LIMESI ♡

1. $$\lim_{x\to-\infty}\left(\sqrt{x^2+x}+x\right)=\lim_{x\to\infty}\left(\sqrt{x^2-x}-x\right)\cdot\frac{\sqrt{x^2-x}+x}{\sqrt{x^2-x}+x}$$ → racio.

$$\lim_{x\to\infty}\frac{\cancel{x^2}-x-\cancel{x^2}}{\sqrt{x^2-x}+x}\;\Big/{:x}=\lim_{x\to\infty}\frac{-1}{\sqrt{1-\frac{1}{x}}+1}=\frac{-1}{2}$$

$$\frac{1}{x}\to 0$$

2. $$\lim_{x\to\infty}\left(\sqrt{x^4+2x^2}-\sqrt{x^4+1}\right)\Big/\cdot\frac{\sqrt{x^4+2x^2}+\sqrt{x^4+1}}{-\|-\;\;-\|-}$$

$$\lim_{x\to\infty}\frac{\cancel{x^4}+2x^2-\cancel{x^4}-1}{\sqrt{x^4+2x^2}+\sqrt{x^4+1}}\;\Big/{:x^2}=\frac{2}{2}=1$$

3. $$\lim_{n\to\infty}\frac{2^n+3\cdot5^n}{3^n-4^n}=\lim_{n\to\infty}\frac{2^n+3\cdot5^n}{4^n\left[\left(\frac{3}{4}\right)^n-1\right]}\;\Big/{:5^n}$$

$$=\lim_{n\to\infty}\frac{\left(\frac{2}{5}\right)^n+3}{\left(\frac{4}{5}\right)^n\left[\left(\frac{3}{4}\right)^n-1\right]}=\frac{3}{0(0-1)}=-\infty\;!$$

PAZI NA $-\infty$

4. $$\lim_{n\to\infty}\left(\frac{3n+7}{8n-2}\right)^{5n}=\lim_{n\to\infty}\left(1+\frac{-5n+9}{8n-2}\right)^{5n}$$

$$=\lim_{n\to\infty}\left(1+\frac{1}{\frac{8n-2}{9-5n}}\right)^{\frac{8n-2}{9-5n}\cdot\frac{9-5n}{8n-2}\cdot 5n}$$

$$\to e$$

$$=e^{\lim_{n\to\infty}\left(\frac{9-5n}{8n-2}\cdot 5n\right)\,/:n^2\,/:n^2}=e^{-\infty}=0$$

5. $\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^{x-1} = \lim_{x\to\infty}\left(1+\frac{2}{x-1}\right)^{x-1} = \lim_{x\to\infty}\left(1+\frac{1}{\frac{x-1}{2}}\right)^{\frac{x-1}{2}\cdot 2} \to e$

$= e^2$

6. $\lim_{x\to 1}\frac{tg(x-1)}{x^2-1} = \lim_{x\to 1}\frac{tg(x-1)}{(x-1)(x+1)} = \lim_{x\to 1}\frac{1}{x+1} = \frac{1}{2}$

$\frac{tg(x-1)}{x-1}\to 1$

UVRSTI

7. $\lim_{x\to 2}\frac{\sin(x-2)}{x^2-4} = \lim_{x\to 2}\frac{\sin(x-2)}{(x-2)(x+2)} = \lim_{x\to 2}\frac{1}{x+2} = \frac{1}{4}$

$\frac{\sin(x-2)}{x-2}\to 1$

UVRSTI

8. a) $\lim_{x\to 2^-} e^{\frac{x-3}{x-2}} = \lim_{x\to 2^-} e^{\frac{2-0-3}{2-0-2}} = \lim_{x\to 2^-} e^{\frac{-1}{0^-}} = e^{\infty} = \infty$

b) $\lim_{x\to 2^+} e^{\frac{x-3}{x-2}} = \lim_{x\to 2^+} e^{\frac{2+0-3}{2+0-2}} = \lim_{x\to 2^+} e^{\frac{-1}{0^+}} = \lim_{x\to 2^+}\frac{1}{e^{\infty}} = 0$

c) $\lim_{x\to 1^-} arc\,tg\left(\frac{x-2}{x-1}\right) = \lim_{x\to 1^-} arc\,tg\left(\frac{1-0-2}{1-0-1}\right)$

$= \lim_{x\to 1^-} arc\,tg\left(\frac{-1}{0^-}\right) = \frac{\pi}{2}$

SUMA GEOMETRIJSKOG REDA

$$\Sigma = \frac{q^{n+1}-1}{q-1}$$

9. $\lim_{n\to\infty}\frac{3^n+3^{n-1}+\dots+3+1}{3^n+3^{n-1}}$

$\Sigma = \frac{3^{n+1}-1}{2}$

$\lim_{n\to\infty}\frac{3^{n+1}-1}{2(3^n+3^{n-1})}\ \frac{/:3^n}{/:3^n} = \frac{1}{2}\lim_{n\to\infty}\frac{3}{1+\frac{1}{3}} = \frac{9}{8}$

10. Odredi parametar a da funkcija bude neprekinuta u točki $x=0$

da bi funkcija bila neprekinuta, limesi obje funkcije moraju biti jednaki

$$f(x)=\begin{cases} x\cdot\frac{1}{tg2x}, & x\neq 0 \\ a+\cos 2x, & x=0 \end{cases}$$

$$f(0)=a+\cos 2x=\lim_{x\to 0}(a+\cos 2x)=\lim_{x\to 0}\frac{x \;/:2x}{tg2x\;/:2x}$$

$$=\lim_{x\to 0}\frac{\frac{x}{2x}}{\underbrace{\frac{tg2x}{2x}}_{\to 1}}=\frac{1}{2}$$

$$\lim_{x\to 0}(a+\cos 2x)=\frac{1}{2}$$

u x smo uvrstili 0
$\cos 0=1$

$$a+1=\frac{1}{2}\Rightarrow a=-\frac{1}{2}$$

11. $$f(x)=\begin{cases}\frac{x}{\sin 2x} \\ m+\sin\left(x+\frac{\pi}{2}\right)\end{cases}$$

$$=m+\sin\left(x+\frac{\pi}{2}\right)=\lim_{x\to 0}\left[m+\sin\left(x+\frac{\pi}{2}\right)\right]$$

$$=\lim_{x\to 0}\left(\frac{x}{\sin 2x}\right)=\frac{1}{2}$$

$$m-1=\frac{1}{2}\longrightarrow m=\frac{3}{2}$$

12. Dokaži da je niz konverg. i nađi limes

$a_1 = \sqrt{6}$

$a_{n+1} = \sqrt{6 + a_n}$

$a_2 = \sqrt{6+\sqrt{6}} = 2.9 > a_1 \Rightarrow$ rastući niz $\uparrow$

$L = \lim a_n = \lim a_{n+1} = \lim \sqrt{6 + a_n}$

$L = \lim \sqrt{6 + a_n}$

$L = \sqrt{6 + \lim a_n} = \sqrt{6 + L}$

$L = \sqrt{6+L} \quad /^2$

$L^2 = 6 + L$

$L^2 - L - 6 = 0$

$L_1 = \cancel{-2}$

$\boxed{L_2 = 3}$

13. $a_1 = \sqrt{3}$

$a_{n+1} = \sqrt{3a_n}$

$a_2 = \sqrt{3 \cdot \sqrt{3}} = 2{,}28 > a_1 \Rightarrow$ rastući niz $\uparrow$

$L = \lim a_n = \lim a_{n+1} = \lim \sqrt{3a_n}$

$L = \sqrt{3L} \quad /^2$

$L^2 = 3L \Rightarrow \boxed{L = 3}$

14. $a_1 = 2$

$$a_{n+1} = \frac{a_n + 1}{2}$$

$a_2 = \frac{3}{2}$, $a_3 = \frac{5}{4}$, $a_4 = \frac{9}{8}$ $\Rightarrow$ $\downarrow$ padajući niz

$$L = \lim a_n = \lim a_{n+1} = \lim \frac{a_n+1}{2}$$

$$L = \frac{L+1}{2}$$

$$2L = L + 1$$

$$\boxed{L = 1}$$

15. $$\lim_{x\to 0} \frac{e^{3x} - e^{2x}}{x} = \lim_{x\to 0} \frac{e^{3x} - e^{2x} + 1 - 1}{x}$$

Pomnožimo i podijelimo da naštimamo

$$= \lim_{x\to 0} \frac{e^{3x}-1}{x} - \lim_{x\to 0} \frac{e^{2x}-1}{x} = 3 \overbrace{\lim_{x\to 0} \frac{e^{3x}-1}{3x}}^{1} - 2 \overbrace{\lim_{x\to 0} \frac{e^{2x}-1}{2x}}^{1}$$

$$= 3\cdot 1 - 2 \cdot 1 = 1$$

16. $$\lim_{x\to\infty} x\left[\ln(x+1) - \ln x\right] = \lim_{x\to\infty}\left(x \cdot \ln \frac{x+1}{x}\right)$$

$\to$ oblik $\infty - \infty$

$$= \lim_{x\to\infty} \frac{\ln \frac{x+1}{x}}{\frac{1}{x}}$$

SUPSTITUCIJA: $y = \frac{1}{x}$

$y \to 0$

ako $x \to \infty$ onda $y \to 0$

$$= \lim_{y\to 0} \frac{\ln(1+y)}{y} = 1$$

$\Downarrow$

def.

17. $\lim_{x\to 1} \operatorname{arctg} \left(\frac{x}{1-x}\right)^3 = ?$

$$\lim_{x\to 1^-} \operatorname{arctg} \left(\frac{1^-}{1-1^-}\right)^3 = \frac{\pi}{2}$$

$$\lim_{x\to 1^+} \operatorname{arctg} \left(\frac{1^+}{1-1^+}\right) = -\frac{\pi}{2}$$

$\Big\}$ ne postoji limes

18. Odredi a da funkc. bude neprekinuta

$$f(x) = \begin{cases} a\cdot th\,\frac{x+3}{x+2}, & x > -2 \\ x^2 + a^2 x - 1, & x \le -2 \end{cases}$$

$$x^2+a^2x-1 \quad \big| \quad a\,th\,\frac{x+3}{x+2}$$
$$-2$$

$$\lim_{x\to -2^-} = f(-2) = 4 - 2a^2 - 1 = 3 - 2a^2$$

$$\lim_{x\to -2^+} a\cdot th\,\frac{x+3}{x+2} = \lim_{x\to -2^+} a\cdot th\,\underbrace{\frac{-2+0+3}{-2+0+2}}_{\to\infty} = \lim_{x\to -2^+} a\cdot \underbrace{th\,\infty}_{\to 1} = a$$

$$3 - 2a^2 = a$$
$$2a^2 + a - 3 = 0$$

$$a_1 = -\frac{3}{2} \qquad a_2 = 1$$

DERIVACIJE ♡

$$\left[f(x) \pm g(x)\right]' = f'(x) \pm g'(x)$$

$$\left[f(x) \cdot g(x)\right]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$ → koliko god članova u naz, svi idu na kvadrat

1. $f(x) = 5 + x + 2\sqrt{x} + x^4$ ♡ derivacija konstante = 0

$f'(x) = 0 + 1 + 2 \cdot \frac{1}{2\sqrt{x}} + 4x^3$

2. $f(x) = \frac{1}{x^4} + x^6 \cdot \sin x - \sin 2$

$f'(x) = \frac{0 \cdot x^4 - 1 \cdot 4x^3}{x^8} + 6x^5 \cdot \sin x + x^6 \cdot \cos x - 0$

...

DZ

1. 4)

$f(x) = \frac{x}{\sqrt{x^2+1}}$ jer je $\sqrt{x^2+1}$ složena funkc. imamo i korjenovanje i potenciranje

$f'(x) = \frac{1 \cdot \sqrt{x^2+1} - x \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{x^2+1}$

2. $f(x) = \text{arc tg } \frac{1}{x^2+1}$

$$f'(x) = \frac{1}{1+\left(\frac{1}{x^2+1}\right)^2} = \dots = \frac{x^4+2x^2+1}{x^4+2x^2+2}$$

$$f''(x) = \frac{(4x^3+4x)\cdot(x^4+2x^2+2) - (x^4+2x^2+1)\cdot(4x^3+4x)}{(x^4+2x^2+2)^2}$$

$f''(1) = \dots \rightarrow$ uvrstiti 1

3. $f(x) = x^3 \cdot 2^x + \underbrace{(x+1)}_{k}{}^x$

$$f'(x) = 2x \cdot 2^x + x^2 \cdot 2^x \cdot \ln 2 + \underbrace{k^x \ln k}_{(x+1)^x \ln(x+1)}$$

↳ zeko kaže da nije siguran
za ovaj, pa si provjerite
u starim zadaćama,
ja nisam našla