FENIL DESAI

RBE 500 — HOMEWORK 3

4.2

verify $S(a)p = a \times p$ $\forall$ a & $p \in R^3$

$a = [a_1 \; a_2 \; a_3]^T$ & $p = [p_1 \; p_2 \; p_3]^T$

$$S(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

$$S(a)P = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$= \begin{bmatrix} -a_3 P_2 + a_2 P_3 \\ P_1 a_3 - a_1 P_3 \\ -a_2 P_1 + P_2 a_1 \end{bmatrix}$$

$$a \times p = \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ P_1 & P_2 & P_3 \end{bmatrix}$$

$$(a_2 P_3 - P_2 a_3)\hat{i} - (a_1 P_3 - P_1 a_3)\hat{j} + (a_1 P_2 - P_1 a_2)\hat{k}$$

$$\therefore \; a \times P = \begin{bmatrix} a_2 P_3 - P_2 a_3 \\ P_1 a_3 - a_1 P_3 \\ a_1 P_2 - P_1 a_2 \end{bmatrix}$$

→ Hence, we can see that $S(a)p = a \times p$

$\boxed{4.3}$ Prove $R(a \times b) = Ra \times Rb$ for $R \in SO(3)$

→ Let us consider $R = (r_1 \; r_2 \; r_3)$ where r_1, r_2 & r_3 are the column vectors of R

&

$$a = (a_1, a_2, a_3)^T$$
$$b = (b_1, b_2, b_3)^T$$

— Calculating the cross product

$$R_a = [a_1 r_1 \quad a_2 r_2 \quad a_3 r_3]$$

$$R_b = [b_1 r_1 \quad b_2 r_2 \quad b_3 r_3]$$

$$R_a \times R_b = \begin{bmatrix} i & j & k \\ a_1 r_1 & a_2 r_2 & a_3 r_3 \\ b_1 r_1 & b_2 r_2 & b_3 r_3 \end{bmatrix}$$

$$(R_a \times R_b)^T = \begin{bmatrix} a_2 b_3 r_2 \times r_3 - a_3 b_2 r_2 \times r_3 \\ -a_1 b_3 r_1 \times r_3 + a_3 b_1 r_1 \times r_3 \\ a_2 b_2 r_2 \times r_3 - a_2 b_1 r_2 \times r_3 \end{bmatrix}$$

$$(R_a \times R_b)^T = \begin{bmatrix} (a_2 b_3 - a_3 b_2) r_2 \times r_3 \\ (a_3 b_1 - a_1 b_3) r_1 \times r_3 \\ (a_1 b_2 - a_2 b_1) r_2 \times r_3 \end{bmatrix}$$

$\because$ $R \in SO(3)$, we can safely state that -

$r_1 \times r_2 = r_3$
$r_1 \times r_3 = r_2$
$r_2 \times r_3 = r_1$

$$Ra \times Rb = [(a_2b_3 - a_3b_2)r_1 \quad (a_3b_1 - a_1b_3)r_2 \quad (a_1b_2 - a_2b_1)r_3]$$

$$= R(a \times b)$$

$\therefore$ We can prove that

$$\boxed{Ra \times Rb = R(a \times b)}$$

$\boxed{4.5}$ $a = (1, -1, 2)$ & $R = R_{x,90°}$

Show $\rightarrow$ $RS(a)R^T = S(Ra)$

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$S(a) = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\Longrightarrow R(S(a))R^T \Rightarrow$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & -1 \\ -1 & -1 & 0 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\rightarrow Ra = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$S(Ra) = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Hence

$$RS(a)R^T = S(Ra)$$

→

	Linear Component	Angular component
Revolute Joint	$J_{V_i} = Z^0_{i-1} \times (O^0_n - O^0_{i-1})$	$J_{\omega_i} = Z^0_{i-1}$
Prismatic Joint	$J_{V_i} = Z^0_{i-1}$	$J_{\omega_i} = 0$

→ As we have this problem, it has assigned frames according to DH conventions & therefore, we can assume that we can easily derive T (homogenous transformation matrix) for the same & also known all the intermediate homogenous transformation matrices.

$$T = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6$$

$$= H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 H_5^6$$

$$J^T = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \begin{bmatrix} J_v & J_\omega \\ Z^0_0 \times (O^0_6 - O^0_0) & Z^0_0 \\ Z^0_1 \times (O^0_6 - O^0_1) & Z^0_1 \\ Z^0_2 & 0 \\ Z^0_3 \times (O^0_6 - O^0_3) & Z^0_3 \\ Z^0_4 \times (O^0_6 - O^0_4) & Z^0_4 \\ Z^0_5 \times (O^0_6 - O^0_5) & Z^0_5 \end{bmatrix} \begin{array}{c} \text{Joint} \\ i=1 \\ i=2 \\ i=3 \\ i=4 \\ i=5 \\ i=6 \end{array}$$

Z_0^0 → Zaxis of the frame 0 ∴ $[0 \; 0 \; 1]$

Z_1^0 → Zaxis of $\underset{(A_1)}{H_1^0} \rightarrow R_1^0 \, [0 \, 0 \, 1]^T$

Z_2^0 → Zaxis of $H_2^0 (A_1 A_2) \rightarrow R_2^0 \, [0 \, 0 \, 1]^T$

Z_3^0 → Zaxis of $\underset{(A_1 A_2 A_3)}{H_3^0} \Rightarrow R_3^0 \, [0 \, 0 \, 1]^T$

Z_4^0 → Zaxis of $H_4^0 (A_1 A_2 A_3 A_4) \Rightarrow R_4^0 [0 \, 0 \, 1]^T$

Z_5^0 → Zaxis of $H_5^0 (A_1 A_2 A_3 A_4 A_5) \Rightarrow R_5^0 [0 \, 0 \, 1]^T$

if $T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$

$\boxed{O_6^0 = t}$

$\boxed{O_0^0 = [0 \; 0 \; 0]^T}$

$$H_1^0 = \begin{bmatrix} R_1^0 & t_1^0 \\ 0 & 1 \end{bmatrix} \rightarrow \boxed{t_1^0 = O_1^0}$$

$$H_2^0 = \begin{bmatrix} R_2^0 & t_2^0 \\ 0 & 1 \end{bmatrix} \rightarrow \boxed{O_2^0 = t_2^0}$$

$$H_3^0 = \begin{bmatrix} R_3^0 & t_3^0 \\ 0 & 1 \end{bmatrix} \rightarrow \boxed{O_3^0 = t_3^0}$$

$$H_4^0 = \begin{bmatrix} R_4^0 & t_4^0 \\ 0 & 1 \end{bmatrix} \rightarrow \boxed{O_4^0 = t_4^0}$$

$$H_5^0 = \begin{bmatrix} R_5^0 & t_5^0 \\ 0 & 1 \end{bmatrix} \rightarrow \boxed{O_5^0 = t_5^0}$$

→ ROS Assignment -

→ I am submiting "forward_kinematics" package as a part of HW3.

→ roscore

→ To run the subscriber use →

```
rosrun forward_kinematics forward kinematics
```

→ In a new terminal publish values of q1, q2 & q3 using

```
rostopic pub FK_topic forward_kinematics/joint_Variabl
"q1:XX q2:XX q3:XX"
```

where XX is a joint variable in "degrees".

→ I have calculated this forward kinematics considering the link lengths (L1 & L2 & L3) to be 1.0 m.