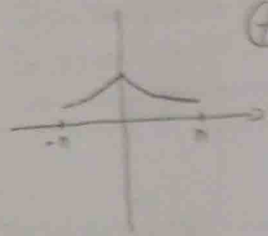
1) $f(x)=e^{-x}$ na $(0,\pi)$ i sumu $\dfrac{e^{\pi}+(-1)^{n+1}}{1+n^2}$ (+) $\boxed{2011}$



- funkcija je parna $\Rightarrow b_n = 0$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} e^{-x}\,dx = \frac{2}{\pi}\cdot(-1)\cdot e^{-x}\Big|_0^{\pi}$$

$$= -\frac{2}{\pi}\left(e^{-\pi}-1\right) = \frac{2\left(1-e^{-\pi}\right)}{\pi}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = \frac{2}{\pi}\int_0^{\pi} e^{-x}\cos nx\,dx = \left|\begin{matrix} u=e^{-x} & dv=\cos nx\,dx \\ du=-e^{-x}dx & v=\frac{1}{n}\sin nx \end{matrix}\right|$$

$$= \frac{2}{\pi}\Bigg(\underbrace{\frac{e^{-x}}{n}\sin nx\Big|_0^{\pi}}_{0} + \frac{1}{n}\int_0^{\pi} e^{-x}\sin nx\,dx\Bigg) = \left|\begin{matrix} u=e^{-x} & dv=\sin nx\,dx \\ du=-e^{-x}dx & v=-\frac{1}{n}\cos nx \end{matrix}\right|$$

$$= \frac{2}{\pi}\left(\frac{-e^{-x}}{n^2}\cdot\cos nx\Big|_0^{\pi} - \frac{1}{n^2}\int e^{-x}\cos nx\,dx\right)$$

$$= \frac{2}{\pi}\left(-\frac{1}{n^2}\left(e^{-\pi}\cos n\pi - 1\right) - \frac{1}{n^2}\cdot I\right)$$

$$I = \frac{1-e^{-\pi}\cos n\pi}{n^2} - \frac{1}{n^2}I$$

$$I\left(1+\frac{1}{n^2}\right) = \frac{1-e^{-\pi}\cos n\pi}{n^2}$$

$$I\left(\frac{n^2+1}{\cancel{n^2}}\right) = \frac{1-e^{-\pi}\cos n\pi}{\cancel{n^2}}$$

$$I = \frac{1-e^{-\pi}\cos n\pi}{n^2+1}$$

$$a_n = \frac{2}{\pi}\cdot\frac{1-e^{-\pi}\cos n\pi}{n^2+1}$$

$$\boxed{S(x) = \frac{\left(1-e^{-\pi}\right)}{\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-e^{-\pi}\cos n\pi}{n^2+1}}$$

$$f(x) = S(x)$$

$$f(0) = S(0)$$

$$1 = \frac{\left(1-e^{-\pi}\right)}{\pi} + \frac{2e^{-\pi}}{\pi}\sum_{n=1}^{\infty}\frac{e^{\pi}+(-1)^{n+1}}{1+n^2}$$

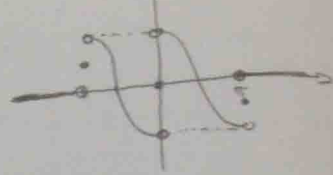
$$\frac{\pi-1+e^{-\pi}}{\cancel{\pi}} = \frac{2e^{-\pi}}{\cancel{\pi}}\cdot\sum_{n=1}^{\infty}\frac{e^{\pi}+(-1)^{n+1}}{1+n^2}$$

$$\boxed{\sum_{n=1}^{\infty}\frac{e^{\pi}+(-1)^{n+1}}{1+n^2} = \frac{e^{\pi}}{2}\left(\pi-1+e^{-\pi}\right)}$$

I školsko zadaća

Grupa A

2.) $f(x) = \begin{cases} \cos x, & x \in [0, \pi] \\ 0, & x \in (\pi, \infty] \end{cases}$



- funkcija je neparna $A(\lambda) = 0$

$$B(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin\lambda x\,dx = \frac{2}{\pi}\int_0^{\pi}\cos x\,\sin\lambda x\,dx = \frac{1}{\pi}\int_0^{\pi}\big(\sin(\lambda x + x) + \sin(\lambda x - x)\big)dx$$

$$= \frac{1}{\pi}\left(\frac{-1}{\lambda+1}\cos(\lambda x + x)\Big|_0^{\pi} + \frac{-1}{\lambda-1}\cos(\lambda x - x)\Big|_0^{\pi}\right)$$

$$= \frac{1}{\pi}\left(\frac{-1}{\lambda+1}(-\cos\lambda\pi - 1) + \frac{-1}{\lambda-1}(-\cos\lambda\pi - 1)\right)$$

$$= \frac{\cos\lambda\pi + 1}{\pi}\left(\frac{1}{\lambda+1} + \frac{1}{\lambda-1}\right) = \frac{\cos\lambda\pi + 1}{\pi}\cdot\frac{\lambda-\cancel{1}+\lambda+\cancel{1}}{\lambda^2-1} = \frac{2\lambda(\cos\lambda\pi + 1)}{\pi(\lambda^2-1)}$$

$$\tilde{f}(x) = \frac{2}{\pi}\int_0^{\infty}\frac{\lambda(\cos\lambda x - 1)}{(\lambda^2-1)}\cdot\sin\lambda x\,d\lambda$$

3.) funkcija je periodična $f: R \to R$ ako postoji nekakav $T > 0$ takav da za sve x vrijedi:

$$f(x) = f(x+T)$$

$$f(x) = 2 + \sin\frac{\pi x}{2} + \cos(4\pi x) + tg(3\pi x)$$

$$T = \frac{2\pi}{\omega} = 4 \qquad T = \frac{2\pi}{\omega} = \frac{1}{2} \qquad T = \frac{\pi}{\omega} = \frac{1}{3}$$

$$p \cdot T_1 = r \cdot T_2 = s \cdot T_3 = T$$

$$1 \cdot 4 = 8 \cdot \frac{1}{2} = 12 \cdot \frac{1}{3} = T$$

$$4 = 4 = 4 = T \Rightarrow \boxed{T = 4}$$

S indeksom "0" u koncentracijama

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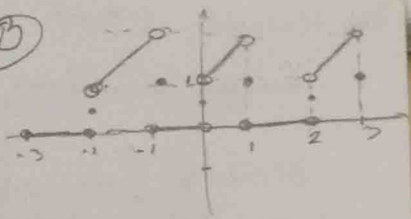
1.) $f(x) = \begin{cases} 0, & x \in [-1,0) \\ x+1, & x \in [0,1] \end{cases}$ i izračunaj $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ B



$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{1}{1}\int_0^1 (x+1)\,dx = \left(\frac{x^2}{2}+x\right)\Big|_0^1 = \frac{3}{2}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\cos\frac{n\pi x}{L}\,dx = 1\cdot\int_0^1 (x+1)\cdot\cos n\pi x\,dx = \int_0^1 x\cdot\cos n\pi x\,dx + \int_0^1 \cos n\pi x\,dx$$

$$= \left(\underbrace{x\cdot\frac{1}{n\pi}\cdot\sin n\pi x\Big|_0^1}_{0} - \frac{1}{n\pi}\int_0^1 \sin n\pi x\,dx + \underbrace{\frac{1}{n\pi}\cdot\sin n\pi x\Big|_0^1}_{0}\right)$$

$$= \frac{1}{n^2\pi^2}\cdot\cos n\pi x\Big|_0^1 = \frac{\cos n\pi - 1}{n^2\pi^2}$$

$$a_{2n} = \frac{\cos 2n\pi - 1}{4n^2\pi^2} = 0$$

$$a_{2n-1} = \frac{\cos(2n-1)\pi - 1}{(2n-1)^2\pi^2} = \frac{-2}{(2n-1)^2\pi^2}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx = 1\cdot\int_0^1 (x+1)\cdot\sin n\pi x\,dx = \int_0^1 x\cdot\sin n\pi x\,dx + \int_0^1 \sin n\pi x\,dx$$

$$= \left(x\cdot\frac{-1}{n\pi}\cdot\cos n\pi x\Big|_0^1 + \frac{1}{n\pi}\int_0^1 \cos n\pi x\,dx + \frac{-1}{n\pi}\cdot\cos n\pi x\Big|_0^1\right)$$

$$= \frac{-1}{n\pi}\cdot\cos n\pi + \underbrace{\frac{1}{n^2\pi^2}\sin n\pi x\Big|_0^1}_{0} - \frac{1}{n\pi}(\cos n\pi - 1)$$

$$= -\frac{1}{n\pi}(\cos n\pi + \cos n\pi - 1) = \frac{1-2\cos n\pi}{n\pi}$$

$$S(x) = \frac{3}{4} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)\pi x + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1-2\cos n\pi}{n}$$

$$f(0) = S(0)$$

$$\frac{1}{2} = \frac{3}{4} - \frac{2}{\pi^2}\cdot\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} + 0$$

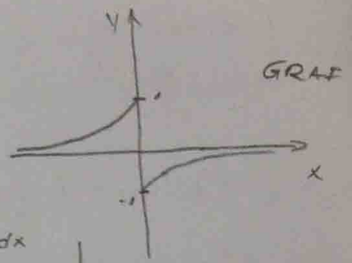
$$\frac{2-3}{4} = -\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

2.) $f(x) = e^x \quad (-\infty, 0)$

- funkcija je neparna $A(\lambda) = 0$

GRAF



$$B(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin \lambda x \, dx$$

$$= \frac{2}{\pi} \int_{-\infty}^{0} e^x \cdot \sin \lambda x \, dx = \left| \begin{matrix} u = e^x & dv = \sin \lambda x dx \\ du = e^x dx & v = -\frac{1}{\lambda} \cos \lambda x \end{matrix} \right|$$

$$= +\frac{2}{\pi} \left(-\frac{e^x}{\lambda} \cdot \cos \lambda x \Big|_{-\infty}^{0} + \frac{1}{\lambda} \int_{-\infty}^{0} e^x \cdot \cos \lambda x \, dx \right) = \left| \begin{matrix} u = e^x & dv = \cos \lambda x dx \\ du = e^x dx & v = \frac{1}{\lambda} \sin \lambda x \end{matrix} \right|$$

$$= \frac{2}{\pi} \left(-\frac{1}{\lambda} + \frac{1}{\lambda} \Big(\underbrace{\frac{e^x}{\lambda} \cdot \sin \lambda x \Big|_{-\infty}^{0}}_{0} - \frac{1}{\lambda} \int_{-\infty}^{0} e^x \cdot \sin \lambda x \, dx \right)$$

$$I = -\frac{1}{\lambda} - \frac{1}{\lambda^2} \cdot I$$

$$I \left(1 + \frac{1}{\lambda^2} \right) = -\frac{1}{\lambda}$$

$$I \left(\frac{\lambda^2 + 1}{\lambda^2} \right) = -\frac{1}{\lambda}$$

$$I = \frac{-\lambda}{\lambda^2 + 1}$$

$$B(\lambda) = -\frac{2}{\pi} \cdot \frac{\lambda}{\lambda^2 + 1}$$

$$f(x) = -\frac{2}{\pi} \int_{0}^{\infty} \frac{\lambda}{\lambda^2 + 1} \cdot \sin \lambda x \, d\lambda$$

3.) Diskretni amplitudni spektar se definira kao $c_n = \sqrt{a_n^2 + b_n^2}$, a fazni spektar kao $tg \varphi = \frac{a_n}{b_n}$.

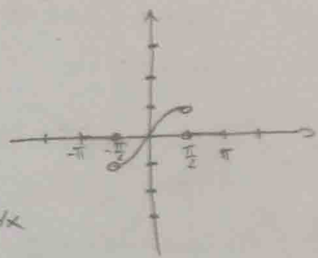
$$x^2 = \frac{\pi^2}{3} + 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2} \cos(nx), \; x \in [-\pi, \pi]$$

$$c_0 = |a_0| = \frac{2\pi^2}{3}$$

$$c_n = \sqrt{\frac{16}{n^4} + 0} = \frac{4}{n^2}$$

1. ŠZ 2011 Grupa A

1.) $f(x) = \begin{cases} \sin x & x \in [0, \frac{\pi}{2}) \\ 0 & x \in [\frac{\pi}{2}, \pi] \end{cases}$



- funkcija je neparna $\Rightarrow a_0, a_n = 0$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin x \sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} (\cos(x-nx) - \cos(x+nx))\,dx = \frac{1}{\pi}\cdot\left(\frac{1}{1-n}\sin(x-nx)\Big|_0^{\frac{\pi}{2}} - \frac{1}{1+n}\sin(x+nx)\Big|_0^{\frac{\pi}{2}}\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{1-n}\sin\left(\frac{\pi}{2}-\frac{\pi}{2}n\right) - \frac{1}{1+n}\sin\left(\frac{\pi}{2}+\frac{\pi}{2}n\right)\right)$$

$$= \frac{1}{\pi}\cdot\cos\frac{\pi}{2}n\cdot\left(\frac{1}{1-n} - \frac{1}{1+n}\right) = \frac{\cos\frac{\pi}{2}n}{\pi}\cdot\frac{\cancel{1}+n-\cancel{1}+n}{1-n^2} = \frac{2n\cdot\cos\frac{\pi}{2}n}{\pi(1-n^2)}$$

$$b_{2n} = \frac{4n\cdot\cos n\pi}{\pi(1-n^2)}$$

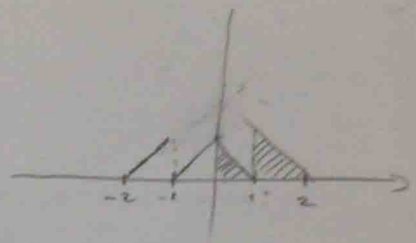
za $n=1$:

$$\Rightarrow \frac{1}{\pi}\int_0^{\frac{\pi}{2}}(\cos 0 - \cos 2x)\,dx = \frac{1}{\pi}\left(x - \underbrace{\frac{1}{2}\sin 2x}_{0}\right)\Big|_0^{\frac{\pi}{2}} = \frac{1}{2}$$

$$\boxed{S(x) = \frac{1}{2}\sin nx + \frac{1}{\pi}\sum_{n=2}^{\infty}\frac{4n\cos n\pi}{1-n^2}\cdot\sin 2nx}$$

2.)

$$f(x) = \begin{cases} 1-|x| , & x\in(-1,1) \\ 2-|x| , & x\in(-2,-1]\cup[1,2) \\ 0 , & \text{inače} \end{cases}$$



- funkcija je parna $\Rightarrow B(\lambda)=0$

$$A(\lambda) = \frac{2}{\pi}\int_0^\infty f(x)\cos\lambda x\,dx = \frac{2}{\pi}\int_0^1 (1-x)\cos\lambda x\,dx + \int_1^2 (2-x)\cos\lambda x\,dx$$

$$= \frac{2}{\pi}\Big(\underbrace{(1-x)\cdot\frac{1}{\lambda}\cdot\sin\lambda x\Big|_0^1}_{0} + \frac{1}{\lambda}\int_0^1 \sin\lambda x\,dx + (2-x)\frac{1}{\lambda}\cdot\sin\lambda x\Big|_1^2 + \frac{1}{\lambda}\int_1^2 \sin\lambda x\,dx$$

$$= \frac{2}{\pi}\Big(-\frac{1}{\lambda^2}\cos\lambda x\Big|_0^1 - \frac{1}{\lambda}\sin\lambda - \frac{1}{\lambda^2}\cos\lambda x\Big|_1^2\Big)$$

$$= \frac{2}{\pi}\Big(-\frac{1}{\lambda^2}(\cancel{\cos\lambda}-1) - \frac{1}{\lambda}\sin\lambda - \frac{1}{\lambda^2}\cos 2\lambda + \cancel{\frac{1}{\lambda^2}\cos\lambda}\Big)$$

$$= \frac{2}{\pi}\cdot\frac{1-\lambda\sin\lambda-\cos 2\lambda}{\lambda^2}$$

$$\boxed{\tilde{f}(x) = \frac{2}{\pi}\int_0^\infty \frac{1-\lambda\sin\lambda-\cos 2\lambda}{\lambda^2}\cdot\cos\lambda x\,d\lambda}$$

$f(0)=S(0)$

$$1 = \frac{2}{\pi}\int_0^\infty \frac{1-\lambda\sin\lambda-\cos 2\lambda}{(\lambda^2)}$$

$$\frac{\pi}{2} = \int_0^\infty \frac{1-\lambda\sin\lambda-\cos^2\lambda+\sin^2\lambda}{\lambda^2}$$

$$\frac{\pi}{2} = \int_0^\infty \frac{2\sin^2\lambda-\lambda\sin\lambda}{\lambda^2}$$

3.) Dirichletovi uvjeti

1.) Funkcija po dijelovima mora biti neprekidna i prekidi moraju biti prve vrste

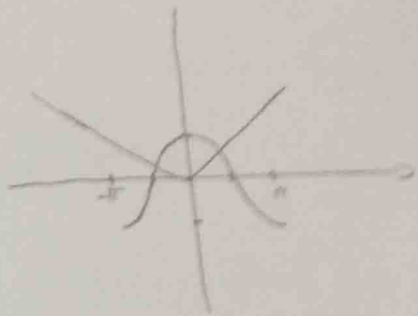
2.) Funkcija mora biti monotona ili mora imati konačan broj ekstrema

$$f(x) = tg\,x \quad na \quad [0,\pi]$$

- ne zadovoljava Dirichletove uvjete ima prekid druge vrste

Grupa B

$f(x) = |x| + \cos x$ na $[-\pi, \pi]$



$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$= \frac{2}{\pi}\left(\int_0^{\pi} x\,dx + \int_0^{\pi}\cos x\,dx\right)$$

$$= \frac{2}{\pi}\left(\frac{x^2}{2}\Big|_0^{\pi} + \sin x\Big|_0^{\pi}\right) = \frac{2\pi}{2} = \pi$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = \frac{2}{\pi}\int_0^{\pi}(x+\cos x)\cdot\cos nx\,dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi} x\cos nx\,dx + \int_0^{\pi}\cos x\cdot\cos nx\,dx\right] = \left|\begin{matrix} u = x & dv = \cos nx\,dx \\ du = dx & v = \frac{1}{n}\sin nx\end{matrix}\right|$$

$$= \frac{2}{\pi}\left[\underbrace{\frac{1}{n}x\sin nx\Big|_0^{\pi}}_{0} - \frac{1}{n}\int_0^{\pi}\sin nx\,dx + \frac{1}{2}\left[\int_0^{\pi}\cos(x+nx)\,dx + \int_0^{\pi}\cos(x-nx)\,dx\right]\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{n^2}\cos nx\Big|_0^{\pi} + \underbrace{\frac{1}{2}\cdot\left(\frac{1}{1+n}\sin(x+nx)\Big|_0^{\pi} + \frac{1}{1-n}\sin(x-nx)\Big|_0^{\pi}\right)}_{0}\right]$$

$$= \frac{2}{n^2\pi}\cdot(\cos n\pi - 1)$$

$$a_{2k+1} = \frac{2(\cos(2k+1)\pi - 1)}{(2k-1)^2\pi} = \frac{2}{\pi}\cdot\frac{-2}{(2k-1)^2}$$

za $n=1$

$$= \frac{2}{\pi}\left[\frac{-1}{\cancel{n}}\int_0^{\pi}\sin 4x\,dx + \frac{1}{2}\left(\int_0^{\pi}\cos 2x\,dx + \int_0^{\pi}1\,dx\right)\right]$$

$$= \frac{2}{\pi}\left[+1\cdot\cos x\Big|_0^{\pi} + \frac{1}{2}(0 + x)\Big|_0^{\pi}\right]$$

$$= \frac{2}{\pi}\left[1\cdot(\cos\pi - 1) + \frac{\pi}{2}\right] = \frac{2}{\pi}\left(-2 + \frac{\pi}{2}\right) = \left(1 - \frac{4}{\pi}\right)$$

$=$

$$\boxed{S(x) = \frac{\pi}{2} + \left(1 - \frac{4}{\pi}\right)\cos x - \frac{4}{\pi}\sum_{k=2}^{\infty}\frac{1}{(2k-1)^2}\cos(2k-1)x}$$

$f(0) = S(0)$

$$1 = \frac{\pi}{2} + \left(1 - \frac{4}{\pi}\right) - \frac{4}{\pi}\sum_{k=2}^{\infty}\frac{1}{(2k-1)^2}$$

$$-\frac{\pi}{2} + \frac{4}{\pi} = -\frac{4}{\pi}\sum_{k=2}^{\infty}\frac{1}{(2k-1)^2} \Big/ \frac{\pi}{4}$$

$$\frac{\pi^2}{8} + 1 = \sum_{k=2}^{\infty}\frac{1}{(2k-1)^2}$$

$$\frac{\pi^2}{8} = \sum_{k=0}^{\infty}\frac{1}{(2k+1)^2}$$

2.) $f(x)=\begin{cases} x^2+1 & x\in[-1,1] \\ 0 & \text{inače}\end{cases}$

$$A(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos\lambda x\, d\lambda$$

$$= \frac{1}{\pi}\int_{-1}^{1}(x^2+1)\cos\lambda x\,dx = \begin{vmatrix} u=x^2+1 & dv=\cos\lambda x\,dx \\ du=2x\,dx & v=\frac{1}{\lambda}\sin\lambda x\end{vmatrix}$$

$$= \frac{1}{\pi}\left[\frac{1}{\lambda}(x^2+1)\sin\lambda x\Big|_{-1}^{1} - \frac{2}{\lambda}\int_{-1}^{1} x\sin\lambda x\,dx\right] = \begin{vmatrix} u=x & dv=\sin\lambda x\,dx \\ du=dx & v=-\frac{1}{\lambda}\cos\lambda x\end{vmatrix}$$

$$= \frac{1}{\pi}\left[\frac{1}{\lambda}(2\sin\lambda - 2\sin-\lambda) + \frac{2}{\lambda^2}x\cos\lambda x\Big|_{-1}^{1} - \frac{2}{\lambda^2}\int_{-1}^{1}\cos\lambda x\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{4\sin\lambda}{\lambda} + \frac{2}{\lambda^2}(\cos\lambda+\cos\lambda) - \frac{2}{\lambda^3}\sin\lambda x\Big|_{-1}^{1}\right]$$

$$= \frac{1}{\pi}\left[\frac{4\sin\lambda}{\lambda} + \frac{4\cos\lambda}{\lambda^2} - \frac{4\sin\lambda}{\lambda^3}\right]$$

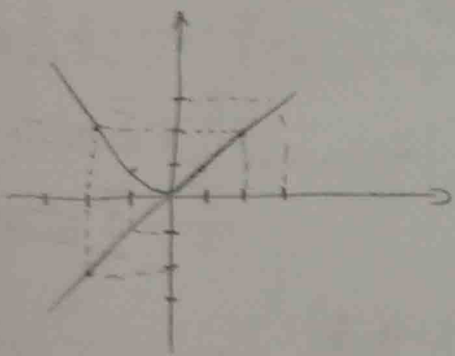
$$f(x) = \int_0^\infty (A(\lambda)\cos\lambda x + B(\lambda)\sin\lambda x)\,d\lambda$$

$$\boxed{= \frac{4}{\pi}\int_0^\infty \frac{1}{\lambda^3}(\lambda^2\sin\lambda + \lambda\cos\lambda - \sin\lambda)\cos\lambda x\, d\lambda}$$

3.) $f(x)=x$

$g(x)=x^2$

jesu li ortogonalne na $[-1,1]$ i na $[-1,2]$

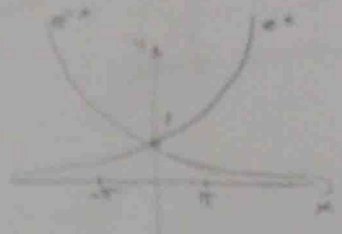


$f(x-a)$

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (b_n \sin nx + a_n \cos nx)$$

1 [illegible] [illegible]

[illegible] A



1) $f(x) = e^{-x}$ en $(0, \pi)$

$$f(x) = \begin{cases} e^{-x} & (0,\pi) \\ e^{x} & (-\pi, 0) \end{cases} \qquad \sum_{n=0}^{\infty} \frac{(-1)^{n+1} + e^{\pi}}{n^2+1}$$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} e^{-x}\,dx = -\frac{2}{\pi}\cdot e^{-\pi} + \frac{2}{\pi} = \frac{2}{\pi}(1-e^{-\pi})$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = \frac{2}{\pi}\int_0^{\pi} e^{-x}\cos nx\,dx = \left| \begin{matrix} u = e^{-x} & dv = \cos nx\,dx \\ du = -e^{-x}dx & v = \frac{1}{n}\sin nx \end{matrix} \right|$$

$$= \frac{2}{\pi}\Big[\underbrace{\frac{1}{n}e^{-x}\sin nx \Big|_0^{\pi}}_{0} + \frac{1}{n}\int_0^{\pi} e^{-x}\sin nx\,dx \Big] = \left| \begin{matrix} u = e^{-x} & dv = \sin nx\,dx \\ du = -e^{-x} & v = -\frac{1}{n}\cos nx \end{matrix} \right|$$

$$= \frac{2}{\pi}\Big[-\frac{1}{n^2}e^{-x}\cos nx\Big|_0^{\pi} - \frac{1}{n^2}\int_0^{\pi} e^{-x}\cos nx\,dx \Big]$$

$$I = \Big[-\frac{1}{n^2}\big[e^{-\pi}\cos n\pi - 1\big] - \frac{1}{n^2} I \Big]$$

$$I = -\frac{1}{n^2}\big[e^{-\pi}\cos n\pi - 1\big] - \frac{1}{n^2} I$$

$$\frac{\pi}{2} - \frac{1}{n^2}$$

$$\frac{\pi n^2 + 2}{2n^2}$$

$$I\left(1 + \frac{1}{n^2}\right) = -\frac{1}{n^2}\big[e^{-\pi}\cos n\pi - 1\big]$$

$$I\left(\frac{n^2+1}{n^2}\right) = -\frac{1}{n^2}\big[e^{-\pi}\cos n\pi - 1\big]$$

$$I = \frac{-1}{n^2+1}\big[e^{-\pi}(-1)^n - 1\big]$$

$$\boxed{S(x) = \frac{1}{\pi}(1-e^{-\pi}) - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{e^{-\pi}(-1)^n - 1}{n^2+1}\cos(nx)}$$

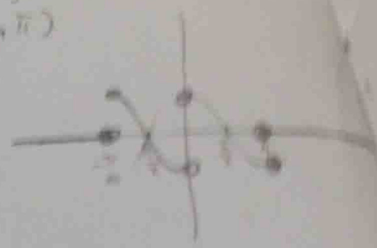
$f(0) = S(0)$

$$\frac{1}{\pi}(1-e^{-\pi}) - \frac{2}{\pi}\frac{(e^{-\pi}(-1)^n - 1)}{n^2+1} = 1$$

$$\frac{2e^{-\pi}}{\pi}\frac{((-1)^{n+1} + e^{\pi})}{n^2+1} = 1 - \frac{1}{\pi}(e^{-\pi} - 1)$$

$$\boxed{\frac{(-1)^{n+1} + e^{\pi}}{n^2+1} = \frac{e^{\pi}}{2}(e^{-\pi} - 1) + \frac{\pi}{2}e^{\pi}}$$

2.) $f(x) = \begin{cases} \cos x & \text{na } [0,\pi] \\ 0 & x \in [\pi, \infty] \end{cases}$ $\quad f'(x) = \begin{cases} -\cos x & [-\pi, 0] \\ 0 & x \in [-\infty, \pi) \end{cases}$



$A(\lambda) = 0$

$$B(\lambda) = \frac{2}{\pi}\int_0^\pi \cos x \cdot \sin \lambda x \, dx = \frac{1}{2\pi}\left[\int_0^\pi \sin(\lambda x + x)\,dx + \int_0^\pi \sin(\lambda x - x)\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{-1}{\lambda+1}\cos(\lambda x + x)\Big|_0^\pi - \frac{1}{\lambda - 1}\cos(\lambda x - x)\Big|_0^\pi\right]$$

$$= \frac{1}{\pi}\left[\frac{-1}{\lambda+1}\left(\cos(\lambda\pi + \pi) - 1\right) - \frac{1}{\lambda-1}\left(\cos(\lambda\pi - \pi) - 1\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{-1}{\lambda+1}\left(-\cos\lambda\pi - 1\right) - \frac{1}{\lambda-1}\left(-\cos\lambda\pi - 1\right)\right]$$

$$= \frac{1}{\pi}\left(\cos\lambda\pi + 1\right) \cdot \frac{\lambda - \cancel{1} + \lambda + \cancel{1}}{\lambda^2 - 1} = \frac{2\lambda((-1)^n + 1)}{\pi(\lambda^2 - 1)} \quad \text{u nj.} \quad \frac{2\lambda(1 + \cos n\pi)}{\pi(\lambda^2 - 1)}$$

$$\tilde{f}(x) = \int_0^\infty \left(A(\lambda)\cos\lambda x + B(\lambda)\sin\lambda x\right)d\lambda = \boxed{\frac{1}{\pi}\int_0^\infty \frac{2\lambda(\cos n\pi + 1)}{\lambda^2 - 1}\sin\lambda x \, d\lambda}$$

3.) funkcija je periodična ako postoji T takav da vrijedi $f(x) = f(x+T)$

$$f(x) = 2 + \underbrace{\sin\frac{\pi x}{2}}_{T_1} + \underbrace{\cos(4\pi x)}_{T_2} + \underbrace{tg(3\pi x)}_{T_3}$$

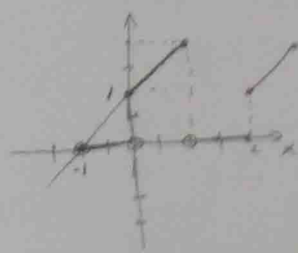
$$T = \frac{2\pi}{\omega}$$

$$T_1 = \frac{2\cancel{\pi}}{\frac{\cancel{\pi}}{2}} = \boxed{4} \quad T_2 = \frac{2\pi}{4\pi} = \boxed{\frac{1}{2}} \quad T_3 = \frac{\pi}{3\pi} = \boxed{\frac{1}{3}}$$

$V(4, \frac{1}{2}, \frac{1}{3})$ = funkcija je periodična s periodom 4 je parna $\Rightarrow B(\lambda)=0$

Grupa B

$$f(x)=\begin{cases}0, & x\in[-1,0)\\ x+1, & x\in[0,1]\end{cases} \quad i \quad \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$



$$a_0=\frac{2}{T}\int_0^2 f(x)\,dx=\frac{\cancel{2}}{\cancel{2}}\int_0^1 (x+1)\,dx=$$

$$=\cdot\frac{(x+1)^2}{2}\Big|_0^1=\left(2-\frac{1}{2}\right)=1-\frac{1}{2}=\frac{3}{2}$$

$$a_n=\frac{2}{T}\int_0^2 f(x)\cos\frac{2n\pi x}{T}\,dx=\int_0^1 (x+1)\cos n\pi x\,dx=\begin{vmatrix}u=x+1 & dv=\cos n\pi x\,dx\\ du=dx & v=\frac{1}{n\pi}\sin n\pi x\end{vmatrix}$$

$$=\Big[\underbrace{\frac{1}{n\pi}(x+1)\sin n\pi x\Big|_0^1}_{0}-\frac{1}{n\pi}\int_0^1 \sin n\pi x\,dx\Big]$$

$$=\Big[\frac{1}{n^2\pi^2}\cos n\pi x\Big|_0^1\Big]=\frac{1}{n^2\pi^2}(\cos n\pi-1)$$

$$a_{2n-1}=\frac{1}{\pi^2}\cdot\frac{1}{(2n-1)^2}(\cos(2n-1)\pi-1)=\frac{-2}{\pi^2(2n-1)^2}=\boxed{-\frac{2}{\pi^2(2n-1)^2}}$$

$$b_n=\frac{2}{T}\int_0^2 f(x)\sin\frac{2n\pi x}{T}\,dx=\int_0^1 (x+1)\sin n\pi x\,dx=\begin{vmatrix}u=x+1 & dv=\sin n\pi x\,dx\\ du=dx & v=\frac{-1}{n\pi}\cos n\pi x\end{vmatrix}$$

$$=\Big[\frac{-1}{n\pi}(x+1)\cos n\pi x\Big|_0^1+\underbrace{\frac{1}{n\pi}\int_0^1 \cos n\pi x\,dx}_{0}\Big]$$

$$=-\frac{1}{n\pi}(2\cos n\pi-1)=\frac{1-2\cos n\pi}{n\pi}$$

$$b_{2n-1}=\frac{1-2\cos(2n-1)\pi}{(2n-1)\pi}=\boxed{\frac{3}{(2n-1)\pi}}$$

- ako nema prekida u x_1 $f(x_1)=S(x_1)$

$$S(x)=\frac{3}{4}-\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos((2n-1)\pi x)+\frac{3}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin((2n-1)\pi x)$$

$$-\frac{1}{2\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\pi x)$$

- ako funkcija ima prekid u x_1 $S(x_1)=\frac{1}{2}(f(x_1^-)+f(x_1^+))\neq f(x_1)$

$$f(0)=S(0)$$

$$\boxed{\frac{1}{2}=\frac{3}{4}-\frac{2}{\pi^2}\cdot\frac{1}{(2n-1)^2}}$$

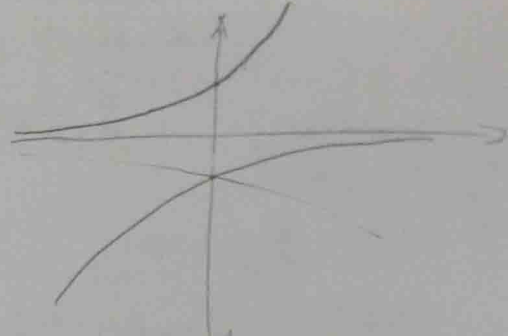
$$-\frac{1}{4}=-\frac{2}{\pi^2}\cdot\frac{1}{(2n-1)^2}\Rightarrow\frac{1}{(2n-1)^2}=\boxed{+\frac{\pi^2}{8}}$$

- funkcija je parna $\Rightarrow B(\lambda)=0$

$A(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cdot \cos\lambda x$ ... grupa A 2006

2.) $f(x) = e^{x}$, $x<0$ $(-\infty, 0)$

$f(x) = -e^{-x}$, $x>0$ $(0,\infty)$



$A(\lambda) = 0$

$B(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin\lambda x dx$

$= \frac{2}{\pi}\int_{-\infty}^{0} e^{x}\sin\lambda x dx = \left| \begin{matrix} u=e^{x} & dv=\sin\lambda x dx \\ du=e^{x}dx & v=\frac{-1}{\lambda}\cos\lambda x \end{matrix} \right|$

$= \frac{2}{\pi}\left[-\frac{1}{\lambda}e^{x}\cos\lambda x \Big|_{-\infty}^{0} + \frac{1}{\lambda}\int_{-\infty}^{0} e^{x}\cos\lambda x dx \right] = \left| \begin{matrix} u=e^{x} & dv=\cos\lambda x dx \\ du=e^{x}dx & v=\frac{1}{\lambda}\sin\lambda x \end{matrix} \right|$

$= \frac{2}{\pi}\left[-\frac{1}{\lambda}(1-0) + \underbrace{\frac{1}{\lambda^2}e^{x}\sin\lambda x \Big|_{-\infty}^{0}}_{0} - \frac{1}{\lambda^2}\int_{-\infty}^{0} e^{x}\sin\lambda x dx \right]$

$I = -\frac{1}{\lambda} - \frac{1}{\lambda^2} I$

$I\left(1+\frac{1}{\lambda^2}\right) = -\frac{1}{\lambda}$

$I\left(\frac{\lambda^2+1}{\lambda^2}\right) = -\frac{1}{\lambda}$

$\boxed{I = \frac{-\lambda}{\lambda^2+1}}$

$\tilde{f}(x) = \int_{0}^{\infty}\left(A(\lambda)\cos\lambda x + B(x)\sin\lambda x\right)d\lambda$

$\boxed{= \int_{0}^{\infty} \frac{-2\lambda}{\pi(\lambda^2+1)}\sin(\lambda x)d\lambda}$

3.) $a_m = c_0 + \sum_{n=1}^{\infty} c_n$

$c_0 = |a_0|$

$c_n = \sqrt{a_n^2 + b_n^2}$ $\quad tg\,\varphi = \frac{a_n}{b_n}$

$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(n\pi x)$ na $(-\pi,\pi)$

$c_0 = |a_0| = \boxed{\frac{\pi^2}{3}}$

$c_n = \sqrt{\frac{16}{n^4}+0} = \boxed{\frac{4}{n^2}}$

- funkcija je parna $\Rightarrow B(\lambda)=0$

$$\ldots = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cdot\cos\lambda\xi\, d\xi = \frac{2}{\pi}\int_0^1 \sin\pi\xi\,\cos\lambda\xi\, d\xi = \frac{1}{\pi}\int_0^1\left(\sin(\pi\xi+\lambda\xi)+\sin(\pi\xi-\lambda\xi)\right)d\xi$$

Grupa A 2006

1.) $T=2\pi$

$f(x-a)$

$$S(x)=\frac{a_0}{2}+\sum_{n=0}^{\infty}(b_n\sin nx + a_n\cos nx)$$

$$S(x-a)=\frac{a_0}{2}+\sum_{n=0}^{\infty}(b_n\cdot\sin n(x-a)+a_n\cos n(x-a))$$

$$S(x-a)=\frac{a_0}{2}+\sum_{n=0}^{\infty}\left(b_n\cdot(\sin nx\cos na-\cos nx\sin na)+a_n(\cos nx\cos na+\sin nx\sin na)\right)$$

$$S(x-a)=\frac{a_0}{2}+\sum_{n=0}^{\infty}\left(b_n\sin nx\cos na-\underline{b_n\cos nx\sin na}+\underline{a_n\cos nx\cos na}+a_n\sin nx\sin na\right)$$

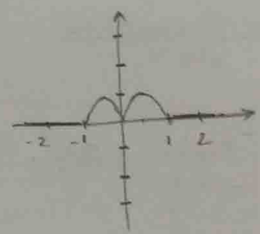
$$S(x-a)=\frac{a_0}{2}+\sum_{n=0}^{\infty}(A_n\cos nx+B_n\sin nx)$$

$$A_n=\cos nx\,(a_n\cos na-b_n\sin na)$$

$$B_n=\sin na\,(a_n\sin na+b_n\cos na)$$

2.)

$$f(x)=\begin{cases}|\sin(\pi x)|, & x\in[-1,1]\\ 0, & \text{inače}\end{cases}$$



- funkcija je parna $B(\lambda)=0$

$$A(\lambda)=\frac{1}{\pi}\int_{-\infty}^{\infty}f(x)\cos\lambda x\,dx=\frac{2}{\pi}\int_0^1\sin(\pi x)\cdot\cos\lambda x\,dx$$

$$=\frac{1}{\pi}\int_0^1(\sin(\pi x+\lambda x)+\sin(\pi x-\lambda x))dx$$

$$=\frac{1}{\pi}\cdot\left(\frac{-1}{\pi+\lambda}\cos(\pi x+\lambda x)\Big|_0^1+\frac{-1}{\pi-\lambda}\cos(\pi x+\lambda x)\Big|_0^1\right)$$

$$=\frac{1}{\pi}\left(-\frac{1}{\pi+\lambda}(\cos(\pi+\lambda)-1)-\frac{1}{\pi-\lambda}(\cos(\pi-\lambda)-1)\right)$$

$$=\frac{1}{\pi}\left(\frac{1}{\pi+\lambda}(\cos\lambda+1)+\frac{1}{\pi-\lambda}(\cos\lambda+1)\right)$$

$$=\frac{\cos\lambda+1}{\cancel{\pi}}\cdot\frac{\overset{2}{\cancel{\pi}-\cancel{\lambda}+\cancel{\pi}+\cancel{\lambda}}}{\pi^2-\lambda^2}=\frac{2(\cos\lambda+1)}{\pi^2-\lambda^2}$$

$$\boxed{f(x)=2\int_0^{\infty}\frac{2(\cos\lambda+1)}{\pi^2-\lambda^2}\cdot\cos\lambda x\,d\lambda}\qquad a_m=|A(\lambda)|=\left|\frac{2(\cos\lambda+1)}{\pi^2-\lambda^2}\right|$$

za $\lambda\neq\pi$ za $\lambda\neq\pi$

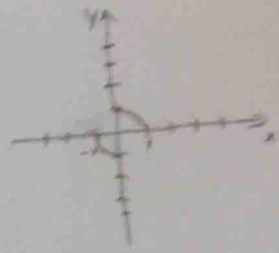
temperatura promijeni s $T_1 = 300$ K na $T_2 = 360$ K

Rješenje:

Primjenom izraza (2.3), intrinzična koncentracija se, prema (2.5), ...

Grupa B

1.) $f(x) = 1 - x^2 \quad x \in \langle 0, 1 \rangle \qquad x^2 - 1$



- razvijena po sinus funkcijama u red $S(x)$

$S(5\pi) = ?$

- funkcija je neparna $a_n = 0$

$a_0 = 0$

$$b_n = 2\int_0^1 (1-x^2)\cdot \sin n\pi x\,dx = 2\left(\int_0^1 \sin n\pi x\,dx - \int_0^1 x^2 \cdot \sin n\pi x\,dx\right)$$

$$= 2\left(\frac{-1}{n\pi}\cos n\pi x\Big|_0^1 - \left(x^2 \cdot \frac{-1}{n\pi}\cos n\pi x\Big|_0^1 + \frac{2}{n\pi}\int_0^1 x\cdot\cos n\pi x\,dx\right)\right)$$

$$= 2\cdot\left(-\frac{1}{n\pi}(\cancel{\cos n\pi} - 1) + \cancel{\frac{1}{n\pi}\cos n\pi} - \frac{2}{n\pi}\Big(\underbrace{x\cdot\frac{1}{n\pi}\cdot\sin n\pi x\Big|_0^1}_{0} - \frac{1}{n\pi}\int_0^1 \sin n\pi x\,dx\right)$$

$$= 2\left(\frac{1}{n\pi} - \frac{2}{n^3\pi^3}\cos n\pi x\Big|_0^1\right) = \frac{2}{\pi}\left(\frac{1}{n\pi} - \frac{2}{n^3\pi^3}(\cos n\pi - 1)\right)$$

$$= 2\cdot\frac{n^2\pi^2 - 2\cos n\pi + 2}{n^3\pi^3}$$

$$\boxed{S(x) = 2\sum_{n=1}^{\infty}\frac{n^2\pi^2 - 2\cos n\pi + 2}{n^3\pi^3}\sin n\pi x}$$

$\frac{313.5}{1930}$

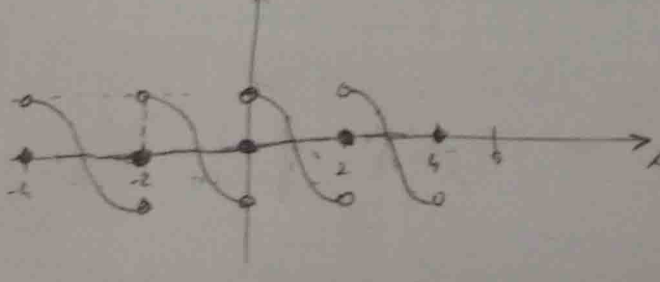
$5\pi \approx 15.7 \qquad f(5\pi) = S(5\pi)$

$T = 2$

$f(5\pi - 8\cdot T) = S(5\pi)$

$f(5\pi - 16) = S(5\pi)$

$$\boxed{S(5\pi) = (5\pi - 16)^2 - 1}$$



PITAT JEL DOBRO?

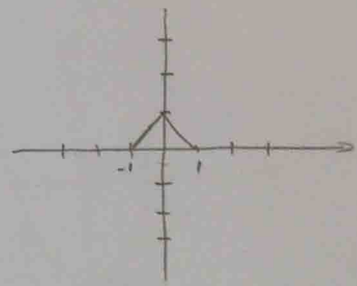
$f(x) = 1 - |x| \quad \langle -1, 1 \rangle$

Primjenom Parsevalo...

- funkcija je p...

a_0 ...

$f(x) = 1 - |x| \quad \langle -1, 1 \rangle$



primjenom Parsevala izračunaj $1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4}$

$\frac{1}{(2n+1)^4}$

- funkcija je parna $b_n = 0$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = 2\cdot\int_0^1 (1-x)\,dx = 2\cdot\left(x - \frac{x^2}{2}\right)\Big|_0^1$$

$$= 2\left(1 - \frac{1}{2}\right) = 1$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = 2\int_0^1 (1-x)\cos n\pi x\,dx = 2\left(\int_0^1 \cos n\pi x\,dx - \int_0^1 x\cdot\cos n\pi x\,dx\right)$$

$$= 2\left(\underbrace{\frac{1}{n\pi}\sin n\pi x\Big|_0^1}_{0} - \left(\underbrace{x\cdot\frac{1}{n\pi}\cdot\sin n\pi x\Big|_0^1}_{0} - \frac{1}{n\pi}\int_0^1 \sin n\pi x\,dx\right)\right)$$

$$= 2\left(-\frac{1}{n^2\pi^2}\cdot\cos n\pi x\Big|_0^1\right) = \frac{-2}{n^2\pi^2}(\cos n\pi - 1) = \frac{2 - 2\cos n\pi}{n^2\pi^2}$$

za $\quad a_{2n} = \dfrac{2 - 2\cos 2n\pi}{4n^2\pi^2} = 0$

za $\quad a_{2n+1} = \dfrac{2 - 2\cos(2n+1)\pi}{(2n+1)^2\pi^2} = \dfrac{4}{(2n+1)^2\pi^2}$

$$\boxed{S(x) = \frac{1}{2} + 4\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2\pi^2}\cos(2n-1)\pi x}$$

$$\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty} a_n^2 + \sum_{n=1}^{\infty} b_n^2 = \frac{2}{T}\int_a^b |f(x)|^2\,dx$$

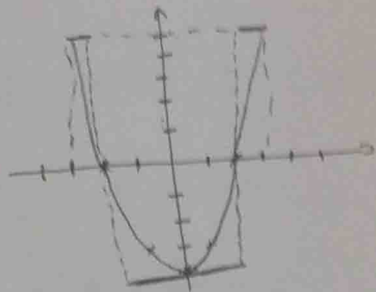
$$\frac{1}{2} + \sum_{n=0}^{\infty}\frac{16}{(2n+1)^4\pi^4} = 2\int_0^1 (1-x)^2\,dx = 2\int_0^1 (1 - 2x + x^2)\,dx = 2\left(x - x^2 + \frac{x^3}{3}\right)\Big|_0^1 = \frac{2}{3}$$

$$\sum_{n=0}^{\infty}\frac{16}{(2n+1)^4\pi^4} = \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$$

$$\boxed{\sum_{n=0}^{\infty}\frac{1}{(2n+1)^4} = \frac{\pi^4}{96}}$$

Grupa A

1) $f(x) = \operatorname{sgn}(x^2-4)$, $x \in \langle -3, 3 \rangle$



$S(\sqrt{111}) = ?$

- funkcija je parna $\Rightarrow b_n = 0$

$$f(x) = \begin{cases} 1 , & [-3,-2] \cup [2,3] \\ -1 , & [-2,2] \end{cases}$$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{3}\left(\int_0^2 -dx + \int_2^3 dx\right) = \frac{2}{3}\left(-x\Big|_0^2 + x\Big|_2^3\right) = \frac{2}{3}(-2+3-2) = -\frac{2}{3}$$

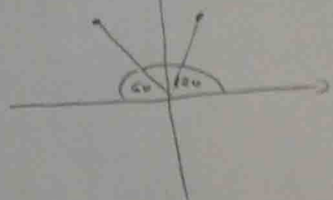
$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = \frac{2}{3}\left(\int_0^2 -\cos\frac{n\pi x}{3}\,dx + \int_2^3 \cos\frac{n\pi x}{3}\,dx\right)$$

$$= \frac{2}{3}\left(-\frac{3}{n\pi}\sin\frac{n\pi x}{3}\Big|_0^2 + \frac{3}{n\pi}\sin\frac{n\pi x}{3}\Big|_2^3\right)$$

$$= \frac{2}{3}\left(-\frac{3}{n\pi}\left(\sin\frac{2n\pi}{3}\right) + \frac{3}{n\pi}\left(\underbrace{\sin n\pi}_{0} - \sin\frac{2n\pi}{3}\right)\right)$$

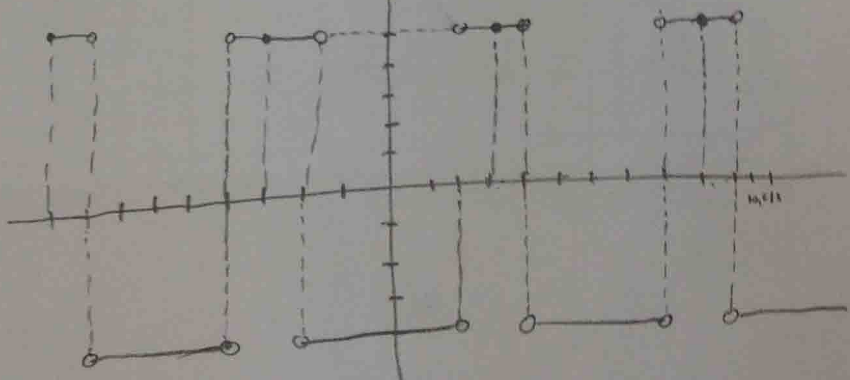
$$= \frac{2}{\cancel{3}}\left(-\frac{\cancel{6}^{\,2}}{n\pi}\sin\frac{2n\pi}{3}\right)$$

$$= -\frac{4}{n\pi}\cdot\sin\frac{2n\pi}{3}$$



$$\boxed{S(x) = -\frac{1}{3} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin\frac{2n\pi}{3}}{n}\cos\frac{n\pi x}{3}}$$

10,5



$$\boxed{S(\sqrt{111}) = -1}$$

Primjer 2.2 Izračunati promjenu intrinzičnih koncentracija u ... temperatura promijeni s $T_1 = 300$ K na $T_2 = 360$ K.

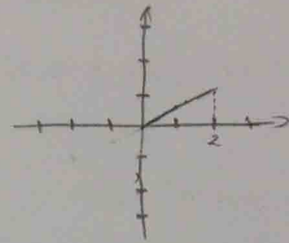
... intrinzična koncentracija se, prema (2.5), može pisati

$n_i \sim \frac{1}{\text{širina zab. pojasa}}$

2) $T = 2$

$f(x) = \frac{x}{2} \quad \langle 0, 2 \rangle$

i izračunaj $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} = \frac{(-1)^n}{2n+1}$



$$a_0 = \frac{2}{T}\int_a^b f(x)\,dx = 1\int_0^2 \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_0^2 = 1$$

$$a_n = \frac{2}{T}\int_a^b f(x)\cos\frac{2n\pi x}{T}\,dx = \frac{1}{2}\int_0^2 x\cdot\cos n\pi x\,dx = \frac{1}{2}\Big(\underbrace{x\cdot\frac{1}{n\pi}\cdot\sin n\pi x\Big|_0^2}_{0} - \frac{1}{n\pi}\int_0^2 \sin n\pi x\,dx\Big)$$

$$= \frac{1}{2}\cdot\frac{1}{n^2\pi^2}\cos n\pi x\Big|_0^2 = \frac{1}{2}\cdot\frac{1}{n^2\pi^2}(\cos 2n\pi - 1) = 0$$

$$b_n = \frac{2}{T}\int_a^b f(x)\sin\frac{2n\pi x}{T}\,dx = \frac{1}{2}\int_0^2 x\cdot\sin n\pi x\,dx = \frac{1}{2}\Big(x\cdot\frac{-1}{n\pi}\cdot\cos n\pi x\Big|_0^2 + \frac{1}{n\pi}\int_0^2 \cos n\pi x\,dx\Big)$$

$$= \frac{1}{\cancel{2}}\Big(\frac{-\cancel{2}}{n\pi}\cos 2n\pi + \underbrace{\frac{1}{n^2\pi^2}\cdot\sin n\pi x\Big|_0^2}_{0}\Big) = -\frac{1}{n\pi}$$

$$S(x) = \frac{1}{2} - \sum_{n=1}^{\infty}\frac{1}{n\pi}\cdot\sin n\pi x$$

$$S\left(\tfrac{1}{2}\right) = f\left(\tfrac{1}{2}\right)$$

$$\frac{1}{4} = \frac{1}{2} - \sum_{n=1}^{\infty}\frac{1}{n\pi}\cdot\sin\frac{\pi}{2}n$$

za $n = 2k+1$

$$\frac{1}{4} = \frac{1}{2} - \sum_{n=0}^{\infty}\frac{1}{(2k+1)\pi}\sin\frac{\pi}{2}(2k+1)$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{(2k+1)\pi} = \frac{1}{2} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}$$

$$\boxed{\sum_{n=0}^{\infty}\frac{(-1)^n}{(2k+1)} = \frac{\pi}{4}}$$

temperatura promijenom s ...

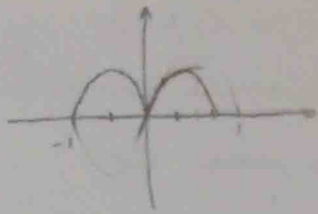
Rješenje:

... primjenom izraza (2.3), intrinzična koncentracija se, p...

$n_i \sim$ ... širina zab. ...

2) $f(x) = \begin{cases} |\sin(\pi x)| & , x \in [-1,1] \\ 0 & , \text{inače} \end{cases}$ (A)

2006



- funkcija je parna => $B(\lambda) = 0$

$$A(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos\lambda\xi \, d\xi = \frac{2}{\pi}\int_0^1 \sin\pi\xi \cos\lambda\xi \, d\xi = \frac{1}{\pi}\int_0^1 \left(\sin(\pi\xi+\lambda\xi) + \sin(\pi\xi-\lambda\xi)\right) d\xi$$

$$= \frac{1}{\pi}\left(\int_0^1 \sin(\pi\xi+\lambda x)\, d\xi + \int_0^1 \sin(\pi\xi-\lambda\xi)\, d\xi\right) = \frac{1}{\pi}\left(\frac{-1}{\pi+\lambda}\cos(\pi\xi+\lambda\xi)\Big|_0^1 + \frac{-1}{\pi-\lambda}\cos(\pi\xi-\lambda\xi)\right|$$

$$= \frac{1}{\pi}\left(\frac{-1}{\pi+\lambda}(-\cos\lambda - 1) + \frac{-1}{\pi-\lambda}(-\cos\lambda-1)\right)$$

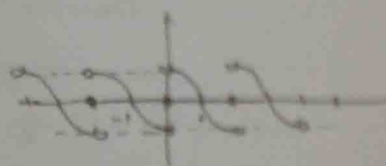
$$= \frac{\cos\lambda+1}{\pi}\left(\frac{\pi-\cancel{\lambda}+\pi+\cancel{\lambda}}{\pi^2-\lambda^2}\right) = \frac{2\cancel{\pi}(\cos\lambda+1)}{\cancel{\pi}(\pi^2-\lambda^2)} = \frac{2\cos\lambda+2}{\pi^2-\lambda^2}$$

$$\boxed{\tilde{f}(x) = 2\int_0^{\infty} \frac{\cos\lambda+1}{\pi^2-\lambda^2}\cdot\cos\lambda x\, d\lambda \quad , \lambda \neq \pi}$$

$$\text{Am}(\lambda) = |A(\lambda)| = 2\cdot\left|\frac{\cos\lambda+1}{\pi^2-\lambda^2}\right| \quad , \lambda \neq \pi$$

(B)

2) $f(x) = 1-x^2 \quad x \in \langle 0,1 \}$ po sinus funkciji



- funkcija je neparna => $a_0, a_n = 0$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx = 2\int_0^1 (1-x^2)\sin n\pi x\, dx = 2\left(\int_0^1 \sin n\pi x\, dx - \int_0^1 x^2 \sin n\pi x\, dx\right) = \left| \begin{matrix} u=x^2 & dv=\sin n\pi x\,dx \\ du=2x\,dx & v=-\frac{\cos n\pi x}{n\pi} \end{matrix} \right.$$

$$= 2\left(-\frac{1}{n\pi}\cos n\pi x\Big|_0^1 - \left(x^2\cdot\frac{-1}{n\pi}\cdot\cos n\pi x\Big|_0^1 + \frac{2}{n\pi}\int_0^1 x\cos n\pi x\, dx\right)\right) = \left| \begin{matrix} u=x & dv=\cos n\pi x\,dx \\ du=dx & v=\frac{1}{n\pi}\sin n\pi x \end{matrix} \right.$$

$$= 2\left(\frac{1-\cos n\pi}{n\pi} - \left(\frac{-\cos n\pi}{n\pi} + \frac{2}{n\pi}\left(\underbrace{x\cdot\frac{1}{n\pi}\sin n\pi x\Big|_0^1}_{0} - \frac{1}{n\pi}\int_0^1 \sin n\pi x\, dx\right)\right)\right)$$

$$= 2\left(\frac{1-\cos n\pi}{n\pi} - \left(\frac{-\cos n\pi}{n\pi} + \frac{2}{n^3\pi^3}\cdot\cos n\pi x\Big|_0^1\right)\right)$$

$$= 2\left(\frac{1}{n\pi} - \cancel{\frac{\cos n\pi}{n\pi}} + \cancel{\frac{\cos n\pi}{n\pi}} - \frac{2}{n^3\pi^3}(\cos n\pi - 1)\right)$$

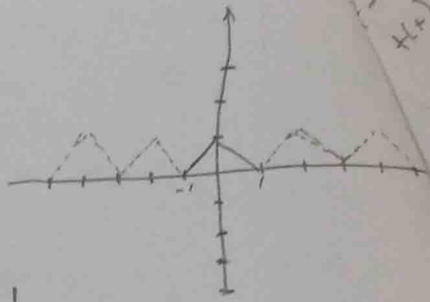
$$= \frac{2\cdot(n^2\pi^2 - 2\cos n\pi + 2)}{n^3\pi^3}$$

$$S(5\pi) = f(5\pi) = f(5\pi - 2\cdot 8) = f(5\pi - 16) = (5\pi-16)^2 = 1$$

$$\boxed{S(x) = \frac{2}{\pi^3}\sum_{n=1}^{\infty}\frac{n^2\pi^2 - 2\cos n\pi + 2}{n^3}\cdot\sin(n\pi x)}$$

2.) $f(x) = 1 - |x|$ na $\langle -1, 1 \rangle$

Primjenom Parsevala izračunaj $1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4}$



- funkcija je parna $b_n = 0$

$$a_0 = \frac{2}{L}\int_0^L f(x)dx = 2\int_0^1 (1-x)dx = 2\left(x - \frac{x^2}{2}\right)\Big|_0^1 = 1$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cdot\cos\frac{n\pi x}{L}dx = 2\int_0^1 (1-x)\cdot\cos n\pi x\, dx = 2\left(\int_0^1 \cos n\pi x\, dx - \int_0^1 x\cdot\cos n\pi x\, dx\right)$$

$$= 2\left(\underbrace{\frac{1}{n\pi}\sin n\pi x\Big|_0^1}_{0} - \left(\underbrace{x\cdot\frac{1}{n\pi}\sin n\pi x\Big|_0^1}_{0} - \frac{1}{n\pi}\int_0^1 \sin n\pi x\, dx\right)\right)$$

$$= 2\left(\frac{1}{n\pi}\cdot\left(\frac{-1}{n\pi}\cdot\cos n\pi x\Big|_0^1\right)\right) = \frac{2(1-\cos n\pi)}{n^2\pi^2}$$

$$a_{2n} = \frac{2(1-\cos 2n\pi)}{4n^2\pi^2} = 0$$

$$a_{2n+1} = \frac{2(1-\cos(2n+1)\pi)}{(2n+1)^2\pi^2} = \cdot\frac{4}{(2n+1)^2\pi^2}$$

$$\boxed{S(x) = \frac{1}{2} + \frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cdot\cos(2n+1)\pi x}$$

$$\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2) = \frac{2}{T}\int_a^b |f(x)|^2 dx$$

$$\frac{1}{2} + \sum_{n=1}^{\infty}\frac{16}{\pi^4(2n+1)^4} = 2\int_0^1 |(1-x)|^2 dx = 2\int_0^1 (1-2x+x^2)dx$$

$$= 2\cdot\left(x - x^2 + \frac{x^3}{3}\right)\Big|_0^1$$

$$= 2\left(1 - 1 + \frac{1}{3}\right) = \frac{2}{3}$$

$$\frac{1}{2} + \sum_{n=1}^{\infty}\frac{16}{\pi^4(2n+1)^4} = \frac{2}{3}$$

$$\sum_{n=1}^{\infty}\frac{16}{\pi^4(2n+1)^4} = -\frac{1}{2} + \frac{2}{3} = \frac{-3+4}{6} = +\frac{1}{6}$$

$$\boxed{\sum_{n=1}^{\infty}\frac{1}{(2n+1)^4} = +\frac{\pi^4}{96}}$$

$T=2$

$f(x) = \frac{x}{2} \quad x \in \langle 0, 2)$ A

Izracunaj $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

$$a_0 = \frac{2}{T}\int_a^b f(x)\,dx = 1\int_0^2 \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_0^2 = 1$$

$$a_n = \frac{2}{T}\int_a^b f(x)\cdot\cos\frac{2n\pi x}{T}\,dx = \frac{1}{2}\int_0^2 x\cos n\pi x\,dx = \left|\begin{matrix} u=x & dv=\cos n\pi x\,dx \\ du=dx & v=\frac{1}{n\pi}\sin n\pi x\end{matrix}\right|$$

$$= \frac{1}{2}\left(\underbrace{\frac{x}{n\pi}\sin n\pi x\Big|_0^2}_{0} - \frac{1}{n\pi}\int_0^2 \sin n\pi x\,dx\right) = \frac{1}{2}\cdot\frac{1}{n^2\cdot\pi^2}\cdot\cos n\pi x\Big|_0^2$$

$$= \frac{1}{2\pi^2}\cdot\frac{\cos 2n\pi - 1}{n^2} = 0$$

$$b_n = \frac{2}{T}\int_a^b f(x)\sin\frac{2n\pi x}{T}\,dx = \frac{1}{2}\int_0^2 x\sin n\pi x\,dx = \left|\begin{matrix} u=x & dv=\sin n\pi x\,dx \\ du=dx & v=\frac{-1}{n\pi}\cos n\pi x\end{matrix}\right|$$

$$= \frac{1}{2}\left(\frac{-x}{n\pi}\cos n\pi x\Big|_0^2 + \frac{1}{n\pi}\int_0^2 \cos n\pi x\,dx\right)$$

$$= \frac{1}{\cancel{2}}\left(\frac{-\cancel{2}}{n\pi}\cos 2n\pi + \underbrace{\frac{1}{n^2\pi^2}\sin n\pi x\Big|_0^2}_{0}\right) = \frac{-1}{n\pi}$$

$$\boxed{S(x) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin n\pi x}{n}} \qquad n\frac{\pi}{2}$$

$$S\left(\frac{1}{2}\right) = f\left(\frac{1}{2}\right)$$

$$\frac{1}{4} = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin\frac{\pi}{2}n}{n}$$

Za $n = 2k$

$$\frac{1}{4} = \frac{1}{2} - \frac{1}{\pi}\underbrace{\sum_{k=1}^{\infty}\frac{\sin k\frac{\pi}{2}}{2k}}_{0} .$$

Za $n = 2k+1$

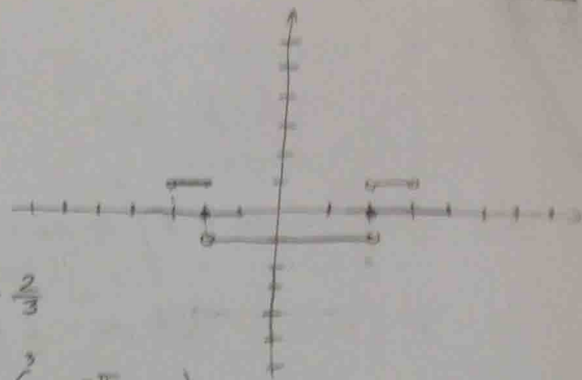
$$\frac{1}{4} = \frac{1}{2} - \frac{1}{\pi}\sum_{k=0}^{\infty}\frac{\sin(2k+1)\frac{\pi}{2}}{2k+1}$$

$$\frac{1-2}{4} = -\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2k+1)\frac{\pi}{2}}{2k+1}$$

$$\boxed{\sum_{n=0}^{\infty}\frac{\sin(2k+1)\frac{\pi}{2}}{2k+1} = \frac{\pi}{4}}$$

1.) $f(x) = sgn(x^2-4)$, $x \in \langle -3, 3 \rangle$

- funkcija je parna $\Rightarrow b_n = 0$



$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{3}\left(\int_0^2 -dx + \int_2^3 dx\right)$$

$$= \frac{2}{3}\left(-x\Big|_0^2 + x\Big|_2^3\right) = \frac{2}{3}(-2+3-2) = -\frac{2}{3}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cdot\cos\frac{n\pi x}{L}\,dx = \frac{2}{3}\left(-\int_0^2 \cos\frac{n\pi x}{3}\,dx + \int_2^3 \cos\frac{n\pi x}{3}\,dx\right)$$

$$= \frac{2}{3}\left(-\frac{3}{n\pi}\cdot\sin\frac{n\pi x}{3}\Big|_0^2 + \frac{3}{n\pi}\sin\frac{n\pi x}{3}\Big|_2^3\right)$$

$$= \frac{2}{3}\left(-\frac{3}{n\pi}\cdot\sin\frac{2n\pi}{3} - \frac{3}{n\pi}\cdot\sin\frac{2n\pi}{3}\right)$$

$$= \frac{2}{\cancel{3}}\left(-\frac{6}{n\pi}\sin\frac{2n\pi}{3}\right) = -\frac{4}{n\pi}\cdot\sin\frac{2n\pi}{3}$$

$107 : 6 = 18$

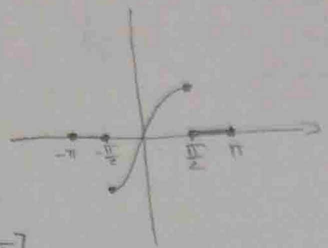
$$\boxed{S(x) = -\frac{1}{3} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin\frac{2n\pi}{3}}{n}\cdot\cos\frac{n\pi x}{3}}$$

$$S(\sqrt{111}) = f(\sqrt{111}) = sgn(111-4) = sgn(107) = -1$$

$$\boxed{S(\sqrt{111}) = -1}$$

$$f(x) = \begin{cases} \sin x & x \in [0, \frac{\pi}{2}] \\ 0 & x \in [\frac{\pi}{2}, \pi] \end{cases}$$ Grupa A

proširimo funkciju sinx



$$f(x) = \begin{cases} \sin x & x \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ 0 & , x \in [-\pi, -\frac{\pi}{2}] \cup [\frac{\pi}{2}, \pi] \end{cases}$$

$$a_0 = \frac{2}{L} \int_0^L f(x)\,dx = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \sin x\,dx = -\frac{4}{\pi} (\cos x \Big|_0^{\frac{\pi}{2}}) = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{L} \int_0^L f(x) \cdot \sin \frac{n\pi x}{L} dx = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin x \cdot \sin 2nx\,dx$$

$$= \frac{1}{\pi} \left[\int_0^{\frac{\pi}{2}} \cos(x-2nx)dx \quad \int_0^{\frac{\pi}{2}} \cos(x+2nx)dx \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{1+2n} \sin(x-2nx) \Big|_0^{\frac{\pi}{2}} - \frac{1}{1+2n} \sin(x+2nx) \Big|_0^{\frac{\pi}{2}} \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{1-2n} \left(\sin\left(\frac{\pi}{2} - \pi n\right) \right) - \frac{1}{1+2n} \left(\sin\left(\frac{\pi}{2} + \pi n\right) \right) \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{1-2n} \cos n\pi - \frac{1}{1+2n} \cos n\pi \right]$$

$$= \frac{1}{\pi} \cos n\pi \left(\frac{\cancel{1}+2n-\cancel{1}+2n}{1-4n^2} \right) = \frac{4n \cos n\pi}{\pi(1-4n^2)}$$

$$b_{2n} = \frac{8k}{\pi(1-16k^2)}$$

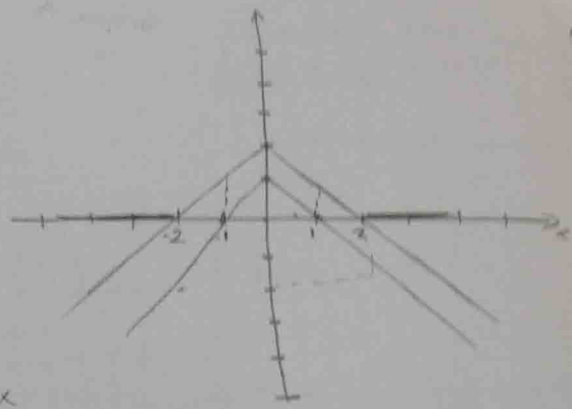
$$b_{2k+1} = \frac{-4(2k+1)}{\pi(1-4(2k+1)^2)}$$

za $n = \frac{1}{2}$

$$= \frac{1}{\pi} \left(\int_0^{\frac{\pi}{2}} \cos 0\,dx - \int_0^{\frac{\pi}{2}} \cos 2x\,dx \right) = \frac{1}{\pi} \left(x - \frac{1}{2} \sin 2x \right) \Big|_0^{\frac{\pi}{2}} = \frac{1}{\pi} \cdot \frac{\pi}{2} - 0 = \frac{1}{2}$$

$$\boxed{S(x) = \frac{1}{2} \sin x - \frac{1}{\pi} \sum_{k=0}^{\infty} \frac{4(2k+1)}{1-4(2k+1)^2} \sin(4k+2)x + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{8k}{1-16k^2} \sin(4kx)}$$

2.) $f(x) = \begin{cases} 1-|x|, & x \in (-1,1) \\ 2-|x|, & x \in (-2,-1] \cup [1,2) \\ 0 & \text{inače} \end{cases}$



$B(\lambda) = 0$

$$A(\lambda) = \frac{2}{\pi}\left(\int_0^1 f(x)\cos\lambda x + \int_1^2 f(x)\cos\lambda x\right)dx$$

$$= \frac{2}{\pi}\left(\int_0^1 (1-|x|)\cos\lambda x\,dx + \int_1^2 (2-|x|)\cos\lambda x\,dx\right) = \left|\begin{matrix} u = 1-|x| & dv = \cos\lambda x\,dx \\ du = -1dx & v = \frac{1}{\lambda}\sin\lambda x \end{matrix}\right|$$

$$= \frac{2}{\pi}\left[\left(\underbrace{\frac{1}{\lambda}(1-|x|)\sin\lambda x\Big|_0^1}_{0 \text{ ili}} + \frac{1}{\lambda}\int_0^1 \sin\lambda x\,dx\right) + \left(\frac{1}{\lambda}(2-|x|)\sin\lambda x\Big|_1^2 + \frac{1}{\lambda}\int_1^2 \sin\lambda x\,dx\right)\right]$$

$$\frac{2\sin\lambda}{\lambda}$$

$$= \frac{2}{\pi}\left[-\frac{1}{\lambda^2}\cos\lambda x\Big|_0^1 - \frac{\sin\lambda x}{\lambda} - \frac{1}{\lambda^2}\cos\lambda x\Big|_1^2\right]$$

$$= \frac{2}{\pi}\left[\cancel{-\frac{1}{\lambda^2}\cos\lambda} + \frac{1}{\lambda^2} - \frac{\sin\lambda x}{\lambda} - \frac{\cos 2\lambda}{\lambda^2} + \cancel{\frac{1}{\lambda^2}\cos\lambda}\right]$$

$$= \frac{2}{\pi}\left[\frac{\cos 2\lambda}{\lambda^2} - \frac{\sin\lambda x}{\lambda} - \frac{1}{\lambda^2}\right] = \frac{2}{\lambda^2\pi}\left[-\cos 2\lambda - \lambda\sin\lambda x + 1\right]$$

$$\boxed{\tilde{f}(x) = \frac{2}{\pi}\int_0^\infty \frac{1-\cos 2\lambda - \lambda\sin\lambda}{\lambda^2}\,d\lambda}$$

$$f(0) = \tilde{f}(0)$$

$$\frac{1+2+0}{3} = \frac{2}{\pi}\int_0^\infty \frac{1-\cos 2\lambda - \lambda\sin\lambda}{\lambda^2}$$

$$1 = \frac{2}{\pi}\int_0^\infty \frac{1-\cos 2\lambda - \lambda\sin\lambda}{\lambda^2} \Rightarrow \boxed{\frac{\pi}{2} = \int_0^\infty \frac{1-\cos 2\lambda - \lambda\sin\lambda}{\lambda^2}}$$

$1 - \cos^2\lambda + \sin^2\lambda$

$$\frac{2\sin^2\lambda - \lambda\sin\lambda}{\lambda^2} = \frac{\sin\lambda}{\lambda}\left(\frac{2\sin\lambda}{\lambda} - 1\right)$$