3. ZADATAK   WIR 2014

S_1: $w(n) + \frac{1}{2} w(n-1) = 2u(n)$ S_2: $y(n) - \frac{1}{5} y(n-1) = 4w(n)$

a) vrlna domena, diskretna, impuls, pravo h_1

$h_1(n) + \frac{1}{2} h_1(n-1) = \delta(n)$

$q + \frac{1}{2} = 0$ $h_1(n) = C_1 \cdot \left(\frac{-1}{2}\right)^n$

$q = -\frac{1}{2}$

$h(0) = 1$

$C_1 = 1 \Rightarrow$

$h_1(n) = 1 \cdot \left(\frac{1}{2}\right)^n \mu(n)$

h_2 z

$\frac{Y(z)}{z^0} - \frac{1}{5} \frac{Y(z)}{z^1} = 4W(z)$

$Y(z)\left(1 - \frac{1}{5} z^{-1}\right) = 4W(z)$

$H(z) = \frac{Y(z)}{W(z)} = \frac{4}{1 - \frac{1}{5} z^{-1}} = \frac{4z}{z - \frac{1}{5}}$

$h(n) = 4 \cdot \left(\frac{1}{5}\right)^n \mu(n)$ ✓

2) ZADATAK! Serija → množimo   impulse od oba

prebacujemo

$2u(n)$

$H_1(z) = 2 \cdot \frac{z}{z + \frac{1}{2}}$ $H_2(z) = \cancel{4} \cdot \frac{4z}{z - \frac{1}{5}}$

već ima

$H_{uk}(z) = H_1(z) \cdot H_2(z) = \frac{2z}{z + \frac{1}{2}} \cdot \frac{4z}{z - \frac{1}{5}} = \frac{8z^2}{\left(z + \frac{1}{2}\right)\left(z - \frac{1}{5}\right)}$

$\frac{H_{uk}(z)}{z} = \frac{8z}{\left(z + \frac{1}{2}\right)\left(z - \frac{1}{5}\right)} \Rightarrow \frac{A}{z + \frac{1}{2}} + \frac{B}{z - \frac{1}{5}} = \frac{40}{7} \cdot \frac{z}{z + \frac{1}{2}} + \frac{16}{7} \cdot \frac{z}{z - \frac{1}{5}}$

$A = \left.\frac{8z}{z - \frac{1}{5}}\right|_{z=-\frac{1}{2}} = \frac{40}{7}$ $B = \frac{8z}{z + \frac{1}{2}} = \frac{16}{7}$ $= \left(\frac{40}{7} \cdot \left(\frac{-1}{2}\right)^n + \frac{16}{7} \cdot \left(\frac{1}{5}\right)^n\right) \mu(n)$

4. $y''(t) + 11y'(t) + 30y(t) = 2u'(t) + 6u(t)$ $\quad u(t) = 3e^{-5t}\mu(t)$

wir zom $y(0^-) = 3,\ y'(0^-) = -40$ $\quad a_0 = 1 \quad a_1 = 11 \quad a_2 = 30$

a) prisilni odziv $\quad b_0 = 0 \quad b_1 = 2 \quad b_2 = 6$

Ce^{at}

$a^2 + 11a + 30 = 0$

$a_1 = -5 \quad a_2 = -6$

$y_h(t) = C_1 e^{-5t} + C_2 e^{-6t}$

$y_p(t) = K \cdot e^{-5t} \cdot t$

$y_p'(t) = Ke^{-5t} - 5Ke^{-5t} \cdot t$

$y_p''(t) = -5Ke^{-5t} - 5Ke^{-5t} + 25Ke^{-5t}t$

upoč!

PAZI unos!

~~$K \cdot e^{-5t} \cdot t + 11Ke^{-5t} - 55Ke^{-5t} \cdot t - 150Ke^{-5t} - 150Ke^{-5t} + 30Ke^{-5t}t = -30e^{-5t} + 18e^{-5t}$~~

$-5K\cancel{e^{-5t}} - 5K\cancel{e^{-5t}} + 25K\cancel{e^{-5t}}t + 11K\cancel{e^{-5t}} - 55K\cancel{e^{-5t}} \cdot t + 30K\cancel{e^{-5t}} \cdot t = -30\cancel{e^{-5t}} + 18\cancel{e^{-5t}}$

$-5K - 5K + 25Kt + 11K - 55Kt + 30Kt = -30 + 18$

$K + 0Kt = -12$ $\quad y_p(t) = -12te^{-5t}$

$K = -12$

$y_{tot} = C_1 e^{-5t} + C_2 e^{-6t} - 12te^{-5t}$ $\quad y(0^+) - y(0^-) = 0 \quad y(0^+) = 3$

$y'_{tot} = -5C_1 e^{-5t} - 6C_2 e^{-6t} - 12e^{-5t} + 60te^{-5t}$ $\quad y'(0^+) - y'(0^-) = b_1 u(0) = 0 + 2 \cdot 3$

$y'(0^+) = 6 - 40 = -34$

$C_1 + C_2 = 3$ $\quad C_1 = 3 - C_2$

$-5C_1 - 6C_2 - 12 = -34$ $\quad C_1 = -4$ $\quad y_{tot} = -4e^{-5t} + 7e^{-6t} - 12te^{-5t}$

$-15 + 5C_2 - 6C_2 = -22$

$-C_2 = -7$ $\quad C_2 = 7$

1) [illegible] $\quad$ [illegible] $u(t)=0$ $\quad$ $Y(0^-)=Y(0^+)=0$

$Y'(0^+)=6$

$Y_{mn}(t)=C_1 e^{-5t}+C_2 e^{-6t}-12t e^{-5t}$

$Y'_{mn}(t)=-5C_1-6C_2-12e^{-5t}+60te^{-5t}$

$Y_{mn}(t)=18e^{-5t}-18e^{-6t}-12te^{-5t}$

$C_1+C_2=0$ $\quad$ $C_1=-C_2$

$-5C_1-6C_2-12=6$ $\quad$ $C_1=18$

$5C_2-6C_2-12=6$

$-C_2=18$

$C_2=-18$

$\mu(t)$

[illegible] $u(t)=0$

$Y_{nf}=C_1 e^{-5t}+C_2 e^{-6t}$

$Y_n'(t)=-5C_1e^{-5t}+-6C_2e^{-6t}$

$Y(0^-)=3$

$Y'(0^-)=-40$

$Y(0^+)=3$

$Y'(0^+)=-40$ $\quad$ $C_1+C_2=3$ $\quad$ $C_1=3-C_2$

$-5C_1-6C_2=-40$ $\quad$ $C_1=-22$

$-15+5C_2-6C_2=-40$

$-C_2=-25$ $\quad$ $Y_n(t)=(-22e^{-5t}+25e^{-6t})\mu(t)$

$C_2=25$

5. c)

$y(n) + a_1 y(n-1) + a_2 y(n-2) = b_0 u(n)$

$u_1(n) = 1^n \quad y_1(n) = 1^n \quad u_2(n) = \left(\frac{1}{2}\right)^n \Rightarrow y_2(n) = 2\left(\frac{1}{2}\right)^n$

$u_3(n) = \left(\frac{1}{4}\right)^n \quad y_3(n) = 4\left(\frac{1}{4}\right)^n$

a) PARAMETRI SUSTAVA $\{a_1, a_2, b_0\}$

$1^n + a_1 \cdot 1^{n-1} + a_2 \cdot 1^{n-2} = b_0 1^n$

$1 + a_1 + a_2 = b_0 \quad (I)$

$\left(\frac{1}{2}\right)^n + a_1 \left(\frac{1}{2}\right)^{n-1} + a_2 \left(\frac{1}{2}\right)^{n-2} = 2b_0 \left(\frac{1}{2}\right)^n$

$1 + a_1 4 + 8a_2 = b_0 \quad (II)$

$4\left(\frac{1}{4}\right)^n + 4a_1 \left(\frac{1}{4}\right)^{n-1} + 4\left(\frac{1}{4}\right)^{n-2} a_2 = b_0 \left(\frac{1}{4}\right)^n$

$4 + 16a_1 + 64a_2 = b_0$

$2 + 4a_1 + 8a_2 = 1 + a_1 + a_2$

$3a_1 = -1 - 4a_2 \;|:3 \qquad a_1 = -\frac{1}{3} - \frac{7}{3}a_2$

$4 - \frac{16}{3} - \frac{112}{3}a_2 + 64a_2 = 1 + a_1 + a_2$

$4 - \frac{16}{3} - \frac{112}{3}a_2 + 64a_2 = 1 - \frac{1}{3} - \frac{7}{3}a_2 + a_2$

$28a_2 = 2$

$a_2 = \frac{1}{14}$

$4 + 16a_1 + \frac{32}{7} = 1 + a_1 + \frac{1}{14}$

$15a_1 = -\frac{15}{2}$

$a_1 = -\frac{1}{2}$

$b_0 = \frac{4}{7}$

$y(n) + -\frac{1}{2}y(n-1) + \frac{1}{14}y(n-2) = \frac{4}{7}u(n)$