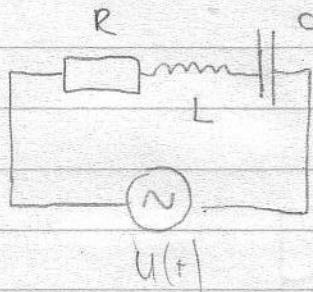


0

@S

$$U(t) = U_m \sin(\omega t + \varphi) \rightarrow 2\pi f$$

$$U_{max} = \sqrt{2} |U_{eff}|$$



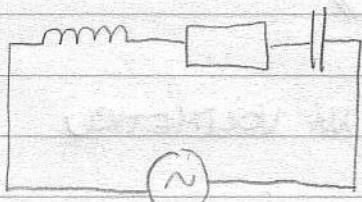
$$|| \quad X_C = \frac{1}{\omega C} \quad \boxed{-90^\circ} \quad \text{kasni napon za } 30^\circ = \frac{-j}{\omega L}$$

$$|| \quad X_L = \omega L \quad \boxed{90^\circ} \quad \text{prethodno stoji za } 30^\circ = j\omega L$$

$$U = i = U_{eff} / \omega \rightarrow *$$

$$= U_{eff} \cdot \cos \varphi + j U_{eff} \cdot \sin \varphi$$

$\underbrace{\quad}_{Re\{U\}}$ $\underbrace{\quad}_{Im\{U\}}$



$$R = 4 \Omega$$

$$X_L = 10 \Omega$$

$$U = 100V$$

$$X_C = 7 \Omega$$

$$Z_{uk} = R + X_L + X_C \Rightarrow Z_{uk} = 4 + j10 - j7 =$$

$$= R + jX_L - jX_C$$

$$= 4 + j3 \Omega$$

$$= Z_{uk} L_f$$

$$= \sqrt{4^2 + 3^2} \quad \boxed{\arctg \frac{3}{4}}$$

$$= 5 \quad \boxed{36,87} \Omega$$

$$i = I \angle 0^\circ$$

$$I \angle 0^\circ = \frac{U}{Z} = \frac{100 \angle 0^\circ}{5 \angle 36,87^\circ}$$

0

$$\star \frac{5 \angle 30^\circ}{10 \angle 45^\circ} = 0.5 \angle -15^\circ$$

$$I = \frac{100}{5} = 20 \text{ A}$$

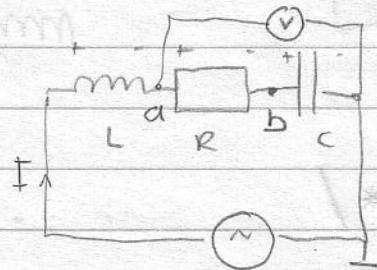
0° = Lu - 36,87°

$$\underline{Lu = 36,87^\circ}$$

$$U_R = I \cdot R$$

$$= 20 \angle 0^\circ \cdot 4 \angle 0^\circ$$

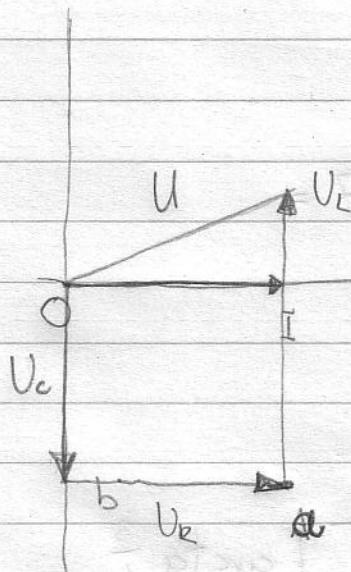
$$= 80 \angle 0^\circ //$$



$$U_L = I \cdot X_L = 20 \angle 0^\circ \cdot 10 \angle 90^\circ = 200 \angle 90^\circ //$$

$$U_C = I \cdot X_C = 20 \angle 0^\circ \cdot 7 \angle -90^\circ = 140 \angle -90^\circ //$$

NAPON NA VOLIMETRU

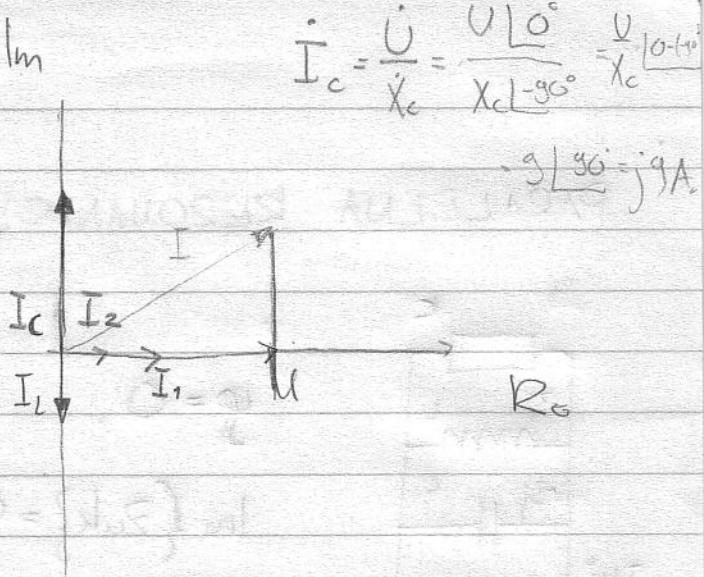
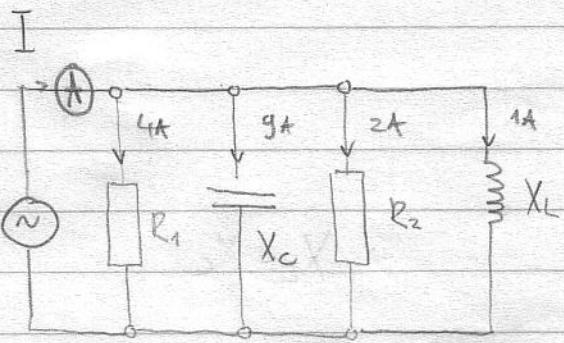


$$U_V = U_R + U_C =$$

$$= 80 \angle 0^\circ + 140 \angle -90^\circ$$

$$= 80 - j 140$$

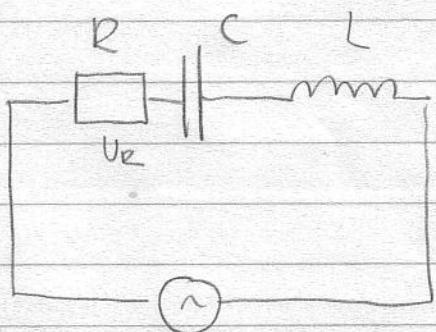
$$U_V = \sqrt{80^2 + 140^2} =$$



STRUJA KROZ OTPORNIK
JE U FAZI S
NAPONOM

$$I = 4 + j + 2 - j = 6 + 8j = 10 \angle 53^\circ$$

$$I = \sqrt{6^2 + 8^2} = 10 \text{ A}$$



$$Y = 0$$

$$X_C = X_L$$

$$\operatorname{Im}\{Z_{lk}\} = 0$$

$$\frac{1}{\omega C} = \omega L$$

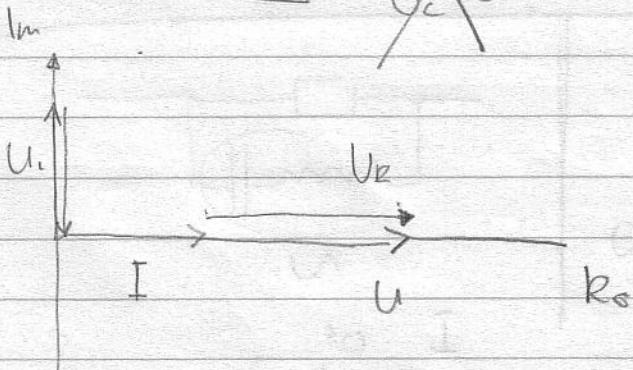
$$\omega = \frac{1}{\sqrt{LC}}$$

$$U = U \angle 0^\circ \quad U = U_R$$

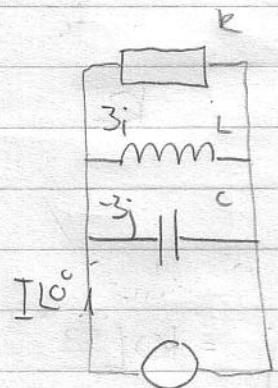
$$Z_{lk} = R$$

$$I = I \angle 0^\circ$$

~~$$U_C = 0$$~~



PARALELNA REZONANCIJA



$$\varphi = 0$$

$$X_L = X_C$$

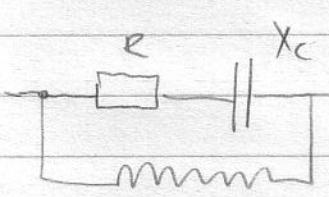
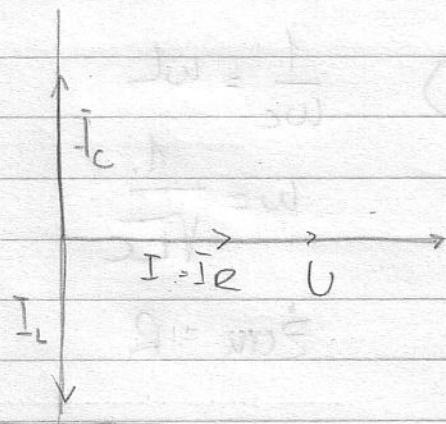
$$\operatorname{Im}\{Z_{uk}\} = 0$$

$$\frac{1}{Z_{uk}} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

~~$$I_L = 0$$~~
~~$$I_C = 0$$~~

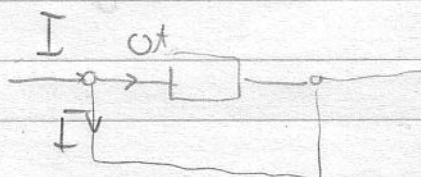
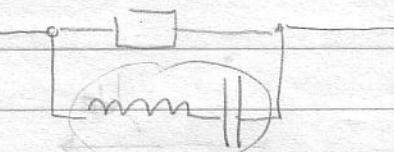
$$Z_{uk} = R$$

$$Z_{LC} = \frac{jX_L jX_C}{jX_L + jX_C} = \frac{3j(-3j)}{3j - 3j} = \infty$$



$$\varphi = 0$$

$$\operatorname{Im}\{Z_{uk}\} = 0$$



VUT IMPEDANCJE

Samo je jedan

$$\dot{S} = \dot{U} \cdot (\dot{I})^* = U \cdot I | \downarrow \text{VA}$$

\downarrow
ukupna
(prirodna)
snaga

$$= U \cdot I \cdot \cos \phi + j U \cdot I \cdot \sin \phi$$

$\underbrace{\quad \quad \quad}_{P} \quad \underbrace{\quad \quad \quad}_{Q}$

$Q > 0 \text{ ind.}$



[W]

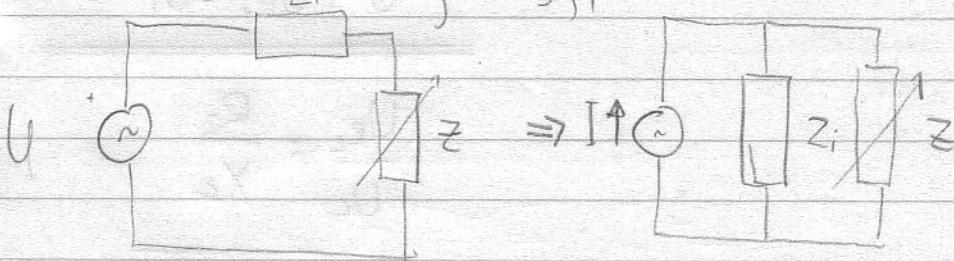
radna
snaga

[Var]

jakači
snaga

$Q < 0 \text{ kap}$

$$Z_i = R + jX = 3 - j4$$

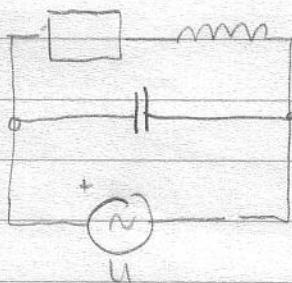


Za max snagu
na Z :
$Z = (Z_i)^* = 3 + j4$

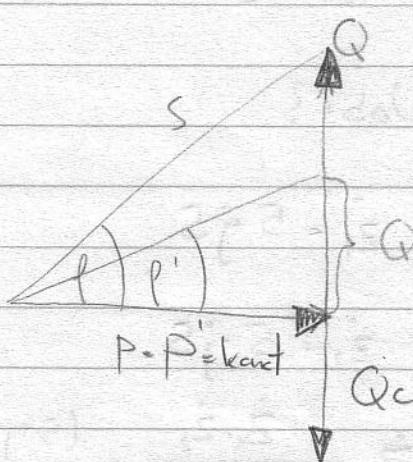
$Z \rightarrow$ radni otpor (R)

P Q P, S, cosφ

$$R = |Z_i| = \sqrt{(Re)^2 + (Im)^2}$$



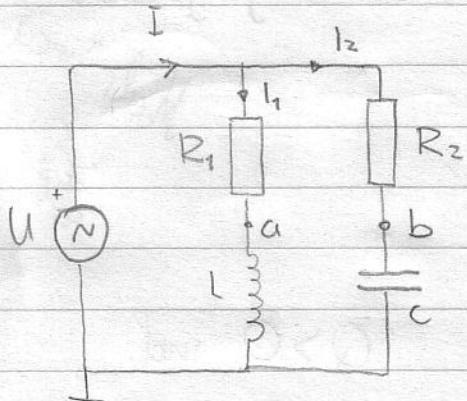
$$C = ? \Rightarrow \cos \phi$$



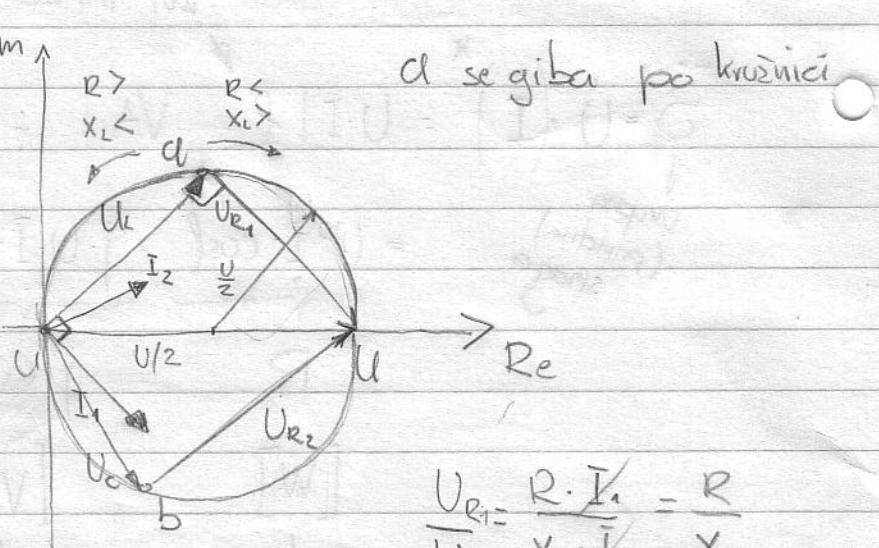
$$= \sqrt{3^2 + 4^2} = 5 \Omega$$

Za kompenzaciju je potrebe

$Q' = Q - Q_c$ snage = kondenzator ide
parallelno



$$U_{ab} = ?$$



$$\frac{U_{R_1}}{U_L} = \frac{R \cdot I_1}{X_L \cdot I_1} = \frac{R}{X_L}$$

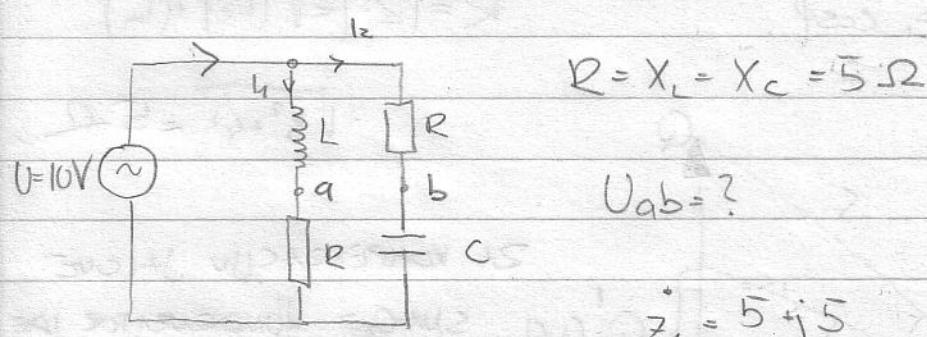
$I_1 \rightarrow RL$ (nalazi se između 0° ; 90°)

I na $R \rightarrow 0^\circ$
 na $L \rightarrow -90^\circ$
 na $C \rightarrow 90^\circ$

$$U = \sqrt{U_{R_1}^2 + U_L^2}$$

$$\frac{U_{R_2}}{U_C} = \frac{R_2}{X_C}$$

$$U = \sqrt{U_{R_2}^2 + U_C^2}$$



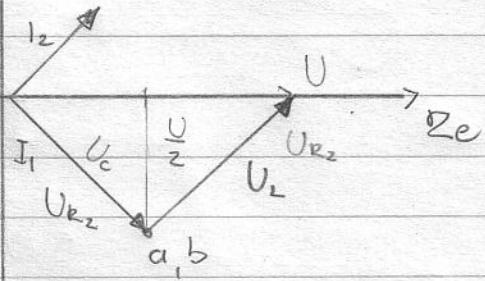
$$R = X_L = X_C = 5 \Omega$$

$$U_{ab} = ?$$

$$z_1 = 5 + j5$$

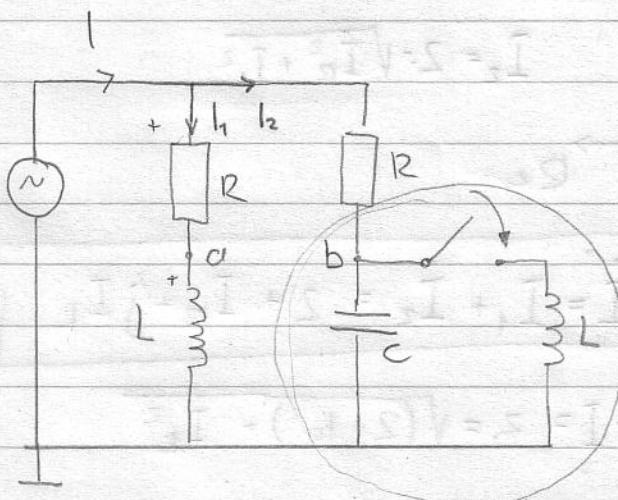
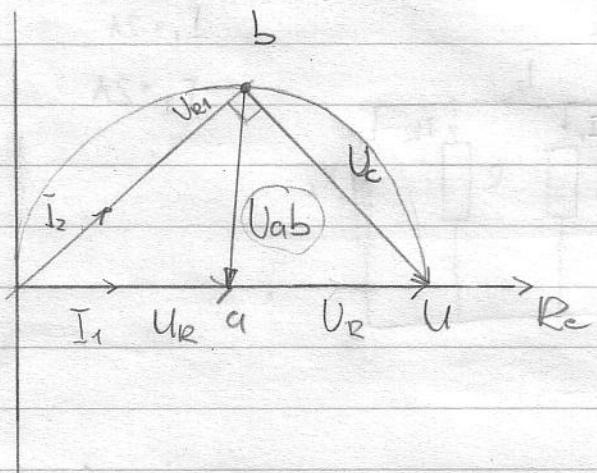
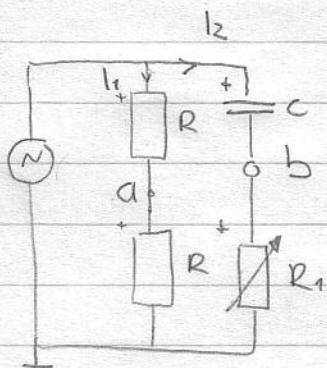
$$z_2 = 5 - j5$$

$$Z_w = \frac{z_1 \cdot z_2}{z_1 + z_2} = \frac{(5+j5)(5-j5)}{5+j5+5-j5} = 5 \Omega$$



$$U_{ab} = U_{R_2} - U_L$$

$$= -U_C + U_{R_1}$$

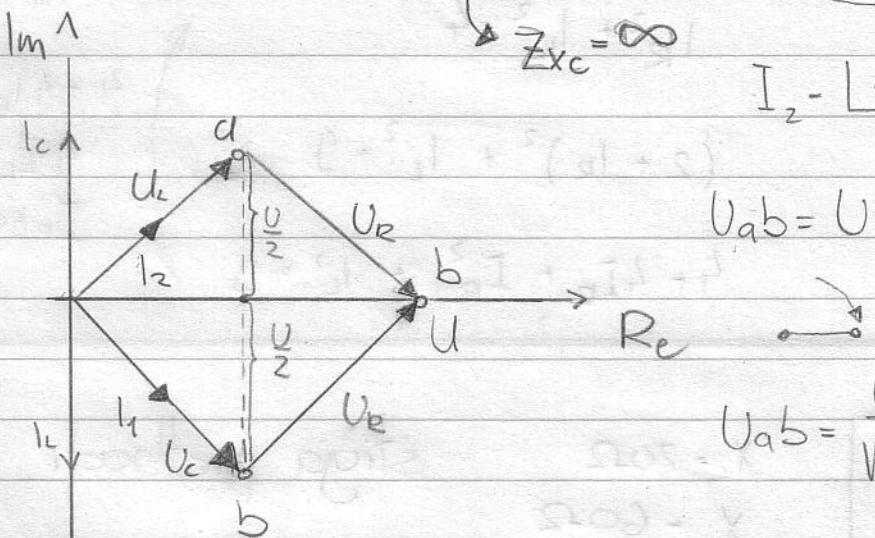


$$R = X_L = X_C = 10 \Omega$$

$U_{ab} = ?$ (kako se pravljeni kad se sklopka zatvori)

$$I_1 = -45^\circ \Rightarrow \text{jer su } R : X_L \text{ jednaki}$$

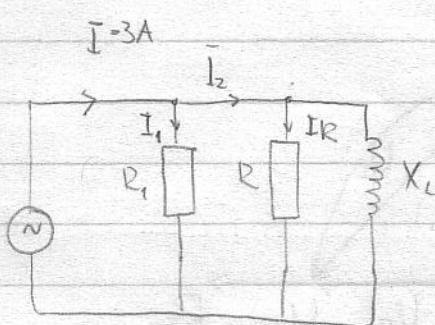
$$I_2 = 45^\circ \Rightarrow \text{jer su } R : X_C \text{ jednaki}$$



$$U_{ab} = \frac{U}{\sqrt{2}}$$

$$I_c = \frac{U}{X_L} = 10A$$

na LC $\Rightarrow U$



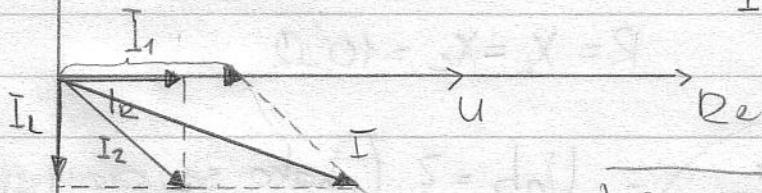
$$I_1 = 2A$$

$$I_2 = 2A$$

$|m_A|$

$$\dot{I}_2 = \dot{I}_R - j \dot{I}_L$$

$$\bar{I}_2 = 2 = \sqrt{\dot{I}_R^2 + \dot{I}_L^2}$$



$$\boxed{\dot{I} = \dot{I}_1 + \dot{I}_2 = 2 + \dot{I}_R - j \dot{I}_L}$$

$$I = 3 = \sqrt{(2 + \dot{I}_R^2) + \dot{I}_L^2}$$

$$\dot{I}_R^2 + \dot{I}_L^2 = 4$$

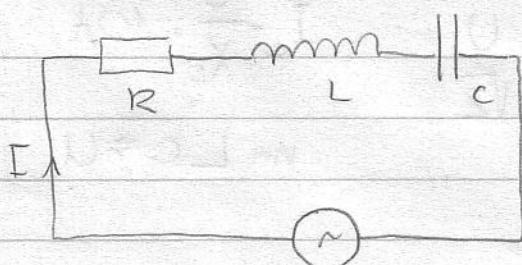
$$(2 + \dot{I}_R)^2 + \dot{I}_L^2 = 9$$

$$4 + 4\dot{I}_R + \dot{I}_R^2 + \dot{I}_L^2 = 9$$

$$4 + 4\dot{I}_R = 9 - 4$$

$$4\dot{I}_R = 1$$

$$\dot{I}_R = 0.25A$$



$$X_L = 20\Omega$$

$$X_C = 60\Omega$$

struja prethodni

ukupnom naponu za

30°

$$I = I_1 \text{ ili}$$

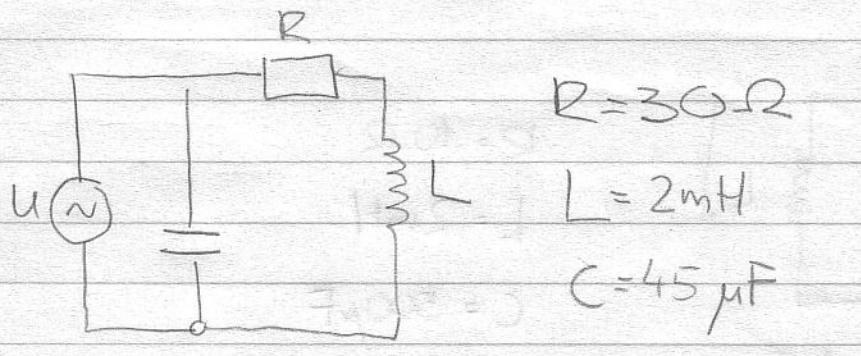
$$\dot{z} = \frac{\dot{u}}{I} = \frac{\dot{u} \angle \dot{u}}{I \angle 30^\circ + \dot{u}} = \dot{z} \angle -30^\circ$$

$$\dot{u} = U \angle \dot{u}$$

$$\dot{u} = 30^\circ + \dot{u}$$

$$\dot{z} = R + jX_L - jX_C = R - j40 \Rightarrow \rho = \arctg \frac{-40}{R}$$

8.



RESONANZ

$$\rho = 0$$

$$\text{Im}\{Z_{\text{Ruk}}\} = 0$$

$$Q = 0$$

(U, I u Fazi)

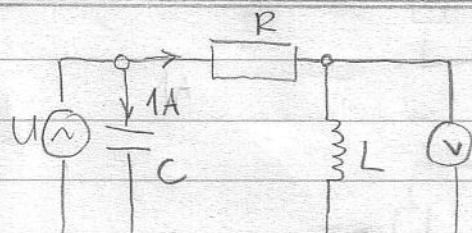
$$Z_{\text{Ruk}} = RL \parallel C = \frac{(R + jX_L)(-jX_C)}{R + jX_L - jX_C} = \frac{X_L X_C - jR X_C}{R + j(X_L - X_C)} \cdot \frac{R - j(X_L - X_C)}{R - j(X_L - X_C)}$$

$$= \frac{R X_L X_C - R X_C (X_L - X_C)}{R^2 + (X_L - X_C)^2}$$

$$- j \frac{R^2 X_C + X_L X_C (X_L - X_C)}{R^2 + (X_L - X_C)^2} = 0$$

$$R^2 X_C + X_L X_C (X_L - X_C) = 0 \quad | : X_C$$

$$30^2 + \omega L \left(\omega L - \frac{1}{\omega C} \right) = 0$$



$$I_C = 1A$$

$$\text{Im}\{Z_{\text{Ruk}}\} = 0$$

$$U_V = 6V$$

$$R^2 + X_L (X_L - X_C) = 0$$

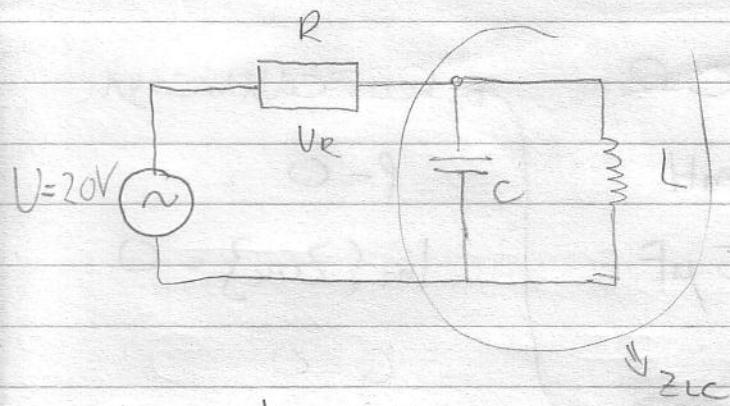
$$U = 10V$$

$$X_L = ?$$

$$X_C = \frac{U}{I_C} = 10\Omega$$

$$U_R = \sqrt{U^2 - U_C^2} = 8V$$

$$\frac{U_R}{U_L} = \frac{I_{RL} \cdot R}{I_{RC} \cdot X_L} = \frac{R}{X_L} = \frac{8}{6}$$



$$R = 10 \Omega$$

$$L = 2 \text{ mH}$$

$$C = 500 \mu\text{F}$$

$$\omega = 1000 \text{ rad/s}$$

$$X_L = \omega L = 2 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{10^3 \cdot 0,5 \cdot 10^{-3}} = 2 \Omega$$

$$Z_{LC} = \frac{j X_L (-X_C)}{j X_L - j X_C} = \infty$$

$$\boxed{Z_{\text{uk}} = R + Z_{LC} = \infty \Omega}$$

$$I_{Uk} = 0$$

$$U_R = 0V$$

$$\dot{I}_L = \frac{\dot{U}_{LC}}{j X_L} = \frac{20}{j 2} = -j 10A$$

$$\dot{U}_{RC} = 20$$

$$\dot{I}_R = \frac{\dot{U}_{LC}}{-j X_C} = j 10A$$

$$u(+)=10 \sin(\omega t + \frac{\pi}{4})$$

$$i(+)=2 \cos(\omega t)$$

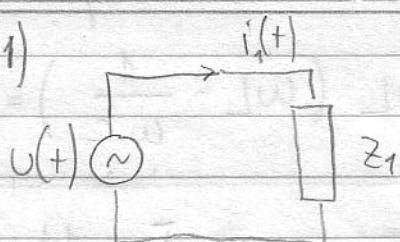
$$i_2(+)$$

$$z_2 = 20 \angle 30^\circ$$

$$\dot{U} = \frac{10}{\sqrt{2}} \angle 45^\circ$$

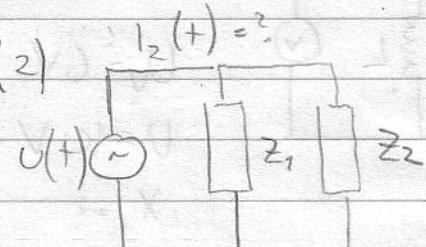
$$\dot{I} = \frac{2}{\sqrt{2}} \angle 90^\circ$$

(1)



$$i(+)=2 \sin(\omega t + \frac{\pi}{2})$$

(2)



$$\dot{Z} = \frac{\dot{U}}{\dot{I}} = \frac{\frac{10}{\sqrt{2}} \angle 45^\circ}{\frac{2}{\sqrt{2}} \angle 90^\circ} = 5 \angle -45^\circ$$

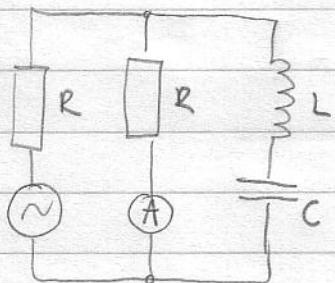
$$\dot{Z}_{Uk} = \frac{\dot{Z}_1 \cdot \dot{Z}_2}{\dot{Z}_1 + \dot{Z}_2} = \frac{100 \angle -15^\circ}{3,53 - j 3,53 + 17,32 + j 10} = \frac{100 \angle -15^\circ}{21 + j 6,5} = \frac{100 \angle -15^\circ}{22 \angle 17^\circ} = 45 \angle -32^\circ$$

10.

$$I_2 = \frac{U}{Z_{lk}} = \frac{\frac{10}{\sqrt{2}} [45^\circ]}{\frac{4\sqrt{2}}{1} [-32^\circ]} = 1.57 [77^\circ]$$

$$i_2(t) = 1.57\sqrt{2} \sin(\omega t + 77^\circ)$$

RADIJANI!



$$U(t) = 10\sqrt{2} \sin(314t) \text{ V}$$

$$R = 100 \Omega$$

$$X_L = 10 \Omega$$

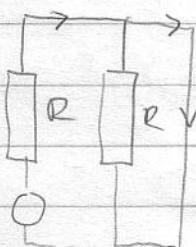
$$C = ?$$

$$X_L = X_C$$

$$I_A = 0 \quad X_C = 10 \Omega$$

$$\frac{1}{\omega C} = 10$$

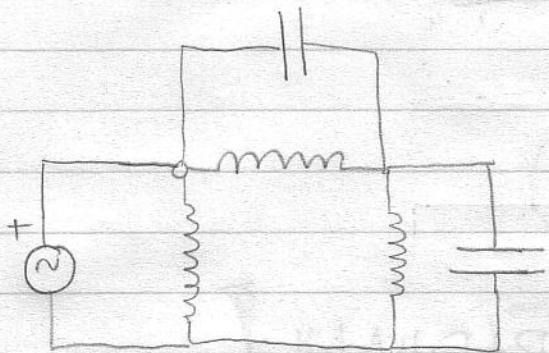
$$Z_{lk} = R$$



$$I_{lk} = \frac{10}{100} = 0.1 \text{ A}$$

$$U_C = I X_C = 1 \text{ V}$$

TYWNA
ZK
PREDSEĐENIKU!



$$L = 4 \text{ mH}$$

$$C = 20 \mu\text{F}$$

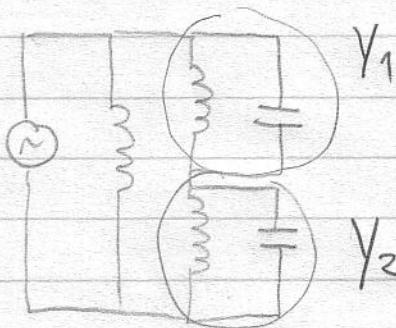
$$Z_{\text{sh}} = \frac{U}{I} = \infty$$

$$f = ?$$

$$\boxed{I = 0}$$

$$Y_{\text{sh}} = 0 \quad G = \frac{1}{R}$$

$$X_C = \frac{-j}{\omega C} = \frac{1}{B_C}$$



$$\underline{B_C = j\omega C}$$

$$\dot{X}_L = j\omega L = \frac{1}{B_L}$$

$$B_L = \frac{-j}{\omega L}$$

$$Y_1 = B_C + B_L = j(B_C - B_L)$$

$$Y_2 = Y_1$$

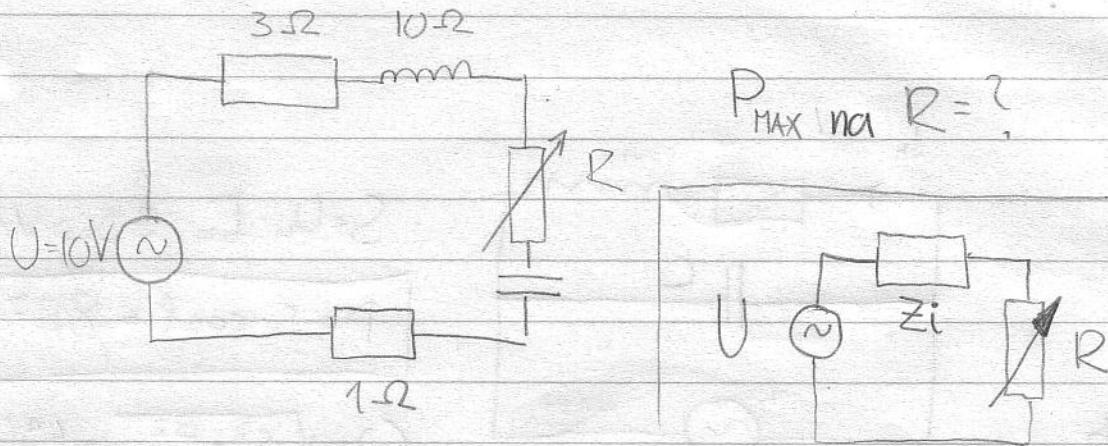
$$Y_{12} = \frac{\dot{Y}_1 \cdot \dot{Y}_2}{\dot{Y}_1 + \dot{Y}_2} = \frac{j(B_C - B_L) \cdot j(B_C - B_L)}{2 \cdot j(B_C - B_L)} = j \frac{B_C - B_L}{2}$$

$$Y_{\text{sh}} = Y_{12} + B_L =$$

$$= j \frac{B_C}{2} - j \frac{B_L}{2} - j \frac{B_L}{2}$$

$$B_C = 3 B_L$$

$$wC = \frac{3}{wL}$$



$$Z_i = 3 + j10 - j7 + 1$$

$$= 4 + j3$$

$$R = |Z_i| = \sqrt{4^2 + 3^2} = 5\Omega$$

$$P = U_R \cdot I$$

$$= I \cdot R \cdot I = I^2 R$$

$$= \frac{100}{50} \cdot \sqrt{50} W$$

$$Z_{uk} = Z_i + R$$

$$= 4 + j3 + 5$$

$$= 9 + j3$$

$$Z_{uk} = \sqrt{9^2 + 3^2} = \sqrt{90}$$

2. slučaj: sve isto; umjesto \boxed{R} je Z

$$Z = Z_i^* = 4 - j3 \Omega$$

$$I = \frac{U}{Z_{uk}} = \frac{10}{\sqrt{90}}$$

$$I = \frac{U}{Z} = \frac{10}{\sqrt{90}}$$

$$Z_{uk} = Z + Z_i$$

$$= 8 \Omega$$

$$P_Z = I^2 \cdot R_{\text{Re}}\{Z\} = \frac{100}{64} \cdot 4 W$$

$$Q = I^2 \cdot \text{Im}\{Z\} = \frac{100}{64} \cdot (-3) \text{ VA}$$

$$S = P + jQ$$

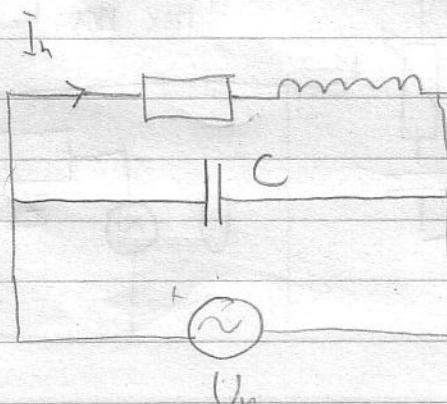
$$S_Z = \sqrt{P_Z^2 + Q_Z^2}$$

$$U_i = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$I_n = 4,5 \text{ A}$$

$$\cos \varphi = 0,866$$



$$S = U_n \cdot I_n = 990 \text{ VA}$$

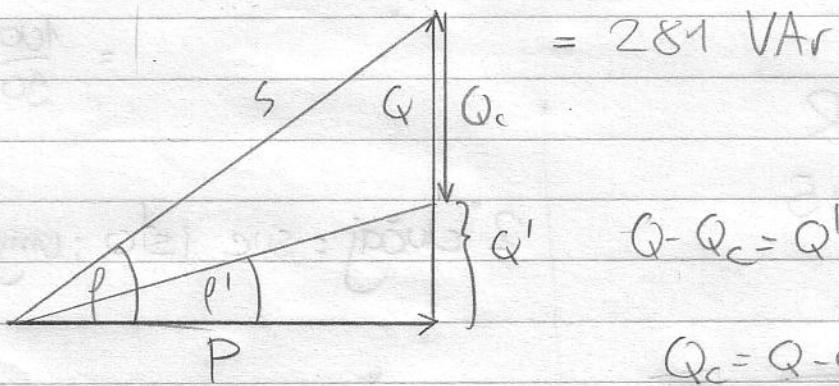
$$P = S \cdot \cos \varphi = 857 \text{ W}$$

$$Q = \sqrt{S^2 - P^2} = 496 \text{ VAr}$$

SA C

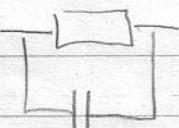
$$P = S' \cdot \cos \varphi \Rightarrow S' = 902 \text{ VA}$$

$$Q' = \sqrt{S'^2 - P^2}$$



$$\cos \varphi - \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$

R C



$$\cos \varphi = 0,5$$

$$U_c = U_R$$

$$\cos \varphi' \Rightarrow \square \parallel$$

$$= 215 \text{ VAr}$$

$$= U \cdot I_c = \frac{U_n^2}{X_C}$$

$$\frac{1}{wC} = X_C = \frac{U_n^2}{Q_c}$$

$$\cos \varphi = \frac{P}{\sqrt{P^2 + Q^2}}$$

$$\dot{S} = \dot{U} (\dot{I})^* = U \cdot I \cdot Z$$

$$= \frac{U^2}{Z} \cos \varphi + j \frac{U^2}{Z} \sin \varphi$$

P

Q

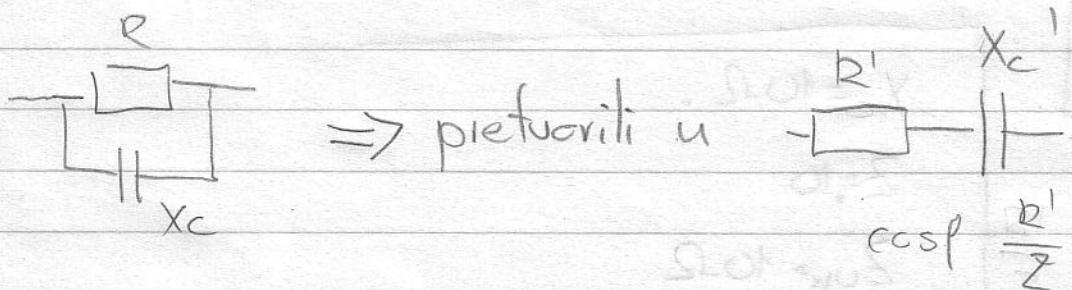
ZADATAK SA
PROŠLOGODIŠNJE
MEĐUISPIT

II. NAČIN ZA RJEŠAVANJE (pretvaranje paralele u seriju)

$$Z_{UK} = \frac{-jR X_C}{R - jX_C} = \frac{R X_C^2 - jR^2 X_C}{R^2 + X_C^2}$$

$$= \left(\frac{R X_C^2}{R^2 + X_C^2} \right) - j \left(\frac{R^2 X_C}{R^2 + X_C^2} \right)$$

$$R' \quad X_C'$$



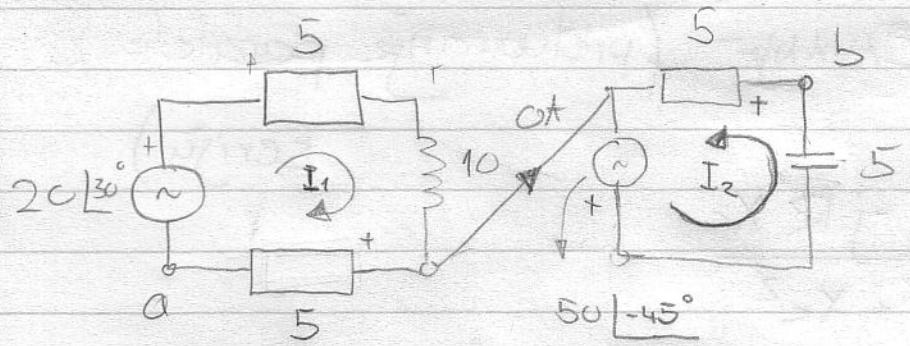
$$\cos \phi = \frac{R'}{Z}$$

$$\phi = 60^\circ$$

$$\operatorname{tg} \phi = \frac{X_C'}{R'} = \frac{R^2 X_C}{R X_C^2} = \frac{R}{X_C}$$

$$\operatorname{tg} \phi' = \frac{X_C}{R} \Rightarrow \phi'$$

$$\Rightarrow \cos \phi'$$



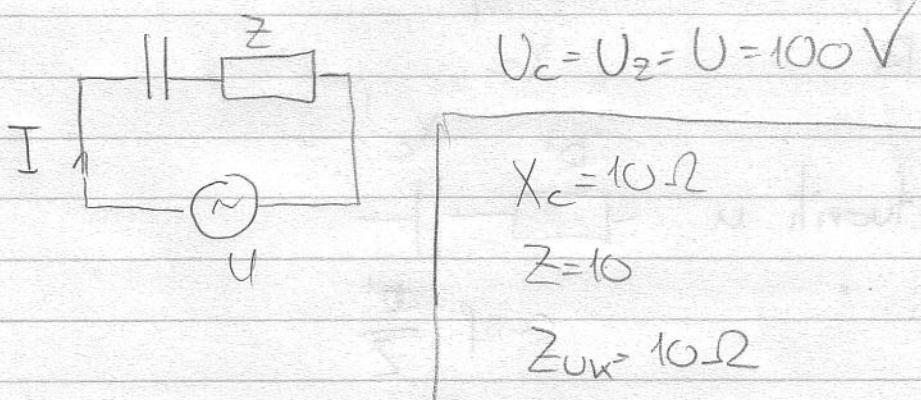
$$U_{ab} = -5 \cdot I_2 - 5 \cdot I_1$$

$$U_{ab} = |-5 I_2 - 5 I_1|$$

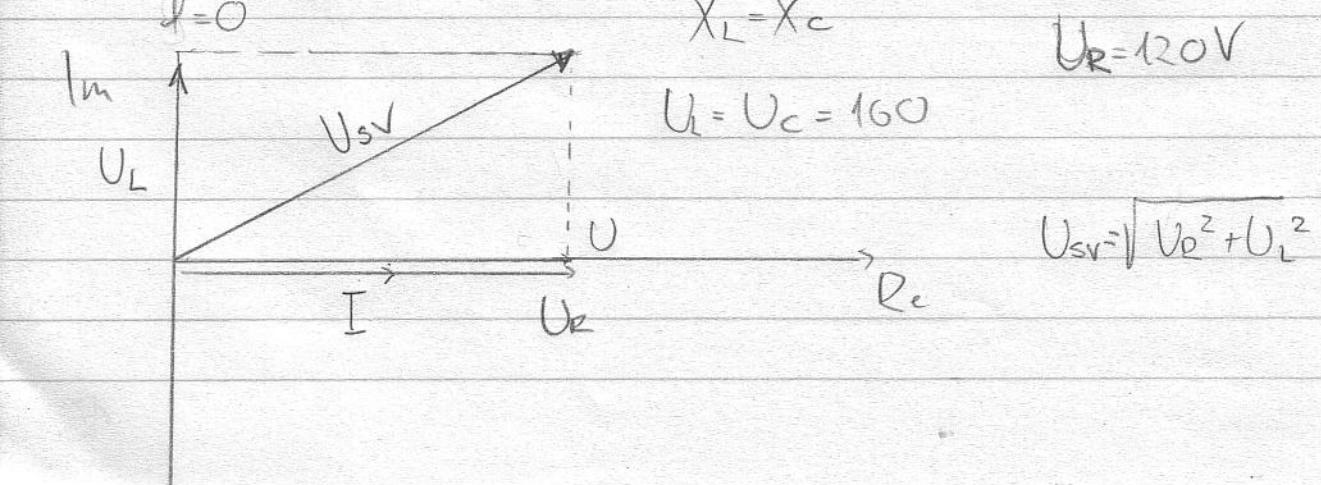
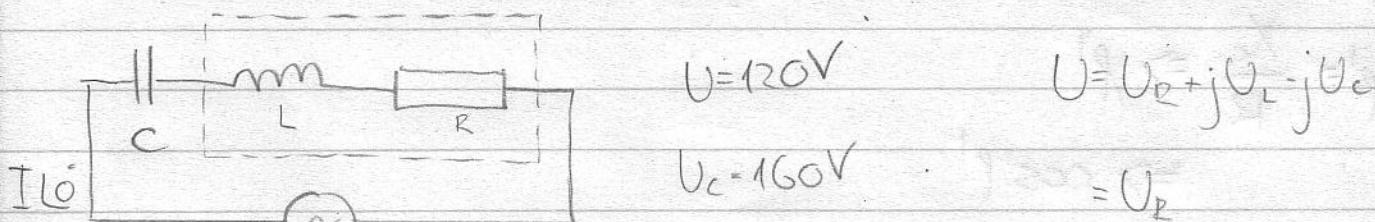
$$U_{ab} = ?$$

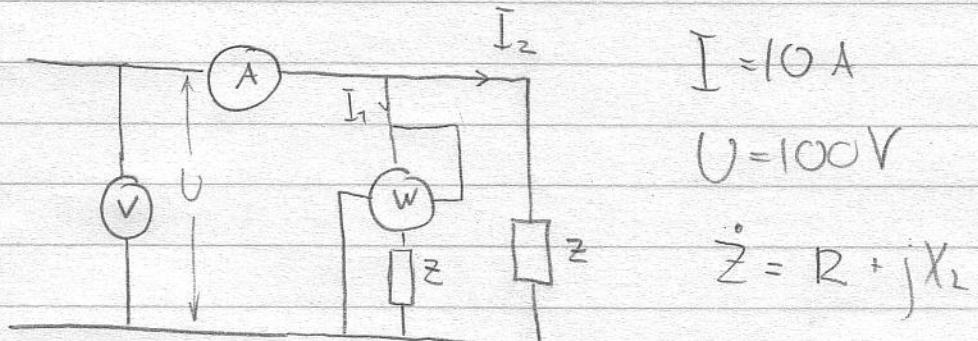
VII. 1-14.

$$I = 10 \text{ A}$$



$$\begin{aligned} Z &= R + jX \\ Z_{uk} &= R + jX - jX_c \end{aligned} \quad \left\{ \begin{aligned} Z &= 10 = \sqrt{R^2 + X^2} \\ Z_{uk} &= 10 = \sqrt{R^2 + (X-10)^2} \end{aligned} \right.$$





$$\vec{Z} = R + jX_L$$

$$R = \frac{1}{4} Z$$

$$I_1 = 5 \text{ A}$$

$$P = U \cdot I_1 \cos \phi$$

$$Z_{\text{th}} = \frac{Z}{2} = 10 \Omega$$

$$Z = 20 \Omega$$



$$R = 5 \Omega$$

$$P = I_1^2 \cdot R = 125 \text{ W}$$

$$X = \sqrt{Z^2 - R^2}$$