Zadatci za vježbu IV

Zad 1. a) $T(2,-1,3)$, $\vec{s_1} = (2,3,-1)$, $\vec{s_2} = (1,-2,1)$

$\vec{s_1} \parallel \Pi$, $\vec{s_2} \parallel \Pi \implies \vec{n} = \vec{s_1} \times \vec{s_2}$

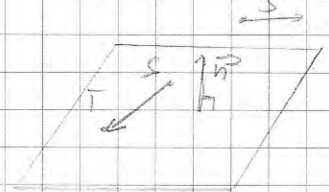
$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 1 & -2 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}$$

$$= \vec{i} - 3\vec{j} - 7\vec{k}$$

$\Pi \ldots \quad 1(x-2) - 3(y+1) - 7(z-3) = 0$

$$\boxed{x - 3y - 7z + 16 = 0}$$

b) $T(2,1,-3)$, $S(0,-1,-2)$, $\vec{s} = (2,-2,1)$



$\vec{s} \parallel \Pi$, $\vec{ST} \parallel \Pi \implies \vec{n} = \vec{s} \times \vec{ST}$

$\vec{ST} = (2,2,-1)$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 1 \\ 2 & 2 & -1 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & 0 \\ 2 & 2 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 4 & 0 \\ 2 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 4 & 0 \\ 2 & 2 \end{vmatrix}$$

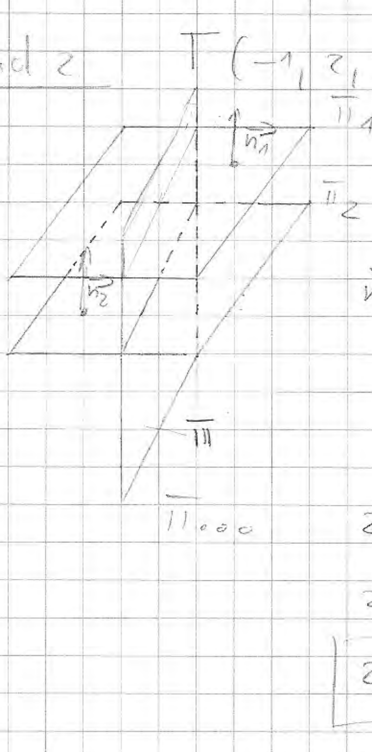
$$= 0\vec{i} + 4\vec{j} + 8\vec{k}$$

$\Pi \ldots \quad 0(x-2) + 4(y-1) + 8(z+3) = 0 \quad /:4$

$$\boxed{y + 2z + 5 = 0}$$

Zad 2. $T(-1,2,3)$ $\quad \Pi_1 \ldots \; x + 2y - z + 3 = 0$

$\Pi_2 \ldots \; 3x + 2y = 0$



$\vec{n_1} \parallel \Pi$, $\vec{n_2} \parallel \Pi \implies \vec{n} = \vec{n_1} \times \vec{n_2}$

$\vec{n_1} \nparallel \vec{n_2}$

$$\implies \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 3 & 2 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$$

$$= 2\vec{i} - 3\vec{j} - 4\vec{k}$$

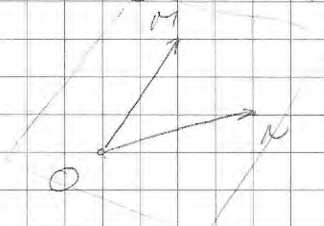
$\Pi \ldots \quad 2(x+1) - 3(y-2) - 4(z-3) = 0$

$2x - 3y - 4z + 2 + 6 + 12 = 0$

$$\boxed{2x - 3y - 4z + 20 = 0}$$

Zad 3.

a) $O(0,0,0)$ $M(2,1,2)$, $N(-1,1,1)$



π: $\vec{OM} \parallel \pi$, $\vec{ON} \parallel \pi \Rightarrow \vec{n} = \vec{OM} \times \vec{ON}$

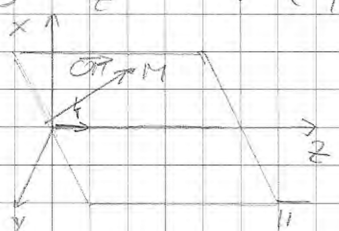
$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 2 \\ -1 & 1 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= -\vec{i} - 4\vec{j} + 3\vec{k}$$

$$-(x-0) - 4(y-0) + 3(z-0) = 0 \quad / \cdot (-1)$$

$$\boxed{x + 4y - 3z = 0}$$

b) Oz $M(1,2,0)$

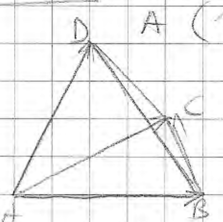


$\vec{OM} \parallel \pi$, $\vec{k} \parallel \pi \Rightarrow \vec{n} = \vec{OM} \times \vec{k}$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \cancel{\vec{k}\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix}}$$

$$= 2\vec{i} - \vec{j} \rightarrow \pi \ldots \boxed{2x - y = 0}$$

Zad 4.



$A(1,2,3)$ $B(2,-1,0)$, $C(2,3,-2)$, $D(1,2,1)$

$\vec{AB} = (1,-3,-3)$ $\vec{AC} = (1,1,-5)$, $\vec{AD}(0,0,-2)$

$\vec{BC} = (0,4,-2)$ $\vec{BD} = (-1,3,1)$

π_{ABC} $\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & -3 \\ 1 & 1 & -5 \end{vmatrix} = 18\vec{i} + 2\vec{j} + 4\vec{k}$

$$9(x-1) + 1(y-2) + 2(z-3) = 0$$

$$\boxed{9x + y + 2z - 17 = 0}$$

$\pi_{ABD} \ldots$ $\vec{n} = \vec{AB} \times \vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & -3 \\ 0 & 0 & -2 \end{vmatrix}$

$\vec{n} = 6\vec{i} + 2\vec{j}$

$$3(x-1) + 1(y-2) + 0(z-3) = 0 \quad \boxed{3x + y - 5 = 0}$$

π_{ACD} $\vec{n} = \vec{AC} \times \vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -5 \\ 0 & 0 & -2 \end{vmatrix} = -2\vec{i} + 2\vec{j} \rightarrow +1(x-1) - 1(y-2) = 0$

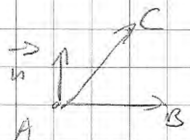
$$\boxed{x - y + 1 = 0}$$

π_{BCD} $\vec{n} = \vec{BC} \times \vec{BD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & -2 \\ -1 & 3 & 1 \end{vmatrix} = 10\vec{i} + 2\vec{j} + 4\vec{k}$

$$5(x-2) + 1(y+1) + 2(z-0) = 0$$

$$\boxed{5x + y + 2z - 9 = 0}$$

Zad 5. $A(2,1,2)$ $B(1,2,2)$ $C(2,2,1)$ $D(3,4,\alpha)$



$\vec{AB} = (-1, 1, 0)$

$\vec{AC} = (0, 1, -1)$

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\vec{i} - \vec{j} - \vec{k}$$

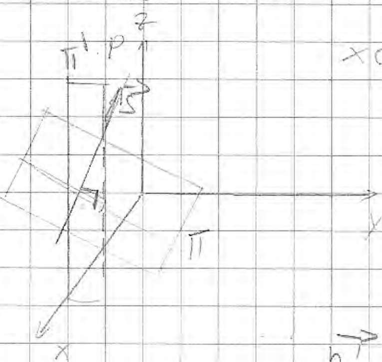
$\Pi \ldots \quad x-2 + y-1 + z-2 = 0$

$$\boxed{x + y + z - 5 = 0} \leftarrow D$$

$3 + 4 + \alpha - 5 = 0$

$\alpha = 5 - 7 \Rightarrow \boxed{\alpha = -2}$

Zad 6 $\Pi \ldots \; x + 2y - 2z + 3 = 0$ $\quad \vec{n} = (1, 2, -2)$



$xOz \quad x + 2\cdot 0 - 2z + 3 = 0$

$$x - 2z + 3 = 0 \quad p \ldots \begin{cases} x = t \\ y = 0 \\ z = \frac{3}{2} + \frac{1}{2}t \end{cases} \Rightarrow \vec{s} = (1, 0, \tfrac{1}{2})$$

$\vec{s} \parallel \Pi'$, $\vec{n} \parallel \Pi'$ $\Rightarrow \vec{n'} = \vec{s} \times \vec{n}$

$$\vec{n'} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 1 \\ 1 & 2 & -2 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & 1 \\ 2 & -2 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix}$$

$$= -2\vec{i} + 5\vec{j} - 4\vec{k}$$

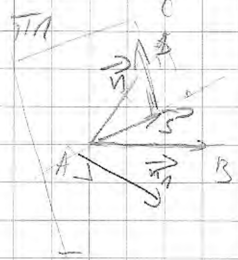
točka .. npr $t = 0$ $T(0, 0, \tfrac{3}{2})$

$\Pi' \ldots \quad 2(x-0) - 5(y-0) + 4(z - \tfrac{3}{2}) = 0$

$$\boxed{2x - 5y + 4z - 6 = 0}$$

?

Zad 7 $A(1,1,1)$ $B(2,3,2)$ $C(-1,3,5)$



$$\vec{n}_{ABC} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ -2 & 2 & 4 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ -2 & 4 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix}$$

$$= 6\vec{i} - 6\vec{j} + 6\vec{k} \rightarrow \vec{i} - \vec{j} + \vec{k}$$

$$\vec{s} = \widehat{AB} + \widehat{AC} = \frac{1}{\sqrt{1+4+1}}(1,2,1) + \frac{1}{\sqrt{4+4+16}}(-2,2,4)$$

$$= \frac{1}{\sqrt{6}}\left[(1,2,1) + \frac{1}{2}(-2,2,4)\right] = \frac{1}{\sqrt{6}}(0,3,3)$$

$$\vec{n}_1 = \vec{s} \times \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 3 \\ 1 & -1 & 1 \end{vmatrix} = 6\vec{i} + 3\vec{j} - 3\vec{k} \rightarrow 2\vec{i} + \vec{j} - \vec{k}$$

$\Pi_1 \ldots \quad 2(x-1) + 1(y-1) - 1(z-1) = 0$

kroz točku A $\quad \boxed{2x + y - z - 2 = 0}$

Zad 8 a) $T(1,2,3)$ $p = q = r = \Delta$

$\pi \ldots \quad Ax + By + Cz + D = 0 \qquad Ax + By + Cz = -D \quad /: -D$

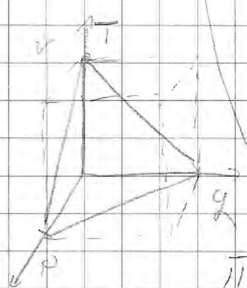
$$\frac{x}{\frac{-D}{A}} + \frac{y}{\frac{-D}{B}} + \frac{z}{\frac{-D}{C}} = 1 \qquad \frac{x}{\Delta} + \frac{y}{\Delta} + \frac{z}{\Delta} = 1 \quad / \cdot \Delta$$

$p \qquad q \qquad r$

$$x + y + z - \Delta = 0 \rightarrow \vec{n} = (1,1,1)$$

$$\pi \ldots \quad 1(x-1) + 1(y-2) + 1(z-3) = \boxed{x + y + z - 6 = 0}$$

b) $T(0,0,1)$ $S(9,4,-1)$ $V_\Delta = 6$



$$V = \frac{\text{baza} \cdot \text{visina}}{3} = \frac{\frac{p \cdot q}{2} \cdot r}{3} = \frac{1}{6} p \cdot q \cdot r$$

$$\boxed{r = 1} \rightarrow p \cdot q = 36$$

$\pi \quad \frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$ 1) $\frac{9}{p} + \frac{4}{q} - 1 = 1 \quad / \cdot pq$

2) $9q + 4p = 2 \cdot p \cdot q = 2 \cdot 36$ $p = \frac{36}{q}$ 3) $q = \frac{8 \pm \sqrt{64 - 64}}{2}$

$$9q + 4 \cdot \frac{36}{q} = 2 \cdot 36 \quad /:9 \cdot q$$

$$q^2 - 8q + 16 = 0$$

$\boxed{q = 4}$

$\boxed{p = 9}$

4) $\pi \ldots \quad \frac{x}{9} + \frac{y}{4} + \frac{z}{1} = 1 \quad / \cdot 36$

$$\boxed{4x + 9y + 36z - 36 = 0}$$

Zad 9 $T_1(2,-1,0)$ $T_2(1,1,2)$

p-anc $\vec{s} = \overrightarrow{T_1T_2} = (-1,2,2)$ $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-0}{2}$

$$\frac{x-2}{-1} = \frac{y+1}{2} \qquad \frac{y+1}{2} = \frac{z-0}{2}$$

$$2x - 4 = -y - 1 \qquad y + 1 = z$$

$\pi_1 \ldots \quad 2x + y - 3 = 0 \qquad \pi_2 \ldots \quad y - z + 1 = 0$

p-onen $\alpha(2x + y - 3) + \beta(y - z + 1) = 0$

Zad 10. $T(2,-1,0)$ $\quad \pi \ldots\ x+y-z+2=0$

$$\frac{x+y-z+2}{\sqrt{3}}=0$$

$$d(T,\pi)=\left|\frac{2-1-0+2}{\sqrt{3}}\right|=\boxed{\sqrt{3}}$$

Zad 11.

a) $\pi_1 \ldots\ 2x+y-2z+3=0$

$\pi_2 \ldots\ 2x+y-2z+9=0$

I) odaberimo bilo koju tačku jedne od ravni

II) računamo udaljenost te tačke od druge ravnine

I) $x=0 \quad z=0$

$0+y-0+3=0$

$y=-3 \quad T(0,-3,0)$

II) $\pi_2 \ldots\ \dfrac{2x+y-2z+9}{\underbrace{\sqrt{4+1+4}}_{=3}}=0$

$$d(T,\pi_2)=\left|\frac{0-3+0+9}{3}\right|=\boxed{2}$$

b) $\pi_1 \quad 4x+3y-5z-8=0$ $\qquad \pi_2 \quad 4x+3y-5z+12=0$

$T \ldots\ y=0 \quad z=0 \quad 4x=8$

$x=2$

$T(2,0,0)$

$$\frac{4x+3y-5z+12}{\underbrace{\sqrt{16+9+25}}_{=\sqrt{50}}}=0$$

$$d(T,\pi_2)=\left|\frac{8+0+0+12}{\sqrt{50}}\right|=\frac{2}{5}\sqrt{50}=\boxed{2\sqrt{2}}$$

Zad 12. $\pi \ldots\ x+3y-2z+1=0$

$\pi_1 \ldots\ x+3y-2z+D=0$

moraju biti paralelne, jer ako nisu sijeku se $d=0$

$T \ldots\ y=0 \quad z=0 \quad x=-1$

$T(-1,0,0)$

$$d(T,\pi_1)=\left|\frac{-1+0+0+D}{\underbrace{\sqrt{1+9+4}}_{=\sqrt{14}}}\right|=3$$

$$|D-1|=3\sqrt{14}$$

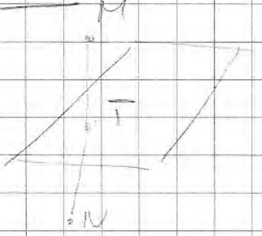
$D-1=3\sqrt{14}$ $\qquad D-1=-3\sqrt{14}$

$\boxed{D=3\sqrt{14}+1}$ $\qquad \boxed{D=1-3\sqrt{14}}$

$\pi_{11} \ldots\ x+3y-2z+1+3\sqrt{14}=0$ $\quad \pi_{12} \ldots\ x+3y-2z+1-3\sqrt{14}=0$

Zad 13.

$M(2,1,-1)$, $N(0,3,3)$



$\vec{n} = \overrightarrow{NM} = (2,-2,-4)$ $\quad \pi \ldots \ 1(x-1) - 1(y-2) - 2(z-1) = 0$

$T = \frac{M+N}{2} \Rightarrow (1,2,1)$ $\quad \boxed{x - y - 2z + 3 = 0}$

Zad 14.

a) $\pi_1 \ldots \ x + 2y + 2 = 0 \rightarrow \vec{n_1} = (1,2,0)$

$\pi_2 \ldots \ 2x - y + 2z = 0 \rightarrow \vec{n_2} = (2,-1,2)$

$\sphericalangle(\pi_1, \pi_2) = \sphericalangle(\vec{n_1}, \vec{n_2})$ → rađeno na satu

$$\cos\varphi = \frac{|\vec{n_1}\cdot\vec{n_2}|}{|\vec{n_1}|\cdot|\vec{n_2}|} = \frac{|2-2+0|}{\sqrt{1+4}\cdot\sqrt{4+4+1}} = 0 \quad \boxed{\varphi = \frac{\pi}{2}}$$

b) $\pi_1 \ldots \ 3x - y - z = 0 \rightarrow \vec{n_1} = (3,-1,-1)$ $\quad \boxed{\varphi = \arccos\frac{\sqrt{33}}{33}}$

$\pi_2 \ldots \ x + y + z = 0 \rightarrow \vec{n_2} = (1,1,1)$ $\quad \approx 79{,}98^\circ$

$$\cos\varphi = \frac{|\vec{n_1}\cdot\vec{n_2}|}{|\vec{n_1}|\,|\vec{n_2}|} = \frac{|3-1-1|}{\sqrt{9+1+1}\cdot\sqrt{1+1+1}} = \frac{1}{\sqrt{11}\,\sqrt{3}} = \frac{\sqrt{33}}{33}$$

c) $\pi_1 \ldots \ x + y + 2z - 3 = 0 \rightarrow \vec{n_1} = (1,1,2)$

$\pi_2 \ldots \ 2x + 2y - 2z + 3 = 0 \rightarrow \vec{n_2} = (1,1,-1)$

$$\cos\varphi = \frac{|1+1-2|}{\sqrt{\ }\cdot\sqrt{\ }} = 0 \rightarrow \boxed{\varphi = \frac{\pi}{2}}$$

Zad 15

$A(2,5,0)$, $B(1,6,2)$ $C(-1,4,1)$ $D(1,4,3)$

$\sphericalangle(\pi_{ABC}, \pi_{ABD}) = \sphericalangle(\vec{n}_{ABC}, \vec{n}_{ABD})$

$\overrightarrow{AB} = (-1,1,2)$ $\quad \overrightarrow{AC} = (-3,-1,1)$ $\quad \overrightarrow{AD} = (-1,-1,3)$

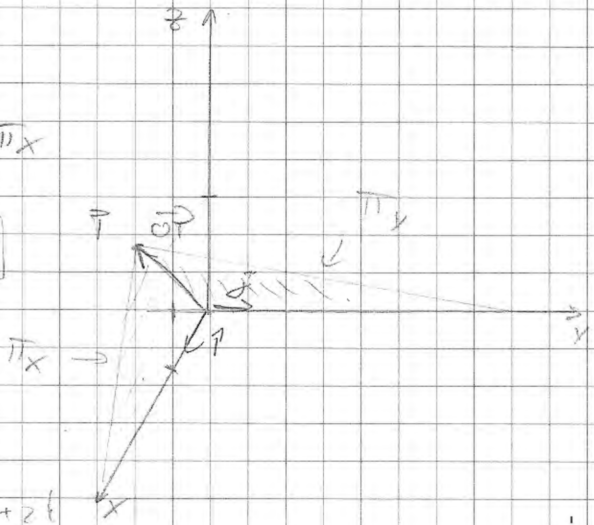
$$\vec{n}_{ABC} = \overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ -3 & -1 & 1 \end{vmatrix} = 3\vec{i} - 5\vec{j} + 4\vec{k}$$

$$\vec{n}_{ABD} = \overrightarrow{AB}\times\overrightarrow{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ -1 & -1 & 3 \end{vmatrix} = 5\vec{i} + \vec{j} + 2\vec{k}$$

$\boxed{\varphi = \arccos\frac{3}{25}\sqrt{15}}$ $\quad \approx 62^\circ\, 18'$

$$\cos\varphi = \frac{|\vec{n}_{ABC}\cdot\vec{n}_{ABD}|}{|\vec{n}_{ABC}|\cdot|\vec{n}_{ABD}|} = \frac{|15-5+8|}{\underbrace{\sqrt{50}\cdot\sqrt{30}}_{=10\sqrt{15}}} = \frac{18}{10\sqrt{15}} = \frac{3\sqrt{15}}{25}$$

Zad 16. $T(2,-1,3)$



$\vec{OT} = (2,-1,3)$ $\quad \vec{OT} \parallel \pi_x$, $\vec{i} \parallel \pi_x$

$$\vec{n_x} = \vec{OT} \times \vec{i} = \begin{vmatrix} \vec i & \vec j & \vec k \\ 2 & -1 & 3 \\ 1 & 0 & 0 \end{vmatrix} = 0\vec{i} + \boxed{3\vec{j} + \vec{k}}$$

$\vec{OT} \parallel \pi_y$, $\vec{j} \parallel \pi_y$

$$\vec{n_y} = \vec{OT} \times \vec{j} = \begin{vmatrix} \vec i & \vec j & \vec k \\ 2 & -1 & 3 \\ 0 & 1 & 0 \end{vmatrix} = -3\vec{i} + 0\vec{j} + 2\vec{k}$$

$$\vec{n_y} = 3\vec{i} - 2\vec{k}$$

$$\cos\varphi = \frac{|\vec{n_x} \cdot \vec{n_y}|}{|\vec{n_x}| \cdot |\vec{n_y}|} = \frac{|0\cdot 3 + 3\cdot 0 + 1\cdot -2|}{\sqrt{9+1}\cdot\sqrt{9+4}} = \frac{2}{\sqrt{10}\cdot\sqrt{13}} = \frac{2}{\sqrt{130}} = \frac{2\sqrt{130}}{130}$$

$$\boxed{\varphi = \arccos \frac{2}{130}\sqrt{130}} \approx 79^\circ\ 53'\ 50{,}1''$$

Zad 17

a) $p \ldots \begin{cases} 2x+3y-z+2=0 \\ 3x-y+2z+1=0 \end{cases}$ $\rightarrow$ 1) $x = t$ $\quad 3y - z + 2t + 2 = z$

$3t - y + 6y + 4t + 4 + 1 = 0$

$5y + 7t + 5 = 0$

$y = -1 - \frac{7}{5}t$

2) $t \to 5t$

$z = -3 - \frac{21}{5}t + \frac{10}{5}t + 2$

$= -1 - \frac{11}{5}t$

$p \ldots \begin{cases} x = 5t \\ y = -1 - 7t \\ z = -1 - 11t \end{cases}$

$$\boxed{\frac{x-0}{5} = \frac{y+1}{-7} = \frac{z+1}{-11}}$$

b) $p \ldots \begin{cases} x + 3y + 2z + 3 = 0 \\ -x - 2y + z = 0 \end{cases}$ $/+$

1) $y = t$

$0 + y + 3z + 3 = 0$

$3z = -3 - t$

$z = -1 - \frac{1}{3}t$

$-x - 2t - 1 - \frac{1}{3}t = 0$

$x = -\frac{7}{3}t - 1$

2) $t \to -3t$

$p \ldots \begin{cases} x = 7t - 1 \\ y = -3t \\ z = t - 1 \end{cases}$ $\rightarrow$ $\boxed{\frac{x+1}{7} = \frac{y-0}{-3} = \frac{z+1}{1}}$

Zad 18. a) $p \ldots \begin{cases} 2x+y=0 \\ x-2z=0 \end{cases}$ $\quad x=t,\; y=-2t,\; z=\frac{1}{2}t \quad \Rightarrow \quad \boxed{\begin{aligned} x&=-2t \\ y&=4t \\ z&=-t \end{aligned}}$

b) $p \ldots \begin{cases} x+2y+z+3=0 \\ x-3y-z+1=0 \end{cases} \Big/ +$ $\quad y=\boxed{2t}$ $\quad z=x-3y+1$

$2x-y+4=0 \qquad = -2+t-6t+1$

$x=\boxed{-2+t} \qquad = \boxed{-1-5t}$

Zad 19.

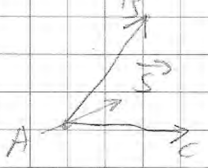
$p_1 \ldots \dfrac{x-1}{1} = \dfrac{y+2}{2} = \dfrac{z-0}{3} \qquad p_2 \ldots \dfrac{x-0}{0} = \dfrac{y}{2} = \dfrac{z+2}{-2}$

$\vec{s_1} = (1,2,3) \qquad \vec{s_2} = (0,2,-2)$

$\cos\varphi = \dfrac{|\vec{s_1}\cdot\vec{s_2}|}{|\vec{s_1}|\,|\vec{s_2}|} = \dfrac{|1\cdot 0+2\cdot 2-3\cdot 2|}{\sqrt{1+4+9}\cdot\sqrt{0+4+4}} = \dfrac{2}{\sqrt{14}\sqrt{8}} = \dfrac{2}{2\sqrt{2}\sqrt{2}\sqrt{7}} = \boxed{\dfrac{1}{2\sqrt{7}}}$

$\boxed{\varphi = \arccos \frac{1}{2}\frac{\sqrt{7}}{7}} \approx 29^\circ\, 6'$

Zad 20. $A(1,1,1) \quad B(3,0,3) \quad C(-2,1,1)$



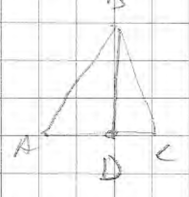
$\vec{s} = \vec{AB} + \vec{AC}, \quad \vec{AB} = (2,-1,2) \quad \vec{AC} = (-3,0,0)$

$\vec{s} = \dfrac{1}{\sqrt{4+1+4}}\cdot(2,-1,2) + \dfrac{1}{\sqrt{9}}(-3,0,0)$

$= \frac{1}{3}\boxed{(-1,-1,2)}$

$p \ldots \boxed{\dfrac{x-1}{-1} = \dfrac{y-1}{-1} = \dfrac{z-1}{2}}$

Zad 21. $A(1,-2,-4), \; B(3,1,-3) \quad C(5,1,-2)$



Određimo pravac kroz AC

$\vec{s} = \vec{AC} = (4,3,-3) \qquad p \ldots \begin{cases} x=1+4t \\ y=-2+3t \\ z=-4-3t \end{cases}$

visina je okomita na pravac ⇒ najkraća udaljenost točke B do pravca $\quad d(B, p_{AC}) = \mathrm{sqrt}\left[(3-1-4t)^2 + (1+2-3t)^2 + (-3+4+3t)^2\right]$

$\downarrow$

$$d(B, p_{AC}) = \operatorname{sqrt}\left[(2-4t)^2 + (3-3t)^2 + (1+3t)^2\right]$$

$$= \operatorname{sqrt}\left[16t^2 + 9t^2 + 9t^2 - 16t - 18t + 6t + 4 + 9 + 1\right]$$

$$= \operatorname{sqrt}\left[\underbrace{34t^2 - 28t + 14}_{g(t)}\right]$$

deriviramo f-ju pod korijenom jer je sqrt rastuća f-ja pa ne utječe na x-koordinate extrema

$$g' = 68t - 28 \Rightarrow 68t - 28 = 0$$

$$t = \frac{7}{17}$$

$B(3, 1, -3)$

$D \ldots \quad x = 1 + 4\frac{7}{17} = \frac{45}{17}$

$y = -2 + \frac{3 \cdot 7}{17} = -\frac{13}{17}$

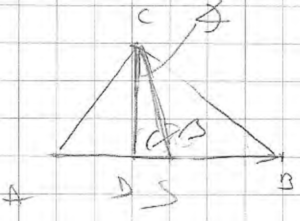
$z = -4 - \frac{3 \cdot 7}{17} = -\frac{89}{17}$

$$\vec{BD} = \vec{s} = \left(-\frac{6}{17}, -\frac{30}{17}, -\frac{38}{17}\right) \Big/ \cdot -\frac{17}{2}$$

$$\vec{s} = (3, 15, 19)$$

$$\boxed{p \ldots \frac{x-3}{3} = \frac{y-1}{15} = \frac{z+3}{19}}$$

Zad 22 $\quad A(2, 1, -1) \quad B(2, 0, 1) \quad C(0, 1, 1)$



težište $\quad S\left(\frac{2+2}{2}, \frac{1+0}{2}, \frac{-1+1}{2}\right)$

$$S\left(2, \frac{1}{2}, 0\right)$$

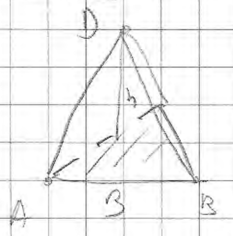
$$\vec{CS} = \left(2, -\frac{1}{2}, -1\right) \qquad \vec{SA} = \left(0, -\frac{1}{2}, -1\right)$$

$$\cos\beta = \frac{|\vec{SC} \cdot \vec{SA}|}{|\vec{SC}| \cdot |\vec{SA}|} = \frac{|-4 \cdot 0 + 1 \cdot -1 + 2 \cdot -2|}{\sqrt{16+1+4} \cdot \sqrt{0+1+4}} = \frac{3}{\sqrt{21} \cdot \sqrt{5}} = \frac{\sqrt{105}}{35}$$

$\vec{CS} \cdot 2 \qquad \vec{SA} \cdot 2$

$$\measuredangle(\vec{CS}, \vec{CD}) = 90^\circ - \beta = 90^\circ - \arccos\frac{\sqrt{105}}{35} = 17^\circ\ 1'$$

Zad 23. $A(-1,-3,-1)$ $B(5,3,8)$ $C(-1,-3,5)$ $D(2,1,-4)$



$$V = \frac{B\cdot h}{3} = \frac{1}{6}\left[\vec{AB},\vec{AC},\vec{AD}\right]$$

$$B = \frac{1}{2}\left|\vec{AB}\times\vec{AC}\right|$$

$$h\cdot\left|\vec{AB}\times\vec{AC}\right| = \left[\vec{AB},\vec{AC},\vec{AD}\right]$$

možda bi je ovo trebalo preko pravca i ravnine..., eto riješeno :)

$\vec{AB}=(6,6,9)$, $\vec{AC}=(0,0,6)$, $\vec{AD}=(3,4,-3)$

$$\vec{AB}\times\vec{AC} = 3\cdot 6\begin{vmatrix} \vec i & \vec j & \vec k \\ 2 & 2 & 3 \\ 0 & 0 & 1\end{vmatrix} = 3\cdot 6\left[\vec i\begin{vmatrix}2 & 3\\ 0 & 1\end{vmatrix} - \vec j\begin{vmatrix}2 & 3\\ 0 & 1\end{vmatrix} + \cancel{\vec k\begin{vmatrix}2 & 2\\ 0 & 0\end{vmatrix}}\right]$$

$$= 18\cdot\left[2\vec i - 2\vec j\right] = 36\left[\vec i - \vec j\right]$$

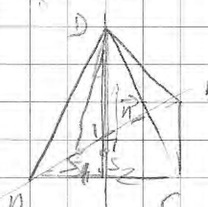
$$\left|\vec{AB}\times\vec{AC}\right| = 36\sqrt{2}$$

$$\left[\vec{AB},\vec{AC},\vec{AD}\right] = 3\cdot 6\begin{vmatrix} 2 & 2 & 3 \\ 0 & 0 & 1 \\ 3 & 4 & -3\end{vmatrix} = 18\left[0\cancel{\begin{vmatrix}2 & 3\\ 4 & -3\end{vmatrix}} - 0\cancel{\begin{vmatrix}2 & 3\\ 3 & -3\end{vmatrix}} + 1\begin{vmatrix}2 & 2\\ 3 & 4\end{vmatrix}\right]$$

$$= 18\cdot 2 = 36$$

$$h = \frac{36}{36\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

Zad 24. $A(0,0,0)$ $B(1,1,0)$ $C(0,3,0)$ $D(1,1,2)$



$S_1 \ldots p_{AB} \ldots \vec s = \vec{AC} = (2,1,0)$ $\quad \frac{x-0}{2} = \frac{y-0}{1} = \frac{z-0}{0}$

$x = 2t$, $y = t$, $z = 0$

$$d(D,S_1)^2 = (2t-1)^2 + (t-1)^2 + 2^2$$

$$= 4t^2 - 4t + 1 + t^2 - 2t + 1 + 2^2$$

$$= \underbrace{5t^2 - 6t + 6}_{g} \qquad g' = 10t - 6 = 0$$

$$t = \frac{3}{5}$$

$$\vec{DS_1} = \left(\frac{1}{5}, -\frac{2}{5}, -2\right) = (1,-2,-10) \qquad S_1\left(\frac{6}{5},\frac{3}{5},0\right)$$

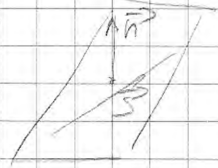
$$\boxed{\varphi = \arccos\frac{2\sqrt{105}}{21}}$$

$$\approx 12^\circ\, 36'$$

$S_2 \ldots \Pi_{ABC} \ldots \vec n = \vec{AB}\times\vec{AC} = \begin{vmatrix}\vec i & \vec j & \vec k\\ 2 & 1 & 0\\ 0 & 3 & 0\end{vmatrix} = 0\vec i + 0\vec j + \boxed{6\vec k}$

$$\sin\varphi = \cos\angle(\vec n,\vec{DS_0}) = \frac{|\vec{DS_0}\cdot\vec n|}{|\vec{DS_0}|\cdot|\vec n|} = \frac{|1\cdot 0 - 2\cdot 0 - 10\cdot 1|}{\sqrt{1+4+100}\cdot 1} = \frac{10}{\sqrt{105}} = \boxed{\frac{2\sqrt{105}}{21}}$$

Zad 25. $\frac{x-2}{m} = \frac{y-1}{4} = \frac{z-3}{-4}$ $\quad \Pi \ldots \ 4x - 3y - 6z + 13 = 0$



$\vec{s} \perp \vec{n}$ – paralelnost $\qquad$ pripada li točka $T_0(2,1,3)$ ravnini

$(m, 4, -4) \cdot (4, -3, -6) = 0$

$4m - 12 + 24 = 0 \qquad\qquad 4 \cdot 2 - 3 \cdot 1 - 6 \cdot 3 + 13 = 0$

$4m = -12 \qquad\qquad 8 - 3 - 18 + 13 = 0$

$\boxed{m = -3} \qquad\qquad 0 = 0$ ✓✓

Zad 26. $T(2,4,3)$ $\quad p \ldots \begin{cases} x = t \\ y = -2t + 1 \\ z = 2t - 2 \end{cases} \rightarrow T_p(t, -2t+1, 2t-2)$

1. način: udaljenost + derivacija

$d(T, T_p)^2 = (2-t)^2 + (4+2t-1)^2 + (3-2t+2)^2$ $\quad$ nije bitno

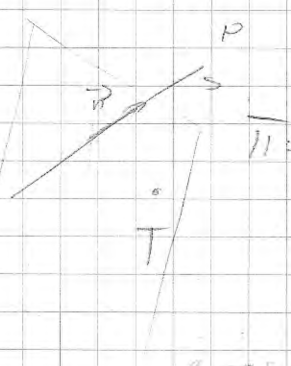
$= t^2 + 4t^2 + 4t^2 - 4t + 12t - 20t + N$

$= 9t^2 - 12t + N$

$d^2 = g, \ g' = 18t - 12 = 0 \Rightarrow t = \frac{12}{18} = \frac{2}{3}$

$$\boxed{T'\left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)}$$

2. način: postavljamo ravninu kroz T, okomito na p



$\vec{n} = \vec{s} = (1, -2, 2) \qquad T(2,4,3)$

$\Pi \ldots \ 1(x-2) - 2(y-4) + 2(z-3) = 0$

$x - 2y + 2z - 2 + 8 - 6 = 0$

$x - 2y + 2z = 0$

presjek pravca i ravnine

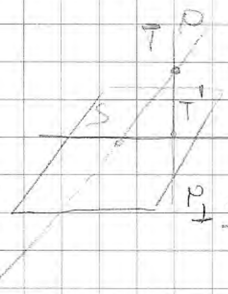
$t - 2(-2t+1) + 2(2t-2) = 0$

$t + 4t - 2 + 4t - 4 = 0$

$9t - 6 = 0$

$t = \frac{6}{9} = \frac{2}{3} \qquad \boxed{T'\left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)}$

Zad 27. $p \ldots \frac{x-9}{-2} = \frac{y+7}{11} = \frac{z-4}{-10} \Rightarrow p \ldots \begin{cases} x = -2t+9 \\ y = 11t-7 \\ z = -10t+4 \end{cases}$



$\pi \ldots 2x + y - 2z - 3 = 0$

$S \ldots 2(-2t+9) + (11t-7) - 2(-10t+4) - 3 = 0$

$-4t + \cancel{18} + 11t - \cancel{7} + 20t - \cancel{8} - \cancel{3} = 0$

$t = 0 \Rightarrow S = S(9, -7, 4)$

$T \to$ npr. $t = 1$ $\quad T(7, 4, -6)$

$p_\perp \ldots \quad \vec{s}_\perp = \vec{n} = (2, 1, -2) \qquad p_\perp \ldots \begin{cases} x = 7 + 2\alpha \\ y = 4 + \alpha \\ z = -6 - 2\alpha \end{cases}$

$p_\perp \cap \pi = T' \ldots \quad 2(7+2\alpha) + (4+\alpha) + 2(+6+2\alpha) - 3 = 0$

$14 + 4\alpha + 4 + \alpha + 12 + 4\alpha - 3 = 0$

$9\alpha + 27 = 0 \Rightarrow \boxed{\alpha = -3}$

$T'(1, 1, 0)$

$T', S \in p' \quad \vec{s} = \overrightarrow{T'S} = (8, -8, 4) = 4(2, -2, 1)$

$p \ldots \boxed{\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z-0}{1}} \qquad p \ldots \frac{x-9}{2} = \frac{y+7}{-2} = \frac{z-4}{1}$

Zad 28. $A(3, 0, 2) \quad p \ldots \frac{x-1}{2} = \frac{y-0}{1} = \frac{z+1}{-1}$

$\pi \ldots 2x + 2y + 2z + 1 = 0 \qquad p \ldots \begin{cases} x = 2t+1 \\ y = t \\ z = -t-1 \end{cases}$

$S \in \pi \quad S = \left(\frac{4+2t}{2}, \frac{t}{2}, \frac{-t+1}{2} \right)$

$4 + 2t + \cancel{t} - \cancel{t} + 1 + 1 = 0$

$2t = -6$

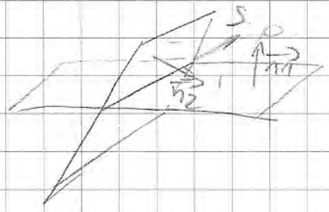
$\boxed{t = -3}$

$\boxed{T(-5, -3, 2)}$

Zad 29. $\pi \ldots\ x - 2y + z = 0 \qquad \rho_1 \ldots\ 2x - 3y + z - 4 = 0$

$\rho_2 \ldots\ x + y - z + 2 = 0$

$p = \rho_1 \cap \rho_2$



$\vec{s} \parallel \vec{n_1}, \quad \vec{s} \parallel \vec{n_2}$

$$\vec{s} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2\vec{i} + 3\vec{j} + 5\vec{k}$$

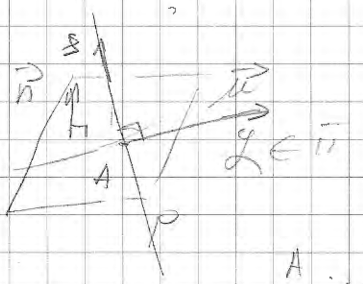
$T_0 \ldots\ x = 0 \qquad -3y + z - 4 = 0$

$y - z + 2 = 0 \quad \Big| + \quad -2y - 2 = 0$

$y = -1 \qquad z = 1$

$T(0, -1, 1)$

$$p \ldots \begin{cases} x = 2t \\ y = -1 + 3t \\ z = 1 + 5t \end{cases}$$



$\vec{n} \perp \vec{u}, \quad \vec{s} \perp \vec{u} \quad \Rightarrow \vec{u} = \vec{n} \times \vec{s}$

$q \in \pi$

$$\vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 5 \\ 1 & -2 & 1 \end{vmatrix} = 13\vec{i} + 3\vec{j} - 7\vec{k}$$

$A \ldots\ A = p \cap \pi \qquad (2t) - 2(-1 + 3t) + (1 + 5t)$

$2t - 6t + 5t + 2 + 1 = 0$

$\boxed{t = -3}$

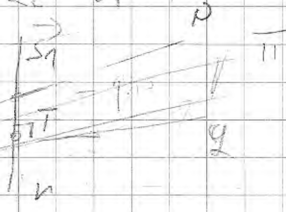
$A(-6, -10, -14)$

$$q \ldots \boxed{\frac{x+6}{13} = \frac{y+10}{3} = \frac{z+14}{-7}}$$

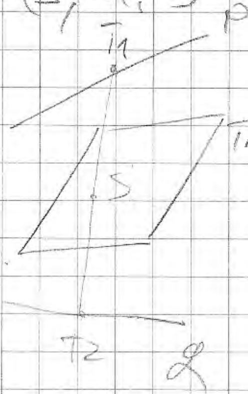
Zad 30. $p \ldots\ \frac{x-1}{1} = \frac{y-1}{2} = \frac{z+1}{1} \qquad q \ldots\ \frac{x+1}{1} = \frac{y-0}{1} = \frac{z+1}{-1}$

$\vec{s_1} = (1, 2, 1) \qquad \vec{s_2} = (1, 1, -1)$

$$p \ldots \cancel{\begin{cases} x = \alpha + 1 \\ y = 2\alpha + 1 \\ z = \alpha - 1 \end{cases}} \qquad q \ldots \cancel{\begin{cases} x = \beta - 1 \\ y = \beta \\ z = -\beta - 1 \end{cases}}$$



$$\vec{n} = \vec{s_1} \times \vec{s_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -3\vec{i} + 2\vec{j} - \vec{k}$$



$\vec{n}=(3,-2,1)$

$p \ldots \begin{cases} \cancel{\alpha+1} = z \\ \cancel{2\alpha+1} = x \\ \cancel{\alpha-1} = z \end{cases} \qquad q \begin{cases} \cancel{\beta-1} = z \\ \cancel{\beta} = x \\ \cancel{-3\beta-1} = z \end{cases}$

$T_1(1,1,-1)$

$T_2(-1,0,-1)$

ako je π jednako udaljen od p i q onda raspolovljuje spojnice bilo kojih točaka

$S=\left(\frac{1-1}{2}, \frac{1+0}{2}, \frac{-1-1}{2}\right)=\left(0, \frac{1}{2}, -1\right)$

$\pi \ldots \; 3(x-0)-2\left(y-\frac{1}{2}\right)+1(z+1)=0$

$\boxed{3x-2y+z+2=0}$

Zad 31

$p \ldots \begin{cases} x+y-3z+6=0 \\ x-y-z=0 \end{cases} \; /+ \qquad x=t \qquad y=\frac{1}{2}t-\frac{3}{2}$

$2t-4z+6=0$

$4z=2t+6$

$z=\frac{1}{2}t+\frac{3}{2}$

$t \to 2t \qquad p \ldots \begin{cases} x=2t \\ y=t-\frac{3}{2} \\ z=t+\frac{3}{2} \end{cases}$

$T(1,0,1) \qquad d=\sqrt{8} \qquad d(T,p)^2=(2t-1)^2+\left(t-\frac{3}{2}\right)^2+\left(t+\frac{1}{2}\right)^2$

$8=4t^2+t^2+t^2-4t-3t+t+1+\frac{9}{4}+\frac{1}{4}$

$8=6t^2-6t+\frac{7}{2} \quad / \cdot 2$

$16=12t^2-12t+7$

$0=12t^2-12t-9 \quad /:3$

$0=4t^2-4t-3$

$t=\frac{4 \pm \sqrt{16+3\cdot 16}}{8}$

$=\frac{4 \pm 2\cdot 4}{8}$

$t_1=\frac{4+8}{8}=\frac{12}{8}=\frac{3}{2}$

$t_2=\frac{4-8}{8}=-\frac{4}{8}=-\frac{1}{2}$

$\boxed{T_1(3,0,3)} \quad \boxed{T_2(-1,-2,1)}$

Zad 37.

$p_1 \ldots \begin{cases} x = t - 3 \\ y = 3t \\ z = t \end{cases}$ $\quad p_2 \ldots \begin{cases} x = 3u + 2 \\ y = 2u + 1 \\ z = u + \alpha \end{cases}$

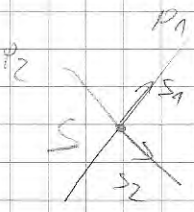
$p_1 \cap p_2$

x: $t - 3 = 3u + 2$, $t = 3u + 5$

y: $3t = 2u + 1$, $9u + 15 = 2u + 1$, $7u = -14$, $u = -2$

$t = -1$

z: $t = u + \alpha$, $\alpha = t - u = -1 - (-2) = \boxed{1}$



$S \ldots (-4, -3, -1)$

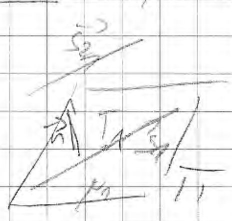
$\vec{s_1} = (1, 3, 1) \quad \vec{s_2} = (3, 2, 1)$

$$\vec{n} = \vec{s_1} \times \vec{s_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 1 \\ 3 & 2 & 1 \end{vmatrix} = \vec{i} + 2\vec{j} - 7\vec{k}$$

$1(x+4) + 2(y+3) - 7(z+1) = 0$

$\boxed{x + 2y - 7z + 3 = 0}$

Zad 38. $p_1 \ldots \frac{x-3}{2} = \frac{y+4}{-1} = \frac{z-2}{-3}$ $\quad p_2 \ldots \frac{x+5}{4} = \frac{y-2}{7} = \frac{z-1}{2}$



$\vec{s_2} \parallel \pi, \ \vec{s_1} \parallel \pi \Rightarrow \vec{n} = \vec{s_1} \times \vec{s_2}$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -3 \\ 4 & 7 & 2 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & -3 \\ 7 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & -3 \\ 4 & 2 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & -1 \\ 4 & 7 \end{vmatrix}$$

$= 19\vec{i} - 16\vec{j} + 18\vec{k}$

$\pi \ldots 19(x - 3) - 16(y + 4) + 18(z - 2) = 0$

$\boxed{19x - 16y + 18z - 157 = 0}$

$3 \cdot 19 = 57 \quad 4 \cdot 16 = 64 \quad 18 \cdot 2 = 36$

$-57 \quad -64 \quad -36$

$57 + 64 + 36 = 157$

Zad 39. $p \ldots \begin{cases} 3x + y - z + \alpha = 0 \\ 2x - 2y + z + 1 = 0 \end{cases} \cap O_x$

$y = z = 0$

$x = t \quad 5t - y + \alpha + 1 = 0$

$y = 5t + \alpha + 1$

$2t - 10t - 2\alpha - 2 + z + 1 = 0$

$z = 8t + 2\alpha + 1$

$y = 0 \quad 5t + \alpha + 1 = 0$

$\alpha = -1 - 5t$

$z = 0$

$0 = 8t - 2 - 10t + 1$

$0 = -2t - 1$

$\boxed{t = -\frac{1}{2}}$

$\alpha = -1 + \frac{5}{2} = \boxed{\frac{3}{2}}$

Zad 35. $\pi \ldots x+2y-z-2=0$

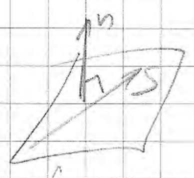
$p \ldots \begin{cases} x-\alpha y+z+1=0 \\ 3x+y-z+\beta=0 \end{cases}$

1.

$\boxed{x=t}$ $\quad t-\alpha y+z+1=0$

$z=\alpha y-1-t$

$\boxed{z=\frac{\alpha}{\alpha-1}(1+\beta+4t)-1-t}$



$3t+y-(\alpha y-1-t)+\beta=0$

$3t+y-\alpha y+1+t+\beta=0$

$y(1-\alpha)=-4t-1-\beta$

$\boxed{y=\frac{1}{\alpha-1}(1+\beta+4t)}$

$\frac{4\alpha}{\alpha-1}-1$

$\frac{4\alpha-\alpha+1}{\alpha-1}$

$\frac{3\alpha+1}{\alpha-1}$

$\vec{s}=\left(1, \frac{4}{\alpha-1}, \frac{3\alpha+1}{\alpha-1}\right)$

$\vec{s}\cdot\vec{n}=0 \quad 1+\frac{8}{\alpha-1}-\frac{3\alpha+1}{\alpha-1}=0 \quad /\cdot(\alpha-1) \quad \alpha\neq 1$

$\alpha-1+8-3\alpha-1=0$

$-2\alpha+6=0$

$\boxed{\alpha=3}$

$t=0$

$0+\frac{2}{2}(1+\beta+0)$

$-\frac{3}{2}(1+\beta+0)+1+0-2=0$

$-\frac{1}{2}(1+\beta)-1=0 \quad /\cdot 2$

$\beta+1+2=0$

$\boxed{\beta=-3}$

<u>Zad 36.</u> $T(1,0,7)$, $p_1 \ldots \frac{x-1}{2} = \frac{y+3}{1} = \frac{z-3}{3}$

$\overset{(1)}{T} \in g$, $\overset{(2)}{p_1 \cap g \neq \emptyset}$ $\quad p_2 \ldots \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+2}{-2}$

(3) $g \perp p_2$

$\vec{s_2} = (1,2,-2)$

$\vec{s_g} = (\alpha, \beta, \gamma)$

(3) $\vec{s_g} \cdot \vec{s_2} = 0$

$\boxed{\alpha + 2\beta - 2\gamma = 0}$

$p_1 \ldots \begin{cases} x = 2t+1 \\ y = t-3 \\ z = 3t+3 \end{cases}$ $\qquad$ (1) $g \ldots \left.\begin{cases} x = 1 + \alpha \cdot \Delta \\ y = 0 + \beta \cdot \Delta \\ z = 7 + \gamma \cdot \Delta \end{cases}\right\}$ $\Delta \in \mathbb{R}$, $\Delta \neq 0$

zapravo može i bez Δ jer se i tako α, β, γ za koje to vrijedi !!!

(2) $2t + \cancel{1} = \cancel{1} + \alpha \cdot \Delta$ $\qquad t - 3 = \beta \cdot \Delta$ $\qquad 3t + 3 = 7 + \gamma\Delta$

$\alpha\Delta = 2t$ $\qquad \beta = \frac{t-3}{\Delta}$ $\qquad 3t - 4 = \gamma\Delta$

$\alpha = \frac{2t}{\Delta}$ $\quad \Delta \neq 0$ $\qquad \gamma = \frac{3t-4}{\Delta}$

$\frac{2t}{\Delta} + 2 \cdot \frac{t-3}{\Delta} - 2 \cdot \frac{3t-4}{\Delta} = 0 \quad / \cdot \Delta$

$2t + 2t - 6 - 6t + 8 = 0$

$-2t + 2 = 0$

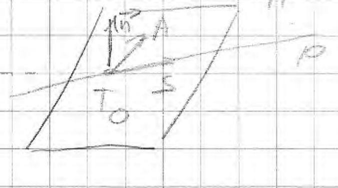
$\boxed{t = 1}$ $\qquad \alpha = \frac{2}{\Delta} \quad \beta = \frac{-2}{\Delta} \quad \gamma = -\frac{1}{\Delta}$

$\Delta = 1 \quad \alpha = 2, \ \beta = -2, \ \gamma = -1$

$g \ldots \frac{x-1}{2} = \frac{y-0}{-2} = \frac{z-7}{-1}$ $\qquad \boxed{\frac{x-1}{2} = \frac{y-0}{-2} = \frac{z-7}{-1}}$

<u>Zad 37</u> $A(-3,1,1)$ $\qquad p \ldots \begin{cases} z = 0 \\ x + y - 2z + 3 = 0 \end{cases}$ $\qquad x = t$, $\ x + y + 3 = 0$, $\ y = -3 - t$

$p \ldots \begin{cases} x = t \\ y = -3 - t \\ z = 0 \end{cases}$ $\qquad \vec{s} = (1,-1,0) \qquad T_0(0,-3,0)$



$\vec{s} \parallel \pi$
$\overrightarrow{T_0A} \parallel \pi$ $\Rightarrow \vec{n} = \vec{s} \times \overrightarrow{T_0A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 4 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$

$\overrightarrow{T_0A} = (-3, 4, 1)$ $\qquad = \vec{i} \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$

$= \vec{i} + \vec{j} - \vec{k}$

$\pi \ldots 1(x+3) + 1(y-1) - 1(z-1) = 0$

$\boxed{x + y - z + 3 = 0}$

14.

Zad 38. $p \ldots \frac{x-1}{2} = \frac{y+1}{2} = \frac{z-0}{-1} \ldots \begin{cases} x = 2t+1 \\ y = 2t-1 \\ z = -t \end{cases}$

$\pi_1 \ldots \; 3x + y + 2z - 1 = 0 \qquad \pi_2 \ldots \; x + 2y + 3z + 1 = 0$

$\frac{3x + y + 2z - 1}{\sqrt{9+1+4}} = 0 \qquad \frac{x + 2y + 3z + 1}{\sqrt{9+4+1}} = 0$

$d(T, \pi_1) = d(T, \pi_2) \quad \frac{|3(2t+1) + (2t-1) + 2(-t) - 1|}{\cancel{\sqrt{14}}} = \frac{|2t + 1 + 2(2t-1) + 3(-t) + 1|}{\cancel{\sqrt{14}}}$

$|6t + 1| = |3t|$

$6t + 1 = 3t \qquad\qquad 6t + 1 = -3t$

$3t = -1 \qquad\qquad 9t = -1$

$t = -\frac{1}{3} \qquad\qquad t = -\frac{1}{9}$

$T_1\left(-\frac{2}{3} + \frac{3}{3}, -\frac{2}{3} - \frac{3}{3}, \frac{1}{3}\right)$

$\boxed{T_1\left(\frac{1}{3}, -\frac{5}{3}, \frac{1}{3}\right)}$

$T_2\left(-\frac{2}{9} + \frac{9}{9}, -\frac{2}{9} - \frac{9}{9}, \frac{1}{9}\right)$

$\boxed{T_2\left(\frac{7}{9}, -\frac{11}{9}, \frac{1}{9}\right)}$

Zad 39. $p \ldots \frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-2}{2} \qquad \vec{s} = (2, -3, 2) \quad T_0(1, -2, 2)$

$\pi \ldots \; 3x + 2y - z - 5 = 0$

$\vec{n} = (3, 2, -1)$

$\vec{n_1} = \vec{n} \times \vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & -1 \\ 2 & -3 & 2 \end{vmatrix} = \vec{i} \begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & -1 \\ 2 & 2 \end{vmatrix} + \vec{k} \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}$

$= \vec{i} - 8\vec{j} - 13\vec{k}$

$\pi \ldots \; 1(x-1) - 8(y+2) - 13(z-2) = 0$

$x - 8y - 13z - 1 - 16 + 26 = 0$

$\boxed{x - 8y - 13z + 9 = 0}$

Zad 40. $p \ldots \frac{x+1}{2} = \frac{y-2}{-3} = \frac{z-2}{-7}$ $T_0(-1, 2, 2)$

$\vec{s} = (2, -3, -7)$

$S\left(\frac{2-2}{2}, \frac{1+1}{2}, \frac{0-2}{2}\right)$ $A(2,1,0)$, $B(-2,1,-2)$

$S(0, 1, -1)$

$\vec{ST_0} = (-1, 1, 3)$ $\vec{n} = \vec{s} \times \vec{ST_0}$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 3 \\ 2 & -3 & -7 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 3 \\ 0 & -1 & -1 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 2 \\ 0 & -1 & -1 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$= 2\vec{i} - \vec{j} + \vec{k}$

$\pi \ldots 2(x-0) - (y-1) + (z+1) = 0$

$\boxed{2x - y + z + 2 = 0}$

Zad 41. $Oy \ldots x = z = 0$ $y = t$

$\pi_1 \ldots \frac{3x - y + 4z - 1}{\sqrt{9+1+16}} = 0$ $\pi_2 \ldots \frac{x - 5z + 2}{\sqrt{1+25}} = 0$

$\frac{|-t-1|}{\sqrt{26}} = \frac{|2|}{\sqrt{26}}$

$-t - 1 = 2$ $t = -3$ $\boxed{T(0, -3, 0)}$

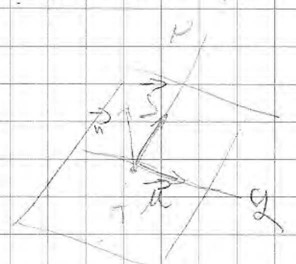
$-t - 1 = -2$ $t = 1$ $\boxed{T(0, 1, 0)}$

Zad 42. $p \ldots \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{3}$ $p \ldots \begin{cases} x = 1 + 2t \\ y = -1 - 3t \\ z = 2 + 3t \end{cases}$

$\pi \ldots 2x + y - z + 5 = 0$ $2 + 4t - 1 - 3t - 2 - 3t + 5 = 0$

$-2t = -4$

$t = 2$ $T(5, -7, 8)$



$\vec{s} \perp \vec{u}$, $\vec{n} \perp \vec{u}$ $\boxed{t = 2}$

$$\vec{u} = \vec{s} \times \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 3 \\ 2 & 1 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} -3 & 3 \\ 1 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & -3 \\ 2 & 1 \end{vmatrix}$$

$= 0\vec{i} + 8\vec{j} + 8\vec{k}$

$p \ldots \boxed{\frac{x-5}{0} = \frac{y+7}{1} = \frac{z-8}{1}}$

Zad 43 $\quad p: \frac{x-1}{1} = \frac{y+2}{5} = \frac{z+3}{-3} \qquad p: \begin{cases} x = t+1 \\ y = 5t-2 \\ z = -3t-3 \end{cases}$

$A(2,1,7),\ B(-4,3,1)$

$d(A,T) = d(B,T) \ /^2$

$(t+1-2)^2 + (5t-2-1)^2 + (-3t-3-7)^2 = (t+1+4)^2 + (5t-2-3)^2 + (-3t-3-1)^2$

$(t-1)^2 + (5t-3)^2 + (3t+10)^2 = (t+5)^2 + (5t-5)^2 + (3t+4)^2$

$-2t - 30t + 60t + 1 + 9 + 100 = 10t - 50t + 24t + 25 + 25 + 16$

$28t + 110 = -16t + 66$

$44t = -44$

$\boxed{t = -1} \qquad T(-1+1, -5-2, 3-3) \quad \boxed{T(0,-7,0)}$

Zad 44 $\quad p: \begin{cases} 2x+3y-z+5=0 \\ x-2y+3z+1=0 \ /\cdot(-2) \end{cases} \ \boxed{y=t}$

$-2x+4y-6z-2=0$

$0+7y-7z+3=0$

$7z = 7t+3$

$\boxed{z = t + \frac{3}{7}}$

$2x + 3t - t - \frac{3}{7} + 5 = 0$

$2x = -2t - \frac{32}{7}$

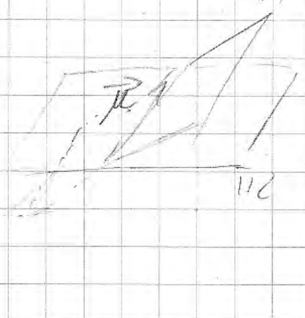
$x = -t - \frac{16}{7}$

$\vec{s} = (-1, 1, 1)$

$q: \boxed{\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z-2}{-1}}$

Zad 45. $\quad \frac{x-0}{3} = \frac{y-1}{m} = \frac{z-0}{2} \qquad \vec{s} = (3, m, 2)$

$\pi_1: x+2y-1=0 \qquad \pi_2: x-2z+2=0$



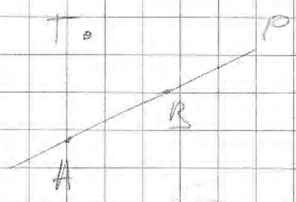
$\vec{n} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 1 & 0 & -2 \end{vmatrix} = -4\vec{i} + 2\vec{j} - 2\vec{k}$

$\vec{n} = 2\vec{i} - \vec{j} + \vec{k}$

$\vec{s} \cdot \vec{n} = 0 \qquad 6 - m + 2 = 0$

$\boxed{m = 8}$

Zad 46. $p \ldots \frac{x-1}{2} = \frac{y+2}{1} = \frac{z+1}{-3}$ $A(1,-2,-1)$ $T(6,1,-6)$



$p \ldots \begin{cases} x = 2t+1 \\ y = t-2 \\ z = -3t-1 \end{cases}$ $d(T,A) = d(T,B)$ $/^2$

$\Rightarrow 5^2 + 3^2 + 5^2 = (2t-5)^2 + (t-3)^2 + (-3t+5)^2$

$\cancel{5^2+3^2+5^2} = 4t^2 + t^2 + 9t^2 - 20t - 6t - 30t + \cancel{5^2+3^2+5^2}$

$0 = t(14t - 56)$

$t = 0 \Rightarrow B(1,-2,-1) = A$ $t = \frac{56}{14} = 4 \Rightarrow \boxed{B(9,2,-13)}$

Zad 47. $A(2,0,1)$ $B(4,2,0)$ $p \ldots \begin{cases} x = t-2 \\ y = 2t-3 \\ z = 3t-2 \end{cases}$ $C \in p$

$\pi \ldots 2x + 3y + 4z - 5 = 0$ $D \in \pi$ $D(\alpha, \beta, \gamma)$

$\vec{AB} = (2, 2, -1)$ $\vec{CD} = (\alpha - t + 2, \beta - 2t + 3, \gamma - 3t + 2)$

$\alpha - t + 2 = 2$ $\beta - 2t + 3 = 2$ $\gamma - 3t + 2 = -1$

$\alpha = t$ $\beta = 2t - 1$ $\gamma = 3t - 3$

$2t + 6t - 3 + 12t - 12 - 5 = 0$

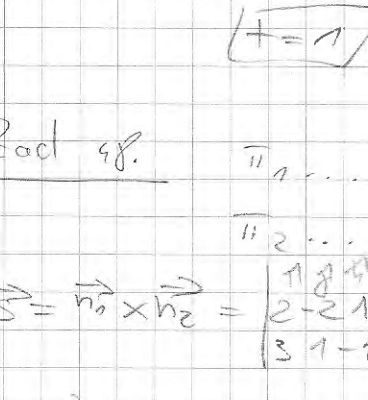
$20t = 20$ $\boxed{C(-1,-1,1)}$

$\boxed{t = 1} \Rightarrow \boxed{D(1,1,0)}$

Zad 48. $\pi_1 \ldots 2x - 2y + z - 1 = 0$ $yOz \Rightarrow x = 0$

$\pi_2 \ldots 3x + y - z - 2 = 0$

$\vec{s} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 1 \\ 3 & 1 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix} = \vec{i} + 5\vec{j} + 8\vec{k}$



$T_0 .$ $x = 0$ $-2y + z - 1 = 0$ / $-y$ ... $= 0$

x ... $- z - 2 = 0$ $y = -3$

$z = -5$

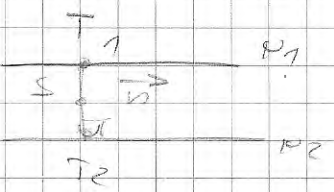
$T_0(0, -3, -5)$

$\vec{s} \mapsto yOz = (0, 5, 8)$

$p \ldots \boxed{\frac{x-0}{0} = \frac{y+3}{5} = \frac{z+5}{8}}$

$p \ldots \begin{cases} x = 0 \\ y = 5t - 3 \\ z = 8t - 5 \end{cases}$ $/ t = 1$ $x = 0$, $y = 2$, $z = 3$ $\frac{x-0}{0} = \frac{y-2}{5} = \frac{z-3}{8}$

Zad 49. $p_1 \ldots \frac{x-1}{2} = \frac{y+2}{1} = \frac{z-3}{-3}$ $p_2 \ldots \frac{x-3}{2} = \frac{y+2}{1} = \frac{z-9}{-3}$



$\vec{s_1} = \vec{s_2} = (2, 1, -3)$ $d(p_1, \pi) = d(p_2, \pi)$

$T_1(1, -2, 3)$ $T_2(2t+3, t-2, -3t+9)$

$\vec{T_1T_2} \cdot \vec{s} = 0$ $(2t+2)\cdot 2 + t - 3\cdot(-3t+6) = 0$

$4t+4+t+9t-18=0$

$14t = 14$ $t = 1$ $T_2(5, -1, 6)$

$\vec{n} = \vec{T_1T_2} = (4, 1, 3)$ $\pi \ldots 4x + y + 3z + D = 0$

$|4 - 2 + 9 + D| = |12 - 2 + 27 + D|$

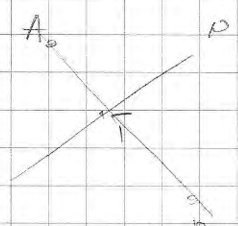
$|11 + D| = |37 + D|$ $\pi \ldots$ $\boxed{4x + y + 3z - 24 = 0}$

$11 + D = -D - 37$

$2D = -48$

$\boxed{D = -24}$

Zad 50. $A(4, 3, 10)$ $p \ldots \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$



$\vec{s} = (3, 4, 5)$ p $\begin{cases} x = 3t+1 \\ y = 4t+2 \\ z = 5t+3 \end{cases}$

$d(A,p)^2 = (3t-3)^2 + (4t-1)^2 + (5t-7)^2$

$= \underbrace{t^2(9+16+25) - t(18+8+70) + C}_{h}$

$h' = 100t - 96 = 0 \Rightarrow t = \frac{24}{25}$

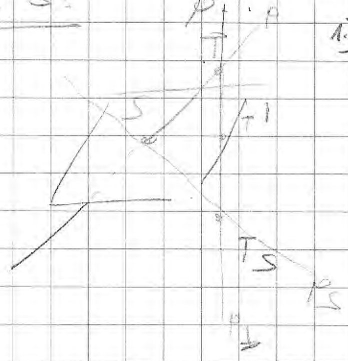
$T\left(\frac{97}{25}, \frac{146}{25}, \frac{39}{5}\right)$ $T = \frac{A+B}{2}$

$B = 2T - A = \left(\frac{94}{25}, \frac{217}{25}, \frac{28}{5}\right)$

Službeno rješenje je $B(4, 9, 0)$ no to ne može biti točno jer AB nije okomit na $\vec{s}$

✓

Zad 31



$$\frac{x-1}{1} = \frac{y-0}{2} = \frac{z+1}{1} \qquad \pi \ldots \quad -2y + z + 5 = 0$$

1.) $S \ldots \quad p \ldots \begin{cases} x = t+1 \\ y = 2t \\ z = t-1 \end{cases}$

$$0 \cdot (t+1) - 2(2t) + (t-1) + 5 = 0$$

$$-4t + t + 4 = 0$$

$$-3t = -4$$

$$t = \frac{4}{3}$$

2.) $S\left(\frac{4}{3} + \frac{3}{3},\ 2 \cdot \frac{4}{3},\ \frac{4}{3} - \frac{3}{3}\right) = \boxed{S\left(\frac{7}{3}, \frac{8}{3}, \frac{1}{3}\right)}$

3.) $p_\perp \quad T(1, 0, -1) \qquad \vec{s}_\perp = \vec{n} = (0, -2, 1)$

$p_\perp \ldots \begin{cases} x = 1 + 0 \cdot \Delta \\ y = 0 - 2 \cdot \Delta \\ z = -1 + \Delta \end{cases}$

4.) $p_\perp \cap \pi \qquad 0 \cdot 1 - 2(-2\Delta) + (\Delta - 1) + 5 = 0$

$$4\Delta + \Delta - 1 + 5 = 0$$

$$5\Delta = -4$$

$$\Delta = -\frac{4}{5}$$

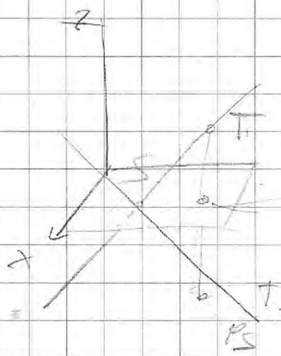
5.) $T'\left(1, \frac{8}{5}, -\frac{9}{5}\right)$ 6.) $\vec{s}_s = \overrightarrow{T_s S} = \left(\frac{7}{3} - 1, \frac{8}{3} - \frac{8}{5}, \frac{1}{3} + \frac{9}{5}\right)$

$$\overrightarrow{T_s S} = \left(\frac{4}{3}, \frac{16}{3 \cdot 5}, \frac{32}{3 \cdot 5}\right) = \frac{4}{3}\left(1, \frac{4}{5}, \frac{8}{5}\right) = \frac{4}{15}(5, 4, 8)$$

$$\vec{p}_s \ldots \boxed{\frac{x - \frac{7}{3}}{5} = \frac{y - \frac{8}{3}}{4} = \frac{z - \frac{1}{3}}{8}}$$

Zad 5? $p \ldots \frac{x-1}{2} = \frac{y+3}{1} = \frac{z-4}{-1}$ $p \ldots \begin{cases} x = 2t+1 \\ y = t-3 \\ z = -t+4 \end{cases}$

A) xOy



$T(1,-3,4)$

$T'(1,-3,0)$

$T_S(1,-3,-4)$

$S \ldots \quad z=0 \quad t=4$

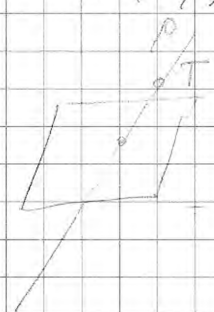
$\Rightarrow y=1, \; x=9$

$S(9,1,0)$

$\vec{s_s} = \overrightarrow{T_S S} = (8,4,4) = 4(2,1,1)$

$p_s \ldots \boxed{\frac{x-1}{2} = \frac{y+3}{1} = \frac{z+4}{1}}$

B) $-x+4y+2z-16=0$ $T(1,-3,4)$

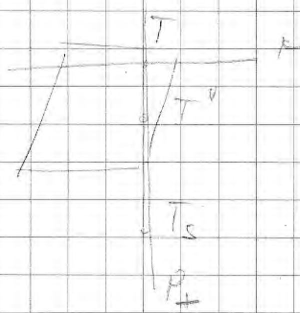


$S \ldots \quad -(2t+1)+4(t-3)+2(-t+4)-16=0$

$-2t-1+4t-12-2t+8-16=0$

$0 \cdot t = 21$ X   podeli:

$\vec{n} \cdot \vec{s} = -2+4-2=0 \quad \vec{n} \perp \vec{s} \Rightarrow p \parallel \pi$



$p_\perp \quad \vec{s_\perp} = \vec{n} = (-1,4,2)$

$p_\perp \ldots \begin{cases} x = 1-t \\ y = -3+4t \\ z = 4+2t \end{cases} \cap \pi$

$-(1-t)+4(-3+4t)+2(4+2t)-16=0$

$t-1+16t-12+4t+8-16=0$

$21t = 21 \quad \boxed{t=1}$

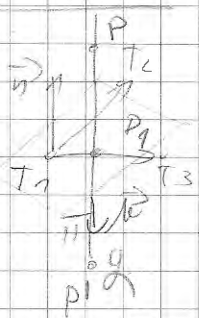
$T'(0,1,6)$

$T^S = (2T'-T) = (-1,5,8)$

$p_S \ldots \boxed{\frac{x+1}{2} = \frac{y-5}{1} = \frac{z-8}{-1}}$

$\vec{s} = \vec{s} = (2,1,-1)$

Zad 32



$P(3,-1,1)$ $T_1(-2,-1,-1)$, $T_2(2,1,-5)$ $T_3(-4,-1,0)$

$\overrightarrow{T_1T_2} = (4,2,-4)$ $\overrightarrow{T_1T_3} = (-2,0,1)$

$\vec{n} = \overrightarrow{T_1T_2} \times \overrightarrow{T_1T_3} = 2\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -2 \\ -2 & 0 & 1 \end{vmatrix} = 2 \cdot \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -1 \\ -2 & 0 & 1 \end{vmatrix}$

$= 2 \cdot \left[\vec{i}\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 0 & -1 \\ -2 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} \right]$

$= 2 \cdot (\vec{i} + 2\vec{j} + 2\vec{k})$

$\pi \ldots \quad 1(x+2) + 2(y+1) + 2(z+1) = 0$

$x + 2y + 2z + 6 = 0$

$\vec{u} = \vec{n} = (1,2,2) \Rightarrow g \ldots \begin{cases} x = 3 + \Delta \\ y = -1 + 2\Delta \\ z = 1 + 2\Delta \end{cases}$

$3 + \Delta \cancel{-2} + 4\Delta \cancel{+2} + 4\Delta + 6 = 0$

$9\Delta = -9$

$\Delta = -1$

$P_1(2,-3,-1)$

$P' = (2P_1 - P) = \boxed{(1,-5,-3)}$

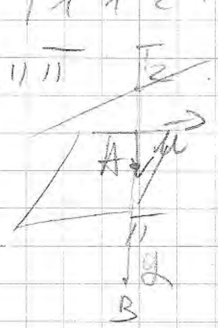
Zad 34

$p_1 \ldots \frac{x-1}{1} = \frac{y-3}{2} = \frac{z-2}{1}$ $p_2 \ldots \frac{x-0}{1} = \frac{y-2}{1} = \frac{z-11}{2}$

$p_2 \parallel \pi$ $p_1 \in \pi$ $\rightarrow \begin{cases} \vec{n} = \vec{s_1} \times \vec{s_2} \\ T_1 \in \pi \quad \underline{T_1(1,3,2)} \end{cases}$ $\pi \ldots 3x - y - z + 2 = 0$

$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 3\vec{i} - \vec{j} - \vec{k}$

$p_2 \parallel \pi$



$T_2(0,2,11)$

$g \ldots \begin{cases} x = 3\Delta \\ y = 2 - \Delta \\ z = 11 - \Delta \end{cases}$ $\cap \pi$

$\vec{n} = \vec{u}$

$A \ldots \quad 9\Delta - 2 + \Delta - 11 + \Delta + 2 = 0$

$11\Delta = 11$

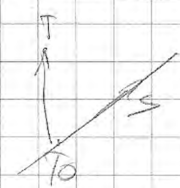
$\Delta = 1$

$A(3,1,10)$

$B \equiv (2A - T_2) = (6,0,9)$

$p_3 \ldots \boxed{\frac{x-6}{1} = \frac{y-0}{1} = \frac{z-9}{2}}$

Zad 55. $A(1,0,2)$ $T(1,2,3)$ $p \ldots \frac{x+1}{1} = \frac{y+2}{3} = \frac{z+3}{2}$



1. $T_0(-1,-2,-3)$ $\vec{s} = (1,3,2)$

$\overrightarrow{T_0T} = 2(1,2,3)$ $\vec{n} = \overrightarrow{T_0T} \times \vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

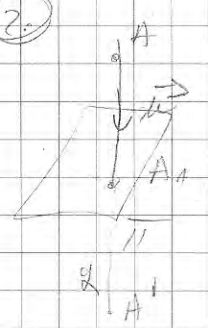
$= \vec{i}\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}$

$= 5\vec{i} - \vec{j} - \vec{k}$

$\pi \ldots 5(x-1) - (y-2) - (z-3) = 0$

$5x - y - z = 0$

2.



$\vec{w} = \vec{n} = (5,-1,-1)$

$q \ldots \begin{cases} x = 1 + 5\lambda \\ y = 0 - \lambda \\ z = 2 - \lambda \end{cases}$

$5 + 25\lambda + \lambda - 2 + \lambda = 0$

$27\lambda = -3$

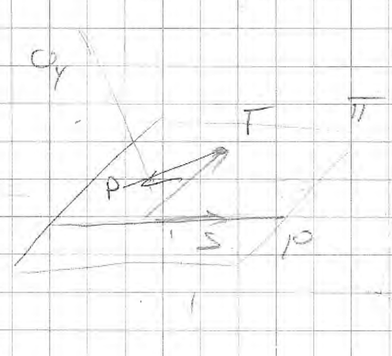
$\lambda = -\frac{1}{9}$

$A_1\left(\frac{4}{9}, \frac{1}{9}, \frac{14}{9}\right)$

3. $A' = (2A_1 - A) = \left(\frac{8}{9} - \frac{9}{9}, \frac{2}{9} - 0, 2\cdot\left(\frac{14}{9} - \frac{9}{9}\right)\right)$

$\boxed{A'\left(-\frac{1}{9}, \frac{2}{9}, \frac{20}{9}\right)}$

Zad 56. $T(2,0,1)$ $p \ldots \frac{x-2}{1} = \frac{y-1}{2} = \frac{z+1}{1}$ O_y $x=0$, $z=0$



1. $\pi \ldots$ $\vec{s} = (1,2,1)$ $T_0(2,1,-1)$

$\overrightarrow{TT_0} = (0,1,-2)$

$\vec{n} = \vec{s} \times \overrightarrow{TT_0} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 0 & 1 & -2 \end{vmatrix} = -5\vec{i} + 2\vec{j} + \vec{k}$

$\pi \ldots 5(x-2) - 2(y-0) - (z-1) = 0$

$5x - 2y - z - 9 = 0$

$0 - 2y - 0 - 9 = 0$

$y = -\frac{9}{2}$

$O_y \ldots x = 0$, $z = 0$

$P\left(0, -\frac{9}{2}, 0\right)$

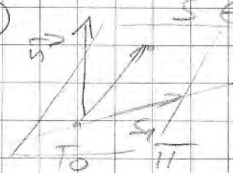
$\overrightarrow{PT} = \left(2, +\frac{9}{2}, 1\right)$

$= \frac{1}{2}(4,9,2)$

$p \ldots \boxed{\frac{x-2}{4} = \frac{y-0}{9} = \frac{z-1}{2}}$

Zad 53. $S(-1,1,-1)$ $p_1 \dots \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-0}{-1}$, $p_2 \dots \begin{cases} x = 4t \\ y = -5t-5 \\ z = 2t+3 \end{cases}$

(1.)



$S \in \pi$ $T_0(1,2,0)$ $\vec{s_1} = (2,3,-1)$

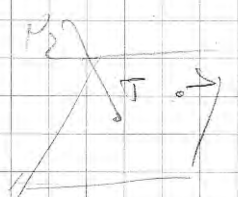
$p_1 \in \pi$ $\vec{ST_0} = (2,1,1)$ $\vec{n} = \vec{ST_0} \times \vec{s_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 2 & 1 & 1 \end{vmatrix}$

$= \vec{i}\begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 4\vec{i} - 4\vec{j} - 4\vec{k}$

$\pi \dots x+1 - y+1 - z - 1 = 0$

$x - y - z + 1 = 0$

(2.) $p_2 \cap \pi$



$4t + 5t + 5 - 2t - 3 + 1 = 0$

$7t = -3$ $T\left(-\frac{12}{7}, -\frac{20}{7}, \frac{15}{7}\right)$

$t = -\frac{3}{7}$

$\vec{ST} = \left(-\frac{5}{7}, -\frac{27}{7}, \frac{22}{7}\right) = \frac{1}{7}(-5, -27, 22)$

$p \dots \boxed{\frac{x+1}{-5} = \frac{y-1}{-27} = \frac{z+1}{22}}$

Zad 54. $S(3,-5,4)$ $p \dots \begin{cases} x = 2z-1 \\ y = -3z+4 \end{cases} \Rightarrow \begin{cases} x = 2\alpha - 1 \\ y = -3\alpha + 4 \\ z = \alpha \end{cases}$

(1.)

$q \dots \begin{cases} y = -4x+1 \\ z = x-2 \end{cases} \Rightarrow \begin{cases} x = \beta \\ y = -4\beta + 1 \\ z = \beta - 2 \end{cases}$

$T_0(-1,4,0)$ $\vec{s} = (2,-3,1)$

$\vec{T_0S} = (4,-9,4)$ $\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 1 \\ 4 & -9 & 4 \end{vmatrix} = -3\vec{i} - 4\vec{j} - 6\vec{k}$

$\pi \dots 3(x-3) + 4(y+5) + 6(z-4) = 0$

$3x + 4y + 6z - 13 = 0$

(2.)

$3\beta - 16\beta + 4 + 6\beta - 12 - 13 = 0$

$-7\beta = 21$ $A(-3, 13, -5)$ $\vec{AS} = (6, -18, 9)$

$\beta = -3$ $= 3(2, -6, 3)$

$p \dots \boxed{\frac{x-3}{2} = \frac{y+5}{-6} = \frac{z-4}{3}}$

Zad 59. $A(4,0,-1)$ $\quad p \ldots \frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{3}$ $\quad q \ldots \begin{cases} x = t \\ y = 6t+2 \\ z = 9t-1 \end{cases}$

1. $T_0(1,3,0)$ $\quad \overrightarrow{T_0A}(3,-3,-6) = 3(1,-1,-2)$ $\quad \vec{s} = (2,4,3)$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 3 \\ 1 & -1 & -2 \end{vmatrix} = -5\vec{i} + 7\vec{j} - 6\vec{k}$$

$$\pi \ldots 5(x-4) - 7(y-0) + 6(z+1) = 0$$

$$5x - 7y + 6z - 14 = 0$$

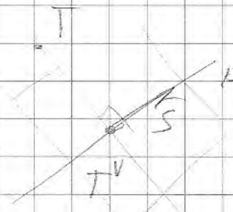
2. $q \cap p$ $\quad 5t - 42t - 14 + 54t - 6 - 14 = 0$

$$17t = 34 \quad t = 2 \qquad T(2,14,17)$$

$$\overrightarrow{AT} = (-2,14,18) = 2(-1,7,9)$$

$$r \ldots \boxed{\frac{x-4}{-1} = \frac{y-0}{7} = \frac{z+1}{9}}$$

Zad 60. $T(3,2,-1)$ $\quad p \ldots \begin{cases} x = t-1 \\ y = t-2 \\ z = t-1 \end{cases}$



$$\vec{n} = \vec{s} = (1,1,1)$$

$$\pi \ldots x-3+y-2+z+1=0$$

$$x+y+z-4=0$$

$$t-1+t-2+t-1-4=0$$

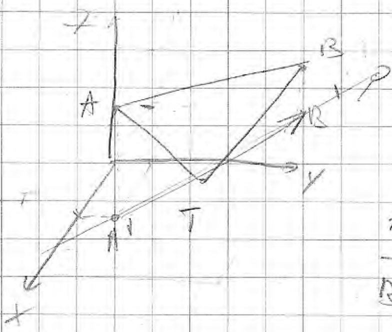
$$3t = 8$$

$$t = \frac{8}{3} \qquad T'\left(\frac{5}{3}, \frac{2}{3}, \frac{5}{3}\right)$$

$$\overrightarrow{T'T} = \left(\frac{4}{3}, \frac{4}{3}, -\frac{8}{3}\right) = \frac{4}{3}(1,1,-2)$$

$$p \ldots \boxed{\frac{x-3}{1} = \frac{y-2}{1} = \frac{z+1}{-2}}$$

Zad 61. $T \in xOy \Rightarrow (x, y, 0)$ $\quad A(2,1,3)$ $B(-2,4,1)$



očito je da će T ležati u p-[illegible].

$\overrightarrow{AB}$ na xOy $(z=0)$

$A'(2,1,0)$ $\quad B'(-2,4,0)$

$\overrightarrow{B'A'} = (4,-3,0)$

$$p \ldots \begin{cases} x = 2 + 4t \\ y = 1 - 3t \\ z = 0 \end{cases} \quad T \in p$$

$$d(A,T) + d(B,T) = f(t)$$

$$f(t) = \sqrt{(4t)^2 + (3t)^2 + 3^2} + \sqrt{(4t+4)^2 + (3t+3)^2 + 1}$$

$$= \sqrt{25t^2 + 9} + \sqrt{25t^2 + 50t + 26}$$

$$f'(t) = \frac{1}{\cancel{2}\sqrt{25t^2+9}} \cdot \cancel{2} \cancel{25}t + \frac{1}{\cancel{2}\sqrt{25t^2+50t+26}} \cdot \cancel{2} \cdot \cancel{25}(t+1) = 0 \quad / : 25 \quad / \cdot \sqrt{\cdot}\sqrt{\cdot}$$

$$t\sqrt{25t^2+50t+26} + (t+1)\cdot\sqrt{25t^2+9} = 0$$

$$t^2(25t^2+50t+26) = (t^2+2t+1)(25t^2+9)$$

$$\cancel{25t^4} + \cancel{50t^3} + 26t^2 = \cancel{25t^4} + 9t^2 + \cancel{50t^3} + 18t + 25t^2 + 9$$

$$8t^2 + 18t + 9 = 0$$

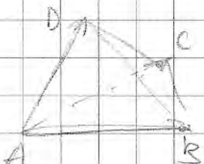
$$t_{1,2} = -\frac{3}{4}, -\frac{3}{2}$$

$T_1\left(-1, \frac{13}{4}, 0\right)$ $\quad$ $T_2\left(-4, \frac{11}{2}, 0\right)$

$f(t_1) = \sqrt{41}$ $\quad f(t_2) = 2\sqrt{29}$

$\approx 6{,}4$ min $\quad = 10{,}8$

Zad 62. $A(2,0,1)$ $B(1,1,-1)$ $C(0,3,1)$



$p \ldots \begin{cases} x = 2\lambda + 1 \\ y = \lambda - 1 \\ z = 2\lambda \end{cases}$ $V = 12$

$V = \frac{1}{6} \left[\vec{AB}, \vec{AC}, \vec{AD}\right]$ $\vec{AB} = (-1, 1, -2)$ $\vec{AC} = (-2, 3, 0)$

$\to \frac{1}{2}$ pola baze paralelepipeda $\vec{AD} = (2\lambda - 1, \lambda - 1, 2\lambda - 1)$

$\frac{1}{3}$ jer je šiljasto tijelo

$\left|\left[\vec{AB}, \vec{AC}, \vec{AD}\right]\right| = 72$ $\left|\begin{vmatrix} -1 & 1 & -2 \\ -2 & 3 & 0 \\ 2\lambda-1 & \lambda-1 & 2\lambda-1 \end{vmatrix}\right| = \left|\begin{vmatrix} 1 & 1 & -2 \\ -2 & 3 & 0 \\ 0 & \lambda-1 & 2\lambda-1 \end{vmatrix}\right|$

$= \left|\begin{vmatrix} 1 & 1 & -2 \\ 0 & 5 & -4 \\ 0 & \lambda-1 & 2\lambda-1 \end{vmatrix}\right| = 1 \left|\begin{vmatrix} 5 & -4 \\ \lambda-1 & 2\lambda-1 \end{vmatrix}\right|$ $|5(2\lambda-1) + 4(\lambda-1)|$

$= |10\lambda - 5 + 4\lambda - 4|$

$= |14\lambda - 9|$

$|14\lambda - 9| = 72$ $14\lambda = 81$, $14\lambda = -63$

$\lambda_1 = \frac{81}{14}$ $\boxed{D_1\left(\frac{88}{7}, \frac{67}{14}, \frac{81}{7}\right)}$

$\lambda_2 = -\frac{9}{2}$ $\boxed{D_2\left(-8, -\frac{11}{2}, -9\right)}$

Zad 63. $A(0,0,1)$ $B(1,0,0)$ $C(0,1,0)$ $\pi \ldots 2x + 3y + z - 1 = 0$

$\vec{n} = (2,3,1)$

$p_A \ldots \begin{cases} x = 2\alpha \\ y = 3\alpha \\ z = 1 + \alpha \end{cases}$ $p_B \ldots \begin{cases} x = 1 + 2\beta \\ y = 3\beta \\ z = \beta \end{cases}$ $p_C \ldots \begin{cases} x = 2\gamma \\ y = 1 + 3\gamma \\ z = \gamma \end{cases}$

$A' \ldots 4\alpha + 9\alpha + \alpha + 1 - 1 = 0$ $B' \ldots 2 + 4\beta + 9\beta + \beta - 1 = 0$ $C' \ldots 4\gamma + 3 + 9\gamma + \gamma - 1 = 0$

$\alpha = 0$ $14\beta = -1$ $14\gamma = -2$

$A'(0,0,1)$ $B'\left(\frac{6}{7}, -\frac{3}{14}, -\frac{1}{14}\right)$ $C'\left(-\frac{2}{7}, \frac{4}{7}, -\frac{1}{7}\right)$

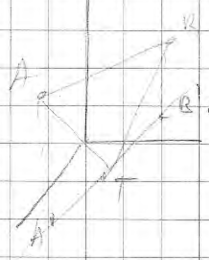
$P = \frac{1}{2}\left|\vec{A'B'} \times \vec{A'C'}\right|$ $\vec{A'B'} = \left(\frac{6}{7}, -\frac{3}{14}, -\frac{15}{14}\right)$ $\vec{A'C'} = \left(-\frac{2}{7}, \frac{4}{7}, -\frac{8}{7}\right)$

$= \frac{3}{14}(4, -1, -5)$ $= -\frac{2}{7}(1, -2, 4)$

$\frac{1}{2}\left|\vec{A'B'} \times \vec{A'C'}\right| = \frac{1}{2}\left|-\frac{6}{14 \cdot 7}\right| \left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & -5 \\ 1 & -2 & 4 \end{vmatrix}\right| = \frac{3}{14 \cdot 7} \left|-14\vec{i} - 21\vec{j} - 7\vec{k}\right|$

$= \boxed{\frac{3\sqrt{14}}{14}}$

Zad 64 $\quad T \in xOy \Rightarrow z=0 \quad A(-2,2,4) \quad B(3,-5,8)$



$A'(-2,2,0) \quad \overrightarrow{A'B'} = (5,-7,0)$

$B'(3,-5,0) \quad p \ldots \begin{cases} x = -2+5t \\ y = 2-7t \\ z = 0 \end{cases}$

$f(t) = d(A,T) + d(B,T)$

$= \sqrt{(5t)^2 + (7t)^2 + 4^2} + \sqrt{(5t-5)^2 + (7t-7)^2 + 8^2}$

$= \sqrt{25t^2 + 49t^2 + 16} + \sqrt{25t^2 + 49t^2 - 50t - 98t + 25 + 49 + 64}$

$= \sqrt{74t^2 + 16} + \sqrt{74t^2 - 148t + 138}$

$f'(t) = \frac{\cancel{2} \cdot \cancel{74} \cdot t}{2\sqrt{74t^2+16}} + \frac{\cancel{2} \cdot \cancel{74}(t-1)}{\cancel{2}\sqrt{74t^2-148t+138}} = 0 \quad / \sqrt{} \cdot \sqrt{}$

$t \cdot \sqrt{74t^2 - 148t + 138} = -(t-1) \cdot \sqrt{74t^2+16} \quad / ()^2$

$t^2(74t^2 - 148t + 138) = (t^2 - 2t + 1) \cdot (74t^2 + 16)$

$\cancel{74t^4} - \cancel{148t^3} + 138t^2 = \cancel{74t^4} + 16t^2 - \cancel{148t^3} - 32t + 74t^2 + 16$

$48t^2 + 32t - 16 = 0$

$3t^2 + 2t - 1 = 0$

$t_{1,2} = \frac{1}{3}, -1$

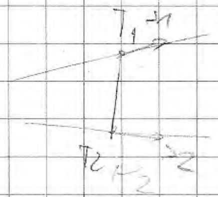
$\boxed{T_1\left(-\frac{1}{3}, -\frac{1}{3}, 0\right)} \quad T_2(-7, 9, 0)$

$f(t_1) = \sqrt{218}$ $\quad f(t_2) = 9\sqrt{10} = \sqrt{810}$

$\uparrow$
min

Zad 65

$p_1 \ldots \begin{cases} x = 2\alpha + 1 \\ y = \alpha \\ z = 3\alpha \end{cases}$ $\qquad$ $p_2 \ldots \begin{cases} x = 3\beta - 5 \\ y = 4\beta - 8 \\ z = \beta + 1 \end{cases}$



$\vec{s_1} = (2, 1, 3)$ $\qquad$ $\vec{s_2} = (3, 4, 1)$

$\overrightarrow{T_1T_2} = (3\beta - 2\alpha - 6,\ 4\beta - \alpha - 8,\ \beta - 3\alpha + 1)$

$\overrightarrow{T_1T_2} \perp \vec{s_1} \Rightarrow 6\beta - 4\alpha - 12 + 4\beta - \alpha - 8 + 3\beta - 9\alpha + 3 = 0$

$13\beta - 14\alpha - 17 = 0 \quad / \cdot 2$

$26\beta = 28\alpha + 34$

$\overrightarrow{T_1T_2} \perp \vec{s_2} \Rightarrow 9\beta - 6\alpha - 18 + 16\beta - 4\alpha - 32 + \beta - 3\alpha + 1 = 0$

$26\beta - 13\alpha - 49 = 0$

$28\alpha + 34 - 13\alpha - 49 = 0$

$15\alpha = 15 \qquad \alpha = 1 \qquad \boxed{T(3, 1, 3)}$

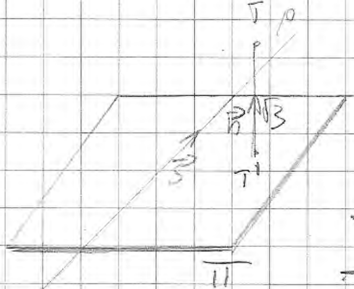
Zad 66

Zad 6.6

$p \ldots \begin{cases} x = 2t+1 \\ y = 3t-1 \\ z = t+1 \end{cases}$ $\quad \vec{s} = (2,3,1) \quad T_0(1,-1,1)$

$T(1,2,1) \qquad d(T,\pi) = \sqrt{3} \qquad \vec{n} = \overrightarrow{T'T} \Rightarrow |\vec{n}| = \Delta \cdot \sqrt{3}$

$\vec{n} = (A, B, C) \qquad \Delta \in \mathbb{R} \setminus \{0\}$



I $\quad \vec{n} \cdot \vec{s} = 0 \Rightarrow \boxed{2A + 3B + C = 0}$

bilo koji osim 0

II $\quad |\vec{n}| = \Delta \cdot \sqrt{3} \quad /()^2 \quad A^2 + B^2 + C^2 = 3 \cdot \Delta \quad / \Delta = 1$

$$\boxed{A^2 + B^2 + C^2 = 3}$$

III $\quad \pi \ldots \quad A(x-x_1) + B(y-y_1) + C(z-z_1) = 0$

$T_0(1,-1,1)$

$$A(x-1) + B(y+1) + C(z-1) = 0$$

$$Ax + By + Cz \underbrace{-(A - B + C)}_{D} = 0$$

$$\sqrt{3} = \frac{|A + 2B + C + D|}{|\vec{n}|}$$

$$\sqrt{3} = \frac{|\cancel{A} + 2B + \cancel{C} - \cancel{A} + B - \cancel{C}|}{\sqrt{3}}$$

$3 = |3B| \Rightarrow |B| = 1 \Rightarrow$ biramo jedan B

npr. $\boxed{B = -1} \xrightarrow{\text{I, II}}$ $\quad 2A - 3 + C = 0 \Rightarrow C = 3 - 2A$

$$A^2 + 1 + C^2 = 3$$

$$A^2 + 1 + 4A^2 - 12A + 9 - 3 = 0$$

$$5A^2 - 12A + 7 = 0$$

$$\boxed{A = \frac{7}{5}, \quad 1}$$

$$\boxed{C = \frac{1}{5}, \quad 1}$$

$\vec{n}_1 = (7, -5, 1)$

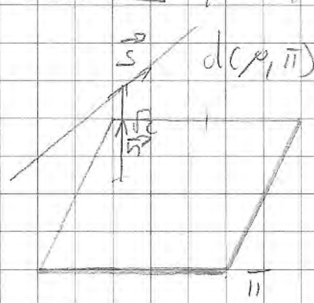
$\vec{n}_2 = (1, -1, 1)$

$\Downarrow$

$$\boxed{\begin{array}{l} \pi_1 \ldots \quad 7x - 5y + z - 13 = 0 \\ \pi_2 \quad x - y + z - 3 = 0 \end{array}}$$

i da smo ostavili Δ kroz cijeli račun na kraju bi odabrali pojedini (5 i 1)

Zad 67. $p \ldots \begin{cases} x = t+1 \\ y = t+2 \\ z = t-5 \end{cases}$ $\vec{s} = (1,1,1)$ $T_0(1,2,-5)$



$d(p,\pi) = \sqrt{2}$ $T(2,1,-2)$ $\vec{n} = (A,B,C)$

I $\vec{n} \perp \vec{s} \Rightarrow \vec{n}\cdot\vec{s} = 0 \Rightarrow \boxed{A+B+C=0}$

II $|\vec{n}| = \Delta \cdot \sqrt{2}$, $\Delta \in \mathbb{R}\setminus\{0\} \Rightarrow |\vec{n}| = \sqrt{2}$

$\boxed{A^2+B^2+C^2=2}$

III $T \in \pi \Rightarrow A(x-2)+B(y-1)+C(z+2)=0$

$Ax+By+Cz \underbrace{-(2A+B-2C)}_{D}=0$

$d(p,\pi) = d(T_0,\pi) = \sqrt{2} = \dfrac{|A+2B-5C+D|}{|\vec{n}|}$

$2 = |A+2B-5C-2A-B+2C|$

$2 = |-A+B-3C| \Rightarrow |A-B+3C| = 2$

biramo jedno

$\boxed{A-B+3C=2}$

I $\overset{1.}{\Rightarrow}$ $A = -B-C$ 2. 3. $\boxed{C = B+1}$ 4. 6. $\Rightarrow$ II

5. $\boxed{A = -2B-1}$

7. $(2B+1)^2+(B+1)^2+B^2=2$

$6B^2+6B+\cancel{2}=\cancel{2}$

$B(B+1)=0$

8.

$$\begin{array}{cc} & \vec{n}_1 \\ A = & \\ B = & \\ C = & \end{array}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \begin{array}{c} \vec{n}_2 \end{array}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

9.

10. π_1 $1(x-2)+0(y-1)-1(z+2)=0$

$\boxed{x-z-4=0}$

π_2 $1(x-2)-1(y-1)+0(z+2)=0$

$\boxed{x-y-1=0}$

<u>Zad 68</u> $p_1 \ldots \begin{cases} x = 10\alpha+1 \\ x = -7\alpha-5 \\ z = 3\alpha+3 \end{cases}$ $\quad p_2 \ldots \begin{cases} x = 10\beta-2 \\ y = -7\beta-4 \\ z = 3\beta-1 \end{cases}$ $\quad \vec{s} = (10,-7,3)$

$T_1(1,-5,3)$

$$d^2(T_1T_2) = (10\beta-3)^2 + (7\beta-1)^2 + (3\beta-4)^2$$
$$= \underbrace{113\beta^2 - 88\beta + C}_{g}$$
$$g' = 226\beta - 88 = 0 \Rightarrow \beta = \frac{44}{113} \Rightarrow T_2\left(\frac{214}{113}, \frac{-500}{113}, \frac{19}{113}\right)$$

$$\overrightarrow{T_1T_2} = \ldots \frac{1}{113}(101, 25, -320)$$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 101 & 25 & -320 \\ 10 & -7 & 3 \end{vmatrix} = \ldots = -565\vec{i} - 3503\vec{j} - 452\vec{k}$$
$$= -113(5\vec{i} + 31\vec{j} + 4\vec{k})$$

$$d(T_1,\pi) = 10 = \frac{|5 - 5\cdot 31 + 4\cdot 3 + D|}{\sqrt{5^2+31^2+4^2}} \Rightarrow 10\sqrt{1002} = |D-138|$$

$$\boxed{\begin{aligned} \pi_1 \ldots\ & 5x+31y+4z+138+10\sqrt{1002} \\ \pi_2 \ldots\ & 5x+31y+4z+138-10\sqrt{1002} \end{aligned}}$$

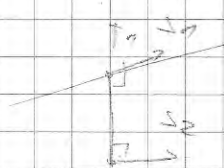
Zad 69. $T(2,1,3)$ $\quad p \ldots \begin{cases} x = 2t-1 \\ y = t+2 \\ z = -2t-2 \end{cases}$

$$d(T,p)^2 = (2t-3)^2 + (t+1)^2 + (2t+5)^2$$
$$= 4t^2 + t^2 + 4t^2 - 12t + 2t + 20t + 9 + 1 + 25$$
$$= \underbrace{9t^2 + 10t + 35}_{g}$$

$$g' = 18t + 10 = 0 \qquad t = -\frac{5}{9}$$
$$g'' = 18 > \quad \text{min}$$

$$d(T,p)^2 = \cancel{9}\cdot\frac{25}{\cancel{9}\cdot 9} - \frac{10\cdot 5}{9} + 35 = \frac{1}{9}(25 - 50 + 35\cdot 9)$$
$$= \frac{1}{9}(290)$$
$$\boxed{d = \frac{1}{3}\sqrt{290}}$$

Zad 70. $\quad p_1 \ldots \begin{cases} x = 2\alpha+3 \\ y = -3\alpha+10 \\ z = 4\alpha+3 \end{cases} \qquad p_2 \ldots \begin{cases} x = 3\beta+1 \\ y = -2\beta+1 \\ z = 3\beta+1 \end{cases}$



$$\overrightarrow{T_1T_2} = (3\beta - 2\alpha - 2,\ -2\beta + 3\alpha - 9,\ 3\beta - 4\alpha - 2)$$

1) $\overrightarrow{T_1T_2} \perp \vec{s_1}$

$$6\beta - 4\alpha - 4 + 6\beta - 9\alpha + 27 + 12\beta - 16\alpha - 8 = 0$$
$$24\beta - 29\alpha + 15 = 0 \qquad 3$$

2) $\overrightarrow{T_1T_2} \perp \vec{s_2}$

$$9\beta - 6\alpha - 6 + 4\beta - 6\alpha + 18 + 9\beta - 12\alpha - 6 = 0$$
$$22\beta - 24\alpha + 6 = 0 \quad /:2$$
$$11\beta - 12\alpha + 3 = 0$$

$$24\beta - 29\alpha + 15 = 0 \quad /\cdot 11$$
$$11\beta - 12\alpha + 3 = 0 \quad /\cdot(-24) \quad +$$
$$\alpha(-29\cdot 11 + 24\cdot 12) + 15\cdot 11 - 3\cdot 24 = 0$$
$$-31\alpha = -93$$
$$\boxed{\alpha = 3} \qquad \boxed{\beta = 3}$$

$T_1(9, 1, 15) \qquad T_2(10, -5, 10)$

$d(A,B) = \sqrt{1^2 + 6^2 + 5^2} = \boxed{\sqrt{62}}$

Zad 71 $\quad p \ldots \begin{cases} x + y + 2z - 9 = 0 \\ 2x + y + 3z - 13 = 0 \end{cases}$ $\quad \underline{x = t}$ $\quad / \cdot -1$ $\quad +$ $\quad -x + 0 - z + 4 = 0$

$\underline{z = -t + 4}$

$2t + y - 3t + 12 - 13 = 0$

$\underline{y = t + 1}$

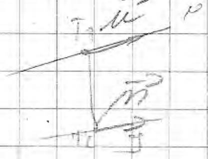
$O(0,0,0)$

$d(O,p)^2 = t^2 + (t+1)^2 + (t-4)^2 = \underbrace{3t^2 - 6t + 17}_{g}$

$g' = 6t - 6 = 0 \quad t = 1$

$g'' = 6 > \quad \min$ $\quad \boxed{A(1,2,3)}$

Zad 72 $\quad p \ldots \begin{cases} x = \Delta - 4 \\ y = -\Delta - 2 \\ z = -\Delta + 10 \end{cases}$ $\quad q \ldots \begin{cases} x = t + 3 \\ y = -4t + 3 \\ z = 2t - 3 \end{cases}$



$\overrightarrow{T_2T_1} = (\Delta - t - 7, -\Delta + 4t - 5, -\Delta - 2t + 13)$

$\vec{u} = (1,-1,-1) \quad \vec{n} = (1,-4,2)$

$\overrightarrow{T_2T_1} \cdot \vec{u} = 0 = \Delta - t - 7 + \Delta - 4t + 5 + \Delta + 2t - 13$

$3\Delta - 3t - 15 = 0$

$\Delta - t - 5 = 0 \quad t = \Delta - 5$

$\overrightarrow{T_2T_1} \cdot \vec{n} = 0 = \Delta - t - 7 + 4\Delta - 16t + 20 - 2\Delta - 4t + 26$

$3\Delta - 21t + 39 = 0$

$3\Delta - 21\Delta + 21 \cdot 5 + 39 = 0$

$-18\Delta = -144$

$\boxed{\Delta = 8}$ $\quad \boxed{A(4,-10,2)}$

Zad 73 $\quad p_1 \ldots \begin{cases} 2x + 2y - z - 10 = 0 \\ x - y - z - 22 = 0 \end{cases}$ $\quad x = 0$ $\quad / \cdot -1$ $\quad +$ $\quad 3y + 12 = 0$

$y = -4$

$4 - z - 22 = 0 \quad z = -18 \quad T(0,-4,-18)$

$p_2 \ldots \begin{cases} x = 3t - 7 \\ y = -t + 5 \\ z = 4t + 9 \end{cases}$ $\quad d(T,p_2)^2 = (3t-7)^2 + (t-9)^2 + (4t+27)^2$

$= \underbrace{26t^2 + 156t + 859}_{g}$

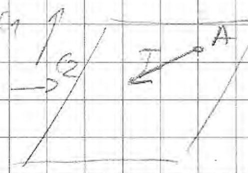
$g' = 52t + 156 = 0$

$\boxed{t = -3}$

$d^2 = 625$

$\boxed{d = 25}$

Zad 74. $T(x_1, y_1, z_1)$ $\quad \vec{c_1}=(l_1, m_1, n_1)$ $\quad c_2(l_2, m_2, n_2)$



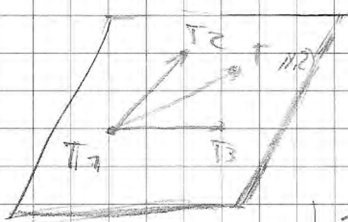
$A(x,y,z)$ $\quad \vec{AT}$, $\vec{c_1}$ i $\vec{c_2}$ moraju biti komplanarni

tj. $[\vec{AT}, \vec{c_1}, \vec{c_2}]=0$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

Q.E.D.

Zad 75 $T_1(x_1, y_1, z_1)$, $T_2(x_2, y_2, z_2)$, $T_3(x_3, y_3, z_3)$



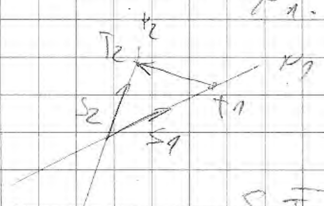
$\vec{T_1T}$, $\vec{T_1T_2}$ i $\vec{T_1T_3}$ moraju biti komplanarni

tj. $[\vec{T_1T}, \vec{T_1T_2}, \vec{T_1T_3}]=0$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

Q.E.D

Zad 76.



$p_1 \ldots \frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ $\quad p_2 \ldots \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$

za $\forall T_1 \in p_1$ i $\forall T_2 \in p_2$

$\vec{T_1T_2}$, $\vec{s_1}$ i $\vec{s_2}$ moraju biti komplanarni

$[\vec{T_1T_2}, \vec{s_1}, \vec{s_2}]=0$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

Q.E.D.

$\overline{n} \ldots$ $\quad l_1l_2(x-x_1)+m_1m_2(y-y_1)+n_1n_2(z-z_1)=0$

$l_1l_2(x-x_2)+m_1m_2(y-y_2)+n_1n_2(z-z_2)=0$