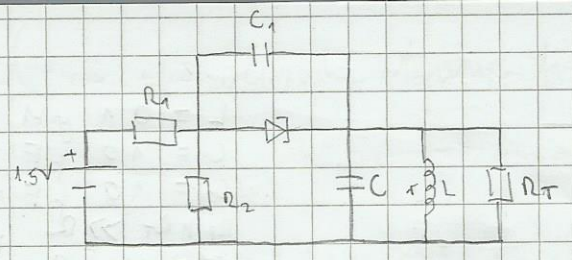
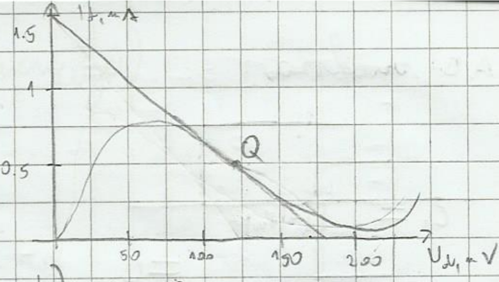
Zadaci:





1. $I_d = 0.5$ mA
$U_d = 120$ mV

a) $r_d = ?$ b) jednadžba pravca c) $f_g = ?$ d) $f = ?$

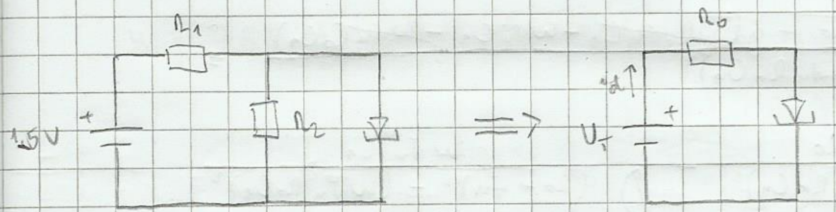
$T_1(\overset{x_1}{0}, \overset{y_1}{1.5})$
$T_2(120, 0.5)$

$$y - 1.5 = \frac{0.5 - 1.5}{120 - 0}(x - 0)$$

$$y - 1.5 = -\frac{1}{120}x \xrightarrow[y = I]{x = U} I = -\frac{1}{120}U + 1.5\,[mA]$$

$$g_d = -\frac{1}{120} \Rightarrow r_d = -120\ \Omega$$

DC analiza: $\left(\begin{matrix} Z_L = 0 \\ Z_C = \infty \end{matrix}\right)$



$$U_T = 1.5 \cdot \frac{R_2}{R_1 + R_2}$$

$$R_0 = R_1 \parallel R_2$$

$$U_T = i_d \cdot R_0 + U_d$$

* u praksi se obično uzima: $R_0 = \frac{1}{3}|r_d|$

$$R_0 = 40\ \Omega$$

$$1.5\frac{R_2}{R_1 + R_2} = 40 i_d + U_d \Rightarrow \frac{R_2}{R_1 + R_2} = \frac{40 i_d + U_d}{1.5}$$

$$\frac{R_1 R_2}{R_1 + R_2} = 40 \Rightarrow R_1 = 40 \cdot \frac{R_1 + R_2}{R_2}$$

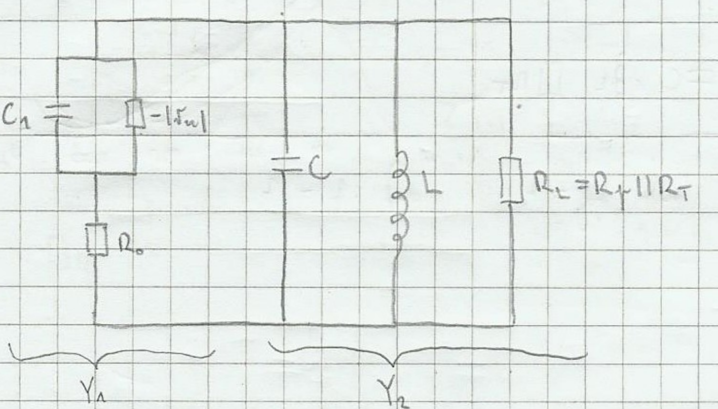
$$\frac{R_2}{R_1 + R_2} = \frac{40 \cdot 0.5 \cdot 10^{-3} + 120 \cdot 10^{-3}}{1.5} = \frac{7}{75}$$

$$R_1 = \frac{40}{\frac{7}{75}} = \frac{3000}{7} = 428.57\ \Omega$$

$$75 R_2 = 7R_1 + 7R_2$$

$$68 R_2 = 7 R_1 \Rightarrow R_2 = \frac{7}{68}R_1 = \frac{750}{17} = 44.12\ \Omega$$

AC analiza:



$L = 0.1\ \mu H = 10^{-7}\ H$
$C = 10\ pF = 10^{-11}\ F$
$C_1 = 10\ pF = 10^{-11}\ F$
$R_p || R_T \gg R_0\ ,\ R_L \gg R_0$
$r_n = 120\ \Omega$
$R_0 = 40\ \Omega$

$$Z_1 = R_0 + r_n \,||\, \frac{1}{sC_1} = R_0 + \frac{\frac{r_n}{sC_1}}{r_n + \frac{1}{sC_1}} = R_0 + \frac{r_n}{1 + s r_n C_1}$$

$$Y_1 = \frac{1}{R_0 + \frac{r_n}{1 + s r_n C_1}} = \frac{1 + s r_n C_1}{R_0 + s r_n R_0 C_1 + r_n} = \frac{1 + s r_n C_1}{(R_0 + r_n) + s r_n R_0 C_1}$$

$$s = j\omega \Rightarrow Y_1 = \frac{1 + j\omega r_n C_1}{(R_0 + r_n) + j\omega r_n R_0 C_1} \cdot \frac{(R_0 + r_n) - j\omega r_n R_0 C_1}{(R_0 + r_n) - j\omega r_n R_0 C_1} =$$

$$= \frac{R_0 + r_n + \omega^2 r_n^2 C_1^2 R_0 + j\left(\cancel{\omega R_0 r_n C_1} + \omega r_n^2 C_1 - \cancel{\omega r_n R_0 C_1}\right)}{(R_0 + r_n)^2 + (\omega r_n R_0 C_1)^2} =$$

$$= \frac{R_0 + r_n + \omega^2 r_n^2 C_1^2 R_0}{(R_0 + r_n)^2 + (\omega r_n R_0 C_1)^2} + j\,\frac{\omega r_n^2 C_1}{(R_0 + r_n)^2 + (\omega r_n R_0 C_1)^2}$$

$$Y_2 = sC + \frac{1}{R_L} + \frac{1}{sL} \xrightarrow{s = j\omega} Y_2 = j\omega C + \frac{1}{R_L} - j\frac{1}{\omega L}$$

$$Y_2 = \frac{1}{R_L} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$Y_{uk} = Y_1 + Y_2$$

$$Y_{uk} = \frac{R_0 + r_n + \omega^2 r_n^2 R_0 C_1^2}{(R_0 + r_n)^2 + (\omega r_n R_0 C_1)^2} + \frac{1}{R_L} + j\left[\frac{\omega r_n^2 C_1}{(R_0 + r_n)^2 + (\omega r_n R_0 C_1)^2} + \omega C - \frac{1}{\omega L}\right]$$

$\mathrm{Re}\{Y_{uk}\} = 0$ (uvjet samopobudnosti; za pronalazak granične frekv.)

$$(R_0 + r_n) R_L + \omega^2 r_n^2 R_0 R_L C_1^2 + (R_0 + r_n)^2 + (\omega r_n R_0 C_1)^2 = 0$$

$$\omega^2 = \frac{-(R_0 + r_n)^2 - (R_0 + r_n) R_L}{r_n^2 R_0 R_L C_1^2 + r_n^2 R_0^2 C_1^2} = -\frac{(R_0 + r_n)(R_L + R_0 + r_n)}{r_n^2 C_1^2 R_0 (R_L + R_0)}$$

$$\omega = \frac{1}{r_n C_1}\sqrt{\frac{(R_0 + r_n)(R_L + R_0 + r_n)}{-R_0 (R_L + R_0)}} \approx \frac{1}{r_n C_1}\sqrt{\frac{(R_0 + r_n)\cdot \cancel{R_L}}{-R_0 \cdot \cancel{R_L}}} = \frac{1}{r_n C_1}\sqrt{\frac{R_0 + r_n}{-R_0}}$$

$$\omega_g = \frac{1}{120 \cdot 10^{-11}} \cdot \sqrt{\frac{40 - 120}{-40}} = 1.1785 \cdot 10^{9} \Rightarrow f_g = \frac{\omega}{2\pi}$$

$$f_g = 187.56\ MHz$$

$\mathrm{Im}\{Y_{ulz}\} = 0$ (za pronalazak strane frekvencije)

$$\frac{\omega r_u^2 C_1}{(R_0 + r_u)^2 + (\omega r_u R_0 C_1)^2} = \frac{1 - \omega^2 LC}{\omega L}$$

$$\omega^2 r_u^2 L C_1 = \left[(R_0 + r_u)^2 + \omega^2 r_u^2 R_0^2 C_1^2\right](1 - \omega^2 LC)$$

$$* \quad \omega^2 = \frac{1}{r_u^2 C_1^2} \frac{R_0 + r_u}{-R_0} \quad / \cdot R_0^2 r_u^2 C_1^2$$

$$\omega^2 r_u^2 R_0^2 C_1^2 = -R_0 (R_0 + r_u)$$

$$\omega^2 r_u^2 L C_1 + \left[\omega^2 LC - 1\right]\left[(R_0 + r_u)^2 - R_0(R_0 + r_u)\right] = 0$$

$$\omega^2 r_u^2 L C_1 + (\omega^2 LC - 1)(R_0 + r_u)(\cancel{R_0} + r_u - \cancel{R_0}) = 0$$

$$\omega^2 r_u^2 L C_1 + (\omega^2 LC - 1)(R_0 r_u + r_u^2) = 0$$

$$\omega^2 r_u^2 L C_1 + \omega^2 R_0 r_u L C + \omega^2 r_u^2 L C - R_0 r_u - r_u^2 = 0$$

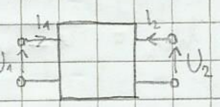
$$\omega^2 = \frac{r_u (R_0 + r_u)}{r_u^2 L C_1 + r_u L C (R_0 + r_u)} = \frac{R_0 + r_u}{r_u L C_1 + LC(R_0 + r_u)}$$

$$\omega = 755.93 \cdot 10^6 \quad \Rightarrow \quad f = \frac{\omega}{2\pi}$$

$$f = 120.31 \text{ MHz}$$

2.

* h-parametri

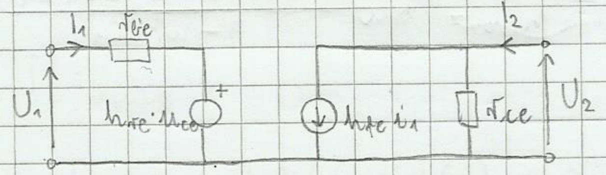


$$U_1 = h_{11} I_1 + h_{12} U_2$$
$$I_2 = h_{21} I_1 + h_{22} U_2$$

$$h_{11} = \frac{U_1}{I_1}\Big|_{U_2=0} \; [\Omega] \qquad h_{21} = \frac{I_2}{I_1}\Big|_{U_2=0} \; [\varnothing] \Rightarrow h_{fe}$$

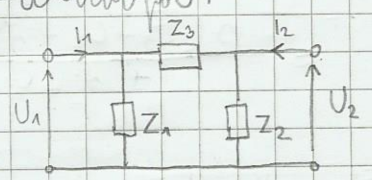
$$h_{12} = \frac{U_1}{U_2}\Big|_{I_1=0} \; [\varnothing] \qquad h_{22} = \frac{I_2}{U_2}\Big|_{I_1=0} \; [S]$$

* tranzistori:



$$h_{11} = h_{ie} = r_{ie}$$
$$h_{12} = h_{re}$$
$$h_{21} = h_{fe}$$
$$h_{22} = \frac{1}{r_{ce}}$$

* π - četveropol:

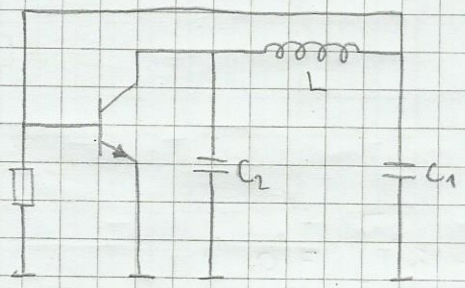


$$h_{11} = Z_1 \parallel Z_3$$
$$h_{12} = \frac{Z_1}{Z_1 + Z_3}$$
$$h_{21} = \frac{-Z_1}{Z_1 + Z_3}$$
$$h_{22} = \frac{Z_1 + Z_2 + Z_3}{Z_2 (Z_1 + Z_3)}$$

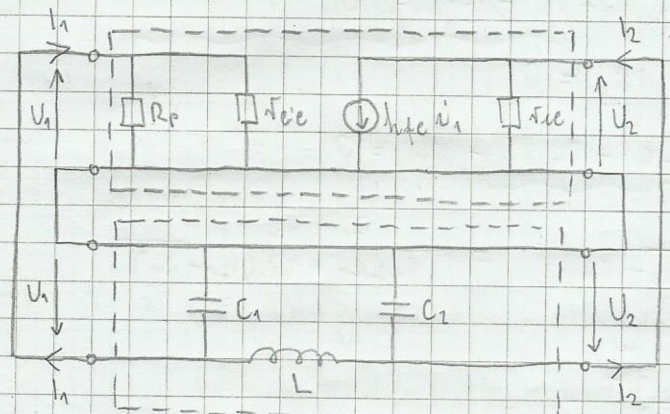
Colpittsov oscilator

$r_{be} = 1\ k\Omega$
$1/r_{ce} = 2.5 \cdot 10^{-4}\ S$
$h_{fe} = 100$
$L = 40\ nF$
$f = 200\ MHz$

$f_x = ?$ uvjet samopobude = ? $\frac{C_2}{C_1} = ?$ $h_{fe\,min} = ?$



$R_p = R_1 \,||\, R_2 \,||\, R_c$



$Z_1 = \frac{1}{sC_1}$

$Z_2 = \frac{1}{sC_2}$

$Z_3 = sL$

$$h'_{11} = \frac{1}{sC_1} \,||\, sL = \frac{\frac{1}{sC_1} \cdot sL}{\frac{1}{sC_1} + sL} = \frac{\frac{L}{\cancel{C_1}}}{\frac{1+s^2LC_1}{s\cancel{C_1}}} = \frac{sL}{1+s^2LC_1} \Rightarrow h'_{11} = \frac{j\omega L}{1-\omega^2 LC_1}$$

$$h'_{12} = \frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + sL} = \frac{\frac{1}{\cancel{sC_1}}}{\frac{1+s^2LC_1}{\cancel{sC_1}}} = \frac{1}{1+s^2LC_1} \Rightarrow h'_{12} = \frac{1}{1-\omega^2 LC_1}$$

$$h'_{21} = -\frac{1}{1-\omega^2 LC_1}$$

$$h'_{22} = \frac{\frac{1}{sC_1} + \frac{1}{sC_2} + sL}{\frac{1}{sC_2}\left(\frac{1}{sC_1} + sL\right)} = \frac{\frac{C_1 + C_2 + s^2 LC_1C_2}{sC_1C_2}}{\frac{1}{s^2C_1C_2} + \frac{L}{C_2}} = \frac{\frac{C_1+C_2+s^2LC_1C_2}{\cancel{sC_1C_2}}}{\frac{1+s^2LC_1}{s^{\cancel{2}}\cancel{C_1C_2}}}$$

$$= \frac{s\left(C_1 + C_2 + s^2 LC_1C_2\right)}{1+s^2LC_1} \Rightarrow h'_{22} = j\omega\,\frac{C_1+C_2-\omega^2 LC_1C_2}{1-\omega^2LC_1}$$

Tranzistor: $u_1 = h_{11} i_1 + h_{12} u_2$ (1)

$i_2 = h_{21} i_1 + h_{22} u_2$ (2)

Π-četveropol: $u_1 = -h'_{11} i_1 + h'_{12} u_2$ (3)

$-i_2 = -h'_{21} i_1 + h'_{22} u_2$ (4)

① - ③ : $(h_{11} + h_{11}')u_1 + (h_{12} - h_{12}')u_2 = 0$

② + ④ : $(h_{21} - h_{21}')u_1 + (h_{22} + h_{22}')u_2 = 0$

~~trivijalno rješenje~~ $\cancel{u_1 = 0}$, $\cancel{u_2 = 0}$ netrivijalno rješenje: $\Delta = 0$

$$\begin{vmatrix} h_{11} + h_{11}' & h_{12} - h_{12}' \\ h_{21} - h_{21}' & h_{22} + h_{22}' \end{vmatrix} = 0$$

$$(h_{11} + h_{11}')(h_{22} + h_{22}') - (h_{12} - h_{12}')(h_{21} - h_{21}') = 0$$

$$\left(h_{11} + \frac{j\omega L}{1-\omega^2 L C_1}\right)\left(h_{22} + j\omega \frac{C_1 + C_2 - \omega^2 L C_1 C_2}{1-\omega^2 L C_1}\right) - \left(h_{12} - \frac{1}{1-\omega^2 L C_1}\right)\left(h_{21} + \frac{1}{1-\omega^2 L C_1}\right) = 0$$

$$a = 1 - \omega^2 L C_1$$

$$h_{11}h_{22} + j\omega h_{11}\frac{C_1 + C_2 \cdot a}{a} + h_{22}\frac{j\omega L}{a} - \frac{\omega^2 L[C_1 + C_2 \cdot a]}{a^2} - h_{12}h_{21} - \frac{h_{12}}{a} + \frac{h_{21}}{a} + \frac{1}{a^2} = 0$$

$$a^2 h_{11}h_{22} + j\omega h_{11} a[C_1 + aC_2] + j\omega L h_{22} a - \omega^2 L[C_1 + aC_2] - a^2 h_{12}h_{21} - a h_{12} + a h_{21} + 1 = 0$$

$\mathrm{Im}\{\text{"izraz"}\} = 0$

$$\omega h_{11} a[C_1 + aC_2] + \omega L h_{22} a = 0 \quad /:\omega a$$

$$h_{11}(C_1 + aC_2) + L h_{22} = 0$$

$$h_{11}C_1 + a h_{11} C_2 + L h_{22} = 0$$

$$h_{11}C_1 + (1 - \omega^2 L C_1) h_{11} C_2 + h_{22} L = 0$$

$$h_{11}C_1 + h_{11}C_2 - \omega^2 L C_1 C_2 h_{11} + h_{22} L = 0$$

$$\omega^2 L C_1 C_2 h_{11} = h_{11}(C_1 + C_2) + h_{22} L \quad \Big| : L C_1 C_2 h_{11}$$

$$\omega^2 = \frac{C_1 + C_2}{L C_1 C_2} + \frac{h_{22}}{h_{11} C_1 C_2}, \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\omega^2 = \underbrace{\frac{1}{L C_{eq}}}_{\omega_0^2} + \underbrace{\frac{h_{22}}{h_{11} C_1 C_2}}_{\text{korekcijski faktor}}$$

$\mathrm{Re}\{\text{"izraz"}\} = 0$

$$a^2 h_{11}h_{22} - \omega^2 L(C_1 + aC_2) - a^2 h_{12}h_{21} - a(h_{12} - h_{21}) + 1 = 0$$

$$a^2 \underbrace{(h_{11}h_{22} - h_{12}h_{21})}_{\Delta h} - a(h_{12} - h_{21}) + 1 - \omega^2 L \cdot (C_1 + aC_2) = 0$$

$$a = 1 - \omega_0^2 L C_1 = 1 - \frac{1}{L C_{eq}} L C_1 = 1 - \frac{C_1}{\frac{C_1 C_2}{C_1 + C_2}} = 1 - \frac{C_1 + C_2}{C_2} = \cancel{1} - \frac{C_1}{C_2} \cancel{-1}$$

$$a = -\frac{C_1}{C_2}$$

$$a^2 \Delta h + a(h_{21} - h_{12}) + 1 - \omega^2 L \underbrace{(\cancel{C_1} - \cancel{C_1})}_{=0} = 0$$

$$a^2 \Delta h + a(h_{21} - h_{12}) + 1 = 0$$

$$\frac{C_2}{C_1} = n = -\frac{1}{a}$$

$$\frac{1}{n^2}\Delta h - \frac{1}{n}(h_{21} - h_{12}) + 1 = 0 \quad / \; n^2$$

$$n^2 - n(h_{21} - h_{12}) + \Delta h = 0 \quad , \quad h_{12} \approx 0$$

$$n^2 - h_{21} n + \Delta h = 0$$

Vieteove formule:

$$\left.\begin{aligned} a x^2 + b x + c &= 0 \\ x_1 + x_2 &= -\frac{b}{a} \\ x_1 \cdot x_2 &= \frac{c}{a} \end{aligned}\right\} \qquad \begin{aligned} n_1 + n_2 &= \frac{h_{21}}{1} = h_{21} \\ n_1 \cdot n_2 &= \frac{\Delta h}{1} = \Delta h \end{aligned}$$

pretpostavka: $n_1 \gg n_2 \implies n_1 \approx h_{21}$

$$h_{21} \cdot n_2 = \Delta h \quad / : h_{21}$$

$$n_2 = \frac{\Delta h}{h_{21}}$$

$$\frac{\Delta h}{h_{21}} < n < h_{21}$$

$$h_{21} = h_{fe} = 100$$

$$\Delta h = h_{11} h_{22} - \underbrace{h_{12} h_{21}}_{\approx 0} = h_{11} h_{22} = \left| \begin{aligned} h_{11} &= r_{be} = 1000 \\ h_{22} &= 1/r_{ce} = 2.5 \cdot 10^{-4} \end{aligned} \right| = 10^3 \cdot 2.5 \cdot 10^{-4} = 0.25$$

$$\frac{\Delta h}{h_{21}} = 0.0025$$

$$n = \frac{C_2}{C_1} \implies 0.0025 < \frac{C_2}{C_1} < 100$$

b) $\frac{C_2}{C_1} = 1$, $h_{fe_{min}} = ?$

$$\frac{C_2}{C_1} = n = 1$$

$$n^2 - h_{21} n + \Delta h = 0$$

$$1 - h_{21} + 0.25 = 0 \implies h_{21} = h_{fe_{min}} = 1.25$$

3.
$C_1 = 91\ pF$ $h_{ie} = 15.2\ \Omega$
$C_2 = 43\ pF$ $h_{fe} = -0.969$
$L = 0.11\ \mu H$ $h_{re} = 0.0672$
$C_4 = 0.1\ \mu F$ $h_{oe} = 2.66\ mS$

$f = ?$ $f_0 = ?$ uvjet samopobude = ?

Colpittsov oscilator

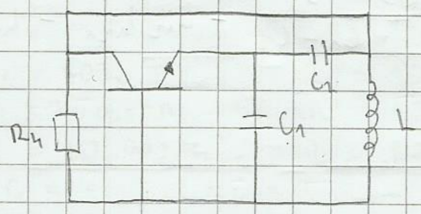
ako je SZB => h..

$h_{11} = h_i$
$h_{12} = h_r$
$h_{21} = h_f$
$h_{22} = h_o$

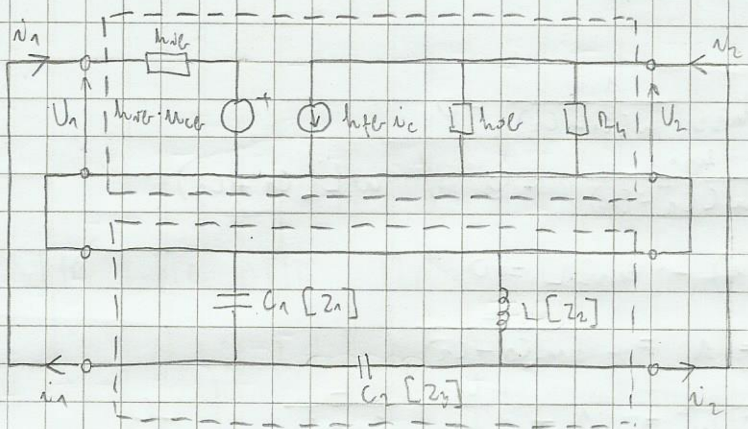
AC analiza:

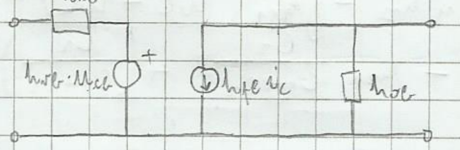
$C_3, C_4 \rightarrow$ kratki spoj

zavojnica $P_r \rightarrow$ prazni hod



linearna nadomjesna shema za mali signal:





$$Z_1 = \frac{1}{sC_1} \Rightarrow Z_1 = \frac{1}{j\omega C_1}$$
$$Z_2 = sL \Rightarrow Z_2 = j\omega L$$
$$Z_3 = \frac{1}{sC_2} \Rightarrow Z_3 = \frac{1}{j\omega C_2}$$

T_2 - četveropol:

$$h_{11}' = Z_1 \| Z_3 = \frac{Z_1 Z_3}{Z_1 + Z_3} = \frac{\frac{1}{j\omega C_1} \cdot \frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{\frac{1}{\cancel{j\omega C_1 j\omega C_2}}}{\frac{j\omega C_1 + j\omega C_2}{\cancel{j\omega C_1 j\omega C_2}}} = \frac{1}{j\omega(C_1 + C_2)}$$

$$h_{12}' = \frac{Z_1}{Z_1 + Z_3} = \frac{h_{11}'}{Z_3} = \frac{1}{j\omega(C_1 + C_2)} \cdot j\omega C_2 = \frac{C_2}{C_1 + C_2}$$

$$h_{21}' = -h_{12}' = -\frac{C_2}{C_1 + C_2}$$

$$h_{22}' = \frac{Z_1 + Z_2 + Z_3}{Z_2(Z_1 + Z_3)} = \frac{\frac{1}{j\omega C_1} + j\omega L + \frac{1}{j\omega C_2}}{j\omega L\left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right)} = \frac{\cancel{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}}{j\omega L\left(\cancel{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}\right)} + \frac{j\omega L}{j\omega L\left(\frac{1}{j\omega C_1} + \dots\right)}$$

$$= \frac{1}{j\omega L} + \frac{1}{\frac{j\omega C_1 + j\omega C_2}{-\omega^2 C_1 C_2}} = \frac{1}{j\omega L} + \frac{-\omega C_1 C_2}{j(C_1 + C_2)} = \frac{1}{j\omega L} + j\omega \frac{C_1 C_2}{C_1 + C_2}$$

tranzistor:
(1) $u_1 = h_{11} i_1 + h_{12} u_2$
(2) $i_2 = h_{21} i_1 + h_{22} u_2$

T_1 - četveropol:
(3) $u_1 = -h_{11}' i_1 + h_{12}' u_2$
(4) $-i_2 = -h_{21}' i_1 + h_{22}' u_2$

(1) - (3): $(h_{11} + h_{11}') i_1 + (h_{12} - h_{12}') u_2 = 0$

(2) + (4): $(h_{21} - h_{21}') i_1 + (h_{22} + h_{22}') u_2 = 0$

$$\Delta h = 0$$

$$(h_{11} + h_{11}')(h_{22} + h_{22}') - (h_{12} - h_{12}')(h_{21} - h_{21}') = 0$$

$$h_{11}h_{22} + h_{11}h_{22}' + h_{11}'h_{22} + h_{11}'h_{22}' - h_{12}h_{21} + h_{12}h_{21}' + h_{12}'h_{21} - h_{12}'h_{21}' = 0$$

$$h_{11}h_{22} + \frac{h_{11}}{j\omega L} + j\omega h_{11}C + \frac{h_{22}}{j\omega(C_1+C_2)} + \frac{1}{j\omega(C_1+C_2)}\left(\frac{1}{j\omega L} + j\omega C\right) - h_{12}h_{21} - h_{12}\frac{C_2}{C_1+C_2} + h_{21}\frac{C_2}{C_1+C_2}$$
$$+ \left(\frac{C_2}{C_1+C_2}\right)^2 = 0$$

$$h_{11}h_{22} + jh_{11}\left(\omega C - \frac{1}{\omega L}\right) - jh_{22}\frac{1}{\omega(C_1+C_2)} - \frac{1}{\omega^2 L(C_1+C_2)} + \frac{C}{C_1+C_2} - h_{12}h_{21} - h_{12}\frac{C_2}{C_1+C_2}$$
$$+ h_{21}\frac{C_2}{C_1+C_2} + \left(\frac{C_2}{C_1+C_2}\right)^2 = 0$$

$$\mathrm{Im}\{\text{"izraz"}\} = 0$$

$$h_{11}\left(\omega C - \frac{1}{\omega L}\right) - h_{22}\frac{1}{\omega(C_1+C_2)} = 0$$

$$h_{11}\cdot\frac{\omega^2 LC - 1}{\omega L} - \frac{h_{22}}{\omega(C_1+C_2)} = 0 \quad / \cdot \omega L(C_1+C_2)$$

$$h_{11}(C_1+C_2)(\omega^2 LC - 1) - h_{22}L = 0$$

$$\omega^2 LC - 1 = \frac{h_{22}L}{h_{11}(C_1+C_2)}$$

$$LC\omega^2 = 1 + \frac{h_{22}}{h_{11}}\frac{L}{C_1+C_2} \quad / : LC$$

$$\omega^2 = \underbrace{\frac{1}{LC}}_{\omega_0^2} + \frac{h_{22}}{h_{11}}\frac{1}{\cancel{C_1+C_2}}\cdot\frac{\cancel{C_1+C_2}}{C_1\cdot C_2} = \frac{1}{LC} + \frac{h_{22}}{h_{11}}\frac{1}{C_1\cdot C_2}$$

$$\omega^2 = \frac{1}{0.11\cdot10^{-6}\cdot\frac{91\cdot43}{91+43}\cdot10^{-12}} + \frac{2.66\cdot10^{-3}}{15.2}\cdot\frac{1}{91\cdot43\cdot10^{-24}} = 3.56\cdot10^{17}$$

$$\omega = 2\pi f \Rightarrow f = 94.97\ \mathrm{MHz}$$

- ako zanemarimo utjecaj parametara tranzistora:

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.11\cdot10^{-6}\cdot\frac{91\cdot43}{91+43}\cdot10^{-12}}} = 557.96\cdot10^6$$

$$f_0 = 88.8\ \mathrm{MHz}$$

$$\mathrm{Re}\{\text{"izraz"}\} = 0$$

$$h_{11}h_{22} + \frac{C}{C_1+C_2} - \frac{1}{\omega^2 L(C_1+C_2)} - h_{12}h_{21} + (h_{21} - h_{12})\frac{C_2}{C_1+C_2} + \left(\frac{C_2}{C_1+C_2}\right)^2 = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \frac{1}{\omega^2 L(C_1+C_2)} = \frac{\cancel{L}C}{\cancel{L}(C_1+C_2)} = \frac{C}{C_1+C_2}$$

$$h_{11}h_{22} + \cancel{\frac{C}{C_1+C_2}} - \cancel{\frac{C}{C_1+C_2}} - h_{12}h_{21} + \frac{C_2}{C_1+C_2}h_{21} - h_{12}\frac{C_2}{C_1+C_2} + \left(\frac{C_2}{C_1+C_2}\right)^2 = 0$$

$$h_{21} = \frac{h_{11}h_{22} - h_{12}\frac{C_2}{C_1+C_2} + \left(\frac{C_2}{C_1+C_2}\right)^2}{h_{12} - \frac{C_2}{C_1+C_2}} = \frac{19.2\cdot 266\cdot 10^{-3} - 0.0672\cdot\frac{43}{91+43} + \left(\frac{43}{91+\dots}\right.}{0.0672 - \frac{43}{91+43}}$$

$$\left.\begin{aligned} h_{21} &= -0.48 \\ h_{fe} &= -0.969 \end{aligned}\right\} \quad |h_{fe}| > |h_{21}| \checkmark \Rightarrow \text{uvjet ravnopolnoće zadovoljen}$$

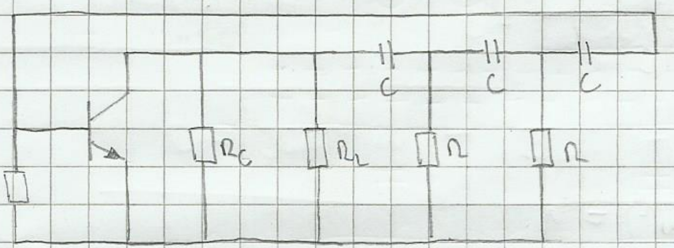
4. $r_{bie} = 1000\ \Omega$ $\qquad R_C = 1000\ \Omega$ $\qquad$ RC-oscilator

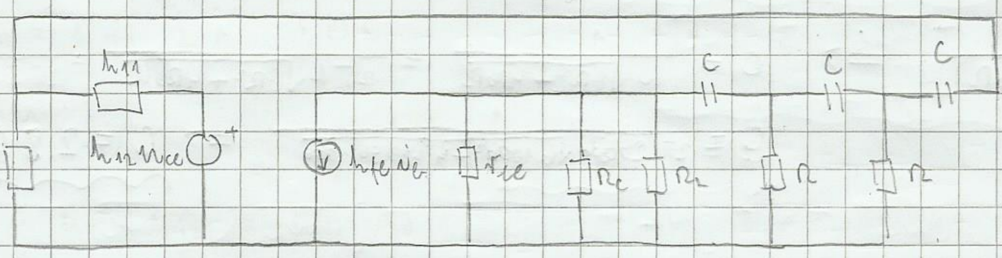
$r_{ce} = 40\,000\ \Omega$ $\qquad R = 1000\ \Omega$

$f = 20\,000\ Hz$ $\qquad R_L = 800\ \Omega$

$C = ?$ $\qquad h_{fe} = ?$

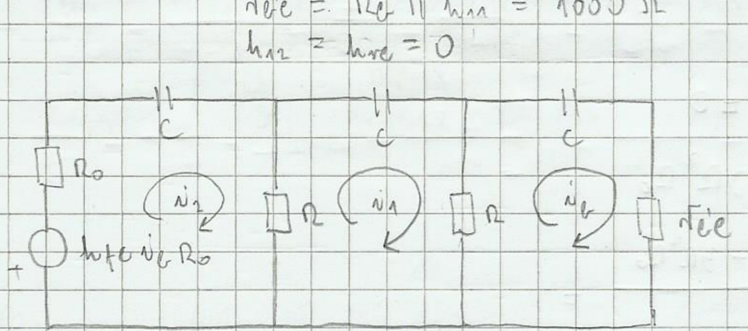
AC-analiza:





$r_{bie} = R_B \parallel h_{11} = 1000\ \Omega$

$h_{12} = h_{re} = 0$



$R_o = r_{ce} \parallel R_C \parallel R_L = 40000 \parallel 1000 \parallel 80\dots$

$R_o = \frac{40000}{91} \approx 439.56\ \Omega$

① $i_b\left(R + r_{bie} + \frac{1}{j\omega C}\right) - i_1 R = 0$

② $i_1\left(R + R + \frac{1}{j\omega C}\right) - i_b R - i_2 R = 0$

③ $i_2\left(R_o + R + \frac{1}{j\omega C}\right) - i_1 R + h_{fe}R_o i_b = 0$

$$\begin{array}{ccc} i_b & i_1 & i_2 \end{array}$$
$$\begin{vmatrix} R + r_{bie} + \frac{1}{j\omega C} & -R & 0 \\ -R & 2R + \frac{1}{j\omega C} & -R \\ h_{fe}R_o & -R & R_o + R + \frac{1}{j\omega C} \end{vmatrix} = 0$$

$$0 + R\begin{vmatrix} R + r_{b'e} + \frac{1}{j\omega C} & -R \\ h_{fe} R_o & -R \end{vmatrix} + \left(R_o + R + \frac{1}{j\omega C}\right)\begin{vmatrix} R + r_{b'e} + \frac{1}{j\omega C} & -R \\ -R & 2R + \frac{1}{j\omega C} \end{vmatrix} = 0$$

$$R\left[-R\left(\underbrace{R + r_{b'e}}_{2R} + \frac{1}{j\omega C}\right) + R h_{fe} R_o\right] + \left(R_o + R + \frac{1}{j\omega C}\right)\left[\left(2R + \frac{1}{j\omega C}\right)\left(\underbrace{R + r_{b'e}}_{2R} + \frac{1}{j\omega C}\right) - R^2\right] = 0$$

$$-2R^3 - \frac{R^2}{j\omega C} + R^2 h_{fe} R_o + \left(R_o + R + \frac{1}{j\omega C}\right)\left(4R^2 + \frac{4R}{j\omega C} - \frac{1}{\omega^2 C^2} - R^2\right) = 0$$

$$-2R^3 - \frac{R^2}{j\omega C} + R^2 h_{fe} R_o + 3R_o R^2 + \frac{4R_o R}{j\omega C} - \frac{R_o}{\omega^2 C^2} + 3R^3 + \frac{4R^2}{j\omega C} - \frac{R}{\omega^2 C^2} + \frac{3R^2}{j\omega C} - \frac{4R}{\omega^2 C^2} + j\frac{1}{\omega^3 C^3} = 0$$

$$R^3 + j\frac{R^2}{\omega C}(1 - 4 - 3) + R^2 R_o (h_{fe} + 3) - j\frac{4R_o R}{\omega C} - \frac{R_o + R + 4R}{\omega^2 C^2} + j\frac{1}{\omega^3 C^3} = 0$$

$Im\{\text{"izraz"}\} = 0$

$$-6\frac{R^2}{\omega C} - 4\frac{R_o R}{\omega C} + \frac{1}{\omega^3 C^3} = 0 \quad / \omega^3 C^3$$

$$-6R^2\omega^2 C^2 - 4R_o R\,\omega^2 C^2 + 1 = 0$$

$$C^2(6R^2\omega^2 + 4R_o R\omega^2) = 1$$

$$C^2 = \frac{1}{\omega^2(6R^2 + 4R_o R)} \Rightarrow C = \frac{1}{\omega\sqrt{6R^2 + 4R_o R}} = \frac{1}{2\pi f\sqrt{6R^2 + 4R_o R}}$$

$$C = \frac{1}{2\pi \cdot 20000\sqrt{6 \cdot 1000^2 + 4 \cdot 439.56 \cdot 1000}} \Rightarrow C = 2.86\ nF$$

$Re\{\text{"izraz"}\} = 0$

$$R^3 + R^2 R_o (h_{fe} + 3) - \frac{R_o + 5R}{\omega^2 C^2} = 0$$

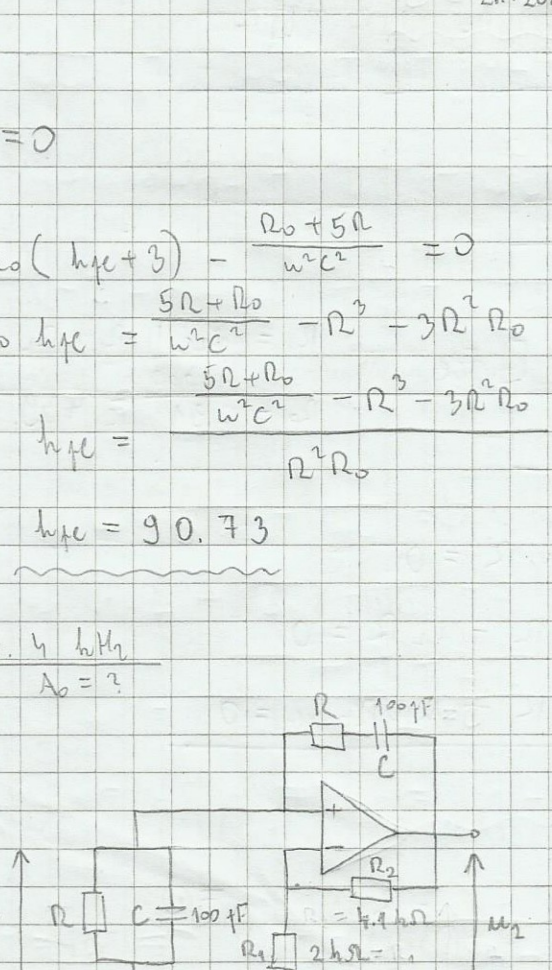
$$R^2 R_o h_{fe} = \frac{5R + R_o}{\omega^2 C^2} - R^3 - 3R^2 R_o$$

$$h_{fe} = \frac{\frac{5R + R_o}{\omega^2 C^2} - R^3 - 3R^2 R_o}{R^2 R_o} =$$

$$h_{fe} = 90.73$$

5. $f = 787.4\ kHz$

$R = ?$ $\quad A_0 = ?$



$$u_1 = u_2 \cdot \frac{R \parallel \frac{1}{j\omega C}}{R \parallel \frac{1}{j\omega C} + \left(R + \frac{1}{j\omega C}\right)} = u_2 \cdot \frac{\frac{R}{1 + j\omega RC}}{\frac{R}{1 + j\omega RC} + \frac{1 + j\omega RC}{j\omega C}}$$

$$\frac{u_2}{u_1} = \frac{\frac{R}{1 + j\omega RC} + \frac{1 + j\omega RC}{j\omega C}}{\frac{R}{1 + j\omega RC}} = 1 + \frac{\frac{1 + j\omega RC}{j\omega C}}{\frac{R}{1 + j\omega RC}} =$$

$$= 1 + \frac{(1 + j\omega RC)^2}{j\omega RC} = 1 + \frac{1 + 2j\omega RC - \omega^2 R^2 C^2}{j\omega RC} =$$

$$= 1 + \frac{1}{j\omega RC} + 2 - \frac{\omega RC}{j} = 3 - j\,\frac{1 - (\omega RC)^2}{\omega RC}$$

$$\mathrm{Im}\left\{\frac{u_2}{u_1}\right\} = 0$$

$$\frac{1 - (\omega RC)^2}{\omega RC} = 0 \Rightarrow (\omega RC)^2 = 1$$

$$\omega RC = 1 \Rightarrow R = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 787400 \cdot 1}$$

$$R = 2021.27\ \Omega$$

$$G = 1 + \frac{4.1}{2} = 3.05$$

$$\frac{u_2}{u_1} = 3$$

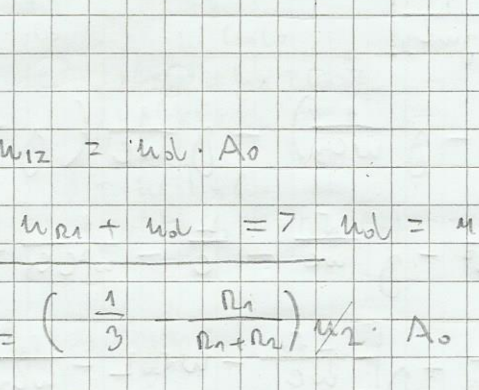
$$u_2 = u_{iz} = u_d \cdot A_0$$

$$u_1 = u_{R1} + u_d \Rightarrow u_d = u_1 - u_{R1} = \frac{1}{3}u_2 - \frac{R_1}{R_1 + R_2}u_2 = \left(\frac{1}{3} - \frac{R_1}{R_1 + R_2}\right)$$

$$\cancel{u_2} = \left(\frac{1}{3} - \frac{R_1}{R_1 + R_2}\right)\cancel{u_2}\, A_0 \Rightarrow A_0 = \frac{1}{\frac{1}{3} - \frac{R_1}{R_1 + R_2}} = \frac{1}{\frac{1}{3} - \frac{2}{4.1 + 2}}$$

$$A_0 = 183$$

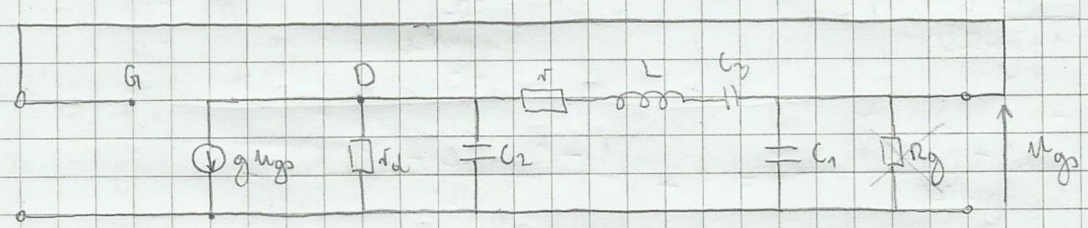
6. Gouriet - Clapp oscillator



$C_1 = C_2 = 500\ pF$ $\qquad r = 15\ \Omega$
$C_3 = 100\ pF$ $\qquad r_d = 5\ k\Omega$
$C_g = 0.1\ \mu F$ $\qquad R_g = 560\ k\Omega$
$L = 500\ \mu H$

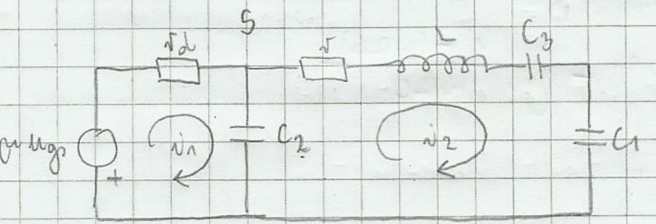
$f = ?$

AC analiza:



$C_G \to$ kratki spoj

$R_g \to \infty$



$u_{gs} = i_2 \cdot \frac{1}{j\omega C_1}$

$C_1 = C_2 = C$

$\mu = g \cdot r_d$ → strmina

$$① \quad i_1\left(r_d + \frac{1}{j\omega C}\right) + \mu u_{gs} - i_2 \frac{1}{j\omega C} = 0$$

$$② \quad i_2\left(\frac{1}{j\omega C} + r + j\omega L + \frac{1}{j\omega C_3} + \frac{1}{j\omega C}\right) - i_1 \cdot \frac{1}{j\omega C} = 0$$

$$\begin{vmatrix} r_d + \frac{1}{j\omega C} & -\frac{1}{j\omega C} + \frac{\mu}{j\omega C} \\ -\frac{1}{j\omega C} & \frac{2}{j\omega C} + r + j\omega L + \frac{1}{j\omega C_3} \end{vmatrix} = 0$$

$$\left(r_d - j\frac{1}{\omega C}\right)\left(-j\frac{2}{\omega C} + r + j\omega L - j\frac{1}{\omega C_3}\right) - j\frac{1}{\omega C}\left(j\frac{1}{\omega C} - j\frac{\mu}{\omega C}\right) =$$

$$-j\frac{2r_d}{\omega C} + r r_d + j\omega r_d L - j\frac{r_d}{\omega C_3} - \frac{2}{\omega^2 C^2} - j\frac{r}{\omega C} + \frac{L}{C} - \frac{1}{\omega^2 C C_3} + \frac{1}{\omega^2 C^2} - \frac{\mu}{\omega^2 C^2}$$

$$r r_d + \frac{1}{\omega^2 C^2}(-2 + 1 - \mu) + \frac{L}{C} - \frac{1}{\omega^2 C C_3} + j\left[-\frac{2r_d}{\omega C} + \omega r_d L - \frac{r_d}{\omega C_3} - \frac{r}{\omega C}\right] = 0$$

$$\text{Im}\{\text{"nazivnik"}\} = 0$$

$$-\frac{2r_d}{\omega C} + \omega r_d L - \frac{r_d}{\omega C_3} - \frac{r}{\omega C} = 0 \quad / \cdot \omega$$

$$-2\frac{r_d}{C} + \omega^2 r_d L - \frac{r_d}{C_3} - \frac{r}{C} = 0$$

$$\omega^2 = \frac{\frac{r_d}{C_3} + \frac{r}{C} + 2\frac{r_d}{C}}{r_d L} = 2.8012 \cdot 10^{13}$$

$$\omega = 2\pi f \Rightarrow f = 842349\ Hz$$

ako je $r = 0 \Rightarrow \omega^2 = \frac{\frac{r_d}{C_3} + 2\frac{r_d}{C}}{r_d L} = 2.8 \cdot 10^{13}$

$$f' = 842169\ Hz$$

$\mathrm{Re}\{\text{"nwaz"}\} = 0$

$$r r_d + \frac{1}{\omega^2 C^2}(-1-\mu) + \frac{L}{C} - \frac{1}{\omega^2 C C_3} = 0$$

$$r r_d - \frac{1}{\omega^2 C^2} - \frac{\mu}{\omega^2 C^2} + \frac{L}{C} - \frac{1}{\omega^2 C C_3} = 0 \quad / \cdot \omega^2 C^2$$

$$\omega^2 r r_d C^2 - 1 - \mu + \omega^2 L C - \frac{C}{C_3} = 0$$

$$\mu = \omega^2 r r_d C^2 + \omega^2 L C - \frac{C}{C_3} - 1 = 2.8012\cdot 10^{13}\cdot 15\cdot 5000\cdot (500\cdot 10^{-12})^2 + 2.8012\cdot 10^{13}\cdot 500\cdot 10^{-6}\cdot 500\cdot 10^{-}$$

$$\mu = 1.528$$

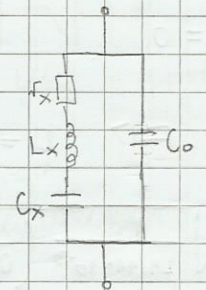
- za $r = 0 \Rightarrow \mu = 1 \Rightarrow g = \frac{\mu}{r_d} = \frac{1}{5000}$

$$g = 0.2 \text{ mS}$$

7.

$C_0 = 0.3\ \mu F$
$C_x = 0.003\ \mu F$
$L_x = 21.11\ mH$
$Q = 20\,000$

$f_s = ?\quad f_p = ?\quad r_x = ?$



$$Z_x = \frac{\left(r_x + j\omega L_x + \frac{1}{j\omega C_x}\right)\cdot \frac{1}{j\omega C_0}}{r_x + j\omega L_x + \frac{1}{j\omega C_x} + \frac{1}{j\omega C_0}} = \left| r_x \approx 0 \right| =$$

$$= \frac{\frac{L_x}{C_0} - \frac{1}{\omega^2 C_x C_0}}{\frac{-\omega^2 L_x C_x C_0 + C_x + C_0}{j\omega C_x C_0}} = \frac{\frac{\omega^2 L_x C_x - 1}{\omega^2 C_x C_0}}{\frac{C_x + C_0 - \omega^2 L_x C_x C_0}{j\omega C_x C_0}} =$$

$$= j\,\frac{1 - \omega^2 L_x C_x}{\omega(\omega^2 L_x C_x C_0 - C_x - C_0)}$$

* szeregowa rezonancja:

$Z_x = 0$

$$1 - \omega_s^2 L_x C_x = 0$$

$$\omega_s = \frac{1}{\sqrt{L_x C_x}}$$

$$f_s = \frac{1}{2\pi\sqrt{L_x C_x}} = \frac{1}{2\pi\sqrt{0.02111\cdot 3\cdot 10^{-15}}}$$

$$f_s = 19\,999\,327.29 \text{ Hz}$$

* paralelna rezonancja:

$Z_x \to \infty \quad (Y_x = 0)$

$$\omega^2 L_x C_x C_0 - C_x - C_0 = 0$$

$$\omega_P = \sqrt{\frac{C_x + C_0}{L_x C_x C_0}}$$

$$f_P = \frac{1}{2\pi}\sqrt{\frac{1}{L_x}\left(\frac{1}{C_0} + \frac{1}{C_x}\right)}$$

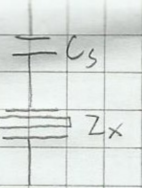
$$f_P = 20\,099\,075.17 \text{ Hz}$$

$$\Delta f = f_P - f_s = 99\,748 \text{ Hz}$$

$$\frac{f_P}{f_s} = 1.005$$

$$Q = \frac{X_L}{r_x} \Rightarrow r = \frac{X_L}{Q} = \frac{2\pi f_s L}{Q} = \frac{2\pi\cdot 19\,999\,327.29\cdot 21.11\cdot 10^{-3}}{20\,000}$$

$$r_x = 132.63\ \Omega$$



$$Z_{uk} = Z_{C_s} + Z_x = \frac{1}{j\omega C_s} + Z_x$$

$f_s = ?$ uz $C_s = 0.3\ \mu F$ i $C_s = 10\ \mu F$

$$Z_{uk} = \frac{1}{j\omega C_s} + j\frac{1-\omega^2 L_x C_x}{\omega(\omega^2 L_x C_x C_0 - C_x - C_0)} = \frac{-j(\omega^2 L_x C_x C_0 - C_x - C_0) + jC_s - j\omega^2 L_x C_x C_s}{\omega C_s(\omega^2 L_x C_x C_0 - C_x - C_0)}$$

$$= j\frac{C_s(1-\omega^2 L_x C_x) + C_x + C_0 - \omega^2 L_x C_x C_0}{\omega C_s(\omega^2 L_x C_x C_0 - C_x - C_0)}$$

* serijska rezonancija: $Z_{uk} = 0$

$$C_s(1-\omega^2 L_x C_x) + C_x + C_0 - \omega^2 L_x C_x C_0 = 0$$

$$C_s - \omega^2 L_x C_x C_s + C_x + C_0 - \omega^2 L_x C_x C_0 = 0$$

$$\omega^2 = \frac{C_s + C_x + C_0}{L_x C_x C_s + L_x C_x C_0} \implies f = \frac{1}{2\pi}\sqrt{\frac{C_s + C_x + C_0}{L_x C_x (C_s + C_0)}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{L_x}\left(\frac{1}{C_x} + \frac{1}{C_s + C_0}\right)} = \frac{1}{2\pi}\sqrt{\frac{1}{21.11\cdot 10^{-3}}\left(\frac{1}{0.003\cdot 10^{-12}} + \frac{1}{0.3\cdot 10^{-12} + C_0}\right)}$$

$$f(C_s = 0.3\ \mu F) = 20\ 049\ 263.26\ Hz$$

$$f(C_s = 10\ \mu F) = 20\ 002\ 239.6\ Hz$$