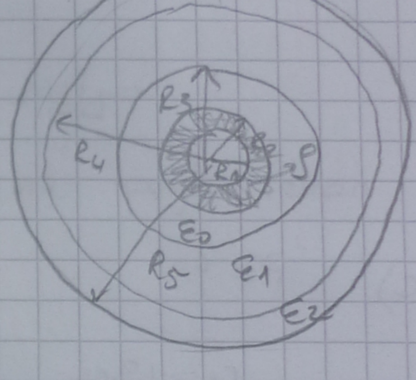
1. ZAD Između sfernih ljuski polumjera R_1 i R_2 nalazi se na

volumne gustoće ρ određene jednadžbom:

$$\rho = \begin{cases} 0, & r < R_1 \\ \dfrac{10^{-6}}{r+1}, & R_1 \le r \le R_2 \quad \left[\dfrac{C}{m^2}\right] \\ 0, & r > R_2 \end{cases}$$

Zadano je $\varepsilon_{r1} = 4$, $\varepsilon_{r2} = 2$, $R_1 = 1cm$, $R_2 = 3cm$, $R_3 = 4{,}5cm$, $R_4 = 10cm$, $R_5 = 14cm$.



a) $E(r_1 = 0{,}02) = ?$

$$\oint \vec{D}\,\vec{n}\,ds = \iiint_V \rho\,dV \qquad \boxed{dV = 4\pi r^2 dr}$$

$$D \cdot \cancel{4} \cdot r_1^2 \cancel{\pi} = \int_{R_1}^{r_1} \frac{10^{-6}}{r+1} \cancel{4\pi} r^2 dr = \qquad \frac{r^2 + r - r}{r+1} = r - \ldots$$

$$= r - \frac{(r+\ldots}{\ldots}$$

$$D \cdot r_1^2 = 10^{-6} \left[\frac{r^2}{2} - r + \ln(r+1) \right]_{R_1}^{r_1} \qquad = r - 1 \ldots$$

$$D = \frac{10^{-6}}{r_1^2} \left[\frac{1}{2}(r_1^2 - R_1^2) - (r_1 - R_1) + \ln\left(\frac{r_1+1}{R_1+1}\right) \right]$$

$$E \Rightarrow \frac{D}{\varepsilon_0} = 648{,}4 \; \frac{V}{m}$$

b) iznos el indukcije

$r_1 = 4\,cm$

$D \sim \frac{Q}{r^2}$

$D\left[\frac{\mu C}{m^2}\right]$

$$D\,4 r_1^2 \pi = \iiint_{R_1}^{R_2} \rho\, dV = 10^{-6}\left[\frac{r^2}{2} - r + \ln(r+1)\right]_{R_1}^{R_2}$$

$D = 5{,}3\,\frac{\mu C}{m^2}$

c) $P = ?$

$r_1 = 13\,cm$

$\varepsilon_{r2} = 2$

$$D \cdot 4 r_1^2 \pi = \left[\quad\right]_{R_1}^{R_2}$$

$P = D\left(1 - \frac{1}{\varepsilon_r}\right)$

$P = D\left(1 - \frac{1}{2}\right) = 251\,\frac{pC}{m^2}$

d) $W = ?$

$D = \varepsilon_0 \vec{E}$

$\vec{E} = \frac{\vec{D}}{\varepsilon_0}$

$R_2 \leq r \leq R_3$

$$D = \frac{10^{-6}}{r^2}\left[\frac{r^2}{2} - r + \ln(r+1)\right]_{R_1=0{,}01}^{R_2=0{,}03} = 10^{-6} \cdot \frac{8{,}47 \cdot 10^{-6}}{r^2}$$

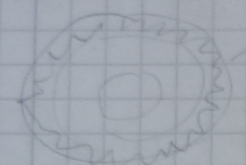
$$W = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}\, dV = \frac{1}{2}\frac{1}{\varepsilon_0}\int_{R_2}^{R_3} \frac{\left[10^{-6} \cdot 8{,}47 \cdot 10^{-6}\right]^2}{r^{4}}\, 4 r^2 \pi\, dr \quad dV$$

$$W = \frac{2\pi}{\varepsilon_0}\left(8{,}47 \cdot 10^{-12}\right)^2 \cdot \left(-\frac{1}{r}\right)_{R_2}^{R_3} = \underline{0{,}57\,\mu J}$$

ALTERNATIVA ① $W = \frac{Q^2}{2C}$ $\quad C = 4\pi\varepsilon \frac{R_2 R_3}{R_3 - R_2} = \frac{4\pi\varepsilon}{\frac{1}{R_2} - \frac{1}{R_3}}$

ALTERNATIVA ② $W = \frac{CU^2}{2}$ $\quad U = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{R_2} - \frac{1}{R_3}\right)$

[u ispitu bilo – isto ali s metalnim obručem



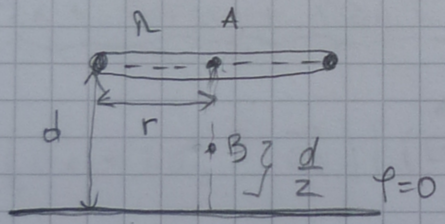
→ metalni dio

– na potencijal ne bi utjecao

– metal → ne utječe na polje izvan

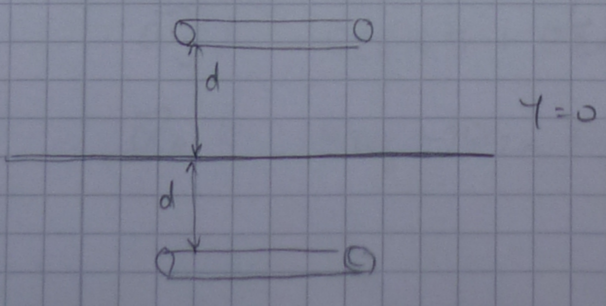
$$\int_{\infty}^{R_3} - \int_{R_2}^{R}$$
]

ZAD 2. Na udaljenosti $d = 1\,m$ od uzemljene metalne ravnine nalazi se prsten zanemarivog poprečnog presjeka, polumjera $r = 0{,}5\,m$ nabijen nabojem $\lambda = 17\,\mu C/m$ prema slici.



a) jakost el. polja u $\left[\frac{V}{m}\right]$ u točki A u središtu prstena

↳ oslikavanje



E_0

→ polje na osi

PRSTEN (izvedeno u zbirci)

$$E = \frac{\lambda \cdot r}{2\varepsilon_0} \frac{z}{(r^2+z^2)^{3/2}}$$

$\vec{E}_A = \underbrace{\vec{E}_{1A}}_{=0} + \vec{E}_{1B}$ - oslikani samo

$$E_A = \frac{\lambda \cdot r \cdot 2d}{2\varepsilon_0 (r^2 + (2d)^2)^{3/2}} = 109,6 \ \frac{V}{m}$$

b) iznos jakosti el. polja $[\frac{V}{m}]$ u točki B na udaljenosti 0,5m od ravnine

$h_1 = 0,5$

$$E_B = \frac{\lambda \cdot r}{2\varepsilon_0} \cdot \left[\frac{0,5}{(r^2 + 0,5^2)^{3/2}} + \frac{1,5}{(r^2 + 1,5^2)^{3/2}} \right] = 861 \ \frac{V}{m}$$

c) Odrediti iznos rada u [J] koji je potrebno utrošiti da se proton u beskonačnosti dovede u točku A.

$W_{p\infty} = 0$

$W_{pA} = Q \cdot \varphi_A \Rightarrow A = W_{pA}$

$$\varphi = \frac{\lambda}{2\varepsilon_0} \cdot \frac{r}{\sqrt{r^2 + z^2}}$$ (φ na osi prstena)
izvod - Berberović

$$\varphi_A = \frac{\lambda}{2\varepsilon_0} \left[\frac{r}{\sqrt{r^2}} - \frac{r}{\sqrt{r^2 + (2d)^2}} \right]$$

$W = \varphi_A \cdot Q = 1,16 \cdot 10^{-16} \ J$

d) $\nabla \times \vec{E} = ?$

$\underline{\nabla \times \vec{E} = 0}$

3. Granica dielektrika i slobodnog prostora određena je jednadžbom $x + 2y + z = 12m$ u području u kojem se nalazi ishodište, relativne dielektričnosti $\varepsilon_r = 4$, zadana je jakost el. polja $\vec{E}_1 = 2\vec{a}_x + 5\vec{a}_z \left[\frac{V}{m}\right]$

a), b) $E = ? \left[\frac{V}{m}\right]$ u slobodnom prostoru u smjeru osi x, E_{2x}

$$\vec{n} = \left(\vec{a}_x + 2\vec{a}_y + \vec{a}_z\right) \frac{1}{\sqrt{6}}$$

1. jdba $\vec{n} \cdot \left(\vec{D}_2 - \vec{D}_1\right) = 0 = \sigma$

$$\vec{D}_1 = \varepsilon_0 \cdot \underset{4}{\varepsilon_r} \cdot \vec{E}_1$$

2. jdba $\vec{n} \times \left(\vec{E}_2 - \vec{E}_1\right) = 0 = \vartheta$

$$\vec{D}_1 = \varepsilon_0 \left(8\vec{a}_x + 20\vec{a}_z\right)$$

$$\vec{E}_2 = E_{2x}\vec{a}_x + E_{2y}\vec{a}_y + E_{2z}\vec{a}_z,$$

$$\vec{D}_2 = \varepsilon_0 \cdot \vec{E}_2 = \varepsilon_0\left(E_{2x}\vec{a}_x + E_{2y}\vec{a}_y + E_{2z}\vec{a}_z\right)$$

1. $$\frac{1}{\sqrt{6}} \cdot \left(\vec{a}_x + 2\vec{a}_y + \vec{a}_z\right)\varepsilon_0 \cdot \left[\left(E_{2x} - 8\right)\vec{a}_x + E_{2y}\vec{a}_y + \left(E_{2z} - 20\right)\vec{a}_z\right] = 0$$

$$E_{2x} - 8 + 2E_{2y} + E_{2z} - 20 = 0$$

$$\underline{E_{2x} + 2E_{2y} + E_{2z} = 28}$$

2. $$\frac{1}{\sqrt{6}}\left(\vec{a}_x + 2\vec{a}_y + \vec{a}_z\right)\left[\left(E_{2x} - 2\right)\vec{a}_x + E_{2y}\vec{a}_y + \left(E_{2z} - 5\right)\vec{a}_z\right] = \vec{0}$$

$$\vec{a}_z E_{2y} - \vec{a}_y\left(E_{2z} - 5\right) - \vec{a}_z 2\left(E_{2x} - 2\right) + \vec{a}_x 2\left(E_{2z} - 5\right) + \vec{a}_y\left(E_{2x} - 2\right) - \vec{a}_x E_{2y} = \vec{0}$$

$\vec{a}_x$: $-E_{2y} + 2E_{2z} - 10 = 0 \Rightarrow \underline{2E_{2z} - E_{2y} = 10}$

$\vec{a}_y$: $-E_{2z} + 5 + E_{2x} - 2 = 0 \Rightarrow \underline{E_{2x} - E_{2z} = -3}$

$\vec{a}_z$: $E_{2y} - 2E_{2x} + 4 = 0 \Rightarrow \underline{2E_{2x} - E_{2y} = 4}$

$$E_{2x} - E_{2y} = 2$$

$$\begin{cases} E_{2y} = 2E_{2z} - 10 \\ E_{2x} = E_{2z} - 3 \end{cases}$$

$(I) \Rightarrow E_{2z} - 3 + 4E_{2z} - 20 + E_{2z} = 28$

$6E_{2z} = 51$

$E_{2z} = 8{,}5$

$E_{2y} = 7$

$\underline{\underline{E_{2x} = 5{,}5}}$

c) iznos integrala $\oint E \cdot dl$ u [V] po krivulji oblika trokuta određenog točkama

$(0;0;0) \rightarrow (0{,}8\,m;0;0) \rightarrow (0;0;0;)$

a), b) DRUGI NAČIN

$n = \frac{1}{\sqrt{6}}(\vec{a_x} + 2\vec{a_y} + \vec{a_z})$

$\vec{n} \cdot \vec{E} = \vec{E_n}$

$D_{1n} = D_{2n}$

$E_{1t} = E_{2t}$

$\vec{E_1} \Rightarrow \begin{cases} \vec{E_{1n}} = (\vec{n} \cdot \vec{E_1}) \cdot \vec{n} \Rightarrow \vec{D_{1n}} = \varepsilon_0 \varepsilon_{r1} \vec{E_{1n}} \\ \vec{E_{2t}} = E_1 - E_{1n} \end{cases}$

$E_1 = \vec{E_{1t}} + \vec{E_{1n}}$

$\vec{E_{1n}} = (2+5) \cdot \frac{1}{6} \cdot (\vec{a_x} + 2\vec{a_y} + \vec{a_z}) = \frac{7}{6}(\vec{a_x} + 2\vec{a_y} + \vec{a_z})$

$\vec{E_{1t}} = 2\vec{a_x} + 5\vec{a_z} - \frac{7}{6}(\vec{a_x} + 2\vec{a_y} + \vec{a_z}) =$
$= \frac{5}{6}\vec{a_x} - \frac{14}{6}\vec{a_y} + \frac{23}{6}\vec{a_z}$

$D = \varepsilon \cdot E$

$\vec{D_{1n}} = \vec{E_{1n}} \cdot \varepsilon_0 \varepsilon_{r1}$

$\vec{E_{2n}} = \frac{\vec{D_{1n}}}{\varepsilon_0 \varepsilon_{r2}} = \vec{E_{1n}} \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = 4E_{1n} = \frac{14}{3}(\vec{a_x} + 2\vec{a_y} + \vec{a_z})$

$\vec{E_2} = \vec{E_{1t}} + \vec{E_{2n}} = \frac{33}{6}\vec{a_x} + \frac{56-14}{6}\vec{a_y} + \frac{28+23}{6}\vec{a_z}$

(4.) U cilindričnom koord. sustavu raspodjela naboja u slobodnom prostoru zadana je s

$$\rho = \begin{cases} 0, & r < 0{,}5\,m \\ \frac{10^{-9}}{r^4}, & 0{,}5\,m \le r \le 1\,m \quad \left[\frac{C}{m^3}\right] \\ \frac{10^{-9}}{r^3}, & 1\,m \le r \le 2\,m \end{cases}$$

Točka referentnog potencijala nalazi se na polumjeru 10 m, $\varphi(r = 10\,m) = 0$

a) Odredite potencijal u točki (3m, 0, 0) u [V]

Linijski naboj na pravcu

$$\sigma = D = \frac{Q}{S}$$

$$E = \frac{D}{\varepsilon_0} = \frac{Q}{\varepsilon \cdot S} = \frac{\lambda \cdot \ell}{\varepsilon \cdot S}$$

$$E = \frac{\lambda}{2\pi \varepsilon r}$$

$$\varphi = -\int \frac{\lambda}{2\pi\varepsilon r} = \frac{\lambda}{2\pi\varepsilon} \ln \frac{R_a}{R_e}$$

b) jakost el. polja u (1,4m, 0, 0)

$$\varepsilon_0 E \cdot 2\pi \cdot r \cdot \ell = 10^{-9} \Big[\int_{r=0,5}^{1} \frac{1}{r^3} \underbrace{2\pi \cdot r \cdot \ell \cdot dr}_{dV} + \int_{r=1}^{2} \frac{1}{r^3} 2\pi \cdot r \cdot \ell \cdot dr$$

$$E = \frac{2 \cdot 10^{-9}}{\varepsilon_0} \cdot \frac{1}{r}$$

$$\varphi(r=3) = -\int_{10}^{3} \frac{2 \cdot 10^{-9}}{\varepsilon_0 \cdot r}\, dr = \underline{\underline{272\,V}}$$

c) $r = 0{,}7$

$$\varepsilon_0 \cdot E \cdot 2\pi \cdot r \cdot \ell = 10^{-9} \Big[\int_{0,5}^{0,7} \frac{1}{r^3} 2\pi \cdot r \cdot \ell \cdot dr \Big] \Rightarrow E = 158{,}1 \frac{V}{m}$$

d) $$\varphi(r=0) = -\int_{r=10}^{2} E' dr - \int_{2}^{1} E'' dr - \int_{1}^{0,5} E''' dr = 575\,V$$