9. Primjene diferencijalnog računa

9.1. Aproksimacija Taylorovom formulom

2.MI-2009-6.) a) Odredi aproksimaciju broja $\sqrt{35}$ koristeći se diferencijalom.

$$\sqrt{35} = ?$$

$\sqrt{35} = 5.91608\ldots$

PODSJETIMO SE:

$$\boxed{df(x) = f'(x)\cdot dx}$$

$f(x+h) - f(x) \longrightarrow$ PRIRAST FUNKCIJE

za $h \to 0$ $\quad f(x+h) - f(x) \approx df(x)$

$$f(x+h) \approx f(x) + f'(x)\cdot h$$

$$\boxed{f(x+h) \approx f(x) + f'(x)\cdot h}$$

x..... najbliži br. kojem možemo izvaditi $\sqrt{\ }$

$x = 36$

$h = -1 \longrightarrow$ razlika do "pravog" broja $\quad 36 \to 35 \longrightarrow -1 = h$

$f(x) = \sqrt{x} \longrightarrow f'(x) = \dfrac{1}{2\sqrt{x}}$

$\sqrt{35} = f(36-1) \approx f(36) + f'(36)\cdot(-1) \approx 6 - \underbrace{\dfrac{1}{12}}_{\text{bez kalk.}} \approx 5{,}9166 \to$ točno na 3 decimale

$$f(x) \approx f(c) + f'(c)\cdot(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

$\longrightarrow$ TAYLOROV POLINOM n-tog STUPNJA oko TOČKE $x=c$.

$\sqrt{35}$, aproksimiraj Taylorovim polinomom $\underline{2.}$ stupnja!

$\underline{c=36}$

$$f'(x) = \frac{1}{2\sqrt{x}} \longrightarrow f''(x) = \left(\frac{1}{2}x^{-\frac{1}{2}}\right)' = -\frac{1}{4}x^{-\frac{3}{2}}$$

$$\sqrt{35} \approx 6 + \frac{1}{12}\cdot(-1) + \frac{-\frac{1}{4\cdot 6^3}}{2!}\cdot(-1)^2 \longrightarrow \text{do tuda u ispitu}$$

$$\approx 5{,}91608 \longrightarrow \text{točno na 5 decimala}$$

TM „TAYLOROV TEOREM"

Neka je zadana funkcija $f: I \to \mathbb{R}$ koja u točki $c \in I$ ima sve derivacije do reda $n+1$. Tada možemo funkciju zapisati kao $\boxed{f(x) = T_n(x) + R_n(x)}$ gdje je

$$\boxed{T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k} \longrightarrow \text{TAYLOROV polinom } n\text{-tog stup.}$$

$$\boxed{R_n(x) = \frac{f^{(n+1)}(x_1)}{(n+1)!}(x-c)^{n+1}}$$

$x_1 \in (x,c)$ ili (c,x)

$\longrightarrow$ OSTATAK u LANGRANGOVOM OBLIKU

$c \dots$ točka oko koje se razvija

2.MI - 2009 - 7.) Napiši $f(x)=e^x$ u obliku Taylorovog polinoma oko točke $x=0$, pritom dokaži da je $\lim_{n\to\infty} R_n(x)=0$.

$\boxed{c=0}$ $f(x)=e^x \longrightarrow f(0)=1$

$f'(x)=e^x \longrightarrow f'(0)=1$

$f''(x)=e^x \longrightarrow f''(0)=1$

$\vdots$

$f^{(n)}(x)=e^x \quad f^{(n)}(0)=1$

$$T_n(x)=\sum_{k=0}^{n}\frac{x^k}{k!} \to c=0$$

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dokaži: $\lim_{n\to\infty} R_n(x)=0$

$$R_n(x)=\frac{e^{x_1}}{(n+1)!}\cdot x^{n+1}$$ ili $x_1\in(x,0)$ ili $x_1\in(0,x)$

$$\lim_{n\to\infty}\frac{e^{x_1}\cdot x^{n+1}}{(n+1)!}=0$$ jer faktorijele brže teže u ∞

def. Taylorov polinom oko točke $x=0$ se zove MACLAURINOV POLINOM.

Zad.) a) $f(x)=\sin x$, razvij oko $x=0$.

1 $f(x)=\sin x$

$f'(x)=\cos x$

$f''(x)=-\sin x$

$f'''(x)=-\cos x$

4 $f^{IV}(x)=\sin x$

$\vdots$

$f^{4k}(x)=\sin x \longrightarrow$ svaka 4. je $\sin x \longrightarrow f^{4k}(0)=0$

$f^{4k+1}(x)=\cos x \longrightarrow f^{4k+1}(0)=1$

$f^{4k+2}(x)=-\sin x \longrightarrow f^{4k+2}(0)=0$

$f^{4k+3}(x)=-\cos x \to f^{4k+3}(0)=-1$ ne treba mat. ind.

$$T_n(x) = \sum_{k=0}^{n} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$$ → kako se mijenjaju predznaci 1 ili −1

samo neparni članovi "prežive" → $f^{(2k+1)} = 1$ ili -1, ostali 0 !!

$$T_n(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$k=0$, $k=1$, $k=2$, $k=3$

$$R_{2n+1}(x) = \frac{f^{(2n+2)}(x_1)}{(2n+2)!} \cdot x^{2n+2}$$

sazna pot. kada uvrstiš $k=n$

razvoj $\cos x$

$$\cos x = \sum_{k=0}^{n} \frac{(-1)^k}{(2k)!} + R_{2n}(x)$$ D.Z.

koji stupanj T_n treba uzeti da bi $\sin x \in \langle 0, \frac{\pi}{4} \rangle$ izračunali s točnošću 10^{-3} ?

c, $x = \frac{\pi}{4}$

$$|R_{2n+1}(x)| \leq 10^{-3}$$ → jel greška može biti $\pm$

$$\left| \frac{f^{(2n+2)}(x_1)}{(2n+2)!} \cdot \left(\frac{\pi}{4}\right)^{2n+2} \right| \leq \frac{\left(\frac{\pi}{4}\right)^{2n+2}}{(2n+2)!} \leq 10^{-3}$$

< 1

uvrštavati : $n=0$ → nije $\leq 10^{-3}$

$n=1$ → nije $\leq 10^{-3}$

$\boxed{n=2}$ → JE $\leq 10^{-3}$ ✓

dakle : $\boxed{T_n(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}}$ nam daje $\sin x$ aproksimativno s točnošću 10^{-3}

9.DZ−10.) Prikaži $f(x)=x^3+2x^2-x+1$ kao polinom u varijabli $(x-1)$.

$\boxed{c=1}$ $\qquad$ $\boxed{c=1}$

$$f'(x)=3x^2+4x-1 \rightarrow f'(1)=6$$
$$f''(x)=6x+4 \rightarrow f''(1)=10$$
$$f'''(x)=6 \rightarrow f'''(1)=6$$

$$f(x)=3+\frac{6}{1!}(x-1)+\frac{10}{2!}(x-1)^2+\frac{6}{3!}(x-1)^3$$

$$\boxed{f(x)=3+6(x-1)+5(x-1)^2+(x-1)^3}$$

9.ZZV−2.) $f(x)=\ln\frac{1+x}{1-x}$ napiši u formi $T_6(x)+R_6(x)$ oko $c=0$.

!! $f(x)=\ln(1+x)-\ln(1-x)$

$$f'(x)=\frac{1}{1+x}-\frac{1}{1-x}\cdot(-1)=\frac{1}{1+x}+\frac{1}{1-x} \rightarrow 2$$

$$f''(x)=\frac{-1}{(x+1)^2}+\frac{1}{(1-x)^2} \rightarrow 0$$

$$f'''(x)=\frac{2}{(x+1)^3}+\frac{2}{(1-x)^3} \rightarrow 4$$

$$f^{IV}(x)=\frac{-6}{(1+x)^4}+\frac{6}{(1-x)^4} \rightarrow 0$$

$$f^{V}(x)=\frac{24}{(1+x)^5}+\frac{24}{(1-x)^5} \rightarrow 48$$

$$f^{VI}(x)=\frac{-120}{(1+x)^6}+\frac{120}{(1-x)^6} \rightarrow 0$$

$$\ln\frac{1+x}{1-x}=2x+\frac{4}{3!}x^3+\frac{48}{5!}x^5+R_6(x)$$

$$=2x+\frac{2}{3}x^3+\frac{2}{5}x^5+R_6(x)$$

još jednom derivirati $f^{VI}(x)$

$$R_6(x)=\left(\frac{720}{(1+x_1)^7}+\frac{720}{(1-x_1)^7}\right)\frac{x^7}{7!}$$

9.2. L'HOSPITALOVO PRAVILO

„L'Hospitalov teorem"

Neka su f i g diferencijabilne na (a,b), $g'(x) \neq 0$. Ako za $x_0 \in (a,b)$ vrijedi

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x) = 0 \quad \text{ili} \quad \lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x) = \pm\infty \quad \text{i ako}$$

postoji $\lim_{x\to x_0} \frac{f'(x)}{g'(x)}$, tada je

$$\boxed{\lim_{x\to x_0} \frac{f'(x)}{g'(x)} = \lim_{x\to x_0} \frac{f(x)}{g(x)}}.$$

NAPOMENA: Teorem vrijedi i kada $x \to \infty$!

—6.)
$$\lim_{x\to\frac{\pi}{4}} \frac{\ln tgx}{\cos 2x} = \left(\frac{0}{0}\right) \underset{\text{L'Hospital}}{=} \lim_{x\to\frac{\pi}{4}} \frac{\frac{1}{tgx}\cdot\frac{1}{\cos^2 x}}{-\sin 2x\cdot 2} = \frac{1\cdot\frac{1}{\frac{2}{4}}}{-2} = -1$$

-09-P.)
$$\lim_{x\to\frac{\pi}{2}} \frac{tg5x}{tg3x} = \text{PAZI! kada } x\to 0 \text{ onda možemo } \frac{\sim 5x}{\sim 3x} \ !!$$

$$= \left(\frac{\infty}{\infty}\right) \overset{L'H}{=} \lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\cos^2 5x}\cdot 5}{\frac{1}{\cos^2 3x}\cdot 3} = \lim_{x\to\frac{\pi}{2}} \frac{\cos^2 3x}{\cos^2 5x}\cdot\frac{5}{3} = \frac{5}{3}\cdot\left(\frac{0}{0}\right) \text{ JOŠ JEDNOM L'H !}$$

$$= \frac{\cancel{5}}{\cancel{3}}\cdot\lim_{x\to\frac{\pi}{2}} \frac{\cancel{2}\cos 3x\cdot(-\sin 3x)\cdot\cancel{3}}{\cancel{2}\cos 5x\cdot(-\sin 5x)\cdot\cancel{5}} = \left(\frac{0}{0}\right) =$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos 3x(\sin 3x)}{\cos 5x(\sin 5x)} = \lim_{x\to\frac{\pi}{2}} \frac{-\sin 3x\cdot 3\cdot(-\sin 3x) + \cos 3x\cdot(-\cos 3x)\cdot 3}{-\sin 5x\cdot 5\,(-\sin 5x) + \cos 5x\cdot(-\cos 5x)\cdot 5} = \frac{3}{5}$$

V-4)
$$\lim_{x\to\infty} \frac{\ln^n x}{x} = \left(\frac{\infty}{\infty}\right) \overset{L'H}{=} \lim_{x\to\infty} \frac{n\ln^{n-1}x\cdot\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{n\ln^{n-1}x}{x} = \lim_{x\to\infty} \frac{n(n-1)\ln^{n-2}x}{x}$$

$$= \dots = \lim_{x\to\infty} \frac{\sim n!}{x} = 0$$

2.MI-09-8.) $\lim\limits_{x\to\infty} x\cdot \operatorname{arcctg}(2x) = (\underline{\underline{\infty\cdot 0}})$!! <u>NE MOŽE ODMAH L'H.</u> !

$$\lim_{x\to\infty}\frac{\operatorname{arcctg}(2x)}{\frac{1}{x}} = \left(\frac{0}{0}\right) \overset{L'H}{=} \text{DA}! = \lim_{x\to\infty}\frac{\frac{-1}{1+4x^2}\cdot 2}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to\infty}\frac{2x^2}{1+4x^2} = \frac{\sim 2x^2}{\sim 4x^2} = \frac{1}{2}\ //$$

9.DZ-15.) $\lim\limits_{x\to-\infty}\left(x e^{-\frac{1}{x^2}} - x\right) = (-\infty+\infty) =$ <u>NE MOŽE ODMAH L'H</u>

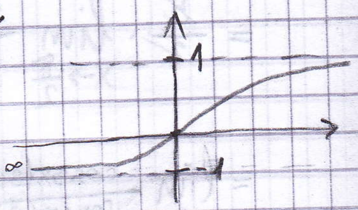
NAMJESTITI NA RAZLOMAK !

$$= \lim_{x\to-\infty}\left(x\cdot\left(e^{-\frac{1}{x^2}}-1\right)\right) = \lim_{x\to-\infty}\frac{e^{-\frac{1}{x^2}}-1}{\frac{1}{x}} = \left(\frac{0}{0}\right) \overset{L'H.}{=} \lim_{x\to-\infty}\frac{e^{-\frac{1}{x^2}}\cdot\left(-\frac{-2}{x^3}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to-\infty}\frac{2e^{-\frac{1}{x^2}}}{-x} = \frac{2}{\infty} = 0\ //$$

9.DZ-14.) $\lim\limits_{x\to-\infty}\frac{\ln \operatorname{ch}(x+3)}{x} = \left(\frac{\infty}{-\infty}\right) \overset{L'H.}{=} \lim\limits_{x\to-\infty}\frac{\frac{1}{\operatorname{ch}(x+3)}\cdot \operatorname{sh}(x+3)\cdot 1}{1} =$

$$= \lim_{x\to-\infty}\frac{\operatorname{sh}(x+3)}{\operatorname{ch}(x+3)} = \lim_{x\to-\infty}\operatorname{th}(x+3) = \operatorname{th}(-\infty) = -1\ //$$



$$\left(\frac{\infty}{\infty}\right) \overset{L'H.}{=} \lim_{x\to-\infty}\frac{\operatorname{ch}(x+3)}{\operatorname{sh}(x+3)} = \left(\frac{\infty}{\infty}\right) \overset{L'H}{=} \lim_{x\to-\infty}\frac{\operatorname{sh}(x+3)}{\operatorname{ch}(x+3)}$$

L'H. NE MORA UVIJEK DOVESTI DO RJEŠENJA !!

2.MI-10.-7.) U ovisnosti o parametru izračunaj:

$$\lim_{x\to 0^+}\frac{x\cdot\cos x + a\sin x}{x^3} = \left(\frac{0}{0}\right) \overset{L'H.}{=} \lim_{x\to 0^+}\frac{\cos x - x\sin x + a\cos x}{3x^2} =$$

$\cos x(1+a)$

(I) za $a=-1$:

$$\lim_{x\to 0^+}\frac{-x\sin x}{3x^2} = -\frac{1}{3}\ //$$

za $a > -1$:

$$\lim_{x\to 0^+} \frac{\cos x\overbrace{(a+1)}^{>0}}{3x^2} = +\infty \quad //$$

za $a < -1$:

$$-\infty \quad //$$

-11.) $\lim\limits_{x\to\infty} \frac{x+\sin x}{x+\cos x}$ može li se upotrijebiti L'Hospital ?

→ $\sin\infty$ i $\cos\infty$ NE POSTOJE ⟹ LIMES NE POSTOJI

→ L'Hospital: „Ako postoji limes..." → NE MOŽEMO L'H.

~ svejedno izračunajte limes:

$$\lim_{x\to\infty} \frac{x+\sin x}{x+\cos x} \cdot \frac{x}{x} = \lim_{x\to\infty} \frac{1+\overbrace{\frac{\sin x}{x}}^{\to 0}}{1+\underbrace{\frac{\cos x}{x}}_{\to 0}} = 1 \quad //$$

-16.)

$$\lim_{x\to\infty} x^{\frac{1}{x}} = (\infty^0) =$$

$$= \lim_{x\to\infty} e^{\frac{1}{x}\ln x} = e^{\lim\limits_{x\to\infty}\frac{1}{x}\ln x} \overset{L'H}{=} e^{\lim\limits_{x\to\infty}\frac{\frac{1}{x}}{1}} = e^0 = 1$$

-06-10.)

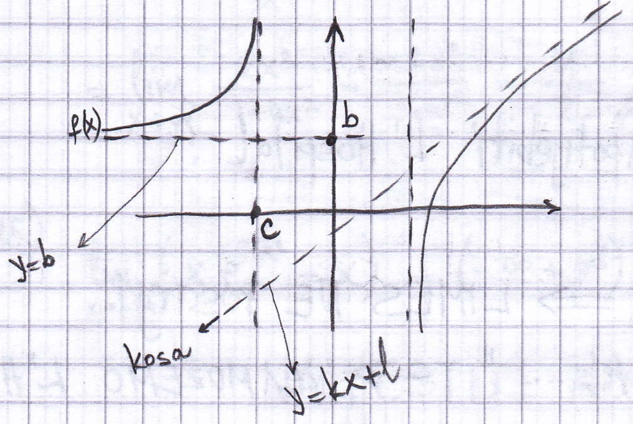
$$\lim_{x\to\pi} \cos(\pi-x)^{\frac{1}{(\pi-x)^2}} = (1^\infty) =$$

$$= e^{\lim\limits_{x\to\pi}\frac{1}{(\pi-x)^2}\ln\cos(\pi-x)} \overset{L'H}{=} = e^{\lim\limits_{x\to\pi}\frac{\frac{1}{\cos(\pi-x)}\cdot(-\sin(\pi-x))\cdot\cancel{(-1)}}{(2\pi-2x)\cdot\cancel{(-1)}}} =$$

$$= e^{\lim\limits_{x\to\pi}\frac{-\overbrace{tg(\pi-x)}^{\to 0\ \sim \pi-x}}{2\pi-2x}} = \frac{-\cancel{(\pi-x)}}{2\cancel{(\pi-x)}} = e^{-\frac{1}{2}} \quad //$$

9.3 ASIMPTOTE

→ pravac čija udaljenost od funkcije teži nuli



def. Pravac $x=c$ je VERTIKALNA ASIMPTOTA ako vrijedi $\lim_{x\to c^-} f(x) = \pm\infty$ ili $\lim_{x\to c^+} f(x) = \pm\infty$.

def. Pravac $y=b$ je HORIZONTALNA ASIMPTOTA ako vrijedi $\lim_{x\to\pm\infty} f(x) = b$.

Za $x\to\infty$ kažemo DESNA HA

Za $x\to-\infty$ kažemo LIJEVA HA

def.

Pravac $y=kl+l$ je KOSA ASIMPTOTA ako vrijedi:

$$\lim_{x\to\pm\infty} (f(x) - kx - l) = 0 \quad //$$

$$\boxed{k = \lim_{x\to\pm\infty} \frac{f(x)}{x}}$$

$$\boxed{l = \lim_{x\to\pm\infty} (f(x) - kx)}$$

2c)

$$y=\frac{x^3+2x^2+1}{2x^2-x} \Rightarrow 2x^2-x\neq 0$$

$$x(2x-1)\neq 0$$

$$x\neq 0\ ,\ x\neq \frac{1}{2}$$

$$D(f)=\mathbb{R}\setminus\left\{0,\frac{1}{2}\right\}$$

VERTIKALNE ASIMP.:

TREBA ODREDITI LIMES I SLIJEVA I SDESNA

$$\lim_{x\to 0^+}\frac{x^3+2x^2+1}{x(2x-1)}=\frac{1}{0^-}=-\infty$$

$$\lim_{x\to 0^-}\frac{x^3+2x^2+1}{x(2x-1)}=\frac{1}{0^+}=+\infty$$

$\boxed{x=0}$ JEST VERT. AS. po def.

$$\lim_{x\to \frac{1}{2}^+}\frac{x^3+2x^2+1}{x(2x-1)}=\frac{\frac{1}{8}+\frac{1}{2}+1}{\frac{1}{2}\cdot 0^+}=+\infty$$

$$\lim_{x\to \frac{1}{2}^-}\frac{x^3+2x^2+1}{x(2x-1)}=\frac{\frac{1}{8}+\frac{1}{2}+1}{\frac{1}{2}\cdot 0^-}=-\infty$$

$\boxed{x=\frac{1}{2}}$ JEST VER. AS.

HORIZONTALNE ASIM.:

$$\lim_{x\to\pm\infty}f(x)=\infty \longrightarrow \text{NEMA HA.}$$

KOSA ASIM.:

$$k=\lim_{x\to\pm\infty}\frac{x^3+2x^2+1}{2x^3-x^2}\ \frac{\sim x^3}{\sim 2x^3}=\frac{1}{2}$$

→ i lijeva i desna, $\pm\infty$ NE UTJEČE
INAČE PROVJERITI OBA SLUČAJA!

$$l=\lim_{x\to\pm\infty}\left(\frac{x^3+2x^2+1}{2x^2-x}-\frac{1}{2}x\right)=$$

$$=\lim_{x\to\pm\infty}\frac{2x^3+4x^2+2-2x^3+x^2}{4x^2-2x}=\frac{5}{4}$$

→ opet ne ovisi o $\pm\infty$

→ $\boxed{y=\frac{1}{2}x+\frac{5}{4}}$ JE KOSA ASIMP.

2.MI-08'-9.) $y = e^{\frac{1}{x^2-4x}}$ $\longrightarrow$ $\left.\begin{array}{l} x^2-4x \neq 0 \\ x(x-4) \neq 0 \end{array}\right\} D(f) = \mathbb{R} \setminus \{0,4\}$

1° VA:

$\lim\limits_{x\to 0^+} e^{\frac{1}{x(x-4)}} = e^{\frac{1}{0^-}} = e^{-\infty} = 0 \longrightarrow$ NIJE VA s desne !

$\lim\limits_{x\to 0^-} e^{\frac{1}{x(x-4)}} = e^{\frac{1}{0^+}} = e^{+\infty} = +\infty \Rightarrow x=0$ je LIJEVA VA !

$\lim\limits_{x\to 4^+} e^{\frac{1}{x(x-4)}} = e^{\frac{1}{0^+}} = e^{+\infty} = +\infty \rightarrow x=4$ je DESNA VA !

$\lim\limits_{x\to 4^-} e^{\frac{1}{x(x-4)}} = e^{\frac{1}{0^-}} = e^{-\infty} = 0 \rightarrow$ NIJE VA s LIJEVE !

2° HA:

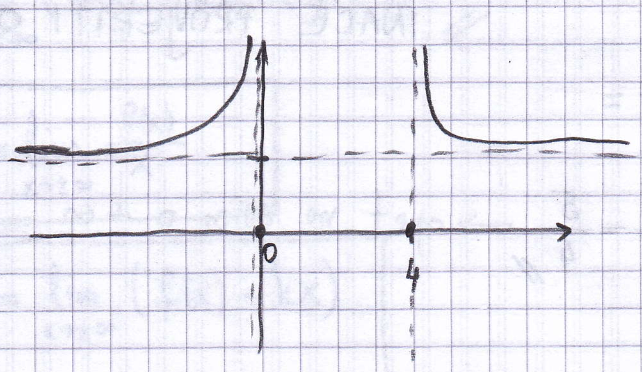
$\lim\limits_{x\to \pm\infty} e^{\frac{1}{x^2-4x}} = e^{\frac{1}{\infty}} = e^{0} = 1 \rightarrow$ nema veze je li $x \to \pm\infty$

s obje strane

$\longrightarrow y=1$ je HA !

$\longrightarrow$ ISTOVREMENO NE MOGU BITI I HA I KA NA ISTOJ STRANI !!!

$\Longrightarrow$ NEMA KA !



$$y = x \cdot \operatorname{arctg} \frac{x^2}{x+1} \qquad \left.\begin{matrix} x+1 \neq 0 \\ \underline{x \neq -1} \end{matrix}\right\} D(f) = \mathbb{R} \setminus \{-1\}$$

$$\left.\begin{matrix} \lim\limits_{x \to -1^+} x \cdot \operatorname{arctg} \overset{+\infty}{\frac{x^2}{x+1}} = -1 \cdot \frac{\pi}{2} = \frac{-\pi}{2} \,// \\ \lim\limits_{x \to -1^-} x \cdot \operatorname{arctg} \overset{-\infty}{\frac{x^2}{x+1}} = -1 \cdot \frac{-\pi}{2} = \frac{\pi}{2} \,// \end{matrix}\right\} \underline{\text{NEMA}} \text{ VERTIKALNE ASIMP.}$$

HA <u>NE POSTOJ</u> $\Longrightarrow$ računamo <u>KOSU</u> ASIMP.

$$k_1 = \lim_{x \to +\infty} \frac{\cancel{x} \operatorname{arctg} \frac{x^2 \sim x^2}{x+1 \sim \cancel{x}}}{\cancel{x}} = \frac{\pi}{2}$$

$$k_2 = \lim_{x \to -\infty} \operatorname{arctg}(-\infty) = -\frac{\pi}{2}$$

$$l_1 = \lim_{x \to +\infty} \left(x \operatorname{arctg} \frac{x^2}{x+1} - \frac{\pi}{2} x \right) = \lim_{x \to +\infty} \left(x \cdot \left(\operatorname{arctg} \frac{x^2}{x+1} - \frac{\pi}{2} \right) \right) =$$

$$= \lim_{x \to +\infty} \frac{\operatorname{arctg} \frac{x^2}{x+1} - \frac{\pi}{2}}{\frac{1}{x}} \overset{L'H}{=} \lim_{x \to +\infty} \frac{\frac{1}{1+\left(\frac{x^2}{x+1}\right)^2} \cdot \frac{2x(x+1) - x^2}{(x+1)^2}}{-\frac{1}{x^2}} = \ldots = -1 \,//$$

$$l_2 = \lim_{x \to -\infty} \ldots\ldots = -1 \,//$$

$\rightarrow$ $y = \frac{\pi}{2} x - 1$ DKA

$\rightarrow$ $y = -\frac{\pi}{2} x - 1$ LKA

DOMAĆA ZADAĆA:

a) $f(x) = tg^3 \frac{\pi x}{3} = \left(tg\frac{\pi x}{3}\right)^3$

$f'(x) = \cancel{3}\left(tg\frac{\pi x}{3}\right)^2 \cdot \frac{1}{\cos^2\frac{\pi x}{3}} \cdot \frac{\pi}{\cancel{3}} = \frac{\pi \cdot tg^2\frac{\pi x}{3}}{\cos^2\frac{\pi x}{3}} = \pi \cdot \frac{\frac{\sin^2\frac{\pi x}{3}}{\cos^2\frac{\pi x}{3}}}{\frac{\cos^2\frac{\pi x}{3}}{1}} = \pi \cdot \frac{\sin^2\frac{\pi x}{3}}{\cos^4\frac{\pi x}{3}}$ //

b) $f(x) = \ln^2(\sin(5x)) = \left(\ln(\sin(5x))\right)^2$

$f'(x) = 2\ln(\sin(5x)) \cdot \frac{1}{\sin(5x)} \cdot \cos 5x \cdot 5 = 10\ln(\sin(5x)) \cdot ctg(5x)$ //

c) $f(x) = x \cdot \cos^2(5x) = x \cdot (\cos(5x))^2$

$f'(x) = \cos^2(5x) + x \cdot 2\cos(5x) \cdot (-\sin(5x)) \cdot 5 = \cos^2(5x) - 10x \cdot \cos(5x) \cdot \sin(5x)$

$= \cos^2(5x) - 5x \cdot \sin(10x)$ //

d) $f(x) = \frac{x}{\sqrt{x^2+1}}$

$f'(x) = \frac{\sqrt{x^2+1} - x \cdot \frac{1}{\cancel{2}\sqrt{x^2+1}} \cdot \cancel{2}x}{x^2+1} = \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} = \frac{\frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2+1}}}{\frac{x^2+1}{1}} = \frac{1}{(x^2+1)(\sqrt{x^2+1})}$ //

$f(x) = arctg\frac{1}{x^2+1}$

$f''(1) = ?$

$f'(x) = \frac{1}{\left(\frac{1}{x^2+1}\right)^2+1} \cdot \frac{-1}{(x^2+1)^2} \cdot 2x = \frac{-2x}{\left(\frac{1}{(x^2+1)^2}+1\right)(x^2+1)^2} = \frac{-2x}{1+(x^2+1)^2}$

$f''(x) = \frac{-2\cdot(1+(x^2+1)^2) + 2x \cdot (2(x^2+1)\cdot 2x)}{(1+(x^2+1)^2)^2}$

$f''(1) = \frac{-2\cdot(1+4)+2\cdot(2\cdot2\cdot2)}{(1+4)^2} = \frac{-10+16}{25} = \frac{6}{25}$ //

$y = xe^{-x}$ $\qquad (e^x)^{-1} = -(e^x)^{-2} \cdot e^x = -\frac{1}{(e^x)^{\cancel{2}}} \cdot \cancel{e^x} = -\frac{1}{e^x} = -e^{-x}$

$xy' = (1-x)y$

$y' = e^{-x} + x \cdot e^{-x} \cdot (-1) = e^{-x} - xe^{-x} = e^{-x}(1-x)$ o.k.

$x \cdot y' = \underbrace{x \cdot e^{-x}}_{y}(1-x) = y \cdot (1-x)$ //

4.) za $\forall x \neq 0$ vrijedi:

$$(\ln|x|)' = \frac{1}{x}$$

1° za $x>0 \longrightarrow (\ln|x|)' = (\ln x)' = \frac{1}{x}$ //

2° za $x<0 \longrightarrow (\ln|x|)' = (\ln(-x))' = \frac{1}{-x} \cdot (-x)' = \frac{1}{-x} \cdot (-1) = \frac{1}{x}$ //

5.)

a) $f(x) = \sin x$

$f^{(n)} = ?$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{IV}(x) = \sin x$

$\vdots$

$$\boxed{\begin{aligned} f^{4k}(x) &= \sin x \\ f^{4k+1}(x) &= \cos x \\ f^{4k+2}(x) &= -\sin x \\ f^{4k+3}(x) &= -\cos x \end{aligned}} \quad k \in \mathbb{N}_0$$

Dokaz mat. ind.? NE!

b) $f(x) = a^x$

$f^{(n)} = ?$

$f'(x) = a^x \cdot \ln a$ c

$f''(x) = \ln a \cdot (a^x)' = \ln a \cdot a^x \ln a = a^x \ln^2 a$

$f'''(x) = a^x \ln^2 a = \ln^2 a \cdot (a^x)' = a^x \ln a \cdot \ln^2 a = a^x \ln^3 a$ c

$\vdots$

$f^k(x) = a^x \ln^k a, \; k \in \mathbb{N}_0$ //

$\underline{k=1}$

$f'(x) = a^x \ln a$ // o.k.

...stavimo da tvrdnja $f^k(x) = a^x \ln^k a$ vrijedi za <u>neki</u> $k \in N$

...ati da tvrdnja vrijedi za $(k+1) \in N$: $f^{k+1}(x) = a^x \ln^{k+1} a$

$f^{k+1}(x) = (f^k(x))' = (a^x \ln^k a)' = \ln^k a \cdot (a^x)' = \ln^k a \cdot a^x \cdot \ln a = \underline{\underline{a^x \cdot \ln^{k+1} a}}$

$f^{k+1}(x) = a^x \ln^{k+1} a$ // $\leftrightarrow$ o.k.

→ ...ema principu mat. ind. dokazali smo da tvrdnja vrijedi za $\forall k \in N$.

$\underline{\underline{\text{NE !}}}$

6.) $f'(0)$ postoji ??

$$f(x)=\sqrt{1-\sqrt{1-x^2}}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\sqrt{1-\sqrt{1-h^2}}-0}{h}=\lim_{h\to 0}\frac{\sqrt{1-\sqrt{1-h^2}}}{h}\cdot\frac{\sqrt{1+\sqrt{1-h^2}}}{\sqrt{1+\sqrt{1-h^2}}}=$$

$$=\lim_{h\to 0}\frac{\sqrt{1-1+h^2}}{h\left(\sqrt{1+\sqrt{1-h^2}}\right)}=\lim_{h\to 0}\frac{\sqrt{h^2}}{h\cdot\left(\sqrt{1+\sqrt{1-h^2}}\right.}$$

$$\longrightarrow \lim_{h\to 0^+}\frac{\sqrt{h^2}}{h\cdot\sqrt{1+\sqrt{1-h^2}}}=\lim_{h\to 0^+}\frac{\cancel{+h}}{\cancel{h}\cdot\sqrt{1+\sqrt{1-h^2}}}=\frac{1}{\sqrt{2}}\,//$$

$$\longrightarrow \lim_{h\to 0^-}\frac{\sqrt{h^2}}{h\cdot\sqrt{1+\sqrt{1-h^2}}}=\lim_{h\to 0^-}\frac{-\cancel{h}}{\cancel{h}\cdot\sqrt{1+\sqrt{1-h^2}}}=\frac{-1}{\sqrt{2}}\,//$$

$$\Longrightarrow \lim_{h\to 0^+}\frac{\sqrt{h^2}}{h\cdot\sqrt{1+\sqrt{1-h^2}}}\neq\lim_{h\to 0^-}\frac{\sqrt{h^2}}{\sqrt{1+\sqrt{1-h^2}}\cdot h}\Longrightarrow \text{NE POSTOJI } f'(0) \;!$$

7.) $f(x)=x\sin\frac{1}{x}$, $x\neq 0$, $f(0)=0$

a) $f'(0)=$ postoji li ?

$$f'(0)=\lim_{h\to 0}\frac{h\sin\frac{1}{h}-0}{h}=\lim_{h\to 0}\frac{\cancel{h}\sin\frac{1}{h}}{\cancel{h}}=\lim_{h\to 0}\sin\frac{1}{h}$$

$$\lim_{h\to 0^+}\sin\frac{1}{0^+}=\lim_{h\to 0^+}\sin(+\infty)^{\to[-1,1]}=\text{NE POSTOJI}$$, isto vrijedi za $h\to 0^-$, $\sin(-\infty)$

PA NE POSTOJI LIMES $\Longrightarrow$ NE POSTOJI $f'(0)$!

b) $g'(0)=$ postoji li ?

$$g(x)=x^2\sin\frac{1}{x},\ x\neq 0,\ g(0)=0$$

$$g'(0)=\lim_{h\to 0}\frac{h^2\sin\frac{1}{h}-0}{h}=\lim_{h\to 0}h\cdot\sin\frac{1}{h}=0\cdot[-1,1]=0\,//\longrightarrow \text{postoji limes}$$

$$g'(0)=\underline{\underline{0}}$$

$$f(x) = \begin{cases} -1+4x-x^2, & x<1 \longrightarrow f(1)=2 \\ ax, & x \geq 1 \longrightarrow f(1)=a \end{cases}$$

... bude neprekinuta ! $a = ?$

$\lim f(x) = f(a)$

$\lim (-1+4x-x^2) = 2 \longrightarrow \longrightarrow I_1$

$\xrightarrow{I} \lim\limits_{x\to 1^+} (f(x)) = \lim\limits_{x \to 1^-} (f(x))$

$\lim (\overset{const}{a}x) = a \cdot \lim\limits_{x\to 1^+} (x) = a \longrightarrow I_2$

$\boxed{a=2}$

$f(x) = 2x, \ x \geq 1$

$$f'(1) = \lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0^-} \frac{-1+4(1+h)-(1+h)^2-2}{h} = \lim_{h\to 0^-} \frac{\cancel{-1+4}+4h\cancel{-1}-2h-h^2\cancel{-2}}{h} =$$

$$= \lim_{h\to 0^-} \frac{2h-h^2}{h} = \lim_{h\to 0^-} \frac{\cancel{h}(2-h)}{\cancel{h}} = \lim_{h\to 0^-} (2-h) = 2$$

O.K. !

$$f'(1) = \lim_{h\to 0^+} \frac{f(1+h)-2}{h} = \lim_{h\to 0^+} \frac{2(1+h)-2}{h} = \lim_{h\to 0^+} \frac{\cancel{2}+2\cancel{h}\cancel{-2}}{\cancel{h}} = 2$$

$f(x) = -1+4x-x^2, \ x<1 \longrightarrow f(1) = 2$

$g(x) = ax^2, \ x \geq 1 \longrightarrow f(1) = a \longrightarrow g(x) = 2x^2$

$\underline{a=2} \longrightarrow g(x) = 2x^2, \ x \geq 1$

$$g'(1) = \lim_{h\to 0^+} \frac{g(1+h)-g(1)}{h} = \lim_{h\to 0^+} \frac{2(1+h)^2-2}{h} = \lim_{h\to 0^+} \frac{2(1+2h+h^2)-2}{h} = \lim_{h\to 0^+} \frac{4h+2h^2}{h} =$$

$$= \lim_{h\to 0^+} \frac{\cancel{h}(4+2h)}{\cancel{h}} = \lim_{h\to 0^+} (4+2h) = 4$$

$$g'(1) = \lim_{h\to 0^-} \frac{-1+4(1+h)-(1+h)^2-2}{h} = \lim_{h\to 0^-} \frac{\cancel{-1+4}+4h\cancel{-1}-2h-h^2\cancel{-2}}{h} = \lim_{h\to 0^-} \frac{2h-h^2}{h} = \frac{\cancel{h}(2-h)}{\cancel{h}}$$

$$= \lim_{h\to 0^-} (2-h) = 2$$

NE POSTOJI !!

9.) $y = x \cdot \sqrt[3]{x-1} = x \cdot (x-1)^{\frac{1}{3}}$

a) $T(0,0)$

$y' = \sqrt[3]{x-1} + x \cdot \frac{1}{3}(x-1)^{-\frac{2}{3}}$

$y - 0 = -1(x-0)$

$\boxed{y = -x}$

b) $T\left(\frac{3}{4}, \frac{-3}{4\sqrt[3]{4}}\right)$ $\frac{3}{4} \cdot \sqrt[3]{\left(-\frac{1}{4}\right)} = -\frac{1}{\sqrt[3]{4}}$

$y + \frac{3}{4\sqrt[3]{4}} = -\frac{1}{\sqrt[3]{4}} + \frac{3}{4} \cdot \frac{1}{3}\left(\frac{3}{4}-1\right)^{-\frac{2}{3}}\Big)\left(x - \frac{3}{4}\right)$

$y + \frac{3}{4\sqrt[3]{4}} = \underbrace{\sqrt[3]{\left(-\frac{1}{4}\right)} + \frac{1}{4} \cdot \frac{1}{\sqrt[3]{\left(-\frac{1}{4}\right)^2}}}_{0} \cdot \left(x - \frac{3}{4}\right)$

$\boxed{y = -\frac{3}{4\sqrt[3]{4}}}$

c) $T(1,0)$

$y - 0 = \frac{1}{3\sqrt[3]{0^2}}(x-1)$

$y = "\infty"(x-1) \longrightarrow \boxed{x=1}$

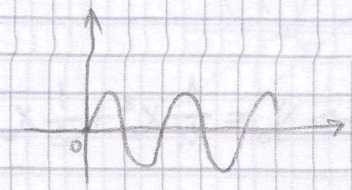
$y = A \cdot \sin(\omega x)$

$k = 1$

$T = 4\pi$

$A = ?\quad \omega = ?$

$\omega = \frac{2\pi}{T} = \frac{2\cancel{\pi}}{4\cancel{\pi}} = \frac{1}{2}$



$k = y'(0) = 1$

$y' = A \cdot \cos \frac{1}{2}x \cdot \frac{1}{2}$

$\frac{1}{2}A = 1 \quad | \cdot 2$

$\boxed{A = 2}$

$P \ldots y = x \ldots t \rightarrow k = 1 \longrightarrow k_2 = -1$

$y = \lambda e^x$

$y' = \lambda e^x = 1 \longrightarrow \lambda = \frac{1}{e^x} = e^{-x}$

$y = e^{-x} \cdot e^x = 1 = x \longrightarrow \boxed{\lambda_1 = e^{-1}}$

$y' = \lambda e^x = -1$

$\lambda = \frac{-1}{e^x} = -e^{-x}$

$y = -e^{-x} \cdot e^x = -1 = x$

$\Rightarrow \boxed{\lambda_2 = -e^1 = -e}$

$f_1(x) = \ln x$

$f_2(x) = ax^2$

$k_1 = k_2 \quad f_1'(x) = f_2'(x)$

$\frac{1}{x_0} = \frac{a \cdot 2x_0}{1}$

$1 = 2ax_0^2$

$a = \frac{1}{2x_0^2}$

$\boxed{a = \frac{1}{2e}}$

$\ln x_0 = a x_0^2$

$a = \frac{\ln x_0}{x_0^2} = \frac{1}{2x_0^2}$

$\cancel{x_0^2} = 2\cancel{x_0^2} \cdot \ln x_0$

$\ln x_0 = \frac{1}{2} \quad | e$

$\boxed{x_0 = e^{\frac{1}{2}}}$

9 13.)

$M(-2,-3)$

$y = \frac{1}{4}x^2 + \frac{1}{2}x - \frac{3}{4} \longrightarrow y_0 = \frac{1}{4}x_0^2 + \frac{1}{2}x_0 - \frac{3}{4}$

$t \ldots \; \underline{\underline{y}} - y_0 = y'(x_0)(\underline{\underline{x}} - x_0)$

M

$y' = \frac{1}{2}x + \frac{1}{2} \longrightarrow y'(x_0) = \frac{1}{2}x_0 + \frac{1}{2} = \frac{1}{2}(x_0+1)$

$-3 + \frac{1}{4}x_0^2 + \frac{1}{2}x_0 - \frac{3}{4} = \frac{1}{2}(x_0+1)(-2-x_0) \quad / \cdot 4$

$-12 + x_0^2 + 2x_0 - 3 = 2(x_0+1)(-2-x_0)$

$x_0^2 + 2x_0 - 15 = 2(-2x_0 - x_0^2 - 2 - x_0)$

$x_0^2 + 2x_0^2 + 2x_0 + 6x_0 - 15 + 4 = 0$

$3x_0^2 + 8x_0 - 11 = 0$

$x_{0_{12}} = \frac{-8 \pm \sqrt{64 + 12 \cdot 11}}{6} = \frac{-8 \pm 14}{6}$

$x_{0_1} = 1 \qquad x_{0_2} = -\frac{11}{3}$

$y_{0_1} = \frac{1}{4} + \frac{1}{2} - \frac{3}{4} = 0 \qquad y_{0_2} = \frac{1}{4} \cdot \frac{11^2}{3^2} + \frac{1}{2} \cdot \frac{-11}{3} - \frac{3}{4} = \frac{7}{9}$

$y'(x_{0_1}) = \underline{\underline{1}} \qquad y'(x_{0_2}) = \frac{1}{2} \cdot \frac{-11}{3} + \frac{1}{2} = -\frac{4}{3}$

$t_1 \ldots \; y = x - 1$

$t_2 \ldots \; y - \frac{7}{9} = -\frac{4}{3}\left(x + \frac{11}{3}\right) \quad \cdot 9$

$y - 7 = -12\left(x + \frac{11}{3}\right)$

$y = -12\left(x + \frac{11}{3}\right) + 7$

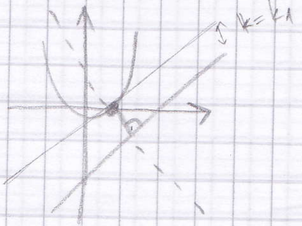
$y = -12x - 44 + 7$

$t_2 \ldots \; y = -12x - 37$

$y = x^2 + x - 2$

$y = 5x - 10$

$k = k_1 = 5$

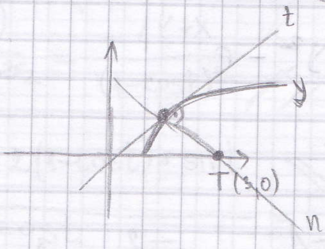


$y' = 2x + 1 = 5 \longrightarrow 2x = 4$

$\boxed{x = 2} \Longrightarrow \boxed{y = 4} \quad T(2,4)$

$y = \sqrt{x^2+1}$

$T(3,0)$



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$y - 0 = \frac{-1}{k}(x - 3)$

$y' = \frac{1}{\cancel{2}\sqrt{x^2+1}} \cdot \cancel{2}x = k$

$y = \frac{-1}{k}(x-3)$

$y = \sqrt{x^2+1}$

$k = \frac{x}{\sqrt{x^2+1}}$

$\sqrt{x^2+1} = \frac{-1}{\frac{x}{\sqrt{x^2+1}}} \cdot (x-3)$

$\sqrt{x^2+1} = \frac{-\sqrt{x^2+1} \cdot (x-3)}{x} \quad , x \neq 0$

$x \cdot \sqrt{x^2+1} = -(x-3) \cdot \sqrt{x^2+1} \quad |^2$

$x^2 \cdot \cancel{(x^2+1)} = (x-3)^2 \cdot \cancel{(x^2+1)}$

$\cancel{x^2} = \cancel{x^2} - 6x + 9$

$6x = 9 \quad | : 6$

$\boxed{x = \frac{3}{2}} \longrightarrow y = \sqrt{\frac{9}{4} + 1} = \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}$

$\boxed{T\left(\frac{3}{2}, \frac{\sqrt{13}}{2}\right)}$

16.) $T(1,-1)$

$$xy^2 + e^{x+y} = 2$$

$$y', y''\Big|_{T(1,-1)} = ?$$

$$xy^2 + e^{x+y} = 2 \quad /'$$

$$y^2 + x\cdot 2y\cdot y' + e^{x+y}\cdot(1+y') = 0$$

$$y'\cdot(2xy + e^{x+y}) = -y^2 - e^{x+y}$$

$$y' = \frac{-y^2 - e^{x+y}}{2xy + e^{x+y}}$$

$$y'\Big|_{T(-1,1)} = \frac{-1-1}{-2+1} = \frac{-2}{-1} = 2$$

$$y'' = \frac{\left(-2y\cdot y' - e^{x+y}\cdot(1+y')\right)\cdot(2xy+e^{x+y}) - (-y^2 - e^{x+y})\cdot\left(2(y+x\cdot y') + e^{x+\text{[illegible]}}\right.}{(2xy+e^{x+y})^2}$$

$$y''\Big|_{T(-1,1)} = \frac{\left(-2\cdot 2 - 1\cdot(1+2)\right)\cdot(-2+1) - \overbrace{(-1-1)}^{-2}\cdot\left(2\cdot\overbrace{(1-2)}^{-2} + 1\cdot\overbrace{(1+2)}^{3}\right)}{(-2+1)^2}$$

$$= -7\cdot(-1) + 2 = 7+2 = 9$$

17.) $T(1,1)$

$$y^x + \sin(x-1) = \sqrt{y} \quad /' \qquad y^x = e^{x\ln y}$$

t.... ? n..... ?

$$e^{x\ln y}\cdot\left(\ln y + x\cdot\frac{1}{y}\cdot y'\right) + \cos(x-1)\cdot 1 - \frac{1}{2\sqrt{y}}\cdot y' = 0$$

$$y'\cdot\left(y^x\cdot\frac{x}{y} - \frac{1}{2\sqrt{y}}\right) = -\cos(x-1) - y^x\cdot\ln y$$

$$y' = \frac{-(\cos(x-1) + y^x\cdot\ln y)}{\frac{y^x\cdot x}{y} - \frac{1}{2\sqrt{y}}} = \frac{-2y\cdot\sqrt{y}\cdot(\cos(x-1) + \text{[illegible]}}{2x\cdot y^x\sqrt{y} - y\ \text{[illegible]}}$$

$$y'\Big|_{T(1,1)} = \frac{-2\cdot(1+0)}{2-1} = -2$$

t.... $y-1 = -2(x-1) \longrightarrow y = -2x+2+1 = -2x+3$ n.... $y = \frac{1}{2}x + \frac{1}{2}$ [illegible]

$$x = t - \sin t$$
$$y = 1 - \cos t$$

$$y' = \frac{\dot{y}}{\dot{x}} = \frac{\sin t}{1-\cos t}$$

$$y'' = \frac{\left(\frac{\sin t}{1-\cos t}\right)'}{1-\cos t} = \frac{\frac{\cos t\cdot(1-\cos t) - \sin t\cdot \sin t}{(1-\cos t)^2}}{1-\cos t} = \frac{\frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^2}}{1-\cos t} = \frac{\cos t - (\cos^2 t + \sin^2 t)}{(1-\cos t)^3}$$

$$y'' = \frac{\cos t - 1}{(1-\cos t)^3} = \frac{-\cancel{(1-\cos t)}}{(1-\cos t)^{\cancel{3}\,2}} = \frac{-1}{(1-\cos t)^2}$$

$$y''' = \frac{\left(\frac{-1}{(1-\cos t)^2}\right)'}{1-\cos t} = \frac{\frac{1}{(1-\cos t)^{\cancel{4}\,3}} \circ 2\cancel{(1-\cos t)} \circ \sin t}{1-\cos t} = \frac{\frac{2\sin t}{(1-\cos t)^3}}{1-\cos t} = \frac{2\sin t}{(1-\cos t)^4}$$

$$x = \sin^2 t - \cos t \quad \rightarrow \quad x = 1$$
$$y = \ln\left(\operatorname{tg}\frac{t}{2}\right) \quad \rightarrow \quad y = 0$$
$$t = \frac{\pi}{2} \longrightarrow T(1,0)$$

t... ? n..... ?

$$y' = \frac{\frac{1}{\operatorname{tg}\frac{t}{2}} \circ \frac{1}{\cos^2\frac{t}{2}} \circ \frac{1}{2}}{2\sin t\cdot\cos t + \sin t} = \frac{\frac{1}{2\cdot\frac{\sin\frac{t}{2}}{\cancel{\cos\frac{t}{2}}}\cdot\cos^{\cancel{2}}\frac{t}{2}}}{\sin 2t + \sin t} = \frac{1}{\sin t(\sin 2t + \sin t)}$$

$$y'(t) = \frac{1}{\sin^2\frac{\pi}{2} + \sin\frac{\pi}{2}\circ\sin\pi} = \frac{1}{1} = 1$$

t..... $y - 0 = 1\cdot(x-1)$

t... $y = x - 1$

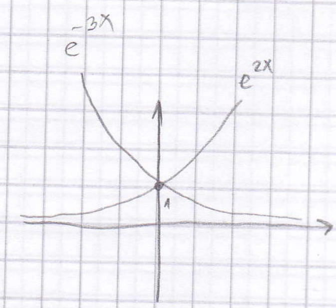
n..... $y = -x + 1$

20.) $y = e^{2x}$

$y = e^{-3x}$

$\alpha = ?$

$$\boxed{tg\alpha = \frac{k_2 - k_1}{1 + k_1 \cdot k_2}}$$



$e^{-3x} = e^{2x} \quad / \ln$

$-3x = 2x$

$-5x = 0$

$\boxed{x = 0} \rightarrow \boxed{y = 1}$

a i sa slike $D(0,1)$

$D(0,1)$

$y' = e^{2x} \cdot 2 \longrightarrow k_2 = 2$

$y' = e^{-3x} \cdot (-3) \longrightarrow k_1 = -3$

$$tg\alpha = \frac{2+3}{1-6} = -1$$

$$\alpha = -\frac{\pi}{4} \longrightarrow \boxed{\alpha = \frac{3\pi}{4}}$$

PREDAVANJA *

Odredi sve asimptote :

$f(x) = 2x - 1 - \sqrt{x^2-x+1}$

$D(f)$: $x^2 - x + 1 \geq 0$

$x_{1,2} = \frac{1 \pm \sqrt{1-4}}{2}$ $\Big\}$ $D(f) = \mathbb{R}$ $\longrightarrow$ NEMA VERTIKALNIH ASIMP.

$\Rightarrow$ NE MOŽEMO ODREDITI IMA LI H.A. ILI K.A PRVO RAČUNAMO KOSU A. !!!

$\longrightarrow$ ako nam ispadne $k=0 \Rightarrow$ H.A.

$$k = \lim_{x\to+\infty} \frac{2x-1-\sqrt{x^2-x+1}}{x} = \lim_{x\to+\infty}\Big(2 - \underbrace{\frac{1}{x}}_{\to 0} - \underbrace{\frac{\sqrt{x^2-x+1}}{x}}_{1}\Big) = 1 = k_1$$

$$k_2 = \lim_{x\to-\infty}\Big(2 - \frac{1}{x} - \frac{\sqrt{x^2-x+1}}{x}\Big) = 3 = k_2$$

PAZI !!!

$$l_1 = \lim_{x\to+\infty}\left(x-1-\sqrt{x^2-x+1}\right)\cdot\frac{(x-1)+(\sqrt{x^2-x+1})}{(x-1)+\sqrt{x^2-x+1}} =$$

$$= \lim_{x\to+\infty}\frac{\cancel{x^2}-2x+1-\cancel{x^2}+x-1}{(x-1)+\sqrt{x^2-x+1}} \begin{matrix}\sim -x\\ \sim 2x\end{matrix} = -\frac{1}{2} = l_1$$

NIJE NEODREĐEN OBLIK !

$$l_2 = \lim_{x\to-\infty}\left(2x-1-\sqrt{x^2-x+1}-3x\right) = \lim_{x\to-\infty}\left(-x-1-\sqrt{x^2-x+1}\right) = (\infty+\infty) = \infty$$

$\Rightarrow$ nema LIJEVE KOSE ASIMP.

D.K.A : $y = x - \frac{1}{2}$

9.zzv. -25.) $y=\dfrac{(\sqrt{x}+a)^3}{\sqrt{x}+2}$

→ za koji a će fja imati desnu kosu asimptotu?

$D(f)=[0,+\infty\rangle$ // → zato nema LIJEVE KOSE A.

$k=\lim\limits_{x\to+\infty}\dfrac{x\sqrt{x}+3xa+3\sqrt{x}\cdot a^2+a^3}{x\sqrt{x}+2x}=1$ → bez obzira na a!

$l=\lim\limits_{x\to+\infty}\left(\dfrac{(\sqrt{x}+a)^3}{\sqrt{x}+2}-x\right)=\lim\limits_{x\to+\infty}\dfrac{\cancel{x\sqrt{x}}+3xa+3\sqrt{x}\cdot a^2+a^3-\cancel{x\sqrt{x}}-2x}{\sqrt{x}+2}=$

$=\lim\limits_{x\to+\infty}\dfrac{x(3a-2)+3\sqrt{x}\cdot a^2+a^3}{\sqrt{x}+2}=\begin{cases}3\cdot\left(\frac{2}{3}\right)^2, & a=\frac{2}{3}\\ \pm\infty, & a\neq\frac{2}{3}\end{cases}$

→ za $a=\frac{2}{3}$ postoji D.K.A!

D.K.A... $y=x+\frac{4}{3}$

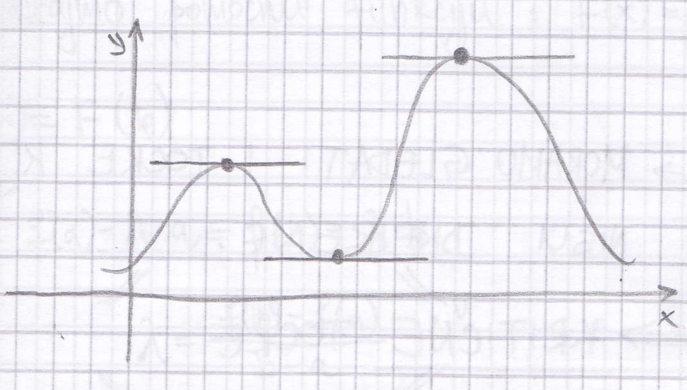
9.4. OSNOVNI TEOREMI DIF. RAČUNA

Za točku $a \in S$ kažemo da je točka maksimuma od funkcije f ako za sve $x \in S$ vrijedi $f(x) \leq f(a)$.

Za točku $a \in S$ kažemo da je točka minimuma od funkcije f ako za sve $x \in S$ vrijedi $f(x) \geq f(a)$.

$$f(a) = \max_{x \in S} f(x) \quad , \quad f(a) = \min_{x \in S} f(x)$$

$\Rightarrow$ jednim imenom: ekstremi funkcije f



$$\boxed{f'(x) = 0}$$

TM „FERMATOV TEOREM"

Neka je I otvoreni interval u $\mathbb{R}$ i neka je $f: I \to \mathbb{R}$ diferencijabilna. Ako je $a \in I$ točka LOKALNOG EKSTREMA, tada je $f'(a) = 0$.

Dokaz: (sa slike)

Neka je a lok. max $\Rightarrow$ tada je $f(x) \leq f(a)$. Ako je $x < a$:

$\dfrac{f(x) - f(a)}{x-a} \geq 0 \; \Big| \lim_{x \to a-}$, ako je $x > a$: $\dfrac{f(x) - f(a)}{x-a} \leq 0 \; \Big| \lim_{x \to a+}$

$f'(a) \geq 0 \qquad\qquad f'(a) \leq 0$

$$f'(a) = 0 \quad \text{Ⓓ}$$

def.

Za diferencijabilnu funkciju $f: I \to R$ točke koje su rješenje jednadžbe $f'(x)=0$ zovemo STACIONARNE TOČKE.

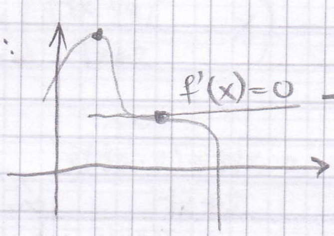
• NAPOMENA:

Obrat Fermatovog teorema NE VRIJEDI.

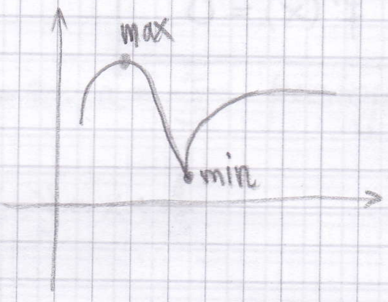
⇒ ako postoji derivacija $=0$ ne mora imati ekstreme

npr. x^3

ili:



$f'(x)=0$ → a nije ekstrem!



→ MORAMO GLEDATI I TOČKE KOJE NISU DIFIRENC.

⇒ KRITIČNE TOČKE

def. KRITIČNE TOČKE su točke u kojima je $f'(x)=0$ ili $f'(x)$ NE POSTOJI.

$\boxed{\text{TM}}$ „ROLLEOV TEOREM"

Neka je $f: [a,b] \to R$ neprekinuta funkcija i diferencijabilna na (a,b). Ako je $f(a)=f(b)$, tada postoji neka točka takva da je $f'(c)=0$.

Dokaz:

→ ako je f konstanta, onda je $f'(x)=0$ za $\forall x$

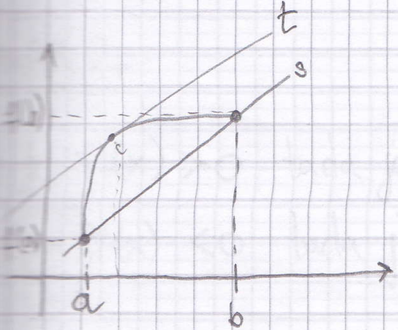
→ ako f nije konstanta, onda postoji min ili max tada po Fermatovom t. ⇒ da je $f'(x)=0$

ISPIT

„LAGRANGEOV TEOREM SREDNJE VRIJEDNOSTI DIF. RAČUNA"

Neka je $f:[a,b]\longrightarrow \mathbb{R}$ neprekinuta i diferencijabilna na (a,b). Tada postoji $c\in(a,b)$ tako da

$$\boxed{f(b)-f(a)=f'(c)\cdot(b-a)}$$



koefic. smjera $s=\frac{f(b)-f(a)}{b-a}=f'(c)$

LAGRANGE: UVIJEK POSTOJI $t\parallel s$.

$f'(c)=k \dots t$

DOKAZ:

definirajmo pomoćnu funkciju: $F(x)=f(x)-\lambda x$ takvu da je

$F(a)=F(b)$

$\Rightarrow f(a)-\lambda\cdot a=f(b)-\lambda\cdot b$

$$\lambda=\frac{f(b)-f(a)}{b-a}$$

$\Rightarrow$ po Rollovom TM postoji $c\in(a,b)$ takav da je $F'(c)=0$.

$F'(x)=f'(x)-\lambda$

$f'(c)=\lambda=\frac{f(b)-f(a)}{b-a}$

9.DZ - 7.) $f(x) = \arcsin x$, primjeni Lagrangeov teorem srednje vrij. dif. računa, odredi potreban c i geom. interpretiraj.

$$f'(c) = \frac{f(b) - f(a)}{b-a} \qquad f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-c^2}} = \frac{f(1) - f(-1)}{1-(-1)} \qquad D(f) = [-1,1]$$

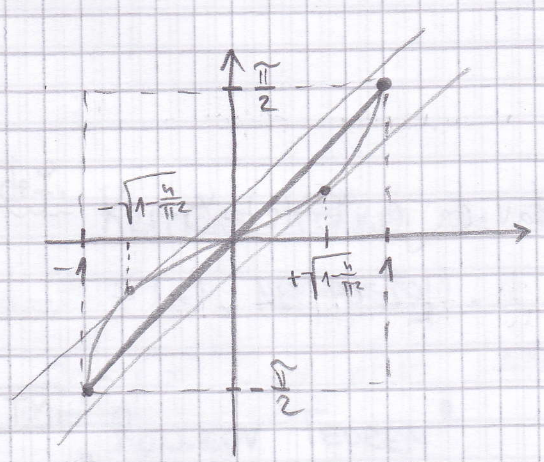
$$\frac{1}{\sqrt{1-c^2}} = \frac{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)}{1+1} = \frac{\pi}{2} \quad \Big|^2$$

$$\frac{1}{1-c^2} = \frac{\pi^2}{4}$$

$$1 - c^2 = \frac{4}{\pi^2}$$

$$c^2 = 1 - \frac{4}{\pi^2}$$

$$\boxed{c = \pm\sqrt{1-\frac{4}{\pi^2}}}$$



→ prema Lagrangeu postoje tangente paralelne pravcu koji spaja točke

KOROLAR 1.

Ako je $f'(x)=0$ za sve x, tada je f konstanta.

Ako je $f'(x)=g'(x)$, tada je $f(x)=g(x)+c$, tj. razlikuju se za konstantu.

KOROLAR 2.

Ako je $f'(x)>0$ tada je f STROGO RASTUĆA.

Ako je $f'(x)<0$ tada je f STROGO PADAJUĆA.

Dokaz:

Uzmemo $x_1 < x_2$ i neka je $f'(x)>0$

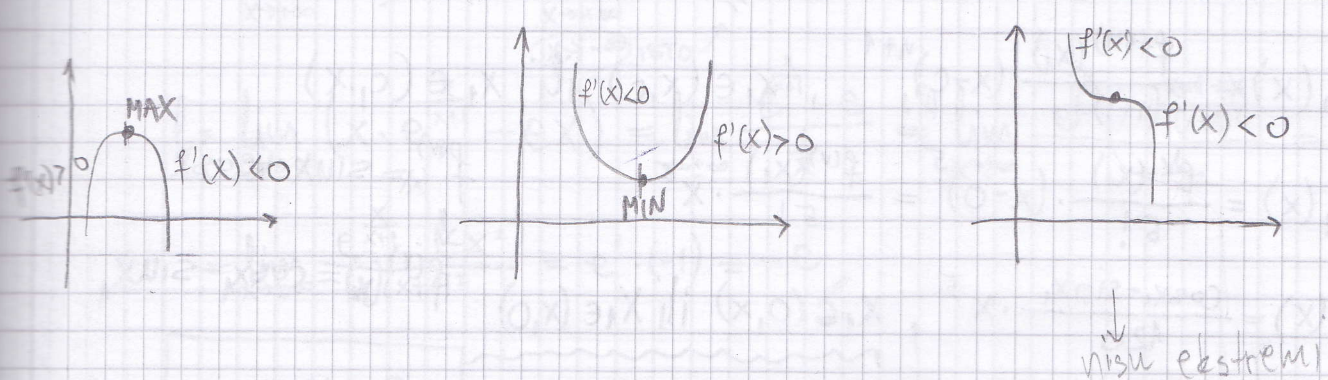
$$f(x_2)-f(x_1) = \underbrace{f'(c)}_{>0}\,\underbrace{(x_2-x_1)}_{>0} > 0 \quad f(x_2)>f(x_1) \Rightarrow f \text{ raste!}$$

10.1. INTERVALI MONOTONOSTI I EKSTREMI

Def. Intervali monotonosti su intervali na kojima je funkcija rastuća ili padajuća.

→ Krajnje točke intervala su RUBOVI DOMENE ili KRITIČNE TOČKE.

Kako odrediti karakter ekstrema?



→ AUDITORNE ←

2MI. - 2010 - 6.)

$f(x) = \sin x + \cos x$ prikaži u obliku $f(x) = T_4(x) + R_4(x)$ oko točke $c=0$

Rj.:

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

$$c=0 \longrightarrow T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} \cdot x^k$$

$n=4$

$$T_4(x) = \sum_{k=0}^{4} \frac{f^{(k)}(0)}{k!} \cdot x^k = \frac{f^{(0)}(0)}{0!} \cdot x^0 + \frac{f^{(1)}(0)}{1!} \cdot x^1 + \frac{f^{(2)}(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} \cdot x^4$$

$$T_4(x) = f(0) + \frac{f'(0)}{1!} \cdot x + \frac{f''(0)}{2!} \cdot x^2 + \frac{f'''(0)}{3!} \cdot x^3 + \frac{f^{IV}(0)}{4!} \cdot x^4$$

$f(x) = \sin x + \cos x \longrightarrow f(0) = 1$

$f'(x) = \cos x - \sin x \longrightarrow f'(0) = 1$

$f''(x) = -\sin x - \cos x \longrightarrow f''(0) = -1$

$f'''(x) = -\cos x + \sin x \longrightarrow f'''(0) = -1$

$f^{IV}(x) = \sin x + \cos x \longrightarrow f^{IV}(0) = 1$

$T_4(x) = 1 + \frac{1}{1!} x - \frac{1}{2!} x^2 - \frac{1}{3} x^3 + \frac{1}{4!} x^4 \longrightarrow$ možemo ostaviti tako !!

$T_4(x) = 1 + x - \frac{1}{2} x^2 - \frac{1}{6} x^3 + \frac{1}{8} x^4$ //

$R_n(x) = \frac{f^{(n+1)}(x_1)}{(n+1)!}(x-c)^{n+1}$, $x_1 \in (x,c)$ ili $x_1 \in (c,x)$

$R_4(x) = \frac{f^{V}(x_1)}{5!} \cdot (x-0)^5 = \frac{f^{(V)}(x_1)}{5!} \cdot x^5$

$f^{(IV)}(x) = \sin x + \cos x$

$\Downarrow$

$f^{(V)}(x_1) = \cos x_1 - \sin x_1$

$R_4(x) = \frac{\cos x_1 - \sin x_1}{120} \cdot x^5$, $x_1 \in (0,x)$ ili $x_1 \in (x,0)$

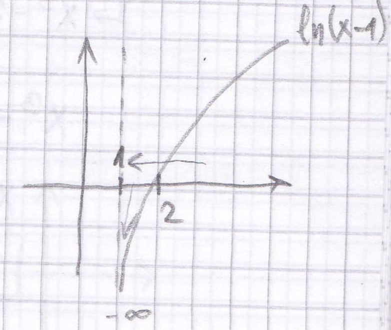
PIŠI ! ! ! !

$x^x = e^{x\ln x}$

$$\lim_{x\to 1} \frac{x^x - x}{1-x+\ln x} = \left(\frac{0}{0}\right) \overset{L'H}{\underset{\text{o.k.}}{=}} \lim_{x\to 1} \frac{e^{x\ln x}\cdot\left(\cancel{x}\cdot\frac{1}{\cancel{x}}+\ln x\right)-1}{-1+\frac{1}{x}} = \lim_{x\to 1} \frac{x^x\cdot(1+\ln x)-1}{\frac{1}{x}-1} \overset{L'H}{=} \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 1} \frac{e^{x\ln x}\cdot(\ln x+1)\cdot(\ln x+1) + e^{x\ln x}\left(\frac{1}{x}-0\right)-0}{0-\frac{1}{x^2}} = \frac{2}{-1} = -2$$

$$\lim_{x\to 1^+} \ln x\cdot \ln(x-1) = (0\cdot(-\infty)) =$$



$$= \lim_{x\to 1^+} \frac{\ln(x-1)}{\frac{1}{\ln x}} = \left(\frac{\infty}{\infty}\right) \overset{L'H}{=} \lim_{x\to 1^+} \frac{\frac{1}{x-1}}{-\frac{1}{\ln^2 x}\cdot\frac{1}{x}} =$$

$$= \lim_{x\to 1^+} \frac{-x\ln^2 x}{x-1} = \left(\frac{0}{0}\right) \overset{L'H}{\underset{\text{o.k.}}{=}} \lim_{x\to 1^+} \frac{-\ln^2 x - \cancel{x}\cdot 2\ln x\cdot\frac{1}{\cancel{x}}}{1} = \lim_{x\to 1^+} \frac{-\ln^2 x - 2\ln x}{1} = 0$$

Odrediti sve asimptote krivulje: $y = x\cdot e^{\frac{x}{x+1}}$.

R: $D(f) = \mathbb{R}\setminus\{-1\}$

VA:

$$\left.\begin{aligned} &\lim_{x\to -1^+} x\cdot e^{\frac{x}{x+1}} = -1\cdot e^{-\infty} = 0 \\ &\lim_{x\to -1^-} x\, e^{\frac{x}{x+1}} = (-1\cdot e^{+\infty}) = -\infty \end{aligned}\right\} \quad \frac{x=-1}{\text{(lijeva)}}$$

KA: $y = kx + l$

$$k = \lim_{x\to+\infty} \frac{\cancel{x}\cdot e^{\frac{x}{x+1}}}{\cancel{x}} = \lim_{x\to+\infty} e^{\frac{x}{x+1}} = e^1 = e$$

$x\to-\infty$ ISTO

$$l = \lim_{x\to\pm\infty}\left(x\cdot e^{\frac{x}{x+1}} - ex\right) = \lim_{x\to\pm\infty} \frac{e^{\frac{x}{x+1}}-e}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\pm\infty} \frac{e^{\frac{x}{x+1}}\cdot\frac{x+1-x}{(x+1)^2}}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to\pm\infty} \frac{e^{\frac{x}{x+1}}\cdot(-x^2)}{(x+1)^2} = e^1\cdot(-1) = -e$$

$$\boxed{y = ex - e}$$

2.MI - 2009 - 5.) Jednadžbe tangente i normale na krivulju

$$x^3 - 2e^x(1-y)^2 - y^2 \ln y + 8 = 0$$ u točki s ordinatom 1

Rj.:

$\underline{y=1}$

$$\hookrightarrow x^3 - 2e^x(1-1)^2 - \ln 1 + 8 = 0$$

$$x^3 + 8 = 0$$

$$x^3 = -8$$

$$\boxed{x=-2} \qquad T(-2,1)$$

$$x^3 - 2e^x(1-y)^2 - y^2 \ln y + 8 = 0 \quad /'$$

$$3x^2 - 2e^x(1-y)^2 - 2e^x \cdot 2 \cdot (1-y) \cdot (-y)' - 2y \cdot y' \cdot \ln y - y^2 \cdot \frac{1}{y} \cdot y' = 0$$

uvrstimo točku:

$$12 - 0 - 0 - 0 - 1 \cdot y' = 0 \Rightarrow y' = 12$$

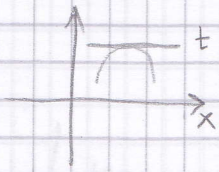
$$t \ldots\ldots y - 1 = 12(x+2)$$

$$y = 12(x+2) + 1$$

$$t \ldots\ldots y = 12x + 25$$

$$n \ldots\ldots y = -\frac{1}{12}x -$$

DOMAĆA ZADAĆA



$y = 3x^4 - 4x^3 - 12x^2 + 20$

$t \parallel x \longrightarrow \underline{k = 0}$

$y' = 12x^3 + 12x^2 - 24x = 0$

$x(12x^2 + 12x - 24) = 0$

$\boxed{x_1 = 0}$

$\downarrow$

$\boxed{y = 20}$

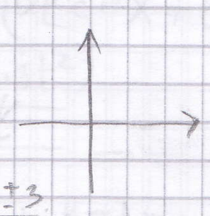
$T_1(0, 20)$

$T_2(-2, -12)$

$T_3(1, 15)$

$12x^2 + 12x - 24 = 0 \quad | : 12$

$x^2 + x - 2 = 0$



$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$

$\boxed{x_2 = -2}$

$\downarrow$

$y_2 = 3 \cdot 2^4 - 4 \cdot 2^3 - 12 \cdot 2^2 + 20$

$y_2 = 2^2 \cdot (3 \cdot 2^2 - 8 - \cancel{12}) + 20$

$\boxed{y_2 = -12}$

$\boxed{x_3 = 1}$

$\downarrow$

$y_3 = 3 + 4 - 12 + 20$

$\boxed{y_3 = 15}$

$y^4 = 4x^4 + 6xy \quad |'$

$A(1, 2)$

$t \ldots ?$

$4y^3 \cdot y' = 16x^3 + 6 \cdot (y + x \cdot y')$

$y'(4y^3 - 6x) = 16x^3 + 6y$

$y' = \frac{16x^3 + 6y}{4y^3 - 6x} = \frac{8x^3 + 3y}{2y^3 - 3x}$ //

$y'\Big|_{A(1,2)} = \frac{8+6}{16-3} = \frac{14}{13}$ //

$y - 2 = \frac{14}{13}(x - 1)$

$y = \frac{14}{13}x - \frac{14}{13} + 2$

$\boxed{y = \frac{14}{13}x + \frac{12}{13}} \ldots t$

3.) $T(1,1)$

$$xy^2 + x^4y^3 = 2 \quad /'$$

$$y', y''\Big|_{T(1,1)} = ?$$

$$y^2 + x\cdot 2y\cdot y' + 4x^3\cdot y^3 + x^4\cdot 3y^2\cdot y' = 0$$

$$y'\cdot(2xy + x^4\cdot 3y^2) = -y^2 - 4x^3y^3$$

$$y' = \frac{-y^2-4x^3y^3}{2xy+x^4\cdot 3y^2} = \frac{y^{\cancel{2}}(-1-4x^3y)}{\cancel{y}(2x+x^4\cdot 3y)} = \frac{-y-4x^3y^2}{2x+x^4\cdot 3y}$$

$$y'\Big|_{T(1,1)} = \frac{-1-4}{2+3} = -1$$

$$y'' = \frac{\left(-y'-4(3x^2\cdot y^2+x^3\cdot 2y\cdot y')\right)(2x+x^4\cdot 3y) - (-y-4x^3y^2)(2+4x^3\cdot 3y+x^4\cdot 3y')}{(2x+x^4\cdot 3y)^2}$$

$$y''\Big|_{T(1,1)} = \frac{\left(1-4\overbrace{(3-2)}^{1}\right)(2+3) - \overbrace{(-1-4)}^{-5}\overbrace{(2+12-3)}^{11}}{(2+3)^2} = \frac{-3\cdot 5+55}{25} = \frac{\cancel{40}^{\,8}}{\cancel{25}_{\,5}} = \frac{8}{5}$$

4.) $x = a\cos^3 t$

$y = a\sin^3 t$

$$t = \frac{\pi}{4} \nearrow x = a\cdot\left(\frac{\sqrt{2}}{2}\right)^3 = a\cdot\frac{\cancel{2}^{1}\sqrt{2}}{\cancel{8}_{4}} = \frac{a\sqrt{2}}{4} \qquad T\left(\frac{a\sqrt{2}}{4}, \frac{a\sqrt{2}}{4}\right)$$

$$\searrow y = x = \frac{a\sqrt{2}}{4}$$

$t \ldots ?$

$$y' = \frac{\dot y}{\dot x} = \frac{\cancel{a\cdot 3}\sin^{\cancel{2}}t\cdot\cancel{(\cos t)}}{\cancel{a\cdot 3}\cos^{\cancel{2}}t\cdot\cancel{(-\sin t)}} = \frac{-\sin t}{\cos t} = -\operatorname{tg} t = -\operatorname{tg}\frac{\pi}{4} = -1 = k$$

$$y - \frac{a\sqrt{2}}{4} = -1\left(x - \frac{a\sqrt{2}}{4}\right)$$

$$y = -x + \frac{a\sqrt{2}}{4} + \frac{a\sqrt{2}}{4}$$

$$\boxed{y = -x + \frac{a\sqrt{2}}{2}} \quad \ldots t$$

$= \ldots + 1$

$= 3t + e^{t}$

$y', y'' = ?$

$= \frac{\dot{y}}{\dot{x}} = \frac{3+e^{t}}{2t}$

KRIVO

$-2\cdot 3 - 2\cdot e^{t}$

$= \frac{\left(\frac{3+e^{t}}{2t}\right)'}{2t} = \frac{\frac{e^{t}\cdot 2t-(3+e^{t})\cdot 2}{(2t)^{2}}}{2t} = \frac{2e^{t}\cdot t - 2\cdot(3+e^{t})}{(2t)^{3} = 8t^{3}} = \frac{2\cdot e^{t}(t-1)-6}{8t^{3}}$

$= \frac{\left(\frac{2te^{t}-2(e^{t}+3)}{8t^{3}}\right)'}{2t} = \frac{\frac{(2e^{t}-2\cdot e^{t})8t^{3}-(2te^{t}-2(e^{t}+3))(8\cdot 3t^{2})}{64t^{6}}}{2t} =$

$= \frac{-8t^{3}e^{t}-24t^{2}\cdot(2te^{t}-2e^{t}-6)}{128t^{7}} = \frac{-\cancel{8t^{2}}(te^{t}+3(2te^{t}-2e^{t}-6)}{\cancel{128}_{16}\,t^{\cancel{7}\,5}} =$

$= \frac{-te^{t}-6te^{t}+6e^{t}+18}{16t^{5}} = \frac{-7t\cdot e^{t}+6e^{t}+18}{16t^{5}} = \frac{-7\cancel{t}e^{t}}{16t^{\cancel{5}}} + \frac{\overbrace{6e^{t}+18}^{\cancel{2}\cdot(3e^{t}+9)}}{\cancel{16}_{8}t^{5}} =$

$= \frac{-7e^{t}}{16t^{4}} + \frac{3(e^{t}+3)}{8t^{5}}$ NE

$f(x) = \sin x$

$I = \left[0, \frac{\pi}{2}\right]$

$c = ?$

$f(b) - f(a) = f'(c)\cdot(b-a)$ LAGRANGE

$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{1-0}{\frac{\pi}{2}-0} = \frac{2}{\pi}$ $\qquad f'(x) = \cos x$

$\cos c = \frac{2}{\pi}$

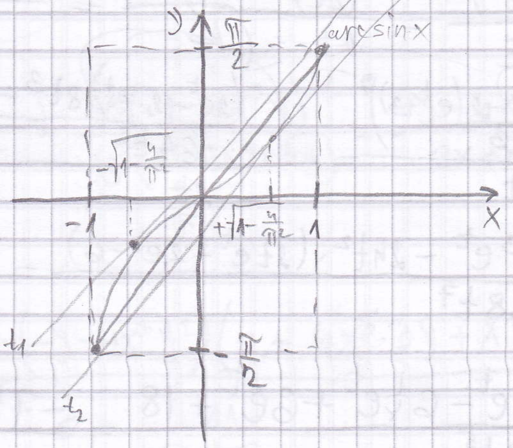
$c = \cos^{-1}\frac{2}{\pi} = \arccos\frac{2}{\pi}$

7.) $f(x) = \arcsin x$

$I = [-1, 1]$

$$f'(c) = \frac{f(b) - f(a)}{b-a} = \frac{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)}{1+1} = \frac{\pi}{2}$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$



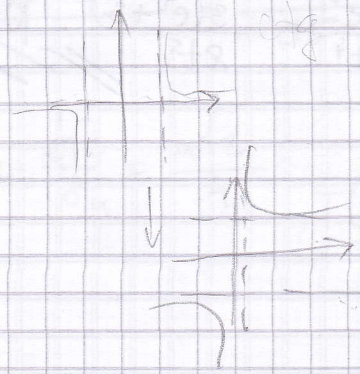
$$\frac{1}{\sqrt{1-c^2}} = \frac{\pi}{2}$$

$$\sqrt{1-c^2} = \frac{2}{\pi} \quad \Big|^2$$

$$1 - c^2 = \frac{4}{\pi^2}$$

$$c^2 = 1 - \frac{4}{\pi^2}$$

$$\boxed{c = \pm\sqrt{1 - \frac{4}{\pi^2}}}$$



8.) $f(x) = \operatorname{ch} x$

$c = 0$

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

$$R_n(x) = \frac{f^{n+1}(x_1)}{(n+1)!} \cdot (x-c)^{n+1} \qquad x_1 \in (c, x) \text{ ili } x_1 \in (x, c)$$

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k + \frac{f^{n+1}(x_1)}{(n+1)!} \cdot x^{n+1} \quad \text{za } x_1 \in (0, x) \text{ ili } x_1 \in (x, 0)$$

$f'(x) = \operatorname{sh} x \longrightarrow f^{(2k)}(x) = \operatorname{sh} x \longrightarrow \frac{e^x - e^{-x}}{2} = \frac{1-1}{2} = 0$

$f''(x) = \operatorname{ch} x \longrightarrow f^{2k}(x) = \operatorname{ch} x \longrightarrow 1$

$f'''(x) = \operatorname{sh} x \longrightarrow f^{2k+1}(x) = \operatorname{sh} x$