ZI - 2011

1. $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$

$Ax = b \Rightarrow x = A^{-1} \cdot b$

$\det A = (2 \cdot 3 - 4 \cdot 1)_{\text{mod} 5} = 2$

$A^{-1} = \frac{1}{\det A} \cdot \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}^T = \frac{1}{2} \cdot \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = 3 \cdot \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ 12 & 6 \end{bmatrix}_{\text{mod} 5} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

$x \cdot 2 = 1 \Rightarrow x = 3$ (jer $3 \cdot 2 = 6_{\text{mod} 5} = 1$)

$\Rightarrow \underbrace{\boxed{\frac{1}{2} = 3 \text{ u modulo } 5}}$

$x + 4 = 0 \Rightarrow x = 1$ (jer $1 + 4 = 5_{\text{mod} 5} = 0$)

$-4 = 1$

$x + 1 = 0 \Rightarrow -1 = 4$

$x = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

2. $A = \begin{bmatrix} a+b & a-b \\ a+2b & b-2c \end{bmatrix} \in X$

a) (1) $A_1 + A_2 = \begin{bmatrix} (a_1+a_2)+(b_1+b_2) & a_1+a_2-(b_1+b_2) \\ (a_1+a_2)+2(b_1+b_2) & (b_1+b_2)-2(c_1+c_2) \end{bmatrix} \in X$

(2) $\alpha \cdot A = \begin{bmatrix} \alpha a + \alpha b & \alpha a - \alpha b \\ \alpha a + 2\alpha b & \alpha b - 2\alpha c \end{bmatrix} \in X$

(1) i (2) $\Rightarrow$ $X$ je v. potpr. od $M_{2,2}$

b) $A = a \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}_{E_1} + b \underbrace{\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}}_{E_2} + c \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}}_{E_3}$

$X = L(E_1, E_2, E_3)$ (3)

$a \cdot E_1 + b \cdot E_2 + c \cdot E_3 = 0 \Rightarrow \left.\begin{matrix} a+b=0 & a+2b=0 \\ a-b=0 & b-2c=0 \end{matrix}\right\}$ $a = b = c = 0$ (4)

$E_1, E_2$ i $E_3$ su l. nez.

(3) i (4) $\Rightarrow$ $E_1, E_2$ i $E_3$ su baza od $X$, $\dim X = 3$ !

c) $U: X \to \mathbb{R}^4$

$U(A) = (a+b,\ a+2b,\ a+3b,\ a+4b)$

$$\left.\begin{aligned} U(E_1) &= U(1,0,0) = (1,1,1,1)\\ U(E_2) &= U(0,1,0) = (1,2,3,4)\\ U(E_3) &= U(0,0,1) = (0,0,0,0)\end{aligned}\right\} \quad U = \begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 0\\ 1 & 3 & 0\\ 1 & 4 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 2 & 0\\ 0 & 3 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$\Rightarrow$ rang $(U) = 2 \Rightarrow$ rang operatora $\boxed{r = 2}$

$\dim(\ker U) = d = n - r = \dim X - 2 = 3 - 2 = 1$ ... defekt

3. $A: P_3 \to M_{2,2}$

$$A(a_0 + a_1 x + a_2 x^2 + a_3 x^3) = \begin{bmatrix} a_0 + 2a_1 & a_1 - a_2\\ a_1 & a_3 - a_1\end{bmatrix}$$

kanonska baza u $M_{2,2}$

a) $A(1) = |a_0 = 1,\ a_i = 0,\ i \neq 0| = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = 1 \cdot E_1 + 0E_2 + 0E_3 + 0E_4$

$A(x) = \begin{bmatrix}2 & 1\\ 1 & -1\end{bmatrix} = 2E_1 + 1E_2 + 1E_3 - 1E_4$

$A(x^2) = \begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix} = 0E_1 - 1E_2 + 0E_3 + 0E_4$

$A(x^3) = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} = 0E_1 + 0E_2 + 0E_3 + 1E_4$

$$A = \begin{bmatrix}1 & 2 & 0 & 0\\ 0 & 1 & -1 & 0\\ 0 & 1 & 0 & 0\\ 0 & -1 & 0 & 1\end{bmatrix}$$

b) $A: X \to Y$ je izomorfizam ako postoji $A^{-1}$ t.d. $A^{-1}: Y \to X$

($\dim X = \dim Y = n$ i $\ker(A) = \{0\}$).

$A$ je izomorf. onda i samo onda kada $\det A \neq 0$.

$$\det A = 1 \cdot \begin{vmatrix}1 & -1 & 0\\ 1 & 0 & 0\\ -1 & 0 & 1\end{vmatrix} = 1 \cdot 1 \cdot \begin{vmatrix}1 & -1\\ 1 & 0\end{vmatrix} = 1 \cdot 1 \cdot 1 = 1 \neq 0.$$

$A$ je izomorfizam!

4)

4. $A = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$ $e_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $e_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$T = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \Rightarrow A' = T^{-1} \cdot A \cdot T$

$A' = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & -8 \\ -4 & 5 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{3} & -\frac{8}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{bmatrix}$

5. $A(a,b,c) = (a+b, a-2c, a-3b, 2a+c)$

$B(a,b,c) = A(c,a,b) = (c+a, c-2b, c-3a, 2c+b)$

$A(1,0,0) = (1,1,1,2)$

$A(0,1,0) = (1,0,-3,0)$

$A(0,0,1) = (0,-2,0,1)$

$\Rightarrow A = \begin{array}{c} \begin{matrix} a & b & c \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -2 \\ 1 & -3 & 0 \\ 2 & 0 & 1 \end{bmatrix} \end{array}$

$B = \begin{array}{c} \begin{matrix} a & b & c \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 1 \\ -3 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \end{array} \Rightarrow A - B = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & -3 \\ 4 & -3 & -1 \\ 2 & -1 & -1 \end{bmatrix}$

$\|A-B\|_\infty = \max\{2, 6, 8, 4\} = 8$ ✓

6. $A = \begin{bmatrix} 2/7 & 0 & 0 \\ 0 & 1/3 & 2/3 \\ 0 & 2/5 & 4/5 \end{bmatrix}$

$\sum_{k=2}^{\infty} A^k$ konvergira ako $r(A) < 1$

$\|A\|_1 = \max\left\{\frac{2}{7}, \frac{11}{15}, \frac{13}{15}\right\} = \frac{13}{15} < 1$, $r(A) \leq \|A\|_1$

$\Rightarrow \boxed{r(A) < 1}$

$\sum_{k=0}^{\infty} A^k = I + A + \sum_{k=2}^{\infty} A^k = (I-A)^{-1} \Rightarrow \boxed{\sum_{k=2}^{\infty} A^k = (I-A)^{-1} - I - A}$

4. A stabilna ako $\forall \lambda$ $Re(\lambda) < \emptyset$!

$\Leftrightarrow e^{At} \to \emptyset$ kada $t \to \infty$

$$A = \begin{bmatrix} -3-2a+ai & a^3+2i \\ 0 & 3a+4-i \end{bmatrix}$$

$$\Rightarrow \sigma(A) = \{-3-2a+ai,\ 3a+4-i\}$$

$$\Rightarrow \left.\begin{array}{ll} -3-2a < 0 \Rightarrow & 2a > -3 \\ & a > -\frac{3}{2} \\ 3a+4 < \emptyset \Rightarrow & 3a < -4 \\ & a < -\frac{4}{3} \end{array}\right\} a \in \left(-\frac{3}{2}, -\frac{4}{3}\right)$$

8. a) $D_i = \{z \in \mathbb{C} : |z - a_{ii}| < r_i\}$, gdje

$$r_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|$$

b)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\|A\|_2^2 = r(A^T \cdot A)$$

$$A^T \cdot A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 10 & -7 \\ 1 & -7 & 6 \end{bmatrix}$$

$A^T A$ je simetrična pa $\lambda \in \sigma(A^T A)$ je realan broj.

$A^T A$ je i semidefinitna: $(A^T A x | x) = (Ax | Ax) = \|Ax\|^2 \geq 0$, $\forall x$

$\Rightarrow \lambda \geq 0$

za $A^T A$ su Geršgorinovi krugovi zapravo intervali na $x$-osi:

$D_1$: $|z-1| \leq 1 \Rightarrow [0, 2]$

$D_2$: $|z-10| \leq 7 \Rightarrow [3, 17]$

$D_3$: $|z-6| \leq 8 \Rightarrow [-2, 14]$

$\Rightarrow \sigma(A) \in [0, 17] \Rightarrow r(A^T A) \leq 17 \Rightarrow \|A\|_2^2 \leq 17$

$\|A\|_2 \leq \sqrt{17} = c$

b) $\lim_{k\to\infty}\left(\frac{A}{5}\right)^k$

U prošlom zadatku smo vidjeli da je $\|A\|_2 \le \sqrt{17} \approx 4,123$

$\Rightarrow \quad r(A) < \sqrt{17}$

za matricu $\frac{A}{5}$ vrijedi $r\left(\frac{A}{5}\right) = \frac{r(A)}{5}$

$$\Rightarrow r\left(\frac{A}{5}\right) \le \frac{\sqrt{17}}{5} \approx 0,8246 < 1$$

$\Rightarrow \lim_{k\to\infty}\left(\frac{A}{5}\right)^k$ teži u nul-matricu jer je $r\left(\frac{A}{5}\right) < 1$!

9.
$$\left.\begin{aligned} -2x_1 + 4x_2 &= b_1 \\ x_1 + 3x_2 &= b_2 \end{aligned}\right\} A = \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$x_1 = -\frac{4}{-2}x_2 + \frac{b_1}{-2} = 2x_2 - \frac{1}{2}b_1$$

$$x_2 = -\frac{1}{3}x_1 + \frac{1}{3}b_2$$

$$T = \begin{bmatrix} 0 & 2 \\ -\frac{1}{3} & 0 \end{bmatrix} \qquad b' = \begin{bmatrix} -\frac{b_1}{2} \\ \frac{b_2}{3} \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, U = \begin{bmatrix} 0 & 4 \\ 0 & 0 \end{bmatrix}$$

$$B = D^{-1}(L+U) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ \frac{4}{3} & 0 \end{bmatrix}$$

$$k_B(\lambda) = \begin{vmatrix} \lambda & 2 \\ -\frac{1}{3} & \lambda \end{vmatrix} = \lambda^2 + \frac{2}{3} \rightarrow \lambda_{1,2} = \pm 0,8165 j \Rightarrow r(B) < 1$$

$\Rightarrow$ Jacobijeva metoda konvergira!

$$x^{k+1} = Tx^k + b' \qquad b' = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$$

$$x^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x^1 = Tx^0 + b' = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$$

$$x^2 = Tx^1 + b' = (T + I)b' = \begin{bmatrix} 1 & 2 \\ -\frac{1}{3} & 1 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} \\ \frac{1}{2} \end{bmatrix}$$

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(10) $A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ -1 & -1 \end{bmatrix}$

a) $A^T \cdot A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$

$k(\lambda) = \begin{vmatrix} \lambda-5 & -1 \\ -1 & \lambda-5 \end{vmatrix} = (\lambda-5)^2 - 1 = \lambda^2 - 10\lambda + 24$

$\Rightarrow \lambda_1 = 6 \quad \lambda_2 = 4$

!

$\sigma_1 = \sqrt{6} \quad \sigma_2 = 2$

$\lambda_1 = 6 \quad \left[\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow x_1 = x_2 \Rightarrow v_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \rightarrow$ moraju biti jedinične dužine

$\lambda_2 = 4 \quad \left[\begin{array}{cc|c} -1 & -1 & 0 \\ -1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow x_1 = -x_2 \Rightarrow v_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$u_1 = \frac{1}{\sigma_1} \cdot A v_1 = \frac{1}{\sqrt{6}} \cdot \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{3}} \cdot \begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$u_2 = \frac{1}{\sigma_2} \cdot A v_2 = \frac{1}{2} \cdot \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

$$\boxed{A = \sigma_1 \cdot u_1 \cdot v_1^T + \sigma_2 \cdot u_2 \cdot v_2^T = \sqrt{6} \cdot \frac{1}{\sqrt{3}} \cdot \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 1 \end{bmatrix} \frac{1}{\sqrt{2}}}$$

b) $A^+ = \frac{1}{\sigma_1} \cdot v_1 \cdot u_1^T + \frac{1}{\sigma_2} \cdot v_2 \cdot u_2^T$

$= \frac{1}{\sqrt{6}} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & -1 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{3}} + \frac{1}{2} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{6} \cdot \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{12} & \frac{1}{12} & -\frac{1}{6} \\ -\frac{1}{12} & -\frac{5}{12} & -\frac{1}{6} \end{bmatrix}$

c) $x = A^+ \cdot b = \begin{bmatrix} 5/3 \\ 1 \\ 6 \end{bmatrix}$

11. $y'(t) = A(t)\, y(t) + f(t)$ , $y(0) = y_0$

a) $$y(t) = X(t)\left[y_0 + \int_0^t X^{-1}(s) f(s)\, ds\right]$$

$$y(t) = X(t)\, z(t)$$

$$y' = X' z + X z' = A(t)\, y(t) + f(t)$$

$$X z' = f(t)$$

$$z'(t) = X^{-1}(t) f(t) \quad / \int$$

$$z(t) - z_0 = \int_0^t X^{-1}(s)\, f(s)\, ds$$

$$y(0) = X(0) \cdot z(0) = I \cdot z(0) = z(0)$$

$$\Rightarrow z = y_0 + \int_0^t X^{-1}(s) f(s)\, ds$$

$$y = X(t) \cdot \left[y_0 + \int_0^t X^{-1}(s) f(s)\, ds\right]$$

b) $$\dot{x}_1 = 2x_1 - 2x_2 + t$$

$$\dot{x}_2 = 2x_2 + 1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -2 \\ 0 & 2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} t \\ 1 \end{bmatrix}}_{f(t)}$$

$$\dot{x} = A x + f(t)$$

$$\left|\begin{matrix} \lambda - 2 & 2 \\ 0 & \lambda - 2 \end{matrix}\right]$$

$$\left(\begin{array}{cc|c} 0 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right) \sim \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow x_2 = 0 \quad v = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$(\lambda I - A) v = -v_1$$

$$\left[\begin{array}{cc|c} 0 & 2 & -1 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \quad x_2 = -\frac{1}{2} \qquad v_2 = \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 \\ 0 & -\frac{1}{2} \end{bmatrix} \Rightarrow T^{-1} \cdot A \cdot T = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} = J \Rightarrow \text{Jordanova forma}$$

$$A = T J T^{-1}$$

$$e^{At} = T \cdot e^{Jt} T^{-1} = T \cdot \begin{bmatrix} e^{2t} & t e^{2t} \\ 0 & e^{2t} \end{bmatrix} \cdot T^{-1} = \begin{bmatrix} e^{2t} & -2t e^{2t} \\ 0 & e^{2t} \end{bmatrix}$$

d) $x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$x(t) = e^{At} \cdot x_0 + \int_0^t e^{At} \cdot e^{-As} \cdot f(s)\, ds = e^{At} x_0 + \int_0^t e^{A(t-s)} f(s)\, ds$$

$$e^{At} \cdot x_0 = \begin{bmatrix} e^{2t} & -2t e^{2t} \\ 0 & e^{2t} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2t e^{2t} \\ e^{2t} \end{bmatrix}$$

$$e^{A(t-s)} \cdot f(s) = \begin{bmatrix} e^{2(t-s)} & -2(t-s) e^{2(t-s)} \\ 0 & e^{2(t-s)} \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix} = \begin{bmatrix} s\, e^{2t} \cdot e^{-2s} - 2(t-s) e^{2(t-s)} \\ e^{2t} \cdot e^{-2s} \end{bmatrix}$$

$$\int_0^t e^{A(t-s)} f(s)\, ds = \begin{bmatrix} e^{2t} \int_0^t s e^{-2s}\, ds - 2 \int_0^t (t-s) e^{2(t-s)}\, ds \\ e^{2t} \int_0^t e^{-2s}\, ds \end{bmatrix} \Rightarrow \text{nedamise integrirat} \; ☺$$