ELASTIČNOST

δ - relativna deformacija

$$\delta = \frac{\text{PROMJENA}}{\text{RAVNOTEŽNO STANJE}}$$

σ - naprezanje

$$\sigma = \frac{\text{SILA}}{\text{POVRŠINA}} = \frac{F}{S}\ [Pa]$$

$$\text{MODUL ELASTIČNOSTI} = \frac{\text{NAPREZANJE}}{\text{REL. DEF.}} = \frac{\sigma}{\delta}$$

VLAK

$$\delta_L = \frac{\Delta L}{L}$$

$$\sigma = \frac{F}{S}$$

YUNGOV MODUL ELASTIČNOSTI

$$E = \frac{\sigma}{\delta_L} = \frac{\frac{F}{S}}{\frac{\Delta L}{L}}\ [Pa]$$

HOOKEOV ZAKON

- vrijedi u lin. području
- sila kojom se pokušava sabiti ili produljiti šipka proporcionalna je produljenju

$$F = ES\frac{\Delta L}{L} = k\Delta L = kx$$

POISSONOV OMJER

- govori kako i koliko će se šipka stanjiti

$$\delta_L = \frac{\Delta L}{L} \text{ - longitudino produljenje}$$

$$\delta_x = \frac{\Delta x}{x} = \frac{\Delta y}{y} \text{ lateralno sažimanje}$$

$$\mu = -\frac{\frac{\Delta x}{x}}{\frac{\Delta L}{L}} = \frac{\delta_x}{\delta_L}$$

TLAK

- sila iz svih smjerova, djeluje na sve površine

VOLUMNI MODUL ELASTIČNOSTI

$$B = -\frac{\sigma}{\delta_V} = -\frac{F/S}{\frac{\Delta V}{V}} \rightarrow p$$

KOMPRESIBILNOST: $\varkappa = B^{-1} = -\frac{1}{V}\frac{\Delta V}{\Delta p}$

VEZA E, B i μ:

- tlak je vlak u svim smjerovima
- djelujmo na jednu stranicu

$$V \rightarrow V + \Delta V_a = a\left(1+\frac{\Delta a}{a}\right)b\left(1-\mu\frac{\Delta a}{a}\right)c\left(1-\mu\frac{\Delta a}{a}\right)$$

$$= abc\left(1+\frac{\Delta a}{a}\right)\left(1-\mu\frac{\Delta a}{a}\right)^2$$

$$= V\left(1+\frac{\Delta a}{a}\right)\left(1-2\mu\frac{\Delta a}{a}+\ldots\right)$$

$$= V\left(1+(1-2\mu)\frac{\Delta a}{a}+\ldots\right)$$

$$= V\left(1+(1-2\mu)\frac{\sigma}{E}+\ldots\right)$$

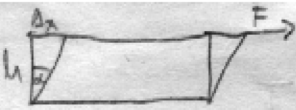
$$V+\Delta V = V\left(1+(1-2\mu)\frac{\sigma}{E}+\ldots\right)^3$$

$$= V\left(1+3(1-2\mu)\frac{\sigma}{E}+\ldots\right)$$

$$\frac{\Delta V}{V} = 3(1-2\mu)\frac{\sigma}{E} = -\frac{\sigma}{B} \text{ razlažanje}$$

$$\Rightarrow E = 3B(1-2\mu)$$

SMIK



TORZIJA ŠIPKE $\Delta D = \frac{\Delta M}{\varphi}$

- izvodimo za cijev pa ju „popunimo"

Modul smicanja:

$$G = \frac{\sigma}{\delta_x} = \frac{F/S}{\frac{\Delta x}{h}}$$

$$G \simeq \frac{E}{3}$$

KUT ZAKRETA

$$r\varphi = L\delta = \Delta x$$

$$\varphi = \frac{L}{r}\delta$$

$$\varphi = \frac{L}{r}\cdot\frac{\sigma}{G}$$

MOMENT SILE

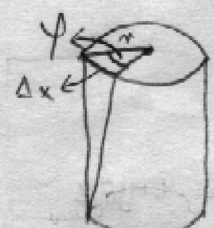
$$\Delta M = \Delta F \cdot r = \sigma \Delta A \cdot r = \sigma 2\pi r^2 \Delta r$$

KONSTANTA TORZIJE:

$$\Delta D = \frac{\cancel{\sigma} 2\pi \Delta r\, r^2}{\frac{L}{r}\cdot\frac{\cancel{\sigma}}{G}} = \frac{2\pi r^3 \Delta r\, G}{L}$$

Punjenje šipke:

$$D = \int_{r_0}^{r=R} \Delta D = \int_{r_0}^{R} \frac{2\pi G}{L} r^3 \Delta r = \frac{\pi R^4 G}{2L}$$



$$\delta_x = \frac{\Delta x}{L}$$

$$M = D\phi$$

ENERGIJA (vlak)

Sila $F = F(\Delta L) = SE\frac{\Delta L}{L}$

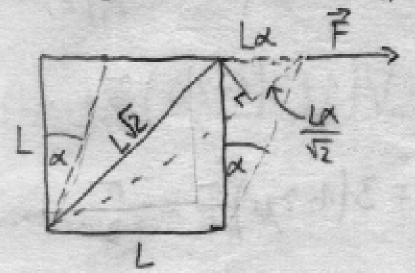
$$dW = F(\Delta L)\cdot d(\Delta L) = \frac{SE}{L}\Delta L\, d(\Delta L)$$

Ukupan rad: $\int dW = \int_0^{\Delta L} \frac{SE}{L}\Delta L\, d(\Delta L) = \frac{SE(\Delta L)^2}{2L} \Rightarrow$ potencijalna

Volumna gustoća energije:

$$\varepsilon = \frac{E_{pot.}}{V} = \frac{E_{pot.}}{SL} = \frac{1}{\cancel{S}L}\frac{\cancel{S}E(\Delta L)^2}{2L} \qquad \varepsilon = \frac{E}{2}\left(\frac{\sigma}{E}\right)^2 = \frac{\sigma^2}{2E}$$

SMIK - veza modula elastičnosti



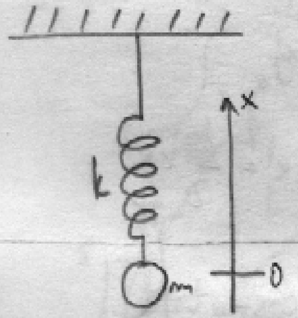
$$\delta_L = \frac{\delta L}{L} = \frac{\theta}{2}$$

$$\frac{\Delta L}{L} = \frac{\theta}{2} + \mu\frac{\sigma}{E} = \frac{\sigma}{E} + \mu\frac{\sigma}{E}$$

$$= \frac{\sigma}{E}(1+\mu) = \frac{\sigma}{2G}$$

$$\boxed{\frac{1}{E} = \frac{1}{3G} + \frac{1}{9B}} \longleftarrow \boxed{E = 2G(1+\mu)} \qquad \boxed{G = (1-2\mu)\frac{3B}{2}}$$

TITRANJE



$F=-kx$

j. gibanja:

$$m\ddot{x}(t)=-kx(t)\ ,\ \cancel{x(0)=A\ ,\ \dot{x}(0)=0}$$

$$m\ddot{x}(t)+kx(t)=0 \qquad \omega_0^2=\frac{k}{m}$$

$$\ddot{x}(t)+\omega_0^2x(t)=0$$

$$r^2+\omega_0^2=0$$

$$r=\pm\omega i$$

$$x_1(t)=e^{\omega_0 i t} \qquad x_2(t)=e^{-\omega_0 i}$$

$$x(t)=C_1x_1(t)+C_2x_2(t) \Rightarrow \text{početni uvjeti}$$

$$\cancel{x(0)=A=C_1+C_2}$$

$$\cancel{\dot{x}(0)=0=i\omega_0C_1-i\omega_0C_2}$$

JEDNOSTAVNIJI PUT IZ KNJIGE

$\to\ x(t)\in\mathbb{R}$

$$C_1=C_2^*=\frac{A}{2}e^{i\phi} \text{ gdje je } A,\phi\in\mathbb{R}$$

$$x(t)=\frac{A}{2}e^{i\phi}e^{\omega_0 i t}+\frac{A}{2}e^{-i\phi}e^{-\omega_0 i t}$$

$$x(t)=\frac{A}{2}\left(e^{(\phi+\omega_0t)i}+e^{(-\phi-\omega_0t)i}\right)$$

$$x(t)=\frac{A}{2}\left(\cos(\phi+\omega_0t)+i\sin(\phi+\omega_0t)+\cos(-\phi-\omega_0t)+i\sin(-\phi-\omega_0t)\right)$$

$$=\frac{A}{2}\left(\cos(\phi+\omega_0t)+\cancel{i\sin(\phi+\omega_0t)}+\cos(\phi+\omega_0t)-\cancel{i\sin(\phi+\omega_0t)}\right)$$

$$=A\cos(\phi+\omega_0t)$$

PERIOD

$$\omega_0T=2\pi \Rightarrow \omega_0=\frac{2\pi}{T}=\sqrt{\frac{k}{m}}$$

$$\Rightarrow T=2\pi\sqrt{\frac{m}{k}}$$

početni uvjeti $t=t_0$

$$x_0=x(t_0)=A\cos(\phi+\omega_0t)$$

$$v_0=\dot{x}(t_0)=-\omega_0A\sin(\phi+\omega_0t)$$

$$t_0=0$$

$$x(0)=A\cos(\phi)=x_0 \Rightarrow A=\frac{x_0}{\cos(\phi)}$$

$$\dot{x}(0)=-\omega_0A\sin(\phi)=v_0 \Rightarrow A=\frac{v_0}{-\omega_0\sin(\phi)}$$

$$A=\sqrt{x_0^2-\left(\frac{v_0}{\omega_0}\right)^2}$$

$$\frac{x_0}{\cos(\phi)}=\frac{v_0}{-\omega_0\sin(\phi)}$$

$$\mathrm{tg}\,\phi=\frac{v_0}{-\omega_0x_0}$$

ENERGIJA TITRANJA

$$\Delta U + \Delta T = \text{konst.}$$

$$\Delta U \equiv \Delta W = -\int_p^x F\,dx = -\int_p^x m\frac{dv}{dt}\,dx$$

$$= -\int_p^k m\,v\,dv = -m\frac{v^2}{2}\Big|_p^k = -\frac{m}{2}\left(v_k^2 - v_p^2\right) = -\Delta T$$

$$E = U + T = \ldots = \frac{1}{2}kA^2$$

$$U(x=0) \equiv 0$$

$$U(x) = \int_0^x F(x')\,dx'$$

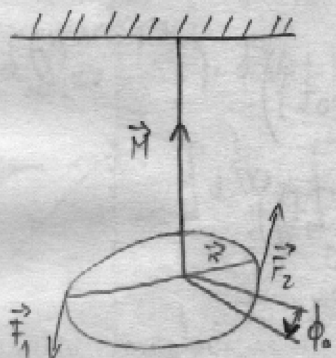
$$= \int_0^x kx'\,dx' = \frac{1}{2}kx^2$$

$$U(t) = \frac{1}{2}k\left[A\cos(\omega t + \phi)\right]^2$$

$$T(t) = \frac{1}{2}m(v(t))^2$$

$$= \frac{1}{2}m\left(-\omega A\sin(\omega t + \phi)\right)^2$$

TORZIONO NJIHALO



$$\vec{M} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$$

$$\vec{M} = -\vec{M}_E \sim \phi$$

konst. torzije:

$$D = \frac{\pi R^4 G}{2L}$$

jed. gib. $M_E = I\alpha = -D\phi$

$$\alpha = \frac{d^2\phi}{dt^2}$$

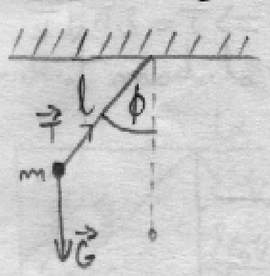
$$\ddot{\phi} + \omega_T^2\phi = 0 \qquad \omega_T = \sqrt{\frac{D}{I}}$$

$$\phi(0) = 0$$

$$\phi(t) = \phi\cos\omega_T t$$

$$T = 2\pi\sqrt{\frac{I}{D}} \Leftarrow \omega_T T = 2\pi$$

MATEMATIČKO NJIHALO



$$I = ml^2$$

$$G = mg$$

$$M = -mgl\sin\phi$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

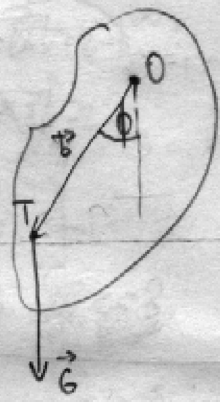
$$M = I\alpha = ml^2\frac{d^2\phi}{dt^2} = -mgl\sin\phi$$

$$\ddot{\phi} + \omega_M^2\sin\phi = 0 \qquad \omega_M = \sqrt{\frac{g}{l}}$$

$\sin\phi = \phi$ za male ϕ

$$\ddot{\phi} + \omega_M^2\phi = 0 \Rightarrow \phi(t) = \phi_0\cos(\omega_M t)$$

FIZIČKO NJIHALO



$$\vec{M} = I\vec{\alpha} \qquad I\frac{d^2\phi}{dt^2} = -bmg\sin\phi$$

$$\vec{M} = \vec{b} \times \vec{G}$$

$$\ddot{\phi} + \frac{bmg}{I}\sin\phi = 0$$

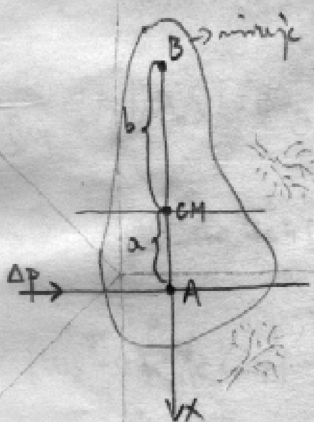
$$M = bmg\sin\phi$$

$$\ddot{\phi} + \omega_F^2\phi = 0 \qquad \omega_F^2 = \frac{bmg}{I}$$

$$\phi(t) = \phi_0\cos(\omega_F t + \varphi) \qquad \omega_F T = 2\pi$$

$$T = 2\pi\sqrt{\frac{bmg}{I}}$$

CENTAR UDARA



$$\Delta p = m v_{CM}$$

$$a\Delta p = I_{CM}\omega$$

$$v(x) = v_{CM} + \omega x = \frac{\Delta p}{m} + \frac{a\Delta p}{I_{CM}}x$$

$$v(x) = 0 \Rightarrow$$

$$\Delta p\left(\frac{1}{m} + \frac{a}{I_{CM}}x\right) = 0$$

$$x = -\frac{I_{CM}}{ma} = -b$$

$$l = a + b = l_{reducirana}$$

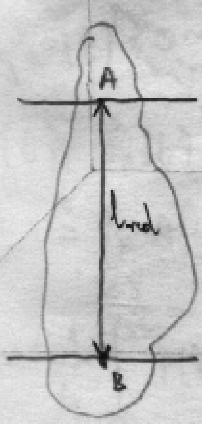
$$l = a + b = l_{red}$$

$$l = \frac{I_{CM}}{mb} + b = \overbrace{\frac{I_{CM} + mb^2}{mb}}^{\text{Steiner}} = \frac{I_B}{mb} = l_{red}$$

REDUCIRANA DULJINA ŠTAPA: $b = \frac{l}{2}$

$$l_{red} = \frac{I_B}{mb} = \frac{I_{CM} + mb^2}{mb} = \frac{\frac{ml^2}{12} + \frac{ml^2}{4}}{m\left(\frac{l}{2}\right)} = \frac{\frac{4\cancel{m}l^2}{12}}{\cancel{m}\frac{l}{2}} = \frac{2}{3}l$$

REVERZIONO NJIHALO



$$I_A = K_A^2 m = (K_0^2 + a^2)m$$

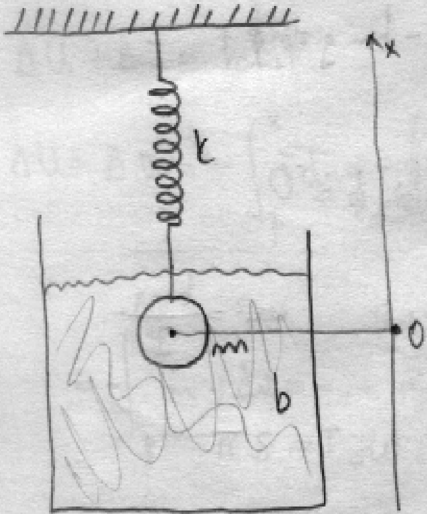
$$I_B = K_B m = (K_0^2 + b^2)m$$

$$\omega_a^2 = \frac{mga}{I_A} = \frac{ga}{K^2 + a^2} = \frac{g}{l_{red}}$$

$$b = l_{red} - a = \left(\frac{K_0^2}{a} + a\right) - a = \frac{K_0^2}{a}$$

$$\omega_b = \frac{mga}{I_B} = \frac{ga}{(K_0^2 + b^2)} = \ldots = \frac{ga}{K_0^2 + a^2} = \omega_a^2$$

PRIGUŠENO TITRANJE



jed. gibanja:

$$m\ddot{x}(t) = -kx(t) - b\dot{x}(t)$$

$$2\delta = \frac{b}{m} \qquad \omega_0^2 = \frac{k}{m}$$

$$\ddot{x}(t) + 2\delta\dot{x}(t) + \omega_0^2 x(t) = 0$$

$$x(t) = e^{\alpha t}$$

$$r^2 + 2\delta r + \omega_0^2 = 0$$

$$r = \frac{-2\delta \pm \sqrt{4\delta^2 - 4\omega_0^2}}{2}$$

$$r = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

$$x_1 = e^{(-\delta + \sqrt{\delta^2 - \omega_0^2})t}$$

$$x_2 = e^{(-\delta - \sqrt{\delta^2 - \omega_0^2})t}$$

$\sqrt{\ }$ - im. slabo gušenje

$\delta \gg \omega_0$ jako gušenje e^{-t}

Slučaj 1. $\delta < \omega_0$

$$\alpha_{1,2} = -\delta \pm i\omega \qquad \omega = \sqrt{\omega_0^2 - \delta^2}$$

$$x(t) = a_1 e^{\alpha t} + a_2 e^{\alpha t} \quad \in \mathbb{C}$$

$x \in \mathbb{R}$

$$a_1 = a_2^* = \frac{A}{2} e^{i\phi}$$

$$x(t) = \underbrace{Ae^{-\delta t}}_{\text{envelopa}} \cos(\omega t + \phi)$$

Slučaj 2. $\delta = \omega_0$

$$\alpha_{1,2} = -\delta$$

$$x(t) = (a_1 + a_2 t)e^{-\delta t}$$

$$x_0 = x(t=0)$$

$$v_0 = \dot{x}(t=0)$$

$$x(t) = e^{-\delta t}(x_0 + (v_0 + x_0\delta)t)$$

Slučaj 3.

$\delta > \omega_0$

$$\alpha_{1,2} = \delta \pm g \qquad g = \sqrt{\delta^2 - \omega_0^2}$$

$$x(t) = e^{-\delta t}(b_1 \operatorname{ch}(gt) + b_2 \operatorname{sh}(gt))$$

$$x_0 = x(t=0)$$

$$v_0 = \dot{x}(t=0)$$

$$x(t) = e^{-\delta t}\left(x_0 \operatorname{ch}(gt) + \frac{v_0 + x_0\delta}{g} \operatorname{sh}(gt)\right)$$

$$P = \frac{dW}{dt} = Fv = bvv$$

$$\frac{d}{dt}E = \frac{d}{dt}(T + U) = \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right)$$

$$= m\dot{x}\ddot{x} + k\dot{x}x$$

$$= \dot{x}(m\ddot{x} + kx)$$

$$= \dot{x}(-b\dot{x}) = -bv^2$$

$$= -b\frac{2}{m}\frac{mv^2}{2} = -4\delta T$$

logaritamski faktor prigušenja:

$$\lambda = \ln\left(\frac{x(t)}{x(t+\tau)}\right) \left(\frac{A(t)}{A(t+\tau)}\right)$$

$$= \ln\frac{Ae^{-\delta t}}{Ae^{-\delta(t+\tau)}} = \ln e^{\delta\tau} = \delta\tau$$

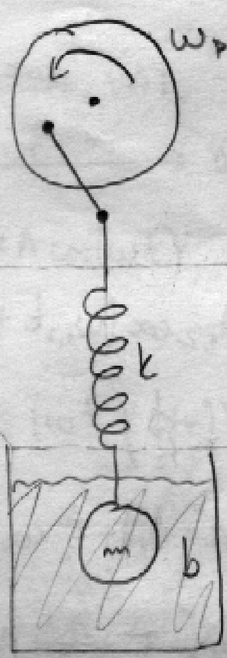
$$= 2\pi\frac{\delta}{\omega} \simeq 2\pi\frac{\delta}{\omega_0}$$

Q-faktor

$$Q^{-1} = -\frac{1}{2\pi}\left(\frac{\Delta E}{E}\right)_{\text{ciklus}} = \frac{2\delta}{\omega}$$

Izvod?!

PRISILNO TITRANJE



j. gib.

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F_p\cos(\omega_p t)$$

$$2\delta = \frac{b}{m} \quad \omega_0^2 = \frac{k}{m} \quad f_p = \frac{F_p}{m}$$

$$\ddot{x}(t) + 2\delta\dot{x}(t) + \omega_0^2 x(t) = f_p\cos(\omega_p t)$$

rješenje oblika: $x(t) = A(\omega_p)\cos(\omega_p t + \phi(\omega_p)) = A(\omega_p)e^{i(\omega_p t + \phi(\omega_p))}$

$$\ddot{x}(t) + 2\delta\dot{x}(t) + \omega_0^2 x(t) = f_p e^{i\omega_p t}$$

$$\Rightarrow \left(-\omega_p^2 + i2\delta\omega_p + \omega_0^2\right)A(\omega_p)\cancel{e^{i\omega_p t}}e^{-i\phi(\omega_p)} = f_p\cancel{e^{i\omega_p t}}$$

$$\left(-\omega_p^2 + \omega_0^2 + i2\delta\omega_p\right)A(\omega_p) = f_p e^{i\phi(\omega_p)}$$

$$A(\omega_p) = \frac{f_p}{\sqrt{(\omega_0^2+\omega_p^2)+(2\delta\omega_p)^2}}$$

$$\mathrm{tg}\,\phi(\omega_p) = \frac{2\delta\omega_p}{\omega_0^2-\omega_p^2}$$

$\omega_p^2 > \omega_0^2$ - u protufazi (negativno)

$\omega_p^2 < \omega_0^2$ - u fazi (pozitivno)

Max. amplitude

$$\frac{d}{d\omega_p}A(\omega_p) = 0$$

$$\Rightarrow \frac{d}{d\omega_p}\left((\omega_0^2-\omega_p^2)^2 + 4\delta^2\omega_p^2\right)$$

$$= 2(\omega_0^2-\omega_p^2)(-2\omega_p) + 8\delta^2\omega_p = 0$$

$$-\omega_0^2 + \omega_p^2 + 2\delta^2 = 0$$

$$\boxed{\omega_p^2 = \omega_0^2 - 2\delta^2}$$

Snaga (t)

$$P(t) = F_p(t)v(t) = F_p\cos(\omega_p t)\,A(\omega_p)\left[-\omega_p\sin(\omega_p t - \phi)\right]$$

$$= F_p A(\omega_p)\,\omega_p\left[\cos^2(\omega_p t)\sin\phi + \sin(\omega_p t)\cos(\omega_p t)\cos\phi\right]$$

$$\sin\phi = \frac{2\delta\omega_p A(\omega_p)}{f_p}$$

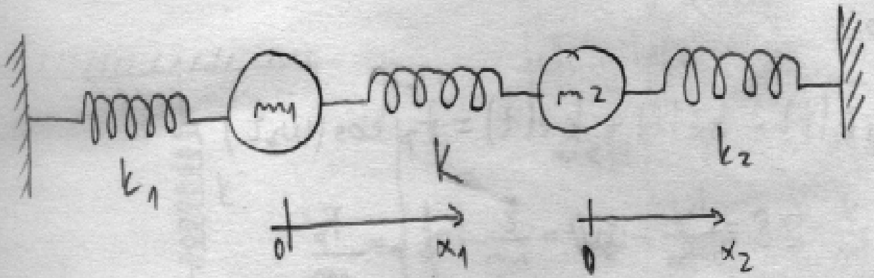
$$= m\delta f_p^2\frac{\omega_p^2}{(\omega_0^2-\omega_p^2)-4\delta^2\omega_p^2}$$

$$\bar{P} = \frac{1}{\tau}\int_0^{\tau}P(t)dt$$

$$= \frac{\omega_p}{2\pi}F_p A(\omega_p)\int_0^{\tau}[\ldots]\omega_p dt \quad x = \omega_p t$$

$$= \frac{\omega_p}{2\pi}F_p A(\omega_p)\int_0^{2\pi}\left[\cos^2 x\sin\phi + \sin x\cos x\cos\phi\right]dx = \frac{\omega_p}{2\pi}F_p A(\omega_p)\,\pi\sin\phi$$

OBERBECKOVA NJIHALA



$m_1 = m_2$

$k_1 = k_2$

j. gib.:

$$\left.\begin{aligned} m\ddot{x}_1 &= -kx_1 - K(x_1 - x_2) \\ m\ddot{x}_2 &= -kx_2 - K(x_1 - x_2) \end{aligned}\right\} (1)$$

$$\zeta_{1,2} = x_1 \pm x_2 \qquad m\zeta_1 = -k\zeta_1$$

$$\ddot{\zeta}_{1,2} = \ddot{x}_1 \pm \ddot{x}_2 \qquad m\zeta_2 = -k\zeta_2 - 2K\zeta_2$$

$$= -(k + 2K)\zeta$$

$$\omega_1^2 = \frac{k}{m}$$

$$\omega_2^2 = \frac{k+2K}{m} \qquad \zeta_{1,2} + \omega_{1,2}^2 \zeta_{1,2} = 0$$

rješenja: $\zeta_{1,2}(t) = A_{1,2} \cos(\omega_{1,2} t + \phi_{1,2})$

$$x_{1,2} = \frac{1}{2}(\zeta_1 + \zeta_2)$$

$$x_1 = \frac{A}{2}\cos(\omega_1 t + \phi_1) + \frac{A}{2}\cos(\omega_2 t + \phi_2)$$

$$\omega_1 \simeq \omega_2, \ \omega_{1,2} = \overline{\omega} + \frac{\Delta\omega}{2}, \ A_1 = A_2, \ \phi_1 = \phi_2 = 0$$

$$x = \ldots = A\cos(\overline{\omega} t)\cos\left(\frac{\Delta\omega}{2} t\right)$$

VALOVI

PROGRESIVNI VAL

izvor $\vec{r}$ detektor

$f(t)=A\cos(\omega t)$ $\quad$ $A(r)\cos(\omega t-\phi(r))$

$\phi(r)=kr$

valni vektor

$$A(r)\cos(\omega t+\phi(r)) = A(r)\cos(\omega t-kr)$$

$$= A(r)\cos(k(vt-r)) \qquad k=\frac{2\pi}{\lambda} \qquad \omega=\frac{2\pi}{T}$$

$$= A(r)\cos\left(2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\right) \qquad \omega=\frac{2\pi}{\lambda}v$$

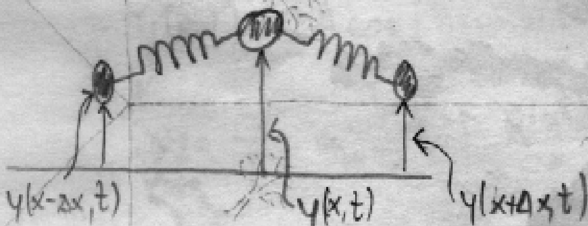
period $\quad$ valna duljina

$$\frac{2\pi}{T}=\frac{2\pi}{\lambda}v$$

$$T=\frac{\lambda}{v}$$

VALNA JEDNADŽBA

TRANSFERZALNI VAL



jed. gib.

$$\Delta m\,\frac{\partial^2}{\partial t^2}y(x,t)=T\left(\frac{y(x-\Delta x,t)-y(x,t)}{\Delta x}+\frac{y(x+\Delta x,t)-y(x,t)}{\Delta x}\right)$$

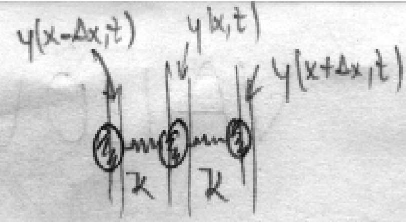
$$\frac{\partial^2}{\partial t^2}y(x,y)=\frac{T}{\Delta m}\left(\frac{y(x-\Delta x,t)-2y(x,t)+y(x+\Delta x,t)}{\Delta x}\right) \qquad \Delta m=\frac{M}{N} \quad \Delta x=\frac{L}{N}$$

$\mu=\frac{M}{L}$ lin. gust. mase.

$$=\frac{TL}{M}\left(\frac{y(x-\Delta x,t)+y(x+\Delta x,t)-2y(x,y)}{(\Delta x)^2}\right)$$

$$\frac{\partial^2}{\partial t^2}y(x,y)-\underbrace{\frac{LT}{M}}_{=\frac{T}{\mu}}\frac{\partial^2}{\partial x^2}y(x,t)=0$$

LONGITUDINALNI VAL



j. gib.

$$\Delta m \frac{\partial^2}{\partial t^2} y(x,t) = \left(\cancel{\pm T} + \mathcal{K}\left(y(x+\Delta x,t) - y(x,t)\right)\right)$$

$$+ \left(\cancel{\mp T} + \mathcal{K}\left(y(x-\Delta x,t) - y(x,t)\right)\right)$$

$$\frac{\partial^2}{\partial t^2} y(x,t) = \frac{\mathcal{K}}{\Delta m}\left[y(x+\Delta x,t) - 2y(x,t) + y(x-\Delta x,t)\right]$$

$$\mathcal{K} = Nk = \frac{L}{\Delta x}\frac{SE}{L} = \frac{SE}{\Delta x}$$

$$\Delta m = \frac{M}{N} \quad \Delta x = \frac{L}{N}$$

$$\frac{\partial^2}{\partial t^2} y(x,t) = \frac{SEL}{M(\Delta x)^2}\left[\ldots\right] = \frac{SEL}{M} \frac{\partial^2}{\partial x^2} y(x,t)$$

$$\rho = \frac{M}{V} = \frac{M}{SL}$$

$$\frac{\partial^2}{\partial t^2} y(x,t) - \frac{E}{\rho}\frac{\partial^2}{\partial x^2} y(x,t) = 0$$

Rješenje:

$$\ddot{y} - v^2 y'' = 0$$

$$y(x,t) = f(x \pm vt)$$

$$= \partial_t f'(x \pm vt)(\pm v) - v^2 \partial_x\left(f'(x \pm vt)\right)$$

$$= f''(x \pm vt)(\pm v)(\pm v) - v^2 f''(x \pm vt)$$

$$= f''(x \pm vt) v^2 - v^2 f''(x \pm vt)$$

SUPERPOZICIJA VALOVA

$$y_1(x,t) = A_1 \cos(k_1 x - \omega_1 t + \phi_1)$$

$$y_2(x,t) = A_2 \cos(k_2 x - \omega_2 t + \phi_2)$$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$y(x,t) = 2A\cos(\bar{k}x + \bar{\omega}t + \bar{\phi})\cos\left(\frac{\Delta k}{2}x + \frac{\Delta\omega}{2} + \frac{\Delta\phi}{2}\right)$$

$$\bar{k}_1 = \frac{k_1+k_2}{2} \qquad \bar{\omega} = \frac{\omega_1+\omega_2}{2}$$

$$\bar{\phi} = \frac{\phi_1+\phi_2}{2} \qquad A_1 = A_2 = A$$

VAŽNI SLUČAJEVI

$$\left.\begin{array}{ll} \omega_1 = \omega_2 = \omega & \Delta\omega = 0 \\ k_1 = k_2 = k & \Delta k = 0 \\ \phi_1 = \phi_2 = \phi & \Delta\phi = 0 \end{array}\right\}$$ konstruktivna interferencija

$$\left.\begin{array}{l} \Delta\omega = 0 \\ \Delta k = 0 \\ \frac{\Delta\phi}{2} = \frac{\pi}{2} \Rightarrow \Delta\phi = \pi \end{array}\right\}$$ destruktivna interferencija

$$y(x,t) = 2A\underbrace{\cos(\bar{k}x - \bar{\omega}t)}_{\text{brzina oscilirajа}} \cdot \overbrace{\cos\left(\frac{\Delta k}{2}\left(x - \underset{\downarrow\ \text{grupna brzina}}{\frac{\Delta\omega}{\Delta k}}t\right)\right)}^{\text{envelopa}} \qquad v_g < v_f$$

$$v_f = \frac{\bar{\omega}}{\bar{k}}$$

REFLEKSIJA VALOVA

Na čvrstom kraju val se invertira (III. NZ. čvrsti kraj djeluje na uže istom silom, ali suprotnog predznaka.

Faza se promijeni za π

$$y_{\rightarrow}(x,t) = A\cos(\omega t - kx) \Rightarrow y_{\leftarrow} A\cos(\omega t + kx)$$

Na slobodnom kraju nema promjene u fazi.

ENERGIJA I SNAGA VALOVA

$$\Delta E_k = \frac{1}{2}\Delta m v_y^2 = \frac{1}{2}\mu\Delta x\left(\frac{\partial y}{\partial t}\right)^2$$

$$\frac{dE}{dt} = \frac{dE_k}{dx} + u(x)$$

$$\frac{dE_k}{dx} = u(x)$$

$$v_y = \frac{\partial y}{\partial t} = \frac{\partial}{\partial t}\left[A_0 \cos(\omega t - kt)\right]$$

$$= -A_0 \omega \sin(\omega t - kt)$$

$$\frac{dE_k}{dx} = \frac{1}{2}\mu A_0^2 \omega^2 \sin^2(\omega t - kx)$$

$$\left\langle \frac{dE_k}{dx} \right\rangle = \frac{1}{4}\mu\omega^2 A_0^2 = \langle u(x) \rangle$$

$$\frac{dE}{dx} = 2\,\frac{dE_k}{dx}$$

$$\bar{P} = \left\langle \frac{dE}{dt} \right\rangle$$

$$\bar{P} = \frac{1}{2}\mu\omega^2 A_0 v$$

STOJNI VALOVI

$$y(t,x) = y_{\rightarrow}(t,x) + y_{\leftarrow}(t,x)$$

$$= A\cos(\omega t - kx) + A\cos(\omega t + kx + \pi)$$

$$= 2A\cos\left(\omega t + \frac{\pi}{2}\right)\cos\left(kx + \frac{\pi}{2}\right)$$

$y(t,x) = 2A\sin(\omega t)\sin(kx) \Rightarrow$ opis stojnog vala

$\rightarrow$ superpozicija dvaju progresivnih valova suprotnih smjerova

$$y(t,0) = 0 \quad y(t, x = l) = 0 \quad y(t = 0, x) = 0$$

$$y(t,l) = 0 = 2A\sin(kl)\sin(\omega t) \Rightarrow k_m \cdot l = m\pi$$

$$\frac{2\pi l}{\lambda_m} = m\pi \qquad \lambda_m = \frac{2l}{m} \quad m \in \mathbb{N}$$

LONGITUDINALNI VALOVI

$$\xi(t,x) = \xi_0 \cos(\omega t - kx + \phi)$$

$$v = \sqrt{\frac{E}{\rho}}$$

STOJNI VAL ZA PRIDRŽAVANI ŠTAP U SREDINI: $\cos\left(k_m \frac{l}{2}\right) = 0 \Rightarrow k_m \frac{l}{2} = (2m+1)\frac{\pi}{2}$

$$\lambda_m = \frac{2l}{2m+1}$$

"DOZVOLJENE" λ

$$\lambda_0 = \frac{2l}{1} \quad \lambda_1 = \frac{2l}{3}$$

slobodan - čvrst

$$\xi(t,0) = 2\xi_0$$

$$\xi(t,l) = 0$$

$$\lambda_m = \frac{4l}{2m-1}$$

$$\nu_m = (2m-1)\nu_1$$

POLUOTVORENE SVIRALE

$$\lambda_m = \frac{4l}{2m-1}$$

slobodan - slobodan	čvrst - čvrst
$\xi(t,0) = 2\xi_0$	0
$\xi(t,l) = 2\xi_0$	0

$$\lambda = \frac{2l}{m}$$

$$\nu_m = m \cdot \nu_1$$

INTENZITET ZVUKA

$$\Delta E = \frac{1}{2}\Delta m (v(t))^2$$

$$\xi(t,x) = \xi_0 \cos(\omega t - kx + \phi)$$

$$\Delta E = \frac{1}{2}\Delta m (\omega \xi_0)^2$$

$$\Delta E = \frac{1}{2}(\rho S \Delta x)(\omega \xi_0)^2$$

$$P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\rho S v (\omega \xi_0)^2$$

$$I = \frac{\text{Snaga}}{\text{Površina}} = \frac{1}{2}\rho v (\omega \xi_0)^2$$

DOPPLEROV EFEKT

- kod zvuka je bitno gibanje u odnosu na medij

$$\lambda_{ispred} = \frac{\overline{BC}}{m_i} = \frac{\overline{BC}\, T_i}{t_i} = \frac{\overline{BC}}{\nu_i t_i}$$

$$\overline{BC} = \overline{AC} - \overline{BC} = v_0 t_i - v_i t_i = (v_0 - v_i) t_i$$

$$\lambda_{ispred} = \frac{v_0 - v_i}{\nu_i}$$

$$\lambda_{iza} = \frac{\overline{BD}}{\nu_i} = \frac{v_0 + v_i}{\nu_i}$$

$$\nu_{ispred} = \nu_i \frac{v_0}{v_0 - v_i}$$

$$\nu_{iza} = \nu_i \frac{v_0}{v_0 + v_i}$$

$$\nu_s = \nu_i \frac{v_0 - v_s}{v_0 - v_i}$$