KOROLAR 1.

Ako je $f'(x) = 0$ za sve x, tada je f konstanta.

Ako je $f'(x) = g'(x)$, tada je $f(x) = g(x) + c$, tj. razlikuju se za konstantu.

KOROLAR 2.

Ako je $f'(x) > 0$ tada je f STROGO RASTUĆA.

Ako je $f'(x) < 0$ tada je f STROGO PADAJUĆA.

Primjeti:

Ako uzmemo $x_1 < x_2$ i neka je $f'(x) > 0$

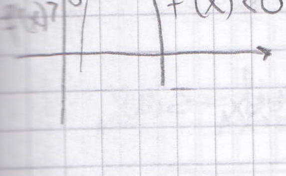
$$f(x_2) - f(x_1) = \underbrace{f'(c)}_{>0} \underbrace{(x_2 - x_1)}_{>0} > 0 \quad f(x_2) > f(x_1) \Rightarrow f \text{ raste!}$$

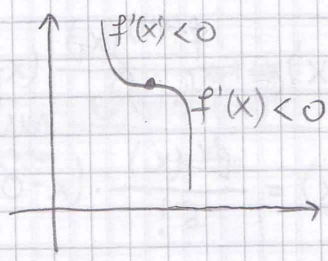
10.1. INTERVALI MONOTONOSTI I EKSTREMI

Def. Intervali monotonosti su intervali na kojima je funkcija rastuća ili padajuća.

→ Krajnje točke intervala su RUBOVI DOMENE ili KRITIČNE TOČKE.

Kako odrediti karakter ekstrema?





nisu ekstremi

* NASTAVAK PREDAVANJA *

Odredi intervale monotonosti, ekstreme i skiciraj:

1) $f(x) = x^5 - 2x^4 + x^3$

1. KORAK: <u>DOMENA</u>

$D(f) = \mathbb{R}$

2. KORAK: <u>STACIONARNE TOČKE</u>

$f'(x) = 0$

$f'(x) = 5x^4 - 8x^3 + 3x^2 = 0$

$x^2(5x^2 - 8x + 3) = 0$

$\boxed{x_1 = 0}$ $\qquad x_{2,3} = \dfrac{8 \pm \sqrt{64-60}}{10}$

$\boxed{x_2 = 1}$ $\qquad \boxed{x_3 = \frac{3}{5}}$

3. KORAK: <u>TABLICA</u>

	$-\infty$ ... 0	0 ... 3/5	3/5 ... 1	1 ... $+\infty$
$f'(x)$	+	+	−	+
$f(x)$	↗	↗	↘	↗

MAX (u 3/5), MIN (u 1)

INTERVALI RASTA: $\langle -\infty, \frac{3}{5} \rangle \cup \langle 1, +\infty \rangle$

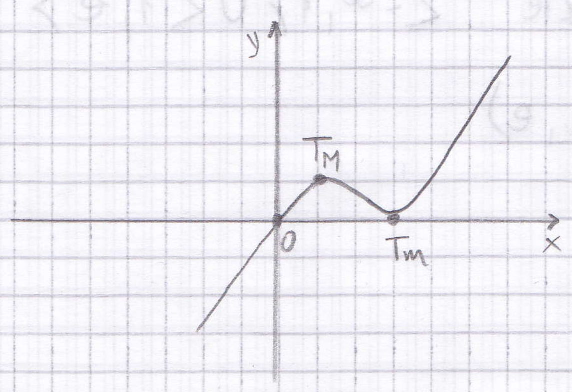
INTERVAL PADA: $\langle 3/5, 1 \rangle$

EKSTREMI:

$\max \rightarrow T_M\left(\frac{3}{5}, 0.034\right)$

$\min \rightarrow T_m(1, 0)$

4. KORAK: <u>GRAF</u>



b) PZI-06-3)

$$f(x) = \frac{x}{\ln x}$$ → NAJČEŠĆE NA ISPITU, ILI tg ... ZANIMLJIVE f

1.) DOMENA

$$\ln x \neq 0$$

$$\underbrace{x \neq 1 \quad i \quad x > 0}$$

$$D(f) = \mathbb{R}^+ \setminus \{1\} = \langle 0, +\infty \rangle \setminus \{1\}$$

2.) STAC. TOČKE

$$f'(x) = 0$$

$$f'(x) = \frac{\ln x - x \cdot \frac{1}{x}}{\ln^2 x} = \frac{\ln x - 1}{(\ln x)^2} = 0$$

$$\ln x - 1 = 0$$

$$\ln x = 1$$

$$\boxed{x = e}$$

3.) TABLICA

NE ZABORAVI V.A. !!!

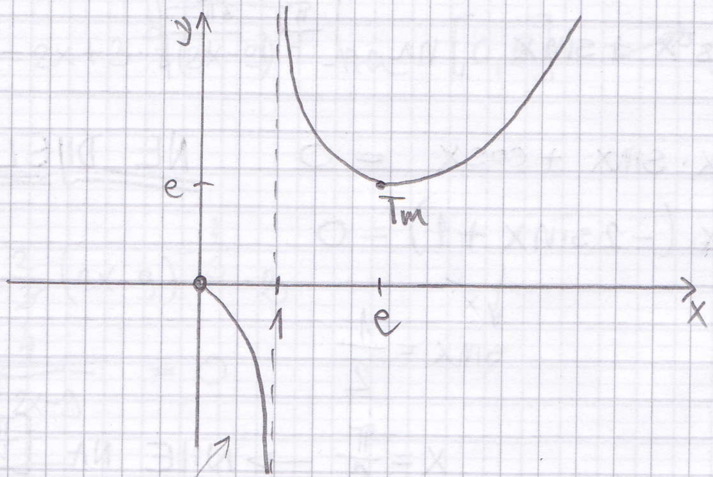
	$-\infty$ … 0	0 … 1	1 … e	e … $+\infty$
$f'(x)$	−	−	−	+
$f(x)$	↘	↘	↘	↗

MIN

PAZI : INTERVALI PADA : BEZ 1 !!

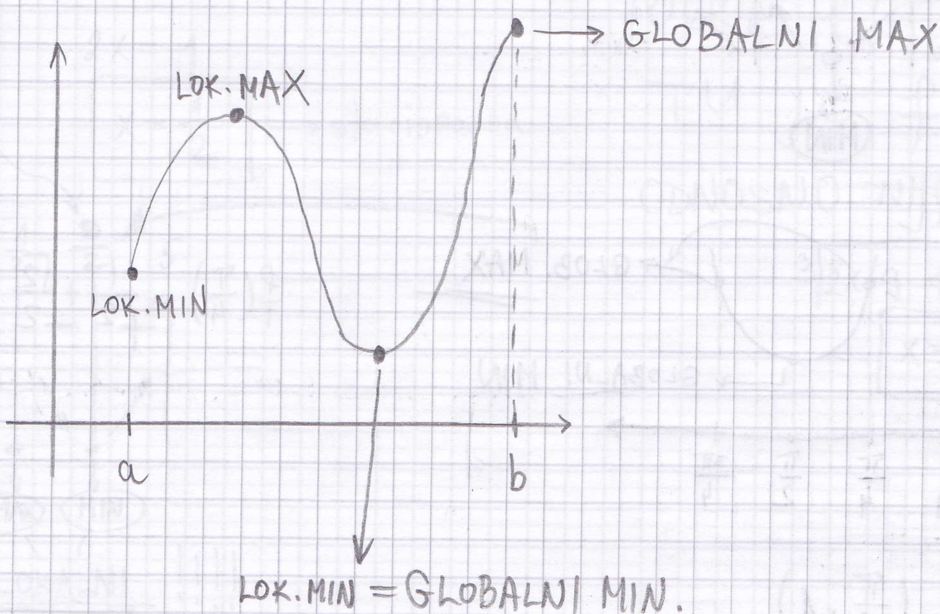
dakle $\langle -\infty, 1 \rangle \cup \langle 1, e \rangle$

min → $T_m(e, e)$



$\lim\limits_{x \to 0^+} \frac{x}{\ln x} = 0$ → DA VIDIMO KAKO SE GRAF PONAŠA LIJEVO OD V.A.

Odredi lok. i globalne ekstreme f. na <u>INTERVALU</u>:



→ Dakle, trebamo vidjeti kako se ponašanaju na RUBOVIMA INTERVALA

Z1 - 06. - 2.)

$$f(x) = \cos^2 x + \sin x \quad , \text{na} \quad \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

$$f'(x) = -2\cos x \cdot \sin x + \cos x = 0$$

NE DIJELI S $\cos x$!!

$$\cos x \, (-2\sin x + 1) = 0$$

$\cos x = 0$

$$\boxed{x = \frac{\pi}{2}}$$

samo na $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

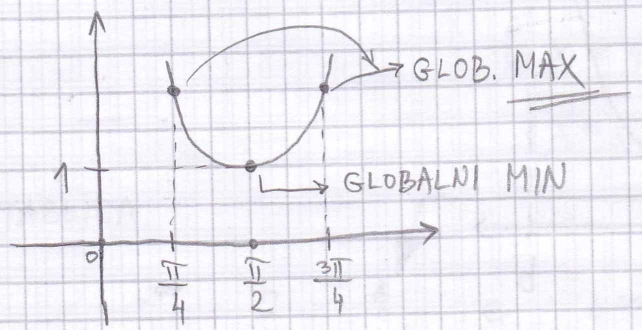
$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} \longrightarrow$ NIJE NA $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$!!

NE

	$-\infty$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$+\infty$
$f'(x)$	$-$	$+$	$+$	
$f(x)$	$\searrow$	$\nearrow$	$\nearrow$	

MIN



$$f\left(\frac{\pi}{4}\right) = \frac{1}{2} + \frac{\sqrt{2}}{2} = f\left(\frac{3\pi}{4}\right)$$

$T_m\left(\frac{\pi}{2}, 1\right)$

$T_M\left(\frac{\pi}{4}, \frac{1}{2} + \frac{\sqrt{2}}{2}\right)$ i za $\frac{3\pi}{4}$!!

1-09.-13) $f(x)=2x+3\cdot\sqrt[3]{(2x-9)^2}$ na $[0,5]$ → OVAKAV TIP VRLO LAKO NA ISPITU!!! BHHH

→ nema V.A.

$f'(x)=2+\cancel{3}\cdot\frac{2}{\cancel{3}}(2x-9)^{-\frac{1}{3}}\cdot 2$

$f'(x)=2+\frac{4}{\sqrt[3]{2x-9}}=0$

$\frac{2\cdot\sqrt[3]{2x-9}+4}{\sqrt[3]{2x-9}}=0$ → NE ZABORAVI!!! IDE U TABLICU, SVE KRITIČNE TOČKE!!! TU JE f ŠILJAK, ODNOSNO NIJE DIF.!

$\sqrt[3]{2x-9}\neq 0$

$\boxed{x\neq\frac{9}{2}}$

$\sqrt[3]{2x-9}=-2 \quad |^3$

$2x-9=-8$

$2x=1$

$\boxed{x=\frac{1}{2}}$ → stacionarna

	0	$\frac{1}{2}$	$\frac{9}{2}$	5
$f'(x)$	+	−	+	
$f(x)$	↗	↘	↗	
		MAX	MIN	

LOKALNI !!!!

$T_M\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$ → $T_M\left(\frac{1}{2}, 13\right)$ lokalni maximum

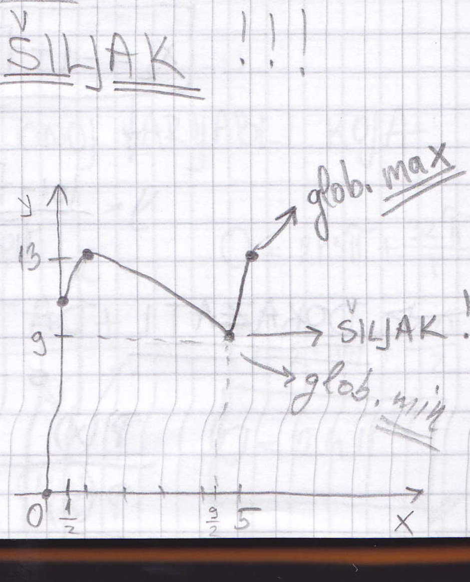
$T_m\left(\frac{9}{2}, f\left(\frac{9}{2}\right)\right)$ → $T_m\left(\frac{9}{2}, 9\right)$ lokalni minimum → ŠILJAK !!!

sada vidjeti je li globalno, RUBOVI:

$f(0)=9\sqrt[3]{3}$ $\quad f(5)=13$

⇒ globalni minimum: $T_m\left(\frac{9}{2}, 9\right)$

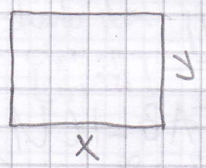
globalni maximum: $(5, 13)$



ZADACI S „RJEČIMA" → SIGURNO NA ISPITU !!

→ ZNATI SVE FORMULE ZA OPLOŠJA ....

Z1-10'-1.) Pravokutna livada, ograda 100 kn/m, pokriti podlogom livadu 200 kn/m². Ako imamo 20 000 kn kolike moraju biti dimenzije ove livade da ogradimo maksimalno veliku površinu?



$2x+2y$ ograde

$(2x+2y)\cdot 100 \rightarrow$ novci ograde

$x\cdot y\cdot 200 \rightarrow$ novci za podlogu

$$(2x+2y)\cdot 100 + x\cdot y\cdot 200 = 20\,000$$

$$\vdots$$

$$\underline{x+y+xy=100}$$

$$\rightarrow y=\frac{100-x}{1+x}$$

$P = x\cdot y$

$$P = x\cdot\frac{100-x}{1+x} = \frac{100x-x^2}{1+x}$$ → NAĆI MAX OVE FUNKCIJE JER SE TRAŽI max velika površina!

$$P'(x) = \frac{100-x^2-2x}{(1+x)^2} = 0$$

$$x_{1,2} = \frac{2\pm\sqrt{404}}{-2} = \frac{2\pm 2\sqrt{101}}{-2}$$

$x_1 = -1-\sqrt{101}$ NE

$\boxed{x_2 = -1+\sqrt{101}}$

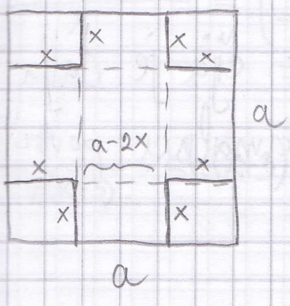
$$y = \frac{101-\sqrt{101}}{\sqrt{101}} = \sqrt{101}-1$$

$\underline{x = y = \sqrt{101}-1}$ → KVADRAT

→ DOKAZATI DA SE RADI O MAX !!

	0		$\sqrt{101}-1$	
$P'(x)$		$+$		$-$
$P(x)$		↗	MAX	↘

Karton, kvadrat, a stranica. Želimo napraviti kutiju (otvorenu) maksimalnog volumena.



$$V=\underbrace{(a-2x)^2}_{\text{BAZA}} \cdot \underbrace{x}_{\text{VISINA}}$$

konstanta!!

$$V = a^2 x - 4ax^2 + 4x^3 \quad /'$$

$$V'(x) = a^2 - 8ax + 12x^2 = 0$$

NE $\boxed{x_1 = \frac{1}{2}a}$ $\boxed{x_2 = \frac{1}{6}a}$ DA!!

NEMA SMISLA JER : $x = \frac{a}{2} \to$ BAZA:

$$a - 2x = a - 2\cdot\frac{a}{2} = 0$$

	0		$\frac{1}{6}a$		$\frac{1}{2}a$
$V'(x)$		+		−	
$V(x)$		↗	MAX	↘	

MAX → DOKAZALI!

$$V\left(\frac{1}{6}a\right) = \frac{4}{9}a^2 \cdot \frac{1}{6}a$$

$$\boxed{V\left(\frac{1}{6}a\right) = \frac{2a^3}{27}}$$

2.) Koliko je minimalno oplošje valjka ako mu je volumen 16π?

$$V = 16\cancel{\pi} = B \cdot v = r^2\cancel{\pi} \cdot v \to v = \frac{16}{r^2}$$

$$O = 2B + P = 2r^2\pi + 2r\pi \cdot v$$

$$O = 2r^2\pi + \frac{32\pi}{r}$$

→ DERIVIRATI PO ONOJ VARIJABLI KOJA OSTANE!!!

$$O'(r) = 4r\pi - \frac{32\pi}{r^2} = 0 \quad /\cdot \frac{r^2}{4\pi}$$

$$r^3 - 8 = 0$$

$$r^3 = 8 \to \boxed{r = 2}$$

	0		2	
$O'(r)$		−		+
$O(r)$		↘	MIN	↗

$$O = 2r^2\pi + \frac{32\pi}{r}$$

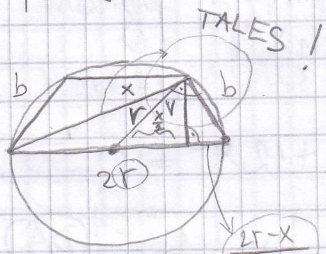
$$O = 2\cdot 4\pi + \frac{32\pi}{2}$$

$$\boxed{O = 24\pi}$$

→ Z1 → SVI ĆE BITI SA RIJEČIMA ! ?? BUREK ?

→ 6 ZAD, 3 SA DERIVACIJAMA I SVA 3 SA RIJEČIMA !

10.DZ-8.) U kružnicu polumjera r upisan je trapez čija je dulja osnovnica promjer kružnice. Kada će imati maximalnu površinu?



$\frac{2r-x}{2} = r - \frac{x}{2}$

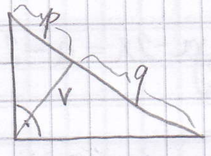
$$P = \frac{a+c}{2} \cdot v$$

$$\underline{a = 2r}$$

$$c = x$$

$$P = \frac{2r+x}{2} \cdot v$$

ILI $v = \sqrt{r^2 - \left(\frac{x}{2}\right)^2}$



$$\boxed{v = \sqrt{p \cdot q}} \longrightarrow v = \sqrt{\left(r - \frac{x}{2}\right)\left(r + \frac{x}{2}\right)}$$

$$P = \frac{2r+x}{2} \cdot \sqrt{\left(r-\frac{x}{2}\right)\left(r+\frac{x}{2}\right)} = \left(r + \frac{x}{2}\right)\sqrt{r^2 - \frac{x^2}{4}}$$

$$P'(\underline{\underline{x}}) = \frac{1}{2}\sqrt{r^2 - \frac{x^2}{4}} + \left(r + \frac{x}{2}\right) \cdot \frac{1}{2\sqrt{r^2 - \frac{x^2}{4}}} \cdot \frac{-2x}{4} = 0$$

$$\frac{1}{2}r^2 - \frac{1}{8}x^2 - \frac{1}{4}rx - \frac{1}{8}x^2 = 0 \quad | \cdot (-8)$$

$$2x^2 + 2rx - 4r^2 = 0$$

$$x_{1,2} = \dots$$

$x_1 = -2r$ NE $\qquad \boxed{x_2 = r}$

	0		r	
$P'(x)$		+		−
$P(x)$		↗	MAX	↘

$$\boxed{P = \frac{3r^2\sqrt{3}}{4}}$$

* NASTAVAK PREDAVANJA *

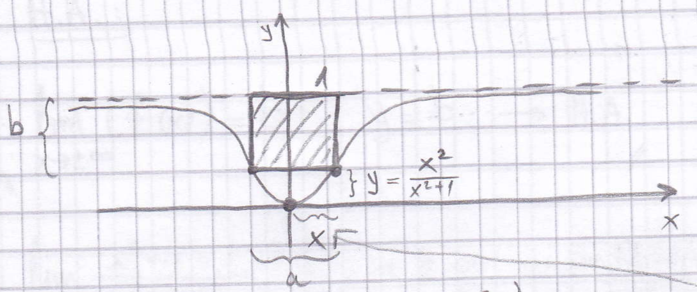
21-09-13.) Naći pravokutnik max površine čija se 2 vrha nalaze na $y=\frac{x^2}{x^2+1}$, a jedna stranica na horizontalnoj asimptoti te krivulje.

H.A.

$$\lim_{x\to\pm\infty}\frac{x^2}{x^2+1}\begin{matrix}/:x^2\\/:x^2\end{matrix}=1\longrightarrow\boxed{y=1}\quad \text{H.A.}$$

→ parna je ⟹ simetrična, zbog kvadrata

za $x=0\to y=0$, sve vrijednosti su između 0 i 1 ! kvadr



$$P(x)=\underbrace{2x}_{a}\cdot\underbrace{\left(1-\frac{x^2}{x^2+1}\right)}_{b}$$

$$P(x)=\frac{2x}{x^2+1}\longrightarrow P'(x)=\frac{2-2x^2}{(x^2+1)^2}=0$$

$x^2=1$

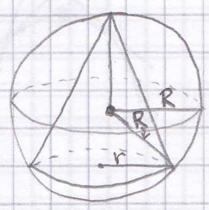
$x=\pm1$ → zbog slike

$\boxed{x=1}$

	0	1	
$P'(x)$	+		−
$P(x)$	↗	MAX	↘

$$P(1)=\frac{2}{1+1}=1$$

Z-4.) Kugli polumjera R upiši uspravan kružni stožac.

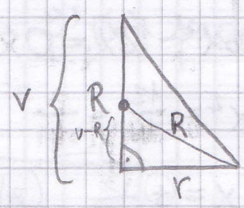


$$V=\frac{1}{3}r^2\pi\cdot v$$

$$V=\frac{\pi}{3}\left(R^2-(v-R)^2\right)\cdot v$$

$$\vdots$$

$$V=\frac{\pi}{3}\left(2v^2R-v^3\right)$$



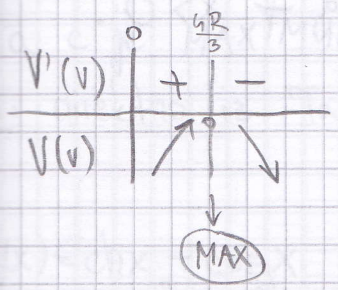
$$r=\sqrt{R^2-(v-R)^2}$$

$$V'(v)=\frac{\pi}{3}\left(4vR-3v^2\right)=0$$

ZADAN !! R je zadan da je R !!! kečap?

$$\boxed{v=\frac{4R}{3}}$$

$$\boxed{V\left(\frac{4R}{3}\right)=\frac{32R^3\pi}{81}}$$

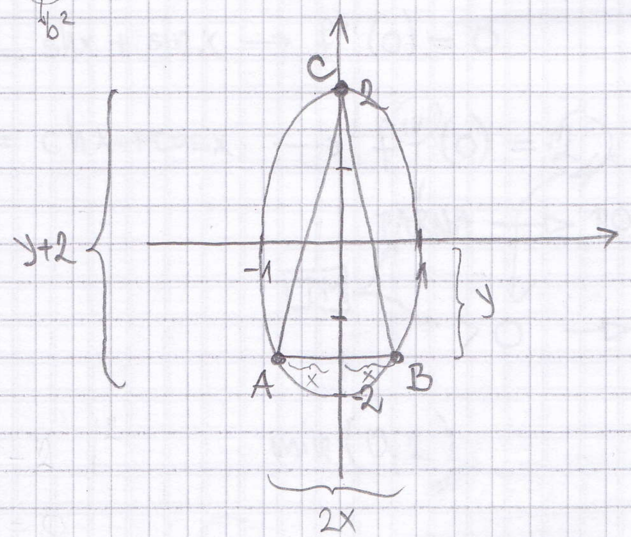


Z-9.) Jednakokračan trokut ABC upiši u elipsu $4x^2+y^2=4$ tako da mu je vrh $C(0,2)$ i nasuprot AB.

$$y=\sqrt{4-4x^2}$$

$$x^2+\frac{y^2}{4}=1$$

$1 \to a^2$, $4 \to b^2$



$$P=\frac{1}{2}a\cdot v_a=\frac{1}{2}\cdot 2x\cdot(y+2)$$

$$P=x\left(2+\sqrt{4-4x^2}\right)$$

$$P'(x)=2+\sqrt{4-4x^2}+x\cdot\frac{1}{2\sqrt{4-4x^2}}\cdot(-8x)$$

$$\vdots$$

$$P'(x)=\frac{\sqrt{4-4x^2}+2-4x^2}{\frac{1}{2}\sqrt{4-4x^2}}=0$$

$$\sqrt{4-4x^2} = 4x^2-2 \quad \Big|^2$$

$$16x^4 - 12x^2 = 0$$

$$x^2(16x^2-12) = 0$$

$$x^2 = \frac{12}{16} = \frac{3}{4}$$

$$x = \pm\frac{\sqrt{3}}{2} \quad \text{(zbog slike)}$$

	0		$\frac{\sqrt{3}}{2}$	
$P'(x)$		$+$		$-$
$P(x)$		$\nearrow$	MAX	$\searrow$

$$\boxed{P = \frac{3\sqrt{3}}{2}}$$

10.DZ-6.) Dokaži da je $\underbrace{x-\frac{x^2}{2} < \ln(1+x)}_{\text{I}} \underbrace{< x}_{\text{II}}$, za $\forall\ x>0$.

I $\quad \underbrace{\ln(1+x) - x + \frac{x^2}{2}}_{f(x)} > 0$

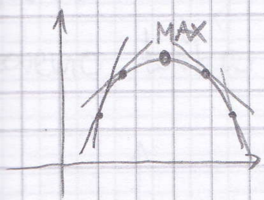
$f(0)=0 \rightarrow$ DOKAZATI DA JE f RASTUĆA! (poslije nule sve veće od

$f'(x) = \frac{1}{1+x} - 1 + x = \frac{x^2}{1+x}$ za $x>0$ $f'(x)$ UVIJEK JE VEĆE OD 0

FUNKCIJA JE RASTUĆA KADA JOJ JE DERIVACIJA > 0

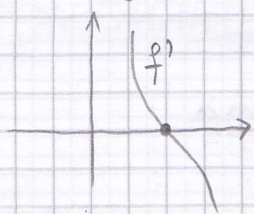
II $\quad x - \ln(1+x) > 0$

10.2. KONVEKSNOST I KONKAVNOST



$\longrightarrow$ koeficijenti se smanjuju $\rightarrow f'(x)$ pada

nagib pada



$\Rightarrow \boxed{f''(x) < 0}$

$\boxed{TM}$

Ako je $f'(c)=0$, $f''(c)=0$, $\dots$, $f^{(n-1)}(c)=0$ i ako je $f^{(n)}(c)\neq 0$ tada c NIJE EKSTREM ako je n neparan, a JE EKSTREM ako je n paran i to c je MINIMUM ako je $f^{(n)}(c)>0$, a c je MAXIMUM ako je $f^{(n)}(c)<0$.

Dokaz: direktno iz Taylora II

$f(x)=\operatorname{ch}x+\cos x$. Ekstremi?

$f'(x)=\operatorname{sh}x-\sin x=0$

$\operatorname{sh}x=\sin x$

$\boxed{x=0}$

$f''(x)=\operatorname{ch}x-\cos x \longrightarrow f''(0)=0 \longrightarrow$ tražimo prvu različitu od 0

$f'''(x)=\operatorname{sh}x+\sin x \rightarrow f'''(0)=0$

$f^{IV}(x)=\operatorname{ch}x+\cos x \longrightarrow f^{IV}(0)=2$

PARNA $\rightarrow$ POSTOJI EKSTREM!

$\boxed{TM}$

$>0 \rightarrow$ MINIMUM

min(0, 2)

ŠK

→ der. po def.

→ Postoji li der. ?

→ implic. i param. →

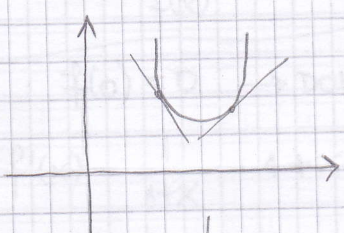
→ tangente, s riječima zadaci, kut između krivulja

→ divljački L'H

→ sve cakice sa asimptotama

def. • Ako je $f''(x) > 0$ na (a, b), prva derivacija raste (raste nagib tangente) i tada kažemo da je funkcija **KONVEKSNA**.

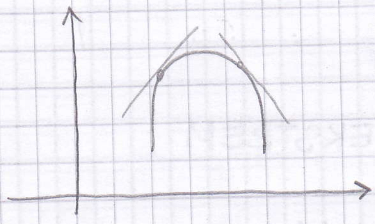
• Ako je $f''(x) < 0$, prva derivacija pada i tada kažemo da je f **KONKAVNA**.



$f''(x) > 0$

↓

KONVEKSNA FUNKCIJA JE **SRETNA** !!! ☺

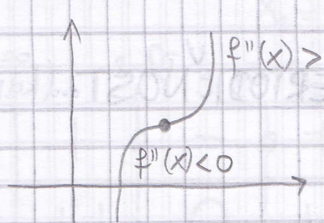


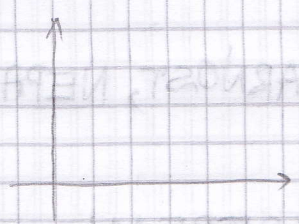
$f''(x) < 0$

↓

KONKAVNA FUNKCIJA JE **TUŽNA** !!! ☹

Točka kod koje dolazi do promjene predznaka druge derivacije zovemo točka **INFLEKSIJE** (pregiba).





ODZ - 13_B)

$$f(x) = (1+x^2)e^x \implies D(f) = \mathbb{R}$$

$$f'(x) = 2xe^x + (1+x^2)e^x$$

$$f''(x) = 2e^x + 2xe^x + 2xe^x + (1+x^2)e^x$$

$$= e^x(3+4x+x^2) = 0$$

e^x >0 UVIJEK!

$$x^2+4x+3=0$$

$$x_1 = -1$$

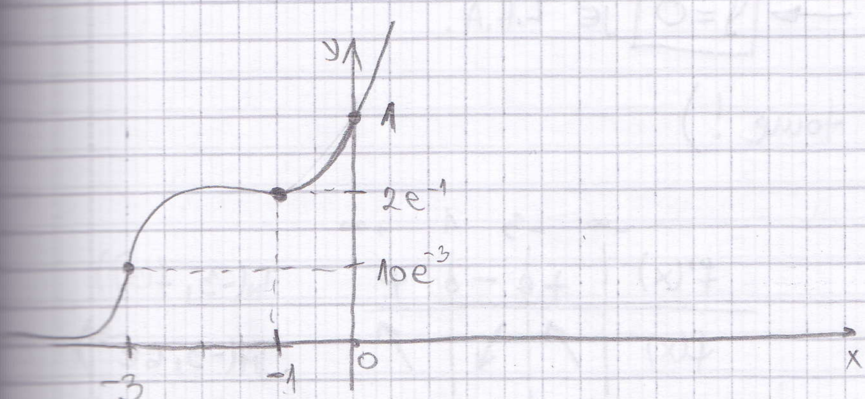
$$x_2 = -3$$

	$-\infty$	-3	-1	$+\infty$
$f''(x)$	+	−	+	
$f(x)$	∪	∩	∪	

T_{i_1} T_{i_2}

UVRSTITI U f(x) IZVORNU FUNKCIJU !! NE U f''(x) ili f'(x) !

$T_{i_1}(-3, 10e^{-3})$ $T_{i_2}(-1, 2e^{-1})$



10.3. KVALITATIVNI GRAF FUNKCIJE

→ sigurno na ispitu !!

1. DOMENA
2. NULTOČKE, PARNOST, NEPARNOST, PERIODIČNOST... (karakteristike)
3. ASIMPTOTE
4. RAST, PAD, EKSTREMI
5. KIKI (KONVEKSNOST, KONKAVNOST, INFLEKSIJA)
6. GRAF

Z1-07-3.)

$$f(x) = (x^2-3)e^x$$

1. $D(f) = \mathbb{R}$

2. $x^2 - 3 = 0$

$x^2 = 3 \longrightarrow x_{12} = \pm\sqrt{3}$ // → NULTOČKE

3.

V.A. → nema (po $D(f)$)

H.A.

$\lim\limits_{x\to+\infty} (x^2-3)e^x = (\infty\cdot\infty) = +\infty$ → NEMA D.H.A.

$\lim\limits_{x\to-\infty} (x^2-3)e^x = (\infty\cdot 0) = \lim\limits_{x\to-\infty} \dfrac{x^2-3}{\frac{1}{e^x}} = \left(\dfrac{\infty}{\infty}\right)$ DEREMO L'H $=$

$= \lim\limits_{x\to-\infty} \dfrac{2x}{-e^{-x}}$ = DEREMO JOŠ JEDNOM L'H $=$

$= \lim\limits_{x\to-\infty} \dfrac{2}{e^{-x}} = \dfrac{2}{\infty} = 0$ // ⟶ $\boxed{y=0}$ JE L.H.A.

→ KOSE NEMA (razmisliti o tome !)

4.

$f'(x) = e^x(x^2+2x-3) = 0$

$x^2+2x-3=0$

$x_1 = 1$

$x_2 = -3$

	$-\infty$	-3		1	$+\infty$
$f'(x)$	+	o	−	o	+
$f(x)$	↗		↘		↗
		MAX		MIN	

$T_M(-3, f(3))$

$T_M(-3, 6e^{-3})$

$T_m(1, -2e)$

$f''(x) = e^x(x^2+4x-1) = 0$

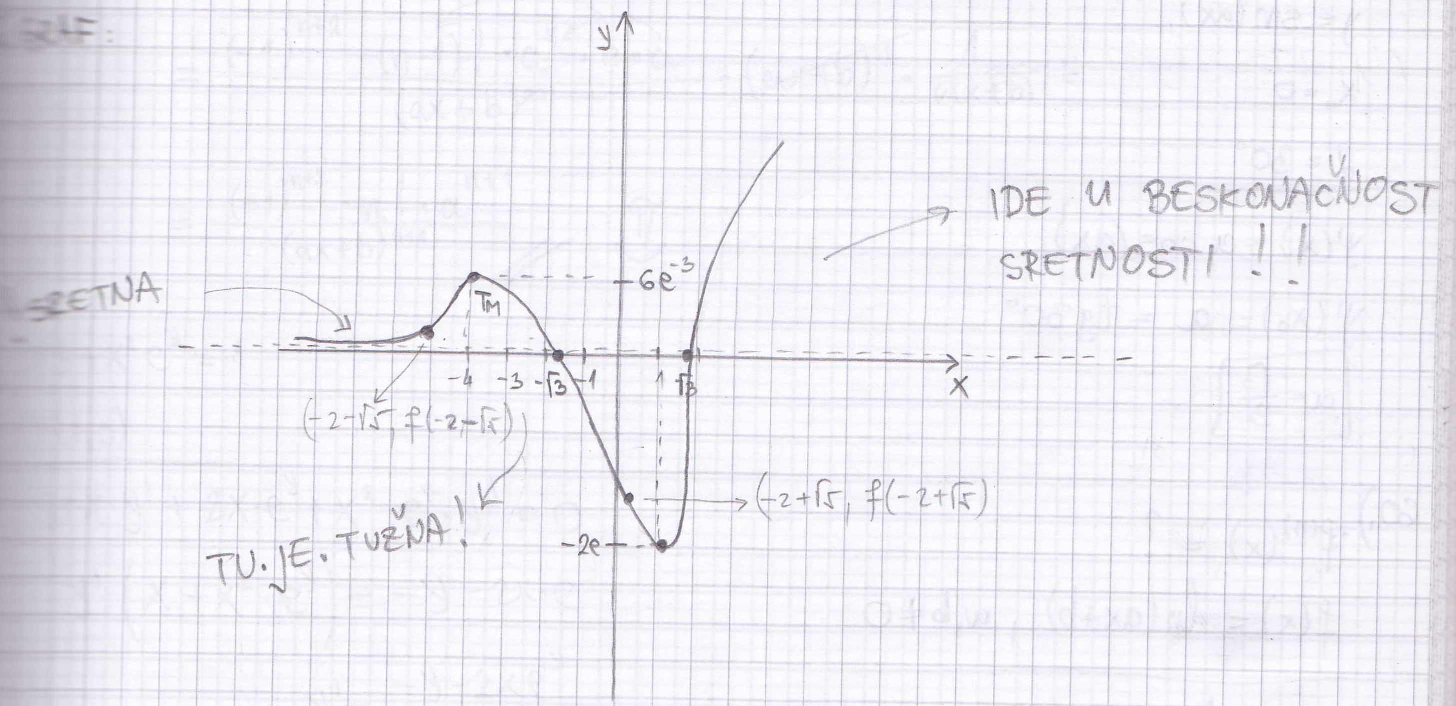
$x_1 = -2-\sqrt{5}$

$x_2 = -2+\sqrt{5}$

} NAJČEŠĆE „RUŽNI" BR. jer je teško namjestiti

	$-\infty$	$-2-\sqrt{5}$		$-2+\sqrt{5}$	$+\infty$
$f''(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\cup$	$\downarrow T_{i_1}$	$\cap$	$\downarrow T_{i_2}$	$\cup$

GRAF:



* NASTAVAK PREDAVANJA *

Z1-10-2.) Izračunaj sve i nacrtaj $f(x) = x + 2\operatorname{arctg}\frac{1}{x}$.

1.) $\underline{D(f)} = \mathbb{R} \setminus \{0\}$

2.) nultočke $\rightarrow$ nema smisla, nisu potrebne

3.) asimptote

1° <u>V.A.</u>

$$\lim_{x\to 0^+}\left(x + 2\operatorname{arctg}\overset{+\infty}{\frac{1}{x}}\right) = \pi$$

$$\lim_{x\to 0^-}\left(x + 2\operatorname{arctg}\overset{-\infty}{\frac{1}{x}}\right) = -\pi$$

} <u>NEMA</u> V.A (nije $\pm\infty$)

2° <u>H.A</u>

$\rightarrow$ nema ! (očito da je ∞)

3° <u>K.A</u>

$$k = \lim_{x\to\infty}\left(1 + \frac{2\operatorname{arctg}\frac{1}{x}}{x}\right) = \lim_{\substack{x\to+\infty \\ \ominus \rightarrow \text{ISTO}}}(1+0) = 1$$

$$l = \lim_{\substack{x\to+\infty \\ \ominus \rightarrow \text{ISTO}}}\left(\cancel{x} + 2\operatorname{arctg}\frac{1}{x} - \cancel{x}\right) = 0$$

$y = x \rightarrow$ kosa i sa lijeve i sa desne

4.) rast, pad, ekstremi

$$f'(x) = 1 + \frac{}{1+(\frac{1}{x})^2}\cdot\frac{-1}{x^2} = \frac{x^2-1}{x^2+1} = 0$$

$x^2 - 1 = 0$

$\underline{x = \pm 1}$

	$-\infty$	-1	0	1	
$f'(x)$	+	−	−	+	
$f(x)$	↗	↘	↘	↗	
		M		m	

$T_{MAX}\left(-1, -\frac{\pi}{2}\right)$

$T_{min}\left(1, 1+\frac{\pi}{2}\right)$

$$f''(x) = \dots = \frac{4x}{(x^2+1)^2} = 0$$

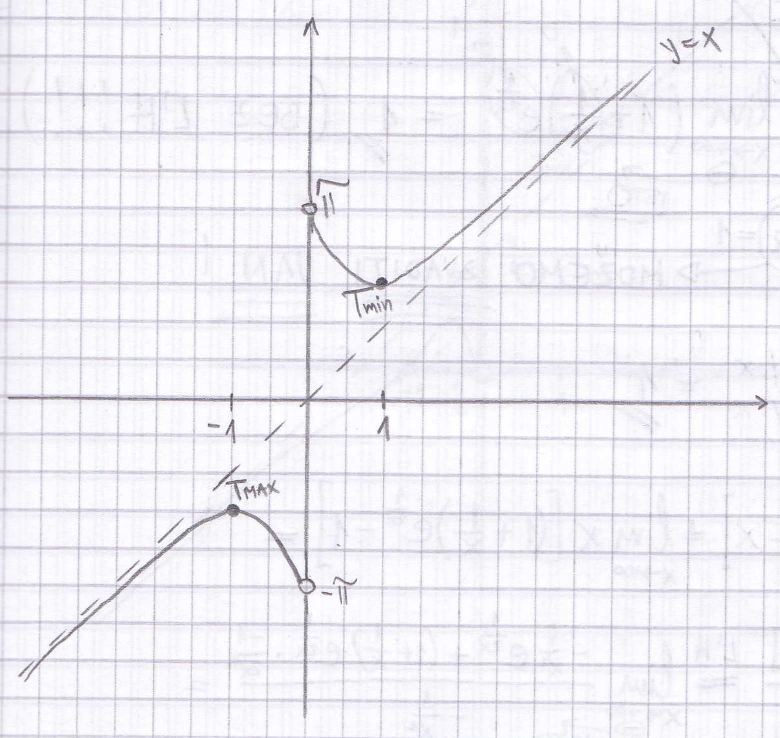
$\boxed{x=0}$ ⟶ NEMA INFLEKSIJE !!

	$-\infty$	0	$+\infty$
$f''(x)$	$-$		$+$
$f(x)$	$\cap$		$\cup$

PAZI !!

!! NIJE TOČKA INFLEKSIJE

JER NIJE U DOMENI FUNKCIJE !



10.DZ-20.) Izračunaj sve i nacrtaj $f(x) = (x+1)e^{\frac{1}{2x}}$. (bez kiki).

ILKO DOBRE VOLJE ???

1.) $D(f) = \mathbb{R} \setminus \{0\}$

2.) nultočke:

$\boxed{x=-1}$

3.) asimptote:

1° V.A.

NEĆE TU BITI NIKAKVOG L'HOSPITALA SHOE!! UVRSTI, PROVJERI !!!

BAŠ KAO NA KPZ...

$\lim\limits_{x\to 0^+} (x+1)e^{\frac{1}{2x}} = +\infty$ DA

$\lim\limits_{x\to 0^-} (x+1)e^{\frac{1}{2x}} = 0$ NE

$\boxed{x=0} \longrightarrow$ D.V.A.

2° H.A

$\hookrightarrow$ NEMA

3° K.A.

TRIK! SJETI SE BEZ L'H !!!

$k = \lim\limits_{x\to +\infty} \frac{(x+1)e^{\frac{1}{2x}}}{x} = \lim\limits_{x\to+\infty} \left(1+\frac{1}{x}\right)e^{\frac{1}{2x}} = 1$ (BEZ L'H !!!)

$\ominus$ ISTO

ILI $\lim\limits_{x\to+\infty} \frac{(x+1)e^{\frac{1}{2x}}}{x}$, $e^{\frac{1}{2x}} = 1 \longrightarrow$ MOŽEMO IZVADITI VAN!

$= \lim\limits_{x\to+\infty} \frac{x+1}{x} \begin{vmatrix} :x \\ :x \end{vmatrix} = 1$

$l = \lim\limits_{x\to+\infty}\left[(x+1)e^{\frac{1}{2x}} - x\right] = \lim\limits_{x\to+\infty} x\left[\left(1+\frac{1}{x}\right)e^{\frac{1}{2x}} - 1\right] =$

$= \lim\limits_{x\to+\infty} \frac{\left[\left(1+\frac{1}{x}\right)e^{\frac{1}{2x}}-1\right]}{\frac{1}{x}} \overset{L'H}{=} \lim\limits_{x\to+\infty} \frac{-\frac{1}{x^2}e^{\frac{1}{2x}} + \left(1+\frac{1}{x}\right)\cdot e^{\frac{1}{2x}}\cdot\frac{-1}{2x^2}}{-\frac{1}{x^2}} =$

$\ominus$ ISTO

$\longrightarrow$ BEZ L'H PAMETNA KOLEGICA !???

$= \frac{1+\frac{1}{2}}{1} = \frac{3}{2}$

$\longrightarrow y = x + \frac{3}{2}$ K.A. s obje strane

$= \{$ KOLEGICA BEZ DERIVIRANJA NARAVNO $\}$

$$= e^{\frac{1}{2x}} + (x+1)e^{\frac{1}{2x}} \cdot \frac{-1}{2x^2} = 0 \quad /: e^{\frac{1}{2x}}$$ UVIJEK $\neq 0$ PA MOŽEMO !

$$1 - \frac{1}{2x} - \frac{1}{2x^2} = 0 \quad / \cdot 2x^2$$

$$2x^2 - x - 1 = 0$$

$$x_{1,2} = \frac{1 \pm 3}{4} \quad \begin{array}{l} \nearrow x_1 = 1 \\ \searrow x_2 = -\frac{1}{2} \end{array}$$

	$-\infty$	$-\frac{1}{2}$	0	1	$+\infty$
$f'(x)$	+	−	−	+	
$f(x)$	↗	↘	↘	↗	
		MAX		MIN	

$T_{MAX}\left(-\frac{1}{2}, \frac{1}{2}e^{-1}\right)$

$T_{MIN}\left(1, 2e^{\frac{1}{2}}\right)$

$\Sigma x_1 \rightarrow$ ne treba $\widehat{\approx}$ PROKLETI ILKO!!!!