Auditorne vježbe

19. siječnja 2018.

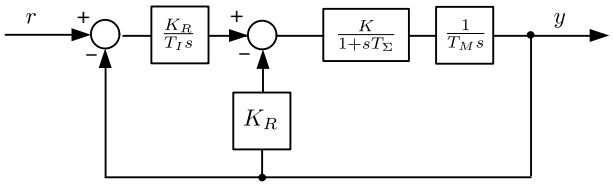
Modulni optimum

Jednadžbe modulnog optimuma:

$$a_i^2 + 2\sum_{j=1}^{j}(-1)^j a_{i-j}a_{i+j} = 0 \tag{1}$$

1. zadatak

Za sustav prikazan Slikom 1 potrebno je izvesti parametre modificiranog PI regulatora koristeći modulni optimum.



Slika 1: Proces s astatizmom prvog reda upravljan modificiranim PI regulatorom

Rješenje: $K_R = \frac{1}{2K}\frac{T_M}{T_\Sigma}$, $T_I = 4T_\Sigma$.

Prošireni modulni optimum

Jednadžbe proširenog modulnog optimuma:

$$a_i^2 + 2\sum_{j=1}^{i}(-1)^j a_{i-j}a_{i+j} = b_i^2 + 2\sum_{j=1}^{i}(-1)^j b_{i-j}b_{i+j} \tag{2}$$

2. zadatak

Za sustav drugog reda dan prijenosnom funkcijom:

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}. \tag{3}$$

potrebno je izvesti izraze za prošireni modulni optimum.

Rješenje: $a_0^2 = b_0^2$, $a_1^2 - 2a_0a_2 = b_1^2 - 2b_0b_2$.

3. zadatak

Izvesti optimalni faktor prigušenja ζ za sustav drugog reda dan prijenosnom funkcijom:

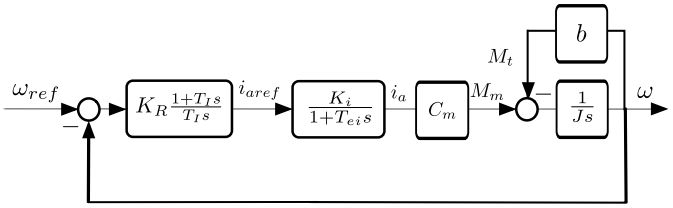
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{4}$$

koristeći modulni optimum.

Rješenje: $\zeta = \frac{\sqrt{2}}{2}$.

4. zadatak

Nadređena petlja upravljanja brzinom vrtnje istosmjernog motora s nezavisnom i konstantnom uzbudom prikazana je blokovskom shemom na Slici 2. Pritom su: $K_i = 1$, $T_{ei} = 5\,\text{ms}$, $C_m = 1.33\,\text{Nm/A}$, i $J = 3\,\text{kgm}^2$, $b = 1\,\text{Nms/rad}$. Potrebno je odrediti parametre PI regulatora koristeći modulni optimum.



Slika 2: Nadređena petlja upravljanja brzinom vrtnje s izraženim viskoznim trenjem

Rješenje:

Jednadžbe:

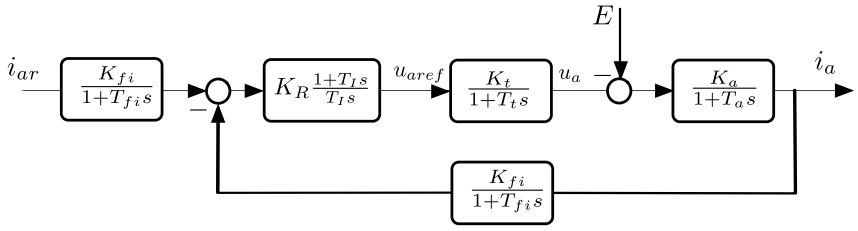
$$a_1^2 - 2a_0a_2 = b_1^2 \tag{5}$$

$$a_2^2 - 2a_1a_3 = 0 \tag{6}$$

Odakle slijedi: $K_R = 225.56$, $T_I = 3$ s.

5. zadatak

Petlja regulacije struje armature istosmjernog motora sa konstantnom i nezavisnom uzbudom prikazana je na Slici 3. Zadani su sljedeći parametri: $K_a = 4\,\text{A/V}$, $T_a = 0.03$ s, $K_t = 0.25$, $T_{mi} = 0.001$ s , $K_{fi} = 1$, $T_{fi} = 0.002$ s. Potrebno je odrediti parametre PI regulatora koristeći modulni optimum.



Slika 3: Podređena petlja upravljanja strujom armature istosmjernog motora s nezavisnom i konstantnom uzbudom

Rješenje:

$$G_{cl(s)} = \frac{KK_R + KK_RT_Is}{T_IT_1T_2T_3s^4 + T_I(T_1T_2 + T_1T_3 + T_2T_3)s^3 + T_I(T_1 + T_2 + T_3)s^2 + T_I(1 + KK_R)s + KK_R} \tag{7}$$

Jednadžbe:

$$a_1^2 - 2a_0a_2 = b_1^2 \tag{8}$$

$$a_2^2 - 2a_1a_3 + 2a_0a_4 = 0 \tag{9}$$

Slijedi: $K_R = 5.03$, $T_I = 0.03$ s.

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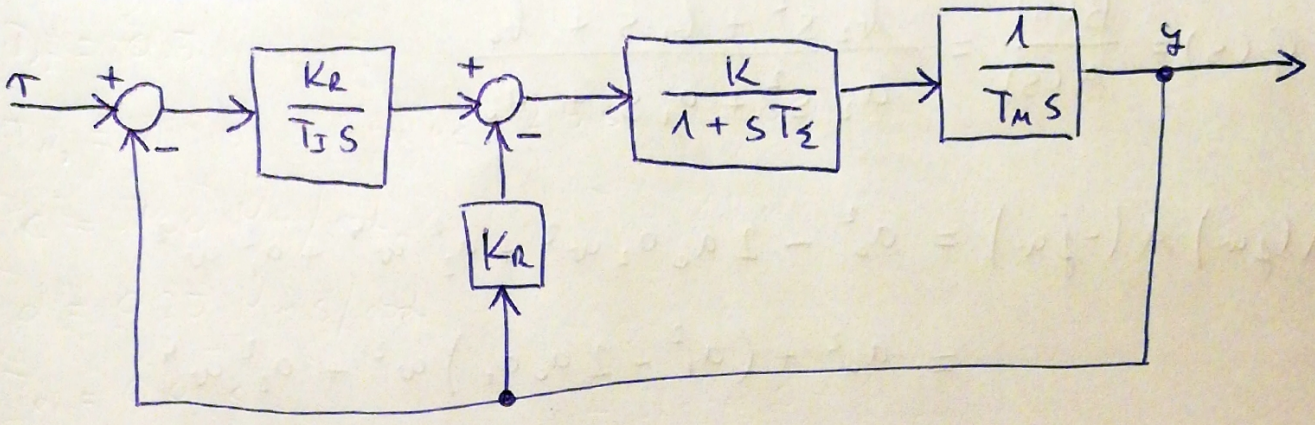
① 

$$G_r(s) = \frac{1}{\frac{T_I T_M T_\Sigma}{K K_R} s^3 + \frac{T_I T_M}{K K_R} s^2 + T_I s + 1}$$

$$\rightarrow \quad a_1 = T_I$$

$$a_2 = \frac{T_I T_M}{K K_R}$$

$$a_3 = \frac{T_I T_M T_\Sigma}{K K_R}$$

$$\rightarrow \quad a_2^2 - 2 a_1 a_3 + 2 a_0 a_4 = 0 \quad (a_4 = 0)$$

$$\frac{T_I^2 T_M^2}{(K K_R)^2} = 2 T_M T_\Sigma \frac{T_I^2}{K K_R} \quad \Rightarrow \quad \boxed{K_R = \frac{1}{2K} \frac{T_M}{T_\Sigma}}$$

$$\rightarrow \quad a_1^2 = 2 a_0 a_2$$

$$T_I^2 = \frac{2 T_I T_M}{K K_R} \quad \Rightarrow \quad \boxed{T_I = 4 T_\Sigma}$$

(2.)

$$G(s) = \frac{B(s)}{A(s)} = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

$$A(j\omega)\,A(-j\omega) = a_0^2 - 2a_0 a_2 \omega^2 + a_2^2 \omega^4 + a_1^2 \omega^2$$

$$= a_0^2 + (a_1^2 - 2a_0 a_2)\omega^2 + a_2^2 \omega^4$$

$$B(j\omega)\,B(-j\omega) = b_0^2 + (b_1^2 - 2b_0 b_2)\omega^2 + b_2^2 \omega^4$$

$$\rightarrow A(j\omega)\,A(-j\omega) = B(j\omega)\,B(-j\omega)$$

$$a_0^2 = b_0^2$$

$$a_1^2 - 2a_0 a_2 = b_1^2 - 2b_0 b_2$$

$$a_2^2 = b_2^2$$

(3.)

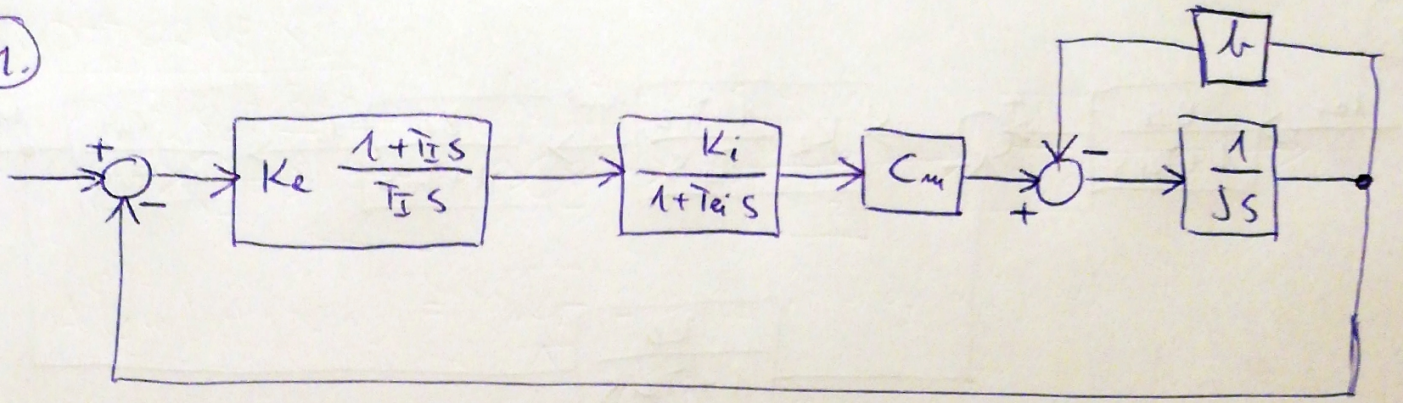
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{\frac{1}{\omega_n^2} s^2 + \frac{2\zeta}{\omega_n} s + 1}$$

$$\rightarrow a_1 = \frac{2\zeta}{\omega_n} \qquad a_2 = \frac{1}{\omega_n^2}$$

$$\rightarrow a_1^2 - 2a_0 a_2 = 0$$

$$\left(\frac{2\zeta}{\omega_n}\right)^2 = \frac{2}{\omega_n^2} \rightarrow \boxed{\zeta = \frac{\sqrt{2}}{2}}$$

4.



$K_i = 1$, $T_{ei} = 5\,ms$

$C_m = 1.33\ Nm/A$

$J = 3\ kg\,m^2$

$b = 1\ Nms/rad$

$$G_r(s) = \frac{T_I s + 1}{\underbrace{\frac{J T_{ei} T_I}{K_e K_i C_m}}_{a_3} s^3 + \underbrace{\frac{T_I (J + b T_{ei})}{K_e K_i C_m}}_{a_2} s^2 + \underbrace{\frac{T_I (b + K_e K_i C_m)}{K_e K_i C_m}}_{a_1} s + 1}$$

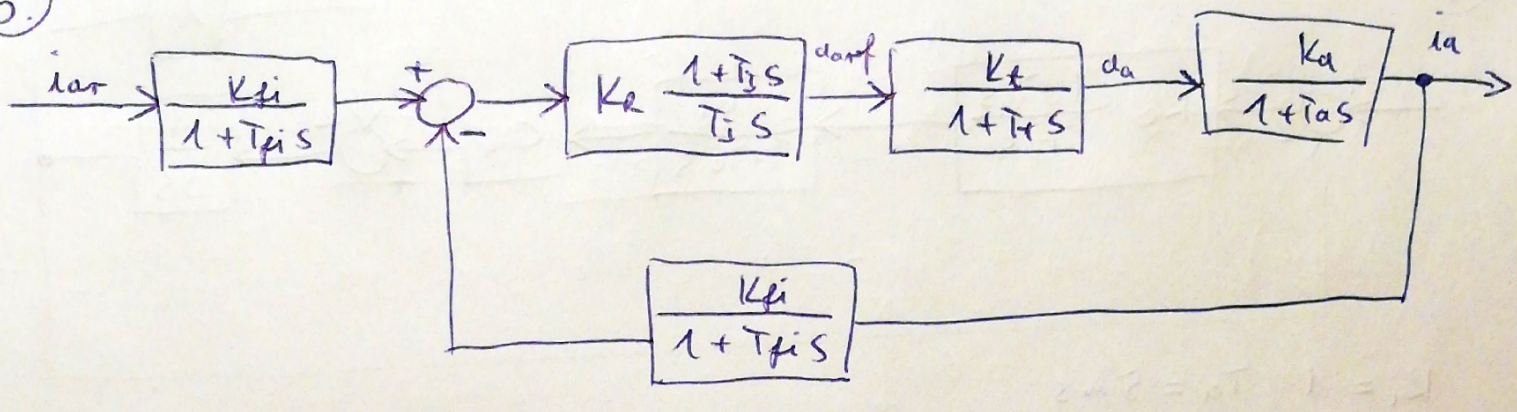
$$\rightarrow \quad a_2^2 - 2 a_1 a_3 = 0$$

$$K_e = \frac{(J + b T_{ei})^2 - J b T_{ei}}{2 K_i C_m J T_{ei}} \Rightarrow \boxed{K_e = 225.9405}$$

$$\rightarrow \quad a_1^2 - 2 a_0 a_2 = b_1^2$$

$$T_I = \frac{2 K_e K_i C_m (J + b T_{ei})}{b^2 + 2 K_e K_i C_m} \Rightarrow \boxed{T_I = 3\,s}$$

5.



$K_a = 4\ A/V$, $T_a = 0.03\ s$

$K_t = 0.25$, $T_t = 1\ ms$

$K_{fi} = 1$, $T_{fi} = 2\ ms$

$$G_r(s) = \frac{T_I K_R K s + K_R K}{T_I T_1 s^4 + T_I T_2 s^3 + T_I T_3 s^2 + T_I (1 + K_R K) s + K_R K}$$

$\rightarrow K = K_{fi} K_t K_a = 1$

$T_1 = T_{fi} T_t T_a = 6 \cdot 10^{-8}$

$T_2 = T_{fi} T_t + T_{fi} T_a + T_t T_a = \cancel{2.00012 \cdot 10^{-6}}\ 9.2 \cdot 10^{-5}$

$T_3 = T_t + T_a + T_{fi} = 0.033$

$\rightarrow a_1^2 - 2 a_0 a_2 = b_1^2$

$a_2^2 - 2 a_1 a_3 + 2 a_0 a_4 = 0$

$$K_R = \frac{T_3^3 + T_1 - 2 T_2 T_3}{2K (T_2 T_3 - T_1)} \Rightarrow \boxed{K_R = 5.0277}$$

$$T_I = \frac{2 K_R K T_3}{1 + 2 K_R K} \Rightarrow \boxed{T_I = 0.03\ s}$$