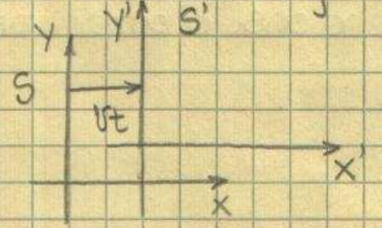
Specijalna teorija relativnosti (A. Einstein, 1905)

1. Neovisnost zakona fizike o izboru koordinatnog sustava

- Galileieve transformacije i Newtonova jednadžba gibanja



$$x = x' + vt$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

$$\frac{d^2x'}{dt'^2} = \frac{d^2x}{dt^2}$$ (akceleracije su jednake u sustavu S i S')

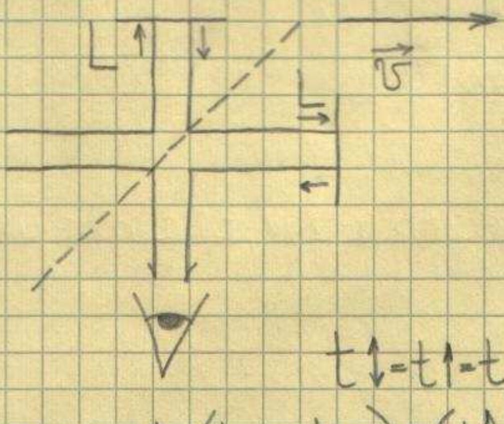
- Galileieve transformacije i elektromagnetizam, "Bljesak" u $\vec{r}=0, t=0$

S: $x^2+y^2+z^2=c^2t^2$ (sfera)

S': $(x'+vt)^2+y'^2+z'^2=c^2t'^2$ (nije sfera, nije OK)

Rješenja: eter ili odustajanje od Galileievih transformacija

2. Michelson-Morleyev eksperiment



$t_{\rightarrow}$: $L + vt_{\rightarrow} = ct_{\rightarrow}$

$$t_{\rightarrow} = \frac{L}{c-v}$$

$$t_{\leftarrow} = \frac{L}{c+v}$$

$t\updownarrow = t\uparrow = t\downarrow$: $\sqrt{L^2+(vt\updownarrow)^2} = ct\updownarrow$, $t\updownarrow = \frac{L}{\sqrt{c^2-v^2}}$

kašnjenje: $\Delta t = (t_{\rightarrow} + t_{\leftarrow}) - (t\uparrow + t\downarrow)$

$$\Delta t = \frac{2L}{c} \cdot \frac{1}{1-v^2/c^2}\left[1-\sqrt{1-v^2/c^2}\right]$$

Michelson i Morley dobiju $\Delta t_{exp.}=0$ (ne mogu izmjeriti), ubijena ideja etera, čeka se nova teorija koja će pokazati pogrešku Michelsona i Morleya. (Njihov se korak u gibanju skratio, što nisu znali, dokazala spec. teorija relat.)

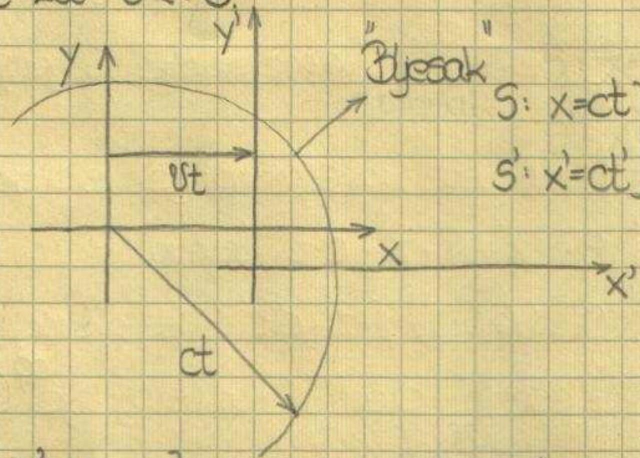
3. Einsteinovi postulati

1EP: Svi fizikalni zakoni imaju isti oblik u svim koordinatnim sustavima koji se jedan u odnosu na drugi gibaju jednoliko po pravcu.

2EP: Brzina svjetlosti je u svim sustavima ista, i ne ovisi o brzini izvora ili promatrača.

4. Lorentzove transformacije

1.) Prelaze u Galileieve transformacije za $v \ll c$.
2.) Simetrične u odnosu na $\pm v$.
3.) Linearne u x, y, z, t.
4.) Sukladne sa 2EP.



$$\left.\begin{array}{l} S: x = ct \\ S': x' = ct' \end{array}\right\} \text{u (1)}$$

$$\left.\begin{array}{l} x' = \gamma(x - vt) \\ x = \gamma(x' + vt') \end{array}\right\} (1)$$

eliminacijom x i x' slijedi:

$$\left.\begin{array}{l} t' = \gamma t + \dfrac{\gamma^2 - 1}{\gamma v} x \\ t = \gamma t' - \dfrac{\gamma^2 - 1}{\gamma v} x' \end{array}\right\} (2) \qquad \begin{array}{l} t' = \gamma\left(t - \dfrac{vx}{c^2}\right) \\ t = \gamma\left(t' + \dfrac{vx'}{c^2}\right) \end{array}$$

$$\left.\begin{array}{l} t' = \dfrac{\gamma t}{c}(c - v) \\ t = \dfrac{\gamma t'}{c}(c + v) \end{array}\right\} \Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

(γ uvrstimo u x, x', t i t' gore u (1) i (2))

Lorentzove transformacije i Bljesak

$S: x^2 + y^2 + z^2 = c^2 t^2$

$S': x'^2 + y'^2 + z'^2 = c^2 t'^2$ (kada uvrstimo gornje izraze za x i t) ✓

5. Transformacije brzina

$S: \vec{u} = \dfrac{d\vec{r}}{dt}$

$S': \vec{u'} = \dfrac{d\vec{r'}}{dt'}$

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

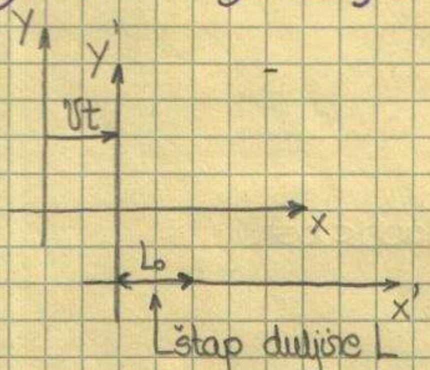
$$u_x' = \frac{dx'}{dt'} = \gamma\left(\frac{dx}{dt}\cdot\frac{dt}{dt'} - \frac{v\,dt}{dt'}\right) = \gamma(u_x - v)\frac{dt}{dt'} \qquad \frac{dt'}{dt} = \gamma\left(1 - \frac{vu_x}{c^2}\right)$$

$$u_x' = \gamma(u_x - v)\cdot\frac{1}{\gamma\left(1 - \frac{vu_x}{c^2}\right)} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{dt}\cdot\frac{dt}{dt'} = u_y \frac{1}{\gamma\left(1 - \frac{vu_x}{c^2}\right)} = \frac{u_y\sqrt{1 - v^2/c^2}}{1 - \frac{vu_x}{c^2}}$$

$$u_z' = \frac{u_z\sqrt{1 - v^2/c^2}}{1 - \frac{vu_x}{c^2}}$$

6. Kontrakcija duljine



S': $L_0 = x_2' - x_1'$ @t'

S: $L = x_2 - x_1$ @t

$$L_0 = x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1) = \gamma L, \quad L \neq L_0, \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} > 1$$

7. Dilatacija vremena

S: $\Delta t = t_2 - t_1$ @x

S': $\Delta t' = t_2' - t_1'$ @x'

$$\Delta t' = \gamma\left(t_2 - \frac{vx}{c^2}\right) - \gamma\left(t_1 - \frac{vx}{c^2}\right) = \gamma(t_2 - t_1) = \gamma\Delta t$$

8. Relativistička dinamika

Količina gibanja: $\vec{p} = \gamma m \vec{v}$

Energija: $E = \gamma m c^2$

Newtonov aksiom: $\frac{d\vec{p}}{dt} = \frac{d}{dt}\frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \vec{F}$

Rad i energija: $W=\int_{poč.}^{kon.} \vec F \cdot d\vec s = \int_{p.}^{k.} \frac{d\vec p}{dt}\cdot d\vec s = \int_{p.}^{k.} \vec v \, d\vec p$

$d(\vec v \cdot \vec p) = \vec v d\vec p + \vec p d\vec v$ ... parcijalna integracija

$$\int_{p.}^{k.} \vec v d\vec p - \int_{p.}^{k.} \vec p d\vec v = \vec v \cdot \vec p \Big|_{p.}^{k.} - m\int \gamma \vec v d\vec v$$

Zbog jednostavnosti: 1D, $v_p = 0$

$$W = \gamma_{kon.} \cdot m \cdot v_{kon.}^2 - m\int_0^{v_{kon.}} \frac{v\,dv}{\gamma}$$

$$= \gamma_{kon.} m v_{kon.}^2 + mc^2 \gamma \Big|_0^{v_{kon.}} = \gamma_{kon.} m v_{kon.}^2 + mc^2\left(\frac{1}{\gamma} - 1\right)$$

$$= \gamma m v^2 + \frac{mc^2}{\gamma} - mc^2 = \gamma mc^2 \underbrace{\left(\frac{v^2}{c^2} + \frac{1}{\gamma^2}\right)}_{1} - mc^2 = \boxed{\gamma mc^2 - mc^2 \equiv E_{kin.}}$$

energija mirovanja

$$E = \gamma mc^2 = mc^2 + E_{kin.}$$

Mehanika fluida

1. Fluid

Tekućina $\underbrace{B = \frac{1}{\kappa} = V\frac{\partial P}{\partial V}}$ ≫ → volumni modul elastičnosti

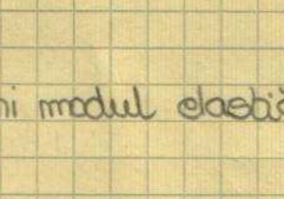
Plin "idealni plin"

$\underbrace{\kappa = \frac{1}{B} = -\frac{1}{V}\frac{\Delta V}{\Delta p} = \frac{1}{P}}$ → koeficijent izotermne kompresije (adijabatska kom

2. Statika fluida

Tlak: $p = \frac{|\vec F_\perp|}{S} = \frac{|d\vec F_\perp|}{dS}$

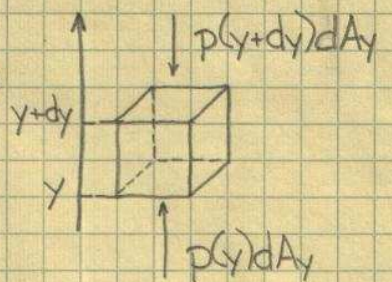
Jednadžba hidrostatske ravnoteže:



Pretpostavke:
- dm u statičkoj ravnini
- tlak ovisi samo o y
- fluid se nalazi u polju gravitacijske sile $\vec g = -g\vec j$

Sile u x i z smjeru se dokidaju.

Sile u y smjeru:



$$dF_y = -g\,dm + p(y)dA_y - p(y+dy)dA_y$$
$$= -g\rho\,dx\,dy\,dz + p(y)\,dx\,dz - p(y+dy)\,dx\,dz$$
$$= 0$$

$$g\rho\,dy = p(y) - p(y+dy)$$

$$-g\rho = \frac{p(y+dy)-p(y)}{dy}$$, lim $dy \rightarrow 0$, kockica je malena

$$-\rho g = \frac{dp}{dy}$$

Hidrostatski tlak

Pretpostavke:

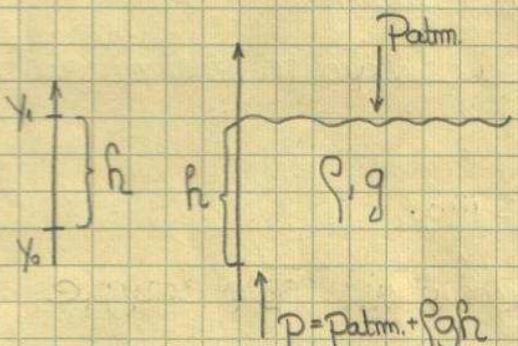
- ispunjena jednadžba hidrostatske ravnoteže
- fluid je nestlačiv
- g je konstanta

$$\frac{dp}{dy} = -\rho g \;/\cdot dy, \int$$

$$\int_{y_0}^{y_1} dp = -\rho g \int_{y_0}^{y_1} dy$$

$$p(y_1) - p(y_0) = -\rho g (y_1 - y_0)$$

$$p(y_0) = p(y_1) + \rho g h$$



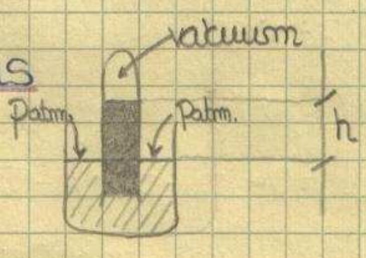
Pascalov princip - u nestlačivom, mirnom fluidu Δp se prenosi na čitav flui

Hidraulički tijesak (hidraulička preša)

$$\Delta p = \frac{F_1}{S_1} = \frac{F_2}{S_2}$$ → Pascalov princip

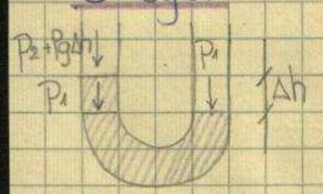
Isti rezultat dobijemo i principom virtualnog rada, $\delta W = -F_1 \Delta y_1 + F_2 \Delta y_2$ te očuvanjem količine fluida, $\Delta V = -S_1 \Delta y_1 = S_2 \Delta y_2$.

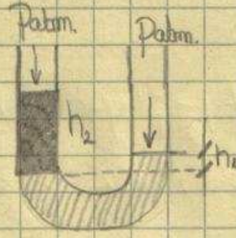
Torricellijev pokus



$$p_{atm.} = \rho(\text{tekućine})\, g h$$

U cijev





$\rho g \Delta h = \Delta p$ $\qquad$ $\rho_1 h_1 = \rho_2 h_2$

Barometarska formula

Pretpostavke:

- T=konstanta
- idealni plin, $pV=nRT$, $n=\frac{m}{M}$
- hidrostatska ravnoteža
- g=konstanta

Jednadžba hidrostatske ravnoteže:

$$\frac{dp}{dh} = -\rho g = -\rho_0 g \frac{p}{p_0} \Big/ \cdot \frac{dh}{p}, \int_{h=0}^{h=H}$$

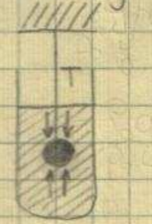
$$\int_0^H \frac{dp}{p} = -\frac{\rho_0}{p_0} g \int_0^H dH$$

$$\ln p \Big|_{p_0}^{p_H} = -\frac{\rho_0}{p_0} gH \Big/ e^{\wedge}$$

$$\boxed{p_H = p_0 \cdot e^{-\frac{\rho_0}{p_0}gH}}$$

Arhimedov zakon (uzgon)

Tijelo uronjeno u tekućinu gubi na težini za težinu istisnute tekućine.



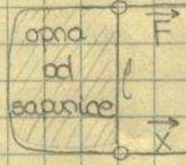
$T = mg - V\rho g$

ρ – tekućine

(3) Površinska napetost

$$\sigma = \frac{\Delta W}{\Delta S} = \frac{\Delta F}{\Delta l}$$

σ – napetost površine (sigma)

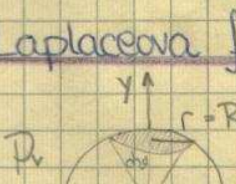


Princip virtuelnog rada: povećanje površine - $\Delta S = 2l\Delta x$

obavljeni rad - $\Delta W = F\Delta x$

$\sigma = \frac{\Delta W}{\Delta S} = \frac{F}{2l} \Rightarrow F = \sigma \cdot 2l$ jer opna ima dvije strane

Laplaceova formula



Površina: $dA = (Rd\vartheta)^2 \pi$

Opseg: $dl = 2(Rd\vartheta)\pi$

Ravnoteža: $dF_y = 0 = (p_u - p_v)dA - \sigma\, dl\, d\vartheta$

$\boxed{p_u - p_v = \frac{2\sigma}{R}}$ voda u zraku, zrak u vodi $\qquad$ $\boxed{p_u - p_v = \frac{4\sigma}{R}}$ zrak u zraku

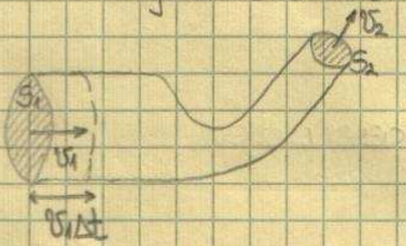
4. Dinamika fluida

Pretpostavke:

- fluid je nestlačiv, izoterman, nije viskozan
- tok je stacionaran i laminaran

Jednadžba kontinuiteta:

- količina fluida u volumenu je stalna



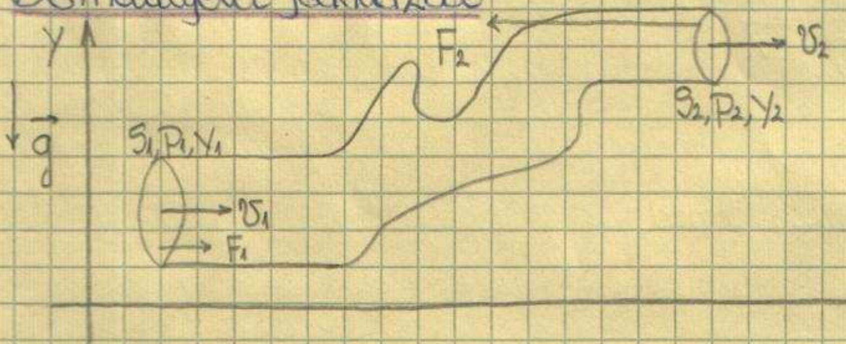
ulaz: $\Delta V_1 = S_1 v_1 \Delta t$

izlaz: $\Delta V_2 = S_2 v_2 \Delta t$

$\Delta V_1 + \Delta V_2 = 0$

$\Rightarrow S_1 v_1 = S_2 v_2$

Bernoullijeva jednadžba



Iz jedn. kontinuiteta:

$S_1 v_1 = S_2 v_2 = S v$

RAD SILA: $\Delta W_1 = F_1 \Delta x_1 = S_1 p_1 v_1 \Delta t = S p_1 v \Delta t$

$\Delta W_2 = -F_2 \Delta x_2 = -S_2 p_2 v_2 \Delta t = -S p_2 v \Delta t$

UKUPNI RAD: $\Delta W = \Delta W_1 + \Delta W_2 = S v (p_1 - p_2) \Delta t$

OČUVANJE ENERGIJE: $\Delta W = \Delta E_{kin.} + \Delta E_{pot.}$

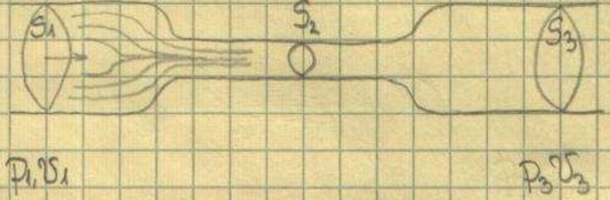
$\Delta E_{kin.} = \frac{1}{2} \Delta m (v_2^2 - v_1^2) = \frac{1}{2} \rho S v \Delta t (v_2^2 - v_1^2)$

$\rho \Delta V, \Delta V = S v \Delta t$

$\Delta E_{pot.} = \Delta m g (y_2 - y_1) = \rho S v \Delta t g (y_2 - y_1)$

$$\boxed{p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g (y_2 - y_1)}$$ ili $$\boxed{p + \frac{1}{2}\rho v^2 + \rho g y = \text{konst.}}$$

5. Viskoznost

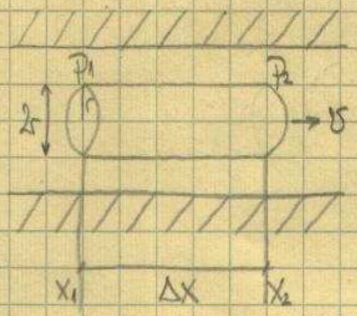


$p_3 = p_1$ $\quad$ $p_3 < p_1$

$v_3 = v_1$

SILA: $F_\eta = \eta S \frac{v}{t} = \eta S \frac{dv}{dt}$ $\quad$ η... koef. viskoznosti

Poisuelleov zakon



SILA NA VALJAK:

$$F_\eta = \eta\, 2r\pi (x_2 - x_1)\frac{dv}{dr} = r^2\pi(p_1 - p_2)$$

$$2\eta(x_2 - x_1)\frac{dv}{dr} = r(p_1 - p_2)$$

$$dv = \frac{-r\Delta p}{2\eta \Delta x}\, dr \quad // \int_R^r$$

$$v(r) - v(R) = -\frac{1}{2\eta}\frac{\Delta p}{\Delta x}\cdot \frac{r^2}{2}\Bigg|_R^r$$

$$v(r) = \frac{R^2}{4\eta}\cdot\frac{\Delta p}{\Delta x}\left(1 - \frac{r^2}{R^2}\right)$$

↳ brzina kojom tekućina teče u centru cijevi

Tok:

$$q = \frac{\Delta V}{\Delta t}, \quad \Delta V = Av\Delta t$$

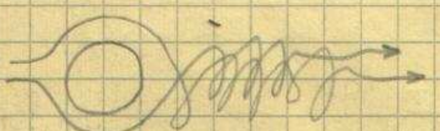
$$q = \int v\, da = \int_0^R v(r)\overbrace{2r\pi\, dr}^{da} = \frac{\pi}{8\eta}\cdot\frac{\Delta p}{\Delta x}\cdot R^4$$

Turbulencija:

Laminarna:

mala brzina

Turbulentna:



velika brzina

Prelaz laminarnog u turbulentno strujanje

Reynoldsov broj

$$Re = \frac{\rho v l}{\eta} = \frac{v l}{\nu}$$

$\nu \to$ kinetička viskoznost

Stokesov zakon

- kugla, sporo gibanje: $F_\eta = 6\pi\eta R v$
- općenitije: $F_\eta = \frac{1}{2} C_d \rho S v^2$

↳ koef. otpora

$$C_d = \frac{24}{Re} + \frac{6}{1+\sqrt{Re}} + \frac{2}{5}$$

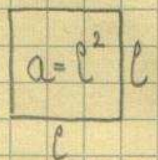
Toplina

1. **Termodinamička temperatura** je mjera kinetičke energije.
"apsolutna nula" (kada se čestice u plinu prestanu gibati)

2. **Toplinsko širenje**

DULJINA: $\Delta l = \alpha l \Delta t$ → promjena temperature
↳ koef. linearnog rastezanja

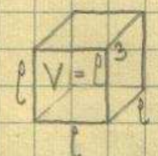
POVRŠINA:



$$\frac{\Delta a}{a} = \frac{a'-a}{a} = \frac{(l+\Delta l)^2 - l^2}{l^2} = \frac{1}{l^2}\left(2l\Delta l + \Delta l^2\right) = \frac{2\Delta l}{l} = 2\alpha \Delta t = \beta \Delta t$$

↳ koef. površinskog rastezanja $\beta = 2\alpha$

VOLUMEN:



$$\frac{\Delta V}{V} = \frac{V'-V}{V} = \frac{(l+\Delta l)^3 - l^3}{l^3} = \frac{1}{l^3}\left(l^3 + 3l^2\Delta l + 3l\Delta l^2 + \Delta l^3 - l^3\right) =$$

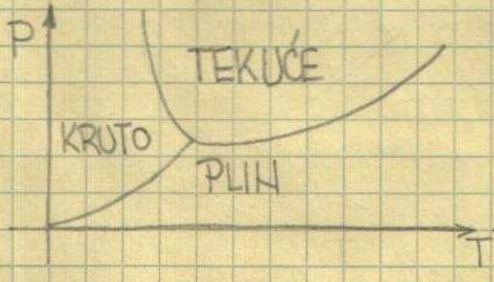
$$= \frac{3\Delta l}{l} + \cancel{\frac{3\Delta l^2}{l^2}} + \cancel{\frac{\Delta l^3}{l^3}} = \frac{3\Delta l}{l} = 3\alpha\Delta t = \gamma \Delta t$$

↳ koef. volumnog rastezanja $\gamma = 3\alpha$

3. **Plinski zakoni**

Jednadžba stanja idealnog plina

$pV = nRT$, $R = 8.314\ J/molK$ ili $pV = NkT$, $k = 1.381 \cdot 10^{-23}\ J/K$, Boltzmannova konstanta



... fazni dijagram vode

Avogadrov zakon: Pri jednakom tlaku i temperaturi, isti volumeni različitih plinova sadrže jednak broj čestica.

3 empirijska zakona: (oni su međusobno ovisni)

1) Boyle-Mariotteov zakon $T = $ konst., $pV = $ konst. (izotermno)
2) Gay-Lussacov $p = $ konst., $\frac{V}{T} = $ konst. (izobarno)
3) Charles-Gay-Lussac $V = $ konst. $\frac{p}{T} = $ konst. (izohorno)

Izvod za izohorni proces (analogno i za sve ostale procese): →

$$(p_0,V_0,T_0) \xrightarrow[\text{izobarno}]{} (p_0,V_1,T_1) \xrightarrow[\text{izotermno}]{} (p_1,V_0,T_1)$$

$$\frac{V_0}{T_0}=\frac{V_1}{T_1} \qquad p_0V_1=p_1V_0$$

$$T_0=\frac{V_0}{V_1}T_1 \qquad p_0=\frac{p_1}{V_1}V_0$$

$$\frac{p_0}{T_0}=\frac{p_1}{T_1}$$

Standardni uvjeti: $p_0 = 1\ \text{atm} = 101\ 325\ \text{Pa}$, $T_0 = 0^\circ C - 273.15\ \text{K}$, $V_0 = 22.91383\ \text{L}$

Izvod jednadžbe stanja:

$$(p_0,V_0,T_0) \xrightarrow[\text{izobarno}]{} (p_0,V',T) \xrightarrow[\text{izotermno}]{} (p,V,T)$$

$$\frac{V_0}{T_0}=\frac{V'}{T} \qquad p_0V'=pV$$

$$V'=V_0\frac{T}{T_0} \qquad pV=\frac{p_0V_0}{T_0}T=RT \Rightarrow \text{općenito: } pV=nRT=N\frac{R}{N_A}T$$

$$n=\frac{N}{N_A} \qquad \frac{R}{N_A}\to k$$

Daltonov zakon parcijalnih tlakova

$$pV=nRT \Rightarrow \begin{array}{l} p_1V=n_1RT \\ p_2V=n_2RT \\ \vdots \end{array}$$

$$p=p_1+p_2+\dots \qquad n=n_1+n_2+\dots$$

4. Kalorimetrija

Specifični toplinski kapacitet $\qquad c_{H_2O}=4190\ \text{J K}^{-1}\text{kg}^{-1}$

$\Delta Q = cm\Delta t$ $\qquad$ $\Delta Q = Cn\Delta t$

↳ spec. toplinski kapacitet $[\text{JK}^{-1}\text{kg}^{-1}]$ $\qquad$ ↳ molarni topl. kapac. $[\text{JK}^{-1}\text{mol}^{-1}]$

Dulong Petitovo pravilo

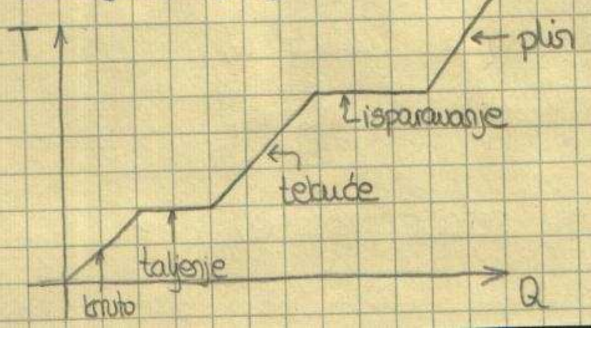
Pravilo tvrdi da je za krute tvari $c_p \approx c_v \approx 3R$ tj. $C_p \approx C_v \approx 3R$.

Meyerova relacija (vrijedi samo za plinove)

$C_p - C_v = R$

Adijabatski koeficijent plina $\kappa = \frac{c_p}{c_v}$

Promjene agregatnih stanja, latentne topline



$Q_{taljenja} = m l_t$ → spec. laten. toplina taljenja $[\text{J kg}^{-1}]$

$Q_{isparavanja} = m l_i$ → spec. laten. toplina isparavanja $[\text{J kg}^{-1}]$

5. Vođenje topline

KONVEKCIJA: medij, gibanje

RADIJACIJA: elektro-magnetno zračenje

KONDUKCIJA: medij, mirovanje

Toplinski tok: $\Phi = \frac{dQ}{dt} = \underbrace{\lambda S \frac{\Delta T}{\Delta x}}$ koef. toplinske vodljivosti

"Ohmov zakon" $\Phi = \frac{\Delta T}{R}$, $R = \frac{\Delta x}{\lambda S}$

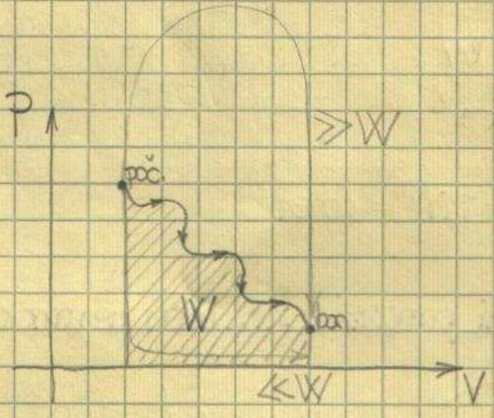
6. Prvi zakon termodinamike

$d'Q = dU + d'W$ → funkcija procesa

dU → funkcija stanja

$dW = pdV$

$W = \int pdV$



Izvod Meyerove relacije:

$d'Q = nC_v dT = dU \longrightarrow \quad d'Q = dU + d'W \quad \longleftarrow dW = pdV = nRdT$

$d'Q = nC_p dT \quad\quad \cancel{n}C_p d\cancel{T} = \cancel{n}C_v d\cancel{T} + \cancel{n}R d\cancel{T}$

$$C_p - C_v = R$$

1-atomni plinovi: $C_p = \frac{5}{2}R$, $C_v = \frac{3}{2}R$, $\kappa = \frac{5}{3}$

2-atomni plinovi: $C_p = \frac{7}{2}R$, $C_v = \frac{5}{2}R$, $\kappa = \frac{7}{5}$

3-atomni plinovi: $C_p = 4R$, $C_v = 3R$, $\kappa = \frac{4}{3}$

Adijabata

$d'Q = 0$ (sustav ne može ili ne stigne razmijeniti toplinu s okolinom)

$d'Q = dU + d'W = 0 \quad\quad pV = nRT$

$= nC_v dT + pdV \quad\quad pdV + Vdp = nRdT$

$= \frac{\cancel{n}C_v}{\cancel{n}R}(pdV + Vdp) + pdV \quad\quad dT = \frac{(pdV + Vdp)}{nR}$

$= \left(\frac{C_v}{R} + 1\right)pdV + \frac{C_v}{R}Vdp \quad / \cdot \frac{R}{pV}$

$$\frac{C_V}{R}\frac{dp}{p} = -\left(\frac{C_V}{R}+1\right)\frac{dV}{V}$$

$$\frac{dp}{p} = -\left(\frac{C_V+R}{C_V}\right)\frac{dV}{V} = \frac{C_p dV}{C_V V} = \boxed{-\kappa\frac{dV}{V}}$$

$$\frac{dp}{p} = -\kappa\frac{dV}{V}$$

$$\ln p\Big|_1^2 = -\kappa\ln V\Big|_1^2$$

$$\ln\frac{p_2}{p_1} = -\kappa\ln\frac{V_2}{V_1}$$

$$\frac{p_2}{p_1} = \left(\frac{V_2}{V_1}\right)^{-\kappa}$$

$$\frac{p_2}{p_1}\cdot\frac{V_2^{\kappa}}{V_1^{\kappa}} = 1$$

$$p_1V_1^{\kappa} = p_2V_2^{\kappa} = konst.$$

Rad pri reverzibilnim procesima:

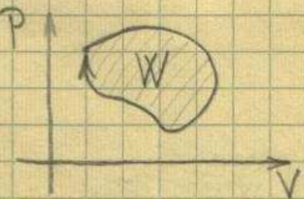
Po izotermi: $W_T = nRT\ln\frac{V_{kon.}}{V_{poč.}}$

Po izobari: $W_p = p(V_{kon.}-V_{poč.})$

Po izohori: $W_V = 0$

Po adijabati: $W_{Ad.} = \frac{nR}{\kappa-1}(T_{poč.}-T_{kon.})$

7. Kružni procesi, 2. zakon termodinamike

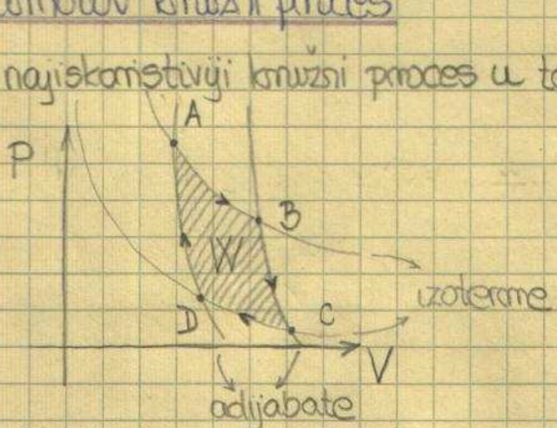


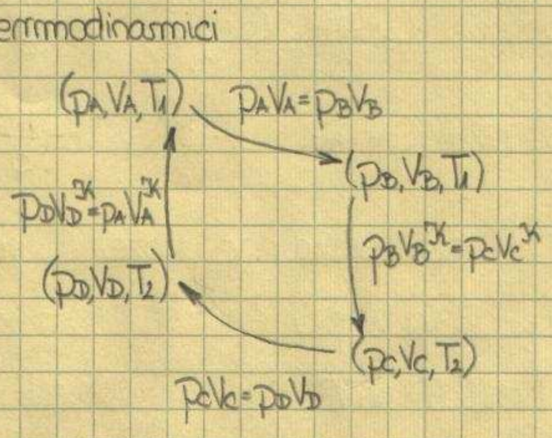
$$W = \oint dW = \oint p\,dV = Q$$

2. zakon termodinamike: Nemoguće je napraviti toplinski stroj koji bi svu toplinu iz jednog spremnika pretvorio u rad.

Carnotov kružni proces

- najiskoristiviji kružni proces u termodinamici





Iskoristivost:

$$\eta = \frac{Dobiveno}{Uloženo} = \frac{Rad}{Toplina\ iz\ T_1}$$

$$Rad = \oint dW = W_{AB}+W_{BC}+W_{CD}+W_{DA}$$

$$W_{AB} = nRT_1\ln\frac{V_B}{V_A} = Q_1 > 0$$

$$W_{BC} = \frac{nR}{\kappa-1}(T_1-T_2) = -W_{DA}$$

$$W_{CD} = nRT_2\ln\frac{V_D}{V_C} = Q_2 < 0$$

$$\eta = \frac{W_{AB}+W_{BC}+W_{CD}+W_{DA}}{Q_1} = 1 + \frac{W_{CD}}{W_{AB}} = 1 + \frac{T_2 \ln\frac{V_D}{V_C}}{T_1 \ln \frac{V_B}{V_A}} = 1 - \frac{T_2}{T_1} = \eta_{Carnot}$$

- Kod inverznog Carnotovog procesa (obrnute strelice) rad je negativan jer ga mi dobavljamo.

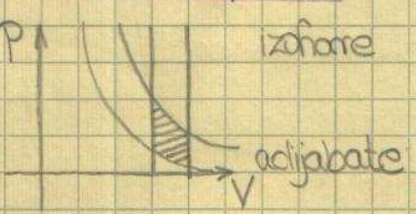
Toplinska pumpa (∇... efikasnost)

$$\nabla_{TP} = \frac{|Q_1|}{|W|}$$

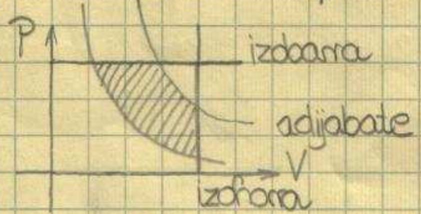
Hladnjak

$$\nabla_H = \frac{|Q_2|}{|W|}$$

Otto kružni proces



Dieselov kružni proces



8. Entropija - funkcija stanja $\boxed{dS = \frac{d'Q}{T}}$

Kako se mijenja entropija u jednom ciklusu Carnotovog kružnog procesa:

$$\frac{|Q_2|}{Q_1} = \frac{T_2}{T_1}$$

$$\Delta S_1 = \int_A^B dS = \int_A^B \frac{d'Q}{T_1} = \frac{Q_1}{T_1}, \quad \Delta S_2 = \int_D^C dS = \int_D^C \frac{d'Q}{T_2} = \frac{|Q_2|}{T_2}, \quad \Delta S = \Delta S_1 + \Delta S_2$$

Clausiusov teorem $\rightarrow \oint dS = 0$

Entropija idealnog plina

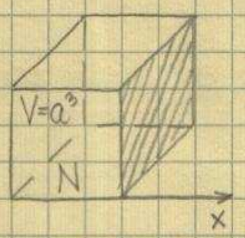
$$\Delta S = S_2 - S_1 = \int_1^2 \frac{d'Q}{T} = \int_1^2 \frac{dU + pdV}{T} = \int_1^2 \frac{dU}{T} + \int_1^2 \frac{pdV}{T} = \int_1^2 \frac{nC_v dT}{T} + \int_1^2 \frac{nR}{V} dV = nC_v \ln T \Big|_1^2 + nR \ln V \Big|_1^2$$

$$= nC_v \ln \frac{T_{kon.}}{T_{poč.}} + nR \ln \frac{V_{kon.}}{V_{poč.}}$$

Molekularno-kinetička teorija plinova

1. Model idealnog plina

2.) Tlak plina



- pri udaru i-te čestice o desni zid na zid se prenosi: $\Delta p_{x/i} = 2p_{x/i} = 2m_i v_{x/i}$
- vjerojatnost da čestica s $v_{x/i} > 0$ udari u desni zid u Δt: $q_i = \frac{v_{x/i}\Delta t}{a}$

$$\text{Tlak} = \frac{\text{Sila}}{\text{Površina}} = \frac{\Delta p/\Delta t}{\text{Površina}}$$

$$p = \frac{\frac{1}{\Delta t}\sum^{*} \Delta p_{x/i}\, q_i}{a^2} = \frac{1}{a^2\Delta t}\sum^{*} 2m_i v_{x/i}\frac{v_{x/i}\Delta t}{a} = \frac{2}{a^3}\sum^{*} m_i \overbrace{v_{x/i}^2}^{\frac{1}{3}v_i^2} = \frac{2}{3a^3}\sum_i \frac{m_i v_i^2}{2} = \frac{2}{3V}\sum_i E_{kin/i} = \frac{2N}{3V}\overline{E_{kin}}$$

3.) Ekviparticijski teorem

- 1-atomna mol. ima 3 stupnja slobode, 2-atomna 5, 3 i više atomna 6

i... stupanj slobode

$$U = Ni\frac{kT}{2} = ni\frac{RT}{2}, \quad \overline{E}_{kin.} = \frac{3}{2}kT$$

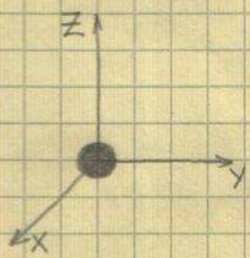
4.) Toplinski kapacitet plina

$$C_V = \frac{i}{2}R$$

$$C_p = \left(1 + \frac{i}{2}\right)R$$

$$\kappa = 1 + \frac{2}{i}$$

5.) Toplinski kapacitet krutih tvari



$$E_{kin.} = \frac{m}{2}\left(v_x^2 + v_y^2 + v_z^2\right)$$

$$E_{pot.} = \frac{k_x}{2}x^2 + \frac{k_y}{2}y^2 + \frac{k_z}{2}z^2$$

$i = 6$

$\Rightarrow \quad C_V = 3R$