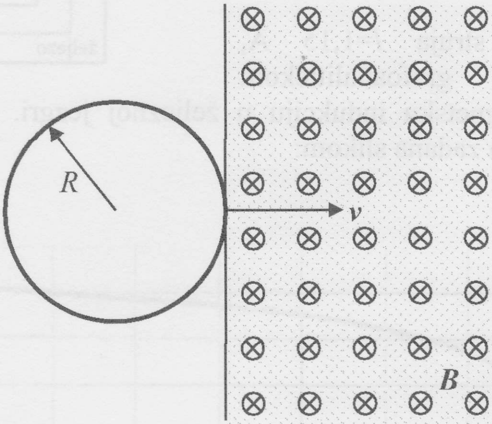
ZI 2015

Zadatke je rješio prof. Dadić.
NE ODGOVARAM za moguće pogreške u prepisivanju !

Ime i prezime____Matični broj________________________

1. Vodljiva okrugla petlja polumjera R giba se konstantnom brzinom v prema slici. U trenutku $t=0$ počinje ulaziti u dio prostora u kojemu vlada homogeno magnetsko polje indukcije B prema slici. Izvan omeđenog prostora $B=0$. Odredite u vremenskom intervalu $0<t<R/v$ izraz za napon induciran u petlji $e(t)=?$

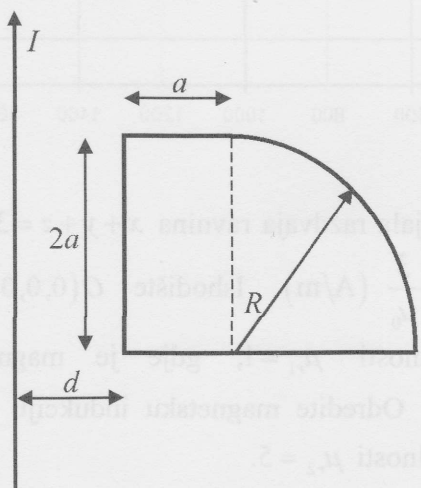


2. Jakost električnog polja sinusno promjenjivog ravnog vala koji se prostire u idealnom dielektriku zadana je s $\vec{E}=(\vec{a}_x-\vec{a}_y)\cos\left[2\cdot10^8 t-2(x+y)\right]$ V/m. Zadano je $\mu_r=1$.

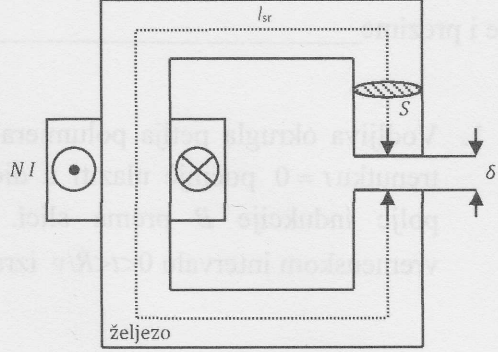
   Odredite:

   a. vektor magnetskogpolja,
   b. Poyntingov vektor,
   c. valnu impedanciju sredstva,
   d. relativnu dielektričnu konstantu sredstva.

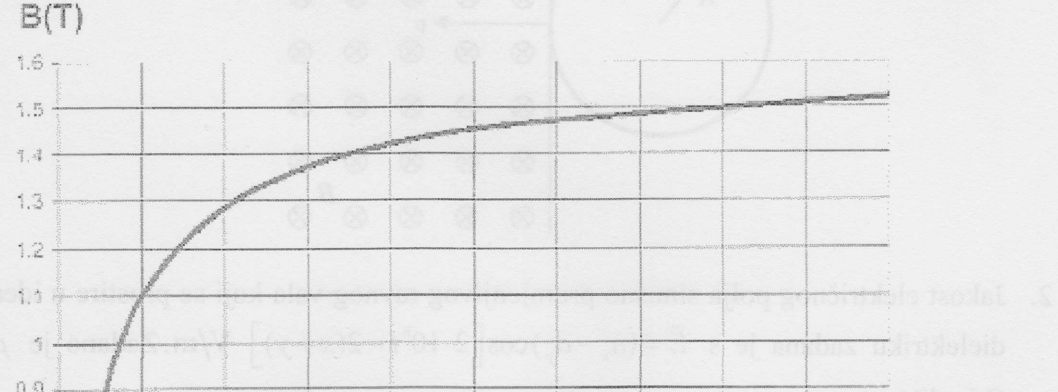
3. U ravnini beskonačno dugog ravnog vodiča nalazi se petlja prema slici. Odredite međuinduktivitet vodiča i petlje $a=d=1$ m , $R=2$ m.



a. Neka je zadana magnetska indukcija u feromagnetskom materijalu (željezu) $B_{Fe} = 1{,}1\ \mathrm{T}$. Odredite potrebnu struju I uz pretpostavku da je relativna magnetska permeabilnost feromagnetskog materijala $\mu_r = 4000$.



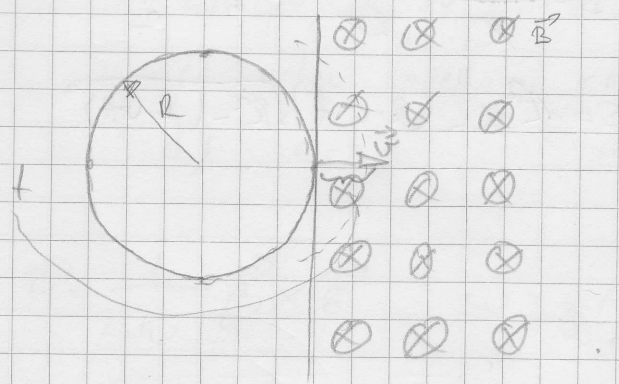
b. Ukoliko je struja I=1,15 A, odredite grafoanalitičkom metodom magnetsku indukciju u željeznoj jezgri. Pritom je krivulja magnetiziranja zadana slikom:



5. Dva magnetska materijala razdvaja ravnina $x + y + z = 3$. Na granici se nalazi strujni oblog $\vec{K} = \vec{a}_x \dfrac{3}{\mu_0} - \vec{a}_y \dfrac{3}{\mu_0}$ (A/m). Ishodište $O(0,0,0)$ nalazi se u sredstvu s relativnom permeabilnosti $\mu_{r1} = 1$ gdje je magnetska indukcija zadana s

(1.) Vodljiva okrugla petlja polumjera r giba se konstantnom brzinom v prema slici. U trenutku $t=0$ počinje ulaziti u dio prostora indukcije B prema slici. Izvan označenog prostora $B=0$. Odredite u vremenskom intervalu $0<t<\frac{R}{v}$ izraz za napon induciran u petlji $e(t)$

$x = v \cdot t$



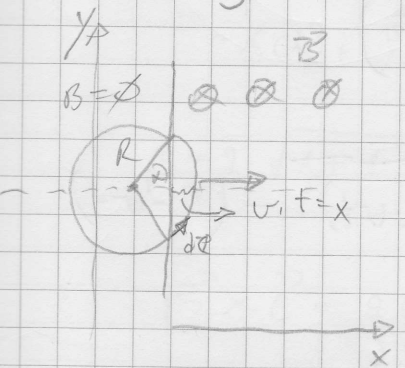
$$e = -\frac{d\phi}{dt}$$

$$e = -\iint_S \frac{\partial \vec{B}}{\partial t}\vec{n}\,ds + \oint_c (\vec{v}\times\vec{B})\,d\vec{l}$$

$$B = konst. \Rightarrow e = \oint_c (\vec{v}\times\vec{B})\,d\vec{l}$$

1.)

$$e = \oint (\vec{v}\times\vec{B})\,d\vec{l}$$



$$x\text{-}y:\ d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y$$

$$\vec{v}\times\vec{B} = vB\,\vec{a}_y$$

$$e = \int vB\vec{a}_y(dx\,\vec{a}_x + dy\,\vec{a}_y)$$

$$e = \int vB\,dy = vB\cdot 2\cdot\int_{y=0}^{R\sin\alpha} dy$$

$B = konst.$

$B \neq B(y)$

$$e = R2vB\cdot\sin\alpha$$

$x = v \cdot t$

$$e = R\cdot 2vB\sqrt{1-\frac{x^2}{R^2}} = R2vB\sqrt{1-\frac{R-vt}{R^2}}$$

$$\cos\alpha = \frac{x}{R}$$

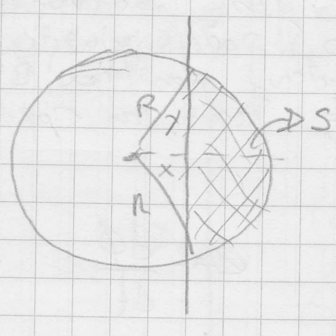
$$e = 2vB\sqrt{R^2-(R-v\cdot t)^2}$$

$$e = 2vB\sqrt{t(2Rv - v^2 t)}$$

$$\sin\alpha = \sqrt{1-\frac{x^2}{R^2}}$$

$$e = Blv = B\cdot v\cdot 2y$$

2.) $e = -\frac{d\phi}{dt}$



$$e = -\frac{d\phi}{dt} = -B\frac{dS}{dt}$$

$$S = \frac{R^2 \cancel{\pi}}{\cancel{2}} \cdot \frac{\cancel{2}\alpha}{\cancel{\pi}} - 2 \cdot \frac{x \cdot y}{2}$$

$$S = R^2 \alpha - (R - vt)\sqrt{R^2 - (R - vt)^2}$$

$$\cos\alpha = \frac{R - vt}{R}$$

$$\alpha = \arccos\frac{R - vt}{R}$$

$$S = R^2 \arccos\frac{R - vt}{R} - (R - vt)\cdot\sqrt{R^2 - (R - vt)^2}$$

$$e = -B\frac{d}{dt}\left(R^2 \arccos\frac{R - vt}{R} - (R - vt)\sqrt{R^2 - (R - vt)^2}\right)$$

$$e = -B\left\{\frac{-R^2}{\sqrt{1 - \left(\frac{R - vt}{R}\right)^2}}\left(-\frac{v}{R}\right) - (-v)\sqrt{R^2 - (R - vt)^2} - (R - vt)\right.$$

$$\left.\cdot\frac{1}{2\sqrt{R^2 - (R - vt)^2}}\cdot(-2)(R - vt)(-v)\right\}$$

$$e = B\left\{\frac{v^2 t(2R - vt)}{\sqrt{R^2 - (R - vt)^2}} + v\sqrt{R^2 - (R - vt)^2}\right\}$$

Jakost el. polja sinusno promjenjivog ravnog vala koji se prostire u idealnom dielektriku zadana je s:

$$\vec{E} = (\vec{a_x} - \vec{a_y}) \cos[2 \cdot 10^8 t \underbrace{- 2(x+y)}_{-\vec{\beta}\vec{r}}] \frac{V}{m}, \quad \mu$$

Odredite: a) vektor magn. polja? $\vec{H} = ?$

b) poyntingov vektor? $\vec{N} = ?$

c) valna impedancija sredstva? $Z = ?$

d) $\varepsilon_r = ?$

a)

$$\vec{H} = \frac{1}{\mu\omega} \vec{\beta} \times \vec{E}, \quad \omega = 2 \cdot 10^8 \text{ Rad/s}$$

$$\left.\begin{aligned} \vec{\beta} &= \beta_x \vec{a_x} + \beta_y \vec{a_y} + \beta_z \vec{a_z} \\ \vec{r} &= x\vec{a_x} + y\vec{a_y} + z\vec{a_z} \end{aligned}\right\} \Rightarrow \vec{\beta} = +2(\vec{a_x} + \vec{a_y})$$

$$\beta = 2 \cdot \sqrt{1+1} = 2\sqrt{2} \left(\frac{r}{m}\right)$$

$$\vec{\beta} \times \vec{E} = 2(\vec{a_x} + \vec{a_y}) \times (\vec{a_x} - \vec{a_y}) \cdot \cos[\ ]$$

$$\vec{\beta} \times \vec{E} = 2[-\vec{a_z} - \vec{a_z}] \cdot \cos[\ ]$$

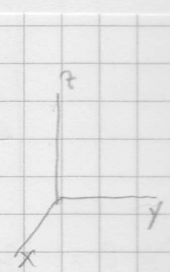
$$\vec{H} = \frac{1 \cdot (-4)}{2 \cdot 10^8 \cdot 4 \cdot \pi \cdot 10^{-7}} \cos[\ ]$$

$$\vec{H} = \frac{-\vec{a_z}}{20\pi} \cos(2 \cdot 10^8 t - 2(x+y)) \left(\frac{A}{m}\right)$$

b) $\vec{N} = ?$

$$\vec{N} = \vec{E} \times \vec{H} = (\vec{a_x} - \vec{a_y}) \times (-\vec{a_z}) \cdot \frac{1}{20\pi} \cdot \cos^2(\ )$$

$$\vec{N} = (\vec{a_y} + \vec{a_x}) \frac{1}{20\pi} \cos^2(\omega t - 2(x+y))$$



d)

$$c = \frac{c_0}{\sqrt{\varepsilon_r}}, \quad \beta = \frac{\omega}{c}, \quad c = \frac{\omega}{\beta} = \frac{2 \cdot 10^8}{2\sqrt{2}} = \frac{10^8}{\sqrt{2}} \text{ m/s}$$

4. Za magnetski krug prema slici zadano je $S = 3\ cm^2$, $L = 25$ cm, $\delta = 0.27$ mm

$$\varepsilon_r = \left(\frac{c_0}{v}\right)^2 = \left(\frac{3\cdot 10^8}{10^8}\cdot\sqrt{2}\right)^2 = 18$$

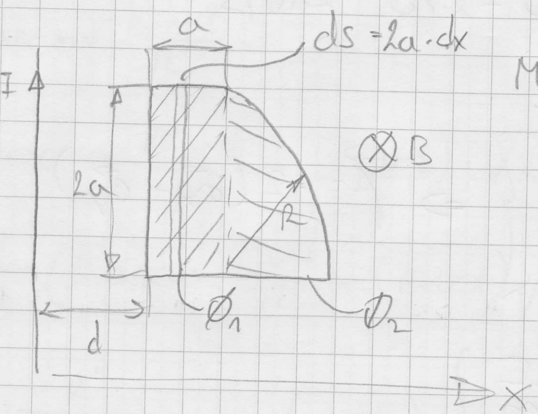
c)

$$Z = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{Z_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{3\sqrt{2}} = \frac{40\pi}{\sqrt{2}}\ (\Omega)$$

3. U ravnini beskonačno dugog ravnog vodiča nalazi se petlja prema slici. Odredite međuinduktivitet M.

$a = d = 1\,m$
$R = 2\,m$

$M = ?$



$$M = \frac{\Phi}{I} = \frac{\Phi_1 + \Phi_2}{I}$$

1°)

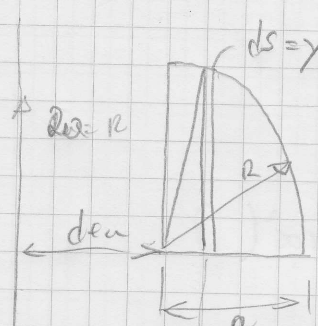
$\Phi_1 = ?$

$$\Phi_1 = \int_{x=d}^{d+a} \frac{\mu_0 I}{2\pi x}\cdot 2a\cdot dx \qquad \Phi_1 = \frac{\mu_0 I\cdot 2a}{2\pi}\ln\frac{d+a}{d}$$

2°)

$\Phi_2 = ?$

$$d+a = R = \overset{d}{1} + \overset{a}{1} = \overset{R}{2}$$



$$ds = y\cdot dx = \sqrt{R^2 - (x-R)^2}\cdot dx$$
$$= \sqrt{\cancel{R^2} - x^2 + 2xR - \cancel{R^2}}\ dx$$
$$ds = \sqrt{2Rx - x^2}\ dx$$

$$\Phi_2 = \int_{x=R}^{2R} \frac{\mu_0 I}{2\pi x}\sqrt{2Rx - x^2}\,dx$$

$$\Phi_2 = \frac{\mu_0 I}{2\pi}\int_{x=2}^{4}\frac{\sqrt{4x - x^2}}{x}\,dx = \left| \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right| = \frac{\mu_0 I}{2\pi}\cdot 2\int_{t_1}^{t_2}\frac{\sqrt{t^2(4-t^2)}}{t^2}\cdot t\,dt$$

$$\Phi_2 = \frac{\mu_0 I}{2\pi} \cdot 2 \int_{t_1}^{t_2} \sqrt{4 - t^2}\, dt = \left| \begin{matrix} t = 2\sin k \\ dt = 2\cos k\, dk \end{matrix} \right| = 2 \cdot \frac{\mu_0 I}{2\pi} \cdot \int_{k_1}^{k_2} 2\cos^2 k \cdot 2\, dk$$

$$= \frac{\mu_0 I}{2\pi} \cdot 4 \cdot \int_{k_1}^{k_2} (1 + \cos 2k)\, dk = \frac{\mu_0 I}{2\pi} \left(4k + 2\sin 2k\right) \Big|_{k_1}^{k_2}$$

$x_1 = 2 \quad , \quad x_2 = 4$

$t_1 = \sqrt{2} \qquad t_2 = 2$

$k_1 = \arcsin \frac{\sqrt{2}}{2} \qquad k_2 = \arcsin 1$

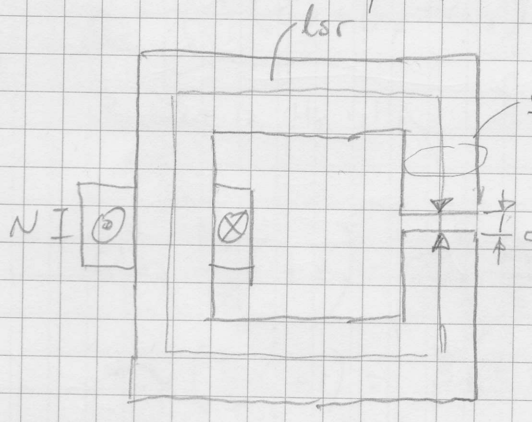
$k_1 = \frac{\pi}{4} \qquad k_2 = \frac{\pi}{2}$

u B.P. str. 425. tip 26c

$$\Phi_2 = \frac{\mu_0 I}{2\pi} \cdot (\pi - 2) \Rightarrow M = \frac{\mu_0}{2\pi} \left[2\ln 2 + \pi - 2\right]$$

2. Magnetski krug prema slici zadano je $S = 3\,cm^2$, $l_{sr} = 25\,cm$, $\delta = 0{,}27\,m$ $N = 280$, $S_{želj.} = S_\delta$. Neka je zadana magn. indukcija u željezu $B_{Fe} = 1.1\,T$. Odredite potrebnu struju I uz pretpostavku da je relativna permeabilnost feromagnetskog materijala $\mu_r = 4000$.

b) ukoliko je $I = 1.15\,A$ odredite graf analitičkom metodom magn. indukciju u jezgri. Krivulja magn. zadana slikom



a) $\oint \vec{H}\, d\vec{l} = NI$

$H_{Fe} \cdot l_{sr} + H_\delta \cdot \delta = NI$

$B = \mu H$

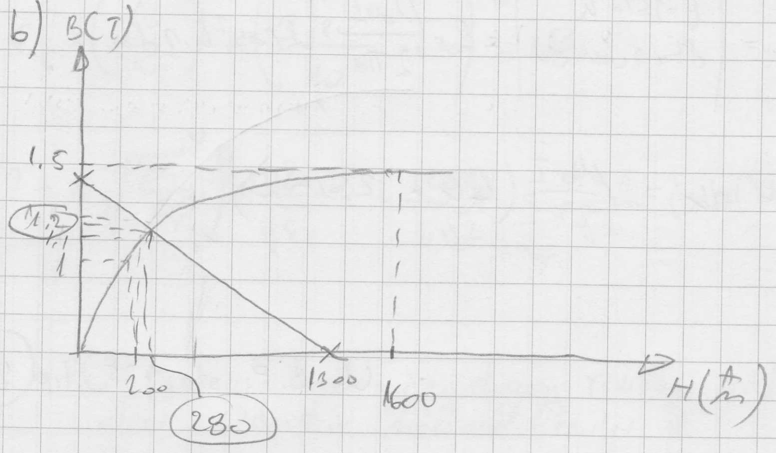
isti jer su $\Phi_{Fe} = \Phi_\delta \Rightarrow \cancel{S}_{Fe} \cdot B_{Fe} = B_\delta \cdot \cancel{S}_\delta$

$B_{Fe} = B_\delta$

$H_{Fe} = \frac{B}{\mu_0 \cdot 4000} \; ; \; H_\delta = \frac{B}{\mu_0}$

$I = \frac{1}{N}\left[\frac{1.1 \cdot 0.25}{\mu_0 \cdot 4000} + \frac{1.1}{\mu_0} \cdot 0{,}27 \cdot 1\right.$

b)



$I = 1{,}15\,A$, $H_{Fe} \cdot l_{sr} + H_\delta \cdot \delta = NI$

$$H_{Fe} \cdot l_{sr} + \frac{B_{Fe}}{\mu_0} \cdot \delta = NI$$

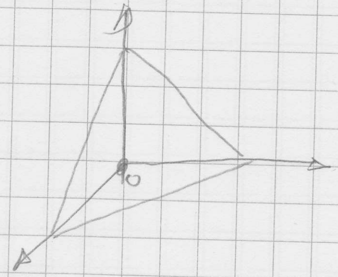
TOČKA PRESJECIŠTA:

1) $H = 0 \quad B = \frac{\mu_0 N I}{\delta} = 1{,}498\,T$

2) $B = \phi \Rightarrow H = \frac{NI}{l_{sr}} = 1288\ A/m$

Dva magn. materijala razdvaja ravnina $x+y+z=3$. Na granici se nalazi strujni oblog $\vec{k}=\vec{a}_x\frac{3}{\mu_0}-\vec{a}_y\frac{3}{\mu_0}\left(\frac{A}{m}\right)$.
Ishodište $O(0,0,0)$ nalazi se u sredstvu s relativnom permeabilnosti $\mu_r=1$, gdje je magn. indukcija $\vec{B}_1=\vec{a}_x-\vec{a}_y+3\vec{a}_z$ (T)
Odredite magn. indukciju $\vec{B}_2=?$ u sredstvu s $\mu_{r2}=5$

$$\vec{n}=\frac{\vec{a}_x+\vec{a}_y+\vec{a}_z}{\sqrt{3}}$$



$$\vec{B}_2=B_{2x}\vec{a}_x+B_{2y}\vec{a}_y+B_{2z}\vec{a}_z$$

(I) $\vec{n}\cdot(\vec{B}_2-\vec{B}_1)=0$

(II) $\vec{n}\times(\vec{H}_2-\vec{H}_1)=\vec{k}$

(I)

$$\frac{1}{\sqrt{3}}(\vec{a}_x+\vec{a}_y+\vec{a}_z)\cdot\left[(B_{2x}-1)\vec{a}_x+(B_{2y}+1)\vec{a}_y+(B_{2z}-3)\vec{a}_z\right]=0$$

(I) $B_{2x}+B_{2y}+B_{2z}=3$

(II)

$$\cancel{\frac{1}{\mu_0}}\cdot\frac{1}{\sqrt{3}}(\vec{a}_x+\vec{a}_y+\vec{a}_z)\times\left[\left(\frac{B_{2x}}{5}-1\right)\vec{a}_x+\left(\frac{B_{2y}}{5}+1\right)\vec{a}_y+\left(\frac{B_{2z}}{5}-3\right)\vec{a}_z\right]=$$

izvučen iz H_2 i H_1

$$=\frac{3\vec{a}_x}{\cancel{\mu_0}}-\frac{3\vec{a}_y}{\cancel{\mu_0}}$$

$$\vec{a}_z\left(\frac{B_{2y}}{5}+1\right)-\vec{a}_y\left(\frac{B_{2z}}{5}-3\right)-\vec{a}_z\left(\frac{B_{2x}}{5}-1\right)+\vec{a}_x\left(\frac{B_{2z}}{5}-3\right)+\vec{a}_y\left(\frac{B_{2x}}{5}-1\right)$$

$$+\vec{a}_x\left(\frac{B_{2y}}{5}+1\right)=3\sqrt{3}\,(\vec{a}_x-\vec{a}_y)\ \cancel{0\cdot\vec{a}_z}$$

izjednačit komponente

$\vec{a}_z$: $\frac{B_{2y}}{5}+1-\frac{B_{2x}}{5}-1=0$ $\qquad$ $\vec{a}_x$: $\frac{B_{2z}}{5}-3-\frac{B_{2y}}{5}+1=3\sqrt{3}$

$\vec{a}_y$: $-\frac{B_{2z}}{5}+3+\frac{B_{2x}}{5}-1=-3\sqrt{3}$

$$\begin{cases} B_{2x}=1-3\sqrt{3}\ (T)\\ B_{2z}=11+10\sqrt{3}\ (T)\\ B_{2y}=-9+5\sqrt{3}\ (T)\end{cases}$$