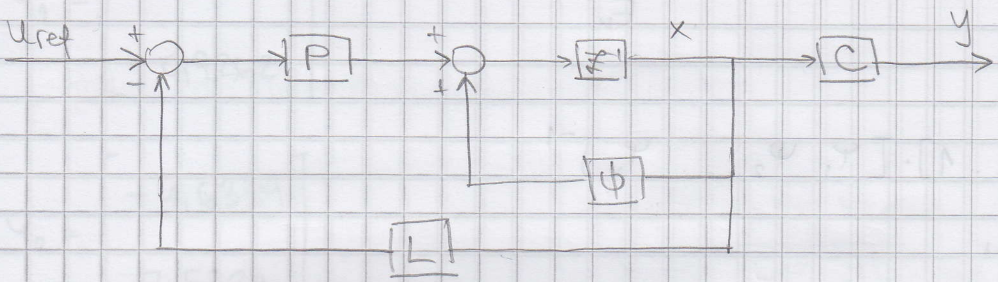


$$x(k+1) = \underbrace{(\Phi - \Gamma \cdot L)}_{\Phi_{cl}} x(k)$$

$$\Delta_{cl}(z) = |zI - \Phi + \Gamma L| = 0 \longrightarrow z_1, z_2 \ldots z_n$$

$$\frac{Y}{U_{ref}} = C(zI - \Phi_{cl})^{-1}\Gamma$$

$$|zI_n - \Phi + \Gamma L| = |(zI_n - \Phi)(I_n + (zI_n - \Phi)^{-1}\Gamma \cdot L)|$$

$$= |zI_n - \Phi| \cdot \underbrace{|I_n + (zI_n - \Phi)^{-1}\Gamma \cdot L|}_{\substack{0 \\ \Delta_{cli}}} = 0 \rightarrow \text{svojstvene vrijednosti}$$

$$|I_n + \underbrace{(zI_n - \Phi)^{-1}\Gamma}_{\substack{M_1 \\ (n \times p)}} \underbrace{L}_{\substack{M_2 = L \\ (p \times n)}})| = 0$$

$$|I_n + M_1 \cdot M_2| = |I_p + M_2 \cdot M_1|$$

$$= |I_p + L \cdot (zI_n - \Phi)^{-1}\Gamma)| = 0$$

$\underline{p=1}$ $\quad L = [l_1\, l_2 \ldots, l_n] = ?$

$$1 + L \underbrace{(zI_n - \Phi)^{-1}\Gamma}_{\varphi_i} = 0$$

z_{cli} – i-to rješenje

$$\underset{n \times p}{\varphi_i} = (z_{cli} I_n - \Phi)^{-1}\Gamma$$

$$\boxed{L \cdot \varphi_i = -1}$$

$$L \cdot [\varphi_1 \mid \varphi_2 \; \ldots \; \varphi_n] = -\underbrace{[1 \; 1 \; \ldots \; 1]}_{E_n} \quad / \; [\varphi_1 \mid \varphi_2 \ldots \varphi_n]^{-1}$$

$n \times n$

$$L = -[1 \; 1 \; \ldots \; 1] \cdot [\varphi_1 \; \varphi_2 \ldots \; \varphi_n]^{-1}$$

$$L = -E_n \cdot \Psi^{-1}$$

Primjer

$$\phi = \begin{bmatrix} 0{,}9909 & 0{,}0861 \\ -0{,}1772 & 0{,}7526 \end{bmatrix} \qquad |z_{oL} I_n - \phi| = 0$$

$$\Gamma = \begin{bmatrix} 0{,}6045 \\ 0{,}0561 \end{bmatrix}$$

$$z_{oL} = 0{,}9048 \; ; \; 0{,}8187$$

$$z_{oL}^2 - 1{,}7235 z_{oL} + 0{,}7408 = 0$$

$$(z - z_{oL1})(z - z_{oL2}) = 0$$

$$\mathcal{C} = [\Gamma \mid \phi\Gamma] = \begin{bmatrix} 0{,}0045 & 0{,}0119 \\ 0{,}0861 & 0{,}0623 \end{bmatrix} \quad \text{rang}\,\mathcal{C} = 2 \; ! \; \text{upravljivo!}$$

a) realni, različiti

$$\left.\begin{aligned} z_{d1} = z_{z1} &= 0{,}95 \\ z_{d2} = z_{z2} &= 0{,}85 \end{aligned}\right\} \text{željena rješenja}$$

$$\varphi_1 = \begin{bmatrix} 1{,}4167 \\ -0{,}7262 \end{bmatrix}$$

$$\varphi_2 = \begin{bmatrix} -4{,}6339 \\ 7{,}5324 \end{bmatrix}$$

$$L = \begin{bmatrix} -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \varphi_1 & \varphi_2 \end{bmatrix}^{-1}$$

$$L = \begin{bmatrix} -1{,}1304 & -0{,}8282 \end{bmatrix}$$

b) konj. komplex

$$z_{1,2} = 0{,}95 \pm j0{,}1$$

$$\varphi_1 = \begin{bmatrix} -0{,}0799 - j0{,}4576 \\ 0{,}5168 + j0{,}1248 \end{bmatrix}$$

$$\varphi_2 = \begin{bmatrix} -0{,}0779 + j0{,}4576 \\ 0{,}5168 - j0{,}1248 \end{bmatrix}$$

$$L = \begin{bmatrix} -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \varphi_1 & \varphi_2 \end{bmatrix}^{-1}$$

c) $p = 1$

$z_{\check{z}_1} = z_{\check{z}_2}$

$\psi_1 = \psi_2$

$[\psi_1 \ \ \psi_2]^{-1}$ – invert ne postoji !

$(\underline{\det = 0})$

$z_{\check{z}1}$

$\hookrightarrow \psi_1 = (z_{\check{z}1} \cdot I_n - \Phi)^{-1} \Gamma$

$L \cdot \psi_i = -1 \ \Big/ \frac{d}{dz} \Big|_{z = z_d}$

$L \cdot \underbrace{\left[\frac{\partial}{\partial z} (z I_n - \Phi)^{-1} \cdot \Gamma \right]}_{\psi_2'}\Bigg|_{z = z_d} = 0$

$L \cdot [\psi_1 \ \ \psi_2'] = [-1 \ \ 0]$

c) $z_{\check{z}_1} = z_{\check{z}2} = 0{,}95$

$\psi_1 = \begin{bmatrix} 1{,}4167 \\ -0{,}7262 \end{bmatrix}$

$\psi_2' = \frac{\partial}{\partial z} (z I_2 - \Phi)^{-1} \Gamma \Big|_{z = z_{\check{z}2} = 0{,}95}$

$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \qquad \Gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$

$$\begin{bmatrix} \gamma_1(z-\Phi_{22}) + \gamma_2\Phi_{12} \\ \\ \gamma_2(z-\Phi_{11}) + \gamma_1\Phi_{21} \end{bmatrix}$$

?

$$z^2 - (\Phi_{11}+\Phi_{22})z + \Phi_{11}\Phi_{22} - \Phi_{12}\Phi_{21} = |zI_2 - \Phi|$$

$$\Psi_2' = \begin{bmatrix} -41{,}3963 \\ \\ 36{,}1366 \end{bmatrix}$$

$$L = [-1 \quad 0]\,[\Psi_1 \quad \Psi_2']^{-1} = [-1{,}1701 \quad -1{,}9591]$$

Drugi način (provjera)

$$\Phi_{cl} = \Phi - \Gamma L$$

$$|zI_2 - \Phi_{cl}| = 0$$

ACKERMANNOVA FORMULA – 2 postupak

$$\boxed{L = \underbrace{[0 \quad 0 \quad \dots \quad 1]}_{n} \cdot \mathcal{C}^{-1} \cdot \alpha_c(\Phi)}$$

→ i za diskr.
i za kont.

polinom željene dinamike

$$\alpha_c = (z - z_1) \cdot (z - z_2) = z^2 + \beta_1 z + \beta_2 z$$

$$\alpha_c(\Phi) = \Phi^2 + \beta_1 \Phi + \beta_2 \cdot I_n$$

DIGRESIJA

$$\alpha_c(z_i) = 0$$

Cayley-Hamilton-ov teorem

$$\boxed{\alpha_c(\Phi) = 0_{n \times n}}$$

$$|zI - \Phi| = : \alpha(z)$$

matrica sustava zadovoljava svoju karakt. jedn.

kont: $\Phi \to A$

$\Gamma \to B$

$L = acker(A, B, \lambda)$

↑ vektor željenih svojstvenih vrijednosti

$$\begin{bmatrix} z_{ž1} \\ \vdots \\ z_{žn} \end{bmatrix}$$

(+) → nema problema sa $z_{ž1} = z_{ž2}$

(−) → numerički nepouzdano za sustave $n \geq 10$

$cond(\mathcal{C}) = ?$ kondicijski broj

>> LOŠE! $(10^8 \,!\,)$

Primjer

$$\Phi = \begin{bmatrix} 0{,}25 & 0{,}5 \\ 1 & 2 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \qquad \mathcal{C} = \begin{bmatrix} 1 & 2{,}25 \\ 4 & 9 \end{bmatrix}$$

rang $\mathcal{C} = 1 \neq 2$

↳ nije upravljiv!

$z_o = 0 \;,\; 2{,}25$

$z_ž = 0 \;,\; 0{,}5$

$acker\left(\Phi, \Gamma, \begin{bmatrix} 0 \\ 0{,}5 \end{bmatrix}\right)$

$L = [inf \;\; inf]$ ili NaN

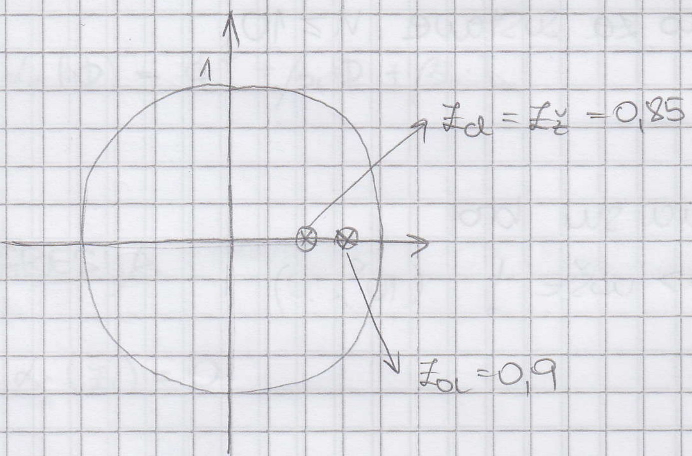
$L = \text{place}(\Phi, \Gamma, z_{\check{z}})$

(+) → numerička stabilnost

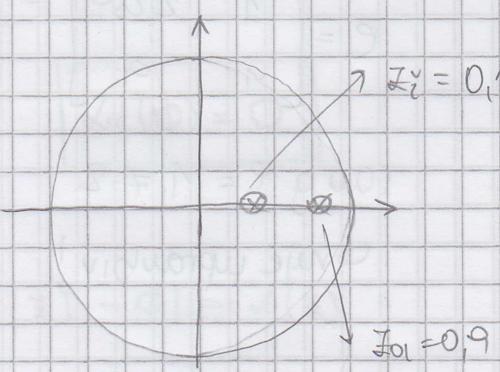
(+) → MIMO sustavi

(−) → broj višestrukih željenih svojstvenih vrijednosti ≤ broj ulaza

$L = [\,0{,}1944 \quad 0{,}3809\,]$



$l_1 = 0{,}05$



$l_2 = 0{,}8$

Primjer

$$\Phi = \begin{bmatrix} 0{,}9 & 0{,}95 \\ -0{,}1 & 0{,}6 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 0{,}5 \\ -0{,}5 \end{bmatrix}$$

$$z_{ol} = 0{,}75 \mp j0{,}27$$

a) $z_{\check{z}1,2} = 0{,}7 \ ; \ 0{,}8$

$$\Delta_{ol}(z) = (z - z_{ol1})(z - z_{ol2}) = z^2$$

$$\Delta_{cl}(z) = (z - z_{\check{z}1})(z - z_{\check{z}2}) = z^2 - 1{,}5z + 0{,}56 \quad = \alpha_c$$

$$\alpha_c(\Phi) = \Delta_{cl}(\Phi) \quad = \Phi^2 - 1{,}5\Phi + 0{,}56$$

$$\alpha_c(\Phi) = \begin{bmatrix} -0{,}075 & 0 \\ 0 & -0{,}075 \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} 0{,}5 & -0{,}025 \\ -0{,}5 & -0{,}35 \end{bmatrix} \Rightarrow \text{rang}(\mathcal{C}) = 2$$

$$L = [0 \ 0 \ \dots \ 1] \cdot \mathcal{C}^{-1} \cdot \alpha_c(\Phi)$$

$$L_a = [0{,}2 \quad 0{,}2]$$

$$\|L_a\| = 0{,}283$$

b) $z_{\check{z}1} = 0,1$

$z_{\check{z}2} = 0,2$

$L_b = [\,0,92 \quad -3,32\,]$

$\|L_b\| = 3,45$

↳ energija

c) DEAD BEAT REGULATOR

$z_{\check{z}1} = 0$

$z_{\check{z}2} = 0$

$\alpha_c(z) = (z - z_{\check{z}1})(z - z_{\check{z}2}) = z^2$

$\alpha_c(\Phi) = 0$

$\Phi_{cl}^2 = 0$

$x(k+1) = \Phi_{cl}\, x(k)$

$$G(s) = \frac{1}{(s+1)(s+2)}$$

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = 0$$

ZOH

sys = ss(A, B, C, D)

sysd = c2d (sys, T, 'ZOH')

a) $T = 1\ s$

$$L_1 = \begin{bmatrix} 2{,}7385 & -1{,}8178 \end{bmatrix}$$

$$\phi = \begin{bmatrix} 12{,}06 & -9{,}342 \\ 4{,}671 & -1{,}952 \end{bmatrix} \quad \Gamma = \begin{bmatrix} 4{,}671 \\ 1{,}476 \end{bmatrix}$$

b) $T = 0{,}1\ s$

$$L_2 = \begin{bmatrix} 16{,}025 & 83{,}8917 \end{bmatrix}$$

c) $T = 0{,}01\ s$

$$L_3 = \begin{bmatrix} 151 & 9848{,}9 \end{bmatrix}$$

MINIMUM NORM UPRAVLJANJE

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$

$$x(k) = \underbrace{\Phi^k \cdot x(0)}_{x_{zi}} + \underbrace{\sum_{i=0}^{k-1} \Phi^i \cdot \Gamma \cdot u(k-1-i)}_{x_{zs}}$$

$$x(k) - x_{zi} = \underbrace{\begin{bmatrix} \Gamma & \Phi\Gamma & \dots & \Phi^{k-1}\Gamma \end{bmatrix}}_{M} \cdot \underbrace{\begin{bmatrix} u(k-1) \\ u(k-2) \\ \vdots \\ u(0) \end{bmatrix}}_{u}$$

$$x(k) - x_{zi}(k) = \underbrace{M}_{n \times (p \cdot k)} \cdot \underbrace{u}_{(p \cdot k) \times 1}$$

$$u = M^T (M M^T)^{-1} \cdot [x(k) - x_{zi}(k)]$$