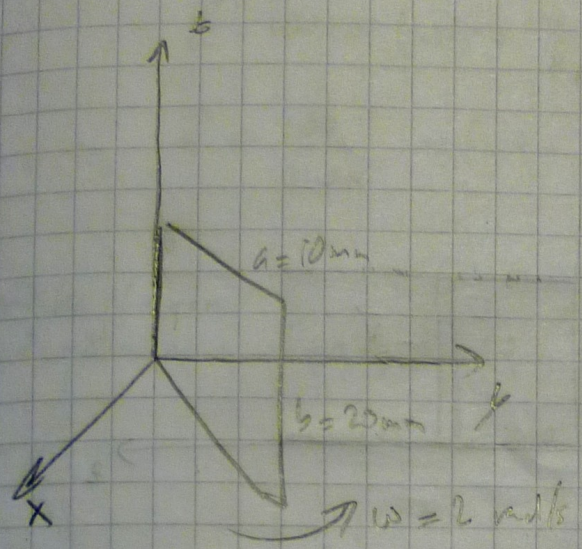
Školski rad

AUDITORNE ETF, 30. 5. 2011.

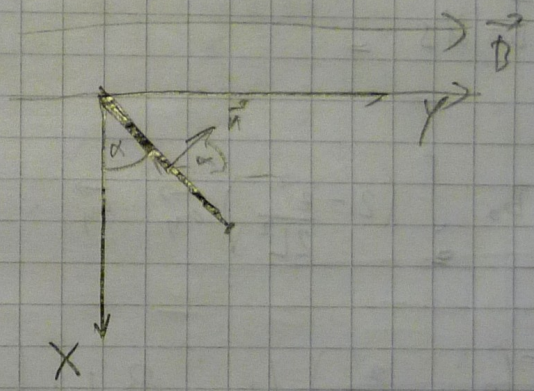
1.



$\vec{B} = 10\,\vec{a_y}\ \frac{mWb}{m^2}$

$i(t) = ?$

$R = 20\ m\Omega$



$\vec{n} = \cos\alpha\,\vec{a_y} - \sin\alpha\,\vec{a_x}$

$$\Phi = \int_S \vec{B}\,\vec{n}\,dS = 10\cdot 10^{-3}\cdot a\cdot b\cdot \underbrace{\vec{n}\cdot\vec{a_y}}_{\cos\alpha} = 2\cdot 10^{-6}\cos\alpha$$

$$\alpha = \alpha_0 + \omega t$$

$$u_{ind} = -\frac{d\Phi}{dt}$$

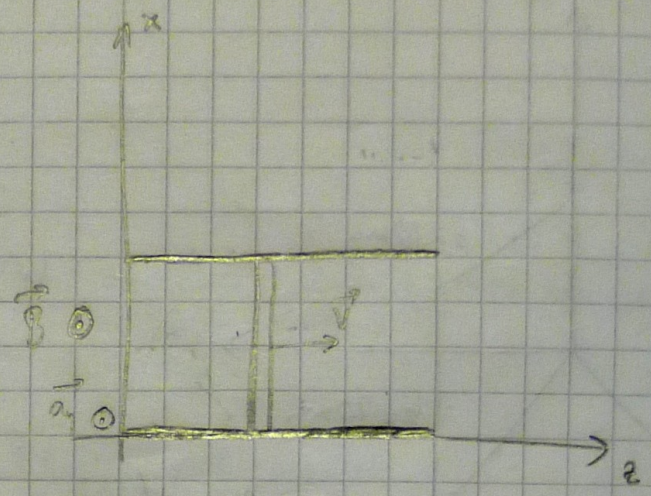
$$\Phi = 2\cdot 10^{-6}\cdot\cos(\alpha_0 + \omega t)$$

$$u_{ind} = +4\cdot 10^{-6}\sin(\alpha_0 + \omega t)$$

$$i(t) = \frac{u(t)}{R} = \underline{0.2\cdot 10^{-3}\sin(\alpha_0 + \omega t)\ A}$$

2.) Vodljivi štap duljine L giba se po vodljivim tračnicama u polju indukcije $\vec{B}$ prema slici.

$$\vec{B} = B_0\left(2 - \frac{z^2}{L^2}\right)\vec{a_y}$$



Odredite inducirani napon u štapu u trenutku $z = \frac{L}{2}$.

ze $\vec{n} = \vec{a_y}$

$$\phi = \int_S \vec{B}\vec{n}\, dS =$$

$$= B_0 \int_z \left(2 - \frac{z^2}{2L^2}\right)\vec{a_y}\cdot\vec{a_y}\cdot L\,dz =$$

$$= B_0 L \int_z \left(2 - \frac{z^2}{2L^2}\right) dz = B_0 L \cdot \left(2z - \frac{1}{2L^2}\cdot\frac{z^3}{3}\right)$$

$$z = v \cdot t$$

$$\phi(t) = B_0 L\left(2\cdot\vec{v}\cdot t - \frac{1}{6L^2}\cdot(\vec{v}\cdot t)^3\right)$$

$$U_{ind} = -\frac{d\phi}{dt} = -\frac{d\phi}{dz}\cdot\underbrace{\left(\frac{dz}{dt}\right)}_{v} =$$

$$= \underline{-B_0 L v\left(2 - \frac{z^2}{2L^2}\right)}$$

uvrstimo $z = \frac{L}{2} \Rightarrow U_{ind} = -B_0 L v \cdot \frac{15}{8}$

(3.) zadan je val:

$$\vec{E} = 100 \cos\left(\omega t + \frac{4}{3}\pi x\right) \vec{a_z} \ \frac{V}{m}$$

$$\vec{H} = \frac{100}{120\pi} \cos\left(\omega t + \frac{4}{3}\pi x\right) \vec{a_y} \ \frac{A}{m}$$

Odredite fazore $\underline{\vec{E}}$ i $\underline{\vec{H}}$. Kolika je prosječna snaga koja prolazi pravokutnikom određenim s $(0,0,0)$, $(0,2,0)$, $(0,2,2)$ i $(0,0,2)$, $\vec{n} = \vec{a_x}$?

$$e^{jx} = \cos x + j \sin x$$

$$e^{j\left(\omega t + \frac{4}{3}\pi x\right)} = e^{j\omega t} + e^{j\frac{4}{3}\pi x}$$

$$\underline{\vec{E}} = 100 \, e^{j\frac{4}{3}\pi x} \, \vec{a_z}$$

↑ amplituda ista

$$\underline{\vec{H}} = \frac{100}{120\pi} e^{j\frac{4}{3}\pi x} \, \vec{a_y}$$

- snaga ide preko Poyntingovog vektora

$$\vec{N} = \vec{E} \times \vec{H}$$

↳ za fazore: $\vec{N} = \vec{E} \times \vec{H}^*$

$$N_{sr} = \frac{1}{2} \frac{E_m^2}{|z|} = \frac{1}{2} \frac{100^2}{(120\pi)^2}$$

↳ z = valna impedancija, $z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

$$P = -N_{sr} \cdot S = \underline{-53\,W}$$

↳ normala na plohu suprotnog smjera u odnosu na tok snage

(4) Jakost električnog polja ravnog vala koji se širi u dielektriku s $\mu_r = 1$

Zadano je:

$$\vec{E} = 3\sin(\overbrace{2\cdot 10^8 t - 2x}^{\omega t - \beta x})\,\vec{a_y}$$

Smjer u kojem putuje val? → iz 2 ovaj u zagradama ⇒ putuje u smjeru $\vec{a_x}$

$\lambda = ?$ $\qquad \lambda = \frac{2\pi}{\beta} = \pi$

$\varepsilon_r = ?$

$-\beta x \quad \vec{a_x}$

$+\beta x \quad -\vec{a_x}$

$-\beta y \quad \vec{a_y}$

$+\beta y \quad -\vec{a_y}$

$\varepsilon_r \rightarrow$ iz

$$\boxed{c_v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{\omega}{\beta}}$$

$$c_v = 10^8$$

$$\varepsilon_r = \frac{c^2}{c_v^2 \cdot \mu_r} = 9$$

$\vec{H} = ?$

$$\vec{H} = \frac{1}{\omega\mu}\,\vec{\beta} \times \vec{E}$$

↳ ima smjer gibanja vala, dakle $\vec{a_x}$

$$\vec{H} = \frac{1}{2\cdot 10^8 \cdot 4\pi\cdot 10^{-7}}\, 2\,\vec{a_x} \times 3\sin(\omega t - \beta x)\,\vec{a_y} =$$

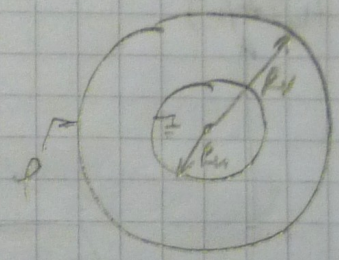
$$= 0.024 \sin(2\cdot 10^8 t - 2x)\,\vec{a_z}$$

(5.) Cilindrični kondenzator duljine 5 m ispunjen je dielektrikom s $\varepsilon_r = 6.7$.

Ako je unutrašnja elektroda uzemljena, a vanjska ima potencijal

$\varphi = 250 \sin(377t)$ V, odredite:

$\vec{E}(r = 5.5\,\text{mm})$ od osi



$R_u = 5.5$ mm

$R_v = 6$ mm

$\varphi = \ln r \cdot C_1 + C_2$

→ iz rubnih uvjeta

$$\varphi(r) = \frac{250}{\ln \frac{R_v}{R_u}} \ln\left(\frac{r}{R_u}\right) \sin\left(377\, t\right)$$

$$\vec{E} = -\frac{\partial \varphi}{\partial r} \vec{a_r} = -250 \sin(377t) \cdot \frac{1}{\ln\left(\frac{R_v}{R_u}\right)} \cdot \frac{1}{r}$$

$$\vec{E}(r = 0.0055) = -249310 \sin(377t)$$

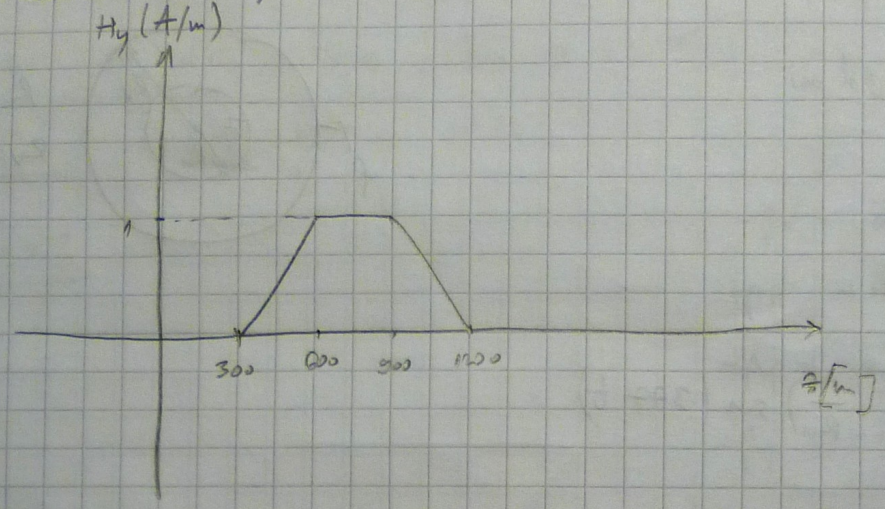
Odredite gustoću struje pomaka na $r = 5.2$ mm od osi.

$$\vec{J}_{pom} = \frac{\partial \vec{D}}{\partial t} = \varepsilon \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{r} \cdot 8.854 \cdot 10^{-12} \cdot 6.7 \cdot \left(-1371.2 \cdot 377 \cdot \cos(377t)\right)$$

$$= -\frac{30.67 \cdot 10^{-6}}{r} \cos(377t)$$

$$\Rightarrow \vec{J}(r = 0.0052) = -5.9 \cdot 10^{-3} \cos(377t)\, \vec{a_r}$$

(6.) Ravni val gibe se u +z smjeru u prostoru ($\mu_r = 1$, $\varepsilon_r = 4$). $E \to +x$. Prostorna promjena magnetskog polja (koje ima samo y-komponentu) zadana je slikom u trenutku $t = 1\,\mu s$



Odredite $E(t = -1\,\mu s,\ z = 0)$

$E(t = -1.5\,\mu s,\ z = 0)$.

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_r}} \cdot \underbrace{\sqrt{\frac{\mu_0}{\varepsilon_0}}}_{120\pi} = \sqrt{\frac{\mu_r}{\varepsilon_r}} \cdot 120\pi = 60\pi = 188.56$$

$$E_m = H_m \cdot Z = \underbrace{1}_{\text{sa slike}} \cdot 188.56 \ \frac{V}{m}$$

$$z_2 - z_1 = c_v (t_2 - t_1) = \frac{c}{\sqrt{\mu_r \varepsilon_r}} (t_2 - t_1) = 1.5 \cdot 10^8 \ \frac{m}{s} \cdot (t_2 - t_1)$$

$$z_2 = 1.5 \cdot 10^8 (1\,\mu s - t)$$

a) $t = -1\,\mu s \Rightarrow z_2 = 300$

$H = 0 \to E = 0$

b) $t = -1.5\,\mu s \to z = 375$

$H = 0.25 \to E = \frac{188.56}{4} = 47.13$

c) $t = -4\,\mu s \to z = 750$

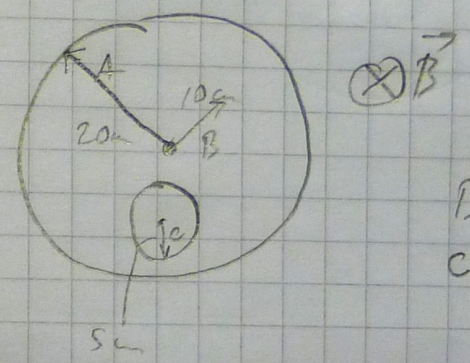
$H = 1 \to E = 188.56$

d) $t = -6\,\mu s \quad z = 1050$

$H = 0.5 \to E = 94.3$

(7) Sst s metelkou kovové kotouče leží v x-y rovině a nachází se v homogenním poli indukcí $B = 0.5\,T$ mimo stíní. Ve prostoru kde se kotouče nachází rotují a do ní osovou svíslou kovovou elektrickou spojeny, určete:

U_{AB} v $t = 2h\,15min\,30s$.



A – min
B – h
C – sec

r – dolyní kotouče

$$U_{ind} = \int_0^r \omega \cdot r \cdot B \cdot dr = \omega B \frac{r^2}{2}$$

$$\omega_{min} = \frac{2\pi}{3600} \qquad \omega_{sst} = \frac{2\pi}{12 \cdot 3600}$$

$$U_{AB} = B \cdot \left(\frac{2\pi}{3600} \cdot r_{min}^2 - \frac{2\pi}{12 \cdot 3600} \cdot r_{sec}^2 \right) \cdot \frac{1}{2} = 17.1\,\mu V$$