ELEKTRIČNE POJAVE

ELEKTRIČNO POLJE

- elementarni naboj: $Q(e^-) \equiv e = -1{,}6 \cdot 10^{-19}\,C$

Coulombov zakon: - sila na naboj u polju → polje oko točkastog naboja

$$\vec{F} = k\frac{Q_1 Q_2}{r^2}\vec{r} = Q_2\vec{E} \Rightarrow E = k\frac{Q_1}{r^2}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}\vec{r} \quad \left[\frac{N}{C}\right]$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8{,}9875\cdot 10^{9}\,\frac{Nm^2}{C^2}$$

$$\varepsilon_0 = 8{,}8542\cdot 10^{-12}\,\frac{C^2}{Nm^2}$$

Princip superpozicije:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{r_i^2}\hat{r}_i$$

linearna gust. naboja: $\lambda = \frac{dQ}{dl}$

$$\Downarrow$$

$$\vec{E}_l = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dl\, \vec{r}}{r^2}$$

površinski raspored naboja $\sigma = \frac{dQ}{dS}$

$$\Downarrow$$

$$\vec{E}_\sigma = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, dS\, \vec{r}}{r^2}$$

prostorni raspored naboja $\rho = \frac{dQ}{dV}$

$$\Downarrow$$

$$\vec{E}_V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, dV\, \vec{r}}{r^2}$$

TOK EL. POLJA:

$$d\phi_E = \vec{E}\,d\vec{S} = E\cdot dS\cos\left(\frac{\vec{E}}{\vec{n}}\right) = E\,dS\cos\phi$$

$$\phi_E = \int_S \vec{E}\,\vec{n}\,dS \Rightarrow \text{ZATVORENA POVRŠINA} \quad \phi_E = \oint \vec{E}\,\vec{n}\,dS = \oint E\,dS\cos\phi$$

$$\phi_E = \oint E\,dS, \; E = \frac{1}{4\pi\varepsilon}\frac{Q}{R^2} \Rightarrow \phi_E = E\oint dS \Rightarrow 4R^2\pi$$

$$\phi_E = \frac{Q}{\varepsilon_0} \Rightarrow \text{za bilo koju zatvorenu površinu}$$

GAUSOV ZAKON ZA ELEKTRIČNO POLJE I PRVA MAXWELOVA JEDNADŽBA

$$\vec{D} = \varepsilon\cdot\vec{E}$$

$$\oint \vec{D}\,d\vec{S} = \iiint_V (\vec{\nabla}\vec{D})\,dx\,dy\,dz = \iiint_V (\vec{\nabla}\vec{D})\,dV = \iiint_V \rho\,dV \qquad \boxed{\vec{\nabla}\vec{D} = \rho}$$

ELEKTRIČNI POTENCIJAL

$$\int_A^B \vec{E}d\vec{l} = \int_A^M \vec{E}d\vec{l} + \underbrace{\int_M^{M'} \vec{E}d\vec{l}}_{0} + \int_{M'}^{N} \vec{E}d\vec{l} + \underbrace{\int_N^{N'} \vec{E}d\vec{l}}_{0} + \int_{N'}^{B} \vec{E}d\vec{l}$$

$$\int_A^B \vec{E}d\vec{l}$$

$d\vec{l} \parallel \vec{r}$ $\quad \vec{E}d\vec{l} = Edl = Edr$

$d\vec{l} \perp \vec{r}$ $\quad \vec{E}d\vec{l} = 0$

$$= \int_A^M \vec{E}d\vec{r} + \int_{M'}^{N} \vec{E}d\vec{r} + \int_{N'}^{B} \vec{E}d\vec{r} = \frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\Big|_A^M - \frac{1}{r}\Big|_{M'}^{N} - \frac{1}{r}\Big|_{N'}^{B}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(-\cancel{\frac{1}{r_M}} + \frac{1}{r_A} - \cancel{\frac{1}{r_N}} + \cancel{\frac{1}{r_{M'}}} - \frac{1}{r_B} + \cancel{\frac{1}{r_{N'}}}\right)$$

$$\oint \vec{E}d\vec{l} = 0, \; r_A = r_B \quad \Leftarrow \quad = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

SILA

$$\vec{F} = Q' \cdot \vec{E}$$

$$W = \oint \vec{F}d\vec{l} = Q'\oint \vec{E}d\vec{l} = 0 \qquad \int_A^B \vec{E}d\vec{l} = \int_A^B -dV(r) = -\left(V(r_B) - V(r_A)\right)$$

$$\left.\begin{aligned} \int_A^B \vec{E}dl &= -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \\ Edl &= -dV(r) \end{aligned}\right\} \Rightarrow \quad V(r_B) - V(r_A) = -\int_A^B \vec{E}d\vec{l}$$

$$r_A \to \infty \quad V(r_B) = -\int_\infty^{r_B} \vec{E}d\vec{l}$$

$$\vec{F} = Q\vec{E} \qquad \boxed{V(r_B) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r_B}} \qquad E = -\vec{\nabla}V$$

$W = -\int_\infty^{r_B} \vec{F}d\vec{l} = -Q\int_\infty^{r_B} \vec{E}d\vec{l} = Q'\left[V(r_B) - V(\infty)\right] = Q'V(r_B)$ - el. pot. energija u toč. B

POISSONOVA I LAPLACEOVA JEDNADŽBA

$$\left.\begin{aligned} \nabla\vec{E} &= \frac{\rho}{\varepsilon_0} \\ \vec{E} &= -\nabla V \end{aligned}\right\} \quad \nabla(-\nabla V) = -\Delta V = \frac{\rho}{\varepsilon_0} \qquad \Delta - \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$\boxed{\Delta V = -\frac{\rho}{\varepsilon_0}}$ POISSONOVA $\qquad \Rightarrow \rho = 0 \; ; \; \boxed{\Delta V = 0}$ LAPLACEOVA

RASPOREDI NABOJA

$$V(r') = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r})dV}{|\vec{r} - \vec{r}'|}$$

Linijski naboj na tankom prstenu pol. R na udaljenosti r od središta prstena: ⊥os kroz središte

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{r^2 + R^2}}$$

Površinski raspored naboja Q na ploči R na udaljenosti r od površine ploče, na okomitoj osi kroz središte (σ pov. gust. naboja).

$$V(r) = \frac{\sigma}{4\pi\varepsilon_0}\left[\sqrt{r^2 + R^2} - r\right]$$

Nevodljiva kugla:	Vodljiva kugla:
$V(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}, & r \geq R \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right), & r < R \end{cases}$	$V(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}, & r \geq R \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}, & r < R \end{cases}$

EL. POLJE U IZOLATORU (KONDENZATORU)

$E = \vec{E}_0 + \vec{E}_{ind}$ $\quad \frac{E_0}{\varepsilon_r} = E_0 - E_{ind} \Rightarrow E_{ind} = E_0\left[1 - \frac{1}{\varepsilon_r}\right]$, $E_0 = \frac{\sigma}{\varepsilon_0}$, $E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0}$

$\vec{E} = \frac{1}{\varepsilon_r}\vec{E}_0$ $\quad \sigma_{ind} = \sigma\left(1 - \frac{1}{\varepsilon_r}\right)$ $\quad Q_{ind} = Q\left(1 - \frac{1}{\varepsilon_r}\right)$

ELEKTRIČNA STRUJA

$$\left.\begin{aligned} \Delta Q &= n_0 S \Delta x q \\ \Delta x &= v_d \cdot \Delta t \end{aligned}\right\} \quad \Delta Q = n_0 S v_d \Delta t q \;\; /:\Delta t \qquad I = \frac{\Delta Q}{\Delta t} = n_0 S v_d q$$

$$\Rightarrow J = \frac{I}{S} = n_0 v_d q \qquad \boxed{\vec{J} = \sigma \vec{E}}$$ Ohmov zakon

σ → vodljivost $[S]$ spec.

$\rho = \frac{1}{\sigma}$ → specifičan otpor

vodič duljine l:

$$E = \frac{\Delta V}{l} \Rightarrow J = \sigma\frac{\Delta V}{l} = \frac{I}{S}, \; R = \frac{l}{\sigma S} \Rightarrow R = \frac{\Delta V}{I} \Rightarrow I = \frac{\Delta V}{R}$$

ENERGIJA I SNAGA

$$P = \frac{dW}{dt} = \frac{\Delta V dq}{dt} = \underline{\underline{UI}}$$

EL. MAGNETSKE POJAVE

OERSTEDOV POKUS

- pravilo desne ruke
- struja u gibanju mag. polje.

LORENZOVA SILA

$|\vec{F}_m| \propto Q v$

$|\vec{F}_m| \propto |\vec{B}|$

$\vec{F}_m \perp \vec{v}$

$\vec{F}_m \perp \vec{B}$

$|\vec{F}_m| \propto \sin\theta \equiv \sin(\vec{v}, \vec{B})$

$\vec{F}_m = Q\vec{v} \times \vec{B}$

$F_m = Q v B \sin\theta$

$\vec{F}_L = Q\vec{E} + Q\vec{v}\times\vec{B} = \vec{F}_{Le} + \vec{F}_{Lm}$

$a_{cp} = \frac{F_m}{m} = \frac{qvB}{m} = \frac{v^2}{R}$

$2R\pi = vT$

$T = \frac{2R\pi}{v} = \frac{2\pi m}{qB}$

$\nu_c = \frac{1}{T} = \frac{qB}{2\pi m}$

$\boxed{\omega_c = \frac{qB}{m}}$

SILA NA VODIČ U POLJU

$\vec{F}_m = Q\vec{v}\times\vec{B}$ $\quad dQ = n_0 q S \Delta l$

$d\vec{F} = n_0 S \Delta l q \vec{v} \times \vec{B} = I\Delta\vec{l} \times \vec{B}$

u smjeru $\vec{v}$

$$\vec{F} = I\int_A^B d\vec{l} \times \vec{B} = \int (\vec{I} \times \vec{B})\, dl$$

BIOT-SAVARTOV ZAKON

$d\vec{B} \perp d\vec{s}$

$d\vec{B} \perp d\vec{r}$

$|d\vec{B}| \propto \frac{1}{r^2}$

$|d\vec{B}| \propto I, ds$

$|d\vec{B}| \propto \sin\angle(ds, r)$

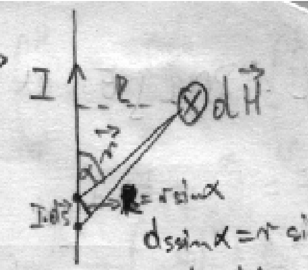
$$d\vec{B} = \frac{I\, d\vec{s} \times d\vec{r}}{r^2}\,\frac{\mu_0}{4\pi}$$

permitivnost vakuuma

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s} \times d\vec{r}}{r^2}$$

POLJE TANKOG RAVNOG VODIČA:

$$B = \frac{\mu_0 I}{2\pi r}$$



$ds \sin\alpha = r \sin\alpha\, d\alpha$

$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{I\, d\vec{s} \times d\vec{r}}{r^2}$$

$$|d\vec{s} \times d\vec{r}| = ds\,\frac{|\vec{r}|}{r}\sin\alpha$$

$\frac{|\vec{r}|}{r} \to 1$

$$|d\vec{B}| = \frac{\mu_0}{4\pi} I \frac{ds \sin\alpha}{r^2} = \frac{\mu_0}{4\pi} I \frac{r\, d\alpha}{r^2}$$

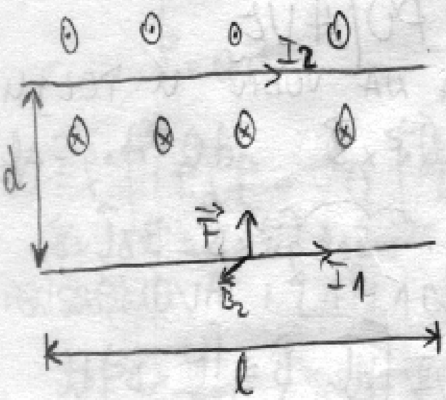
$$|d\vec{B}| = \frac{\mu_0}{4\pi} I \frac{d\alpha}{\frac{R}{\sin\alpha}} = \frac{\mu_0}{4\pi}\frac{I}{R}\sin\alpha\, d\alpha$$

$$B = \frac{\mu_0}{4\pi}\cdot\frac{I}{R}\int_0^{\pi}\sin\alpha\, d\alpha = \frac{\mu_0}{2\pi}\frac{I}{R}$$

POLJE KRUŽNE PETLJE POLUMJERA R NA OKOMITOJ OSI:

$$B = \frac{\mu_0 I R^2}{2(r^2+R^2)^{3/2}}$$

DEFINICIJA AMPERA



$$F = I_1 B_2 l = I_1 l \frac{2\mu_0 I_2}{4\pi d}$$

$$= 2\frac{\mu_0}{4\pi} l \frac{I_1 I_2}{d}$$

U istom smjeru sila je privlačna

$I_1 = I_2 = I$

$d = 1\,m$

$$\frac{F}{l} = 2\frac{\mu_0}{4\pi} I^2 = 2\cdot 10^{-7}\,\frac{N}{m}$$

$$\boxed{I = 1\,A}$$

HALLOV EFEKT

$V_H = E_t h = v_d B h$

$v_d = \frac{I}{n_0 q S}$

$V_H = \frac{IBh}{n_0 q S} = \frac{1}{n_0 q}\frac{IBh}{S}$

$= R_H \frac{IBh}{S}$

MAGNETSKI TOK, 2. MAXWELLOVA JEDNADŽBA

$$d\phi_B = B \cdot d\vec{S} = B\,dS\cos\left(\frac{\vec{B}}{d\vec{S}}\right) = B\,dS\cos\theta$$

$$\phi_B = \iint_S \vec{B}\,d\vec{S} \qquad [\phi_B] = [T \cdot m^2]$$

zatvorena povr.: $\phi_B = \oiint_S \vec{B}\,d\vec{S}$

$$\boxed{\oiint_S \vec{B}\,d\vec{S} = 0} \qquad \boxed{\nabla \cdot \vec{B} = 0}$$

$\phi_B = N_i - N_u \Rightarrow N_i = N_u$ jer se mag. silnice zatvaraju u same sebe

~~AMPEROV i AMPERE-MAXWELLOV ZAKON, 3. MAXWELOVA JEDNADŽBA~~

$$\underbrace{\oint_l \vec{B}\,d\vec{l}}_{\text{cirkulacija vektora } \vec{B} \text{ po krivulji } d\vec{l}} = \mu_0 I$$

AMPEROV ZAKON: $\oint_l \vec{B}\,d\vec{l} = \mu_0 \sum_i I_i \quad , \nabla \times \vec{B} = \mu_0 \vec{J}$

$$\oint \vec{B}\,d\vec{l} = \mu_0 \iint_S \vec{J} \cdot d\vec{S} \xRightarrow{\text{Stokes}} \oint_l \vec{B}\,d\vec{l} = \iint_S \nabla \times \vec{B}\,d\vec{S} = \mu_0 \int \vec{J} \cdot d\vec{S}$$

Ukupni tok iz svake zatvorene površine mora iščezavati:

KONT.:

$$\oint \vec{J} \cdot d\vec{S} = 0 = -\frac{dq}{dt}$$

$$\nabla \vec{J} = -\frac{\partial}{\partial t}\rho$$

$$\nabla \vec{J} = -\frac{\partial}{\partial t}\varepsilon_0 \nabla E = -\nabla\left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

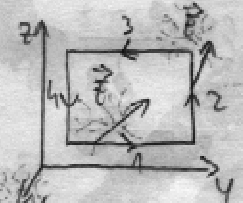
$$\oiint \vec{J}_p\,d\vec{S} = -\frac{\partial}{\partial t} Q_{unutar\,S}$$

NA NASTAVI: 3. MAXWELL, INDUKCIJA

$$\Phi_B = \iint_S B\,dS = Blx \qquad \frac{dx}{dt} = v$$

$$\frac{d\Phi_B}{dt} = Blv \qquad \varepsilon = \frac{1}{q}\oint \vec{F}\,d\vec{l} = \oint \vec{E}\,d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\boxed{\oint \vec{E}\,d\vec{l} = -\frac{d}{dt}\iint \vec{B}\,d\vec{S}}$$



$E_y(1), \Delta y$

$$\oint_k \vec{E}\,d\vec{l} = E_y(1)\cdot\Delta y + E_z(2)\Delta z - E_y(3)\Delta y - E_z(4)\Delta z$$

$$E_y(3) = E_y(1) + \frac{\partial E_y}{\partial z}\Delta z$$

$$E_z(2) = E_z(4) + \frac{\partial E_z}{\partial y}\Delta y$$

$$\oint_k \vec{E}\,d\vec{l} = \frac{\partial E_z}{\partial y}\Delta y \Delta z - \frac{\partial E_y}{\partial z}\Delta z \Delta y$$

$$= \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\Delta y \Delta z = (\vec{\nabla} \times \vec{E})_x \Delta y \Delta z$$

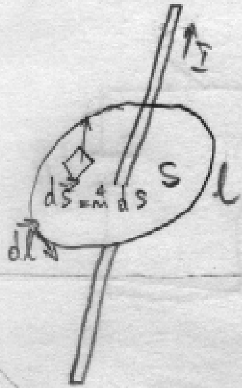
$$= (\vec{\nabla} \times \vec{E})_m \Delta S = (\vec{\nabla} \times \vec{E}) \cdot \vec{m}\,\Delta S$$

$$\oint \vec{E}\,d\vec{l} = \iint_S (\vec{\nabla} \times \vec{E})\vec{m} \cdot dS$$

$$= -\frac{\partial}{\partial t}\iint_S \vec{B}\,\vec{m}\,dS$$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}}$$

AMPEROV; AMPERÉ MAXWELOV ZAKON; 4. MAXWELL



Zbog protoka struje: $\oint_l \vec{B}\, d\vec{l} = \mu_0 I$

cirkulacija $\vec{B}$ po krivulji l

Dva vodiča:

a) $\oint \vec{B}\, d\vec{l} = \mu_0 (I_1 + I_2)$ b) $\oint \vec{B}\, d\vec{l} = \mu_0 (I_1 - I_2)$

$I_1 = I_2$ $I_1 = -I_2$

AMPEROV ZAKON:

$$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 \sum_i I_i \qquad I = \int \vec{J} \cdot d\vec{S}$$

$$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{S} \xRightarrow{\text{stokes}} \oint_l \vec{B}\, d\vec{l} = \int_S \nabla \times \vec{B}\, d\vec{S} = \mu_0 \int \vec{J} \cdot d\vec{S}$$

Polje za ravni vodič:

$$B = \mu_0 I / 2 r \pi$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$ diff. Amperov

AMPERE - MAXWELL

Ako gledamo samo jednu ploču kondenzatora tada vrijedi:

$$\oint \vec{J} \cdot d\vec{S} = 0 = -\frac{dq}{dt} \Rightarrow \nabla \vec{J} = -\frac{\partial}{\partial t} \rho \quad (1. \text{ Maxwell})$$

poopćenje

$$\Rightarrow \nabla \vec{J} = -\frac{\partial}{\partial t} (\varepsilon_0 \nabla \vec{E}) = -\nabla \left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

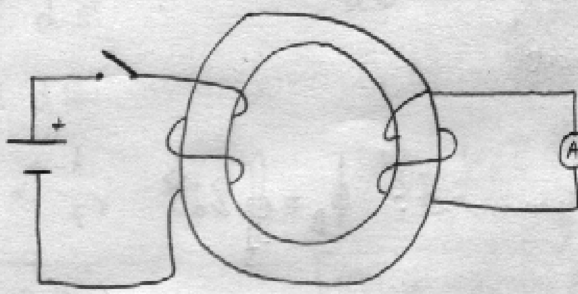
$$\oint \vec{J}_p \cdot d\vec{S} = -\frac{\partial}{\partial t} Q_{\text{unutar } S}$$

$$-\frac{\partial}{\partial t} Q_{\text{unutar } S} = -\varepsilon_0 \frac{\partial}{\partial t} \oint \vec{E}\, d\vec{S}$$

$$\oint \vec{B}\, d\vec{l} = \mu_0 (I - I_p) = \mu_0 \int (\vec{J} - \vec{J}_p)\, d\vec{S} = \mu_0 \int \vec{J} \cdot d\vec{S} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{S}$$

$$\boxed{\oint \vec{B}\, d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{S} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{S}} \xrightarrow{\text{Stokes}} \boxed{\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{E}}$$

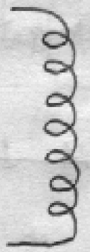
ELEKTROMAGNETSKA INDUKCIJA



$$\Phi_B = \iint \vec{B}\, d\vec{S}$$

$$\mathcal{E}_i \propto \frac{d\Phi_B}{dt}$$

$$\mathcal{E}_i = -\frac{d\Phi_B}{dt}$$



$$\mathcal{E}_{iN} = -N\frac{d\Phi_B}{dt}$$

$$\vec{F}_m = q\vec{v}\times\vec{B} \qquad qvB = qE$$

$$F_m = F_e \qquad \Rightarrow E = v\cdot B$$

$$U = E\cdot l = Bvl$$

$$W = F\cdot l = qvB\cdot l \qquad \mathcal{E} = \frac{W}{q} = vBl \qquad \mathcal{E} = \frac{1}{q}\oint \vec{F}d\vec{l} = \oint(\vec{v}\times\vec{B})d\vec{l}$$

$$\Phi_B = \iint B\,ds = Blx \qquad \frac{dx}{dt} = v$$

$$\frac{d\Phi_B}{dt} = Blv \qquad \mathcal{E} = \boxed{\oint \vec{E}\,d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{\partial}{\partial t}\iint_S \vec{B}\cdot d\vec{S}}$$

IZMJENIČNA STRUJA

$$\mathcal{E}(t) = \mathcal{E}_0 \cos\omega t = \mathcal{E}_0 \cos\frac{2\pi t}{T} \qquad \overline{I(t)^2} = \frac{1}{2}I_0^2 \qquad P = I_{ef}^2 R = \frac{1}{2}I_0^2 R = \frac{1}{2}I_0\mathcal{E}_0$$

$$I(t) = I_0 \cos\omega t = \mathcal{E}_0 \cos\frac{2\pi t}{T} \qquad I_{ef} = \frac{I_0}{\sqrt{2}}$$

ELEKTROMAGNETSKI VALOVI

VALNA JEDNADŽBA:

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\phi(x,t)=0$$

$$\phi(x,t)=f(x+vt)$$

MAXWELL:

$$\vec{\nabla}\vec{E}=\frac{\rho}{\varepsilon_0}=0 \quad (M1)$$

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \quad (M2)$$

$$\vec{\nabla}\vec{B}=0 \quad (M3)$$

$$c^2\vec{\nabla}\times\vec{B}=\cancel{\frac{\vec{J}}{\varepsilon_0}}+\frac{\partial\vec{E}}{\partial t} \quad (M4)$$

IDENTITET: $\vec{\nabla}\times(\vec{\nabla}\times\vec{A})=\vec{\nabla}(\vec{\nabla}\cdot\vec{A})-\nabla^2\vec{A}$

Postavimo se u prostor gdje se osjete samo djelovanja polja, dakle ρ i $J=0$.

EL. POLJE:

$(\vec{\nabla}\times M2)$ $\quad \vec{\nabla}\times(\vec{\nabla}\times\vec{E})=\vec{\nabla}(\vec{\nabla}\cdot E)-\nabla^2\vec{E}$

$(M1=0)$ $\quad \vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)=-\vec{\nabla}^2\vec{E}$

$$\vec{\nabla}^2\vec{E}-\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})=0$$

$(M4)$ $\quad \vec{\nabla}^2\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=0$

MAGNETSKO POLJE:

$(\vec{\nabla}\times M4)$ $\quad \vec{\nabla}\times(\vec{\nabla}\times\vec{B})=\vec{\nabla}(\vec{\nabla}\vec{B})-\nabla^2\vec{B}$

$(M3=0)$ $\quad \vec{\nabla}\times\left(\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\right)=-\nabla^2\vec{B}$

$$\vec{\nabla}^2\vec{B}+\frac{1}{c^2}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})=0$$

$(M2)$ $\quad \vec{\nabla}^2\vec{B}-\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}=0$

RJEŠENJE:

$$\vec{E}=\vec{E}(x,t)=\vec{j}E_{0y}\cos(kx-\omega t) \qquad \frac{\omega}{k}=c$$

$$\boxed{B(x,t)=\frac{k}{\omega}E_{0y}\cos(kx-\omega t)}$$

$$\vec{\nabla}\times\vec{E}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & \underbrace{E_y}_{\cos(\omega t-kx)} & 0\end{vmatrix}=-E_{0y}\cos(kx-\omega t)\left(\cancel{\vec{i}\frac{\partial}{\partial z}}-\vec{k}\frac{\partial}{\partial x}\right)=\vec{k}\left(-kE_{0y}\sin(kx-\omega t)\right)$$

$$=-\frac{\partial\vec{B}}{\partial t}=-\vec{k}\frac{\partial B_z}{\partial t}$$

(M2)

POYNTINGOV TEOREM

Mehanički rad (W_{meh}) izvršen od Lorenzove sile F_L na nabojima jednak smanjenju spremljene energije u em. polju (w_{EM}) minus energija koja je iscurila kroz površinu.

$$\partial W = \vec{F} \cdot d\vec{l} = dq\left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot d\vec{l}$$

$$= dq\left(\vec{E} + \vec{v} \times \vec{B}\right) \vec{v} \cdot dt$$

$$= dq \, \vec{E}\vec{v} \, dt \qquad dq \cdot \vec{v} = \underbrace{\rho}_{\text{gustoća naboja}} dV \cdot \vec{v} = \vec{J} \, dV$$

$$= \vec{E} \cdot \vec{J} \, dV dt$$

$$\boxed{P = \frac{dW}{dt} = \int E \cdot \vec{J} \, dV = ?}$$

$W = \int_V e_{meh} dV$ - uk. meh. prob gustoće

$$\frac{dW}{dt} = \frac{\partial}{\partial t}\int_V e_{meh} dV \Rightarrow \frac{\partial}{\partial t}\int_V e_{meh} \, dV = \frac{\partial}{\partial t}\int_V w_{EM} dV$$

$$\frac{\partial}{\partial t}\int_V \left(e_{meh} - w_{EM}\right) dV = -\int \nabla \vec{S} \, dV \quad - \int_S \vec{S} \, d\vec{a}$$

$$\nabla S = -\frac{\partial}{\partial t}\left(e_{meh} - w_{EM}\right) \Rightarrow \text{Poyntingov teorem}$$

$$(M4) \quad \vec{E} \cdot \vec{J} = \vec{E}\left(\frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$= \frac{1}{\mu_0} \vec{E}\left(\vec{\nabla} \times \vec{B}\right) - \varepsilon_0 \vec{E} \frac{\partial \vec{E}}{\partial t} \qquad \left[\vec{\nabla} \cdot \left(\vec{a} \times \vec{B}\right) = \vec{b}\left(\vec{\nabla} \times \vec{a}\right) - \vec{a}\left(\vec{\nabla} \times \vec{b}\right)\right.$$

?

$$= \frac{1}{\mu_0}\left[\vec{B} \cdot \underbrace{\left(\vec{E} \times \vec{\nabla}\right)}_{\Downarrow (M3)} - \vec{\nabla}\left(\vec{E} \times \vec{B}\right)\right] - \varepsilon_0 \vec{E} \frac{\partial \vec{E}}{\partial t}$$

$$(M3) \quad = \frac{1}{\mu_0}\left[\vec{B} \cdot \left(-\frac{\partial \vec{B}}{\partial t}\right) - \vec{\nabla}\left(\vec{E} \times \vec{B}\right)\right] - \varepsilon_0 \vec{E} \frac{\partial \vec{E}}{\partial t}$$

$$= -\frac{1}{\mu_0} \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} - \varepsilon_0 \vec{E} \frac{\partial \vec{E}}{\partial t} - \frac{1}{\mu_0} \vec{\nabla}\left(\vec{E} \times \vec{B}\right)$$

$$= -\frac{1}{2} \frac{\partial}{\partial t}\left(\frac{1}{\mu_0} B^2 - \varepsilon_0 E^2\right) - \frac{1}{\mu_0} \vec{\nabla}\left(\vec{E} \times \vec{B}\right)$$

$$w_{EM} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

$$w_{EM} = \varepsilon_0 E_{0y}^2 \cos^2(\omega t - kx)$$

$$\overline{w_{EM}} = \frac{1}{2} \varepsilon_0 E_{0y}^2$$

$$\vec{S} = \hat{i} c \, w_{EM}$$

$$P = -\frac{\partial}{\partial t}\int_V \frac{1}{2}\left(\frac{1}{\mu_0} B^2 - \varepsilon_0 E^2\right) dV - \int_V \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right) dV$$

$$= -\frac{\partial}{\partial t}\int_V \overbrace{\frac{1}{2}\left(\frac{1}{\mu_0} B^2 - \varepsilon_0 E^2\right)}^{w_{EM} \text{ - gust EM zračenja}} dV - \int_S \overbrace{\frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)}^{\text{Poyntingov vektor } \left[\frac{\text{snaga}}{\text{površina}}\right] \text{ - intenzitet}} d\vec{a} = -\frac{\partial}{\partial t} W_{em} - \int_S \vec{S} \, d\vec{a}$$

FOTOMETRIJA

KANDELA - izvor ima jakost kandele u nekom pravcu ako on emitira zračenje
(1 cd) frekvencije $\nu = 5{,}4\cdot 10^{14}$ Hz i ako je energetska jakost u tom pravcu jednaka $\frac{1}{683}$ W po steradijanu.

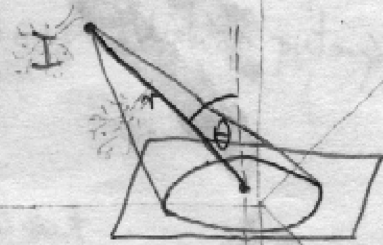
Svjetlosni tok: $d\phi = I\,d\Omega$ $\quad d\Omega = \frac{dS}{r^2}$ kut odgovara pov S na udaljenosti r

$$d\phi = I\frac{dS}{r^2} \quad [lm]$$

svjetlosna efikasnost: $\eta = \frac{\phi_{uk}}{P}$

Osvijetljenost površine: $E = \frac{d\phi}{dS} = \frac{d\phi}{dS_0}\cos\theta \quad [lx]$ → Lambertov kosinusni zakon

točkasti izvor: $\left.\begin{aligned}\int d\phi &= 4\pi I\\ \int dS &= 4R^2\pi\end{aligned}\right\} E = \frac{I}{r^2}\cos\theta$



Plošni izvori:

Iluminacija: $L = \frac{dI}{dS\cos\theta} = \frac{d^2\phi}{d\Omega\, dS\cos\theta} \quad \left[\frac{cd}{m^2}\right]$

$$d\phi = L\,dS\int\cos\theta\, d\Omega = L\,dS\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\, d\theta\int_0^{2\pi} d\varphi$$

$$= \pi L\, dS$$

Svjetljivost (egzitancija):

$$M = \frac{d\phi}{dS} = \pi L \quad \left[\frac{lm}{m^2}\right]$$

GEOMETRIJSKA OPTIKA

ZAKONI

1. Zakon pravocrtnog širenja.
2. Zakon nezavisnosti snopova
3. Zakon refleksije $\vartheta_1 = \vartheta_2$
4. Zakon loma (Snell) $\frac{\sin\vartheta_1}{\sin\vartheta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$ $\qquad n = \frac{c}{v}$

PRINCIP NAJKRAĆEG VREMENA (Fermatov princip)

Iz jedne točke u drugu svjetlost putuje najkraćim putem.

Onim putem kojim će stići u najkraćem vremenu (među bliskim putanjama)

DULJINA OPTIČKOG PUTA:

el. vremena $dt = \frac{1}{v} dl$ el. geo. puta

brzina svjetla u sredstvu

$dt = \frac{n}{c} dl$

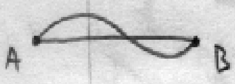
optički put

$ds = c\,dt$

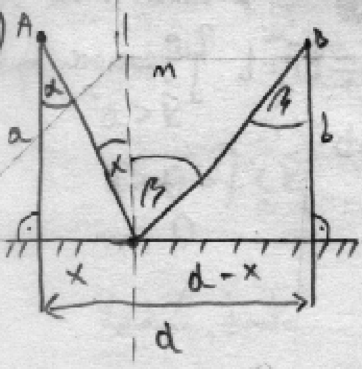
$ds = n\,dl$

FERMATOV PRINCIP DOKAZUJE SVE

1.



3.

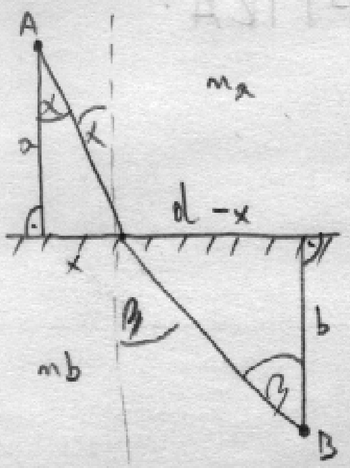


$s(x) = \sqrt{a^2 + x^2} + \sqrt{(b)^2 + (d-x)^2}$

$$0 = \frac{ds}{dx} = \frac{2x}{2\sqrt{a^2+x^2}} + \frac{2(d-x)(-1)}{2\sqrt{b^2+(d-x)^2}} = \frac{x}{\sqrt{a^2+x^2}} - \frac{(d-x)}{\sqrt{b^2+(d-x)}}$$

$$= \sin\alpha - \sin\beta \Rightarrow \boxed{\alpha = \beta}$$

4.



$$S(x) = n_a\sqrt{a^2+x^2} + n_b\sqrt{b^2+(d-x)^2}$$

$$\frac{dS(x)}{dx} = 0 = \frac{n_a x}{\sqrt{a^2+x^2}} - \frac{n_b(d-x)}{\sqrt{b^2+(d-x)^2}}$$

$$= n_a \sin\alpha - n_b \sin\beta = 0$$

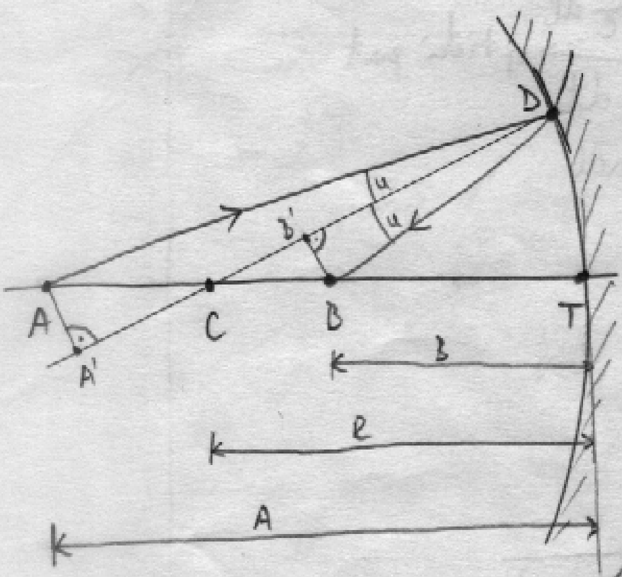
$$\boxed{\frac{n_a}{n_b} = \frac{\sin\beta}{\sin\alpha}} = \frac{\frac{c}{v_a}}{\frac{c}{v_b}} = \frac{v_b}{v_a}$$

RAVNO ZRCALO

stigmatičnost - daje vjernu oštru sliku

$a = b$

SFERNO ZRCALO



$$\Delta(AA'C) \cong \Delta(BB'C)$$

$$\Delta(AA'D) \cong \Delta(BB'D)$$

$$\frac{\overline{AA'}}{\overline{AC}} = \frac{\overline{BB'}}{\overline{BC}} \quad ; \quad \frac{\overline{AA'}}{\overline{AD}} = \frac{\overline{BB'}}{\overline{BD}}$$

$$\frac{\overline{AD}}{\overline{AC}} = \frac{\overline{BD}}{\overline{BC}}$$

Ako se radi o paraksijalnim zrakama tada vrijedi:

$\overline{AD} \simeq \overline{AT} = a$ i $\overline{BD} \simeq \overline{BT} = b$ } Gaussove aproksimacije

$$\frac{a}{\overline{AC}} = \frac{b}{\overline{BC}}$$

$$\overline{AC} = a - R \qquad \overline{BC} = R - b$$

$$b\overline{AC} = a\overline{BC}$$

$$\frac{\overline{AC}}{a} = \frac{\overline{BC}}{b} \Rightarrow \frac{a-R}{a} = \frac{R-b}{b} \quad /\cdot ab \quad /: abR$$

$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{2}{R}}$$

ŽARIŠTA SFERNOG ZRCALA

F_a - predmetno žarište - Ono mjesto na optičkoj osi iz kojeg zrake kada izlaze nakon refleksije na zrcalu imaju smjer paralelan s optičkom osi.

$\overline{F_aT} = f_a$

$a \to f_a \quad b \to \infty$

$$\frac{1}{f_a} + \frac{1}{\infty} = \frac{2}{R} \Rightarrow f_a = \frac{R}{2}$$

$$\boxed{\frac{f}{a} + \frac{f}{b} = 1 \text{ ili } \frac{1}{a} + \frac{1}{b} = \frac{1}{f}}$$

F_b - slikovno žarište $\quad \overline{F_bT} = f_b$

$f_b = f_a \equiv f$

(Za ravno vrijedi $a = -b$ i $R = \infty$)

KONVENCIJA O PREDZNACIMA

zrake dolaze s lijeva na desno

- i A, B, C lijevo od tjemena onda $a>0,\ b>0,\ R>0$
- i ako je B s desne strane tjemena onda $b<0$
- i ako je C s desne strane tjemena onda $c<0$

TRANSFERZALNO POVEĆANJE

$$m = -\frac{b}{a}$$

$m<0$ slika je obrnuta

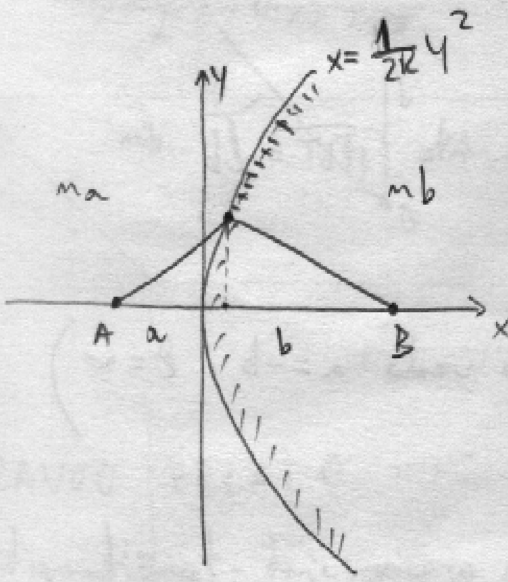
$|m|>1$ slika je uvećana

UDUBLJENO $(R>0)$ — ISPUPČENO $(R<0)$

$a>R$	$a=R$	$f<a<R$	$a=f$	$a<f$	Bilo gdje
$b>0>f<R$	$b=a$	$b>R$	$b=+\infty$	$b<0$	$b<0$
$m<0$	$m=-1$	$m<-1$	—	$m>0$	$m>0$
Realna, obrnuta umanjena	Realna obrnuta	Realna obrnuta povećana	=	Virtualna uspravna uvećana	Virtualna uspravna smanjena

Totalna refleksija $\beta_g = \arcsin \frac{n_1}{n_2}$

LOM SVJETLOSTI NA SFERNOJ GRANICI



približni zakrivljenosti parabole: $x = \frac{1}{2R}y^2$

$$S(y) = n_a\sqrt{(a+x)^2+y^2} + n_b\sqrt{(b-x)^2+y^2}$$

$$= n_a\sqrt{\left(a+\frac{y^2}{2R}\right)^2+y^2} + n_b\sqrt{\left(b-\frac{y^2}{2R}\right)^2+y^2}$$

$$\sqrt{1+\varepsilon} \simeq 1+\frac{1}{2}\varepsilon$$

razlika duljina optičkog puta:

$$\Delta S(y) = S(y) - S(0)$$

$$= S(y) - (n_a a + n_b b)$$

$$= n_a a\left[\sqrt{\left(1+\frac{y^2}{2Ra}\right)^2+\frac{y^2}{a^2}} - 1\right]$$

$$+ n_b b\left[\sqrt{\left(1+\frac{y^2}{2Rb}\right)^2+\frac{y^2}{b^2}} - 1\right]$$

za male $\frac{y}{a}, \frac{y}{b}, \frac{y}{R}$

$$\Delta S(y) = n_a a\left(\cancel{1}+\frac{1}{2}\frac{y^2}{Ra}+\frac{1}{2}\frac{y^2}{a^2}-\cancel{1}\right)$$

$$+ n_b b\left(1+\frac{1}{2}\frac{y^2}{Rb}+\frac{1}{2}\frac{y^2}{b^2}-1\right)$$

$$= \frac{y^2}{2}\underbrace{\left(\frac{n_a}{R}+\frac{n_a}{a}-\frac{n_b}{R}+\frac{n_b}{b}\right)}_{=0}$$

$\Delta S(y) = 0$

$$\Rightarrow \boxed{\frac{n_a}{a}+\frac{n_b}{b}=\frac{n_b-n_a}{R}}$$

jdžb. sfernog dioptra.

ŽARIŠTA

$a \to f_a \quad b \to \infty \qquad b \to f_b \quad a \to \infty$

$$f_a = \frac{n_a}{n_b-n_a}R \qquad f_b = \frac{n_b}{n_b-n_a}R$$

$$\frac{f_b}{f_a} = \frac{n_2}{n_1} \qquad \frac{f_a}{a}+\frac{f_b}{b}=1$$

PREDZNACI:

$a>0$ predmet lijevo od tjemena

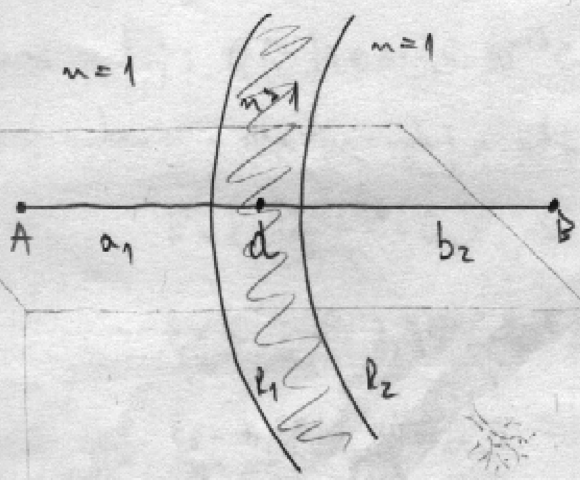
$b>0$ slika desno od tjemena

$R>0$ središte zakrivljenosti desno od tjemena

$$m = -\frac{n_1 b}{n_2 a}$$

LEĆA

- dva sferna dioptra



$$\frac{1}{a_1}+\frac{n}{b_1}=\frac{n-1}{R_1} \quad \text{(prvi dioptar)}$$

$$\frac{n}{a_2}+\frac{1}{b_2}=\frac{1-n}{R_2} \quad \text{(drugi dioptar)}$$

$d=b_1+a_2 \Rightarrow$ izvor drugog je točka fokusa prvog zrcala

Tanka leća $d \to 0$

$$\left.\begin{aligned} \frac{1}{a_1}+\frac{n}{b_1}&=\frac{n-1}{R_1} \\ \frac{n}{-b_1}+\frac{1}{b_2}&=\frac{1-n}{R_2} \end{aligned}\right\}$$

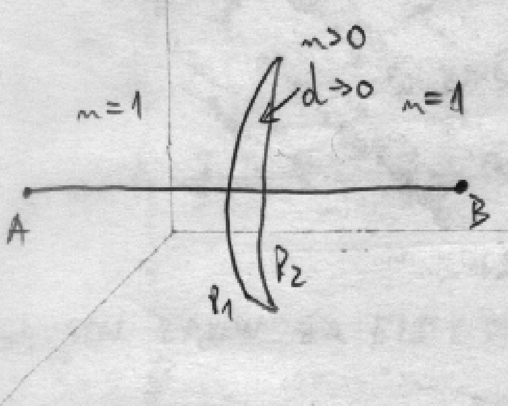
$$\frac{1}{a_1}+\frac{1}{b_2}=\frac{n-1}{R_1}+\frac{1-n}{R_2}$$

$$=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(n-1)\left(\frac{R_2-R_1}{R_1R_2}\right)$$

$$\frac{1}{a_1}+\frac{1}{b_2}=(n-1)\left(\frac{R_2-R_1}{R_1R_2}\right)$$

$$=\frac{1}{f}=J \text{ - jakost povećanja}$$



$$\frac{f}{a}+\frac{f}{b}=1 \qquad m=-\frac{b}{a}$$

PREDZNACI:

$a>0$ predmet lijevo od tjemena

$b>0$ slika desno od tjemena

$R>0$ centar zakrivljenosti desno od tjemena