S_2C_3$

$r_{13} = -S_1$

$d_x = L_3C_1C_2C_3 - L_3C_1S_2S_3 + L_2C_1C_2$

$r_{21} = S_1C_2C_3 - S_1S_2S_3$

$r_{22} = -S_1C_2S_3 - S_1S_2C_3$

$r_{23} = C_1$

$d_y = L_3S_1C_2C_3 - L_3S_1S_2S_3 + L_2S_1C_2$

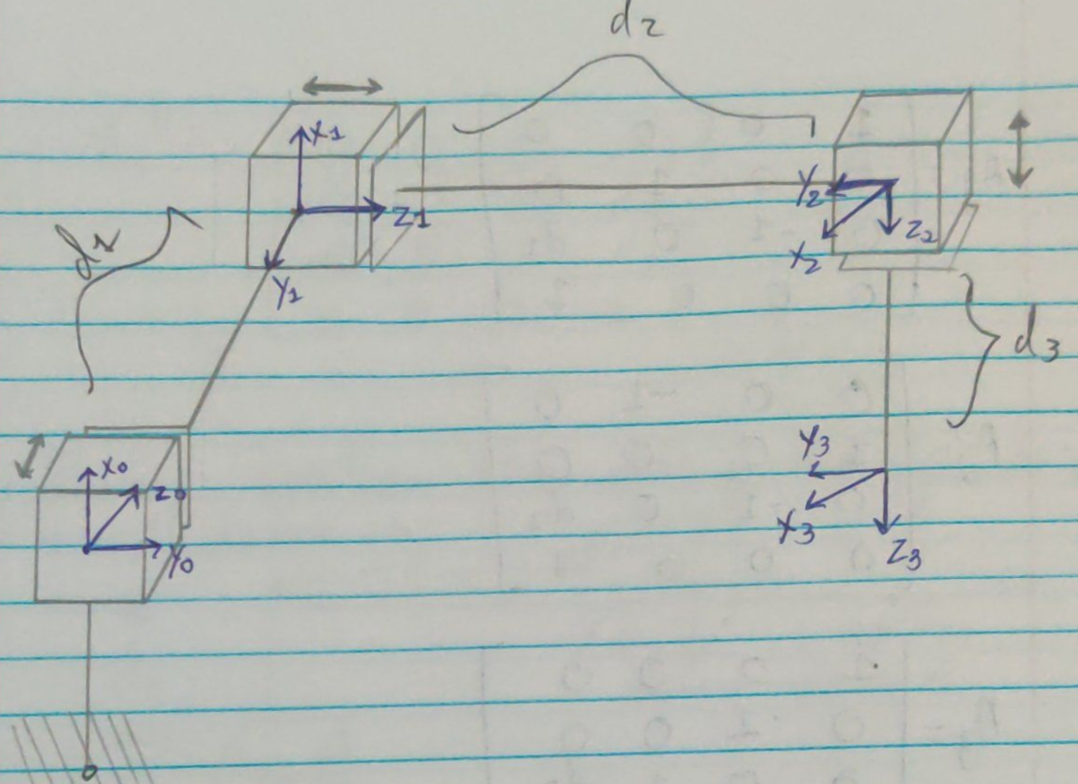
$r_{31} = -S_2C_3 - C_2S_3$

$r_{32} = S_2S_3 - C_2C_3$

$r_{33} = 0$

$d_z = -L_3S_2C_3 - L_3C_2S_3 - L_2S_2 + L_1$

3.6



→ For the problem in consideration, the reference frames are assigned according to the D-H convention.

→ The D-H parameter table for the given kinematic chain is —

a	θ	d	α
0	0°	d_1	$-90°$
0	90°	d_2	$-90°$
0	0°	d_3	0°

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3$$

$$\therefore A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

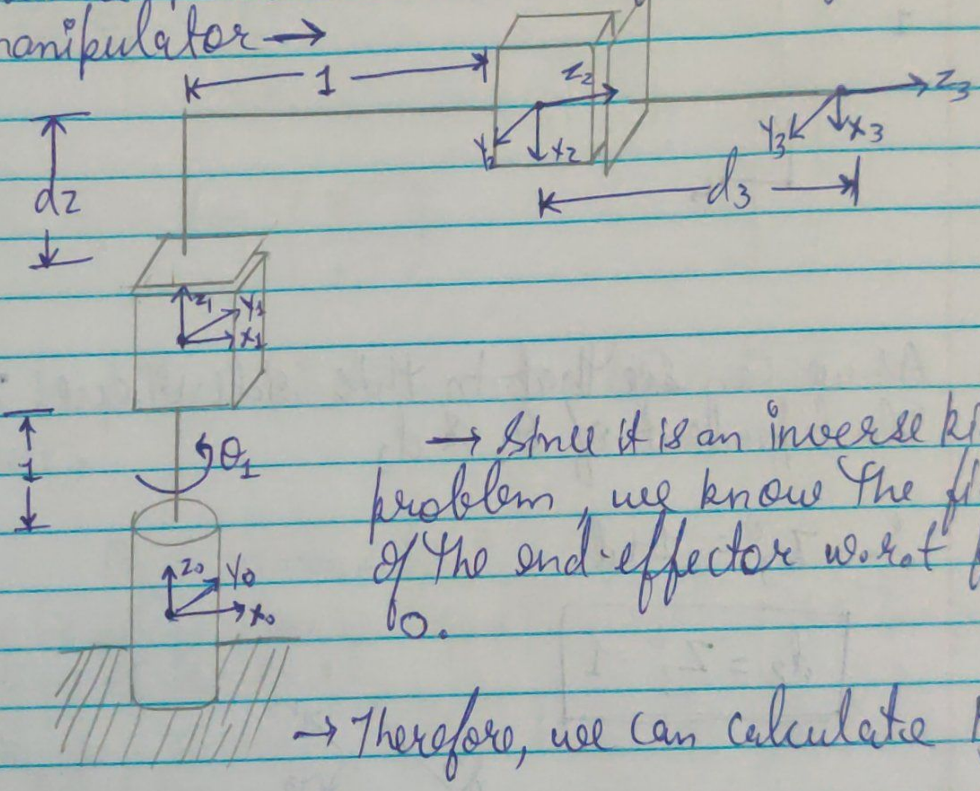
$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans.

5.3 Inverse position kinematics for the Cylindrical manipulator →



→ Since it is an inverse kinematics problem, we know the final pose of the end-effector w.r.t frame 0.

→ Therefore, we can calculate H_3^0.

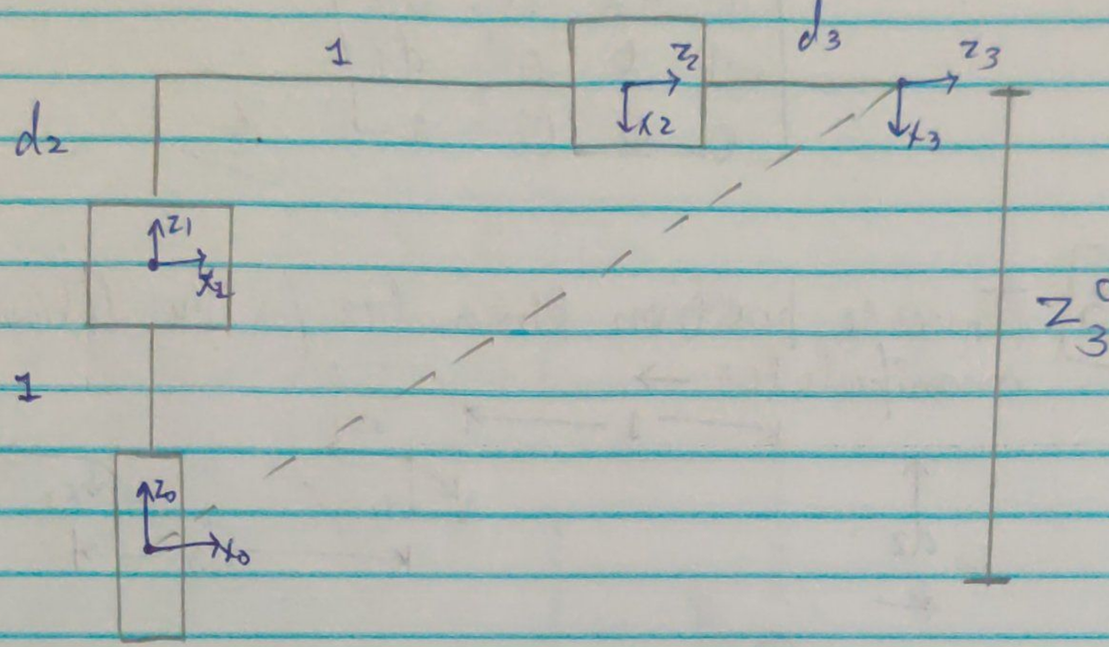
Now, let us assume that we are known the homogenous transformation matrix $H_3^0 = \begin{bmatrix} R_3^0 & T_3^0 \\ 0 & 1 \end{bmatrix}$

∴ We are also knowing x_3^0, y_3^0, z_3^0 (end-effector position coordinates w.r.t the base frame).

→ To solve this problem, our aim is to find the θ_1, d_2 & d_3 variables given the end-effector position coordinates.

→ P.T.O.

★→ Side View

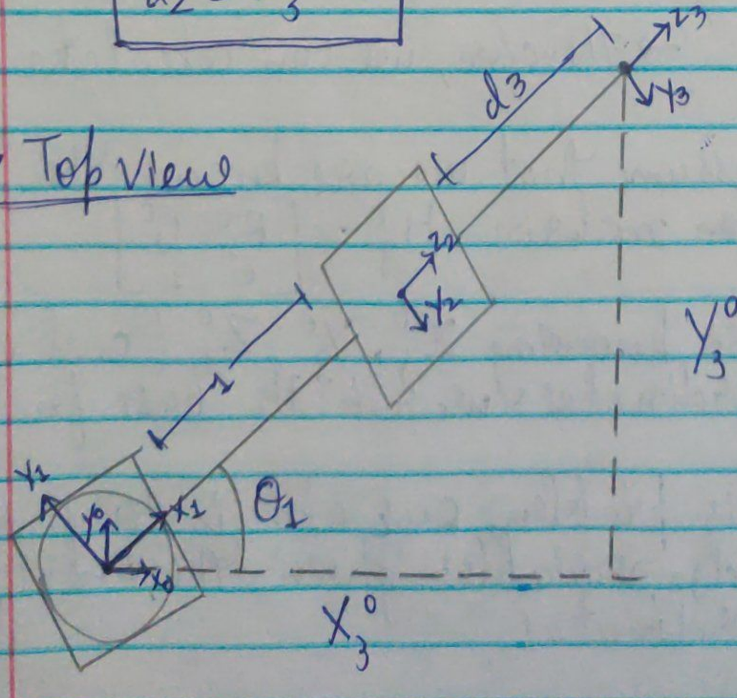


As we can see that in this side view, Z_3^0 is idendependent of θ_1 & d_3

$\therefore\ Z_3^0 = 1 + d_2$

$$\boxed{d_2 = Z_3^0 - 1}$$

★→ Top View



→ From this view we can identify two relations

→ $(1+d_3)^2 = (x_3^0)^2 + (y_3^0)^2$

$$\therefore \boxed{d_3 = \sqrt{(x_3^0)^2 + (y_3^0)^2} - 1}$$

→ $\tan\theta_1 = \dfrac{y_3^0}{x_3^0}$

$$\therefore \theta_1 = \tan^{-1}\left(\frac{y_3^0}{x_3^0}\right)$$

To keep the solution more consistend we will use,

$$\theta_1 = \text{atan2}\,(x_3^0, y_3^0)$$

– it gives values of θ & compansates for all the quadrants & signs of $(x_3^0$ & $y_3^0)$.

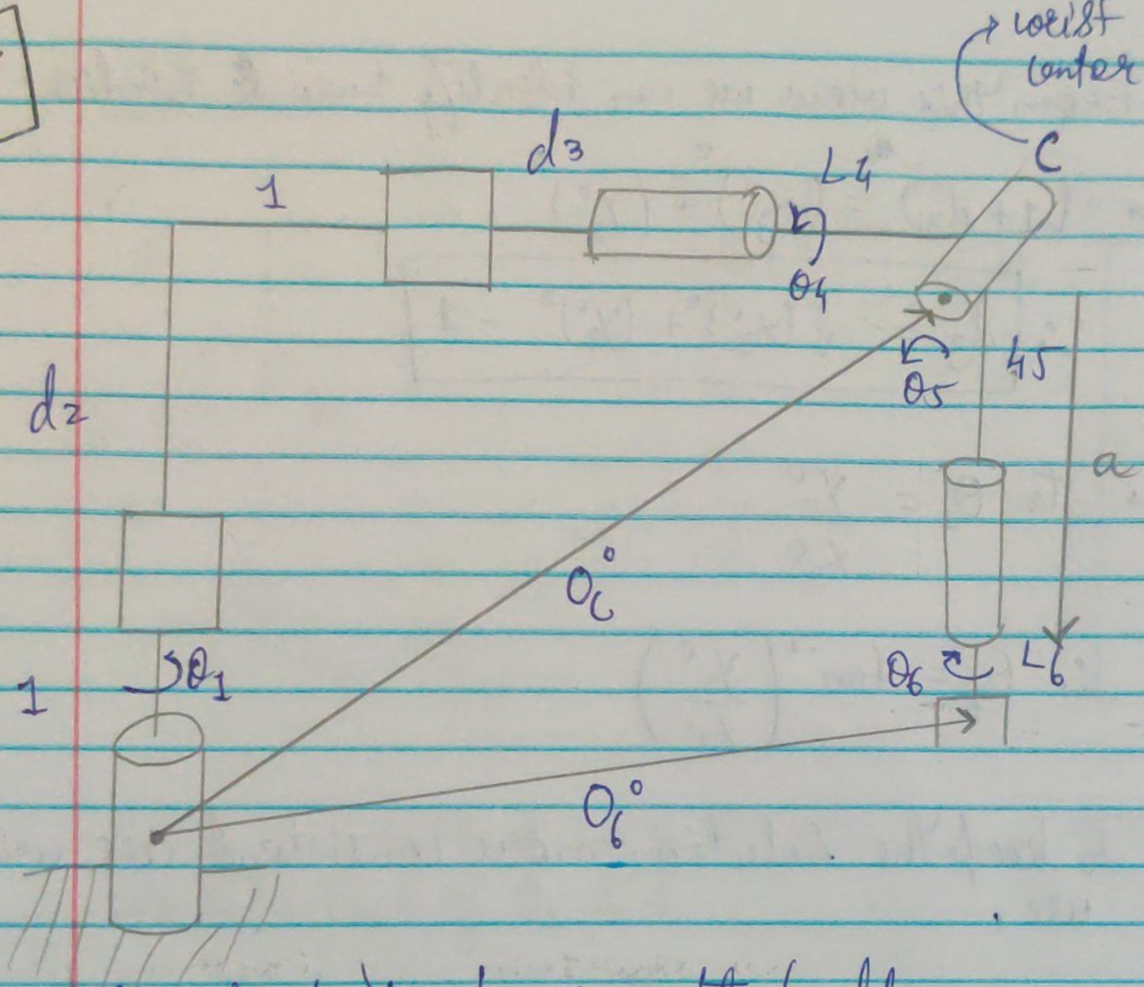
Therefore, Inverse position kinematic for the cylindrical manipulator

$$\theta_1 = \text{atan2}\,(x_3^0, y_3^0) \quad \text{or} \quad \text{atan2}\left((T_3^0)^T\begin{bmatrix}1\\0\\0\end{bmatrix}, (T_3^0)^T\begin{bmatrix}0\\1\\0\end{bmatrix}\right)$$

$$d_2 = z_3^0 - 1 \quad \text{or} \quad (T_3^0)^T\begin{bmatrix}0\\0\\1\end{bmatrix} - 1$$

$$d_3 = \sqrt{(x_3^0)^2 + (y_3^0)^2} - 1 \quad \text{or} \quad \sqrt{\left((T_3^0)^T\begin{bmatrix}1\\0\\0\end{bmatrix}\right)^2 + \left((T_3^0)^T\begin{bmatrix}0\\1\\0\end{bmatrix}\right)^2} - 1$$

$\boxed{5.5}$



Similar to the previous ~~the~~ problem,

→ Let us assume that we are known the homogenous transformation matrix $H_6^0 = \begin{bmatrix} R_6^0 & T_6^0 \\ 0 & 1 \end{bmatrix}$

$$\therefore O_6^0 = T_6^0$$

→ now we also know the direction & magnitude of a.

$$\therefore a = (L_5 + L_6) \cdot R_6^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore O_c^0 = O_6^0 - (L_5 + L_6) R_6^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

→ <u>side view</u>

$z \rightarrow$ component of $O_c^0 \Rightarrow (O_c^0)^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\therefore \boxed{d_2 = (O_c^0)^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 1}$$

→ <u>Top view</u>

~~$(1 + d_3 + L_4)$~~

$$(1 + d_3 + L_4)^2 = \underbrace{\left((O_c^0)^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)^2}_{x \text{ component of } O_c^0} + \underbrace{\left((O_c^0)^T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)^2}_{Y \text{ component of } O_c^0}$$

$$\therefore \boxed{d_3 = \sqrt{\left((O_c^0)^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)^2 + \left((O_c^0)^T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)^2} - 1 - L_4}$$

$$\therefore \boxed{\theta_1 = \operatorname{atan2}\left((O_c^0)^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, (O_c^0)^T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)}$$

→ As we already found out θ_1, d_2 & d_3 we can write the homogenous transformation matrix H_3^0 using the forward kinematics concepts of D-H conventions

$$\therefore H_3^0 = \begin{bmatrix} R_3^0 & T_3^0 \\ 0 & 1 \end{bmatrix}$$

∴ we can write Rotation matrix R_6^3

$$\therefore R_6^3 = (R_3^0)^T R_6^0 \qquad \text{let } R_6^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

→ Now, we can observe that solving for the spherical wrist is essentially same as solving for the Euler angles

$$\therefore R_6^3 = R_{z,\theta_4} \cdot R_{y,\theta_5} \cdot R_{z,\theta_6}$$

$$\therefore R_6^3 = \begin{bmatrix} C\theta_4 C\theta_5 C\theta_6 - S\theta_4 S\theta_6 & -C\theta_4 C\theta_5 S\theta_6 - S\theta_4 C\theta_6 & C\theta_4 S\theta_5 \\ S\theta_4 C\theta_5 C\theta_6 + C\theta_4 S\theta_6 & -S\theta_4 C\theta_5 S\theta_6 + C\theta_4 C\theta_6 & S\theta_4 S\theta_5 \\ -S\theta_5 C\theta_6 & S\theta_5 S\theta_6 & C\theta_5 \end{bmatrix}$$

→ now comparing R_6^3 element by element

→ Here S & C means Sine & Cosine respectively.

→ Using atan2(x, y) to avoid singularities

→ $$\frac{+\cancel{\sin(\theta_5)}\sin(\theta_6)}{-\cancel{\sin(\theta_5)}\cos(\theta_6)} = \frac{r_{32}}{r_{31}}$$

$$\frac{\sin(\theta_6)}{\cos(\theta_6)} = -\frac{r_{32}}{r_{31}} \quad \therefore \tan(\theta_6) = -\frac{r_{32}}{r_{31}}$$

$$\therefore \boxed{\theta_6 = \operatorname{atan2}(r_{31}, -r_{32})}$$

→ $$\frac{\sin(\theta_4)\cancel{\sin(\theta_5)}}{\cos(\theta_4)\cancel{\sin(\theta_5)}} = \frac{r_{23}}{r_{13}}$$

$$\tan(\theta_4) = \frac{r_{23}}{r_{13}}$$

$$\boxed{\theta_4 = \operatorname{atan2}(r_{13}, r_{23})}$$

→ $$\frac{S\theta_4 S\theta_5}{\cos\theta_5} = \frac{r_{23}}{r_{33}}$$

$$S\theta_4 \tan\theta_5 = \frac{r_{23}}{r_{33}}$$

$$\tan\theta_5 = \frac{r_2}{r_{33} \times \sin(\operatorname{atan2}(r_{13}, r_{23}))}$$

$$\therefore \boxed{\theta_5 = \operatorname{atan2}(r_{33} \times \sin(\operatorname{atan2}(r_{13}, r_{23})), r_2)}$$