1. Blic

$q = 1nC$

$\vec{v} = \vec{a}_x - 2\vec{a}_y + 2\vec{a}_z \quad [\frac{m}{s}]$

$\vec{E} = E_y \vec{a}_x + E_y \vec{a}_y + E_z \vec{a}_z \quad [\frac{V}{m}]$

$\vec{B} = -\vec{a}_x + \vec{a}_y + 2\vec{a}_z \quad [T]$

$\vec{F} = 3\vec{a}_x - 2\vec{a}_y + 5\vec{a}_z \quad [nN]$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 1 & -2 & 2 \\ -1 & 1 & 2 \end{vmatrix} = \vec{a}_x(-4-2) - \vec{a}_y(2+2) + \vec{a}_z(1-1)$$

$$= -6\vec{a}_x - 4\vec{a}_y - \vec{a}_z$$

$$\vec{E} + \vec{v} \times \vec{B} = E_x\vec{a}_x + E_y\vec{a}_y + E_z\vec{a}_y - 6\vec{a}_x - 4\vec{a}_y - \vec{a}_z$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$(3\vec{a}_x - 2\vec{a}_y + 5a_z)[nN] = [1\cancel{n}C]\left[\vec{a}_x(E_x - 6) + \vec{a}_y(E_y - 4) + \vec{a}_z(E_z - 1)\right]$$

$3 = E_x - 6 \Rightarrow \boxed{E_x = 9}$

$-2 = E_y - 4 \Rightarrow \boxed{E_y = 2}$

$5 = E_z - 1 \Rightarrow \boxed{E_z = 6}$

$$\boxed{\vec{E} = 9\vec{a}_x + 2\vec{a}_y + 6\vec{a}_z}$$

1. Blic

Polje je aproksimirano izrazom $\vec{E} = \left(\frac{1}{x^2+y^2+z^2}\right)(\vec{a}_x + \vec{a}_y + \vec{a}_z)$.

Odredi gustoću u točki $T(1,2,3)$.

$$\rho = \varepsilon_0 \nabla \vec{E} = \varepsilon_0 \left[\frac{\partial}{\partial x}\left(\frac{1}{x^2+y^2+z^2}\right) + \frac{\partial}{\partial y}\left(\frac{1}{x^2+y^2+z^2}\right) + \right.$$

$$\left. + \frac{\partial}{\partial z}\left(\frac{1}{x^2+y^2+z^2}\right)\right]$$

$$\rho = \varepsilon_0 \left[\frac{-2x}{(x^2+y^2+z^2)^2} + \frac{-2y}{(x^2+y^2+z^2)^2} + \frac{-2z}{(x^2+y^2+z^2)^2} \right]\Bigg|_T$$

$$\rho = \varepsilon_0 \left[\frac{-2\cdot 1}{(1^2+2^2+3^2)^2} \quad \frac{-2\cdot 2}{(1^2+2^2+3^2)^2} \quad - \frac{2\cdot 3}{(1^2+2^2+3^2)^2} \right]$$

$$\rho = \varepsilon_0 \left[\frac{-2}{196} - \frac{4}{196} - \frac{6}{196} \right]$$

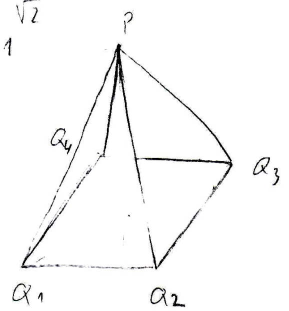
$$\rho = \varepsilon_0 \cdot \left(-\frac{12}{196}\right) \qquad \varepsilon_0 = 8{,}85 \cdot 10^{-12}$$

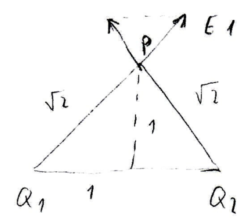
$$\boxed{\rho = -0{,}06\,\varepsilon_0}$$

2. Blic

Izračunat jačinu polja u točki A koje se nalazi iznad sjecišta dijagonala kvadrata stranice a na visini h. U svakom kutu kvadrata je po jedan naboj od 1nC.

$a = \sqrt{2}$
$h = 1$





$$\vec{r}_1 = \vec{r}_2 = \vec{r}_3 = \vec{r} = \left(1 - \frac{\sqrt{2}}{2}\right)\vec{a}_y$$

$$R_1 = \vec{a}_y + \frac{\sqrt{2}}{2}\vec{a}_x$$

$$R_2 = -\frac{\sqrt{2}}{2}\vec{a}_x + \vec{a}_y$$

$$R_3 = (1-\sqrt{2})\vec{a}_y - \frac{\sqrt{2}}{2}\vec{a}_x$$

$$R_4 = (1-\sqrt{2})\vec{a}_y + \frac{\sqrt{2}}{2}\vec{a}_x$$

$$E = \frac{4Q}{4\pi\varepsilon}\left(\frac{\vec{R}_1}{|\vec{R}_1|^3} + \frac{\vec{R}_2}{|\vec{R}_2|^3} + \frac{R_3}{|\vec{R}_3|^3}\right) = 12.1\ V$$

2. Blic

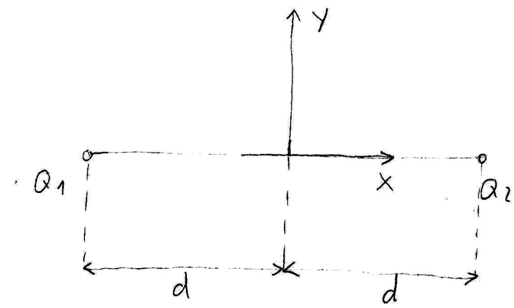
Dva su točkasta naboja postavljena na udaljenost 2d prema slici. Odredite iznos naboja Q_2 takav da je potencijal u točki A (X, Y) jednak nuli.

$Q_1 = 1\,nC$

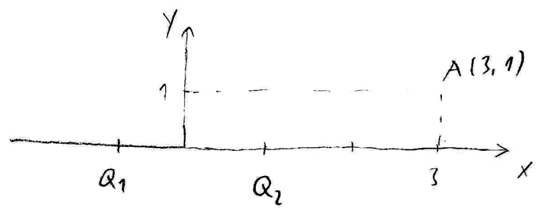
$d = 1\,m$

$X = 3\,m$

$Y = 1\,m$



$$\varphi(r) = \frac{Q}{4\pi \varepsilon_0 r}$$



$$\frac{Q_1}{\cancel{4\pi\varepsilon_0} r_1} = \frac{Q_2}{\cancel{4\pi\varepsilon_0} r_2}$$

$$Q_2 = Q_1 \frac{r_2}{r_1}$$

$$r_1 = \sqrt{4^2+1^2} = \sqrt{17}$$

$$r_2 = \sqrt{2^2+1^2} = \sqrt{5}$$

$$Q_2 = 1 \cdot \frac{\sqrt{5}}{\sqrt{17}} = \boxed{0.542\,nC}$$

2. Blic

$\varphi = 3x^2y - y^3$

$\vec{E} = ?$

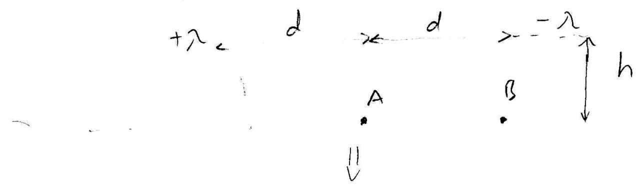
$\vec{E} = -\nabla\varphi$

$$\vec{E} = -\frac{\partial}{\partial x}\left(3x^2y - y^3\right)\vec{a}_x - \frac{\partial}{\partial y}\left(3x^2y - y^3\right)\vec{a}_y$$

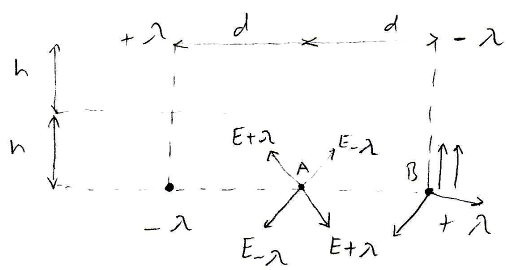
$$\boxed{\vec{E} = (-6xy)\vec{a}_x + \left(-3x^2 - 3y^2\right)\vec{a}_y}$$

3. Blic

Beskonačno dugi dvožični vod nabijen nabojem linijske gustoće $\pm\lambda = 10\,nC/m$ nalazi se na visini $h = 1m$ iznad zemlje. Vodiči su razmaknuti za $2d = 2m$. Odredite jakost električnog polja u točkama A i B prema slici



$\Downarrow$



$E_A, E_B = ?$

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$\boxed{E_A = 0}$$

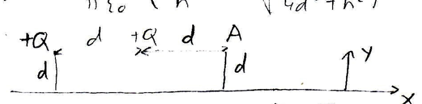
$$E_{BX} = \frac{\lambda}{2\pi\varepsilon_0}(\cos\alpha - \cos\alpha)\vec{a}_y = 0$$

$$\sin\alpha = \frac{h}{R}$$

$$R = \sqrt{4d^2 + h^2}$$

$$E_{BY} = \left[\frac{2\lambda}{2\pi\varepsilon_0 h} - \frac{2\lambda}{2\pi\varepsilon_0 R}\sin\alpha\right]\vec{a}_y$$

$$E_{BY} = \frac{\lambda}{\pi\varepsilon_0}\left(\frac{1}{h} - \frac{1}{\sqrt{4d^2+h^2}}\right)\vec{a}_y = \boxed{288\ V/m}$$

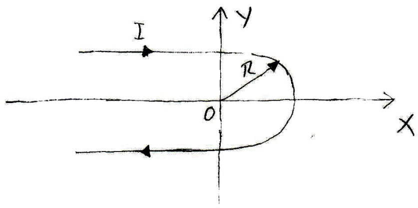


$$\vec{E} = \frac{Q}{4\pi\varepsilon d^2}\left[\vec{a}_x\left(1 + \frac{1}{4} - \frac{1}{8\sqrt{2}} - \frac{1}{5\sqrt{5}}\right) + \vec{a}_y\left(-\frac{1}{8\sqrt{2}} - \frac{2}{5\sqrt{5}}\right)\right]$$

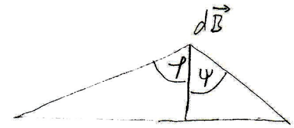
4. blic

Kroz beskonačno dugu tanku žicu teče struja I. Ako je žica savijena prema slici odredite gustoću magnetskog toka u ishodištu po iznosu i smjeru.

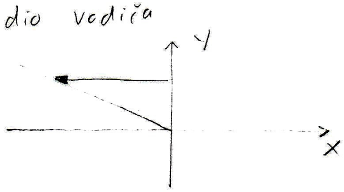
$\vec{B}$ u $(0,0,0) = ?$



$$\vec{B} = \vec{a}_\alpha \frac{\mu_0 I}{4\pi}(\sin\varphi + \sin\psi)$$

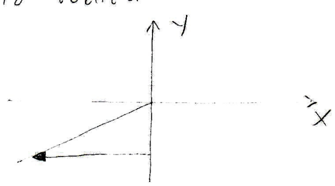


1. dio vodiča



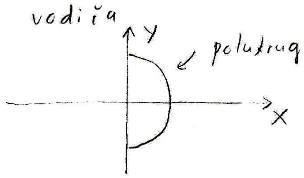
$$\varphi = \frac{\pi}{2},\ \psi = 0 \Rightarrow \boxed{\vec{B}_1 = -\vec{a}_z \frac{\mu_0 I}{4\pi}\cdot 1}$$

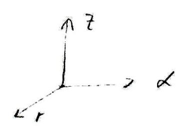
2. dio vodiča

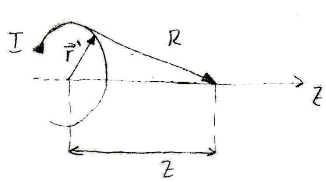


$$\varphi = 0,\ \psi = \frac{\pi}{2} \Rightarrow \boxed{\vec{B}_2 = -\vec{a}_z \frac{\mu_0 I}{4\pi}\cdot 1}$$

3. dio vodiča





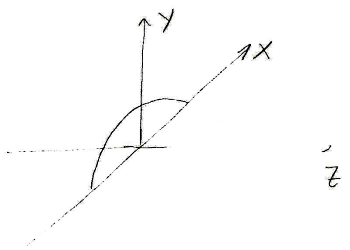


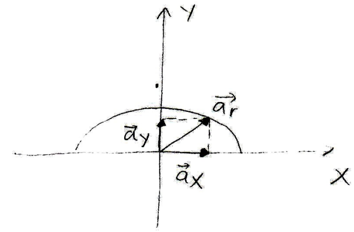
$$d\vec{l} = r\,d\alpha \cdot \vec{a}_\alpha$$

$$\left.\begin{aligned}\vec{r}\,' &= r_0 \vec{a}_r\\ \vec{r} &= z\vec{a}_z\end{aligned}\right\}\quad \vec{R} = \vec{r} - \vec{r}\,' \quad \boxed{\vec{R} = z\vec{a}_z - r_0\vec{a}_r}$$

$$d\vec{l}\times\vec{R} = \vec{a}_r \cdot r z\,d\alpha + \vec{a}_z r\,d\alpha \cdot r_0$$

$$= (z\vec{a}_r + r_0\vec{a}_z)\, r_0\, d\alpha$$





$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{(z\vec{a}_r + r_0\vec{a}_z) r_0 \, d\alpha}{(z^2 + r_0^2)^{\frac{3}{2}}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{\pi} \frac{z r_0 \vec{a}_r \, d\alpha}{(z^2 + r_0^2)^{\frac{3}{2}}} + \frac{\mu_0 I}{4\pi} \int_0^{\pi} \frac{r_0^2 \, d\alpha \, \vec{a}_z}{(z^2 + r_0^2)^{\frac{3}{2}}}$$

$$\vec{a}_r = \cos\alpha \, \vec{a}_x + \sin\alpha \, \vec{a}_y$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{z r_0}{(z^2 + r_0^2)^{\frac{3}{2}}} \int_0^{\pi} (\cos\alpha \, \vec{a}_x + \sin\alpha \, \vec{a}_y) \, d\alpha + \frac{\mu_0 I}{4\pi} \frac{r_0^2}{(z^2 + r_0^2)^{\frac{3}{2}}} \vec{a}_z \int_0^{\pi} d\alpha$$

$z = 0$

$$\boxed{\vec{B}_3 = -\vec{a}_z \frac{\mu_0 I}{4R}}$$

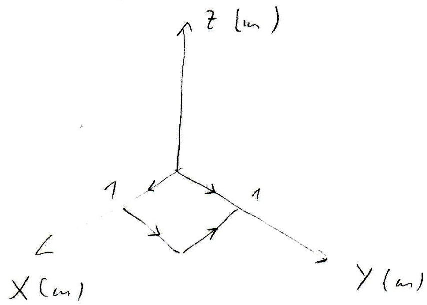
$$\boxed{\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{4\pi R} (2 + \pi)(-\vec{a}_z)}$$

5. blic

Zadan je vektor jakosti magi polja u pravocrtnom koordinatnom sustavu kao

$$\vec{H} = \frac{1}{x+3}\vec{a}_y \left(\frac{A}{m}\right)$$

Odredite vektor gustoće struje u ishodištu i ukupnu struju kroz kvadratnu površinu prema slici!



$$\vec{H} = \vec{a}_y \frac{1}{x+3}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{1}{x+3} & 0 \end{vmatrix} = \vec{a}_x \cdot 0 - \vec{a}_y \cdot 0 + \frac{\partial}{\partial x}\left(\frac{1}{x+3}\right)\vec{a}_z$$

$$\boxed{\vec{J} = -\frac{1}{(x+3)^2}\vec{a}_z} \qquad \vec{J}(0,0,0) = \boxed{-\frac{1}{9}\vec{a}_z}$$

$$I = \int \vec{J}\vec{n}\, dS, \quad dS = dx\,dy, \quad \vec{n} = \vec{a}_z$$

$$I = \iint_S \frac{1}{(x+3)^2}\, dx\,dy = -y\Big|_{y=0}^{1} \int_{x=0}^{1} \frac{1}{(x+3)^2}\, dx = \boxed{-\frac{1}{12}\, A}$$

6 blic

Odredi induktivitet torusa, ako je $\mu_r = 1000$, $N = 1000$, $R_{sr} = 15cm$, $S = 2cm^2$, $I = 1A$.

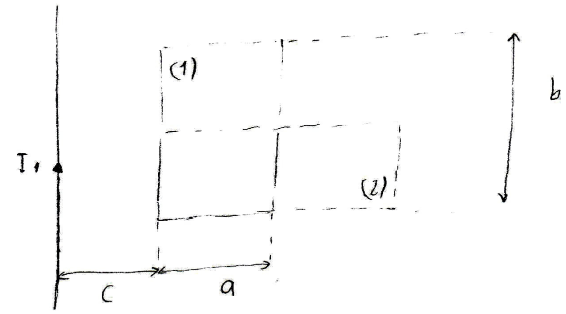
$$\boxed{l_{sr} = 2\pi R_{sr}} = 2 \cdot 3.14 \cdot 0.15 = \boxed{0.942\ m}$$

$$\boxed{L = \frac{\mu N^2 S}{l_{sr}}} = \boxed{\frac{\mu_0 \mu_r N^2 S}{l_{sr}}} = \frac{4 \cdot 3.14 \cdot 10^{-7} \cdot 1000 \cdot 1000^2 \cdot 2 \cdot 10^{-4}}{0.942}$$

$$\boxed{L = 0.267\ H}$$

6. blic

Odredi meduinduktivitet u položaju (1) i (2) (napomena $I_1 = I_2$)



položaj (1)

$$L_{12} = \frac{\phi_{12}}{I_2}$$

$$\phi_{12} = \mu_0 \int_c^{c+a} dx \int_0^b \frac{I_1}{2\pi x} dy = \frac{\mu_0 I_1 b}{2\pi} \int_c^{c+a} \frac{dx}{x} = \boxed{\frac{\mu_0 I_1 b}{2\pi} \ln \frac{c+a}{c}}$$

$$\boxed{L_{12} = \frac{\phi_{12}}{I_2} = \frac{\mu_0 b I_1}{2\pi I_2} \ln \frac{c+a}{c}}$$

položaj (2)

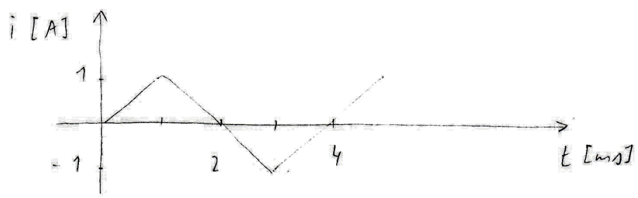
$$L_{12} = \frac{\phi_{12}}{I_2}$$

$$\phi_{12} = \mu_0 \int_c^{c+b} dx \int_0^a \frac{I_1}{2\pi x} dy = \frac{\mu_0 I_1 a}{2\pi} \int_c^{c+b} \frac{dx}{x} = \boxed{\frac{\mu_0 I_1}{2\pi} a \ln \frac{c+b}{c}}$$

$$\boxed{L_{12} = \frac{\phi_{12}}{I_2} = \frac{\mu_0 a I_1}{2\pi I_2} \ln \frac{c+b}{c}}$$

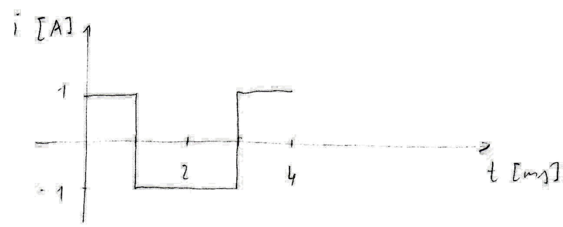
7. blic

Kroz kvadratnu petlju induktiviteta $L = 1H$ teče trokutna vremenski promjenjiva struja prema slici. Odrediti i skicirati valni oblik induciranog napona u petlji.



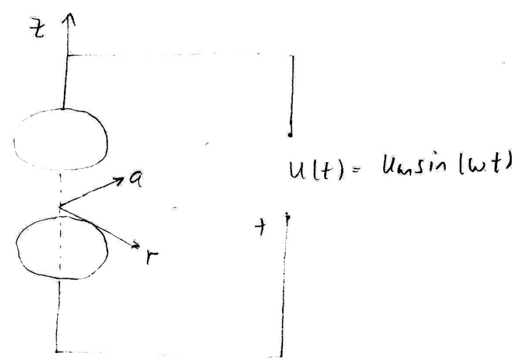
$$\varepsilon_{ind} = L \frac{di}{dt}$$

$$\varepsilon_{ind} = 1 . \frac{d\,(\text{trokutasti napon})}{dt} = \text{pravokutni napon}$$



9. blic

Pločasti kondenzator čine 2 kružne ploče razmaknute za d, između kojih je dielektrik s ε i $\kappa = 0$. Kondenzator je priključen na izmjenični napon $u(t) = U_m \sin(\omega t)$. Odrediti jakost magnetskog polja i iznos Poyntingovog vektora između ploča kondenzatora



$$\vec{E} = \frac{u}{d}\vec{a}_z = \boxed{\vec{a}_z \frac{U_m}{d} \sin(\omega t)}$$

$$\vec{D} = \varepsilon_0 \vec{E} = \vec{a}_z \varepsilon_0 \frac{U_m}{d} \sin(\omega t)$$

$$\nabla \times \vec{H} = \underbrace{\vec{J}}_{0} + \frac{\partial \vec{D}}{\partial t} = \vec{a}_z \frac{\omega \varepsilon_0 U_m \cos(\omega t)}{d}$$

cilindrični $\nabla \times \vec{H} = \vec{a}_z \frac{1}{r}\left[\frac{\partial}{\partial r}(r H_\alpha) - \underbrace{\frac{\partial H_r}{\partial \alpha}}_{0}\right]$

$$\vec{a}_z \cdot \frac{1}{r}\frac{\partial}{\partial r}(r H_\alpha) = \vec{a}_z \frac{\omega \varepsilon_0 U_m}{d} \cos(\omega t) \quad / \cdot r \quad / \int$$

$$r H_\alpha = \frac{\omega \varepsilon_0 U_m}{d} \cdot \frac{r^2}{2} \cos(\omega t) \quad / : r$$

$$\vec{H} = H \cdot \vec{a}_\alpha = \vec{a}_\alpha \frac{\omega \varepsilon_0 U_m r \cos(\omega t)}{2d}$$

$$\boxed{\vec{N} = \vec{E} \times \vec{H} = -\vec{a}_r \frac{\omega \varepsilon U_m^2}{2d} r \sin(\omega t) \cos(\omega t)}$$