A1

Zadana je nelinearna diferencijalna jednadžba prvog reda:

$$\dot{y}(t) + [y(t)]^2 = \frac{u^2(t)}{y(t)}$$

Potrebno je:

a) Linearizirati nelinearnu diferencijalnu jednadžbu u okolini radne točke određene s (u_0, y_0) i odrediti prijenosnu funkciju $G(s) = \frac{Y(s)}{U(s)}$.

b) Odrediti za koju radnu točku (u_0, y_0) će nagib odziva na pobudu $\Delta u(t) = S(t)$ u trenutku $t = 0^+$ biti jednak 8.

c) Nacrtati blokovsku shemu kojoj je ulaz stvarni u, a izlaz stvarni y. Koristite samo blokove prijenosne funkcije $G(s)$ iz b) dijela zadatka, sumatore i konstante. Tom se shemom treba aproksimirati ponašanje nelinearnog modela u okolini radne točke.

Rješenje:

Za radnu točku (u_0, y_0) linearizirana diferencijalna jednadžba je:

$$\Delta\dot{y} + 2y_0\Delta y = \frac{2u_0}{y_0}\Delta u - \frac{u_0^2}{y_0^2}\Delta y.$$

Prijenosna funkcija određuje se iz:

$$sY(s) + 2y_0 Y(s) = \frac{2u_0}{y_0}U(s) - \frac{u_0^2}{y_0^2}Y(s).$$

Prijenosna funkcija je:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\frac{2u_0}{y_0}}{s + 2y_0 + \frac{u_0^2}{y_0^2}}.$$

Računanje nagiba odziva na pobudu $\Delta u(t) = S(t)$ u trenutku $t = 0^+$:

$$k = \lim_{s\to\infty} s^2 Y(s) = \lim_{s\to\infty} s^2 G(s)\frac{1}{s} = \lim_{s\to\infty} \frac{\frac{2u_0}{y_0}s}{s + 2y_0 + \frac{u_0^2}{y_0^2}} = \frac{2u_0}{y_0}.$$

Imamo dvije jednadžbe s dvije nepoznanice:

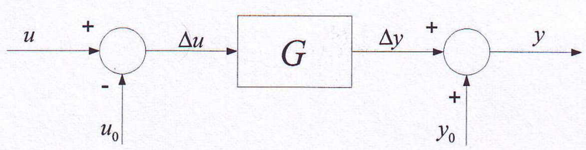
$$\frac{2u_0}{y_0} = 8$$

i iz početne diferencijalne jednadžbe:

$$y_0^2 = \frac{u_0^2}{y_0} \Rightarrow y_0^3 = u_0^2.$$

Konačni rezultat je: $y_0 = 16$ i $u_0 = 64$.

Blokovska shema:



2. IZLAZNI TEST
GRUPA A1

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3.AUT1

$$\dot{y}(t) + [y(t)]^2 = \frac{u^2(t)}{y(t)}$$

(a) $\dot{y}(t) = 0 \rightarrow y_0^2 = \frac{u_0^2}{y_0} \rightarrow u_0^2 = y_0^3$

$y = y_0 + \Delta y, \quad \dot{y} = \Delta\dot{y}$

$u = u_0 + \Delta u$

$$\dot{y}(t) = \frac{u^2(t)}{y(t)} - y^2(t) = f(u, y)$$

$$\dot{y}(t) \approx \underset{0}{\underbrace{f(u_0, y_0)}} + \left.\frac{\partial f}{\partial u}\right|_{S.T.} \Delta u + \left.\frac{\partial f}{\partial y}\right|_{S.T.} \Delta y$$

$$\Delta\dot{y} = \left(-\frac{u_0^2}{y_0^2} - 2y_0\right)\Delta y + \frac{2u_0}{y_0}\Delta u$$

$$\Delta\dot{y} + \left(\frac{u_0^2}{y_0^2} + 2y_0\right)\Delta y = \frac{2u_0}{y_0}\Delta u$$

$\mathcal{L}(\Delta y) = Y(s)$

$\mathcal{L}(\Delta u) = U(s)$

$$sY(s) + \left(\frac{u_0^2}{y_0^2} + 2y_0\right)Y(s) = \frac{2u_0}{y_0}U(s)$$

$$Y(s)\left(s + \frac{u_0^2}{y_0^2} + 2y_0\right) = \frac{2u_0}{y_0}U(s)$$

$$\frac{Y(s)}{U(s)} = G(s) = \frac{2u_0 y_0}{s y_0^2 + u_0^2 + 2y_0^3}$$

(b) $\Delta u(t) = S(t) \circ\!\!-\!\!\bullet\ U(s) = \frac{1}{s}$

$$y(s) = G(s) \cdot U(s)$$

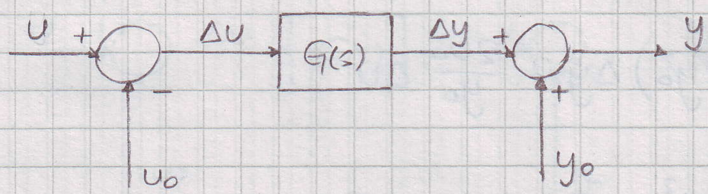
$$\dot{y}(0^+) = \lim_{s\to\infty} s^2 y(s)$$

$$\dot{y}(0^+) = \lim_{s\to\infty} s^2 \cdot \frac{1}{s} \frac{2u_0y_0}{s y_0^2 + u_0^2 + 2y_0^3}$$

$$\dot{y}(0^+) = \lim_{s\to\infty} \frac{2u_0y_0 s}{s y_0^2 + u_0^2 + 2y_0^3}$$

$$\dot{y}(0^+) = \frac{2u_0}{y_0}, \quad \dot{y}(0^+) = 8 \quad \rightarrow \quad \left.\begin{matrix} u_0 = 4y_0 \\ u_0^2 = y_0^3 \end{matrix}\right\} u_0 = 64 \quad y_0 = 16$$

(c)



A2

Zadana je nelinearna diferencijalna jednadžba prvog reda:

$$\dot{y}(t) + [y(t)]^4 = u^2(t)y(t)$$

Potrebno je:

a) Linearizirati nelinearnu diferencijalnu jednadžbu u okolini radne točke određene s (u_0, y_0) i odrediti prijenosnu funkciju $G(s) = \dfrac{Y(s)}{U(s)}$.

b) Odrediti za koju radnu točku (u_0, y_0) će nagib odziva na pobudu $\Delta u(t) = S(t)$ u trenutku $t = 0^+$ biti jednak 2.

c) Nacrtati blokovsku shemu kojoj je ulaz stvarni u, a izlaz stvarni y. Koristite samo blokove prijenosne funkcije $G(s)$ iz b) dijela zadatka, sumatore i konstante. Tom se shemom treba aproksimirati ponašanje nelinearnog modela u okolini radne točke.

Rješenje:
Za radnu točku (u_0, y_0) linearizirana diferencijalna jednadžba je:

$$\Delta\dot{y} + 4y_0^3\Delta y = 2u_0y_0\Delta u + u_0^2\Delta y.$$

Prijenosna funkcija određuje se iz:

$$sY(s) + 4y_0^3Y(s) = 2u_0y_0U(s) + u_0^2Y(s).$$

Prijenosna funkcija je:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2u_0y_0}{s + 4y_0^3 - u_0^2}.$$

Računanje nagiba odziva na pobudu $\Delta u(t) = S(t)$ u trenutku $t = 0^+$:

$$k = \lim_{s\to\infty} s^2Y(s) = \lim_{s\to\infty} s^2G(s)\frac{1}{s} = \lim_{s\to\infty} \frac{2u_0y_0s}{s + 4y_0^3 - u_0^2} = 2u_0y_0.$$

Imamo dvije jednadžbe s dvije nepoznanice:

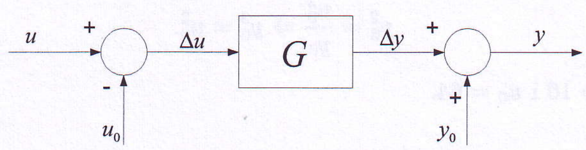
$$2u_0y_0 = 2$$

i iz početne diferencijalne jednadžbe:

$$y_0^4 = u_0^2y_0 \Rightarrow y_0^3 = u_0^2.$$

Konačni rezultat je: $y_0 = 1$ i $u_0 = 1$.

Blokovska shema:



2. IZLAZNI TEST
GRUPA A2

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$$\dot{y}(t) + [y(t)]^4 = u^2(t)\, y(t)$$

(a) $\dot{y}(t) = 0 \longrightarrow y_0^4 = u_0^2 y_0 \longrightarrow u_0^2 = y_0^3$

$$y = y_0 + \Delta y \quad , \quad \dot{y} = \dot{\Delta y}$$
$$u = u_0 + \Delta u$$

$$\dot{y}(t) = u^2(t)\, y(t) - y^4(t) = f(u, y)$$

$$\dot{y}(t) \approx \underbrace{f(u_0, y_0)}_{0} + \frac{\partial f}{\partial y}\Big|_{S.T.} \Delta y + \frac{\partial f}{\partial u}\Big|_{S.T.} \Delta u$$

$$\dot{\Delta y} = (u_0^2 - 4y_0^3)\Delta y + 2u_0 y_0 \Delta u$$

$$\dot{\Delta y} + (4y_0^3 - u_0^2)\Delta y = 2u_0 y_0 \Delta u$$

$$\mathcal{L}(\Delta y) = Y(s)$$
$$\mathcal{L}(\Delta u) = U(s)$$

$$sY(s) + (4y_0^3 - u_0^2)Y(s) = 2u_0 y_0 U(s)$$

$$Y(s)\left(s + 4y_0^3 - u_0^3\right) = 2u_0 y_0 U(s)$$

$$\frac{Y(s)}{U(s)} = G(s) = \frac{2u_0 y_0}{s + 4y_0^3 - u_0^3}$$

(b) $\Delta u(t) = S(t) \circ\!\!-\!\!\bullet\; U(s) = \frac{1}{s}$

$$y(s) = G(s)\, U(s)$$

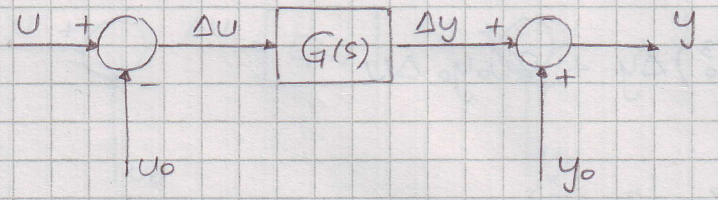
$$\dot{y}(0^+) = \lim_{s\to\infty} s^2 y(s)$$

$$\dot{y}(0^+) = \lim_{s\to\infty} s^{\cancel{2}} \cdot \frac{1}{\cancel{s}} \; \frac{2u_0y_0}{s+4y_0^3-u_0^3}$$

$$\dot{y}(0^+) = \lim_{s\to\infty} \frac{2u_0y_0 s}{s+4y_0^3-u_0^3}$$

$$\dot{y}(0^+) = 2u_0y_0 \;,\quad \dot{y}(0^+) = 2 \;\rightarrow\; \left.\begin{matrix} u_0y_0 = 1 \\ u_0 = \frac{1}{y_0} \end{matrix}\right\} u_0 = 1,\; y_0 = 1$$

(c)



B1

Zadana je nelinearna diferencijalna jednadžba drugog reda:

$$\ddot{y}(t) + \dot{y}(t) + [y(t)]^3 = [u(t)]^3 - \dot{u}(t)y(t).$$

Potrebno je:

a) Linearizirati nelinearnu diferencijalnu jednadžbu u okolini radne točke određene s $u_0 = 1$ i odrediti prijenosnu funkciju $G(s) = \frac{Y(s)}{U(s)}$.

b) Odrediti nagib odziva u trenutku $t = 0^+$ i stacionarno stanje uz pobudu $\Delta u(t) = S(t)$.

c) Na osnovu određenog nagiba i stacionarnog stanja skicirajte odziv sustava.

Rješenje:

Zadana je radna točka određena s $u_0 = 1$ iz čega slijedi da je: $y_0^3 = u_0^3 \Rightarrow y_0 = 1$.

Za radnu točku (u_0, y_0) linearizirana diferencijalna jednadžba je:

$$\Delta\ddot{y} + \Delta\dot{y} + 3y_0^2\Delta y = 3u_0^2\Delta u - y_0\Delta\dot{u}.$$

Prijenosna funkcija određuje se iz:

$$s^2Y(s) + sY(s) + 3y_0^2Y(s) = 3u_0^2U(s) - y_0sU(s).$$

Prijenosna funkcija je:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3u_0^2 - y_0s}{s^2 + s + 3y_0^2}.$$

Nagib u trenutku $t = 0^+$:

$$k = \lim_{s\to\infty} s^2Y(s) = \lim_{s\to\infty} s^2G(s)\frac{1}{s} = \lim_{s\to\infty} \frac{3u_0^2s - y_0s^2}{s^2 + s + 3y_0^2} = -y_0 = -1$$

Stacionarno stanje:

$$ss = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sG(s)\frac{1}{s} = \lim_{s\to 0} \frac{3u_0^2 - y_0s}{s^2 + s + 3y_0^2} = \frac{u_0^2}{y_0} = 1$$

Skica odziva. Dovoljno je bilo ucrtati odgovarajući početni nagib, valjano stacionarno stanje i odgovarajuću duljinu prijelazne pojave (red velicine, preko mjesta polova prijenosne funkcije).

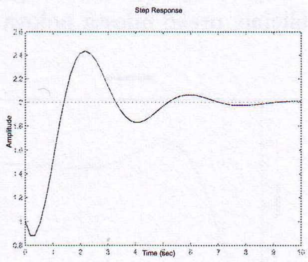


Figure 1: Skica odziva sustava

2. IZLAZNI TEST
GRUPA B1

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$$\ddot{y}(t) + \dot{y}(t) + [y(t)]^3 = [u(t)]^3 - \dot{u}(t)\,y(t)$$

(a) $\ddot{y}(t) = \dot{y}(t) = 0,\ \dot{u}(t) = 0 \rightarrow y_0^3 = u_0^3 \longrightarrow u_0 = y_0 = 1$

$$y = y_0 + \Delta y,\ \dot{y} = \Delta\dot{y},\ \ddot{y} = \Delta\ddot{y}$$
$$u = u_0 + \Delta u,\ \ \dot{u} = \Delta\dot{u}$$

$$\ddot{y}(t) = u^3(t) - \dot{u}(t)y(t) - \dot{y}(t) - y^3(t) = f(u, y, \dot{u}, \dot{y})$$

$$\ddot{y}(t) \approx \underset{0}{\underbrace{f(u_0, y_0, 0, 0)}} + \left.\frac{\partial f}{\partial u}\right|_{S.T.}\Delta u + \left.\frac{\partial f}{\partial y}\right|_{S.T.}\Delta y + \left.\frac{\partial f}{\partial \dot{u}}\right|_{S.T.}\Delta\dot{u} + \left.\frac{\partial f}{\partial \dot{y}}\right|_{S.T.}\Delta\dot{y}$$

$$\Delta\ddot{y} = (3u_0^2)\Delta u + (-3y_0^2)\Delta y - \Delta\dot{y} - y_0\Delta\dot{u}$$

$$\Delta\ddot{y} + \Delta\dot{y} + 3\Delta y = 3\Delta u - \Delta\dot{u}$$

$$\mathcal{L}(\Delta y) = Y(s)$$
$$\mathcal{L}(\Delta u) = U(s)$$

$$s^2Y(s) + sY(s) + 3Y(s) = 3U(s) - sU(s)$$
$$Y(s)(s^2+s+3) = U(s)(3-s)$$

$$\frac{Y(s)}{U(s)} = G(s) = \frac{3-s}{s^2+s+3}$$

(b) $\Delta u(t) = S(t) \circ\!\!-\!\!\bullet \; U(s) = \frac{1}{s}$

$$y(s) = U(s) \cdot G(s)$$

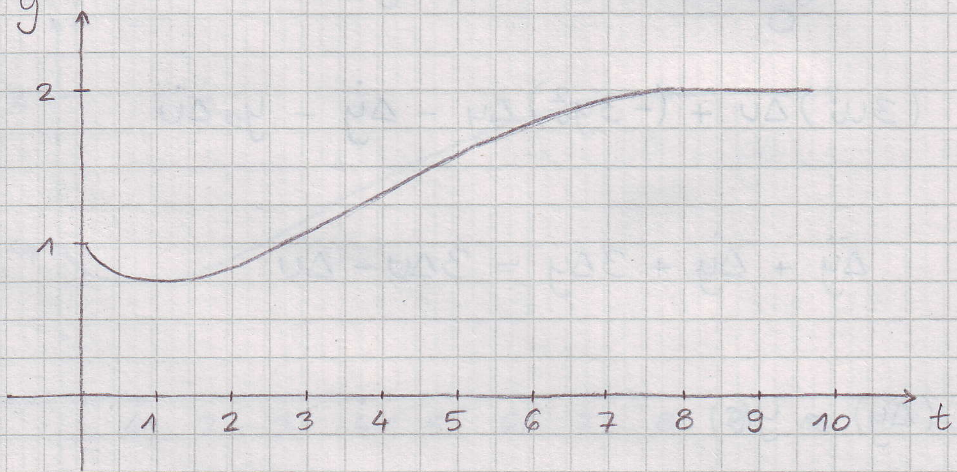
$$\dot{y}(0^+) = \lim_{s\to\infty} s^2 y(s) \qquad y(\infty) = \lim_{s\to 0} s\, y(s)$$

$$\dot{y}(0^+) = \lim_{s\to\infty} s^2 \cdot \frac{1}{\cancel{s}} \frac{3-s}{s^2+s+3} \qquad y(\infty) = \lim_{s\to 0} s \cdot \frac{1}{\cancel{s}} \frac{3-s}{s^2+s+3}$$

$$\dot{y}(0^+) = \lim_{s\to\infty} \frac{3s-s^2}{s^2+s+3} \qquad y(\infty) = \lim_{s\to 0} \frac{3-s}{s^2+s+3}$$

$$\dot{y}(0^+) = -1 \qquad y(\infty) = 1$$

(c)



B2
Zadana je nelinearna diferencijalna jednadžba drugog reda:

$$\ddot{y}(t) + \dot{y}(t) - \frac{1}{y(t)} = u(t) - y(t)\dot{u}(t).$$

Potrebno je:

a) Linearizirati nelinearnu diferencijalnu jednadžbu u okolini radne točke određene s $y_0 = 1$ i odrediti prijenosnu funkciju $G(s) = \frac{Y(s)}{U(s)}$.

b) Odrediti nagib odziva u trenutku $t = 0^+$ i stacionarno stanje uz pobudu $\Delta u(t) = S(t)$.

c) Na osnovu određenog nagiba i stacionarnog stanja skicirajte odziv sustava.

Rješenje:
Zadana je radna točka određena s $y_0 = 1$ iz čega slijedi da je: $-\frac{1}{y_0} = u_0 \Rightarrow u_0 = -1$. Za radnu točku (u_0, y_0) linearizirana diferencijalna jednadžba je:

$$\Delta\ddot{y} + \Delta\dot{y} + \frac{1}{y_0^2}\Delta y = \Delta u - y_0\Delta\dot{u}.$$

Prijenosna funkcija određuje se iz:

$$s^2Y(s) + sY(s) + \frac{1}{y_0^2}Y(s) = U(s) - y_0 sU(s).$$

Prijenosna funkcija je:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1 - y_0 s}{s^2 + s + \frac{1}{y_0^2}}.$$

Nagib u trenutku $t = 0^+$:

$$k = \lim_{s\to\infty} s^2Y(s) = \lim_{s\to\infty} s^2G(s)\frac{1}{s} = \lim_{s\to\infty} \frac{s - y_0 s^2}{s^2 + s + \frac{1}{y_0^2}} = -y_0 = -1$$

Stacionarno stanje:

$$ss = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sG(s)\frac{1}{s} = \lim_{s\to 0} \frac{1 - y_0 s}{s^2 + s + \frac{1}{y_0^2}} = \frac{1}{\frac{1}{y_0^2}} = 1$$

Skica odziva. Dovoljno je bilo ucrtati odgovarajući početni nagib, valjano stacionarno stanje i odgovarajuću duljinu prijelazne pojave (red velicine, preko mjesta polova prijenosne funkcije).

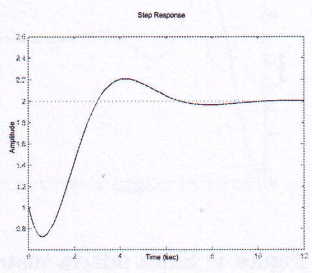


Figure 2: Skica odziva sustava

2. IZLAZNI TEST
GRUPA B2

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$$\ddot{y}(t) + \dot{y}(t) - \frac{1}{y(t)} = u(t) - y(t)\dot{u}(t)$$

(a) $\ddot{y}(t) = \dot{y}(t) = 0$, $\dot{u}(t) = 0 \rightarrow -\frac{1}{y_0} = u_0 \rightarrow u_0 y_0 = -1$

$u_0 = -1$, $y_0 = 1$

$y = y_0 + \Delta y$, $\dot{y} = \Delta\dot{y}$, $\ddot{y} = \Delta\ddot{y}$

$u = u_0 + \Delta u$, $\dot{u} = \Delta\dot{u}$

$$\ddot{y}(t) = u(t) - y(t)\dot{u}(t) - \dot{y}(t) + \frac{1}{y(t)} = f(u, y, \dot{u}, \dot{y})$$

$$\ddot{y}(t) \approx \underbrace{f(u_0, y_0, 0, 0)}_{=0} + \frac{\partial f}{\partial u}\Big|_{S.T.}\Delta u + \frac{\partial f}{\partial y}\Big|_{S.T.}\Delta y + \frac{\partial f}{\partial \dot{u}}\Big|_{S.T.}\Delta\dot{u} + \frac{\partial f}{\partial \dot{y}}\Big|_{S.T.}\Delta\dot{y}$$

$$\Delta\ddot{y} = \left(-\frac{1}{y_0^2}\right)\Delta y + \Delta u - \Delta\dot{y} - y_0\Delta\dot{u}$$

$$\Delta\ddot{y} + \Delta\dot{y} + \Delta y = \Delta u - \Delta\dot{u}$$

$\mathcal{L}(\Delta y) = Y(s)$

$\mathcal{L}(\Delta u) = U(s)$

$s^2 Y(s) + sY(s) + Y(s) = U(s) - sU(s)$

$Y(s)(s^2 + s + 1) = U(s)(1 - s)$

$$\frac{Y(s)}{U(s)} = G(s) = \frac{1-s}{s^2+s+1}$$

(b) $\Delta u(t) = S(t) \;\circ\!\!-\!\!\bullet\; U(s) = \frac{1}{s}$

$$y(s) = G(s) \cdot U(s)$$

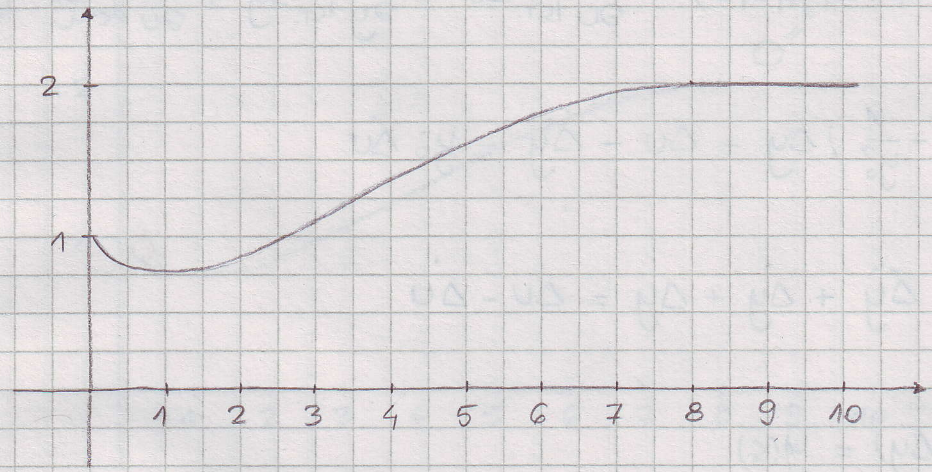
$$\dot{y}(0^+) = \lim_{s \to \infty} s^2 y(s) \qquad y(\infty) = \lim_{s \to 0} s\, y(s)$$

$$\dot{y}(0^+) = \lim_{s \to \infty} s^{\cancel{2}} \cdot \frac{1}{\cancel{s}} \frac{1-s}{s^2+s+3} \qquad y(\infty) = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \frac{1-s}{s^2+s+3}$$

$$\dot{y}(0^+) = \lim_{s \to \infty} \frac{s-s^2}{s^2+s+3} \qquad y(\infty) = \lim_{s \to 0} \frac{1-s}{s^2+s+3}$$

$$\dot{y}(0^+) = -1 \qquad y(\infty) = 1$$

(c)



B3

Zadana je nelinearna diferencijalna jednadžba drugog reda:

$$\ddot{y}(t) + \dot{y}(t) + \ln y(t) = u(t) - y(t)e^{\dot{u}(t)}.$$

Potrebno je:

a) Linearizirati nelinearnu diferencijalnu jednadžbu u okolini radne točke određene s $y_0 = 1$ i odrediti prijenosnu funkciju $G(s) = \dfrac{Y(s)}{U(s)}$.

b) Odrediti nagib odziva u trenutku $t = 0^+$ i stacionarno stanje uz pobudu $\Delta u(t) = S(t)$.

c) Na osnovu određenog nagiba i stacionarnog stanja skicirajte odziv sustava.

Rješenje:

Zadana je radna točka određena s $y_0 = 1$ iz čega slijedi da je: $\ln y_0 = u_0 - y_0 \Rightarrow u_0 = 1$.

Za radnu točku (u_0, y_0) linearizirana diferencijalna jednadžba je:

$$\Delta\ddot{y} + \Delta\dot{y} + \frac{1}{y_0}\Delta y = \Delta u - e^{\dot{u}_0}\Delta y - y_0 e^{\dot{u}_0}\Delta\dot{u}.$$

Prijenosna funkcija određuje se iz:

$$s^2Y(s) + sY(s) + \frac{1}{y_0}Y(s) = U(s) - Y(s) - y_0 sU(s).$$

Prijenosna funkcija je:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1 - y_0 s}{s^2 + s + \frac{1}{y_0} + 1}.$$

Nagib u trenutku $t = 0^+$:

$$k = \lim_{s\to\infty} s^2Y(s) = \lim_{s\to\infty} s^2G(s)\frac{1}{s} = \lim_{s\to\infty} \frac{s - y_0 s^2}{s^2 + s + \frac{1}{y_0} + 1} = -y_0 = -1$$

Stacionarno stanje:

$$ss = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sG(s)\frac{1}{s} = \lim_{s\to 0} \frac{1 - y_0 s}{s^2 + s + \frac{1}{y_0} + 1} = \frac{1}{\frac{1}{y_0} + 1} = \frac{1}{2}$$

Skica odziva. Dovoljno je bilo ucrtati odgovarajući početni nagib, valjano stacionarno stanje i odgovarajuću duljinu prijelazne pojave (red velicine, preko mjesta polova prijenosne funkcije).

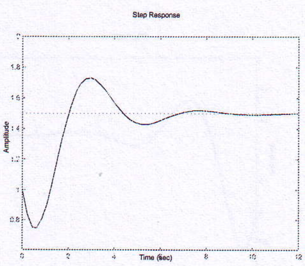


Figure 3: Skica odziva sustava

2. IZLAZNI TEST

GRUPA B3

MARKO GULIN

0036428227

3.AUT 1

$$\ddot{y}(t) + \dot{y}(t) + \ln y(t) = u(t) - y(t)e^{\dot{u}(t)}$$

(a) $\ddot{y}(t) = \dot{y}(t) = 0$, $\dot{u}(t) = 0 \rightarrow \ln y_0 = u_0 - y_0 \rightarrow u_0 = y_0 = 1$

$y = y_0 + \Delta y$, $\dot{y} = \Delta\dot{y}$, $\ddot{y} = \Delta\ddot{y}$

$u = u_0 + \Delta u$, $\dot{u} = \Delta\dot{u}$

$$\ddot{y}(t) = u(t) - y(t)e^{\dot{u}(t)} - \dot{y}(t) - \ln y(t) = f(u, y, \dot{u}, \dot{y})$$

$$\ddot{y}(t) \sim \underbrace{f(u_0, y_0, 0, 0)}_{=0} + \left.\frac{\partial f}{\partial u}\right|_{S.T.}\Delta u + \left.\frac{\partial f}{\partial y}\right|_{S.T.}\Delta y + \left.\frac{\partial f}{\partial \dot{u}}\right|_{S.T.}\Delta\dot{u} + \left.\frac{\partial f}{\partial \dot{y}}\right|_{S.T.}\Delta\dot{y}$$

$$\Delta\ddot{y} = \left(-1 - \frac{1}{y_0}\right)\Delta y + \Delta u - \Delta\dot{y} - y_0\Delta\dot{u}$$

$$\Delta\ddot{y} + \Delta\dot{y} + 2\Delta y = \Delta u - \Delta\dot{u}$$

$\mathcal{L}(\Delta y) = Y(s)$

$\mathcal{L}(\Delta u) = U(s)$

$s^2 Y(s) + sY(s) + 2Y(s) = U(s) - sU(s)$

$Y(s)(s^2 + s + 2) = U(s)(1 - s)$

$$\frac{Y(s)}{U(s)} = G(s) = \frac{1-s}{s^2+s+2}$$

(b) $\Delta u(t) = \delta(t) \circ\!\!-\!\!\bullet\ U(s) = \frac{1}{s}$

$$y(s) = U(s)\,G(s)$$

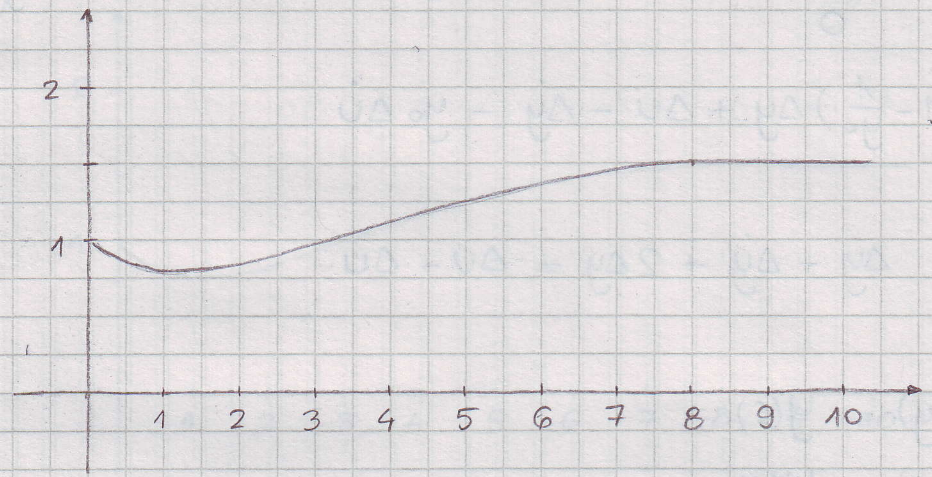
$$\dot{y}(0^+) = \lim_{s\to\infty} s^2\, y(s) \qquad y(\infty) = \lim_{s\to 0} s\, y(s)$$

$$\dot{y}(0^+) = \lim_{s\to\infty} s^{\cancel{2}} \cdot \frac{1}{\cancel{s}}\ \frac{1-s}{s^2+s+2} \qquad y(\infty) = \lim_{s\to 0} \cancel{s}\cdot\frac{1}{\cancel{s}}\ \frac{1-s}{s^2+s+2}$$

$$\dot{y}(0^+) = \lim_{s\to\infty} \frac{s-s^2}{s^2+s+2} \qquad y(\infty) = \lim_{s\to 0} \frac{1-s}{s^2+s+2}$$

$$\dot{y}(0^+) = -1 \qquad y(\infty) = \frac{1}{2}$$

(c)



B4

Zadana je nelinearna diferencijalna jednadžba drugog reda:

$$\ddot{y}(t) + \dot{y}(t) - \frac{1}{y(t)} = u(t) - y(t)e^{\dot{u}(t)}.$$

Potrebno je:

a) Linearizirati nelinearnu diferencijalnu jednadžbu u okolini radne točke određene s $y_0 = 1$ i odrediti prijenosnu funkciju $G(s) = \frac{Y(s)}{U(s)}$.

b) Odrediti nagib odziva u trenutku $t = 0^+$ i stacionarno stanje uz pobudu $\Delta u(t) = S(t)$.

c) Na osnovu određenog nagiba i stacionarnog stanja skicirajte odziv sustava.

Rješenje:

Zadana je radna točka određena s $y_0 = 1$ iz čega slijedi da je: $-\frac{1}{y_0} = u_0 - y_0 \Rightarrow u_0 = 0$.

Za radnu točku (u_0, y_0) linearizirana diferencijalna jednadžba je:

$$\Delta\ddot{y} + \Delta\dot{y} + \frac{1}{y_0^2}\Delta y = \Delta u - e^{\dot{u}_0}\Delta y - y_0 e^{\dot{u}_0}\Delta\dot{u}.$$

Prijenosna funkcija određuje se iz:

$$s^2Y(s) + sY(s) + \frac{1}{y_0^2}Y(s) = U(s) - Y(s) - y_0 sU(s).$$

Prijenosna funkcija je:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1 - y_0 s}{s^2 + s + \frac{1}{y_0^2} + 1}.$$

Nagib u trenutku $t = 0^+$:

$$k = \lim_{s\to\infty} s^2Y(s) = \lim_{s\to\infty} s^2G(s)\frac{1}{s} = \lim_{s\to\infty} \frac{s - y_0 s^2}{s^2 + s + \frac{1}{y_0^2} + 1} = -y_0 = -1$$

Stacionarno stanje:

$$ss = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sG(s)\frac{1}{s} = \lim_{s\to 0} \frac{1 - y_0 s}{s^2 + s + \frac{1}{y_0^2} + 1} = \frac{1}{\frac{1}{y_0^2} + 1} = \frac{1}{2}$$

Skica odziva. Dovoljno je bilo ucrtati odgovarajući početni nagib, valjano stacionarno stanje i odgovarajuću duljinu prijelazne pojave (red velicine, preko mjesta polova prijenosne funkcije).

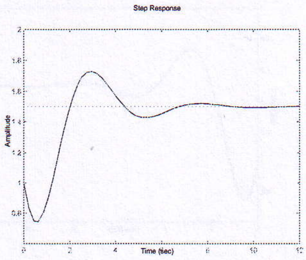


Figure 4: Skica odziva sustava

2. IZLAZNI TEST
GRUPA B4

MARKO GULIN
0036428227
3.AUT1

$$\ddot{y}(t) + \dot{y}(t) - \frac{1}{y(t)} = u(t) - y(t)e^{\dot{u}(t)}$$

(a) $\ddot{y}(t) = \dot{y}(t) = 0, \ \dot{u}(t) = 0 \rightarrow -\frac{1}{y_0} = u_0 - y_0 \rightarrow u_0 = 0, \ y_0 = 1$

$$y = y_0 + \Delta y, \ \dot{y} = \Delta\dot{y}, \ \ddot{y} = \Delta\ddot{y}$$

$$u = u_0 + \Delta u, \ \dot{u} = \Delta\dot{u}$$

$$\ddot{y}(t) = u(t) - y(t)e^{\dot{u}(t)} - \dot{y}(t) + \frac{1}{y(t)} = f(u, y, \dot{u}, \dot{y})$$

$$\ddot{y}(t) \approx \underbrace{f(u_0, y_0, 0, 0)}_{0} + \frac{\partial f}{\partial u}\Big|_{S.T.}\Delta u + \frac{\partial f}{\partial y}\Big|_{S.T.}\Delta y + \frac{\partial f}{\partial \dot{u}}\Big|_{S.T.}\Delta\dot{u} + \frac{\partial f}{\partial \dot{y}}\Big|_{S.T.}\Delta\dot{y}$$

$$\Delta\ddot{y} = \left(-1 - \frac{1}{y_0^2}\right)\Delta y + \Delta u - \Delta\dot{y} - y_0\Delta\dot{u}$$

$$\Delta\ddot{y} + \Delta\dot{y} + 2\Delta y = \Delta u - \Delta\dot{u}$$

$$\mathcal{L}(\Delta y) = Y(s)$$

$$\mathcal{L}(\Delta u) = U(s)$$

$$s^2Y(s) + sY(s) + 2Y(s) = U(s) - sU(s)$$

$$Y(s)(s^2 + s + 2) = U(s)(1 - s)$$

$$\frac{Y(s)}{U(s)} = G(s) = \frac{1 - s}{s^2 + s + 2}$$

(b) $\Delta u(t) = S(t) \circ\!\!-\!\!\bullet\; U(s) = \frac{1}{s}$

$$Y(s) = U(s) \cdot G(s)$$

$$\dot{y}(0^+) = \lim_{s\to\infty} s^2 Y(s) \qquad y(\infty) = \lim_{s\to 0} s Y(s)$$

$$\dot{y}(0^+) = \lim_{s\to\infty} s^{\cancel{2}} \cdot \frac{1}{\cancel{s}} \frac{1-s}{s^2+s+2} \qquad y(\infty) = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \frac{1-s}{s^2+s+2}$$

$$\dot{y}(0^+) = \lim_{s\to\infty} \frac{s-s^2}{s^2+s+2} \qquad y(\infty) = \lim_{s\to 0} \frac{1-s}{s^2+s+2}$$

$$\dot{y}(0^+) = -1 \qquad y(\infty) = \frac{1}{2}$$

(c)