

### ZADATAK 1

$$h(x) = \underset{C_i}{\operatorname{argmax}} P(x|C_i) \cdot P(C_i)$$

a) Bayesov klasif. temelji se na Bayesovom pravilu

$$P(C_j|x) = \frac{P(x|C_j) \cdot P(C_j)}{\sum_{k=1}^K P(x|C_k) \cdot P(C_k)}$$

$P(C_j)$  - apriorna vj. klase

$P(x|C_j)$  - izglednost klase

↳ klasom ujetovana gustoća

↳ zajednička gustoća  $P(x, C_j)$

$P(C_j|x)$  - aposteriorna vj. da primjer  $x$  pripada klasi  $C_j$

↳ marginalizacija  $\Rightarrow$  marginalna gustoća  $p(x)$  je jednaka za sve klase  $C_j$ ; služi samo za normalizaciju na jedinični interval

b) Model je parametarski budući da se podaci pokravaju teorijskim razdiobama, pa se učenje svodi na izračunavanje parametara tih razdioba na temelju podataka

↳ binaрни  
parametri  
izraz  
funkcija

c) To je generativni model jer modeliramo zajedničku vj. koja opisuje način generiranja podataka.

$$P(x, C_j) = P(x|C_j) \cdot P(C_j)$$

$$P(C_j|x) = \frac{P(x|C_j) \cdot P(C_j)}{P(x)} \quad \rightarrow \text{opisuje generiranje}$$

## ZADATAK 2

$$\begin{aligned} P(C_1) &= 0.2 \\ P(C_2) &= 0.05 \\ P(C_3) &= 0.75 \end{aligned}$$

$$P(\vec{x}|C_1) = 0.8$$

$$\stackrel{\rightarrow}{P}(\vec{x}|C_2) = 0.5$$

$$\stackrel{\rightarrow}{P}(\vec{x}|C_3) = 0.5$$

$$a) \quad P(C_1|\vec{x}) = \frac{0.8 \cdot 0.2}{0.8 \cdot 0.2 + 0.05 \cdot 0.5 + 0.75 \cdot 0.5} = \frac{2}{7} \approx 0.286$$

$$P(C_2|\vec{x}) = \frac{0.05 \cdot 0.5}{0.8 \cdot 0.2 + 0.05 \cdot 0.5 + 0.75 \cdot 0.5} = \frac{5}{112} \approx 0.045$$

$$P(C_3|\vec{x}) = \frac{0.75 \cdot 0.5}{0.8 \cdot 0.2 + 0.05 \cdot 0.5 + 0.75 \cdot 0.5} = \frac{75}{112} \approx 0.669$$

$$h(\vec{x}) = \arg \max_{C_k} p(\vec{x}|C_k) \cdot p(C_k) = \underline{\underline{C_3}}$$

b)

$$L_1 = \begin{pmatrix} 0 & 5 & 1 \\ 100 & 0 & 5 \\ 10 & 1 & 0 \end{pmatrix} \quad \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} \quad \text{strana}$$

$$C_2 = C_3 \quad \text{od} \quad C_1 = C_3$$

$$C_3 = C_1$$

rat. p. kao span

$$R(C_j|x) = \sum_{k=1}^K L_{kj} \cdot P(C_k|x)$$

$$\begin{aligned} R(C_1|x) &= \sum_{k=1}^K L_{kj} \cdot P(C_k|x) = 0 \cdot P(C_1|x) + 100 \cdot P(C_2|x) + 10 \cdot P(C_3|x) \\ &= 100 \cdot \frac{5}{112} + 10 \cdot \frac{75}{112} = \frac{625}{56} = 11.16 \end{aligned}$$

$$R(C_2|x) = 5 \cdot P(C_1|x) + 0 \cdot P(C_2|x) = \frac{5 \cdot 2}{7} + \frac{75}{112} = \frac{235}{112} \approx 2.098$$

$$R(C_3|x) = P(C_1|x) + 2 \cdot P(C_2|x) = \frac{2}{7} + 2 \cdot \frac{5}{112} = \frac{3}{8} = 0.375$$

$$h_j(x) = \operatorname{argmin}_{C_k} R(C_k|x) = \underline{\underline{C_3}}$$

$$c) P^l(C_1) = 0.3$$

$$P^l(C_2) = 0.3$$

$$\underline{P^l(C_3) = 0.4}$$

posterior  
ij. 1.2e

$$P^u(C_1|x) = \frac{P(C_1) \cdot P(x)}{P(C_1)} = 0.19$$

$$P^u(C_2|x) = 0.0075$$

$$P^u(C_3|x) = 1.256$$

$$P(C_1|x) = \frac{0.19}{1.435} = 0.13$$

$$P(C_2|x) = 0.006$$

$$P(C_3|x) = 0.864$$

$$P(C_3|x) = 0.864$$

$$L_2 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$P(C_1|x) = \frac{P(x|C_1) \cdot P(C_1)}{\sum_i P(x|C_i) \cdot P(C_i)} = \frac{0.3 \cdot 0.8}{0.3 \cdot 0.8 + 0.3 \cdot 0.5 + 0.4 \cdot 0.5} = \frac{24}{59} = 0.407$$

$$P(C_2|x) = \frac{0.3 \cdot 0.5}{-11} = \frac{15}{59} = 0.259$$

$$P(C_3|x) = \frac{0.4 \cdot 0.5}{-11} = \frac{20}{59} = 0.338$$

+ rechts je ut ist i.yf.

$$2) R(C_1|x) = P(C_1|x) + P(C_3|x) = \frac{35}{59}$$

$$R(C_2|x) = \frac{24}{59} + \frac{20}{59} = \frac{44}{59}$$

$$R(C_3|x) = \frac{39}{59}$$

$$h_2(x) = \underset{C_2}{\operatorname{argmin}} R(C_2|x) = \underline{\underline{C_1}}$$

$$3) L_3 = L_1 - \begin{pmatrix} 0 & 5 & 1 \\ 100 & 0 & 5 \\ 10 & 1 & 0 \end{pmatrix}$$

$$R(C_1|x) = 100 \cdot \frac{15}{59} + 10 \cdot \frac{20}{59} = \frac{1200}{59} = 20.34$$

$$R(C_2|x) = 5 \cdot \frac{24}{59} + \frac{39}{59} = \frac{159}{59} = 2.695$$

$$R(C_3|x) = \frac{24}{59} + 5 \cdot \frac{15}{59} = \frac{99}{59} = 1.678$$

$$h_3(x) = \underset{C_3}{\operatorname{argmin}} R(C_3|x) = \underline{\underline{C_3}}$$

### ZADATAK 3

#### a) Naivni Bayes

$$D = \{x^{(i)}, y^{(i)}\}_{i=1}^n$$

$$y^{(i)} \in \{c_1, \dots, c_k\}$$

N.B.

$$h(\vec{x}) = \operatorname{argmax}_{j'} \prod_{k=1}^n P(x_k | c_j)$$

$$h(x_1, \dots, x_n) = \operatorname{argmax}_j P(x_1, \dots, x_n | c_j) \cdot P(y=c_j)$$

b)  $\rightarrow$  pretpostavka o ujetnoj nez. značajki za klasu

$$P(x_1, \dots, x_n) = P(x_1) \cdot P(x_2 | x_1) \cdot P(x_3 | x_1, x_2) \cdots P(x_n | x_1, \dots, x_{n-1})$$

$$\begin{aligned} P(x_1, \dots, x_n | c_j) &= P(x_1 | c_j) \cdot P(x_2 | x_1, c_j) \cdots P(x_n | x_1, \dots, x_{n-1}, c_j) \\ &= \prod_{k=1}^n P(x_k | x_1, \dots, x_{k-1}, c_j) \quad x_i \perp x_j | c_j \text{ i.d.u.} \end{aligned}$$

$$\Rightarrow P(x_i | x_j, c) = P(x_i | c)$$

$$\begin{aligned} P(x_1, \dots, x_n | c_j) &= P(x_1 | c_j) \cdot P(x_2 | x_1, c_j) \cdots P(x_n | x_1, \dots, x_{n-1}, c_j) \\ &= P(x_1 | c_j) \cdot P(x_2 | c_j) \cdots P(x_n | c_j) = \prod_{k=1}^n P(x_k | c_j) \end{aligned}$$

c) Zbog velikog broja parametara imat će vrlo visoku varijancu

i bit će stolje prenaučenosti. On sa vremenom klasif. primjere

iz skupa za učenje, ali svim nevidenim daje greš. O.

Pretpostavka nam treba kako bi smanjili broj par modela, a time i složenost modela. Priпадa pristranosti jezika, odnosno pristranosti ograničavanjem.

d) 10 klasa

matrica  $32 \times 32$

broj parametara?

$$10 \text{ klasa} \rightarrow K=10 \Rightarrow K-1 = 9 + 32^2 \cdot 10$$

$$+ \sum_{k=1}^n (K_k - 1) \cdot K$$

?

$10240$

$$K=10$$

$$n = 32 \times 32 = 1024$$

$$h(\vec{x}) = \operatorname{argmax}_c P(c)$$

$$P(c=0), \dots, P(c=9) \Rightarrow \text{jedna nula}$$

$$\prod_{k=1}^n P(x_k | c)$$

$$10240$$

$$\prod_{k=1}^n P(x_k | c)$$

čim se bira  
složenost  
i pristranost

kompat  
s obzirom

binarni  
obziri

10240

## ZADATEK 4

a) HL. PROG.

$$P(\text{dal} | \text{istra, ne, kamp, bus})$$

$$= P(\text{Istra}|\text{da}) \cdot P(\text{ne}|\text{da}) \cdot P(\text{kamp}|\text{da}) \cdot P(\text{bus}|\text{da})$$

$$\cdot P(\text{da})$$

$$= \frac{2}{2} \cdot \frac{1}{4} \cdot \frac{0}{2} \cdot \frac{0}{2} \cdot \frac{4}{7} = 0$$

$$P(\text{ne} | \text{istra, ne, kamp, bus}) = P(\text{Istra}|\text{ne}) \cdot P(\text{ne}|\text{ne}) \cdot P(\text{kamp}|\text{ne})$$

$$P(\text{bus}|\text{ne}) = \frac{0}{2} \cdot \frac{3}{4} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{3}{7} = \frac{9}{56} = 0$$

$$h(\text{Istra, ne, kamp, bus}) = \underset{\substack{\text{argmax} \\ \text{C} \in \{\text{da}, \text{ne}\}}}{P(C|x)} = \begin{cases} \text{ne} & \text{maji} \\ \text{da} & \text{progoniti} \end{cases}$$

$$P(\text{dal} | \text{Dalmacija, da, hotel, bus}) = P(\text{Dalm.}|\text{da}) \cdot P(\text{da}|\text{da})$$

$$P(\text{hotel}|\text{da}) \cdot P(\text{bus}|\text{da}) = \frac{2}{3} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{0}{2} = 0$$

$$P(\text{ne} | \text{Dalm.}, \text{da}, \text{hotel}, \text{bus}) = P(\text{Dalm}|\text{ne}) \cdot P(\text{da}|\text{ne}) \cdot P(\text{h}|\text{ne}) \cdot P(\text{bus}|\text{ne})$$

$$P(\text{h}) = \frac{1}{3} \cdot \frac{0}{3} \cdot \frac{0}{2} \cdot \frac{2}{2} = 0$$

$$\rightarrow \text{ne može progoniti} \quad P(\text{h} = \text{da}) = \frac{2+1}{4+3}$$

b) LAPLACEOV PROG.

$$P(X_k|G) = \frac{N_{kj} + 1}{N + k}$$

?

$$P(\text{rijest} = \text{istra})$$

NE

VAYA

U - bosi mog. vij.

majice x

LAPLACE ?

$$P(\text{dal} | \text{istra, ne, kamp, bus}) =$$

$$= P(\text{Istra}|\text{da}) \cdot P(\text{ne}|\text{da}) \cdot P(\text{kamp}|\text{da}) \cdot P(\text{bus}|\text{da}) \cdot P(\text{da})$$

$$= \frac{2+1}{2+3} \cdot \frac{1+1}{4+2} \cdot \frac{0+1}{2+3} \cdot \frac{0+1}{2+3} \cdot \frac{4}{7} = \frac{4}{875} = 4,57 \cdot 10^{-3}$$

$$0,001662$$

$$P(\text{ne} | \text{istra, ne, kamp, bus}) = \frac{2}{5} \cdot \frac{4}{6} \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{3}{7} = 0,073$$

$$0,0143$$

$$h(\text{Istra, ne, kamp, bus}) = \underline{\text{ne}}$$

$$0,0099$$

$$P(\text{dal} | \text{Dalm.}, \text{da}, \text{hotel}, \text{bus}) = \frac{2+1}{3+3} \cdot \frac{3+1}{3+2} \cdot \frac{2+1}{2+3} \cdot \frac{0+1}{2+3} \cdot \frac{4}{7} = 0,027$$

$$P(\text{ne} | \text{Dalm.}, \text{da}, \text{hotel}, \text{bus}) = \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{3}{7} = 0,027428$$

$$0,0011$$

$$h(\text{Dalm.}, \text{da}, \text{hotel}, \text{bus}) = \underline{\text{ne}} \text{ može procj. } \checkmark \text{ DNA}$$

ZADANJE 5

a) entropija  $E(x) = -\sum_x p(x) \cdot \ln p(x)$   
 $E_R(x) = -\sum_x p(x) \ln_2(x)$

$$D_{KL}(p \parallel g) = E_R(x) - E(x)$$

$$= -\sum_x p(x) \ln g(x) + \sum_x p(x) \cdot \ln p(x)$$

$$= \sum_x p(x) (\ln p(x) - \ln g(x))$$

$$= \sum_x p(x) \ln \frac{p(x)}{g(x)}$$

$$D_{KL}(p(x,y) \parallel p(x) \cdot p(y)) = \sum_y \sum_x p(x,y) \ln \frac{p(x,y)}{p(x)p(y)} = I(x,y)$$

b)

x\y	1	2	3	
1	0.2	0.05	0.3	0.55
2	0.05	0.3	0.1	0.4
	0.25	0.35	0.4	

$$I(x,y) = \sum_y \sum_x \ln \frac{p(x,y)}{p(x) \cdot p(y)} = 0.2 \ln \frac{0.2}{0.55 \cdot 0.75} + 0.05 \ln \frac{0.05}{0.55 \cdot 0.25}$$

$$+ 0.05 \ln \frac{0.05}{0.55 \cdot 0.35} + 0.3 \ln \frac{0.3}{0.45 \cdot 0.35} + 0.3 \ln \frac{0.3}{0.45 \cdot 0.35}$$

$$+ 0.1 \ln \frac{0.1}{0.45 \cdot 0.1} = 0.1945$$

Nisu nezavisne?

vrijeme info nije nezavisno!

da je O  $\Rightarrow$  onda su potpuno  
nezavisne

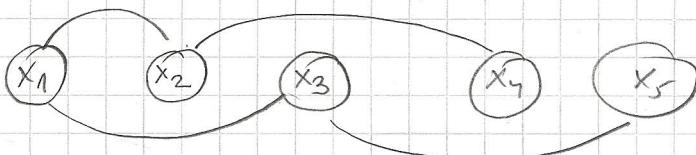
ZADATAK 0

40, 13<sup>5</sup>

$$\rightarrow I(x_3, C) > I(x_1, C) > I(x_2, C) > I(x_4 | C) > I(x_5 | C)$$

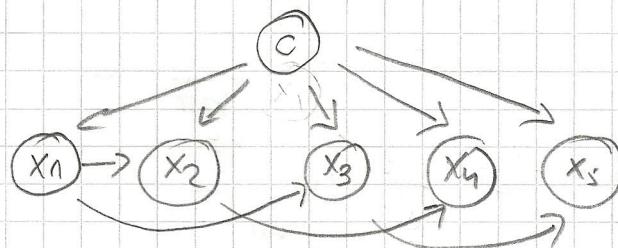
$$\begin{aligned} \rightarrow I(x_1, x_3 | C) &> I(x_2, x_4 | C) > I(x_1, x_2 | C) > I(x_3, x_4 | C) > I(x_1, x_4 | C) \\ &> I(x_3, x_5 | C) > I(x_1, x_5 | C) > I(x_2, x_3 | C) > I(x_2, x_5 | C) > I(x_3, x_5 | C) \end{aligned}$$

a) ALGORITAM TAN (Bayesova mreža)



n-1 bridom

~~do k ne deljimo~~  
povezan graf



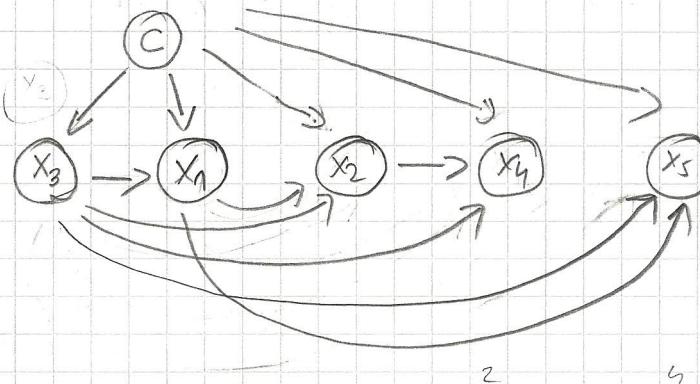
broj parametara:  
 $X = 60, 13^5$   
 $K = 2$

$$P(C|x_1, x_2, x_3, x_4, x_5) \propto P(y_1|C) \cdot P(x_2|x_1, C) \cdot P(x_3|x_1, C) \cdot P(x_4|x_2, C) \cdot P(x_5|x_3, C) \cdot P(C)$$

broj kom.s. = 1

$$= 13$$

b) algoritam 2-DB



P  
depo  
zhanj!

$$P(C|x_1, x_2, x_3, x_4, x_5) \propto P(y_1|C) \cdot P(x_1|x_3, C) \cdot P(x_2|x_1, x_3, C) \cdot P(x_4|x_2, x_3, C) \cdot P(x_5|x_1, x_3, C) \cdot P(C)$$

n.k

$$= 21$$

c) NAIVAN  $K \leq 1$   $\therefore 1 + 2 \cdot 5 = 11$

~~$TAN = 1 + 2 \cdot 2 + 4 \cdot 3 \cdot 2 = 1 + 4 + 24 = 29$~~

~~$DB = 1 + 2 \cdot 2 + 2 \cdot 8 \cdot 2 + 2 \cdot 3 \cdot 4 = 5 + 12 + 24 = 41$~~

## ZADATAK 7

$X = \mathbb{R}$  3 klase

$$P(C_1) = 0.3$$

$$P(C_2) = 0.2$$

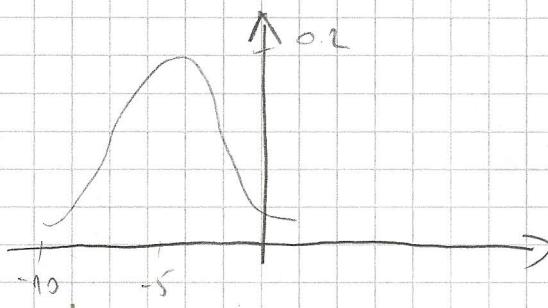
$$P(C_3) = 1 - 0.3 - 0.2 = 0.5$$

$$\mu_1 = -5, \mu_2 = 0, \mu_3 = 5 \\ \sigma_1^2 = 5 \quad \sigma_2^2 = 1 \quad \sigma_3^2 = 10$$

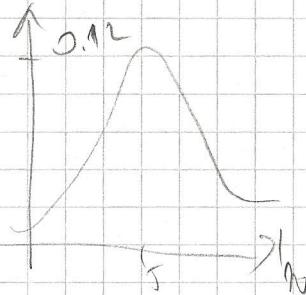
$p(x|g), p(x|c_1), p(x), p(g|x)$

$$p(x|g) = \frac{1}{\sqrt{2\pi}\sigma_g} \exp \left\{ -\frac{(x-\mu_g)^2}{2\sigma_g^2} \right\}$$
$$p(x|C_1) = \frac{1}{\sqrt{2\pi}\sqrt{5}} e^{-\frac{(x+5)^2}{2 \cdot 5}} = \frac{1}{\sqrt{10\pi}} e^{-\frac{(x+5)^2}{10}}$$
$$p(x|C_2) = \frac{1}{\sqrt{2\pi \cdot 1}} e^{-\frac{x^2}{2}} = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$
$$p(x|C_3) = \frac{1}{\sqrt{2\pi \cdot 10}} e^{-\frac{(x-5)^2}{2 \cdot 10}} = \frac{1}{\sqrt{20\pi}} \cdot e^{-\frac{(x-5)^2}{20}}$$

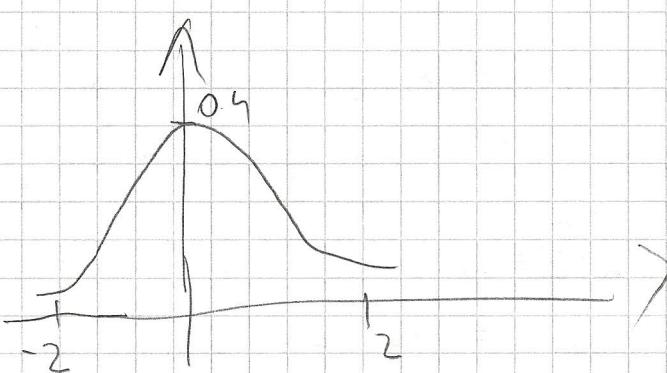
$p(x|C_1)$



$p(x|C_2)$



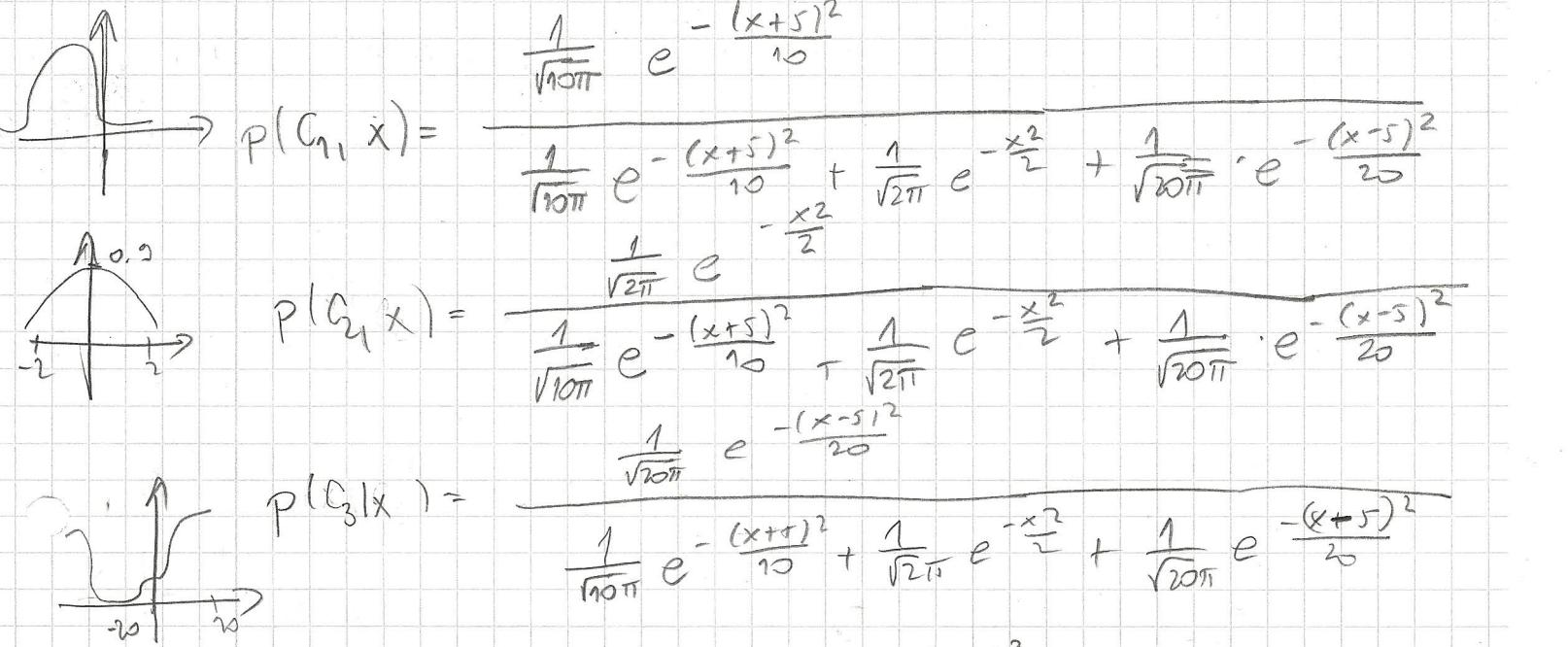
0.5



0.5 je gustiče vjerojatnost.

$$p(x) = p(x|C_1) \cdot P(C_1) + p(x|C_2) \cdot P(C_2) + p(x|C_3) \cdot P(C_3)$$

$$= \frac{1}{\sqrt{10\pi}} e^{-\frac{(x+5)^2}{10}} + \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} + \frac{1}{\sqrt{20\pi}} \cdot e^{-\frac{(x-5)^2}{20}}$$



$$p(C_1|x) = \frac{\frac{1}{\sqrt{10\pi}} e^{-\frac{(x+5)^2}{10}}}{\frac{1}{\sqrt{10\pi}} e^{-\frac{(x+5)^2}{10}} + \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} + \frac{1}{\sqrt{20\pi}} \cdot e^{-\frac{(x-5)^2}{20}}}$$

$$p(C_2|x) = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}{\frac{1}{\sqrt{10\pi}} e^{-\frac{(x+5)^2}{10}} + \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} + \frac{1}{\sqrt{20\pi}} \cdot e^{-\frac{(x-5)^2}{20}}}$$

$$p(C_3|x) = \frac{\frac{1}{\sqrt{20\pi}} \cdot e^{-\frac{(x-5)^2}{20}}}{\frac{1}{\sqrt{10\pi}} e^{-\frac{(x+5)^2}{10}} + \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} + \frac{1}{\sqrt{20\pi}} \cdot e^{-\frac{(x-5)^2}{20}}}$$

$$p(x|C_1) = p(x|C_1) \cdot p(C_1) = \frac{0.3}{\sqrt{10\pi}} \cdot e^{-\frac{(x+5)^2}{10}}$$

$$p(x|C_2) = \frac{0.2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$p(x|C_3) = \frac{0.5}{\sqrt{20\pi}} e^{-\frac{(x-5)^2}{20}}$$

## ZADATAK 8

$$X = \mathbb{R}^n, \mathcal{N}(\mu_j, \Sigma_j)$$

OPĆENITI BAYES

$$p(x|g_j) = \frac{1}{(2\pi)^{n/2} |\Sigma_j|^{1/2}} \cdot \exp \left\{ -\frac{1}{2} (x - \mu_j)^T \Sigma_j^{-1} (x - \mu_j) \right\}$$

$$h_j(x) = \ln p(x|g_j) + \ln P(g_j)$$

$$= \ln (2\pi)^{-n/2} + \ln |\Sigma_j|^{-1/2} + \ln e^{-\frac{1}{2} (x - \mu_j)^T \Sigma_j^{-1} (x - \mu_j)} + \ln P(g_j)$$

$$= -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln |\Sigma_j| - \frac{1}{2} (x - \mu_j)^T \Sigma_j^{-1} (x - \mu_j) + \ln P(g_j)$$

konst.

$$= -\frac{1}{2} \ln |\Sigma_j| - \frac{1}{2} (x - \mu_j)^T \Sigma_j^{-1} (x - \mu_j) + \ln P(g_j)$$

$$\begin{aligned} &= -\frac{1}{2} \ln |\Sigma_j| - \frac{1}{2} x^T \Sigma_j^{-1} x + \frac{1}{2} x^T \Sigma_j^{-1} \mu_j + \frac{1}{2} \mu_j^T \Sigma_j^{-1} x - \frac{1}{2} \mu_j^T \Sigma_j^{-1} \mu_j + \ln P(g_j) \\ &= -\frac{1}{2} \ln |\Sigma_j| - \frac{1}{2} x^T \Sigma_j^{-1} x + x^T \Sigma_j^{-1} \mu_j - \frac{1}{2} \mu_j^T \Sigma_j^{-1} \mu_j + \ln P(g_j) \end{aligned}$$

Dijeyenata

$$\hat{\Sigma} = \sum_j \hat{P}(g_j) \cdot \hat{\Sigma}_j$$

$$h_j(x) = -\frac{1}{2} \ln \hat{\Sigma} - \frac{1}{2} x^T \hat{\Sigma}^{-1} x + x^T \Sigma_j^{-1} \mu_j - \frac{1}{2} \mu_j^T \Sigma_j^{-1} \mu_j + \ln P(g_j)$$

konst.      konst.

$$= x^T \Sigma_j^{-1} \mu_j - \frac{1}{2} \mu_j^T \Sigma_j^{-1} \mu_j + \ln P(g_j)$$

Uz pretpostavku kako var. nisu korelirane  $\Rightarrow$  dijagonalna kov. matrica

$$\Sigma = \text{diag}(\sigma_i^2) \Rightarrow \text{invert } \hat{\Sigma} = 1/\sigma_i^2 I$$

$$h_j(x) = \sum_{i=1}^n \frac{x_i \cdot \mu_{ij}}{\sigma_i^2} - \frac{1}{2} \sum_{i=1}^n \frac{\mu_{ij}^2}{\sigma_i^2} + \ln P(g_j)$$

$$= \sum_{i=1}^n \left( \frac{x_i \cdot \mu_{ij}}{\sigma_i^2} - \frac{\mu_{ij}^2}{2\sigma_i^2} \right) + \ln P(g_j)$$

$$= \sum_{i=1}^n \frac{\mu_{ij}}{\sigma_i^2} (-2x_i + \mu_{ij}) + \ln P(g_j)$$

$\Rightarrow$  dodaje se redundantičan član  $x^2$  (jednako u svim hipotezama)

pa nam nista ne znaci za maksim.

$$h_j(x) = -\frac{1}{2} \sum_{i=1}^n \frac{x_i^2 - 2x_i \mu_{ij} + \mu_{ij}^2}{\sigma_i^2} + \ln P(g)$$

$$= -\frac{1}{2} \sum_{i=1}^n \left( \frac{x_i - \mu_{ij}}{\sigma_i} \right)^2 + \ln P(g)$$

### ZADATAK 9

$G_1$  = "završava FER u roku"

$G_2$  = "produžuje studij"

$\rightarrow$  svaki primjer ima 6 ulaznih varijabli  $\Rightarrow 1. - 4.$

$H_1 \Rightarrow$  dijeljena kov. m.

$H_2 \Rightarrow$  dijagonalna kov.m.

$H_3 \Rightarrow$  izotropna kov.m.

a)  $H_1 \Rightarrow K_n \Rightarrow$  vektori srednjih vrijednosti  $6 \cdot 2 = 12$

$\frac{n}{2}(n+1) \Rightarrow$  kovarijacijska  $\frac{6}{2} \cdot 7 = 21$

$K-1 \Rightarrow$  apriorne

1

34 parametara

$H_2 \Rightarrow K_n \Rightarrow$  srednje v.  $6 \cdot 2 = 12$

$n \Rightarrow$  kovarij. 6

$K-1 \Rightarrow$  apriori 1

19 parametara

$H_3 \Rightarrow K_n \Rightarrow$  srednje 12

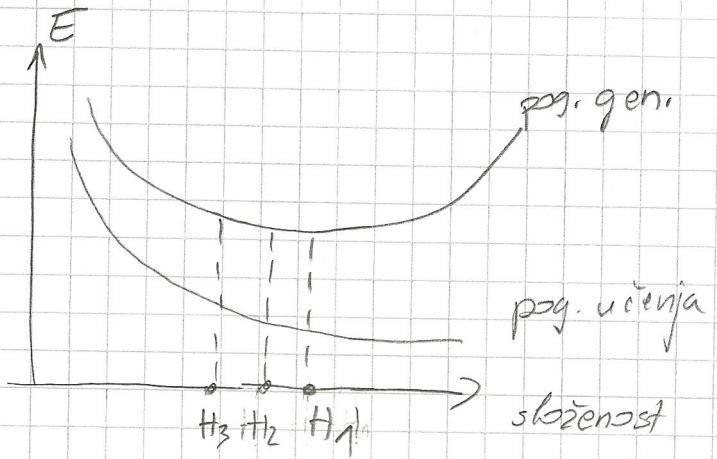
$n \Rightarrow$  kov. 1

$K-1 \Rightarrow$  apriorne 1

14 parametara

b) Razumno je pretpostaviti da postoji korelacija između prosjeku u sr. šk. i uspjeha na drž. mat; uz to i da će uspjeh učenika rasti kroz razrede,  $\Rightarrow$  veća var. u 1. i 2., a manja u 3. i 4. zbog prilagođbe

Model  $H_1$  bio bi najbolji jer druga dva uvođe previše pretpostavki i postaju prejednostavni.



d) Na odluku utječe više faktora

→ ravnaju li se primjeri prema nekoj dist. i boji?

→ želimo li provjeriti točnost zbog memorije / vremena  
ali najviše ovisi o samom problemu.

Isprobati više različitih modela, usporediti empiriske,  
pog. gen., usporediti perf.

→ eksperimentalno UNAPREDJENA provjera

## ZADATAK 10

$$L(y, h(x|\theta)) \quad , \quad \in (\theta, D)$$

a) Izrađiti f-ju qubitku B.k. preko f-je izglednog:

$$\ln L(y | h(x|\theta))$$

$$Rj: L(y, h(x|\theta)) = -N \ln p(x|\theta)$$

na slajdu!!! qf 6

b)

