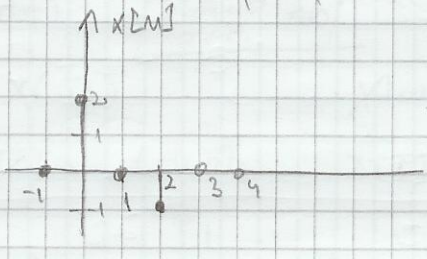
Vježba 1.

Zadatak za pripremu



$t_1 = 0.5$

$t_2 = 1.5$

$x[n] = \{..0, \underline{2}, 1, 0, -1, 0, 0..\}$

Interpolacija

$h_{II}(t) = \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$ $\quad T_S = 1$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]\,\text{sinc}(t-n)$$

$$= 2\,\text{sinc}(t-0) - 1\cdot\text{sinc}(t-2)$$

$$x_1(t=0.5) = 2\,\text{sinc}(0.5) - \text{sinc}(-1.5)$$

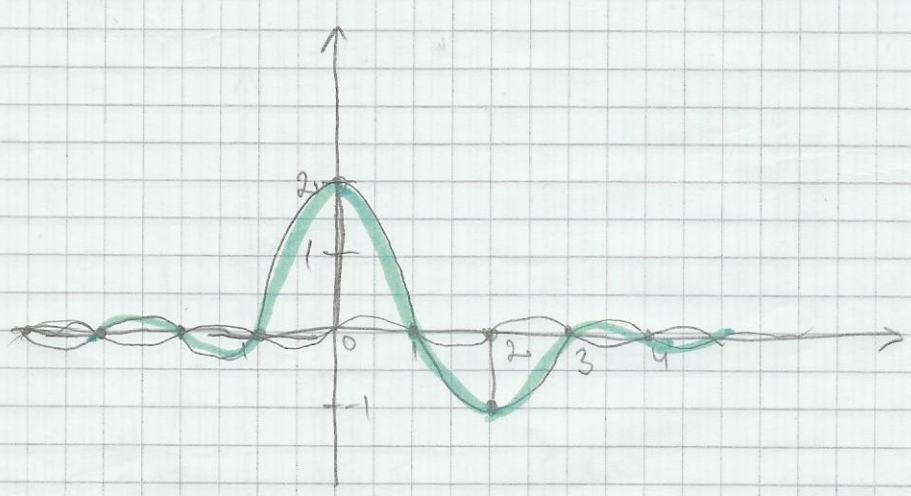
$$= 1.2732 + 0.2122$$

$$= 1.4854$$

$$x_2(t=1.5) = 2\,\text{sinc}(1.5) - \text{sinc}(-0.5)$$

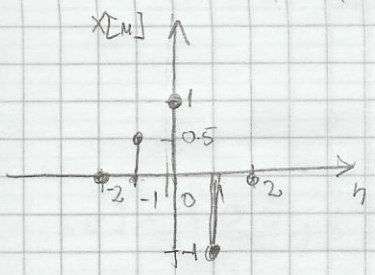
$$= -0.4244 - 0.6366$$

$$= -1.061$$



Vježba 2.

Zadatak za pripremu



$$x[n] = \{0.5, \underset{\uparrow}{1}, -1, 0\}$$

1) $X(e^{j\omega}) = ?$ $\quad X(e^{j\omega})$ za $\omega = \frac{\pi}{4} = ?$

$$X(\omega) = X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-1}^{2} x[n] e^{-j\omega n} = 0.5 e^{-j\omega(-1)} + 1 \cdot e^{-j\omega 0} - 1 \cdot e^{-j\omega 1} + 0 \cdot e^{-j\omega 2}$$

$$= 0.5 e^{j\omega} + 1 - e^{-j\omega}$$

$$\left[e^{-jx} = \cos x - j \sin x \quad e^{jx} = \cos x + j \sin x \right]$$

$$X\left(\frac{\pi}{4}\right) = 0.5 e^{j\frac{\pi}{4}} + 1 - e^{-j\frac{\pi}{4}}$$

$$= 0.5\left(\cos\tfrac{\pi}{4} + j \sin\tfrac{\pi}{4}\right) + 1 - \left(\cos\left(\tfrac{\pi}{4}\right) - j \sin\left(\tfrac{\pi}{4}\right)\right)$$

$$= 1. = 0.5 \cos\tfrac{\pi}{4} + 1.5 j \sin\tfrac{\pi}{4}$$

$$= 1 - \frac{\sqrt{2}}{4} + \frac{3\sqrt{2}}{4} j$$

2) $DFT_4\{0.5, \underset{\uparrow}{1}, -1, 0\} = \sum_{n=-1}^{2} x[n] \cdot W_4^{nk}$ $\qquad W_N^{nk} = e^{-j2\pi \frac{nk}{N}}$

$$= 0.5 W_4^{-1k} + W_4^{0k} - W_4^{1k} \qquad k = 0, 1, 2, 3$$

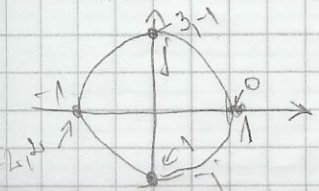
$$= 0.5 W_4^{-k} + 1 - W_4^{k}$$

$$X[0] = 0.5 W_4^{-1\cdot 0} + 1 \cdot W_4^{0\cdot 0} - W_4^{1\cdot 0} + 0 \cdot W_4^{2\cdot 0} = 0.5 + 1 - 1 + 0 = 0.5$$

$$X[1] = 0.5 W_4^{-1\cdot 1} + 1 \cdot W_4^{0\cdot 1} - W_4^{1\cdot 1} + 0 \cdot W_4^{2\cdot 1} = 0.5j + 1 + j + 0 = 1 + 1.5j$$

$$X[2] = 0.5 W_4^{-1\cdot 2} + 1 \cdot W_4^{0\cdot 2} - W_4^{1\cdot 2} + 0 \cdot W_4^{2\cdot 2} = -0.5 + 1 + 1 + 0 = 1.5$$

$$X[3] = 0.5 W_4^{-1\cdot 3} + 1 \cdot W_4^{0\cdot 3} - W_4^{1\cdot 3} + 0 \cdot W_4^{2\cdot 3} = -0.5j + 1 - j + 0 = 1 - 1.5j$$



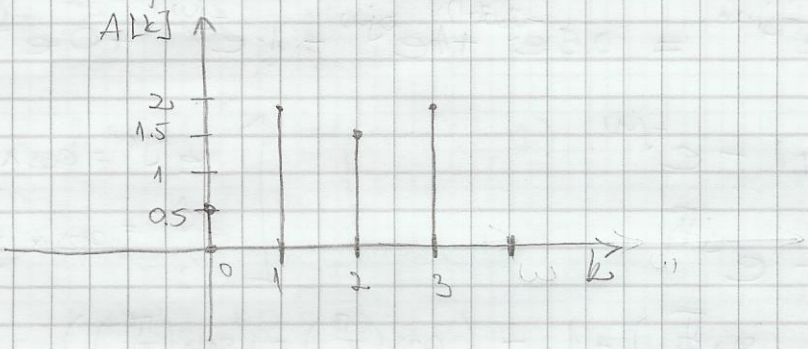
$X[0] = 0.5$

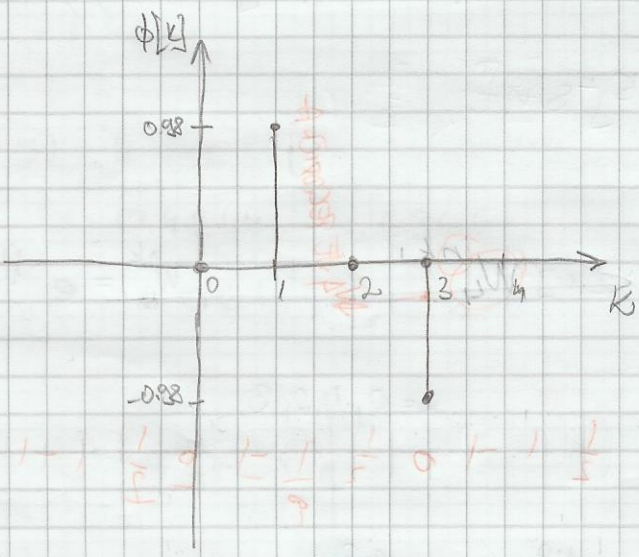
$X[1] = 1 + 1.5j$

$X[2] = 1.5$

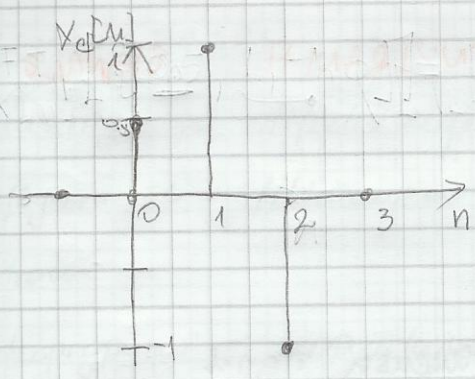
$X[3] = 1 - 1.5j$

k	X[k]	A[k]	$\phi[k]$
0	0.5	0.5	0
1	$1+1.5j$	1.8	0.982 (56°18'36")
2	1.5	1.5	0
3	$1-1.5j$	1.8	-0.982 (-56°18'36")





3)



$$x_c[n] = \{\underline{0.5}, 1, -1, 0\}$$

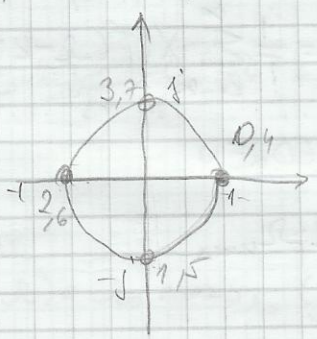
$$DFT_4\{x_c[n]\} = \sum_{n=0}^{3} x[n]\, W_4^{nk} = 0.5W_4^{0k} + W_4^{1k} - W_4^{2k} + 0 \cdot W_4^{3k}$$

$$X[0] = 0.5 + 1 - 1 + 0 = 0.5$$

$$X[1] = 0.5 - j + 1 + 0 = 1.5 - j$$

$$X[2] = 0.5 - 1 - 1 + 0 = -1.5$$

$$X[3] = 0.5 + j + 1 + 0 = 1.5 + j$$



k	X[k]	A[k]	b[k]
0	0.5	0.5	0
1	$1.5 - j$	1.8	-0.588
2	-1.5	1.5	0
3	$1.5 + j$	1.8	0.588

$$W_4^k = \{1, -j, -1, j\} \quad k = 0,1,2,3$$

$$x[(n-n_0)] \circ\!\!-\!\!\bullet\, X[k]\, W_N^{kn_0}$$

$$x_c[(n-1)] \circ\!\!-\!\!\bullet\, X[k]\, W_4^{1k} = \{0.5, -j + 1.5, -1.5, j + 1.5\}$$

$$= \{0.5, 1.5 - j, -1.5, 1.5 + j\}$$

Vrijedi svojstvo pomaka

Vježba 3.

$w_c = 0.125 \cdot 2\pi = 0.25\pi$

$N = 5$

$L = N + 1 = 6$

HAMMINGOV OTVOR

$$w_{ha}[n] = 0.54 + (1 - 0.54)\cos\left(\frac{2\pi(n+\frac{1}{2})}{L-1}\right)$$

$$= 0.54 + 0.46\cos\left(\frac{2\pi(n+\frac{1}{2})}{5}\right) \quad ; \; n \in \left[-\frac{N+1}{2}, \frac{N-1}{2}\right]$$

$$h_{NP}[n] = \frac{\sin\left[(n+\frac{1}{2})w_c\right]}{(n+\frac{1}{2})\pi}$$

$h'[n] = w_{ha}[n] \cdot h_{NP}[n]$

N	-3	-2	-1	0	1	2
$w_{ha}[n]$	0.08	0.3978	0.912	0.912	0.3978	0.08
$h_{NP}[n]$	0.11763	0.19605	0.24362	0.24362	0.19605	0.11763
$h'[n]$	$9.4104 \cdot 10^{-3}$	0.0779	0.2222	0.2222	0.0779	$9.4104 \cdot 10^{-3}$

$h'[n] = \{ 9{,}4104 \cdot 10^{-3}, 0{,}0779, \underline{0.2222}, 0.2222, 0.0779, 9.4104 \cdot 10^{-3} \}$

↓ nekauzalan

$h_c[n] = \{ \underline{9.4104 \cdot 10^{-3}}, 0.0779, 0.2222, 0.2222, 0.0779, 9.4104 \cdot 10^{-3} \}$

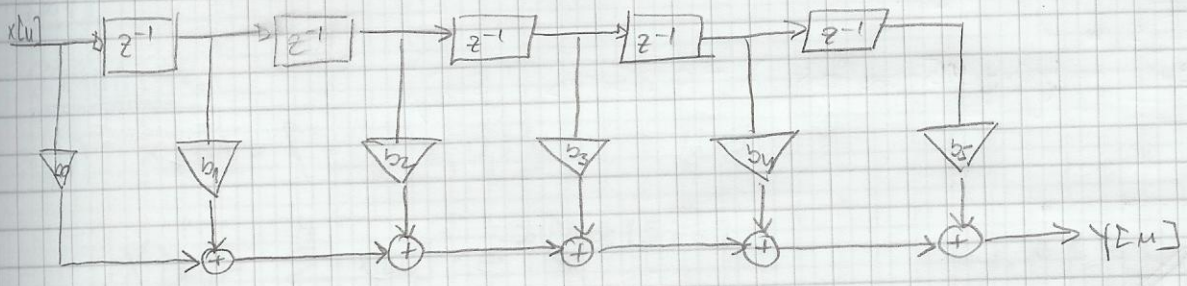
↓ kauzalan

$$H_c(z) = \sum_{n=0}^{5} h_c(n) z^{-n} = 0.0094 + 0.0779 z^{-1} + 0.2222 z^{-2} + 0.2222 z^{-3} + 0.0779 z^{-4} + 0.0094 z^{-5}$$

KVANTIZACIJA KOEFICIJENATA

$$h_c'[n] = \frac{\text{round}(2^5 \cdot h_c[n])}{2^5}$$

$h_c'[n] = \{ \underline{0}, 0{,}0625, 0{,}21875, 0.21875, 0.0625, 0 \}$



$$\text{FRAKCIJA} = \frac{x}{2^{B-1}} = \frac{x}{2^5} \qquad \text{CJELOBROJAN} = x = \text{round}\left(2^{B-1} \cdot h_c[n]\right)$$

$h_c[n]$ (cjelobrojan) $= \{0, 2, 7, 7, 2, 0\}$

$h_c[n]$ (frakcija) $= \{0, 0.0625, 0.21875, 0.21875, 0.0625, 0\}$

FREKVENCIJSKA KARAKTERISTIKA

$$H_c(e^{j\omega}) = e^{-j\frac{N}{2}\omega} \left\{ \sum_{n=1}^{\frac{N+1}{2}} b(n) \cos\left(\omega\left(n-\tfrac{1}{2}\right)\right) \right\}$$

$$b(n) = 2h_c\left(\frac{N+1}{2} - n\right) \quad ; \quad 1 \le n \le \frac{N+1}{2} \quad ; N=5$$

$$1 \le n \le 3$$

$$H_c(e^{j\omega}) = \underbrace{e^{-j\frac{5}{2}\omega}}_{f(\omega)} \underbrace{\left\{ \sum_{n=1}^{3} b(n) \cos\left(\omega\left(n-\tfrac{1}{2}\right)\right) \right\}}_{A(\omega)}$$

$$A(\omega) = b_1 \cos(0.5\omega) + b_2 \cos(1.5\omega) + b_3 \cos(2.5\omega)$$

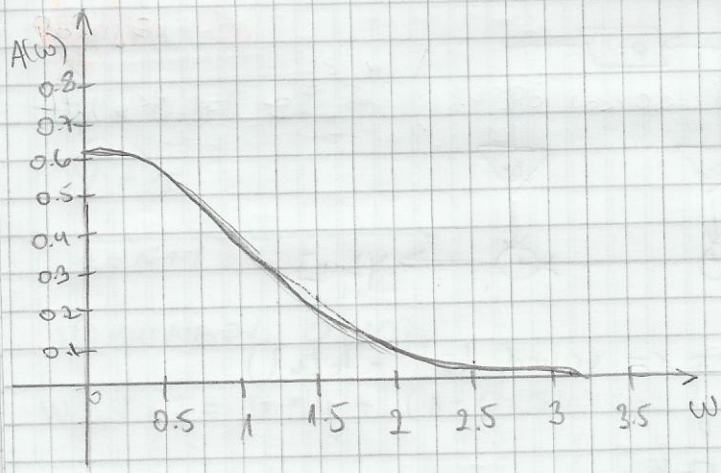
	Prije zaokruživanja	Poslije zaokruživanja
b_1	0.4444	0.4375
b_2	0.1556	0.125
b_3	0.0188	0

PRIJE

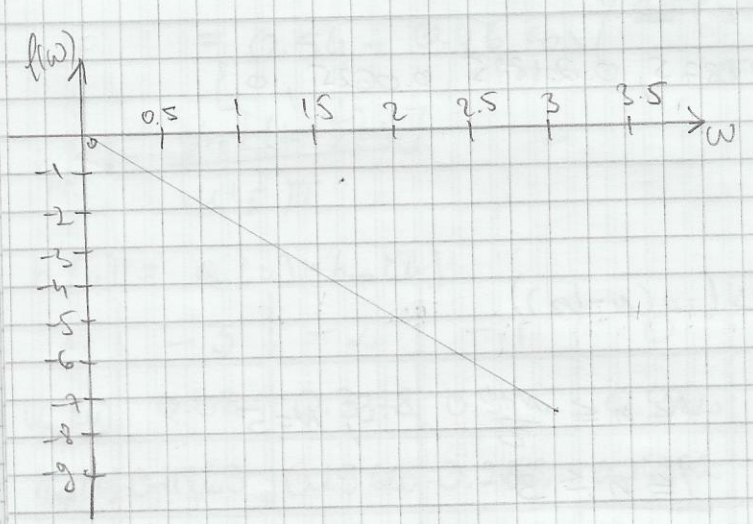
$$H_c(e^{j\omega}) = \underbrace{e^{-j\frac{5}{2}\omega}}_{e^{f(\omega)}} \underbrace{\left(0.4444\cos(0.5\omega) + 0.1556\cos(1.5\omega) + 0.0188\cos(2.5\omega)\right)}_{A(\omega) = |H_c(e^{j\omega})|}$$

POSLIJE

$$H_c(e^{j\omega}) = e^{-j5/2\omega} \cdot \left(0.4375\cos(0.5\omega) + 0.125\cos(1.5\omega)\right)$$


A(ω)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
1
1.5
2
2.5
3
3.5
ω

Loonusõlde


f(ω)
0.5
1
1.5
2
2.5
3
3.5
ω
0
-1
-2
-3
-4
-5
-6
-7
-8
-9

Vježba 4. PROJEKTIRANJE OPTIMALNIH FIR FILTARA

PARKS-McCLELLANOVA METODA

PARAMETRI:
- frekvencija w_p - propuštanje
- frekvencija w_s - gušenje
- red filtra N
- odstupanje u području propuštanja - δ_1
- odstupanje u području gušenja - δ_2

$$K = \frac{\delta_1}{\delta_2}$$

OPTIMIZIRANJE:
- Optimizira se amplitudno-frekv. karakteristika $A_e(e^{jw})$ koja aproksimira frekvencijsku karakteristiku $D(e^{jw})$ pomoću težinske funkcije $W(w)$

- TEŽINSKA FUNKCIJA:

$W(w)$ omogućava različite težinske faktore pogreške aproksimacije za različita frekvencijska područja:

$$W(w) = \begin{cases} \frac{1}{K}, & 0 \le w \le w_p \\ 1, & w_s \le w \le \pi \end{cases}$$

- omogućava različitu valovitost za područje propuštanja i gušenja, tj. valovitost nije jednaka za cijela frekvencijska područja

RED FILTRA $N = 12$

$M = N/2 = 6$

$$A_e(e^{jw}) = h_e[0] + \sum_{n=1}^{6} 2 h_e[n] \cos(wn)$$

$$h[n] = h_e[n-6], \quad n = 0, 1, \ldots N$$

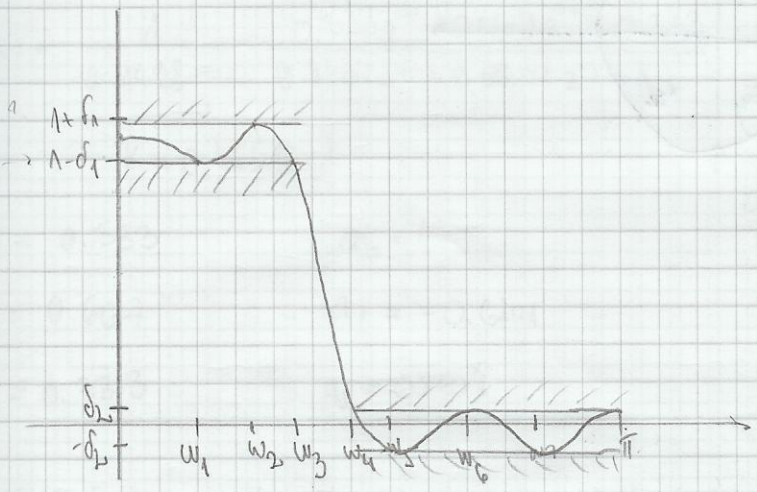
$$H(e^{jw}) = A_e(e^{jw})\, e^{-jwM}$$

$$\cos(wn) = T_n(\cos w)$$

$$A_e(e^{jw}) = \sum_{n=0}^{M} a_n (\cos w)^n = P_M(\cos w)$$

$w_i \in F \quad w_1 < w_2 < \dots < w_{M+2}$

$E(w_i) = -E(w_{i+1}) = \pm |E| \quad , i = 1,2,\dots,(M+1)$



Vježba 5. PROJEKTIRANJE IIR FILTARA BILINEARNOM TRANSFORMACIJOM I NJIHOVA REALIZACIJA DIREKTNOM FORMOM II

- digitalni NP filtar reda $N=2$, $w_g = \pi/3$
- period otipkavanja je $T=2$
- Butterworthova aproksimacija

Bilinearna transformacija $\Rightarrow \quad s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

preslikavanje s-ravnine u z-ravninu

$\Omega = \frac{2}{T} tg\left(\frac{w}{2}\right) \quad i \quad w = 2 \operatorname{arctg}\left(\frac{\Omega T}{2}\right)$

$\Omega_g = \frac{2}{T} tg\left(\frac{w_g}{2}\right) = \frac{2}{2} tg\left(\frac{\pi/3}{2}\right) = \frac{\sqrt{3}}{3} = \Omega_c$

$|H_c(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}} = H_c(j\Omega)\cdot H_c(-j\Omega) \qquad \Omega = s/j \;,\; s = j\Omega$

$H_c(s)\cdot H_c(-s) = \frac{1}{1+\left(\frac{s/j}{\Omega_c}\right)^{2N}} = \frac{1}{1+\left(\frac{-s}{\frac{\sqrt{3}}{3}}\right)^4} = \frac{1}{1+\frac{9s^4}{9}} = \frac{1}{1+9s^4}$

$1+9s^4 = 0$

$(1+3s^2) - 6s^2 = 0$

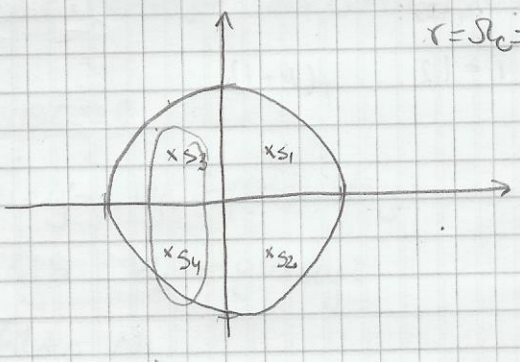
$(1+3s^2-\sqrt{6}s)(1+3s^2+\sqrt{6}s) = 0$

$$3s^2 - \sqrt{6}s + 1 = 0$$

$$s_{1,2} = \frac{\sqrt{6} \pm \sqrt{6-12}}{6}$$

$$s_1 = \frac{\sqrt{6} + \sqrt{6}j}{6}$$

$$s_2 = \frac{\sqrt{6} - \sqrt{6}j}{6}$$



$r = s_c = \sqrt{3}/3$

$$3s^2 + \sqrt{6}s + 1 = 0$$

$$s_{3,4} = \frac{-\sqrt{6} \pm \sqrt{6-12}}{6}$$

$$s_3 = \frac{-\sqrt{6} + \sqrt{6}j}{6}$$

$$s_4 = \frac{-\sqrt{6} - \sqrt{6}j}{6}$$

$$H_c(s) = s_c^2 \frac{1}{s - s_3} \cdot \frac{1}{s - s_4} = \left(\frac{\sqrt{3}}{3}\right)^2 \cdot \frac{1}{s - \left(\frac{-\sqrt{6} + \sqrt{6}j}{6}\right)} \cdot \frac{1}{s - \left(\frac{-\sqrt{6} - \sqrt{6}j}{6}\right)}$$

$$H_c(s) = \frac{1}{3} \cdot \frac{1}{\left(s + \frac{\sqrt{6}}{6}\right) - \frac{\sqrt{6}}{6}j} \cdot \frac{1}{\left(s + \frac{\sqrt{6}}{6}\right) + \frac{\sqrt{6}}{6}j}$$

$$H_c(s) = \frac{1}{3} \cdot \frac{1}{s^2 + 2s\frac{\sqrt{6}}{6} + \frac{1}{6} + \frac{1}{6}}$$

$$H_c(s) = \frac{1}{3} \cdot \frac{1}{s^2 + \frac{\sqrt{6}}{3}s + \frac{1}{3}}$$

Bilineama $\quad s = \frac{2}{2}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = \frac{1 - z^{-1}}{1 + z^{-1}}$

$$H(z) = \frac{1/3}{\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)^2 + \frac{\sqrt{6}}{3}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) + \frac{1}{3}}$$

$$H(z) = \frac{1/3\,(1 + z^{-1})^2}{(1 - z^{-1})^2 + \frac{\sqrt{6}}{3}(1 - z^{-1})(1 + z^{-1}) + \frac{1}{3}(1 + z^{-1})^2}$$

$$H(z) = \frac{1/3\,(1 + 2z^{-1} + z^{-2})}{1 - 2z^{-1} + z^{-2} + \frac{\sqrt{6}}{3} - \frac{\sqrt{6}}{3}z^{-2} + \frac{1}{3} + \frac{2}{3}z^{-1} + \frac{1}{3}z^{-2}}$$

$$H(z) = \frac{1/3 + 2/3\,z^{-1} + 1/3\,z^{-2}}{\frac{4 - \sqrt{6}}{3}z^{-2} - \frac{4}{3}z^{-1} + \frac{4 + \sqrt{6}}{3}}$$

$$H(z) = \frac{0.333 z^{-2} + 0.667 z^{-1} + 0.333}{0.5168 z^{-2} - 1.333 z^{-1} + 2.1498}$$

Skaliranje

$$H(z) = \underbrace{\frac{1}{2.1498}}_{K = 0.46516} \frac{0.333 z^{-2} + 0.667 z^{-1} + 0.333}{0.2404 z^{-2} - 0.6201 z^{-1} + 1}$$

$$K = \frac{b_{max}}{a_{max}}$$

$b_0 = 0.333$ $\quad a_0 = 1$

$b_1 = 0.667$ $\quad a_1 = -0.6201$

$b_2 = 0.333$ $\quad a_2 = 0.2404$

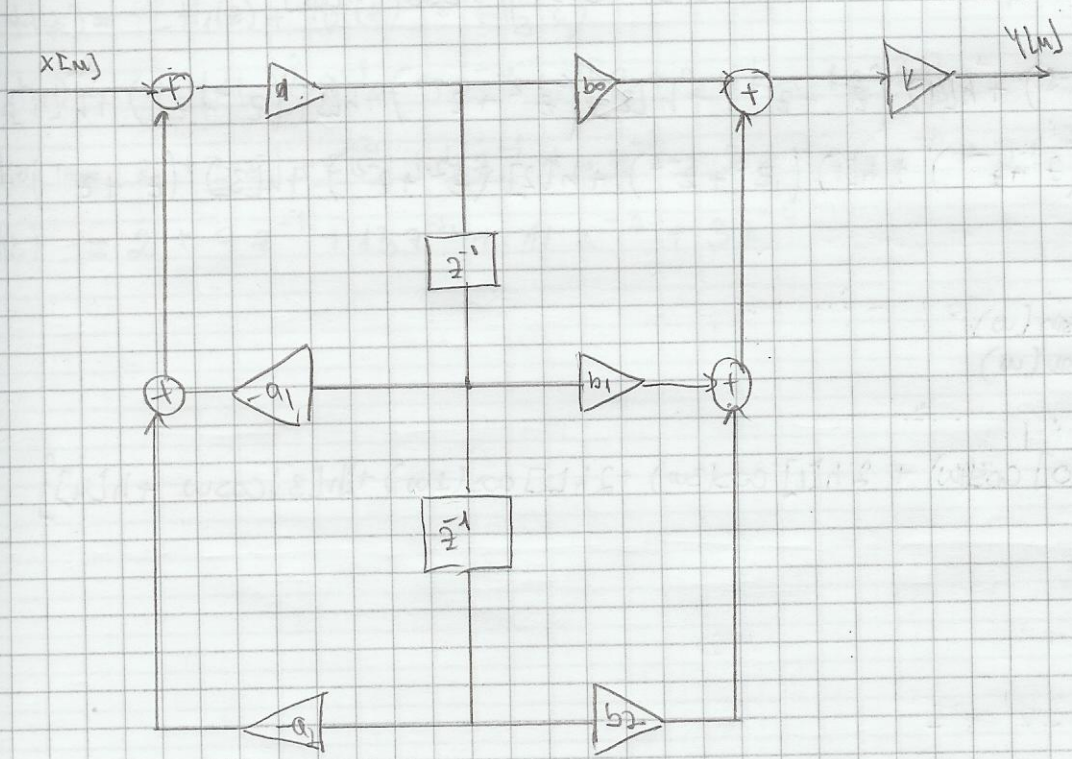
Kvantizacija zaokruživanjem u oblik frakcija, $B = 6$

$$\frac{\text{round}\left[2^{B-1} \cdot x\right]}{2^{B-1}}$$

$b_0 = 0.3125$ $\quad a_0 = 1$ $\quad K = 0.4375$

$b_1 = 0.65625$ $\quad a_1 = -0.625$

$b_2 = 0.3125$ $\quad a_2 = 0.21875$



6. VJEŽBA

direktna II i kaskadna realizacija ; 4 bita (3 b + 1 predznak)

$$H(z) = \frac{(0.53 + 1.54 z^{-1})(1.89 + 2z^{-1} + z^{-2})}{(0.35 + 0.045 z^{-1} - 0.7 z^{-2})(1 - 0.23 \cdot z^{-1})}$$

$$= \frac{1.0017 + 1.06 z^{-1} + 0.53 z^{-2} + 2.9106 z^{-1} + 3.08 z^{-2} + 1.54 z^{-3}}{0.35 - 0.0805 z^{-1} + 0.045 z^{-1} - 0.01035 z^{-2} - 0.7 z^{-2} + 0.161 z^{-3}}$$

$$= \frac{1.0017 + 3.9706 z^{-1} + 3.61 z^{-2} + 1.54 z^{-3}}{0.35 - 0.0355 z^{-1} - 0.71035 z^{-2} + 0.161 z^{-3}}$$

$$= 3.9706 \quad \frac{0.2523 + z^{-1} + 0.9092 z^{-2} + 0.3879 z^{-3}}{0.35 - 0.0355 z^{-1} - 0.71035 z^{-2} + 0.161 z^{-3}}$$

I) $1 - 0.23 z^{-1} = 0 \;/ \cdot z$

$p_1 = z_1 = 0.23$

II) $0.35 + 0.045 z^{-1} - 0.7 z^{-2} = 0 \;/ \cdot z^2$

$0.35 z^2 + 0.045 z - 0.7 = 0$

$$p_{2,3} = z_{2,3} = \frac{-0.045 \mp \sqrt{0.045^2 - 4 \cdot 0.35 \cdot (-0.7)}}{2 \cdot 0.35}$$

$p_2 = -1.4799$

$p_3 = 1.3514$ → postoje polovi izvan jedinične kružnice

⇒ filtar nije stabilan

DIREKTNA II REALIZACIJA

$b_0 = \frac{1}{8} \cdot \text{round}(8 \cdot 0.2523) = 0.25$

$b_1 = 1$

$b_2 = 0.875$

$b_3 = 0.375$

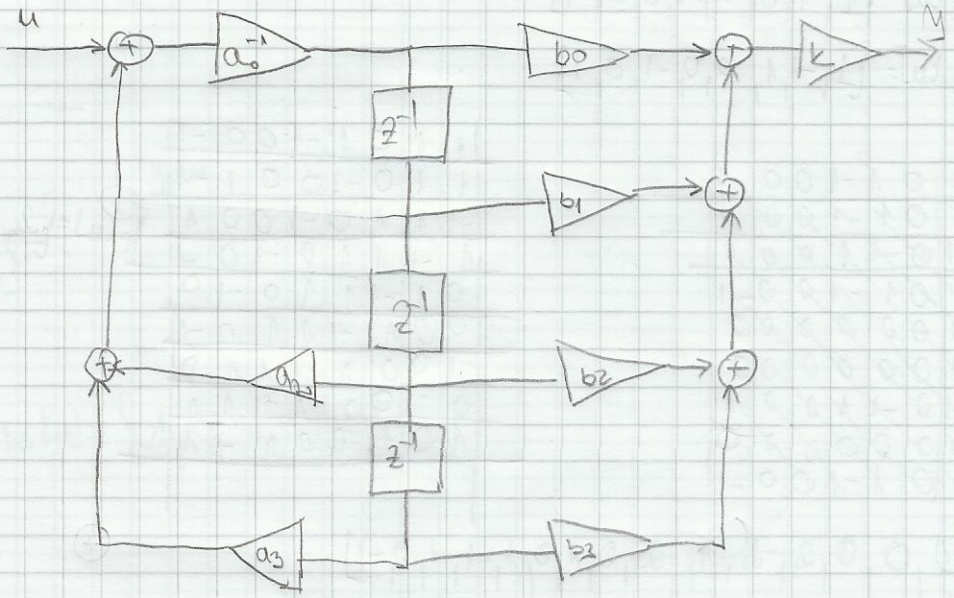
$a_0 = 0.375 ; \; a_0^{-1} = 2.675$

$a_1 = 0$

$a_2 = -0.75$

$a_3 = 0.125$

$k = 4$



KASKADNA REALIZACJA

$$H(z) = H_1(z) \cdot H_2(z) = \frac{0.53 + 1.54 z^{-1}}{1 - 0.23 z^{-1}} \cdot \frac{1.89 + 2z^{-1} + z^{-2}}{0.35 + 0.045 z^{-1} - 0.7 z^{-2}}$$

$$= \underbrace{0.53 \cdot \frac{1 + 2.9057 z^{-2}}{1 - 0.23 z^{-1}}}_{H_1(z)} \cdot \underbrace{5.4 \cdot \frac{1 + 1.0582 z^{-1} + 0.5291 z^{-2}}{1 + 0.1286 z^{-1} - 2 z^{-2}}}_{H_2(z)}$$

$k_1 = 0.5$

$b_{10} = 1$

$b_{11} = 2.875$

$b_{12} = 0$

$a_{10} = 1$

$a_{11} = -0.25$

$a_{12} = 0$

$k_2 = 5.375$

$b_{20} = 1$

$b_{21} = 1$

$b_{22} = 0.5$

$a_{20} = 1$

$a_{21} = 0.125$

$a_{22} = -2$