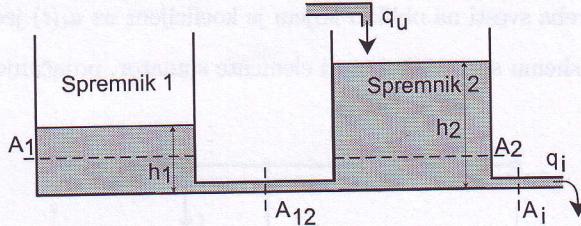


A1

Na slici je prikazan sustav skladištenja fluida u dva spojena spremnika (spremnik 1, spremnik 2). Razina fluida u spremnicima regulira se promjenom ulaznog protoka $q_u(t)$. Potrebno je:



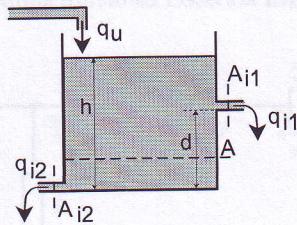
1. Napisati diferencijalne jednadžbe koje opisuju ponašanje razine fluida u spremnicima 1 i 2.
2. Skicirati blokovsku shemu sustava skladištenja fluida koristeći elemente sumator, pojačanje, integrator, matematički blok sqrt.

Napomena: Prilikom rješavanja zadatka prepostaviti da vrijedi $A_i, A_{12} \ll A_1, A_2$ te $h_2 > h_1$.

$$\begin{aligned} q_u - q_i - q_{12} &= \rho A_2 \frac{dh_2}{dt} \\ q_{12} &= \rho A_1 \frac{dh_1}{dt} \\ q_i &= A_i \rho \sqrt{2gh_2} \\ q_{12} &= A_{12} \rho \sqrt{2g(h_2 - h_1)} \\ \frac{dh_2}{dt} &= \frac{1}{\rho A_2} q_u - \frac{A_i}{A_2} \sqrt{2gh_2} - \frac{A_{12}}{A_2} \sqrt{2g(h_2 - h_1)} \\ \frac{dh_1}{dt} &= \frac{A_{12}}{A_1} \sqrt{2g(h_2 - h_1)} \end{aligned}$$

A2

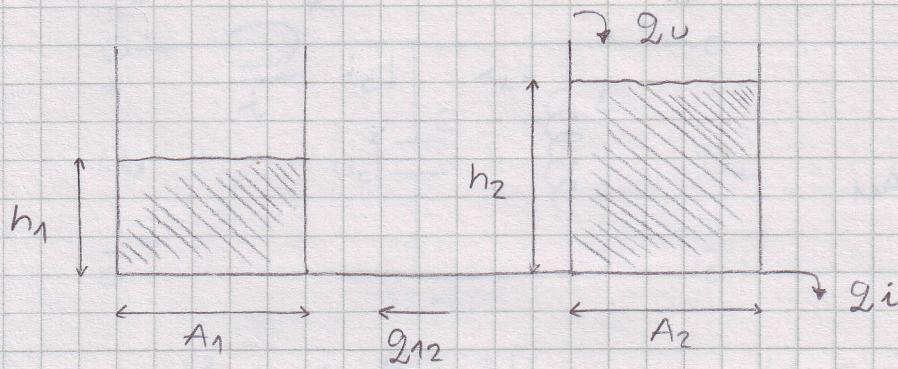
Na slici je prikazan sustav skladištenja fluida u spremniku. Razina fluida u spremniku regulira se promjenom ulaznog protoka $q_u(t)$. Potrebno je:



1. Napisati diferencijalnu jednadžbu koja opisuje ponašanje razine fluida u spremniku.
2. Skicirati blokovsku shemu sustava skladištenja fluida koristeći elemente sumator, pojačanje, integrator, matematički blok sqrt.

Napomena: Prilikom rješavanja zadatka prepostaviti da vrijedi $A_{i1}, A_{i2} \ll A$ te $h > d$.

$$\begin{aligned} q_u - q_{i1} - q_{i2} &= \rho A \frac{dh}{dt} \\ q_{i1} &= A_{i1} \rho \sqrt{2g(h - d)} \\ q_{i2} &= A_{i2} \rho \sqrt{2gh} \\ \frac{dh}{dt} &= \frac{1}{\rho A} q_u - \frac{A_{i1}}{A} \sqrt{2g(h - d)} - \frac{A_{i2}}{A} \sqrt{2gh} \end{aligned}$$



$$(1) \quad q_{12} = A_{12} \rho \frac{dh_1}{dt}$$

$$\frac{dh_1}{dt} = \frac{A_{12}}{A_1} \sqrt{2g} \sqrt{h_2 - h_1}$$

$$q_{12} = A_{12} \rho v_{12}$$

$$q_{12} = A_{12} \rho \sqrt{2g} \sqrt{h_2 - h_1}$$

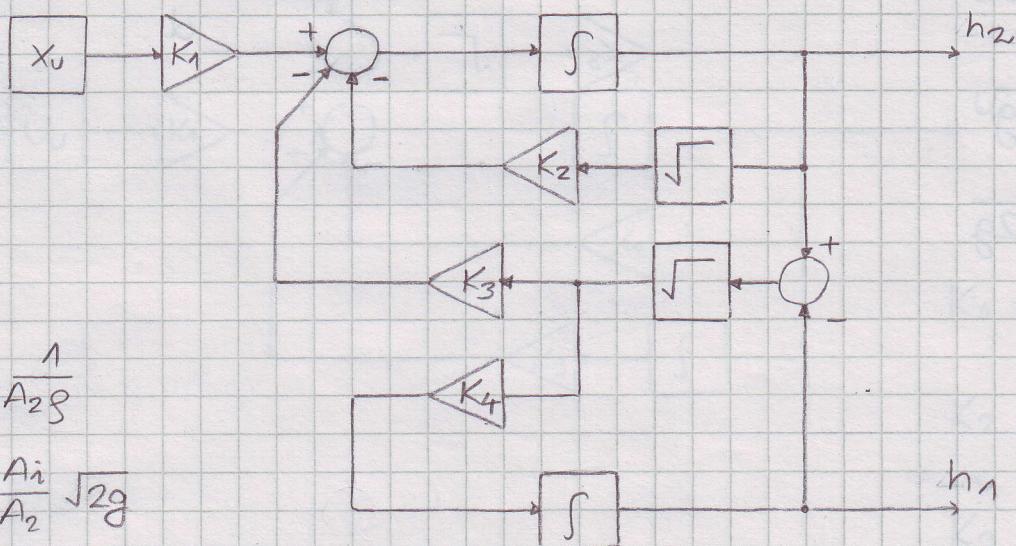
$$(2) \quad q_u - (q_i + q_{12}) = A_2 \rho \frac{dh_2}{dt}$$

$$q_i = A_i \rho v_i$$

$$q_i = A_i \rho \sqrt{2g} \sqrt{h_2}$$

$$q_u - A_i \rho \sqrt{2g} \sqrt{h_2} - A_{12} \rho \sqrt{2g} \sqrt{h_2 - h_1} = A_2 \rho \frac{dh_2}{dt}$$

$$\frac{dh_2}{dt} = \frac{q_u}{A_2 \rho} - \frac{A_i}{A_2} \sqrt{2g} \sqrt{h_2} - \frac{A_{12}}{A_2} \sqrt{2g} \sqrt{h_2 - h_1}$$

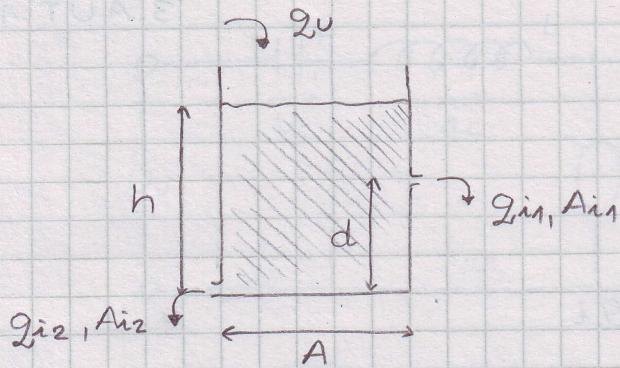


$$K_1 = \frac{1}{A_2 \rho}$$

$$K_2 = \frac{A_i}{A_2} \sqrt{2g}$$

$$K_3 = \frac{A_{12}}{A_2} \sqrt{2g}$$

$$K_4 = \frac{A_{12}}{A_1} \sqrt{2g}$$



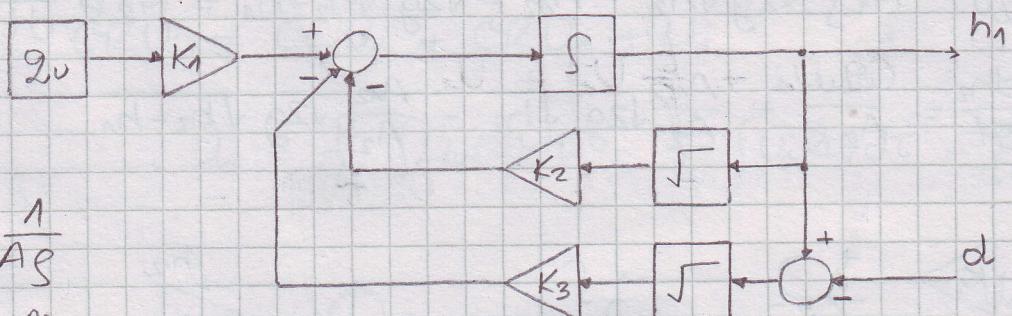
$$Q_U - (Q_{i1} + Q_{i2}) = A \beta \frac{dh}{dt}$$

$$Q_{i1} = A_{i1} \beta \sqrt{2g} \sqrt{h-d}$$

$$Q_{i2} = A_{i2} \beta \sqrt{2g} \sqrt{h}$$

$$Q_U - A_{i1} \beta \sqrt{2g} \sqrt{h-d} - A_{i2} \beta \sqrt{2g} \sqrt{h} = A \beta \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{Q_U}{A \beta} - \frac{A_{i1}}{A} \sqrt{2g} \sqrt{h-d} - \frac{A_{i2}}{A} \sqrt{2g} \sqrt{h}$$



$$K_1 = \frac{1}{A \beta}$$

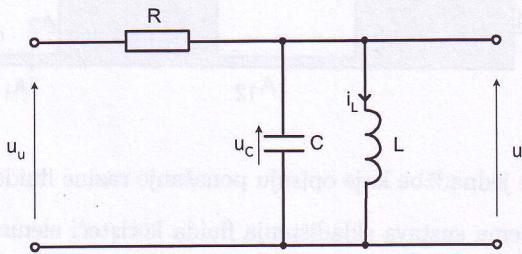
$$K_2 = \frac{A_{i2}}{A} \sqrt{2g}$$

$$K_3 = \frac{A_{i1}}{A} \sqrt{2g}$$

B1

Na slici je prikazana shema električnog kruga. Potrebno je:

- Napisati diferencijalnu jednadžbu koja opisuje ovisnost izlaznog napona $u_i(t)$ o ulaznom naponu $u_u(t)$. Jednadžbu treba svesti na oblik u kojem je koeficijent uz $u_i(t)$ jednak 1.
- Skicirati blokovsku shemu sustava koristeći elemente sumator, pojačanje i integrator.

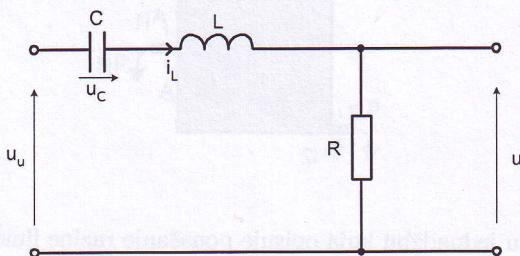


$$\begin{aligned}
 u_u &= iR + u_i \\
 i &= i_C + i_L \\
 i_C &= C \frac{du_i}{dt} \\
 i_L &= \frac{1}{L} \int_0^t u_i d\tau + i_{L0} \\
 u_u &= RC \frac{du_i}{dt} + \frac{R}{L} \left(\int_0^t u_i d\tau + i_{L0} \right) + u_i \quad / \frac{d}{dt} \quad / \frac{L}{R} \\
 \frac{R}{L} \frac{du_u}{dt} &= LC \frac{d^2 u_i}{dt^2} + \frac{L}{R} \frac{du_i}{dt} + u_i
 \end{aligned}$$

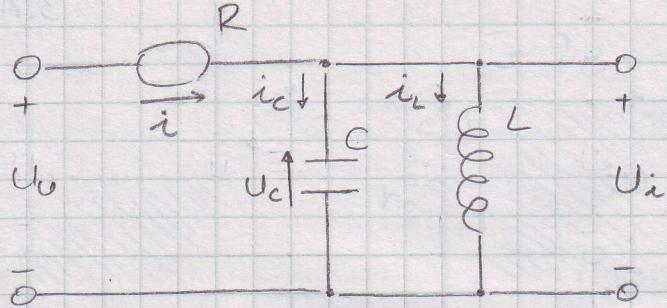
B2

Na slici je prikazana shema električnog kruga. Potrebno je:

- Napisati diferencijalnu jednadžbu koja opisuje ovisnost izlaznog napona $u_i(t)$ o ulaznom naponu $u_u(t)$. Jednadžbu treba svesti na oblik u kojem je koeficijent uz $u_i(t)$ jednak 1.
- Skicirati blokovsku shemu sustava koristeći elemente sumator, pojačanje i integrator.



$$\begin{aligned}
 u_u &= \frac{1}{C} \int_0^t id\tau + u_{C0} + L \frac{di}{dt} + u_i \quad / \frac{d}{dt} \\
 i &= \frac{u_i}{dt} \\
 \frac{du_u}{dt} &= \frac{1}{RC} u_i + \frac{L}{R} \frac{d^2 u_i}{dt^2} + \frac{du_i}{dt} \quad / RC \\
 RC \frac{du_u}{dt} &= LC \frac{d^2 u_i}{dt^2} + RC \frac{du_i}{dt} + u_i
 \end{aligned}$$



$$U_U = i \cdot R + U_i$$

$$i = i_C + i_L$$

$$U_C = \frac{1}{C} \int i_C(\tau) d\tau$$

$$\dot{i}_C = C \ddot{U}_C$$

$$U_L = L \frac{di_L}{dt}$$

$$i = C \ddot{U}_i + \frac{1}{L} \int U_i(\tau) d\tau \quad i_L = \frac{1}{L} \int U_L(\tau) d\tau$$

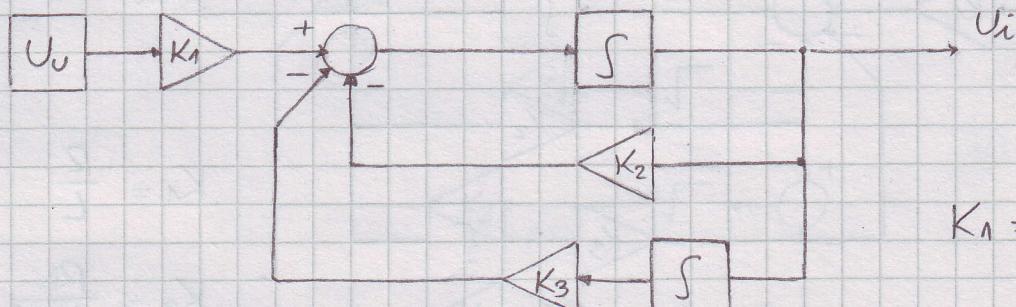
$$U_i = U_C = U_L$$

$$U_U = (C \ddot{U}_i + \frac{1}{L} \int U_i(\tau) d\tau) R + U_i / \frac{d}{dt}$$

$$\ddot{U}_U = CR \ddot{U}_i + \frac{R}{L} U_i + \dot{U}_i / \cdot \frac{L}{R}$$

$$\frac{L}{R} \ddot{U}_U = CL \ddot{U}_i + \frac{L}{R} \dot{U}_i + U_i$$

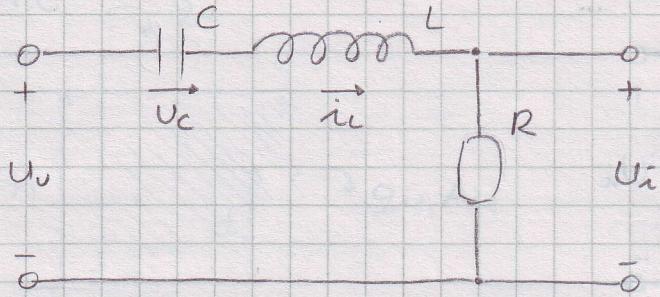
$$U_i = \frac{1}{RC} \int U_U(\tau) d\tau - \frac{1}{RC} \int U_i(\tau) d\tau - \frac{1}{LC} \int \int U_i(\tau) d\tau d\tau$$



$$K_1 = \frac{1}{RC}$$

$$K_2 = \frac{1}{RC}$$

$$K_3 = \frac{1}{LC}$$



$$U_o = U_C + U_L + U_i$$

$$U_i = i_L \cdot R \rightarrow i_L = \frac{1}{R} U_i$$

$$(i_L)' = \frac{1}{R} \dot{U}_i \quad (i_L)'' = \frac{1}{R} \ddot{U}_i$$

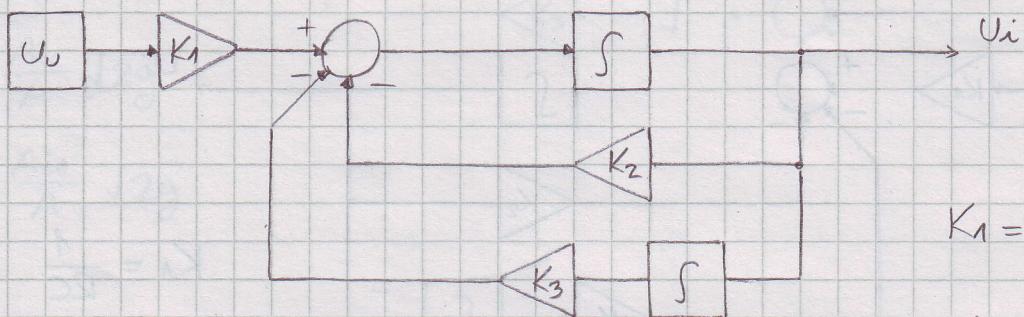
$$U_o = \frac{1}{C} \int i_L(\tau) d\tau + L \frac{di_L}{dt} + U_i / \frac{d}{dt}$$

$$\dot{U}_o = \frac{1}{C} i_L + L \frac{d^2 i_L}{dt^2} + \dot{U}_i$$

$$\ddot{U}_o = \frac{1}{CR} U_i + \frac{L}{R} \ddot{U}_i + \ddot{U}_i$$

$$CR \dot{U}_o = CL \ddot{U}_i + CR \ddot{U}_i + U_i$$

$$U_i = \frac{R}{L} \int U_o(\tau) d\tau - \frac{R}{L} \int U_i(\tau) d\tau - \frac{1}{CL} \iint U_i(\tau) d\tau dt$$



$$K_1 = \frac{R}{L}$$

$$K_2 = \frac{R}{L}$$

$$K_3 = \frac{1}{CL}$$