1. Sume

I

1. $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)}$

$\frac{1}{(n-1)(n+1)} = \frac{A}{n-1} + \frac{B}{n+1} \quad / \cdot (n-1)(n+1)$

$1 = A(n+1) + B(n-1)$

$1 = An + A + Bn - B$

$1 = n(A+B) + A - B$

$\left.\begin{matrix} A+B=0 \\ A-B=1 \end{matrix}\right\} + \qquad A = \frac{1}{2}$

$B = -\frac{1}{2}$

$\sum_{n=2}^{\infty} \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right) = \frac{1}{2}\left[\left(1 - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{5}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right)\right.$

$\left. \dots \right] = \frac{1}{2}\left[1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}\right] =$

$= \frac{1}{2}\left(\frac{3}{2} - \frac{n+1+n}{n(n+1)}\right) = \frac{3}{4} - \frac{1}{2}\,\frac{2n+1}{n^2+n}$

$S = \lim_{n\to\infty}\left(\frac{3}{4} - \frac{1}{2}\,\frac{2n+1}{n^2+n}\right) = \frac{3}{4}$ //

2. $\sum_{n=1}^{\infty} \frac{2^{n-1}}{(e-1)^{2n}}$

$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x|<1$

$= \sum_{n=1}^{\infty} \frac{2^n \cdot \frac{1}{2}}{\left[(e-1)^2\right]^n} = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{2}{(e-1)^n}\right)^n$

$= \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{2}{(e-1)^2}\right)^{n+1} = \frac{1}{2}\sum_{n=0}^{\infty}\frac{2}{(e-1)^2}\left(\frac{2}{(e-1)^2}\right)^n$

$= \frac{1}{\cancel{2}}\frac{\cancel{2}}{(e-1)^2}\sum_{n=0}^{\infty}\left(\frac{2}{(e-1)^2}\right)^n = \frac{1}{(e-1)^2}\cdot\frac{1}{1-\frac{2}{(e-1)^2}}$

$= \frac{1}{\cancel{(e-1)^2}}\cdot\frac{1}{\frac{(e-1)^2-2}{\cancel{(e-1)^2}}} = \frac{1}{e^2-2e-1}$

→ da je $|x|>1$ onda divergira

→ u onom slučaju konvergira

2.) Kriteriji konvergencije

a) Cauchy: za $\sum a_n$ vrijedi:

$$\lim_{n\to\infty} \sqrt[n]{a_n} = L \quad \ldots \quad \begin{array}{ll} L<1 & \text{konv} \\ L>1 & \text{div.} \\ L=1 & \text{nema odluke} \end{array}$$

1. $\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n-2}\right)^n$

$$\lim_{n\to\infty} \sqrt[\cancel{n}]{\left(\frac{2n+1}{3n-2}\right)^{\cancel{n}}} = \lim_{n\to\infty} \frac{2n+1}{3n-2} = \frac{2}{3} < 1 \quad \text{konv}$$

2. $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n(n+1)}$

$$\boxed{\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e}$$

$$\lim_{n\to\infty} \sqrt[\cancel{n}]{\left(\left(\frac{n-1}{n+1}\right)^{n+1}\right)^{\cancel{n}}} = \lim_{n\to\infty} \left(\frac{n-1}{n+1}\right)^{n+1} = \lim_{n\to\infty} \left(1 - \frac{2}{n+1}\right)^{n+1}$$

$$= e^{-2} < 1 \quad \text{konv}$$

3. $\sum_{n=1}^{\infty} \frac{1}{2^n} \left(\frac{n}{n+1}\right)^{-n^2}$

$$\lim_{n\to\infty} \sqrt[\cancel{n}]{\left(\frac{1}{2}\left(\frac{n}{n+1}\right)^{-n}\right)^{\cancel{n}}} = \lim_{n\to\infty} \frac{1}{2} \left(\frac{n}{n+1}\right)^{-n}$$

$$= \frac{1}{2} \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{\frac{-n+2n+1}{2n+1}} \quad \ldots \quad \frac{e}{2} > 1 \quad \text{div}$$

4. $\sum_{n=1}^{\infty} \frac{2n-1}{\sqrt{2^n}}$

$$\boxed{e^{\ln a} = a}$$

$$\lim_{n\to\infty} \sqrt[\cancel{n}]{\left(\frac{\sqrt[n]{2n-1}}{\sqrt{2}}\right)^{\cancel{n}}} = \lim_{n\to\infty} \frac{\sqrt[n]{2n-1}}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \lim_{n\to\infty} e^{\ln \sqrt[n]{2n-1}} = \frac{1}{\sqrt{2}} e^{\lim_{n\to\infty} \frac{1}{n} \ln(2n-1)}$$

$$= \frac{1}{\sqrt{2}} \lim_{n\to\infty} \frac{\ln(2n-1)}{n} = \frac{\infty}{\infty} = \frac{1}{\sqrt{2}} e^{\lim_{n\to\infty} \overbrace{\frac{1}{2n-1}\cdot 2}^{0}} = \frac{1}{\sqrt{2}} < 1$$

konv

b) D'Alembertov: za $\sum a_n$ vrijedi:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$$

$L < 1$ konv.
$L > 1$ div.
$L = 1$ nema odluke

1. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

$$\lim_{n\to\infty} \frac{\frac{((n+1)!)^2}{(2\cdot(n+1))!}}{\frac{(n!)^2}{(2n)!}} = \lim_{n\to\infty} \frac{\frac{\cancel{(n!)^2}(n+1)^2}{\cancel{(2n)!}\cdot(2n+1)\cdot(2n+2)}}{\frac{\cancel{(n!)^2}}{\cancel{(2n)!}}}$$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{(2n+1)(2n+2)} = \frac{1}{4} < 1 \text{ konv}$$

2. $\sum_{n=1}^{\infty} \frac{n!}{2^n+1}$

$$\lim_{n\to\infty} \frac{\frac{(n+1)!}{2^{n+1}+1}}{\frac{n!}{2^n+1}} = \lim_{n\to\infty} \frac{\frac{\cancel{n!}(n+1)}{2\cdot 2^n+1}}{\frac{\cancel{n!}}{2^n+1}}$$

$$= \lim_{n\to\infty} \frac{(n+1)(2^n+1)}{2\cdot 2^n+1} = \lim_{n\to\infty} \underbrace{(n+1)}_{\downarrow \, +\infty} \cdot \frac{2^n+1}{2\cdot 2^n+1}$$

$$= +\infty \rightarrow \text{div.}$$

c) Poredbeni: $\sum a_n \sim \sum b_n$

III

$a_n \leq k \cdot b_n \quad , k>0$

1) ako b_n konv. onda konv. i početni red

2) ako a_n div. onda div. i b_n

ili $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = L \quad , \quad L \in [1, +\infty\rangle$

1. $\sum\limits_{n=1}^{\infty} \frac{1}{2n-1} \sim \sum\limits_{n=1}^{\infty} \frac{1}{n}$

$\lim\limits_{n\to\infty} \left(\frac{\frac{1}{2n-1}}{\frac{1}{n}}\right) = \lim\limits_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2}$

$\boxed{\sum \frac{1}{n^r} \quad \begin{matrix} r>1 \text{ konv} \\ r\leq 1 \text{ div} \end{matrix}}$

znači div

2. $\sum\limits_{n=1}^{\infty} \frac{\ln n}{n^3+n+1} \leq \frac{n}{n^3+n+1} \leq \frac{n}{n^3} \leq \frac{1}{n^2}$

konv. $\qquad$ konv.

3. $\sum\limits_{n=1}^{\infty} (\ln(n+1) - \ln n) = \sum\limits_{n=1}^{\infty} \ln\frac{n+1}{n}$

$= \sum\limits_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right) \sim \frac{1}{n}$

$\boxed{\begin{matrix} \ln(1+x) \sim x \;, \; x\to 0 \\ e^x+1 \;, \; \sin x \; tgx \end{matrix}}$

div

$\lim\limits_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n}} = 1$

3. $\sum_{n=1}^{\infty} \frac{2\cdot 5\cdot 8\cdot \ldots (3n-1)}{1\cdot 5\cdot 9\cdot \ldots (4n-3)}$

$$\lim_{n\to\infty} \frac{\frac{\cancel{2\cdot 5\cdot 8\cdot \ldots (3n-1)}(3n+2)}{\cancel{1\cdot 5\cdot 9\cdot \ldots (4n-3)}(4n+1)}}{\frac{\cancel{2\cdot 5\cdot 8\cdot \ldots (3n-1)}}{\cancel{1\cdot 5\cdot 9\cdot \ldots (4n-3)}}} = \lim_{n\to\infty} \frac{3n+2}{4n+1} = \frac{3}{4} < 1$$

konv

d) Integralni kriterij: $f : [a, \infty\rangle \to [0, \infty\rangle$, $a > 0$ IV

ako $\int_a^\infty f(x)\,dx$ konv. $\to \sum f(n)$

1. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$f(x) = \frac{1}{x \ln x}$

$f'(x) = -\frac{\ln x - x\frac{1}{x}}{x^2 \ln^2 x} = -\frac{\ln x + 1}{x^2 \ln^2 x} < 0, \quad x \geq 2$

$\int_2^\infty \frac{1}{x \ln x}\,dx = \begin{bmatrix} \ln x = t \\ dx\frac{1}{x} = dt \end{bmatrix} = \int_{\ln 2}^{\infty} \frac{dt}{t} = \ln|t| \Big|_{\ln 2}^{\infty}$

$= \underset{\downarrow \atop \infty}{\ln \infty} - \underbrace{\ln \ln 2}_{\text{mali broj}} = +\infty \to \underline{\text{div}}$

2. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ $\sim \underbrace{\frac{1}{n^2} \to \underline{\text{konv.}}}$

jednostavniji postupak po poredbenom kriteriju

$f(x) = \frac{1}{x(x+1)}$

$f'(x) = \quad \ldots < 0, \quad x \geq 1$

$\int_1^\infty \frac{dx}{x(x+1)} = \qquad \frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$1 = Ax + A + Bx$

$1 = x(A+B) + A$

$A + B = 0 \quad A = 1 \quad B = -1$

$\int_1^\infty \frac{dx}{x} - \int_1^\infty \frac{dx}{x+1} = \left(\ln|x| - \ln|x+1|\right)\Big|_1^\infty = \ln\left|\frac{x}{x+1}\right|\Big|_1^\infty$

$= \underbrace{\lim_{x \to \infty} \ln\left|\frac{x}{x+1}\right|}_{0} - \underbrace{\ln\left(\frac{1}{1+1}\right)}_{-\ln\frac{1}{2}} = \ln 2 \to \underline{\text{konv}}$

e) Leibnitzov: $\sum (-1)^n a_n$

1. $a_n \to$ padajući niz

2. $\lim\limits_{n\to\infty} a_n = 0$

1. $\sum\limits_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n}$

padajući ? → je

$\lim\limits_{n\to\infty} \frac{1}{n} = 0$ → konv.

aps. konv. → uvjetna konvergencija

$\sum\limits_{n=1}^{\infty} \frac{1}{n}$ → div

2. $\sum\limits_{n=1}^{\infty} (-1)^n \frac{1}{e^n}$

aps. konv.

$\sum\limits_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$ → geometrijski red

$\sum\limits_{n=0}^{\infty} \left(\frac{1}{e}\right)^{n+1} = \sum\limits_{n=0}^{\infty} \frac{1}{e}\left(\frac{1}{e}\right)^n = \frac{1}{e} \sum\limits_{n=0}^{\infty} \left(\frac{1}{e}\right)^n$ → konv.

3. Konv. u rubnim točkama

V

1. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n\sqrt{n}-n} \rightarrow$ A'Ae

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(x-3)^{n+1}}{(n+1)\sqrt{n+1}-(n+1)}}{\frac{(x-3)^n}{n\sqrt{n}-n}}$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{(x-3)^n}(x-3)}{\cancel{(x-3)^n}} \cdot \frac{n\sqrt{n}-n}{n\sqrt{n+1}+\sqrt{n+1}-n-1} \right|$$

$$= |x-3| \lim_{x\to\infty} \left| \frac{1-\frac{1}{\sqrt{n}}\to 0}{\sqrt{1+\frac{1}{n}}\to 0 + \dots} \right|$$

$$= |x-3| < 1$$

$$-1 < x-3 < 1$$

$$2 < x < 4$$

1. $x=2$

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n\sqrt{n}-n} \rightarrow$ padajući

$\lim_{n\to\infty} \frac{1}{n\sqrt{n}-n} = 0$ } konv.

2. $x=4$

$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}-n} \sim \frac{1}{n\sqrt{n}} \sim \frac{1}{n^{\frac{3}{2}}} \quad \frac{3}{2}>1$ konv.

$K=[2,4]$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n (x-1)^n}{(3n-2)\, 2^n} \rightarrow$ Cauchy

$$\lim_{n\to\infty} \sqrt[\cancel{n}]{\left|\frac{3\cdot(x-1)}{\sqrt[n]{3n-2}\cdot 2}\right|^{\cancel{n}}} = \lim_{n\to\infty} \left|\frac{3(x-1)}{\sqrt[n]{3n-2}\cdot 2}\right|$$

$$= \frac{3}{2}|x-1| \lim_{n\to\infty} \left|\underbrace{\frac{1}{\sqrt[n]{3n-2}}}_{\text{ovako nešto je uvijek } 1}\right| = \frac{3}{2}|x-1| < 1$$

$$\frac{3}{2}|x-1| < 1$$

$$|x-1| < \frac{2}{3}$$

$$-\frac{2}{3} < x-1 < \frac{2}{3}$$

$$\frac{1}{3} < x < \frac{5}{3}$$

1. $x = \frac{1}{3}$

$$\sum_{n=1}^{\infty} \frac{3^n \cdot \left(\frac{1}{3}-1\right)^n}{(3n-2)\, 2^n} = \sum_{n=1}^{\infty} \frac{\cancel{3^n}\, \frac{(-2)^n}{\cancel{3^n}}}{(3n-2)\cdot 2^n}$$

$$= \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{3n-2} \rightarrow \left.\begin{array}{l}\text{padajući}\\ =0\end{array}\right\} \text{konv.}$$

2. $x = \frac{5}{3}$

$$\sum_{n=1}^{\infty} \frac{3^n \left(\frac{5}{3}-1\right)^n}{(3n-2)\cdot 2^n} = \sum_{n=1}^{\infty} \frac{\cancel{3^n}\, \frac{\cancel{2^n}}{\cancel{3^n}}}{(3n-2)\cancel{2^n}} = \sum_{n=1}^{\infty} \frac{1}{3n-2}$$

$$\sim \frac{1}{n} \rightarrow \text{div.}$$

$$K \left[\frac{1}{3}, \frac{5}{3}\right\rangle$$

Suma geometrijskog reda → der. / int.

VI

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots = \frac{1}{1-x}$$

1. $$\sum_{n=1}^{\infty} \frac{n^2}{5^n} = \sum_{n=1}^{\infty} n^2 \cdot \left(\frac{1}{5}\right)^n$$

$$\sum_{n=1}^{\infty} n^2 \cdot x^n = ?$$

kod prve derivacije mijenja se početni indeks

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots = \frac{1}{1-x} \quad /'$$

$$\sum_{\boxed{n=1}}^{\infty} n x^{n-1} = 0 + 1 + 2x + \dots = \frac{1}{(1-x)^2} \quad / \cdot x$$

pazi na ovo

→ izgubili smo jedan član

$$\sum_{n=1}^{\infty} n x^n = x + 2x^2 + \dots = \frac{x}{(1-x)^2} \quad /'$$

$$\sum_{n=1}^{\infty} n^2 x^{n-1} = 1 + 4x + \dots = \frac{(1-x)^2 + 2x(1-x)}{(1-x)^4}$$

$$\sum_{n=1}^{\infty} n^2 x^{n-1} = \frac{1 - \cancel{2x} + x^2 + \cancel{2x} - 2x^2}{(1-x)^4} = \frac{1-x^2}{(1-x)^4} \quad / \cdot x$$

$$\boxed{\sum_{n=1}^{\infty} n^2 x^n = \frac{x - x^3}{(1-x)^4}}$$

$$\sum_{n=1}^{\infty} n^2 \left(\frac{1}{5}\right)^n = \frac{\frac{1}{5} - \frac{1}{125}}{\left(1 - \frac{1}{5}\right)^4} = \frac{24 \cdot 5}{4^4} = \frac{15}{32} //$$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{\left(\frac{2}{3}\right)^n}{n(n+1)} = ?$

$$\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$$

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots = \frac{1}{1-x} \quad / \int$$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots = -\ln|1-x| \quad / \int$$

$$\sum_{n=0}^{\infty} \frac{x^{n+2}}{(n+1)(n+2)} = -\int \ln|1-x|\, dx$$

nije potrebno apsolutno jer je $\left(1-\frac{2}{3}\right)>0$

$$= \begin{bmatrix} u = \ln(1-x) & du = dx \\ du = -\frac{dx}{1-x} & u = x \end{bmatrix}$$

$$= -\left(x\ln(1-x) - \int \frac{-x\,dx}{1-x}\right)$$

$$= -\left(x\ln(1-x) - \int \frac{1-x-1}{1-x}\, dx\right)$$

$$= -\left(x\ln(1-x) - \int dx + \int \frac{dx}{1-x}\right)$$

$$= -\left(x\ln(1-x) - x - \ln(1-x)\right)$$

$$= -x\ln(1-x) + x + \ln(1-x)$$

$$\sum_{n=0}^{\infty} \frac{x^{n+2}}{(n+1)(n+1)} = -x\ln(1-x) + x + \ln(1-x)$$

$$\sum_{n=1}^{\infty} \frac{x^{n+1}}{n(n+1)} = -x\ln(1-x) + x + \ln(1-x) \quad / : x$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)} = -\ln(1-x) + 1 + \frac{\ln(1-x)}{x}$$

$$\sum_{n=1}^{\infty} \frac{\left(\frac{2}{3}\right)^n}{n(n+1)} = -\ln\left(1-\frac{2}{3}\right) + 1 + \frac{1}{\frac{2}{3}}\ln\left(1-\frac{2}{3}\right)$$

$$= -\ln\left(\frac{1}{3}\right) + 1 + \frac{3}{2}\ln\left(\frac{1}{3}\right)$$

$$= \ln 3 + 1 - \frac{3}{2}\ln 3 = 1 - \frac{1}{2}\ln 3 \;//$$

Taylor i Maclauren

VII

$$T: f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$M: f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \to \text{Taylorov red za } a=0$$

1. $f(x) = \frac{1}{3+2x}$ u Macl. red

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_{n=0}^{\infty} (-1)^n x^n = \frac{1}{1+x}$$

$$= \frac{1}{3\left(1+\frac{2x}{3}\right)} = \frac{1}{3} \cdot \frac{1}{1+\frac{2x}{3}}$$

$$= \frac{1}{3} \sum_{n=0}^{\infty} (-1)^n \left(\frac{2x}{3}\right)^n //$$

$$= \sum_{n=0}^{\infty} \frac{(-2)^n x^n}{3^{n+1}} //$$

2. $f(x) = e^{5x}$, Macl

$f(0) = 1$

$f'(0) = 5$

$f'(0) = 25$

$f'(0) = 125$

$\vdots$

$$f^{(n)}(0) = 5^n$$

$$e^{5x} = \frac{5^n \cdot x^n}{n!} = \sum_{n=0}^{\infty} \frac{(5x)^n}{n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

3. $f(x) = \frac{1}{(3+x)^2}$, Mac.

$$\frac{1}{(3+x)^2} = \frac{1}{\left(3\left(1+\frac{x}{3}\right)\right)^2} = \frac{1}{9\left(1+\frac{x}{3}\right)^2} = \frac{1}{9} \cdot \frac{1}{\left(1+\frac{x}{3}\right)^2} \to \text{ovo nas smeta}$$

$$\sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^n = \frac{1}{1+\frac{x}{3}} \quad /'$$

$$\sum_{n=1}^{\infty} (-1)^n \cdot n \left(\frac{x}{3}\right)^{n-1} = \frac{-\frac{1}{3}}{\left(1+\frac{x}{3}\right)^2} = \frac{-\frac{1}{3}}{\left(\frac{3+x}{3}\right)^2} = -\frac{1}{\cancel{3}} \cdot \frac{1}{\frac{(3+x)^2}{\cancel{9}\,3}} = -3 \cdot \frac{1}{(3+x)^2}$$

$$\sum_{n=1}^{\infty} (-1)^n \cdot \left(-\frac{1}{3}\right) \cdot n \left(\frac{x}{3}\right)^{n-1} = \frac{1}{(3+x)^2} //$$

$$= \sum_{n=0}^{\infty} (-1) \ldots n \cdot \frac{x^{n}}{3^n} //$$

4. $f(x) = \frac{a}{2x-x^2}$ oko točke 1 tj. $a=1$

$$f(x) = \frac{1}{-(x^2-2x)} = \frac{1}{-(\underbrace{x^2-2x+1}_{(x-1)^2}-1)} = \frac{1}{1-(x-1)^2} \rightarrow \text{geom. red}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_{n=0}^{\infty} (x-1)^{2n} = \frac{1}{1-(x-1)^2} \quad //$$