

# KOMPLEKSNA ANALIZA

Zadaci za vježbu

Konformna preslikavanja – II. dio

(32) A)  $G = \{0 < \operatorname{Re} z < \pi, \operatorname{Im} z > 0\}$ ,  $w = e^{iz}$

$w = e^{iz} = e^{ix+iy} = e^{ix}e^{-y} =$   
 $= e^{-y}(\cos x + i \sin x) =$   
 $= \underbrace{e^{-y} \cos x}_u + i \underbrace{e^{-y} \sin x}_v$

a...  $x=0, y \in (0, \infty)$   
 d...  $u = e^{-y} \rightarrow u \in (0, 1)$   
 $v = 0$

→ područje je: lijeve strane

b...  $y=0, x \in (0, \pi)$   
 c...  $u = -e^{-y} \rightarrow u \in (-1, 0)$   
 $v = 0$

$u^2 + v^2 = 1 \rightarrow$  polukružnica  $u^2 + v^2 = 1$

a...  $x=\pi, y \in (0, \infty)$   
 d...  $u = -e^{-y} \rightarrow u \in (-1, 0)$   
 $v = 0$

$w = 0$

$G^* = \{|w| < 1, \operatorname{Im} w > 0\}$

→ područje je: sljevice strane

35) B)  $G = \{ z \in \mathbb{C} : z > 0, -\pi < \operatorname{Im} z < \pi \}, \psi = e^y z$

$$w = \psi(z) = e^y (x + iy) = \frac{e^x e^{iy} - e^{-x} e^{-iy}}{2} =$$

$$= \frac{1}{2} \left( e^x (\cos y + i \sin y) - e^{-x} (\cos y - i \sin y) \right) =$$

$$= \frac{1}{2} \left( \cos y (e^x - e^{-x}) + i \sin y (e^x + e^{-x}) \right) =$$

$$= \cos y \frac{e^x - e^{-x}}{2} + i \sin y \frac{e^x + e^{-x}}{2} =$$

$$= \underbrace{\sinh x \cos y}_u + i \underbrace{\cosh x \sin y}_v$$

a.  $y = \pi, x \in (0, \infty)$   
 $z^* = u = -e^x \rightarrow u \in (-\infty, 0)$

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 $z^* = u = -e^x \rightarrow u \in (-\infty, 0)$   
 $v = 0$

b.  $x = 0, y \in (-\pi, \pi)$   
 $z^* = u = 0$   
 $v = \sin y \rightarrow v \in (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

c.  $y = \pi, x \in (0, \infty)$   
 $z^* = u = -e^x \rightarrow u \in (-\infty, 0)$   
 $v = 0$

$\Rightarrow$  područje je zbijave stope

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$\Rightarrow$  cijelo dijeljenje je prekriveno  
 $(-\infty, 0) \cup [0, 1]$

(37) A)  $S = \{ \operatorname{Im} z > 0 \}$ ,  $w = \ln z$ ,  $W(\mathbb{D}) = \frac{i\pi}{2}$

$$\begin{aligned} \ln(i) &= \ln|1| + i(\arg(i) + 2k\pi) = \\ &= i\frac{\pi}{2} + i \cdot 2k\pi = \frac{i\pi}{2} \Rightarrow \boxed{b=0} \end{aligned}$$

$w = \ln|z| + i\arg(z)$

$\rightarrow$  područje je s lijeve strane

$x = -\infty \Rightarrow \ln|-\infty| + i\arg(-\infty) = \infty + i \cdot \pi$   
 $x = -e \Rightarrow \ln|-e| + i\arg(-e) = 1 + i \cdot \pi$   
 $x = -1 \Rightarrow \ln|-1| + i\arg(-1) = 0 + i \cdot \pi$

$\rightarrow$  područje je s lijeve strane

$x = -\infty \Rightarrow \ln|-\infty| + i\arg(-\infty) = \infty + i \cdot \pi$   
 $x = -e \Rightarrow \ln|-e| + i\arg(-e) = 1 + i \cdot \pi$   
 $x = -1 \Rightarrow \ln|-1| + i\arg(-1) = 0 + i \cdot \pi$

$\lim_{x \rightarrow 0^-} \ln|x| = -\infty \Rightarrow x \rightarrow 0^- \Rightarrow -\infty + i \cdot \pi$

$\lim_{x \rightarrow 0^+} \ln|x| = -\infty \Rightarrow x \rightarrow 0^+ \Rightarrow -\infty + i \cdot 0$

$x = 1 \Rightarrow \ln|1| + i\arg(1) = 0 + i \cdot 0$   
 $x = e \Rightarrow \ln|e| + i\arg(e) = 1 + i \cdot 0$   
 $x = \infty \Rightarrow \ln|\infty| + i\arg(\infty) = \infty + i \cdot 0$

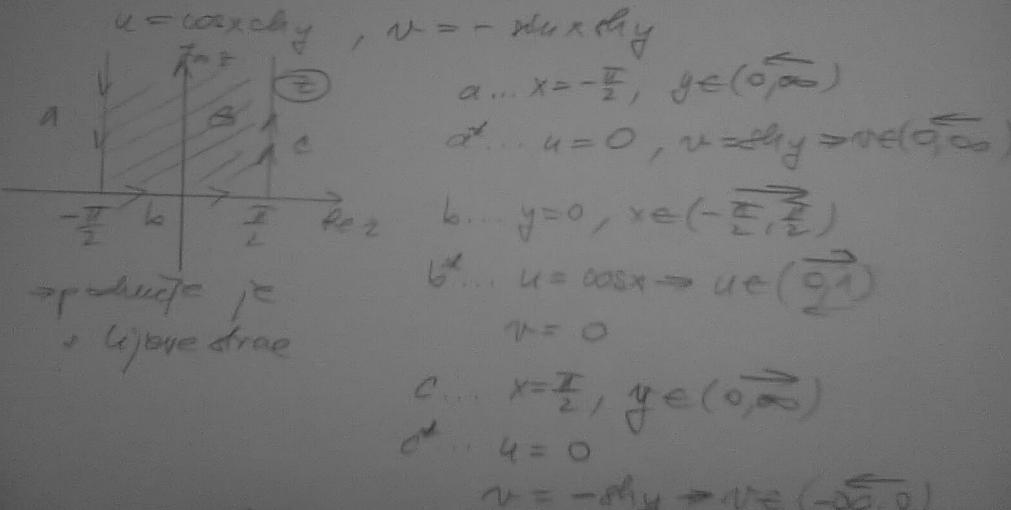
$\rightarrow$  pruga

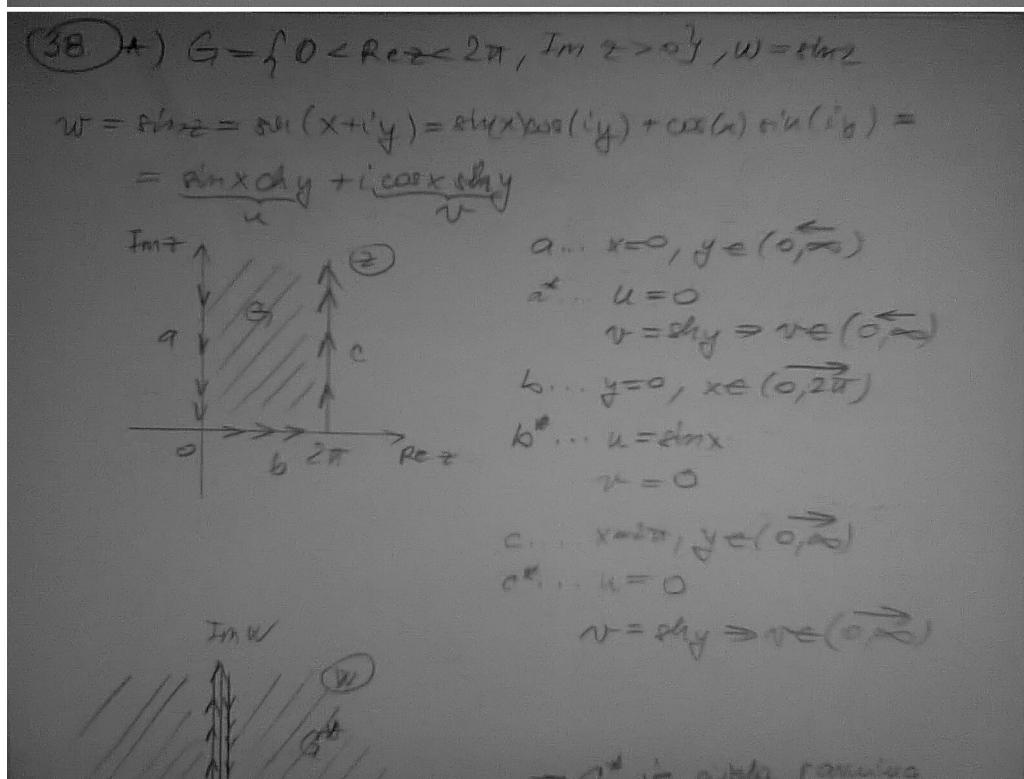
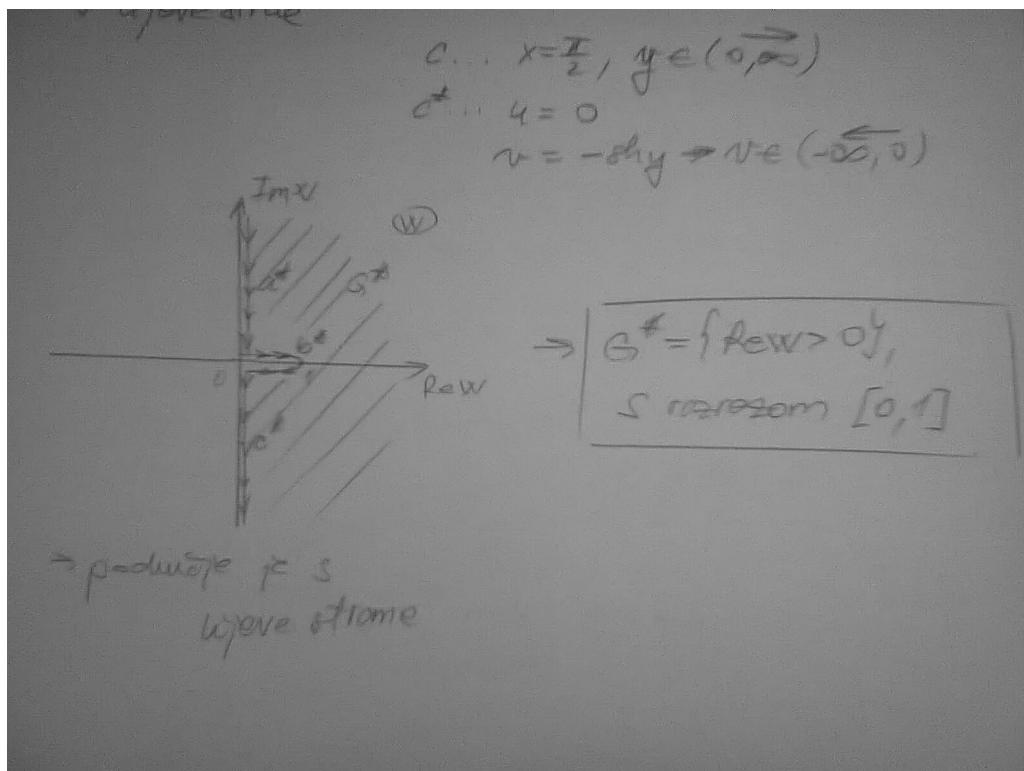
$$\begin{aligned}
 x = -e &\Rightarrow \ln|1-e| + i\arg(-e) = 1+i\pi \\
 x = -1 &\Rightarrow \ln|1-1| + i\arg(-1) = 0+i\pi \\
 \lim_{x \rightarrow 0^-} \operatorname{Im} z &= -\infty \Rightarrow z = 0 \Rightarrow -\infty + i\cdot\pi \\
 \lim_{x \rightarrow 0^+} \operatorname{Im} z &= -\infty \Rightarrow z = 0 \Rightarrow -\infty + i\cdot 0 \quad \text{pretki smr na } x > 0!!!
 \end{aligned}$$

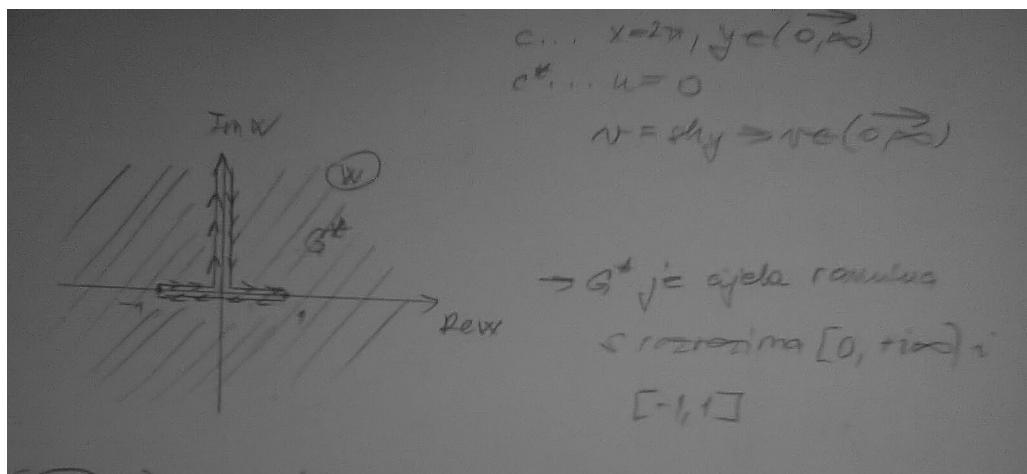
$$\begin{aligned}
 x = 1 &\Rightarrow \ln|1| + i\arg(1) = 0+i\cdot 0 \\
 x = e &\Rightarrow \ln|e| + i\arg(e) = 1+i\cdot 0 \\
 x = \infty &\Rightarrow \ln|\infty| + i\arg\infty = \infty + i\cdot 0
 \end{aligned}$$

3)  $G = \left\{ -\frac{\pi}{2} < \operatorname{Re} z < \frac{\pi}{2}, \operatorname{Im} z > 0 \right\}, w = \cos z$

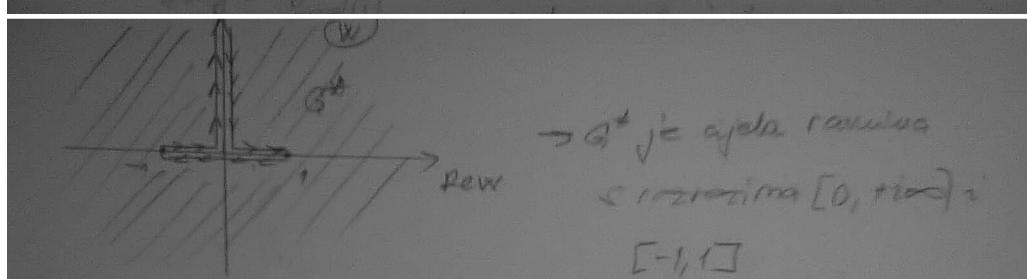
$$\begin{aligned}
 w = \cos z &= \cos(x+iy) = \cos x \cos(iy) - \sin x \sin(iy) = \\
 &= \cos x \cosh y - i \sin x \sinh y
 \end{aligned}$$



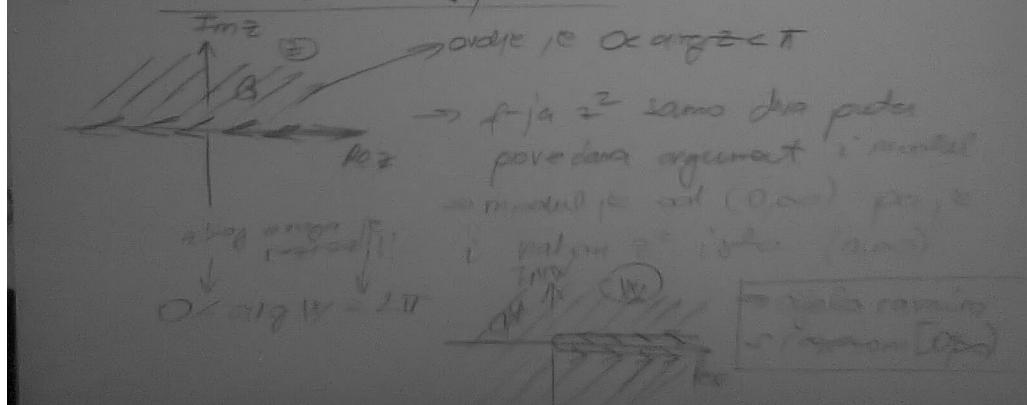




(35. B)  $G = \{\text{Im } z \geq 0\}, w = z^2$



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③ a)  $\Omega = \{0 < \operatorname{Re} z < \pi, \operatorname{Im} z > 0\}, W = \mathbb{H}$

$$w = f(z) = \frac{\sin x \cos y}{\sin^2 y - \sin^2 x} + i \frac{\sin x \sin y}{\sin^2 y - \sin^2 x}$$

a...  $x=0, y \in (0, \infty)$   
a...  $u=0$   
 $v = \operatorname{thy} \Rightarrow v \in (0, 1)$

b...  $y=0, x \in (0, \pi)$

b...  $u = \operatorname{tg} x = u \in (0, \infty) \cup (-\infty, 0)$

c...  $x=\pi, y \in (0, \infty)$

c...  $u=0$   
 $v = \operatorname{thy} \Rightarrow v \in (0, 1)$

b...  $y=0, x \in (0, \pi)$

b...  $u = \operatorname{tg} x = u \in (0, \infty) \cup (-\infty, 0)$

c...  $x=\pi, y \in (0, \infty)$

c...  $u=0$   
 $v = \operatorname{thy} \Rightarrow v \in (0, 1)$

③ b)  $\Omega = \left\{ y > \frac{\pi}{6} \right\}, W = \mathbb{H}$

