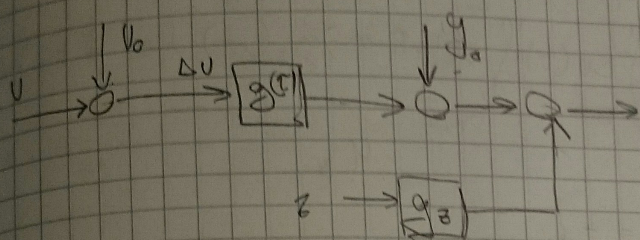
1. MI 14/15

Korištenjem kor. analize odrediti.



$R_{\Delta u z}(\tau) = 0, \forall \tau$

$$\boxed{R_{uy}(\tau) = \int_0^\infty R_{uu}(\tau-\sigma)\, g(\sigma)\, d\sigma}$$

$\overline{\Delta y} = \bar{z} = 0$

$u_0 y_0 = -4$ $\quad S_{\Delta u \Delta u}(\omega) = 4$

$R_{uy}(\tau) = \int e^{-\tau}\cos(\tau)\, S(\tau) - 4 \quad (1)$

$$\boxed{R_{uy}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u(t)\, y(t+\tau)\, dt}$$

$$R_{uy}(\tau) = R_{\Delta u + u_0,\, \Delta y - y_0 + y_z}(\tau) = R_{\Delta u \Delta y}(\tau) + R_{\Delta u y_z}(\tau) + R_{\Delta u y}(\tau) + R_{u_0 \Delta y}(\tau) + R_{u_0 y_0}(\tau) + R_{u_0 y_z}(\tau)$$

ukupni izraz

$R_{\Delta u \Delta u}(\tau) = 4\,\delta(\tau)$

$$R_{\Delta u \Delta y}(\tau) = \int_0^\infty R_{\Delta u \Delta u}(\tau-\sigma)\, g(\sigma)\, d\sigma = \int_0^\infty 4\,\delta(\tau-\sigma)\, g(\sigma)\, d\sigma = 4\, g(\tau)$$

$$R_{\Delta u y_0}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \Delta u(t) \underbrace{y_0(t+\tau)}_{\text{konst}} \, dt = y_0 \cdot \overline{\Delta u} = 0$$

$$R_{\Delta u y_z}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \Delta u(t) \underbrace{\int_0^{\infty} z(t+\tau-\sigma) g_z(\sigma) \, d\sigma}_{y_z} \, dt = 0$$

$$R_{u_0 y_0}(\tau) = u_0 y_0 = -4$$

Natrag na (4):

$$4g(\tau) - 4 = 2e^{-\tau} \cos(\tau) S(\tau)$$

$$\Rightarrow g(\tau) = \frac{1}{2} e^{-\tau} \cos(\tau) S(\tau)$$

2.

M1 14/15

$$S_{uu}(\omega) = \frac{1}{2} \frac{9\omega^2+1}{\omega^2+4}$$

$$S_{uy}(j\omega) = \frac{9\omega^2+1}{\omega^2+4} \cdot \frac{1}{3j\omega+1}$$

a) $S_{yy}(\omega) = ?$

$$S_{uy}(j\omega) = S_{uu}(\omega) \cdot G(j\omega) \quad (1)$$

$$|G(j\omega)|^2 = \frac{S_{yy}(\omega)}{S_{uu}(\omega)} \leftarrow \text{gV0} \quad (2)$$

$$G(j\omega) = \frac{2}{3j\omega+1}$$

$$|G(j\omega)|^2 = \frac{4}{9\omega^2+1}$$

iz (2):

$$\boxed{S_{yy}(\omega) = \frac{2}{\omega^2+4}}$$

b) $\overline{P}_y = ?$

$$\overline{P}_y = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y^2(t)\, dt = R_{yy}(0) \quad (3)$$

$$\underset{\uparrow \atop \tau}{R_{yy}(0)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{yy}(\omega)\, e^{j\omega \cdot \overset{\tau}{0}}\, d\omega \sim \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{yy}(\omega)\, d\omega = \boxed{\frac{1}{2}}$$

(3.) M1 12/13

$$f(t) = \lambda e^{-\lambda t}, \quad t > 0$$

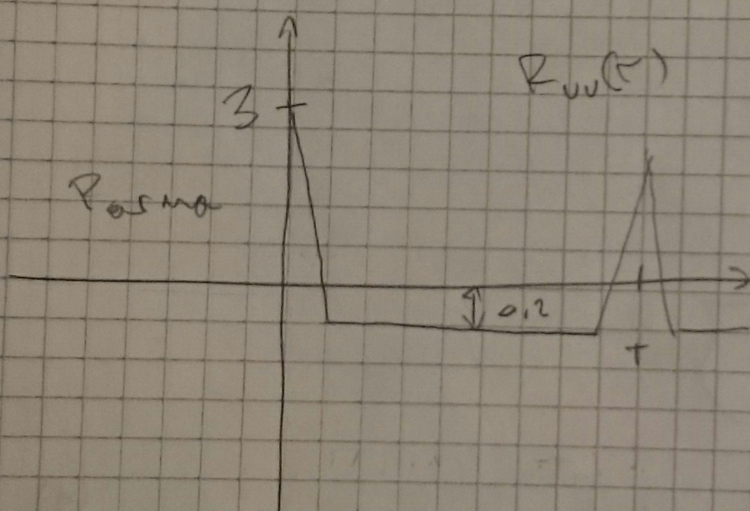
$$\{t_1, t_2, \dots, t_m\}$$

$$L = \prod_{i=1}^{M} \lambda e^{-\lambda t_i} = \lambda^N e^{-\lambda \sum_i t_i}$$

$$\ln(L) = N \ln(\lambda) + (-\lambda) \sum_i t_i$$

$$\frac{\partial \ln L}{\partial \lambda} = \frac{N}{\lambda} - \sum_i^N t_i \Rightarrow \lambda = \frac{N}{\sum t_i}$$

(4.) M1 12/13



PRBS:

T - period

$T = N \cdot \Delta t$

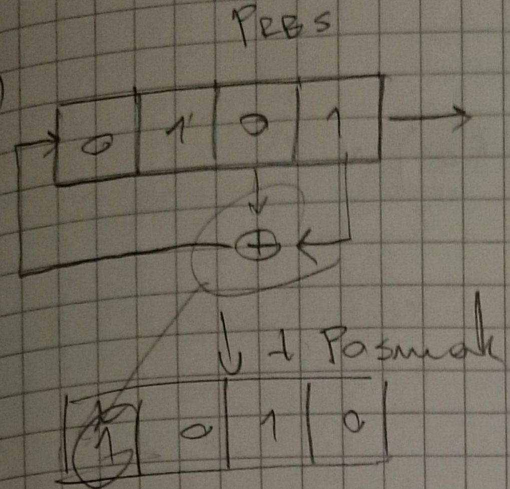
Δt - vrijeme uzorkovanja

a) $c^2 = 3$; $\Delta t = \frac{1}{15}$

$$R_{uu}(\Delta t) = 0.2 = -\frac{c^2}{N} \Rightarrow N = 15$$

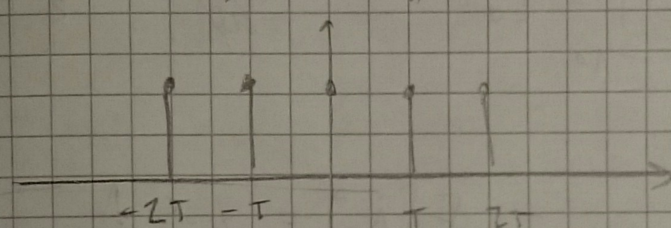
$T = 1$

b)



c) $N \gg \;,\; \Delta t \approx 0$ (PRBS)

$$R_{uy}(\tau) = \cancel{\lim_{T \to \infty} \frac{1}{2T}} \int_{0}^{\infty} R_{uu}(\tau - \sigma)\, g(\sigma)\, d\sigma$$



$$R_{uu}(\tau) = \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} c^2 \Delta t\, \delta(\tau - nT - \sigma)\, g(\sigma)\, d\sigma$$

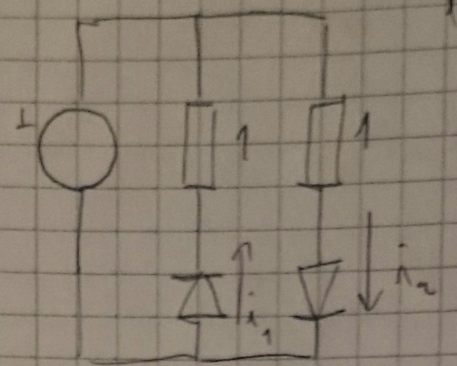
$$= c^2 \Delta t \left[g(\tau + \infty T) + \dots g(\tau + T) + g(\sigma) + g(\tau - T) + \dots + g(\tau - \infty T) \right]$$

$$\forall \tau < T \Rightarrow g(\tau) = 0$$

$$= c^2 \Delta t \sum_{0}^{\Delta} g(\tau + nT)$$

5. zad

$u \sim U(-10, 10)\ V$



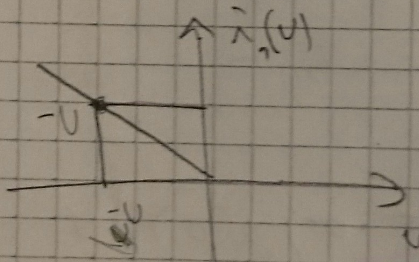
gustoća v.v.

a)

$$E[u] = \int_{-10}^{10} u \cdot f(u)\, du = 0$$

b)

$$E[i_1] = \int_{-10}^{0} -u \frac{1}{20}\, du = \cancel{\frac{1}{20}} = +\frac{5}{2}\ A$$



$E[i_2] = 2.5\ A$

c)

$E[i_1 \cdot i_2] = 6.25$

AEX

$$u \longrightarrow \boxed{B} \longrightarrow \bigcirc \xrightarrow{} \boxed{A^{-1}} \longrightarrow y, \qquad \varepsilon \downarrow +$$

$$A(z^{-1})\, y(z) = B(z^{-1})\, u(z) + \varepsilon(z)$$

$$\left.\begin{aligned} &E[\varepsilon(k)] = 0 \\ &E[u(k)\varepsilon(k)] = 0 \\ &E[y(k)\varepsilon(k)] = 0 \end{aligned}\right\} \text{stvar}$$

6. <u>MI 12/13</u>

$$y(k) = a\, y(k-1) + b\, u(k-1) + v(k)$$

$$v(k) = 0.3\, u(k) + \varepsilon(k)$$

k	$u(k)$	$y(k)$
0	-1	1
1	1	0.76
2	-1	0.86
3	-1	0.02
4	-1	-0.72
5	1	-0.85

$\hat{a}, \hat{b} = ?$

$$y(k) = a\, y(k-1) + b\, u(k-1) + 0.3\, u(k) + \varepsilon(k)$$

$$\hat{\Theta} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}$$

$$y(1) - 0.3\, u(1) = a\, y(0) + b\, u(0) + \varepsilon(1)$$

$\vdots$

$$\hat{\Theta} = (\Phi^T \Phi)^{-1} \Phi^T y \qquad \Phi = \begin{pmatrix} y(0) & u(0) \\ \vdots & \vdots \\ y(4) & u(4) \end{pmatrix}$$

b)

$$\hat{\Theta} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \qquad \hat{\Theta} = (W^T \phi)^{-1} W^T y$$

$$W = \cancel{\begin{pmatrix} y_H(0) & u(1) \\ \vdots & \\ y_H(4) & u(4) \end{pmatrix}}$$

$$y_H(k) = u(k-1)$$

$$\begin{pmatrix} u(0) & u(1) \\ \vdots & \\ u(3) & u(4) \end{pmatrix}$$

7.

$$G_p(z) = \frac{b_1 + b_2 z^{-1}}{z^2 + a_1 z - a_2} \qquad T_p = 30\ [s] \qquad T_s = 0.15\ [s]$$

a) ξ, ξ^2, ξ^3

$$G_p(z) = \frac{b_1 z^{-2} + b_2 z^{-3}}{1 + a_1 z^{-1} - a_2 z^{-2}}$$

$$y(k) + a_1 y(k-1) - a_2 y(k-2) = b_1 u(k-2) + b_2 u(k-3)$$

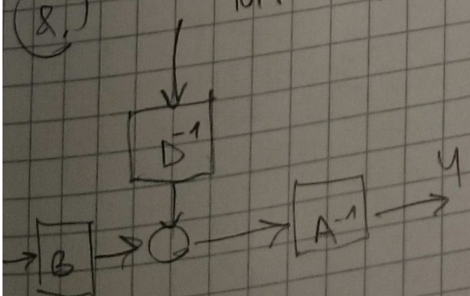
8. MI 14/15

$$Ay = v(z) + Bu$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2}$$

$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2}$$

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2}$$



$$E[y(8)] = E\{-a_1 y(7) - a_2 y(6) + b_1 u(7) + b_2 u(6) + v(8)\}$$

$$v = \frac{\varepsilon}{D} \Rightarrow D(z^{-1})\, v(z) = \varepsilon(z)$$