ELEKTROMAGNETSKA POLJA

- *Riješeni svi zadatci za vježbu, kronološki poredani*
- *Točnost potvrđena i usklađena s ostalim dostupnim rješenjima na FER2 te od Trkulje osobno ☺*
- *Skenirane stranice iz bilježnice, stoga se unaprijed ispričavam na (eventualno) nečitkom rukopisu*
- *Sva pitanja slobodno na PM*
- *Usput, položio sam predmet i bilježnica je pomogla nekolicini kolega u spremanju ispita - stoga i motivacija za dijeljenje s budućim generacijama*

uredio:

Schmendrick

ELEKTRO-MAGNETSKA POLjA

SADRŽAJ:

1. ZADACI ZA VJEŽBU

- VEKTORSKA ANALIZA
- ELEKTROSTATIKA
- MAGNETOSTATIKA
- ELEKTROMAGNETIZAM

1. VEKTORSKA ANALIZA

1.1.

$$\vec{r} = 2\vec{a}_x + 2\vec{a}_y + 2\vec{a}_z$$

$$\vec{r}' = 4\vec{a}_x + 3\vec{a}_y + 3\vec{a}_z$$

$$\boxed{\vec{R}'} = \vec{r} - \vec{r}' = \boxed{-2\vec{a}_x - \vec{a}_y - \vec{a}_z}$$

$$\boxed{\vec{a}_r =} \frac{\vec{R}}{|\vec{R}|} = \frac{-2\vec{a}_x - \vec{a}_y - \vec{a}_z}{\sqrt{2^2+1^2+1^2}} = \boxed{-\sqrt{\frac{2}{3}}\,\vec{a}_x - \frac{1}{\sqrt{6}}\,\vec{a}_y - \frac{1}{\sqrt{6}}\,\vec{a}_z}$$

1.2. $T_{KART}(2, 1, 3)$

CILINDRIČNE KOORDINATE: $\boxed{T_{CIL}(2.236,\ 0.464,\ 3)}$

$$\boxed{\rho} = \boxed{\sqrt{x^2+y^2}} = \sqrt{4+1} = \sqrt{5} = \boxed{2.236}$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{1}{2} \Rightarrow \phi = 26.56505^\circ$$

$$\phi[rad] = \phi[^\circ] \cdot \frac{\pi}{180} \Rightarrow \boxed{\phi[rad] = 0.464}$$

$$\boxed{z = 3}$$

SFERNE KOORDINATE: $\boxed{T_{SFE}(3.742,\ 0.641,\ 0.464)}$

$$\boxed{\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}} = \cos^{-1}\frac{3}{\sqrt{4+1+9}} = \cos^{-1}\frac{3}{\sqrt{14}}$$

$$\theta = 36.69922[^\circ] \Rightarrow \boxed{\theta = 0.641\,[rad]}$$

$$\boxed{r =}\ \boxed{\sqrt{x^2+y^2+z^2}} = \sqrt{14} = \boxed{3.742} \qquad \phi_{CIL} = \phi_{SFE}$$

$\boxed{1.3.}$ $\vec{A} = yz\,\vec{a_x} - 2x\,\vec{a_y} + \vec{a_z}$ $\qquad T(3,1,1)$

$\vec{B} = \vec{a_x} + z\,\vec{a_y}$

a) $\vec{A} - \vec{B} = (yz-1)\,\vec{a_x} + (-2x-z)\,\vec{a_y} + \vec{a_z}$

u $T(3,1,1) \Rightarrow \boxed{\vec{A} - \vec{B} = -7\vec{a_y} + \vec{a_z}}$

b) $\vec{A} + \vec{B} = (yz+1)\,\vec{a_x} + (-2x+z)\,\vec{a_y} + \vec{a_z}$

u $T(3,1,1) \Rightarrow \boxed{\vec{A} + \vec{B} = 2\vec{a_x} - 5\vec{a_y} + \vec{a_z}}$

c) $\vec{A} \cdot \vec{B} = yz\,\overbrace{\vec{a_x} \cdot \vec{a_x}}^{1} + (-2x \cdot z)\,\overbrace{\vec{a_y} \cdot \vec{a_y}}^{1}$

u $T(3,1,1) \Rightarrow \boxed{\vec{A} \cdot \vec{B}} = yz - 2xz = 1 - 6 = \boxed{-5}$

d) $\vec{A} \times \vec{B} = yz \cdot z\,\vec{a_x} \times \vec{a_y} - 2x\,\vec{a_y} \times \vec{a_x} + \vec{a_z} \times \vec{a_x}$

$+ z\,\vec{a_z} \times \vec{a_y} = yz^2\,\vec{a_z} + 2x\,\vec{a_z} + \vec{a_y} - z\,\vec{a_x}$

u $T(3,1,1) \Rightarrow \boxed{\vec{A} \times \vec{B} = -1\,\vec{a_x} + \vec{a_y} + 7\vec{a_z}}$

$\boxed{1.4.}$ $\boxed{dV = r^2 \sin\theta \, dr \, d\theta \, d\alpha}$ SFERNI KOORDINATNI SUSTAV

za r=konst. $dS = r^2 \sin\theta \, d\theta \, d\alpha$

$$P = \iint_S r^2 \sin\theta \, d\theta \, d\alpha = r^2 \int_{\pi/3}^{\pi/2} \sin\theta \, d\theta \int_0^{2\pi} d\alpha$$

$$P = 1.5^2 \left(-\cos\theta \Big|_{\theta=\pi/3}^{\pi/2}\right) 2\pi = 2\pi \cdot 1.5^2 \Big(\underbrace{-\cos\frac{\pi}{2}}_{0} + \underbrace{\cos\frac{\pi}{3}}_{\frac{1}{2}}\Big)$$

$\boxed{P = 7.07\ m^2}$

$\boxed{1.5.}$ $\boxed{dV = r\,dr\,d\alpha\,dz}$

CILINDRIČNI KOORDINATNI SUSTAV

$$V = \int_1^3 r\,dr \int_{\pi}^{3\pi/2} d\alpha \int_1^2 dz = \frac{r^2}{2}\Big|_{r=1}^{3} \; \alpha\Big|_{\alpha=\pi}^{3\pi/2} \; z\Big|_{z=1}^{2}$$

$$V = \frac{3^2-1}{2} \cdot \left(\frac{3\pi}{2} - \pi\right) \cdot 1 = 4 \cdot \frac{\pi}{2} = 6.28\ m^3$$

$\boxed{V = 6.28\ m^3}$

1.6.

$\vec{A} = 2\vec{a}_x - 3\vec{a}_y + 2\vec{a}_z$

$\vec{B} = \vec{a}_x + 2\vec{a}_y + \vec{a}_z$

PROJEKCIJA VEKTORA $\vec{B}$ NA VEKTOR $\vec{A}$ JE UMNOŽAK $\vec{B} \cdot \vec{A}_0$, GDJE JE $\vec{A}_0 = \dfrac{\vec{A}}{|\vec{A}|}$

$$A_0 = \frac{2\vec{a}_x - 3\vec{a}_y + 2\vec{a}_z}{\sqrt{2^2+3^2+2^2}} = 0.48507\,\vec{a}_x - 0.7276\,\vec{a}_y + 0.48507\,\vec{a}_z$$

$\vec{B} \cdot \vec{A}_0 = 0.48507 - 2 \cdot 0.7276 + 0.48507$

$\boxed{\vec{B} \cdot \vec{A}_0 = -0.485}$

$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos\measuredangle(\vec{A}, \vec{B})$

$$\cos\measuredangle(\vec{A}, \vec{B}) = \frac{-2}{\sqrt{17} \cdot \sqrt{6}} \Rightarrow \boxed{\measuredangle(\vec{A}, \vec{B}) = 101.42^\circ}$$

1.7.

$|\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \cdot |\sin\measuredangle(\vec{A}, \vec{B})|$

$$|\vec{A} \times \vec{B}| = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 2 & -3 & 2 \\ 1 & 2 & 1 \end{vmatrix} = |-7\vec{a}_x + 7\vec{a}_z| = \sqrt{98}$$

$$|\sin\varphi| = \frac{\sqrt{98}}{\sqrt{102}} = 0.980196058$$

$\sin\varphi = \pm 0.980196058$

$\varphi_1 = 78.57824^\circ$

$\boxed{\varphi_2 = 101.42176^\circ}$ – provjeri se skiciranjem

1.8. $\vec{A} = (y+2)\vec{a_x} + (x-1)\vec{a_y}$ u $T(7,5,1)$

$\boxed{\vec{A}(T)} = (5+2)\vec{a_x} + (7-1)\vec{a_y} = \boxed{7\vec{a_x} + 6\vec{a_y}}$

$\vec{A}\cdot\vec{B_0} \Rightarrow \vec{B_0} = \frac{2}{\sqrt{17}}\vec{a_x} - \frac{3}{\sqrt{17}}\vec{a_y} + \frac{2}{\sqrt{17}}\vec{a_z}$

$$\boxed{\vec{A}\cdot\vec{B_0} = -\frac{4}{\sqrt{17}}}$$

1.9. $dS = r\,d\alpha\,dz$

$P = r\int_{\frac{10\pi}{180}}^{\frac{100\pi}{180}} d\alpha \int_0^2 dz = 3\cdot\frac{90\pi}{180}\cdot 2$

$$\boxed{P = 3\pi\ m^2}$$

1.10. $\vec{A} = 5\vec{a_x}$, $\vec{B} = 4\vec{a_x} + B_y\vec{a_y}$ $B_y = ?$ $\sphericalangle \vec{A},\vec{B} = 45^\circ$

$\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|\cos 45^\circ$

$20 = 5\sqrt{16+B_y^2}\cdot\frac{\sqrt{2}}{2} \,/\cdot 2$

$40 = 5\sqrt{2}\sqrt{16+B_y^2} \,/:5$

$8 = \sqrt{32+2B_y^2} \,/^2 \Rightarrow 64 = 32 + 2B_y^2$ $B_y^2 = 16$ $\boxed{B_y = \pm 4}$

1.11. $T_1(2,-5,-2)$, $T_2(14,-5,3)$

$\vec{r_1} = 2\vec{a_x} - 5\vec{a_y} - 2\vec{a_z}$ $\vec{r_2} = 14\vec{a_x} - 5\vec{a_y} + 3\vec{a_z}$

$\vec{R} = \vec{r_2} - \vec{r_1} = 12\vec{a_x} + 5\vec{a_z}$ $|\vec{R}| = \sqrt{169} = 13$

$\boxed{\vec{R_0}} = \frac{\vec{R}}{|\vec{R}|} = \boxed{\frac{12}{13}\vec{a_x} + \frac{5}{13}\vec{a_z}}$

1.12. $\vec{A} = \vec{u_x} + \pi \vec{u_\alpha} + 3\vec{u_z}$

$\vec{B} = \alpha \vec{u_x} + \beta \vec{u_\alpha} - 6\vec{u_z}$

UVJET ZA PARALELNOST VEKTORA:

$\sin \sphericalangle \vec{A}, \vec{B} = 0$

$|\vec{A} \times \vec{B}| = 0$

$$\begin{vmatrix} \vec{u_x} & \vec{u_\alpha} & \vec{u_z} \\ 1 & \pi & 3 \\ \alpha & \beta & -6 \end{vmatrix} = \vec{u_x}(-6\pi - 3\beta) - \vec{u_\alpha}(-6 - 3\alpha) + \vec{u_z}(\beta - \alpha\pi) = 0$$

$-6\pi - 3\beta = 0$

$-3\beta = 6\pi$

$\boxed{\beta = -2\pi}$

$-6 - 3\alpha = 0$

$-3\alpha = 6$

$\boxed{\alpha = -2}$

$\beta - \alpha\pi = 0$

$-\alpha\pi = -\beta$

$\boxed{\alpha = \frac{\beta}{\pi}}$

1.13. $\vec{A} = \vec{u_x} + 2\vec{u_y} - \vec{u_z}$

$\vec{B} = \alpha\vec{u_x} + \vec{u_y} + 3\vec{u_z}$

UVJET ZA OKOMITOST VEKTORA:

$\cos \sphericalangle \vec{A}, \vec{B} = 0$

$\vec{A} \cdot \vec{B} = 0$

$\vec{A} \cdot \vec{B} = 0 \Rightarrow \alpha + 2 - 3 = 0 \Rightarrow \boxed{\alpha = 1}$

1.14. $\varphi = 2xy - 5z$

$\nabla\varphi = \frac{\partial\varphi}{\partial x}\vec{u_x} + \frac{\partial\varphi}{\partial y}\vec{u_y} + \frac{\partial\varphi}{\partial z}\vec{u_z}$

$\nabla\varphi = 2y\vec{u_x} + 2x\vec{u_y} - 5\vec{u_z}$

$\boxed{\nabla\varphi(1,2,3) = 4\vec{u_x} + 2\vec{u_y} - 5\vec{u_z}}$

$\boxed{1.15.}$ $\vec{E} = \frac{50}{r}\vec{a_r} - 4\vec{a_z}$ $\vec{a_E}$ u $T(10, 20°, 2)$

$\vec{E}(T) = 5\vec{a_r} - 4\vec{a_z}$

$\alpha = \tan^{-1}\frac{y}{x} \;/ \tan$

$\tan 20° = \frac{y}{x} \Rightarrow y = 0.36397 \cdot x$

$r = 5 \Rightarrow 5 = \sqrt{x^2 + y^2} \Rightarrow 5 = \sqrt{x^2 + 0.13247x^2}$

$5 = \sqrt{1.13247x^2} \;/^2$ $\quad 25 = 1.13247x^2$

$x^2 = \frac{25}{1.13247}$ $\quad \boxed{x = 4.69846}$

$\boxed{y = 1.71009}$

$|\vec{E}| = \sqrt{x^2 + y^2 + z^2}$ $\quad \boxed{z = -4}$

$|\vec{E}| = 6.40312$

$$\boxed{\vec{E_0} = 0.73377\vec{a_x} + 0.26707\vec{a_y} - 0.625\vec{a_z}}$$

$\boxed{1.16.}$ $|\vec{E}| = 10$ $\quad \frac{50^2}{r^2} = 84$

$\sqrt{\left(\frac{50}{r}\right)^2 + 16} = 10 \;/^2$ $\quad \boxed{r}^2 = \frac{50^2}{84} \boxed{= 5.45}$

$\boxed{1.17.}$ $\vec{A} = 10\vec{a_r} - 3\vec{a_\theta} + 5\vec{a_\alpha}$ $\quad |\vec{A}| = \sqrt{134} = 11.57583$

$\boxed{\vec{B} = 2\vec{a_r} + 5\vec{a_\theta} + 3\vec{a_\alpha}}$

$\vec{A_0} = \frac{\vec{A}}{|\vec{A}|} = 0.86386\vec{a_r} - 0.25916\vec{a_\theta} + 0.43193\vec{a_\alpha}$

$\vec{B} \cdot \vec{A_0} = 2 \cdot 0.86386 - 5 \cdot 0.25916 + 3 \cdot 0.43193$

$$\boxed{\vec{B} \cdot \vec{A_0} = 1.72771}$$

1.18. $\vec{n} = \vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & \vec{a}_\alpha \\ 10 & -3 & 5 \\ 2 & 5 & 3 \end{vmatrix} = \vec{a}_r(-34) - \vec{a}_\theta(20) + \vec{a}_\alpha(56)$

$$\vec{n}_0 = \frac{\vec{n}}{|\vec{n}|} = \frac{-34\vec{a}_r - 20\vec{a}_\theta + 56\vec{a}_\alpha}{\sqrt{1256 + 900 + 3136}}$$

$$\boxed{\vec{n}_0 = -0.496\,\vec{a}_r - 0.292\,\vec{a}_\theta + 0.818\,\vec{a}_\alpha}$$

1.19. $\vec{A}_{KAR} = (x^2 - y^2)\,\vec{a}_y + (x \cdot z)\,\vec{a}_z$

$P(6, 60^\circ, -4)$

$6 = \sqrt{x^2 + y^2}$ $\quad \frac{60\pi}{180} = \tan^{-1}\frac{y}{x}$

$6 = \sqrt{4x^2}$ $\quad y = 1.73205\,x$ $\quad \boxed{y = 5.19615}$

$\boxed{x = 3}$

$A_y = -18$ $\quad \boxed{A_z = -12}$

$\boxed{A_\rho} = -18 \cdot \vec{a}_y \cdot \vec{a}_\rho = -18 \cdot \sin\phi = -18 \cdot \frac{\sqrt{3}}{2} = \boxed{-9\sqrt{3}}$

$\boxed{A_\alpha} = -18 \cdot \vec{a}_y \cdot \vec{a}_\alpha = -18 \cos\phi = -18 \cdot \frac{1}{2} = \boxed{-9}$

$$\boxed{\vec{A}_{CIL} = -9\sqrt{3}\,\vec{a}_r - 9\,\vec{a}_\alpha - 12\,\vec{a}_z}$$

1.20. $dQ = \rho\, dV \;/\!\!/ \int$ $\quad r = 0.2\,m,\ \rho = \frac{1}{\sqrt{x^2 + y^2}}$

$$Q = \iiint_V \frac{1}{\sqrt{x^2 + y^2}}\, r^2 \sin\theta\, dr\, d\theta\, d\alpha$$

SFERNE KOORDINATE:

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$x^2 + y^2 = r^2 \sin^2\theta\, \underbrace{(\sin^2\phi + \cos^2\phi)}_{1}$

$\sqrt{x^2 + y^2} = r \sin\theta$

$$Q = \frac{1}{\cancel{r \cdot \sin\theta}} \int_0^{0.2} \cancel{r^2} \cdot \cancel{\sin\theta}\, dr \int_0^{\pi} d\theta \int_0^{2\pi} d\alpha$$

$$Q = \frac{1}{\cancel{2}} r^2 \Big|_{r=0}^{0.2} \cdot \cancel{2}\pi \cdot \pi$$

$$Q = 0.2^2 \cdot \pi^2$$

$$\boxed{Q = 0.39478\ C}$$

1.21. $f(x,y,z) = 5x + 10x\cdot z - x\cdot y + 6$

$$\nabla f = \frac{\partial f}{\partial x}\vec{a_x} + \frac{\partial f}{\partial y}\vec{a_y} + \frac{\partial f}{\partial z}\vec{a_z}$$

$$\nabla\cdot f = (5+10z-y)\vec{a_x} + (-x)\vec{a_y} + 10x\,\vec{a_z}$$

1.22. $f(x,y,z) = 2\sin(\alpha) - r\cdot z + 4$

$$\nabla f = -z\vec{a_r} + \frac{2}{r}\cos\alpha\,\vec{a_\alpha} - r\vec{a_z}$$

1.23. $f(x,y,z) = 2r\cos\theta - 5\alpha + 2$

$$\nabla f = 2\cos\theta\,\vec{a_r} + (-2\sin\theta)\vec{a_\theta} - \frac{5}{r\sin\theta}\vec{a_\alpha}$$

1.24. $f(x,y,z) = x\cdot y + 2z^2$

$$\nabla f = y\vec{a_x} + x\vec{a_y} + 4z\vec{a_z}$$

$$\nabla f(1,1,1) = \vec{a_x} + \vec{a_y} + 4\vec{a_z}$$

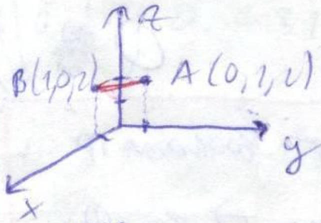
$$\vec{A} = \vec{a_x} - 2\vec{a_y} + \vec{a_z} \qquad |\vec{A}| = \sqrt{1+4+1} = \sqrt{6}$$

$$\vec{A_0} = \frac{1}{\sqrt6}\vec{a_x} - \frac{2}{\sqrt6}\vec{a_y} + \frac{1}{\sqrt6}\vec{a_z}$$

$$\boxed{\nabla f\cdot\vec{A_0} =} \frac{1}{\sqrt6} - \frac{2}{\sqrt6} + \frac{4}{\sqrt6} = \boxed{\frac{3}{\sqrt6}}$$

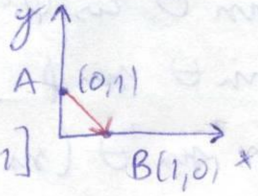
1.25. $\vec{F}(x,y,z) = (x+y)\vec{a_x} - x\vec{a_y} + z\vec{a_z}$

od $A(0,1,2)$ do $B(1,0,2)$



$$I = \int_A^B \vec{F}\cdot d\vec{r} = \int_A^B \left[(x+y)dx - x\,dy + z\,dz\right]$$

PARAMETRIZACIJA DUŽINE $\overline{AB}$:

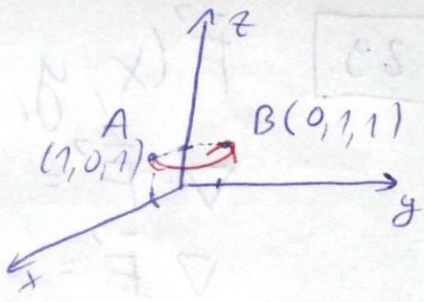


$$\left.\begin{array}{l} x = t \longleftrightarrow dx = dt \\ y = 1-t \longleftrightarrow dy = -dt \\ z = 2 \longleftrightarrow dz = 0 \end{array}\right\} t\in[0,1]$$

$$I = \int_0^1 (t+1-t)\,dt - t(-dt) + 2\cdot 0 = \int_0^1 (1+t)\,dt =$$

$$I = t\Big|_0^1 + \frac{t^2}{2}\Big|_0^1 = 1 + \frac{1}{2} = \frac{3}{2} = 1.5$$

1.26. $\vec{F}(x,y,z) = \vec{a_x} + 2\vec{a_y} + \vec{a_z}$



$x = \cos t$

$y = \sin t \qquad t \in \left(0, \frac{\pi}{2}\right)$

$z = 1$

$dx = -\sin t \, dt$

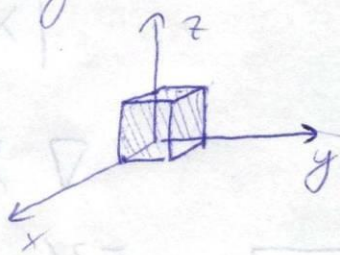
$dy = \cos t \, dt$

$dz = 0$

$$I = \int_A^B dx + 2dy + dz = \int_0^{\frac{\pi}{2}} (-\sin t \, dt + 2\cos t \, dt)$$

$$\boxed{I =} (\cos t + 2\sin t)\Big|_{t=0}^{\frac{\pi}{2}} = (0 - 1 + 2 - 0) = \boxed{1}$$

1.27. $\vec{F}(x,y,z) = 2x^2 y \, \vec{a_x} + z \, \vec{a_y} + y \, \vec{a_z}$



$$\boxed{\oiint_S \vec{F} \cdot \vec{n} \, dS = \iiint_V \operatorname{div} \vec{F} \, dx\,dy\,dz}$$

$$\Phi = \int_V (4xy + 0 + 0) \, dx\,dy\,dz$$

$$\boxed{\Phi =} \, 4 \int_0^1 x \, dx \int_0^1 y \, dy \int_0^1 dz = 4 \left(\frac{x^2}{2} \cdot \frac{y^2}{2} \, z\right)\Big|_0^1 = \boxed{1}$$

1.28. $\vec{F}(x,y,z) = e^{-\alpha y}(\cos(3x)\,\vec{a_x} - \sin(3x)\,\vec{a_y})$

$$\nabla \vec{F} = -e^{-\alpha y} \sin(3x) \cdot 3 - (-\alpha \, e^{-\alpha y} \sin(3x))$$

$$\boxed{\nabla \vec{F} = e^{-\alpha y} \sin 3x \, (-3 + \alpha)}$$

1.29. $\vec{F}(x,y,z) = 5x^2 \cdot \sin(\pi \cdot x)\vec{a_x}$ $T(x=0.5)$

$$\nabla \cdot \vec{F} = 5 \cdot 2x \cdot 1 \cdot \sin(\pi x) + 5x^2 \cos(\pi x) \cdot \pi$$

$$\nabla \cdot \vec{F} = 10x \sin(\pi x) + 5x^2 \cdot \pi \cdot \cos(\pi x)$$

$$\nabla \cdot \vec{F}(T) = \underbrace{5 \cdot \sin\frac{\pi}{2}}_{1} + \underbrace{\frac{5}{4}\pi \cdot \cos\frac{\pi}{2}}_{0}$$

$$\nabla \cdot \vec{F}(T) = 5$$

1.30. $\vec{F}(x,y,z) = x y \cdot \vec{a_x} + 2y \cdot z \cdot \vec{a_y} - \vec{a_z}$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & -1 \end{vmatrix} = \vec{a_x}(0-2y) - \vec{a_y}(0-0) + \vec{a_z}(0-x)$$

$$\nabla \times \vec{F} = -2y\,\vec{a_x} - x\,\vec{a_z}$$

1.31. $\vec{F}(r,\alpha,z) = 2\vec{a_r} + \sin(\alpha)\vec{a_\alpha} - z \cdot \vec{a_z}$

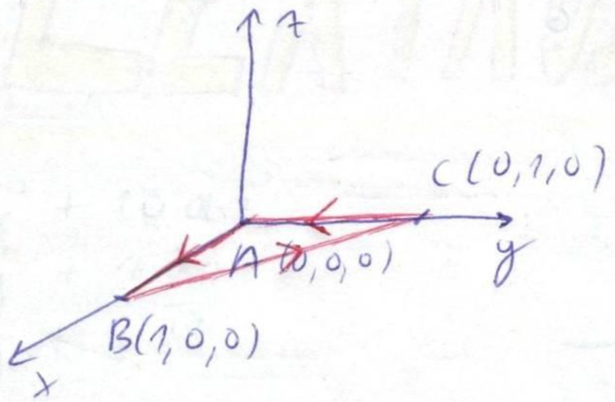
$$\boxed{\nabla \times \vec{F} = \left(\frac{1}{r} \cdot \frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z}\right)\vec{a_r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{a_\alpha} + \frac{1}{r}\left(\frac{\partial}{\partial r}(r \cdot A_\alpha) - \frac{\partial A_r}{\partial \alpha}\right)\vec{a_z}}$$

$$\nabla \times \vec{F} = \left(\frac{1}{r} \cdot 0 - 0\right)\vec{a_r} + (0-0)\vec{a_\alpha} + \frac{1}{r}(\sin\alpha - 0)\vec{a_z} =$$

$$\boxed{\nabla \times \vec{F} = \frac{1}{r}\sin\alpha \cdot \vec{a_z}}$$

1.32. $\vec{F}(x, y, z) = x \cdot y\, \vec{a_x} + 2\vec{a_y}$

$A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$



$\overline{AB}$ $\quad t = x \quad t \in (0,1) \quad dx = dt$

$y = 0 \quad dy = 0$

$z = 0 \quad dz = 0$

$$\boxed{\oint \vec{F}\, d\vec{r} = \oint xy\, dx + 2dy}$$

$$I_1 = \int_A^B 0$$

$\overline{BC}$ $\quad x = -t + 1 \quad t \in (0,1) \quad dx = -dt$

$y = t \quad dy = dt$

$z = 0 \quad dz = 0$

$$= \int_B^C \left[(-t+1) \cdot t\,(-dt) + 2dt\right]$$

$$= \left[\int_0^1 (t^2 - t)\, dt + 2\int_0^1 dt\right] = \left(\frac{1}{3} t^3 - \frac{1}{2} t^2\right)\Big|_0^1 + 2$$

$$\boxed{I_2 =} \frac{2 - 1 + 12}{6} = \boxed{\frac{13}{6}}$$

$\overline{CA}$ $\quad x = 0 \quad t \in (1,0) \quad dx = 0$

$y = t \quad dy = dt$

$z = 0 \quad dz = 0$

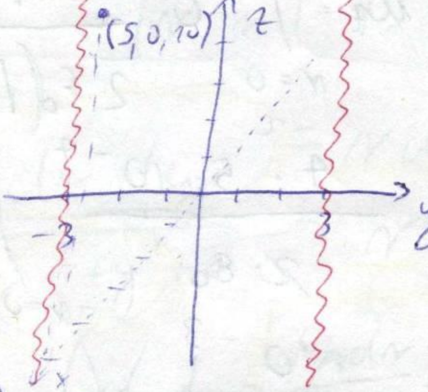
$$\boxed{I_3 =} \int_C^A \left[0 \cdot t \cdot 0 + 2dt\right] = 2\int_1^0 dt = \boxed{-2}$$

$$\oint \vec{F}\cdot d\vec{r} = I_1 + I_2 + I_3$$
$$= 0 + \frac{13}{6} - 2$$

$$\boxed{\oint \vec{F}\cdot d\vec{r} = \frac{1}{6}}$$

II. ELEKTROSTATIKA

2.1.



$$\vec{r} = 5\vec{a_x} + 10\vec{a_z}$$

$$\vec{r_1'} = 3\vec{a_y} + z\vec{a_z}$$

$$\vec{r_2'} = -3\vec{a_y} + z\vec{a_z}$$

$$dl = dz$$

$$\lambda = 2 \cdot 10^{-9}\ \frac{C}{m}$$

$$\vec{R_1} = \vec{r} - \vec{r_1'} = 5\vec{a_x} - 3\vec{a_y} + (10-z)\vec{a_z}$$

$$\vec{E_1} = \frac{1}{4\pi\varepsilon_0}\int_z \frac{\lambda\ \vec{R_1}}{|\vec{R_1}|^3}\, dz$$

$$\vec{E_1} = \frac{2\cdot 10^{-9}}{4\pi\varepsilon_0}\int_z \frac{\left[5\vec{a_x} - 3\vec{a_y} + (10-z)\vec{a_z}\right] dz}{\left(\sqrt{z^2 - 20z + 134}\right)^3}$$

$$E_{1x} = \frac{10^{-8}}{4\pi\varepsilon_0}\int_z \frac{dz}{\left(\sqrt{z^2-20z+134}\right)^3}$$

$$E_{1x} = \frac{10^{-8}}{4\pi\varepsilon_0}\cdot\left(\frac{2z-20}{68\cdot\sqrt{z^2-20z+134}}\right)\Bigg|_{-\infty}^{+\infty}$$

$$E_{1x} = \frac{10^{-8}}{4\pi\varepsilon_0}\cdot\left(\frac{2\cdot\infty-20}{68\sqrt{\dots}} - \frac{2(-\infty)-20}{68\sqrt{\dots}}\right)$$

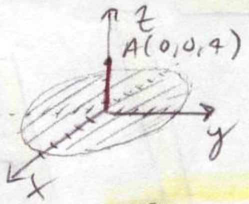
$$E_{1x} = \frac{10^{-8}}{4\pi\varepsilon_0}\cdot\frac{4}{68} = 5.2869$$

$$E_{2x} = E_{1x} \qquad \boxed{\vec{E_{uk}}} = 2\cdot E_{1x}\vec{a_x} = \boxed{10.57\vec{a_x}}$$

$E_{1y} = -E_{2y}$ — PONIŠTAVA SE ZBOG SIMETRIJE

$E_{1z} = \boxed{}\ E_{2z}$ — PONIŠTAVA SE ZBOG INTEGRACIJE

2.2 $\sigma = \frac{5 \cdot 10^{-4}}{r}$



$\lambda = \sigma \cdot dr$

$$\vec{E} = \vec{u}_z \int_{r=0}^{5} \sigma \frac{r \cdot z}{2\varepsilon_0 \left(\sqrt{r^2+z^2}\right)^3} \cdot dr \qquad \int \frac{dr}{\left(\sqrt{r^2+4^2}\right)^3} = \frac{r}{4^2\sqrt{r^2+4^2}}$$

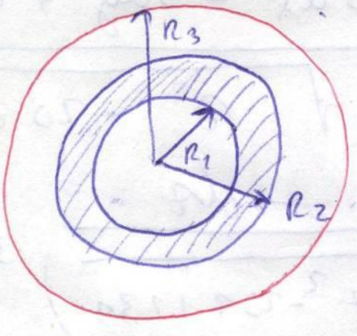
$$E_z = \frac{4 \cdot 5 \cdot 10^{-4}}{\cancel{2} \cdot \varepsilon_0} \int_{r=0}^{5} \frac{1}{\cancel{r}} \cdot \frac{\cancel{r}}{\left(\sqrt{r^2+4^2}\right)^3} dr = \frac{10^{-3}}{\varepsilon_0} \frac{1}{16} \left(\frac{r}{\sqrt{r^2+4^2}}\right)\Bigg|_{r=0}^{5}$$

$$E_z = \frac{10^{-3}}{\varepsilon_0} \cdot \left(\frac{5}{16\sqrt{5^2+4^2}} - 0\right) = \frac{10^{-3}}{\varepsilon_0} \cdot \frac{5}{16\sqrt{41}}$$

$$\boxed{E_z = 5.51211 \ \frac{MV}{m}}$$

2.3

$$\int \sin^2 \alpha \, d\alpha = \frac{\alpha}{2} - \frac{1}{4} \sin 2\alpha$$



$R_1 = 1m$, $R_2 = 2m$, $R_3 = 3m$

$\rho = \frac{5 \sin^2(\alpha)}{r^2} \left[\frac{C}{m^3}\right]$

$dV = r^2 \sin\theta \, dr \, d\theta \, d\alpha$

$$\Phi = \oint\!\!\oint \vec{D} \, \vec{n} \, dS = \int_V \rho \, dV = \int_V \frac{5 \sin^2(\alpha)}{\cancel{r^2}} \cancel{r^2} \sin\theta \, dr \, d\theta \, d\alpha$$

$$\Phi = 5 \int_1^2 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} \sin^2(\alpha) \, d\alpha =$$

$$\Phi = 5 \cdot 2 \cdot \left(\frac{2\pi}{2} - \frac{1}{4}\sin 4\pi - \frac{0}{2} + \frac{1}{4}\sin 0\right)$$

$$\Phi = 5 \cdot 2 \cdot \pi$$

$$\Phi = 10\pi$$

$$\boxed{\Phi = 31{,}42 \ [C]}$$

2.4. $Q = 30\cdot10^{-9}[C]$ $\quad r_0 = 1m$

$$Q = \int_S \sigma\, dS = \sigma \cdot r_0^2\pi = \sigma\pi$$

$$\vec r = 2\vec a_z$$
$$\vec r' = x\vec a_x + y\vec a_y$$
$$|\vec r - \vec r'| = \sqrt{x^2+y^2+4}$$

$$\boxed{\sigma = \frac{30\cdot10^{-9}}{\pi}}$$

$$\varphi(\vec r) = \frac{1}{4\pi\varepsilon}\int_S \frac{\sigma\, dS}{|\vec r - \vec r'|}$$

$$\varphi = \frac{1}{4\pi\varepsilon}\cdot\frac{30\cdot10^{-9}}{\pi}\int_S \frac{dx\cdot dy}{\sqrt{x^2+y^2+4}} \qquad dx\,dy = r\,dr\,d\alpha, \quad x^2+y^2 \to r^2$$

$$\varphi = \frac{30\cdot10^{-9}}{4\pi^2\varepsilon}\int_0^1 \frac{r\,dr}{\sqrt{r^2+2^2}}\cdot\int_0^{2\pi} d\alpha \qquad \boxed{\int\frac{r\,dr}{\sqrt{r^2+a^2}} = \sqrt{r^2+a^2}}$$

$$\varphi = \frac{\overset{15}{\cancel{30}}\cdot10^{-9}}{\cancel{2}\,4\pi^2\varepsilon}\cdot\cancel{2\pi}\left(\sqrt{r^2+4}\right)\Big|_{r=0}^{1}$$

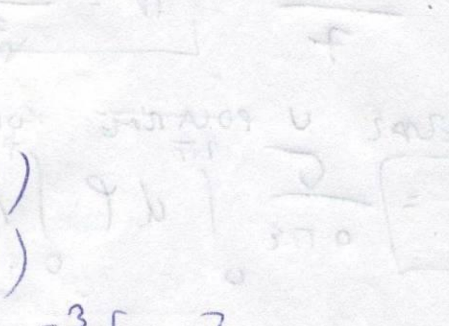
$$\varphi = \frac{15\cdot10^{-9}}{\pi\cdot\varepsilon}\left(\sqrt{1+4}-\sqrt{4}\right) = \frac{15\cdot10^{-9}}{\pi\varepsilon}\left(\sqrt5-\sqrt4\right)$$

$$\boxed{\varphi = 127{,}30309\,[V]}$$

2.5 $q_A = 300\cdot10^{-9}[C]$, $A(1,-1,3)$

$q_B = ?$ $\quad B(3,-3,2)$

$$\vec F_{BA} = (8\vec a_x - 8\vec a_y - 4\vec a_z)\cdot10^{-3}[N]$$



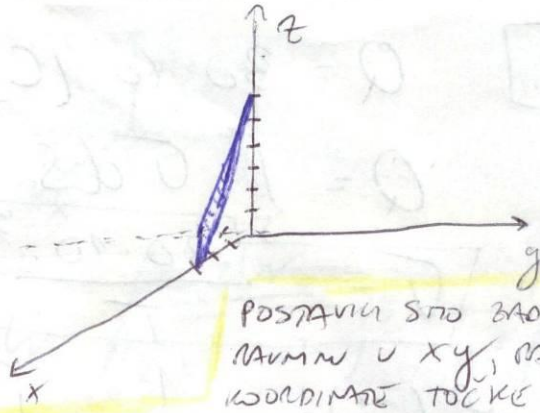
$$\vec r_A = \vec a_x - \vec a_y + 3\vec a_z$$
$$\vec r_B = 3\vec a_x - 3\vec a_y + 2\vec a_z$$
$$\vec R_{BA} = \vec r_A - \vec r_B = -2a_x + 2a_y + a_z$$
$$|\vec R_{BA}| = \sqrt9 = 3 \qquad |\vec R_{BA}|^3 = 27$$

$$\vec F_{BA} = \frac{Q_B\cdot Q_A}{4\pi\varepsilon}\cdot\frac{(-2\vec a_x + 2\vec a_y + \vec a_z)}{27}$$

$$(8\vec a_x - 8\vec a_y - 4\vec a_i)\,10^{-3} = \frac{Q_B\cdot300\cdot10^{-9}}{4\pi\varepsilon}\left(\frac{-2\vec a_x + 2\vec a_y + \vec a_z}{27}\right)$$

$$\boxed{Q_B} = \frac{8\cdot10^{-3}\cdot4\pi\cdot\varepsilon\cdot27}{-2\cdot300\cdot10^{-9}} = \frac{36\pi\varepsilon}{-25\cdot10^{-6}} = \boxed{-40.05\,\mu C}$$

$\boxed{2.6.}$ $\sigma = 0.3 \cdot 10^{-9} \left[\frac{C}{m^2}\right]$



$2x - 3y + z = 6$

$\vec{n}_{12} = 2\vec{a}_x - 3\vec{a}_y + \vec{a}_z$

$\vec{n} = \frac{\vec{n}_{12}}{|\vec{n}_{12}|} = \frac{2\vec{a}_x - 3\vec{a}_y + \vec{a}_z}{\sqrt{14}}$

POSTAVIMO ŠTO ZADANU RAVNINU U xy, RAČUNAMO KOORDINATE TOČKE KOJA PREDSTAVLJA UDALJENOST RAVNINE OD ISHODIŠTA

$\vec{N} = x\vec{a}_x + y\vec{a}_y$

$\vec{N} = a\,\vec{a}_z$

$\frac{x}{2} = \frac{y}{-3} = \frac{z}{1} = t$

$x = 2t \quad y = -3t \quad z = t$

UDALJENOST TOČKE T OD RAVNINE JE a

$2 \cdot 2t - 3(-3t) + t = 6$

$4t + 9 + t = 6$

$14t = 6$ $\quad \boxed{t = \frac{3}{7}}$

KOORDINATE TE TOČKE SU: $\boxed{T\left(\frac{6}{7}, -\frac{9}{7}, \frac{3}{7}\right)}$

$a = \sqrt{\left(\frac{6}{7}\right)^2 + \left(-\frac{9}{7}\right)^2 + \left(\frac{3}{7}\right)^2}$

$a = \frac{3\sqrt{14}}{7}$ $\quad \boxed{\vec{N} = \frac{3\sqrt{14}}{7}\vec{a}_z}$

$\vec{E} = \frac{1}{4\pi\varepsilon} \iint \sigma\, dS \frac{\frac{3\sqrt{14}}{7}\vec{a}_z - x\vec{a}_x - y\vec{a}_y}{\left(\sqrt{x^2 + y^2 + \frac{9 \cdot 14}{49}}\right)^3}$

PRIJELAZ U POLARNE KOORDINATE (ZANEMARUJEMO KOMPONENTE UZ x i y)

$\boxed{E_z} = \frac{\sigma}{4\pi\varepsilon} \int_0^{2\pi} d\varphi \int_0^{\infty} r\,dr \frac{\frac{3\sqrt{14}}{7}}{\left(\frac{9 \cdot 14}{49} + r^2\right)^{1.5}} = \boxed{\frac{-\sigma}{2\varepsilon}}$

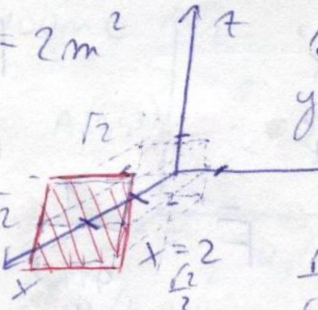
OVO JE IZNOS POLJA U z SMJERU POMAKNUTOG KOORDINATNOG SUSTAVA POTREBNO JE PREBACITI SE U POČETNI (normalni) KOORDINATNI SUSTAV. VEKTOR JAKOSTI POLJA MORA DJELOVATI U SMJERU VEKTORA OKOMITOG NA RAVNINU STOGA JE POTREBNO POMNOŽITI IZNOS POLJA E_z S JEDINIČNIM VEKTOROM U SMJERU VEKTORA ZADANE RAVNINE $\vec{n}$.

$\vec{E} = \frac{-\sigma}{2\varepsilon} \cdot \frac{2\vec{a}_x - 3\vec{a}_y + \vec{a}_z}{\sqrt{14}} = -16.94149 \frac{1}{\sqrt{14}}(2\vec{a}_x - 3\vec{a}_y + \vec{a}_z)$

$\vec{E} = -9{,}05561\vec{a}_x + 13.58341\vec{a}_y - 4.5278\vec{a}_z$

$\boxed{E_x = -9.05 \left[\frac{V}{m}\right]}$

$\boxed{2.7.}$ $\vec D = 10x^3\,\vec a_x \left[\frac{C}{m^2}\right]$ $\quad S = 2\,m^2$ $\quad z \in \left(-\frac{\sqrt2}{2}, \frac{\sqrt2}{2}\right)$, $\; y \in \left(-\frac{\sqrt2}{2}, \frac{\sqrt2}{2}\right)$



$$\Phi = \iint_S \vec D \cdot \vec n \cdot dS \qquad \vec n = \vec a_x \qquad dS = dy\,dz$$

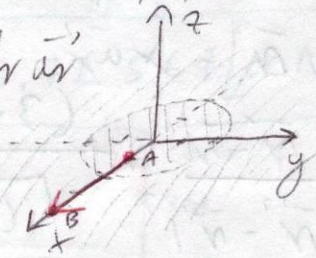
$$\Phi = \iint_S 10x^3\,\vec a_x \cdot \vec a_x \cdot dy\,dz = 10x^3\Big|_{x=2} \int_{-\frac{\sqrt2}{2}}^{\frac{\sqrt2}{2}} dy \int_{-\frac{\sqrt2}{2}}^{\frac{\sqrt2}{2}} dz$$

$$\boxed{\Phi} = 10 \cdot 2^3 \cdot \frac{\sqrt2}{2} \cdot 2 \cdot \frac{\sqrt2}{2} \cdot 2 = 10 \cdot 2^4 = \boxed{160\,C}$$

$\boxed{2.8.}$ $\vec E_1 = \frac{5}{r^2}\,\vec a_r \quad (0 < r \le 2) \qquad A(1,0,0)$

$\vec E_2 = \frac{2.5}{r}\,\vec a_r \quad (r > 2) \qquad B(4,0,0)$

$d\vec l = dr\,\vec a_r$



$$U_{AB} = \varphi(A) - \varphi(B) = \int_a^b \vec E\, d\vec l$$

$$U_{AB} = \int_1^2 \vec E_1\, dr + \int_2^4 \vec E_2\, dr$$

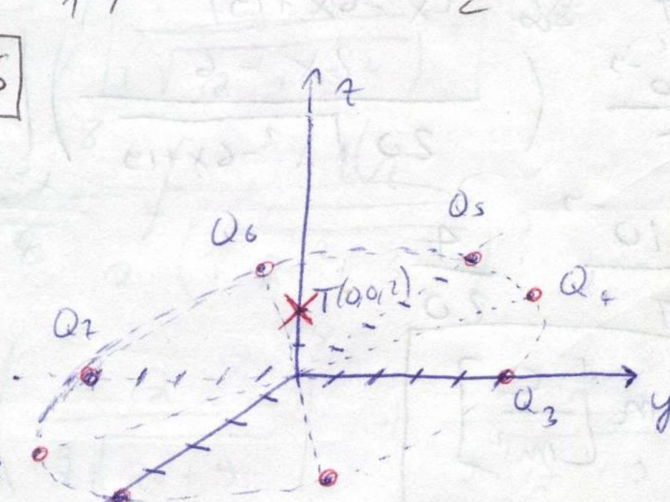
$$U_{AB} = \int_1^2 \frac{5}{r^2}\, dr + \int_2^4 \frac{2.5}{r}\, dr$$

$$U_{AB} = -5\left(\frac{1}{r}\right)\Big|_{r=1}^{2} + 2.5 \ln r \Big|_{r=2}^{4}$$

$$U_{AB} = -5\left(\frac12 - \frac11\right) + 2.5 \ln\frac42$$

$$\boxed{U_{AB} = 4.23286}$$

$\boxed{2.9.}$



$\vec{r}_1' = 5\vec{a_x}$

$\vec{r}_T = 2\vec{a_z}$

$|R_{1T}| = \sqrt{29}$

$\vec{R}_{1T} = 2\vec{a_z} - 5\vec{a_x}$

$Q_1 = Q_2 = \dots = Q_8 = 100\,nC$

$Q_T = 20\,mC$

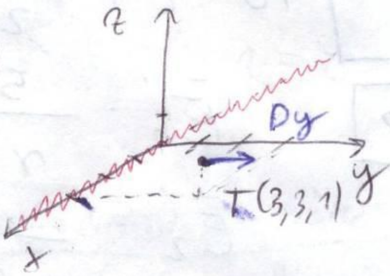
sile u x i y smjeru će se poništiti zbog simetrije

$$\vec{F} = \frac{1}{4\pi\varepsilon} Q_1 \cdot Q_2 \, \frac{2\vec{a_z} - 5\vec{a_x}}{(\sqrt{29})^3}$$

$$F_z = \frac{1}{4\pi\varepsilon} Q_1 \cdot Q_2 \cdot \frac{2}{29^{1.5}} = 2.3 \cdot 10^{-7}\,N$$

$$\boxed{F_{uk} = 8 \cdot F_z = 1.84\,[\mu N]}$$

$\boxed{2.10.}$ $\lambda = 5 \cdot 10^{-9} \left[\frac{C}{m}\right]$ $T(3,3,1)$



$\vec{r} = 3\vec{a_x} + 3\vec{a_y} + \vec{a_z}$

$\vec{r}' = x\vec{a_x}$

$\vec{D} = \varepsilon \cdot \vec{E}$

$$\frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} = \frac{(3-x)\vec{a_x} + 3\vec{a_y} + \vec{a_z}}{\sqrt{(3-x)^2 + 9 + 1}^{\,3}}$$

$$\vec{D} = \frac{1}{4\pi} \int_x \frac{\lambda \, dl \, (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{D} = \frac{5 \cdot 10^{-9}}{4\pi} \int_{-\infty}^{+\infty} \left[\frac{(3-x)\vec{a_x} + 3\vec{a_y} + \vec{a_z}}{(x^2 - 6x + 19)^{\frac{3}{2}}}\right] dx$$

$$D_y = \frac{15 \cdot 10^{-9}}{4\pi} \int_{-\infty}^{+\infty} \frac{dx}{(x^2 - 6x + 19)^{\frac{3}{2}}}$$

$$D_y = \frac{15 \cdot 10^{-9}}{4\pi} \left(\frac{2x - 6}{20\sqrt{x^2 - 6x + 19}} \right) \Bigg|_{x=-\infty}^{+\infty}$$

$$D_y = \frac{15 \cdot 10^{-9}}{4\pi} \cdot \frac{4}{20}$$

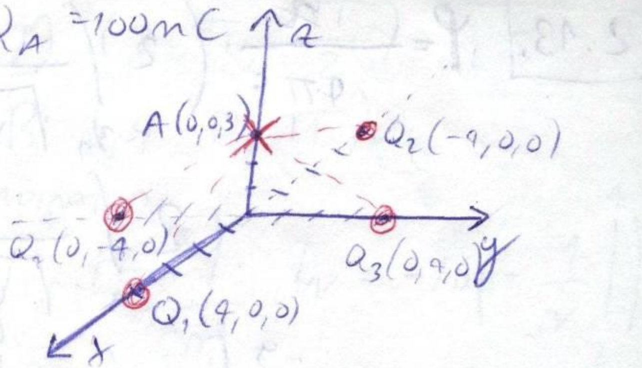
$$\boxed{D_y = 0.239\,n \left[\frac{C}{m^2}\right]}$$

$\boxed{2.11.}$ $\vec{r}_A' = 3\vec{a}_z$ $\quad Q_n = 20\,nC \quad Q_A = 100\,nC$

$\vec{r}_1' = 4\vec{a}_x$, $\vec{r}_2' = -4\vec{a}_x$

$\vec{r}_3' = 4\vec{a}_y$, $\vec{r}_4' = -4\vec{a}_y$

$|\vec{R}_{nA}| = \sqrt{25} = 5$



RADI SIMETRIJE PONIŠTAVAJU SE KOMPONENTE SILE U Y I X SMJERU

$$\vec{F}_{uk} = \frac{Q_n Q_A}{4\pi\varepsilon}\left(\frac{3}{5^3}\cdot 4\right)\vec{a}_z$$

$$\vec{F} = \frac{20\cdot 100\cdot 10^{-18}}{4\pi\varepsilon}\cdot\frac{12}{5^3} = \frac{48\cdot 10^{-18}}{\pi\varepsilon}\vec{a}_z$$

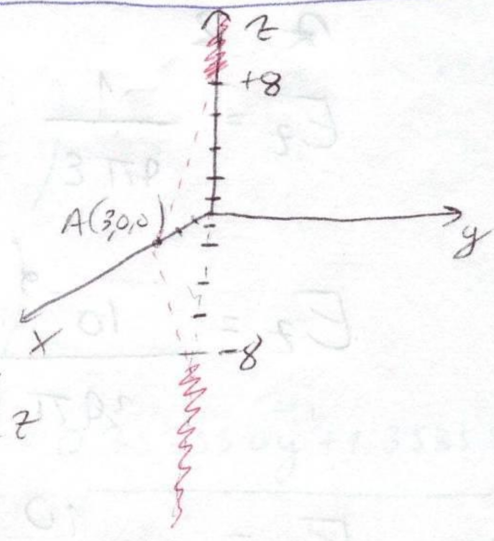
$$\boxed{F = 1.72564\,[\mu N]}$$

$\boxed{2.12.}$ $\lambda = 30\cdot 10^{-9}\left[\frac{C}{m}\right]$

$z \in (-\infty, -8) \cup (8, +\infty)$

$\vec{r} = 3\vec{a}_x$

$\vec{r}' = z\vec{a}_z$



$$\vec{E} = \frac{1}{4\pi\varepsilon}\int \lambda\left(\frac{3\vec{a}_x - z\vec{a}_z}{\left(\sqrt{9+z^2}\right)^3}\right)dz$$

$$\vec{E} = \frac{3\cdot 10^{-8}}{4\pi\varepsilon}\int\left[\frac{3\vec{a}_x - z\vec{a}_z}{\left(\sqrt{z^2+3^2}\right)^3}\right]dz$$

KOMPONENTE U Z SMJERU SE PONIŠTAVAJU RADI SIMETRIJE

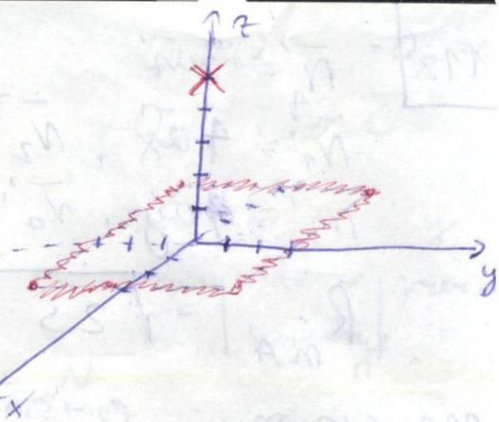
$$E_x = \frac{9\cdot 10^{-8}}{4\pi\varepsilon}\int_z \frac{dz}{\left(\sqrt{z^2+3^2}\right)^3}$$

$$\int\frac{dz}{\left(\sqrt{z^2+a^2}\right)^3} = \frac{z}{a^2\sqrt{z^2+a^2}}$$

$$E_x = \frac{9\cdot 10^{-8}}{4\pi\varepsilon}\left(\frac{z}{9\sqrt{z^2+9}}\Bigg|_{z=-\infty}^{-8} + \frac{z}{9\sqrt{z^2+9}}\Bigg|_{z=8}^{+\infty}\right)$$

$$E_x = \frac{9\cdot 10^{-8}}{4\pi\varepsilon}\left(-\frac{8}{9\sqrt{8^2+9}}\cdot 2 + \frac{1}{9}\cdot 2\right) \Rightarrow \boxed{E_x = 11.45\left[\frac{V}{m}\right]}$$

$\boxed{2.13.}$ $$\varphi = \frac{x}{4\pi\varepsilon}\left(2\int_{-3}^{3}\frac{dx}{\sqrt{x^2+3^2+5^2}} + 2\int_{-3}^{3}\frac{dy}{\sqrt{y^2+34}}\right)$$



$$\varphi = \frac{10^{-9}}{4\pi\varepsilon}\left[4\cdot\left(\ln\left|x+\sqrt{x^2+34}\right|\right)\Big|_{-3}^{3}\right]$$

$$\varphi = \frac{10^{-9}}{4\pi\varepsilon}\left[4\left(\ln\frac{3+\sqrt{9+34}}{-3+\sqrt{43}}\right)\right] \Rightarrow \boxed{\varphi = 35.52958\,V}$$

$\boxed{2.14.}$ $$\sigma = 10^{-9}\cos^2(\alpha)\left[\frac{C}{m^2}\right]$$

$\vec{r} = 2\vec{a_z}$ $\quad \vec{r'} = x\vec{a_x} + y\vec{a_y}$

$\vec{R} = (-x\vec{a_x} - y\vec{a_y}) + 2\vec{a_z}$

$|\vec{R}|^3 = \left(\sqrt{x^2+y^2+4}\right)^3$

$r = 4$

$z = 2$

$$E_z = \frac{1}{4\pi\varepsilon}\iint_S \sigma\cdot\frac{2}{\left(\sqrt{x^2+y^2}+4\right)^3}\underbrace{r\,dr\,d\varphi}_{dS}$$

$$E_z = \frac{10^{-9}\cdot 2}{2\cdot 4\pi\varepsilon}\int_0^4\frac{r}{\left(\sqrt{r^2+2^2}\right)^3}\int_0^{2\pi}\cos^2\alpha\,d\alpha$$

$$E_z = \frac{10^{-9}}{2\pi\varepsilon}\left[\left(-\frac{1}{\sqrt{r^2+4}}\right)\Bigg|_{r=0}^{4}\left(\frac{\alpha}{2}+\frac{1}{4}\sin 2\alpha\right)\Bigg|_{\alpha=0}^{2\pi}\right]$$

$$E_z = \frac{10^{-9}}{2\pi\varepsilon}\left[-\left(\frac{1}{\sqrt{4^2+4}} - \frac{1}{\sqrt{4}}\right)\cdot\left(\frac{2\pi}{2}+\frac{1}{4}\sin 4\pi - 0\right)\right]$$

$$E_z = \frac{10^{-9}}{2\cancel{\pi}\varepsilon}\cdot 0.27639\cdot\cancel{\pi}$$

$$\boxed{E_z = 15.60838\left[\frac{V}{m}\right]}$$

2.15. $\vec{E} = \frac{-16}{r^2}\vec{a_r}$ $A(2, \pi, \frac{\pi}{2})$ $B(4, 0, \pi)$

$\vec{dl} = dr\vec{a_r}$ POLJE SE MIJENJA S r KOORDINATOM

$$U_{AB} = \int_A^B \vec{E}\cdot\vec{dl} = -\int_2^4 16\cdot\frac{1}{r^2}\,dr = -16\left(-\frac{1}{r}\right)\Big|_2^4$$

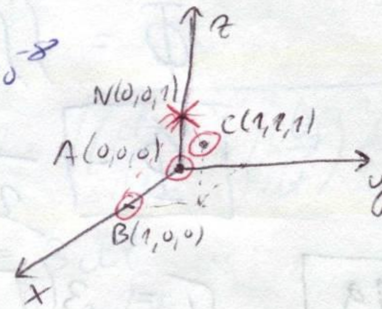
$$\boxed{U_{AB}} = 16\left(\frac{1}{4} - \frac{1}{2}\right) = \boxed{-4}$$

2.16. $\vec{r_m} = \vec{a_z}$ $Q_1 = Q_2 = Q_3 = 10\cdot 10^{-9}$, $Q_N = 2\cdot 10^{-8}$

$\vec{r_1} = 0$ $\vec{R_1} = \vec{a_z}$

$\vec{r_2} = \vec{a_x}$ $\vec{R_2} = -\vec{a_x} + \vec{a_z}$

$\vec{r_3} = \vec{a_x} + \vec{a_y} + \vec{a_z}$ $\vec{R_3} = -\vec{a_x} - \vec{a_y}$



$$\vec{F_1} = \frac{Q_1\cdot Q_n}{4\pi\varepsilon}\cdot\frac{\vec{a_z}}{1}$$

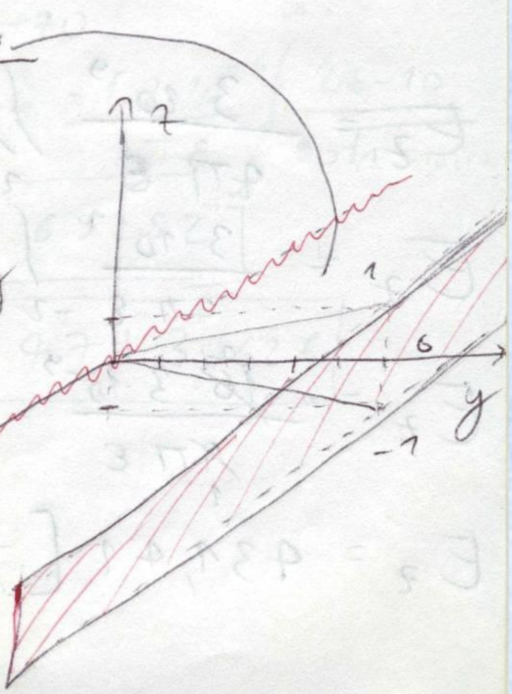
$$\vec{F_2} = \frac{Q_2\cdot Q_n}{4\pi\varepsilon}\cdot\left(\frac{-\vec{a_x} + \vec{a_z}}{(\sqrt{2})^3}\right)$$

$$\vec{F_3} = \frac{Q_3\cdot Q_n}{4\pi\varepsilon}\left(\frac{-\vec{a_x} - \vec{a_y}}{(\sqrt{2})^3}\right)$$

$$\vec{F_{UK}} = \frac{2\cdot 10^{-16}}{24\pi\varepsilon}\left(-0.7071\vec{a_x} - 0.35355\vec{a_y} + 1.35355\vec{a_z}\right)$$

$$\boxed{F_{UK}} = \frac{10^{-16}}{2\pi\varepsilon}\cdot 1.56751 = \boxed{2.81768\,\mu N}$$

2.17. ZADATAK SE RJEŠAVA GRAFIČKI. RAČUNA SE KUT PRI KOJEM LINIJSKI NABOJ "GLEDA" NA DIO RAVNINE $y = 6$, $-1 < z < 1$



$$\alpha = \tan^{-1}\frac{1}{6} = 0.16514\ [rad]$$

$$2\alpha = 0.33029\ [rad] = X$$

$$\boxed{y[\%]} = \frac{X}{2\pi} = \boxed{5.25684}$$

2.18. $Q = 18 \cdot 10^{-9}\ C$, $r = 3$

$$\oint\!\!\oint_S \vec{D} \cdot \vec{n}\, dS = Q$$

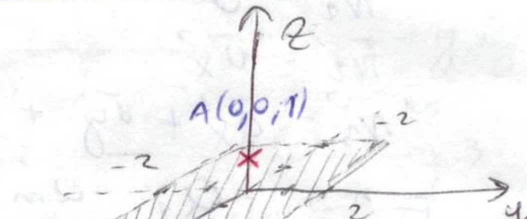
$$D \cdot 4 \cdot r^2 \cdot \pi = 18 \cdot 10^{-9}$$

$$D = \frac{\cancel{18}}{\cancel{9} \cdot \cancel{4} \cdot \pi} \cdot 10^{-9} = \frac{1}{2\pi} \cdot 10^{-9}$$

$$\Phi = \iint \vec{D} \cdot \vec{n}\, dS \quad dS = 4\pi\, m^2$$

$$\boxed{\Phi} = \frac{1}{\cancel{2\pi}} \cdot 10^{-9} \cdot \cancel{4\pi} = \boxed{2\, nC}$$

2.19. $\sigma = 3\left(\sqrt{x^2+y^2+1}\right)^3 \cdot 10^{-9}$



$T(0,0,1)$

$\vec{r} = \vec{a_z}$

$\vec{r}' = x\vec{a_x} + y\vec{a_y}$

$\vec{R} = \vec{r} - \vec{r}' = -x\vec{a_x} - y\vec{a_y} + \vec{a_z}$

$|\vec{R}| = \sqrt{x^2+y^2+1}$

$$\vec{E} = \frac{1}{4\pi\varepsilon} \iint_S \sigma\, dS \cdot \frac{\vec{R}}{|\vec{R}|^3}$$

POMISTAVAJU SE RADI SIMETRIJE

$$\vec{E} = \frac{1}{4\pi\varepsilon} \iint \frac{3 \cdot \cancel{\left(\sqrt{x^2+y^2+1}\right)^3} \cdot 10^{-9} \cdot (\underbrace{-x\vec{a_x} - y\vec{a_y}} + \vec{a_z})}{\cancel{\left(\sqrt{x^2+y^2+1}\right)^3}}$$

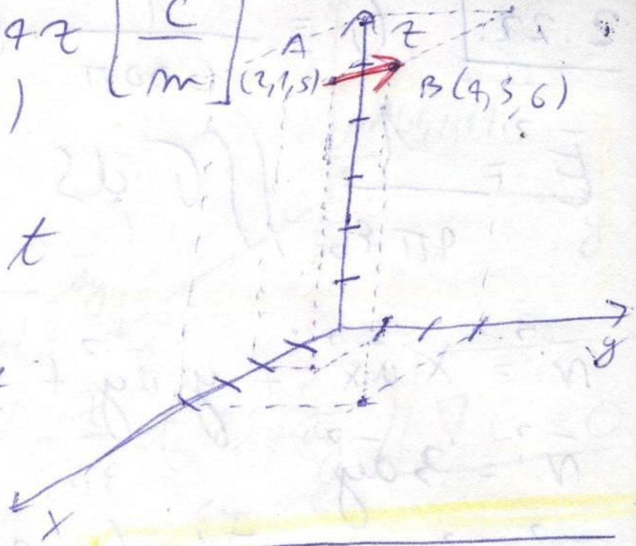
$$E_z = \frac{3 \cdot 10^{-9}}{4\pi\varepsilon} \iint dx\, dy$$

$$E_z = \frac{3 \cdot 10^{-9}}{4\pi\varepsilon} \int_{-2}^{2} dx \int_{-2}^{2} dy$$

$$E_z = \frac{\overset{4}{\cancel{16}} \cdot 3 \cdot 10^{-9}}{\underset{1}{\cancel{4}}\pi\varepsilon} = \frac{12 \cdot 10^{-9}}{\pi \cdot \varepsilon}$$

$$E_z = 431{,}41 \left[\frac{V}{m}\right]$$

2.20. $\lambda(x,y,z) = 2x + 3y - 4z \left[\frac{C}{m}\right]$



PARAMETRIZACIJA PRAVCA $\quad t \in (0,1)$

$$\frac{x - x_A}{x_B - x_A} = \frac{y - y_A}{y_B - y_A} = \frac{z - z_A}{z_B - z_A} = t$$

$$\frac{x-2}{2} = \frac{y-1}{2} = \frac{z-5}{1} = t$$

$x = 2t + 2 \qquad \frac{dx}{dt} = 2$

$y = 2t + 1 \qquad \frac{dy}{dt} = 2 \qquad \frac{dz}{dt} = 1$

$z = t + 5$

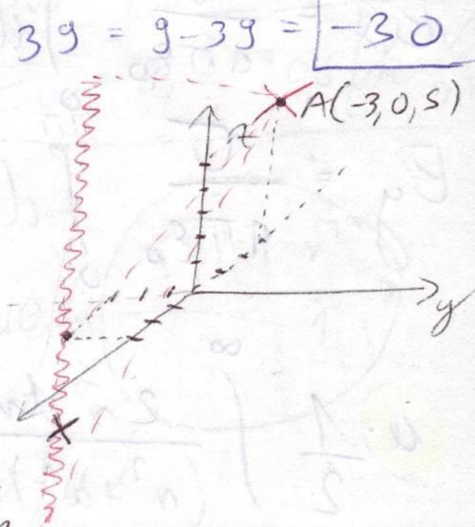
$$dl = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$dl = 3dt$

$$Q = \int_l \lambda(l)\, dl = \int_0^1 (4t + 4 + 6t + 3 - 4t - 20) \cdot 3dt$$

$$\boxed{Q =} 18 \int_0^1 t\, dt - 39 \int_0^1 dt = 18 \cdot \frac{1}{2} \cdot 1^2 - 39 = 9 - 39 = \boxed{-30}$$

2.21. $\lambda = 0.4 \cdot 10^{-6} \left[\frac{C}{m}\right]$



$\vec{r} = -3\vec{a_x} + 5\vec{a_z}$

$\vec{r}' = 3\vec{a_x} - 3\vec{a_y} + z\vec{a_z}$

$\vec{R} = \vec{r} - \vec{r}' = -6\vec{a_x} + 3\vec{a_y} + (5-z)\vec{a_z}$

$$\vec{E} = \frac{1}{4\pi\varepsilon} \int_l \frac{\lambda\, dl}{|\vec{r} - \vec{r}'|^3} \cdot (\vec{r} - \vec{r}') \qquad dl = dz$$

$$\vec{E} = \frac{1}{4\pi\varepsilon} \cdot 0.4 \cdot 10^{-6} \int_z \frac{-6\vec{a_x} + 3\vec{a_y} + (5-z)\vec{a_z}}{\left(\sqrt{z^2 - 10z + 70}\right)^3} dz$$

$$E_x = \frac{-6 \cdot 0.4 \cdot 10^{-6}}{4\pi\varepsilon} \int_{-\infty}^{+\infty} \frac{dz}{\sqrt{z^2 - 10z + 70}^3} \qquad E_x = \frac{-1.2 \cdot 10^{-6}}{2\pi\varepsilon} \left.\frac{2z - 10}{90\sqrt{z^2 - 10z + 70}}\right|_{-\infty}^{\infty}$$

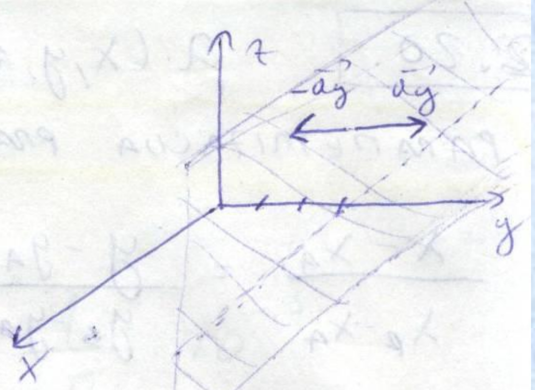
$$E_x = \frac{-1.2 \cdot 10^{-6}}{2\pi\varepsilon} \cdot \frac{4}{90} \qquad \boxed{E_x = -958.69252}$$

$$E_y = \frac{3 \cdot 0.4 \cdot 10^{-6}}{4\pi\varepsilon} \cdot \frac{4}{90} \Rightarrow \boxed{E_y = 479.39626}$$

$$\boxed{\vec{E} = -960\vec{a_x} + 480\vec{a_y}}$$

2.22. $\sigma = \frac{1}{600\pi} \cdot 10^{-6}$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \iint \sigma \cdot dS \cdot \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}}$$



$\vec{r} = x\vec{a_x} + y\vec{a_y} + z\vec{a_z}$

$\vec{r}' = 3\vec{a_y}$

$\vec{r} - \vec{r}' = x\vec{a_x} + (y-3)\vec{a_y} + z\vec{a_z}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \iint \sigma \cdot \frac{[x\vec{a_x} + (y-3)\vec{a_y} + z \cdot \vec{a_z}]}{(x^2 + z^2 + (y-3)^2)^{\frac{3}{2}}} \, dx\, dz$$

$$E_x = \frac{\sigma}{4\pi\varepsilon_0} \int_0^{2\pi} d\varphi \int_0^{\infty} \frac{r\cos\varphi \cdot r\,dr}{(r^2 + (y-3)^2)^{\frac{3}{2}}} = 0 \qquad E_z = 0$$

$$E_y = \frac{\sigma}{4\pi\varepsilon_0} \int_0^{2\pi} d\varphi \int_0^{\infty} \frac{(y-3)\, r\,dr}{(r^2 + (y-3)^2)^{\frac{3}{2}}} \Rightarrow \left| \begin{array}{c} y-3 = a \\ u = r^2 + a^2 \\ du = 2r\,dr \\ r\Big|_0^{\infty} \approx u\Big|_{a^2}^{\infty} \end{array} \right|$$

$$a \cdot \frac{1}{2} \int_0^{\infty} \frac{2r\,dr}{(r^2+a^2)^{\frac{3}{2}}} = \frac{1}{2} \int_{a^2}^{\infty} u^{-\frac{3}{2}}\, du = \frac{1}{2} \frac{1}{\sqrt{u}} (-2) \Big|_{a^2}^{\infty} = \frac{1}{|a|}$$

$$E_y = \frac{\sigma}{\overset{2}{4}\pi\varepsilon_0} \cdot 2\pi \cdot \frac{a}{|a|} = \frac{\sigma}{2\varepsilon_0} \frac{y-3}{|y-3|} = 29.96 \frac{y-3}{|y-3|}$$

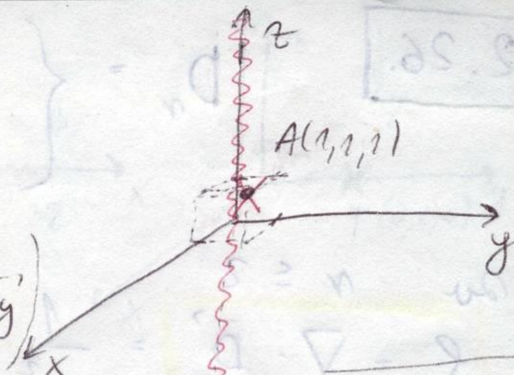
$$\vec{E_y} = \begin{cases} -29{,}96\,\vec{a_y}, & y < 3 \\ 29{,}96\,\vec{a_y}, & y > 3 \end{cases}$$

$\boxed{2.23.}$

$\vec{r}' = z\,\vec{u}_z$

$\vec{r} = \vec{u}_x + \vec{u}_y + \vec{u}_z$

$\vec{R} = \vec{u}_x + \vec{u}_y + (1-z)\,\vec{u}_z$



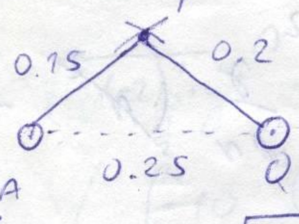
$$\vec{E} = \frac{\lambda}{4\pi\varepsilon}\left(\int \frac{dz}{(z^2-2z+3)^{\frac{3}{2}}}\,\vec{u}_x + \int \frac{dz}{(z^2-2z+3)^{\frac{3}{2}}}\,\vec{u}_y\right)$$

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon}\left(2\cdot\frac{2z-2}{4\sqrt{z^2-2z+3}}\Bigg|_{z=-\infty}^{+\infty}\right)^{1}(\vec{u}_x+\vec{u}_y) = \frac{\lambda}{4\pi\varepsilon}\left(\vec{u}_x+\vec{u}_y\right) \quad \boxed{\nabla\cdot\vec{E}=0}$$

$\boxed{2.24.}$

$Q_A = 20\cdot 10^{-9}\ C$

$Q_B = 10\cdot 10^{-9}\ C$



$$E_A = \frac{Q_A}{4\pi\varepsilon d^2} = \frac{20\cdot 10^{-9}}{4\pi\varepsilon(0.15)^2} = 7989.10438$$

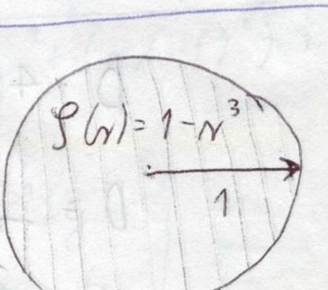
$$E_{uk} = \sqrt{E_A^2 + E_B^2}$$

$$E_B = \frac{Q_B}{4\pi\varepsilon d^2} = \frac{10\cdot 10^{-9}}{4\pi\varepsilon(0.2)^2} = 2296.9356$$

$$\boxed{E_{uk} = 8{,}29906\ \left[\frac{kV}{m}\right]}$$

$\boxed{2.25.}$

$$\oiint \vec{D}\cdot\vec{n}\,dS = \int_V \rho\,dV \qquad R = 4$$



$$\varepsilon\cdot E\cdot 4R^2\pi = \int_0^1 (1-r^3)\,r^2\,dr \int_0^\pi \sin\theta\,d\theta \int_0^{2\pi} d\alpha$$

$$\varepsilon\cdot E\cdot 4\cdot 4^2\cdot\pi = \int_0^1 (r^2 - r^5)\,dr \int_0^\pi \sin\theta\,d\theta \int_0^{2\pi} d\alpha$$

$$\varepsilon\cdot E\cdot 4^2\cdot \cancel{4\cdot\pi} = \left[\left(\frac{1}{3}r^3\Big|_0^1 - \frac{1}{6}r^6\Big|_0^1\right)\right]\cdot \cancel{2\cdot 2\pi}$$

$$\varepsilon\cdot E\cdot 16 = \left(\frac{1}{3} - \frac{1}{6}\right) = \frac{1}{6}$$

$$\boxed{E =}\ \frac{1}{\varepsilon\cdot 16\cdot 6} = \boxed{\frac{1}{96\,\varepsilon_0}}$$

2.26.

$$D_r = \begin{cases} \frac{5r^2}{4}, & r \le 2 \\ \frac{20}{r^2}, & r > 2 \end{cases}$$

za $r \le 2$

$$\rho_1 = \nabla \cdot \vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_r\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{5r^2}{4}\right)$$

$$\boxed{\rho_1} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{5}{4}r^4\right) = \frac{1}{\cancel{r^2}} \cdot \frac{5}{\cancel{4}} \cdot \cancel{4} \cdot r^{\cancel{3}} = \boxed{5r}$$

za $r > 2$

$$\boxed{\rho_2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\cancel{r^2}\frac{20}{\cancel{r^2}}\right) = \boxed{0}$$

$$\boxed{\rho = \begin{cases} 5r, & r \le 2 \\ 0, & r > 2 \end{cases}}$$

2.27.

$r_0 = 5\ m \qquad r = 100\ m$

$$\rho = 1.2(5-2r)\cdot 10^{-6}\left[\frac{C}{m^3}\right]$$

$$D \cdot 4 \cdot r^2 \pi = \int_0^{r_0} 1.2(5-2r)\cdot 10^{-6} \cdot r^2\,dr \int_0^{\pi} \sin\theta\,d\theta \int_0^{2\pi} d\phi$$

$$D = \frac{1.2\cdot 10^{-6} \cdot 2 \cdot 2\cancel{\pi}}{100^2\,\cancel{\pi}} \int_0^5 (5r^2 - 2r^3)\,dr$$

$$D = 4.8\cdot 10^{-10}\left(5\cdot\frac{1}{3}r^3\Big|_{r=0}^{5} - 2\,\frac{1}{4}r^4\Big|_{r=0}^{5}\right)$$

$$D = 4.8\cdot 10^{-10}\left(5\cdot\frac{1}{3}5^3 - \cancel{2}\cdot\frac{1}{\cancel{4}_2}5^4\right)$$

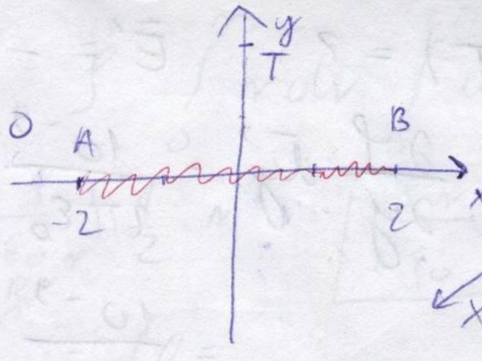
$$D = 4.8\cdot 10^{-10}\left(\frac{5^4}{3} - \frac{5^4}{2}\right)$$

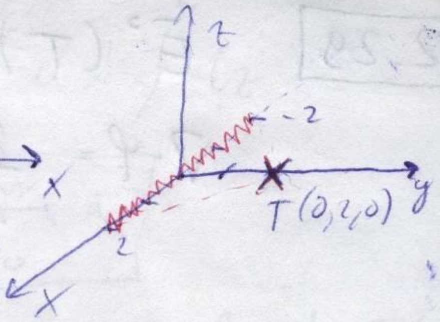
$$D = 4.8\cdot 10^{-10}(-104.16667)$$

$$\boxed{D = -12.5\cdot 10^{-9}\ \frac{C}{m^2}}$$

2.28 $\lambda = |x| \cdot 10^{-9}$

$$\lambda = \begin{cases} -10^{-9} \cdot x, & x<0 \\ 10^{-9} \cdot x, & x>0 \end{cases}$$





PARAMETRIZACIJA PRAVCA

$$\frac{x+2}{4} = \frac{y}{0} = \frac{z}{0} = t$$

$x = 4t-2$ $\quad \frac{dx}{dt} = 4 \quad dl = 4\,dt \quad \vec{r} = y\,\vec{a_y}$

$y = 0 \quad |\vec{r} - \vec{r}\,'| = \sqrt{x^2+y^2} = \sqrt{(4t-2)^2+y^2} \quad \vec{r}\,' = (4t-2)\vec{a_x}$

$z = 0 \quad t \in (0,1)$

$$\varphi = \frac{1}{4\pi\varepsilon_0}\int_0^1 \frac{\lambda\, 4\,dt}{\left(y^2+(4t-2)^2\right)^{\frac12}} = \frac{1}{\pi\varepsilon_0}\left(\int_0^{\frac12}\frac{-10^{-9}(4t-2)\,dt}{\left(y^2+(4t-2)^2\right)^{\frac12}} + \int_{\frac12}^{1}\frac{10^{-9}(4t-2)\,dt}{\left(y^2+(4t-2)^2\right)^{\frac12}}\right) =$$

$$\int_a^l \frac{(4t-2)\,dt}{\left(y^2+(4t-2)^2\right)^{\frac12}} = \left| \begin{array}{l} u = 4t-2 \\ du = 4\,dt \\ u_1 = 4a-2 \\ u_2 = 4l-2 \end{array} \right| =$$

$$= \int_{4a-2}^{4l-2} \frac{\frac14 u\,du}{(y^2+u^2)^{\frac12}} = \frac14 \int_{4a-2}^{4l-2}\frac{u\,du}{(u^2+y^2)^{\frac12}} = \left| \begin{array}{l} x = u^2+y^2 \\ dx = 2u\,du \\ x_1 = (4a-2)^2+y^2 \\ x_2 = (4l-2)^2+y^2 \end{array}\right| =$$

$$= \frac14 \cdot \frac12 \int_{4a-2}^{4l-2}\frac{2u\,du}{\sqrt{x}} = \frac18 \int_{(4a-2)^2+y^2}^{(4l-2)^2+y^2}\frac{dx}{\sqrt{x}} = \frac{1}{\cancel{8}_4}\,\cancel{2}\sqrt{x}\,\Big|_{x=(4a-2)^2+y^2}^{(4l-2)^2+y^2}$$

$$= \frac14\left(\sqrt{(4l-2)^2+y^2} - \sqrt{(4a-2)^2+y^2}\right)$$

$$\varphi = \frac{10^{-9}}{\pi\varepsilon_0}\cdot\left[\frac14\left(\sqrt{(4\cdot 0-2)^2+y^2} - \sqrt{(4\cdot\tfrac12-2)^2+y^2}\right) + \frac14\left(\sqrt{(4-2)^2+y^2} - \sqrt{(4\cdot\tfrac12-2)^2+y^2}\right)\right]$$

$$\varphi = \frac{10^{-9}}{\cancel{4}_2\pi\varepsilon_0}\left[\cancel{2}\sqrt{4+y^2} - \cancel{2}y\right] = \frac{10^{-9}}{2\pi\varepsilon_0}\left(\sqrt{4+y^2}-y\right)$$

za $y=2$

$$\boxed{\varphi(2) = \frac{10^{-9}}{2\pi\varepsilon_0}}\cdot\left(2\sqrt2 - 2\right) = \frac{10^{-9}}{\pi\varepsilon_0}\left(\sqrt2-1\right) = \boxed{14.89\,V}$$

2.29 $\vec{E}(T) = ?$ $\quad \vec{E} = -\nabla \cdot \varphi \quad \varphi = \frac{10^{-9}}{2\pi\varepsilon_0}\left(\sqrt{4+y^2} - y\right)$

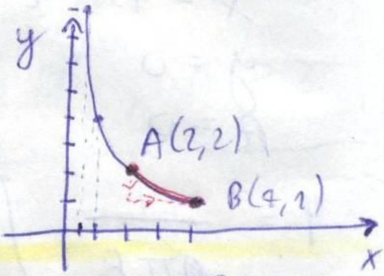
$$\nabla \cdot \varphi = \frac{\partial \varphi}{\partial y}\vec{a_y} = \frac{10^{-9}}{2\pi\varepsilon_0}\left(\frac{1}{\cancel{2}\sqrt{4+y^2}} \cdot \cancel{2}y - 1\right)$$

$$= \frac{10^{-9}}{2\pi\varepsilon_0}\left(\frac{y}{\sqrt{y^2+4}} - 1\right)$$

$$\boxed{\vec{E}} = -\frac{10^{-9}}{2\pi\varepsilon_0}\left(\frac{2}{\sqrt{8}} - 1\right) = -\frac{10^{-9}}{2\pi\varepsilon_0}\left(\frac{\sqrt{2}}{2} - 1\right) = \boxed{5.26\ \frac{V}{m}}$$

2.30 $\vec{E} = 4x\vec{a_x} + 2\vec{a_y}\left[\frac{V}{m}\right]$

$y = \frac{4}{x} \quad A(2,2) \quad B(4,1)$



$$W(a) - W(b) = -\int_b^a q \cdot \vec{E} \cdot \vec{dl}$$

$$A = -q\int_{(4,1)}^{(2,2)} 4x\,dx + 2\,dy = -q\left(4\int_4^2 x\,dx + 2\int_1^2 dy\right)$$

$$\boxed{A} = -q\left(4 \cdot \frac{1}{2}x^2\Big|_{x=4}^{2} + 2\right) = -q(-22) = \boxed{22eV}$$

ZA OBA PUTA INTEGRALNE DOBIJE SE ISTI REZULTAT. POTENCIJALNA ENERGIJA NABOJA U STATIČKOM ELEKTRIČNOM POLJU NE OVISI O PUTU, VEĆ SAMO O POČETNOJ I KRAJNJOJ TOČKI TOG PUTA

2.31 $\varepsilon \cdot E \cdot r^2\pi = \sigma \cdot \int_0^R r\,dr \int_0^{2\pi} d\varphi$

$$\varepsilon \cdot E \cdot r^2 \cancel{\pi} = \sigma \ \frac{1}{2}r^2\Big|_{r=0}^{R} \cdot 2\cancel{\pi} = \frac{1}{\cancel{2}}\sigma \cdot R^2 \cdot \cancel{2}$$

$$\boxed{\vec{E} = \frac{\sigma \cdot R^2}{\varepsilon \cdot r^2}\vec{a_r}}$$

2.32 $\varphi = -\int_{T_{ref}}^{P} \vec{E} \cdot \vec{dl}$

za $r > R$: $\left(-\frac{1}{r}\right)\Big|_{r=\infty}^{r}$

za $r > R$ $\quad \varphi = -\int_\infty^r \frac{\sigma \cdot R^2}{\varepsilon \cdot r^2}dr = -\frac{\sigma \cdot R^2}{\varepsilon}\int_\infty^r \frac{dr}{r^2}$ $\quad \boxed{\frac{1}{r}} - \frac{1}{\infty} \to 0$

$$\boxed{\varphi = \frac{\sigma \cdot R^2}{\varepsilon \cdot r}}$$

$$\boxed{\varphi = \begin{cases} \frac{\sigma \cdot R}{\varepsilon}, & r < R \\ \frac{\sigma R^2}{\varepsilon r}, & r > R \end{cases}}$$

za $r < R$: $\left(-\frac{1}{r}\right)\Big|_{r=\infty}^{R}$ $\quad \boxed{\frac{1}{R}}$

za $r < R$

$$\boxed{\varphi = \frac{\sigma \cdot R}{\varepsilon}}$$

2.33. $\varepsilon_0 \cdot E \cdot 2r\pi \cdot L = \rho \cdot \int_0^r r\,dr \int_0^{2\pi} d\alpha \int_0^L dz$

$$\boxed{\vec{E}} = \frac{\rho}{\varepsilon_0 \cdot r}\cdot\frac{1}{2} r^2 = \boxed{\frac{\rho r}{2\varepsilon_0}\vec{a_r}}$$

$$\overbrace{\varphi(R)}^{1V} - \varphi(r) = -\int_r^R \vec{E}\cdot d\vec{l} \qquad d\vec{l} = dr\,\vec{a_r}$$

$$\varphi(r) = 1 + \int_r^R \frac{\rho r}{2\varepsilon_0}\, dr$$

$$\varphi(r) = 1 + \frac{\rho}{2\varepsilon_0}\left(\frac{1}{2} r^2\Big|_{r=r}^{R}\right) = 1 + \frac{\rho}{2\varepsilon_0}\cdot\frac{1}{2}(R^2-r^2)$$

$$\boxed{\varphi(r) = 1 + \frac{\rho}{4\varepsilon_0}(1-r^2)}$$

2.34.

$A(1,\pi,4)$
$B(3,\pi,4)$

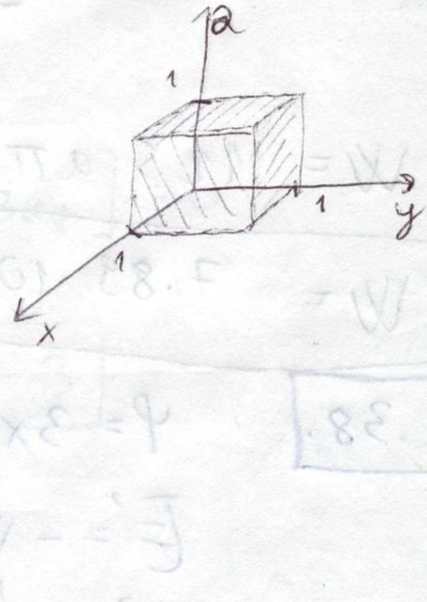
$$\vec{E} = \frac{\tau}{2\pi\varepsilon_0}\cdot\frac{x\vec{a_x}+y\vec{a_y}}{\sqrt{x^2+y^2}}$$

$$\vec{E} = \frac{\tau}{2\pi\varepsilon_0}\cdot\frac{\vec{r}}{r}$$

$$U_{BA} = \varphi(B)-\varphi(A) = -\int_A^B \vec{E}\,\vec{a_r}\,dr = -\int_1^3 \frac{\tau}{2\pi\varepsilon_0}\cdot\frac{1}{r}\,dr$$

$$U_{BA} = -\frac{\tau}{2\pi\varepsilon_0}\ln r\Big|_{r=1}^{3} = -\frac{\tau}{2\pi\varepsilon_0}\cdot\ln\frac{3}{1} \Rightarrow \boxed{U_{BA} = -98{,}74\,kV}$$

2.35. $\varphi = 2x+4y$



$$\vec{E} = -\nabla\cdot\varphi = -(2\vec{a_x}+4\vec{a_y})$$

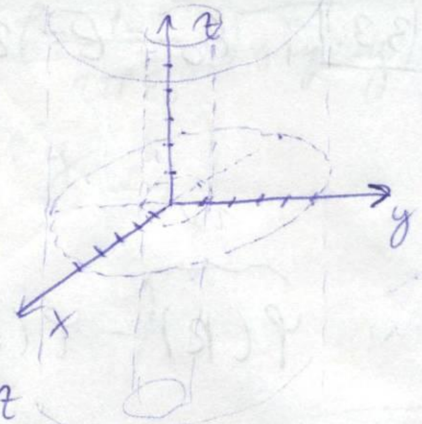
$$|\vec{E}| = \sqrt{4+16} = \sqrt{20} \qquad |E|^2 = 20$$

$$\boxed{W = \frac{1}{2}\int_V \varepsilon\cdot|\vec{E}|^2\,dV}$$

$$W = \cancel{\frac{1}{2}}\cdot\varepsilon\cdot \overset{10}{\cancel{20}}\int_0^1 dx\int_0^1 dy\int_0^1 dz$$

$$\boxed{W =}\ 10\,\varepsilon_0 = \boxed{8.859\cdot 10^{-11}}$$

2.36. $\vec{E} = \frac{10^5}{r}\vec{a_r}$ $\quad |\vec{E}|^2 = \frac{10^{10}}{r^2}$



$dV = r\,dr\,d\alpha\,dz$

$$W = \frac{1}{2}\int_V \varepsilon\cdot|\vec{E}|^2\,dV$$

$$W = \frac{10^{10}}{2}\cdot\varepsilon\int_{0.01}^{0.05}\frac{1}{r^2}\cdot r\,dr\int_0^{2\pi}d\alpha\int_0^{0.5}dz$$

$$\boxed{W =} \frac{\varepsilon\cdot 10^{10}}{\cancel{2}}\ln\frac{0.05}{0.01}\cdot\cancel{2}\pi\cdot 0.5 = \boxed{0.22383\,J}$$

2.37. $\vec{E} = -5e^{-\frac{r}{a}}\vec{a_r}$ $\quad 0\le r\le 2a$

$|\vec{E}|^2 = 25e^{-2\cdot\frac{r}{a}}$ $\quad 0\le z\le 5a$

$$W = \frac{1}{2}\varepsilon\cdot 25\int_0^{2a}e^{-\frac{2r}{a}}r\,dr\int_0^{2\pi}d\alpha\int_0^{5a}dz$$

$$W = \frac{25}{\cancel{2}}\cdot\varepsilon\cdot\cancel{2}\pi\cdot 5a\left(\frac{a^2\cdot e^{-\frac{2}{a}r}}{4}\left(-\frac{2}{a}r-1\right)\right)\Bigg|_0^{2a}$$

$\int x\cdot e^{cx}dx = \frac{e^{cx}}{c^2}(cx-1)$

$\int r\cdot e^{-\frac{2}{a}r}dr = \frac{e^{-\frac{2}{a}r}}{\frac{4}{a^2}}\left(-\frac{2}{a}r-1\right)$

$$W = 125a\pi\varepsilon\left[\left(\frac{a^2e^{\frac{-2\cdot 2a}{a}}}{4}\cdot\left(\frac{-2\cdot 2a}{a}-1\right)\right) \quad r=0 \quad -\left(\frac{a^2}{4}e^0\left(-1\right)\right)\right]$$

$$W = 125a\pi\varepsilon\left[\frac{a^2}{4}\left(1-e^{-4}\cdot 5\right)\right]$$

$$\boxed{W = 7.89\cdot 10^{-10}a^3}$$

2.38. $\varphi = 3x^2+4y^2$ [V]

$\vec{E} = -\nabla\cdot\varphi = -(6x\vec{a_x}+8y\vec{a_y})$

$|\vec{E}| = \sqrt{36x^2+64y^2}$ $\quad |\vec{E}|^2 = 36x^2+64y^2$

$$W = \frac{\varepsilon}{2}\left(\int_V 36x^2\,dx\,dy\,dz + \int_V 64y^2\,dx\,dy\,dz\right)$$

$$W = \frac{\varepsilon}{2}\left(36\int_1^3 x^2 dx \int_1^3 dy \int_1^3 dz + 64\int_1^3 y^2 dy \int_1^3 dx \int_1^3 dz\right) =$$

$$W = \frac{\varepsilon}{2}\left(36 \cdot \frac{1}{3} x^3 \Big|_{x=1}^{3} \cdot 2 \cdot 2 + 64 \cdot \frac{1}{3} y^3 \Big|_{y=1}^{3} \cdot 2 \cdot 2\right)$$

$$W = \frac{\varepsilon}{2}\left(\frac{36}{3}(3^3-1)\cdot 4 + \frac{64}{3}(3^3-1)\cdot 4\right)$$

$$W = \frac{\varepsilon}{2}\left(1248 + 2218.66667\right)$$

$$\boxed{W = 15.34693\ nJ}$$

$\boxed{2.39.}$ $\vec{E_2} = 2\vec{a_x} - 3\vec{a_y} + 5\vec{a_z}$, $\varepsilon_{r2} = 2$, $z > 0$

$\vec{E_1} = ?$, $\varepsilon_{r1} = 5$, $z < 0$

$\vec{n_{12}} = \vec{a_z}$

$$\vec{n_{12}} \cdot (\varepsilon_{r2} \vec{E_2} - \varepsilon_r \vec{E_1}) = 0$$

$$\vec{a_z} \cdot (4\vec{a_x} - 6\vec{a_y} + 10\vec{a_z} - 5E_{1x}\vec{a_x} - 5E_{1y}\vec{a_y} - 5E_{1z}\vec{a_z}) = 0$$

$$10 - 5E_{1z} = 0$$

$$-5E_{1z} = -10$$

$$\boxed{E_{1z} = 2}$$

$$\vec{n_{12}} \times (\vec{E_2} - \vec{E_1})$$

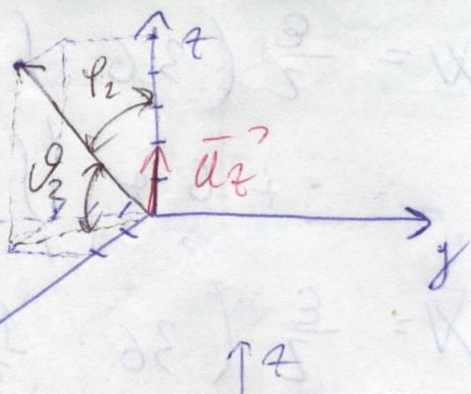
$$\vec{a_z} \times \left[(2 - E_{1x})\vec{a_x} + (-3 - E_{1y})\vec{a_y} + 3\vec{a_z}\right] = 0$$

$$\vec{a_y}(2 - E_{1x}) - \vec{a_x}(-3 - E_{1y}) = 0$$

$\boxed{E_{1x} = 2}$ $\boxed{E_{1y} = -3}$

$$\boxed{\vec{E_1} = 2\vec{a_x} - 3\vec{a_y} + 2\vec{a_z}}$$

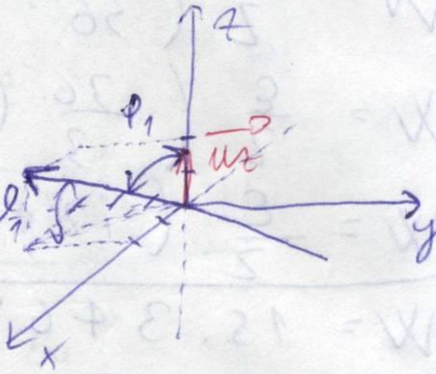
2.40. $\vec{E}_2 = 2\vec{a}_x - 3\vec{a}_y + 5\vec{a}_z$



$$\cos\varphi_2 = \frac{\vec{E}_2 \cdot \vec{u}_z}{|\vec{E}_2| \cdot |\vec{u}_z|} = \frac{(2\vec{a}_x - 3\vec{a}_y + 5\vec{a}_z)\cdot \vec{a}_z}{\sqrt{4+9+25}\,\sqrt{1}}$$

$$\cos\varphi_2 = \frac{5}{\sqrt{38}} \Rightarrow \varphi_2 = 35.8^\circ$$

$$\boxed{\vartheta_2 =} \; 90^\circ - \varphi_2 = \boxed{54.2^\circ}$$



$$\cos\varphi_1 = \frac{\vec{E}_1 \cdot \vec{u}_z}{|\vec{E}_1| \, |\vec{u}_z|} = \frac{2}{\sqrt{17}} \Rightarrow \varphi_1 = 61^\circ$$

$$\boxed{\vartheta_2 =} \; 90^\circ - \varphi_2 = \boxed{29^\circ}$$

2.41. $\varepsilon_r = 3.2$

$$\vec{D} = 2\cdot 10^{-6}\,\vec{a}_x \left[\frac{C}{m^2}\right]$$

$$\vec{P} = (\varepsilon_r - 1)\cdot\varepsilon_0\cdot\vec{E}$$

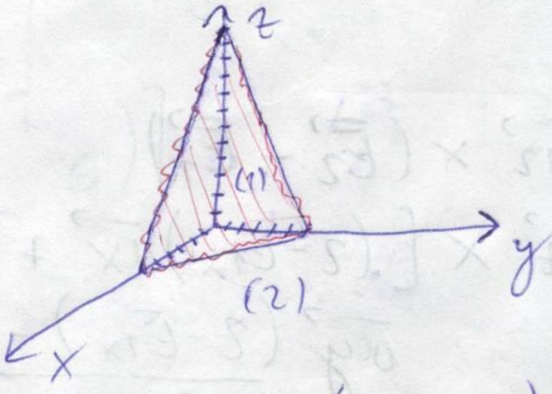
$$\vec{P} = \left(1 - \frac{1}{\varepsilon_r}\right)\cdot\vec{D}$$

$$\vec{P} = (1 - 0.3125)\cdot 2\cdot 10^{-6}\,\vec{a}_x$$

$$\boxed{\vec{P} = 1.375\cdot 10^{-6}\,\vec{a}_x}$$

2.42. $3x + 2y + z = 12$ $\quad \vec{E}_1 = 2\vec{a}_x + 5\vec{a}_z$

$\varepsilon_{r1} = 3 \quad \varepsilon_{r2} = 1$



$$\vec{n}_{12} = \frac{3\vec{a}_x + 2\vec{a}_y + \vec{a}_z}{\sqrt{14}}$$

$$\vec{n}_{12}(\varepsilon_0\,\varepsilon_{r2}\,\vec{E}_2 - \varepsilon_0\,\varepsilon_{r1}\,\vec{E}_1) = 0$$

$$\varepsilon_0 \cdot \frac{1}{\sqrt{14}}(3\vec{a}_x + 2\vec{a}_y + \vec{a}_z)\left[(E_{2x} - 6)\vec{a}_x + E_{2y}\vec{a}_y + (E_{2z} - 15)\vec{a}_z\right]$$

$$\varepsilon_0 \cdot \frac{1}{\sqrt{14}}\left[3(E_{2x} - 6) + 2E_{2y} + E_{2z} - 15\right] = 0$$

$$\frac{\varepsilon_0}{\sqrt{14}}(3E_{2x} + 2E_{2y} + E_{2z} - 33) = 0 \quad (4)$$

$$\vec{n}_{12} \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\frac{1}{\sqrt{14}} \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 3 & 2 & 1 \\ E_{2x}-2 & E_{2y} & E_{2z}-5 \end{vmatrix} = \frac{1}{\sqrt{14}}\vec{a}_x\left[2\cdot(E_{2z}-5)-E_{2y}\right] \quad (1)$$

$$-\frac{1}{\sqrt{14}}\vec{a}_y\left[3(E_{2z}-5)-E_{2x}+2\right] \quad (2)$$

$$+\frac{1}{\sqrt{14}}\vec{a}_z\left(3E_{2y}-2E_{2x}+4\right) \quad (3)$$

$$\left.\begin{aligned} E_{2z} &= 5+\frac{1}{2}E_{2y} \\ E_{2x} &= \frac{3}{2}E_{2y}+2 \end{aligned}\right\} \to (4) \quad \frac{9}{2}E_{2y} + 6 + \frac{5}{2}E_{2y} - 28 = 0$$

$$E_{2y}\left(\frac{9}{2}+\frac{5}{2}\right) = 22 \qquad \boxed{E_{2y}}\ \frac{9+5}{2} = 22 = \boxed{3.14}$$

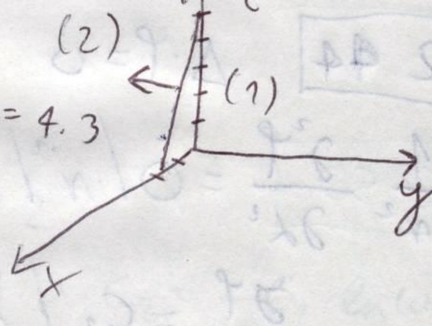
$$\boxed{E_{2x} = 6.71} \qquad \boxed{E_{2z} = 6.57} \qquad \boxed{\vec{E}_2 = 6.71\vec{a}_x + 3.14\vec{a}_y + 6.57\vec{a}_z}$$

$\boxed{2.43.}$

$$3x + z = 5$$

$$\vec{D}_1 = (4.5\vec{a}_x + 3.2\vec{a}_z)\cdot 10^{-7}, \quad \varepsilon_{r1} = 4.3$$

$$\vec{D}_2 = ?, \quad \varepsilon_{r2} = 1.8$$



$$\vec{n}_{12} = \frac{3\vec{a}_x + \vec{a}_z}{\sqrt{10}}$$

$$\vec{n}_{12}\cdot(\vec{D}_2 - \vec{D}_1) = 0$$

$$\frac{1}{\sqrt{10}}(3\vec{a}_x + \vec{a}_z)\cdot\left[(D_{2x} - 4.5\cdot 10^{-7})\vec{a}_x + D_{2y}\vec{a}_y + (D_{2z} - 3.2\cdot 10^{-7})\vec{a}_z\right] = 0$$

$$3D_{2x} - 3\cdot 4.5\cdot 10^{-7} + D_{2z} - 3.2\cdot 10^{-7} = 0 \quad (4)$$

$$\vec{n}_{12} \times \left[\left(\frac{D_{2x}}{\varepsilon_0 \cdot \varepsilon_{r2}} - \frac{4.5 \cdot 10^{-7}}{\varepsilon_0 \cdot \varepsilon_{r1}} \right) \vec{a_x} + \vec{a_y} \left(\frac{D_{2y}}{\varepsilon_0 \cdot \varepsilon_{r2}} \right) + \left(\frac{D_{2z}}{\varepsilon_0 \cdot \varepsilon_{r2}} - \frac{3.2 \cdot 10^{-7}}{\varepsilon_0 \cdot \varepsilon_{r1}} \right) \vec{a_z} \right]$$

$$\frac{1}{\sqrt{10}} \cdot \frac{1}{\varepsilon_0} \cdot \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 3 & 0 & 1 \\ \frac{D_{2x}}{1.8} - \frac{4.5 \cdot 10^{-7}}{4.3} & \frac{D_{2y}}{1.8} & \frac{D_{2z}}{1.8} - \frac{3.2 \cdot 10^{-7}}{4.3} \end{vmatrix} = \vec{a_x} \left(-\frac{D_{2y}}{1.8} \right) - \vec{a_y} \left(\frac{3 \cdot D_{2z}}{1.8} - 3 \cdot \frac{3.2 \cdot 10^{-7}}{4.3} - \frac{D_{2x}}{1.8} + \frac{4.5 \cdot 10^{-7}}{4.3} \right) + \vec{a_z} \left(\frac{3}{1.8} D_{2y} \right)$$

$D_{2z} = 0.33334 D_{2x} + 0.71162 \cdot 10^{-7}$ (1)

(1) => (4)

$3 D_{2x} + 0.33334 D_{2x} + [0.71162 - (3 \cdot 4.5 + 3.2)] \cdot 10^{-7}$

$\boxed{D_{2x} = 4.79651 \cdot 10^{-7}}$

$\boxed{D} = \sqrt{D_{2x}^2 + D_{2z}^2} = \boxed{5.32398 \cdot 10^{-7}}$

$\boxed{D_{2z} = 2.31046 \cdot 10^{-7}}$

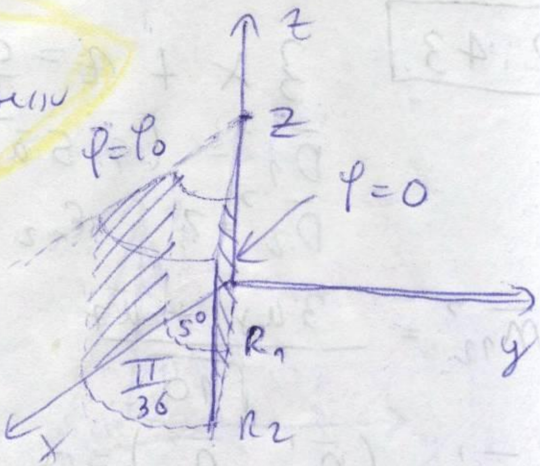
$\boxed{2.44}$ $\Delta \cdot \varphi = 0$

KUT JE FIKSAN, IZJEDNAČAVAMO DERIVACIJU PO KUTU S NULOM

$$\frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \alpha^2} = 0 \Big/ \cdot r^2 \Big/ \int$$

$$\frac{\partial \varphi}{\partial \alpha} = C_1 \Big/ \int \qquad \varphi(\alpha) = C_1 \alpha + C_2$$

$$\varphi(0) = C_2 = \varphi_0$$



$$\varphi(\alpha) = -\frac{36 \varphi_0}{\pi} \alpha + \varphi_0 \qquad \varphi\left(\frac{\pi}{36}\right) = 0$$

$$\vec{E} = -\nabla \varphi = -\frac{1}{r} \frac{\partial \varphi}{\partial \alpha} \vec{a_\alpha} \qquad C_1 \cdot \frac{\pi}{36} + C_2 = 0 \qquad C_1 = -\frac{36}{\pi} \cdot C_2$$

$$\frac{\partial \varphi}{\partial \alpha} = -\frac{36 \varphi_0}{\pi} \qquad \vec{E} = \frac{36 \varphi_0}{r \cdot \pi} \vec{a_\alpha} \qquad |\vec{E}|^2 = \left(\frac{36 \varphi_0}{r \pi} \right)^2$$

$$W = \frac{1}{2} \varepsilon_0 \int_0^{\frac{\pi}{36}} d\alpha \int_{R_1}^{R_2} r \cdot \frac{1}{r^2} dr \int_0^z dz \cdot \frac{36^2 \varphi_0^2}{\pi^2}$$

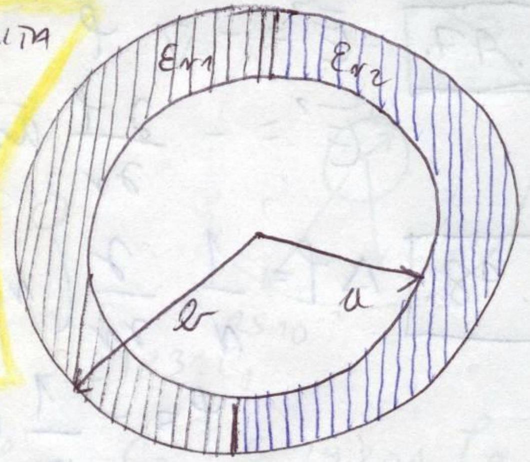
$\alpha = 5°$

$$W = \frac{18 \varepsilon_0 \varphi_0^2}{\pi} \cdot z \cdot \ln \frac{R_2}{R_1} \qquad W = \frac{1}{2} C \cdot U^2 \qquad \boxed{C} = \frac{2 \cdot W}{U^2} = \boxed{\frac{36}{\pi} \cdot \varepsilon_0 \cdot z \ln \frac{R_2}{R_1}}$$

$$U^2 = \varphi_0^2$$

2.45.

AKO JE GRANICA DIELEKTRIKA OKOMITA NA PLOČE KONDENZATORA ONDA SU KOMPONENTE E_1 i E_2 JEDNAKE, A UKUPAN NABOJ NA KONDENZATORU JE PODIJELJEN NA DIJELOVE U OMJERIMA OVISNO O POVRŠINI KOJU ZAUZIMAJU



$$E_1 = E_2 \qquad \varepsilon_{r1} = 1.3 \qquad \varepsilon_{r2} = 3.7$$

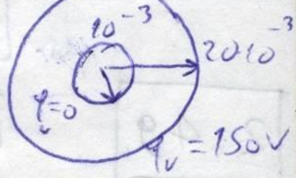
$$\oint_S \vec{D}\cdot\vec{n}\,dS = Q = D_1 S_1 + D_2 S_2$$

$$\cancel{\varepsilon_0}\,\varepsilon_{r1}\,\cancel{E\cdot 2\pi L(r_B - r_A)}\cdot\frac{1}{2} +$$

$$\cancel{\varepsilon_0}\,\varepsilon_{r2}\,\cancel{E\cdot 2\pi L(r_B - r_A)}\cdot\frac{1}{2} = Q = \cancel{\varepsilon_0}\,\varepsilon_{r3}\,\cancel{E\,2\pi L(r_B - r_A)}$$

$$\boxed{\varepsilon_{r3}} = \frac{(\varepsilon_{r1} + \varepsilon_{r2})}{2} = \frac{1.3+3.7}{2} = \boxed{2.5}$$

2.46.



$$\Delta\varphi = 0$$

$$\Delta\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) = 0 \;/\cdot r /\int$$

$$r\frac{\partial\varphi}{\partial r} = C_1 \;/ : r /\int$$

$$\varphi(r) = C_1 \ln r + C_2$$

$$\varphi(10^{-3}) = C_1\cdot\ln 10^{-3} + C_2 = 0 \implies C_2 = -C_1\cdot\ln 10^{-3}$$

$$\varphi(20\cdot 10^{-3}) = C_1\cdot\ln 2\cdot 10^{-2} + C_2 = 150$$

$$C_1 \ln 2\cdot 10^{-2} - C_1\cdot\ln 10^{-3} = 150$$

$$C_1(\ln 2\cdot 10^{-2} - \ln 10^{-3}) = 150$$

$$\boxed{\begin{aligned} C_1 &= 50.07123 \\ C_2 &= 345.8798 \end{aligned}}$$

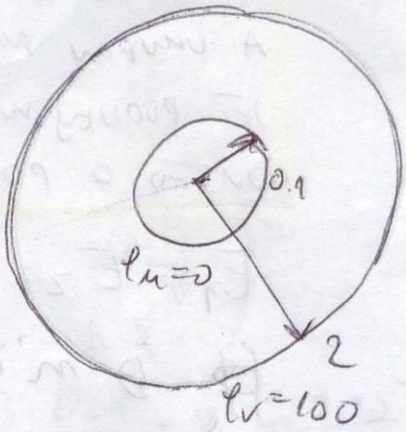
$$\boxed{\varphi(r) = 50.07123\ln r + 345.8798}$$

2.47.

$$\vec{E} = -\nabla \cdot \varphi$$

$$\vec{E} = -\frac{\partial \varphi}{\partial r}\vec{u_r} = -\frac{50.07123}{r}\vec{u_r}$$

2.48.



$$\Delta \varphi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi}{\partial r}\right) = 0$$

$$\varphi = -\frac{1}{r}C_1 + C_2$$

$$\varphi(0.1) = -\frac{1}{0.1}C_1 + C_2 = 0$$

$$C_2 = 10C_1$$

$$\varphi(2) = -\frac{1}{2}C_1 + 10C_1 = 100$$

$$\boxed{C_1 = 10.52631} \quad \boxed{C_2 = 105.26315}$$

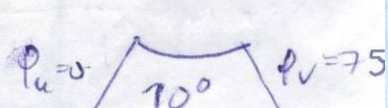
$$\boxed{\varphi(r) = -\frac{1}{r}10.52631 + 105.26315}$$

2.49.

$$\vec{E} = -\nabla\varphi = -\frac{\partial \varphi}{\partial r}\vec{u_r} = -\left(\frac{1}{r^2}10.52631\right)\vec{u_r}$$

$$\vec{E} = \varepsilon_0 \vec{D} \Rightarrow \boxed{\vec{D} = \frac{-9.31999 \cdot 10^{-11}}{r^2}\vec{u_r}}$$

2.50.



$$\varepsilon_r = 1.65$$

$$\varphi(\alpha) = C_1 \cdot \alpha + C_2$$

$$\varphi(0) = C_2 = 0$$

$$\varphi\left(\frac{\pi}{18}\right) = C_1 \cdot \frac{\pi}{18} = 75 \Rightarrow C_1 = \frac{1350}{\pi}$$

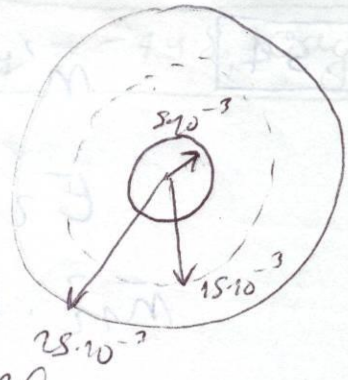
$$\varphi(\alpha) = \frac{1350}{\pi} \cdot \alpha$$

$$\vec{E} = -\nabla\varphi = -\left(\frac{1}{r}\frac{1350}{\pi}\vec{u_\alpha}\right)$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \cdot \left(-\frac{1}{r}\frac{1350}{\pi}\right)\vec{a_\alpha}$$

$$\boxed{\vec{D} = -\frac{6.27779}{r} \cdot 10^{-9}\vec{u_\alpha}\left[\frac{C}{m^2}\right]}$$

2.51. $\vec{E}(0.015) = -8.28\cdot10^{3}\,\vec{u_r}$



$\varphi = C_1 \ln r + C_2$

$\varphi(0.005) = C_1\cdot\ln 0.005 + C_2 = 0$

$C_2 = 5.29831\,C_1$

$\varphi(0.025) = \varphi_0 = C_1\cdot\ln 0.025 + 5.29831\,C_1$

$C_1 = 0.62133\,\varphi_0 \qquad C_2 = 3.29204\,\varphi_0$

$\varphi(r) = \varphi_0(0.62133\ln r + 3.29204)$

$\vec{E} = -\nabla\cdot\varphi$

$\cancel{-}8.28\cdot10^{3}\,\vec{u_r} = \cancel{-}\frac{\partial\varphi}{\partial r}\vec{u_r}$

$8.28\cdot10^{3} = \frac{\varphi_0\cdot 0.62133}{r} \Rightarrow \boxed{\varphi_0 = 199.89377\,V}$

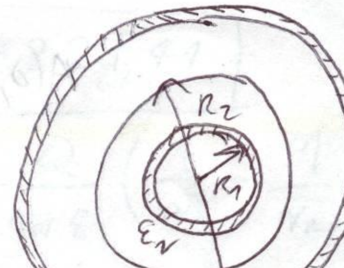
2.52. $Q = \oint_S \varepsilon\cdot\vec{E}\,\vec{n}\,dS = \iint_S \sigma\,dS \qquad \vec{n} = \vec{u_r}$

$\varepsilon\cdot E\cdot\cancel{2r\pi L} = \sigma\cdot\cancel{2r\pi L}$

$\boxed{\sigma} = \varepsilon\cdot E = \boxed{43.98667\,n\frac{C}{m^2}}$

$\vec{E} = -\nabla\varphi = -\frac{124.2}{r}\vec{u_r} \qquad E(0.025) = 4967.99984$

2.53. $R_1 = 0.08,\ +Q \qquad R_3 = 0.24,\ -Q$



$R_2 = 0.16 \qquad \varepsilon_r = 4 \qquad Q = 15\cdot10^{-9}$

за $r < R_1$, $\vec{E} = 0$

за $R_1 < r < R_2$

$\oint \vec{D}\,\vec{n}\,dS = Q$

$\varepsilon_0\,\varepsilon_r\,E\cdot4\cdot0.14^2\pi = Q$

$\boxed{E = \frac{Q}{\varepsilon_0\,\varepsilon_r\,4\cdot0.14^2\pi} = 1719.59357\frac{V}{m}}$

2.54.

$\vec{n} = \vec{a_r}$ $\quad E_1 = \frac{Q}{\varepsilon_0 \cdot \varepsilon_r \cdot 4 \cdot r^2 \pi} \cdot \vec{a_r}$

$E_2 = E_{2r} \cdot \vec{a_r}$

$\vec{n_{12}} \cdot \left(\varepsilon_0 \cdot E_{2r} \vec{a_r} - \frac{Q}{4 r^2 \pi} \vec{a_r} \right) = 0$

$\varepsilon_0 E_{2r} - \frac{Q}{4 r^2 \pi} = 0 \quad E_{2r} = \frac{Q}{\varepsilon_0 4 r^2 \pi}$

$$\boxed{E_{2r}(0.18) = 4160.99186 \frac{V}{m}}$$

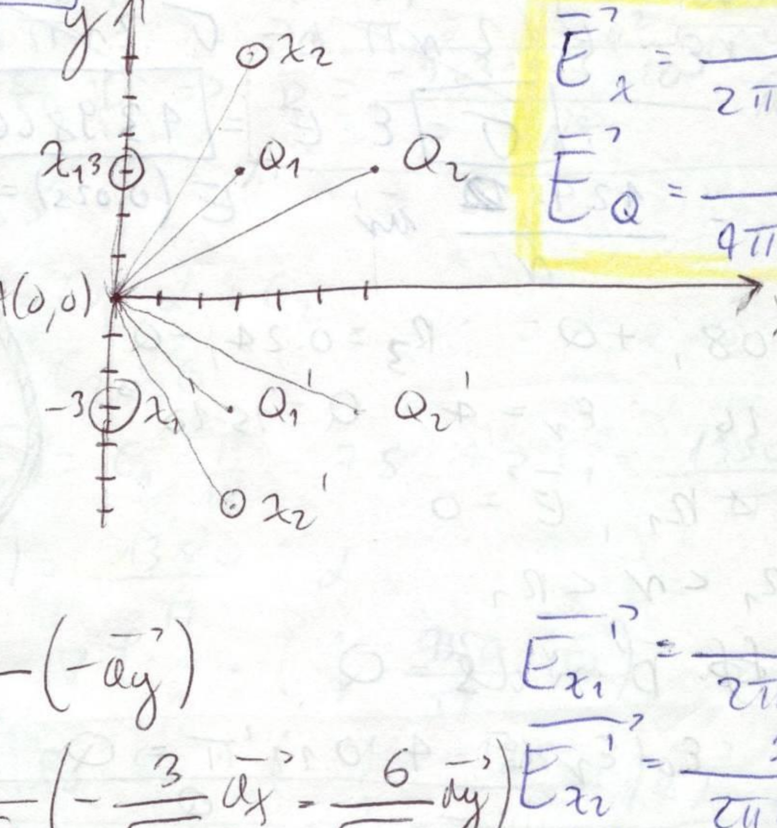
2.55. $\vec{P} = (\varepsilon_r - 1) \cdot \varepsilon_0 \cdot \vec{E}$

$$\boxed{\vec{P} = 34.97056\ n \frac{C}{m^2}}$$

2.56. $Q_{POL} = \oiint \vec{P} \cdot \vec{n} \cdot dS$ za $r = 0.16$

$\boxed{Q_{POL}} = 34.97056 \cdot 10^{-9} \cdot 4 r^2 \pi = \boxed{11.25\ nC}$

2.57.



$\vec{E}_\lambda = \frac{\lambda}{2\pi\varepsilon_0 R}(\cos\alpha \vec{a_x} + \sin\alpha \vec{a_y})$

$\vec{E}_Q = \frac{Q}{4\pi\varepsilon_0 R^2}(\cos\alpha \vec{a_x} + \sin\alpha \vec{a_y})$

ORIGINALI:

$\vec{E_{\lambda_1}} = \frac{\lambda_1}{2\pi\varepsilon_0 \cdot 3}(-\vec{a_y})$

$\vec{E_{\lambda_2}} = \frac{\lambda_2}{2\pi\varepsilon_0 \sqrt{45}}\left(-\frac{3}{\sqrt{45}}\vec{a_x} - \frac{6}{\sqrt{45}}\vec{a_y}\right)$

$\vec{E_{Q_1}} = \frac{Q_1}{4\pi\varepsilon_0 (3\sqrt{2})^2}\left(-\frac{\sqrt{2}}{2}\vec{a_x} - \frac{\sqrt{2}}{2}\vec{a_y}\right)$

$\vec{E_{Q_2}} = \frac{Q_2}{4\pi\varepsilon_0 (\sqrt{45})^2}\left(-\frac{6}{\sqrt{45}}\vec{a_x} - \frac{3}{\sqrt{45}}\vec{a_y}\right)$

SLIKE:

$\vec{E'_{\lambda_1}} = \frac{\lambda_1'}{2\pi\varepsilon_0 \cdot 3} \cdot \vec{a_y}$

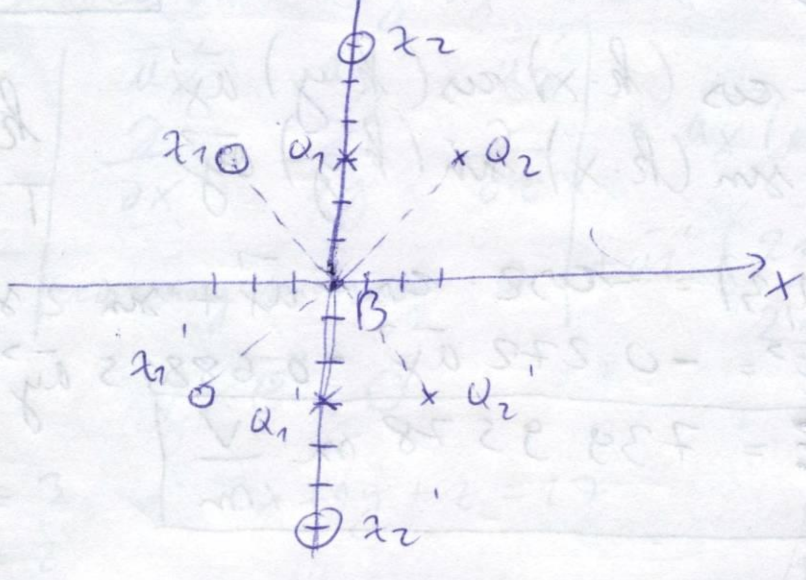
$\vec{E'_{\lambda_2}} = \frac{\lambda_2'}{2\pi\varepsilon_0 \sqrt{45}}\left(-\frac{3}{\sqrt{45}}\vec{a_x} + \frac{6}{\sqrt{45}}\vec{a_y}\right)$

$\vec{E'_{Q_1}} = \frac{Q_1'}{4\pi\varepsilon_0 (3\sqrt{2})^2}\left(-\frac{\sqrt{2}}{2}\vec{a_x} + \frac{\sqrt{2}}{2}\vec{a_y}\right)$

$\vec{E'_{Q_2}} = \frac{Q_2'}{4\pi\varepsilon_0 (\sqrt{45})^2}\left(-\frac{6}{\sqrt{45}}\vec{a_x} + \frac{3}{\sqrt{45}}\vec{a_y}\right)$

komponente u smjeru $\vec{a_x}$ se ponište, A $\boxed{E_y = -498.30\,\vec{a_y}}$

$\boxed{2.58.}$



ORIGINALI:

$$\vec{E}_{\lambda_1} = \frac{\lambda_1}{2\pi\varepsilon_0 \cdot 3\sqrt{2}}\left(\frac{\sqrt{2}}{2}\vec{a_x} - \frac{\sqrt{2}}{2}\vec{a_y}\right)$$

$$\vec{E}_{\lambda_2} = \frac{\lambda_2}{2\pi\varepsilon_0 \cdot 6}\left(-\vec{a_y}\right)$$

$$\vec{E}_{Q_1} = \frac{Q_1}{4\pi\varepsilon_0 \cdot 3^2}\left(-\vec{a_y}\right)$$

$$\vec{E}_{Q_2} = \frac{Q_2}{4\pi\varepsilon_0 (3\sqrt{2})^2}\left(-\frac{\sqrt{2}}{2}\vec{a_x} - \frac{\sqrt{2}}{2}\vec{a_y}\right)$$

$$\vec{E}_{uk} = \emptyset \cdot \vec{a_x} - 221{,}41376\,\vec{a_y}$$

SLIKE:

$$\vec{E}_{\lambda_1}' = \frac{\lambda_1'}{2\pi\varepsilon_0 3\sqrt{2}}\left(\frac{\sqrt{2}}{2}\vec{a_x} + \frac{\sqrt{2}}{2}\vec{a_y}\right)$$

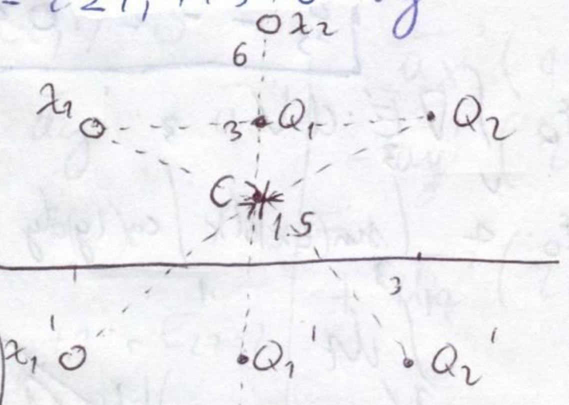
$$\vec{E}_{\lambda_2}' = \frac{\lambda_2'}{2\pi\varepsilon_0 \cdot 6} \cdot \vec{a_y}$$

$$\vec{E}_{Q_1}' = \frac{Q_1'}{4\pi\varepsilon_0 \cdot 3^2} \cdot \vec{a_y}$$

$$\vec{E}_{Q_2}' = \frac{Q_2'}{4\pi\varepsilon_0 (3\sqrt{2})^2}\left(-\frac{\sqrt{2}}{2}\vec{a_x} + \frac{\sqrt{2}}{2}\vec{a_y}\right)$$

$$\boxed{E_{uk} = 221{,}41}$$

$\boxed{2.59.}$



$$\boxed{\varphi_Q = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{1}{r_{REF}}\right) \qquad \varphi_\lambda = \frac{\lambda}{2\pi\varepsilon}\ln\frac{r_{REF}}{r}}$$

ORIGINALI:

$$\varphi_1 = \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{1.5} - \frac{1}{3}\right)$$

$$\varphi_{Q_2} = \frac{Q_2}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{1.5^2+3^2}} - \frac{1}{3}\right)$$

$$\varphi_{\lambda_1} = \frac{\lambda_1}{2\pi\varepsilon_0}\ln\frac{3}{\sqrt{3^2+1.5^2}}$$

$$\varphi_{\lambda_2} = \frac{\lambda_2}{2\pi\varepsilon_0}\ln\frac{6}{4.5}$$

$$\boxed{\varphi_{UKC} = 333.81103}$$

SLIKE:

$$\varphi_{Q_1}' = \frac{Q_1'}{4\pi\varepsilon_0}\left(\frac{1}{4.5} - \frac{1}{3}\right)$$

$$\varphi_{Q_2}' = \frac{Q_2'}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{4.5^2+3^2}} - \frac{1}{3}\right)$$

$$\varphi_{\lambda_1}' = \frac{\lambda_1'}{2\pi\varepsilon}\ln\frac{3}{\sqrt{4.5^2+3^2}}$$

$$\varphi_{\lambda_2}' = \frac{\lambda_2'}{2\pi\varepsilon}\ln\frac{6}{7.5}$$

$\boxed{2.60.}$ $A = Q \cdot (\varphi_C - \varphi_0)$

$A = 15 \cdot 10^{-12} \cdot 333.81103$ $\boxed{A = 5\,n\,J}$

$\boxed{2.61.}$ $\vec{E} = -\cos(k \cdot x)\cos(k \cdot y)\,\vec{a_x} + \sin(k \cdot x)\sin(k \cdot y)\,\vec{a_y}$

$k = 2$ rad/m

$T(1, 2, 3)$

$\vec{E}(1,2,3) = -\cos 2 \cdot \cos 4\,\vec{a_x} + \sin 2 \sin 4\,\vec{a_y}$

$\vec{E} = -0.272\,\vec{a_x} - 0.68815\,\vec{a_y}$

$\boxed{E = 739.9578\ m\frac{V}{m}}$

$\boxed{2.62.}$ $E_x = -\cos 2x \cos 2y$

$E_y = \sin 2x \sin 2y$

$\frac{dE_x}{dx} = -(-\sin(2x) \cdot \cos(2y) \cdot 2) = 2\sin(2x)\cos(2y)$

$\frac{dE_y}{dy} = \sin(2x)\cos(2y) \cdot 2 = 2\sin(2x)\cos(2y)$

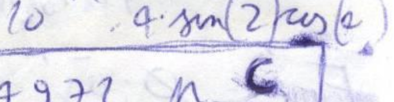
$\nabla \cdot \vec{E} = 4\sin(2x)\cos(2y)$

$\rho_s = \varepsilon_0 \nabla \cdot \vec{E}(1,2,4) = 8.854 \cdot 10^{-12} \cdot 4 \cdot \sin(2)\cos(4)$

$\boxed{\rho_s = -21.04972\ p\frac{C}{m^3}}$

$\boxed{2.63.}$ $\oint \vec{D} \cdot \vec{n}\, dS = \varepsilon_0 \int_V \nabla \cdot \vec{E}\, dV$

$\Phi = \varepsilon_0 \cdot 4 \int_1^3 \sin(2x)\,dx \int_1^3 \cos(2y)\,dy \int_1^3 dz$



$\Phi = 2 \cdot \varepsilon_0 \cdot 4\left(-\cos 2x \cdot \frac{1}{2}\right)\Big|_{x=1}^{3} \cdot \frac{1}{2}(\sin(2y))\Big|_{y=1}^{3}$

$\Phi = 2\,\varepsilon_0\,(-1.37631) \cdot (-1.18871)$

$\boxed{\Phi = 28.97095\ p\ C}$

$\boxed{2.64.}$ $\nabla \times \vec{E}$ u $T(2,1,2)$

$$\vec{E} = -\cos(2x)\cos(2y)\,\vec{a_x} + \sin(2x)\sin(2y)\,\vec{a_y}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\cos 2x \cos 2y & \sin 2x \sin 2y & 0 \end{vmatrix} = \vec{a_x}(0) - \vec{a_y}(0) + \vec{a_z}\big(2\cos(2x)\sin(2y) - 2\cos(2x)\sin(2y)\big)$$

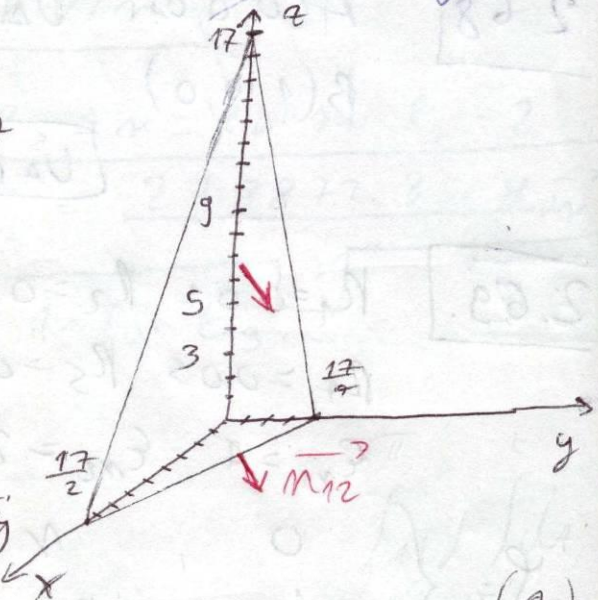
$$\nabla \times \vec{E} = 0$$

$\boxed{2.65.}$ $\varepsilon_{r1} = 3$ $\quad 2x + 4y + z = 17$

$\varepsilon_{r2} = 2$

$$\vec{E_1} = -2\vec{a_x} - \vec{a_y} + 2\vec{a_z}$$

$$\vec{n_{12}} = \frac{2\vec{a_x} + 4\vec{a_y} + \vec{a_z}}{\sqrt{21}}$$



$$\vec{n_{12}} \cdot \Big[\big(D_{2x} - (-2\cdot 3\cdot \varepsilon_0)\big)\vec{a_x} + \big(D_{2y} - (-1\cdot 3\cdot\varepsilon_0)\big)\vec{a_y} + \big(D_{2z} - (2\cdot 3\cdot \varepsilon_0)\big)\vec{a_z}\Big] = 0 \qquad (4)$$

$$\frac{2}{\sqrt{21}}(D_{2x} + 6\varepsilon_0) + \frac{4}{\sqrt{21}}(D_{2y} + 3\varepsilon_0) + \frac{1}{\sqrt{21}}(D_{2z} - 6\varepsilon_0) = 0 \,\Big|\, \frac{\sqrt{21}}{4}$$

$$\vec{n_{12}} \times (\vec{E_2} - \vec{E_1}) = 0$$

$$\frac{1}{\sqrt{21}}\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 2 & 4 & 1 \\ E_{2x}+2 & E_{2y}+1 & E_{2z}-2 \end{vmatrix} = \vec{a_x}(4E_{2z} - 8 - E_{2y} - 1) - \vec{a_y}(2E_{2z} - 4 - E_{2x} - 2) + \vec{a_z}(2E_{2y} + 2 - 4E_{2x} - 8) = 0$$

$E_{2y} = 4E_{2z} - 9 \;(1)$; $\quad 2(\varepsilon_{r2}E_{2x} + 6) + 4(\varepsilon_{r2}E_{2y} + 3) + (\varepsilon_{r2}E_{2z} - 6) = 0$

$E_{2x} = 2E_{2z} - 6 \;(2)$ $\quad 8E_{2z} - 24 + 12 + 32E_{2z} - 72 + 12 + 2E_{2z} - 6 = 0$

$(1),(2) \Rightarrow (4)$ $\quad E_{2x} = -2.28571$

$$\boxed{D_{2x} = -40.47542\ \mu\frac{C}{m^2}}$$

2.66. $E_{2y} = -1.57142$

$$\boxed{D_{2y} = -27.82685\ \mu \frac{C}{m^2}}$$

2.67. $E_{2z} = 1.85714$

$$\boxed{D_{2z} = 32.88628\ \mu \frac{C}{m^2}}$$

2.68 $A(0,0,0)$ $\quad U_{AB} = -\int_B^A \vec{E}_1' \, d\vec{l}$

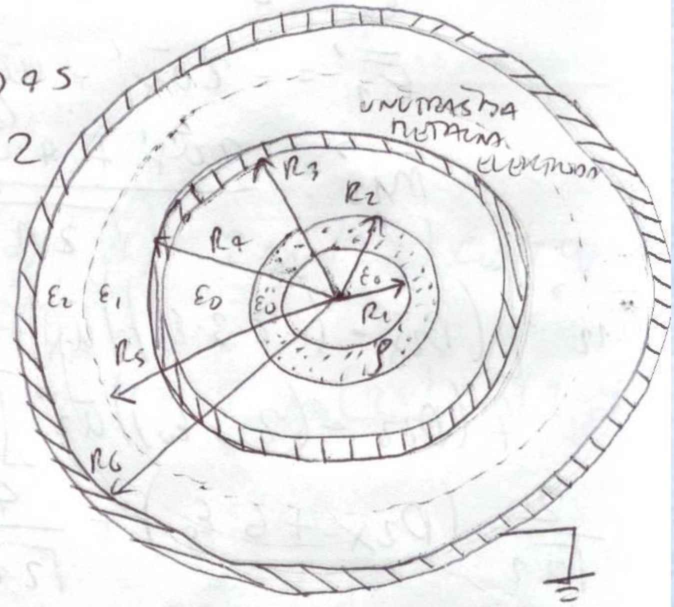
$B(1,0,0)$

$$\boxed{U_{AB} =} -\int_1^0 (-2)\, dx = \boxed{-2\,V}$$

2.69. $R_1 = 0.01 \quad R_2 = 0.03 \quad R_3 = 0.045$

$R_4 = 0.05 \quad R_5 = 0.1 \quad R_6 = 0.2$

$\varepsilon_{r1} = 4 \quad \varepsilon_{r2} = 2$



$$\rho = \begin{cases} 0 , & r < R_1 \\ \dfrac{1}{r^4 + 81} , & R_1 \le r \le R_2 \\ 0 , & r > R2 \end{cases}$$

za $\quad r < R_1 , \quad \vec{E} = 0$

za $\quad R_1 \le r \le R_2 \quad$; za $r < R_3$

$$\varepsilon_0 \cdot E \cdot 2r\pi \cdot L = \int_0^{2\pi} d\alpha \int_0^{L} dz \int_{0.01}^{0.03} \frac{r}{r^4 + 3^4}\, dr$$

$$\varepsilon_0 \cdot E \cdot \cancel{2} r \cancel{\pi} \cdot \cancel{L} = 2\pi \cdot L \cdot \frac{1}{2 \cdot 3^2} \operatorname{arctg} \left(\frac{r}{a}\right)^2 \Bigg|_{r=0.01}^{0.03}$$

$$\varepsilon_0 \cdot E \cdot r = \frac{1}{2 \cdot 9} \cdot \left(\operatorname{arctg} \frac{0.03^2}{9} - \operatorname{arctg} \frac{0.01^2}{9} \right)$$

$$\boxed{\vec{E}_1 = \frac{557749.7014}{r}\, \vec{a}_r}$$

$\vec{n}_{12} = \vec{a}_r$ zapravo ε_{r1} u zadatku jer je $\varepsilon_{r1} = 1$

$\vec{n}_{12}(\varepsilon_0 \cdot \varepsilon_{r2} \cdot \vec{E}_2 - \varepsilon_0 \cdot \varepsilon_{r1} \vec{E}_1) = 0$

$$4 \cdot E_2 - \frac{557749.7014}{r} = 0$$

$$E_2 = \frac{139436.1754}{r} \quad \text{za } r = 0.09$$

$$\boxed{E_2 = 1549.29083 \ kV/m}$$

2.70. traži se $\vec{P}$ u prostoru $R_5 \le r \le R_6$ za $\varepsilon_r = 2$

$$\vec{E}_3 = \frac{\varepsilon_{r2}}{\varepsilon_{r3}} \cdot \vec{E}_2 \Rightarrow \boxed{\vec{E}_3 = \frac{278872.3508}{r} \vec{a}_r}$$

$\varepsilon_{r2}\ \varepsilon_{r3}$

$$\vec{P} = (\varepsilon_{r3} - 1) \cdot \varepsilon_0 \cdot \vec{E}_3 = 1 \cdot \varepsilon_0 \cdot \vec{E}_3$$

za $r = 0.12$ $\boxed{P = 20.57613 \ \mu \frac{C}{m^2}}$

$$\int_0^{2\pi} d\alpha \int_0^{L} dz$$

2.71. $W = \frac{1}{2} C \cdot U^2$

$$W = \frac{1}{2} \varepsilon_0 \int_V (\varepsilon_{r2} E_2^2 + \varepsilon_{r3} E_3^2) dV$$

$$W = \frac{1}{2} \varepsilon_0 \left(\varepsilon_{r2} \int_{0.05}^{0.1} \frac{1.94424 \cdot 10^{10}}{r^2} \cdot r\,dr + \varepsilon_{r3} \int_{0.1}^{0.2} \frac{7.77697 \cdot 10^{10}}{r^2} r\,dr \right)$$

TRAŽI SE ENERGIJA PO JEDINICI DULJINE KONDENZATORA

$$W = \frac{1}{2} \varepsilon_0 \left(\varepsilon_{r2} \cdot 1.94424 \cdot 10^{10} \ln \frac{0.1}{0.05} + \varepsilon_{r3} \cdot 7.77697 \cdot 10^{10} \ln \frac{0.2}{0.1} \right)$$

$$\boxed{W} = \frac{1}{2} \cdot 2\pi \cdot \cancel{L} \cdot \varepsilon_0 (5.39057 \cdot 10^{10} + 1.07811 \cdot 10^{11}) = \boxed{4.49827 \frac{J}{m}}$$

2.72. $U = -\int_B^A \vec{E} \cdot d\vec{l} = \int_{R_4}^{R_5} E_2 \cdot dr + \int_{R_5}^{R_6} E_3 \, dr$

$$\boxed{U =} \ 96649.79185 + 193299.5837 = \boxed{289949.3755}$$

$$\boxed{C =} \ \frac{2W}{U^2} = \frac{9}{8.40706 \cdot 10^{10}} = \boxed{107.05283 \ p \frac{F}{m}}$$

III. MAGNETOSTATIKA

$\boxed{3.1.}$ $\vec{A} = 2.5\,\vec{a}_\theta + 5\,\vec{a}_z$ $\qquad T(2, \frac{\pi}{6}, 0)$

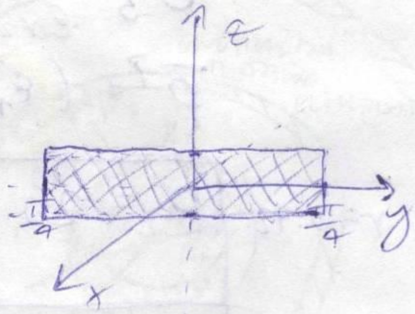
$A_\theta = 2.5 \qquad A_z = 5$

$\boxed{\vec{B} = \nabla \times \vec{A}}$ $\qquad \boxed{B_z = \frac{1}{r}\left(\frac{\partial}{\partial r}(r \cdot A_\theta) - \frac{\partial A_r}{\partial \theta}\right)}$

$B_z = \frac{1}{r} \cdot 2.5 \qquad \boxed{B_z(T) = 1.25}$

$\boxed{3.2}$ $\vec{J} = 100y\sin(2y)\,\vec{a}_x \left[\frac{A}{m^2}\right]$

$\boxed{I = \iint_S \vec{J} \cdot \vec{n} \cdot dS}$ $\qquad \vec{n} = \vec{a}_x$



$I = \iint_S 100y\sin(2y)\,\vec{a}_x \vec{a}_x\,dy\,dz$

$I = 100 \int_{-0.01}^{0.01} dz \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} y \cdot \sin(2y)\,dy$

$\boxed{\int u \cdot dv = u \cdot v - \int v \cdot du}$ $\qquad u = y \quad du = dy$

$v' = \sin(2y) \quad dv = \sin(2y)\,dy$

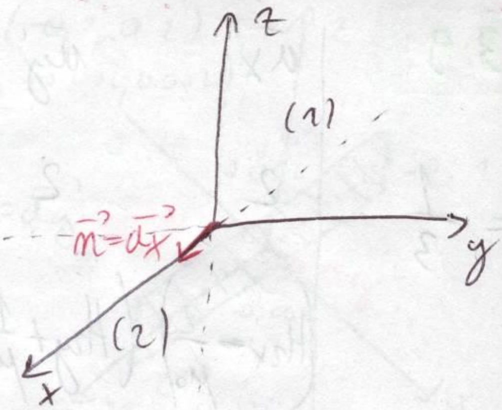
$v = -\frac{1}{2} \cdot \cos(2y)$

$\boxed{\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} y \cdot \sin(2y)\,dy} = y \cdot \left(-\frac{1}{2} \cdot \cos 2y\right)\Big|_{y=-\frac{\pi}{4}}^{\frac{\pi}{4}} - \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} -\frac{1}{2}\cos(2y)\,dy = \underbrace{\frac{\pi}{4}\left[-\frac{1}{2} \cdot \cos\left(2 \cdot \frac{\pi}{4}\right)\right] - \left(-\frac{\pi}{4}\right)\left[-\frac{1}{2} \cdot \cos\left(2 \cdot \left(-\frac{\pi}{4}\right)\right)\right]}_{0}$

$+ \frac{1}{2}\left(\frac{1}{2}\sin 2y\right)\Big|_{y=-\frac{\pi}{4}}^{y=\frac{\pi}{4}}$

$= \frac{1}{4}\left(\sin\frac{\pi}{2} - \left(\sin\left(-\frac{\pi}{2}\right)\right)\right) = \frac{1}{4} \cdot 2 = \boxed{\frac{1}{2}}$

$$\boxed{I} = 100 \cdot 2 \cdot 0.01 \cdot \frac{1}{2} = \boxed{1\,[A]}$$



3.3. $\vec{K} = 6.5\,\vec{a_z}\left[\frac{A}{m}\right]$

(1) za $x<0$, $H_1 = 10\,\vec{a_y}\left[\frac{A}{m}\right]$ $\quad \vec{n} = \vec{a_x}$

(2) za $x>0$, $H_2 = ?\ \left[\frac{A}{m}\right]$

$$\boxed{\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{K}}$$

$$\vec{a_x} \times \left[H_{2x}\vec{a_x} + (H_{2y} - 10)\vec{a_y} + H_{2z}\vec{a_z}\right] = 6.5\,\vec{a_z}$$

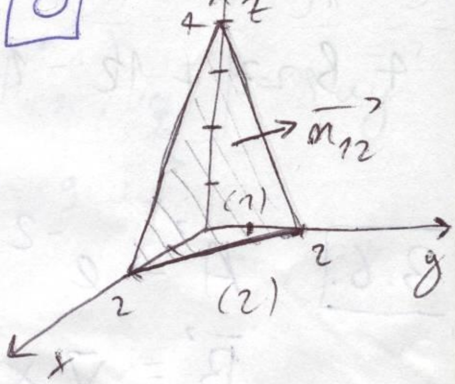
$$\vec{a_z}(H_{2y} - 10) - H_{2z}\vec{a_y} = 6.5\,\vec{a_z}, \quad H_{2x} = H_{2z} = 0$$

$$H_{2y} - 10 = 6.5 \qquad \boxed{H_{2y} = 16.5}$$

3.4. $\vec{A} = \cos x \sin y\,\vec{a_x} + \sin x \cos y\,\vec{a_y}\ [Tm]$

$$\boxed{\vec{B}} = \nabla \times \vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos x \sin y & \sin x \cos y & 0 \end{vmatrix} = \vec{a_z}(\cos x \cos y - \cos x \cos y) = \boxed{0}$$

u $T(1,1,1)$



3.5. $4x + 4y + 2z = 8$

(1) $\vec{H_1} = \frac{1}{\mu_0}(2\vec{a_x} - \vec{a_y})$, $\mu_{r1} = 4$

(2) $\vec{H_2} = ?$, $\mu_{r2} = 3$

$$\vec{n} = 4\vec{a_x} + 4\vec{a_y} + 2\vec{a_z}$$

$$\vec{n}_{12} = \frac{4\vec{a_x} + 4\vec{a_y} + 2\vec{a_z}}{\sqrt{36}} = \frac{1}{3}(2\vec{a_x} + 2\vec{a_y} + \vec{a_z})$$

$$\boxed{H = \frac{B}{\mu_0 \cdot \mu_r}}$$

$$\vec{n}_{12} \times (\vec{H_2} - \vec{H_1}) = 0$$

$$\frac{1}{3}(2\vec{a_x} + 2\vec{a_y} + \vec{a_z}) \times \left[\left(H_{2x} - \frac{1}{\mu_0}\cdot 2\right)\vec{a_x} + \vec{a_y}\left(H_{2y} + \frac{1}{\mu_0}\right) + \vec{a_z}(H_{2z})\right]$$

$$= \frac{1}{3} \cdot \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 2 & 2 & 1 \\ \left(H_{2x} - \frac{2}{\mu_0}\right) & \left(H_{2y} + \frac{1}{\mu_0}\right) & H_{2z} \end{vmatrix} = \frac{1}{3}\left[\vec{a}_x\left(2H_{2z} - H_{2y} - \frac{1}{\mu_0}\right) + -\vec{a}_y\left(2H_{2z} - H_{2x} + \frac{2}{\mu_0}\right) + \vec{a}_z\left(2H_{2y} + \frac{2}{\mu_0} - 2H_{2x} + \frac{4}{\mu_0}\right)\right] = 0$$

$$(-1-) \quad 2H_{2z} - H_{2y} - \frac{1}{\mu_0} = 0 \Rightarrow H_{2z} = \frac{1}{2}\left(H_{2y} + \frac{1}{\mu_0}\right) \Rightarrow \boxed{B_{2z} = \frac{B_{2y}}{2} + \frac{3}{2}}$$

$$(-2-) \quad 2H_{2z} - H_{2x} + \frac{2}{\mu_0} = 0 \Rightarrow H_{2x} = 2\left(H_{2z} + \frac{1}{\mu_0}\right) \Rightarrow \boxed{B_{2x} = 2B_{2z} + 6}$$

$$(-3-) \quad 2H_{2y} + \frac{6}{\mu_0} - 2H_{2x} = 0$$

$$\vec{n}_{12} \cdot (\vec{B}_2 - \vec{B}_1) = \frac{1}{3}(2\vec{a}_x + 2\vec{a}_y + \vec{a}_z) \cdot \left[\left(B_{2x} - \frac{2}{\mu_0}\mu_0 \cdot 4\right)\vec{a}_x + \left(B_{2y} + \frac{1}{\mu_0}\mu_0 \cdot 4\right)\vec{a}_y + B_{2z}\vec{a}_z\right] = 0$$

$$(-4-) \quad 2B_{2x} - 16 + 2B_{2y} + 8 + B_{2z} = 0$$

$$(-1-), (-2-) \Rightarrow (-4-)$$

$$4B_{2z} + 12 - 16 + 2B_{2y} + 8 + \frac{B_{2y}}{2} + \frac{3}{2} = 0$$

$$B_{2y}\left(\frac{9}{2}\right) = -\frac{23}{2} \qquad \boxed{B_{2y}} = -\frac{23}{9} = \boxed{-2.56}$$

3.6. $\vec{A} = e^{-2z}[\sin(0.5\alpha)]\vec{a}_\alpha$

$\vec{B} = \nabla \times \vec{A} \qquad B_r\left(0.8, \frac{\pi}{3}, 0.5\right) = ?$

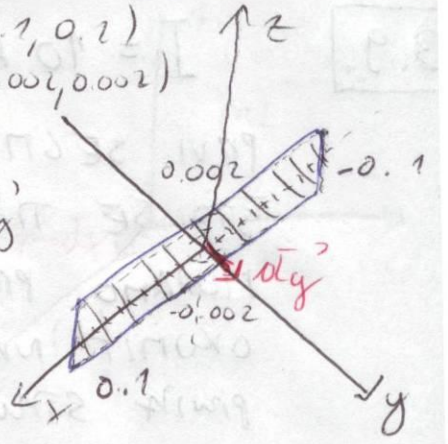
$$B_r = \frac{1}{r} \cdot \frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z}$$

$$B_r = -\left(-2e^{-2z}\sin(0.5\alpha)\right)$$

$$B_r = 2e^{-2\cdot 0.5}\sin\frac{\pi}{6} = \frac{1}{2}$$

$$\boxed{B_r} = e^{-1} = \boxed{0.36787}$$

3.7. $\vec{J} = 100|x|\vec{a_y}$ $\quad x\in(-0.1,\,0.1)$, $z\in(-0.002,\,0.002)$



$$\vec{J} = \begin{cases} -100x, & x<0 \\ 100x, & x>0 \end{cases} \cdot \vec{a_y} \qquad \vec{n} = \vec{a_y}$$

$$I = \iint_S 100|x|\vec{a_y}\cdot\vec{a_y}\,dx\,dz$$

$$I = 100\int_{-0.1}^{0.1}|x|\,dx\int_{-0.002}^{0.002}dz$$

$$I = 100\left(\int_{-0.1}^{0}(-x)\,dx + \int_{0}^{0.1}x\,dx\right)\cdot 2\cdot 0.002$$

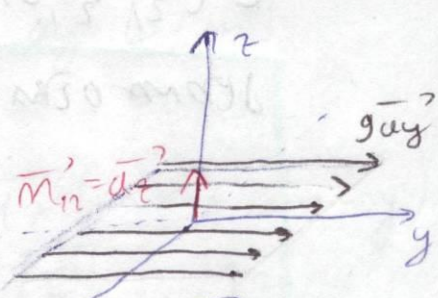
$$I = 100\left(-\frac{1}{2}x^2\Big|_{x=-0.1}^{0} + \frac{1}{2}x^2\Big|_{x=0}^{0.1}\right)\cdot 0.004$$

$$I = 0.4\cdot\left(\frac{1}{2}0.1^2 + \frac{1}{2}0.1^2\right) = 0.004 \qquad \boxed{I[mA] = 4}$$

3.8. $\vec{K} = 9\vec{a_y}$

$z<0$ $\quad \vec{H_1} = ?$, $\mu_{r1} = 4$

$z>0$ $\quad \vec{H_2} = 14.5\vec{a_x} + 8\vec{a_z}$, $\mu_{r2} = 3$



$$\vec{m_{12}}\times\left[(14.5 - H_{1x})\vec{a_x} + (-H_{1y})\vec{a_y} + (8 - H_{1z})\vec{a_z}\right] = 9\vec{a_y}$$

$$(14.5 - H_{1x})\vec{a_y} - \vec{a_x}(-H_{1y}) = 9\vec{a_y}$$

$$14.5 - H_{1x} = 9$$

$$\boxed{H_{1x} = 5.5}$$

$$\vec{m_{12}}\cdot(\vec{B_2} - \vec{B_1}) = \vec{a_z}\cdot(\vec{H_2}\mu_0\mu_{r2} - \vec{H_1}\mu_0\mu_{r1})$$

$$= \mu_0\vec{a_z}\cdot\left[(14.5\cdot 3 - 5.5\cdot 4)\vec{a_x} + (-H_{1y}\mu_{r1})\vec{a_y} + (8\cdot 3 - H_{1z}\cdot 4)\vec{a_z}\right] = 0$$

$$8\cdot 3 - H_{1z}\cdot 4 = 0$$

$$\boxed{H_{1z} = 6}$$

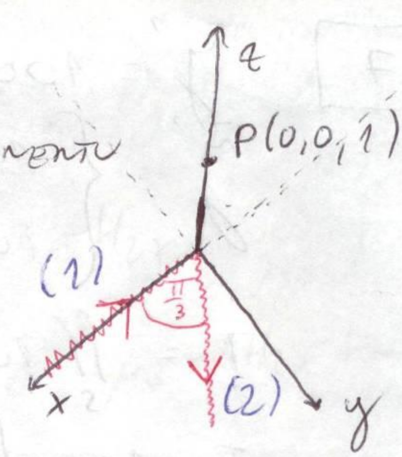
$$\boxed{\vec{H_1} = 5.5\vec{a_x} + 6\vec{a_z}}$$

$\boxed{\vec{H_{1n}} = H_{1n}\vec{n}}$ $\qquad$ $\boxed{\vec{H_{1t}} = \vec{H_1} - \vec{H_{1n}}}$

$\vec{H_{1n}} = 6\vec{a_z}$ $\qquad$ $\vec{H_{1t}} = 5.5\vec{a_x}$

3.11. $\boxed{H_{1n} = 6}$ $\qquad$ $\boxed{H_{1t} = 5.5}$

3.9. $I = 10\,A$, $R = 1\,m$, $\alpha = \frac{\pi}{3}$



PRVI SEGMENT SADRŽI SAMO y-KOMPONENTU

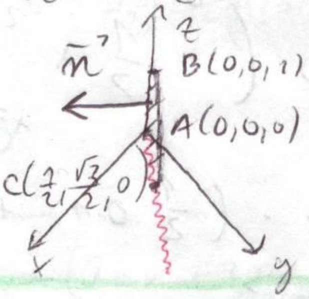
ŠTO SE TIČE SEGMENTA (2), MORAMO PRONAĆI JEDINIČNI VEKTOR OKOMIT NA PRAVAC KOJIM SE PRUŽA STRUJNICA I NA PRAVAC NA KOJEM SE NALAZI TOČKA U KOJOJ TRAŽIMO x-KOMPONENTU JAKOSTI MAGNETNOG POLJA

IZABEREMO TRI TOČKE KOJE SE NALAZE NA ISTOJ RAVNINI I TRAŽIMO JEDINIČNI VEKTOR OKOMIT NA TU RAVNINU

$A(0,0,0)$

$B(0,0,1)$

$C\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$



JEDNADŽBA RAVNINE ODREĐENA S TRI TOČKE:

$$(2) \quad \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = x\left(-\frac{\sqrt{3}}{2}\right) - y\left(-\frac{1}{2}\right) + z \cdot 0 = 0$$

$\varphi = \frac{\pi}{2}, \psi = 0$ → KUTEVI KOJE ZATVARAJU SPOJNICE $\overline{PA}$ i $\overline{PZ}$, GDJE JE A ISHODIŠTE, A Z TOČKA U BESKONAČNOSTI

$$-\frac{\sqrt{3}}{2} x + \frac{1}{2} y = 0$$

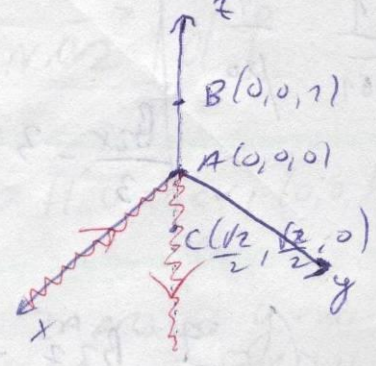
VIDIMO DA BI JEDINIČNI VEKTOR S OVIM KOEFICIJENTIMA VRIJEDIO DA JE STRUJNICA S DRUGIM SMJEROM STRUJE (SUPROTNIM).

$$\vec{n}' = \frac{\sqrt{3}}{2}\vec{a_x}' - \frac{1}{2}\vec{a_y}'$$

$$\vec{H_2}' = \frac{I}{4\pi\cdot r}\cdot(\sin 0^\circ + \sin 90^\circ)\cdot\left(\frac{\sqrt{3}}{2}\vec{a_x}' - \frac{1}{2}\vec{a_y}'\right)$$

$$\boxed{H_{2x}} = \frac{5\sqrt{3}}{4\pi} = \boxed{0.69}$$

3.10.



$$\boxed{H_{1y} = \frac{10}{4\pi\cdot 1}\cdot 1\,\cancel{(\vec{a_y})}} = \boxed{\frac{10}{4\pi}}$$

A(0,0,0)

B(0,0,1)

$C\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

$$\begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y$$

$$\vec{n}' = \frac{\sqrt{2}}{2}\vec{a_x}' - \frac{\sqrt{2}}{2}\vec{a_y}'$$

$$\vec{H_2}' = \frac{10}{4\pi\cdot 1}\cdot 1\left(+\frac{\sqrt{2}}{2}\vec{a_x}' - \frac{\sqrt{2}}{2}\vec{a_y}'\right)$$

$$\boxed{H_{2y} = -\frac{5\sqrt{2}}{4\pi}} \qquad \boxed{H_y = \cancel{H_{1y}+}\; H_{2y}} = \boxed{0.23307}$$

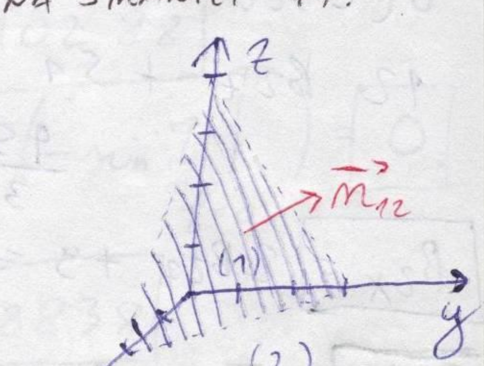
3.11. RJEŠENJE U SKLOPU ZADATKA 3.8. NA STRANICI 47.

3.12.

$$6x + 4y + 3z = 12$$

$$\vec{H_1}' = \frac{1}{\mu_0}(3\vec{a_x}' - 0.5\vec{a_y}'),\ \mu_{r1} = 5,\ \mu_{r2} = 3$$

$$B_{2x} = ?$$



$$\vec{n}_{12}' = \frac{1}{\sqrt{61}}(6\vec{a_x}' + 4\vec{a_y}' + 3\vec{a_z}')$$

$$\vec{n}_{12}' \times \left[\left(\frac{B_{2x}}{\mu_0\cdot\mu_{r2}} - H_{1x}\right)\vec{a_x}' + \left(\frac{B_{2y}}{\mu_0\cdot\mu_{r2}} - H_{1y}\right)\vec{a_y}' + \frac{B_{2z}}{\mu_0\cdot\mu_{r2}}\vec{a_z}'\right] = 0$$

$$\frac{1}{\sqrt{61}}\cdot\frac{1}{\mu_0}\cdot(6\vec{a_x}' + 4\vec{a_y}' + 3\vec{a_z}') \times \left[\left(\frac{B_{2x}}{3} - 3\right)\vec{a_x}' + \left(\frac{B_{2y}}{3} + \frac{1}{2}\right)\vec{a_y}' + \frac{B_{2z}}{3}\vec{a_z}'\right] = 0$$

$$\frac{1}{\sqrt{61}}\cdot\frac{1}{\mu_0}\cdot\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 6 & 4 & 3 \\ \frac{B_{2x}}{3}-3 & \frac{B_{2y}}{3}+\frac{1}{2} & \frac{B_{2z}}{3} \end{vmatrix} = \left[\left(\frac{4}{3}B_{2z}-B_{2y}-\frac{3}{2}\right)\vec{a_x} + \left(2B_{2z}-B_{2x}+9\right)\vec{a_y} + \left(2B_{2y}+3-\frac{4}{3}B_{2x}+12\right)\vec{a_z} \right]$$

$$= \begin{cases} \frac{4}{3}B_{2z}-B_{2y}-\frac{3}{2}=0 \Rightarrow B_{2y}=\frac{4}{3}B_{2z}-\frac{3}{2} \\ 2B_{2z}-B_{2x}+9=0 \Rightarrow B_{2x}=2B_{2z}+9 \end{cases}$$

$$\vec{n_{12}}\cdot(\vec{B_2}-\vec{B_1}) = \frac{1}{\sqrt{61}}\left(6\vec{a_x}+4\vec{a_y}+3\vec{a_z}\right)\cdot\left[(B_{2x}-15)\vec{a_x}+(B_{2y}+2.5)\vec{a_y}+B_{2z}\vec{a_z}\right]=0$$

$$6B_{2x}-90+4B_{2y}+10+3B_{2z}=0$$

$$6(2B_{2z}+9)-90+4\left(\frac{4}{3}B_{2z}-\frac{3}{2}\right)+10+3B_{2z}=0$$

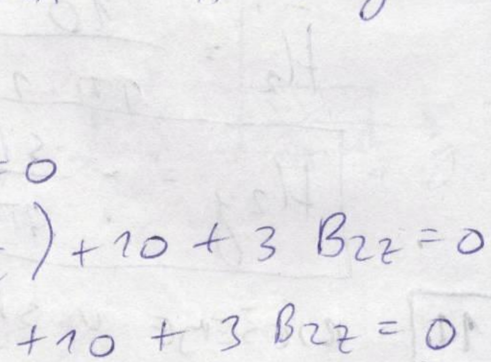
$$12B_{2z}+54-90+\frac{16}{3}B_{2z}-6+10+3B_{2z}=\boxed{0}$$

$$\frac{45}{3}B_{2z}+\frac{16}{3}B_{2z}-32=0 \Rightarrow B_{2z}=\frac{96}{91}$$

$$\boxed{B_{2x}}=2B_{2z}+9=2\cdot\frac{96}{61}+9=\boxed{12.14754}$$

3.13.

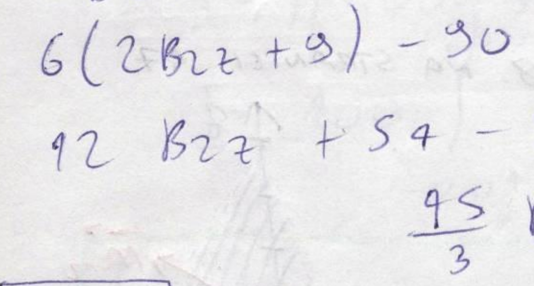
$I = 1\,A \qquad R = 1$



STRUJNICU DIJELIMO NA TRI SEGMENTA.
UDJELAS SEGMENATA (1) i (3) JE JEDNAK

$H_1 = H_3 \qquad \vec{n} = \vec{a_x}$

(1), (3)



$$\boxed{H_1 = H_3 = \frac{I}{4\pi r}\left(\sin\varphi+\sin\psi\right)\vec{n}}$$

JEDINIČNI VEKTOR SMJER JAKOSTI MAG. POLJA U PROMATRANOJ TOČKI

$\varphi = \frac{\pi}{2} \qquad \psi = 360^\circ - \alpha$

$\psi = 360^\circ - 45^\circ$

$\psi = 315^\circ$

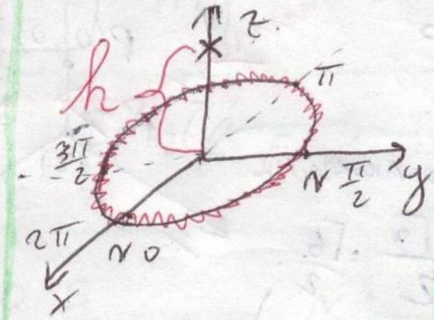
$$H_1 = \frac{1}{4\pi\cdot 1}\cdot\left(\sin 90^\circ + \sin 315^\circ\right)\vec{a_x}$$

$$H_1 = \frac{1}{4\pi}\left(1 - \frac{\sqrt{2}}{2}\right)\vec{a_x}$$

$$\boxed{H_1 = 0.0233 = H_3}$$

(2)

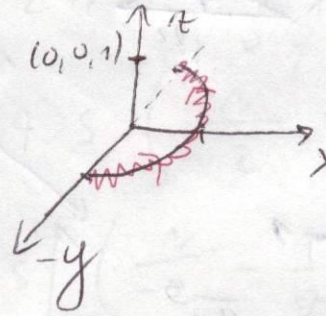
PRIKAZ STRUJNICE KOJA ZATVARA PUNI KRUG



$$\vec{H} = \frac{I \cdot N}{4\pi\left(\sqrt{h^2+N^2}\right)^3}\int_{\alpha_1 + N\vec{a_z}}^{\alpha_2}\left[h\cos\varphi\,\vec{a_x} + h\sin\varphi\,\vec{a_y}\right] d\varphi$$

FORMULA ZA IZRAČUN $\vec{H}$ U TOČKI $(0,0,h)$

U ZADATKU



X-KOMPONENTA JE ZAPRAVO Y-KOMPONENTA IZ ORIGINALNE FORMULE

$$H_{2x} = \frac{1 \cdot 1}{4\pi\left(\sqrt{1+1}\right)^3}\int_0^{\pi} \sin\varphi\, d\varphi$$

$$H_{2x} = \frac{1}{4\pi 2^{\frac{3}{2}}}\left(-\cos\varphi\Big|_{\varphi=0}^{\pi}\right) = \frac{1}{4\pi 2^{\frac{3}{2}}}\left(-\left(\cos\pi - \cos 0\right)\right)$$

$$\boxed{H_{2x}} = \frac{1}{4\pi 2^{\frac{3}{2}}} \cdot 2 = \frac{2^{-\frac{1}{2}}}{4\pi} = \boxed{0.05626}$$

$$\boxed{H_{UKx}} = H_1 + H_{2x} + H_3 = \boxed{0.10288}$$

3.14. $\boxed{H_{2y}} = \frac{1}{4\pi 2^{1.5}}\int_0^{\pi} -\cos\varphi = \frac{1}{2^{3.5}\pi}\left(-\sin\varphi\Big|_{\varphi=0}^{\pi}\right) = \boxed{0}$

3.15. $\boxed{H_{2z}} = \frac{1}{2^{3.5}\pi}\int_0^{\pi} 1 \cdot d\varphi = \boxed{0.08838}$

3.16. $\vec{H} = (y\cos(ax))\vec{a_x} + (y+e^x)\vec{a_z}$

$\vec{J} = \nabla \times \vec{H}$

$$\boxed{\vec{J}} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\cos(ax) & 0 & y+e^x \end{vmatrix} = \boxed{\vec{a_x} - e^x\vec{a_y} - \cos(ax)\vec{a_z}}$$

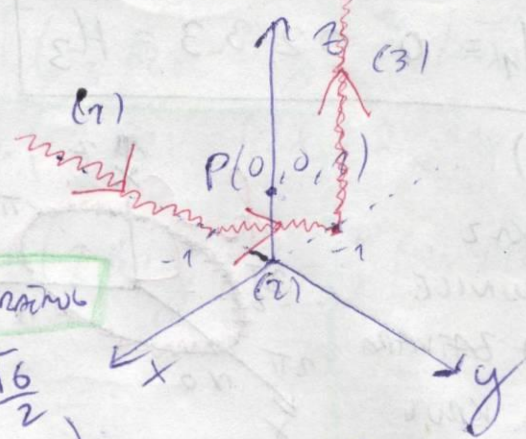
3.17. $\vec{J}$ u yz-ravnini $\Rightarrow$ $\boxed{\vec{J}(x=0)} = \sqrt{3} = \boxed{1.73}$

$\boxed{3.18.}$ $I = 10\,A$

$H_{UKx} = ?$

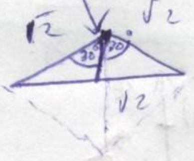
(1) $H_1 = \frac{10}{4\pi}\left(1 - \frac{\sqrt{2}}{2}\right)\cdot \vec{a_x}$

$\boxed{H_{1x} = 0.233}$



VISINA JEDNAKOSTRANIČNOG TROKUTA

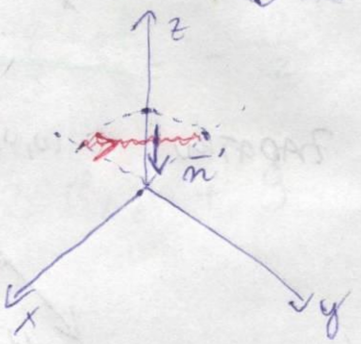
$r = \frac{a\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$



(2) $A(0, -1, 0)$

$B(-1, 0, 0)$ $\quad \varphi = \psi = 30^\circ$

$C(0, 0, 1)$



$$\begin{vmatrix} x & y+1 & z \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = x + y - z = -1$$

$$\vec{n} = \frac{\vec{a_x} + \vec{a_y} - \vec{a_z}}{\sqrt{3}}$$

$$H_{2x} = \frac{\cancel{10}^{5}}{\cancel{4}_{2}\pi \cdot r}\left(\sin 30^\circ + \sin 30^\circ\right)\cdot\frac{1}{\sqrt{3}}$$

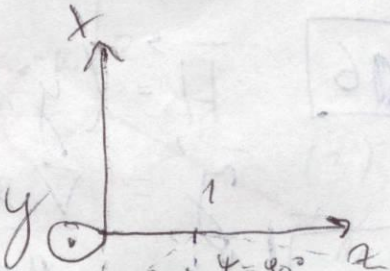
$$\boxed{H_{2x}} = \frac{5}{\cancel{2}\pi\cdot\frac{\sqrt{6}}{\cancel{2}}}\cdot 1\cdot\frac{1}{\sqrt{3}} = \frac{5}{\pi\sqrt{18}} = \boxed{0.375}$$

SEGMENT (3) NEMA X KOMPONENTU U TOČKI $P(0, 0, 1)$ $\boxed{H_{3x} = 0}$

$$\boxed{H_{UKx}} = H_{1x} + H_{2x} = \boxed{0.608}$$

$\boxed{3.19.}$ (2) $\boxed{H_{2y}} = H_{2x} = \boxed{0.375}$ $\quad$ (1) SEGMENT NE SADRŽI Y-KOMPONENTU $\boxed{H_{1y} = 0}$

(3) $H_3 = \frac{10}{4\pi\cdot r}\left(\sin\varphi + \sin\psi\right)\cdot\vec{a_y}$



$r = 1$

$H_{3y} = \frac{5}{2\pi}\cdot\left(\frac{2+\sqrt{2}}{2}\right)$

$H_{3y} = \frac{10 + 5\sqrt{2}}{4\pi}$

$\boxed{H_{3y} = 1.358}$ $\quad$ $\boxed{H_{UKy}} = H_{2y} + H_{3y} = \boxed{1.733}$

3.20. $H_{1z} = H_{3z} = 0$

$$\boxed{H_{2z} =} -H_{2y} = -H_{2x} = \boxed{-0.375}$$

3.21. $$\boxed{\oint_c \bar{H}' \cdot d\bar{l} = \sum I = \int_S \bar{J}_s' \cdot \bar{n}' \cdot dS}$$

za $r < 0.5$

$$H_1 \cdot 2r\pi = \int_0^r 4.5\, r \cdot e^{-2r}\, dr \int_0^{2\pi} d\varphi =$$

$$\boxed{\int r \cdot e^{-2r} = \left(\frac{r}{-2} - \frac{1}{(-2)^2}\right) \cdot e^{-2r}}$$

$$H_1 \cancel{2r\pi} = 4.5 \left(\frac{r}{-2} - \frac{1}{4}\right) e^{-2r} \Big|_{r=0}^{r} \cancel{2\pi}$$

$$H_1 \cdot r = 4.5 \left[\left(\frac{r}{-2} - \frac{1}{4}\right) e^{-2r} - \left(-\frac{1}{4}\right) \cdot e^{0}\right] =$$

$$H_1 \cdot r = 4.5 \left(\frac{1}{4} - \frac{2r+1}{4}\, e^{-2r}\right)$$

$$\boxed{\bar{H}_1' =} \frac{4.5}{r} \cdot \frac{1}{4} \left(1 - 2r e^{-2r} - e^{-2r}\right) = \boxed{\frac{1.125}{r}\left(1 - e^{-2r} - 2r e^{-2r}\right) \bar{u}_\varphi'}$$

za $r > 0.5$

$$H_2 \cdot \cancel{2r\pi} = 4.5 \int_0^{0.5} r e^{-2r}\, dr \cdot \cancel{2\pi}$$

$$H_2 \cdot r = 4.5 \left[\left(\frac{0.5}{-2} - \frac{1}{4}\right) e^{-2 \cdot 0.5} + \frac{1}{4}\right]$$

$$H_2 \cdot r = 4.5 \left(-\frac{1}{2}\, e^{-1} + \frac{1}{4}\right)$$

$$\boxed{H_2 = \frac{0.297}{r}\, \bar{u}_\varphi'}$$

3.22. $\bar{H}' = 3r\, \bar{u}_\alpha'$

$$\bar{J}' = \nabla \times \bar{H}' = \frac{1}{r}\left(\frac{\partial}{\partial r}(r A_\alpha) - \frac{\partial A_r}{\partial \alpha}\right) \bar{u}_z'$$

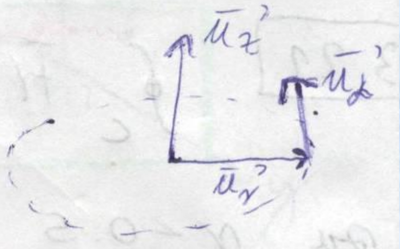
$$\bar{J}' = \frac{1}{r}\left(\frac{\partial}{\partial r}(3r^2) - 0\right) \bar{u}_z'$$

$$\bar{J}' = \frac{6r}{r}\, \bar{u}_z' \qquad \boxed{\bar{J}' = 6\, \bar{u}_z'}$$

3.23. $\vec{H} = k \cdot \sin x \, \vec{a_y}$ $\qquad \vec{J} = \nabla \times \vec{H}$

$$\vec{J} = \vec{a_z}\left[\frac{\partial}{\partial x}(A_y) - \frac{\partial}{\partial y}(A_x)\right]$$

$$\boxed{\vec{J} = k \cos x \, \vec{a_z}}$$



3.24. $\oint_C \vec{H} \cdot \vec{dl} = \iint_S \vec{J} \cdot \vec{n} \, dS$

$$H \cdot 2r\pi = \int_0^r k e^{-a \cdot r} \cdot r \, dr \int_0^{2\pi} d\varphi$$

$$H \cdot \cancel{2} r \cancel{\pi} = k\left(\frac{r}{-a} - \frac{1}{a^2}\right) e^{-ar} \Big|_{r=0}^{r} \cancel{2\pi}$$

$$H \cdot r = k\left[\left(\frac{r}{-a} - \frac{1}{a^2}\right) e^{-ar} - \left(-\frac{1}{a^2}\right) \cdot 1\right]$$

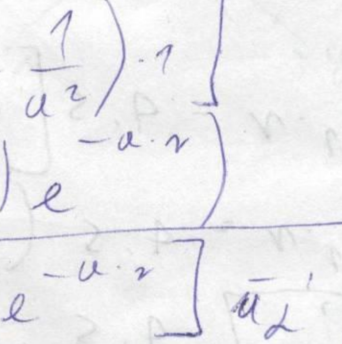
$$H \cdot r = k \cdot \frac{1}{a^2}\left(1 - (1 + a \cdot r) e^{-a \cdot r}\right)$$

$$\boxed{\vec{H} = \frac{k}{a^2 \cdot r}\left[1 - (1 + a \cdot r) e^{-a \cdot r}\right] \vec{a_\varphi}}$$

3.25. ISTI REZULTAT KAO U PRETHODNOM ZADATKU, SAMO SE U ODREĐENOM INTEGRALU ZAMIJENI r SA r_0 = RADIJUS VODIČA

$$\boxed{\vec{H} = \frac{k}{a^2 \cdot r}\left[1 - (1 + a \cdot r_0) e^{-a \cdot r_0}\right] \vec{a_\varphi}}$$

3.26. $J = J_0 e^{-(r_1 - r)}$ $\qquad \int r e^r = (r-1) e^r$



za $r < r_1$

$$H \cdot 2r\pi = J_0 \int_0^r e^{-r_1} \cdot e^r \cdot r \, dr \int_0^{2\pi} d\varphi$$

$$H \cdot \cancel{2} r \cancel{\pi} = J_0 \cdot e^{-r_1}\left[(r-1) e^r \Big|_{r=0}^{r}\right] \cancel{2\pi}$$

$$H \cdot r = J_0 e^{-r_1}\left[(r-1) e^r - (-1) \cdot 1\right]$$

$$\boxed{H = \frac{J_0 \cdot e^{-r_1}}{r}\left[(r-1) e^r + 1\right]}$$

3.27. za $r_1 < r < r_2$

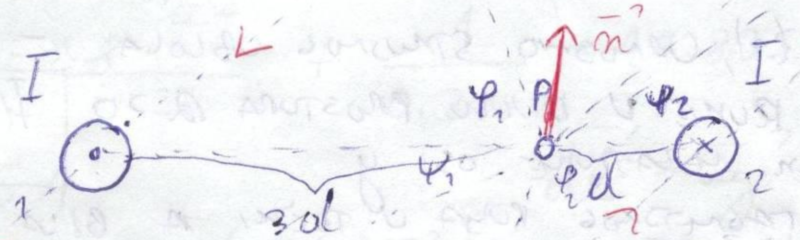
$$H \cdot 2r\pi = J_0 \cdot e^{-r_1} \int_0^{r_1} r e^{r}\, dr \cdot 2\pi$$

$$H \cdot r = J_0 \cdot e^{-r_1}\left[(r_1 - 1) e^{r_1} + 1\right]$$

$$\boxed{H = \frac{J_0}{r}\left(r_1 - 1 + e^{-r_1}\right)}$$

3.28. $\boxed{H = 0}$ za $r > r_2$ poništavaju se struja unutrašnjeg vodiča (r_1) i povratna struja beskonačne tanke ljuske radijusa r_2.

3.29.



$\varphi_1 = \psi_1 = 90^\circ$

$\varphi_2 = \psi_2 = 90^\circ$

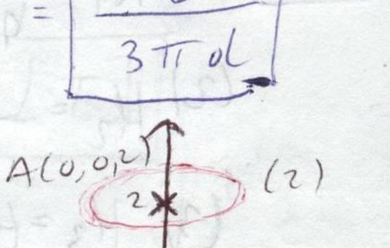
Doprinos jakosti magnetskog polja u točki P od obje strujnice je u istom smjeru

$$H_1 = \frac{I}{4\pi \cdot 3d}\left(\sin 90^\circ + \sin 90^\circ\right) = \frac{I}{2\pi 3d}$$

$$H_2 = \frac{I}{4\pi \cdot d}\left(\sin 90^\circ + \sin 90^\circ\right) = \frac{I}{2\pi d}$$

$$\boxed{H_{uk}} = H_1 + H_2 = \frac{I + 3I}{2\pi 3d} = \frac{4I}{2\pi 3d} = \boxed{\frac{2I}{3\pi d}}$$

3.30. Računamo doprinos mag. polju u točki A od obje kružne strujnice



(1) $$\vec{H}_1 = \frac{I \cdot 1}{4\pi\left(\sqrt{4+1}\right)^3} \int_0^{2\pi} \left(\underbrace{2\cos\varphi\, \vec{a}_x + 2\sin\varphi\, \vec{a}_y}_{0} + 1 \cdot \vec{a}_z\right) d\varphi$$

$$\boxed{\vec{H}_1} = \frac{I}{4\pi \cdot 5^{1.5}} \cdot 2\pi\, \vec{a}_z = \boxed{0.04472\, \vec{a}_z}$$

(2) $$\vec{H}_2 = \frac{I \cdot 1}{4\pi\left(\sqrt{0+1}\right)^3} \int_0^{2\pi} \left(0 \cdot \cos\varphi\, \vec{a}_x + 0 \cdot \sin\varphi\, \vec{a}_y + 1 \cdot \vec{a}_z\right) d\varphi$$

$$\boxed{H_2 = 0.5\, \vec{a}_z} \qquad \boxed{\vec{H}_{uk}} = \vec{H}_1 + \vec{H}_2 = \boxed{0.54472\, \vec{a}_z}$$

3.31. $H_1 = H_2$

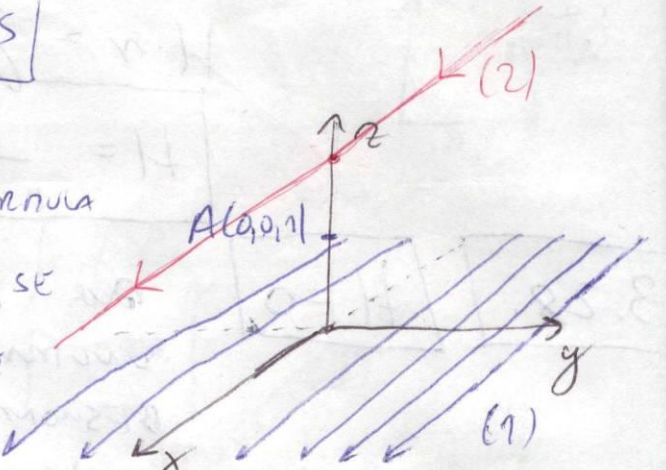
$$H_1 = \frac{I \cdot 1}{2 \cdot 4\pi \left(\sqrt{1+1}\right)^3} \cdot 2\pi \, \bar{a}_z = \frac{1}{2^{2.5}} = 0.17677$$

$$\boxed{H_{uk}} = 2 \cdot H_1 = \boxed{0.35355}$$

3.32. $\bar{K} = k \cdot \bar{a}_x$

$$\boxed{\bar{H} = \frac{|K|}{2} \, \bar{a}_n}$$

IZVEDENA FORMULA ZA STRUJNI OBLOG, $\bar{a}_n$ SE ODREĐUJE PRAVILOM DESNE RUKE



SMJER SEGMENTA (1), ODNOSNO STRUJNOG OBLOGA, JE ZADAN $\bar{a}_x$ PRAVILOM DESNE RUKE U DIJELU PROSTORA $z > 0$ $\boxed{\bar{H}_1 = -\frac{k}{2}\bar{a}_y}$
DAKLE IMA SMJER NEGATIVNE OSI y.
DA BI JAKOST MAGNETSKOG POLJA U TOČKI A BILA JEDNAKA NULI STRUJA JAKOSTI $|H_1| \cdot 2\pi$ MORA TEĆI U SMJERU X OSI DA BI SE PONIŠTILE JAKOSTI U TOČKI A.

ZA SEGMENT (2) $\bar{H}_2 = \frac{I}{4\pi \cdot 1}\left(\sin 90° + \sin 90°\right) = \frac{I}{2\pi}\bar{a}_y$

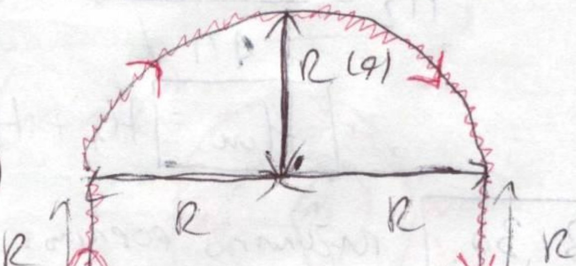
$$|H_1| = |H_2| \qquad \frac{k}{2} = \frac{I}{2\pi} \qquad \boxed{I = \pi \cdot k}$$

3.33.

(1) $H_1 = \frac{I}{4\pi \cdot R}\cdot\left(0 + \frac{\sqrt{2}}{2}\right)(-\bar{a}_z)$

(2) $H_2 = \frac{I}{4\pi R}\cdot\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)(-\bar{a}_z)$

(3) $H_3 = H_1$



(4) $H_4 = \frac{I \cdot R}{4\pi\left(\sqrt{R^2}\right)^3}\int_{\pi}^{0}\left(R \cdot \bar{a}_z\right) d\varphi$

$$H_4 = \frac{I}{4\pi}(-\pi)\,\bar{a}_z = \frac{I}{4}(-\bar{a}_z)$$

$$H_{uk} = H_1 \cdot 2 + H_2 + H_4 = -\frac{1}{4\pi}\cdot\frac{\sqrt{2}}{2}\cdot 2 - \frac{\sqrt{2}}{4\pi} - \frac{1}{4}$$

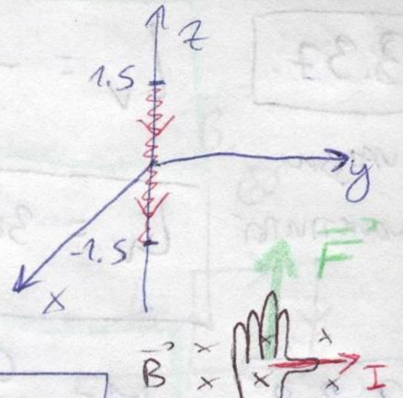
$$H_{uk} = -\frac{1}{4}\left(1 + \frac{2\sqrt{2}}{\pi}\right) \qquad \boxed{|H_{uk}| = \frac{1}{4}\left(1 + \frac{2\sqrt{2}}{\pi}\right)}$$

3.34. $I = 10\,A \quad \vec{B} = 3 \cdot 10^{-4} e^{-0.2x} \vec{a}_y$

MAGNETSKA SILA NA VODIČ PROTJECAN STRUJOM

$$\boxed{\vec{F} = \int_l I (d\vec{l} \times \vec{B})} \qquad d\vec{l} = -\vec{a}_z$$

SMJER STRUJNICE



$$\vec{F} = \int 10 \cdot 3 \cdot 10^{-4} e^{-0.2x} dz$$

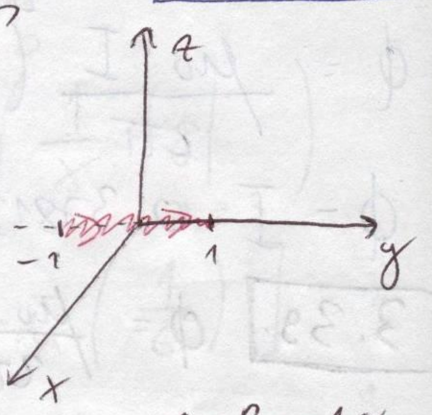
$$\boxed{\vec{F}} = 3 \cdot 10^{-3} e^{-0.2x} \int_{-1.5}^{1.5} dz = \boxed{9 \cdot 10^{-3} e^{-0.2x} \vec{a}_x}$$

$$\boxed{W = \int_s F dS} = \int_0^2 9 \cdot 10^{-3} e^{-0.2x} dx = 9 \cdot 10^{-3} \left(-\frac{1}{0.2} e^{-0.2x} \right) \Big|_{x=0}^{2}$$

$$W = 9 \cdot 10^{-3} \left(-\frac{e^{-0.4}}{0.2} + \frac{1}{0.2} \right) = 14.83559 \cdot 10^{-3} \quad \boxed{W = 0.015\,J}$$

3.35. $I = 5\,A \qquad d\vec{l} = \vec{a}_y$

$$\vec{F} = 1.0607 (\vec{a}_x + \vec{a}_z)$$



$$\vec{F} = \int_y I \cdot \vec{a}_y \times (B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z) dy$$

$$\vec{F} = 5 \int_{-1}^{1} (-\vec{a}_z B_x + \vec{a}_x B_z) dy$$

$10 B_z = 1.0607$

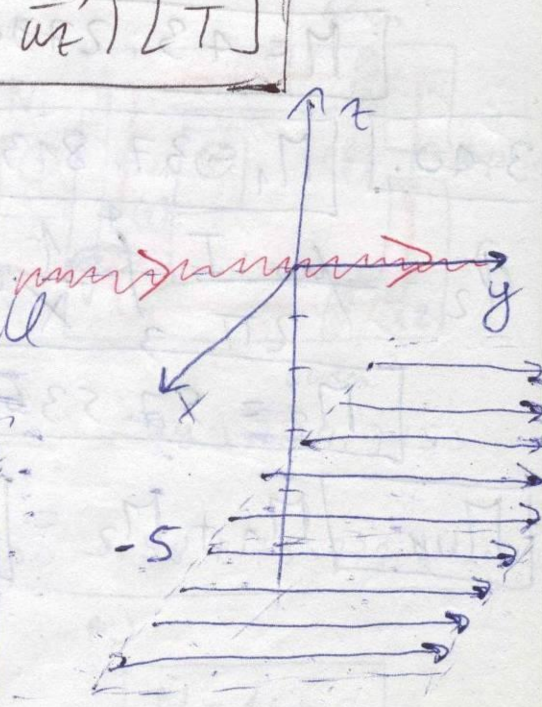
$-10 B_x = 1.0607$

$$1.0607 (\vec{a}_x + \vec{a}_z) = 10 (B_z \vec{a}_x - B_x \vec{a}_z)$$

$$\boxed{\vec{B} = 0.10607 (-\vec{a}_x + \vec{a}_z) \, [T]}$$

3.36. $\vec{K} = 30 \vec{a}_y \quad I = 5\,A \quad d\vec{l} = \vec{a}_y$

$$\vec{B} = \mu_0 \frac{|\vec{K}|}{2} \vec{a}_x \; (\text{za } z > -5)$$



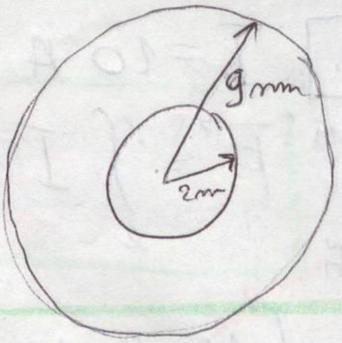
$$\vec{F} = \int_l I \, d\vec{l} \times \vec{B} = \int_l I \, \vec{a}_y \times (\mu_0 15 \cdot \vec{a}_x) dl$$

$$\vec{F} = \int_l 5 \cdot \mu_0 15 \cdot (-\vec{a}_z) dl = -75 \mu_0 l \vec{a}_z$$

$$\frac{\vec{F}}{l} = -75 \mu_0 \vec{a}_z \qquad \boxed{\left| \frac{\vec{F}}{l} \right| = 94.2 \mu \frac{N}{m}}$$

$\boxed{3.37.}$ $\boxed{L_v = \frac{\mu_0}{2\pi} \ln \frac{r_2}{r_1}}$

VANJSKI INDUKTIVITET $\boxed{L_v = 300,81547\ m\frac{H}{m}}$



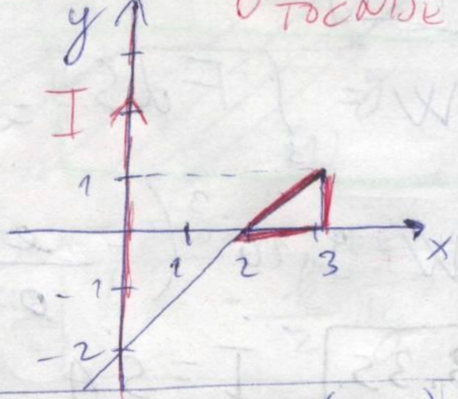
$\boxed{3.38.}$ $\boxed{M = \frac{\Phi}{I}}$ $\boxed{\Phi = \int_x \frac{\mu_0 I}{2\pi x} \cdot dS, \quad dS = y \cdot dx}$ $\boxed{dS = dy\, dx}$ TOČNIJE

$$\Phi = \int_x \frac{\mu_0 \cdot I}{2\pi} \cdot \frac{1}{x} \cdot (x-2)\, dx =$$

$$\Phi = \frac{\mu_0 \cdot I}{2\pi} \int_2^3 \left(1 - \frac{2}{x}\right) dx =$$

$$\Phi = \frac{\mu_0 \cdot I}{2\pi} \left(1 - 2 \ln \frac{3}{2}\right)$$

$$\Phi = I \cdot 0.37813 \cdot 10^{-7} \quad \boxed{M = 37.81395\ nH}$$



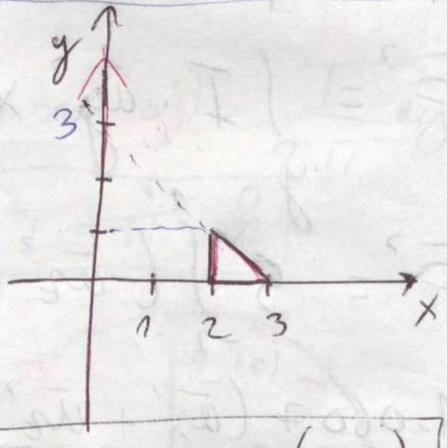
$\boxed{y = x - 2,\ x \in (2,3)}$

$\boxed{3.39.}$ $$\Phi = \frac{\mu_0 \cdot I}{2\pi} \int_2^3 \left(\frac{3}{x} - 1\right) dx$$

$$\Phi = \frac{\mu_0 \cdot I}{2\pi} \cdot \left(3 \ln 1.5 - 1\right)$$

$$\Phi = 0.43279 \cdot 10^{-7} \cdot I$$

$$\boxed{M = 43.27906\ nH}$$



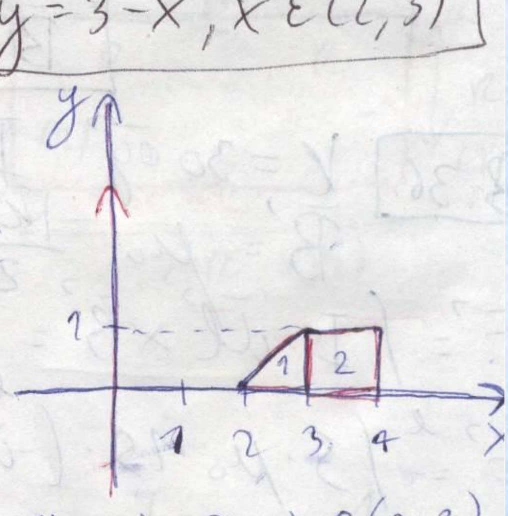
$\boxed{y = 3 - x,\ x \in (2,3)}$

$\boxed{3.40.}$ $\boxed{M_1 = 37.81395\ nH}$

$$\Phi_2 = \frac{\mu_0 \cdot I}{2\pi} \int_3^4 \frac{1}{x} dx = 2 \cdot 10^{-7} \cdot \ln \frac{4}{3} \cdot I$$

$$\boxed{M_2 = 57.53644\ nH}$$

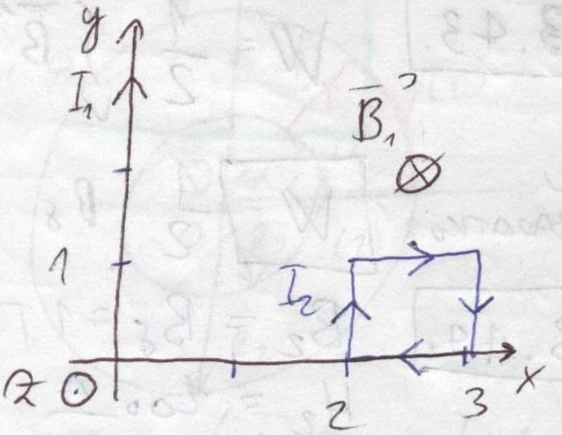
$$\boxed{M_{UK}} = M_1 + M_2 = \boxed{95,35036\ nH}$$



$y_1 = x - 2,\ x \in (2,3)$

$y_2 = 1,\ x \in (3,4)$

$\boxed{3.41.}$ $\vec{F} = I \cdot \int_l (\vec{dl} \times \vec{B})$



$$\vec{B} = \frac{\mu_0 I_1}{2\pi x} (-\vec{a_z})$$

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

$$\vec{dl} \times \vec{B} = \vec{a_y}\left(\frac{\mu_0 I_1}{2\pi x} dx\right) - \vec{a_x}\left(\frac{\mu_0 I_1}{2\pi x} dy\right)$$

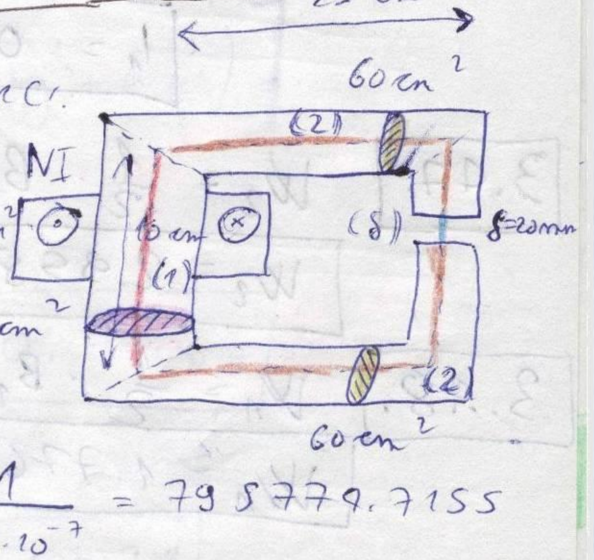
$$\vec{F_{12}} = I_2 \left(\int_x \frac{\mu_0 I_1}{2\pi x} dx\,\vec{a_y} - \int_y \frac{\mu_0 I_1}{2\pi x} dy\,\vec{a_x} \right)$$

$$\vec{F_{12}} = \frac{\mu_0 \cdot I_1 \cdot I_2}{2\pi} \left[\vec{a_y}\left(\int_2^3 \frac{1}{x} dx + \int_3^2 \frac{1}{x} dx \right) - \vec{a_x}\left(\int_0^1 \frac{1}{x} dy \Big|_{x=2} + \int_1^0 \frac{1}{x} dy \Big|_{x=3} \right) \right]$$

$$\vec{F_{12}} = 2 \cdot 10^{-7} \left[\underbrace{\vec{a_y}\left(\ln\frac{3}{2} + \ln\frac{2}{3} \right)}_{0} - \vec{a_x}\left(\frac{1}{6}\right) \right]$$

$$\boxed{\vec{F_{12}} = 33.33\,nN\,(-\vec{a_x})}$$

$\boxed{3.42.}$ KRIVULJA MAGNETIZIRANJA NA SLICI U ZBIRCI.



$B_\delta = 1\,T$ $\quad I = 10\,A$ $\quad N = ?$

$\boxed{\oint_c \vec{H} \cdot \vec{dl} = N \cdot I}$ $\quad S_1 = 0.01\,m^2 \quad S_2 = 0.006\,m^2$

$H_1 \cdot l_1 + H_2 \cdot l_2 + H_\delta \cdot l_\delta = N \cdot I$

IZ GRAFA $\Rightarrow B_\delta = B_2 = 1T \Rightarrow H_2 = 200\,\frac{A}{m}$

$\boxed{B_1 \cdot S_1} = \boxed{B_2 \cdot S_2}$ $\quad H_\delta = \frac{B_\delta}{\mu_0 \mu_r} = \frac{1}{4\pi \cdot 10^{-7}} = 795779.7155$

$B_1 = \frac{1 \cdot 0.006}{0.01} = 0.6$

$N = \frac{95 \cdot 0.1 + 200 \cdot 0.58 + 795774.7155 \cdot 0.02}{10}$

IZ GRAFA:

$H_1 = 95$

$\boxed{N = 1603}$

$\boxed{3.43.}$ $\boxed{W = \frac{1}{2} \int_V \vec{B} \cdot \vec{H} \cdot dV}$ - ENERGIJA MAGNETSKOG POLJA SADRŽANA U VOLUMENU V

U ZADATKU: $\boxed{W} = \frac{1}{2} \cdot B_\delta \cdot H_\delta \cdot \underbrace{S_\delta \cdot \delta}_{dV} = \boxed{47.74648 \text{ J}}$

$\boxed{3.44.}$ $B_2 = B_\delta = 1T$
$H_2 = 200 \frac{A}{m}$
$H_2 = \frac{B_2}{\mu_0 \cdot \mu_r} \Rightarrow \mu_r = \frac{B_2}{H_2} \cdot \frac{1}{\mu_0}$
$\boxed{\mu_r} = \frac{1}{8\pi \cdot 10^{-5}} = \boxed{3978.87}$

$\boxed{3.45.}$ $W_2 = \frac{1}{2} \cdot B_2 \cdot H_2 \cdot S_2 \cdot l_2$
$\boxed{W_2} = \frac{1}{2} \cdot 1 \cdot 200 \cdot 0.006 \cdot 0.25 = \boxed{0.15 \text{ J}}$

$\boxed{3.46.}$ $I_1 = ?$, $N_1 = 200$ $\quad \phi = konst = 120 \cdot 10^{-6}$
$l_2 = 0.5$, $N_2 = 100$
$S_1 = 4 \cdot 10^{-4}$, $l_1 = 16 \cdot 10^{-2}$
$S_2 = 6 \cdot 10^{-4}$, $l_2 = 34 \cdot 10^{-2}$

$\phi = B_1 \cdot S_1 \Rightarrow \boxed{B_1} = \frac{\cancel{120}^{30} \cdot 10^{-6-2}}{\cancel{4} \cdot 10^{-4}} \boxed{= 0.3}$ iz tablice: $\boxed{H_1 = 185}$

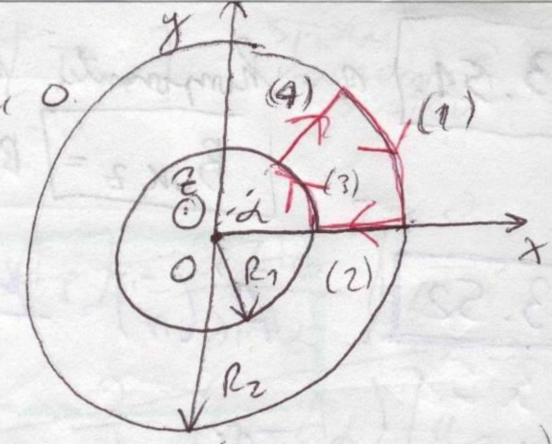
$B_1 \cdot S_1 = B_2 \cdot S_2 \Rightarrow \boxed{B_2} = \frac{0.3 \cdot \cancel{4}^2 \cdot \cancel{10^{-4}}}{3\,\cancel{6} \cdot \cancel{10^{-4}}} = \boxed{0.2}$ $\boxed{H_2 = 145}$

$N_1 I_1 - N_2 \cdot I_2 = H_1 \cdot l_1 + H_2 \cdot l_2$
$\boxed{I_1 = 0.6495 \text{ A}}$

$\boxed{3.47.}$ $W_2 = \frac{1}{2} \cdot B_2 \cdot H_2 \cdot S_2 \cdot l_2 = \frac{1}{2} \cdot 0.2 \cdot 145 \cdot 6 \cdot 10^{-4} \cdot 34 \cdot 10^{-2}$
$\boxed{W_2 = 2.958 \text{ mJ}}$

$\boxed{3.48.}$ $W_1 = \frac{1}{2} \cdot B_1 \cdot H_1 \cdot S_1 \cdot l_1 = \frac{1}{2} \cdot 0.3 \cdot 185 \cdot 4 \cdot 10^{-4} \cdot 16 \cdot 10^{-2}$
$\boxed{W_1 = 1.776 \text{ mJ}}$

$\boxed{3.49.}$ SEGMENTI (2) i (4) NEMAJU UTJECAJ NA JAKOST MAGNETSKOG POLJA U TOČKI O.



$$\vec{H_1} = \frac{I\cdot R_2}{4\pi\left(\sqrt{R_2^2}\right)^3}\int_\alpha^0 R_2\,\vec{a_z}$$

$$\vec{H_1} = \frac{I\cdot\alpha}{4\pi R_2}\left(-\vec{a_z}\right)$$

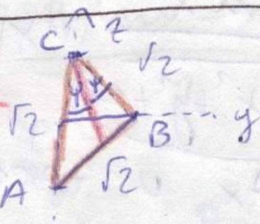
$$\vec{H_3} = \frac{I\cdot\alpha}{4\pi R_1}\vec{a_z} \qquad H_{uk} = |\vec{H_3}| - |\vec{H_1}| = \left(\frac{I\cdot\alpha}{4\pi R_1} - \frac{I\cdot\alpha}{4\pi R_2}\right)$$

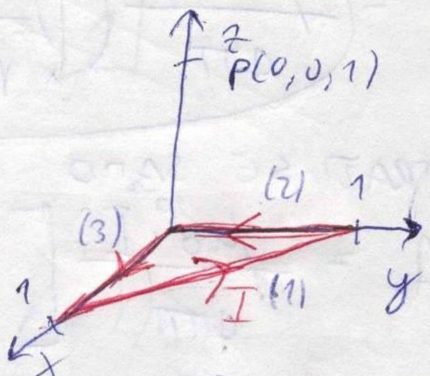
$$\boxed{H_{uk} = \frac{\alpha}{4\pi} I\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$\boxed{3.50.}$ (1) $r = r = \frac{a\sqrt{3}}{2} = \frac{\sqrt{2}\sqrt{3}}{2}$

$a = \sqrt{2}$

$\varphi = \psi = 30^\circ$





$$\begin{array}{l} A(1,0,0) \\ B(0,1,0) \\ C(0,0,1) \end{array} \quad \begin{vmatrix} x-1 & y & z \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0 \qquad I = 1A$$

$$\vec{n} = \frac{\vec{a_x} + \vec{a_y} + \vec{a_z}}{\sqrt{3}}$$

$$(x-1)(1) - y(-1) + z(1) = 0 \Rightarrow x + y + z = 1$$

$$\vec{B_1} = \frac{\mu_0\cdot I}{4\pi\,\frac{\sqrt{6}}{2}}\left(2\cdot\frac{1}{2}\right)\left[\frac{1}{\sqrt{3}}\left(\vec{a_x} + \vec{a_y} + \vec{a_z}\right)\right]$$

$$\boxed{B_{1x} =} \frac{4\pi\cdot 10^{-7}}{2\pi\cdot\sqrt{18}} = \boxed{47.14045\ nT}$$

(2)

$$\boxed{B_2 = B_{2x} =} -\frac{\mu_0\cdot I}{4\pi\cdot 1}\cdot\frac{\sqrt{2}}{2} = \boxed{-70.71067}$$

(3) KOMPONENTA IMA SAMO Y-SMJER.

$$\boxed{B_{ukx} =} B_{1x} + B_{2x} = \boxed{-23.57021}$$

3.51. z - komponentu ima samo prvi segment stranice, dakle:

$$\boxed{B_{UK\,z}} = B_{1x} = \boxed{97.14095\ nT}$$

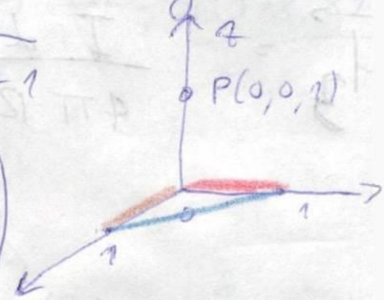
3.52. $$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int_C \frac{d\vec{l}}{|\vec{r}-\vec{r}\,'|}}$$ JEDNADŽBA VEKTORSKOG MAGNETSKOG POTENCIJALA ZA LINIJSKE STRUJE

$d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y + dz\,\vec{a}_z$, $dz=0$ jer z-konst.

$\vec{r} = \vec{a}_z$

$\vec{r}\,' = x\vec{a}_x + y\vec{a}_y$

$|\vec{r}-\vec{r}\,'| = \sqrt{x^2+y^2+1}$



$$\vec{A} = \frac{\mu_0 I}{4\pi}\left(\int_x \frac{dx}{\sqrt{x^2+y^2+1}}\,\vec{a}_x + \int_y \frac{dy}{\sqrt{x^2+y^2+1}}\,\vec{a}_y\right)$$

$y = -x+1$

TRAŽI SE SAMO A_x U ZADATKU

x = konst. = 0

$$A_x = \frac{\overset{10^{-7}}{\mu_0 I}}{4\pi}\left[\int_1^0 \frac{dx}{\sqrt{x^2+(-x+1)^2+1}} + 0 + \int_0^1 \frac{dx}{\sqrt{x^2+0^2+1}}\right]$$

$$\int_1^0 \frac{dx}{\sqrt{2(x^2-x+1)}} = \frac{1}{\sqrt{2}}\int_1^0 \frac{dx}{\sqrt{(x-\frac{1}{2})^2+\frac{3}{4}}} = \left| \begin{array}{l} z = x-\frac{1}{2} \\ \frac{dz}{dx} = 1 \Rightarrow dx = dz \\ a_0 = \frac{1}{2},\ z_0 = -\frac{1}{2} \end{array} \right|$$

$$= \int_{\frac{1}{2}}^{-\frac{1}{2}} \frac{dz}{\sqrt{z^2+\left(\frac{\sqrt{3}}{2}\right)^2}} = \ln\left|z+\sqrt{z^2+\frac{3}{4}}\right|\Bigg|_{z=\frac{1}{2}}^{-\frac{1}{2}} = \ln\frac{-\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{3}{4}}}{\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{3}{4}}}$$

$$\boxed{\int_1^0 \frac{dx}{\sqrt{2(x^2-x+1)}}} = \frac{1}{\sqrt{2}}\ln\frac{\frac{1}{2}}{\frac{3}{2}} = \boxed{-0.77683}$$

$$\boxed{\int_0^1 \frac{dx}{\sqrt{x^2+1}}} = \ln\left|x+\sqrt{x^2+1}\right|\Bigg|_{x=0}^{1} = \ln\frac{1+\sqrt{2}}{1} = \boxed{0.88137}$$

$$\boxed{A_x} = 10^{-7}\left[-0.77683+0.88137\right] = \boxed{10.45337}$$

[nTm]

3.53 $\boxed{A_z = 0}$

$\boxed{3.54}$ $\vec{A} = r^2 \vec{a}_r + 2\sin\theta \, \vec{u}_\theta \; [Tm]$ STERNI KOORDINATNI SUSTAV

$|\vec{B}(1, \frac{\pi}{6}, \frac{\pi}{2})| = ?$

$$\vec{B} = \nabla \times \vec{A} = \cancel{\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{\partial A_\theta}{\partial A_\varphi}\right]}^{\,0}\vec{u}_r$$

$$+ \cancel{\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(r \cdot A_\varphi)\right]}^{\,0}\vec{u}_\theta$$

$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \cancel{\frac{\partial A_r}{\partial\theta}}^{\,0}\right]\vec{u}_\varphi$$

$$\vec{B} = \vec{u}_\varphi\left(\frac{2\sin\theta}{r}\right) \quad \boxed{B} = \frac{2\cdot\sin\frac{\pi}{6}}{1} = \boxed{1}$$

$\boxed{3.55}$ $\vec{H} = \frac{2\cdot\sin\theta}{\mu_0 \cdot r}\vec{a}_\varphi$

$$\vec{J} = \nabla \times \vec{H} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta \cdot A_\varphi)\right)\vec{u}_r$$

$$+ \frac{1}{r}\left(-\frac{\partial}{\partial r}(r \cdot A_\varphi)\right)\vec{u}_\theta$$

$$J_r = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\frac{2\sin^2\theta}{\mu_0 \cdot r}\right)\right) = \frac{1}{r\cancel{\sin\theta}}\cdot\frac{2}{\mu_0\cdot r}\cdot 2\cdot\cancel{\sin\theta}\cos\theta$$

$$J_r = \frac{4\cos\theta}{\mu_0 \cdot r^2} \quad J_\theta = \frac{1}{r}\left(-\frac{\partial}{\partial r}\left(\cancel{r}\cdot\frac{2\cdot\sin\theta}{\mu_0\cdot\cancel{r}}\right)\right) = 0$$

$$\boxed{|\vec{J}| =} J_r = \frac{4\cdot\cos 0}{\mu_0\cdot 0^2} = \boxed{3183.0988\,6\,k\,\frac{A}{m^2}}$$

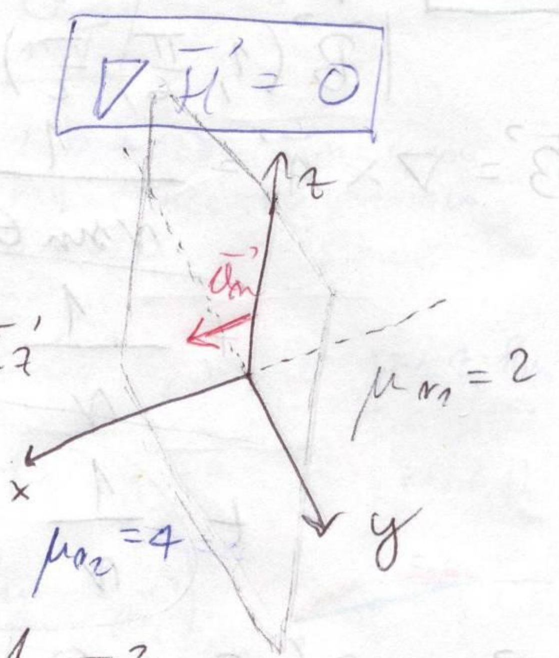
$\boxed{3.56}$ $\boxed{w_M = \frac{1}{2\mu}B^2 = \frac{1}{2}\mu\cdot H^2 \left[\frac{J}{m^3}\right]}$ → GUSTOĆA MAGNETSKE ENERGIJE

$$w_M = \frac{1}{2}\mu_0\cdot|\vec{H}|^2 = \frac{1}{2}\mu_0\left(\frac{2\cdot\sin\theta}{\mu_0\cdot r}\right)^2$$

μ $T(1, 0, \frac{\pi}{2})$ $\boxed{w_M = 0}$

$\boxed{3.57}$ $\vec{H} = \frac{2 \cdot \sin\theta}{\mu_0 \cdot r} \vec{a_\alpha}$

$$\nabla \cdot \vec{H} = \frac{1}{r\sin\theta} \cdot \underbrace{\frac{\partial A_\alpha}{\partial \alpha}}_{=0} \Rightarrow \boxed{\nabla \cdot \vec{H} = 0}$$



$\boxed{3.58.}$ $\vec{B_1} = 0.5\vec{a_x} + \vec{a_y} + \vec{a_z}$

$\vec{n} = \vec{a_x}$ $\vec{K} = -\frac{1}{\mu_0 \cdot 2}\vec{a_y} + \frac{1}{\mu_0 \cdot 2}\vec{a_z}$

$\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{K}$

$$\vec{a_x} \times \left[(H_{2x} - H_{1x})\vec{a_x} + (H_{2y} - H_{1y})\vec{a_y} + (H_{2z} - H_{1z})\vec{a_z} \right] = -\frac{1}{\mu_0 \cdot 2}\vec{a_y} + \frac{1}{\mu_0 \cdot 2}\vec{a_z}$$

$$\vec{a_z}(H_{2y} - H_{1y}) - \vec{a_y}(H_{2z} - H_{1z}) = -\frac{1}{\mu_0 \cdot 2}\vec{a_y} + \frac{1}{\mu_0 \cdot 2}\vec{a_z}$$

$$H_{2y} - H_{1y} = \frac{1}{2\mu_0} \Rightarrow H_{2y} = \frac{1}{2\mu_0} + H_{1y}$$

$\boxed{3.59}$

$$H_{2z} - H_{1z} = \frac{1}{2\mu_0} \quad \frac{B_{2y}}{\mu_0 \cdot \mu_{r2}} = \frac{1}{2\mu_0} + \frac{1}{\mu_0 \cdot \mu_{r1}} \Rightarrow B_{2y} = \frac{\mu_{r2}}{2} + \frac{\mu_{r2}}{\mu_{r1}}$$

$\boxed{B_{2y}} = 2+2 = \boxed{4}$

$\boxed{3.60.}$

$\boxed{B_{2z}} = B_{2y} = \boxed{4}$

$$\vec{n}(\vec{B_2} - \vec{B_1}) = 0 \Rightarrow \vec{a_x} \cdot \left[(B_{2x} - B_{1x})\vec{a_x} + \dots \right] = 0$$

$$B_{2x} - 0.5 = 0 \Rightarrow \boxed{B_{2x} = 0.5}$$

$\boxed{3.61.}$ $\vec{B_2} = 0.5\vec{a_x} + 4\vec{a_y} + 4\vec{a_z}$

$\vec{B_n} = (\vec{B_2} \cdot \vec{n}) \cdot \vec{n} = 0.5\vec{a_x}$

$\vec{B_t} = \vec{B_2} - \vec{B_n} = 4\vec{a_y} + 4\vec{a_z}$ $\boxed{B_t} = \sqrt{32} = \boxed{5.65}$

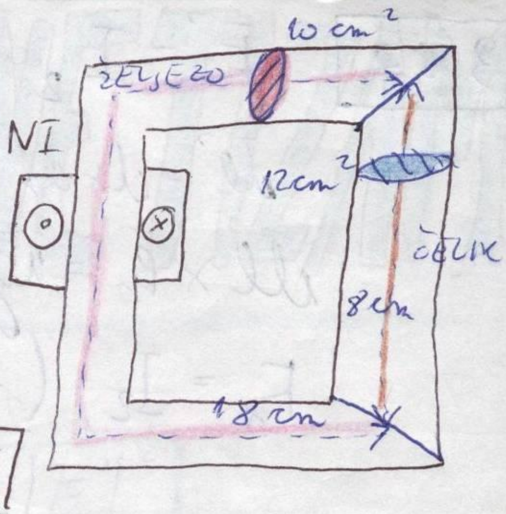
3.62.

$N = 100$

$B_č = 1\,T$

$S_č = 12 \cdot 10^{-4}$ $\qquad S_ž = 10 \cdot 10^{-4}$

$l_č = 8 \cdot 10^{-2}$ $\qquad l_ž = 18 \cdot 10^{-2}$



$$B_č \cdot S_č = B_ž \cdot S_ž$$

$$\boxed{B_ž} = \frac{B_č \cdot S_č}{S_ž} = \frac{1 \cdot 12 \cdot 10^{-4}}{10 \cdot 10^{-4}} = \boxed{1.2\,T}$$

3.63. $H_č$ iz grafa očitan $\Rightarrow$ $\boxed{H_č = 100\,\frac{A}{m}}$

$$W_č = \frac{1}{2} \cdot B_č \cdot H_č \cdot S_č \cdot l_č$$

$$\boxed{W_č} = \frac{1}{2} \cdot 1 \cdot 100 \cdot 12 \cdot 10^{-4} \cdot 8 \cdot 10^{-2} = \boxed{4.8\,mJ}$$

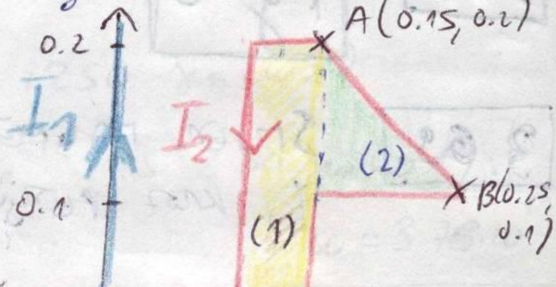
3.64. $H_ž$ iz grafa očitan $\Rightarrow$ $\boxed{H_ž = 400\,\frac{A}{m}}$

3.65. $N \cdot I = H_č \cdot l_č + H_ž \cdot l_ž$

$$I = \frac{100 \cdot 8 \cdot 10^{-2} + 400 \cdot 18 \cdot 10^{-2}}{100} \qquad \boxed{I = 0.8}$$

3.66. $I_1 = 1A \quad I_2 = 2A$

$$\boxed{M = \frac{\Phi_{12}}{I_2} = \frac{\Phi_{21}}{I_1}} \qquad \boxed{\Phi_{21} = \iint_{S_2} \vec{B}_1 \cdot \vec{n}\, dS}$$



$$B_1 = \frac{\mu_0}{2\pi} \frac{I_1}{x} \Rightarrow \Phi_{21} = \iint_{S_2} \frac{\mu_0 \cdot I_1}{2\pi} \frac{1}{x}\, dy\, dx$$

$$\Phi_{21} = \frac{\mu_0 \cdot I_1}{2\pi}\left[\int_{0.1}^{0.15} \frac{1}{x} dx \int_{0}^{0.2} dy + \int_{0.15}^{-y+0.35} \frac{1}{x} dx \int_{0.1}^{0.2} dy\right]$$

$$\Phi_{21} = \frac{\mu_0 \cdot I_1}{2\pi}\left[0.2 \cdot \ln\frac{0.15}{0.1} + \int_{0.1}^{0.2} \ln(0.35 - y)\, dy - \ln 0.15 \int_{0.1}^{0.2} dy\right]$$

$$\Phi_{21} = \frac{\mu_0 \cdot I_1}{2\pi}\left[0.08109 + \left(-\left(\frac{0.35-y}{1} \ln(0.35-y)\right) + y\right)\Bigg|_{y=0.1}^{0.2} + 0.18971\right]$$

$$\Phi_{21} = 2 \cdot 10^{-7} \cdot 0.10879 \cdot I_1$$

$$\boxed{\int \ln(ax+b)\, dx = \left[\frac{ax+b}{a} \ln(ax+b)\right] - x}$$

$$\boxed{M = \frac{\Phi_{21}}{I_1}} \qquad \boxed{M = 21.75988\,mH}$$

JEDNADŽBA PRAVCA KROZ DVIJE TOČKE ZA SEGMENT (2)

$A(0.15, 0.2)$, $B(0.25, 0.1)$

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

$$y - 0.2 = \frac{0.1 - 0.2}{0.25 - 0.15}(x - 0.15)$$

$$y - 0.2 = -x + 0.15$$

$$\boxed{y = -x + 0.35}$$

$\boxed{3.67.}$ $\vec{F} = I_2 \cdot \int_l d\vec{l} \times \vec{B}_1$, $\vec{B}_1 = \frac{\mu_0 \cdot I_1}{2\pi} \cdot \frac{1}{x} \cdot (-\vec{a}_z)$

$$d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y$$

$$d\vec{l} \times \vec{B}_1 = \vec{a}_y\left(\frac{\mu_0 \cdot I_1}{2\pi} \cdot \frac{1}{x}\,dx\right) - \vec{a}_x\left(\frac{\mu_0 \cdot I_1}{2\pi} \cdot \frac{1}{x}\,dy\right)$$

$$F_x = I_2 \cdot \left(-\int_y \frac{\mu_0 \cdot I_1}{2\pi} \cdot \frac{1}{x}\,dy\right)$$

$$F_x = -\frac{I_1 \cdot I_2 \cdot \mu_0}{2\pi}\left(\int_{0.2}^{0} \frac{1}{0.1}\,dy + 0 + \int_{0}^{0.1} \frac{1}{0.15}\,dy + 0 + \int_{0.1}^{0.2} \frac{1}{0.35-y}\,dy + 0\right)$$

$$F_x = -\frac{I_1 I_2 \mu_0}{2\pi}\left(\frac{1}{0.1}\cdot(-0.2) + \frac{1}{0.15}\cdot 0.1 - \ln|y-0.35|\Big|_{y=0.1}^{0.2}\right)$$

$$F_x = -\frac{I_1 \cdot I_2 \cdot \mu_0}{2\pi}\left(-2 + \frac{0.1}{0.15} - \ln\frac{0.15}{0.25}\right) = -\frac{I_1 I_2 \mu_0}{\pi}\left(-0.82250770y\right)$$

$$\boxed{F_x} = 4 \cdot 0.82250 \cdot 10^{-7} = \boxed{329\ nN}$$

$\boxed{3.68.}$ $\boxed{F_y = 0}$

$\boxed{3.69.}$ SMJER MAGNETSKE INDUKCIJE PROIZVEDENE STRUJOM STRUJNIC I_1 KROZ PETLJU JE $\boxed{-\vec{a}_z}$

IV. ELEKTROMAGNETIZAM

4.1. $\vec{E} = 2x \cdot 10^{3}\, \vec{a_y}$ $T(4,5,7)$

GUSTOĆA POHRANJENE ENERGIJE U ELEKTRIČNOM POLJU

$$\boxed{w_E = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}\varepsilon\,|\vec{E}|^2}$$

$|\vec{E}|^2 = 4 \cdot 10^{6} \cdot x^2$

$\varepsilon = 8.859 \cdot 10^{-12}$

$w_E = \frac{1}{2} \cdot 8.859 \cdot 10^{-12} \cdot 4 \cdot 10^{6} \cdot x^2$

$\boxed{w_E} = 17.708 \cdot 10^{-6} \cdot 4^2 = \boxed{283.328\, \mu \frac{J}{m^3}}$

4.2. $K = 5\left[\frac{S}{m}\right]$ $\varepsilon_r = 1.5$

$E = 250 \sin(\underbrace{10^{10}}_{\omega} t)\left[\frac{V}{m}\right]$ $f = \frac{\omega}{2\pi}$

$$\boxed{\begin{aligned} \vec{J} &= K \cdot \vec{E}, \\ \vec{J}_p &= \frac{\partial \vec{D}}{\partial t} \end{aligned}}$$

$\rightarrow$ GUSTOĆA PROVODNE STRUJE

$\rightarrow$ GUSTOĆA STRUJE POMAKA (pomačna struja)

$J = K \cdot E = 5 \cdot 250 \sin(\omega t) = 1250 \sin(\omega t)$

$J_p = \frac{\partial}{\partial t}(\varepsilon_0 \cdot \varepsilon_r \cdot E) = \frac{\partial}{\partial t}\left[8.859 \cdot 10^{-12} \cdot 1.5 \cdot 250 \sin(\omega t)\right]$

$J_p = 3.32025 \cdot 10^{-9} \cdot \omega \cos(\omega t)$

$J_0 = J_{p0} \Rightarrow 1250 = \omega \cdot 3.32025 \cdot 10^{-9}$

$\omega = 3.76 \cdot 10^{11}$

$\boxed{f = 59.91\,[GHz]}$

4.3. $\vec{E} = E_m \sin(\omega t - 3z)\,\vec{a_y}$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

FARADAYEV ZAKON

$\nabla \times \vec{E} = -\vec{a_x}\left(\frac{\partial E_y}{\partial z}\right) = -\vec{a_x}\left[E_m \cdot (-3) \cdot \cos(\omega t - 3z)\right]$

$\frac{\partial \vec{B}}{\partial t} = -\vec{a_x}\left[3 \cdot E_m \cos(\omega t - 3z)\right] \Big/ \int dt$

$B_x = -3 \cdot E_m \cdot \frac{1}{\omega} \cdot \sin(\omega t - 3z)$ $B_{x0} = 0.6 \cdot 10 \cdot \frac{1}{10000}$

$\boxed{B_{x0} = 0.6\ mT}$

4.4. $\vec{E}(z,t) = 50\cos(\omega t - \beta z)\,\vec{a}_x$

$$\boxed{\nabla\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t}} \quad \nabla\times\vec{E} = -\vec{a}_y\left(-\frac{\partial E_x}{\partial z}\right)$$

$$= \vec{a}_y \cdot 50 \cdot \beta \cdot \sin(\omega t - \beta z)$$

$$\nabla\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \,\Big|\int dt \Rightarrow \vec{H} = \left[-\frac{50\cdot\beta}{\mu}\int_t \sin(\omega t - \beta z)\,dt\right]\vec{a}_y$$

$$\vec{H} = \frac{50\cdot\beta}{\mu}\cdot\frac{1}{\omega}\cdot\cos(\omega t - \beta z)\,\vec{a}_y$$

$$\vec{N} = \vec{E}\times\vec{H} = \vec{a}_z\left[\frac{50^2\beta}{\mu\omega}\cdot\cos^2(\omega t - \beta z)\right] \quad \boxed{\beta = \omega\sqrt{\mu\varepsilon}}$$

$$\boxed{\vec{N}_{sr} = \frac{1}{2}\vec{N}} = \frac{1}{2}\cdot\frac{50^2\cdot\omega\cdot\sqrt{\mu\varepsilon}}{\mu\,\omega}\,\vec{a}_z \quad \boxed{P_{sr} = \oiint \vec{N}_{sr}\cdot\vec{n}\cdot dS}$$

$$\boxed{P_{sr}} = N_{sr}\cdot r^2\pi = \frac{50^2}{2}\cdot\frac{\sqrt{8.859\cdot10^{-12}}}{\sqrt{4\pi\cdot10^{-7}}}\cdot 2.5^2\cdot\pi = \boxed{65.14\,W}$$

4.5. $\vec{E} = x\cdot z\,\vec{a}_y\cdot 10^3 \quad w_E = \frac{1}{2}\cdot\varepsilon\cdot|\vec{E}|^2 = \frac{1}{2}\cdot\varepsilon\cdot x^2\cdot z^2\cdot 10^6$

$$\boxed{w_E(1,5,7)} = 4.427\cdot10^{-6}\cdot1^2\cdot7^2 = \boxed{0.000216923\left[\frac{J}{m^3}\right]}$$

4.6. $\vec{E}(z,t) = 10\cos(\omega t - \beta z)\,\vec{a}_x$ (kao u 4.4)

$$\vec{H} = \frac{10\cdot\beta}{\mu\cdot\omega}\cos(\omega t - \beta z)\,\vec{a}_y$$

$$\vec{N} = \vec{E}\times\vec{H} = \vec{a}_z\,\frac{10^2\beta}{\mu\cdot\omega}\cos^2(\omega t - \beta z)$$

$$\vec{N}_{sr} = \frac{1}{2}\vec{N} = \frac{1}{2}\,\frac{10^2\,\omega\sqrt{\mu\varepsilon}}{\mu\,\omega}\,\vec{a}_z$$

$$\vec{N}_{sr} = \frac{1}{2}\cdot10^2\cdot\sqrt{\frac{\varepsilon}{\mu}}\cdot\vec{a}_z \qquad n = \vec{a}_z$$

$$P_{sr} = \oiint \vec{N}_{sr}\cdot\vec{n}\cdot dS = \frac{1}{2}\cdot10^2\cdot\sqrt{\frac{\varepsilon}{\mu}}\cdot1.5^2\cdot\pi$$

$$\boxed{P_{sr} = 0.938\,[W]}$$

4.7. $\vec{H} = 200\, x\, \vec{a_y}$ $\boxed{w_m = \frac{1}{2}\mu |\vec{H}|^2}$ GUSTOĆA POHRANJENE ENERGIJE U MAGNETSKOM POLJU

$$\boxed{w_m(2,5,1)} = \frac{1}{2}\cdot 4\pi\cdot 10^{-7}\cdot 200^2\cdot 2^2 = \boxed{100.53\left[\frac{mJ}{m^3}\right]}$$

4.8. $K = 10^{-3}\left[\frac{S}{m}\right]$ $\varepsilon_r = 2.5$

$E = 6\cdot 10^{-6}\sin(9\cdot 10^9 t)\left[\frac{V}{m}\right]$ $\vec{J} = K\cdot\vec{E}$

$$\boxed{J_0} = 10^{-3}\cdot 6\cdot 10^{-6} = \boxed{6\left[n\frac{A}{m^2}\right]}$$

4.9. $\vec{E}(z,t) = 10\sin(\omega t - \beta z)\vec{a_x} - 15\sin(\omega t - \beta z)\vec{a_y}$

$\vec{E}(0, 0.75\lambda) = 10\sin(-\beta\cdot 0.75\,\lambda)\vec{a_x}$ $\boxed{\lambda = \frac{2\pi}{\beta}}$

$-15\sin(-\beta\cdot 0.75\,\lambda)\vec{a_y}$

$\vec{E}(0, 0.75\lambda) = 10\sin\left(-\cancel{\beta}\cdot\frac{3}{\cancel{4}_2}\cdot\frac{\cancel{2}\pi}{\cancel{\beta}}\right)\vec{a_x} - 15\sin\left(-\cancel{\beta}\cdot\frac{3}{\cancel{4}_2}\cdot\frac{\cancel{2}\pi}{\cancel{\beta}}\right)\vec{a_y}$

$\vec{E}(0, 0.75\lambda) = -10\sin\left(\frac{3\pi}{2}\right)\vec{a_x} + 15\sin\left(\frac{3\pi}{2}\right)\vec{a_y}$

$\vec{E}(0, 0.75\lambda) = 10\,\vec{a_x} - 15\,\vec{a_y}$ $\boxed{|\vec{E}|} = \sqrt{10^2 + 15^2} = \boxed{18.02775}$

4.10. $\vec{E}(z,t) = 150\sin(\omega t - \beta z)\vec{a_x}$, $a = 3\cdot 10^{-2}$, $b = 1.5\cdot 10^{-2}$

$P_{SR} = ?$ $\vec{n} = \vec{a_z}$ $dS = a\cdot b = 4.5\cdot 10^{-4}$

$$\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \qquad \nabla\times\vec{E} = -\vec{a_y}\left[150\cdot\beta\cdot\cos(\omega t - \beta z)\right]$$

$$\frac{\partial\vec{H}}{\partial t} = \frac{150\cdot\beta}{\mu}\cos(\omega t - \beta z)\vec{a_y} \;/\!\int dt \Rightarrow \vec{H} = \frac{150\cdot\beta}{\mu\cdot\omega}\cdot\sin(\omega t - \beta z)\vec{a_y}$$

$$\vec{N} = \vec{E}\times\vec{H} = \vec{a_z}\left[\frac{150^2\cdot\beta}{\mu\cdot\omega}\sin^2(\omega t - \beta z)\right] \quad \vec{N_{SR}} = \frac{1}{2}\cdot\frac{150^2\cdot\beta}{\mu\cdot\omega}\vec{a_z}$$

$$P_{SR} = \oint \vec{N_{SR}}\cdot\vec{n}\cdot dS = \frac{1}{2}\,\frac{150^2\cdot\cancel{\omega}\cdot\sqrt{\mu\cdot\varepsilon}}{\mu\cdot\cancel{\omega}}\cdot 4.5\cdot 10^{-4}$$

$$\boxed{P_{SR} = 0.0134378 52\ [W]}$$

$\boxed{4.11.}$ $\vec{H} = 100\, x y\, \vec{a_z}$

$$w_m = \frac{1}{2} \cdot \mu \cdot |\vec{H}|^2 = \frac{1}{2} \cdot 4\pi \cdot 10^{-7} \cdot 100^2 \cdot x^2 \cdot y^2$$

$$w_m(2,5,1) = \frac{1}{2} \cdot 4\pi \cdot 10^{-7} \cdot 100^2 \cdot 2^2 \cdot 5^2$$

$$\boxed{w_m} = \pi \cdot 10^{-3} \cdot 2^3 \cdot 5^2 = 5^{-1} \cdot \pi = 0.2\pi = \boxed{628.31 \frac{m J}{m^3}}$$

$\boxed{4.12.}$ $K = 10^{-3}$ $\quad E = 6 \cdot 10^{-6} \sin(9 \cdot 10^9 t)$

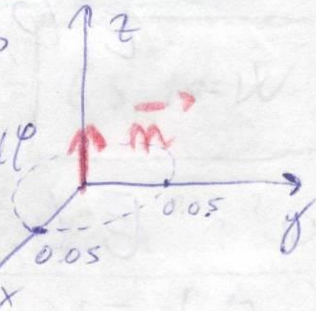
$\varepsilon_r = 2.5$

$$J_P = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \varepsilon_r \cdot 6 \cdot 10^{-6} \cdot \frac{\partial}{\partial t}\left[\sin(9 \cdot 10^9 t)\right]$$

$$J_P = \varepsilon_0 \cdot \varepsilon_r \cdot 6 \cdot 10^{-6} \cdot 9 \cdot 10^9 \cdot \cos(9 \cdot 10^9 t)$$

$$\boxed{J_{P0} = 1.19529 \cdot 10^{-6} \left[\frac{A}{m^2}\right]}$$

$\boxed{4.13.}$ $\vec{B} = 0.5 \cos(377t)(4\vec{a_y} + 4\vec{a_z}) \quad \vec{n} = \vec{a_z}$



$$\phi = \iint_S \vec{B} \cdot \vec{n}\, dS = \iint_S 0.5 \cos(377t)(\cancel{4\vec{a_y}} + 4\vec{a_z}) \cdot \vec{a_z} \cdot r\, dr\, d\varphi$$

$$\phi = 0.5 \cos(377t) \cdot 4 \int_0^{0.05} r\, dr \int_0^{2\pi} d\varphi$$

$$\phi = 0.5 \cos(377t) \cdot 4 \cdot \frac{1}{2} r^2 \Big|_{r=0}^{0.05} \cdot 2\pi = 2\pi \cos(377t) \cdot 0.05^2$$

$$\boxed{u_{ind} = -\frac{d\phi}{dt}} \qquad u = -2\pi \cdot 0.05^2 \cdot 377 \cdot (-\sin(377t))$$

$$u = 4 \cdot 1.48047 \sin(377t) \Rightarrow \boxed{u_0 = 5.92}$$

INDUCIRANI NAPON

$\boxed{4.14.}$ $\vec{E} = E_m \sin(x) \sin(t)\, \vec{a_y}$

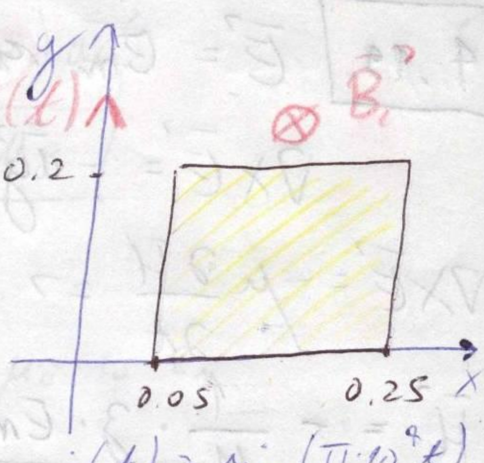
$$\vec{H} = \frac{E_m}{\mu_0} \cos(x) \cos(t)\, \vec{a_z}$$

TREBA PROVJERITI JE LI POLJE ZADOVOLJAVA MAXWELLOVE JEDNADŽBE

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \rightarrow \text{ZADOVOLJAVA}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \rightarrow \boxed{\text{NE ZADOVOLJAVA}}$$

4.15. $\vec{B} = \frac{\mu_0 \cdot i(t)}{2\pi x}\left(-\vec{a_z}\right)$



$i(t) = \sin(\pi \cdot 10^4 t)$

$$\phi = -\iint_S \frac{\mu_0 \cdot i(t)}{2\pi x} \cdot dx \cdot dy$$

$$\phi = -\frac{\mu_0 \cdot i(t)}{2\pi} \int_{0.05}^{0.25} \frac{1}{x}\, dx \int_0^{0.2} dy$$

$$\phi = \frac{-4\pi \cdot 10^{-7} \cdot i(t)}{2\pi} \cdot \ln\frac{0.25}{0.05} \cdot 0.2$$

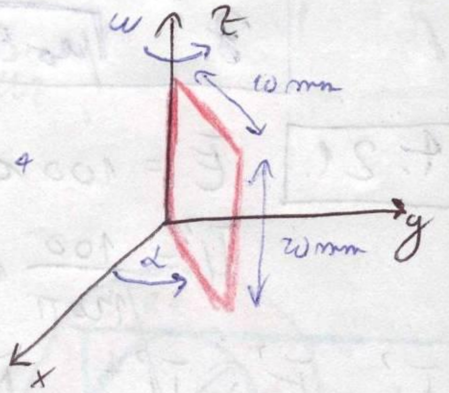
$\phi = -0.64377 \cdot 10^{-7} \sin(\pi \cdot 10^4 t)$ $\quad u_{ind} = -\frac{d\phi}{dt} = 0.64 \cdot 10^{-3} \cdot \pi \cos(\pi 10^4 t)$

$u_0 = 0.64377 \cdot \pi \cdot 10^{-3}$ $\quad \boxed{u = 2.02247\ mV}$

4.16. $R = 20 \cdot 10^{-3}$ $\quad \vec{B} = 10 \cdot 10^{-3}\, \vec{a_y}$

$\omega = 2$ $\quad \alpha = 0.25\pi$

$S = 20 \cdot 10 \cdot 10^{-6} = 2 \cdot 10^{-4}$



$$\phi = B \cdot S \cos \omega t = 2 \cdot 10^{-6} \cos(\underset{0.25\pi}{\omega t})$$

$$u_{ind} = -\frac{d\phi}{dt} = 2 \cdot 10^{-6} \cdot \omega \cdot \sin(\omega t)$$

$u = 2\sqrt{2} \cdot 10^{-6}$ $\quad i_{ind} = \frac{u_{ind}}{R} = \frac{2\sqrt{2} \cdot 10^{-6}}{2 \cdot 10^{-2}} = \sqrt{2} \cdot 10^{-4}$

$\boxed{i_{ind} = 0.14142\ [mA]}$

4.17. $\vec{E} = E_m \cos(\omega t - k_2 \cdot z)\, \vec{a_x}$

$\vec{H} = \frac{E_m}{k_1} \cos(\omega t - k_2 \cdot z)\, \vec{a_y}$

$$\boxed{\frac{E_0}{H_0} = Z = \sqrt{\frac{\mu}{\varepsilon}}} \qquad \frac{\cancel{E_m}}{\frac{\cancel{E_m}}{k_1}} = \boxed{\sqrt{\frac{\mu_0}{\varepsilon_0}} = k_1}$$

4.18. isti podatci kao u zadatku 4.17, traži se k_2

iz zapisa se vidi da je $k_2 = \beta$, a $\boxed{\beta = \omega\sqrt{\mu\varepsilon}}$

$$\boxed{k_2 = \omega \cdot \varepsilon_0 \sqrt{\frac{\mu_0}{\varepsilon_0}}}$$

4.19. $\vec{E} = E_m \cos(\beta \cdot x) \cos(\omega t) \vec{a_z}$

$$\nabla \times \vec{E} = -\vec{a_y}\left[E_m \cdot \beta \cdot (-\sin(\beta x)) \cdot \cos(\omega t)\right]$$

$$\nabla \times \vec{E} = -\mu \frac{\partial H}{\partial t} \Rightarrow \frac{\partial H}{\partial t} = -\frac{1}{\mu}\beta \cdot E_m \sin\beta x \cdot \cos(\omega t)\vec{a_y}$$

$$\vec{H} = -\frac{1}{\mu} \cdot \beta \cdot E_m \cdot \sin(\beta x) \cdot \frac{1}{\omega} \cdot \sin(\omega t) \cdot \vec{a_y}$$

$$\boxed{\vec{H} = -\frac{\sqrt{\varepsilon_0}}{\sqrt{\mu_0}} \cdot E_m \sin(\beta x) \sin(\omega t) \vec{a_y}}$$

4.20 tvorzi se omjer $\frac{\omega}{\beta}$ iz postavljeny sluzbeny rješenja

$$\boxed{\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}}}$$

4.21. $\vec{E} = 100 \cos(\omega t + \frac{2\pi}{5}x)\vec{a_z}$ $\quad \underline{E} = 100 e^{j\frac{2\pi}{5}x}\vec{a_z}$

$\vec{H} = \frac{100}{120\pi}\cos(\omega t + \frac{2\pi}{5}x)\vec{a_y}$ $\quad \underline{\vec{H}}^* = \frac{100}{120\pi} e^{-j\frac{2\pi}{5}x}\vec{a_y}$

$$\boxed{\vec{\underline{N}} = \underline{\vec{E}} \times \underline{\vec{H}}^*} \quad \vec{\underline{N}} = -\vec{a_x}\left(\frac{100^2}{120\pi} e^0\right) \Rightarrow \boxed{\vec{\underline{N}} = -26.52582\,\vec{a_x}}$$

4.22. $\vec{E} = 100\cos(\omega t + \frac{4\pi}{3}x)\vec{a_z}$ $\quad \vec{H} = \frac{100}{120\pi}\cos(\omega t + \frac{4\pi}{3}x)\vec{a_y}$

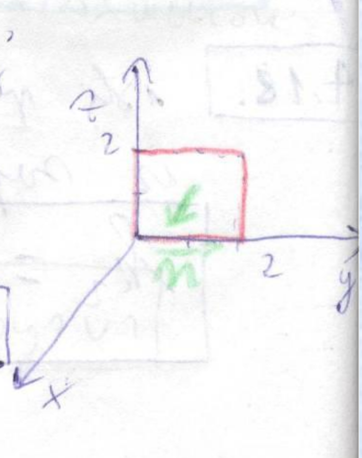
a) $\underline{\vec{E}} = 100\, e^{j\frac{4\pi}{3}x}\vec{a_z}$ $\quad$ b) $\underline{\vec{H}} = \frac{100}{120\pi} e^{j\frac{4\pi}{3}x}\vec{a_y}$

c) $\vec{N} = \vec{E} \times \vec{H} = -\vec{a_x}\left[\frac{100^2}{120\pi}\cos^2\left(\omega t + \frac{4\pi}{3}x\right)\right]$ $\quad \omega = 2 \cdot 10^5$

$$\boxed{N(x=1, t=2)} = -\vec{a_x}\left[\frac{100^2}{120\pi}\cos^2\left(2 \cdot 10^5 \cdot 2 + \frac{4\pi}{3}\right)\right] = \boxed{-10.14\,\vec{a_x}}$$

d) $\vec{N}_{sr} = \frac{1}{2}\vec{N} = \frac{1}{2}\frac{100^2}{120\pi}(-\vec{a_x})$ $\quad \vec{n} = \vec{a_x}$

$$P_{sr} = \iint_S \vec{N}_{sr}\,\vec{n}\,dS = -\frac{1}{2}\frac{100^2}{120\pi}\int_0^2 dy \int_0^2 dz$$



$$\boxed{P_{sr} =} -2 \cdot \frac{100^2}{\cancel{120\pi}_{60\pi}} = -\frac{100^2}{60\pi} = \boxed{-53.05164\ W}$$

$\boxed{4.23.}$ $\mu_r = 1$, $\vec{E} = 3\sin(2\cdot10^8 t - 2x)\,\vec{a}_y$

$\boxed{a)}$ SMJER U KOJEM SE GIBA VAL $\boxed{\vec{u}_x}$

$\boxed{b)}$ $\boxed{\lambda =} \dfrac{2\pi}{\beta} = \dfrac{2\pi}{2} = \boxed{\pi}$

$\boxed{c)}$ $\beta = \omega\cdot\sqrt{\mu_0\cdot\mu_r\cdot\varepsilon_0\cdot\varepsilon_r}\,/^2 \Rightarrow \beta^2 = \omega^2\mu_0\cdot\varepsilon_0\cdot\varepsilon_r$

$\boxed{\varepsilon_r =} \dfrac{\beta^2}{\omega^2\mu_0\varepsilon_0} = \boxed{8.98774}$

$\boxed{d)}$ $\nabla\times\vec{E} = \vec{u}_z\left(-6\cos(2\cdot10^8 t - 2x)\right) = -\mu\dfrac{\partial\vec{H}}{\partial t}$

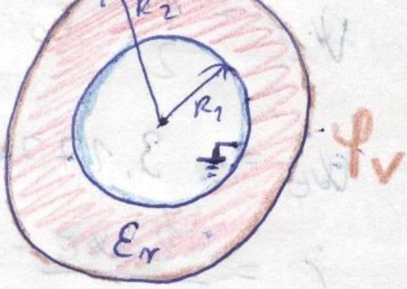
$\dfrac{\partial\vec{H}}{\partial t} = \dfrac{6}{\mu}\cdot\cos(2\cdot10^8 t - 2x)\,\vec{u}_z \,/\!\int dt$

$\vec{H} = \dfrac{6}{\mu}\cdot\dfrac{1}{2\cdot10^8}\sin(2\cdot10^8 t - 2x)\,\vec{u}_z$

$\boxed{\vec{H} = 0.024\sin(2\cdot10^8 t - 2x)\,\vec{u}_z}$

$\boxed{4.24.}$ $R_1 = 5\cdot10^{-3}$, $R_2 = 6\cdot10^{-3}$, $l = 500\cdot10^{-3}$

$\varepsilon_r = 6.7$, $\varphi_V = 250\sin(377)$



$\boxed{a)}$ $\Delta\varphi(r) = 0 \Rightarrow \varphi = \ln r\cdot C_1 + C_2$

$\varphi(R_1) = 0 \Rightarrow \ln R_1\cdot C_1 + C_2 = 0$

$\boxed{C_2 = 5.29831\,C_1} \Rightarrow \boxed{C_1 = 1367.67681\sin(377t)}$

$\varphi(R_2) = \varphi_V \Rightarrow \varphi_V = \ln R_2\cdot C_1 + C_2$

$\boxed{C_2 = 250\sin(377t) + 5.11599\,C_1}$

$\varphi(r) = \ln r(1367.67681\sin(377t)) + C_2$

$\vec{E} = -\nabla\varphi(r) \Rightarrow \nabla\varphi(r) = \dfrac{1}{r}\cdot 1367.67681\sin(377t)\,\vec{u}_r$

$\vec{E}(r = 5.5\cdot10^{-3}) = -\dfrac{1}{5.5\cdot10^{-3}}\cdot 1367.67681\sin(377t)\,\vec{u}_r$

$\boxed{\vec{E} = -248668.5109\sin(377t)\,\vec{u}_r}$

b) $\vec{J}_p = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{\partial \vec{E}}{\partial t} = -8.854 \cdot 10^{-12} \cdot 6.7 \cdot \frac{1367.67}{r} \cdot 377 \cdot \cos(377t)\,\vec{a}_r$

$$\boxed{\vec{J}_p(r = 5.2 \cdot 10^{-3}) = -5.87923 \cdot 10^{-3} \cos(377t)\,\vec{a}_r}$$

c) $I_p = \iint_S \vec{J}_p \cdot \vec{n} \cdot dS \qquad \vec{n} = -\vec{a}_r$

$I_p = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{1367.67}{r} \cdot 377 \cdot \cos(377t) \cdot r 2\pi l$

$$\boxed{I_p = 9.60923 \cdot 10^{-5} \cos(377t)}$$

$dS = r\, d\varphi\, dz$

$l = 500 \cdot 10^{-3}$

d) $W_E = \frac{1}{2} \iiint \varepsilon\, |\vec{E}|^2\, dV$

$|\vec{E}|^2 = \frac{1870539.857}{r^2} \cdot \sin^2(377t)$

$W_E = \frac{1}{2} \cdot \varepsilon_0 \cdot \varepsilon_r \cdot 1870539.857 \cdot \sin^2(377t) \int_{r_1}^{r_2} \frac{r}{r^2}\, dr \int_0^{2\pi} d\varphi \int_0^{l} dz$

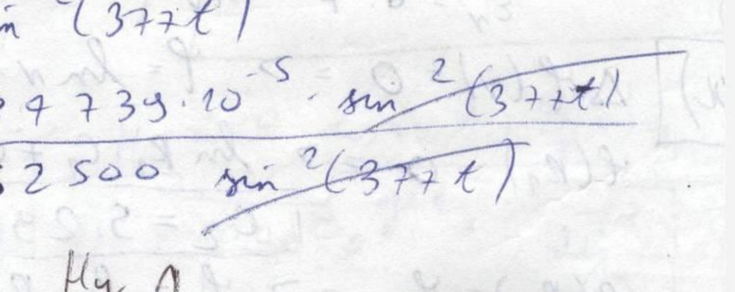
$W_E = 3.17789237 \cdot 10^{-5} \sin^2(377t)$

$C = \frac{2 W_E}{U^2} = \frac{6.355784739 \cdot 10^{-5} \cdot \cancel{\sin^2(377t)}}{62500\, \cancel{\sin^2(377t)}}$

$$\boxed{C = 1.01692 \cdot 10^{-9}\ F}$$

4.25. a) $\frac{E_m}{H_m} = \boxed{Z = \sqrt{\frac{\mu}{\varepsilon}}}$

$\varepsilon_r = 4$



$Z = \sqrt{\frac{\mu_0 \cdot \mu_r}{\varepsilon_0 \cdot \varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r}} = \frac{1}{2} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$Z = 188.36715$

$\boxed{v = \frac{1}{\sqrt{\mu \cdot \varepsilon}}} = 1.49897 \cdot 10^{8} \frac{m}{s}$

$\boxed{v = \frac{z_2 - z_1}{t_2 - t_1}} \qquad z_1 = 0$

$t_2 = 1 \mu s$

$z_2 - 0 = v(t_2 - t_1)$ u $\boxed{a)}$ drugi slučaj $t_1 = -\mu s$

$z_2 = 1.99897 \cdot 10^8 (1\mu s - (-1\mu s)) = 299.79563$

iz grafa $z = 299.79563 \Rightarrow H = 0 \Rightarrow \boxed{E = 0}$

$\boxed{b)}$ $t_1 = -1.5\mu s$

$z_2 = 1.49897 \cdot 10^8 (1\mu s + 1.5\mu s) = 374.7425$

jednadžba pravca: $y - 0 = \frac{1-0}{600-300}(x - 300)$

$H = 0.24914$

$E = H \cdot Z$

$\boxed{E = 46.9301}$

$y = \frac{1}{300}x - 1$

$y = \frac{1}{300} \cdot 374.7425 - 1 = 0.24914$

$\boxed{c)}$ $t_1 = -4\mu s$

$z_2 = 1.9987 \cdot 10^8 \cdot (5\mu s) = 749.35$ $H = 1$

$\boxed{E = 188.36715}$

$\boxed{d)}$ $t_1 = -6\mu s$

$z_2 = 1.9987 \cdot 10^8 \cdot 7\mu s = 1049.09$

jednadžba pravca $y - 1 = \frac{0-1}{1200-900}(x - 900)$

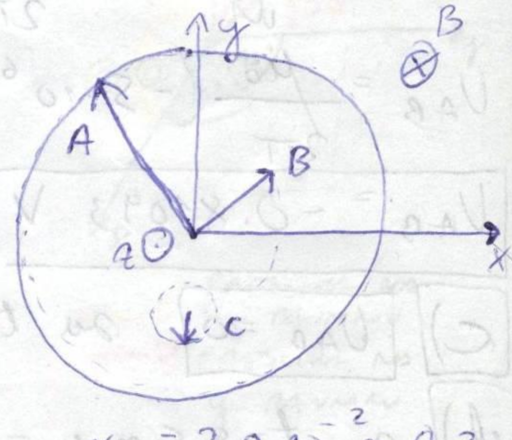
$H = 0.50303$

$\boxed{E = 94.75495}$

$y = -\frac{1}{300}x + 3 + 1 = -\frac{1}{300}x + 4$

$\boxed{4.26.}$ $\vec{B} = -0.5\,\vec{a}_z$ $\vec{v} = -\omega \cdot r \cdot \vec{a}_\phi$

$d\vec{l} = dr\,\vec{a}_r$ $\vec{v} \times \vec{B} = \omega r B \vec{a}_r$



$u_{ind} = \int_r (\vec{v} \times \vec{B}) \cdot d\vec{l} = \omega \cdot B \int_0^r r\,dr$

$\omega_A = \frac{2\pi}{3600}$ $\omega_B = \frac{2\pi}{12 \cdot 3600}$ $\omega_C = \frac{2\pi}{60}$

$\boxed{a)}$

$u_{ind\,A} = \omega_A B \cdot \frac{1}{2} r_A^2 = 0.17453 \cdot 10^{-4}$

$r_A = 20 \cdot 10^{-2} = 0.2$

$r_B = 10 \cdot 10^{-2} = 0.1$

$u_{ind\,B} = \omega_B B \cdot \frac{1}{2} r_B^2 = 0.36361 \cdot 10^{-6}$

$r_C = 5 \cdot 10^{-2} = 0.05$

$\boxed{u_{AB}} = u_{ind\,A} - u_{ind\,B} = \boxed{17.08938\,\mu V}$

b) $u_{ind\,A} = \omega_c \cdot B \cdot \frac{1}{2} \cdot r_c^2 \qquad \omega_c = \frac{2\pi}{60}$

$u_{ind\,C} = 0.654498469 \cdot 10^{-4}$

$\boxed{U_{AC}} = u_{ind\,A} - u_{ind\,C} = \boxed{-47.99684\ \mu V}$

c), d) SAT POČINJE ROTIRATI BRZINOM $\omega_y = \frac{\pi}{2}$ rad/s.
ZA DVIJE SEKUNDE (c) ILI 4 SEKUNDE (d)
SAT NAPRAVI POLA ILI CIJELI KRUG TAKO DA
KOLIČINA MAGNETSKOG TOKA U OBA SLUČAJA
OSTAJE JEDNAKA.

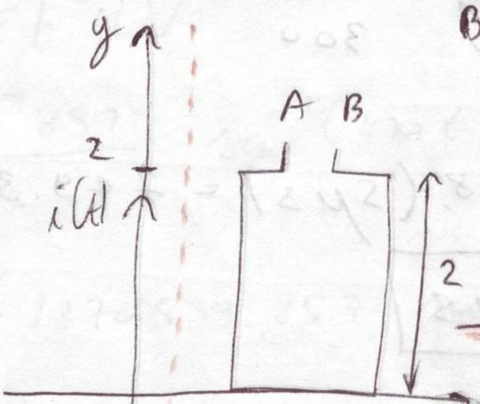
c) $\vec{n} = \omega n \cdot \vec{a}_z \Rightarrow \boxed{U_{AC} = -47.99684\ \mu V}$ d) $U_{AC} = -47.99\ \mu V$

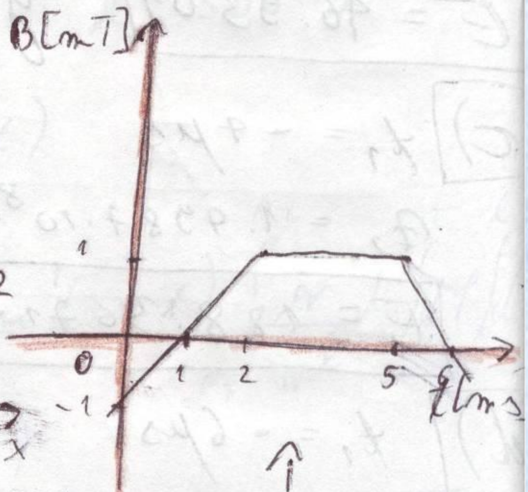
4.27. a) za $t = 4\,ms$

$B = 10^{-3}\ T$

$B = \frac{\mu_0 \cdot i(t)}{2\pi \cdot 1}$

$\boxed{i(t)} = \frac{10^{-3}}{2 \cdot 10^{-7}} = \boxed{5\ kA}$





b) za $t = 1\,ms$

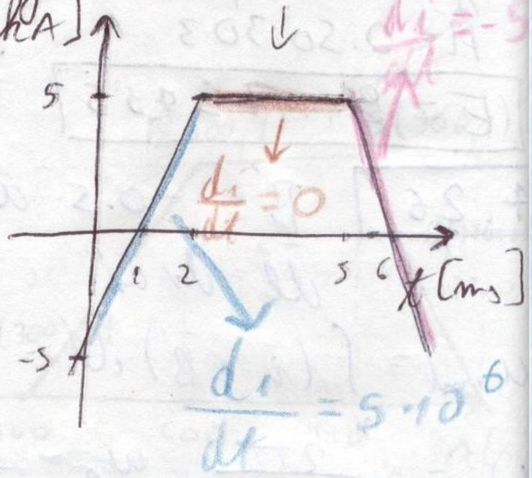
- na udaljenosti 1 m od vodiča
- na udaljenosti 2-3m od vodiča

$U_{AB} = -\frac{d}{dt}\iint_S \vec{B} \cdot \vec{n} \cdot dS$

$U_{AB} = -\frac{d}{dt}(i(t)) \cdot \frac{\mu_0}{2\pi}\int_2^3 \frac{1}{x}\,dx \int_0^2 dy$

$U_{AB} = \frac{\mu_0}{2\pi}(-5 \cdot 10^6) \cdot 2 \cdot 10^{-7} \ln(1.5) \cdot 2$

$\boxed{U_{AB} = -0.81093\ V}$



c) $\boxed{U_{AB} = 0}$ za $t = 3\,ms$ jer je $\frac{di(t)}{dt} = 0$

d) za $t = 5.5\,ms \Rightarrow \frac{di}{dt} = -5 \cdot 10^6$ tako da $\boxed{U_{AB} = 0.81093\,V}$

$\boxed{4.28.}$ $\vec{B} = \begin{cases} -B_0 \dfrac{a \cdot t}{\sqrt{a^2+r^2}} \vec{a_z}, & t > 0 \\ 0, & t < 0 \end{cases}$ $a = 0.1$, $B_0 = 1$, $\kappa = 0$, $\varepsilon_r = 1$

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot \vec{n} \cdot dS$$

$$E_\alpha \cdot 2\pi r = -\frac{\partial}{\partial t} \int_0^r r\, dr \int_0^{2\pi} d\varphi \left(-\frac{B_0 \cdot a \cdot t}{\sqrt{r^2+a^2}} \right)$$

$$E_\alpha \cdot \cancel{2\pi}\, r = B_0 \cdot a \cdot \cancel{2\pi} \int_0^r \frac{r}{\sqrt{r^2+a^2}} dr$$

$$E_\alpha \cdot r = B_0 \cdot a \left(\sqrt{r^2+a^2} - a \right)$$

a) za $r = 0.05$ $\Rightarrow$ $\boxed{E_\alpha = 23.60679\ \mathrm{m}\frac{V}{m}}$

b) za $r = 0.1$ $\Rightarrow$ $\boxed{E_\alpha = 41.42135\ \mathrm{m}\frac{V}{m}}$

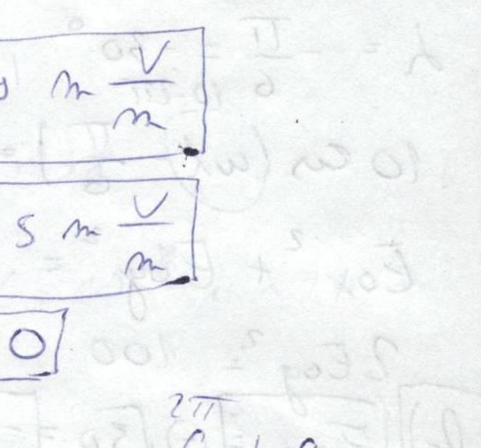
c) za $t = -1\,\mathrm{ms}$ $\Rightarrow$ $B = 0$ $\Rightarrow$ $\boxed{E = 0}$

d) $$\phi = \iint_S \vec{B} \cdot \vec{n} \cdot dS = \int_0^a B_0 \frac{a \cdot t}{\sqrt{r^2+a^2}} r\, dr \int_0^{2\pi} d\varphi$$

$$\boxed{\phi} = 2\pi \cdot 1 \cdot 0.1 \cdot 10^{-3} \left(\sqrt{2}a - a \right) = \boxed{26.0258 \cdot 10^{-6}\ \mathrm{Wb}}$$

$\boxed{4.29.}$ $f = 10^7$, $\omega = 2\pi f$

$$\vec{\beta} = \beta_x \vec{a_x} + \beta_y \vec{a_y} + \beta_z \vec{a_z}$$



$$\vec{\beta} = \left[\cos 45^\circ \cdot \cos 60^\circ \vec{a_x} + \sin 45^\circ \cos 60^\circ \vec{a_y} + \cos 30^\circ \vec{a_z} \right] \cdot \beta$$

$$\beta = \omega \cdot \sqrt{\mu_0 \varepsilon_0} = 0.20958$$

$$\vec{\beta} = 0.20958 \left(\frac{\sqrt{2}}{4} \vec{a_x} + \frac{\sqrt{2}}{4} \vec{a_y} + \frac{\sqrt{3}}{2} \vec{a_z} \right)$$

PRESLIKA NA x RAVNINU

PRESLIKA NA y RAVNINU

U OBA SLUČAJA KUT IZMEĐU PRESLIKA I Z-OSI JE 30° (IZMEĐU PRESLIKA I X I Y OSI 60°)

$$\vec{E} = \vec{E_0} e^{-j \vec{\beta} \cdot \vec{r}}$$

u zadatku: $\underline{E_{0z}} = 0$

$$\vec{E_0} = \underline{E_{0x}} \vec{a_x} + \underline{E_{0y}} \vec{a_y}$$

$$\vec{E} \perp \vec{\beta} \rightarrow \vec{\beta} \cdot \vec{E_0} = 0$$

$$\beta_x \underline{E_{0x}} + \beta_y \underline{E_{0y}} = 0$$

$$\underline{E_{0x}} = E_{0x} e^{j\alpha} \qquad \underline{E_{0y}} = E_{0y} e^{j\alpha}$$

$$\underline{\vec{E}_0} = (E_{0x} \vec{a}_x + E_{0y} \vec{a}_y) e^{j\alpha} \qquad \beta_x = \beta_y = \frac{\sqrt{2}}{4}$$

$$\beta_x E_{0x} + \beta_y E_{0y} = 0 \qquad E_{0x} = -E_{0y}$$

$$|\vec{E}(0,0,0)| = |\underline{\vec{E}_0}| = |E_{0x} \vec{a}_x + E_{0y} \vec{a}_y| e^{j\alpha} = 10\cos(\omega t - \frac{\pi}{6})$$

$$\alpha = -\frac{\pi}{6} = -30^\circ, \quad \omega = 2\pi \cdot 10^7 \cdot 10^{-6} = 20\pi = 2\pi \cdot 10$$
$$10 \cdot 2\pi$$

$$10\cos(\omega t - \frac{\pi}{6}) = 10 e^{-\frac{\pi}{6}} \qquad \boxed{a)} \ \boxed{E_{0x} =} \oplus \sqrt{50} = \boxed{7.1}$$

↓ ZADANO U ZADATKU $E_{0x} > 0$.

$$E_{0x}^2 + E_{0y}^2 = 100$$

$$2E_{0y}^2 = 100$$

$$\boxed{b)} \ \boxed{E_{0y}} = \ominus\sqrt{50} = \boxed{-7.1} \qquad \vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$$

$$\vec{E} = \vec{E}_0 \cdot e^{-j \cdot \vec{\beta} \cdot \vec{r}}$$

$$\vec{E} = e^{-j\frac{\pi}{6}} (\sqrt{50}\,\vec{a}_x - \sqrt{50}\,\vec{a}_y) e^{-j \cdot \beta \left(\frac{\sqrt{2}}{4}x + \frac{\sqrt{2}}{4}y + \frac{\sqrt{3}}{2}z\right)}$$

$$\vec{E} = \sqrt{50}(\vec{a}_x - \vec{a}_y)\cos\left(\omega t - 0.20958\left(\frac{\sqrt{2}}{4}x + \frac{\sqrt{2}}{4}y + \frac{\sqrt{3}}{2}z\right) - \frac{\pi}{6}\right)$$

$$\vec{H}_0 = \frac{1}{\omega\mu} \cdot \vec{\beta} \times \vec{E}_0$$

$$\vec{H}_0 = \frac{1}{\omega\mu} \cdot \overset{\omega\sqrt{\mu\varepsilon}}{\beta} \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{3}}{2} \\ \sqrt{50} & -\sqrt{50} & 0 \end{vmatrix} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \left[\begin{array}{l} \vec{a}_x\left(\frac{\sqrt{3}}{2}\cdot\sqrt{50}\right) \\ -\vec{a}_y\left(-\frac{\sqrt{3}}{2}\cdot\sqrt{50}\right) \\ +\vec{a}_z\left(-2\cdot\frac{\sqrt{2}}{4}\sqrt{5}\right. \end{array} \right.$$

$$\vec{H}_0 = \sqrt{\frac{\varepsilon}{\mu}}\left[\vec{a}_x \cdot \frac{\sqrt{150}}{2} + \vec{a}_y \cdot \frac{\sqrt{150}}{2} - 5\vec{a}_z\right]$$

$$\boxed{c) \ H_{0x} = 16.25475 \ mA/m} = \boxed{H_{0y} \ d)}$$

4.30. $\mu_r = 1$, $\varepsilon_r = 2$ $\quad \vec{H} = \vec{a_y}\, 10 \cos(\omega t - 3x)$

a) $\boxed{\beta = 3}$ npr. $\vec{A} = a \cos(\omega t - b \cdot x) \cdot \vec{n}$

OVA VRIJEDNOST JE β

BRZINA ŠIRENJA VALA:

b) $\boxed{v = \frac{1}{\sqrt{\mu \varepsilon}}} = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r}} = \boxed{2.11987 \cdot 10^8 \frac{m}{s}}$

c) $Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r}} = 266.3913$

$\omega = \frac{\beta}{\sqrt{\mu_0 \varepsilon_0} \cdot \sqrt{\varepsilon_r}} = 6.35962 \cdot 10^8$

$\frac{E_0}{H_0} = Z \Rightarrow E_0 = 2663.9138$

$E = E_0 \cos(\omega t - 3x)$ za $t = 10^{-8}$ i $x = 0.4$

$\boxed{E = 1.15207\ kV/m}$

d) SMJER ŠIRENJA VALA ODREĐUJEMO IZ IZRAZA ZA $\vec{E}$ ili $\vec{H}$ (ili $\vec{N}$)

npr. $\vec{A} = a \cdot \cos(\omega t - b \cdot c) \cdot \vec{n}$

GDJE c MOŽE BITI x, y, z, r, φ ili θ

AKO JE $-$ ISPRED ONDA JE SMJER U $+\vec{c}$,

AKO JE $+$ ISPRED ONDA JE SMJER U $-\vec{c}$.

U OVOM ZADATKU TU JE $\boxed{+\vec{a_x}}$

4.31. $\vec{E} = 100 \sin(\theta) \cdot \frac{1}{r} \cos(10^{10} t - r)\, \vec{a_\theta}$

$\vec{H} = \frac{100}{120\pi} \sin(\theta) \cdot \frac{1}{r} \cdot \cos(10^{10} t - r)\, \vec{a_\varphi}$

a) SMJER PROSTIRANJA VALA JE $\vec{a_r}$

b) $\vec{N} = \vec{E} \times \vec{H} = \vec{a_r} \frac{100^2}{120\pi} \sin^2(\theta) \frac{1}{r^2} \cos^2(10^{10} t - r)$

$$\bar{N}'\left(r=2,\ t=10^{-9},\ \theta=\frac{\pi}{6}\right) = \frac{100^2}{120\pi}\sin^2\frac{\pi}{6}\cdot\frac{1}{2^2}\cos^2\left(10^{10}\cdot 10^{-9}-2\right)$$

$$\boxed{\bar{N}'} = 1.65786\cdot\cos^2 8 = \boxed{35.09741\ m\frac{W}{m^2}}$$

za $r=4\ m,\ \theta=\frac{\pi}{4}$

c) $$N_{sr} = \frac{1}{2}|\bar{N}'| = \frac{1}{2}\cdot\frac{100^2}{120\pi}\left(\frac{\sqrt{2}}{2}\right)^2\cdot\frac{1}{4^2}$$

$$\boxed{N_{sr} = 414,46599\ m\frac{W}{m^2}}$$

d) $$P_{sr} = \oint_S \bar{N}_{sr}\cdot\bar{n}\,dS = \frac{1}{2}\cdot\frac{100^2}{120\pi}\sin^2(\theta)\cdot\cancel{\frac{1}{r^2}}\cdot\cancel{r^2}\int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\alpha$$

$$P_{sr} = \frac{250}{3}\int_0^{\pi}\sin^3\theta\,d\theta = \frac{250}{3}\left(-\underbrace{\frac{\sin^2\theta\cos\theta}{3}\Big|_{\theta=0}^{\pi}}_{0} + \frac{2}{3}\int_0^{\pi}\sin\theta\,d\theta\right)$$

$$\boxed{P_{sr}} = \frac{250}{3}\cdot\frac{2}{3}\cdot(-\cos\theta)\Big|_{\theta=0}^{\pi} = \frac{500}{9}\cdot 2 = \boxed{111.111\ W}$$

Sretno na ispitima !!