$$\varphi(r) = \frac{q}{4\pi\varepsilon_0 r}$$

$$\varphi(\bar{r}) = \sum_{i=1}^{N} \varphi(\bar{r}) = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_0} \frac{1}{|\bar{r} - \bar{r}_i'|}$$

POTENCIJAL

Volumni potencijal

$$\varphi(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \frac{dq}{|\bar{r} - \bar{r}'|} = \frac{1}{4\pi\varepsilon_0} \iiint_V \frac{\rho(\bar{r}')\,dV}{|\bar{r} - \bar{r}'|}$$

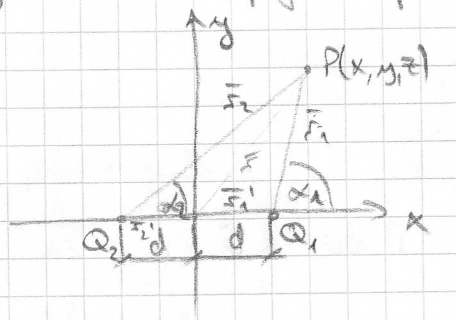
Plošni potencijal

$$\varphi(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \iint_S \frac{\sigma(\bar{r}')\,dS}{|\bar{r} - \bar{r}'|}$$

Linijski potencijal

$$\varphi(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \int_l \frac{\lambda(\bar{r}')\,dl}{|\bar{r} - \bar{r}'|}$$

Zad) Odredite el. polje i potencijal 2 točkasta naboja



$$\bar{r} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

$$\bar{r}_1' = d\bar{a}_x$$

$$\bar{r}_2' = -d\bar{a}_x$$

$$\bar{r}_1 = \bar{r} - \bar{r}_1' = (x-d)\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

$$\bar{r}_2 = \bar{r} - \bar{r}_2' = (x+d)\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

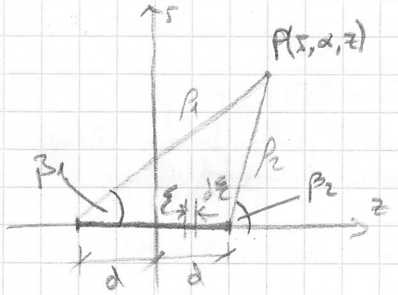
$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{(x-d)\bar{a}_x + y\bar{a}_y + z\bar{a}_z}{\left[(x-d)^2 + y^2 + z^2\right]^{3/2}} \cdot Q_1 + \frac{(x+d)\bar{a}_x + y\bar{a}_y + z\bar{a}_z}{\left((x+d)^2 + y^2 + z^2\right)^{3/2}} Q_2\right]$$

$$\varphi_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1}$$

$$\varphi_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2}$$

$$\varphi = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{(x-d)^2 + y^2 + z^2}} + \frac{Q_2}{\sqrt{(x+d)^2 + y^2 + z^2}}\right]$$

Zad) Odredite el. polje i potencijal jednoliko nabijene dužine



α - zbog simetričnosti se može zanemariti

$\bar{r}$ - promatrana točka
$\bar{r}'$ - izvor

ξ - duljina štapa

$$\left.\begin{aligned} \bar{r} &= \bar{a}_s s + \bar{a}_z z \\ \bar{r}' &= \xi \bar{a}_z \end{aligned}\right\} \quad \bar{R} = \bar{r} - \bar{r}' = \bar{a}_s s + (z-\xi)\bar{a}_z$$

$$\bar{E} = \int \frac{dq\,\bar{R}}{4\pi\varepsilon_0 |\bar{R}|^3} = \int_{\xi=-d}^{d} \frac{\bar{a}_s s + (z-\xi)\bar{a}_z}{4\pi\varepsilon_0 \left[s^2+(z-\xi)^2\right]^{3/2}} \lambda\, d\xi = E_s \bar{a}_s + E_z \bar{a}_z$$

$dq = \lambda d\xi$

!!!

$$E_s = \frac{\lambda}{4\pi\varepsilon_0} \int \frac{s}{[\quad]^{3/2}} d\xi = \frac{\lambda}{4\pi\varepsilon_0 s}\left[\frac{d+z}{\sqrt{s^2+(d+z)^2}} + \frac{d-z}{\sqrt{s^2+(d-z)^2}}\right]$$

$$E_s = \frac{\lambda}{4\pi\varepsilon_0 s}\left(\cos\beta_1 + (-\cos\beta_2)\right)$$

naknadno dodano

$$E_z = \frac{\lambda}{4\pi\varepsilon_0} \int \frac{z-\xi}{[\quad]^{3/2}} d\xi = \frac{\lambda}{4\pi\varepsilon_0} \frac{-s}{s} \left(\frac{1}{\sqrt{(d-z)^2+s^2}} - \frac{1}{\sqrt{(d+z)^2+s^2}}\right)$$

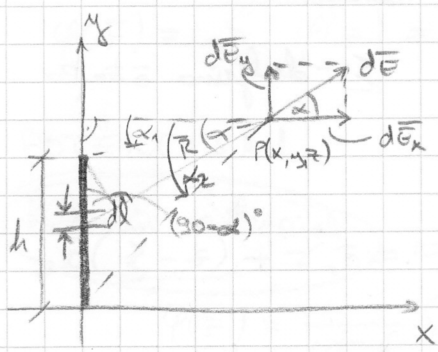
$$E_z = \frac{\lambda}{4\pi\varepsilon_0 s}\left(\sin\beta_2 - \sin\beta_1\right)$$

$$\varphi = \int \frac{d\xi}{4\pi\varepsilon_0 |\bar{R}|} = \frac{\lambda}{4\pi\varepsilon_0} \int_{\xi=-d}^{d} \frac{d\xi}{\sqrt{s^2+(z-\xi)^2}} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{d+z+\xi_1}{z-d+\xi_2}\right)$$

$$\left.\begin{aligned} & d \to \infty \\ & \beta_1 \to 0^\circ \\ & \beta_2 \to 180^\circ \end{aligned}\right] \Rightarrow \left.\begin{aligned} E_s &= \frac{\lambda}{2\pi\varepsilon_0 s} \\ E_z &= 0 \end{aligned}\right\} \quad E = \frac{\lambda}{2\pi\varepsilon_0 s}\bar{a}_s$$

beskonačna nabijena linija

Zad) Odredite el. polje beskonačno duge trake nabijene jednoliko nabojem gustoće σ.

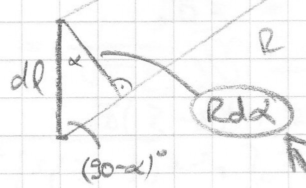


$$E = \frac{\lambda \bar{a}_r}{2\pi\varepsilon_0 r}$$

polje beskonačno duge dužine

$$dE = \frac{\lambda}{2\pi\varepsilon_0 R}$$

$$\lambda = \sigma \cdot dl$$



za α vrlo mali $\sin\alpha \approx \alpha$

$$\sin d\alpha = \frac{x}{R}$$

$$x = \sin d\alpha \cdot R$$

$$x = R\,d\alpha$$

$$\cos\alpha = \frac{R\,d\alpha}{dl}$$

$$dl = \frac{R\,d\alpha}{\cos\alpha}$$

$$dE = \frac{\sigma\,dl}{2\pi\varepsilon_0 R} = \frac{\sigma \cancel{R}\,d\alpha}{2\pi\varepsilon_0 \cancel{R}\cos\alpha} = \frac{\sigma\,d\alpha}{2\pi\varepsilon_0\cos\alpha}$$

$$dE_x = \cos\alpha \cdot dE \qquad d\bar{E}_y = \sin\alpha \cdot dE$$

$$d\bar{E}_x = \frac{\sigma\,d\alpha}{2\pi\varepsilon_0} \qquad d\bar{E}_y = \frac{\sigma\,\mathrm{tg}\alpha}{2\pi\varepsilon_0}\,d\alpha$$

$$E_x = \int_{\alpha_1}^{\alpha_2} d\bar{E}_x = \frac{\sigma}{2\pi\varepsilon_0}(\alpha_2 - \alpha_1)$$

$$E_y = \int_{\alpha_1}^{\alpha_2} d\bar{E}_y = \frac{\sigma}{2\pi\varepsilon_0}\ln\left(\frac{\cos\alpha_1}{\cos\alpha_2}\right)$$

$h \to \infty$

$$\alpha_2 = +\frac{\pi}{2}$$

$$\alpha_1 = -\frac{\pi}{2}$$

$$\boxed{E_x = \frac{\sigma}{2\varepsilon_0}}$$

$$E_y = 0$$

beskonačna nabijena ravnina

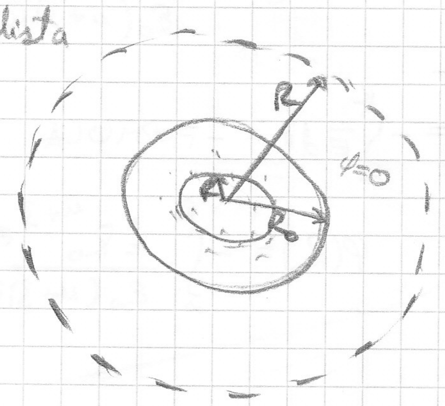
Zad) Tipični za ispit

U vakuumu je raspoređen naboj čija je prostorna gustoća mijenja kao

$$\rho_s = \begin{cases} k r^n & r \le R_0 \\ 0 & r > R \end{cases}$$

gdje je R polumjer zamišljene kugle oko ishodišta

Odredi omjer $\varphi(r=0) / \varphi(r=R_0)$



$$\varepsilon_0 \oiint_S \bar{E}\bar{n}\, dS = \iiint_V \rho\, dV$$

S tako da je r-konst.

① $R \le R_0$

$$\varepsilon_0 \cdot E \underbrace{\oiint_S dS}_{4\pi R^2} = \int_{r=0}^{R} \rho \cdot \underbrace{4\pi r^2 dr}_{dV}$$

$$= \int_{r=0}^{R} k r^n 4\pi r^2 dr$$

$$\varepsilon_0 E \cancel{4\pi} R^2 = k \cancel{4\pi} \frac{R^{(n+3)}}{n+3}$$

$$E = k \frac{R^{n+1}}{\varepsilon_0 (n+3)}$$

$$\vec{E} = E \cdot \bar{a}_r$$

② $R > R_0$

$$\varepsilon_0 \oiint_S \bar{E}\bar{n}\, dS = \int_{r=0}^{R_0} k r^n 4\pi r^2 dr$$

$$\varepsilon_0 \cancel{4\pi} R^2 E = k \frac{R_0^{(n+3)}}{n+3} \cancel{4\pi}$$

$$E = \frac{k \cdot R_0^{n+3}}{\varepsilon_0 (n+3) R^2}$$

$$\bar{E} = E\, \bar{a}_r$$

$r \le R_0 \qquad E = \dfrac{k \cdot R^{n+1}}{\varepsilon_0 (n+3)} \bar{a}_r$

$r > R_0 \qquad E = \dfrac{k \cdot R_0^{n+3}}{\varepsilon_0 (n+3) R^2} \bar{a}_r$

$\varphi = -\int_{\infty}^{R} \bar{E}\, d\bar{l}$ - FORMULA

ZAŠTO BAŠ DO R_0 - ZADAN U ZADATK

$$\varphi(R_0) = -\int_{\infty}^{R} \frac{k R_0^{n+3}}{\varepsilon_0 (n+3) R^2} \bar{a}_r \, dR = -\frac{k R_0^{n+3}}{\varepsilon_0 (n+3)} \left(-\frac{1}{R}\right)\Bigg|_{\infty}^{R_0} = \frac{k R_0^{n+3}}{\varepsilon_0 (n+3) R_0}$$

$$= \frac{k \cdot R_0^{n+2}}{\varepsilon_0 (n+3)}$$

$$\varphi_0 = -\int_{\infty}^{0} \bar{E}\, d\bar{l} = -\int_{\infty}^{R_0} \bar{E}\, d\bar{l} - \int_{R_0}^{0} \bar{E}\, d\bar{l}$$

$$\varphi_0 = -\int_{R_0}^{0} \frac{k R^{n+1}}{\varepsilon_0 (n+3)} dR + \varphi(R_0) = -\frac{k R^{n+2}}{\varepsilon_0 (n+3)(n+2)}\Bigg|_{R_0}^{0} + \varphi(R_0)$$

$$\varphi_0 = \frac{k R_0^{n+2}}{\varepsilon_0 (n+3)(n+2)} + \varphi(R_0)$$

$$\frac{\varphi(0)}{\varphi(R_0)} = 1 + \frac{k \cancel{R_0^{n+2}}}{\cancel{\varepsilon_0 (n+3)}(n+2)} \cdot \frac{\cancel{\varepsilon_0 (n+3)}}{\cancel{k R_0^{n+2}}} = 1 + \frac{1}{n+2} = \frac{n+3}{n+2}$$

$A_{12} = W_{p1} - W_{p2}$ - rad konzervativne sile

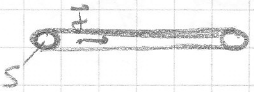
iz $1 \to 2$

razlika potencijala ne ovisi o putu kojim smo došli iz jedne točke u drugu

$$\bar{E} = -\nabla \varphi$$

$\nabla \times E = 0$

$\vec{j} = K \cdot \vec{E}$



$U = E \cdot l$

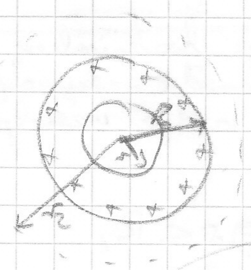
$I = \vec{j} \cdot S$

$R = \frac{1}{K} \cdot \frac{l}{S}$

$$\left.\begin{aligned} \nabla\vec{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla\vec{j} &= K\nabla\vec{E} \end{aligned}\right\} \Rightarrow \rho_s = \rho_{s0}\, e^{-\frac{t}{\tau}}$$

konstanta relaksacije - određuje vrijeme potrebno da se naboji rasporede po površini tijela

Zad) Odredi el. polje jednoliko nabijene metalne kugle primjenom Gaussovog zakona



$\boxed{\text{I} \quad r<R}$

$Q = 0$

$\varepsilon_0 \oint\!\!\oint \vec{E}\, d\vec{S} = 0$

$E = 0$

$$\oint\!\!\oint_S \vec{E}\, dS = \iiint_V \rho\, dV = \sum Q$$

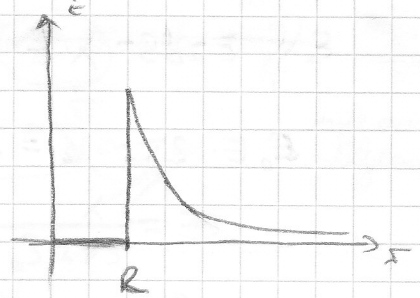
$\boxed{\text{II} \quad r>R}$

$\varepsilon_0 \oint\!\!\oint \vec{E}\, dS = Q$

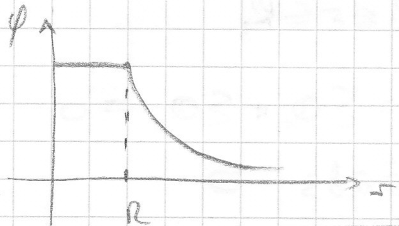
$E \cdot \varepsilon_0 \cdot 4\pi r^2 = Q$

$E(r) = \frac{Q}{\varepsilon_0 4\pi r^2}$

$\vec{E}(r) = \frac{Q}{\varepsilon_0 4\pi r^2}\vec{a}_r$



$\varphi(r) = -\int_\infty^r \vec{E}(r)\, d\vec{r}$



I $r<R$

$\varphi(r) = -\int_\infty^{R_0} \frac{Q}{4\pi\varepsilon r^2} dr + \left(-\int_R^r \vec{E}\, d\vec{r}\right)$

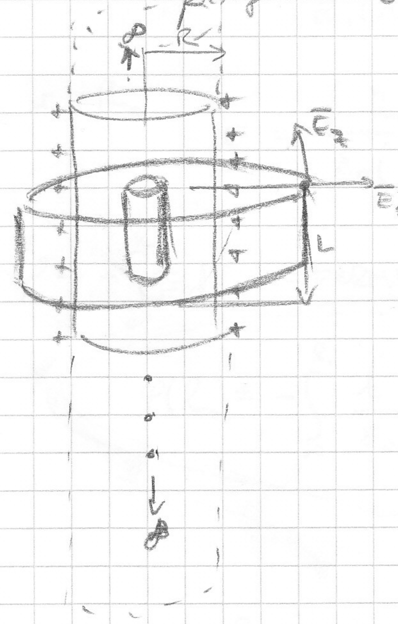
$E=0$

od referentne točke do neke određene
referentna je ∞

II $r>R$

$\varphi(r) = -\int_\infty^R \frac{Q}{4\pi\varepsilon_0 r^2} dr = +\frac{Q}{4\pi\varepsilon_0 r}\Big|_\infty^R = \frac{Q}{4\pi\varepsilon_0 R}$

Zad) Odredite el. polje jednoliko nabijenog metalnog valjka (beskonačnog) primjenom Gaussa



Ⓘ $r < R$

$$\varepsilon_0 \oiint \bar{E} d\bar{S} = Q$$

$Q = 0$ - zbog raspodjele po površini

Ⓘⓘ $r \geq R$

$$\varepsilon_0 \iint_P \bar{E}\bar{n} dS + \overbrace{\iint_{B_1} \bar{E}\bar{n} dS}^{=0} + \overbrace{\iint_{B_2} \bar{E}\bar{n} dS}^{=0} = Q$$

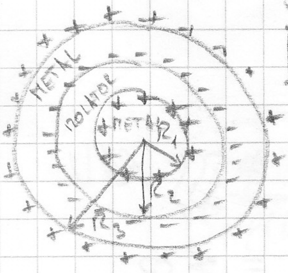
$$\varepsilon_0 E \cdot 2\pi r \cdot \cancel{L} = \lambda \cdot \cancel{L}$$

$$\lambda = \frac{\Delta Q}{\Delta l}$$

ako računamo φ dolazimo do problema → referentna točka u ∞

$$\bar{E} = \frac{\lambda}{2\pi \varepsilon_0 r} \bar{a}_r$$

Zad. Odredi el. polje koaksijalnog kabela primjenom Gaussa
a) vanjski vodič neuzemljen
b) vanjski vodič uzemljen



$\boxed{a)}$

Ⓘ $r < R_1$

$$\varepsilon_0 \oiint_P \bar{E}\bar{n} dS = 0$$

$$\Rightarrow E = 0$$

Ⓘⓘ $R_1 \leq r < R_2$

$$\varepsilon_0 \oiint_P \bar{E}\bar{n} dS = \lambda \cdot L \to Q$$

$$\varepsilon_0 E \cdot 2\pi r \cdot \cancel{L} = \lambda \cdot \cancel{L}$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

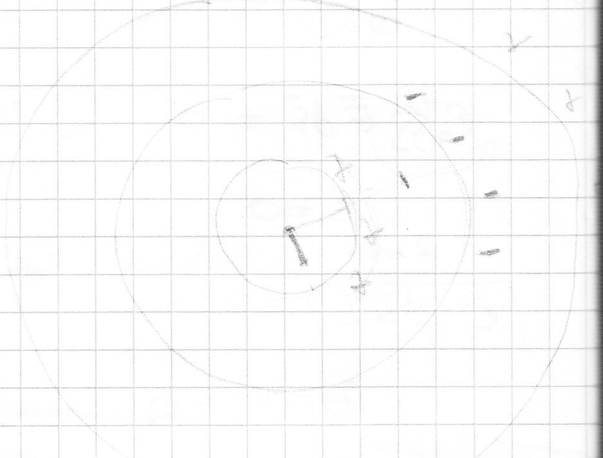
Ⓘⓘⓘ $R_2 \leq r \leq R_3$

$$\Sigma Q = \Sigma Q_+ + \Sigma Q_- = 0$$

$$\varepsilon_0 \oiint \bar{E}\bar{n} dS = 0$$

$$\Rightarrow E = 0$$

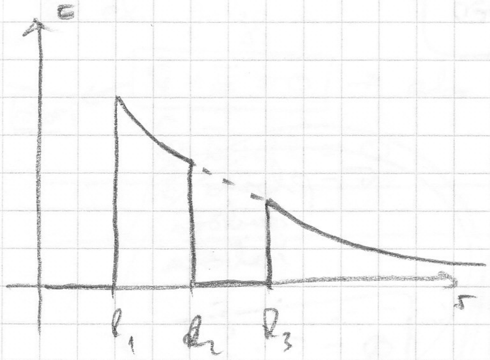
INFLUENCIJA
razdvajanje naboja unutar prethodno nenabijenog metalnog tijela



ⒾⓋ $r \geq R_3$

$$\Sigma Q = \Sigma Q_+ + \cancel{\Sigma Q_-} + \cancel{\Sigma Q_+} = Q$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$



b) VANJSKI VODIČ UZEMLJEN

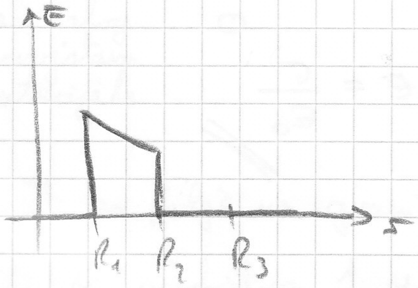
I ISTO

II ISTO

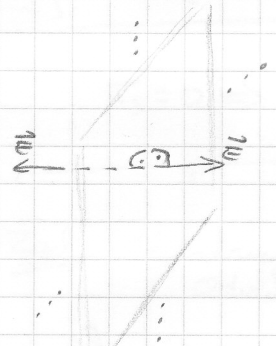
III ISTO

IV $\sum Q = 0$

$\Rightarrow E = 0$



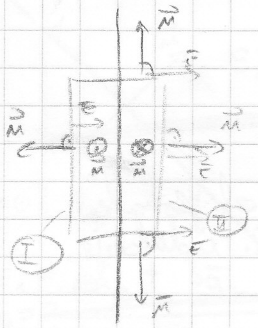
Zad. Odredite el. polje jednoliko nabijene ravnine primjenom Gaussa.



$$\varepsilon_0 \oiint \vec{E}\,\vec{n}\, dS = \sum Q$$

σ - plošna gustoća naboja

$$\underbrace{\varepsilon_0 \cdot E \cdot a \cdot b}_{I} + \underbrace{\varepsilon_0 E \cdot a \cdot b}_{II} = \sigma \cdot a \cdot b$$

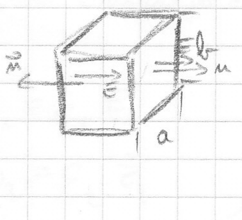


$$2\varepsilon_0 E \cancel{ab} = \sigma \cancel{ab}$$

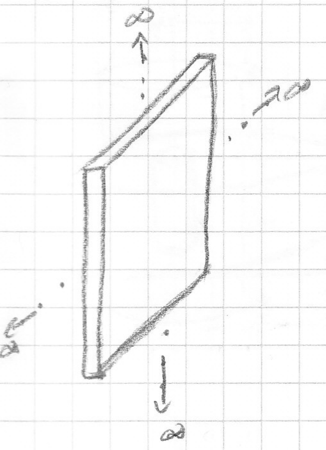
$$E = \frac{\sigma}{2\varepsilon_0}$$

nije ovisno o udaljenosti od ploče

$\varphi \rightarrow$ problem
referentna točka u ∞



Zad. Odredi el. polje jednoliko nabijene metalne $\boxed{\text{Ploče}}$ primjenom Gaussa.



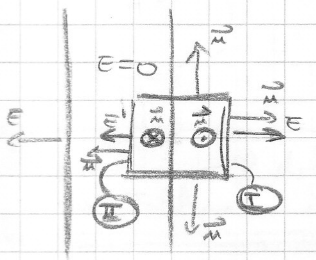
$$\varepsilon_0 \oiint \vec{E}\cdot\vec{n}\, dS = \sum Q$$

σ - plošna gustoća naboja

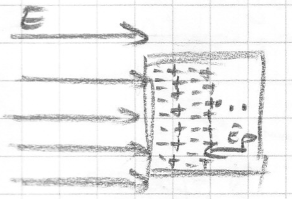
$$\boxed{Q = \sigma \cdot S}$$

$$E' = 0$$

$$\varepsilon_0 E \cdot ab + \underbrace{\varepsilon_0 E' ab}_{0} = \sigma \cdot \cancel{ab}$$



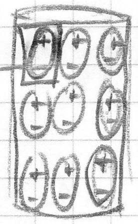
$$E = \frac{\sigma}{\varepsilon_0}$$



P - polarizacija

$$P = \frac{p}{V}$$

p → polarizacija jedne čestice unutar volumena



javlja se polarizirano polje $\vec{E}_p$

$$\vec{E} = \vec{E}_0 + \vec{E}_p$$

$$\nabla\vec{E} = \nabla\vec{E}_0 + \nabla\vec{E}_p = \frac{\rho_s}{\varepsilon_0} + \frac{\rho_p}{\varepsilon_0}$$

$$\nabla(\varepsilon_0\vec{E} + \vec{P}) = \rho_s$$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ - vektor el. pomaka - vektor el. indukcije

$$\vec{D} = \varepsilon_0(1 + \chi_e)\vec{E} = \varepsilon_0\varepsilon_r\vec{E}$$

$\nabla\cdot\vec{D} = \rho_s \;\rightarrow\; \boxed{\oiint_S \vec{D}\cdot\vec{n}\, dS = \iiint_V \rho_s\, dV}$ modificirani Gauss

ako imamo izolator ili tijelo koje teško otpusta elektrone svi zakoni do sad isti

samo umjesto ε_0 ide $\varepsilon_0 \cdot \varepsilon_r$

za el. polje

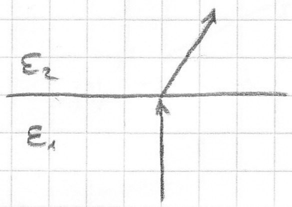
$\boxed{\text{rot}=0}$ ← $\boxed{\nabla \times \bar E = 0}$ → ako to ne vrijedi el. polje ne može biti

$$\oint_c \bar E \cdot dl = 0$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial E}{\partial x} & \frac{\partial E}{\partial y} & \frac{\partial E}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\boxed{\phi = \iint_S E \cdot n \cdot dS}$$

UVJETI NA GRANICI



skretanje silnica zbog različitih svojstava materijala

UVJET PROVODLJIVOSTI $\boxed{n \times (\bar E_2 - \bar E_1) = 0}$ uvjet za polje na granici sredstava iz sredstva 1 u sredstvo 2

$E_{t1} = E_{t2}$

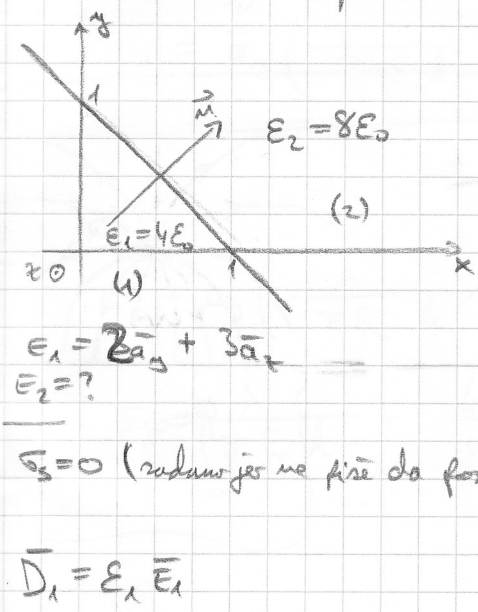
$\boxed{\bar n \, (\bar D_2 - \bar D_1) = \sigma_S}$

$D = E \cdot \varepsilon$

$\sigma_S = 0 \Rightarrow D_{n1} = D_{n2}$

Zad. U području 1 vlada homogeno el. polje $E_1 = 2\bar a_y + 3\bar a_z$ (V/m). Odredite jakost el. polja u području 2.

$\bar n = \frac{\bar n}{|\bar n|}$



$\varepsilon_1 = 2\bar a_y + 3\bar a_z$

$E_2 = ?$

$\sigma_S = 0$ (zadano jer ne piše da postoji)

$\bar D_1 = \varepsilon_1 \bar E_1$

$\bar D_2 = \varepsilon_2 \bar E_2$

$\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$

$\varepsilon_2 = \varepsilon_{r2}\varepsilon_0$

$$\bar E_2 = E_{2x}\bar a_x + E_{2y}\bar a_y + E_{2z}\bar a_z$$

$$\bar n = \frac{\bar a_x + \bar a_y}{\sqrt 2} = (\bar a_x + \bar a_y)\frac{\sqrt 2}{2}$$ jedinični vektor

I $\quad \bar n \, (\bar D_2 - \bar D_1) = \sigma_S = 0$

$$\cancel{\frac{\sqrt2}{2}}(\bar a_x + \bar a_y)(\bar a_x \varepsilon_2 E_{2x} + \bar a_y \varepsilon_2 E_{2y} + a_z \varepsilon_2 E_{2z} - 2\varepsilon_1 \bar a_y - 3\varepsilon_1 \bar a_z) = 0$$

$$\varepsilon_2 E_{2x} + \varepsilon_2 E_{2y} - 2\varepsilon_1 = 0$$

$$\cancel{8\varepsilon_0} E_{2x} + \cancel{8\varepsilon_0} E_{2y} - \underset{1}{\cancel{8\varepsilon_0}} = 0$$

$$E_{2x} + E_{2y} = 1$$

(II) $\bar{n} \times (\bar{E}_2 - \bar{E}_1) = 0$

$$\cancel{\frac{\sqrt{2}}{2}} (\bar{a}_x + \bar{a}_y) \times (E_{2x}\bar{a}_x + E_{2y}\bar{a}_y + E_{2z}\bar{a}_z - 2\bar{a}_y - 3\bar{a}_z) = 0$$

$$E_{2y}\bar{a}_z - E_{2z}\bar{a}_y - 2\bar{a}_z + 3\bar{a}_y - E_{2x}\bar{a}_z + E_{2z}\bar{a}_x - 3\bar{a}_x = 0$$

$$a_x(E_{2z} - 3) + \bar{a}_y(-E_{2z} + 3) + \bar{a}_z(E_{2y} - 2 - E_{2x}) = 0$$

$E_{2z} = 3$

$E_{2y} - 2 - E_{2x} = 0$

$$\begin{cases} E_{2y} - E_{2x} = 2 \\ E_{2x} + E_{2y} = 1 \end{cases} \quad +$$

(I)

$2E_{2y} = 3$

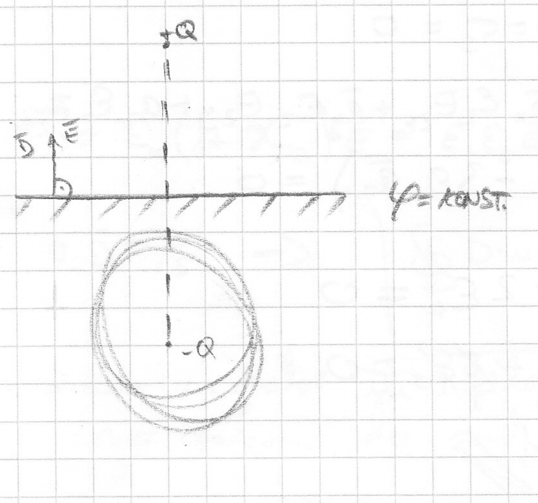
$E_{2y} = \frac{3}{2}$ $\qquad$ $E_{2x} = -\frac{1}{2}$

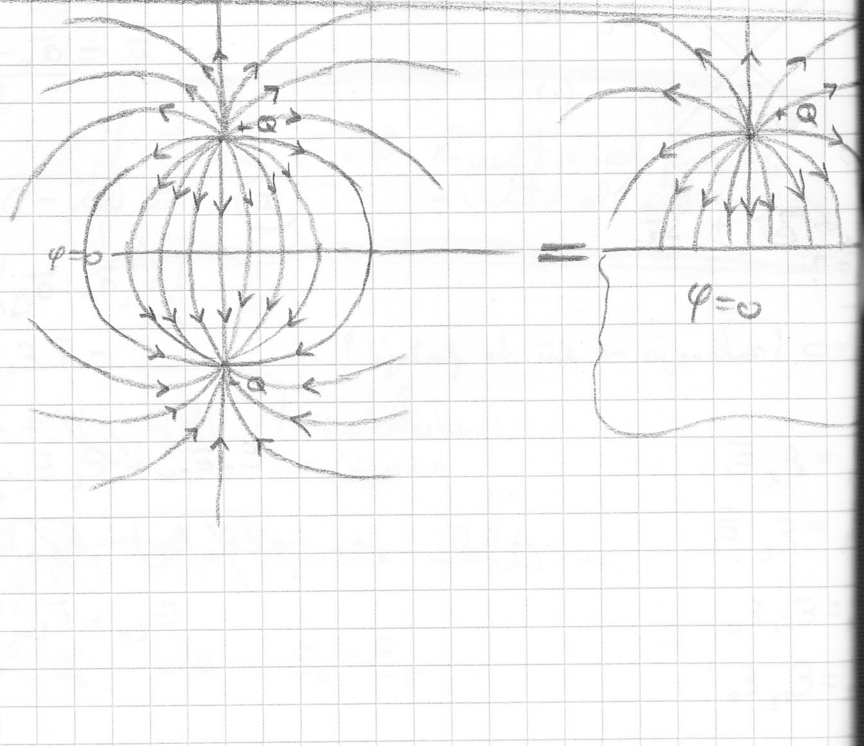
$$\boxed{\bar{E}_2 = -\frac{1}{2}\bar{a}_x + \frac{3}{2}\bar{a}_y + 3\bar{a}_z}$$

a_z je tangencijalna obzirom na granicu između sredstava pa se ne mijenja iz jednog sredstva u drugo.

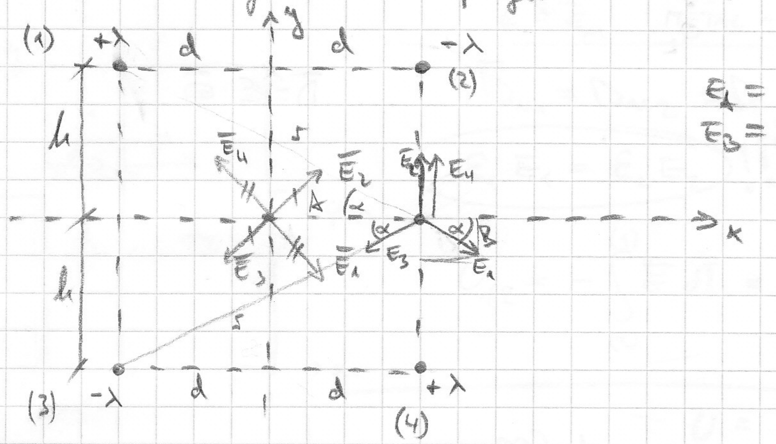
METODA ODSLIKAVANJA

- moraju biti zadovoljeni uvjeti na granici





Zad 2 Beskonačno dugi dvožični vod nabijen nabojem lin. gustoće $\pm\lambda = 10\,\mathrm{nC/m}$ nalazi se na visini $h = 1\,\mathrm{m}$ iznad zemlje. U ISPITU
Vodiči su razmaknuti za $2d = 2\,\mathrm{m}$
Odredi jakost el. polja u točkama A i B prema slici



$E_A = ?$
$E_B = ?$

$$\overline{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\,\vec{a}_r$$

A) $E_A = 0$

B) $$\overline{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 h}\,\overline{a}_y$$

$$\cos\alpha = \frac{E_x}{E} \qquad \sin\alpha = \frac{E_y}{E}$$

$$E_y = \sin\alpha \cdot E$$

$$\overline{E}_4 = \frac{\lambda}{2\pi\varepsilon_0 h}\cdot\overline{a}_y$$

$$E_x = \cos\alpha \cdot E$$

$$\overline{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 r}\left[\cos\alpha\, a_x - \sin\alpha\,\overline{a}_y\right]$$

$$\overline{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 r}\left[-\cos\alpha\,\overline{a}_x - \sin\alpha\,\overline{a}_y\right]$$

$$r^2 = h^2 + (2d)^2$$
$$\sin\alpha = \frac{h}{r}$$

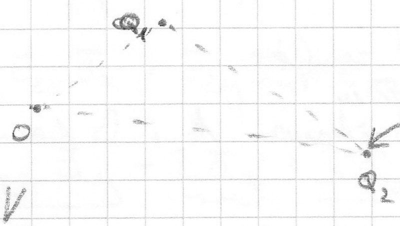
$$\overline{E}_B = \overline{E}_1 + \overline{E}_2 + \overline{E}_3 + \overline{E}_4$$
$$= \overline{a}_y\left(\frac{\lambda}{2\pi\varepsilon_0 h} + \frac{\lambda}{2\pi\varepsilon_0 h} - \sin\alpha\frac{\lambda}{2\pi\varepsilon_0 r} - \sin\alpha\frac{\lambda}{2\pi\varepsilon_0 r}\right)$$
$$+ a_x\left(\cancel{\cos\alpha\frac{\lambda}{2\pi\varepsilon_0 r}} - \cancel{\cos\alpha\frac{\lambda}{2\pi\varepsilon_0 r}}\right)$$

$$= a_y\left(\frac{\lambda}{\pi\varepsilon_0 h} - \sin\alpha\frac{\lambda}{\pi\varepsilon_0 r}\right)$$

$$\overline{E}_B = \left(\frac{\lambda}{\pi\varepsilon_0 h} - \frac{h}{r}\frac{\lambda}{\pi\varepsilon_0 r}\right)\overline{a}_y = \left(\frac{\lambda}{\pi\varepsilon_0 h} - \frac{h\lambda}{\pi\varepsilon_0[h^2+(2d)^2]}\right)\overline{a}_y$$
$$= \frac{\lambda}{\pi\varepsilon_0}\left[\frac{1}{h} - \frac{h}{h^2+4d^2}\right]\overline{a}_y$$
$$= 288\,\mathrm{V/m}$$

ENERGIJA I KAPACITET

$W = \varphi_{21} \cdot Q$



dovođenje naboja Q_2 u polje naboja Q_1 zahtjeva utrošak energije W

$$W = \frac{1}{2} \iiint \varphi(\bar r')\rho(\bar r')\,dV$$

$\bar D = \varepsilon \cdot \bar E$

$dq = \rho \cdot dV$

$$\boxed{W_e = \frac{1}{2} \sum_{k=1}^{N} Q_k \varphi_k}$$ - N idealno vodljivih tijela

$\bar E = -\nabla \varphi$

$\nabla \bar D = \rho_s$

$$W = \frac{1}{2} \oint\!\!\oint \varphi \bar D \bar n \, dS + \frac{1}{2} \iiint \bar D \cdot \bar E \, dV$$

- energija kod sustava vodiča sadržana je gdje je polje aktivno
To je najčešće u području izolatora

$$W = \frac{1}{2} \iiint \bar D \cdot \bar E \, dV = \frac{1}{2} \iiint_V \varepsilon |\bar E|^2 \, dV$$ - ako je obuhvaćen cijeli prostor u kojem vlada polje

$$w_e = \frac{\varepsilon |\bar E|^2}{2}$$ - gustoća energije el. polja

$$W = \iiint_V w_e \, dV$$

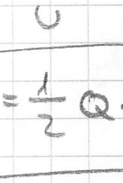
KAPACITET

$C = \frac{Q}{\varphi}$ - jedno usamljeno metalno tijelo

$C = \frac{Q}{U}$ - kondenzator

$$\boxed{W_e = \frac{1}{2} Q \cdot U = \frac{1}{2} \cdot C \cdot U^2 = \frac{1}{2} \frac{Q^2}{C}}$$ - pohranjena el. energija u kapacitetu

$$\frac{1}{2} \sum_{n=1}^{2} \varphi_n Q_n = \frac{1}{2} (\varphi_1 Q - \varphi_2 Q) = \frac{1}{2} QU$$



$$W = \frac{1}{2} \iiint_V \varepsilon E^2 \, dV$$

$$W = \frac{1}{2} \varepsilon E^2 \cdot S \cdot d$$

$E = \frac{U}{d}$

$C = \varepsilon \cdot \frac{S}{d}$

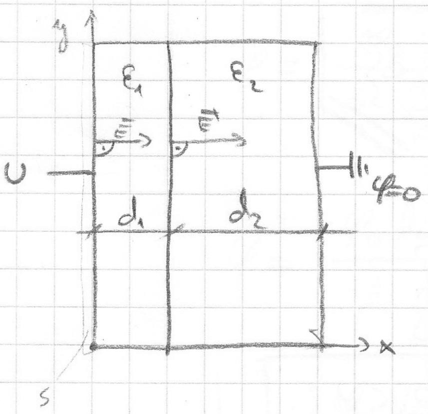
$$W = \frac{CU^2}{2} = \varepsilon \frac{S}{\cancel{d} \cdot 2} E^2 \cdot d^{\cancel{2}} = \frac{1}{2} \varepsilon E^2 \cdot S \cdot d$$

Zad Odredite kapacitet dvoslojnog pločastog kond. prema slici. Površina ploča je S.

$\varepsilon_1 > \varepsilon_2$
$d_2 = 2d_1$

$\bar{D}, \bar{E}$

$U > 0$



$\bar{E} = \bar{E}_M$ (METAL - IZOLATOR)

$\bar{D}_{M1} = \bar{D}_{M2} = D$

$\varepsilon_1 E_1 = \varepsilon_2 E_2$!!!

$\bar{E}_2 = E_2 a_x$
$d\bar{l} = \bar{a}_x dx$

$$U_{12} = -\int_{(2)}^{(1)} \bar{E}\, d\bar{l} = -\int_{d_1+d_2}^{d_1} E_2\, dx - \int_{d_1}^{0} E_1\, dx$$

$$U = E_2 d_2 + E_1 d_1$$
$$= \frac{D_2}{\varepsilon_2} d_2 + \frac{D_1}{\varepsilon_1} d_1$$

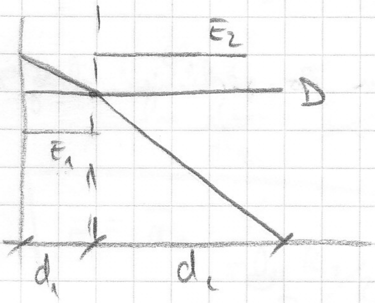
$D_2 = D_1 = D$

$$\oint_S \bar{D}\bar{n}\, dS = Q$$

$D \cdot S = Q$

$$D = \frac{Q}{S} = \sigma$$

$$C = \frac{Q}{U} = \frac{\cancel{D} \cdot S}{\frac{\cancel{D}}{\varepsilon_2} d_2 + \frac{\cancel{D}}{\varepsilon_1} d_1} = \frac{S}{\frac{d_2}{\varepsilon_2} + \frac{d_1}{\varepsilon_1}} = \frac{1}{\frac{d_2}{\varepsilon_2 S} + \frac{d_1}{\varepsilon_1 S}}$$

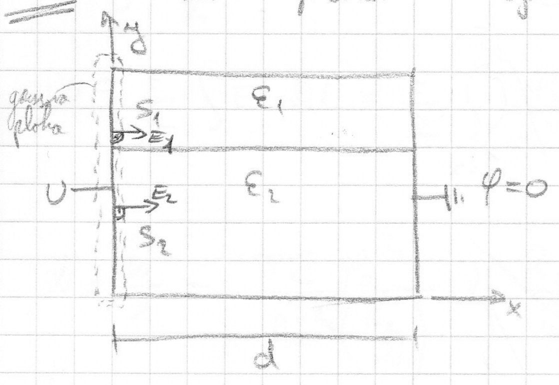


$$C = \frac{1}{\frac{1}{C_2} + \frac{1}{C_1}}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$ - serijski spoj kondenzatora

$E = -\nabla \varphi$

Zad Odredite kapacitet dvoslojnog pločastog kond. prema slici.
Pov. ploča je $S_1 + S_2$
$\varepsilon_1 > \varepsilon_2$



$E_1 = E_{t1}$
$E_2 = E_{t2}$ u odnosu na granicu dielektrik1 - dielektrik2

$E_{t1} = E_{t2}$ - granica sredstava

$E_1 = E_2$

$$U = -\int_d^0 E\, dl = -\int_d^0 E\, dx = E \cdot d$$

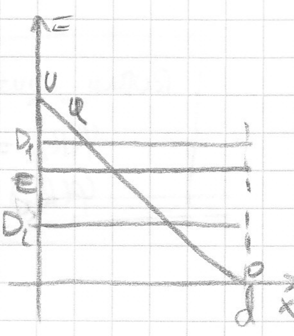
$$E = \frac{U}{d}$$

$$C = \frac{Q}{U}$$

$U = E \cdot d$

$$\oint \bar{E}\bar{n}\, dS = D_1 S_1 + D_2 S_2 = Q$$

$D_1 = \varepsilon_1 \cdot E$
$D_2 = \varepsilon_2 \cdot E$

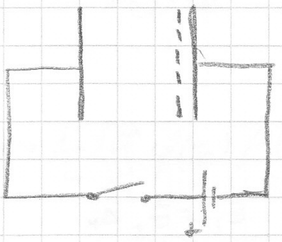


$$C = \frac{\cancel{E}(\varepsilon_1 S_1 + \varepsilon_2 S_2)}{\cancel{E} \cdot d}$$

$$\boxed{C = \frac{\varepsilon_1 S_1 + \varepsilon_2 S_2}{d}}$$

PARALELNI SPOJ KOND.

Sile u električnom polju



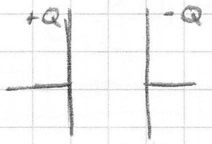
$U = konst.$

$Q = C \cdot U$

↑ mijenja se

$$W = \frac{\partial}{\partial s}\left\{\frac{CU^2}{2}\right\}$$

$$\boxed{W = \frac{U^2}{2}\frac{\partial}{\partial s}\left\{C\right\}}$$



Q - konst.

izolirani sustav

$U = \frac{Q}{C}$

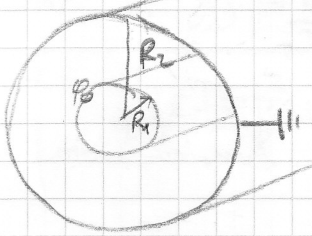
↑ mijenja se

$$W = -\frac{\partial}{\partial s}\left\{\frac{Q^2}{2C}\right\}$$

$$\boxed{W = -\frac{Q^2}{2}\frac{\partial}{\partial s}\left\{\frac{1}{C}\right\}}$$

Zad. Unutarnji vodič cilindričnog kond. polumjera R_1 nalazi se na pot. φ_0 a vanjski polumjera R_2 je uzemljen

Odredi energiju, kapacitet, silu na vanjsku elektrodu po jedinici duljine



$W, C, F = ?$

$$\Delta\varphi = 0$$

$$\Delta\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\alpha^2}}^{\,0} + \cancel{\frac{\partial^2\varphi}{\partial z^2}}^{\,0} = 0$$

CIL. K.S.

$E_z = 0$, $\frac{\partial\varphi}{\partial z} = 0$ } zbog "beskonačnosti" vodiča - duljina

$E_\alpha = 0$, $\frac{\partial\varphi}{\partial\alpha} = 0$ } zbog simetričnosti

$$\cancel{\frac{1}{r}}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) = 0$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) = 0 \quad /\int$$

$$r\frac{\partial\varphi}{\partial r} \quad C_1$$

$$\frac{\partial\varphi}{\partial r} = C_1 \cdot \frac{1}{r} \quad /\int$$

$$\varphi = C_1 \ln r + C_2$$

RUBNI UVJETI

$\varphi(R_1) = \varphi_0$

$\varphi(R_2) = 0$

→

$$\varphi_0 = C_1 \ln R_1 + C_2$$
$$0 = C_1 \ln R_2 + C_2$$

$$C_2 = -C_1 \ln R_2$$

$$\varphi_0 = C_1 \ln R_1 - C_1 \ln R_2 \Rightarrow C_1 \ln\frac{R_1}{R_2} = \varphi_0$$

$$\boxed{C_1 = \frac{\varphi_0}{\ln \frac{R_1}{R_2}}} \qquad \boxed{C_2 = -\frac{\varphi_0 \ln R_2}{\ln \frac{R_1}{R_2}}}$$

$$\varphi = \frac{\varphi_0}{\ln \frac{R_1}{R_2}} \ln r - \frac{\varphi_0 \ln R_2}{\ln \frac{R_1}{R_2}}$$

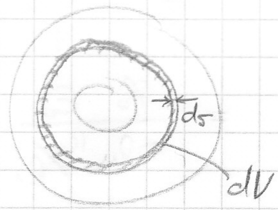
$$\varphi = \frac{\varphi_0 \ln\left(\frac{r}{R_2}\right)}{\ln \frac{R_1}{R_2}}$$

$$\boxed{\varphi = \frac{\varphi_0 \ln\left(\frac{R_2}{r}\right)}{\ln\left(\frac{R_2}{R_1}\right)}}$$ POT.

$$\bar{E} = -\nabla\varphi = -\bar{a}_r \frac{\partial \varphi}{\partial r} \cancel{- a_\alpha \frac{1}{r} \frac{\partial \varphi}{\partial \alpha}}^{0} \cancel{- \bar{a}_z \frac{\partial \varphi}{\partial z}}^{0}$$

$$\bar{E} = +\bar{a}_r \frac{\varphi_0}{\ln \frac{R_2}{R_1}} \cdot \frac{\cancel{r}}{\cancel{R_2}} \cdot \cancel{R_2} \cdot \frac{+1}{r^{\cancel{2}}}$$

$$\boxed{\bar{E} = \bar{a}_r \frac{\varphi_0}{r \ln R_2/R_1}}$$ POLJE



$$dV = 2\pi r \, dr \cdot l$$

$$W = \int_V \frac{D \cdot E}{2} dV$$

$$W = \frac{\varepsilon}{2} \int E^2 dV = \frac{\varepsilon}{\cancel{2}} \int_{r=R_1}^{R_2} \frac{\varphi_0^2}{r^2 \ln^2(R_2/R_1)} \cdot \cancel{2}\pi \cancel{r} \cdot l \, dr$$

$$W' = \frac{W}{l} = \frac{\pi \varepsilon \varphi_0^2}{\ln^{\cancel{2}}\left(\frac{R_2}{R_1}\right)} \cancel{\ln \frac{R_2}{R_1}}$$

$$\boxed{W = \frac{\pi \varepsilon \varphi_0^2}{\ln(R_2/R_1)}}$$ ENERGIJA

$$\cancel{W} = \frac{CU^2}{2}$$

$$U = \varphi_0 \qquad \begin{array}{l} \varphi(R_2) = 0 \\ \varphi(R_1) = \varphi_0 \\ U_1 = \varphi(R_1) - \varphi(R_2) \end{array}$$

$$\frac{C' \cdot \cancel{\varphi_0^2}}{2} = \frac{\pi \varepsilon \cancel{\varphi_0^2}}{\ln \frac{R_2}{R_1}}$$

$$\boxed{C' = \frac{2\pi\varepsilon}{\ln R_2/R_1}}$$

KAPACITET

$$Q = \oint_S \bar{D}\bar{n}\, dS = \varepsilon \oint \bar{E}\bar{n}\, dS$$

$$Q = \oint_S \bar{a}_r \frac{\varphi_0 \varepsilon_0}{r \ln \frac{R_2}{R_1}} \bar{a}_r \, dS$$

$$= \frac{\varphi_0 \varepsilon_0}{r \ln R_2/R_1} \oint_S dS = \frac{\varphi_0 \varepsilon_0 \cdot 2\pi \cancel{r} \cdot l}{\cancel{r} \ln \frac{R_2}{R_1}}$$

$$C = \frac{Q}{U}$$

$$\boxed{C = \frac{2\pi \varepsilon_0 l}{\ln R_2/R_1}}$$

NEIZOLIRANI SUSTAV → STALNI NAPON

$$(\ln x)' = \frac{1}{x}$$

$$\bar{F}' = \bar{a}_{R_2} \frac{U^2}{2} \frac{\partial}{\partial R_2}(C')$$

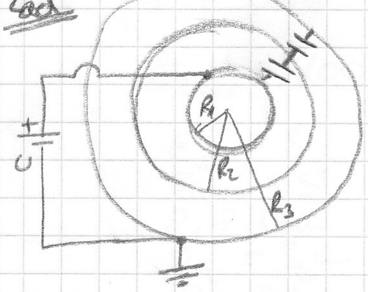
$$\bar{F}' = \bar{a}_{R_2} \frac{1}{2} U^2 \frac{\partial}{\partial R_2}\left(\frac{2\pi\varepsilon}{\ln(R_2/R_1)}\right)$$

$$\bar{F}' = \bar{a}_{R_2} \pi \varepsilon_0 U^2 \frac{-1}{\ln^2\left(\frac{R_2}{R_1}\right)} \cdot \frac{R_1}{R_2} \cdot \frac{1}{R_1}$$

$$\boxed{\bar{F}' = -\bar{a}_{R_2} \frac{\pi \varepsilon \varphi_0^2}{R_2 \ln^2(R_2/R_1)}}$$

Odredite kapacitet kuglastog dvoslojnog kondenzatora prema slici

Zad



$$C = \frac{Q}{U}$$

$$U_{AB} = -\int_B^A \vec{E}\, d\vec{l}$$

$$= -\int_{R_3}^{R_2} \frac{Q}{4\pi\varepsilon_2 r^2} dr - \int_{R_2}^{R_1} \frac{Q}{4\pi\varepsilon_1 r^2} dr$$

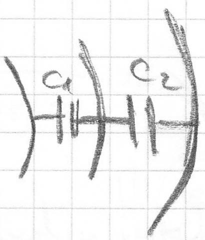
$$= \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{R_2} - \frac{1}{R_3}\right) + \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = \frac{Q}{U} = \frac{4\pi}{\frac{1}{\varepsilon_2}\left(\frac{1}{R_2} - \frac{1}{R_3}\right) + \frac{1}{\varepsilon_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$ KAPACITET DVOSLOJNOG KOND.

R_2 teži k R_3 $\Rightarrow R_2 = R_3$

$$\boxed{C = \frac{4\pi\varepsilon}{\frac{1}{R_1} - \frac{1}{R_2}}}$$ KAPACITET JEDNOSLOJNOG KOND.

$$C_1 = \frac{4\pi\varepsilon_1}{\frac{1}{R_1} - \frac{1}{R_2}}$$



$$C_2 = \frac{4\pi\varepsilon_2}{\frac{1}{R_2} - \frac{1}{R_3}}$$

---

kapacitet samostalne metalne kugle

$R_2 \to \infty$

$$C = \frac{4\pi\varepsilon}{\frac{1}{R_1} - \frac{1}{\infty}} \Rightarrow \boxed{C = 4\pi\varepsilon R_1}$$

MOODLE PITANJA/ZADACI

#1

Potencijal statičkog el. polja u prostoru je

$$\varphi = \frac{A}{y^3+a} \qquad \begin{matrix} A=1 \\ a=1 \end{matrix} \qquad \varepsilon = \varepsilon_0$$

a) $\overline{E}(1,1,1) = ?$

$$\overline{E} = -\nabla\varphi = \frac{\partial}{\partial x}\varphi\,\overline{a}_x + \frac{\partial}{\partial y}\varphi\,\overline{a}_y + \frac{\partial}{\partial z}\varphi\,a_z$$

$$\overline{E} = A\cdot\frac{3y^2}{(y^3+a)^2}\,\overline{a}_y$$

$$|\overline{E}| = 0.75$$

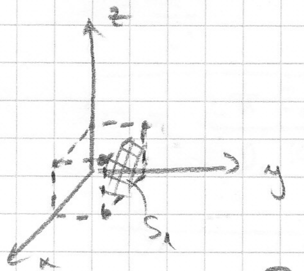
b) $\rho(2,2,2) = ?$

$$\rho = \varepsilon_0\,\mathrm{div}\,\overline{E} = \varepsilon_0\left(\frac{\partial}{\partial x}E + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}E\right)$$

$$= \varepsilon_0\,\frac{3A(-4y^4+2ya)}{(y^3+a)^3}$$

$$\rho = -0.247\,\varepsilon_0$$

c)



Ukupan naboj sadržan u kocki

$T(0,0,0)$

$T_1(1,1,1)$

$$Q = \varepsilon_0 \oiint_S \overline{E}\,\overline{n}\,dS$$

$$Q = \varepsilon_0\cdot E\cdot S_1 = \varepsilon_0\cdot\overset{S_1}{(1\cdot 1)}\cdot A\cdot\frac{3y^2}{(y^3+a)^2}$$

$$Q = \varepsilon_0\cdot 1\cdot 1\cdot A\cdot\frac{3\cdot 1^2}{(1^3+\underbrace{a}_{1})^2} = \varepsilon_0\cdot A\cdot\frac{3}{4} = 0.75\,\varepsilon_0$$

d) $W = 0$ jer kroz zatvorenu krivulju

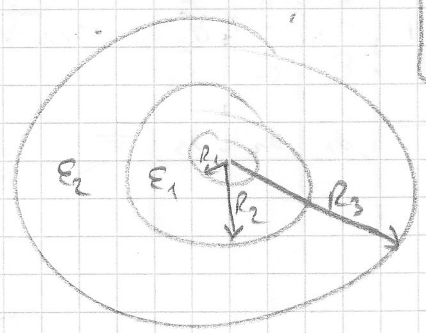
pomicanje nekog naboja po zatvorenoj krivulji

Zadan je 2-slojni kuglasti kondenzator s 2 sloja izolacije prema skici

KUGLA

$E_{2MAX} = 100 \text{ V/m}$

$\varepsilon_{r1} = 3$
$\varepsilon_{r2} = 5$
$R_1 = 1\text{cm}$
$R_2 = 3\text{cm}$
$R_3 = 5\text{cm}$



* $E_{1MAX} = ?$

$$E = \frac{Q}{4\pi \varepsilon_0 \varepsilon_r r^2}$$

$$E_{2MAX} = \frac{Q}{4\pi \varepsilon_0 \varepsilon_{r2} R_2^2}$$

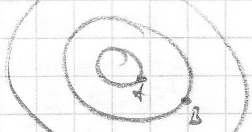
$$Q = E_{2MAX} \cdot 4\pi \varepsilon_0 \varepsilon_{r2} R_2^2$$

$$E_{1MAX} = \frac{Q}{4\pi \varepsilon_0 \varepsilon_{r1} R_1^2}$$

$$E_{1MAX} = \frac{E_{2MAX}\, \varepsilon_{r2} R_2^2}{\varepsilon_{r1} R_1^2}$$

Zadan je cilindrični kondenzator sa 2 sloja izolacije

CILINDAR



$E_{2MAX} = 100 \text{ V/m}$
$\varepsilon_{r1} = 3$
$\varepsilon_{r2} = 5$
$R_1 = 1\text{cm}$
$R_2 = 3\text{cm}$
$R_3 = 5\text{cm}$

$$E = \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_r \cdot r}$$

$$E_{2MAX} = \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_{r2} R_2}$$

$$\lambda = E_{2MAX}\, 2\pi \varepsilon_0 \varepsilon_{r2} R_2$$

!! e)

$$E_{1MAX} = \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_{r1} R_1}$$

$$E_{1MAX} = \frac{E_{2MAX} \cdot \varepsilon_{r2} \cdot R_2}{\varepsilon_{r1} \cdot R_1}$$

f) $\lambda = ?$ naboj po metru duljine na unutarnjem cilindru

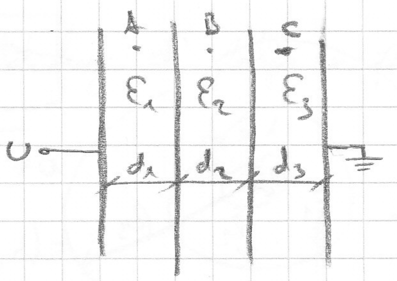
g) $U_{AB} = ?$

$$U_{AB} = -\int_{R_2}^{R_1} \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_{r1} r} = \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_{r1}} \ln \frac{R_2}{R_1} = 5.49 \text{ V}$$

h) $\frac{1}{C'} = \frac{1}{C_1'} + \frac{1}{C_2'}$ $\qquad C_1' = \frac{2\pi \varepsilon_1}{\ln R_2/R_1}$ $\qquad C_2' = \frac{2\pi \varepsilon_2}{\ln R_3/R_2}$

Zadan je troslojni pločasti kond.

$d_1 = 1cm$
$d_2 = 2cm$
$d_3 = 3\,cm$



$\varepsilon_{r1} = 2$
$\varepsilon_{r2} = 3$
$\varepsilon_{r3} = 5$

$S = 0.05\,m^2$

$U = 100V$

1) $E_2 = ?$

$$U = E_1 d_1 + E_2 d_2 + E_3 d_3$$

$$D_1 = D_2 = D_3 \longrightarrow$$ zbog normalne komponente

$$\varepsilon_1 E_1 = \varepsilon_2 E_2 = \varepsilon_3 E_3$$

tang komp. od E i norm. od D prolaze nepromijenjene

$$E_1 = \frac{\varepsilon_2}{\varepsilon_1} E_2$$

$$E_3 = \frac{\varepsilon_2}{\varepsilon_3} E_2$$

$$U = \frac{\varepsilon_2}{\varepsilon_1} E_2 \cdot d_1 + E_2 d_1 + \frac{\varepsilon_2}{\varepsilon_3} E_2 d_3$$

$$E_2 = \frac{U}{[\quad]} = 1886.8\ V/m$$

2) $\varphi(x = d_1)$

$$\varphi = U - E_1 \cdot d_1 = U - \frac{\varepsilon_2}{\varepsilon_2} E_2 d_1 = 71.7V$$

3) $D_c = ?$

$$D_c = D_3 = D_2 = D_1$$

$$D_c = \varepsilon_2 \cdot E_2 = 50.1\ C/m^2$$

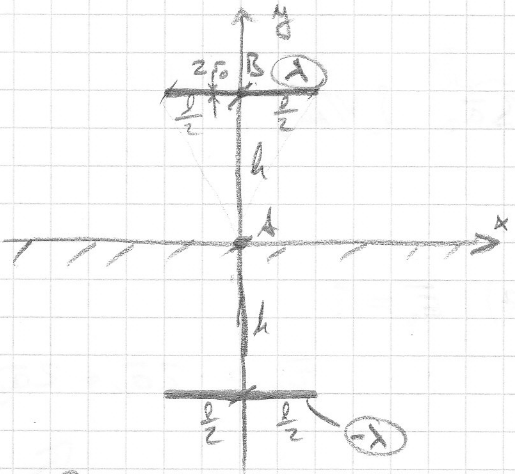
4) $C = ?$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_1 = \varepsilon_0 \varepsilon_{r1} \frac{S}{d_1} \qquad C_2 = \varepsilon_0 \varepsilon_{r2} \frac{S}{d_2} \qquad C_3 = \varepsilon_0 \varepsilon_{r3} \frac{S}{d_3}$$

Zad) Vodič nabijen nabojem

$L = 1\,m$
$\lambda = 10^{-9}\,C/m$
$r_0 = 1\,cm$
$\varepsilon = \varepsilon_0$
$h = 1\,m$



5) $E_y(A) = ?$

zato što su 2 štapa

$$E_A = 2 \cdot \frac{\lambda}{4\pi\varepsilon_0 h} \frac{L}{\sqrt{\left(\frac{L}{2}\right)^2 + h^2}} (-\bar{a}_y)$$

$$E_A = -16.1\ V/m$$

6) $\varphi(A) = ?$

$$\varphi_A = \varphi_+ + \varphi_- = 0$$

---

7) $\varphi(B) = ?$ na polumjeru štapa

$$\varphi(B) = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + r_0^2}}{-\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + r_0^2}}$$

POTENCIJAL KRATKOG ŠTAPA

$$- \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + (2h - r_0)^2}}{-\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + (2h - r_0)^2}}$$

ZBOG SUPERPOZICIJE

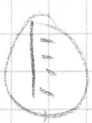
$$= 78.33\,V$$

8) odredite ukupni influencirani naboj na pov. zemlje

$Q_{INF} = ?$

$$|Q_{INF}| = \lambda \cdot L$$
$$= -1\,nC$$

+λ

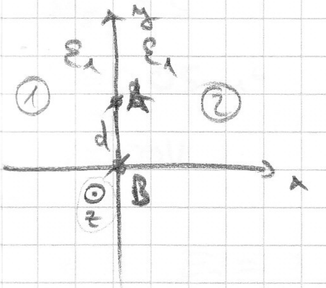


zato je minus

Uvjet na granici

$$\overline{E}_1 = \overbrace{3\,\overline{a}_x}^{E_n} + \overbrace{2\,\overline{a}_y}^{E_t} + \overbrace{3\,\overline{a}_z}^{E_t}\ [V/m]$$

$\varepsilon_{r1} = 2$
$\varepsilon_{r2} = 3$



17) $\overline{E}_2 = ?$

$$\overline{E}_2 = 2\overline{a}_x + 2\overline{a}_y + 3\overline{a}_z$$

$D_{x1} = D_{x2}$

$\varepsilon_1 E_{x1} = \varepsilon_2 E_{x2}$

$$E_{x2} = \frac{\varepsilon_1}{\varepsilon_2} E_{x1} = \frac{2}{3} \cdot 3 = 2$$

18) vektor polarizacije $P_1 = ?$

$$\overline{P}_1 = \overline{D}_1 - \varepsilon_0 \overline{E}_1 = \varepsilon_0 (\underbrace{\varepsilon_{r1}}_{2} - 1)\overline{E}_1 = \varepsilon_0 \overline{E}_1$$

19) $P_2 = ?$

$$\overline{P}_2 = \overline{D}_2 - \varepsilon_0 \overline{E}_2 = \varepsilon_0 ((\varepsilon_{r2} - 1)\overline{E}_2) = 2\varepsilon_0 \overline{E}_2$$

20) $U_{AB} = ?$
$d = 1m$

$U_{AB} = -E_y \cdot d =$ → zbog povećanja polja u smjeru +y