MI 12/13

1. DLTI: $\Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $\Gamma = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$

T – vrijeme uzorkovanja

a) za zadani proces: vrem. optimalan reg. po varijablama stanja

Dinamika procesa:

$$\Delta(z;\Phi) = \det\left\{\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}\right\}$$

$$= \det\left\{\begin{bmatrix} z-1 & -T \\ 0 & z-1 \end{bmatrix}\right\} = z^2 - 2z + 1 + T$$

$\alpha(z) = z^2$ – željena dinamika

$$\Rightarrow \Delta(z;\Phi - \Gamma \cdot K_{DB}) = z^2$$

$$\Phi - \Gamma \cdot K_{DB} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} \cdot \begin{bmatrix} K_1 & K_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} K_1 \cdot \frac{T^2}{2} & K_2 \frac{T^2}{2} \\ K_1 T & K_2 T \end{bmatrix}$$

$$= \begin{bmatrix} 1 - K_1 \frac{T^2}{2} & T\left(1 - K_2 \frac{T}{2}\right) \\ -K_1 T & 1 - K_2 T \end{bmatrix}$$

$$\det\left\{zI-\left[\phi-\Gamma K_{OB}\right]\right\}=\det\left\{\begin{bmatrix} z-1+K_1\frac{T^2}{2} & T\left(K_2\cdot\frac{T}{2}-1\right) \\ K_1 T & z+K_2T-1 \end{bmatrix}\right\}$$

$$= z^2 - z + zK_1\frac{T^2}{2} + zK_2T - K_2T + K_1K_2\frac{T^3}{2} - z + 1 - K_1\frac{T^2}{2} -$$

$$- K_1T^2\left(K_2\frac{T}{2}-1\right)$$

$$= z^2 + z\left[K_1\frac{T^2}{2}+K_2T-2\right] + 1 + K_1\frac{T^2}{2}\left[K_2T-1\right] - K_2T -$$

$$- K_1T^2\left(K_2\frac{T}{2}-1\right)$$

$$\det\left\{zI-\left[\phi-\Gamma K_{OB}\right]\right\}=z^2$$

$$\Rightarrow \quad K_1\frac{T^2}{2}+K_2T-2=0 \qquad (1)$$

$$1+K_1\frac{T^2}{2}\left[K_2T-1\right]-K_2T-K_1T^2\left(K_2\frac{T}{2}-1\right)=0 \qquad (2)$$

$$(1) \Rightarrow K_2=\frac{2}{T}-K_1\frac{T}{2}$$

$$\rightarrow (2) \Rightarrow 1+K_1\frac{T^3}{2}\cdot\left(\frac{2}{T}-K_1\frac{T}{2}\right)-\cancel{K_1\frac{T^2}{2}}-2+\cancel{K_1\frac{T^2}{2}}-K_1\frac{T^3}{2}\cdot\left(\frac{2}{T}-K_1\frac{T}{2}\right)+$$

$$+K_1T^2=0$$

$$1+\cancel{K_1T^2}-\cancel{K_1^2\frac{T^4}{4}}-2-\cancel{K_1T^2}+\cancel{K_1^2\frac{T^4}{4}}+K_1T^2=0$$

$$\Rightarrow \underline{K_1=\frac{1}{T^2}}$$

$$\Rightarrow \underline{K_2}=\frac{2}{T}-\frac{1}{2T}=\underline{\frac{3}{2T}}$$

$$\Rightarrow K_{DB} = \begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix}$$

b) Odredi $u(0)$ i $u(T)$ za $T_1 = 0{,}1\,s$ i $T_2 = 1\,s$ uz zadane poč. uvjete:

$$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$u(t) = r(t) - K_{DB} \cdot x(t)$$

$$r(t) = 0$$

I. $T_1 = 0{,}1\,s$

$$\boxed{K_{DB}\Big|_{T=T_1} = \begin{bmatrix} 100 & 15 \end{bmatrix}}$$

$$u(0) = -K_{DB} \cdot x(0)$$

$$u(0) = -\begin{bmatrix} \frac{1}{0{,}1^2} & \frac{3}{2 \cdot 0{,}1} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\underline{u(0)} = \begin{bmatrix} -100 & -15 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \underline{= -115}$$

$$u(T) = -K_{DB} \cdot x(T)$$

$$\boxed{x(t) = (A_{CL})^t \cdot x(0) \quad , \quad t = 0, 1, 2, \ldots}$$

$$\Rightarrow x(T_1) = (A_{CL})^1 \cdot x(0) = \begin{bmatrix} 1 - K_1 \frac{T_1^2}{2} & T_1\left(1 - K_2 \frac{T_1}{2}\right) \\ -K_1 T_1 & 1 - K_2 T_1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(T_1) = \begin{bmatrix} 1 - 100 \cdot \frac{0{,}1^2}{2} & 0{,}1 \cdot \left[1 - 15 \cdot \frac{0{,}1}{2}\right] \\ -100 \cdot 0{,}1 & 1 - 15 \cdot 0{,}1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(T_1) = \begin{bmatrix} \frac{1}{2} & \frac{1}{40} \\ -10 & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{21}{40} \\ -\frac{21}{2} \end{bmatrix}$$

$$\Rightarrow u(T_1) = -K_{DB} \cdot x(T_1)$$

$$\underline{u(T_1)} = \begin{bmatrix} -100 & -15 \end{bmatrix} \cdot \begin{bmatrix} \frac{21}{40} \\ -\frac{21}{2} \end{bmatrix} = \underline{105}$$

$\underline{\text{II. } T_2 = 1\ s}$

$$u(0) = -K_{DB} \cdot x(0)$$

$$\boxed{K_{DB}\Big|_{T=T_2} = \begin{bmatrix} 1 & \frac{3}{2} \end{bmatrix}}$$

$$u(0) = -\begin{bmatrix} 1 & \frac{3}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -2{,}5$$

$$x(T_2) = (A_{CL})^1 \cdot x(0)$$

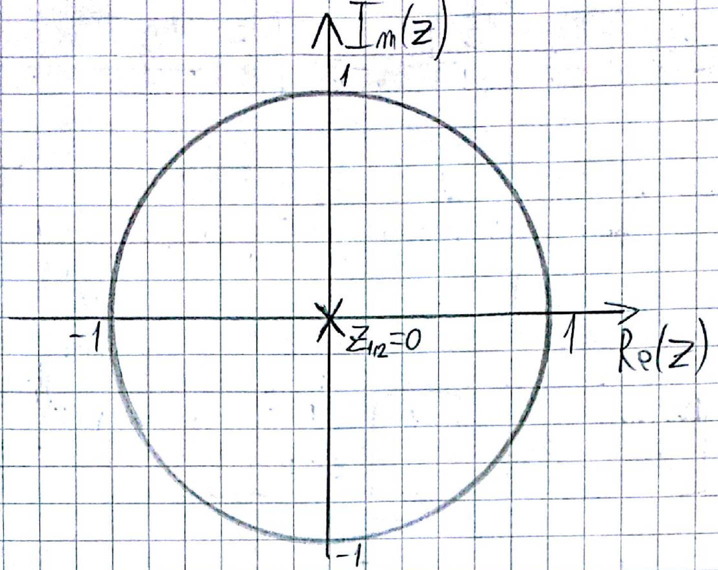
$$x(T_2) = \begin{bmatrix} 1 - 1 \cdot \frac{1^2}{2} & 1 \cdot \left(1 - \frac{3}{2} \cdot \frac{1}{2}\right) \\ -1 \cdot 1 & 1 - \frac{3}{2} \cdot 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(T_2) = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ -1 & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ -\frac{3}{2} \end{bmatrix}$$

$$\Rightarrow u(T_2) = -K_{DB} \cdot x(T_2)$$

$$\underline{u(T_2)} = \begin{bmatrix} -1 & -\frac{3}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{3}{4} \\ -\frac{3}{2} \end{bmatrix} = \underline{1{,}5}$$

c) Skiciraj polove projektiranog zatvorenog sustava upravljanja



d) Je li moguće projektirati regulator koji će sustav dovesti iz $x(0)=\begin{bmatrix}1\\1\end{bmatrix}$ u bilo koje drugo željeno stanje?

Matrica dohvatljivosti:

$$\mathcal{C}=\begin{bmatrix}\Gamma & \Phi\cdot\Gamma\end{bmatrix}$$

$$\mathcal{C}=\begin{bmatrix}\frac{T^2}{2} & \frac{3T^2}{2}\\ T & T\end{bmatrix}$$

$$\det\left(\mathcal{C}\cdot\mathcal{C}^T\right)=\det\left\{\begin{bmatrix}\frac{T^2}{2} & \frac{3T^2}{2}\\ T & T\end{bmatrix}\cdot\begin{bmatrix}\frac{T^2}{2} & T\\ \frac{3T^2}{2} & T\end{bmatrix}\right\}=$$

$$=\det\left\{\begin{bmatrix}\frac{T^4}{4}+\frac{9T^4}{4} & \frac{T^3}{2}+\frac{3T^3}{2}\\ \frac{T^3}{2}+\frac{3T^3}{2} & T^2+T^2\end{bmatrix}\right\}=\det\left\{\begin{bmatrix}\frac{5T^4}{2} & 2T^3\\ 2T^3 & 2T^2\end{bmatrix}\right\}$$

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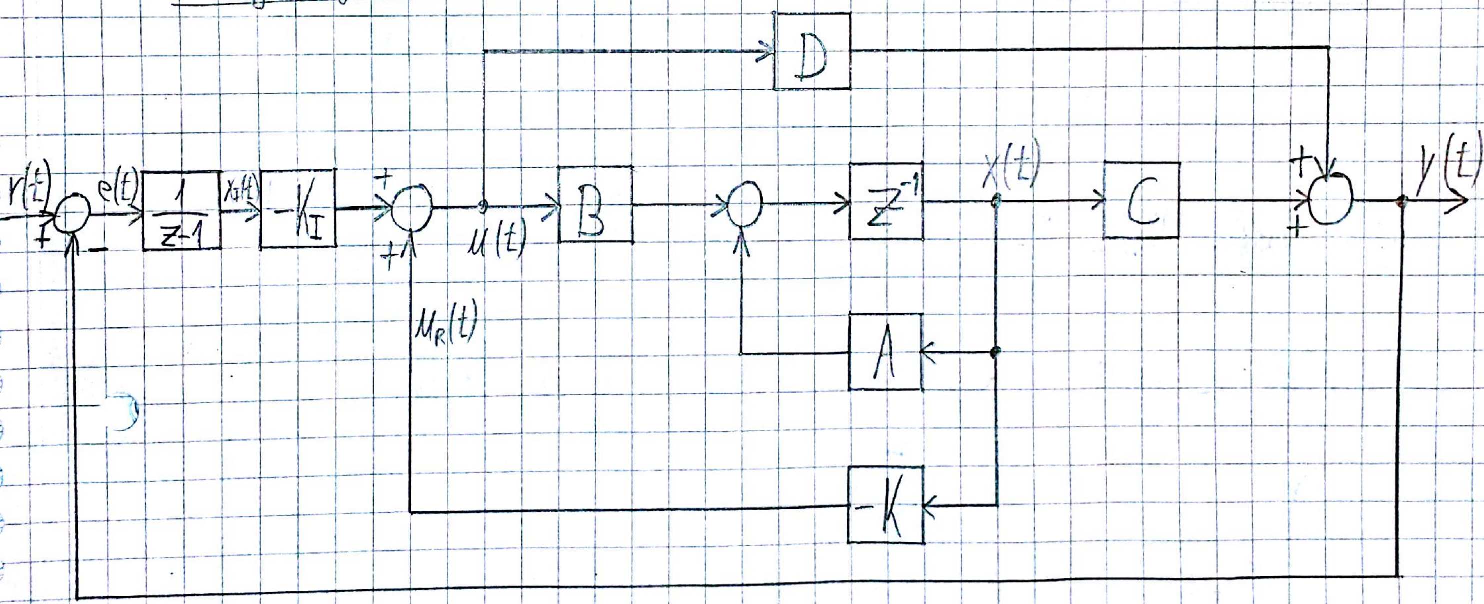
$$= 5T^6 - 4T^6 = T^6 \neq 0, \forall T \neq 0$$

$\Rightarrow$ sustav je dohvatljiv za svaki $T \neq 0$

(za DLTI sustave: dohvatljiv $\Rightarrow$ upravljiv)

$\Rightarrow$ odgovor na pitanje: da, moguće je, uz uvjet $T \neq 0$.

d) Skiciraj shemu diskretnog sustava upravljanja s regulatorom koji omogućava slijeđenje reference.



② CLTI proces:

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -5 & -6 & 0\end{bmatrix},\quad B=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad C=\begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

a) Odredi observer punog reda čija je dinamika određena polovima:

$$p_{1/2}=-10,\ p_3=-15$$

$$\Rightarrow \alpha_{OBS}(s)=(s+10)^2(s+15)=(s^2+20s+100)(s+15)=s^3+15s^2+20s^2+300s+100s+1500$$

$$\alpha_{OBS}(s)=s^3+35s^2+400s+1500$$ – željena dinamika

$$K_{OBS}=\begin{bmatrix}K_{OBS1} & K_{OBS2} & K_{OBS3}\end{bmatrix}^T$$

– sustav je zadan u kanoničkom upravljivom obliku (nije relevantno?)

$$\mathcal{O}=\begin{bmatrix}C\\CA\\CA^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\text{rang}(\mathcal{O})=3 \Rightarrow$ sustav je osmotriv, znači možemo projektirati rekonstruktor

$$\Delta(s; A_{OB}) = \det(s \cdot I_3 - A + K_{OBS} \cdot C)$$

$$A_{OBS} = A - K_{OBS} \cdot C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -6 & 0 \end{bmatrix} - \begin{bmatrix} K_{OBS1} \\ K_{OBS2} \\ K_{OBS3} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$A_{OBS} = \begin{bmatrix} -K_{OBS1} & 1 & 0 \\ -K_{OBS2} & 0 & 1 \\ -5-K_{OBS3} & -6 & 0 \end{bmatrix}$$

$$\Delta(s; A_{OBS}) = \det\left\{ \begin{bmatrix} s+K_{OBS1} & -1 & 0 \\ K_{OBS2} & s & -1 \\ 5+K_{OBS3} & 6 & s \end{bmatrix} \right\} = (s+K_{OBS1})(s^2+6) + 1 \cdot [s \cdot K_{OBS2} + 5 + K_{OBS3}]$$

$$= s^3 + s^2 K_{OBS} + 6s + 6K_{OBS1} + sK_{OBS2} + 5 + K_{OBS3}$$

$$\Delta(s; A_{OBS}) = s^3 + s^2 K_{OBS1} + s(6 + K_{OBS2}) + 6K_{OBS1} + K_{OBS3} + 5$$

$$\alpha_{OBS}(s) = s^3 + 35s^2 + 400s + 1500$$

$$\left.\begin{array}{l} \Rightarrow K_{OBS1} = 35 \\ K_{OBS2} = 394 \\ K_{OBS3} = 1285 \end{array}\right\} \Rightarrow K_{OBS} = \begin{bmatrix} 35 \\ 394 \\ 1285 \end{bmatrix}$$

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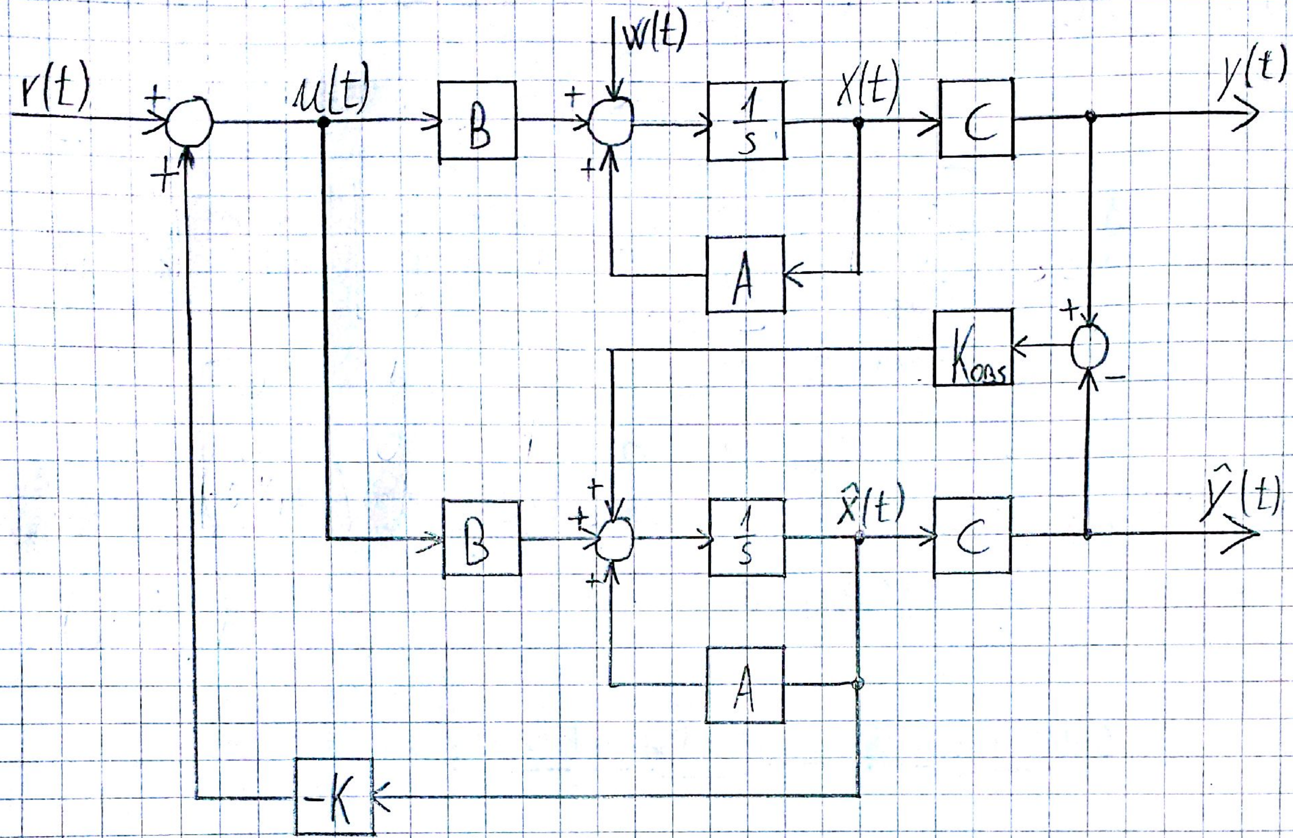
b) Stanja procesa dobivena observerom se koriste kao ulaz u regulator po varijablama stanja – skicirajte blok shemu takvog sustava



c) Odredi dinamiku sustava pod b). Kako observer djeluje na dinamiku regulatora?

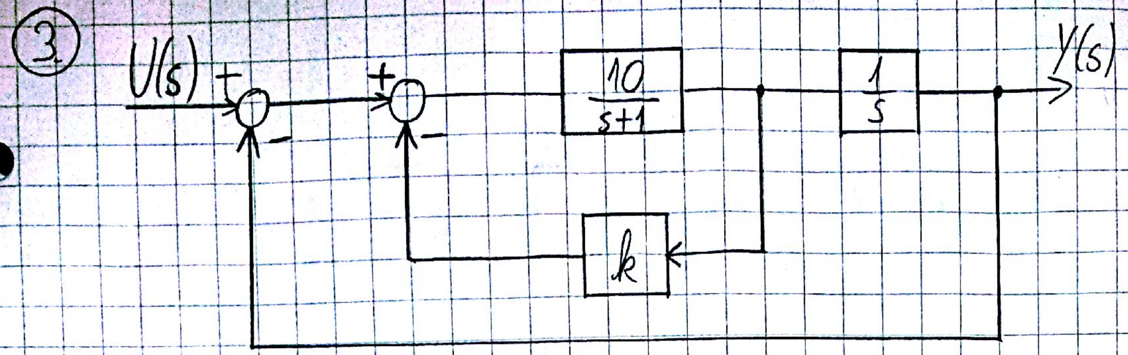
$$K = [K_1 \quad K_2 \quad K_3]$$

$$A_{CL} = A - BK = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5-K_1 & -6-K_2 & -K_3 \end{bmatrix}$$

$$\Delta(s; A_{CL}) = -5 - K_1$$ (to je valjda to, nije nam zadana željena dinamika zatvorenog kruga)

Činjenica da koristimo observer <u>ne utječe</u> na dinamiku regulatora

(SLSU predavanja, str. 144)



a) KMK u ovisnosti o $k>0$

$$G_0(s) = \frac{10}{s(s+1)+10ks+10}$$

IDENTIČAN PRIMJER: OGATA, „MODERN CONTROL ENGINEERING", STR. 389-391

- kar. jednadžba:

$$s^2+s+10ks+10=0$$

- supstitucija: $10k=K$

$$\Rightarrow s^2+s+Ks+10=0 \; / : (s^2+s+10)$$

$$1+\frac{Ks}{s^2+s+10}=0$$

$$z_1=0$$

$$p_{1/2}=-\frac{1}{2}\pm j\cdot\frac{\sqrt{39}}{2}\approx -0{,}5\pm j3{,}1225$$

- fazni uvjet: $\frac{Ks}{s^2+s+10}=\pm 180°(2k+1)$

$$\arg(s-z_1)-\arg(s-p_1)-\arg(s-p_2)=180°(2k+1)$$

$$\arg(s-0)-\arg\left(s+\frac{1}{2}-j\,3{,}1225\right)-\arg\left(s+\frac{1}{2}+j\,3{,}1225\right)=180°(2k+1)$$

$$s = \sigma + j\omega$$

$$\Rightarrow \arg(\sigma + j\omega) - \arg\left[\sigma + \frac{1}{2} + j(\omega - 3{,}1225)\right] - \arg\left[\sigma + \frac{1}{2} + j(\omega + 3{,}1225)\right]$$

$$= \pm 180^\circ (2k+1)$$

$$\arg(\sigma + j\omega) \pm 180^\circ(2k+1) = \arg\left[\sigma + \frac{1}{2} + j(\omega - 3{,}1225)\right] + \arg\left[\sigma + \frac{1}{2} + j(\omega + 3{,}1225)\right]$$

$$\operatorname{arctg}\left(\frac{\omega}{\sigma}\right) \pm 180^\circ(2k+1) = \operatorname{arctg}\left(\frac{\omega - 3{,}1225}{\sigma + \frac{1}{2}}\right) + \operatorname{arctg}\left(\frac{\omega + 3{,}1225}{\sigma + \frac{1}{2}}\right) \Big/ tg$$

$$\boxed{tg(x+y) = \frac{tg(x) + tg(y)}{1 - tg(x) \cdot tg(y)}}$$

$$\frac{\omega}{\sigma} = \frac{\frac{\omega - 3{,}1225}{\sigma + \frac{1}{2}} + \frac{\omega + 3{,}1225}{\sigma + \frac{1}{2}}}{1 - \frac{\omega - 3{,}1225}{\sigma + \frac{1}{2}} \cdot \frac{\omega + 3{,}1225}{\sigma + \frac{1}{2}}}$$

$$\frac{2\omega\left(\sigma + \frac{1}{2}\right)}{\left(\sigma + \frac{1}{2}\right)^2 - \left(\omega^2 - 3{,}1225^2\right)} - \frac{\omega}{\sigma} = 0$$

$$\frac{2\omega\left(\sigma + \frac{1}{2}\right) - \omega \cdot \left(\sigma + \frac{1}{2}\right)^2 + \omega \cdot \left(\omega^2 - 3{,}1225^2\right)}{\sigma \cdot \left(\sigma + \frac{1}{2}\right)^2 - \sigma \cdot \left(\omega^2 - 3{,}1225^2\right)} = 0$$

sređivanje brojnika $\Rightarrow \omega \cdot \left(\sigma^2 - 10 + \omega^2\right) = 0$

$\Rightarrow \omega = 0$ ili $\sigma^2 + \omega^2 = 10$

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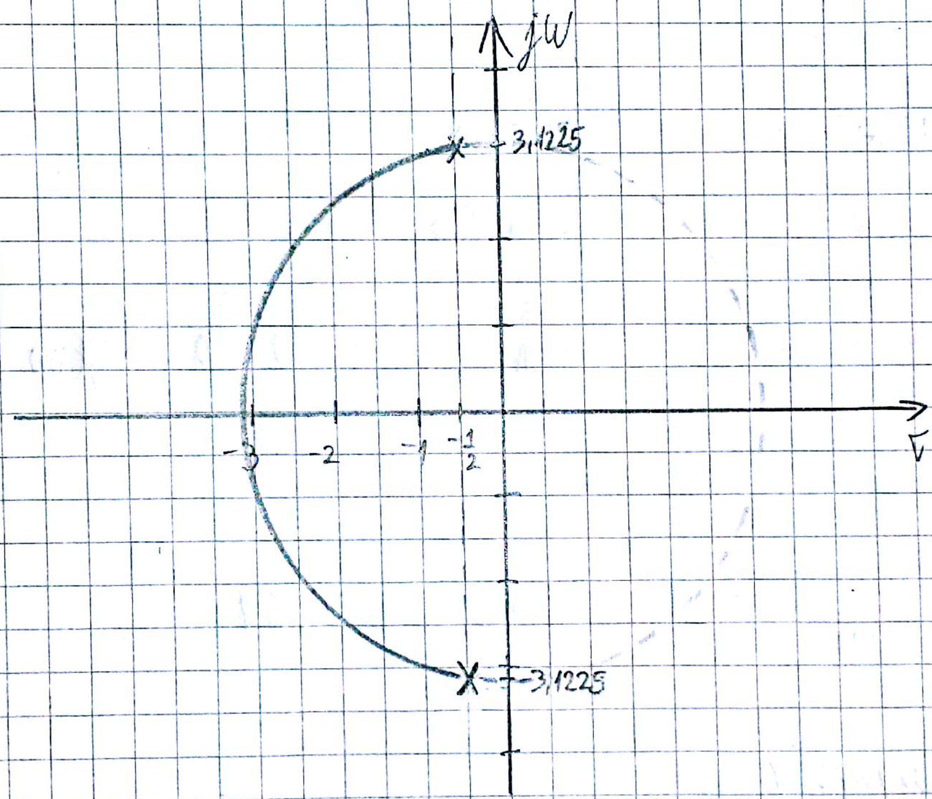
$\omega = 0 \Rightarrow$ cijela realna os

$K > 0$ – negativni dio realne osi

$\sigma^2 + \omega^2 = 10 \Rightarrow$ krug u $(0,0)$, radijusa $\sqrt{10}$

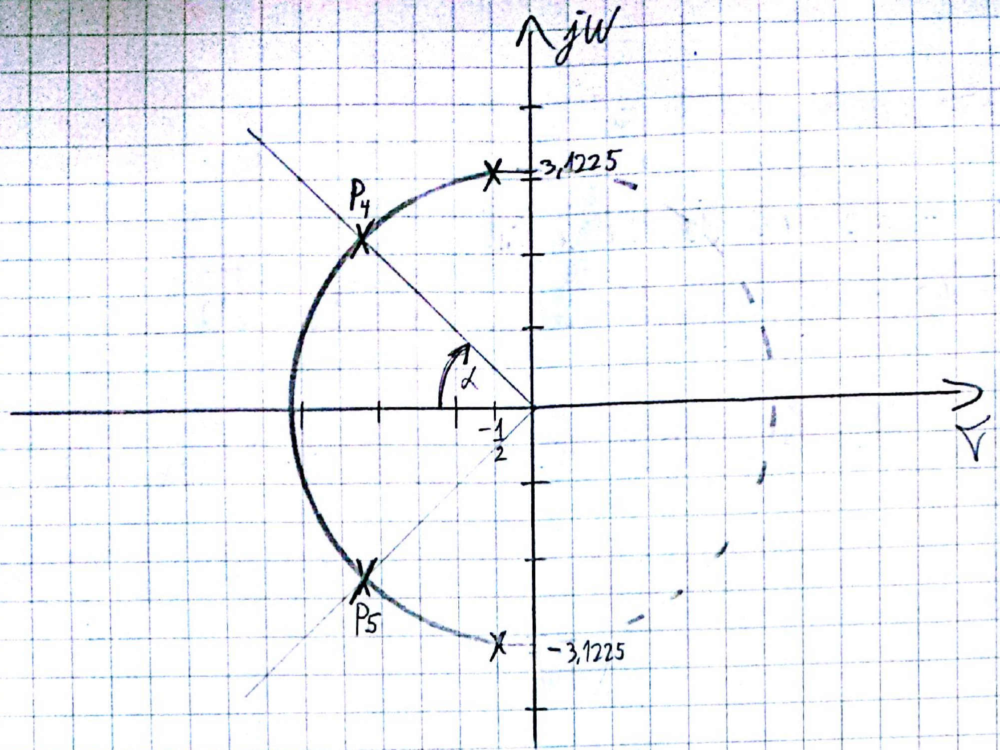
– dio kruga koji leži lijevo od kompleksnih polova odgovara pojačanju $K > 0$.



b) Za koji k polovi zatvorenog kruga imaju prigušenje $\zeta = 0{,}707$?

$\alpha = \arccos(\zeta)$

$\alpha = \arccos\left(\frac{\sqrt{2}}{2}\right) = 45°$



- normiramo kružnicu na jediničnu, tj. da ima radijus 1.
- znamo da je $\cos(45°) = \frac{\sqrt{2}}{2}$. To pomnožimo s $\sqrt{10}$ da dobijemo realni dio polova P_4 i P_5.

$$\Rightarrow \quad \sigma_{4,5} = -\frac{\sqrt{2}}{2} \cdot \sqrt{10} = -\sqrt{5} \approx -2{,}2361$$

- isto tako za imaginarni dio:

$$\Rightarrow \quad \omega_{4,5} = \pm \sin(45°) \cdot \sqrt{10} = \pm\sqrt{5} \approx \pm 2{,}2361$$

$$\Rightarrow \quad P_{4,5} = -\sqrt{5} \pm j\sqrt{5}$$

$$K(p_4) = \frac{|s^2+s+10|}{|s|}\Bigg|_{s=p_4} = \frac{|0+j10-\sqrt{5}-j\sqrt{5}+10|}{|-\sqrt{5}+j\sqrt{5}|} = \frac{10{,}9799}{3{,}1623}$$

$$K(p_4) = 3{,}4721 = K(p_5)$$

$$\Rightarrow k = \frac{K}{10} = 0{,}3472$$

c) Skicirajte položaj polova iz b) na KMK

d) Odredi $k \in \mathbb{R}$ za koji se sustav nalazi na rubu stabilnosti.

– kar. jedn.:

$$s^2 + s + 10ks + 10 = 0$$

$$s^2 + s(1 + 10k) + 10 = 0$$

Hurwitzov kriterij:

$$a_2 = 1, \quad a_1 = 1 + 10k_{kr} \quad a_0 = 10$$

1. $a_0 > 0 \Rightarrow 10 > 0$ ✓

2. $D_1 = \det(a_1) > 0 \Rightarrow 1 + 10k_{kr} > 0$

$$\Rightarrow \underline{\underline{k_{kr} > -0{,}1}}$$

Za $k > -0{,}1$ sustav je stabilan

Drugi način:

$s = j\omega_{kr}$ – rub stabilnosti $\Rightarrow$ nalazimo se na imaginarnoj osi

$$s^2 + s(1 + 10k_{kr}) + 10 = 0$$

$$-\omega_{kr}^2 + 10 + j(\omega_{kr} + 10\omega_{kr}k_{kr}) = 0$$

$$\Rightarrow -\omega_{kr}^2 + 10 = 0 \Rightarrow \omega_{kr} = \pm\sqrt{10}$$

$$\omega_{kr} + 10\omega_{kr} \cdot k_{kr} = 0 \Rightarrow \boxed{k_{kr} = -0{,}1}$$