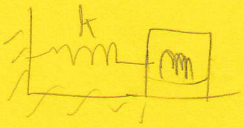
① Napiši jedn. gibanja za masu na opruzi i izvedi njeno opće rj. Napiši izraze za brzinu i akc. masu



$\omega_0^2 = \frac{k}{m} \Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$

$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{k}}}$

2NA:

$m\vec{a} = \sum_i \vec{F}_i$

$m\frac{d^2x}{dt^2} = F_{el}$ $\quad \vec{F}_{el} = -k\vec{x}$

$m\frac{d^2x}{dt^2} = -kx$

$\frac{d^2x}{dt^2} + \underbrace{\frac{k}{m}}_{\omega_0^2} x = 0$

$\boxed{\ddot{x} + \omega_0^2 x = 0}$

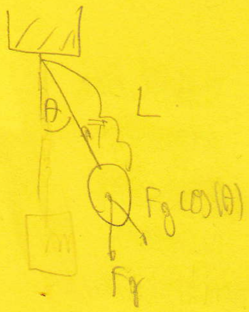
$x(t) = A\cos(\omega_0 t + \varphi_0)$

$v(t) = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t + \varphi_0)$

$a(t) = \frac{dv}{dt} = -A\omega_0^2\cos(\omega_0 t + \varphi_0) = -\omega_0^2 A\cos(\omega_0 t + \varphi_0)$

$\boxed{\ddot{x} = -\omega_0^2 x}$

② Izvedi jedn. gibanja mat. njihala pri malim otklonima od ravnoteže, napiši rj. te odredi period T



2NA

$m\vec{a} = \sum_i \vec{F}_i$

$I\vec{\alpha} = \sum_i \vec{M}_i$ $\qquad I = mL^2$

$mL^2 \cdot \ddot{\theta} = -mg\sin(\theta) \cdot L$ $\qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$

$\ddot{\theta} + \frac{g}{L}\sin(\theta) = 0$

Taylorov razvoj: $\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$

Egzaktno rj.

$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1^2}{2^2}\sin^2\left(\frac{\theta_0}{2}\right) + \dots\right)$

za mali kut: $\sin(\theta) \approx \theta$

$\ddot{\theta} + \underbrace{\frac{g}{L}}_{\omega_M^2}\theta = 0$ $\qquad \theta(t) = \theta_0\cos(\omega_M t + \varphi)$

$\omega_M^2 = \frac{g}{L} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$

③ Izvedi jedn. gibanja fizičkog njihala pri malim otklonima od ravnoteže, napiši rj. te odredi period titranja.

$I\alpha = M$

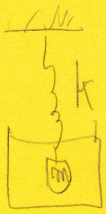
$I\ddot{\theta} = -bmg\sin(\theta)$

za mali kut: $\sin(\theta) \approx \theta$

$\ddot{\theta} + \frac{bmg}{I}\theta = 0$

$\omega_F^2 = \frac{bmg}{I} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{bmg}}}$

4 Napiši jedn. gibanja oscilatora prigušenom silom razmjernom brzini, te izvedi njena 3 rješenja



2.NA

$$m\cdot\vec{a} = \sum_i \vec{F}_i = \vec{F}_{el} + \vec{F}_{tr}$$

$$m\cdot\ddot{x} = -kx - b\dot{x} \;/:m$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0 \qquad \frac{b}{m} = 2\delta,\ \omega_0^2 = \frac{k}{m}$$

$$\boxed{\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0}$$

$$x(t) = Ae^{-\delta t}\cos(\omega t+\varphi) \quad , \; Ae^{-\delta t} = A(t)$$

$$\frac{dx}{dt} = A(-\delta)e^{-\delta t}\cos(\omega t+\varphi) - Ae^{-\delta t}\sin(\omega t+\varphi)\omega$$

$$\frac{d^2x}{dt^2} = A(-\delta)^2 e^{-\delta t}\cos(\omega t+\varphi) - A\omega(-\delta)e^{-\delta t}\sin(\omega t+\varphi) - A(-\delta)\omega e^{-\delta t}\sin(\omega t+\varphi) - A\omega^2 e^{-\delta t}\cos(\omega t+\varphi)$$

$\boxed{\text{uvrsti}}$

$$Ae^{-\delta t}\cos(\omega t+\varphi)\left[\delta^2 - \omega^2 - 2\delta^2 + \omega_0^2\right] + Ae^{-\delta t}\sin(\omega t+\varphi)\left[2\omega\delta - 2\omega\delta\right] = 0$$

$$Ae^{-\delta t}\cos(\omega t+\varphi)\underbrace{\left[\omega_0^2 - \omega^2 - \delta^2\right]}_{=0} = 0 \qquad \boxed{\omega^2 = \omega_0^2 - \delta^2}$$

a) $\omega_0^2 - \delta^2 > 0$ podkritično prigušenje

b) $\omega_0^2 - \delta^2 = 0$ kritično prigušenje

c) $\omega_0^2 - \delta^2 < 0$ nadkritično prigušenje

5 Krenuvši od izraza za ukupnu energiju prigušenog oscilatora, pokaži da energija u vremenu otpada s kvadratom brzine.

$$\frac{dE_{uk}}{dt} = \frac{d}{dt}\left[E_k(\dot{x}) + E_p(x)\right] = \frac{d\dot{x}}{dt}\cdot\frac{dE_k}{d\dot{x}} + \frac{dx}{dt}\cdot\frac{dE_p}{dx} = \frac{d\dot{x}}{dt}\cdot\frac{d(\frac{1}{2}m\dot{x}^2)}{d\dot{x}} + \frac{dx}{dt}\cdot\frac{d(\frac{1}{2}kx^2)}{dx}$$

$$= \ddot{x}\cdot\frac{1}{2}(m\dot{x}\cdot 2) + \dot{x}\cdot\frac{1}{2}k\cdot x\cdot 2 = \underbrace{(m\ddot{x} + kx)}_{-b\dot{x}}\dot{x}$$

$$\boxed{\frac{dE}{dt} = -bv^2}$$

6 Krenuvši od njegove općenite def., izvedi izraz za Q-faktor prigušenog oscilatora

Q-faktor (faktor kvalitete)

$$Q = \frac{\pi}{\delta T} = \frac{\omega}{2\delta} \qquad Q = 2\pi\frac{\bar{E}}{\Delta E} \qquad \bar{E} = \frac{1}{4}k(A_1^2 + A_2^2) \qquad \Delta E = \frac{1}{2}k(A_1^2 - A_2^2)$$

7. Napiši jedn. gibanja prisilnog titranja, izvedi njeno rj. i izraz za rezonantnu frekv. (NAJVEĆA AMPLITUDA)

$$m\ddot{x} = -kx - b\dot{x} + F_p \cos(\omega_p t) \quad / : m$$

$$\frac{b}{m} = 2\delta \qquad \omega_0^2 = \frac{k}{m} \qquad f_p = \frac{F_p}{m}$$

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = f_p \cos(\omega_p t)$$

jer je $OH = OH + PH$

$$x_{OH}(t) = Ae^{-\delta t}\cos(\omega t + \phi)$$

$$x_{PH}(t) = A(\omega_p)\cos(\omega_p t - \phi)$$

$$\dot{x}(t) = -A\omega_p \sin(\omega_p t - \phi) = A\omega_p \cos(\omega_p t - \phi + \tfrac{\pi}{2})$$

$$\ddot{x}(t) = -A\omega_p^2 \cos(\omega_p t - \phi) = A\omega_p^2 \cos(\omega_p t - \phi + \pi)$$

Uvrstimo

$$A\omega_p^2 \cos(\omega_p t - \phi + \pi) + 2\delta A\omega_p \cos(\omega_p t - \phi + \tfrac{\pi}{2}) + \omega_0^2 A \cos(\omega_p t - \phi) = f_p \cos(\omega_p t)$$

$$f_p^2 = (2\delta A\omega_p)^2 + (A\omega_0^2 - A\omega_p^2)^2 \Rightarrow A(\omega_p) = \frac{f_p}{\sqrt{(\omega_0^2 - \omega_p^2)^2 + 4\delta^2\omega_p^2}}, \quad tg(\phi) = \frac{2\delta\omega_p}{\omega_0^2 - \omega_p^2}$$

Rezonancija

$$\frac{d}{d\omega_p}\left[(\omega_0^2 - \omega_p^2)^2 + 4\delta^2\omega_p^2\right] = 2(\omega_0^2 - \omega_p^2)(-2\omega_p) + 4\delta^2 2\omega_p = 0$$

$$0 = -\omega_0^2 + \omega_p^2 + 2\delta^2 \Rightarrow \omega_p^2 = \omega_0^2 - 2\delta^2$$

$$\boxed{\omega_{rez} = \omega_{p0} = \sqrt{\omega_0^2 - 2\delta^2}}$$

Uvrstimo u $A(\omega_p)$

$$A_{max} = \frac{f_p}{2\delta\sqrt{\omega_0^2 - \delta^2}} \qquad \text{za } \delta \to 0,\ A \to \infty$$

8. Napiši jedn. gibanja simetričnog vezanog oscilatora |-k-m-K-m-k-|, izvedi frekv. (vlastitih modova) titranja te napiši opća rj. $x_1(t)$ i $x_2(t)$.

①

$$m_1\ddot{x}_1 = -k_1x_1 + K(x_2 - x_1) \qquad m_1 = m_2 = m$$

$$m_2\ddot{x}_2 = -k_2x_2 - K(x_2 - x_1) \qquad k_1 = k_2 = k$$

$$m\ddot{x}_1 = -kx_1 - K(x_1 - x_2) \quad / : m \qquad \boxed{K = \frac{mg}{L}}$$

$$m\ddot{x}_2 = -kx_2 - K(x_2 - x_1) \quad / : m$$

$$\ddot{x}_1 = -\omega_0^2 x_1 - \Omega^2(x_1 - x_2) \qquad \omega_0^2 = \frac{k}{m}$$

$$\ddot{x}_2 = -\omega_0^2 x_2 - \Omega^2(x_2 - x_1) \qquad \Omega^2 = \frac{K}{m}$$

② i) Titranje u fazi

$$x_1(t) = A_1 \cos(\omega_f t + \varphi_1)$$

$$x_2(t) = A_1 \cos(\omega_f t + \varphi_1)$$

$$\omega^2 - (\omega_0^2 + \Omega^2) + \Omega^2 = 0$$

$$\omega^2 = \omega_0^2 \Rightarrow \boxed{\omega_f = \omega_0 = \sqrt{\frac{k}{m}}}$$

③ ii) Titranje u protufazi

$$x_1(t) = A_2 \cos(\omega_{pf} t + \varphi_2)$$

$$x_2(t) = -A_2 \cos(\omega_{pf} + \varphi_2)$$

$$\omega^2 - (\omega_0^2 + \Omega^2) - \Omega^2 = 0$$

$$\omega^2 = \omega_0^2 + 2\Omega^2 \Rightarrow \boxed{\omega_{pf}} = \sqrt{\omega_0^2 + 2\Omega^2} = \boxed{\sqrt{\frac{k}{m} + \frac{2K}{m}} > \omega_f}$$

④ Opće rješenje

$$x_1(t) = A_1 \cos(\omega_f t + \varphi_1) + A_2 \cos(\omega_{pf} t + \varphi_2)$$

$$x_2(t) = A_1 \cos(\omega_f t + \varphi_1) - A_2 \cos(\omega_{pf} t + \varphi_2)$$

(9) Krenuvši od rješenja za titranje sim. vezanog oscilatora, izvedi osnovnu frekv. i frekv. udara za gibanje s poč. uvjetima $x_1(0)\neq 0$, $v_1(0)=0$, $x_2(0)=0$, $v_2(0)=0$.

Početni uvjeti

$x_1(0)\neq 0 \quad x_2(0)=0$ npr. $x_1(0)=A_0$, $A_0\neq 0$

$v_1(0)=0 \quad v_2(0)=0$

$\dot{x}_1(t)=-A_1\omega_f\sin(\omega_f t+\varphi_1)-A_2\omega_{pf}\sin(\omega_{pf}t+\varphi_2)$ $\quad A_1=A_2=\frac{A_0}{2} \quad \varphi_1=\varphi_2=0$

$\dot{x}_2(t)=-A_1\omega_f\sin(\omega_f t+\varphi_1)+A_2\omega_{pf}\sin(\omega_{pf}t+\varphi_2)$

$x_1(t)=\frac{A_0}{2}\cos(\omega_f t)+\frac{A_0}{2}\cos(\omega_{pf}t)$

$x_2(t)=\frac{A_0}{2}\cos(\omega_f t)-\frac{A_0}{2}\cos(\omega_{pf}t)$

$x_1(t)=A_0\cos\left(\frac{\omega_{pf}+\omega_f}{2}t\right)\cos\left(\frac{\omega_{pf}-\omega_f}{2}t\right)$ i $x_2(t)=A_0\sin\left(\frac{\omega_{pf}+\omega_f}{2}t\right)\sin\left(\frac{\omega_{pf}-\omega_f}{2}t\right)$

$\boxed{f_{udara}=f_{pf}-f}$

(10) Izvedi jedn. gibanja (valnu jedn.) transverzalnog vala na napetom užetu

pomak y

$y=y(x,t)$

$y(x=0,t)=A\sin\left(2\pi\frac{t}{T}\right)=A\sin(\omega t)$

$y(x,t=0)=A\sin\left(2\pi\frac{x}{\lambda}\right)=A\sin(kx)$ $\quad k=\frac{2\pi}{\lambda}$ (valni broj)

faza $\omega t-kx=konst.$

$y(x,t)=A\sin\left(2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\right)=A\sin(\omega t-kx)$ gibanje u +x smjeru

$y(x,t)=A\sin(\omega t+kx)$ — " — $-x$ — " —

$\sum F_x=T\sin(\theta_B)-T\sin(\theta_A)=T\,tg(\theta_B)-T\,tg(\theta_A)$

$\sum F_y\approx T\left[\left(\frac{\partial y}{\partial x}\right)_B-\left(\frac{\partial y}{\partial x}\right)_A\right]$ $\quad \sum F_y=ma_y=\mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right)$

$\frac{\partial^2 y}{\partial x^2}-\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}=0$ $\quad v=\sqrt{\frac{T}{\mu}}$

$\boxed{\frac{\partial^2 y}{\partial x^2}-\frac{1}{v^2}\cdot\frac{\partial^2 y}{\partial t^2}=0}$

$y(x,t)=A\sin(\omega t-kx)$

$\frac{\partial^2 y}{\partial t^2}=-\omega^2A\sin(\omega t-kx)$

$\frac{\partial^2 y}{\partial x^2}=-k^2A\sin(\omega t-kx)$

(11) Napiši jedn. gibanja (valnu jedn.), dokaži da su f-je oblika $f(x-vt)$ i $g(x+vt)$ rješenja opće rj.

$y(x,t)=f\left(t-\frac{x}{v}\right)+g\left(t+\frac{x}{v}\right)$ ili $y(x,t)=f(x-vt)+g(x+vt)$

$y(x,t)=f(x-vt)$

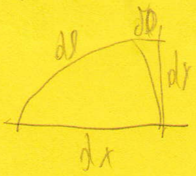
$\frac{\partial^2 y}{\partial x^2}=f'' \quad \frac{\partial^2 y}{\partial t^2}=v^2f'' \Rightarrow f(x-vt)$

$f\left(t_1-\frac{x_1}{v}\right)=f\left(t_2-\frac{x_2}{v}\right)$

$t_1-\frac{x_1}{v}=t_2-\frac{x_2}{v}$

$x_2=x_1+v(t_2-t_1) \Rightarrow f(x-vt)$ opisuje val u smjeru osi +x

(12) Izvedi izraz za prosječnu pot., kin i ukupnu eng. transverzalnog harmoničkog progresivnog vala



$$\delta l = dl - dx = \sqrt{(dx)^2+(dy)^2} - dx$$

$$\delta l \approx \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 dx$$

$$dE_p = T\,\delta l = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2 dx$$

$$\frac{dE_p}{dx} = \frac{1}{2}\mu A^2\omega^2\cos^2(\omega t - kx)$$

$$\boxed{\left\langle \frac{dE_p}{dx}\right\rangle = \frac{1}{4}\mu A^2\omega^2}$$

$$\frac{dE_k}{dx} = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2$$

$$\frac{dE_k}{dx} = \frac{1}{2}\mu A^2\omega^2\cos^2(\omega t - kx)$$

$$\left\langle \frac{dE_k}{dx}\right\rangle = \frac{1}{4}\mu A^2\omega^2$$

$$\boxed{\left\langle \frac{dE}{dx}\right\rangle} = \left\langle \frac{dE_p}{dx}\right\rangle + \left\langle \frac{dE_k}{dx}\right\rangle = \boxed{\frac{1}{2}\mu A^2\omega^2}$$

$$\bar{P} = \left\langle \frac{dE}{dt}\right\rangle$$

$$\boxed{\bar{P} = \frac{1}{2}\mu A^2\omega^2 v}$$

(13) Za progresivni transverzalni val izvedi izraze za amplitude transmitiranog i reflektiranog vala

$$y_u(x,t) = A_u \sin\omega\left(t - \frac{x}{v_1}\right)$$

$$y_r(x,t) = A_r \sin\omega\left(t + \frac{x}{v_1}\right)$$

$$y_t(x,t) = A_t \sin\omega\left(t - \frac{x}{v_2}\right)$$

Rubni uvjeti $x=0$:

$$y_u + y_r = y_t$$

$$\frac{\partial}{\partial x}(y_u + y_r) = \frac{\partial y_t}{\partial x}$$

$$\boxed{A_r = \frac{v_2 - v_1}{v_1 + v_2}\cdot A_u \qquad A_t = \frac{2v_2}{v_1+v_2}\cdot A_u}$$

(14) Pokaži da superpozicijom dvaju progresivnih harmoničkih valova može nastati stojni val

$y_u = A\sin(\omega t + kx)$ upadni val giba se ulijevo

$$y_r = A\sin(\omega t - kx + \pi) = -A\sin(\omega t - kx)$$

$$y = y_u + y_r = 2A\sin(kx)\cos(\omega t)$$

$\sin kx_n = 0 \Rightarrow$ čvorovi

$$x_n = \frac{n\pi}{k} = n\frac{\lambda}{2}, \quad n = 0,1,2,\dots$$

$\sin(kx_n) = \pm 1 \Rightarrow$ trbusi

$$x_n = (2n+1)\frac{\lambda}{4}; \quad n = 0,1,2,\dots$$

(15) Izvedi izraze za frekv i valne duljine stojnih valova na užetu linijske gustoće μ, napetosti niti T i duljine L, s učvršćenim krajevima

$y_+(x,t) = A_1\sin(\omega t - kx)$ u $+x$ smjeru

$y_-(x,t) = A_2\sin(\omega t + kx)$ u $-x$ −//−

$$y(x,t) = y_+(x,t) + y_-(x,t)$$

Rubni uvjeti: $y(x=0,t)=0$
$y(x=L,t)=0$

$$y(x,t) = -2A\sin(kx)\cos(\omega t)$$

$\sin kL = 0 \Rightarrow kL = n\pi$

$$\boxed{\lambda_n = \frac{2L}{n}}\ ; \quad n = 0,1,2,\dots$$

$$\boxed{f_n} = \frac{v}{\lambda_n} = \boxed{\frac{n}{2L}\sqrt{\frac{T}{\mu}}}$$

(16) Pomoću Gaussovog zakona izvedi polje unutar i izvan jednoliko nabijene kugle, polje jednoliko nabijene ravne tanke žice, polje jednoliko nabijene plohe. (6)

Gaussov zakon: $\Phi_E = \oint_{S_1} \vec{E}\cdot d\vec{S} = \oint_{S_1} \frac{Q}{4\pi\varepsilon_0 r^2} dS = \frac{Q}{4\pi\varepsilon_0 r^2}\oint_{S_1} dS = \frac{Q}{4\pi\varepsilon_0 r^2}\cdot 4\pi r^2 \Rightarrow \Phi_E = \frac{Q}{\varepsilon}$ ili $\Phi_E = \frac{Q_{unutra}}{\varepsilon}$

Izvan nabijene kugle: $\oint \vec{E}\cdot d\vec{S} = \frac{Q_{un}}{\varepsilon_0}$

$E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow \boxed{E = \frac{1}{4\pi r^2}\cdot\frac{Q}{r^2}}$

Unutar kugle: $\oint \vec{E}\cdot d\vec{S} = \frac{Q_{un}}{\varepsilon_0}$ $\quad Q_s = Q\cdot\frac{r^3}{R^3}$

$E\cdot\oint d\vec{S} = Q\cdot\frac{r^3}{\varepsilon_0 \cdot R^3}$

$E\cdot 4\pi r^2 = Q\cdot\frac{r^3}{\varepsilon_0 R^3} \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\cdot r}{R^3}}$

Ravna tanka žica: $\oint \vec{E}\cdot d\vec{S} = \frac{Q_{un}}{\varepsilon_0}$

$\int_{baza}\vec{E}\cdot d\vec{S} + \int_{plašt}\vec{E}\cdot d\vec{S} = \frac{\lambda\cdot l}{\varepsilon_0} \Rightarrow E\cdot\int dS = \frac{\lambda l}{\varepsilon_0}$

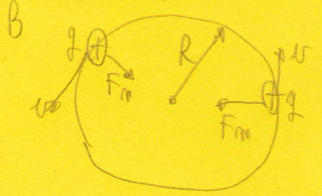
$E\cdot 2\pi r l = \frac{\lambda\cdot l}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{r}$

Jednoliko nabijena ploha: $\oint \vec{E}\cdot d\vec{S} = \frac{Q_{un}}{\varepsilon_0}$

$\int_{baza}\vec{E}\cdot d\vec{S} + \int_{plašt}\vec{E}\cdot d\vec{S} = \frac{\sigma\cdot S}{\varepsilon_0} \Rightarrow 2\cdot\int E\, dS = \frac{\sigma\cdot S}{\varepsilon_0}$

$2\cdot E\cdot S = \frac{\sigma\cdot S}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$

(17) Pokaži da se nabijena čestica u homogenom mag. polju može gibati po kružnici, odredi polumjer kružnice.



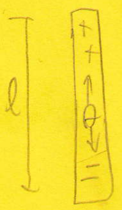
$\vec{F}_m = Q\,\vec{v}\times\vec{B}$

$dW = \vec{F}_m\cdot d\vec{l} = Q(\vec{v}\times\vec{B})\cdot\vec{v}\,dt = 0$

$a_{cp} = \frac{F_m}{m} = \frac{q v B}{m}$ $\quad \vec{v}\perp\vec{B}$

$\boxed{R = \frac{mv}{qB}}$

(18) Izvedi izraz za elektromotornu silu gibanja vodiča u mag. polju



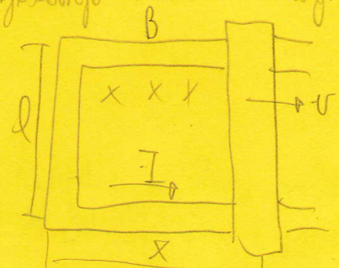
B

$F_e = qE \quad F_m = q v B$

$F_e = F_m$

$E = vB$

$U = El = Blv$



$\Phi_B = Blx$

$\boxed{\varepsilon_i = -\frac{d\Phi_B}{dt} = -Blv}$

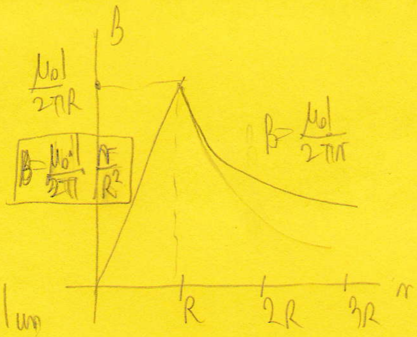
Općenito: $\varepsilon_i = \vec{l}\cdot(\vec{v}\times\vec{B})$

(19) Koristeći Ampère–Maxwellov zakon, izračunaj mag. polje beskon. ravnog tankog vodiča

Izvan vodiča: $\oint_l \vec{B} \cdot d\vec{l} = \mu_0 I$

$B \cdot 2\pi r = \mu_0 \cdot I \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$

$\boxed{B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}}$

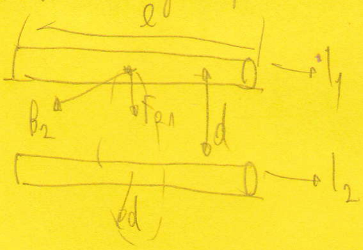


Unutar vodiča: $\oint_l \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{un}$

$B \cdot 2\pi r = \mu_0 \cdot \frac{r^2}{R^2} \cdot I \qquad I_{un} = \frac{r^2}{R^2} \cdot I$

$\boxed{B = \frac{\mu_0 \cdot I}{2\pi R^2} \cdot r}$

(20) Izvedi izraz za silu kojom ravni vodič kojim teče struja I_1 djeluje na jediničnu duljinu paralelnog vodiča na udaljenosti d kojim teče struja I_2. Definiraj jedinicu za jakost struje amper.



$F_{21} = I_1 \cdot l \cdot B_2 = I_1 \cdot l \cdot \frac{\mu_0 I_2}{2\pi d}$

$F_{12} = F_{21} = \frac{\mu_0}{2\pi} \cdot l \cdot \frac{I_1 \cdot I_2}{d} \; / : l \Rightarrow \boxed{\frac{F_{12}}{l} = \frac{F_{21}}{l} = \frac{\mu_0}{2\pi} \cdot \frac{I_1 \cdot I_2}{d}}$

Definicija ampera:

$I_1 = I_2 = I = 1\,A$ i $d = 1\,m$ $\quad \boxed{\frac{F}{l} = 2 \cdot 10^{-7} \frac{N}{m}}$

(21) Krenuvši od Maxwellovih jedn. u vakuumu (u dif. obliku) izvedi valnu jedn za $\vec{E}$ ili $\vec{B}$

Maxwellove jedn. u prostoru bez naboja i struja

$$\nabla \times \boxed{\begin{aligned} &\nabla \cdot \vec{E} = 0 \\ &\nabla \cdot \vec{B} = 0 \\ &\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ &\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \Delta \vec{E}$

$\Delta = \nabla \cdot \nabla = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$-\Delta \vec{E} = -\nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$

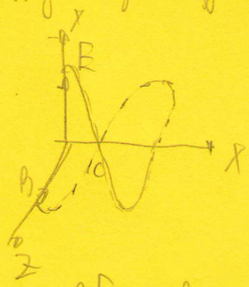
$\Delta \vec{E} = \frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \cdot \frac{\partial \vec{E}}{\partial t}\right)$

$$\boxed{\begin{aligned} &\Delta \vec{E} - \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ &\Delta \vec{B} - \frac{1}{v^2} \cdot \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \end{aligned}} \Rightarrow v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\nabla^2 \vec{E} - \frac{1}{c^2} \cdot \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \Rightarrow \vec{E}(x,t) = \vec{j} E_0 \cos(kx - \omega t)$

$\nabla^2 \vec{B} - \frac{1}{c^2} \cdot \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \Rightarrow \vec{B}(x,t) = \vec{k} B_0 \cos(kx - \omega t)$

(22) Napiši izraz za vektore $\vec{E}$ i $\vec{B}$ ravnog linearno polariziranog elektromag. vala te pokaži da su oni rješenja odgovarajućih valnih jdb. Skiciraj vektore $\vec{E}$ i $\vec{B}$ i smjer njihovog širenja.



$$\vec{E}(x,t) = \vec{j}\, E_0 \cos(kx - \omega t)$$
$$\vec{B}(x,t) = \vec{k}\, B_0 \cos(kx - \omega t)$$

$$\frac{\partial^2 E}{\partial x^2} = -k^2 E_0 \cos(kx - \omega t)$$
$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E_0 \cos(kx - \omega t)$$

$$-k^2 E_0 \cos(kx-\omega t) + \frac{1}{c^2}\omega^2 E_0 \cos(kx-\omega t) = 0$$
$$\frac{k^2}{\omega^2} = \frac{1}{c^2} \Rightarrow \left(\frac{k}{\omega}\right)^2 = \left(\frac{\frac{2\pi}{\lambda}}{2\pi f}\right)^2 = \left(\frac{1}{\lambda f}\right)^2 = \left(\frac{1}{c}\right)^2 = \frac{1}{c^2}$$
$$\boxed{\frac{\omega}{k} = c}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$
$$\frac{\partial E}{\partial x} = -k E_0 \sin(kx - \omega t) \qquad \frac{\partial B}{\partial t} = \omega B_0 \sin(kx - \omega t)$$

$$-k E_0 = -\omega B_0$$
$$\frac{E_0}{B_0} = \frac{\omega}{k} = c \Rightarrow \text{tj.} \boxed{\frac{E}{B} = c}$$

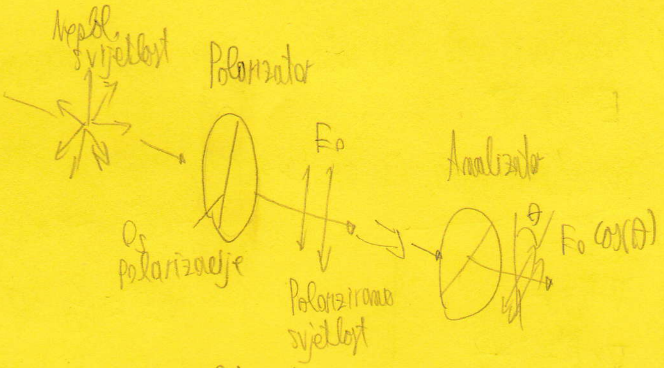
(23) Opiši polarizaciju elektromag. vala (koje se polje koristi za opis, uloga polarizatora) i izvedi Malusov zakon.

Uloga polarizatora: npr. sunčane naočale. Da smanji svjetlost

Horizontalna polarizacija: koristi se B

Vertikalna polarizacija: koristi se E

Malusov zakon $\quad \boxed{I_{trans} = I_0 \cdot \cos^2(\theta)}$



Za nepolariziranu svjetlost

$$\langle \cos^2(\theta) \rangle = \frac{1}{2} \Rightarrow \boxed{I_{trans} = \frac{1}{2} I_0}$$

(24) Napiši Poyntingov vektor ravnog vala čije je el. polje dano izrazom $\vec{E}(x,t) = E_0 \vec{j} \cos(\omega t - kx)$.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\left.\begin{aligned} \vec{E}(x,t) &= \vec{j}\, E_0 \cos(\omega t - kx) \\ \vec{B}(x,t) &= \vec{k}\, B_0 \cos(\omega t - kx) \end{aligned}\right\} \vec{S}(x,t) = \vec{i} \cdot c \cdot \varepsilon_0 \cdot E_0^2 \cos^2(\omega t - kx)$$

$$\boxed{I = \frac{\text{Snaga}}{\text{Površina}} = \frac{P}{4\pi r^2} = \langle \vec{S} \rangle = \vec{i} \cdot \frac{1}{2} \cdot c \cdot \varepsilon_0 \cdot E_0^2}$$