Zadaci iz 2. ciklusa

Primjeri/Kulišić: Gravitacija

$\boxed{8.1}$

$m = 20\,kg$ $\quad R_Z = 6{,}4 \cdot 10^6\,m$

$F = 50\,N$

$a = 0{,}5\,\frac{m}{s^2}$

$a' = a \cdot 1{,}001$

$h = ?$

$$ma = F - \mu m g$$

$$ma' = F - \mu m g'$$

$$g = G\frac{m_Z}{R_Z^2} \qquad g' = G\frac{m_Z}{(R_Z + h)^2} = G\frac{m_Z}{R_Z^2}\left(\frac{R_Z}{R_Z + h}\right)^2 = g\left(\frac{R_Z}{R_Z + h}\right)^2$$

$$ma' = F - \mu m g\left(\frac{R_Z}{R_Z + h}\right)^2$$

$$\boxed{\mu = \frac{F - ma}{mg}}$$

$$ma' = F - (F - ma)\left(\frac{R_Z}{R_Z + h}\right)^2$$

$$\left(\frac{R_Z}{R_Z + h}\right)^2 = -\frac{ma' - F}{F - ma}$$

$$\left(\frac{R_Z}{R_Z + h}\right)^2 = \frac{F - ma'}{F - ma}$$

$$(R_Z + h)^2 = R_Z^2\,\frac{F - ma}{F - ma'}$$

$$\boxed{h = R_Z\left(\sqrt{\frac{F - ma}{F - ma'}} - 1\right) = 800{,}15\,m}$$

$\boxed{8.2}$

Satelit se giba blizu površine planeta gustoće ρ. Kolika je ophodno vrijeme satelita?

$$\rho = \frac{m_1}{V}, \quad V = \frac{4}{3}R^3\pi \Rightarrow \rho = \frac{m_1}{\frac{4}{3}R^3\pi}$$

$T = ?$

$$F = G\frac{m_1 m_2}{R^2}$$

$$\vec{F} + \vec{F}_{cp} = 0 \Rightarrow \vec{F} = -\vec{F}_{cp}$$

$$\boxed{F = F_{cp}} \qquad \omega = \frac{2\pi}{T}$$

$$F_{cp} = m_2\omega^2 R$$

$$G\frac{m_1 \cancel{m_2}}{R^2} = \cancel{m_2}\omega^2 R$$

$$G\frac{m_1}{R^3} = \omega^2$$

$$m_1 = \rho\frac{4}{3}R^3\pi$$

$$\frac{4\pi^2}{T^2} = G\frac{m_1}{R^3}$$

$$T^2 = \cancel{4}\pi^{\cancel{2}}\frac{\cancel{R^3}}{\rho\frac{\cancel{4}}{3}\cancel{R^3}\cancel{\pi}}\frac{1}{G}$$

$$\boxed{T = \sqrt{\frac{3\pi}{\rho G}}}$$

$\boxed{8.34}$

Satelit je lansiran s ekvatora i kreće se po kružnoj putanji u ekvatorijalnoj ravnini u smjeru vrtnje Zemlje. Potrebno je odrediti omjer između polumjera putanje satelita i polumjera Zemlje ako satelit periodički prolazi iznad mjesta lansiranja, s periodom od 2 dana.

$$\frac{T = 2T_z}{\frac{r}{R_z}}$$

$$T_z = \frac{2\pi}{\omega_z} \qquad (\omega - \omega_z)T_z = 2\pi$$

$$\omega_z = \frac{2\pi}{T_z} \qquad (\omega - \omega_z)T_z = \pi$$

$$\left(\omega - \frac{2\pi}{T_z}\right)T_z = \pi$$

$$\boxed{\omega = \frac{3\pi}{T_z}}$$

$$m\omega^2 r = G\frac{m_z m}{r^2}$$

$$g = G\frac{m_z}{R_z^2} \qquad \omega^2 r = gR_z^2\frac{1}{r^2}$$

$$Gm_z = gR_z^2 \qquad \frac{R_z^2}{r^2} = \frac{\omega^2 r}{g}$$

$$\frac{r^3}{R_z^2} = \frac{g}{\omega^2} \quad / : R_z$$

$$\frac{r^3}{R_z^3} = \frac{g}{R_z\omega^2}$$

$$\boxed{\frac{r}{R_z} = \sqrt[3]{\frac{g}{R_z\omega^2}} = \sqrt[3]{\frac{gT_z^2}{R_z 9\pi^2}} = 5{,}058}$$

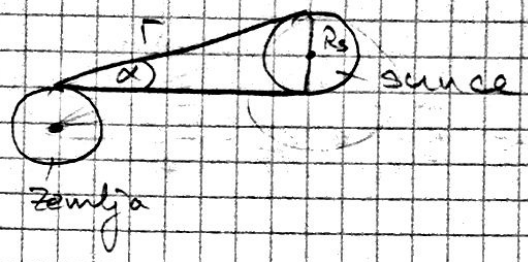
8.5

Kut pod kojim se Sunce vidi sa Zemlje iznosi približno 10^{-2} rad. Iz tog podatka valja naći odnos prosječne gustoće Zemlje i Sunca. Pretpostavlja se da su Zemlja i Sunce jednolike gustoće, a $g = 9{,}81\ \frac{m}{s^2}$.

$\alpha = 10^{-2}$ rad $\quad R_Z = 6{,}37 \cdot 10^6$ m

$g = 9{,}81\ \frac{m}{s^2}$

$R_S = 9{,}95 \cdot 10^8$ m



$$2R_S = r\alpha$$

$$\frac{\rho_Z}{\rho_S} = ?$$

$$F = G\frac{m_Z m_S}{r^2}$$

$$m_Z \omega_Z^2 R_Z = G\frac{m_Z m_S}{r^2}$$

$$\omega = \frac{2\pi}{T_Z} \qquad m = \rho V = \rho \frac{4}{3} R^3 \pi$$

$$m_Z \frac{4\pi^2}{T_Z^2} r = G\frac{m_Z m_S}{r^2}$$

$$g = G\frac{m_Z}{R_Z^2}$$

$$m_Z \frac{4\pi^2}{T_Z^2} = g\frac{R_Z^2 m_S}{r^3} \qquad \frac{R_S^3}{r^3} = \frac{\alpha^3}{2^3}$$

$$\frac{m_Z}{m_S} = \frac{g T_Z^2 R_Z^2}{4\pi^2 r^2}$$

$$\frac{\rho_Z \frac{4}{3} R_Z^3 \pi}{\rho_S \frac{4}{3} R_S^3 \pi} = \frac{g T_Z^2 R_Z^2}{4\pi^2 r^2}$$

$$\boxed{\frac{\rho_Z}{\rho_S} = \frac{g T_Z^2}{4\pi^2 r^3} R_S^3 = \frac{g \alpha^3 T_Z^2}{32\pi^2 R_Z} = 4{,}86}$$

Zadaci Kulišić: Gravitacija

8.1

$h = 10^6 m$ $\quad m_s = 2 \cdot 10^{39} kg$

$\gamma_z, \varphi, \gamma_s = ?$

$$\gamma = \frac{F}{m_2} = G \frac{m_1}{r^2}$$

$$g = G \frac{m_z}{R_z^2}$$

$$m_z = \frac{g R_z^2}{G}$$

$$\gamma_z(T) = G \frac{m_z}{(R_z + h)^2}$$

$$\gamma_z(T) = G \frac{\frac{g R_z^2}{G}}{(R_z + h)^2}$$

$$\gamma_z(T) = \frac{g R_z^2}{(R_z + h)^2} = 7,328 \frac{m}{s^2}$$

$$\varphi = -G \frac{m_z}{R_z + h} = \frac{g R_z^2}{R_z + h} = -5,4 \cdot 10^7 \frac{J}{kg}$$

$$m_z \omega_z^2 r = G \frac{m_z m_s}{r^2}$$

$$r^3 = G \frac{m_s}{\omega_z^2}$$

$$r = \sqrt[3]{G \frac{m_s}{4\pi^2} \cdot T_z^2} = 1,5 \cdot 10^{11} m$$

$$\gamma_s(T) = G \frac{m_s}{(r - h)^2} = 5,93 \cdot 10^{-3} \frac{m}{s^2}$$

$\boxed{8.2}$

$m = 1000\,kg \qquad R_z = 6{,}37 \cdot 10^6$

$h = 10^6\,m$

$v, T, E_k, E_p, E = ?$

$$g = G\frac{m_z}{R_z^2}$$

$$m_z = \frac{g R_z^2}{G}$$

$$F = G\frac{m_z m}{(R_z+h)^2}$$

$$m\omega^2(R_z+h) = \frac{g R_z^2 m}{(R_z+h)^2}$$

$$\omega = \sqrt{\frac{g R_z^2}{(R_z+h)^3}} = 1 \cdot 10^{-3}\,s^{-1}$$

$$\boxed{v = \omega(R_z+h) = 7{,}35\,\frac{km}{s}}$$

$$\omega = \frac{2\pi}{T}$$

$$\boxed{T = \frac{2\pi}{\omega} = 6{,}35 \cdot 10^3\,s}$$

$$E_k = \frac{mv^2}{2} = 2{,}7 \cdot 10^{10}\,J$$

$$E_p = m\varphi$$

$$\varphi = -G\frac{m_z}{(R_z+h)} = -54 \cdot 10^6\,J$$

$$E_p = -54 \cdot 10^9\,J$$

$$\boxed{E = E_k + E_p = -27 \cdot 10^9\,J}$$

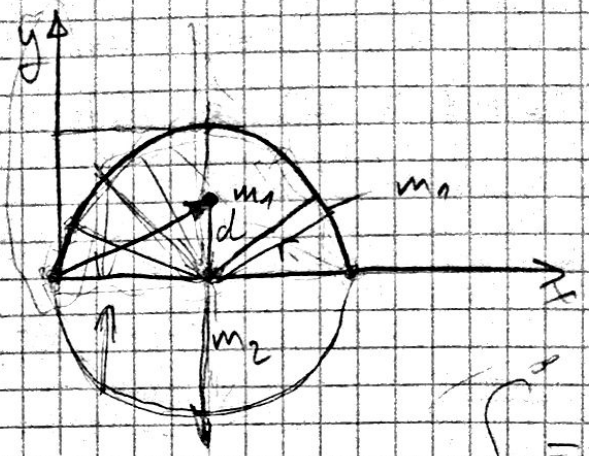
$\boxed{8.3}$

$r = 1\,m$

$m_1 = 10^{-2}\,kg$

$m_2 = 10^{-3}\,kg$

$F = ?$



$$\int = \sum_{i=0}^{\infty}$$

$$\boxed{l = r\pi}$$

$r_{cm} = ?$

$x_{cm} = r$

$y_{cm} = \frac{r}{2}$

$\vec{r}_{cm} = r\vec{i} + \frac{r}{2}\vec{j}$

$|\vec{r}_{cm}| = \sqrt{r^2 + \frac{r^2}{4}} = \frac{r}{2}\sqrt{5}$

$$dF = G\frac{dm_1 m_2}{r^2} \qquad m_1 = dm$$

$F = G$

$$dF = 2Gm_2\int_{\frac{r\pi}{2}}^{r\pi}\int_0^m dm\,\frac{dr}{r^2}$$

$$F = 2Gm_1m_2\int_0^m dm\int_{\frac{r\pi}{2}}^{r\pi}\frac{dr}{r^2} = 2Gm_1m_2\left(-\frac{1}{r\pi} + \frac{2}{r\pi}\right) =$$

$$= Gm_1m_2\left(\frac{2}{r\pi}\right) = G\frac{m_1m_2}{r\,\frac{\pi}{2}} = \underbrace{4{,}25\cdot 10^{-16}\,N}$$

Treba integrirati od polovice kružnog isječka pa do kraja i pomnožiti s dva $\Rightarrow$ 0 jer $\frac{1}{0} = \infty$

$\boxed{8.5}$

$m = 100\,kg$

$M_1 = 1{,}74 \cdot 10^{23}\,kg$

$M_2 = 2{,}85 \cdot 10^{24}\,kg$

$R_1 = 2{,}4 \cdot 10^{6}\,m$

$R_2 = 6{,}1 \cdot 10^{6}\,m$

$W = ?$

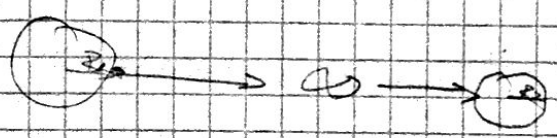
$$W_1 = E_{p2} - E_{p1} = +\int_{R_1}^{\infty} G \frac{m_1 m_2}{r^2}\,dr = +G m M_1 \left(\frac{1}{\infty} - \frac{1}{R_1}\right) =$$

$$= +G \frac{m M_1}{R_1}$$

$$W_2 = \int_{\infty}^{R_2} G \frac{m_1 m_2}{r^2}\,dr = G m M_2 \left(\frac{1}{R_2} - \frac{1}{\infty}\right) = G \frac{m M_2}{R_2}$$

$$\Delta W = W_2 - W_1 = G \frac{m M_2}{R_2} - G \frac{m M_1}{R_1} =$$

$$= G m \left(\frac{M_2}{R_2} - \frac{M_1}{R_1}\right) = 2{,}634 \cdot 10^{8}\,J$$



$\boxed{8.6}$

$$g = G\frac{M_Z}{R_Z^2}$$

$$\boxed{M_Z = \frac{g R_Z^2}{G}}$$

$M_Z = 81 \cdot M_M$ $\quad R_Z = 6{,}37 \cdot 10^6\,m$

$R_Z = 3{,}7\,R_M$

$v_1, v_2 = ?$

$v_1', v_2' = ?$

prva kozmička brzina:

$$\frac{m v_1^2}{R_Z} = mg$$

$$v_1 = \sqrt{R_Z g} = 7{,}9\,\frac{km}{s}$$

druga kozmička brzina

$$W = \int_{R_Z}^{\infty} G\frac{M_Z m}{r^2}\,dr = G\frac{M_Z m}{R_Z} = g m R_Z$$

$$g m R_Z = \frac{m v_2^2}{2}$$

$$v_2 = \sqrt{2 g R_Z} = 11{,}2\,\frac{km}{s}$$

Primjeri/Kulišić: Inercija

$\boxed{7.1}$

Promatrač A ispusti kamen s vrha nebodera. Promatrač B počinje se spuštati dizalom s vrha nebodera u trenutku kada je kamen ispušten. Traže se položaj, brzina i ubrzanje kamena u odnosu prema promatraču B, i to 3s nakon ispuštanja kamena ako se dizalo giba stalnom brzinom $5\,\frac{m}{s}$.

A – sustav S

B – sustav S'

$v_0 = 5\,\frac{m}{s}$

$t = 3s$

$x', v', a' = ?$

$$x' = x - v_0 t$$

$$x = \frac{1}{2}at^2 \qquad a = g$$

$$x = \frac{1}{2}gt^2$$

$$x' = \frac{1}{2}gt^2 - v_0 t = 29{,}15\,m$$

$$v' = v - v_0 = \frac{dx'}{dt} = gt - v_0 = 24{,}4\,\frac{m}{s}$$

$$a' = \frac{dv'}{dt} = g = 9{,}81\,\frac{m}{s^2}$$

$\boxed{7.3}$

Disk se vrti frekvencijom 30 okr./min oko vlastite vertikalne osi. Sitno je tijelo položeno na plohu diska na udaljenosti 20 cm od osi rotacije. Izračunajte najmanji faktor trenja potreban da sitno tijelo ne isklizne s diska?

$$f = 30 \frac{\text{okr.}}{\text{min}} = \frac{1}{2}\,\text{Hz}$$

$$\vec{F}_{cf} = \vec{F}_{tr}$$

$$m\omega^2 r = \mu m g$$

$$\omega^2 r = \mu g$$

$$\mu = \frac{\omega^2 r}{g}$$

$$\omega = 2\pi f = \pi\ \text{rad}\cdot\text{s}^{-1}$$

$$\mu = \frac{(2\pi f)^2\, r}{g} = 0{,}2$$

$\boxed{7.4}$

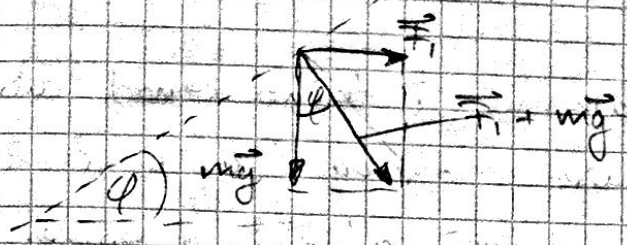
Približujući se semaforu, automobil se usporava decelerацijom $|\vec{a}| = 4\,\text{m/s}^2$. Pretpostavivši stolić i na njemu posudu s vodom u sustavu automobila, pod kojim će se kutom u odnosu prema horizontali postaviti razina vode u posudi?

$a = 4\,\frac{\text{m}}{\text{s}^2}$

$\varphi = ?$

$$F\sin\varphi - ma = 0$$

$$F\cos\varphi - mg = 0$$



$$F\sin\varphi - ma = F\cos\varphi - mg$$

$$F = \frac{ma}{\sin\varphi} \qquad F = \frac{mg}{\cos\varphi}$$

$$\frac{ma}{\sin\varphi} = \frac{mg}{\cos\varphi}$$

$$\frac{\sin\varphi}{\cos\varphi} = \frac{a}{g}$$

$$\operatorname{tg}\varphi = \frac{a}{g}$$

$$\varphi = 22{,}18^\circ$$

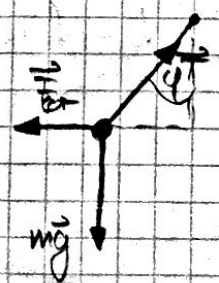
7.6

Na horizontalnoj ploči koja rotira konstantnom kutnom brzinom $\omega = 2\pi\ s^{-1}$ radijalno su postavljene tračnice po kojima se gibaju kolica konstantnom brzinom $v = 0{,}5\ \frac{m}{s}$. Na unutrašnjoj strani prednje stranice kolica obješeno je matematičko njihalo. Kolki kut zatvara to matematičko njihalo s vertikalom?

$\omega = 2\pi\ s^{-1}$

$v = 0{,}5\ \frac{m}{s}$

$\varphi = ?$



$$\vec{F}_{cor} + m\vec{g} + \vec{T} = 0$$

$$T\sin\varphi = 2mv\omega$$

$$T\cos\varphi = mg$$

$$\operatorname{tg}\varphi = 2\frac{v\omega}{g}$$

$$\varphi = 32{,}64^\circ$$

$\boxed{7.7}$

Čovjek stoji na rubu platforme i točno iznad ruba drži matematičko njihalo mase $m = 100g$ i duljine $l = 80cm$. Ako se pri rotaciji nit otkloni za $25°$, a pritom je kinetička energija njihala $1,25J$, kolki je polumjer platforme?

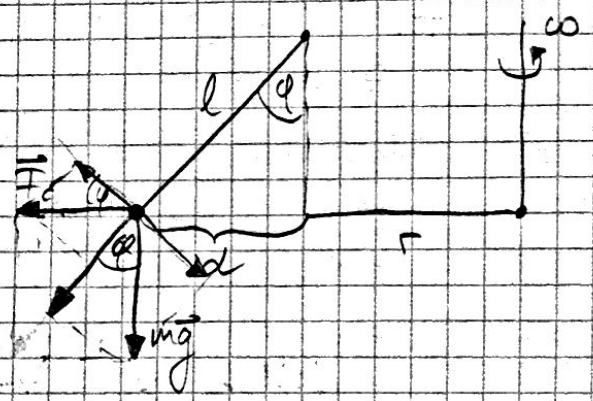
$m = 0,1kg$

$l = 0,8m$

$\varphi = 25°$

$E_k = 1,25J$

$R = ?$



$$F_c \cos\varphi = mg \sin\varphi$$

$$F_c = 2mv\omega \qquad E_k = \frac{mv^2}{2}$$

$$\omega = \frac{v}{R+d} \qquad v^2 = \frac{2E_k}{m}$$

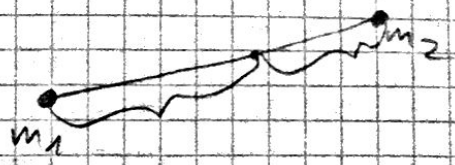
$$F_c = 2m\frac{v^2}{R+d} \qquad v = \sqrt{\frac{2E_k}{m}}$$

$$2\cancel{m}\frac{v^2}{R+d} = \cancel{m}g\,tg\varphi$$

$$R+d = \frac{v^2}{g\,tg\varphi} \qquad d = l\sin\varphi$$

$$R = \frac{v^2}{g\,tg\varphi} - l\sin\varphi = 5,13m$$

$$\frac{m_1}{m_2} = \frac{4}{17}$$



$$\frac{m_1}{r_1} = \frac{m_2}{r_2}$$

$$m_1 = m_2 \frac{4}{17}$$

$$\frac{m_1}{m_2} = \frac{r_1}{r_2} = \frac{4}{17}$$

$$\frac{13}{17} : \frac{17}{17} = \frac{13}{17}$$

$$\frac{m_2}{m_1} = \frac{r_2}{r_1} = \frac{17}{4}$$

$$m_2 = m_1 \frac{r_2}{r_1}$$

7.8 Staza za motociklističke akrobacije prozvana „zidom smrti" oblikovana je u vertikalni cilindar polumjera R i izrađena od materijala koji omogućuje relativno velik faktor trenja. Ako je faktor trenja $\mu = 0{,}75$, a polumjer zida $R = 10\,m$, izračunajte:

a) kojom se najmanjom brzinom motociklist mora vozati da ne klizne dolje?

b) motociklist i cilindar čine rotacijski referentni sustav koji se vrti kutnom brzinom ω oko vertikalne simetrične osi. Za koju najmanju frekvenciju vrtnje sustava, motociklist neće kliznuti prema dolje?

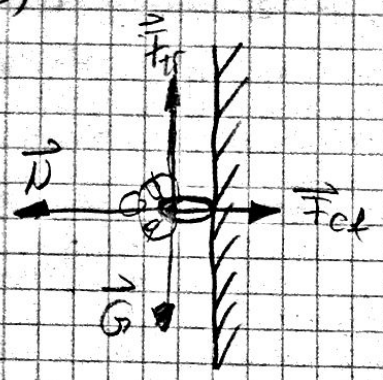
a)

$$\vec{F}_{tr} + m\vec{g} = 0 \qquad F_{tr} \geq mg$$

$$\vec{F}_{tr} = \mu N = \mu m \frac{v^2}{R} \qquad \mu m \frac{v^2}{R} \geq mg$$

$$v \geq \sqrt{\frac{Rg}{\mu}} = 11{,}44\ \frac{m}{s}$$

b)



$$\vec{N} + \vec{F}_{cf} = 0$$

$$\vec{F}_u + \vec{G} = 0$$

$$F_u = \mu N = \mu m \omega^2 R$$

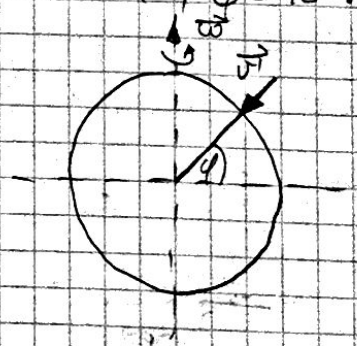
$$F_u \geq mg$$

$$\mu m \omega^2 R \geq mg$$

$$\omega \geq \sqrt{\frac{g}{\mu R}} = 1{,}144 \ \frac{rad}{s}$$

$$f = \frac{\omega}{2\pi} = 0{,}182 \ Hz$$

$\boxed{7.12}$ Kolika Coriolisova sila djeluje na tijelo koje slobodno pada s visine h na zemljopisnoj širini φ? Kolika je otklon i u kojem je smjeru? Potrebno je izračunati otklon za $h = 24$ m, $\varphi = 45°$.



$$\vec{F}_c = m\vec{a}_c = 2m\vec{v}' \times \vec{\omega}$$

$$a = 2\omega v' \cos\varphi$$

$$v' = gt$$

$$\frac{d^2y}{dt^2} = 2\omega v' \cos\varphi = 2\omega g t \cos\varphi \quad \Big\|\int$$

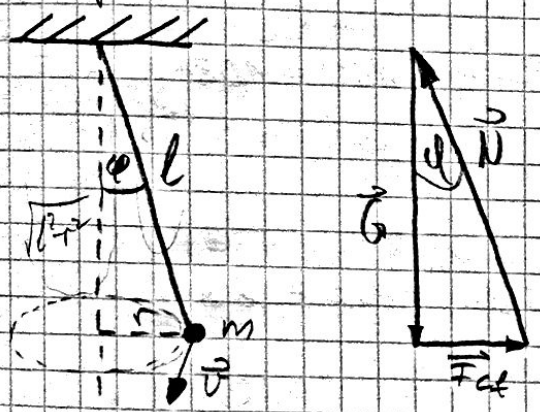
$$\frac{dy}{dt} = \omega g t^2 \cos\varphi \quad \Big\|\int$$

$$y = \frac{1}{3}\omega g t^3 \cos\varphi$$

$$t = \sqrt{\frac{2h}{g}}$$

$$y = \frac{1}{3}\omega g \left(\sqrt{\frac{2h}{g}}\right)^3 \cos\varphi = 0{,}002\,m$$

7.13 Stožasto se njihalo sastoji od kuglice mase $m = 1g$ obješene na nit duljine $l = 1m$, koja se giba jednoliko po kružnici polumjera $0{,}5\,m$. Primjenom D'Alembertova načela valja naći brzinu, period kruženja i napetost niti.



$$\vec{G} + \vec{F}_{cf} + \vec{N} = 0$$

$$tg\varphi = \frac{r}{\sqrt{l^2-r^2}} = \frac{m\frac{v^2}{r}}{mg}$$

$$\frac{v^2}{g} = \frac{r^2}{\sqrt{l^2-r^2}}$$

$$v = r\sqrt{\frac{g}{\sqrt{l^2-r^2}}} = 1{,}7\,\frac{m}{s}$$

$$N = \frac{G}{\cos\varphi} = \frac{mg}{\cos\varphi}$$

$$\cos\varphi = \frac{\sqrt{l^2-r^2}}{l}$$

$$N = \frac{Gl}{\sqrt{l^2-r^2}} = 0{,}0113\,N$$

$$v = \omega r$$

$$\omega = \frac{v}{r}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = 1{,}85\,s$$

Zadaci / Kulišić: Inercija

7.1

$m = 100\,kg$

a)

$\vec{F}' = \vec{F} + \vec{F}_1 = mg - (-ma_0)$

$a_0 = 0$

$F' = mg = 981\,N$

b) $a_0 = 1\,\frac{m}{s^2}$

$\vec{F}' = \vec{F} + \vec{F}_1 = mg - (-ma_0) = 981 + 100 = 1081\,N$

c) $a_0 = -1\,\frac{m}{s^2}$

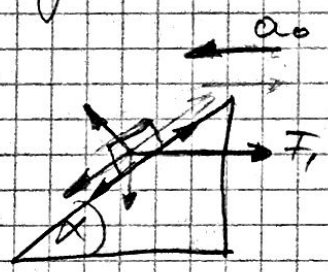
$\vec{F}' = \vec{F} + \vec{F}_1 = mg - (-ma_0) = 981 - 100 = 881\,N$

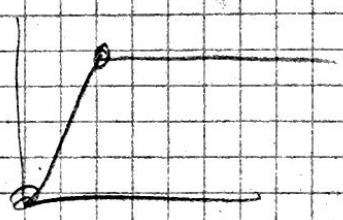
7.2

$\alpha = 30°$

$\mu = 0{,}36$

$a_0 = ?$





$ma = mg\sin\alpha - \mu mg\cos\alpha$

$m\vec{a} + m\vec{a}_0 = 0$

$\vec{F} + \vec{F}_1 = 0$

$\vec{F}_1 = -m\vec{a}_0$

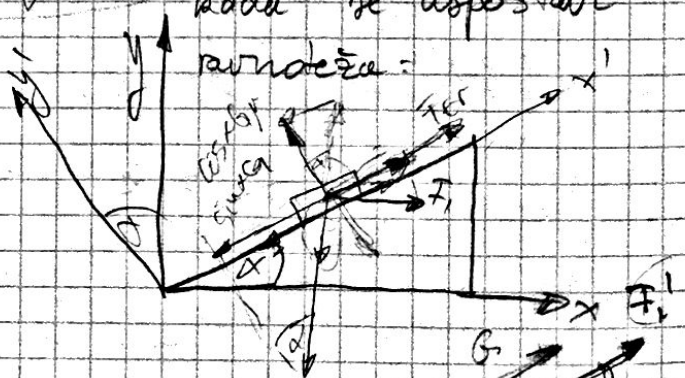
$ma - ma_0 = 0$

$a \leq a_0 \geq 1{,}85\,\frac{m}{s^2}$

$a_0 \geq 1{,}85$

donja granica

kada je uspostavi ravnoteža:



$\cos\alpha = \frac{k}{h}$

$h\cos\alpha = k$

$ma_0 = mg\cos\alpha + \mu mg\cos\alpha$

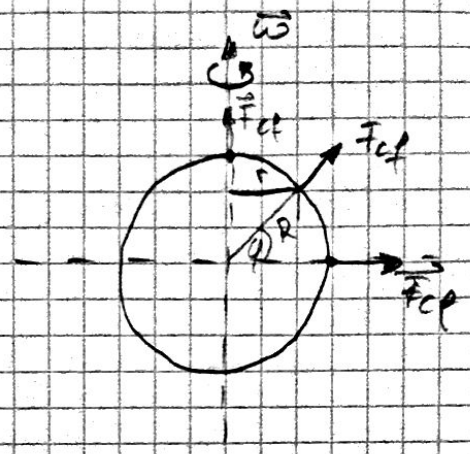
$a_0 = g\cos\alpha(1 + \mu) = 11{,}6\,\frac{m}{s^2}$

$11{,}6\ \frac{m}{s^2} > a_0 > 1{,}85\ \frac{m}{s^2}$

$\boxed{7.3}$

$m = 1kg$

a) na ekvatoru



$F_{cf} = m\omega^2 R =$

$= 0{,}034\ N$

b) $F_{cf} = m\omega^2 r$

$r = R\cos\varphi$

$F_{cf} = m\omega^2 R\cos\varphi = 9024\ N$

c) $F_{cf} = 0$

$\boxed{7.4}$

$m_1 = 0{,}18kg$

$m_2 = 0{,}22kg$

$m_D = 0{,}3kg$

$R = 0{,}1m$

$\alpha = ?$

$F = (m_2 + m_1)g$

$M = \alpha I = RF$

$\alpha \frac{1}{2} m_D R^2 = R(m_2 - m_1)g - Rm$

$\alpha = \frac{2(m_2 - m_1)g}{m_D R} =$

Prinjeri/Kulišić: Relativistička mehanika

9.1 U odnosu prema mirnom promatraču gibaju se dva koordinatna sustava u istom smjeru. Brzina prvog je $v_1 = 0{,}93c$, a drugog $v_2 = 0{,}9c$. Koji kut između dijagonala kvadrata koji miruje u drugom sustavu mjeri promatrač u prvom? Kvadrat je orijentiran tako da su mu dvije stranice paralelne sa smjerom gibanja.

$$v' = \frac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}} = 0{,}184c$$

za S_2 $a = b$, $\alpha = 90°$

za S_1 $a = \dfrac{a' + vt}{\sqrt{1 - \frac{v'^2}{c^2}}}$

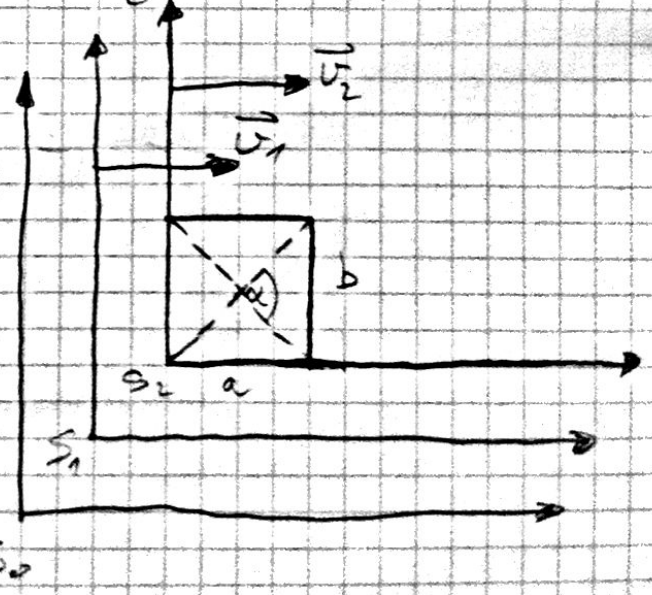
$$a(0) \cdot \sqrt{1 - \frac{v'^2}{c^2}} = a'$$

$$b' = b = a$$

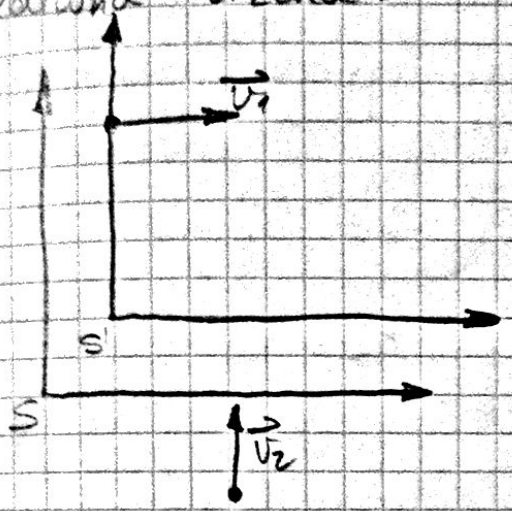
$$tg\left(\frac{\alpha'}{2}\right) = \frac{b'}{a'} = \frac{a}{a\sqrt{1 - \frac{v'^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}}$$

$$\frac{\alpha'}{2} = 45{,}5°$$

$$\alpha' = 91°$$



9.2 Dvije čestice se u mirnom sustavu gibaju brzinama $v_1 = 0{,}6c$, odnosno $v_2 = 0{,}7c$, tako da im se (prividno) putanje sijeku pod pravim kutom. Kolika je njihova relativna brzina?



2. čestica

u S $\quad u_x = 0 \quad u_y = v_2$ $\qquad$ u S' $\quad u_x = u_x' \quad u_y = u_y'$

$$u_x' = \frac{u_x - v_1}{\sqrt{1 - \frac{u_x v_1}{c^2}}} = -v_1$$

$$u_y' = \frac{v_2\sqrt{1-\beta^2}}{1 - \frac{u_x v_1}{c^2}} = v_2\sqrt{1-\left(\frac{v_1}{c}\right)^2}$$

$$u' = \sqrt{u_x'^2 + u_y'^2} = \sqrt{v_1^2 + v_2^2 - \frac{v_1^2 v_2^2}{c^2}} =$$

$$= 2{,}46 \cdot 10^8 \ \frac{m}{s}$$

9.3 U trenutku kada svemirski brod brzinom $v = 0{,}6c$ prolazi pokraj satelita smještenog u blizini Marsa, sa satelita je poslan radio-signal prema Zemlji. Signal stiže na Zemlju nakon $t_1 = 1250\,s$. Koliko traje put od Zemlje do satelita za posadu svemirskog broda?

$$d_z = c t_1$$

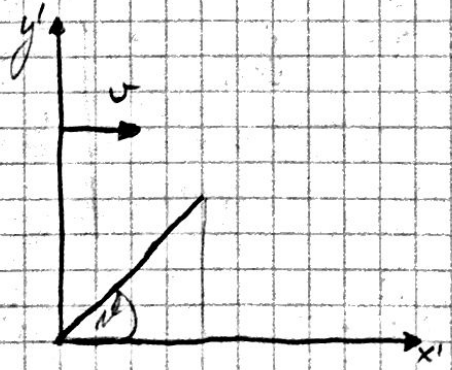
$$t = \frac{d}{v} = \frac{c t_1}{0{,}6c} = 2083\,s$$

$$t' = t\sqrt{1 - \frac{v^2}{c^2}} = 1667\,s$$

9.4 Inercijski sustav S' kreće se brzinom $v = 0{,}6c$ u odnosu prema inercijskom sustavu S. U sustavu S' nalazi se štap koji zatvara kut $\alpha' = 45°$ sa smjerom kretanja. Koji kut zatvara taj štap sa smjerom kretanja u sustavu S?

$v = 0{,}6c$

$\alpha' = 45°$



$S' \rightarrow tg\alpha' = \frac{\Delta y'}{\Delta x'}$

$S \rightarrow tg\alpha = \frac{\Delta y}{\Delta x}$

$\Delta x = \Delta x' \sqrt{1 - \beta^2}$

$$tg\alpha = \frac{1}{\sqrt{1-\beta^2}}\left(\frac{\Delta y}{\Delta x}\right)$$

$$\alpha = 51{,}34°$$

3.6 Izvedite vezu:

a) između ukupne energije i količine gibanja

b) između kinetičke energije i količine gibanja za relativističku česticu mase mirovanja m_0 i brzine v

c) Također valja pokazati da je $\vec{v} = \frac{c^2}{E}\vec{p}$

d) Kolika je količina gibanja i brzina protona kinetičke energije 563 GeV?

a)

$$E = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$$

$$\frac{E^2}{c^2} = \frac{m_0^2 c^2}{1-\frac{v^2}{c^2}} + m_0^2 c^2 - m_0^2 c^2 = m_0^2 c^2 + \frac{m_0^2 v^2}{1-\frac{v^2}{c^2}} =$$

$$= m_0^2 c^2 + p^2$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

b)

$$\left.\begin{aligned} E &= m_0 c^2 + E_k \\ E^2 &= p^2 c^2 + m_0^2 c^4 \end{aligned}\right\} \Rightarrow E^2 = m_0^2 c^4 + 2 m_0 c^2 E_k + E_k^2$$

c) $p = \frac{m_0 v}{\sqrt{1-\beta^2}}$ $\qquad E = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$

$$\boxed{E} = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = \frac{m_0 c^2}{\sqrt{1-\beta^2}} \cdot \frac{v}{v} = \frac{m_0 v}{\sqrt{1-\beta^2}} \frac{c^2}{v} = \boxed{p \frac{c^2}{v}}$$

$$\frac{E}{p} = \frac{c^2}{v} \quad ; \quad \vec{v} = \frac{c^2}{E}\vec{p}$$

9.8 Odredite energiju mirovanja, ukupnu energiju, brzinu i količinu gibanja elektrona kinetičke energije 4,6 MeV.

$m_e = 9,1 \cdot 10^{-31}$ kg

$E_K = 4,6$ MeV

$$E_0 = m_0 c^2 = 8,19 \cdot 10^{-14} J = 0,511 \text{ MeV}$$

$$E = E_0 + E_K = 8,18 \cdot 10^{-13} J = 5,11 \text{ MeV}$$

$$E_K = \frac{E_0}{\sqrt{1-\beta^2}} - E_0$$

$$\beta = \frac{\sqrt{E_K(2E_0 + E_K)}}{E_0 + E_K} = 0,995$$

$$v = 0,995c$$

$$p = \frac{1}{c}\sqrt{E_K(2E_0 + E_K)} = 2,7 \cdot 10^{-21} \text{ kg m s}^{-1}$$

9.12 Čestica koja je masa m_0 kreće se duž osi x tako da je njezin položaj u svakom trenutku zadan izrazom $x = \sqrt{b^2 + c^2t^2}$, pri čemu je b neka konstanta. Čemu je jedna sila pod djelovanjem koje se čestica giba ovako?

$$F = \frac{dp}{dt} = \frac{d}{dt}\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = m_0\left[\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} + \frac{v^2}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\right]\frac{dv}{dt}$$

$$v = \frac{dx}{dt} = \frac{c^2 t}{\sqrt{b^2 + c^2t^2}} \qquad 1 - \frac{v^2}{c^2} = \frac{b^2}{b^2 + c^2t^2}$$

$$c^2 - v^2 = \frac{c^2 b^2}{b^2 + c^2t^2}$$

$$v^2 = \frac{c^2(b^2 + c^2t^2) - c^2b^2}{b^2 + c^2t^2}$$

$$v^2 = \frac{c^4t^2}{b^2 + c^2t^2}$$

Zadania / Kulišić : Relativistika

9.1

$l = 100\,m$

$\Delta l = 99{,}999\,m$

v

$v = ?$

$$\Delta l = l\sqrt{1-\frac{v^2}{c^2}}$$

$$1-\frac{v^2}{c^2} = \left(\frac{\Delta l}{l}\right)^2$$

$$v^2 = \left(1-\left(\frac{\Delta l}{l}\right)^2\right)c^2$$

$$v = c\sqrt{1-\left(\frac{\Delta l}{l}\right)^2}$$

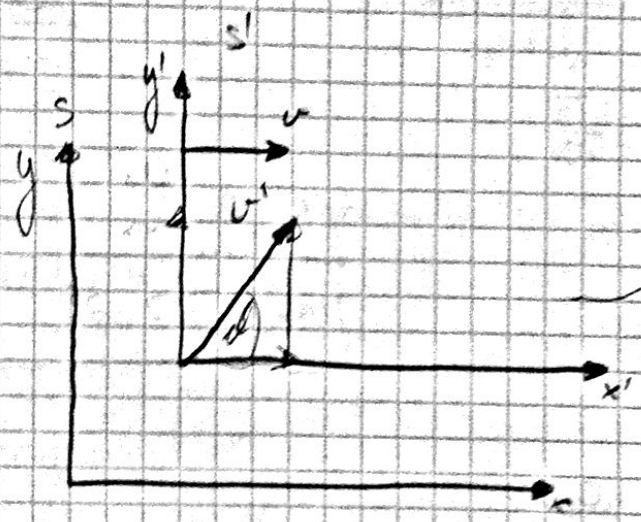
$$v = 1{,}34\cdot 10^6\ \frac{m}{s}$$

9.2

$S \to v = 0{,}6c\ ,\ \alpha' = 60°$

$v' = 0{,}1c$

$\alpha = ?$



S'

$$v'\cdot\cos\alpha' = v_x' = 0{,}05c$$

$$v_y' = \frac{\sqrt{3}}{20}c$$

$$\operatorname{tg}\alpha' = \frac{v_y'}{v_x'}$$

S

$$u_x = \frac{65}{103}c$$

$$u_y = 0{,}0673c$$

$$\operatorname{tg}\alpha = \frac{u_y}{u_x}$$

$$\alpha = 6{,}09°$$

$$u_x' = \frac{u_x - v}{1-\frac{v u_x}{c^2}}$$

$$u_x - u_x'\frac{v u_x}{c^2} = u_x - v$$

$$u_x\left(1 + \frac{u_x' v}{c^2}\right) = u_x' + v$$

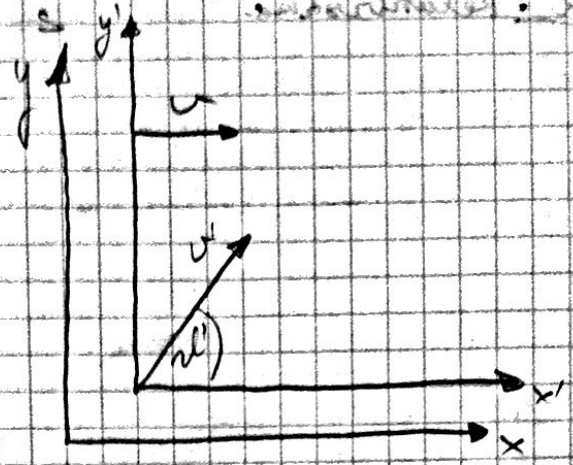
$$u_x = \frac{u_x' + v}{1 + \frac{v u_x' v}{c^2}} = \frac{65}{103}c$$

$$u_y' = \frac{u_y\sqrt{1-\beta^2}}{1-\frac{v u_x}{c^2}}$$

$$u_y'\left(1-\frac{v u_x}{c^2}\right) = u_y\sqrt{1-\beta^2}$$

$$u_y = \frac{u_y'\left(1-\frac{v u_x}{c^2}\right)}{\sqrt{1-\beta^2}} = 0{,}0673c$$

2a. $\underline{v' = c}$



$v_x' = v' \cos\alpha' = \frac{1}{2} c$

$v_y' = \frac{\sqrt{3}}{2} c$

$$\boxed{tg\,\alpha' = \frac{v_y'}{v_x'}}$$

$$v_x' = \frac{v_x - v}{1 - \frac{v_x v}{c^2}}$$

$$v_x' - v_x' \frac{v_x v}{c^2} = v_x - v$$

$$v_x' + v = v_x\left(1 + \frac{v_x' v}{c^2}\right)$$

$$v_x = \frac{v_x' + v}{1 + \frac{v_x' v}{c^2}} = \frac{11}{13} c$$

$$v_y' = \frac{v_y \sqrt{1 - \beta^2}}{1 - \frac{v v_x}{c^2}}$$

$$v_y = \frac{v_y' \left(1 - \frac{v v_x}{c^2}\right)}{\sqrt{1 - \beta^2}}$$

$$v_y = \frac{4\sqrt{3}}{13}$$

$$tg\,\alpha = \frac{v_y}{v_x}$$

$$\boxed{\alpha = 32{,}2^\circ}$$

3.3

$v_1 = 0{,}8c$

$v_2 = 0{,}6c$

$t_1 = 1\,s$

$t_2 = ?$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \beta^2}}$$

$$v' = \frac{v_2 - v_1}{1 - \frac{v_2 v_1}{c^2}} = \frac{0{,}2c}{1 - 0{,}48} = 0{,}385c$$

$$\Delta t = \frac{\Delta t_1}{\sqrt{1 - \frac{v'^2}{c^2}}} = 1{,}084\,s$$

34

$t = 0$

$v = \frac{3}{5}c$

$v_{Lt} = 7c$

$t_L = 32$

$t_L', t_D', t_D = ?$

Pretpostavimo da jedan promatrač miruje dok se drugi gibe od njega relativnom brzinom v

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\beta^2}}$$

$$t_D = \frac{t_L}{\sqrt{1-\frac{v^2}{c^2}}} = 40 \text{ dana}$$

$$s = v\, t_L = 7 \cdot 3 \cdot 10^8 \cdot 32 \cdot 24 \cdot 60 \cdot 60 = 5{,}81 \cdot 10^{15} \text{ m}$$

$$t_L' = \frac{s}{v'} \qquad v' = v_{Lt} - v = 6{,}4c$$

$$t_L' = \frac{s}{v'} = 35 \text{ dana}$$

vrijeme dolaska taliona do opažača D je kraće
$\Rightarrow$ kontrakcija vremena

$$\Delta t_L' = \frac{\Delta t_D'}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow \text{rel. brzina gibanja opažača L i D}$$

$$\Delta t_D' = \Delta t_L' \sqrt{1-\frac{v^2}{c^2}}$$

$$\Delta t_D' = 28 \text{ dana}$$

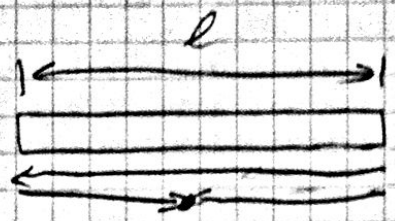
$\boxed{9.5}$

$l = 70\,m$

$v = \frac{c}{2}$

$v_1 = \frac{3}{4}c$

$d = ?$



$$\Delta l = l\sqrt{1-\frac{v^2}{c^2}} = 60{,}62\ m$$

$$v_{\check{c}}' = \frac{v_1 - v}{1-\frac{v_1 v}{c^2}} = 0{,}4c$$

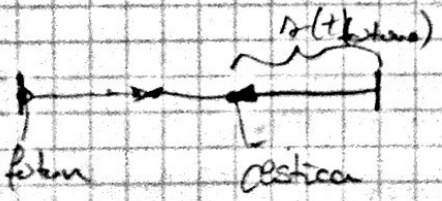
relativna brzina čestice (masivne) naspram promatrača

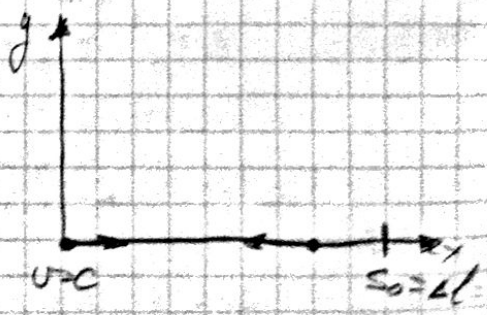
$v_{fotona} = c$

relativna brzina fotona naspram promatrača je $c \Rightarrow$ 1. postulat posebne relativnosti

$$t'_{fotona} = \frac{\Delta l}{c} = 0{,}2\cdot 10^{-6}\,s$$ → foton prijeđe Δl

$$s_0(t'_{fotona}) = v_{\check{c}}'\cdot t'_{fotona} = 24{,}248\ m$$





$$c\cdot t = v_{\check{c}}'\cdot t + s_0$$

$$t = \frac{s_0}{(c - v_{\check{c}}')} = 0{,}135\cdot 10^{-6}\,s$$

$$\boxed{d = ct = 40{,}41\,m}$$ → tražena udaljenost od kraja cevi

9.7

$t = 12\,min = 720\,s$ $\quad m_n = 1{,}674 \cdot 10^{-27}\,kg$

$E = 1\,GeV$

$s = ?$

$$E = E_0 + E_K$$

$$E_K = E - E_0 = \gamma m_n c^2 - m_n c^2 = m_n c^2 (\gamma - 1)$$

$$E_K = \frac{m_n v^2}{2} = m_n c^2 \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$$

$$E_K = E - E_0$$

$$E - E_0 = m_n c^2 \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E - E_0}{m_n c^2} + 1$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E - E_0 + m_n c^2}{m_n c^2}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{m_n c^2}{E}\right)^2$$

$$v^2 = \left[1 - \left(\frac{m_n c^2}{E}\right)^2\right] \cdot c^2$$

$$v = c\sqrt{1 - \left(\frac{m_n c^2}{E}\right)^2}$$

$$v = 0{,}336\,c$$

$$s = v \cdot t = 7{,}27 \cdot 10^{10}\,m$$

$\boxed{9.8}$

$v_1 = 0{,}57c$ $\quad m_e = 9{,}109 \cdot 10^{-31}\,kg$

$v_2 = 0{,}99c$

$U = \frac{\Delta E}{Q}$

$Q = e = 1{,}602 \cdot 10^{-19}\,C$

$U = ?$

$\Delta E = m_e c^2$

Primjeri/Kuliščić: Statika fluida

$\boxed{10.1}$ Na kojoj je dubini pod vodom polumjer mjehura zraka dvostruko manji ($r' = \frac{r}{2}$) od onog pri površini ($p_{pov} = 10^5\,Pa$). (Neka se računa sa stalnom temperaturom i napetost površine vode neka se zanemari.)

$$p_1 V_1 = p_2 V_2$$

$$p_1 = p_{pov} \qquad p_2 = p_{dub} = p_{pov} + \rho g h$$

$$V_1 = \frac{4}{3} r^2 \pi$$

$$V_2 = \frac{4}{3}\left(\frac{r}{2}\right)^3 \pi$$

$$p_1 V_1 = (p_{pov} + \rho g h) V_2$$

$$\rho g h = \frac{p_1 V_1}{V_2} - p_1$$

$$h = \frac{p_1\left(\frac{V_1}{V_2} - 1\right)}{\rho g} = 71{,}36\,m$$

10.3 Kako glasi barometarska formula za standardni atmosferu, pretpostavljajući da temperatura linearno opada s visinom h prema zakonu $T = T_0 - \delta h$, gdje je $T_0 = 288\,K$ ($= 15\,°C$), a $\delta = 6{,}5\,°C/km$. (Slobodno je koristiti za zrak $p = \rho R' T$, gdje je tzv. Dižengirska plinska konstanta za zrak $R' = 287\,J\,kg^{-1}K^{-1}$)

$$dp = -\rho g\,dh$$

$$p = \rho \frac{R}{M} T = \rho R' T$$

$$dp = -\frac{p g}{R' T}\,dh = -\frac{p g\,dh}{R'(T_0 - \delta h)}$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{R'} \int_0^h \frac{dh}{T_0 - \delta h}$$

$$\ln \frac{p}{p_0} = \frac{g}{R'\delta} \ln\left(\frac{T_0 - \delta h}{T_0}\right)$$

$$\frac{p}{p_0} = \left(1 - \frac{\delta h}{T_0}\right)^{\frac{g}{R'\delta}}$$

$$\frac{p}{p_0} = \left(1 - \frac{6{,}5\,\frac{h}{km}}{288}\right)^{5{,}26}$$

10.4 Gustoća tekućine u posudi dubine 1m jest linearna funkcija dubine: Gustoća je na dnu 1,13 $\frac{g}{cm^3}$ a na površini je 10% manja. Koliki je rad potreban da se predmet mase 0,1kg i volumena 5cm³ podigne s dna na površinu tekućine?

$$\rho(x) = kx + l$$

$$\rho(x) = \frac{\rho_1 - \rho_0}{d} x + \rho_0 \qquad \rho_0 = \rho(0), \quad \rho_1 = \rho(d)$$

$$F(x) = mg - U(x) = mg - \left(\frac{\rho_1 - \rho_0}{d} x + \rho_0\right) Vg$$

$$dW = F(x)\,dx$$

$$W = \int_0^d \left[mg - \left(\frac{\rho_1 - \rho_0}{d} x + \rho_0\right) Vg\right] dx = V \frac{gd}{2}(2\rho_1 - \rho_1 - \rho_0)$$

$$\rho_1 = \frac{m}{V}$$

$$W = 4{,}27 \cdot 10^{-2}\,J$$

10.5 U valjkastu posudu čiji je polumjer dna 10cm nalivena je 1 litra vode. U posudu se spušta staklena kocka ($\rho_s = 4 \cdot 10^3 \frac{kg}{m^3}$). Koliku je silu potrebno upotrijebiti da se kocka podigne sa dna, ako je masa kocke 0,75 kg?

$$h = \frac{V_v}{r^2\pi - a^2} = 0{,}0355\,m$$

$$a = \sqrt[3]{\frac{m}{\rho_s}} = 0{,}0572\,m$$

$$F = mg - U = mg - \rho_v g h a^2 = 6{,}22\,N$$

10.6 Okrugli je balon napunjen plinom gustoće $1\,kg\,m^{-3}$. Površinska gustoća materijala od kojega je balon načinjen je $0{,}04\,kg\,m^{-2}$. Kolikim najmanjim polumjerom mora imati balon da bi lebdio u zraku? Za gustoću zraka valja uzeti $1{,}3\,kg\,m^{-3}$.

$$U = mg + V\rho_p g$$

$$V\rho_z g = 3Sg + V\rho_p g$$

$$\frac{4}{3}r^3\pi\rho_z g = 3 4r^2\pi g + \frac{4}{3}r^3\pi\rho_p g$$

$$\frac{r}{3}(\rho_z - \rho_p) = 3$$

$$r = \frac{3S}{\rho_z - \rho_0} = 0{,}4\,m$$

10.7 Tijelo pliva na površini žive tako da mu je 25% volumena potopljeno u živu. Kolikim je postotkom volumena potopljen u živu ako se preko tijela prelije voda tako da tijelo bude potpuno pokriveno? ($\rho_ž = 13{,}6\cdot 10^3\,kg/m^3$)

$$mg = U = \rho_ž \cdot 0{,}25V \cdot g$$

$$\rho_t V g = \rho_ž\, 0{,}25 V g$$

$$\rho_t = \rho_ž\, 0{,}25 = 3{,}4\cdot 10^3\,kg/m^3$$

$$mg = U_v + U_ž = V_v' g \rho_v + V_ž' g \rho_ž = \rho_t V g$$

$$V_v'\rho_v + V_ž'\rho_ž = 0{,}25\rho_ž V \qquad V = V_v' + V_ž'$$

$$V_v' = V - V_ž'$$

$$V\rho_v - V_ž'\rho_v + V_ž'\rho_ž = 0{,}25\rho_ž V$$

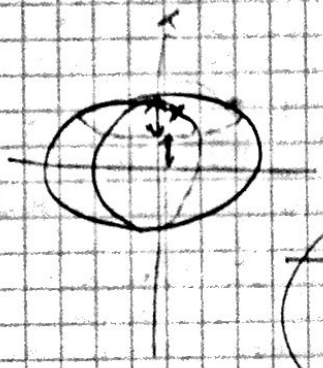
$$V_ž'(\rho_ž - \rho_v) = V(0{,}25\rho_ž - \rho_v)$$

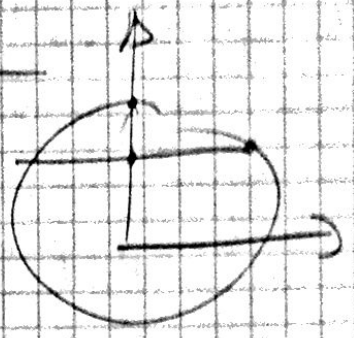
$$V_ž' = V\,\frac{\rho_ž - \rho_v}{\rho_ž - \rho_v} = 0{,}19V \Rightarrow V_ž' = 19\%\ od\ V$$

10.8 Kugla je sastavljena od materijala različitih gustoća. Vanjski sloj kugle polumjera 15 cm napravljen je od aluminija ($\rho_1 = 2700\ kg/m^3$), a unutrašnji je dio polumjer 14 cm ispunjen homogenim materijalom gustoće $\rho_2 = 250\ kg/m^3$. Ako je kugla položena u vodi, kolika je visina dijela kugle iznad razine vode?

$$\boxed{\sum_i G_i = 0}$$ uvjet ravnoteže

$$\rho_1 V_1 g + \rho_2 V_2 g = g \rho_v V - g \rho_v$$





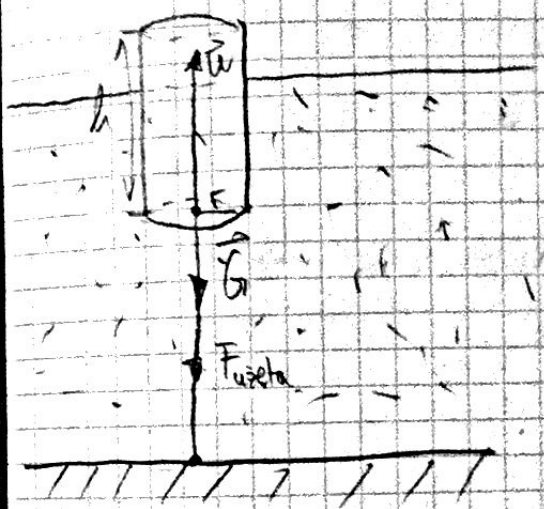
$$V = \frac{4}{3} R^3 \pi$$

$x = r \cos\varphi$

$y = r \sin\varphi$

trostruki integrali

10.11 Cilindrična metalna plutača promjera $2r = 1m$ i visine $h_0 = 2,6m$ pričvršćena je užetom za dno tako da su dvije trećine visine plutače uronjene u vodu. Kolika je sila kojom uže vuče plutaču, ako je debljina stijenki plutače $d = 0,01m$, gustoća metala $\rho = 7,8 \cdot 10^3 \frac{kg}{m^3}$?



$$\vec{F} + \vec{G} = \vec{u}$$

$$\boxed{F + G = u}$$

$$F + mg = \rho_v V' g$$

$$G = mg = \rho_t V g = \rho_t \left(r^2 \pi h_0 - (r-d)^2 \pi (h_0 - d)\right) g$$

$$u = \rho_v V' g = \rho_v \frac{2}{3} r^2 \pi h_0 g$$

$$F = u - G = \frac{2}{3} r^2 \pi h_0 \rho_v g - \rho_t g \left(r^2 \pi h_0 - (r-d)^2 \pi (h_0 - d)\right)$$

$$\boxed{F = 499,7\ N}$$

$\boxed{10.12}$ Koliki je rad potreban da bi se kapljica žive poluprečnika 1 mm razbila na kapljice dvostruko manjeg poluprečnika? ($\sigma = 0{,}48 \frac{N}{m}$)

$$W = \sigma \Delta S = \sigma (8 r_1^2 \pi - 4 r_1^2 \pi) = 4 r_1^2 \pi \sigma = 6 \cdot 10^{-6} J$$

Zadaci/Kolokvij: Statika fluida

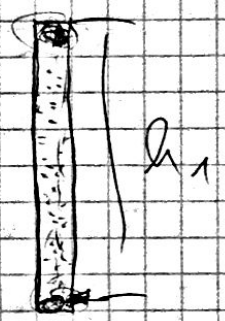
$\boxed{10.1}$

$\rho = 0{,}8 \cdot 10^3 \frac{kg}{m^3}$

$h_1 = 10 m$

$p = 10^5 Pa$

p', $p'_{h_1} = ?$



$$p'(h_1) = p + \rho g h_1$$

$$p'(h_1) = 1{,}78 \cdot 10^5 Pa$$

$$\boxed{p'(h_1) = p'}$$

$$p(h_1) = p' + \rho g h_1 =$$

$$= 2{,}57 \cdot 10^5 Pa$$

$\boxed{10.3}$

$h_0 = 300 m$

$p_0 = 101 kPa$

$h = 1000 m$

$T = konst.$

$p = ?$

$$p(h) = p_0 e^{-\frac{\rho_0}{p_0} g (h - h_0)} =$$

$$= 92{,}46 kPa$$

10.5

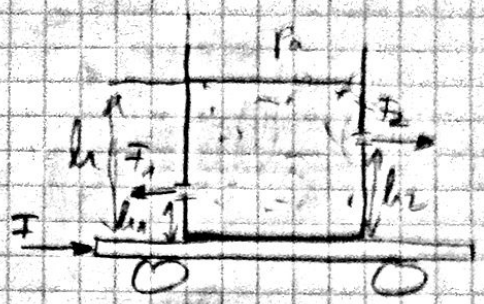
$h = 1\,m$

$S = 10^{-3}\,m^2$

$h_1 = 0{,}25\,m$

$h_2 = 0{,}5\,m$

$F = ?$



$v = \sqrt{2gh}$

$\frac{dv}{dt}$

$$F_2 + F = F_1$$

$$F_1 = p_1 \cdot S \qquad p_1 = p_a + \rho g(h - h_1)$$

$$F_2 = p_2 S \qquad p_2 = p_a + \rho g(h - h_2)$$

$$F = F_2 - F_1$$

$$F = S(p_a + \rho g(h - h_2) - p_a - \rho g(h - h_1)) = 2{,}45\,N$$

$$F_1 = \rho S v_1^2 \qquad v_1 = \sqrt{\frac{F}{\rho S}} =$$

$$p_a + \rho g(h - h_1) + \frac{1}{2}\rho v_1^2 = p_a + \rho g(h - h_2) + \frac{1}{2}\rho v_2^2$$

$$v_2 = \sqrt{2g[(h - h_1) - (h - h_2)] + v_1^2} =$$

$$\boxed{F = v\left(\frac{dm}{dt}\right) = v\rho\frac{dV}{dt} = v\rho S\frac{dh}{dt}}$$

$$F_1 = \rho S \cdot 2g(h - h_2) = 9{,}81$$

$$F_2 = \rho S \cdot 2g(h - h_1) = 14{,}715$$

$$F = F_2 - F_1 = 4{,}9\,N$$

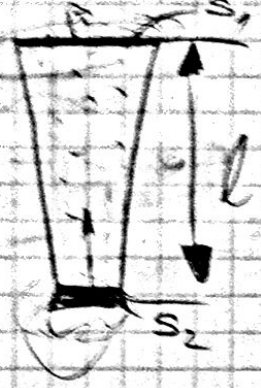
10.6

$l = 5m$

$2r_1 = 10cm = 0{,}1m$

$2r_2 = 5cm = 0{,}05m^2$

$F_{tep} = 200N$



$F_1 = p_1 S_1$

$F_1 = ?$

$S_1 = r_1^2 \pi = 0{,}0031 m^2$

$S_2 = r_2^2 \pi = 0{,}00196 m^2$

$$\frac{F_1 + p_a \cdot S_1}{S_1} = \frac{F_{tep} + S_2 \cdot g \rho l}{S_2}$$

$$F_1 = \frac{F_{tep} + S_2 \rho g l}{S_2} \cdot S_1 - p_a S_1 = \boxed{415{,}95\ N}$$

10.7

$m_{brod} = 100t = 1 \cdot 10^5 kg$

$m_{jen} = 2t = 2 \cdot 10^3 kg$

$\rho_{ž} = 7{,}8 \cdot 10^3 \frac{kg}{m^3}$

$\rho_{jen} = 5 \cdot 10^3 \frac{kg}{m^3}$

$V_{plen} = ?$

$G = U$

$G = (m_b + m_{ju}) g$

$U = \rho_v (V_{plen} + V_{ju} + V_{brod}) g$

$m_b + m_{ju} = \rho_v (V_{plen} + V_{ju} + V_{brod})$

$V_{plen} = \frac{m_b + m_{ju}}{\rho_v} - V_{ju} - V_{brod} =$

$= \boxed{88{,}8\ m^3}$

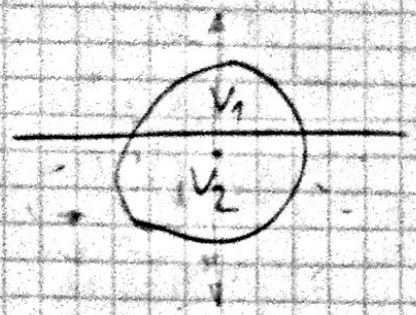
10.8

$V_1 = 195\,m^3$

$\rho_v = 1{,}03\,\frac{g}{cm^3}$

$\rho_{led} = 0{,}9\,\frac{g}{cm^3}$

$V = ?$



$V = V_1 + V_2$

$\boxed{G_{sante} = U_{V_2}}$ — uvjet plavanja

$\rho_{led} V g = \rho_v V_2 g$

$\rho_{led}(V_1 + V_2) = \rho_v V_2$

$\rho_{led} V_1 = V_2(\rho_v - \rho_{led})$

$V_2 = \frac{\rho_{led} V_1}{\rho_v - \rho_{led}}$

$V_2 = 1350$

$V = V_1 + V_2 = 1350 + 195 = 1545\,m^3$

$V_k = \frac{m_m + \rho_L (V_2 + V_1)}{\rho_m}$

$V_2 \rho_m - V_2 \rho_L = m_m + \rho_L V_1$

$V_2 = \frac{m_m + \rho_L V_1}{\rho_m - \rho_L}$

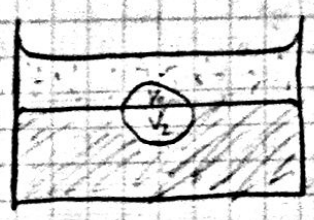
10.9

$\rho_{živa} = 13{,}6 \cdot 10^3\,\frac{kg}{m^3}$

$\rho_{željezo} = 7{,}8 \cdot 10^3\,\frac{kg}{m^3}$

$\rho_{voda} = 1 \cdot 10^3\,\frac{kg}{m^3}$

$V_1, V_2 = ?$



$\boxed{G = U}$ — uvjet lebdenja u fluidu

$\rho_{željezo} V g = \rho_v V_1 g + \rho_{živa} V_2 g$

$\rho_{željezo} V g = \rho_v V_1 g + \rho_{živa}(V - V_1) g$

$V(\rho_{željezo} - \rho_{živa}) = \rho_v V_1 - \rho_{živa} V_1$

$\boxed{V_1 = 0{,}46\,V}$

$\boxed{V_2 = 0{,}54\,V}$

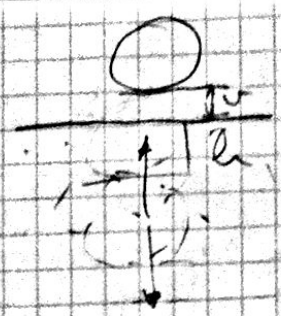
10.10

$\rho_{stijelo} = 800 \frac{kg}{m^3}$

$\rho_v = 1000 \frac{kg}{m^3}$

$h = 1m$

$v = ?$



$$U = \rho_v V g$$

$$E_p = mg(v+h) = \rho_{stijelo} V g (v+h) = U \cdot h$$

$$U = \rho_v V g$$

$$\rho_{stijelo} V g (v+h) = \rho_v V g h$$

$$v = \left(\frac{\rho_v}{\rho_{stijelo}} - 1\right) h$$

$$\boxed{v = 0{,}25 m}$$

10.11

$l = 5m$

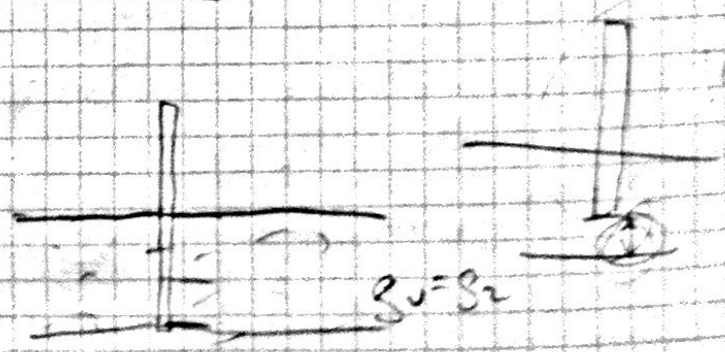
$G = 40N$

$\rho_1 = 796 \frac{kg}{m^3}$

$H = 4{,}75m$

$\rho_2 = 10^3 \frac{kg}{m^3}$

$W = ?$



$$\rho_v g l_1 S = \rho_1 g l S$$

$$\boxed{l_1 = l \frac{\rho_1}{\rho_v}} = 3{,}98 m$$

$$U = \rho_v g S y$$

$$W = \int_0^{H-l_1} U dy = \rho_v g S \int_0^{H-l_1} y dy = \frac{1}{2} \rho_v g S (H-l_1)^2 = 2{,}35 J$$

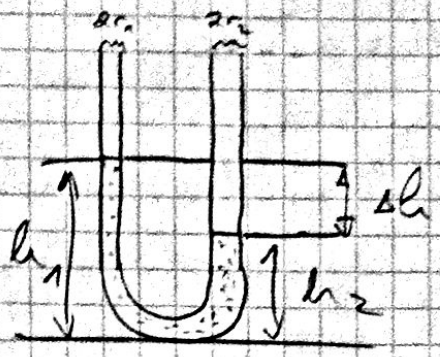
10.12

$r_1 = 4\,mm = 4\cdot 10^{-3}\,m$

$r_2 = 1{,}6\,mm = 1{,}6\cdot 10^{-3}\,m$

$\sigma = 0{,}073\,\frac{N}{m}$

$\Delta h = ?$



$\Delta h = h_1 - h_2$

$\boxed{\theta = 0^\circ} \Rightarrow \cos\theta = 1$

jer potpuno moči stijenku cijevi

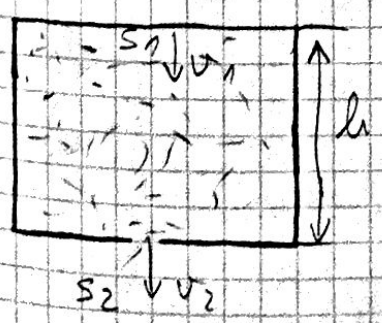
$$h_1 = \frac{2\sigma\cos\theta}{\rho g r_1} = 0{,}0248\,m$$

$$h_2 = \frac{2\sigma}{\rho g r_2} = 0{,}00372\,m$$

$$\Delta h = 0{,}021\,m = 2{,}1\,cm$$

Kulišić/Primjer : Dinamika fluida

11.1 Posuda cilindričnog oblika visine 1,5m ispunjena je vodom do vrha. U kojem će vremenu voda potpuno isteći kroz otvor na dnu posude ako je površina otvora $\frac{1}{350}$ površine dna?



$$p_a + \frac{1}{2}\rho v_1^2 + \rho g h = \frac{1}{2}\rho v_2^2 + p_a$$

$$S_1 v_1 = S_2 v_2 \qquad v_2 = v_1\frac{S_1}{S_2}$$

$$\frac{1}{2}\rho v_1^2 + \rho g h = \frac{1}{2}\rho v_1^2\left(\frac{S_1}{S_2}\right)^2$$

$$v_1^2\left(\frac{1}{2} - \frac{1}{2}\left(\frac{S_1}{S_2}\right)^2\right) = -gh$$

$$v_1^2 = \frac{2gh}{\left(\frac{S_1}{S_2}\right)^2 - 1}$$

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{S_1}{S_2}\right)^2 - 1}} = K\sqrt{h}$$

$$K = \frac{S_2\sqrt{2g}}{\sqrt{S_1^2 - S_2^2}}$$

$$-dy = v_1 dt = k\sqrt{y}\,dt$$

$$-\int_0^t dt = \int_h^0 \frac{1}{k}\frac{dy}{\sqrt{y}}$$

$$\int_0^t dt = \frac{1}{k}\int_0^h \frac{dy}{\sqrt{y}}$$

$$t = \frac{1}{k}\,2\sqrt{h} = \sqrt{\frac{2h\left[\left(\frac{S_1}{S_2}\right)^2-1\right]}{g}}$$

$$t = 193{,}55\,s$$

11.2 Vodoravnom cijevi protječe voda. Na mjestima gdje su presjeci cijevi $S_1 = 1\,cm^2$ i $S_2 = 3\,cm^2$ okomito su spojene dvije manometarske cijevi. Odredite protok vode kroz vodoravnu cijev ako je razlika razina vode u manometrima $\Delta h = 10\,cm$.

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$v_1 S_1 = v_2 S_2 \qquad v_2 = v_1\frac{S_1}{S_2}$$

$$p_2 - p_1 = \rho g \Delta h = \Delta p$$

$$p_2 - p_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$\rho g \Delta h = \frac{1}{2}\rho\left(v_1^2 - v_1^2\left(\frac{S_1}{S_2}\right)^2\right)$$

$$v_1^2 = \frac{2\rho g \Delta h}{\rho\left(1-\left(\frac{S_1}{S_2}\right)^2\right)}$$

$$v_1 = \sqrt{\frac{2g\Delta h}{1-\left(\frac{S_1}{S_2}\right)^2}}$$

$$Q_V = S_1 v_1 = S_1 \sqrt{\frac{2gh}{1-\left(\frac{S_2}{S_1}\right)^2}} = 1{,}5 \cdot 10^{-4}\ \frac{m^3}{s}$$

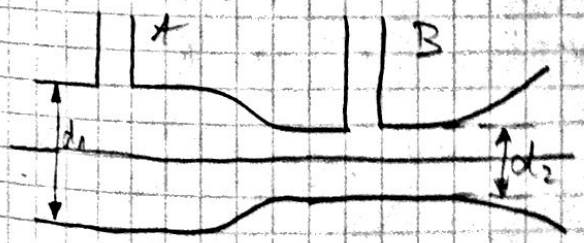
11.3 Kroz cijev duljine 100m teče voda brzinom $1\frac{m}{s}$. Pri zatvaranju ispusnog ventila tlak se povećava za $2 \cdot 10^5$ Pa. Koliko je trajalo zatvaranje ventila?

$$F\Delta t = \Delta(mv) = m\Delta v = mv$$

$$\Delta p = \frac{F}{S} = \frac{m\Delta v}{S\Delta t}$$

$$\Delta t = \frac{m\Delta v}{S\Delta p} = \frac{\rho l v}{\Delta p} = 0{,}5\,s$$

11.4 Količina plina koji struji plinovodom mjeri se sustavom prikazanim na slici. O brzini protjecanja sudi se na osnovi razlike tlakova u točkama A i B. Potrebno je odrediti masu plina koja prostruji kroz jedan sat ako su zadani: $d_1 = 50mm$, $d_2 = 40mm$, $\Delta p = 0{,}188$ Pa i gustoća plina $\rho = 1{,}6\ \frac{kg}{m^3}$.



$$r_1^2 \pi v_1 = r_2^2 \pi v_2$$

$$\left(\frac{v_1}{v_2}\right) = \left(\frac{r_2}{r_1}\right)^2$$

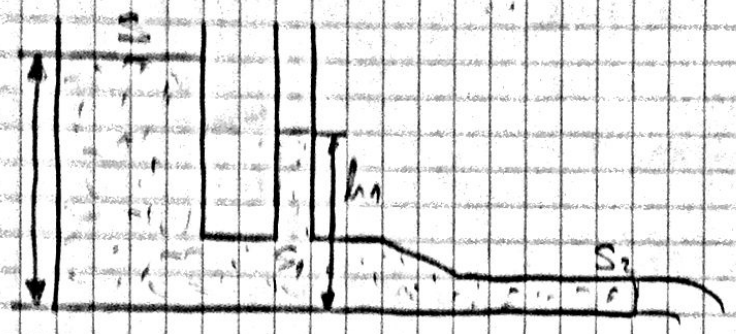
$$p_1 - p_2 = \frac{1}{2}\rho v_2^2\left(1 - \frac{v_1^2}{v_2^2}\right)$$

$$\Delta p = p_1 - p_2$$

$$v_2 = \sqrt{\frac{2\Delta p}{\rho\left(1-\left(\frac{r_2}{r_1}\right)^4\right)}} = 0{,}63\,ms^{-1}$$

$$m = \rho S_2 v_2 t = \rho r_2^2 \pi v_2 t = 4{,}56\,kg$$

11.6 U posudi poprečnog presjeka $300\,cm^2$ nalazi se 12 litara vode. Voda istječe kroz vodoravnu cijev iz otvora pri dnu posude. Ta je cijev na početku presjeka $3\,cm^2$ a zatim se suužava u cijev presjeka $1{,}5\,cm^2$. Koliki je tlak, odnosno koliko je visina stupca vode u manometru u užem dijelu vodoravne cijevi? (Voda je idealan fluid).



$$q = S_1 v_1 = S_2 v_2$$

$$h = \frac{V}{S} = 0{,}4\,m$$

$$p_a + \rho g h + \frac{1}{2}\rho v^2 = p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_2 = p_a \qquad v \approx 0$$

$$\rho g h = \frac{1}{2}\rho v_2^2 \qquad v_2 = \sqrt{2gh} \qquad v_1 = \frac{S_2}{S_1} v_2 = \frac{S_2}{S_1}\sqrt{2gh}$$

$$p_1 = p_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = p_a + \frac{1}{2}\rho v_2^2\left(1 - \frac{S_2^2}{S_1^2}\right) = p_a + \rho g h\left(1 - \frac{S_2^2}{S_1^2}\right)$$

$$p_1 = \rho g h_1 + p_a$$

$$h_1 = h\left(1 - \frac{S_2^2}{S_1^2}\right) = \frac{3}{4} h = 0{,}3\,m$$

Kuloše/Zadaci: Dinamika fluida

11.1

$S_1 = 1{,}6\,cm^2$

$S_2 = 1\,mm^2 = 1\cdot 10^{-6}\,m^2$

$F = 5\,N$

$l = 4\,cm$

$\tau = ?$

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$p_1 - p_2 = \frac{1}{2}\rho v_2^2$$

$$v_2 = \sqrt{\frac{2\Delta p}{\rho}}$$

$$\boxed{\Delta p = \frac{F}{S_1}}$$

$$v_2 = 7{,}905\,\frac{m}{s}$$

$$q_V = \frac{\Delta V}{\Delta t} = S_2 v_2$$

$$\Delta V = S_2 v_2 \Delta t$$

$$\Delta t = \frac{\Delta V}{S_2 v_2} = \frac{S_1 \Delta l}{S_2 v_2} = 0{,}81\,s$$

$\boxed{11.2}$

$2r_1 = 0{,}04\,m$

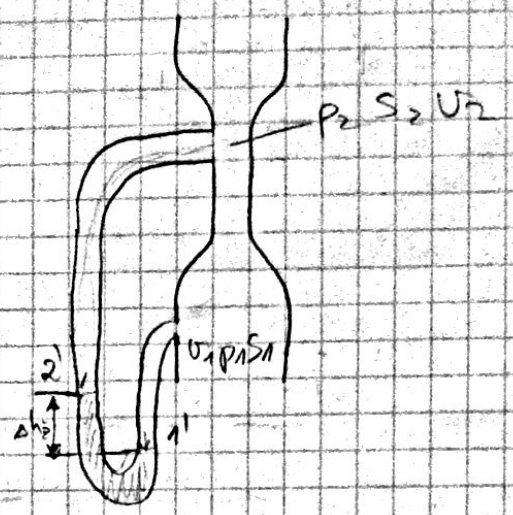
$\Delta h = 0{,}5\,m$

$2r_2 = 0{,}02\,m$

$p_1 = 160\,kPa$

$p_2 = 80\,kPa$

$v_1, g, \Delta h_z = ?$



$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g \Delta h + \frac{1}{2}\rho v_2^2$$

$$S_1 v_1 = S_2 v_2$$

$$v_2 = \frac{S_1}{S_2} v_1$$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g \Delta h + \frac{1}{2}\rho \left(\frac{S_1}{S_2}\right)^2 v_1^2$$

$$v_1^2 \rho \left(1 - \left(\frac{S_1}{S_2}\right)^2\right) = 2(p_2 - p_1 + \rho g \Delta h)$$

$$v_1 = \sqrt{\frac{2(p_2 - p_1 + \rho g \Delta h)}{\rho\left(1 - \left(\frac{S_1}{S_2}\right)^2\right)}} = 3{,}16\ \frac{m}{s}$$

$$q_V = S_1 v_1 = 0{,}00397\ \frac{m^3}{s} = 3{,}97\ \frac{l}{s}$$

$$p_1' = p_1 + \rho g (y_1 - h_1') = p_2 + \rho g (y_2 - h_2') + \rho_{Z} g \Delta h'$$

$$p_1 - p_2 + \rho g (\Delta h) = g(\rho_{Z} - \rho)\Delta h'$$

$$\Delta h' = \frac{\Delta p + \rho g \Delta h}{g(\rho_{Z} - \rho)} = 0{,}69\,m$$

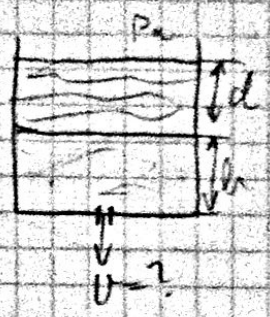
11.3

$h = 2\,m$

$\rho_u = 850\ \frac{kg}{m^3}$

$d = 1\,m$

$v = ?$



$$p_a + g\rho_v h + g\rho_u l = p_a + \frac{1}{2}\rho_v v^2$$

$$v = \sqrt{\frac{2g(\rho_v h + \rho_u l)}{\rho_v}}$$

$$v = 7{,}5\ \frac{m}{s}$$

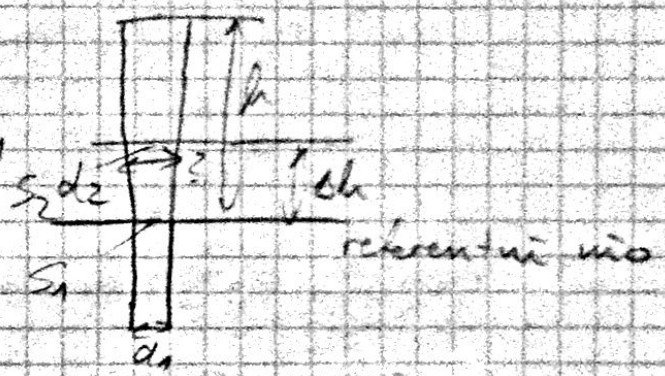
11.4

$2r = 20\,cm = 0{,}02\,m = d_1$

$h = 4{,}1\,m$

$\Delta h = 1\,m$

$d_2 = ?$



$$p_a + \rho g h + \frac{1}{2}\rho v_1^2 = p_a + \rho g h + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho v_1^2 = \rho g h$$

$$v_1 = \sqrt{2gh} = 8{,}97\ \frac{m}{s}$$

$$v_2 = \sqrt{2g(h-\Delta h)} = 7{,}8\ \frac{m}{s}$$

$$S_1 v_1 = S_2 v_2$$

$$S_2 = S_1 \frac{v_1}{v_2} = 3{,}61 \cdot 10^{-3}\ m^2$$

$$r_2^2 \pi = S_2$$

$$r_2 = \sqrt{\frac{S_2}{\pi}} = 0{,}0302\,m$$

$$d_2 = 2r_2 = 2{,}140\,cm$$

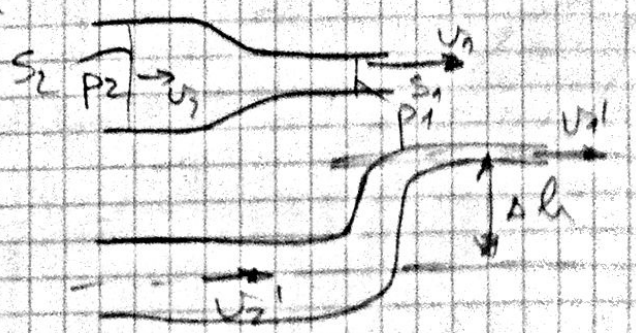
11.5

$\rho_1 = 0{,}9 \frac{g}{cm^3} = 900 \frac{kg}{m^3}$

$v_1 = 5 \frac{m}{s}$

$\Delta p = 5 kPa$

$\Delta h = ?$



$v' = \frac{v_1}{2}$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$S_1 v_1 = S_2 v_2$$

$$\frac{1}{2}\rho v_2^2 = \frac{1}{2}\rho v_1^2 + p_1 - p_2$$

$$v_2 = \sqrt{\frac{\rho v_1^2 + 2\Delta p}{\rho_1}} = 3{,}73 \frac{m}{s}$$

$$S_1 v_1 = S_2 v_2$$

$$\frac{S_1}{S_2} = \frac{v_2}{v_1} = 0{,}746$$

$$S_1 v_1' = S_2 v_2'$$

$$v_2' = v_1' \frac{S_1}{S_1} = \frac{v_1}{2} \cdot \frac{S_1}{S_2} = 1{,}865$$

$$p_2 + \frac{1}{2}\rho v_2'^2 = p_1 + \rho g \Delta h + \frac{1}{2}\rho v_1'^2$$

$$\Delta p + \frac{1}{2}\rho\left((v_2')^2 - (v_1')^2\right) = \rho g \Delta h$$

$$\Delta h = \frac{\Delta p + \frac{1}{2}\left(v_2'^2 - v_1'^2\right)}{\rho g} = 0{,}43 m$$

10 $\boxed{11.6}$

$\rho_z = 1{,}2 \frac{kg}{m^3}$

$h_2 = 0{,}06 m$

$v = ?$

$$v = \sqrt{2g\Delta h \frac{\rho_v}{\rho_z}}$$

$p_1 = p_2$

$$\cancel{p_1} + \rho_v g(h_1) + \frac{1}{2}\rho_z v_1^2 = \cancel{p_2} + \rho_v g h_2 + \frac{1}{2}\rho_z v_2^2$$

$$\frac{1}{2}\rho_z v_1^2 = \rho_v g \underbrace{(h_2 - h_1)}_{\Delta h}$$

$$\frac{1}{2}\rho_z v_1^2 = \rho_v g \Delta h$$

$$v_1 = \sqrt{2g\Delta h \frac{\rho_v}{\rho_z}}$$

$$\boxed{v_1 = 31{,}3 \frac{m}{s}}$$

$\boxed{11.7}$

$p_3 = 80 kPa$

$p_4 = 100 kPa$

$\rho_z = 1 \frac{kg}{m^3}$

$v = ?$

$$p_1 + \frac{1}{2}\rho_z v_1^2 = p_2$$

$$v_1 = \sqrt{\frac{2\Delta p}{\rho_z}}$$

$$v_1 = 200 \frac{m}{s}$$

11.9

$d = 2{,}54\,cm = 0{,}0254\,m$

$l = 18\,m$

$v = 0{,}1\,\frac{m}{s}$

$\eta = 0{,}1\,Pa\,s$

$Re = ?$

$$\rho_n = 850\,\frac{kg}{m^3}$$

$$Re = \frac{\rho v l}{\eta} = 22 \rightarrow \text{laminarno strujanje}$$

$$q_v = Sv = 5{,}67 \cdot 10^{-5}$$

$$q_v = Sv = \frac{7}{5} Sv = 7v$$

$$q_v = \frac{\pi \Delta p}{8\eta l} R^4$$

$$R^2 \pi v = \frac{\pi \Delta p}{8\eta l} R^4$$

$$\Delta p = \frac{8\eta l v}{R^2} = 8\,928{,}017\,Pa$$

11.10

$\eta = 0{,}1\,Pas$ $\quad d' = 0{,}0002\,m$

$d = 0{,}02\,m$

$l = 12\,m$

$\Delta p = 60\,kPa$

$v_1, v_2 = ?$

$v = ?$

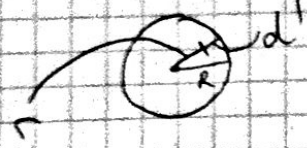
$$q_v = \frac{\pi \Delta p}{8\eta l} R^4$$

$$\bar{v} = \frac{\pi \Delta p}{8\eta l S_r} R^4 = 0{,}1785\,\frac{m}{s} = 17{,}2\,\frac{cm}{s}$$

$$v_1(0) = \frac{\Delta p}{4\eta l}(R^2 - r^2) = \frac{\Delta p}{4\eta l} R^2 =$$

$$= \frac{\Delta p}{4\eta l}\left(\frac{d}{2}\right)^2 = 0{,}357\,\frac{m}{s} = 35{,}7\,\frac{cm}{s}$$

$d' = 0{,}2\,cm$



$$v(R-d') = v(r) = v(0{,}008) = \frac{\Delta p}{4\eta l}(R^2 - r^2) =$$

$$= 0{,}1285\,\frac{m}{s} = 12{,}9\,\frac{cm}{s}$$

11.11

$d_1 = 10\,cm = 0{,}1\,m$ $\quad \eta_{zrak} = 1{,}8 \cdot 10^{-5}\,Pas$

$v_{kzrak} = 0{,}33\,\frac{m}{s}$ $\quad \eta_{voda} = 1 \cdot 10^{-3}\,Pa\,s$

$d_2 = 0{,}0254\,m$

$v_{kvoda} = ?$

$Re = ?$

karakteristična dužina za ajer kružnog presjeka je promjer

$$Re = \frac{\rho v_k l}{\eta} = \frac{\rho\, v_{kzrak} \cdot d}{\eta} =$$

$\approx 2370 \rightarrow$ turbulentno strujanje

$$Re = \frac{\rho v_{kvoda}\, l}{\eta}$$

$Re(\text{kritično}) \approx 2100$

$$Re = \frac{\rho_v v_{kvoda}\, d_2}{\eta_{voda}}$$

$$v_{kvoda} = \frac{Re\, \eta_{voda}}{\rho_v\, d_2} = 0{,}08267\,\frac{m}{s} = 8{,}27\,\frac{cm}{s}$$

$\boxed{11.12}$

$R = 5 \cdot 10^{-5}\,m$ $\qquad \rho_v = 1000\,\frac{kg}{m^3}$

$\eta_z = 1{,}8 \cdot 10^{-5}\,Pa\,s$ $\qquad \rho_z = 1{,}293\,\frac{kg}{m^3}$

$v_{max} = ?$

laminarno strujanje $\Rightarrow$ Stokesov zakon

$$F_{tr} = 6\pi\eta R v \qquad k = 6\pi\eta R$$

$$F_{tr} = kv$$

jednadžba gibanja kapljice

$$ma = mg - F_u - F_{tr}$$

$$\rho_v V a = \rho_v V g - \rho_z V g - kv \quad / : (\rho_v V)$$

$$a = g - \frac{\rho_z}{\rho_v} g - \frac{k}{\rho_v V} v$$

$$\frac{dv}{dt} = g\left(1 - \frac{\rho_z}{\rho_v}\right) - \frac{k}{\rho_v V} v$$

$$A = g\left(1 - \frac{\rho_z}{\rho_v}\right)$$

$$B = \frac{k}{\rho_v V}$$

$$\frac{dv}{dt} = A - Bv$$

$$dt = \frac{dv}{A - Bv} \quad / \int$$

$$\int dt = \int \frac{dv}{A - Bv} \qquad \left| \begin{array}{l} \text{supst.} \\ u = A - Bv \\ du = -B\,dv \end{array} \right|$$

$$\int dt = -\frac{1}{B} \int \frac{du}{u}$$

$$-B \int dt = \int \frac{du}{u}$$

$$\ln C - Bt = \ln|A - Bv|$$

$$A - Bv = Ce^{-Bt}$$

$$v = -\frac{1}{B}\left(Ce^{-Bt} - A\right)$$

$$\boxed{v(0) = 0 = -\frac{1}{B}(C - A)}$$ — početni uvjet iz kojeg se određuje konstanta C

$$-\frac{1}{B}(C - A) = 0 \quad /(-B)$$

$$C - A = 0$$

$$\boxed{C = A}$$

$$C = g\left(1 - \frac{\rho_z}{\rho_v}\right)$$

$$A - Bv = Ae^{-Bt}$$

$$-Bv = A\left(e^{-Bt} - 1\right)$$

$$\boxed{v = \frac{A}{B}\left(1 - e^{-Bt}\right)}$$

$$v = \frac{g\left(1 - \frac{\rho_z}{\rho_v}\right)}{\frac{6\pi\eta R}{\rho_v V}}\left(1 - e^{-\frac{6\pi\eta R}{\rho_v V}t}\right)$$

$$v = \frac{\rho_v V g\left(1 - \frac{\rho_z}{\rho_v}\right)}{6\pi\eta R}\left(1 - e^{-\frac{6\pi\eta R}{\rho_v V}t}\right)$$

$$v_{max} = \lim_{\Delta t \to \infty} v(t) = \lim_{\Delta t \to \infty}\left[\frac{\rho_v V g\left(1 - \frac{\rho_z}{\rho_v}\right)}{6\pi\eta R}\left(1 - e^{-\frac{6\pi\eta R}{\rho_v V}t}\right)\right]$$

$$= \lim_{\Delta t \to \infty}\left[\frac{\rho_v V g\left(1 - \frac{\rho_z}{\rho_v}\right)}{6\pi\eta R}\left(1 - 0\right)\right] = 0{,}302\ \frac{m}{s}$$

slično kao i prethodni

11.13

$r = 4 \cdot 10^{-4} m$

$\eta = 18{,}2 \cdot 10^{-6} Pa\, s$

$v_{tlo} = ?$

$\rho_z = 1{,}293 \frac{kg}{m^3}$

$\rho_v = 1000 \frac{kg}{m^3}$

brzina kapljice u trenutku kada padne na tlo jednaka je maksimalnoj brzini.

$$\boxed{v_{tlo} = v_{max}}$$

ponovno imamo laminarno strujanje, dakle vrijedi

$$\boxed{F_{tr} = 6\pi \eta R v}$$

Jednadžba gibanja:

$$ma = mg - F_u - F_{tr}$$

$$\rho_v V a = \rho_v V g - \rho_z V g - 6\pi \eta R v \quad /:(\rho_v V)$$

$$\boxed{k = 6\pi \eta R}$$

$$a = g - g\frac{\rho_z}{\rho_v} - \frac{k}{\rho_v V} v$$

$$\frac{dv}{dt} = g\left(1 - \frac{\rho_z}{\rho_v}\right) - \frac{k}{\rho_v V} v$$

$$A = g\left(1 - \frac{\rho_z}{\rho_v}\right)$$

$$B = \frac{k}{\rho_v V}$$

$$\frac{dv}{dt} = A - Bv$$

$$dt = \frac{dv}{A - Bv} \quad / \int$$

$$\int dt = \int \frac{dv}{A - Bv} \quad \left| \begin{array}{l} \text{supst} \\ u = A - Bv \\ du = -B dv \end{array} \right|$$

$$t = -\frac{1}{B} \int \frac{du}{u} + \ln C$$

$$-Bt + \ln C = \ln|A - Bv|$$

$$A - Bv = Ce^{-Bt}$$

$\boxed{v(0) = 0}$ — Početni uvjet iz kojeg odredujemo konstantu C

$$A - B \cdot 0 = Ce^{-B \cdot 0}$$

$$\boxed{A = C}$$

$$A - Bv = Ae^{-Bt}$$

$$v(t) = \frac{A}{B}\left(1 - e^{-Bt}\right)$$

$$v(t) = \frac{g\left(1 - \frac{\rho_z}{\rho_v}\right)}{\frac{6\pi\eta R}{\rho_v V}}\left(1 - e^{-\frac{6\pi\eta R}{\rho_v V}t}\right)$$

$$v(t) = \frac{\rho_v V g\left(1 - \frac{\rho_z}{\rho_v}\right)}{6\pi\eta R}\left(1 - e^{-\frac{6\pi\eta R}{\rho_v V}t}\right)$$

$$v_{max} = \lim_{t \to \infty} v(t) = \lim_{t \to \infty}\left[\frac{\rho_v V g\left(1 - \frac{\rho_z}{\rho_v}\right)}{6\pi\eta R}\left(1 - 0\right)\right] =$$

$$= 19{,}14 \ \frac{m}{s}$$

Primjeri/Kušišić: Toplina

$\boxed{12.2}$ Tijelo od bakra i tijelo od aluminija jednaka su volumena pri $T_1 = 273\,K$. Tijelo od aluminija zagrijemo na $373\,K$ i zatim oba tijela spojimo. Kolika će biti relativna promjena volumena novonastalog tijela?

$\rho_{bakar} = 8500\,\frac{kg}{m^3}$; $c_{bakar} = 376\,\frac{J}{kg\,K}$; $\alpha_{bakar} = 1{,}7\cdot 10^{-5}\,\frac{1}{K}$

$\rho_{aluminij} = 2600\,\frac{kg}{m^3}$; $c_{aluminij} = 960\,\frac{J}{kg\,K}$; $\alpha_{aluminij} = 2{,}3\cdot 10^{-5}\,\frac{1}{K}$

$$T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$$

// iz Richmannova zakona

$$m_1 c_1 (T_1 - T) = m_2 c_2 (T - T_2)$$

$$T(m_1 c_1 + m_2 c_2) = m_1 c_1 T_1 + m_2 c_2 T_2$$

$$T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$$

$$T = \frac{\rho_1 c_1 T_1 + \rho_2 c_2 T_2}{\rho_1 c_1 + \rho_2 c_2}$$

$$T = 315{,}8\,K$$

$$\Delta V_1 = V_0\, 3\alpha_1 (T - T_1)$$

$$\Delta V_2 = V_0\, 3\alpha_2 (T - T_2)$$

$$\frac{\Delta V_1 + \Delta V_2}{2V_0} = \frac{3}{2}\left[T(\alpha_1 + \alpha_2) - \alpha_1 T_1 - \alpha_2 T_2\right] = -8{,}8\cdot 10^{-4}$$

12.4 Plin je u posudi od 1L uz pomoć adijabatskog klipa podijeljen na dva jednaka dijela koji su zatim zagrijani do 373K, odnosno do 473K. Kolikі su odgovarajući volumeni nakon pomicanja klipa?

$$p_1 V_1 = nRT_1$$
$$p_1 V_2 = nRT_2$$

$$V = V_1 + V_2$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$V_1 = V_2 \frac{T_1}{T_2}$$

$$V = V_2\left(1 + \frac{T_1}{T_2}\right)$$

$$V = V_2\left(\frac{T_2 + T_1}{T_2}\right)$$

$$V_2 = V \cdot \left(\frac{T_2}{T_2 + T_1}\right)$$

$$V_2 = 0{,}559\,V = 0{,}559\,L$$

$$V_1 = V \frac{T_1}{T_2 + T_1}$$

$$V_1 = 0{,}441\,V = 0{,}441\,L$$

12.5 U zatvorenoj posudi se nalazi dušik na sobnoj temperaturi (20°C) i pod tlakom 10^5 Pa. U posudu se doda određena količina tekućeg dušika temperature vrenja (−196°C) koji ispari, pa temperatura naraste na −140°C. Kad temperatura ponovno naraste na sobnu temperaturu, izmjeri se tlak $1{,}5 \cdot 10^5$ Pa. Izračunajte molarnu toplinu isparavanja ($C_V = 20{,}8 \frac{J}{mol\,K}$).

$$p_0 V = n_0 R T_0$$

$$p_3 V = (n_0 + \Delta n) R T_0$$

$$\Delta n = 0{,}5 n_0$$

$$C_V n_0 (T_0 - T_2) = \Delta n L_i + C_V \Delta n (T_3 - T_1)$$

$$L_i = C_V \left[\frac{n_0}{\Delta n} (T_0 - T_2) - (T_2 - T_1) \right]$$

$$L_i = 5491{,}2 \frac{J}{mol}$$

12.6 Dva staklena balona volumena $V = 10^{-3}\,m^3$ spojena su sa cijevi zanemarivog volumena i sadrže vodik u normirane uvjete gustoće $\rho = 0{,}09\,\frac{kg}{m^3}$. Jedan se balon stavi u tekući kisik na $t_1 = -190°C$, a drugi u vodenu paru na $t_2 = 100°C$. Kolika masa vodika prođe kroz cijev koja spaja balone do izjednačenja tlaka u balonima? Kolikt je taj tlak?

$$p_0 V = \frac{1}{2}(n_1 + n_2) R T_0 \quad , \quad T_0 = 273K$$

$$pV = n_1 R T_1 = n_2 R T_2 \quad \Rightarrow \quad \frac{n_1}{n_2} = \frac{T_2}{T_1}$$

$$\frac{p}{p_0} = \frac{2 n_1 R T_1}{(n_1 + n_2) R T_0} = \frac{2T_1}{\left(1 + \frac{n_2}{n_1}\right) T_0} = \frac{2T_1}{\left(1 + \frac{T_1}{T_2}\right) T_0}$$

$$\frac{p}{p_0} = 0{,}497$$

$$p = 50397\ Pa$$

$$\frac{\frac{1}{2}(n_1 - n_2)}{\frac{1}{2}(n_1 + n_2)} = \frac{T_2 - T_1}{T_2 + T_1} = 0{,}636$$

$$m = \rho V = 9 \cdot 10^{-5}\,kg$$

$$\Delta m = 0{,}636\,m = 5{,}72 \cdot 10^{-5}\,kg$$

Zadaci / Kulišić : Toplina

12.1

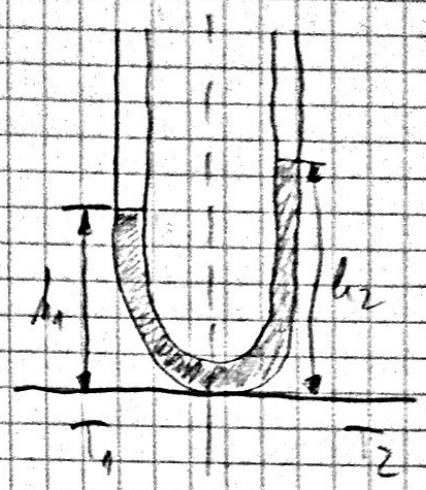
$h_1 = 250\,mm = 0{,}25\,m$

$t_1 = 10°C = 283K$

$h_2 = 300\,mm = 0{,}3\,m$

$t_2 = 80°C = 353K$

$\gamma = ?$



$$V = V_0(1 + \gamma \Delta T)$$

$$\boxed{\begin{aligned} V_0 &= r^2 \pi h_0 \\ T_0 &= 0°C = 273K \end{aligned}}$$

$$\left.\begin{aligned} r^2 \pi h_1 &= r^2 \pi h_0 (1 + \gamma \Delta T) \\ r^2 \pi h_2 &= r^2 \pi h_0 (1 + \gamma \Delta T) \end{aligned}\right\}$$

$$\frac{h_1}{h_2} = \frac{(1 + \gamma(T_1 - T_0))}{(1 + \gamma(T_2 - T_0))}$$

$$\frac{h_1}{h_2} + \gamma(T_2 - T_0)\frac{h_1}{h_2} = 1 + \gamma(T_1 - T_0)$$

$$\gamma\left[(T_1 - T_0) - (T_2 - T_0)\frac{h_1}{h_2}\right] = \frac{h_1}{h_2} - 1$$

$$\gamma = \frac{\frac{h_1}{h_2} - 1}{T_1 - T_2 \frac{h_1}{h_2}}$$

$$\gamma = 1{,}03 \cdot 10^{-3}\ \frac{1}{K}$$

12.2

čvrsta posuda ⇒ $\boxed{V = \text{konst.}}$

$\Delta T = 1K$; $T' = T + 1$

$p' = 1{,}01p$

$T = ?$

$$pV = nRT$$

$$p'V = nRT'$$

$$V = \frac{nRT}{p}$$

$$V = \frac{nRT'}{p'} = \frac{nR(T+1)}{1{,}1p}$$

$$\frac{\cancel{n}\cancel{R}(T+1)}{1{,}01\cancel{p}} = \frac{\cancel{n}\cancel{R}T}{\cancel{p}}$$

$$T + 1 = 1{,}01T$$

$$T(1{,}01 - 1) = 1$$

$$T = \frac{1}{1{,}01-1} = 100K$$

12.3

$V = \text{konst.}$

$T' = 1{,}01T$

$p' = p + 10^3 Pa$

$p = ?$

$$pV = nRT$$

$$V = \frac{nRT}{p}$$

$$\frac{\cancel{nR}T}{p} = \frac{\cancel{nR}\cdot 1{,}01T}{p + 10^3}$$

$$p + 10^3 = 1{,}01p$$

$$p(1{,}01 - 1) = 10^3$$

$$p = \frac{10^3}{1{,}01 - 1} = 10^5 Pa$$

12.6

$V = 10\,L$; $\rho_z = 1{,}293\ \frac{kg}{m^3}$

$p_1 = 10^5\,Pa$; $c_z = 720\ \frac{J}{kgK}$ $\boxed{V = \text{konst.}}$

$p_1' = 5 \cdot p_1$

$Q = ?$

$$n = \frac{m}{M} = \frac{N}{N_A} = \frac{V}{V_{mo}}$$

$$\frac{p_1}{T_1} = \frac{5p_1}{T_2}$$

$$\boxed{T_2 = 5T_1}$$

$$p_1 V = \frac{V}{V_{mo}} R T_1$$

$$T_1 = \frac{p_1 V_{mo}}{R} = 269{,}43\,K$$

$$Q = mc\Delta T$$

$$Q = \rho_z V c_z (T_2 - T_1)$$

$$Q = \rho_z V c_z (4T_1)$$

$$Q = 10033{,}14\,J \approx 10^4\,J$$

12.5

$m_1' = 21\,kg$ // masa He i posude $m_1' = m_1 + m$

$t_1 = -3°C$

$p_1 = 6{,}5 \cdot 10^6\,Pa$

$t_1 = t_2 = -3°C$

$p_2 = 2 \cdot 10^6\,Pa$

$m_2' = 20\,kg$ // masa He i posude $m_2' = m_2 + m$

$p_3 = 1{,}5 \cdot 10^7\,Pa$

$t_3 = 27°C$

$m_3 = ?$

$$pV = nRT = \frac{m}{M} RT$$

(1) → (2) izotermni proces $\boxed{T_1 = T_2}$

$$p_1 V = \frac{m_1}{M} R T_1$$

$$p_2 V = \frac{m_2}{M} R T_2$$

$$\frac{p_1}{p_2} = \frac{m_1}{m_2}$$

→

$$\boxed{m_1 = m_2 \frac{p_1}{p_2}}$$

$$m_1' = m_1 + m$$

$$m_2' = m_2 + m$$

$$m_1' - m_2' = (m_1 + m) - (m_2 + m) = m_1 - m_2$$

$$m_1' - m_2' = m_2 \frac{p_1}{p_2} - m_2$$

$$m_1' - m_2' = m_2 \left(\frac{p_1}{p_2} - 1\right)$$

$$m_2 = \frac{m_1' - m_2'}{\left(\frac{p_1}{p_2} - 1\right)}$$

12.7

$r = 1\,cm = 0{,}01\,m$

$T = 393\,K$

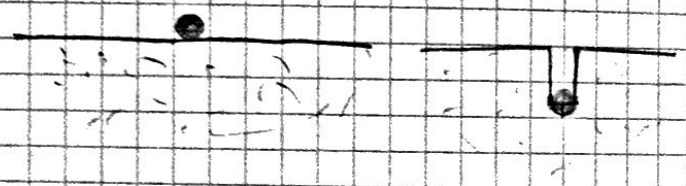
$c_s = 475 \frac{J}{kgK}$

$\rho_0 = 0{,}9 \cdot 10^3 \frac{kg}{m^3}$

$\rho_s = 7{,}9 \cdot 10^3 \frac{kg}{m^3}$

$L_t = 3{,}34 \cdot 10^5 \frac{J}{kg}$

$h = ?$



$$Q_s = Q_1 + Q_2$$

$$m_s c_s \Delta T = L_t m_\ell + m_\ell c_\ell \Delta T$$

$$\rho_s \left(\frac{4}{3} r^3 \pi\right) c_s (T - 0) = L_t \rho_\ell \left(r^2 \pi h' + \frac{2}{3} r^3 \pi\right)$$

$$h' = \left[\frac{\rho_s \left(\frac{4}{3} r^3 \pi\right) c_s T}{L_t \rho_\ell} - \frac{2}{3} r^3 \pi\right] \frac{1}{r^2 \pi} =$$

$$= 0{,}0133\,m$$

$$h = h' + r = 0{,}0233\,m = 2{,}33\,cm$$

12.10

$m = 5\,kg$

$t_1 = 700°C$

$Q = 5 \cdot 10^4\,J$

$c = (231{,}2 + 0{,}1042t)\,\frac{J}{kgK}$

$t_2 = ?$

$T = t_2 + 273$

$$Q = m\,c(t_1)(T - (t_1 + 273))$$

$$m\,c(t_2)(T) = m\,c(t_1)(T - t_1 - 273)$$

$$231{,}2 + 0{,}1042t_2 = (231{,}2 - 0{,}1042t_1)$$

12.8

$V_v = 200\,cm^3 = 2 \cdot 10^{-2}\,m^3$

$T_v = 303\,K$

$m_l = 10\,g = 0{,}01\,kg$

$T_l = 273\,K$

$L_t = 3{,}34 \cdot 10^5\,\frac{J}{kg}$

$T_{konačno} = ?$

$m_v = \rho_v V_v = 0{,}2\,kg$

$$m_v c_v (T_v - T) = m_l L_t + m_l c_v (T - T_l)$$

$$m_l c_v T_l + m_v c_v T_v - m_l L_t = (m_v c_v + m_l c_v) T$$

$$T = \frac{m_l c_v T_l + m_v c_v T_v - m_l L_t}{m_v c_v + m_l c_v}$$

$$T = 297{,}8\,K$$

12.9

$m_l = 0{,}1\,kg$

$m_v = 0{,}1\,kg$

$T_v = 273\,K$

$T_p = 373\,K$

$L_t = 3{,}35 \cdot 10^5\,\frac{J}{kg}$

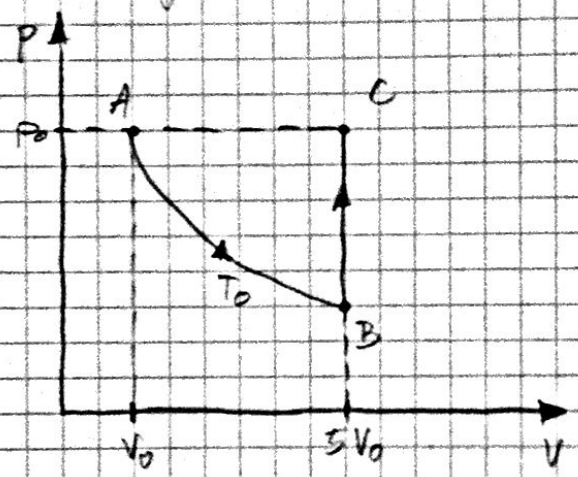
$L_i = 22{,}6 \cdot 10^5\,\frac{J}{kg}$

$c = 4{,}13 \cdot 10^3\,\frac{J}{kg\,K}$

$\Delta m = ?$

Prenjeri/Kulišić: Termodinamika

13.1 Tri mola idealnog plina volumena V_0 na početnoj temperaturi $T_0 = 273\,K$ izotermno se šire do konačnog volumena $5V_0$, a nakon toga se izohorno griju sve dok se tlakovi konačnog i početnog stanja ne izjednače. Ukupna količina topline predana plinu za vrijeme procesa $Q = 80\,kJ$. Odredite adijabatski koeficijent plina.



$$p_0 V_0 = nRT_0$$

$$p_0 5V_0 = nRT_1$$

$$T_1 = \frac{p_0 5V_0}{nR} = 5T_0$$

$$W = nRT_0 \ln\frac{V}{V_0} = nRT_0 \ln 5$$

$$Q' = nC_V(T_1 - T_0)$$

$$Q = W + Q'$$

$$Q = nRT_0 \ln 5 + 4nC_V T_0$$

$$C_V = \frac{Q - nRT_0 \ln 5}{4nT_0}$$

$$C_p = C_V + R$$

$$\kappa = \frac{C_p}{C_V} = 1 + \frac{4}{\frac{Q}{nRT_0} - \ln 5}$$

$$\kappa = 1{,}39$$

13.3 Jednoatomni idealni plin na temperaturi 27°C pri tlaku $p_1 = 1{,}25 \cdot 10^6$ Pa ima volumen $V_1 = 4 \cdot 10^{-3}$ m³. Ako se plinu izobarno dovede količina topline $2 \cdot 10^4$ J, kolika će biti konačna temperatura i volumen plina? Koliko se pritom povećala unutrašnja energija plina? Koliki je rad obavio plin?

$$c_p = \frac{5}{2}\frac{R}{M}$$

$$\Delta T = \frac{\Delta Q}{m c_p} = \frac{2 \Delta Q T_1}{5 p_1 V_1} = 480\,K$$

$$p_1 V_1 = \frac{m}{M} R T_1$$

$$T_2 = T_1 + \Delta T = 780\,K = 507°C$$

$$p_1 V_1 = n R T_1$$

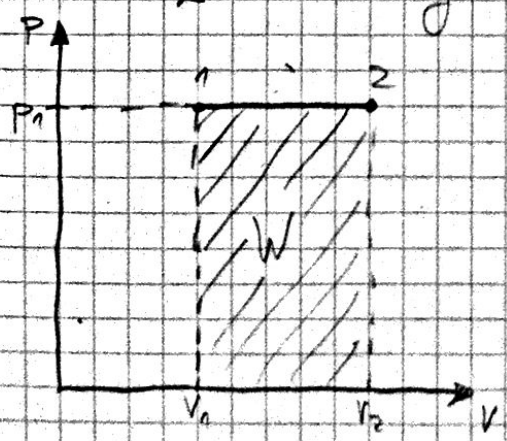
$$p_1 V_2 = n R T_2$$

$$V_2 = \frac{T_2}{T_1} V_1 = 1{,}04 \cdot 10^{-2}\ m^3$$

$$\Delta U = \frac{3}{2} n R \Delta T = \frac{3}{2} \frac{p_1 V_1}{R T_1} R \Delta T = \frac{3}{2} \frac{p_1 V_1}{T_1} \Delta T = 1{,}2 \cdot 10^4\,J$$

$$\Delta W = \int_{V_1}^{V_2} p_1 dV = p_1 (V_2 - V_1)$$

$$\Delta W = 8 \cdot 10^3\,J$$



$$\Delta Q = \Delta U + \Delta W = 12000\,J + 8000\,J$$

$$\Delta Q = 20000\,J$$

13.4 Jedan mol dvoatomnog plina nalazi se na temperaturi 10°C. Adijabatskom se ekspanzijom volumen plina udvostručuje i zatim se izotermnom kompresijom dovodi na svoju početnu vrijednost. Koliku količinu topline plin mora dobiti da bi se vratio u početno stanje?

$$V_1 T_1^{\frac{1}{\kappa-1}} = V_2 T_2^{\frac{1}{\kappa-1}}$$

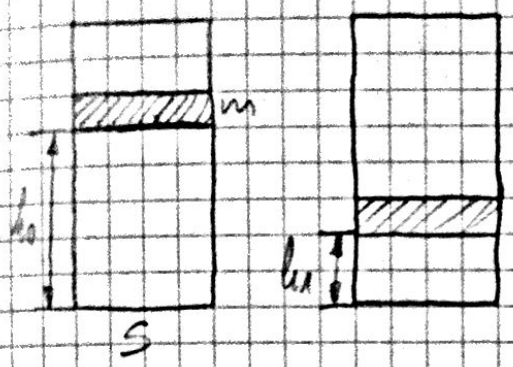
$$\kappa = 1 + \frac{2}{i} = \frac{i+2}{i} = \frac{5+2}{5} = 1{,}4$$

$$T_2 = 2^{1-\kappa} T_1$$

$$Q = \frac{nR}{\kappa - 1} \Delta T$$

$$|Q| = 1424\,J$$

13.5 U vertikalno postavljenoj cijevi adijabatskih zidova, zatvorenoj na oba kraja, učvršćen je teret mase 40kg na visini 1m od dna cijevi. Ispod njega nalazi se 0,16g He na temperaturi 27°C. Unutrašnjost klipa je vakuum. Na kojoj se visini zaustavlja klip pošto se oslobodi?



$$dQ = 0$$

$$\kappa = 1 + \frac{2}{i} = \frac{5}{3}$$

$$W = \frac{1}{\kappa - 1}(p_1 V_1 - p_2 V_2) = mg(h_1 - h_0)$$

$$p_2 = \frac{mg}{S}$$

$$mg(h_1 - h_0) = \frac{3}{2}\left(nRT_1 - \frac{mg}{S} h_1 S\right)$$

$$h_1 = \frac{2}{5} h_0 + \frac{3}{5}\frac{nRT_1}{mg} = 0{,}5526\,m$$

13.15 Carnotov stroj radi s koeficijentom iskorištenja $\eta_1 = 40\%$. Ako se temperaturu hladnijeg spremnika $T_2 = 3°C$ drži konstantnom, koeficijent iskorištenja poraste na $\eta_2 = 50\%$ uz povišenja temperature toplijeg spremnika. Koliko iznosi to povišenja temperature?

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$T_1 = \frac{T_2}{1-\eta_1} \qquad T_1' = \frac{T_2}{1-\eta_2}$$

$$\Delta T = T_1' - T_1 = \frac{T_2(\eta_2 - \eta_1)}{(1-\eta_1)(1-\eta_2)}$$

$$\Delta T = 34\,K$$

Sudac/Kukić | Termodinamika

13.1

$\kappa = 1{,}4$

$p = \text{konst.}$

$V = \text{konst.}$

$C_p - C_v = R$

$C_v, C_p = ?$

$$\kappa = \frac{C_p}{C_v}$$

$$C_v = \frac{1}{n}\left(\frac{dQ}{dT}\right)_{V=\text{konst.}} \quad C_p = \frac{1}{n}\left(\frac{dQ}{dT}\right)_{p=\text{konst.}}$$

$$dQ = dU + dW$$

$$n C_p dT = n C_v dT + p dV$$

$$p dV = nR dT$$

$$\cancel{n} C_p \cancel{dT} = \cancel{n} C_v \cancel{dT} + \cancel{n} R \cancel{dT}$$

$$C_p - C_v = R \qquad C_v \kappa = C_p$$

$$C_v = \frac{C_p}{\kappa}$$

$$C_v(\kappa - 1) = R$$

$$C_v = \frac{R}{\kappa - 1}$$

$$C_p = \frac{\kappa R}{\kappa - 1}$$

$$C_v = 20{,}8 \ \frac{J}{mol\,K}$$

$$C_p = 29{,}1 \ \frac{J}{K}$$

$\boxed{13.2}$

$n = 1000\,mol$

$T_1 = 20°C$

$T_2 = 35°C$

$i = 3$

$p = konst.$

$\Delta S = ?$

$$Q = mc\Delta T$$

$$(m_1 + m_2)c\,T = m_1 c T_1 + m_2 c T_2$$

$$T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2}$$

$$m = \rho V \qquad pV_k = nRT_k$$

$$n = \frac{m}{M}$$

$$m = nM$$

$$\left.\begin{aligned} pV_1 &= nRT_1 \\ pV_2 &= nRT_2 \end{aligned}\right\} \qquad \frac{V_1}{V_2} = \frac{T_1}{T_2} = \boxed{\frac{V_k}{V_{1/2}} = \frac{T_k}{T_{1/2}}}$$

$$V_1 = V_2 \frac{T_1}{T_2}$$

$$p = \frac{nRT_1}{V_1}$$

$$T = \frac{\rho V_1 T_1 + \rho V_2 T_2}{\rho V_1 + \rho V_2} = \frac{\rho V_2 \frac{T_1}{T_2} T_1 + \rho V_2 T_2}{\rho V_2 \frac{T_1}{T_2} + \rho V_2} =$$

$$= \frac{\frac{T_1^2}{T_2} + T_2}{\frac{T_1}{T_2} + 1} = 300{,}5\,K$$

$$c_p = \frac{i+2}{2}R \qquad c_p - c_v = R \qquad c_v = c_p - R = R\left(\frac{i+2}{2} - 1\right)$$

$$dQ = dU + dW$$

$$dQ = mc_v\,dT + p\,dV$$

$$\Delta S_1 = \int_{T_1}^{T} mc_v \frac{dT}{T} + \int_{V_1}^{V_k} p\frac{dV}{T} = m\frac{c_v}{M}\int_{T_1}^{T}\frac{dT}{T} + nR\int_{V_1}^{V_k}\frac{dV}{V} =$$

$$= nc_v \ln\frac{T}{T_1} + nR\ln\frac{V}{V_1} = nc_v\ln\frac{T}{T_1} + nR\ln\frac{T}{T_1}$$

$$\Delta S_2 = \int_{T_2}^{T} mc_v\frac{dT}{T} + \int_{V_2}^{V} nR\frac{dV}{V} = nc_v\ln\frac{T}{T_2} + nR\ln\frac{V}{V_2} =$$

$$= nc_v\ln\frac{T}{T_2} + nR\ln\frac{T}{T_2}$$

$$\Delta S = \Delta S_2 + \Delta S_1 = \ln\frac{T}{T_2}\left[nC_V + nR\right] + \ln\frac{T}{T_1}\left[nC_V + nR\right] =$$

$$= 12{,}25 \ \mathrm{J\,K^{-1}}$$

13.3

$m_1 = 10\,g = 0{,}01\,kg$ $\quad C_V = 4186 \ \frac{J}{kg\,K}$ $\quad nR = \frac{m}{M}R$

$t_1 = 800\,°C$

$m_2 = 20\,g = 0{,}02\,kg$

$t_2 = 15\,°C$

$$Q = mc\Delta T$$

$$(m_1 + m_2)C_V T = m_1 C_V T_1 + m_2 C_V T_2$$

$$T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} =$$

$$= 316{,}33\,K$$

$$\Delta S = \int \frac{dQ}{T} = m_1 c \int_{T_1}^{T} \frac{dT}{T} + m_2 c \int_{T_2}^{T} \frac{dT}{T} =$$

$$= m_1 c \ln\frac{T}{T_1} + m_2 c \ln\frac{T}{T_2} = 0{,}95 \ \mathrm{J\,K^{-1}}$$

13.5

$P = 600\,MW$

$t_1 = 3000\,°C$

$t_2 = 20\,°C$

η, $Q_{odvedena} = ?$

$$\eta = 1 - \frac{T_2}{T_1} = 0{,}488$$

$$Q = Q_1 + Q_{odvedena}$$

$$\eta = \frac{W}{Q}$$

$$Q_{odvedena} = Q - W = 625\,J$$

$$Q = \frac{W}{\eta} = 1225\,J$$

13.8 Dvije litre dušika nalaze se pod tlakom 10^5 Pa. Koji iznos topline valja predati dušiku za udvostručenje njegova volumena? Pri tom je pritisak stalan.

$$W = p\Delta V \quad , \quad p = \text{konst.}$$

$$pV_1 = nRT_1$$

$$pV_2 = nRT_2$$

$$p = \frac{nRT_1}{V_1} = \frac{nRT_2}{V_2} = \frac{nR\Delta T}{\Delta V} \qquad \Delta T = \frac{p\Delta V}{nR}$$

$$Q = nC_p\Delta T = C_p\frac{p\Delta V}{R}$$

$$C_p = \frac{i+2}{2}R = \frac{7}{2}R$$

$$Q = \frac{7}{2}p\Delta V = 7000\ \text{J}$$

13.9 Zrak mase 1kg prevodi se iz početnog stanja ($p_1 = 1$ bar, $T_1 = 300$K) u konačno stanje ($T_2 = 600$K):

a) izohornim zagrijavanjem

b) izobarnim zagrijavanjem i

c) adijabatskom kompresijom ($\kappa = 1{,}4$)

$c_p = 1000\ \text{J kg}^{-1}\text{K}^{-1}$

$c_v = 713\ \text{J kg}^{-1}\text{K}^{-1}$

a)

(1) $T_1 = 300$K, $p_1 = 1$ bar $= 10^5$ Pa, $V_1 = \text{konst.}$ $\rightarrow$ (2) $T_2 = 600$K, $p_2 = ?$, $V_2 = V_1$

$$p_1V_1 = nRT_1$$

$$V_1 = \frac{nRT_1}{p_1} = 0{,}216\ \text{m}^3$$

$$\frac{T_2}{p_2} = \frac{T_1}{p_1} \qquad p_2 = p_1\frac{T_2}{T_1} = 2\ \text{bar}$$

$$\Delta S = S_2 - S_1 = m c_v \ln \frac{T_2}{T_1} = 0{,}5\ kJ\,K^{-1}$$

$$dQ = dU + \cancel{dW} = 0 \quad \text{izovolumno}$$

$$dQ = dU = m c_v dT \,/ : T$$

$$\frac{dQ}{T} = dS = \frac{m c_v dT}{T} \quad / \int_{T_1}^{T_2}$$

$$\Delta S = m c_v \ln \frac{T_2}{T_1} = 0{,}5\ kJ\,K^{-1}$$

b) $T_2 = 600\,K$, $p_2 = 1\,bar$

$$V_3 = \frac{V_1 T_2}{T_1} = 1{,}72\ m^3$$

$$\Delta S = S_2 - S_1 = m c_p \ln \frac{T_2}{T_1} = 0{,}7\ kJ\,K^{-1}$$

13.12 Kolika je promjena entropije pri miješanju 1 kg vode temperature 17°C i 0,5 kg vode temperature 60°C?

$t_1 = 12°C$ $\quad c_v = 4186 \frac{J}{kg\,K}$

$t_2 = 60°C$

$m_1 = 1\,kg$

$m_2 = 0{,}5\,kg$

$\Delta S = ?$

$$(m_1 + m_2) c_v T = m_1 c_v T_1 + m_2 c_v T_2$$

$$T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = 304{,}2\,K$$

$$dQ = dU = m c_v \Delta T$$

$$\Delta S = \Delta S_1 + \Delta S_2 = m_1 c \int_{T_1}^{T} \frac{dT}{T} + m_2 c \int_{T_2}^{T} \frac{dT}{T} =$$

$$= m_1 c \ln \frac{T}{T_1} + m_2 c \ln \frac{T}{T_2} = 12{,}846\ J\,K^{-1}$$

13.13 Specifični i molarni toplinski kapacitet plinova o temperaturi. Ta se ovisnost u prvoj aproksimaciji može predočiti linearnom funkcijom. Ako je za zrak ta ovisnost dana izrazom $C_p = (27,12 + 0,006T)\,J/(mol\,K)$, valja izračunati promjenu ~~entalpije~~ i entropije pri izobarnom zagrijavanju 1 mola zraka od 273K do 820K.

$$\Delta S = \int \frac{dQ}{T} = \int \frac{dU + dW}{T} = \int \frac{nC_V dT + nRdT}{T} =$$

$$= n\int_{T_1}^{T_2} C_p \frac{dT}{T} \qquad C_p = a + bT$$

$$\Delta S = na \ln \frac{T_2}{T_1} + nb(T_2 - T_1) = 33,1\,J/K\,mol$$

13.16 Odredite ukupnu promjenu entropije idealnog plina u Carnotovu procesu.

① U početnom stanju entropija plina jest S_1
Nakon izotermnog širenja plina promjena je entropije

$$S_2 - S_1 = \int \frac{dQ}{T} = \int \frac{pdV}{T} = \frac{m}{M} R \int \frac{dV}{V} = \frac{m}{M} R \ln \frac{V_2}{V_1}$$

② Nakon adijabatskog širenja

$$S_3 - S_2 = \int \frac{dQ}{T} = 0, \text{ jer je } dQ = 0$$

③ Nakon izotermnog sabijanja

$$S_4 - S_3 = \frac{m}{M} R \ln \frac{V_4}{V_3} = -\frac{m}{M} R \ln \frac{V_2}{V_1}$$

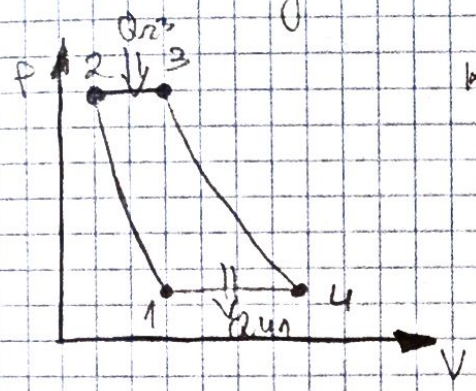
$$S_4 - S_3 = -\frac{m}{M} R \ln \frac{V_2}{V_1}, \text{ jer je } \boxed{\frac{V_3}{V_4} = \frac{V_2}{V_1}}$$

Tijekom adijabatskog sabijanja $dQ = 0$, pa je promjena entropije $S_1 - S_4 = 0$ = nula. Ukupna promjena entropije

$$\Delta S = \frac{m}{M} R \ln \frac{V_2}{V_1} - \frac{m}{M} R \ln \frac{V_2}{V_1}$$

$$\Delta S = 0$$

13.8 Joulev se kružni proces sastoji od dviju adijabata. Kolika je koeficijent iskorištenja ako je omjer tlakova $\frac{p_2}{p_1} = 10$, a adijabatski koeficijent $\kappa = 1{,}4$?



$\kappa = 1{,}4$

① ①→②

$p_1 = \frac{p_2}{10}$ $dQ = 0$

T_1, V_1

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\kappa} = \frac{1}{10} \Rightarrow \frac{V_2}{V_1} = \sqrt[\kappa]{10}$$

T_2

②→③

$Q_{23} = n C_p (T_3 - T_2)$ (+) uložena toplina

④→①

$Q_{41} = n C_p (T_1 - T_4)$ (−) odvedena toplina

$$\eta = \frac{W}{Q_{23}} = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}} = 1 + \frac{T_1 - T_4}{T_3 - T_2}$$

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\kappa-1}{\kappa}}$$

$$\eta = 1 + \frac{\left(\frac{T_1}{T_2}\right) T_2 - \left(\frac{T_4}{T_3}\right) T_3}{T_3 - T_2}$$

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\kappa-1}{\kappa}} = \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}$$

$$= 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\kappa-1}{\kappa}} = 0{,}410$$

Zadaná data:

Příklad 10.1

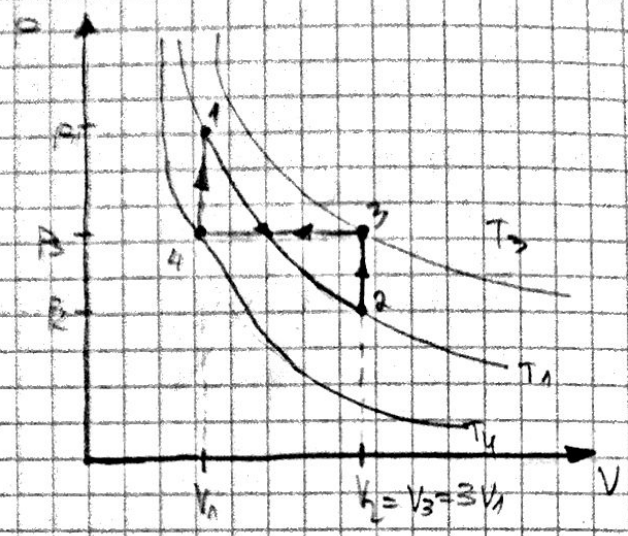
p_1

$T_1 = 350K$

$V_2 = 3V_1$

$T_3 = 400K$

$Q_{12} = ?$



①

$p_1, T_1 = 350K, V_1$

① → ② Izotermo

②

$T_1 = T_2$

$p_2, V_2 = 3V_1$

② → ③ Izobarno

③

$T_3 = 400K$

$p_3, V_3 = V_2 = 3V_1$

③ → ④ Izotermo

$p_4 = p_1$

$T_4, V_4 = V_1$

④ → ① Izobarno

$V_1, p_1, T_1 = 350K$

$$p_1 V_1 = nRT_1$$

$$V_1 = nR\frac{T_1}{p_1}$$

$$p_2 V_2 = nRT_1$$

$$V_2 = nR\frac{T_1}{p_2}$$

$$3V_1 = nR\frac{T_1}{p_2}$$

$$3nR\frac{T_1}{p_1} = nR\frac{T_1}{p_2}$$

$$3p_2 = p_1$$