Magnetostatika

Zadatci za vježbu 2010./11.

by: Steel

ZADATCI ZA VJEŽBU

(1)

$A = 2{,}5\,\vec{a}_\theta + 5\,\vec{a}_\alpha \quad [\mathrm{Tm}]$

$T\left(2\mathrm{m}, \frac{\pi}{6}, 0\right) \qquad [r, \theta, \alpha]$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$ MAGNETSKA INDUKCIJA

$$\nabla \times \vec{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(A_\phi \sin\theta\right) - \frac{\partial A_\theta}{\partial\phi}\right)\vec{r}$$

$$+ \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r A_\phi\right)\right)\vec{\theta}$$

$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r A_\theta\right) - \frac{\partial A_r}{\partial\theta}\right)\vec{\phi} \qquad \vec{\phi} = \alpha$$

TRAŽI SE U α SMJERU PA UZIMAMO SAMO DIO UZ $\vec{\phi}$

$$\vec{B} = \frac{1}{r}\left(2{,}5 - 0\right)\vec{a}_\alpha = \frac{2{,}5}{r}\,\vec{a}_\alpha \qquad r = 2\,\mathrm{m}$$

$$\vec{B}_T = 1{,}25\,\vec{a}_\alpha$$

(2)

ravnina $x = 0$, $-0{,}25\pi\,\mathrm{m} \le y \le 0{,}25\pi\,\mathrm{m}$

$\vec{n} = \vec{a}_x \qquad -0{,}01\,\mathrm{m} \le z \le 0{,}01\,\mathrm{m}$

$$J = 100\,y\,\sin(2y)\,\vec{a}_x \quad [\mathrm{Am}^{-2}]$$

JAKOST STRUJE (uz zadan J):

$$\boxed{I = \iint_S \vec{J}\,\vec{n}\cdot ds} = \int_{-0{,}25\pi}^{0{,}25\pi} dy \int_{-0{,}01}^{0{,}01} 100\,y\,\sin(2y)\,\underbrace{\vec{a}_x\cdot\vec{a}_x}_{1} =$$

$$= \int_{-0{,}25\pi}^{0{,}25\pi} 100\,y\,\sin(2y)\cdot 0{,}02\,dy = 2\int_{-0{,}25\pi}^{0{,}25\pi} y\,\sin(2y)\,dy$$

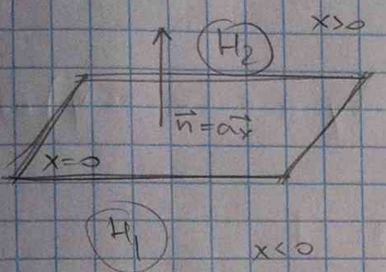
$$= \begin{bmatrix} y = u & \sin(2y)\,dy = dv \\ dy = du & -\frac{1}{2}\cos(2y) = v \end{bmatrix} =$$

$$= 2\left[\cancel{-\frac{y}{2}\cos(2y)\Big|_{-0,25\pi}^{0,25\pi}} + \frac{1}{2}\int_{-0,25\pi}^{0,25\pi}\cos(2y)\,dy\right] =$$

$$= \frac{1}{2}\sin(2y)\Big|_{-0,25\pi}^{0,25\pi} = 1$$

(3)

$K = 6,5\,\vec{a_z}$ $[Am^{-1}]$; $x = 0$



$\vec{H_1} = 10\,\vec{a_y}$

UVJETI NA GRANICI

$$\boxed{\begin{aligned}\vec{n}\times\left(\vec{H_2}-\vec{H_1}\right) &= \vec{K}\\ \vec{n}\cdot\left(\vec{B_2}-\vec{B_1}\right) &= 0\end{aligned}}$$

$$\vec{n}\times\left((H_{2x}-0)\,\vec{a_x} + (H_{2y}-10)\,\vec{a_y} + (H_{2z}-0)\,\vec{a_z}\right) = 6,5\,\vec{a_z}$$

$$(H_{2y}-10)\,\vec{a_z} - (H_{2z}-0)\,\vec{a_y} = 6,5\,\vec{a_z}$$

$H_{2y} - 10 = 6,5$ $\qquad$ $H_{2z} = 0$

$H_{2y} = 16,5$

$\vec{H_2} = 16,5\,\vec{a_y}$ $\qquad$ $|H_2| = 16,5$

$$\left(\begin{matrix}\vec{a_x}\times\vec{a_y} = \vec{a_z} & \vec{a_y}\times\vec{a_z} = \vec{a_x} & \vec{a_z}\times\vec{a_x} = \vec{a_y}\\ \vec{a_x}\times\vec{a_z} = -\vec{a_y} & \vec{a_y}\times\vec{a_x} = -\vec{a_z} & \vec{a_z}\times\vec{a_y} = -\vec{a_x}\end{matrix}\right)$$

④

$$\vec{A} = \cos x \sin y \, \vec{a_x} + \sin x \cos y \, \vec{a_y}$$

$$T(1m, 1m, 1m)$$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{a_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{a_y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{a_z}$$

$$\nabla \times \vec{A} = (0-0)\vec{a_x} + (0-0)\vec{a_y} + (\cos x \cos y - \cos x \cos y)\vec{a_z}$$

$$\vec{B} = \vec{0}$$

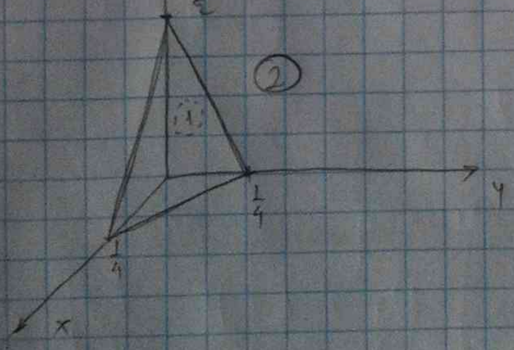
⑤ $\mu_{r1} = 4$

ravnina $\rightarrow$ $4x + 4y + 2z = 8$

$\mu_{r2} = 3$

$\vec{H_1} = \frac{1}{\mu_0}(2\vec{a_x} - 1\vec{a_y})\ [Am^{-1}]$

$B_1 = ?$



MORAMO ODREDITI JEDINIČNU NORMALU

$$\vec{n} = \frac{1}{2}\vec{a_x} + \frac{1}{2}\vec{a_y} + \frac{1}{4}\vec{a_z}$$

$$\vec{n_{12}} = \frac{\frac{1}{2}\vec{a_x} + \frac{1}{2}\vec{a_y} + \frac{1}{4}\vec{a_z}}{\sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{16}}}$$

$$\vec{n_{12}} = \frac{2}{3}\vec{a_x} + \frac{2}{3}\vec{a_y} + \frac{1}{3}\vec{a_z}$$

$$\vec{B_1} = \mu_0 \mu_{r1} \vec{H_1} = 8\vec{a_x} - 4\vec{a_y}$$

UVJETI NA GRANICI

$$\boxed{\begin{aligned} \vec{n} \times (\vec{H}_2 - \vec{H}_1) &= \vec{K} \\ \vec{n} \cdot (\vec{B}_2 - \vec{B}_1) &= 0 \end{aligned}}$$

$$\left(\frac{2}{3}\vec{a_x} + \frac{2}{3}\vec{a_y} + \frac{1}{3}\vec{a_z}\right) \cdot \left[(B_{2x} - 8)\vec{a_x} + (B_{2y} + 4)\vec{a_y} + (B_{2z} - 0)\vec{a_z} \right] = 0$$

$$\frac{2}{3}(B_{2x} - 8) + \frac{2}{3}(B_{2y} + 4) + \frac{1}{3}B_{2z} = 0 \qquad / \frac{3}{2}$$

(a) $B_{2x} + B_{2y} + \frac{1}{2}B_{2z} = 4$

$$\left(\frac{2}{3}\vec{a_x} + \frac{2}{3}\vec{a_y} + \frac{1}{3}\vec{a_z}\right) \times \left[\frac{\vec{B}_2}{\mu_0 \mu_{r2}} - \frac{\vec{B}_1}{\mu_0 \mu_{r1}} \right] = \vec{K}$$

$$\left(\frac{2}{3}\vec{a_x} + \frac{2}{3}\vec{a_y} + \frac{1}{3}\vec{a_z}\right) \times \left[\left(\frac{B_{2x}}{3\mu_0} - \frac{8}{4\mu_0}\right)\vec{a_x} + \left(\frac{B_{2y}}{3\mu_0} + \frac{4}{4\mu_0}\right)\vec{a_y} + \left(\frac{B_{2z}}{3\mu_0} - 0\right)\vec{a_z} \right] = \vec{0}$$

$$\frac{2}{3}\left(\frac{B_{2y}}{3\mu_0} + \frac{1}{\mu_0}\right)\vec{a_z} - \frac{2}{3}\left(\frac{B_{2z}}{3\mu_0}\right)\vec{a_y} - \frac{2}{3}\left(\frac{B_{2x}}{3\mu_0} - \frac{2}{\mu_0}\right)\vec{a_z} + \frac{2}{3}\left(\frac{B_{2z}}{3\mu_0}\right)\vec{a_x}$$

$$+ \frac{1}{3}\left(\frac{B_{2x}}{3\mu_0} - \frac{2}{\mu_0}\right)\vec{a_y} - \frac{1}{3}\left(\frac{B_{2y}}{3\mu_0} + \frac{1}{\mu_0}\right)\vec{a_x} = \vec{0}$$

$$\frac{2}{3}\frac{B_{2z}}{3\mu_0} - \frac{1}{3}\left(\frac{B_{2y}}{3\mu_0} + \frac{1}{\mu_0}\right) = 0 \qquad / \cdot 9\mu_0$$

(1) $2B_{2z} - B_{2y} - 3 = 0$

$$\frac{1}{3}\left(\frac{B_{2x}}{3\mu_0} - \frac{2}{\mu_0}\right) - \frac{2}{3}\left(\frac{B_{2z}}{3\mu_0}\right) = 0 \qquad / 9\mu_0$$

(2) $B_{2x} - 6 - 2B_{2z} = 0$

$$\frac{2}{3}\left(\frac{B_{2y}}{3\mu_0} + \frac{1}{\mu_0}\right) - \frac{2}{3}\left(\frac{B_{2x}}{3\mu_0} - \frac{2}{\mu_0}\right) = 0 \qquad / 9\mu_0$$

(3) $2B_{2y} + 6 - 2B_{2x} + 12 = 0$

(1) $2 B_{2z} - B_{2y} = 3 \quad \Longrightarrow \quad B_{2z} = \frac{3}{2} + \frac{1}{2} B_{2y}$

(2) $B_{2x} - 2 B_{2z} = 6 \qquad B_{2x} - 3 - B_{2y} = 6$

(3) $2 B_{2y} - 2 B_{2x} = -18 \quad \Longleftrightarrow \quad B_{2x} - B_{2y} = 9$

(a) $B_{2x} + B_{2y} + \frac{1}{2} B_{2z} = 4$

$9 + B_{2y} + B_{2y} + \frac{3}{4} + \frac{1}{4} B_{2y} = 4$

$B_{2y} = -2{,}56$

6

$\vec{A} = e^{-2z} \left(\sin(0{,}5\alpha) \right) \vec{a_\alpha} \quad [\mathrm{Tm}]$

$T(0{,}8\,m, \; \pi/3, \; 0{,}5\,m) \qquad [r, \alpha, z]$

$\nabla \times \vec{A}_r = \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \vec{a_r} =$

$= \left(0 + 2 e^{-2z} \sin(0{,}5\alpha) \right) \vec{a_r} =$

$= 0{,}37$

7

$y = \emptyset \qquad -0{,}1\,m \leq x \leq 0{,}1\,m$

$\vec{n} = \vec{a_y} \qquad -0{,}002\,m \leq z \leq 0{,}002\,m$

$\vec{J} = 100\,|x|\,\vec{a_y}$

JAKOST STRUJE

$$\boxed{I = \iint_S \vec{J} \cdot \vec{n} \cdot dS}$$

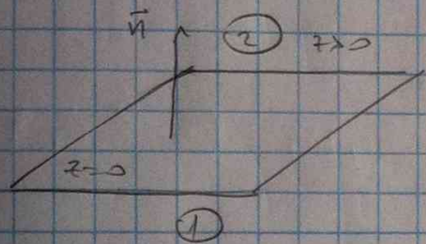
$$I = \iint_S 100|x|\, \underbrace{\vec{a_y} \cdot \vec{a_n}}_{1} \cdot dS$$

rastavljamo na 2 integrala ovisno o x !

$$I = \int_{-0,1}^{0} dx \int_{-0,002}^{0,002} (-100x)\, dz + \int_{0}^{0,1} dx \int_{-0,002}^{0,002} 100x\, dz =$$

$$= 4 \cdot 10^{-3}\ A$$

8) $K = 9\,\vec{a_y}\ [A m^{-1}]$ $\quad z = 0$



$z<0$

$\mu_{r1} = 4$

$\mu_{r2} = 3$

$$\vec{H_2} = 14,5\,\vec{a_x} + 8\,\vec{a_z}\ [A m^{-1}]$$

TANGENCIJALNA KOMPONENTA

$$\boxed{\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{K}}$$

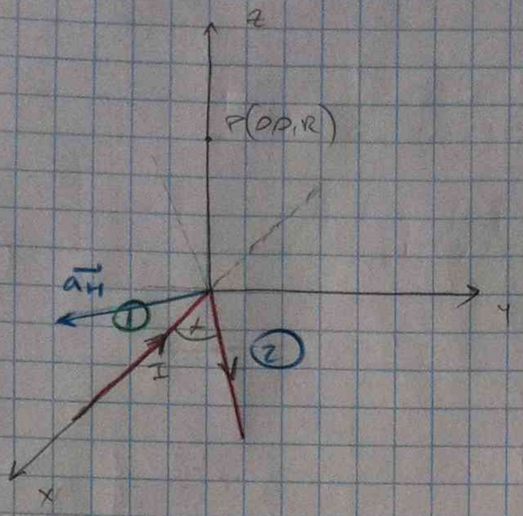
$$\vec{a_z} \times \left[(14,5 - H_{1x})\vec{a_x} + (0 - H_{1y})\vec{a_y} + (8 - H_{1z})\vec{a_z} \right] = \vec{K}$$

$$(14,5 - H_{1x})\vec{a_y} + H_{1y}\vec{a_x} = 9\,\vec{a_y}$$

$$H_{1x} = 5,5$$

$$H = 5,5\,\vec{a_x}$$

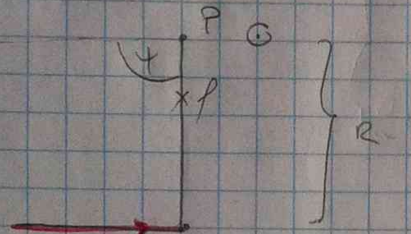
9



$I = 10\,A$

$R = 1\,m \quad \alpha = \frac{\pi}{3}$

1



$\theta = 0° \quad \psi = 90°$

$\vec{a_H} = \vec{a_y}$ NORMALA JE OKOMITA NA IZVOR

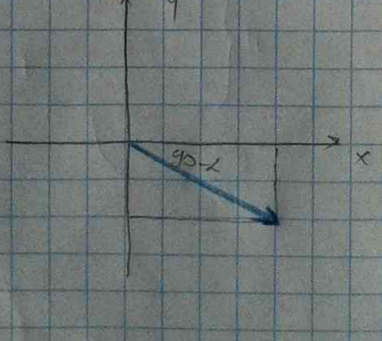
JAKOST MAGNETSKOG POLJA ZA RAVNI VODIĆ

$$\boxed{\vec{H} = \frac{I}{4\pi r}\left(\sin\theta + \sin\psi\right)\cdot \vec{a_H}}$$

udaljenost od izvora

$\vec{H}_1 = 0{,}796\,\vec{a_y}$

2

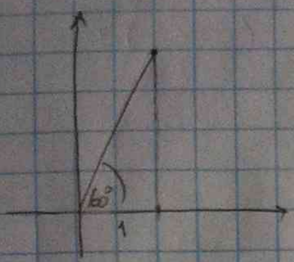


$\vec{a_H} = \frac{\sqrt{3}}{2}\vec{a_x} - \frac{1}{2}\vec{a_y}$

ili: $P(0,0,1)$

$\boxed{\vec{a_n} \text{ preko 3 točke}}$

$A(0,0,0)$
$B(1, \sqrt{3}, 0)$



$$tg\,60° = \frac{y}{1} \Rightarrow y = \sqrt{3}$$

$$\vec{a_n} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_2-x_3 & y_2-y_3 & z_2-z_3 \end{vmatrix}$$

$A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$
$T(x_3, y_3, z_3)$

$$\vec{n} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ -1 & -\sqrt{3} & 0 \\ 1 & \sqrt{3} & -1 \end{vmatrix} =$$

$$= +1\left(-\vec{a_y} - \sqrt{3}\,\vec{a_z}\right) - \sqrt{3}\left(-\vec{a_x} - \vec{a_z}\right) =$$

$$= \sqrt{3}\,\vec{a_x} - \vec{a_y}$$

$$\vec{a_n} = \frac{\sqrt{3}\,\vec{a_x} - \vec{a_y}}{\sqrt{3+1}} = \frac{\sqrt{3}}{2}\vec{a_x} - \frac{1}{2}a_y$$ } mora biti jedinična vrijednost

$r = 1\,m$ $\rho = 0$ $\psi = 90°$

$$\vec{H_2} = 0{,}69\,\vec{a_x} - 0{,}398\,\vec{a_y}$$

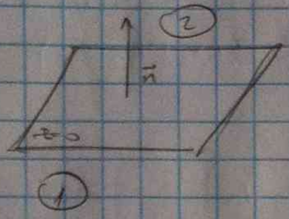
$$\vec{H} = 0{,}69\,\vec{a_x} - 0{,}398\,\vec{a_y}$$

(10)

Jednak zadatak kao i 9, samo je

$$\vec{a_H} = \frac{\sqrt{2}}{2}\vec{a_x} - \frac{\sqrt{2}}{2}\vec{a_y}$$

(11) $\vec{K} = 9\,\vec{a_y}$ $\quad z=0 \quad \vec{n} = \vec{a_z}$ $\quad \mu_{r2} = 3 \quad \mu_{r1} = 4$



$$\vec{H_2} = 14{,}5\,\vec{a_x} + 8\,\vec{a_z} \; [Am^{-1}]$$

OKOMITA KOMPONENTA

$$\boxed{\vec{n}\cdot(\vec{B_2}-\vec{B_1}) = 0}$$

$$\vec{a_z}\left[(B_{2x}-B_{1x})\vec{a_x} + (B_{2y}-B_{1y})\vec{a_y} + (B_{2z}-B_{1z})\vec{a_z}\right] = 0$$

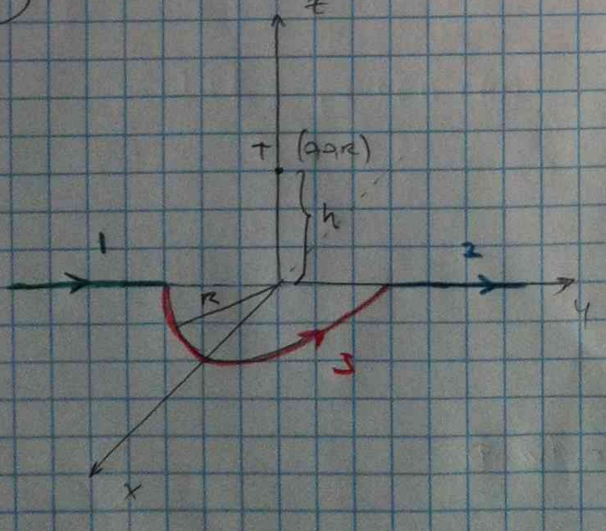
$$B_{2z} = B_{1z}$$

$$\cancel{\mu_0}\,\cancel{\mu_{r2}}\,H_{2z} = \cancel{\mu_0}\,\cancel{\mu_{r1}}\,H_{1z}$$

$$H_{1z} = 6\;[Am^{-1}]$$

(12) pogledati 5 zadatak

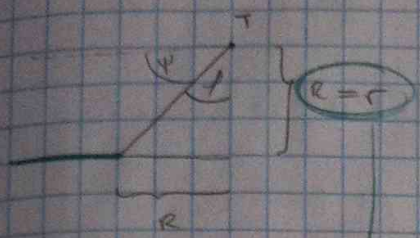
(13)



$R = 1m \qquad I = 1A$

- $\vec{a_H} = \vec{a_x}$
- $\vec{a_H} = \vec{a_x}$
- $\vec{a_H} = \vec{a_z}$

1



$\varphi = -45^\circ \quad \psi = 90^\circ$

↓

MANJI KUT MORA BITI NEGATIVAN

$$\boxed{\vec{H} = \frac{I}{4\pi r}\left(\sin\varphi + \sin\psi\right)\vec{a_H}}$$

ZA RAVNI VODIĆ

$\vec{H}_1 = 0{,}0233\,\vec{a_x}$

2

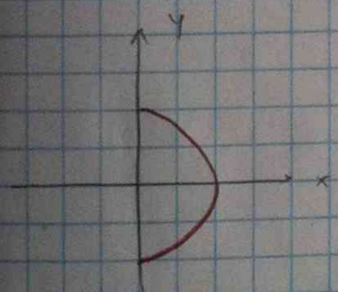
$\vec{H}_2 = \vec{H}_1 = 0{,}0233\,\vec{a_x}$

r – radijus h – udaljenost točke

3

$$\boxed{\vec{H} = \frac{I \cdot r}{4\pi\sqrt{h^2+r^2}^{\,3}} \int \left[h\cos\varphi\,\vec{a_x} + h\sin\varphi\,\vec{a_y} + r\,\vec{a_z}\right] d\varphi}$$

ZA KRUŽNE DIJELOVE



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\ldots]\, d\varphi$$

granice moraju biti pozitivne jer je $\vec{a_H} = \vec{a_z}$ pozitivan

$$\vec{H}_3 = \frac{1 \cdot 1}{4\pi\sqrt{1^2+1^2}^{\,3}}\left[1\sin\varphi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\vec{a_x} - \cancel{1\cos\varphi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\vec{a_y}} + 1\,\varphi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\vec{a_z}\right]$$

$= 0{,}0563\,\vec{a_x} + 0{,}0884\,\vec{a_z}$

$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 = 0{,}104\,\vec{a_x} + 0{,}0884\,\vec{a_z}$

$H_x = 0{,}104$

(14) rješeno je u 13. zadatku $H_y = 0$

(15) rješeno je u 13. zadatku $H_z = 0{,}0884$

(16)

$$\vec{H} = \left(y\left(\cos(ax)\right)\right)\vec{a_x} + \left(y + e^x\right)\vec{a_z} \quad \left[Am^{-1}\right]$$

GUSTOĆA STRUJE

$$\boxed{\vec{J} = \nabla \times \vec{H}}$$

$$\vec{J} = \left(\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right) \times \left[y\cos(ax)\vec{a_x} + \left(y + e^x\right)\vec{a_z}\right] =$$

$$= -\left(e^x\right)\vec{a_y} - \cos(ax)\vec{a_z} + 1\vec{a_x} + 0\vec{a_y} =$$

$$= \vec{a_x} - e^x\vec{a_y} - \cos(ax)\vec{a_z}$$

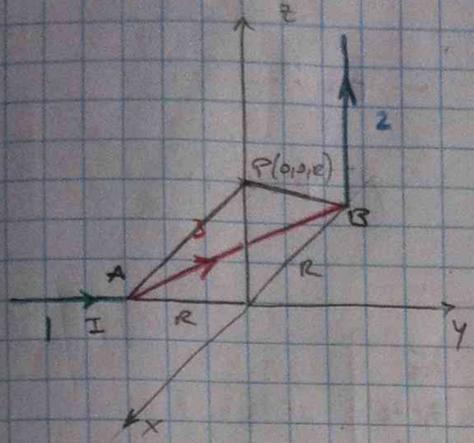
(17) gustoća $\vec{J}$ u yz ravnini $\Rightarrow$ $x = 0$

$$\vec{J}_{ryz} = \vec{a_x} - e^0\vec{a_y} - \cos(a \cdot 0)\vec{a_z} =$$

$$= \vec{a_x} - \vec{a_y} - \vec{a_z}$$

$$|\vec{J}| = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3} = 1{,}73 \ Am^{-2}$$

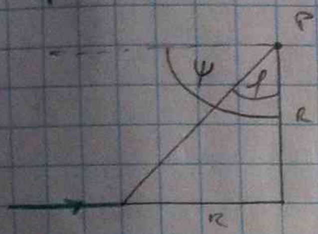
18



- $\vec{a_H} = \vec{a_x}$
- $\vec{a_H} = \vec{a_y}$

$R = 1m$
$I = 10A$

1



$\varphi = -45°$ $\quad \psi = 90°$

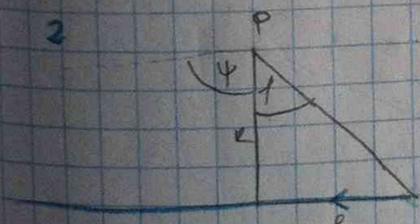
MANJI KUT MORA BITI NEGATIVAN (točka nije iznad vodiča)

$$\boxed{\vec{H} = \frac{I}{4\pi r}(\sin\varphi + \sin\psi)\,\vec{a_H}}$$ ZA RAVNI VODIČ

$r = R$

$\vec{H_1} = 0{,}233\,\vec{a_x}$

2

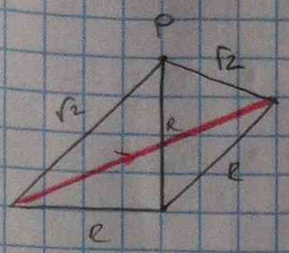


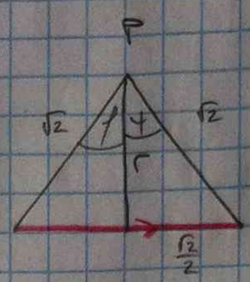
$r = R$

$\psi = 90°$

$\varphi = +45°$ KUT JE POZITIVAN kada točka je iznad vodiča

$\vec{H_2} = 1{,}36\,\vec{a_y}$





$r = \frac{\sqrt{6}}{2}$

$\varphi = \psi = 30°$

(1) $A(0,-1,0)$

(2) $B(-1,0,0)$

(3) $P(0,0,1)$

$$\vec{n} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_2-x_3 & y_2-y_3 & z_2-z_3 \end{vmatrix}$$

$$\vec{n} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{vmatrix} = -1\left(0+\vec{a_z}\right) - 1\left(-\vec{a_x} - \vec{a_y}\right) =$$

$$= \vec{a_x} + \vec{a_y} - \vec{a_z}$$

$$\vec{a_n} = \frac{\vec{a_x} + \vec{a_y} - \vec{a_z}}{\sqrt{3}}$$

$$\vec{H_3} = 0{,}375\,\vec{a_x} + 0{,}375\,\vec{a_y} - 0{,}375\,\vec{a_z}$$

$$\vec{H} = \vec{H_1} + \vec{H_2} + H_3 = 0{,}608\,\vec{a_x} + 1{,}735\,\vec{a_y} - 0{,}375\,\vec{a_z}$$

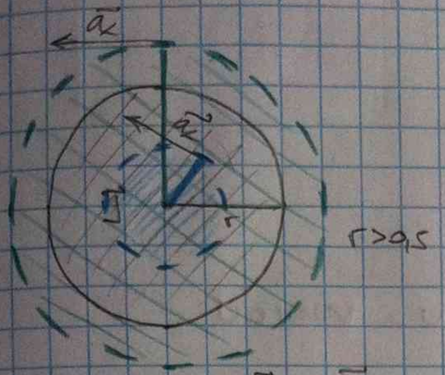
$$H_x = 0{,}608$$

(19) riješeno u 18. zadatku $H_y = 1{,}735$

(20) riješeno u 18. zadatku $H_z = -0{,}375$

(21) $0 < r < 0{,}5\ m$

$\vec{J} = 4{,}5\, e^{-2r}\, \vec{a_z}\ \ Am^{-2}$



ZA ZATVORENU KRIVULJU

$$\boxed{\oint \vec{H}\, d\vec{l} = \sum I = I = \iint_D \vec{J}\vec{n}\, ds}$$

- OBUHVAĆA SAMO DIO GUSTOĆE STRUJE
- OBUHVAĆA CIJELU GUSTOĆU

$$\rightarrow \quad \oint \vec{H}\, d\vec{l} = \vec{H} \cdot \underbrace{2\pi r}_{opseg} = \iint_D 4{,}5\, e^{-2r}\, ds = \int_0^{2\pi} d\varphi \int_0^r r\, 4{,}5\, e^{-2r}\, dr$$

$$= \left[\begin{matrix} r = u & e^{-2r} dr = dv \\ dr = du & -\frac{1}{2} e^{-2r} = v \end{matrix} \right] =$$

$$= 4{,}5 \left[-\frac{r}{2} e^{-2r} \Big|_0^r + \frac{1}{2} \int_0^r e^{-2r}\, dr \right] 2\pi$$

$$\vec{H} \cdot r = \left(-\frac{r}{2} e^{-2r} - \frac{1}{4} e^{-2r} \Big|_0^r \right) 4{,}5$$

$$\vec{H} \cdot r = \left(-\frac{r}{2} e^{-2r} + \frac{1}{4} - \frac{1}{4} e^{-2r} \right) 4{,}5$$

$$\vec{H} = \left(\frac{1}{4r} - \frac{1}{4r} e^{-2r} - \frac{1}{2} e^{-2r} \right) 4{,}5\, \vec{a_\varphi} \qquad 0 < r < 0{,}5\ m$$

$$\longrightarrow \quad \vec{H}\cdot 2\pi r = 4{,}5\left[-\frac{r}{2}e^{-2r}\Big|_0^{0,5} - \frac{1}{4}e^{-2r}\Big|_0^{0,5}\right]\cancel{2\pi}$$

$$\vec{H} = \frac{0{,}297}{r}\,\vec{a_\alpha}$$

22.

$$\vec{H} = 3r\,\vec{a_\alpha}\quad [Am^{-1}]$$

$$\boxed{\vec{J} = \nabla\times\vec{H}}$$ GUSTOĆA STRUJE U VODIĆU

Cilindrični sustav:

$$\vec{J} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z}\right)\vec{a_r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{a_\alpha} + \frac{1}{r}\left(\frac{\partial (rA_\alpha)}{\partial r} - \frac{\partial A_r}{\partial \alpha}\right)\vec{a_z}$$

$$\vec{J} = \frac{1}{r}\left(\frac{\partial(3r^2)}{\partial r}\right)\vec{a_z} = 6\,\vec{a_z}$$

23

$$\vec{H} = k\sin(x)\,\vec{a_y}$$

$$\boxed{\vec{J} = \nabla\times\vec{H}}$$ GUSTOĆA STRUJE U VODIĆU

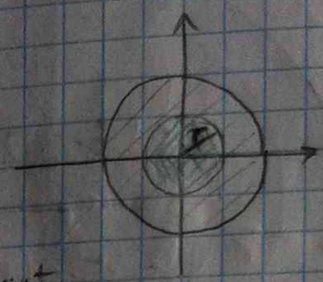
Kartezijev sustav (formule)

$$\vec{J} = k\cos(x)\,\vec{a_z}$$

(24) $\vec{J} = K e^{-ar} \vec{a_z}$, $\vec{H} = ?$ $\vec{n} = \vec{a_z}$

$$\boxed{\oint H d\vec{l} = \Sigma I = I = \iint_S \vec{J}\vec{n}\, dS}$$

$$\vec{H} \cdot 2r\pi = \int_0^{2\pi} d\varphi \int_0^{r} r K e^{-ar}\, dr$$



⋮ (isti integral kao u 25 zad, samo druge granice)

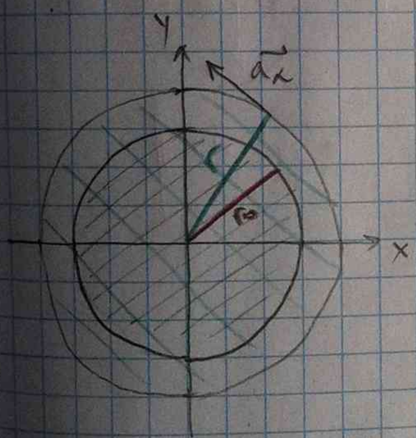
hint

isti r

$$\vec{H} = \frac{K}{a^2 r}\left(1 - (1 + ar) e^{-ar}\right) \vec{a_z}$$

(25)

isti kao 21., za van kružnice



$$\vec{H} \cdot 2r\pi = \int_0^{2\pi} d\varphi \int_0^{r_0} r K e^{-ar}\, dr$$

$$= \begin{bmatrix} r = u & e^{-ar} dr = dv \\ dr = du & -\frac{1}{a} e^{-ar} = v \end{bmatrix}$$

$$\vec{H} \cdot 2r\pi = 2\pi K \left[-\frac{r}{a} e^{-ar} \Big|_0^{r_0} + \frac{1}{a} \int_0^{r_0} e^{-ar} dr \right]$$

$$\vec{H} \cdot r = \left[-\frac{r_0}{a} e^{-ar_0} - \frac{1}{a^2} e^{-ar} \Big|_0^{r_0} \right] K$$

$$\vec{H} = \left(\frac{1}{a^2} - \frac{1}{a^2} e^{-ar_0} - \frac{1}{a} r_0 e^{-ar_0} \right) \frac{K}{r}$$

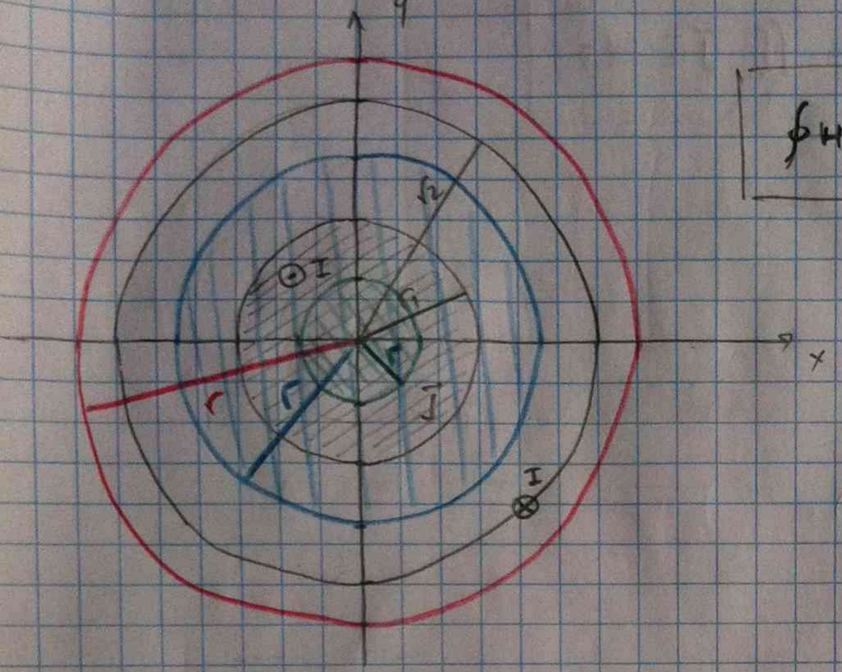
$$\vec{H} = \frac{K}{a^2 r} \left(1 - (1 + ar_0) e^{-ar_0} \right)$$

hint

različiti r-ovi

26, 27, 28

$J = J_0 e^{-(r_1 - r)}$



$$\boxed{\oint H\,d\vec{l} = \Sigma I = I = \iint_S \vec{J}\vec{n}\,ds}$$

$$r < r_1 \Rightarrow \vec{H}\,2\pi r = \int_0^{2\pi} d\varphi \int_0^r r\,J_0\,e^{-(r_1 - r)}\,dr$$

$$\vec{H}\,\cancel{2\pi} r = \cancel{2\pi}\,J_0\,e^{-r_1}\int_0^r r e^r\,dr = \begin{bmatrix} r = u & e^r dr = dv \\ dr = du & e^r = v \end{bmatrix}$$

$$\vec{H} r = J_0 e^{-r_1}\left[r e^r \Big|_0^r - \int_0^r e^r dr \right]$$

$$\vec{H} = \frac{J_0 e^{-r_1}}{r}\left(r e^r - e^r + 1 \right)$$

$$\vec{H} = \frac{J_0 e^{-r_1}}{r}\left(e^r (r-1) + 1 \right)\vec{a}_\varphi$$

Obuhvatili smo samo dio gustoće struje unutrašnjeg vodiča

$r_1 < r < r_2 \Rightarrow \vec{H} 2\pi r = \int_0^{2\pi} d\varphi \int_0^{r_1} r J_0 e^{-(r_1 - r)} dr$

$$\vec{H} = \frac{J_0 e^{-r_1}}{r}\left[r e^{r} \Big|_0^{r_1} - e^{r} \Big|_0^{r_1} \right]$$

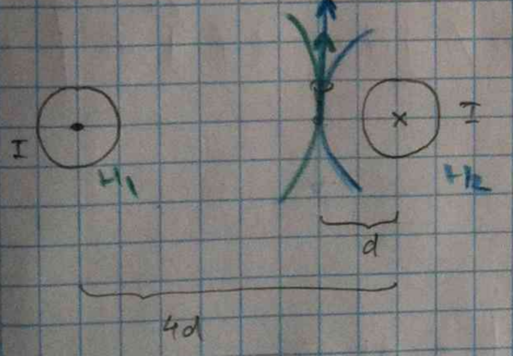
$$\vec{H} = \frac{J_0 e^{-r_1}}{r}\left(r_1 e^{r_1} - e^{r_1} + 1 \right)$$

$$\vec{H} = \frac{J_0}{r}\left(r_1 - 1 + e^{-r_1} \right) \vec{a_z} \qquad (r_1 = a)$$

$r > r_2 \Rightarrow$ poništavaju se struja unutrašnjeg vodiča (r_1) i povratna struja beskonačne tanke ljuske radijusa r_2

$$\vec{H} = 0 \cdot \vec{a_z}$$

(29)



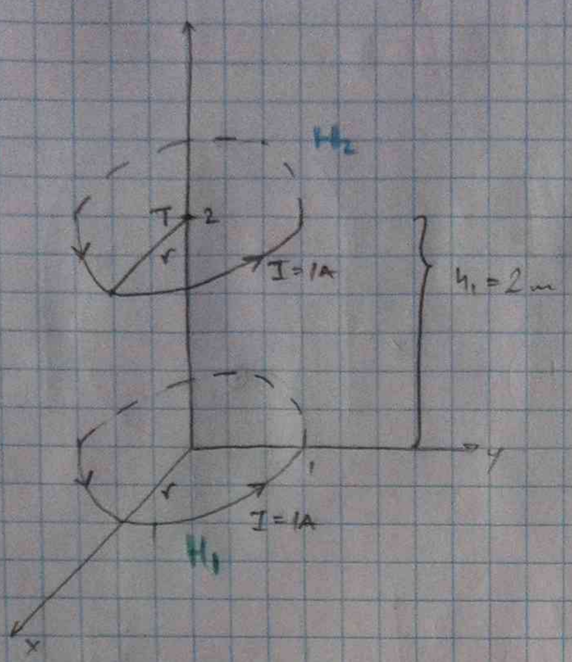
oba vodiča su beskonačno duga, pa su:

$\varphi = 90^\circ \quad \psi = 90^\circ$

$$\boxed{\vec{H} = \frac{I}{4\pi r}\left(\sin\varphi + \sin\psi \right) \vec{a_n}}$$

$$H = H_1 + H_2 = \frac{I}{2\pi 3d} + \frac{I}{2\pi d} = \frac{\cancel{4} I}{\cancel{2}\pi 3d} = \frac{2I}{3\pi d}$$

(30)



$I = 1A$ $T(0,0,2)$
$r = 1m$

$h_2 = 0$

$$\boxed{H = \frac{I \cdot r}{4\pi\sqrt{h^2+r^2}^3} \int \left[h\cos\varphi\, \vec{a_x} + h\sin\varphi\, \vec{a_y} + r_0\, \vec{a_z} \right] d\varphi}$$

ZA KRUŽNI VODIČ

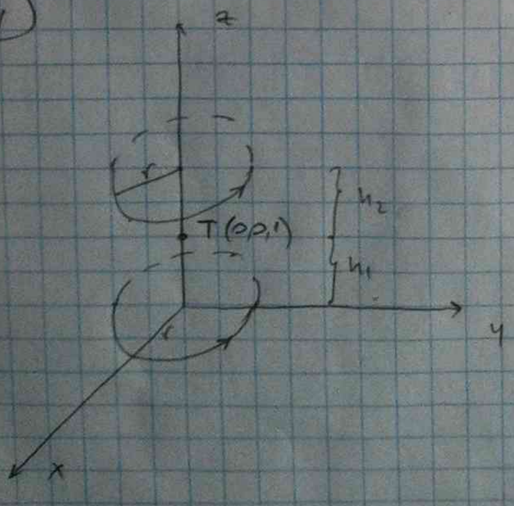
$$\vec{H_1} = \frac{1}{4\pi\sqrt{2^2+1^2}^3} \left[\cancel{2\sin\varphi \Big|_0^{2\pi} \vec{a_x}} - \cancel{2\cos\varphi \Big|_0^{2\pi} \vec{a_y}} + 1\varphi \Big|_0^{2\pi} \vec{a_z} \right] =$$

$$= 0{,}0447\ \vec{a_z}$$

$$\vec{H_2} = \frac{1}{4\pi\sqrt{0^2+1^2}^3} \left[\dots + 1\varphi \Big|_0^{2\pi} \vec{a_z} \right] = 0{,}5\ \vec{a_z}$$

$$\vec{H} = \vec{H_1} + \vec{H_2} = 0{,}5447\ \vec{a_z} \quad [Am^{-1}]$$

(31)



$h_1 = h_2 = 1m$

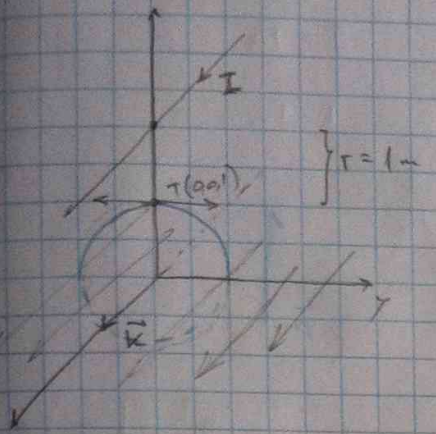
(postupak kao u 30. zadatku)

$$\vec{H_1} = \vec{H_2} = \frac{1}{4\pi\sqrt{1^2+1^2}^3} \left(\dots + 1\varphi \Big|_0^{2\pi} \vec{a_z} \right)$$

$$= 0{,}177\ \vec{a_z}$$

$$\vec{H} = \vec{H_1} + \vec{H_2} = 0{,}354\ \vec{a_z} \quad [Am^{-1}]$$

(37.) $\vec{K} = k\,\vec{a_x} \longrightarrow \vec{n} = \vec{a_z}$



$\vec{a_r}\ \vec{a_\varphi}$

$\vec{a_y} \cdot \vec{a_y} \cdot (-\vec{a_z})$

$$\boxed{\vec{H} = \frac{I}{4\pi r}\left(\sin\varphi + \sin\psi\right)\vec{a_H}}$$

$\vec{a_H} = \vec{a_y}$

$$\vec{H_1} = \frac{I}{2\pi}\,\vec{a_y}$$

$$\oint \vec{H_2}\,ds = \Sigma I = I = \iint_D \vec{K}\,\vec{n}\,ds = \int_0^{2\pi} d\varphi \int_0^1 r\,k\,dr$$

$$\vec{H_2}\,2r\cancel{\pi} = \cancel{\pi}\,k$$

$$\vec{H_2} = \left(\frac{k}{2r}\right)\left(-\vec{a_y}\right)$$

$$|\vec{H_1}| = |\vec{H_2}| \qquad \frac{I}{2\pi} = \frac{k}{2r}$$

$$I = \frac{k\pi}{r} \quad ; \ r = 1\,m$$

$$I = k\pi$$

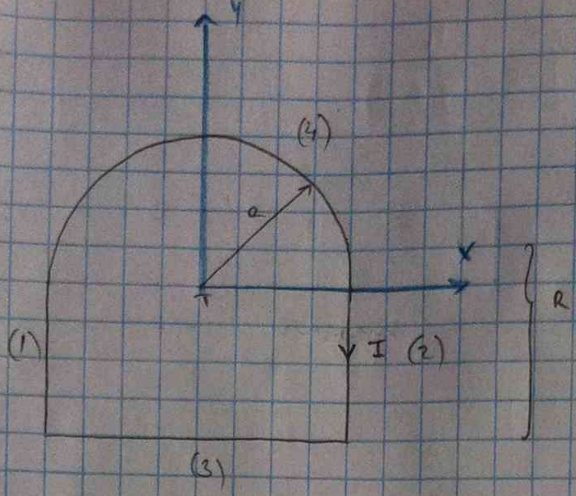
$$\boxed{\vec{B} = \mu_0 \frac{|\vec{K}|}{2}\,\vec{a_H}}$$ izvedena formula za STRUJNI OBLOG

$\vec{a_H}$ se odredi pravilom desne ruke

$$\vec{B_2} = -\mu_0 \frac{k}{2}\,\vec{a_y}$$

$$\vec{H_2} = -\frac{k}{2}\,\vec{a_y}$$ (sada izjednačimo)

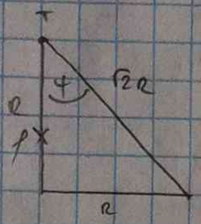
33



$I = 1A$

$R = 1m$

(1)



$$\boxed{\vec{H} = \frac{I}{4\pi r}\left(\sin\varphi + \sin\psi\right)\vec{a_n}}$$

ZA RAVNI VODIČ

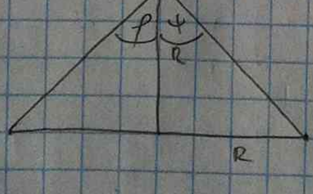
$\vec{a_n} = -\vec{a_z}$ $\quad r = R \quad \varphi = 0^\circ \quad \psi = 45^\circ$

$$\vec{H_1} = -\frac{\sqrt{2}}{8\pi R}\vec{a_z}$$

(2)

$$\vec{H_2} = \vec{H_1} = -\frac{\sqrt{2}}{8\pi R}\vec{a_z}$$

(3)

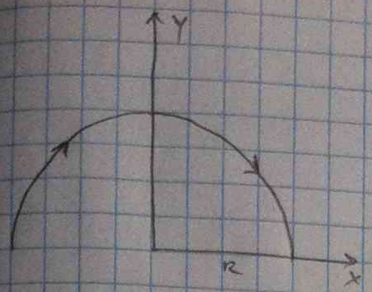


$\varphi = \psi = 45^\circ$

$r = R \quad \vec{a_n} = -\vec{a_z}$

$$\vec{H_3} = -\frac{\sqrt{2}}{4\pi R}\vec{a_z}$$

(4)



$$\boxed{\vec{H} = \frac{I}{4\pi\sqrt{r^2+h^2}^{\,3}} \int [h\cos\varphi\, \vec{a_x} + h\sin\varphi\, \vec{a_y} + r\, \vec{a_z}]\, d\varphi}$$

ZA KRUŽNE VODIĆE

$\vec{a_H} = -\vec{a_z}$, $h = 0$

(granice integracije moraju biti negativne)

$$\vec{H_4} = \frac{I}{4\pi R^{\frac{2}{3}}} \int_{\pi}^{0} [0\vec{a_x} + 0\vec{a_y} + R\vec{a_z}] = - \frac{I}{4R^{\frac{1}{3}}} \vec{a_z}$$

$$\vec{H} = \vec{H_1} + \vec{H_2} + \vec{H_3} + \vec{H_4} = -0{,}4752\, \vec{a_z}$$

(34)

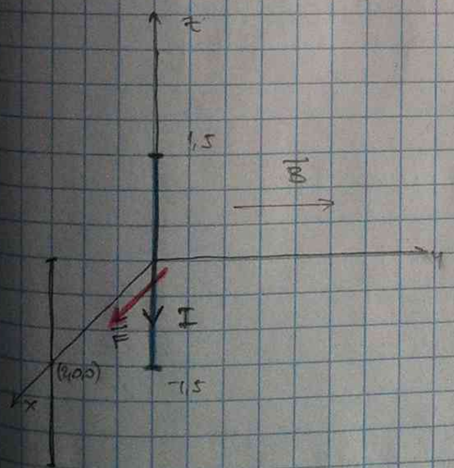
$-1{,}5 \le z \le 1{,}5$ [m]

$I = 10\,A$

$\vec{B} = 3 \cdot 10^{-4} e^{-0{,}2x}\, \vec{a_y}$ [T]

PRAVILO LIJEVE RUKE:

- → silnice ulaze u dlan
- → prsti pokazuju smjer struje
- ⇒ palac – smjer sile



SILA

$$\boxed{\vec{F} = I \int d\vec{l} \times \vec{B}}$$

$d\vec{l}$ određujemo u smjeru struje

ENERGIJA

$$\boxed{W = \int_{x_1}^{x_2} F\, d_x}$$

$$\vec{F} = 10 \int_{-1,5}^{1,5} (-\vec{a_z} \times \vec{a_y})\, 3 \cdot 10^{-4} e^{-0{,}2x}\, dz =$$

$$= 3 \cdot 10^{-3} e^{-0{,}2x} \cdot (1{,}5 + 1{,}5)\, \vec{a_x} =$$

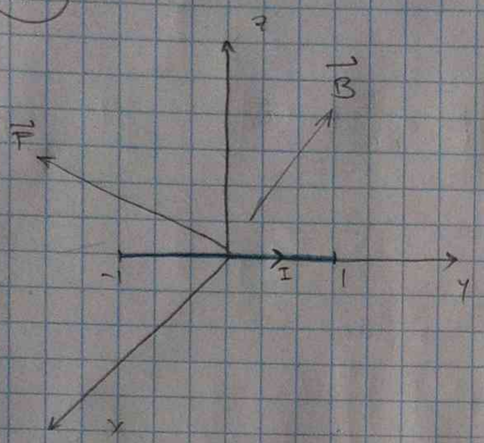
$$= 9 \cdot 10^{-3} e^{-0{,}2x}\, \vec{a_x}$$

$$W = \int_0^2 F\,dx = 9\cdot10^{-3}(-5)\,e^{-\frac{1}{5}x}\Big|_0^2 =$$

$$= -45\cdot10^{-3}\left[e^{-\frac{2}{5}} - 1\right] =$$

$$= 0{,}0148 \text{ J}$$

35



$I = 5A$

$\vec{F} = 1{,}0607\,(\vec{a_x} + \vec{a_z})$

$\vec{dl} = \vec{a_y}$

$$\boxed{\vec{F} = I\int \vec{dl}\times\vec{B}}$$

$$1{,}0607(\vec{a_x} + \vec{a_z}) = 5\int_{-1}^{1} \vec{a_y} \times \left(B_x\vec{a_x} + B_y\vec{a_y} + B_z\vec{a_z}\right)$$

$$1{,}0607(\vec{a_x} + \vec{a_z}) = 5\int_{-1}^{1}\left[-\vec{a_z}B_x + \vec{a_x}B_z\right]$$

$$1{,}0607(\vec{a_x} + \vec{a_z}) = -10B_x\vec{a_z} + 10B_z\vec{a_x}$$

$10B_z = 1{,}0607 \qquad -10B_x = 1{,}0607$

$B_z = 0{,}106 \qquad B_x = -0{,}106$

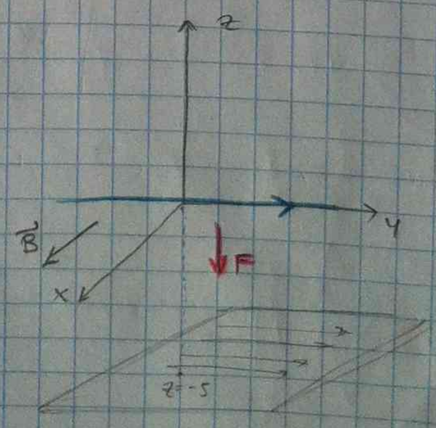
$$\vec{B} = 0{,}106\left(-\vec{a_x} + \vec{a_z}\right)$$

36

$\vec{K} = 30\,\vec{a_y} \quad [A\,m^{-1}]$

$z = -5 \quad [m]$

$I = 5A \qquad \vec{dl} = \vec{a_y}$



$$\boxed{\vec{B} = \frac{|\vec{K}|}{2}\mu_0 \vec{a}_H}$$

IZVEDENA FORMULA ZA MAGNETSKU INDUKCIJU

$\vec{a}_H = \vec{a}_x$ $\qquad \vec{B} = 15\mu_0 \vec{a}_x$

SILA

$$\boxed{\vec{F} = I\int d\vec{l} \times \vec{B}}$$

$\Longrightarrow$ traži se po jedinici duljine

$$F = I \cdot l \cdot B \quad / : l$$

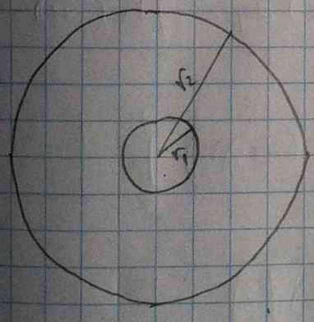
$(\mu_0 = 4\pi 10^{-7})$

$$F = 9{,}42 \cdot 10^{-5}\ N$$

$$\vec{F} = -9{,}42 \cdot 10^{-5}\ \vec{a}_z\ [N]$$

37 $\quad r_1 = 2mm$

$r_2 = 9mm$ $\qquad \mu_0\mu_r = 1$

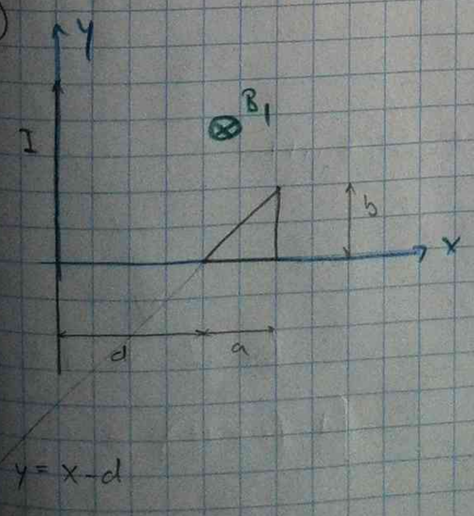


$$L_v = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{r_2}{r_1}\right) = 300{,}8\ nH/m$$

izvedena formula iz Barbarovića...

?! :/

38



$d = 2m \quad a = b = 1m$

MEĐUINDUKTIVITET

$$\boxed{M = \frac{\Phi_{12}}{I_1} = \frac{\Phi_{21}}{I_2}}$$

MAGNETSKI TOK

$$\boxed{\Phi_{12} = \iint_{S_2} \vec{B}_1 \vec{n}\, ds}$$

$\vec{n}$ je UVIJEK u smjeru $\vec{B}_1$ polja

MAGNETSKA INDUKCIJA

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

udaljenost

$$\Phi_{12} = \iint_{2} B_1 ds = \iint_{S_2} \frac{\mu_0 I_1}{2\pi x} ds$$

zamijenili smo s x, jer udaljenost ovisi jedino o x

$$\Phi_{12} = \frac{\mu_0 I_1}{2\pi} \left[\int_0^b dy \int_{y+d}^{d+a} \frac{1}{x} dx \right] =$$

$$= \frac{\mu_0 I_1}{2\pi} \int_0^1 \ln\left(\frac{3}{y+2}\right) dy = \frac{\mu_0 I}{2\pi} \left[\int_0^1 \ln(3)\, dy - \int_0^1 \ln(y+2)\, dy \right] =$$

$$= \frac{\mu_0 I_1}{2\pi} \left[\ln(3) - \left(y \ln(y+2) - y + \frac{2}{1} \ln(y+2) \right) \Big|_0^1 \right] =$$

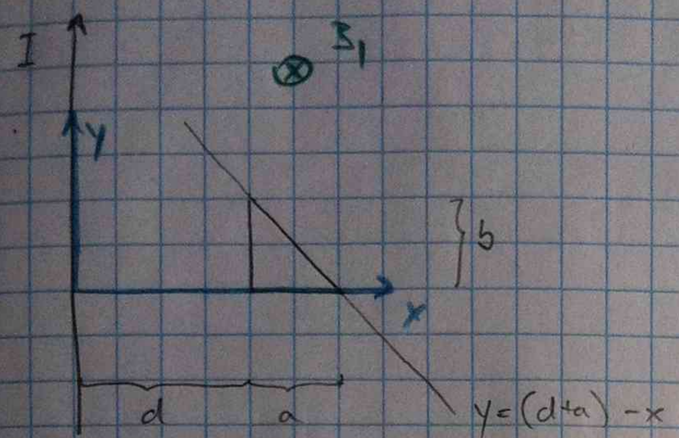
$$= \frac{\mu_0 I_1}{2\pi} \left(\ln(3) - \left[\ln(3) - 1 + 2\ln(3) - 2\ln(2) \right] \right)$$

$$M = \frac{\Phi_{12}}{I_1} = \frac{\mu_0}{2\pi} \left(\ln(3) - 1 \right) = 37{,}8\ nH$$

formula za integral

$$\int \ln(ax+b) = \dots$$

(39)



(postupak kao u 38 zadatku)

$$B = \frac{\mu_0 I}{2\pi x}$$

d=2m a=b=1m

$$\Phi_{12} = \iint_{S_2} \frac{\mu_0 I_1}{2\pi x} dS$$

$$M = \frac{\Phi_{12}}{I_1} = \frac{\mu_0}{2\pi} \iint_{S_2} \frac{1}{x} dS = \frac{\mu_0}{2\pi} \int_0^b dy \int_d^{(d+a)-y} \frac{1}{x} dx =$$

$$= \frac{\mu_0}{2\pi} \int_0^1 dy \ln\left(\frac{3-y}{2}\right) =$$

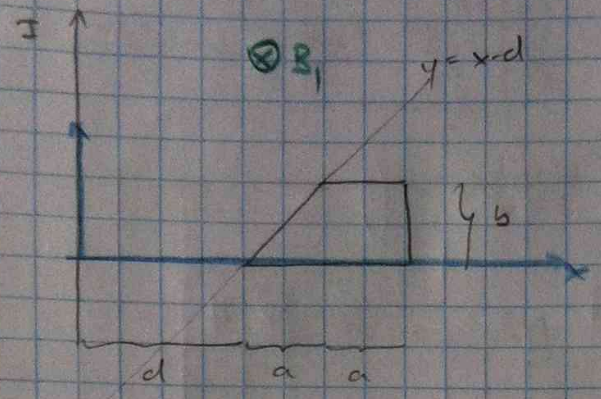
$$= \frac{\mu_0}{2\pi} \left[\int_0^1 \ln(-y+3)\, dy - \int_0^1 \ln(2) \right] =$$

$$= \frac{\mu_0}{2\pi} \left[\left(y \ln(-y+3) - y - 3\ln(-y+3) \right) \Big|_0^1 - \ln(2) \right] =$$

$$= \frac{\mu_0}{2\pi} \left(\cancel{\ln(2)} - 1 - 3\ln(2) + 3\ln(3) - \cancel{\ln(2)} \right) =$$

$$= 43{,}3\ nH$$

(40)



$d = 2m \quad a = b = 1m$

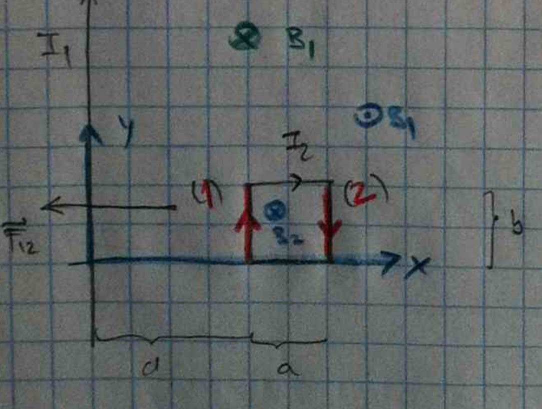
(postupak kao u prethodna 2 zadatka)

$$M = \frac{\mu_0}{2\pi} \iint_{S_2} \frac{1}{x} dS = \frac{\mu_0}{2\pi} \left[\underbrace{\int_0^1 dy \int_{y+2}^{3} \frac{1}{x} dx}_{\text{isto kao u 38.}} + \int_0^1 dy \int_3^4 \frac{1}{x} dx \right] =$$

$$= \frac{\mu_0}{2\pi} \left[0{,}189 + \ln\left(\frac{4}{3}\right) \right] =$$

$$= 95{,}34 \, nH$$

(41.)



SILA SILNICE NA PETLJU

$$\boxed{\vec{F}_{12} = I_2 \int_C d\vec{l} \times \vec{B}_1}$$

$$d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y + dz\,\vec{a}_z$$

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi r} \vec{a}_z$$

$$\vec{dl} \times \vec{B} = -\frac{\mu_0 I_1}{2\pi r}(\vec{a_y})dx - \frac{\mu_0 I_1}{2\pi r}\vec{a_x}dy =$$

$$= -\frac{\mu_0 I_1}{2\pi r}\left(\vec{a_x} - \vec{a_y}\right)$$

$$\vec{F}_{12} = \frac{\mu_0 I_1 I_2}{2\pi}\left[\underbrace{\int_c \frac{1}{x}dx\,\vec{a_y}}_{=\emptyset} - \int_c \frac{1}{x}dy\,\vec{a_x}\right]$$

komponente okomite na strujnica su $\emptyset$

$$\vec{F}_{12} = -\frac{\mu_0 I_1 I_2}{2\pi}\left[\underbrace{\int_0^1 \frac{dy}{2}}_{(1)} + \underbrace{\int_1^0 \frac{dy}{3}}_{(2)}\right]\vec{a_x}$$

→ granice integrala su u smjeru struje

→ umjesto x uvrštavamo njegovu vrijednost

$$\vec{F}_{12} = -\frac{\mu_0 I_1 I_2}{2\pi}\left[\frac{1}{2} - \frac{1}{3}\right]\vec{a_x}$$

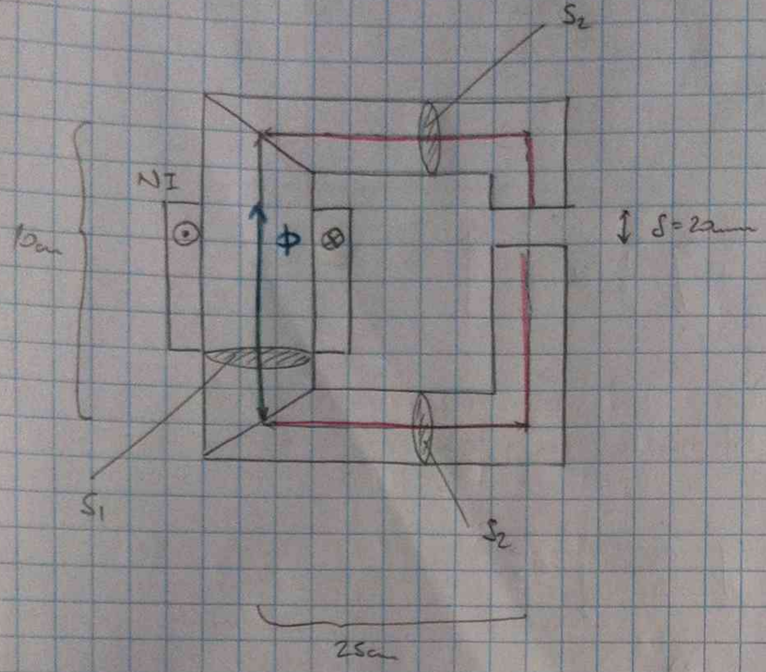
$\vec{F}_{12} = -33,33\,nN\ \vec{a_x}$ $\qquad |F| = 33,33\,nN$

Sila djeluje u smjeru $-\vec{a_x}$, što znači da silnica privlači petlju. To je očigledno jer je dio petlje s strujom u istom smjeru kao silnica bliže nego li dio petlje gdje su struje suprotne.

- Isti smjer struje ⇒ PRIVLAČE SE
- Suprotan smjer struje ⇒ ODBIJAJU SE

Stoga se jače privlače nego li odbijaju pa je smjer uistinu $-\vec{a_x}$.

42, 43, 44, 45

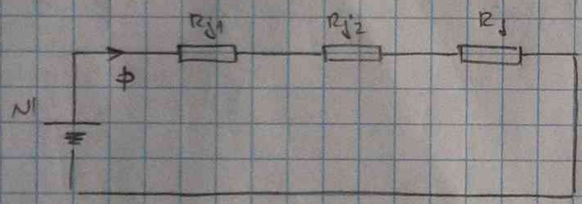


$S_1 = 100\ cm^2$
$S_2 = 60\ cm^2$

$l_1 = 10cm$
$l_2 = 58cm$

$B_\delta = 1T$

$I = 10A$



① postavimo jednadžbu $\boxed{NI = H_i l_i + H_\delta \cdot \delta}$

② Φ = konstantno :

$\Phi = BS \Rightarrow B_1 S_1 = B_2 S_2$ $\quad B_2 = B_\delta$

$$B_1 = \frac{B_\delta S_2}{S_1} = 0{,}6\ T$$

③ $NI = H_1 l_{j1} + H_2 l_{j2} + H_\delta \cdot \delta$

H_1 i H_2 očitamo iz grafa jer μ_r za željezo se mijenja !

$\rightarrow H_1 = 95\ A/m$
$\quad H_2 = 200\ A/m$

H_δ izračunamo preko $B = \mu_0 \mu_r H$ jer je $\mu_r = 1$ (zrak)

$$H_\delta = \frac{B_\delta}{\mu_0 \mu_r}$$

42.) $$N = \frac{H_1 l_{j1} + H_2 l_{j2} + H_\delta \cdot \delta}{I} = 1605$$

ENERGIJA POHRANJENA U PROSTORU

$$\boxed{W_\delta = \frac{1}{2} B_\delta \cdot H_\delta \cdot S \cdot \delta}$$

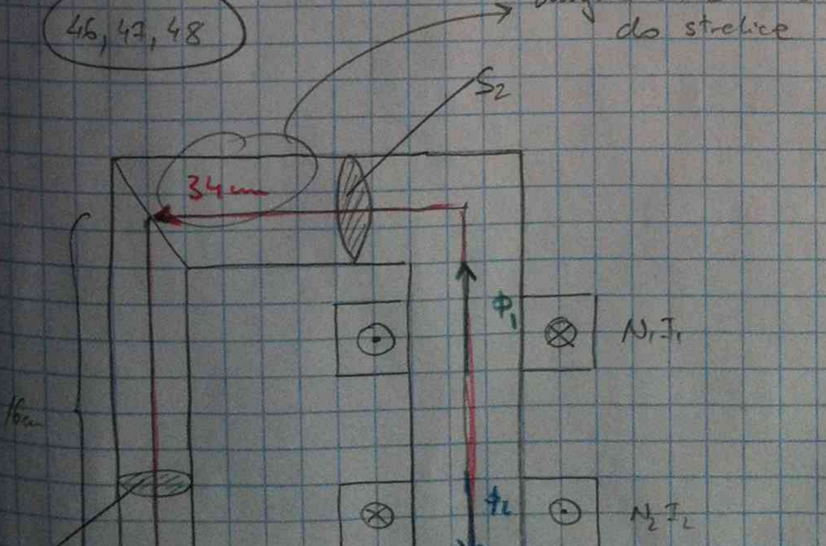
43.) $W = 47{,}77\ J$

44.) $B_2 = \mu_0 \mu_{r_2} H_2 \implies \mu_{r_2} = \frac{B_2}{\mu_0 H_2} = 3980$

45.)

$$W_2 = \frac{1}{2} B_2 H_2 \cdot S_2 \cdot (0{,}25) = 0{,}15\ J$$

46, 47, 48



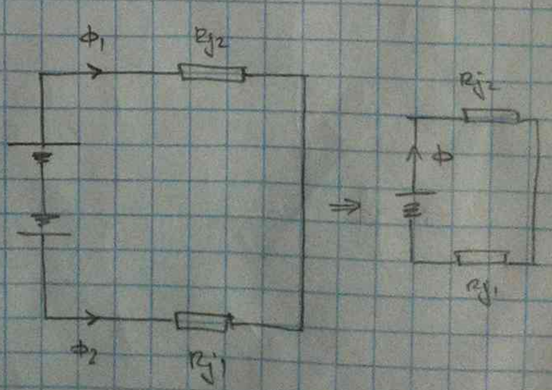
$S_1 = 4 cm^2$
$S_2 = 6 cm^2$

$l_{j_1} = 16 cm$
$l_{j_2} = 34 cm$

$I_2 = 0{,}5 A$

$N_1 = 200$
$N_2 = 100$

$\Phi = 120 \mu Wb$



$$N_1 I_1 - N_2 I_2 = H_1 l_{j1} + H_2 l_{j2}$$

Φ = konstantno $\Rightarrow$ $\Phi = B_1 S_1$

$B_1 = 0{,}3\,T$

$\Phi = B_2 S_2$

$B_2 = 0{,}2\,T$

$\Rightarrow$ iz tablice očitamo:

$H_1 = 185$ A/m

$H_2 = 145$ A/m

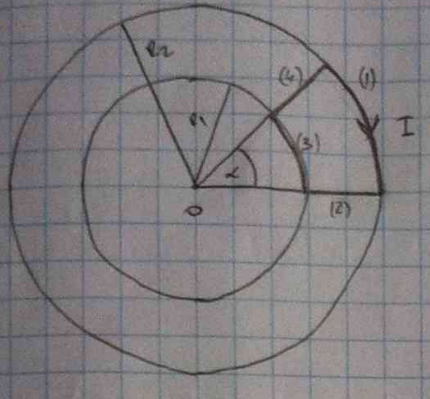
46.) $$I_1 = \frac{H_1 l_{j1} + H_2 l_{j2} + N_2 I_2}{N_1} = 0{,}6445\ A$$

47.) $$\boxed{W_2 = \frac{1}{2} B_2 \cdot H_2 \cdot S_2 \cdot l_2}$$ $l_2 = 34$ cm

$W_2 = 3$ mJ

48.) $W_1 = 1{,}8$ mJ $\quad (l_1 = 16$ cm$)$

49.



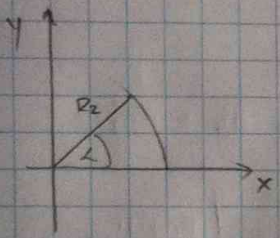
$$\boxed{\vec{H} = \frac{I \cdot r}{4\pi \sqrt{r^2+h^2}^3} \int [h\cos\varphi \vec{a_x} + h\sin\varphi \vec{a_y} + r\vec{a_z}] \, d\varphi}$$

ZA KRUŽNI VODIĆ

$$\boxed{\vec{H} = \frac{I}{4\pi r} (\sin\varphi + \cos\varphi) \vec{a_H}}$$

ZA RAVNI VODIĆ

1) $\vec{a_H} = -\vec{a_z}$



$h = 0$
$r = R_2$

$$\vec{H_1} = \frac{I \cdot R_2}{4\pi R_2^3} \int_{\alpha}^{0} R_2 \vec{a_z} = -\frac{I}{4\pi R_2} \alpha \vec{a_z}$$

granice daju negativnu vrijednost zbog $\vec{a_H} = -\vec{a_z}$

2) i 4)

$\vec{H_2} = \vec{H_4} = \emptyset$ jer vodiči "prolaze" kroz točku i ne tvore mag. polje u njoj ($\varphi = \psi = 0°$, $r = 0$)

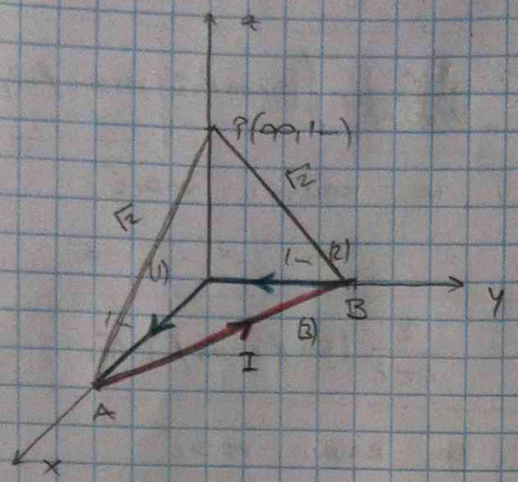
3)

je kao 1) uz $r = R_1$ $\vec{a_H} = \vec{a_z}$

$$\vec{H_3} = \frac{I}{4\pi R_1} \alpha \vec{a_z}$$

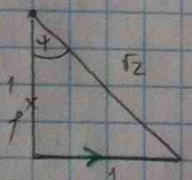
$$\vec{H} = \vec{H_1} + \vec{H_2} + \vec{H_3} + \vec{H_4} = \frac{I}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\vec{a_z}$$

(50)



$I = 1A$

(1)

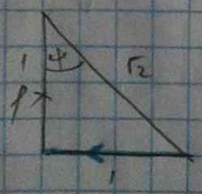


$\varphi = 0^\circ \quad \psi = 45^\circ \quad , \; r = 1m \quad , \; \vec{a}_H = -\vec{a}_y$

$$\boxed{\vec{H} = \frac{I}{4\pi r}\left(\sin\varphi + \sin\psi\right)\vec{a}_H}$$ RAVNI VODIČ

$\vec{H}_1 = -0{,}0563\,\vec{a}_y$

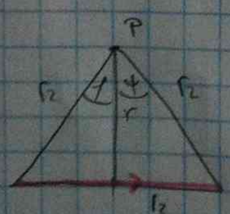
(2)



$\varphi = 0^\circ \quad \psi = 45^\circ \quad , \; r = 1m \, , \; \vec{a}_H = -\vec{a}_x$

$\vec{H}_2 = -0{,}0563\,\vec{a}_x$

(3)



$\varphi = \psi = 30^\circ$

$r = \sqrt{(\sqrt{2})^2 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{6}}{2}$

(1) $A(1,0,0)$

(2) $B(0,1,0)$

(3) $D(0,0,1)$

$$\vec{n}_1 = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_2-x_3 & y_2-y_3 & z_2-z_3 \end{vmatrix}$$

$$\vec{n} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -1\left(0-\vec{a_z}\right)-1\left(-\vec{a_x}-\vec{a_y}\right)$$

$$= \vec{a_x} + \vec{a_y} + \vec{a_z}$$

$$\vec{a_n} = \frac{\vec{a_x} + \vec{a_y} + \vec{a_z}}{\sqrt{3}}$$

$$\vec{H}_3 = \frac{I}{4\pi r}\left(\sin\beta + \sin\psi\right)\vec{a_n} =$$

$$= 0{,}0375\left(\vec{a_x} + \vec{a_y} + \vec{a_z}\right)$$

$$\vec{H} = \vec{H_1} + \vec{H_2} + \vec{H_3} = -18{,}77 \cdot 10^{-3}\,\vec{a_x} - 18{,}77 \cdot 10^{-3}\,a_y + 0{,}0375\,\vec{a_z}$$

$$B_x = \underbrace{-18{,}77 \cdot 10^{-3}}_{H_x}\,\mu_0 = -23{,}57\,nT$$

(51)

rješeno u 50. zadatku

$$B_z = 0{,}0375\,\mu_0 = 47{,}1\,nT$$

52

VEKTORSKI MAGNETSKI POTENCIJAL

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_c \frac{\vec{dl}}{|\vec{r}-\vec{r}'|}}$$

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

udaljenost točke u kojoj tražimo potencijal

udaljenost petlje

$\vec{r} = 1 \cdot \vec{a_z}$

$\vec{r}' = x\,\vec{a_x} + y\,\vec{a_y}$ (jer se mijenja ovisno o x i y)

$$|\vec{r}-\vec{r}'| = \sqrt{1+x^2+y^2}$$

$$A(\vec{r}) = \frac{\mu_0}{4\pi}\left[\int_c \frac{dx}{\sqrt{1+x^2+y^2}}\,\vec{a_x} + \int_c \frac{dy}{\sqrt{1+x^2+y^2}}\,\vec{a_y} + \int \frac{dz}{\sqrt{1+x^2+y^2}}\,\vec{a_z}\right]$$

trenutno se traži samo za $\vec{a_x}$:

hint

INTEGRIRAMO SAMO DIO GDJE SE X MIJENJA, A ZA Y UVRSTIMO VRIJEDNOST

$$\frac{\mu_0}{4\pi}\left[\int_0^1 \frac{dx}{\sqrt{1+x^2+0}} + \int_1^0 \frac{dx}{\sqrt{1+x^2+(1-x)^2}}\right]\vec{a_x} =$$

$$= \frac{\mu_0}{4\pi}\left[\int_0^1 \frac{dx}{\sqrt{1+x^2}} + \int_1^0 \frac{dx}{\sqrt{2-2x+2x^2}}\right]\vec{a_x} =$$

$$= \frac{\mu_0}{4\pi}\left[\ln|x+\sqrt{x^2+1}|\,\Big|_0^1 + \frac{1}{\sqrt{2}}\int_1^0 \frac{dx}{\sqrt{x^2-x+1}}\right] =$$

$$= \frac{\mu_0}{4\pi}\left[\ln(\sqrt{2}+1) + \frac{1}{\sqrt{2}}\left(\ln\left[\frac{2x-1}{1} + 2\sqrt{x^2-x+1}\right]\right)\Big|_1^0\right]$$

$$= \frac{\mu_0}{4\pi}\left[\ln(\sqrt{2}+1) - \frac{1}{\sqrt{2}}\ln(1+2)\right] =$$

$$= 10{,}45\ \mathrm{nTm}$$

(53) $A(\vec{r}) = \frac{\mu_0}{4\pi}\underbrace{\left[\dots \int \frac{dz}{\sqrt{1+x^2+y^2}}\,\vec{a_z} \right]}_{\emptyset}$

jer je z konstantno

MAGNETSKA INDUKCIJA

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

(54)

$$\vec{A} = r^2\,\vec{a_r} + 2\sin(\vartheta)\,\vec{a_\vartheta} \quad [Tm]$$

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{r\sin(\vartheta)}\left(\frac{A_\alpha \sin\vartheta}{\partial \vartheta} - \frac{\partial A_\vartheta}{\partial \alpha}\right)\vec{a_z} +$$

$$\frac{1}{r}\left(\frac{1}{\sin(\vartheta)}\frac{\partial A_r}{\partial \alpha} - \frac{\partial (r A_\alpha)}{\partial r}\right)\vec{a_\vartheta} +$$

$$\frac{1}{r}\left(\frac{\partial (r A_\vartheta)}{\partial r} - \frac{\partial A_r}{\partial \vartheta}\right)\vec{a_\alpha} =$$

$$= 0\,\vec{a_z} + 0\,\vec{a_\vartheta} + \left(\frac{2\sin(\vartheta)}{r} - 0\right)\vec{a_\alpha} =$$

$$= \frac{2\sin(\vartheta)}{r}\,\vec{a_\alpha}$$

$$T\left(r = 1m,\ \vartheta = \frac{\pi}{6},\ \alpha = \frac{\pi}{2}\right)$$

$$|\vec{B}|_T = \frac{2\sin\left(\frac{\pi}{6}\right)}{1} = 1$$

(55) $\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{2\sin(\vartheta)}{r\mu_0}\,\vec{a_\alpha}$

GUSTOĆA STRUJE

$$\boxed{\vec{J} = \nabla \times \vec{H}}$$

$$\vec{J} = \frac{1}{r\sin\vartheta}\left(\frac{\partial(A_\varphi \sin(\vartheta))}{\partial\vartheta} - \frac{\partial A_\vartheta}{\partial\varphi}\right)\vec{a_r} +$$

$$\frac{1}{r}\left(\frac{1}{\sin\vartheta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial(rA_\varphi)}{\partial r}\right)\vec{a_\vartheta} +$$

$$\frac{1}{r}\left(\frac{\partial(rA_\vartheta)}{\partial r} - \frac{\partial A_r}{\partial\vartheta}\right)\vec{a_\varphi}$$

$$\vec{J} = \frac{1}{r\sin\vartheta}\left(\frac{\partial}{\partial\vartheta}\left(\frac{2\sin^2(\vartheta)}{r\mu_0}\right) - 0\right)\vec{a_r} +$$

$$\frac{1}{r}\left(0 - \frac{\partial}{\partial r}\left(\frac{2\sin\vartheta}{\mu_0}\right)\right)\vec{a_\vartheta} +$$

$$0\,\vec{a_\varphi} =$$

$$= \frac{1}{r\cancel{\sin(\vartheta)}} \cdot \frac{2}{r\mu_0}\,2\cancel{\sin(\vartheta)}\cos(\vartheta)\,\vec{a_r}$$

$$= \frac{4\cos(\vartheta)}{r^2\mu_0}\,\vec{a_r}$$

$$T(r = 1m,\ \vartheta = 0,\ \varphi = 0)$$

$$|\vec{J}| = \frac{4\cdot\cos(0)}{1^2\cdot\mu_0} = 3184{,}713\ kA/m^2$$

56.

GUSTOĆA ENERGIJE

$$\boxed{w = \frac{1}{2}\iiint_V \vec{J}\cdot\vec{A}\,dV}$$

$$T(r = 1m,\ \vartheta = 0,\ \varphi = \frac{\pi}{2})$$

$\emptyset$?! (jer je magnetski tok $\emptyset$?)

(57.)

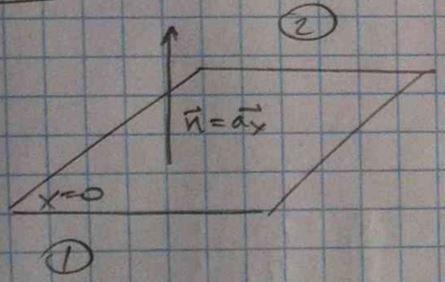
$\nabla \cdot \vec{H} = ?$ $\quad T(r=1m,\ \vartheta=0,\ \varphi=0)$

$$\nabla\cdot\vec{H} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\vartheta}\frac{\partial(A_\vartheta \sin\vartheta)}{\partial\vartheta} + \frac{1}{r\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}$$

(formula sa šalabahtera => Diergence $\nabla\cdot A$)

$\nabla\cdot\vec{H} = 0$

(58, 59, 60)



$\mu_{r1} = 2$
$\mu_{r2} = 4$

$B_1 = 0{,}5\,\vec{a_x} + \vec{a_y} + \vec{a_z}$ [T]

$\vec{K} = \frac{-0{,}5}{\mu_0}\vec{a_y} + \frac{0{,}5}{\mu_0}\vec{a_z}$ [A/m]

$\vec{B} = \mu_0\mu_r\vec{H}$

UVJETI NA GRANICI

$$\boxed{\begin{aligned}&\vec{n}\,(\vec{B}_2-\vec{B}_1) = 0\\ &\vec{n}\times(\vec{H}_2-\vec{H}_1) = \vec{K}\end{aligned}}$$

$\vec{n}\,(\vec{B}_2 - \vec{B}_1) = 0$

$\vec{a_x}\left((B_{2x}-0{,}5)\vec{a_x} + (B_{2y}-1)\vec{a_y} + (B_{2z}-1)\vec{a_z}\right) = 0$

$B_{2x} - 0{,}5 = 0$

$B_{2x} = 0{,}5$

$\vec{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{K}$

$\vec{a_x}\times\left(\left(H_{2x}-\frac{0{,}5}{\mu_0\mu_{r1}}\right)\vec{a_x} + \left(H_{2y}-\frac{1}{\mu_0\mu_{r1}}\right)\vec{a_y} + \left(H_{2z}-\frac{1}{\mu_0\mu_{r1}}\right)\vec{a_z}\right) = \vec{K}$

$\left(H_{2y}-\frac{1}{\mu_0\mu_{r1}}\right)\vec{a_z} - \left(H_{2z}-\frac{1}{\mu_0\mu_{r1}}\right)\vec{a_y} = -\frac{0{,}5}{\mu_0}\vec{a_y} + \frac{0{,}5}{\mu_0}\vec{a_z}$

$$H_{2y} = \frac{0,5}{\mu_0} + \frac{1}{\mu_0 \mu_{r_1}} = \frac{\cancel{2}}{\cancel{2}\mu_0} = \frac{1}{\mu_0}$$

$$B_{2y} = \cancel{\mu_0}\mu_{r_2} H_{2y} = 4$$

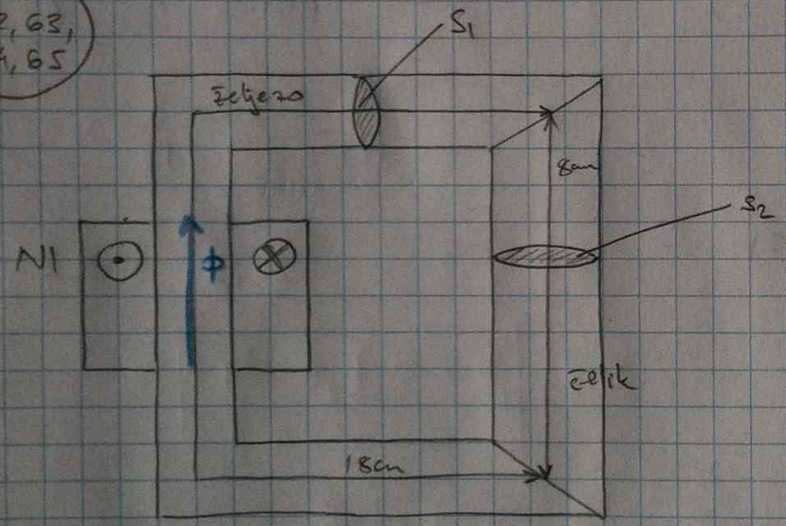
$$H_{2z} = \frac{0,5}{\mu_0} + \frac{1}{\mu_0 \mu_{r_1}} \qquad B_{2z} = 4$$

$$\vec{B}_2 = 0,5\,\vec{a}_x + 4\,\vec{a}_y + 4\,\vec{a}_z$$

61

$$|\vec{B}_2| = \sqrt{0,5^2 + 4^2 + 4^2} = 5,68$$

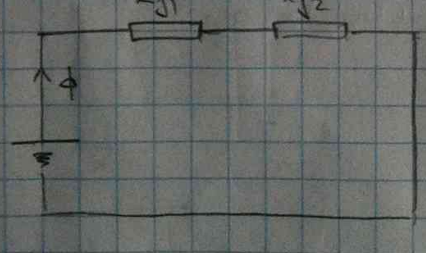
62, 63, 64, 65



$S_1 = 10\,cm^2$
$S_2 = 12\,cm^2$

$B_2 = 1\,T$
$N = 100$

$l_{j_1} = 18\,cm$
$l_{j_2} = 8\,cm$



$$\boxed{\begin{aligned} &\phi = \text{konstantno} = BS \\ &NI = H_1 l_{j_1} + H_2 l_{j_2} \end{aligned}}$$

$B_1 S_1 = B_2 S_2$

62.) $B_1 = 1,2\,T$

očitano iz grafa H_1 za B_1 (željezo), te H_2 za B_2 (čelik)

64.) $H_1 = 400\ A/m$

$H_2 = 100\ A/m$

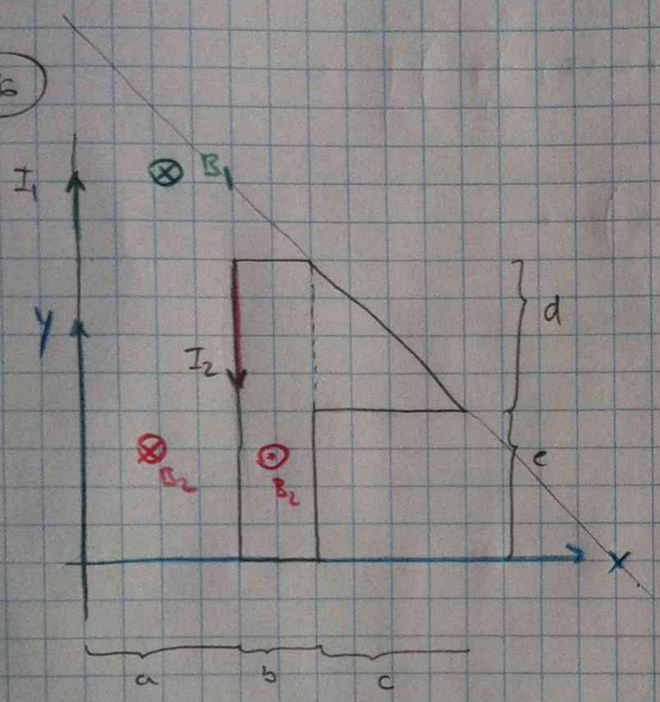
65.) $I = \frac{H_1 l_{j1} + H_2 l_{j2}}{N} = 0{,}8\,A$

63.) MAGNETSKA ENERGIJA

$$\boxed{W = \frac{1}{2} B H S l}$$

$$W_2 = \frac{1}{2} B_2 H_2 S_2 l_{j2} = 4{,}8\,mJ$$

66



$I_1 = 1A$ $\quad a = c = d = e = 10\,cm$

$I_2 = 2A$ $\quad b = 5\,cm$

$y = (e+d) - (x - (a+b)) = e+d+a+b - x$

MEĐUINDUKTIVITET

$$\boxed{M = \frac{\Phi_{12}}{I_1} = \frac{\Phi_{21}}{I_2}}$$

MAGNETSKI TOK

$$\boxed{\Phi_{12} = \iint_{S_2} \vec{B}_1 \vec{n}\, ds}$$

MAG. INDUKCIJA

$$\boxed{B_1 = \frac{\mu_0 I_1}{2\pi r}}$$

$$B_1 = \frac{\mu_0 I_1}{2\pi x}$$

udaljenost ovisi samo o x

$$\Phi_{12} = \frac{\mu_0 I_1}{2\pi} \iint_{S_2} \frac{1}{x}\, ds$$

$$M = \frac{\mu_0}{2\pi} \iint_{S_2} \frac{1}{x}\, ds$$

$$\iint_{S_2} \frac{1}{x}\, ds = \int_0^{d+e} dy \int_a^{a+b} \frac{1}{x}\, dx + \int_e^{d+e} dy \int_{a+b}^{e+d+a+b-y} \frac{1}{x}\, dx =$$

$$= \int_0^{0,2} dy \int_{0,1}^{0,15} \frac{1}{x}\, dx + \int_{0,1}^{0,2} dy \int_{0,15}^{0,35-y} \frac{1}{x}\, dx =$$

$$= 0{,}2 \cdot \ln\left(\frac{0{,}15}{0{,}1}\right) + \int_{0,1}^{0,2} \ln(-y+0{,}35)\, dy - \int_{0,1}^{0,2} \ln(0{,}15)$$

$$= 0{,}271 + \left(\frac{-y+0{,}35}{-1} \ln(-y+0{,}35) - y\right)\Big|_{0,1}^{0,2} =$$

$$= 0{,}271 + (0{,}0846 - 0{,}2466) =$$

$$= 0{,}109$$

$$M = 21{,}8\, nH$$

(67,68) **SILA**

$$\boxed{\vec{F}_{12} = I_2 \int d\vec{l} \times \vec{B}_1}$$

$$\vec{F}_{12} = I_2 \int (dx\, \vec{a}_x + dy\, \vec{a}_y + dz\, \vec{a}_z) \times \left(\frac{\mu_0 I_1}{2\pi r}\right)(-\vec{a}_z)$$

$$\vec{F}_{12} = \frac{\mu_0 I_1 I_2}{2\pi} \left[\underbrace{\int_C \frac{1}{r}\, dx\, \vec{a}_y}_{\text{okomite komponente } = \emptyset} - \int_C \frac{1}{r}\, dy\, \vec{a}_x \right]$$

$$F_y = 0$$

$$\int_C \frac{1}{x}\, dy\, \vec{a_x} = \int_{0,2}^{0} \frac{1}{0,1}\, dy + \int_{0}^{0,1} \frac{1}{0,15}\, dy + \int_{0,1}^{0,2} \frac{1}{0,35-y}\, dy =$$

$$= 10 \cdot (-0,2) + 0,667 + \frac{1}{-1} \ln(-y + 0,35) \Big|_{0,1}^{0,2} =$$

$$= -2 + 0,667 - \left[-1,897 + 1,386\right] =$$

$$= -0,822$$

$$F_x = \frac{\mu_0 I_1 I_2}{2\pi} \cdot 0,822 = 328,8 \text{ nN}$$

(69) **PRAVILO DESNE RUKE**

→ obuhvatimo vodič, palac u smjeru struje

⇒ prsti pokazuju gdje ulazi/izlazi magn. indukcija

$-\vec{a_z}$