① $xyy' = y^2 - x^2 \quad y(1) = \sqrt{2}$

$$xy \frac{dy}{dx} = y^2 - x^2$$

$$xy\,dy = (y^2 - x^2)\,dx$$

$$y' = \frac{y^2 - x^2}{xy} \quad / : \frac{x^2}{x^2} \quad \frac{y}{x} = z$$

$$\frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 - 1}{\frac{y}{x}} = \frac{z^2 - 1}{z} \quad y' = z + z'x$$

$$z + z'x = \frac{z^2 - 1}{z}$$

$$\cancel{z^2} + zxz' = \cancel{z^2} - 1$$

$$\frac{dz}{dx} zx = -1$$

$$z\,dz = \frac{-1}{x}\,dx \quad / \int$$

$$\int z\,dz = -\int \frac{1}{x}\,dx$$

$$\frac{z^2}{2} = -\ln|x| + c$$

$$\frac{z^2}{2} = \ln\frac{c}{x}$$

$$z^2 = \ln\left(\frac{c}{x}\right)^2$$

$$\frac{y^2}{x^2} = \ln\left(\frac{c}{x}\right)^2$$

$$y^2 = x^2 \ln\left(\frac{c}{x}\right)^2 \quad \text{OPĆI OBLIK RJEŠENJA JEDNADŽBE}$$

$$(\sqrt{2})^2 = 1^2 \ln\left(\frac{c^2}{1^2}\right)$$

$$2 = \ln c^2 \quad / e^{\wedge}$$

$$\boxed{c^2 = e^2} \quad / \pm\sqrt{}$$

$$c = \pm e$$

$$\boxed{y^2 = x^2 \ln \frac{e^2}{x^2}}$$

$M(x,y) = xy$

$M(tx,ty) = tx\,ty = t^{2}xy$

$N(x,y) = y^2 - x^2$

$N(tx,ty) = t^2y^2 - t^2x^2 = t^{2}(y^2 - x^2)$

homogena

$$\boxed{\ln = \log_e \quad y = x \\ y = e^x}$$

② ① $y = Ce^x \quad \alpha = 90°$

$k_1 = \frac{-1}{k_2}$ ⇒ uvjet okomitosti pravaca

$y = ax + b$

$y' = Ce^x \rightarrow C = \frac{y'}{e^x}$

① $\rightarrow C = \frac{y}{e^x}$

$\frac{y'}{e^x} = \frac{y}{e^x}$

$\boxed{y = y'}$

$y_1' = \frac{-1}{y_2'}$

$\cancel{y}_2' = \frac{-1}{y_1'}$

$y_2' = \frac{-1}{y}$

$\frac{dy}{dx} = \frac{-1}{y}$

$y\,dy = -1\,dx \;/ \int$

$\int y\,dy = -\int dx$

$\frac{y^2}{2} = -x + C$

$y^2 = -2x + C_1$

$\boxed{y^2 + 2x = C_1}$

(3) $y' + y \underbrace{\operatorname{tg} x}_{p(x)} = \underbrace{2\cos^2 x}_{g(x)}$

$y' + p(x)y = g(x)$ ⇒ opći oblik linearna dif. jed. 1. reda (1.8) ⇒ $\boxed{\text{MAT 2 DIF. JEDNAD.}}$ PDF

(1.) homogena

$$y' + y\operatorname{tg}(x) = 0$$
$$y' = -y\operatorname{tg}(x)$$
$$\frac{dy}{dx} = -y\operatorname{tg}(x)$$
$$\frac{dy}{y} = -\operatorname{tg}(x)\,dx \quad /\int$$
$$\int \frac{1}{y}\,dy = -\int \operatorname{tg}(x)\,dx = -\int \frac{\sin x}{\cos x}\,dx \quad \left|\begin{array}{l} \cos x = u \\ -\sin x\, dx = du \\ dx = \frac{du}{-\sin x} \end{array}\right|$$
$$= -\int -\frac{du}{u}$$
$$\ln|y| = \ln|u| + C =$$
$$\ln|y| = \ln|\cos x| + C$$
$$\ln|y| = \ln|C\cos x| \quad /e^{\wedge}$$
$$y = C\cos x$$

(2.) varijacija konstante $(C \Rightarrow C(x))$

$$y = C(x)\cos x$$

$\boxed{\text{RJEŠENJE:}}$ $\Rightarrow y = (2\sin x \cancel{\cos x} + c)\cos x$

(3.) u 2. u poč. jed.

$$(C(x)\cos x)' + C(x)\cos x \operatorname{tg} x = 2\cos^2 x$$
$$C'(x)\cos x + \cancel{C(x)(-\sin(x))} + \cancel{C(x)\sin x} = 2\cos^2 x$$

C-ovi se uvijek nekako moraju pokratiti

$$C'(x)\cos x = 2\cos^2 x \quad /:\cos x$$
$$C'(x) = 2\cos x \quad /\int$$
$$C(x) = 2\int \cos x\,dx$$
$$\boxed{C(x) = 2\sin x + C}$$

④ $\underbrace{(2x+y+1)}_{P}dx + \underbrace{(x+2y)}_{Q}dy = 0$ $\boxed{(2.1)}$

EGZAKTNA
EULEROV ili
HOMOGENA

$$\left.\begin{array}{l} P'_y = 1 \\ Q'_x = 1 \end{array}\right\} \quad P'_y = Q'_x \Rightarrow \text{egzaktna} \quad \boxed{(2.2)}$$

$$u(x,y) = \int_0^x (2x+y+1)\,dx + \int_0^y (\cancel{2}x_0+2y)\,dy$$

- ako nema razlomaka $x_0=0$ $y_0=0$
- y - razlomak $x_0=1, y_0=1$

REKONSTR. FUNKCIJE IZ POTP. DIF (2.3)

$$= 2\int_0^x x\,dx + \int_0^x y\,dx + \int_0^x dx + \int_0^y 0\,dy + 2\int_0^y y\,dy$$

$$= x^2 + yx + x + y^2$$

$$\boxed{x^2 + yx + x + y^2 = C}$$

⑤ a) $y = axy' + \frac{1}{2}(y')^2$

→ naći singularno rješenje (nema konstanti)
→ uvjeti za singul.

- supstitucija $\boxed{y'=p}$

$F(x,y,p) = axp + \frac{1}{2}p^2$

$$\frac{dF(x,y,y')}{dy'} = 0$$

$$\frac{dF(x,y,p)}{dp} = 0$$

$$ax + p = 0$$

$$\boxed{p = -ax}$$

$$y = axp + \frac{1}{2}p^2$$

$$y = ax(-ax) + \frac{1}{2}a^2x^2 = -a^2x^2 + \frac{1}{2}a^2x^2 = -\frac{1}{2}a^2x^2$$

$$-\frac{1}{2}a^2x^2 = ax\left(-\frac{1}{2}a^2x^2\right)' + \frac{1}{2}\left[\left(-\frac{1}{2}a^2x^2\right)'\right]^2$$

$\Longrightarrow$

$$\dots \quad -\frac{1}{2}a^2x^2 = a\,x\,(-a^2x) + \frac{1}{2}(-a^2x)^2$$

$$-\frac{1}{2}a^2x^2 = -a^3x^2 + \frac{1}{2}a^4x^2 \quad / : a^2x^2$$

$$-\frac{1}{2} = -a + \frac{1}{2}a^2 \quad / \cdot 2$$

$$-1 = -2a + a^2$$

$$\boxed{a^2 - 2a + 1 = 0}$$

$a^2 - 2a + 1 = 0$ kv. jednadžba

$$a_{1,2} = 1 \pm \sqrt{1-1}$$

$$\boxed{a = 1}$$

b) $a = 1$

$y = xy' + \frac{1}{2}(y')^2 \quad ; \quad y' = p \;,\; p = p(x)$

$y = xp + \frac{1}{2}p^2 \quad / \frac{d}{dx}$

$y' = p + xp' + \frac{1}{2}2pp'$

$xp' + pp' = 0$

$p'(x+p) = 0$

① $p' = 0$

$\boxed{p = C}$ $\Rightarrow$ OPĆE RJEŠENJE

② $x + p = 0$

$p = -x$ $\Rightarrow$ NIJE OPĆE

$$\boxed{y = xC + \frac{1}{2}C^2}$$

6 $$yy'' - 2(y')^2 - y^2 = 0$$

wstawiając $y = e^{\int z(x)\,dx}$ 3

$$y' = \left(e^{\int z(x)\,dx}\right)'$$

$$= e^{\int z(x)\,dx} \cdot \left(\int z(x)\,dx\right)'$$

$$\Rightarrow e^{\int z(x)\,dx} \cdot z = y \cdot z$$

1 $$\boxed{y' = y \cdot z}$$

2 $$y'' = (yz)' = y'z + yz'$$

$$\boxed{y'' = yz^2 + yz'}$$

5

$$y(yz^2 + yz') - 2(yz)^2 - y^2 = 0$$

$$y^2z^2 + y^2z' - 2y^2z^2 - y^2 = 0$$

$$y^2z' - y^2z^2 - y^2 = 0 \quad /: y^2$$

$$z' - z^2 - 1 = 0$$

$$z' = z^2 + 1$$

$$\frac{dz}{dx} = z^2 + 1$$

$$\frac{1}{z^2+1}\,dz = dx \quad /\int$$

$$\operatorname{arctg} z = x + c \quad /\cdot \operatorname{tg}$$

$$\boxed{z = \operatorname{tg}(x+c)}$$

4

$$y = e^{\int \operatorname{tg}(x+c)\,dx}$$

$$\int \operatorname{tg}(x+c_1)\,dx = \int \frac{\sin(x+c)}{\cos(x+c)}\,dx = \begin{bmatrix} x+c = u \\ dx = du \end{bmatrix} = \int \frac{\sin u}{\cos u}\,du =$$

$$= \begin{bmatrix} \cos u = t \\ -\sin u\,du = dt \end{bmatrix} = -\int \frac{\cancel{\sin u}}{t}\,\frac{dt}{\cancel{\sin u}} =$$

$$-\ln|t| = -\ln|\cos u| =$$

$$= -\ln|\cos(x+c_1)| + c_2$$

$$y = e^{-\ln|\cos(x+c_1)| + c_2} =$$

$\Longrightarrow$

$$y = e^{-\ln \cos(x+C_1)} \cdot e^{C_2}$$

$$= e^{\ln \frac{1}{\cos(x+C_1)}} \cdot e^{C_2}$$

$$= \frac{1}{\cos(x+C_1)} \cdot e^{C_2}$$

$$y = \frac{e^{C_2}}{\cos(x+C_1)}$$

$$\boxed{y = \frac{C_3}{\cos(x+C_1)}}$$

⑦ $y''' - y'' = 0$; $y(0) = y'(0) = 1$, $y''(0) = 2$

$r^3 - r^2 = 0$ (KARAKT. POLINOM)

$\boxed{(4.4.) \text{ Lujiž str. } 103}$

$r^2(r-1) = 0$

① $r^2 = 0$ ② $r = 1$ ③

$\boxed{r_1 = 0}$ (dvostrukost) $\boxed{r_2 = 1}$ (jednostrukost)

① $y = C_1 e^{r_1 x} + C_2 x e^{r_1 x} + C_3 e^{r_2 x}$

① $\boxed{y = C_1 + C_2 x + C_3 e^x}$ $\rightarrow$ $\boxed{y = -1 - x + 2e^x}$

ZJEŠENJE

② $y' = C_2 + C_3 e^x$ ③

$y'' = C_3 e^x$ ⑤

④

$1 = C_2 + C_3$

$\boxed{2 = C_3}$ $1 = C_2 + 2$

$\boxed{C_2 = -1}$

$1 = C_1 + 2$

$\boxed{C_1 = -1}$ $\rightarrow$ iz uvjeta $y(0) = 1$

(4.3, knjižica 99-100)

8) $y'' + y = \operatorname{ctg} x$

1) homog. jednadžba

$$y'' + y = 0$$

$$r^2 + 1 = 0$$

$$r^2 = -1$$

$$\boxed{r_{1,2} = \pm i}$$

$$y = C_1 e^{\alpha x} \cos \beta x + C_2 e^{\alpha x} \sin \beta x$$

$$r = \alpha \pm \beta i$$

$$\boxed{\begin{array}{l} \alpha = 0 \\ \beta = 1 \end{array}}$$

$$y_H = C_1 \cos x + C_2 \sin x$$

2) varijacija konstanti

$$y_H = C_1(x) \cos x + C_2(x) \sin x$$

$$C_1'(x) \cos x + C_2'(x) \sin x = 0$$

$$C_1'(x)(\cos x)^{(n-1)} + C_2'(x)(\sin x)^{(n-1)} = \operatorname{ctg} x$$

n – broj funkcija (2)
– treba ispasti kv. matrica

$$C_1'(x) \cos x + C_2'(x) \sin x = 0 \quad / : \cos x \quad \text{(jer je zadnji red)}$$

$$-C_1'(x) \sin x + C_2'(x) \cos x = \operatorname{ctg} x \quad / : \sin x$$

$$\left.\begin{array}{l} C_1'(x) + C_2'(x) \frac{\sin x}{\cos x} = 0 \\ -C_1'(x) + C_2'(x) \frac{\cos x}{\sin x} = \frac{\cos x}{\sin^2 x} \end{array}\right\} +$$

1) $C_2'(x) \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) = \frac{\cos x}{\sin^2 x}$

$$C_2'(x) \left(\frac{\overbrace{\sin^2 x + \cos^2 x}^{1}}{\cos x \sin x} \right) = \frac{\cos x}{\sin^2 x} \quad / \cdot \cos x \sin x$$

2) $\boxed{C_2'(x) = \frac{\cos^2 x}{\sin x}}$

$$\boxed{C_1'(x) = -\cos x}$$

$\Rightarrow$

$C_1'(x) = -\cos x \quad / \int$

$C_1(x) = \int -\cos x \, dx$

$\boxed{C_1(x) = -\sin x + C}$

-uvrstiti $C_1(x)$ i $C_2(x)$ u y_H

$C_2'(x) = \dfrac{\cos^2 x}{\sin x} \quad / \int$

$C_2(x) = \int \dfrac{\cos^2 x}{\sin x} \, dx =$

$= \int \dfrac{1 - \sin^2 x}{\sin x} \, dx =$

$= \int \dfrac{1}{\sin x} \, dx - \int \sin x \, dx$

$= \ln |\operatorname{tg} \frac{x}{2}| + \cos x$

$\boxed{C_2(x) = \ln |\operatorname{tg} \frac{x}{2}| + \cos x + D}$

$$\boxed{y_H = (-\sin x + C) \cos x + (\ln |\operatorname{tg} \tfrac{x}{2}| + \cos x + D) \sin x}$$

ZI 2007

① $(y^4 - x^2 y^2) dx + 2x^3 y \, dy = 0$ HOMOG. 4. stupnja

$$2x^3 y \, dy = (x^2 y^2 - y^4) \, dx$$

$$\frac{dy}{dx} = \frac{x^2 y^2 - y^4}{2x^3 y} \quad \begin{matrix} : x^4 \\ : x^4 \end{matrix}$$

$$\frac{dy}{dx} = \frac{\frac{y^2}{x^2} - \frac{y^4}{x^4}}{2 \frac{y}{x}}$$

supstitucija

$z = \frac{y}{x}$

$y' = z + xz'$

$$\frac{dz}{dx} x + z = \frac{z^2 - z^4}{2z} = \frac{z - z^3}{2}$$

$$\frac{dz}{dx} x = \frac{z - z^3}{2} - z = -\frac{z^3 + z}{2}$$

$$\frac{dz}{z^3 + z} = \frac{-1}{2} \frac{dx}{x} \quad / \int$$

$$\int \frac{1}{z^3 + z} \, dz = \frac{-1}{2} \int \frac{1}{x} \, dx$$

$\Longrightarrow$

$$\frac{1}{z(z^2+1)} = \frac{A}{z} + \frac{Bz+C}{z^2+1}$$

$$1 = z^2(A+B) + Cz + A$$

$$A+B=0 \Rightarrow \boxed{B=-1}$$

$$\boxed{C=0}$$

$$\boxed{A=1}$$

$$\int \frac{dz}{z} - \int \frac{z\,dz}{z^2+1} = -\frac{1}{2}\ln|x| + c \qquad \left[\begin{array}{c} z^2+1=u \\ 2z\,dz = du \end{array}\right]$$

$$\ln z - \frac{1}{2}\ln(z^2+1) = -\frac{1}{2}\ln|x| + c \quad /\cdot(-2)$$

$$\ln\frac{z^2+1}{z^2} = \ln cx$$

$$\frac{z^2+1}{z^2} = cx$$

$$\frac{y^2}{x^2}+1 = \frac{y^2}{x^2}\,cx \quad /\cdot x^2$$

$$\boxed{x^2+y^2 = cxy^2}$$ RJEŠENJE

(2) $\underbrace{e^{-y}}_{P}dx + \underbrace{(1-xe^{-y})}_{Q}dy = 0$

$$\left.\begin{array}{l} P'_y = -e^{-y} \\ Q'_x = -e^{-y} \end{array}\right\} P'_y = Q'_x \Rightarrow \text{egzaktna}$$

$$u(x,y) = \int_0^x e^{-y}\,dx + \int_0^y (1 - x_0 e^{-y})\,dy, \quad x_0 \to 0$$

$$= xe^{-y} + y = C$$ UVJET $y(0)=5$

$$0e^{-y} + 5 = C$$

$$\boxed{C=5}$$

$$\boxed{xe^{-y}+y=5}$$ RJEŠENJE

③ $y' + xy = x\cos\left(\frac{x^2}{2}\right), \quad y(\sqrt{\pi}) = \frac{1}{2}$

LDJ 1. reda

① $y' + xy = 0$ (homogena)

$$\frac{dy}{dx} = -xy$$

$$\frac{1}{y}\,dy = -x\,dx \;/\int$$

$$\ln|y| = -\frac{x^2}{2} + c \;/e^{\wedge}$$

$$\boxed{y = Ce^{-\frac{x^2}{2}}}$$

② varijacija konstante

$$y = C(x)e^{-\frac{x^2}{2}}$$

$$y' = C'(x)e^{-\frac{x^2}{2}} + C(x)e^{-\frac{x^2}{2}}\cdot(-x)$$

$$C'(x)e^{-\frac{x^2}{2}} + \cancel{C(x)e^{-\frac{x^2}{2}}\cdot(-x) + x\,C(x)e^{-\frac{x^2}{2}}} = x\cos\left(\frac{x^2}{2}\right) \;/\cdot e^{\frac{x^2}{2}}$$

$$C'(x) = e^{\frac{x^2}{2}}x\cos\left(\frac{x^2}{2}\right) \;/\int$$

$$C(x) = \int e^{\frac{x^2}{2}}x\cos\frac{x^2}{2}\,dx \quad \left[\begin{matrix}\frac{x^2}{2} = t\\ x\,dx = dt\\ dx = \frac{dt}{x}\end{matrix}\right]$$

$$C(x) = \int e^t \cancel{x}\cos t\,\frac{dt}{\cancel{x}} = \int e^t\cos t\,dt$$

$$\left[\begin{matrix}\cos t = u\;/' & e^t\,dt = dv\;/\int\\ -\sin t\,dt = du & e^t = v\end{matrix}\right]$$

$$= e^t\cos t + \int e^t\sin t\,dt = \left[\begin{matrix}\sin t = u & e^t\,dt = dv\\ \cos t\,dt = du & e^t = v\\ \ldots & \ldots\end{matrix}\right]$$

$$= e^t\cos t + e^t\sin t - \int e^t\cos t\,dt$$

$$2\int e^t\cos t\,dt = e^t\cos t + e^t\sin t$$

$$\int e^t\cos t\,dt = \frac{e^t\cos t + e^t\sin t}{2} \rightarrow$$

$$c(x) = \frac{e^{\frac{x^2}{2}}\left(\cos\frac{x^2}{2} + \sin\frac{x^2}{2}\right)}{2} + C$$

- uvrštavamo $C(x)$ u varijaciju konstante

$$y = \frac{1}{2}\left(\cos\frac{x^2}{2} + \sin\frac{x^2}{2}\right) + Ce^{\frac{-x^2}{2}}$$

$$y(\sqrt{\pi}) = \frac{1}{2}$$

$\vdots$

$$C = 0$$

(4) $y = xy' + \frac{1}{y'}$ $\qquad y' = p\ ,\ p = p(x)$

$$y = xp + \frac{1}{p} \Big/ \frac{d}{dx}$$

$$y' = p + xp' - \frac{1}{p^2}p'$$

$$xp' - \frac{p'}{p^2} = 0$$

$$\boxed{p'\left(x - \frac{1}{p^2}\right) = 0}$$

OPĆE RJEŠENJE

① $p' = 0$ opće $\qquad \boxed{y = Cx + \frac{1}{p}}$

$$\boxed{p = C}$$

② $x - \frac{1}{p^2} = 0$ SINGULARNO

$$x = \frac{1}{p^2}$$

$$y = \frac{1}{p^2}\cdot p + \frac{1}{p} = \frac{2}{p}$$

$$p = \frac{2}{y}$$

$$y = x\frac{2}{y} + \frac{y}{2} \Big/ \cdot 2y$$

$$2y^2 = 4x + y^2$$

$$y^2 = 4x$$

$$\boxed{x = \frac{y^2}{4}}$$

SINGULARNO RJEŠENJE

⑤ $yy'' + 2(y')^2 = 0$, $y' = p$ ali $p = p(y)$

kada su sami ipsiloni

$y' = p$

$y'' = p \cdot p'$

$ypp' + 2p^2 = 0 \quad /: p$

$yp' + 2p = 0$

$yp' = -2p$

$y \dfrac{dp}{dy} = -2p$

$\dfrac{dp}{p} = -2\dfrac{dy}{y} \quad /\int$

$\ln p = -2 \ln y + C$

$\boxed{p = \dfrac{C_1}{y^2}}$

$\dfrac{dy}{dx} = \dfrac{C_1}{y^2}$

$\dfrac{1}{C_1} y^2 dy = dx \quad /\int$

$\dfrac{1}{C_1} \cdot \dfrac{y^3}{3} = x + C_2 \quad /\cdot 3C_1$

$y^3 = kx + \epsilon$

$\boxed{y = \sqrt[3]{kx + \epsilon}}$

⑥ $y'' + 5y' + 6y = e^{-x} + e^{-2x}$

① homogena

$y'' + 5y' + 6y = 0$

$r^2 + 5r + 6 = 0$

$r_{1,2} = \dfrac{-5 \pm \sqrt{25 - 24}}{2}$

$\boxed{r_1 = -2}$ $\boxed{r_2 = -3}$

$y_H = C_1 e^{-2x} + C_2 e^{-3x}$

② partikularna rješenja

- koristimo algoritam 4.5 (str. 104 knjižica)

$y_p = y_{p1} + y_{p2}$

a) $y_{p1} \Rightarrow$ oblika $C e^{-x}$

b) $y_{p2} \Rightarrow$ oblika $D e^{-2x}$

→

str. 14

(2a) $y'' + 5y' + 6y = \underline{e^{-x}}$ $\Rightarrow$ $\alpha = -1$, $\beta = 0$ $\Rightarrow$ $\alpha \pm \beta i = -1$

funkc. smetnje $\Rightarrow$ $e^{\alpha x}(Q_1(x)\cos\beta x + Q_2(x)\sin\beta x)$

$$y_{p1} = x^{\overset{0}{m}} e^{\overset{-1}{\alpha} x}(R_1(x)\cos\overset{0}{\beta} x + R_2(x)\sin\overset{0}{\beta} x)$$

-1 nije nultočka korak.
jednadžbe $\Rightarrow m=0$

$$y_{p1} = e^{-x}\cdot \underbrace{R_1(x)}_{=A}$$

$$\boxed{\begin{aligned} y_{p1} &= Ae^{-x} \\ y_{p1}' &= -Ae^{-x} \\ y_{p1}'' &= Ae^{-x} \end{aligned}}$$

(u 2a imamo y'' i y' pa ćemo y_{p1} derivirati)

- uvrstit ćemo ih u 2a

$$Ae^{-x} - 5Ae^{-x} + 6Ae^{-x} = e^{-x}$$

$$2Ae^{-x} = e^{-x}$$

$$\boxed{A = \frac{1}{2}} \Rightarrow \boxed{y_{p1} = \frac{e^{-x}}{2}}$$

(2b) $y'' + 5y' + 6y = e^{-2x}$

f-smetnje $\Rightarrow \alpha = -2, \beta = 0$, $\alpha \pm \beta i = -2 \rightarrow$ nultočka $m=1$

$$y_{p2} = xe^{-2x}\underbrace{R_1(x)}_{=A}$$

$$y_{p2} = Axe^{-2x}$$

$$y_{p2}' = \underbrace{A'xe^{-2x}}_{=0} + \overset{1}{x'}Ae^{-2x} + (e^{-2x})'Ax$$

$$= Ae^{-2x} - 2Axe^{-2x}$$

$$y_{p2}'' = -2Ae^{-2x} - 2Ae^{-2x} + 4Axe^{-2x}$$

$$= 4Axe^{-2x} - 4Ae^{-2x}$$

$\Rightarrow$

$\cancel{4Axe^{-2x}} - 4Ae^{-2x} + 5Ae^{-2x} \cancel{- 10Axe^{-2x}} \cancel{+ 6Axe^{-2x}}$

str. $\boxed{15}$

$= e^{-2x}$

$$Ae^{-2x} = e^{-2x}$$

$$A = 1$$

$$\boxed{y_{p2} = e^{-2x}}$$

RJEŠENJE $\Rightarrow$ $y = y_H + y_{p1} + y_{p2}$

$$\boxed{y = C_1 e^{-2x} + C_2 e^{-3x} + \frac{e^{-x}}{2} + e^{-2x}}$$

7 $y'' + 4y = \dfrac{1}{\cos 2x}$

1 rješavamo homogenu

$y'' + 4y = 0$ $\qquad$ $r = \alpha \pm \beta i = \pm 2i$

$r^2 + 4 = 0$ $\qquad$ $\alpha = 0,\ \beta = 2$

$r^2 = -4 \ / \pm\sqrt{\ }$

$r = \pm\sqrt{-4} = \pm\sqrt{4}\sqrt{-1} = \pm 2i$ rj. su konjugirano-kompleksna

$y_h = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

$$\boxed{y_h = C_1 \cos 2x + C_2 \sin 2x}$$

2 tražimo partikularna rješenja y_p

- imamo samo 1 partikularno jer je funkcija smetnje $\frac{1}{\cos 2x}$ sastoji od 1 člana
- također pošto je 1 član koristimo metodu varijacije konstanti
- da su 2 ili više onda koristimo algoritam sa strane 104. (4.5 u podsjetniku, .pdf)

metoda varijacije konstanti (4.3 podsjetnik, str. 10)

str. $\boxed{16}$

1. $C = C(x)$

$$y_p = \overset{A}{\overset{\parallel}{C_1(x)}}\cos 2x + \overset{B}{\overset{\parallel}{C_2(x)}}\sin 2x$$

$y_p = A\cos 2x + B\sin 2x \Rightarrow$ oblik partik. rješ.

$$A'\cos 2x + B'\sin 2x = 0 \qquad \boxed{n=2}$$
$$A'(\cos 2x)^{(n-1)} + B'(\sin 2x)^{(n-1)} = \frac{1}{\cos 2x}$$

- e sad, kak smo ovaj sustav dobili?
- $n=2$ jer oblik rješenja ima 2 člana
- ~~ako~~ je $n=2$ onda ovo treba imati 2 reda, tj. sustav treba imati 2 jednadžbe
- od prve do ~~pre~~ zadnje jednadžbe s desne strane je uvijek 0
- ~~sam~~ u zadnjoj jednadžbi s desne strane je funkcija smetnje $f(x)$, u našem slučaju $\frac{1}{\cos 2x}$
- u ~~svim~~ redovima s lijeve strane uvijek idu konstante derivirane na prvu
- u 1. redu funkcije uz konstante se ne deriviraju
- u 2. -||- -||- -||- se deriviraju ~~po~~ na $(n-1)$
- u 3. redu se deriviraju na $(n-2)$, itd...

$$A'\cos 2x + B'\sin 2x = 0$$
$$A'(\cos 2x)' + B'(\sin 2x)' = \frac{1}{\cos 2x}$$

$$A'\cos 2x + B'\sin 2x = 0$$
$$-2A'\sin 2x + 2B'\cos 2x = \frac{1}{\cos 2x} \Rightarrow A' = \ldots$$

$\Rightarrow$

$$A' = \frac{-B' \sin 2x}{\cos 2x}$$ (uvrstimo u 2. jednadžbu)

$$2B' \frac{\sin^2 2x}{\cos 2x} + 2B' \cos 2x = \frac{1}{\cos 2x} \quad / \cdot \cos 2x$$

$$2B' \sin^2 2x + 2B' \cos^2 2x = 1$$

$$2B' = 1$$

$$\boxed{B' = \frac{1}{2}}$$

$$\boxed{A' = \frac{-1}{2} tg\, 2x}$$

$$A' = \frac{-tg\, 2x}{2} \quad / \int$$

$$A = \frac{-1}{2} \int tg\, 2x \, dx = \frac{-1}{2} \int \frac{\sin 2x}{\cos 2x} dx = \left| \begin{matrix} \cos 2x = u \\ -2\sin 2x \, dx = du \\ dx = \frac{du}{-2\sin 2x} \end{matrix} \right| =$$

$$= \frac{-1}{2} \int \frac{\cancel{\sin 2x}}{u} \cdot \frac{du}{-2\cancel{\sin 2x}} = \frac{1}{4} \int \frac{1}{u} du = \frac{1}{4} \ln|\cos 2x| + C_1$$

$$B' = \frac{1}{2} \quad / \int$$

$$B = \frac{1}{2} \int dx = \frac{x}{2} + C_2$$

$$\boxed{y_p = \left(\frac{1}{4} \ln|\cos 2x| + C_1\right) \cos 2x + \left(\frac{x}{2} + C_2\right) \sin 2x}$$

RJEŠENJE:

$$y = y_h + y_p$$

$$= C_1 \cos 2x + C_2 \sin 2x + \left(\frac{1}{4} \ln|\cos 2x| + C_3\right) \cos 2x + \left(\frac{x}{2} + C_4\right) \sin 2x$$