10. TJEDAN (SIS AUDITORNE)

10. zadatak

a) homogena jednadžba: $y(n)+4y(n-1)+4y(n-2)=0$

$$Cq^{n}+4Cq^{n-1}+4Cq^{n-2}=0$$

$$Cq^{n}(1+4q^{-1}+4q^{-2})=0 \quad /:Cq^{n} \quad /\cdot q^{2}$$

$$q^{2}+4q+4=0 \qquad m\cdot n=4 < \begin{matrix}2\\2\end{matrix}$$

$$m+n=4$$

$$q^{2}+2q+2q+4=0 \qquad (q+2)(q+2)=0$$

$$q(q+2)+2(q+2)=0 \qquad q_{1,2}=-2 \Rightarrow$$ kada je $q_1=q_2$ (dvostruka multipl.) ispred "2." člana y_H dodaje se "n" pri množe

!

$$y_H(n)=C_1(-2)^{n}+C_2\,n\,(-2)^{n}$$

$$y_H(n)=C_1q^{n}+C_2q^{n}$$

[illegible] ...

impulsni odziv: $(u(n)=\delta(n))$

$$y(n)+4y(n-1)+4y(n-2)=\delta(n)$$

$$y(n)=\delta(n)-4y(n-1)-4y(n-2)$$

$$y_H(n) = C_1(-2)^n + C_2\, n\, (-2)^n$$

kvůli "2" kořenu y_H dodáváme
se "n" při množe

$$y_H(n) = C_1 q^n + C_2 q^n$$

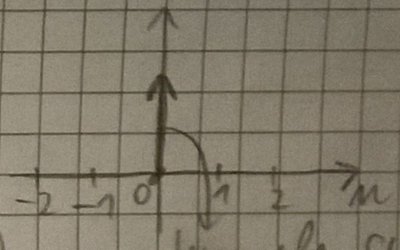
$n q^n \ldots$

Impulsní odezva: $(u(n) = \delta(n))$

$$y(n) + 4y(n-1) + 4y(n-2) = \delta(n)$$

$$y(n) = \delta(n) - 4y(n-1) - 4y(n-2)$$

$(n=0)$ $\quad y(0) = \underbrace{\delta(0)}_{1} - 4\underbrace{y(-1)}_{=0} - 4\underbrace{y(-2)}_{=0} = 1$



Impuls $\delta(n)$

$(n=1)$ $\quad y(1) = \underbrace{\delta(1)}_{=0} - 4\underbrace{y(0)}_{1} - 4\underbrace{y(-1)}_{=0} = -4$

$$y_H(0) = C_1(-2)^0 + C_2 \cdot 0 \cdot (-2)^0 = C_1 = 1$$

$$y_H(1) = C_1(-2)^1 + C_2 \cdot 1 \cdot (-2)^1 = -2C_1 - 2C_2 = -4$$

$$-2 \cdot 1 - 2C_2 = -4$$

$$-2C_2 = -4 + 2 \qquad C_2 = 1$$

$$h(n) = y_H(n) = \underbrace{\left[\overset{1}{C_1}(-2)^n + \overset{1}{C_2}\, n(-2)^n\right]}_{\mu(n)} = \left[(-2)^n(1+n)\right] \cdot \mu(n)$$

c) g od homogenog rješenja ($g = -2$) (dvostruki) podudara se s frekvencijom pobude $u(n) = (-2)^n \mu(n)$ pa u partikularno rješenje dodajemo "n^2":

$y_p(n) = K(-2)^n \cdot n^2$, da su g i frekv. pobude $\neq$ (npr. $g = -3$, $u(n) = (-4)^n \mu(n)$,

y_p bi bio $y_p(n) = K(-4)^n$ bez "n" ili "n^2"

$y_p(n) = K(-2)^n \cdot n^2 \Rightarrow$ u jednadžbu: $K(-2)^n n^2 + 4K(-2)^{n-1}(n-1)^2 + 4K(-2)^{n-2} \cdot (n-2)^2 = (-2)^n \;/ : (-2)^n$

$y_H(n) = C_1(-2)^n + C_2 n(-2)^n$

(da je rješenje homogene jednostruki $g = -2$, a ne $g_{1,2}$ (dvostruki) $= -2$, $y_p(n) = K(-2)^n n$, a ne $K(-2)^n n^2$!)

$$Kn^2 + 4K\underbrace{(-2)^{-1}}_{-\frac{1}{2}}(n^2 - 2n + 1) + 4K\underbrace{(-2)^{-2}}_{\frac{1}{4}}(n^2 - 4n + 4) = 1$$

$$Kn^2 - 2K(n^2 - 2n + 1) + K(n^2 - 4n + 4) = 1$$

$$Kn^2 - 2Kn^2 + 4Kn - 2K + Kn^2 - 4Kn + 4K = 1$$

$$n^2(K - 2K + K) + n(4K - 4K) - 2K + 4K = 1 + n^2 \cdot 0$$

Izjednačavanje koeficijenata uz stupnjeve polinoma:

uz n^2: $K - 2K + K = 0$?

uz n: $4K - 4K = 0$?

uz n^0: $-2K + 4K = 1 \Rightarrow K = \frac{1}{2}$

$$y_{PRISILNI}(n) = \left[\frac{1}{2} n^2 (-2)^n\right] \cdot \mu(n)$$

$\downarrow$

$y_{PART.}$ (y_p)

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(11) a) homogena jednadžba: $y(n) - \ldots$

partikularno rješenje: $y_p(n) = K \ldots$

u zadatku (B.) y_p ima "n" ($y_p = K\left(\frac{1}{4}\right)^n n$), a ovdje y_p nema n ($y_p = K\left(\frac{1}{2}\right)^n$); n se dodaje samo kad se pogode frekv. pobude i hom. rješenja

(B) zad. $u(n) = \left(\frac{1}{4}\right)^n$ $y_H(n) = C\left(\frac{1}{4}\right)^n$

$y_{TOTALNI}(n) = y_H(n) + y_P(n) = \left[C\left(\frac{1}{3}\right)^n \ldots\right.$

$n = 0 \Rightarrow y(n) - \frac{1}{3}y(n-1) = 2^{\ldots}$

$y(0) = C\left(\frac{1}{3}\right)^0 + 3\left(\frac{1}{2}\right)^0$

$4 = C + 3 \Rightarrow C = 1$

uz n^2: $K-2K+K=0$?

uz n: $4K-4K=0$?

uz n^0: $-2K+4K=1 \Rightarrow K=\frac{1}{2}$

$y_{PRISILNI}(n)=\left[\frac{1}{2}n^2(-2)^n\right]\cdot\mu(n)$

$\downarrow$

$y_{PART.}(y_P)$

mirni odziv: početni uvjeti $=0$ $\quad y(-1)=y(-2)=0$

$n=0 \Rightarrow y(0)+4\underbrace{y(0-1)}_{0}+4\underbrace{y(0-2)}_{0}=\underbrace{(-2)^0}_{1} \quad y(0)=1$

$n=1 \Rightarrow y(1)+4\underbrace{y(1-1)}_{y=0=1}+4\underbrace{y(1-2)}_{y(-1)=0}=(-2)^1$

$y(1)+4\cdot 1=-2 \quad y(1)=-6$

$y_{MIRNI}(0)=C_1(-2)^0+\cancel{C_2\cdot 0\cdot(-2)^0}+\cancel{\frac{1}{2}\cdot 0^2\cdot(-2)^0}=1 \quad C_1=1$

$y_{MIRNI}(1)=C_1(-2)^1+C_2\cdot 1\cdot(-2)^1+\frac{1}{2}\cdot 1^2\cdot(-2)^1=-6$

$-2-2C_2-1=-6$

$-2C_2=-3 \quad C_2=\frac{3}{2}$

b) kao c) uz

$y_P(n)=K \ldots$

$y_{MIRNI}(n)=\left[(-2)^n+\frac{3}{2}n^2(-2)^n+\frac{1}{2}n^2(-2)^n\right]\mu(n)$

d) sustav nestabilan $|a|=|-2|=2>1$

$y(0)=\ldots$

4

1

$y(0)=C\left(\frac{1}{3}\right)^0+3\left(\ldots\right.$

$4=C+3 \Rightarrow C=\ldots$

b) mirni sustav $\Rightarrow$ p…

$n=0 \Rightarrow y(n)-\frac{1}{3}y\ldots$

$y(0)=2^{-0}\ldots$

$y(0)=C\left(\frac{1}{3}\right)^0+3\left(\frac{1}{2}\right.\ldots$

$C+3=1$

$C=-2$

$y_{MIRNI}(n)=\left[-2\left(\frac{1}{3}\right)\ldots\right.$

...om pobude

...mo $n^2 n$:

$, u(n) = (-4)^n \mu(n)$

$k(-2)^{n-2} \cdot (n-2)^2 =$

$(-2)^n$

$/ : (-2)^n$

$-4n+4) = 1$

$= 1$

$+ n^2 = 0$

...ugnjitve

$(n) = \left[\frac{1}{2} n^2 (-4)^n\right]$

$\cdot \mu(n)$

(11.) a) homogena jednadžba: $y(n) - \frac{1}{3} y(n-1) = 0 \qquad y_H(n) = Cq^n$

$$Cq^n - \frac{1}{3} Cq^{n-1} = 0 \qquad y_H(n) = C\left(\frac{1}{3}\right)^n$$

$$Cq^n\left(1 - \frac{1}{3} q^{-1}\right) = 0 \quad /: Cq^n \quad /\cdot q$$

$$q - \frac{1}{3} = 0 \quad q = \frac{1}{3}$$

partikularno rješenje: $y_p(n) = K \cdot 2^{-n} = 3 \cdot \left(\frac{1}{2}\right)^n$

u zadatku (8.) y_p ima "n" ($y_p = K\left(\frac{1}{4}\right)^n n$), a ovdje y_p nema n ($y_p = K\left(\frac{1}{2}\right)^n$); n se dodaje samo kad se pogode frekv. pobude i hom. rješenja

(8.) zad. $u(n) = \left(\frac{1}{4}\right)^n \quad y_H(n) = C\left(\frac{1}{4}\right)^n$

$$y_p(n-1) = K \cdot 2^{-n+1}$$

$$K \cdot 2^{-n} - \frac{1}{3} K \cdot 2^{-n+1} = 2^{-n} \quad /: 2^{-n}$$

$$K - \frac{1}{3} K \cdot 2 = 1 \qquad \frac{1}{3} K = 1 \Rightarrow K = 3$$

$$y_{TOTALNI}(n) = y_H(n) + y_p(n) = \left[C\left(\frac{1}{3}\right)^n + 3\left(\frac{1}{2}\right)^n\right] \mu(n)$$

$$n = 0 \Rightarrow y(n) - \frac{1}{3} y(n-1) = 2^{-n}$$

$$y(0) = \underbrace{2^{-0}}_{1} + \frac{1}{3} \underbrace{y(0-1)}_{9} = 1 + \frac{9}{3} = 4$$

ngeve

$y=\left[\frac{1}{2}n^2(-2)^n\right]$

$\cdot\mu(n)$

$y_p)$

o c) MZ

$n)=K$ ...

$]\mu(n)$

$=2>1$

$n=0 \Rightarrow y(n)-\frac{1}{3}y(n-1)=2^{-n}$

$$y(0)=\underbrace{2^{-0}}_{1}+\frac{1}{3}\underbrace{y(0-1)}_{9}=1+\frac{9}{3}=4$$

$$y(0)=C\left(\frac{1}{3}\right)^0+3\left(\frac{1}{2}\right)^0$$

$4=C+3 \Rightarrow C=1$

$$y_{TOTALNI}(n)=\left[\left(\frac{1}{3}\right)^n+3\left(\frac{1}{2}\right)^n\right]\mu(n)$$

b) mirni sustav $\Rightarrow$ poč. uv. $y(-1)=0$

$n=0 \Rightarrow y(n)-\frac{1}{3}y(n-1)=2^{-n}$

$$y(0)=2^{-0}+\frac{1}{3}\underbrace{y(0-1)}_{0}=1$$

$$y(0)=C\left(\frac{1}{3}\right)^0+3\left(\frac{1}{2}\right)^0=1$$

$C+3=1$

$C=-2$

$$y_{MIRNI}(n)=\left[-2\left(\frac{1}{3}\right)^n+3\left(\frac{1}{2}\right)^n\right]\mu(n)$$

nepobuđeni sustav: pobuda $=0$

$u(n)=0$

$y_p(n)=0$

$$y(n)-\frac{1}{3}y(n-1)=0$$

$$y(0)=\frac{1}{3}\underbrace{y(0-1)}_{9}=3$$

$y_H(0)=C\left(\frac{1}{3}\right)^0=3 \Rightarrow C=3$

$$y_{nepobuđeni}(n)=\left[3\left(\frac{1}{3}\right)^n\right]\mu(n)$$