JIP. 2011.

1. $x(t) = \sin\left(\frac{2\pi}{7}t\right)$

a) CTFS

$$x(t) = \frac{1}{2j} e^{j\frac{2\pi}{7}t} - \frac{1}{2j} e^{-j\frac{2\pi}{7}t} = \frac{1}{2} e^{-j\frac{\pi}{2}} e^{j\frac{2\pi}{7}t} - \frac{1}{2} e^{-j\frac{\pi}{2}} e^{-j\frac{2\pi}{7}t}$$

$$X_1 = \frac{1}{2} e^{-j\frac{\pi}{2}} \qquad X_{-1} = \frac{-1}{2} e^{-j\frac{\pi}{2}}$$

b) CTFT → tablica $\delta$ fn.

$$\sin(\omega_0 t) \circ\!\!-\!\!\bullet -j\pi\left(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right)$$

$$\sin\left(\frac{2\pi}{7}t\right) \circ\!\!-\!\!\bullet -j\pi\left(\delta\left(\omega-\frac{2\pi}{7}\right) - \delta\left(\omega+\frac{2\pi}{7}\right)\right)$$

c) pomak u vremenskoj domeni CTFT

$$x(t-t_0) \circ\!\!-\!\!\bullet X(j\omega) e^{-j\omega t_0}$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{+\infty} x(t-t_0) e^{-j\omega t}\,dt = X(j\omega) \cdot e^{-j\omega t_0}$$

Ne pomiču amplitudu, nego fazu

d) $-j\pi\left(\delta\left(\omega-\frac{2\pi}{7}\right) - \delta\left(\omega+\frac{2\pi}{7}\right)\right) \cdot e^{-j\omega 4} = -j\pi\left(\delta\left(\omega-\frac{2\pi}{7}\right) - \delta\left(\omega+\frac{2\pi}{7}\right)\right) e^{-j\frac{4\pi 2}{4}}$

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2. $x(t) = t \cdot [\mu(t+3.5) - \mu(t-3.5)]$

JIR

2011

1) (вмени)

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-3.5}^{3.5} |x(t)|^2 dt = \frac{t^3}{3}\Big|_{-3.5}^{3.5} = \frac{1}{3}(3.5^3 + 3.5^3)$$

$$= \frac{1}{3} \cdot \frac{343}{4} = \frac{343}{12} = 28.583$$

2) $x(n) = x(n \cdot T_s) = n \cdot T_s [\mu(n \cdot T_s + 3) - \mu(n T_s - 3)] =$

$T_s = 1$

$$= n [\mu(n+3) - \mu(n-3)]$$

3) $E = ?$

$$E = \sum_{n=-\infty}^{+\infty} |x(n)|^2 = \sum_{n=-3}^{3} n^2 = \cancel{28} \; 28$$

$$= 9 + 4 + 1 + 0 + 1 + 4 + 9 = 28$$

4) DTFT $X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n) e^{-j\omega n} =$

$$= \sum_{n=-3}^{+3} n \cdot e^{-j\omega n} = -3 \cdot e^{+j\omega 3} - 2e^{j\omega 2} - 1e^{j\omega} + 1e^{-j\omega} + 2e^{-j\omega 2} + 3e^{-j\omega 3}$$

$$= -3(\cancel{\cos(3\omega)} + j\sin(3\omega)) - 2(\cancel{\cos(2\omega)} + j\sin(2\omega)) - 1(\cancel{\cos(\omega)} + j\sin(\omega)) + (\cos(\omega) - j\ldots$$

$$+ 2(\cos(2\omega) - j\sin(2\omega)) + 3(\cos(3\omega) - j\sin(3\omega)) =$$

$$= -6j\sin(3\omega) - 4j\sin(2\omega) - 2j\sin(\omega)$$

3. $y(n) + \frac{1}{16}y(n-2) = v(n) - \frac{1}{4}v(n-1)$

JIR 2014.

a) IMPULSNI U ~~Z DOMENI~~ VREMENSKOJ DOMENI

$\Rightarrow C \cdot a^n$

$a_1 = \frac{1}{4}i = \frac{1}{4}e^{\frac{j\pi}{2}}$

$a^2 + \frac{1}{16} = 0$

$a_2 = -\frac{1}{4}i = -\frac{1}{4}e^{-\frac{j\pi}{2}}$

$a^2 = -\frac{1}{16}$

$y_h(n) = C_1\left(\frac{1}{4}e^{\frac{j\pi}{2}}\right)^n + C_2\left(-\frac{1}{4}e^{-\frac{j\pi}{2}}\right)^n$

$-\cos\pi/2 \mp i\sin\pi/2$

$-i$

IMPULSNI

$y(0) + \frac{1}{16}y(0-2) = \delta(0) - \frac{1}{4}\delta(n-1)$

$y(0) = 1$

$y(1) + \frac{1}{16}y(-1) = \cancel{\delta(1)} - \frac{1}{4}\delta(0)$

$y(1) = -\frac{1}{4}$

$C_1 + C_2 = 1 \qquad C_1 = 1 - C_2$

$C_1\left(\frac{1}{4}e^{\frac{j\pi}{2}}\right) + C_2\left(-\frac{1}{4}e^{-\frac{j\pi}{2}}\right) = -\frac{1}{4}$

$\frac{1}{4}e^{\frac{j\pi}{2}} - \frac{1}{4}e^{\frac{j\pi}{2}}C_2 - \frac{1}{4}e^{-\frac{j\pi}{2}}C_2 = -\frac{1}{4} \Big/ 4$

$e^{\frac{j\pi}{2}} - 2e^{\frac{j\pi}{2}}C_2 = -1$

$-2jC_2 = -1 - j$

$C_2 = \frac{1+j}{2j}\Big|\frac{j}{j}$

$= \frac{j-1}{-2}$

$C_2 = \frac{1-j}{2}$

$C_1 = 1 - \left(\frac{1-j}{2}\right)$

$= \frac{2}{2} - \frac{1-j}{2}$

$= \frac{1+j}{2}$

$y(n) = \frac{1+j}{2}\left(\frac{1}{4}e^{\frac{j\pi}{2}}\right)^n - \frac{1-j}{2}\left(\frac{1}{4}e^{-\frac{j\pi}{2}}\right)^n$

$$y(n) = \frac{1+j}{2}\left(\frac{1}{4}e^{j\frac{\pi}{2}}\right)^n - \frac{1-j}{2}\left(\frac{1}{4}e^{j\frac{\pi}{2}}\right)^n$$

$$c_1 = \frac{1+j}{2}$$

$$|c_1| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \quad c_2 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{2}}{2}$$

$$\measuredangle c_1 = \text{Arctg}\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right) = \text{Arctg}(1) = \frac{\pi}{4} \qquad \measuredangle c_2 = \text{Arctg}(-1) = \frac{-\pi}{4}$$

$$y(n) = \frac{\sqrt{2}}{2} \cdot e^{j\frac{\pi}{4}}\left(\frac{1}{4}e^{j\frac{\pi}{2}}\right)^n - \frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}}\left(\frac{1}{4}e^{j\frac{\pi}{2}}\right)^n =$$

$$= \frac{\sqrt{2}}{2}\left(\frac{1}{4}\right)^n \cdot \left(\frac{e^{j\frac{\pi}{2}n + j\frac{\pi}{4}} + e^{-j\frac{\pi}{2}n - j\frac{\pi}{4}}}{2} \cdot 2\right) =$$

$$= \frac{\sqrt{2}}{2}\left(\frac{1}{4}\right)^n \cdot 2 \cdot \left(\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)\right) = \sqrt{2} \cdot \left(\frac{1}{4}\right)^n \left(\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)\right)$$

$$h(n) = y\,h(n) \cdot \mu(n)$$

$$h(n) = \left[\sqrt{2} \cdot \left(\frac{1}{4}\right)^n \cdot 2\left(\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)\right) - \left[\sqrt{2} \cdot \left(\frac{1}{4}\right)^n \cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right]\right]\mu(n)$$

$$A = \lim_{z \to \frac{1}{4}j} \frac{z - \frac{1}{4}}{z + \frac{1}{4}j} = \frac{1}{2} + \frac{1}{2}j = \frac{\sqrt{2}}{2} e^{j\frac{\pi}{4}}$$

$$B = \lim_{z \to -\frac{1}{4}j} \frac{z - \frac{1}{4}}{z - \frac{1}{4}j} = \frac{1}{2} - \frac{1}{2}j = \frac{\sqrt{2}}{2} e^{-j\frac{\pi}{4}}$$

$$\frac{H(z)}{z} = \frac{\sqrt{2}}{2} e^{j\frac{\pi}{4}} \cdot \frac{1}{z - \frac{1}{4}j} + \frac{\sqrt{2}}{2} e^{-j\frac{\pi}{4}} \cdot \frac{1}{z + \frac{1}{4}j}$$

$$= \frac{\sqrt{2}}{2} e^{j\frac{\pi}{4}} \cdot \left(\frac{1}{4}\right) j^{n} + \frac{\sqrt{2}}{2} e^{-j\frac{\pi}{4}} \cdot \left(\frac{1}{4}\right)^{n} (-j)^{n}$$

$$= \frac{\sqrt{2}}{2} e^{j\frac{\pi}{4}} \left(\frac{1}{4}\right) e^{\frac{j\pi n}{2}} + \frac{\sqrt{2}}{2} e^{-j\frac{\pi}{4}} \left(\frac{1}{4}\right)^{n} e^{-\frac{j\pi n}{2}}$$

$$= \frac{\sqrt{2}}{2} \cdot 2 \cdot \left(\frac{1}{4}\right)^{n} \cdot \left( \cos\left(\frac{\pi}{2} n + \frac{\pi}{4}\right)\right) \mu(n)$$

$$\lim_{x \to -1} \cancel{(x+1)} \cdot \frac{3x^2 + 12x + 11}{\cancel{(x+1)}(x+2)(x+3)}$$

$$\lim_{x \to -1} \frac{3x^2 + 12x + 11}{(x+2)(x+3)} = \frac{3 - 12 + 11}{1 \cdot 2} = \frac{2}{2} = 1 \quad A$$

IBM

c) MIRNI SUSAN

$$U(n) = \left\{ \underline{1}, 0, \frac{1}{16} \right\}$$ → POČ UVJETI = 0

$$\mu(n) = 1 \cdot \delta(n) + \frac{1}{16} \delta(n-2)$$

POČ UVJETI = 0

$$y(n) + \frac{1}{16} y(n-2) = \delta(n) + \frac{1}{16}\delta(n-2) - \frac{1}{4}\left[\cancel{\delta(n-1) + \frac{1}{16}\delta(n-3)}\right]$$

0 NULDA

0

$$y(n) = 1 \cdot h(n) + \frac{1}{16} h(n-2)$$

$$y(n) = 1 \cdot \left[\sqrt{2} \cdot \left(\frac{1}{4}\right)^n \cdot \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)\right] \mu(n) + \frac{1}{16}\left(\sqrt{2}\left(\frac{1}{4}\right)^{n-2} \cdot \cos\left(\frac{\pi}{2} \dots \frac{\pi}{4}\right)\right.$$

d) Z TRANSFOM.

$$U(n) = \delta(n) + \frac{1}{16}\delta(n-2)$$

$$U(z) = 1 + \frac{1}{16} z^{-2}$$

$$Y(z) = U(z) \cdot H(z)$$

$$= \left(1 + \frac{1}{16} z^{-2}\right) \cdot \left(\sqrt{\phantom{x}} H(z)\right)$$

- 4. $y''(t) + 7y'(t) + 10y(t) = u'(t)$

$u(t) = 10\cos(2t)\,\mu(t)$ $\quad y(0^-)=1$

$y'(0^-)=1$

a) PRIJENOSNA + STABILNOST

$$s^2Y(s) + 7sY(s) + 10Y(s) = sU(s)$$

$$H(s) = \frac{s}{s^2+7s+10} \Rightarrow$$ NULA 0, $\infty$

POLOVI

$s_1 = -2 \quad s_2 = -5$ STABILAN, $Re(s_i) < 0$

b) $$H(j\omega) = \frac{j\omega}{10-\omega^2+7j\omega} \Rightarrow |H(j\omega)| = \frac{\omega}{\sqrt{100-20\omega^2+\omega^4+49\omega^2}}$$

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^4+29\omega^2+100}}$$

$$\measuredangle H(j\omega) = \operatorname{Arctg}\left(\frac{\omega}{0}\right) - \operatorname{Arctg}\left(\frac{7\omega}{10-\omega^2}\right) = \frac{\pi}{2} - \operatorname{Arctg}\left(\frac{7\omega}{10-\omega^2}\right)$$

c) PRISILNI ODZIV

$u(t) = 10\cos(2t)$

$Y(s) = H(s)\cdot U(s)$ $\qquad \omega = 2$ $\qquad \dfrac{2}{\sqrt{16+29\cdot4+100}} = \dfrac{\sqrt{58}}{58}$

$$Y(t) = \left(\frac{\sqrt{58}}{58}\cdot 10\cdot\cos(2t+0.405)\right)\mu(t)$$

d) $y_h(t) = C_1e^{-2t} + C_2e^{-5t}$

$$y(t) = C_1e^{-2t} + C_2e^{-5t} + \frac{10\sqrt{58}}{58}\cdot\cos(2t+0.405)$$

$y(t) = c_1 e^{-2t} + c_2 e^{-5t} + \frac{10\sqrt{58}}{58} \cos(2t + 0.405)$

poi wojes

$y(0^-) = 1 \qquad y'(0^-) = 1$

NB

$y'(0^+) \neq y$

$y(0^+) - y(0^-) = 0 \qquad y(0^+) = y(0^-) = 1$

$y'(0^+) + y'(0^-) = 1(u(0^+) - u(0^-))$

$y'(0^+) - y'(0^-) = 10 u(0^+)$

$y'(0^+) = 10 + 1 = 11$

$y(0) = c_1 + c_2 + 1.207 = 1 \qquad c_1 = -0.207 - c_2$

$y'(0) = -2c_1 - 5c_2 - \frac{10\sqrt{58}}{58} \sin(2t + 0.405) \cdot 2 = 11$

$0.414 + c_2 - 5c_2 = 12.035$

$-4c_2 = 11.621$

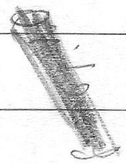
$c_1 = 2.698$

$c_2 = -2.905$

$y(t) = 2.698\, e^{-2t} + -2.905 e^{-5t} + \frac{10\sqrt{58}}{58} \cos(2t + 0.405) u(t)$

5. OD 0

1/12 2014



$$\frac{Y(z)}{z^0} + \frac{1}{2}\frac{Y(z)}{z^1} = \frac{U(z)}{z^0} - \frac{1}{2}\frac{U(z)}{z^{1}}$$

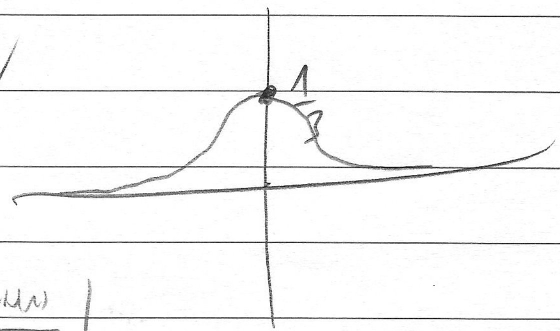
$$Y(z)\left(1+\frac{1}{2}z^{-1}\right) = U(z)\left(1-\frac{1}{2}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}} = \frac{z-\frac{1}{2}}{z+\frac{1}{2}}$$

a) $$|H(e^{j\Omega})| = \left|\frac{e^{j\Omega}-\frac{1}{2}}{e^{j\Omega}+\frac{1}{2}}\right| = \left|\frac{\cos(\Omega)+j\sin(\Omega)-\frac{1}{2}}{\cos(\Omega)+j\sin(\Omega)+\frac{1}{2}}\right|$$

$$= \sqrt{\frac{(\cos(\Omega)-\frac{1}{2})^2+\sin^2(\Omega)}{(\cos(\Omega)+\frac{1}{2})^2+\sin^2(\Omega)}} = \sqrt{\frac{\cos^2 - \cos(\Omega)+\frac{1}{4}+\sin^2(\Omega)}{\cos^2(\Omega)+\cos(\Omega)+\frac{1}{4}+\sin^2(\Omega)}}$$

$$= \sqrt{\frac{\frac{5}{4} \cancel{A\cos} - \cos(\Omega)}{\frac{5}{4}+\cos(\Omega)}} \checkmark$$




$$\sphericalangle H(e^{j\Omega}) = \arctan\left(\frac{\sin(\Omega)}{\cos(\Omega)-\frac{1}{2}}\right) - \arctan\left(\frac{\sin(\Omega)}{\cos(\Omega)+\frac{1}{2}}\right)$$

$$\frac{(\cos(\Omega)+j\sin(\Omega)-\frac{1}{2})}{\cos(\Omega)+j\sin(\Omega)+\frac{1}{2})}$$

c)

$$H(z) = \frac{Y(z)}{U(z)} = \frac{z-\frac{1}{2}}{z+\frac{1}{2}}$$

$$v(n) = \left(2\cos\left(\frac{\pi}{2}n\right)\right)\mu(n)$$

$$\Omega = \frac{\pi}{2}$$

$$H(e^{j\Omega})$$

$$|H(e^{j\frac{\pi}{2}})| = \sqrt{\frac{\frac{5}{4} - \cancel{\cos(\frac{\pi}{2})}}{\frac{5}{4} + \cancel{\cos(\frac{\pi}{2})}}} = 1$$

$$\measuredangle H(e^{j\Omega}) = \text{Arctg}\left(\frac{1}{-\frac{1}{2}}\right) - \text{Arctg}\left(\frac{1}{\frac{1}{2}}\right)$$

$$= \text{Arctg}(-2) - \text{Arctg}(2) =$$

$$= -2.214$$

$$y_p(n) = \left(2 \cdot 1 \cdot \cos\left(\frac{\pi}{2}n + 0.93\right)\right)\mu(n)$$