

ZADACI 1.1

$$1.1-1 \quad Q_1 = 3 \cdot 10^{-8} C \quad |Q_2| = 3 \cdot 10^{-9} C$$

$$n = 10$$

$$d = 10^{-1} m$$

$$\xi_0 = 8.854 \cdot 10^{-12} \frac{As}{Vm}$$

$$a) \quad F = k \frac{Q_1 Q_2}{d^2} = 9 \cdot 10^9 \frac{3 \cdot 10^{-8} \cdot 3 \cdot 10^{-9}}{10^{-2}} \\ = 9 \cdot 10^9 \frac{9 \cdot 10^{-17}}{10^{-2}} \\ = 81 \cdot 10^6 = 81 \mu N$$

b) negativnim je ja sila privlačna

$$c) \quad F = k \frac{Q_1 Q_2}{d^2} \quad F' = k \frac{Q_1 Q_2}{(d/2)^2} = 4F$$

poveća se 4 puta

$$1.1-2 \quad a) \quad Q_3 = +Q_1$$



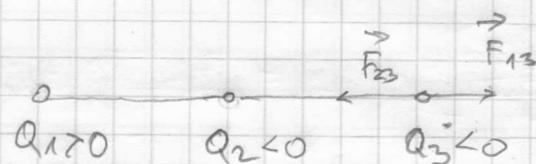
$$F_{12} = 8.1 \cdot 10^{-5} N$$

$$F_{32} = F_{12} + F_{32} \\ = 324 + 81 = 405 \mu N$$

$$F_{32} = k \frac{Q_3 Q_2}{(d/2)^2} = 9 \cdot 10^9 \frac{3 \cdot 10^{-8} \cdot 3 \cdot 10^{-9}}{25 \cdot 10^{-4}} =$$

$$324 \cdot 10^{-4} = 324 \mu N$$

(b)



$$\vec{F}_{\text{uk}} = \vec{F}_{23} + \vec{F}_{13} = 324 - 81 = -243 \text{ MN}$$

odbojna rite

l. 1-3

$$m = 10^{-3} \text{ kg}$$

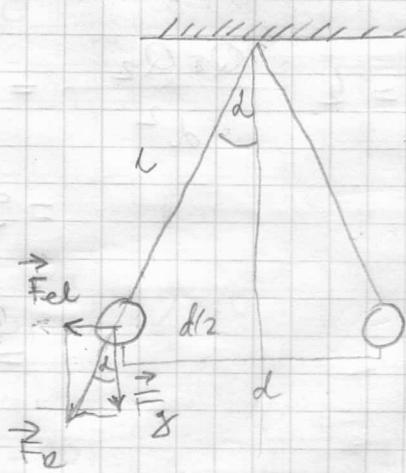
$$l = 1 \text{ m}$$

$$d = 10^{-1} \text{ m}$$

a)

$$\sin \alpha = \frac{d}{2l} = 0.05$$

$$\alpha = 2.87^\circ$$



$$\tan \alpha = \frac{d/2}{\sqrt{l^2 + (\frac{d}{2})^2}} = \frac{d}{2l}$$

$$\tan \alpha = \frac{F_{\text{el}}}{F_g}$$

$$F_{\text{el}} = F_g \tan \alpha = mg \tan \alpha$$

$$= 9.81 \cdot 10^{-3} \tan \alpha$$

$$= 5 \cdot 10^{-4} \text{ N}$$

$$F = k \frac{Q^2}{d^2}$$

$$Q = \sqrt{\frac{Fd^2}{k}} = \sqrt{\frac{5 \cdot 10^{-4} \cdot 10^{-2}}{9 \cdot 10^9}} =$$

$$\sqrt{\frac{5}{9} \cdot 10^{-15}} = \sqrt{5.5 \cdot 10^{-16}} = 2.358 \cdot 10^{-8} \text{ C}$$

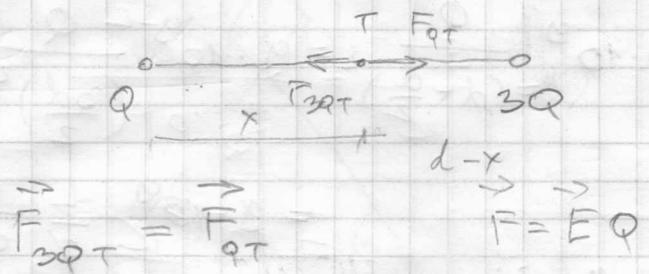
b)

$$Q_{\text{uk}} = Q_1 + Q_2 = 4.672 \cdot 10^{-8}$$

$$F = k \frac{\frac{3}{4} Q_{\text{uk}} \cdot \frac{1}{4} Q_{\text{uk}}}{d^2} = k \frac{\frac{3}{16} Q_{\text{uk}}^2}{d^2} = \frac{kmg}{2l}$$

$$6k Q_{\text{uk}} l = 16 d^3 mg \Rightarrow d = \sqrt[3]{\frac{3k Q_{\text{uk}} l}{16mg}} = 0.09 \text{ m}$$

1.1-4



$$\frac{Q}{x^2} = \frac{3Q}{(d-x)^2}$$

$$3x^2 = d^2 - 2dx + x^2$$

$$2x^2 = d^2 - 2dx$$

$$2x^2 + 2dx - d^2 = 0$$

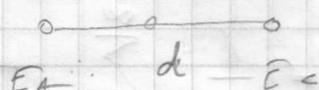
$$x_{1,2} = \frac{-2d \pm \sqrt{4d^2 + 8d^2}}{4}$$

$$= \frac{-2d \pm 2\sqrt{3}d}{4}$$

$$= \frac{-d \pm \sqrt{3}d}{2} = \frac{\sqrt{3}-1}{2}d$$

1.1-5. $E_A = 36 \text{ V/m}$

$$E_C = 9 \frac{V}{m}$$



$$E_A = k \frac{Q}{(x^2)} = 36$$

$$k \frac{Q}{x^2} = 4$$

$$E_B = k \frac{Q}{(x+d)^2} = ?$$

$$4x^2 = x^2 + 2xd + d^2$$

$$E_C = k \frac{Q}{(x+d)^2} = 9$$

$$3x^2 - 2xd - d^2 = 0$$

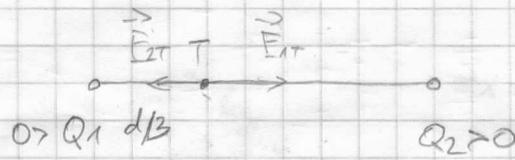
$$x_{1,2} = \frac{2d \pm \sqrt{4d^2 + 12d^2}}{6} = \frac{2d \pm 4d}{6}$$

$$x=d \quad E_B = g \cdot 10^9 \cdot \frac{Q}{(d + \frac{d}{2})^2} = g \cdot 10^9 \cdot \frac{Q}{(\frac{3d}{2})^2} =$$

$$= g \cdot 10^9 \cdot \frac{4}{9} \frac{Q}{d^2} = 4 \cdot 10^9 \frac{Q}{d^2}$$

$$E = \frac{\epsilon Q}{d^2} \quad Q = \frac{Ed^2}{\epsilon} = \frac{g \cdot 10^{-4}}{g \cdot 10^9} = 4 \cdot 10^{-13} \text{ C} = 16 \frac{V}{m}$$

1.1-6



$$Q_1 = 10^{-9} \text{ As}$$

$$d = 10^{-2} \text{ m}$$

$$\vec{E}_{2r} = \vec{E}_{1r}$$

$$\frac{Q_2}{Q_1} = n$$

$$k \frac{Q_1}{(d/3)^2} = k \frac{Q_2}{(2d/3)^2} \quad q_r = ?$$

$$\frac{2Q_1}{d^2} = \frac{gQ_2}{4d^2}$$

$$\frac{Q_2}{Q_1} = 4$$

15)

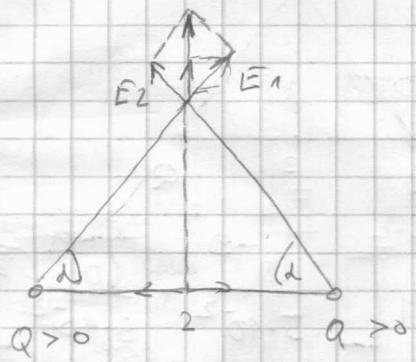
$$V = q_r = \frac{1}{4\pi\epsilon_0} \sum_{n=1}^{\infty} \frac{Q_i}{r_i}$$

$$= 1.2 \cdot \sum_{n=1}^{\infty} \left[\frac{10^{-9}}{10^{-2}} + \frac{4 \cdot 10^{-9}}{2 \cdot 10^{-2}} \right] =$$

$$g \cdot 10^9 \sum_{n=1}^{\infty} \left[3 \cdot 10^{-7} + 6 \cdot 10^{-7} \right] =$$

$$g \cdot 10^9 \cdot 9 \cdot 10^{-7} = 81 \cdot 10^2 = 8.1 \cdot 10^3 \text{ V}$$

b. 1 - 7

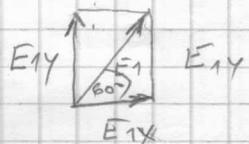


$$|q| = 10^{-9} \text{ A}_{\text{0}}$$

$$d = 5 \cdot 10^{-2}$$

$$\alpha = 60^\circ$$

$$E_1 \parallel \frac{E_2}{E_1} = 2$$



$$\text{a)} \quad E_1 = \frac{F}{Q_1} = k \frac{Q_1 Q_2}{d_1^2} = k \frac{Q}{r^2}$$

$$= k \frac{Q}{r^2}$$

$$= 9 \cdot 10^9 \frac{10^{-9}}{25 \cdot 10^{-4}}$$

$$= 0.36 \cdot 10^4$$

$$= 3.6 \cdot 10^3$$

$$\sin 60^\circ = \frac{E_{1y}}{E_1}$$

$$E_{1y} = E_1 \sin 60^\circ$$

$$= 3.118 \cdot 10^3$$

$$E_{wz} = E_{1y} + E_{2y} = 2 \cdot 3.118 \cdot 10^3$$

$$= 6.235 \cdot 10^3 \frac{\text{V}}{\text{m}}$$

$$E_2 = 0 \quad \frac{E_2}{E_1} = 0$$



$$\cos 60^\circ = \frac{E_x}{E}$$

$$E_x = E \cos 60^\circ$$

$$= 1.8 \cdot 10^3$$

$$E_2 = k \frac{Q}{(d/2)^2}$$

$$= 4 \cdot k \frac{Q}{d^2}$$

$$= 14.4 \cdot 10^3 \frac{\text{V}}{\text{m}}$$

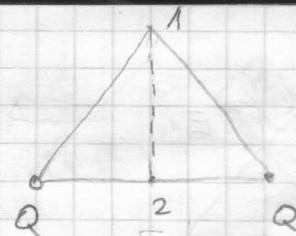
$$E_{wz} = 2E_{1x} = 3.6 \cdot 10^3 \frac{\text{V}}{\text{m}}$$

$$E_{2wz} = 28.8 \cdot 10^3 \frac{\text{V}}{\text{m}}$$

$$\frac{E_2}{E_1} = \frac{28.8}{3.6} = 8$$

$$1.1-8 \quad |Q| = 10^{-9} \text{ As}$$

$$d = 5 \cdot 10^{-2} \text{ m}$$



a) $\varphi_1 = k \sum_{i=1}^2 \frac{Q_i}{r_i} = 9 \cdot 10^9 \left[\frac{10^{-9}}{5 \cdot 10^{-2}} + \frac{10^{-9}}{5 \cdot 10^{-2}} \right] = 9 \cdot 10^9 \cdot \frac{2}{5} \cdot 10^{-7} = 360 \text{ V}$

$$\varphi_2 = 9 \cdot 10^9 \left[\frac{2 \cdot 10^{-9}}{5 \cdot 10^{-2}} + \frac{2}{5} \frac{10^{-9}}{10^{-2}} \right] = 9 \cdot 10^9 \cdot \frac{4}{5} \cdot 10^{-7} = 720 \text{ V}$$

$$U_{12} = \varphi_1 - \varphi_2 = -360 \text{ V}$$

b) $Q = -10^{-9} \text{ As}$

$$\varphi_1 = k \sum_{i=1}^2 \frac{Q_i}{r_i} = 9 \cdot 10^9 \left[\frac{-10^{-9}}{5 \cdot 10^{-2}} - \frac{10^{-9}}{5 \cdot 10^{-2}} \right] = 9 \cdot \left(-\frac{2}{5} \right) \cdot 10^2 = -360 \text{ V}$$

$$\varphi_2 = k \sum_{i=1}^2 \frac{Q_i}{r_i} = 9 \cdot 10^9 \left[\frac{-2 \cdot 10^{-9}}{5 \cdot 10^{-2}} - \frac{2 \cdot 10^{-9}}{5 \cdot 10^{-2}} \right] = -720 \text{ V}$$

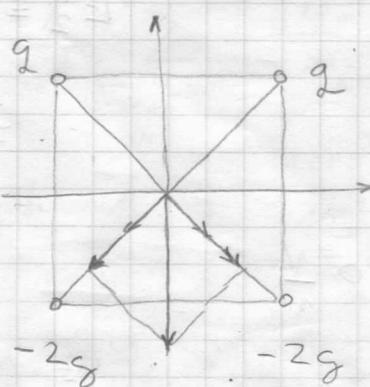
$$U_{12} = \varphi_1 - \varphi_2 = -360 - (-720) = +360 \text{ V}$$

c) $Q_1 = -Q_2 = 10^{-9}$

$$\varphi_1 = k \sum_{i=1}^2 \frac{Q_i}{r_i} = 9 \cdot 10^9 \left[\frac{10^{-9}}{5 \cdot 10^{-2}} - \frac{10^{-9}}{5 \cdot 10^{-2}} \right] = 0 ; \varphi_2 = 0$$

$$U_{12} = \varphi_1 - \varphi_2 = 0 \text{ V}$$

1. 1-9



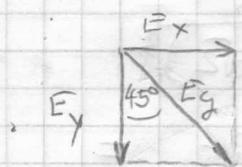
$$a = 10^{-1} \text{ m}$$

$$g = 10^{-9} \text{ As}$$

$$E_g = k \frac{g}{\left(\frac{a\sqrt{2}}{2} \right)^2}$$

$$= k \frac{2g}{a^2} =$$

$$\frac{9 \cdot 10^9 \cdot 2 \cdot 10^{-9}}{10^{-2}} = 18 \cdot 10^2 = 1.8 \cdot 10^3 \frac{V}{m}$$



$$\cos 45^\circ = \frac{E_{2y}}{E_2}$$

$$E_{2y} = E_g \cdot \cos 45^\circ = 1.27 \cdot 10^3$$

$$E_{g\text{ wh}} = 2.545 \cdot 10^3$$

$$E_{-2g} = 3.6 \cdot 10^3 \frac{V}{m}$$

$$E_{2g\text{ y}} = 3.6 \cdot 10^3 \cos 45^\circ = 2.545 \cdot 10^3$$

$$E_{2g\text{ y wh}} = 5.091 \cdot 10^3$$

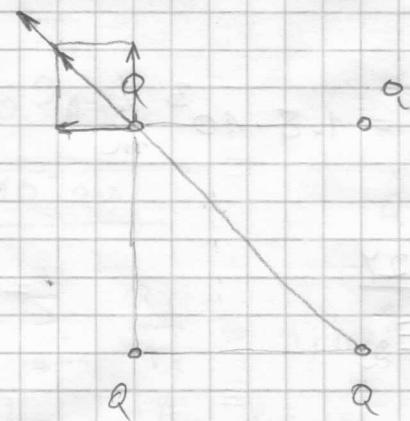
$$E_{\text{wh}} = E_{2y1\text{ wh}} + E_{2y2\text{ y wh}} = \\ = 7.637 \cdot 10^3 \frac{V}{m}$$

$$b) q = q_s = \frac{1}{4\pi\epsilon} \sum_{i=1}^4 \frac{q_i}{r_i} = 9 \cdot 10^9 \left[2 \cdot \frac{\frac{10^{-9}}{10^{-1}\sqrt{2}} - 4 \cdot \frac{10^{-9}}{10^{-1}\sqrt{2}}}{2} \right] =$$

$$9 \cdot 10^9 \left[2 \cdot \frac{2 \cdot 10^{-9}}{10^{-1}\sqrt{2}} - 4 \cdot \frac{2 \cdot 10^{-9}}{10^{-1}\sqrt{2}} \right] =$$

$$9 \cdot 10^9 \cdot (-4) \frac{10^{-8}}{\sqrt{2}} = 9 \cdot (-2)\sqrt{2} \cdot 10 = -254.5 V$$

1. 1.-10.

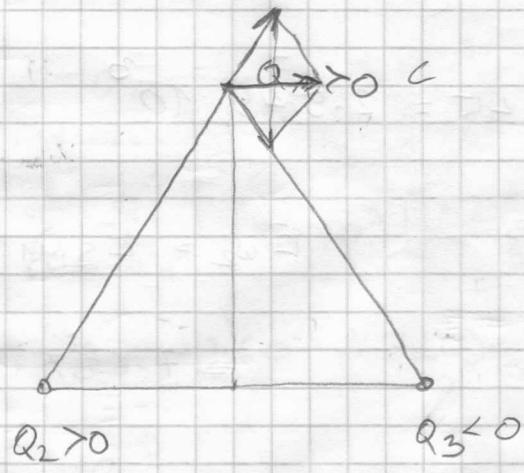


$$F' = k \frac{Q^2}{(d - \sqrt{z^2})^2}$$

$$= \frac{k Q^2}{zd} = \frac{F}{2}$$

$$F_{nk} = F\sqrt{2} + \frac{F}{2} = F\left(\sqrt{2} + \frac{1}{2}\right)$$

1. 1-11



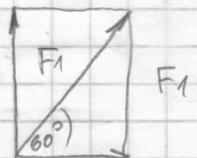
$$Q_1 = Q_2 = 10^{-7} \text{ As}$$

$$Q_3 = 10^{-7} \text{ As}$$

$$d = 2 \cdot 10^{-2}$$

$$F_1 = ?$$

$$F_{21} = k \frac{Q_1 Q_2}{d^2} = 9 \cdot 10^9 \frac{10^{-14}}{4 \cdot 10^{-4}} = 2.25 \cdot 10^{-1}$$



$$F_{1x} = F_1 \cos 60^\circ = \frac{F_1}{2}$$

$$F_{1x}$$

$$F_{nk} = 2 \frac{F_1}{2} = 2.25 \cdot 10^{-1} \text{ N}$$

in richtung C

1. 1-12

$$g = 10^{-6} \text{ C}$$

$$W = 2 \cdot 10^{-5} \text{ J}$$



$$V_{BA} = \varphi_B - \varphi_A = \frac{W}{g} = 20 \text{ V}$$

1 1-13

A
o
QB
o

$$Q = -2.5 \cdot 10^{-7} \text{ As}$$

W_{nA}W_{nB}

$$W_{nA} = -10^{-4} \text{ Ws}$$

$$W_{nB} = -2 \cdot 10^{-4} \text{ Ws}$$

$$\varphi_A = \frac{W_{nA}}{Q} = \frac{-10^{-4}}{-2.5 \cdot 10^{-7}} = 0.4 \cdot 10^3 = 400 \text{ V}$$

$$\varphi_B = \frac{W_{nB}}{Q} = \frac{-2 \cdot 10^{-4}}{-2.5 \cdot 10^{-7}} = 800 \text{ V}$$

$$V_{BA} = \varphi_B - \varphi_A = 400 = \frac{\Delta W}{Q} \Rightarrow \Delta W = 400 Q$$

$$= 4 \cdot 10^2 \cdot 2.5 \cdot 10^{-7}$$

$$= -10^{-4} \text{ Ws}$$

1. 1-14

$$U = 1.7 \cdot 10$$

$$S = 2 \cdot 10^2 \cdot 10^{-4} = 2 \cdot 10^{-4} \text{ m}^2$$

$$d = 10^{-3} \text{ m}$$

$$C_1 Q = C$$

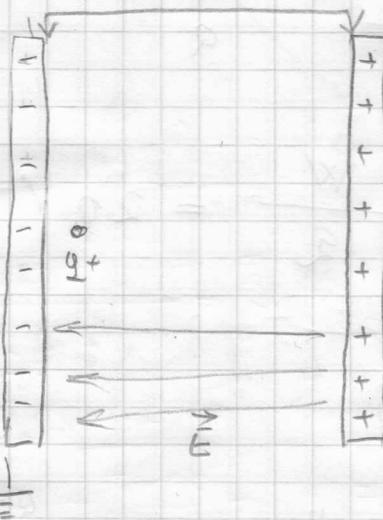
$$C = \epsilon \frac{S}{d} =$$

$$8.854 \cdot 10^{-12} \frac{2 \cdot 10^{-4}}{10^{-3}} = 17.7 \cdot 10^{-13}$$

$$= 177 \text{ nF}$$

$$Q = CV = 1,77 \cdot 10^{-13} \cdot 1,7 \cdot 10^3 = 3 \cdot 10^{-10}$$

1.1-15



$$d = 4 \cdot 10^{-2} \text{ m}$$

$$q = 10^1 \cdot 10^{-9} = 10^{-8} \text{ C}$$

$$W = 2 \cdot 10^{-5} \text{ J}$$

$$U = ? \quad E = ?$$

$$U = \frac{\Delta W}{Q} = 2 \cdot 10^3 \text{ V}$$

$$E = \frac{U}{d} = \frac{2 \cdot 10^3}{4 \cdot 10^{-2}} = 5 \cdot 10^6 \text{ V/m}$$

1.1-16

$$S = 10^{-2} \text{ m}^2$$

$$d = 2 \cdot 10^{-4} \text{ m}$$

$$E_n = \frac{60000}{0,01} = 6 \cdot 10^6 \frac{\text{V}}{\text{m}} = 1,2 \text{ kV}$$

$$\epsilon_r = 0$$

$$C = \epsilon_0 \epsilon_r \frac{S}{d} =$$

$$3,8854 \cdot 10^{-12} \frac{10^{-2}}{2 \cdot 10^{-4}} = 13,28 \cdot 10^{-10}$$

$$1.328 \cdot 10^{-9}$$

$$1.6 \cdot 10^{-9}$$

$$C = \frac{Q}{U}$$

$$Q = CV = 1.328 \cdot 10^{-9} \cdot 1.2 \cdot 10^3$$

$$= 1.6 \cdot 10^{-9} = 1.6$$

$$W = \frac{CV^2}{2} = \frac{1.328 \cdot 10^{-9} \cdot (1.2 \cdot 10^3)^2}{2} =$$

$$\frac{1.328 \cdot 10^{-9} \cdot 1.44 \cdot 10^6}{2} = 0.96 \cdot 10^{-3}$$

$$= 9.6 \cdot 10^{-4}$$

$$1.1-17 \quad U = 400V$$

$$Q = 6 \cdot 10^{-5} A \cdot s$$

$$W = ?$$

$$W = \frac{QV}{2} = \frac{4 \cdot 10^2 \cdot 6 \cdot 10^{-5}}{2}$$

$$= 12 \cdot 10^{-3} V \cdot A \cdot s$$

$$C = \frac{Q}{U} = \frac{6 \cdot 10^{-5}}{4 \cdot 10^2} = 1.5 \cdot 10^{-7} = 0.15 \mu F$$