30.9.2013.

ELASTIČNOST TVARI

- kruto tijelo se može pod utjecajem sile gibati, ali i deformirati

DEFORMACIJE

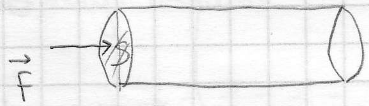
1. ELASTIČNE
2. - da se vrati u poč. položaj - def nije stalna   DJELOMIČNO EL.
3. ne vraća se u poč. pol.   TRAJNE

- mikroskopska građa tijela - utječe na
- makroskopski - mi gledamo

MATERIJALI {
1. svojstva mat. u svim smjerovima jednaka   IZOTROPNI
2. ANIZOTROPNI - npr. drvo - zbog strukture, smjera rasta nema svojstva u svim smjerovima ista

VRSTE DEFORMACIJA

Naprezanje:



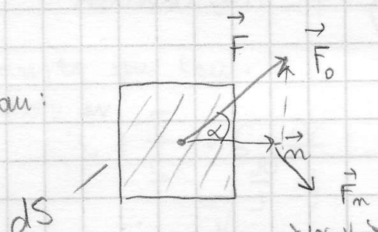
napetost

$$\sigma = \frac{F}{S} \qquad \sigma = \left[\frac{N}{m^2}\right] = Pa$$

$$F \perp S$$

1. tlačno
2. vlačno - drugi smjer sile

ukoliko je sila pod kutom:



jed. vektor u smjeru normale na površinu

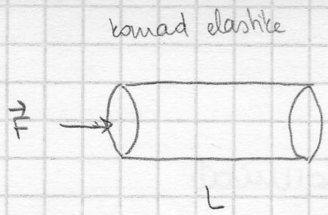
$$d\vec{S} = \vec{n}\, dS$$

1. $\sigma_n = \frac{F_n}{dS} = \frac{F\cos\alpha}{dS}$
2. $\sigma_o = \frac{F_o}{dS} = \frac{F\sin\alpha}{dS}$

$\Longleftarrow$

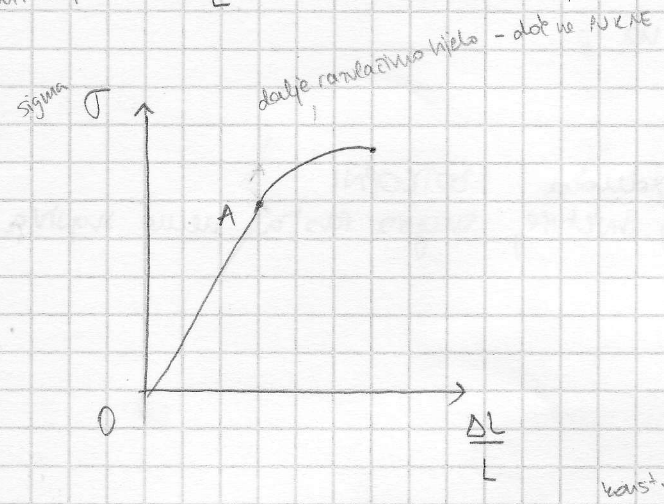
$\sin\alpha = \frac{F_o}{F}$

$\cos\alpha = \frac{F_n}{F}$



Izvlačenje : $\rightarrow L + \Delta L$

kontrakcija štapa, relativno produljenje: $\frac{\Delta L}{L}$

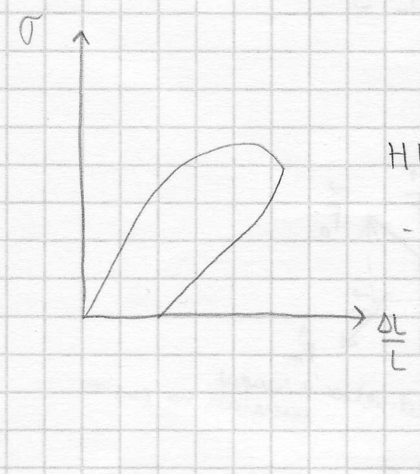


$\sigma \sim \frac{\Delta L}{L}$

$\sigma = E \frac{\Delta L}{L}$ — E: konst. prop.; Youngov modul elastičnosti

$\sigma = \frac{F}{S}$

HOOKEOV ZAKON



HISTEREZA

- mat. nije potpuno elastičan, ne vraća se u poč. pd. i ne vraća se na isti način

POISSONOV BROJ

$\mu = \frac{\frac{\Delta d}{d}}{\frac{\Delta l}{l}}$ → okomito

mora biti pozitivna vel. $\leq 0,5$

negativno - kontrakcija

3D

- djeluju sile u svim smjerovima - uzrokuju promjenu volumena

$$p=\frac{F}{S} = -B\,\frac{\Delta V}{V}$$

$V + \Delta V$ — volumni modul elastičnosti

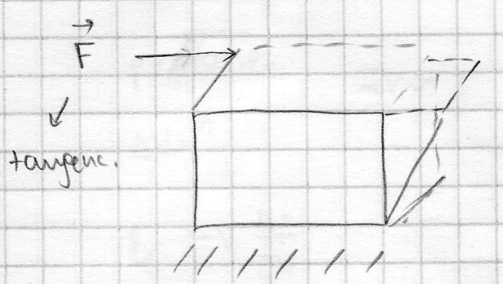
kapa $\kappa = \frac{1}{B}$ kompresibilnost ili stlačivost materijala

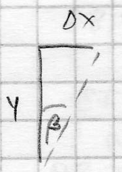
- el. sila je sila prop. pomaku iz pol. ravnoteže
- proizvodi titraje
- zamišljamo da su svi atomi povezani oprugama

$\vec{F}_{el} = -k\Delta\vec{l}$ elastična sila

SMICANJE

- primjer deform.





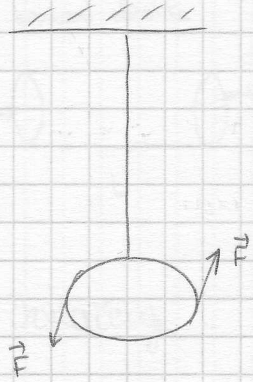
$$\operatorname{tg}\beta = \frac{\Delta x}{y}$$

za male deform. $\operatorname{tg}\beta \approx \beta = \frac{\Delta x}{y}$

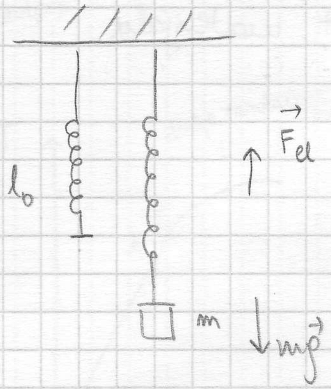
$$\tau = \frac{F}{S} = G\cdot\frac{\Delta x}{y}$$

G — modul smicanja

$$\boxed{\tau = G\cdot\beta}$$



torzija – vrsta smicanja



(ravn. pol. $F_{el} = mg$)

$l = l_0 + \Delta l$

Harmonički oscilator

- idealno titranje pod utjecajem el. sile, da tijelo beskonačno titra

, dulji spiralne opruge

m/kg	k	k/2	k/3	k/4
2				0,488
4				0,73
6				0,889
8	0,503	0,721	0,869	1,021

period

↓ $T \sim \sqrt{m}$

← $T \sim \sqrt{\frac{1}{k}}$ $\quad T \sim \sqrt{\frac{m}{k}}$

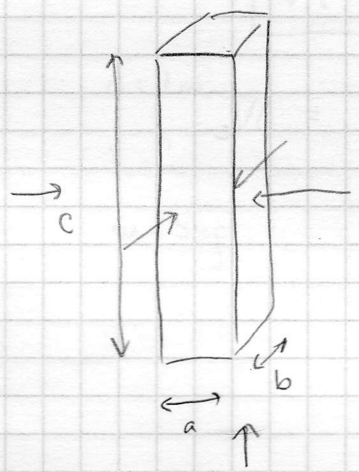
AUDITORNE

teorija

1.2. Prizmatični čelični štap dimenzija 1 m × 0,2 m × 0,1 m opterećen je sa svih strana naprezanjem (tlakom) 10 GPa. Koliko iznosi smanjenje volumena štapa zbog tog opterećenja? Koliki je volumni modul elastičnosti

$$B = -V\frac{\Delta p}{\Delta V}$$

ako je Youngov modul el. $E = \sigma \frac{l}{\Delta l} = 200 \text{ GPa}$, a Poissonov broj, tj. omjer relativne poprečne kontrakcije i relativnog uzdužnog produljenja $\mu = 0{,}3$?



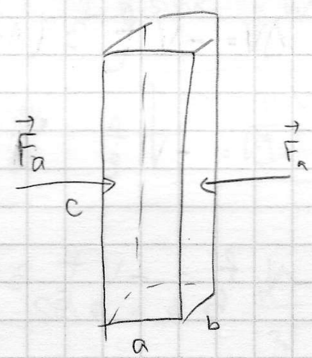
$$E = \sigma\frac{l}{\Delta l} = 200 \text{ GPa}$$

$$\mu = 0{,}3$$

$p = 10 \text{ GPa}$

$\Delta V = ?$

$$B = -V\frac{\Delta p}{\Delta V}$$



$$\mu = -\frac{\frac{\Delta b}{b}}{\frac{\Delta a}{a}} = -\frac{\frac{\Delta c}{c}}{\frac{\Delta a}{a}}$$

$$\frac{\Delta b}{b} = \frac{\Delta c}{c} = -\mu\frac{\Delta a}{a}$$

$$a' = a + \Delta a$$

$$b' = b + \Delta b$$

$$c' = c + \Delta c$$

$$V' = V + \Delta V$$

$$= a'b'c'$$

$$= (a+\Delta a)(b+\Delta b)(c+\Delta c)$$

$$= a\left(1+\frac{\Delta a}{a}\right) b\left(1+\frac{\Delta b}{b}\right) c\left(1+\frac{\Delta c}{c}\right)$$

$$= \underbrace{abc}_{V} \left(1+\frac{\Delta a}{a}\right)\left(1+\frac{\Delta b}{b}\right)\left(1+\frac{\Delta c}{c}\right)$$

$$V' = V\left(1+\frac{\Delta a}{a}\right)\left(1-\mu\frac{\Delta a}{a}\right)\left(1-\mu\frac{\Delta a}{a}\right)$$

$$= V\left[\left(1+\frac{\Delta a}{a}-\mu\frac{\Delta a}{a}-\mu\frac{\Delta a}{a}-\mu\cancelto{0}{\left(\frac{\Delta a}{a}\right)^2}\cdot 2+\mu^2\cancelto{0}{\left(\frac{\Delta a}{a}\right)^3}\right]$$

$$\simeq V + \underbrace{V\frac{\Delta a}{a}(1-2\mu)}_{\equiv \Delta V_a}$$

$$\frac{\Delta a}{a} = -\frac{f}{E}$$

$$\boxed{\Delta V_a = V\cdot\left(-\frac{f}{E}\right)(1-2\mu)}$$

$$\Delta V_b = -V\frac{f}{E}(1-2\mu) = \Delta V_c$$

$$\Delta V = \Delta V_a + \Delta V_b + \Delta V_c$$

$$\Delta V = -V\frac{f}{E}3(1-2\mu)$$

$$\Delta V = -V\frac{f}{B}$$

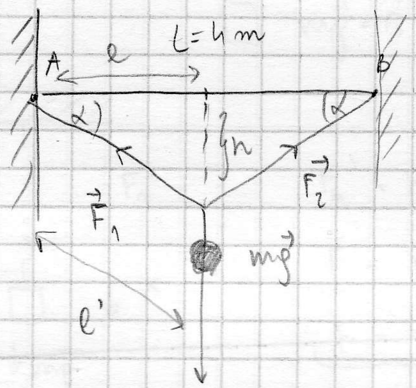
$$-V\frac{f}{B} = -V\frac{f}{E}3(1-2\mu)$$

$$E = 3B(1-2\mu)$$

$$B = \frac{E}{3(1-2\mu)}\,, \quad \underline{\mu < \frac{1}{2}}$$

Horvat, pr. 1.2   str. 1-6

Čelična žica promjera $d = 1\,mm$ razapeta je u horizontalnom smjeru između 2 čvrste točke A i B koje su udaljene $L = 4\,m$. Za koliko će se spustiti središnja točka ako na žicu objesite uteg mase 3 kg?



$d = 1\,mm$

$h = ?$

$m = 3\,kg$

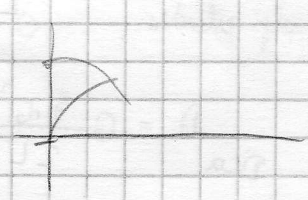
$$\underline{\sum \vec{F} = 0}$$

$$x: \quad F_1 \cos\alpha - F_2 \cos\alpha = 0 \quad \Rightarrow \quad F_1 = F_2 = F$$

$$y: \quad 2F\sin\alpha - mg = 0$$

$$\boxed{F = \frac{mg}{2\sin\alpha}}$$

$$\sin\alpha = \alpha - \frac{\alpha^3}{3!} + \dots$$



$$\cos\alpha = 1 - \frac{\alpha^2}{2} + \dots$$

$$\mathrm{tg}\,\alpha = \alpha + \frac{\alpha^3}{3} + \dots$$

$\alpha \ll 1$

$$\boxed{F \approx \frac{mg}{2\alpha}}$$

$$\underline{l = L/2}$$

$$\Delta L = l' - l \qquad l = l'\cos\alpha$$

$$= \frac{l}{\cos\alpha} - l = l\left(\frac{1}{\cos\alpha} - 1\right) = l\left(1 + \frac{\alpha^2}{2} + \dots - 1\right)$$

$$\boxed{\Delta l \approx l\,\frac{\alpha^2}{2}}$$

$$\frac{\Delta l}{l} E = \frac{F}{S}$$

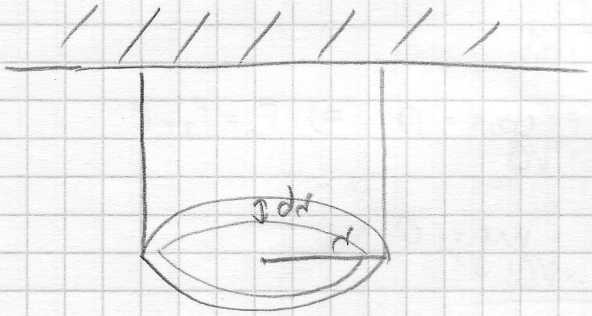
$$S = (d/2)^2 \pi$$

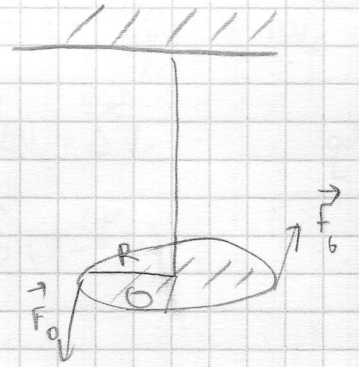
$$\frac{\alpha^2}{2} E = \frac{mg}{2\alpha \quad (d/2)^2 \pi}$$

$$\alpha^3 = \frac{4mg}{Ed^2 \pi}$$

$$\underbrace{\alpha = 0{,}0572}$$

$$h = tg\alpha \cdot l \simeq \alpha \cdot \frac{L}{2} = 11{,}45 \text{ cm}$$





$$D = G \frac{2\pi r^3 dr}{L}$$

šuplji cilindar

$$\int$$

$$M = 2RF_0$$

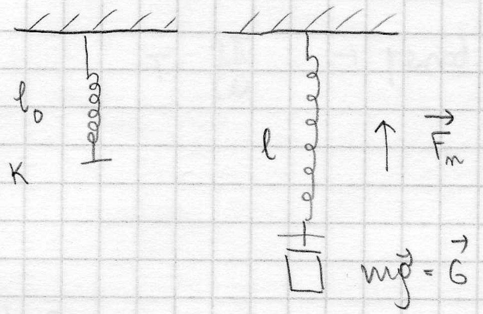
$$M = D\theta$$

$$D = G \frac{\pi r_0^4}{2L}$$

žica

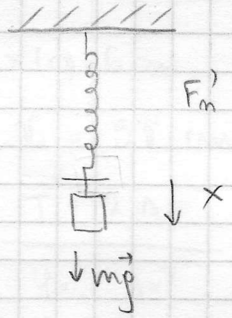
2.10.2013.

Harmonički oscilator



$F_n = k(l - l_0)$

$\vec{G} + \vec{F}_n = 0$ ravnoteža



$F_n' = k(l + x - l_0)$

$F = m\vec{g} - k(l + x - l_0)$

$\underbrace{F = -kx}$ sila pod čijim utjecajem se tijelo giba

$\vec{F}_R = m\vec{a} = m\dfrac{d^2\vec{x}}{dt^2}$

$m\dfrac{d^2x}{dt^2} = -kx$ jednadžba gibanja harm. osc. (jednostavnog titranja)

+poč. uvjeti

$x(t) = ?$ pomak iz položaja ravnoteže ili elongacija

$x(t) = A e^{\alpha t}$

$\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x \quad ; \quad \ddot{x} + \dfrac{k}{m}x = 0$

$$A\alpha^2 e^{\alpha t} + \frac{k}{m} A e^{\alpha t} = 0$$

$$\alpha^2 = -\frac{k}{m}$$

$$\alpha_{1,2} = \pm i \underbrace{\sqrt{\frac{k}{m}}}_{=\omega}$$

$$\alpha_{1,2} = \pm i\omega$$

$$\boxed{x(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} = A_1 e^{-i\omega t} + A_2 e^{i\omega t}}$$

npr. položaj i brzina u trenutku $t = 0$?

$t = 0 \quad x(0) = A \quad v(0) = 0$

$x(0) = A = A_1 + A_2$

$v(t) = \dfrac{dx(t)}{dt} = -i\omega A_1 e^{-i\omega t} + i\omega A_2 e^{i\omega t}$

$v(0) = -A_1 + A_2 = 0 \Rightarrow A_1 = A_2$

$$A_1 = A_2 = \frac{A}{2}$$

$$x(t) = \frac{A}{2}\left(e^{-i\omega t} + e^{i\omega t}\right)$$

$$e^{\pm ix} = \cos x \pm i \sin x$$

$$\underbrace{e^{-i\omega t} + e^{i\omega t}}_{2\cos\omega t}$$

$$\boxed{x(t) = A\cos\omega t}$$

2. $x(0) = 0 \qquad v(0) = v_0$

$A_1 + A_2 = 0 \Rightarrow A_1 = -A_2$

$v(0) = v_0 = -A_1 i\omega + A_2 i\omega \Rightarrow A_2 = \dfrac{v_0}{2i\omega}$

$$x(t) = \frac{v_0}{\omega}\sin\omega t$$

$$x(t) = C(\cos\omega t + \phi)$$

$$x(t) = D\sin(\omega t + \phi)$$

A - amplitude

$\omega t + \phi$ - faza

pr. 3 $x(t) = A\sin(\omega t + \phi)$

$$v(t) = \frac{dx(t)}{dt} = A\omega\cos(\omega t + \phi)$$

$$a(t) = \frac{d^2x(t)}{dt^2} = -A\omega^2\sin(\omega t + \phi)$$

$\omega = \sqrt{\dfrac{k}{m}}$ $\omega = 2\pi f$

↓ ↓

kružna frekvencija frekvencija titranja

$$\omega(t+T)+\phi = \omega t+\phi+2\pi$$

$$\omega T = 2\pi$$

$$T = \frac{2\pi}{\omega} \rightarrow \text{period}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Z 1.16.

$t=0$

$x(0) = 6\ cm$

$v_x = 5\pi\ cm/s$

$T = 2\ s$

$\varphi = ?\quad A = ?$

$$x(t) = A\sin(\omega t+\varphi)$$

$$v(x) = A\omega\cos(\omega t+\varphi)$$

(1) $x(0) = A\sin\varphi = 6\ cm$

(2) $v(0) = A\omega\cos\varphi = 5\pi\ cm/s$

$$\omega = 2\pi f = \frac{2\pi}{T} = \pi\ s^{-1}$$

$$f = \frac{1}{T}$$

$$(1):(2) = \frac{\cancel{A}\sin\varphi}{\cancel{A}\omega\cos\varphi} = \frac{6}{5\pi}$$

$$tg\varphi = \frac{6}{5\pi}\cdot\omega = \frac{6}{5\cancel{\pi}}\cdot\cancel{\pi} \Rightarrow \varphi = 50{,}19^\circ$$

$$A = \frac{6\ cm}{\sin\varphi} = 7{,}81\ cm$$

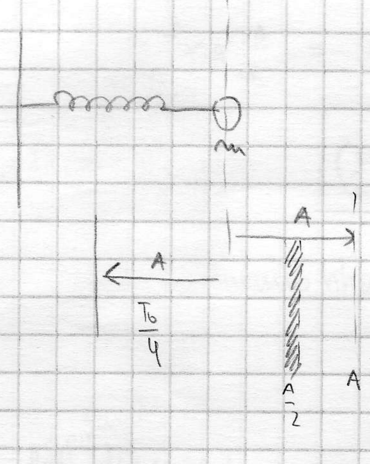
Z 1.21. $m = 2\cdot 10^{-2}\ kg$

$k = 8\ Nm^{-1}$

$T_0 = 2\pi\sqrt{\frac{m}{k}}$

$T_0 = \ldots$

$T = ?$



$$\omega = \frac{2\pi}{T_0}$$

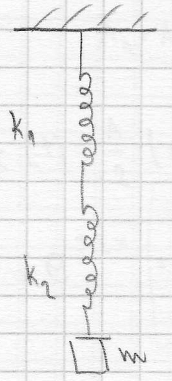
$$t = \frac{T_0}{12}$$

$$T = 2t + 2\cdot\frac{T_0}{4}$$

$$\boxed{T = 2\cdot\frac{T_0}{12} + 2\cdot\frac{T_0}{4} = 0{,}21\ s}$$

$$x(t) = A\sin\omega t \qquad \frac{\cancel{A}}{2} = \cancel{A}\sin\omega t \Rightarrow \sin(\omega t) = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{6}$$

SERIJSKI SPOJ OPRUGE



samo na prvi stavimo teret $mg = k_1 x_1$

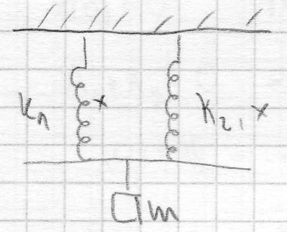
samo na drugi $mg = k_2 x_2$

$$mg = k(x_1 + x_2)$$

$$\cancel{mg} = k\left(\frac{\cancel{mg}}{k_1} + \frac{\cancel{mg}}{k_2}\right)$$

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$

PARALELNI SPOJ



$$mg = kx$$

$$mg = k_1 x + k_2 x$$

$$\boxed{k = k_1 + k_2}$$

Energija titranja

$$E_k = \frac{1}{2} m v^2$$

$$x(t) = A \sin(\omega t + \varphi)$$

$$v(t) = A\omega \cos(\omega t + \varphi)$$

$$E_k = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \varphi)$$

$$\omega^2 = \frac{k}{m}$$

$$\boxed{E_k = \frac{1}{2} k A^2 \cos^2(\omega t + \varphi)}$$

$$E_p = -W = -\int_0^x -kx\,dx = \frac{1}{2} k x^2$$

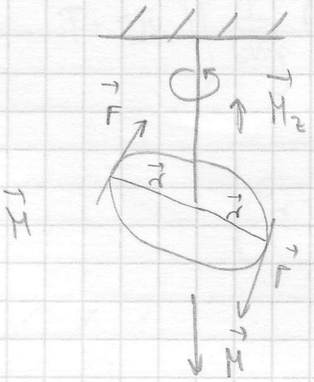
$$\boxed{E_p = \frac{1}{2} k A^2 \sin^2(\omega t + \varphi)}$$

$$E_U = E_k + E_p = \frac{kA^2}{2}$$

Predavanje srijeda, 9. 10. 2013.

Torzioni titraji

smicanje



moment elastičnih sila u niti

tangencijalno se djeluje s dvije sile

svaka sila će proizvesti moment

$M = -D \cdot \theta$ kut

Translacijsko gibanje

$\vec{F} = m \cdot \vec{a}$

Kružno gibanje

$\vec{M} = I \cdot \vec{\alpha}$

$M = I\alpha = I \dfrac{d^2\theta}{dt^2}$

$I\alpha = -D\theta$

$\alpha + \dfrac{D}{I}\theta = 0$

$\ddot{\theta} + \dfrac{D}{I}\theta = 0$

rješenja analogna jednadžbi harm. titranja

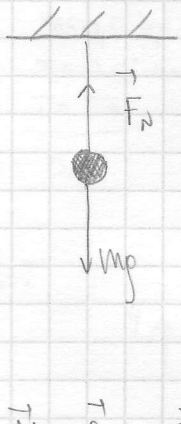
$\omega^2 = \dfrac{D}{I}$

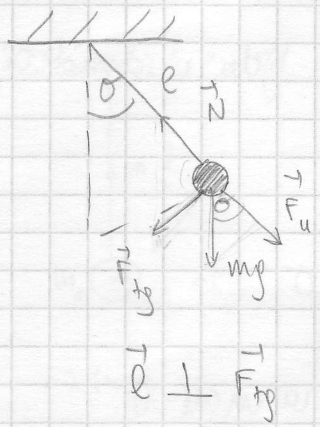
$T = 2\pi\sqrt{\dfrac{I}{D}}$

$\theta = \theta_0 \sin(\omega t + \varphi$

Matematičko njihalo

Tijelo čija je masa zanemariva i obješeno je na nit.





Tijelo se njiše po kružnici zbog tangencijalne sile $\vec{F}_{tg}$

$\sin\theta = \dfrac{F_{tg}}{mg}$ $\quad F_{tg} = (-) mg \sin\theta$

jer djeluje suprotno od smjera povećanja kuta

$\vec{l} \perp \vec{F}_{tg}$

$\vec{M} = \vec{l} \times \vec{F}_{tg}$

$\vec{M} = -lmg\sin\theta$

$I\alpha = -lmg\sin\theta$

$m l^{\cancel{2}} \alpha = -\cancel{l} mg\sin\theta \Rightarrow$

$\boxed{\alpha = -\dfrac{g}{l}\sin\theta}$

$\alpha = \ddot{\theta}$

$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$ $\qquad \sin\theta \approx \theta$, za male θ

$\ddot{\theta} + \frac{g}{l}\theta = 0$ $\qquad \theta(t) = \theta_0 \sin(\omega t + \varphi)$

$$\omega^2 = \frac{g}{l}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

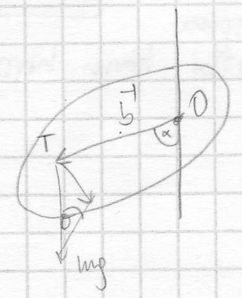
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta_0}{2} + \frac{9}{64}\sin^4\frac{\theta_0}{2} + \dots\right) \leftarrow$$

vrijedi za veće amplitude titraja

Fizičko njihalo

- bilo koje tijelo koje njiše, ali je obješeno u točku koja nije na osi težišta



$\vec{OT} = \vec{b}$

$-mg\sin\theta$ - sila koja vraća tijelo u ravnotežni položaj

b - udaljenost težišta od objesišta

$\vec{M} = I \cdot \vec{\alpha}$

$\vec{M} = \vec{b} \times m\vec{g}$

$M = - bmg\sin\theta$

$\downarrow$ zbog vraćanja tijela u ravnotežni položaj

$I\alpha = -bmg\sin\theta$

$\ddot{\theta} + \frac{bmg}{I}\sin\theta = 0$

$\ddot{\theta} + \frac{bmg}{I}\theta = 0$ za male θ

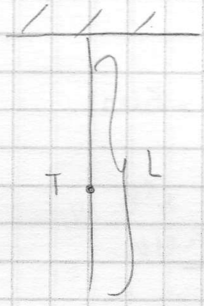
$\theta(t) = \theta_0 \sin(\omega t + \varphi)$

$\omega = \sqrt{\frac{bmg}{I}}$ $\qquad T = 2\pi\sqrt{\frac{I}{bmg}}$

reducirana duljina fizičkog njihala - duljina matematičkog njihala koje ima isto vrijeme titranja kao fizičko njihalo

$$T = 2\pi \sqrt{\frac{I}{bmg}} = 2\pi \sqrt{\frac{l_r}{g}}$$

$$\boxed{l_r = \frac{I}{bm}}$$



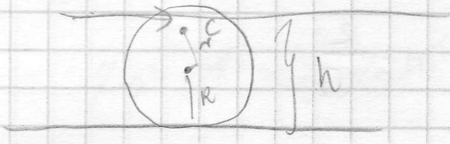
$$I_0 = \frac{ml^2}{12}$$ - moment tromosti štapa kroz os težišta

$$I = I_0 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3} mL^2$$

$$l_r = \frac{\frac{mL^2}{3}}{m\frac{L}{2}} = \frac{2}{3} L \qquad b = \frac{L}{2}$$

središte titranja ili centar udara
(udarom u tu točku štap radi njihanje)

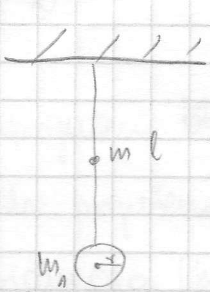
U koju točku treba udariti kuglu da rotira?



$$l_r = \frac{I}{bm} \qquad I_0 = \frac{2}{5} mR^2 \qquad l_r = R + r$$

$$b = R \qquad I_c = I_0 + mr^2$$

1.20. $l = 1\,m$
$m = 0{,}5\,kg$
$r = 0{,}05\,m$
$T = 2\,s$
$\rho = ?$

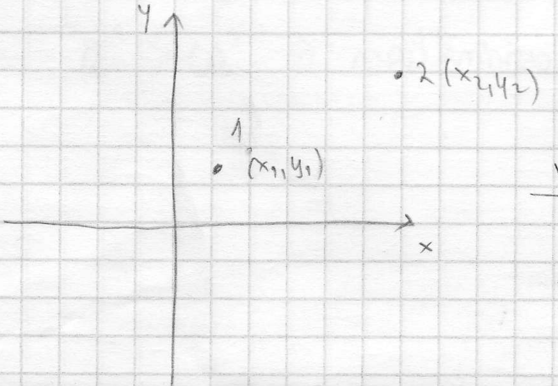


$$\rho = \frac{m_1}{V} = \frac{m_1}{\frac{4}{3} r^3 \pi}$$

$$T = 2\pi \sqrt{\frac{I}{G \cdot b}}$$

težina cijelog njihala, $(m + m_1)g$

b = ?



$$\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = x_{cm}$$

$$\frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = y_{cm}$$

$$b = \frac{m \cdot \frac{l}{2} + m_1 (l + r)}{m_1 + m}$$

$$I = I_5 + I_{kug} = \frac{1}{3} ml^2 + \frac{7}{5} m_1 r^2 + m_1 l^2 = l^2\left(\frac{m}{3} + m_1\right) + \frac{7}{5} m_1 r^2$$

$$I_5 = I_{CM5} + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$

$$I_{kug} = I_{CMr} + m_1 (l+r)^2 = \frac{2}{5} m_1 r^2 + m_1 l^2 + m_1 r^2 = \frac{7}{5} m_1 r^2 + m_1 l^2$$

$$T = 2\pi \sqrt{\frac{I}{Gb}} \Rightarrow m_1 \Rightarrow \rho = 2{,}610\ kg/m^3$$

$$\frac{T^2}{4\pi^2} = \frac{\frac{ml^2}{3} + m_1 l^2 + \frac{7}{5} m_1 r^2}{\cancel{(m_1+m)} g \cdot \frac{m\frac{l}{2} + m_1(l+r)}{\cancel{m_1+m}}} =$$

$$T = 2\pi \sqrt{\frac{I}{Gb}} = 2\pi \sqrt{\frac{\frac{MR^2}{2} + Mr^2}{Mg\, r}} = 2\pi \sqrt{\frac{R^2 + 2r^2}{2g\, r}}$$

1.31.

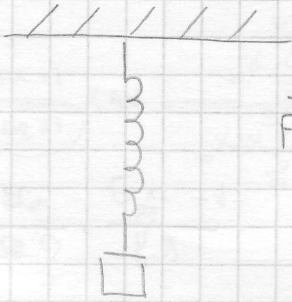
$$G = mg$$

$$b = r$$

$$I = I_0 + Mr^2 = \frac{MR^2}{2} + Mr^2$$

$$\frac{dT(r)}{dr} = 0 \Rightarrow r = \frac{R}{\sqrt{2}}$$

Prigušeno titranje



$\vec{F}_{el} = -k\vec{x}$

$\ddot{x} + \omega_0^2 x = 0$

nepriguseno

$m\ddot{x} = -kx$

odnosi sile otpora

$F_{otpora} \sim v$ ← mi gledamo

$F_{otpora} \sim v^2$

$\vec{F}_o = -b\vec{v}$

konst. sile otpora

$m\vec{a} = \vec{F}_{el} + \vec{F}_{tr}$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \overset{\omega_0^2}{\frac{k}{m}}x = 0$$

2δ konstanta prigušenja

$$x(t) = Ce^{at}$$

$$\frac{dx(t)}{dt} = Cae^{at} \qquad \frac{d^2x}{dt} = Ca^2e^{at}$$

$$\cancel{C}a^2\cancel{e^{at}} + \frac{b}{m}\cancel{C}a\cancel{e^{at}} + \frac{k}{m}\cancel{Ce^{at}} = 0$$

$$a^2 + 2\delta a + \omega_0^2 = 0$$

$$a_{1,2} = \frac{-2\delta \pm\sqrt{4\delta^2 - 4\omega_0^2}}{2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

1. $\delta = 0$ ⇒ nepriguseno titranje
2. $\delta^2 < \omega_0^2$ ⇒ slabo priguseno
3. $\delta^2 = \omega_0^2$ ⇒ granicno (kriticno) prigusenje
4. $\delta^2 > \omega_0^2$ ⇒ nadkriticno ili aperiodicno prigusenje

2. Slabo prigušenje

$\delta^2 < \omega_0^2$

$$a_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -\delta \pm i\omega$$

$$\omega = \sqrt{\omega_0^2 - \delta^2}$$

$x(t) = Ce^{at}$ $\quad x(t) = C_1 e^{a_1 t} + C_2 e^{a_2 t}$

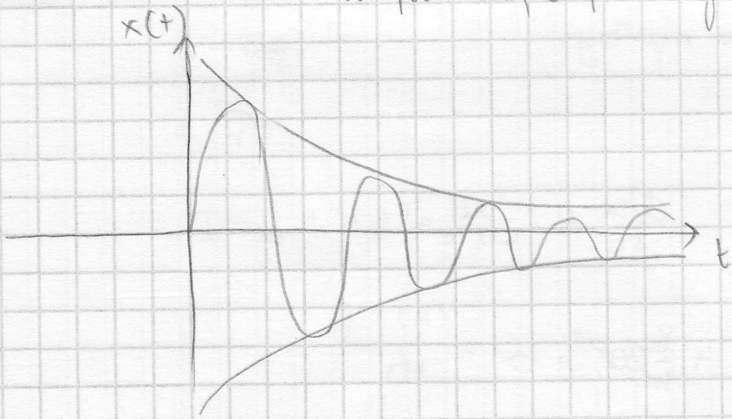
$$x(t) = C_1 e^{(-\delta + i\omega)t} + C_2 e^{(-\delta - i\omega)t}$$

$$e^{\pm i\omega t} = \cos\omega t \pm i\sin\omega t$$

$$x(t) = e^{-\delta t}\left[(C_1 + C_2)\cos\omega t + (C_1 - C_2)\, i\sin\omega t\right]$$

$$x(t) = Ae^{-\delta t}\sin(\omega t + \varphi_0)$$

amplituda, eksponencijalno pada u vremenu



poč. uvjeti

$t = 0$ $\quad x(0) = A_0$

$v(0) = 0$

$x(0) = A_0 = A\sin\varphi_0$

$$v(t) = \frac{dx}{dt} = A(-\delta)e^{-\delta t}\sin(\omega t + \varphi_0) + Ae^{-\delta t}\omega\cos(\omega t + \varphi_0)$$

$$v(0) = 0 \Rightarrow \quad -\delta A\sin\varphi_0 + A\omega\cos\varphi_0 \quad /:A$$

$$-\delta\sin\varphi_0 = -\omega\cos\varphi_0$$

$$\mathrm{tg}\,\varphi_0 = \frac{\omega}{\delta}$$

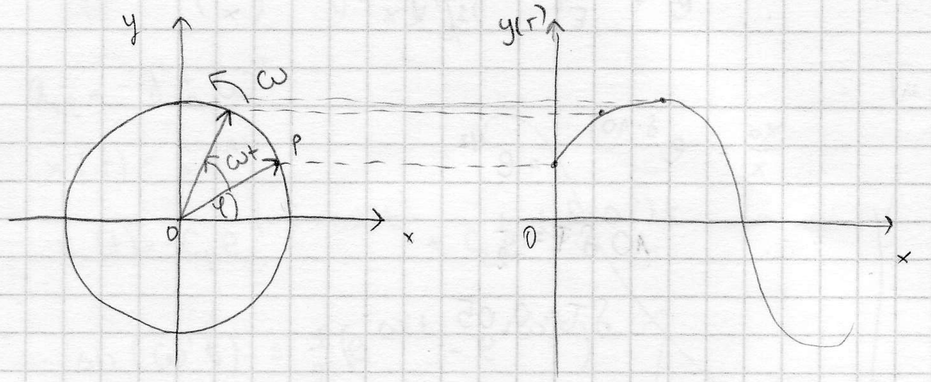
$$T = \frac{2\pi}{\omega} \qquad \varphi_0 = \mathrm{arctg}\frac{\omega}{\delta}$$

logaritamski dekrement prigušenja

$$\frac{A(t)}{A(t+T)} = \frac{Ae^{-\delta t}}{Ae^{-\delta(t+T)}} = e^{\delta T} \rightarrow \delta T = \ln\frac{A(t)}{A(t+T)}$$

FAZORI

- veza između titranja i kružnog gibanja



$\vec{OP}$

$\omega = \frac{kut}{t}$

$\vec{OP} = x(t)\,\vec{i} + y(t)\,\vec{j}$

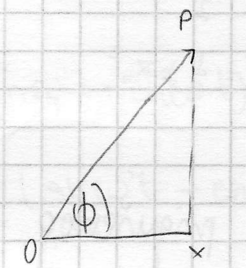
$\phi = \omega t + \varphi$

$x(t) = |\vec{OP}| \cos\phi$

$y(t) = |\vec{OP}| \sin\phi$

$x(t) = |\vec{OP}| \cos(\omega t + \varphi)$

$y(t) = |\vec{OP}| \sin(\omega t + \varphi)$



14.10.2013.
(fale mi i sub)

harmonički oscilator

$\ddot{x} + \omega_0^2 x = 0$

$\vec{F}_tr = -b\vec{v}$

$\frac{d^2x}{dt^2} + 2\delta \frac{dx}{dt} + \omega_0^2 x = 0$ $\qquad \frac{b}{m} = 2\delta$

$x(t) = Ce^{\alpha t}$

$x(t) = a^2 + 2\delta a + \omega_0^2 = 0$

$a_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$

1. $\delta^2 < \omega_0^2$ slabo prigušeno

$x(t) = Ae^{-\delta t} \sin(\omega t + \varphi)$

$\omega = \sqrt{\omega_0^2 - \delta^2}$

imamo titranje

$\frac{A(t)}{A(t+T)} = e^{\delta T}$

$\lambda = \delta \cdot T$ logaritamski dekrement prigušenja

PONAVLJANJE SUBOTA

zad. $t = 10T$

$$E = \frac{E_0}{e}$$

$\lambda = ?$

$$x = x_0 e^{-\delta t}$$

$$\frac{x_0}{x} = e^{\delta t}$$

$$E = \frac{1}{2}kx^2$$

$$E_0 = \frac{1}{2}kx_0^2$$

$$e = \frac{E_0}{E} = \frac{\frac{1}{2}kx_0^2}{\frac{1}{2}kx^2} = \left(\frac{x_0}{x}\right)^2$$

$$\frac{x_0}{x} = e^{\delta \cdot 10T} = e^{1/2}$$

$$10\delta T = \frac{1}{2}$$

$$\delta T = 0{,}05$$

Energija prigušenog harmoničkog oscilatora

$$E = E_k + E_p = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow \frac{dE}{dt} = \frac{d}{dt}(E_k + E_p) = \frac{dE_k}{dv}\cdot\frac{dv}{dt} + \frac{dE_p}{dx}\cdot\frac{dx}{dt}$$

$$v = \frac{dx}{dt}$$

$$= mv\cdot\frac{dv}{dt} + kx\cdot v =$$

$$= v\Big(\overset{\text{rezult. sila}}{m\frac{dv}{dt}} + \overset{\text{elastična sila}}{kx}\Big) = -bv^2$$

$$\underbrace{m\frac{dv}{dt} + kx}_{-bv}$$

Q - kvaliteta ili dobrota (faktor)

gubitak energije u sustavu

$$Q = \frac{\langle E\rangle}{\frac{|\Delta E|}{2\pi}}$$ (srednja vrijednost)

$$\boxed{\langle E\rangle = \frac{1}{T}\int_t^{t+T} E(t)\,dt}$$

$$E \sim x^2$$

$$x \sim e^{-\delta t}$$

$$\langle E\rangle \sim e^{-2\delta t}$$

$$\frac{\langle E\rangle}{t} \sim \frac{d\langle E\rangle}{dt} \sim -2\delta e^{-2\delta t} = k\cdot 2\delta e^{-2\delta t}$$

$$Q = \cancel{2}\pi\,\frac{\cancel{k}\cancel{e^{-2\delta t}}}{T\cancel{k}\cancel{2}\delta \cancel{e^{-2\delta t}}} = \frac{\pi}{\delta\cdot T} = \frac{\pi}{\lambda}$$

neprigušeno - veliki Q faktor
što veće prigušenje, manji

govori o gubicima energije

$a_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$

(2.) $\delta^2 > \omega_0^2$ jako prigušenje

$a_{1,2} = -\delta \pm \omega'$

$x(t) = Ce^{at}$

$x(t) = C_1 e^{(-\delta+\omega')t} + C_2 e^{(-\delta-\omega')t}$

$$\left.\begin{aligned} sh(\omega' t) &= \frac{1}{2}(e^{\omega' t} - e^{-\omega' t}) \\ ch(\omega' t) &= \frac{1}{2}(e^{\omega' t} + e^{-\omega' t}) \end{aligned}\right\} \begin{aligned} e^{\omega' t} &= (sh\omega' t + ch\omega' t) \\ e^{-\omega' t} &= -(sh\omega' t - ch\omega' t) \end{aligned}$$

$x(t) = e^{-\delta t}\left[A sh\omega' t + B ch\omega' t\right]$

$v(t) = \frac{dx(t)}{dt} = -\delta e^{-\delta t}(A sh\omega' t + B ch\omega' t) + e^{-\delta t}(A\omega' ch\omega' t + B\omega' sh\omega' t)$

poč. uvj. $x(0) = x_0$, $v(0) = 0$

$x(0) = x_0 = B$

$v(0) = 0 = -\delta B + \omega' A \Rightarrow A = \frac{\delta B}{\omega'} = \frac{\delta x_0}{\omega'}$

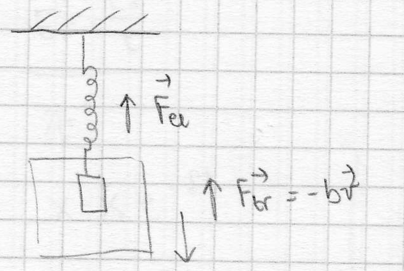
$x(t) = e^{-\delta t}\left[\frac{\delta x_0}{\omega'} sh\omega' t + x_0 ch\omega' t\right]$

- kod jakog prigušenja nema titranja

(3) KRITIČNO PRIGUŠENJE $\delta = \omega_0$

$$\left.\begin{aligned} &\omega' \to 0 \\ &ch\omega' \to 1 \\ &\lim_{\omega' \to 0} \frac{sh\omega' t}{\omega' t} = 1 \end{aligned}\right\} x(t) = x_0 e^{-\delta t} \cdot (1 + \delta t)$$

Prisilno titranje



vanjska

$|\vec{F}_v| = F_0 \sin \omega t$

$A_0 = \frac{F_0}{m}$

$$m\vec{a} = \vec{F}_{el} + \vec{F}_{tr} + \vec{F}_v$$

$$m\frac{d^2x}{dt^2} \qquad -kx \qquad -b\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = A_0 \sin \omega t$$

homogena jedn. $x_h(t) + \underbrace{x_p(t)}_{\text{partikularno}} = x(t)$

$$x_p(t) = a\sin\omega t + b\cos\omega t$$

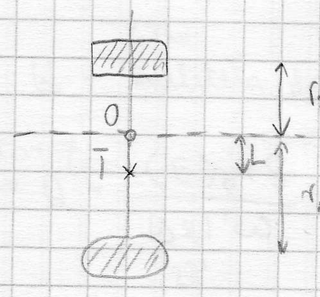
kasni u fazi

$$x_p(t) = A(\omega)\sin(\omega t - \varphi)$$

amplitude ovisi o ω

AUDITORNE

Z. 1.28.



$\frac{k}{m}, \quad \frac{D}{I}$

$x, \dot{x}, \ddot{x} \qquad \theta, \dot{\theta}, \ddot{\theta}$

$s(t), v(t), a(t) \qquad \theta(t), \omega(t), \alpha(t)$

$F = ma \qquad M = I \cdot \alpha$

$v(t) = v_0 + at \qquad \omega(t) = \omega_0 + \alpha t$

$$\omega^2 = \frac{mgL}{I}$$

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$x_1 = r_1$

$x_2 = -r_2$

$$x_{CM} = \frac{m_1 r_1 - m_2 r_2}{m_1 + m_2} = L$$

$$I = \int dm \cdot r^2$$

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2$$

$$\omega^2 = \frac{\cancel{(m_1+m_2)}\, g \cdot (m_1 r_1 \overset{\ominus}{+} m_2 r_2)}{\cancel{(m_1+m_2)}\,(m_1 r_1^2 + m_2 r_2^2)}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m_1 r_1^2 + m_2 r_2^2}{g(m_1 r_1 \underset{\ominus}{+} m_2 r_2)}}$$

r_2 raste, T raste

$m_1 r_1 - m_2 r_2 = 0$

za $CM \to 0$, $T \to \infty$

1.37.

$A_0 = 10\ cm$

$A(t) = A_0 e^{-\delta t}$ prigušeno titranje

$\frac{A(1s)}{A(7s)} = 10$, $A(t_F) = 1\ cm$

$t_F = ?$

$$\frac{\cancel{A_0} e^{-\delta \cdot 1s}}{\cancel{A_0} e^{-\delta \cdot 7s}} = 10$$

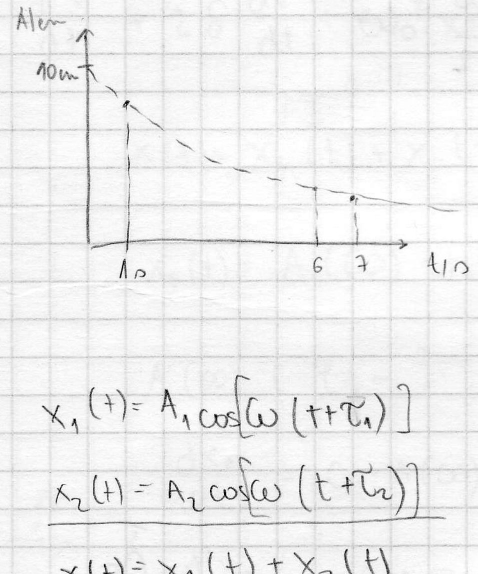
$$e^{6\delta} = 10$$

$$6\delta = \ln 10 \Rightarrow \delta = \frac{\ln 10}{6}$$

$$1\ cm = A_0 e^{-\delta t_F}$$

$$\frac{1\ cm}{10\ cm} = e^{-\delta t_F}$$

$$\cancel{-\ln 10} = -\delta t_F = -\frac{\cancel{\ln 10}}{6} t_F \Rightarrow t_F = 6\ s$$



P. 1.2.

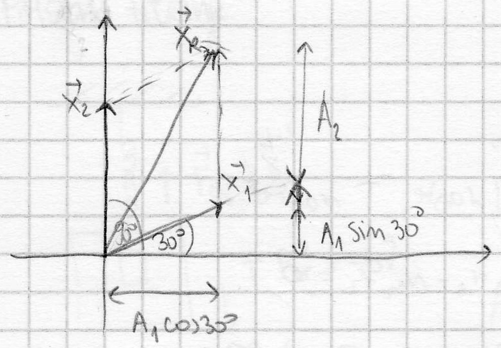
$x_1(t) = A_1 \cos[\omega(t+\tau_1)]$

$x_2(t) = A_2 \cos[\omega(t+\tau_2)]$

$x(t) = x_1(t) + x_2(t)$

$\vec{X}_1$, $|\vec{X}_1| = A_1$, $\varphi_1 = \omega\tau_1 = \frac{\pi}{6}$, rotira frekv. ω

$\vec{X}_2$, $|\vec{X}_2| = 2A_1$, $\varphi_2 = \omega\tau_2 = \frac{\pi}{2}$, rotira frekv. ω



$$|\vec{X}_R| = A_R = \sqrt{(A_2 + A_1 \sin 30°)^2 + (A_1 \cos 30°)^2} = \sqrt{\underbrace{\left(2A_1 + \frac{A_1}{2}\right)^2}_{\frac{5}{2}A_1 \,=\, \frac{25}{4}A_1^2} + \left(A_1 \frac{\sqrt{3}}{2}\right)^2} =$$

$$= \sqrt{A_1^2 \left(\frac{25}{4} + \frac{3}{4}\right)} = A_1\sqrt{7}$$

$$\operatorname{tg}\varphi_R = \frac{\overset{2A_1}{\overset{\|}{A_2}} + \cancel{A_1} \sin 30°}{\cancel{A_1} \cos 30°} = \frac{2 + \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{5}{\sqrt{3}} \Rightarrow \varphi_R = 0{,}39\,\pi = 70{,}9°$$

$$x(t) = A_1\sqrt{7} \cos(\omega t + 0{,}39\pi) = A_1\sqrt{7} \cos\omega(t + 0{,}39)$$

1.36.

$F_0 = 10\ N$, $F(t) = F_0 \sin(\omega t + \varphi_0)$

$s(t) = A \sin(\omega t + \varphi)$, $A = 1\ cm$

$s(t=0) = \frac{A}{2} = A \sin\varphi \Rightarrow \varphi = \frac{\pi}{6}$

$s(t) = A \sin\left(\omega t + \frac{\pi}{6}\right)$

$F(t=0) = F_0 \sin\varphi_0 = F_0 \Rightarrow \varphi_0 = \frac{\pi}{2}$

$F(t) = F_0 \cos \omega t$

$W = ?$ $\qquad T = \frac{2\pi}{\omega}$

$$W = \int_0^T F\,ds = \int_0^T F(t) \cdot \frac{ds}{dt} \cdot dt$$

$$ds = A\omega \cos\left(\omega t + \frac{\pi}{6}\right) dt$$

$$W = \int_0^T F_0 \cos(\omega t) A\omega \cos\left(\omega t + \frac{\pi}{6}\right) dt = \frac{F_0 A \omega}{2} \int_0^T \Big[\underbrace{\cos\left(-\frac{\pi}{6}\right)}_{\frac{\sqrt{3}}{2}} + \cos(2\omega t +$$

$$= \frac{\sqrt{3} F_0 A \omega}{4} \cdot T = \frac{\sqrt{3} F_0 A \cancel{\omega}}{4} \cdot \frac{2\pi}{\cancel{\omega}} = \frac{\sqrt{3} F_0 A \pi}{2} = 0{,}27\ J$$

16.10.2013.

1. $\frac{3}{2} m \frac{d^2x}{dt^2} + kx = 0$ $\qquad \boxed{\omega = \sqrt{\frac{2}{3}\frac{k}{m}}}$

$$\frac{d^2x}{dt^2} + \frac{2}{3}\frac{k}{m} x = 0$$

$$\omega^2 = \frac{2}{3}\frac{k}{m}$$

2. $T = 2\pi\sqrt{\frac{l}{g}}$ ovisi o $\sqrt{l}$, duljini l

torziono: $T_T = 2\pi\sqrt{\frac{I}{D}}$ jedina prava

fizičko: $T_F = 2\pi\sqrt{\frac{I}{mgb}}$ je aproksimativno, i za matematičko isto – vrijede samo za male kutove

DZ Tijelo mase $m = 10\,g$ pričvršćeno je na dvije jednake horizontalno postavljene opruge $k_1 = k_2 = 0,5\, N/m$ kliže po podlozi uz koef. trenja $\mu = 0,1$. Ako se tijelo pomakne u desno za udaljenost $a_0 = 10\,cm$ od ravn. položaja i pusti da titra, nađite njegov položaj kada se ono prvi put zaustavi. Rj: $-8,04$ cm

---

NASTAVAK

$$\frac{d^2x}{dt^2} + 2\delta\underbrace{\frac{dx}{dt}}_{=0} + \omega_0^2 x = \overbrace{A_0 \sin \omega t}^{\text{vanjska sila}}$$

$$x(t) = x_h(t) + x_p(t)$$

jer kada dovedemo vanjsku prisilu, kasni u fazi, ne može istog trenutka se prebaciti s vlastitih titranja na tu

$$x_p(t) = A(\omega) \sin(\omega t - \varphi)$$

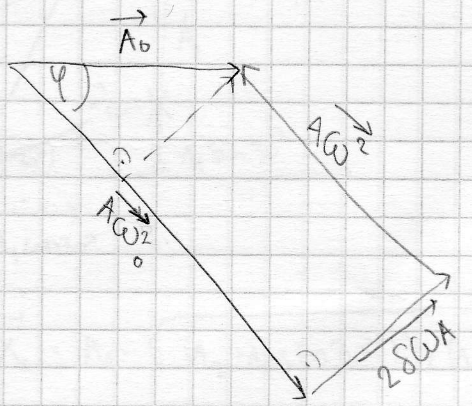
$A(\omega)$ i φ – ?

$$\frac{dx_p}{dt} = \omega \cdot A(\omega) \cos(\omega t - \varphi)$$

$$\frac{d^2x_p}{dt^2} = -\omega^2 A(\omega) \sin(\omega t - \varphi)$$

$$-\omega^2 A(\omega) \sin(\omega t - \varphi) + 2\delta\omega A(\omega) \cos(\omega t - \varphi) + \omega_0^2 A(\omega) \sin(\omega t - \varphi) = A_0 \sin \omega t$$

$$A(\omega)\omega^2 \sin(\omega t - \varphi + \pi) + 2\delta\omega A(\omega)\sin\left(\omega t - \varphi + \frac{\pi}{2}\right) + \omega_0^2 A(\omega)\sin(\omega t - \varphi) = A_0 \sin\omega t$$



$$\vec{A_0}^2 = \overrightarrow{A\omega_0^2} + \overrightarrow{2\delta A\omega} + \overrightarrow{A\omega^2}$$

$$A_0^2 = (2\delta\omega A)^2 + (A\omega_0^2 - A\omega^2)^2$$

$$A = A(\omega) = \frac{A_0^2}{\sqrt{(\omega_0^2-\omega^2)^2 + 4\delta^2\omega^2}}$$

$$\operatorname{tg}\varphi = \frac{2\delta\omega \cancel{A}}{\cancel{A}(\omega_0^2-\omega^2)} = \frac{2\delta\omega}{\omega_0^2-\omega^2}$$

$$\frac{dA(\omega)}{d\omega} = 0$$

$$\frac{dA(\omega)}{d\omega} = \frac{-A_0^2 \cdot \frac{1}{2}\left[(\omega_0^2-\omega^2)^2 + 4\delta^2\omega^2\right]^{-\frac{1}{2}} \cdot \overbrace{\left(2(\omega_0^2-\omega^2)\cdot(-2\omega) + 8\delta^2\omega\right)}^{=0}}{(\omega_0^2-\omega^2)^2 + 4\delta^2\omega^2} = 0$$

$$2A_0\omega\left[(\omega_0^2-\omega^2) - 2\delta^2\right] = 0$$

$$\boxed{\omega = \omega_R = \sqrt{\omega_0^2 - 2\delta^2}}$$

REZONANTNA FREKVENCIJA → amplituda max.

ovisi o faktoru prigušenja δ

$$x_p(t) = A(\omega)\sin(\omega t - \varphi)$$

$$v(t) = \frac{dx_p(t)}{dt} = \omega A(\omega)\cos(\omega t - \varphi) = v_0 \cos(\omega t - \varphi)$$

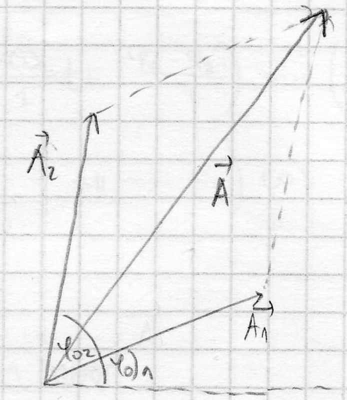
v_0 ↓ amplituda brzine

$$v_0 = \frac{\omega A_0}{\sqrt{(\omega_0^2-\omega^2)^2 + 4\delta^2\omega^2}}$$

Zbrajanje harmoničkih titraja

x, ω_1, ω_2

1. $\omega_1 = \omega_2$



$$\vec{A} = \vec{A}_1 + \vec{A}_2 \quad /^2$$

$$A^2 = A_1^2 + A_2^2 + 2\vec{A}_1\vec{A}_2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_{02} - \varphi_{01})$$

2. $\omega_1 \neq \omega_2$

$$x_1(t) = A\sin(\omega_1 t + \varphi_{01})$$

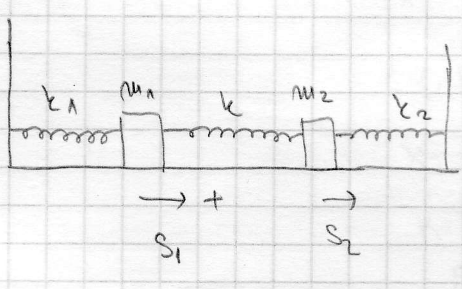
$$x_2(t) = A\sin(\omega_2 t + \varphi_{02})$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$x(t) = x_1(t) + x_2(t) = \underbrace{2A\cos\left[\frac{\omega_1-\omega_2}{2}t + \frac{\varphi_{01}-\varphi_{02}}{2}\right]}_{\text{ponašanje amplitude}} \sin\left[\frac{\omega_1+\omega_2}{2}t + \frac{\varphi_{01}+\varphi_{02}}{2}\right]$$

21.10.2013.

Vezani H.O. → OBERBECK



m_1: $-k_1 s_1$, $-ks_1$, ks_2

$$m_1\frac{d^2s_1}{dt^2} = -k_1s_1 + k(s_2 - s_1) \qquad \text{I}$$

m_2: $-k_2 s_2$, $-ks_2$, ks_1

$$m_2\frac{d^2s_2}{dt^2} = -k_2s_2 - k(s_2 - s_1) \qquad \text{II}$$

$$\left.\begin{aligned} s_1(t) &= A_1\sin(\omega t + \varphi_1) \\ s_2(t) &= A_2\sin(\omega t + \varphi_2) \end{aligned}\right\} \text{uvrstimo}$$

$$A_1 = A_2 = A$$

$$s_1(t) = A\sin(\omega t + \varphi_1)$$

$$s_2(t) = \pm A\sin(\omega t + \varphi_2)$$

Normalne koordinate

① Titranje u fazi

$$q_1(t) = s_1(t) + s_2(t)$$

$$\left.\begin{array}{l} k_1 = k_2 \\ m_1 = m_2 = m \end{array}\right\} \omega_0^2 = \frac{k_1}{m}$$

$$+\left\{\begin{array}{ll} \ddot{s}_1 + \omega_0^2 s_1 - \frac{k}{m}(s_2 - s_1) = 0 & \mathrm{I}' \\ \ddot{s}_2 + \omega_0^2 s_2 + \frac{k}{m}(s_2 - s_1) = 0 & \mathrm{II}' \end{array}\right.$$

$$\underbrace{\ddot{s}_1 + \ddot{s}_2}_{} + \omega_0^2(s_1 + s_2) = 0$$

$$\ddot{q}_1 + \omega_0^2 q_1 = 0 \qquad \omega_1 = \omega_0$$

uopće ne vidimo utjecaj opruge između njih

② Protufazno titranje

$$q_2(t) = s_1(t) - s_2(t)$$

$$\mathrm{I}' - \mathrm{II}' = \ddot{s}_1 - \ddot{s}_2 + \omega_0^2(s_1 - s_2) + \frac{2k}{m}(s_1 - s_2) = 0$$

$$\ddot{q}_2 + \left(\omega_0^2 + \frac{2k}{m}\right) q_2 = 0$$

$$\omega_2^2 = \omega_0^2 + \frac{2k}{m}$$

$$q_1(t) = A\sin(\omega_1 t + \varphi_1)$$

$$q_2(t) = A\sin(\omega_2 t + \varphi_2)$$

$$q_1(t) = s_1(t) + s_2(t)$$

$$q_2(t) = s_1(t) - s_2(t)$$

$$2s_1(t) = q_1(t) + q_2(t) = A\sin(\omega_1 t + \varphi_1) + A\sin(\omega_2 t + \varphi_2)$$

$$2s_2(t) = q_1(t) - q_2(t) = A\sin(\omega_1 t + \varphi_1) - A\sin(\omega_2 t + \varphi_2)$$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$$S_1(t) = 2A\cos\left(\frac{\omega_1-\omega_2}{2}t + \frac{\varphi_1-\varphi_2}{2}\right)\sin\left(\frac{\omega_1+\omega_2}{2}t + \frac{\varphi_1+\varphi_2}{2}\right)$$

$$S_2(t) = 2A\sin\left(\quad -\parallel- \quad\right)\cos\left(\quad -\parallel- \quad\right)$$

Opis valnog gibanja

$s(t) = A\sin\omega t \quad \rightarrow$ izvor

udaljenost

$$s(x,t) = A\sin\omega(t-t') = A\sin\omega\left(t-\frac{x}{v}\right)$$

$x = v\cdot t' \Rightarrow t' = \frac{x}{v}$

λ - valna duljina

$\omega = 2\pi f \quad ; \quad f = \frac{1}{T}$

$s(\vec{r}, t)$ valno gibanje ima ovisnost i o prostoru i o vremenu

v - fazna brzina ne i stvarna brzina

$$\left.\begin{array}{l} x = \lambda \\ t' = T \end{array}\right\} \lambda = v\cdot T = v\cdot\frac{1}{f}$$

$$\boxed{v = \lambda f}$$

$\omega\left(t-\frac{x}{v}\right) = c \qquad c = \text{konst.}$ ravnina nas ovisnost x-a o vremenu - ravnina nas hode li bili - ili +

$t - \frac{x}{v} = \frac{c}{\omega}$

$-\frac{x}{v} = \frac{c}{\omega} - t$

$x = vt + \text{konst.}$

$\Delta\varphi = k\cdot x$

$x = \lambda \qquad \Delta\varphi = 2\pi$

$k\cdot\lambda = 2\pi$

$$\boxed{k = \frac{2\pi}{\lambda}}$$ valni broj

$\vec{k} \rightarrow$ valni vektor

$\vec{k} = k\cdot\vec{n}_0$

- jedinični vektor u smjeru širenja vala

AUDITORNE

Zbirka, Z. 1.39.

$$x = (1\,cm)\cos(\pi s^{-1} t) \Rightarrow \cos(\pi s^{-1} t) = \frac{x}{1\,cm}$$

$$y = (2\,cm)\cos\left(\frac{\pi}{2} s^{-1} t\right) \Rightarrow \cos\left(\frac{\pi}{2} s^{-1} t\right) = \frac{y}{2\,cm}$$

STAZA ? $\rightarrow$ x, y

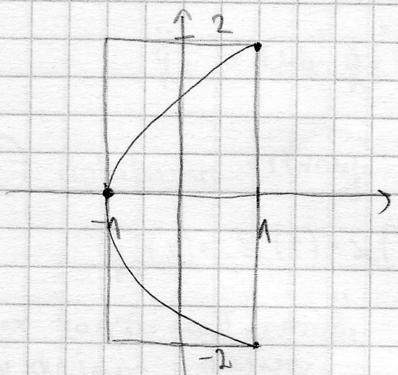
$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\cos(\pi s^{-1} t) = 2\cos^2\left(\frac{\pi}{2} s^{-1} t\right) - 1$$

$$\frac{x}{1\,cm} = 2\left(\frac{y}{2\,cm}\right)^2 - 1$$

$$\left(\frac{y}{2\,cm}\right)^2 = \frac{1}{2}\left(\frac{x}{1\,cm} + 1\right)$$

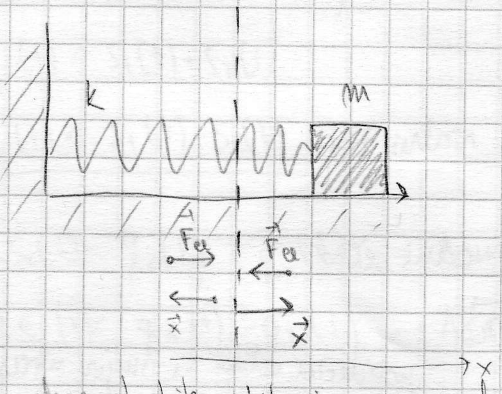
$$y^2 = 2\,cm^2\left(\frac{x}{1\,cm} + 1\right)$$



Aperiodičko gušenje

Horvat, pr. 2.34, str. 2-45

(Coulombov problem)



$m = 0{,}4$

$F = -kx$, $k = 20$ N/m

$F_{tr} = \mu F_0 = \mu m g$, $\mu = 0{,}05$

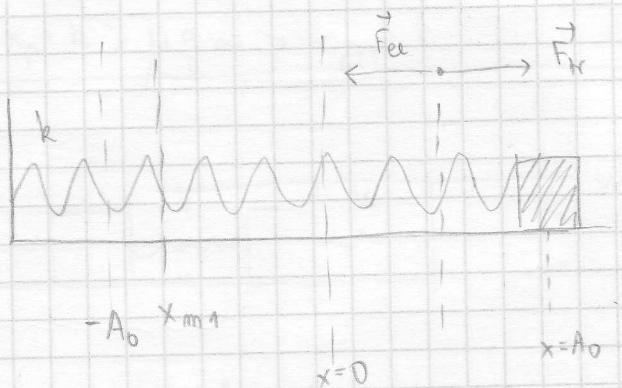
$A_0 = 20$ cm, $x(0) = A_0$, $v(0) = 0$

$kA_0 > \mu m g$ dovoljno velika el. sila da svlada trenje

Kako i nakon koliko titraja se zaustavi?

$\vec{F}_{el} = -k\vec{x}$ za $x<0$ neg., $x>0$ poz.

(1)



$$m\ddot{x} = \mu mg - kx$$

$$\ddot{x} + \frac{k}{m}x = \mu g$$ jdn. gibanja za 1. etapu

$$x(t) = x_{hom}(t) + x_{part.}(t)$$

$$x_{hom}(t) = C\sin(\omega t + \varphi) \qquad \omega^2 = \frac{k}{m}$$

$$x_{par.}(t) = B\mu g$$

$$0 + \omega^2 B\mu g = \mu g \quad \Rightarrow \quad B = \frac{1}{\omega^2}$$

$$x_{part.}(t) = \frac{\mu g}{\omega^2}$$

$$\boxed{x_1(t) = C\sin(\omega t + \varphi) + \frac{\mu g}{\omega^2}}$$ opće rješenje

$$x(0) = C\sin\varphi + \frac{\mu g}{\omega^2} = A_0$$

$$v(0) = C\omega\cos\varphi = 0 \quad \Rightarrow \quad \varphi = \frac{\pi}{2} \quad \Rightarrow \quad x(0) = C + \frac{\mu g}{\omega^2} = A_0$$

$$C = A_0 - \frac{\mu g}{\omega^2}$$

kako: $$x_1(t) = \left(A_0 - \frac{\mu g}{\omega^2}\right)\sin\left(\omega t + \frac{\pi}{2}\right) + \frac{\mu g}{\omega^2}$$

nakon koliko: $$\dot{x}(t) = 0$$

$$\dot{x}_1(t) = \omega\left(A_0 - \frac{\mu g}{\omega^2}\right)\cos\left(\omega t + \frac{\pi}{2}\right) = 0$$

$$\omega t + \frac{\pi}{2} = \frac{3\pi}{2}, \quad \omega t_1 = \pi \qquad t_1 = \frac{\pi}{\omega} = \frac{T}{2}$$

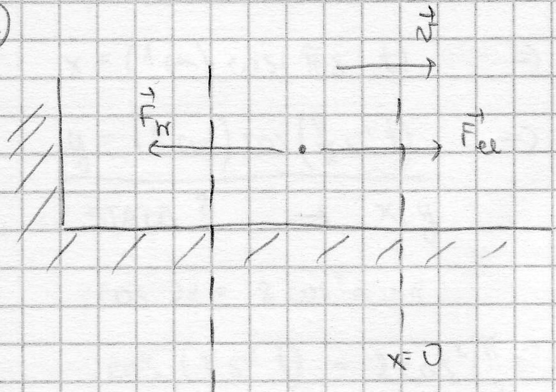
$$T = \frac{2\pi}{\omega}$$

$$\underline{x_1(t_1)} = \left(A_0 - \frac{\mu g}{\omega^2}\right)\sin\Big(\underbrace{\omega\frac{\pi}{\omega} + \frac{\pi}{2}}_{-1}\Big) + \frac{\mu g}{\omega^2}$$

$$= -A_0 + \frac{\mu g}{\omega^2} + \frac{\mu g}{\omega^2} = -A_0 + \frac{2\mu g}{2\omega^2}$$ [illegible]

$$\boxed{x_1(t_1) = -A_0 + \frac{2\mu g}{\omega^2}}$$

(2)



$$-A_0 + \frac{2\mu g}{\omega^2}$$

$$m\ddot{x} = -kx - \mu m g$$

$$\ddot{x} + \frac{k}{m}x = -\mu g$$

$$x_2(t) = D\sin(\omega t + \varphi') - \frac{\mu g}{\omega^2}$$

$$x_2(0) = -A_0 + \frac{2\mu g}{\omega^2}, \quad v_2(0) = 0$$

$$\dot{x}_2(t) = 0 = D\omega\cos\varphi' = 0 \Rightarrow \varphi' = \frac{\pi}{2} \Rightarrow x_2(0) = -A_0 + \frac{2\mu g}{\omega^2} = D - \frac{\mu g}{\omega^2}$$

$$\boxed{D = -A_0 + \frac{3\mu g}{\omega^2}}$$

$$x_2(t) = \left(-A_0 + \frac{3\mu g}{\omega^2}\right)\sin\left(\omega t + \frac{\pi}{2}\right) - \frac{\mu g}{\omega^2}$$

$$\dot{x}_2(t_2) = \omega\left(-A_0 + \frac{3\mu g}{\omega^2}\right)\cos\left(\omega t_2 + \frac{\pi}{2}\right) = 0$$

$$\omega t_2 + \frac{\pi}{2} = \frac{3\pi}{2} \Rightarrow t_2 = \frac{\pi}{\omega} = \frac{T}{2}$$

$$x_2(t) = \left(-A_0 + \frac{3\mu g}{\omega^2}\right)\underbrace{\sin\left(\omega\frac{\pi}{\omega} + \frac{\pi}{2}\right)}_{-1} - \frac{\mu g}{\omega^2}$$

$$= A_0 - \frac{4\mu g}{\omega^2}$$

$$\boxed{x_2(t_2) = A_0 - \frac{4\mu g}{\omega^2}}$$

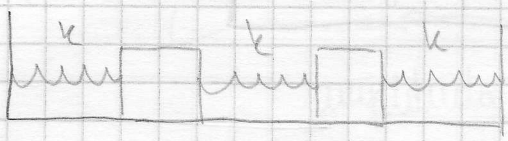
$$x_p(t_p) = (-1)^p A_0 + p\frac{2\mu g}{\omega^2}(-1)^{p+1}$$

$$p=0, \quad x_0(t_0) = A_0 = (-1)^0 A_0 + 0\cdot\frac{2\mu g}{\omega^2} = A_0$$

$$\boxed{x_p(t_p) = (-1)^p\left(A_0 - \frac{2p\mu g}{\omega^2}\right)}$$

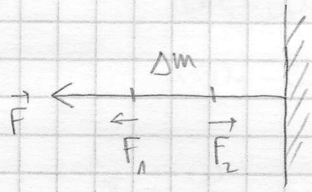
$p \ , \ p+i$

$x_p \cdot k > \mu g m > x_{p+1} \cdot k$



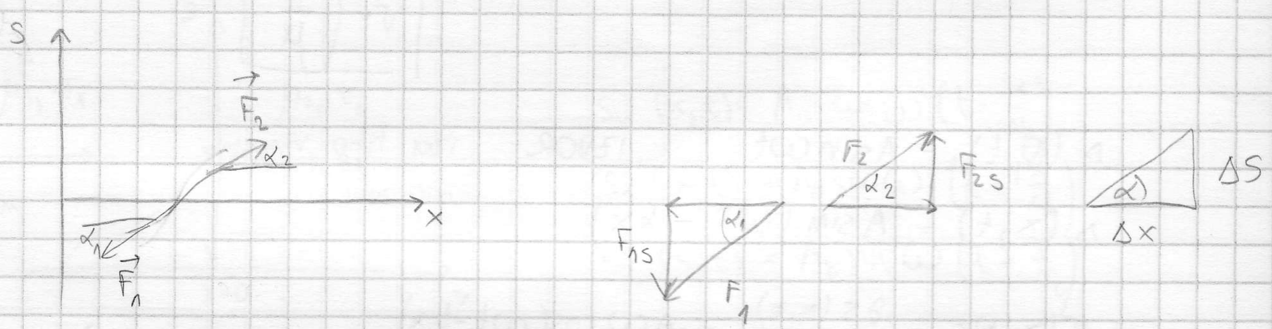
mora hvati u dva normalna

Valna jednadžba transverzalnog vala na žici



$F_1 = F_2 \ , \quad \mu = \frac{m}{l} = \frac{\Delta m}{\Delta l}$

$\Delta l = \Delta x$



$$dF_s = F_{2s} - F_{1s} = F\left(\sin\alpha_2 - \sin\alpha_1\right)$$

$$\sin\alpha \approx \mathrm{tg}\,\alpha \qquad \mathrm{tg}\,\alpha = \frac{\Delta s}{\Delta x} \approx \frac{\partial s}{\partial x}$$

$$dF_s = F\left(\mathrm{tg}\,\alpha_2 - \mathrm{tg}\,\alpha_1\right) = F\left(\underbrace{\left(\frac{\partial s}{\partial x}\right)_2 - \left(\frac{\partial s}{\partial x}\right)_1}\right)$$

$$\left(\frac{\partial s}{\partial x}\right)_2 - \left(\frac{\partial s}{\partial x}\right)_1 = \Delta\left(\frac{\partial s}{\partial x}\right) = \frac{\Delta\left(\frac{\partial s}{\partial x}\right)}{\Delta x} \cdot \Delta x =$$

$$= \underbrace{\frac{\partial}{\partial x}\left(\frac{\partial s}{\partial x}\right)}_{\frac{\partial^2 s}{\partial x^2}} \cdot \Delta x$$

$$\frac{\partial^2 s}{\partial x^2} \, \Delta x$$

$$\boxed{dF_s = F \, \frac{\partial^2 s}{\partial x^2} \, \Delta x}$$

2. N. Z.

$$dF_s = dm \cdot a = dm \cdot \frac{\partial^2 s}{\partial t^2} \Rightarrow F \frac{\partial^2 s}{\partial x^2} \cancel{\Delta x} = \mu \cancel{\Delta x} \cdot \frac{\partial^2 s}{\partial t^2}$$

$$dm = \mu \cdot \Delta x$$

$$\boxed{\frac{\partial^2 s}{\partial x^2} - \frac{\mu}{F} \frac{\partial^2 s}{\partial t^2} = 0} \qquad s = s(x,t)$$

$$s(x,t) = f(x - vt) + g(x + vt)$$

$$f(u) \qquad u = x - vt$$

$$\frac{\partial s}{\partial x} = \underbrace{\frac{\partial f}{\partial u}}_{f'} \cdot \underbrace{\frac{\partial u}{\partial x}}_{1} \qquad \frac{\partial u}{\partial t} = -v$$

$$\frac{\partial^2 s}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial s}{\partial x}\right) = \underbrace{\frac{\partial}{\partial u} f'}_{f''} \cdot \underbrace{\frac{\partial u}{\partial x}}_{1}$$

$$\frac{\partial^2 s}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial s}{\partial t}\right) = \frac{\partial}{\partial u}\left(\frac{\partial s}{\partial t}\right)\frac{\partial u}{\partial t} = \frac{\partial}{\partial u}\left(-v f'\right)(-v) = v^2 f''$$

$$\boxed{v = \sqrt{\frac{F}{\mu}}}$$

$s(0,t) = A \sin \omega t$ IZVOR ima tiho sinusno

$$s(x,t) = A \sin(\omega t - kx)$$

$$v(x,t) = \frac{\partial s(x,t)}{\partial t} = A\omega \cos(\omega t - kx)$$

$$a(x,t) = \frac{\partial v(x,t)}{\partial t} = -A\omega^2 \sin(\omega t - kx)$$

Superpozicija valova

$$\left.\begin{aligned} s_1(x,t) &= A \sin(\omega t - kx) \\ s_2(x,t) &= A \sin(\omega t - kx + \varphi) \end{aligned}\right\} + \qquad \sin\alpha + \sin\beta = 2\cos\frac{\alpha - \beta}{2} \sin\frac{\alpha + \beta}{2}$$

$$= \underbrace{2A\cos\frac{\varphi}{2}}_{\text{amplituda}} \sin\left(\omega t - kx + \frac{\varphi}{2}\right)$$

Maksimalna amplituda: $\cos\frac{\varphi}{2} = \pm 1 \Rightarrow$ KONSTRUKTIVNA INTERFERENCIJA

dva vala su u fazi

$\cos\frac{\varphi}{2} = 0 \Rightarrow$ DESTRUKTIVNA INTERF.

Grupna brzina

$$s(x,t) = s_1(x,t) + s_2(x,t) = A\sin(\omega_1 t - k_1 x) + A\sin(\omega_2 t - k_2 x)$$

$$= \underbrace{2A\cos\left[\frac{(\omega_1-\omega_2)t}{2} - \frac{(k_2-k_1)x}{2}\right]}_{\text{amplituda}} \sin\left[\frac{(\omega_2+\omega_1)t}{2} - \frac{(k_2+k_1)x}{2}\right]$$

$$\frac{(\omega_2-\omega_1)t}{2} - \frac{(k_2-k_1)}{2}x = \text{konst.} \quad / \cdot 2$$

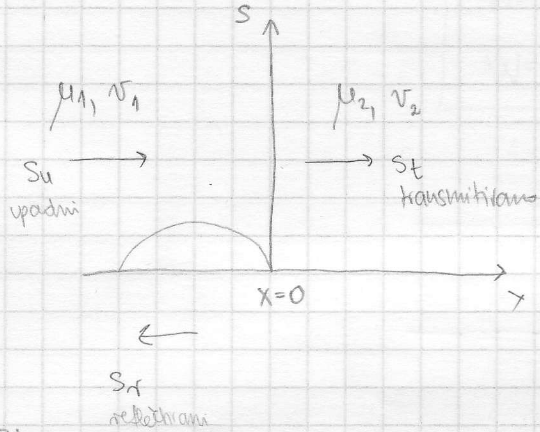
$$(\omega_2-\omega_1)t - (k_2-k_1)x = \text{konst.} \quad /'$$

$$(\omega_2-\omega_1)\,dt - (k_2-k_1)\,dx = 0 \Rightarrow \boxed{\frac{dx}{dt} = \frac{\omega_2-\omega_1}{k_2-k_1}} = v_g$$

$$v_f = \frac{\omega}{k}$$

brzina širenja vala ovisi o → sili napetosti, linearnoj gustoći užeta

Refleksija valova



$$s_u(x,t) = A_u \sin\omega\left(t - \frac{x}{v_1}\right)$$

$$s_r(x,t) = A_r \sin\omega\left(t + \frac{x}{v_1}\right)$$

$$s_t(x,t) = A_t \sin\omega\left(t - \frac{x}{v_2}\right)$$

UVJETI:

1. Kontinuiran: $s_u(x,t) + s_r(x,t) = s_t(x,t)$
2. Nagib konst. $\frac{\partial}{\partial x}(s_u + s_r) = \frac{\partial}{\partial x} s_t$

$\Big\}\ x = 0$

$$A_u \sin(\cancel{\omega t}) + A_r \sin\cancel{\omega t} = A_t \sin\cancel{\omega t}$$

$$\boxed{A_u + A_r = A_t}$$

$$-\frac{A_u}{v_1}\cos\cancel{\omega t} + \frac{A_r}{v_1}\cos\cancel{\omega t} = -\frac{A_t}{v_2}\cos\cancel{\omega t}$$

$$\boxed{\frac{A_u}{v_1} - \frac{A_r}{v_1} = \frac{A_t}{v_2}}$$

$$\frac{A_u}{v_1} - \frac{A_r}{v_1} = \frac{1}{v_2}(A_u + A_r) \Rightarrow$$

$$\boxed{A_t = \frac{2v_2}{v_2+v_1} A_u} \qquad \boxed{A_r = \frac{v_2-v_1}{v_2+v_1} A_u}$$

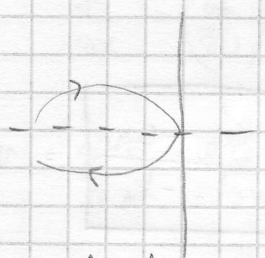
$$v = \sqrt{\frac{F}{\mu}}$$

1. iz rjeđeg u gušće

$\mu_1 < \mu_2$ $\quad v_1 > v_2$ $\quad A_r < 0$ ← mijenja se faza

čvrsti kraj:

$\mu_2 \longrightarrow \infty$ $\quad v_2 \longrightarrow 0$ $\quad A_r = -A_u$



$A_u = A$

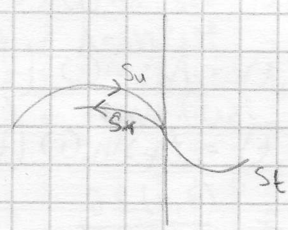
$$s_u(x,t) = A\sin\left(\omega\left(t - \frac{x}{v_1}\right)\right)$$

$$s_r(x,t) = -A\sin\omega\left(t + \frac{x}{v_1}\right) = A\sin\left(\omega\left(t + \frac{x}{v_1}\right) + \pi\right)$$

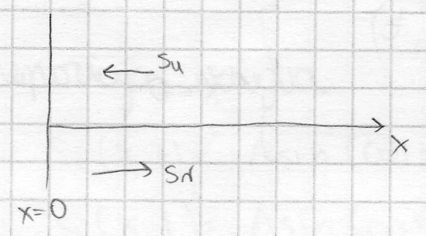
$A_t = 0$

2. iz gušćeg u rjeđe

$\mu_1 > \mu_2$ $\quad v_1 < v_2$



STOJNI VAL:



$$s_u(x,t) = A\sin(\omega t + kx)$$

$$s_r(x,t) = -A\sin(\omega t - kx)$$

$$s_u + s_r = \underline{2A\sin kx\cos\omega t}$$ stojni val

ČVOR: $\sin kx_n = 0$

$kx_n = n\pi \quad , n = 0, 1, 2, \ldots$

$x_n = \frac{n\pi}{k} \qquad k = \frac{2\pi}{\lambda}$

$x_n = \frac{n\cancel{\pi}\lambda}{2\cancel{\pi}} = n\frac{\lambda}{2}$

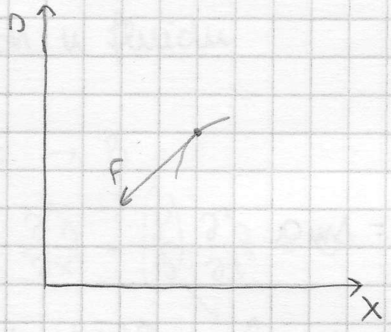
TRBUSI : $\sin kx_m = \pm 1$

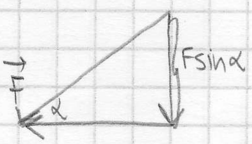
$$kx_m = (2n+1)\frac{\pi}{2}$$

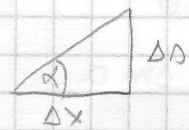
$$x_m = (2n+1)\frac{\lambda}{4}$$

28.10.2013.

Prijenos energije u transverzalnom valu







$$tg\alpha = \frac{\Delta s}{\Delta x}, \quad s(x,t)$$

$$s(x,t) = A\sin(\omega t - kx + \varphi)$$

općenito: $\boxed{W = \vec{F}\cdot\vec{s} \qquad W = Fs\cos\theta}$

$$W = F\sin\alpha\, ds \underbrace{\cos 180°}_{-1}$$

$$P = \frac{dW}{dt} = -\frac{d}{dt}(F\sin\alpha\, ds) =$$

$$P = -F\sin\alpha\frac{\partial s}{\partial t}$$

$$= -F\sin\alpha A\omega\cos(\omega t - kx + \varphi)$$

$$\sin\alpha \approx tg\alpha \approx \frac{\partial s}{\partial x} = -kA\cos(\omega t - kx + \varphi)$$

$$\boxed{P = Fk\omega A^2\cos^2(\omega t - kx + \varphi)}$$

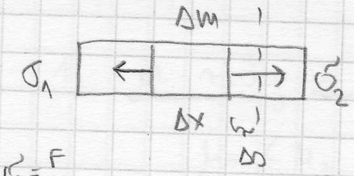
$$\boxed{P \sim A^2}$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\bar{P} = \frac{1}{T}\int_0^T P\,dt = \frac{1}{T}Fk\omega A^2 \underbrace{\int_0^T \cos^2(\omega t - kx + \varphi)\,dt}_{\frac{T}{2}} =$$

$$\boxed{\bar{P} = \frac{1}{2}Fk\omega A^2}$$

Longitudinalni val u štapu



$\sigma = \frac{F}{\cancel{S}\,A}$ (da ne bude ista oznaka)

$$\varepsilon = \frac{\Delta s}{\Delta x} \approx \frac{\partial s}{\partial x}$$

pomicaj, mjerene $s(x,t)$

$$\sigma = \underbrace{E}_{\text{Young}} \cdot \varepsilon \approx E \frac{\partial s}{\partial x}$$

$$\sigma_2 - \sigma_1 \neq 0$$

$$F_2 - F_1 = \Delta m\, a$$

$$\sigma_2 \cdot A - \sigma_1 \cdot A = (\sigma_2 - \sigma_1) A = \Delta\sigma \cdot A = \Delta m\, a$$

$$\frac{\Delta \sigma}{\Delta x} \approx \frac{\partial \sigma}{\partial x} \Rightarrow \Delta\sigma = \frac{\partial \sigma}{\partial x} \cdot \Delta x \qquad \Delta m = \rho \cdot \Delta x \cdot A$$

$$\frac{\partial \sigma}{\partial x} \cdot \cancel{\Delta x} \cdot \cancel{A} = \rho \cancel{\Delta x} \cdot \cancel{A} \cdot \frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial \sigma}{\partial x} = \frac{\partial}{\partial x}\left(E \frac{\partial s}{\partial x}\right) = E \frac{\partial^2 s}{\partial t^2}$$

$$E \frac{\partial^2 s}{\partial x^2} - \rho \frac{\partial^2 s}{\partial t^2} = 0 \quad /:E$$

$$\boxed{\frac{\partial^2 s}{\partial x^2} - \underbrace{\frac{\rho}{E}}_{\frac{1}{v^2}} \frac{\partial^2 s}{\partial t^2} = 0}$$

valna jednadžba long. vala

$$v = \sqrt{\frac{E}{\rho}}, \quad s(x,t) = A \sin(\omega t - kx)$$

Stojni val na štapu

$$s_1(x,t) = A \sin(\omega t - kx)$$

$$s_2(x,t) = A \sin(\omega t + kx)$$

$$s(x,t) = s_1(x,t) + s_2(x,t)$$

$$= \underbrace{2A \cos kx}_{\substack{\text{amplituda} \\ \text{prostorno ponašanje}}} \underset{\substack{\uparrow \\ \text{vremensko ponašanje}}}{\sin \omega t}$$

$\cos kx = \pm 1$ TRBUH

$\cos kx = 0$ ČVOR

$x = L$ trbuh

$\cos k \cdot L = \pm 1$

$k \cdot L = n\pi$

$k = \frac{2\pi}{\lambda} \Rightarrow \frac{2\cancel{\pi}}{\lambda} \cdot L = n\cancel{\pi}$

$$\boxed{\lambda_n = \frac{2L}{n}}$$

Long. Valovi u fluidu

$E \to B$

$$\frac{\partial^2 s}{\partial x^2} - \frac{\rho}{B}\frac{\partial^2 s}{\partial t^2} = 0$$

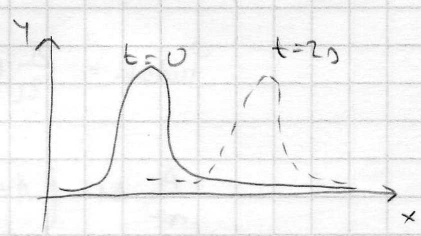
$s(x,t)$ $\quad \frac{1}{v^2} \Rightarrow v = \sqrt{\frac{B}{\rho}}$

- proces širenja zvuka je adijabatski

AUDITORNE

2.2.

$y = \frac{a^3}{a^2 + x^2}$, $a = 1\,m$



$+x$, $v = 2\,m/s$

$y(x, t=2s) = ?$

$x + vt$

$x - vt$

$$y(x,t) = \frac{a^3}{a^2 + (x - vt)^2}$$

$$y(x, t=2s) = \frac{(1m)^3}{(1m)^2 + (x - 2m/s \cdot 2s)^2}$$

$$y(x, t=2s) = \frac{1m}{1 + (\frac{x}{1m} - 4)^2}$$

2.4. $r = 0.1\,mm$

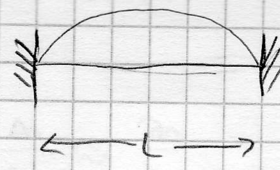
$f_1 = 440\,Hz$

$f_{udari} = \frac{10}{3s}$

$\Delta d = ?$, $\Delta d = 2\Delta r$

$f_{udari} = f_p - f_o = f_1 - f_1'$

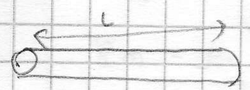
$f_1 = ?$



$\lambda = 2L$

$v = \sqrt{\frac{F}{\mu}}$

Transverzalni val na vretulžici



μ - linearna gustoća mase

$$\left.\begin{aligned} m &= \mu \cdot L \\ m &= \rho \cdot L \underbrace{(r^2\pi)}_{S} \end{aligned}\right\} \quad \mu \rightarrow S \cdot \rho$$

$$v = f \cdot \lambda$$

$$f_1 \cdot 2L = \sqrt{\frac{F}{\rho \cdot S}}$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\rho S}}$$

$$r' = r + \Delta r$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{r^2\pi\rho}} = \frac{1}{2L}\sqrt{\frac{F}{\pi\rho}} \cdot \frac{1}{r} = \frac{A}{r}$$

$$f_{udara} = \frac{1}{2L}\sqrt{\frac{F}{\pi\rho}}\left(\frac{1}{r} - \frac{1}{r+\Delta r}\right) = A\,\frac{r+\Delta r - r}{r(r+\Delta r)} = \frac{A\,\Delta r}{r(r+\Delta r)}$$

$$\frac{A}{r^2}\,\Delta r\left(1 + \frac{\Delta r}{r}\right)^{-1} \simeq \frac{A\,\Delta r}{r^2}\left[1 - \frac{\Delta r}{r} + O\left(\frac{\Delta r}{r}\right)^2 + \dots\right] =$$

$$= \frac{A\,\Delta r}{r^2} = f_1\,\frac{\Delta r}{r}$$

$$\Delta r = \frac{f_{udara}}{f_1} \cdot r = \frac{10}{3s} \cdot \frac{1}{440\,Hz} \cdot 10^{-4}\,m = 7{,}58 \cdot 10^{-7}\,m$$

$$\boxed{\Delta d = 1{,}52 \cdot 10^{-6}\,m}$$

(14)

$$f_1(r) = \frac{A}{r}$$

$$\frac{df_1(r)}{dr} = -\frac{A}{r^2} = -\frac{f_1}{r}$$

$$\Delta f_1(r) = -\frac{f_1}{r} \cdot \Delta r$$

2.6

$$s=s_1+s_2 \quad \begin{cases} s_1(x,t)=(4\,cm)\sin\left(3s^{-1}t-\frac{x}{7cm}\right) \\ s_2(x,t)=(4\,cm)\sin\left(3s^{-1}t+\frac{x}{7cm}+\phi\right) \end{cases}$$

$$s(x=5cm,t)=0$$

ista ω

smjer suprotan propagacija

faza omogućuje "fino podešavanje"

ista amplituda

ista brzina

$$s(x,t)=2\,(4\,cm)\cos\Big(\underbrace{-\frac{2x}{7cm\cdot 2}-\frac{\phi}{2}}_{\text{bavimo se njim}}\Big)\sin\Big(\underbrace{\frac{2\cdot 3s^{-1}t}{2}+\frac{\phi}{2}}_{\text{ne može biti 0 u bilo kojem trenutku}}\Big)$$

$x=5\,cm$:

$$-\frac{5cm}{7cm}-\frac{\phi}{2}=\frac{\pi}{2}$$

$$\boxed{\phi=-\frac{10}{7}-\pi}$$

$$s_2(x,t)=4\,cm\,\sin\left(3s^{-1}t+\frac{x}{7cm}-\frac{10}{7}-\pi\right)$$

2.7.

d, L, F

$f_1(Ag)=?$, $f_1(Fe)=200\,Hz$

$\rho(Ag)=10\,600\,kg/m^3$

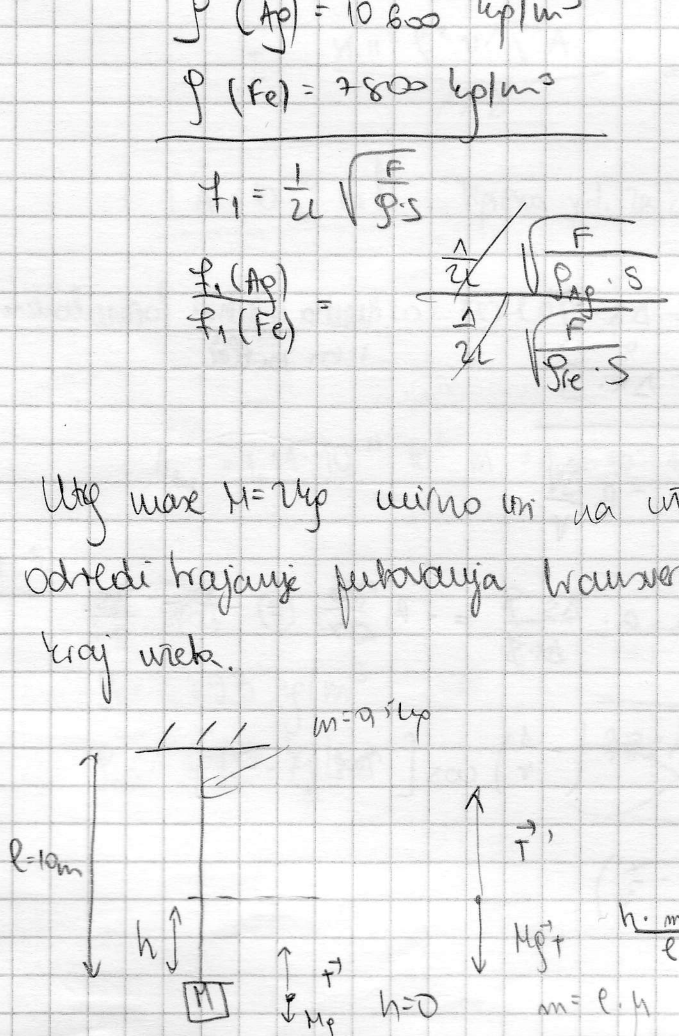
$\rho(Fe)=7800\,kg/m^3$

$$f_1=\frac{1}{2L}\sqrt{\frac{F}{\rho\cdot S}}$$

$$\frac{f_1(Ag)}{f_1(Fe)}=\frac{\cancel{\frac{1}{2L}}\sqrt{\frac{F}{\rho_{Ag}\cdot S}}}{\cancel{\frac{1}{2L}}\sqrt{\frac{F}{\rho_{Fe}\cdot S}}}=\sqrt{\frac{\rho_{Fe}}{\rho_{Ag}}} \Rightarrow f_1(Ag)=f_1(Fe)\cdot\sqrt{\frac{\rho_{Fe}}{\rho_{Ag}}}$$

$$\underline{=171{,}56\,Hz}$$

Uteg mase $M=2\,kg$ visi na užetu duljine $l=10\,m$ i mase $m=0{,}5\,kg$. Odredi trajanje putovanja transverzalnog valnog paketa s jednog na drugi kraj užeta.



$$T(h)=Mg+m\frac{h}{l}g$$

$$=g\left(M+m\frac{h}{l}\right)$$

$$v=\sqrt{\frac{T}{\mu}}, \quad v(h)=\sqrt{\frac{g}{m}\cdot l\left(M+m\frac{h}{l}\right)}$$

$$\frac{dh}{dt} = \sqrt{\frac{g}{m} l}\,\sqrt{M + \frac{mh}{l}}$$

$$dt = \frac{dh}{\sqrt{M + \frac{mh}{l}}} \cdot \sqrt{\frac{m}{gl}}$$

$$t = \sqrt{\frac{m}{gl}} \int_0^l \frac{dh}{\sqrt{M + \frac{mh}{l}}} = \left| \begin{array}{l} M + m\frac{h}{l} = y \\ m\frac{dh}{l} = dy \Rightarrow dh = dy \cdot \frac{l}{m} \end{array} \right| \quad \begin{array}{l} h = 0 \Rightarrow y = M \\ h = l \Rightarrow y = M + m \end{array}$$

$$= \sqrt{\frac{m}{gl}} \frac{l}{m} \int_M^{M+m} \frac{dy}{\sqrt{y}} = \sqrt{\frac{l}{mg}} \cdot 2 \left(\sqrt{M+m} - \sqrt{M} \right) = 2\sqrt{\frac{lM}{mg}} \left(\sqrt{1 + \frac{m}{M}} - 1 \right) = 0{,}477$$

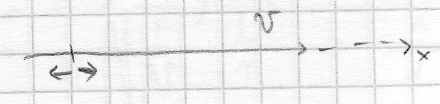
30.10.2013

AUDITORNE

pr. 2.7.

$f = 1000\ Hz$

$A = 2 \cdot 10^{-6}\ m$



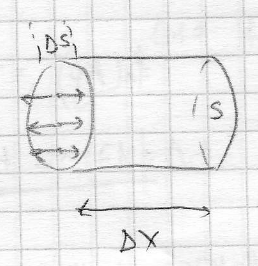
$s(x,t) = A \sin(\omega t - kx)$, $\omega = 2\pi f$, $k = \frac{\omega}{v} = \frac{2\pi f}{v}$

$\omega(t - \frac{k}{\omega} x)$

$t - \frac{x}{v}$

$s(x,t) = A \sin\left[2\pi f \left(t - \frac{x}{v}\right)\right]$

$\Delta p_{max} = ?$



$\Delta V = \Delta s \cdot S$

$V = \Delta x \cdot S$

nema u zbirci formulame izraz za tlak

$$\Delta p = -B \frac{\Delta V}{V}$$

$$\Delta p = -B \cdot \frac{\Delta s \cdot \cancel{S}}{\Delta x \cdot \cancel{S}} = -B \frac{\partial s}{\partial x} = -\rho v^2 \frac{\partial s}{\partial x}$$

$$\frac{\partial s}{\partial x} = A 2\pi f \left(-\frac{1}{v}\right) \cos\left[2\pi f \left(t - \frac{x}{v}\right)\right]$$

$$\Rightarrow \Delta p = -\rho v^{\cancel{2}} A 2\pi f \left(-\frac{1}{\cancel{v}}\right) \cos\left[2\pi f \left(t - \frac{x}{v}\right)\right]$$

$$\Delta p_{max} = 2\pi f v \rho A$$

$\Delta p_{max} = 2\pi f v \rho A = 5{,}51$ Pa

$p_{atm} \simeq 10^5$ Pa

$I = \frac{\bar{P}}{S} = \frac{(\Delta p_{max})^2}{2 v \rho} = 3{,}45 \cdot 10^{-2}$ W/m^2

D, L − razina jakosti buke $\quad L = 10 \log_{10} \frac{I}{I_0} = 10 \log_{10} \left(\frac{3{,}45 \cdot 10^{-2}}{10^{-12}} \right) =$

$= 10\,(0{,}538 + 10) = 105{,}38$ dB

Horvat, pr. 3-13

Mjerenjem je ustanovljeno da ljudsko uho može čuti intenzitet od 10^{-12} W/m^2 vala frekvencije 1000 Hz. To je tzv. "prag čujnosti". Kojom amplitudom titra taj val? ($\rho_{zrak} = 1{,}2$ kg/m^3, $v = 343$ m/s

"bol" 120 dB

prag čujnosti $I_0 = 10^{-12}$ W/m^2

$I = \frac{(\Delta p_{max})^2}{2 v \rho}$, $\quad \Delta p_{max} = 2\pi f v \rho A$

$I = \frac{\cancel{4}^{2}\pi^2 f^2 v^{\cancel{2}} \rho^{\cancel{2}} A^2}{\cancel{2 v \rho}} \Rightarrow A = \frac{1}{\pi f}\sqrt{\frac{I}{2 v \rho}} = 1{,}11 \cdot 10^{-11}$ m $\rightarrow$ prag čujnosti

$\boxed{1\,\text{Å} = 10^{-10}\ \text{m}}$ tipična jed. za atome

$I_0 = 10^{-12}$

$L = 120 = 10 \log \frac{I}{I_0} \rightarrow I_0 10^{\frac{L}{10}} = I$

$I_0 \cdot 10^{12} = I \rightarrow I = 1$ W/m^2

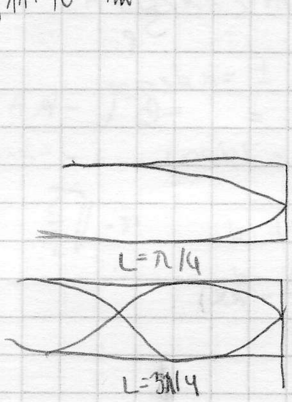
$A_{bol} = 1{,}11 \cdot 10^{-11} \cdot \underbrace{10^6}_{\sqrt{10^{12}}}$ m $= 1{,}11 \cdot 10^{-5}$ m

2.3.

① zrak ② CO_2

$\rho_z = 1{,}293$ kg/m^3

$\rho_{CO_2} = 1{,}977$ kg/m^3



$\lambda = \frac{4L}{N}$

$v = f \cdot \lambda$

$v = \sqrt{\frac{\varkappa p}{\rho}}$

$f = \frac{1}{\lambda}\sqrt{\frac{\varkappa p}{\rho}}$

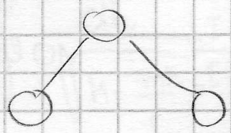
zrak: N_2, O_2

$\sim 10^{-10}$

$i = 5$

$= 3$ translacije $+ 2$ rotacijska

CO_2



$i = 6 = 3$ trans. $+ 3$ rot.

$$\kappa_{zrak} = \frac{i+2}{i} = \frac{7}{5} = 1{,}4$$

$$\kappa_{CO_2} = \frac{8}{6} = \frac{4}{3} = 1{,}33$$

$$\kappa_{zrak}(20°) = 1{,}4 \qquad \kappa_{CO_2}(20°C) = 1{,}3$$

$$\frac{f_1 - f_2}{f_1} = 1 - \frac{f_2}{f_1} \qquad \frac{f_2}{f_1} = \sqrt{\frac{\kappa_{CO_2}}{\rho_{CO_2}} \cdot \frac{\rho_{zrak}}{\kappa_{zrak}}}$$

$$= 1 - \sqrt{\frac{\kappa_2}{\kappa_1}\frac{\rho_1}{\rho_2}} = 0{,}211$$

- nelinearni troatomni CO_2 — 21 %
- linearni — 19 %
- iz tablica — 22 %

$$\frac{\partial^2 s}{\partial x^2} - \frac{\rho}{B}\frac{\partial^2 s}{\partial t^2} = 0 \qquad s(x,t) = A\sin(\omega t - kx)$$

$$\kappa = \frac{1}{B} = -\frac{1}{V}\frac{dV}{dp} \to \frac{\Delta V}{\Delta p}$$

$$\frac{\Delta V}{V} = \frac{S_p \cdot \Delta s}{S_p \cdot \Delta x} = \frac{\Delta s}{\Delta x} \to \frac{\partial s}{\partial x}$$

$$\Delta p = -B\frac{\Delta V}{V} = -B\frac{\partial s}{\partial x} = -B\left(-kA\cos(\omega t - kx)\right)$$

$$= BAk\cos(\omega t - kx) \qquad v = \sqrt{\frac{B}{\rho}} \qquad k = \frac{\omega}{v}$$

$$= v\rho A\omega \cos(\omega t - kx)$$

Δp_{max}

$v \to$ u plinu $\to$ ADIJABATSKI

$$pV^{\varkappa} = \text{konst.} \qquad \varkappa = \frac{C_p}{C_v}$$

$$\varkappa_a = -\frac{1}{V}\frac{dV}{dp} = \frac{1}{B}$$

$$V^{\varkappa} = \frac{\text{kons.}}{p} \quad \Big|'$$

$$\varkappa V^{\varkappa-1}\, dV = -\frac{\text{konst.}\, dp}{p^2} \Rightarrow \frac{dV}{dp} = \frac{-\text{konst.}}{\underbrace{\varkappa V^{\varkappa-1}\cdot p^2}_{pV^{\varkappa}=\text{konst.}}} = \frac{-1}{\varkappa p V^{-1}}$$

pV^{-1}

$$\varkappa_a = \frac{1}{\varkappa p} = \frac{1}{B}$$

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\varkappa \cdot p}{\rho}}$$

$\varkappa = 1,67$ 1-A , $\varkappa = 1,4$ za zrak

$pV = nRT$, $n = \frac{m}{M}$

$$p = \frac{m}{M \cdot V} RT = \frac{\rho RT}{M} \Rightarrow v = \sqrt{\varkappa \frac{RT}{M}}$$

$R = 8,314$ J/molK

$M = 28,84 \cdot 10^{-3}$ kg/mol

$v = 331$ m/s za zrak

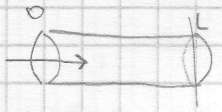
brzina zvučnog vala u plinu

ZRAK - STOJNI VAL

$A \sin(\omega t - kx)$

svirala

1. otvorena



nema skoka u fazi
otvorena - pa je na rubu trbuh

$$s(x,t) = A\sin(\omega t - kx) + A\sin(\omega t + kx)$$
$$= 2A \cos kx \sin \omega t$$

$\cos kx = \pm 1$ maksimumi - trbusi

$x = L$: $\cos kL = \pm 1$

$kL = n\pi \Rightarrow \lambda = \frac{2L}{n}$

$k = \frac{2\pi}{\lambda}$

2. zatvorena

ili $-A$ pa bez $+\pi$

$$s(x,t) = A\sin(\omega t - kx) + A\sin(\omega t + kx + \pi)$$

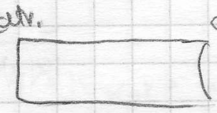
pomak u fazi za π $\quad$ ↑ čvor

$$\Rightarrow \quad \underbrace{\lambda_n = \frac{2L}{n}}$$

$$= 2A\cos\left(kx + \frac{\pi}{2}\right)\sin\omega t$$

3. poluotvorena

zatv. $\quad$ otv.



0 $\qquad$ $x = L$ $\quad$ trbuh

$$\cos\left(kx + \frac{\pi}{2}\right) = \pm 1$$

$$kL + \frac{\pi}{2} = m\pi \qquad k = \frac{2\pi}{\lambda}$$

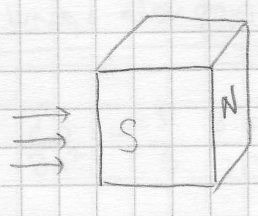
$$\frac{2\cancel{\pi}}{\lambda} \cdot L + \frac{\cancel{\pi}}{2} = m\cancel{\pi}$$

$$\boxed{\lambda_m = \frac{4L}{2m-1}}$$

samo tu imamo otpadanje frekv.

4.11.2013.

Energija mehaničkih valova



$v \cdot \Delta t$

$\Delta V = S \cdot v \cdot \Delta t$

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 = \frac{1}{2}mA^2\omega^2$$

$$s(t) = A\sin\omega t$$

$$v(t) = \underbrace{A\omega}_{v_{max}}\cos\omega t$$

n - koncentracija

$$n = \frac{N}{\Delta V} \quad \text{(N – broj čestica)}$$

$$\Delta E = E \cdot N = n\,\Delta V\,\frac{1}{2}mA^2\omega^2 = \frac{1}{2}\rho A^2\omega^2\Delta V$$

$$\rho = \frac{m \cdot N}{\Delta V} = m \cdot n$$

gustoća energije $\quad w = \dfrac{\Delta E}{\Delta V} = \dfrac{1}{2}\rho A^2\omega^2$

$$\Delta E = w \cdot \Delta V = w S v \Delta t$$

$$P = \frac{\Delta E}{\Delta t} = W S v$$

Intenzitet: $I = \frac{P}{S} = W \cdot v = \frac{1}{2} \rho v A^2 \omega^2 \qquad \left[\frac{W}{m^2}\right]$

Machov broj $= \frac{v_i}{v}$ – avion se giba brzinom 1 Macha ; $v_i = v \Rightarrow$ probij zvučnog zida

Dopplerov efekt

a) izvor $\vec{v}_i = 0$

detektor $\vec{v}_D \neq 0$

$$\vec{v}_{rel} = \vec{v} - \vec{v}_0$$

ovisno dal se približava ili udaljava

$$\lambda \cdot f = v$$

$$f' = \frac{v_{rel}}{\lambda} = \frac{v \pm v_0}{\lambda} \qquad f = \frac{v}{\lambda}$$

$$\boxed{f' = \frac{v \pm v_0}{v} \cdot f}$$

\+ približavanje

– udaljavanje

b) $\vec{v}_d = 0$

$\vec{v}_i \neq 0$

$$\lambda' = \lambda - v_i \cdot T = \frac{v}{f} - \frac{v_i}{f} = \frac{v - v_i}{f}$$

$$\lambda' \cdot f' = v$$

$$f' = \frac{v}{v - v_i} \cdot f$$

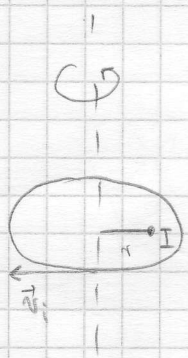
$$f' = \frac{v}{v \pm v_i} f$$

$$\boxed{f' = \frac{v + v_0}{v - v_i} f}$$

v_0, v_i : + približavanje

– udaljavanje

zad 2.17.



$f = 1000$ Hz

$r = 1,5$ m

$\omega = 50\ s^{-1}$

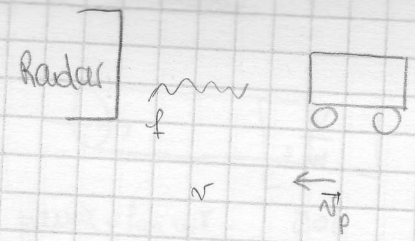
$v = 340$ m/s

$$f' = \frac{v}{v \pm v_i} f$$

$v_i = r\omega$

$f' \to 1283$ Hz

$f' \to 879,28$ Hz



$$f' = \frac{v+v_p}{v} f$$

$$f'' = \frac{v}{v-v_p} f' = \frac{v+v_p}{v-v_p} f$$

$$\Delta f = f'' - f = \frac{2v_p f}{v-v_p}$$

$$v = c = 3\cdot 10^8 \, m/s$$

$$v_p \ll c \Rightarrow \Delta f = \frac{2v_p f}{c} \Rightarrow v_p = \frac{\Delta f c}{2f}$$

ELEKTROMAGNETIZAM

6. 11. 2013.

$\vec{E}$

$\vec{F} = q \cdot \vec{E}$

MATEMATIČKI DIO

$\vec{F}_R = \sum_i \vec{F}_i \Rightarrow \vec{E}_R = \sum_i \vec{E}_i$

skalarne veličine

skalarno polje

$\varphi(x, y, z)$

gradijent φ: $\quad \mathrm{grad}\,\varphi = \frac{\partial \varphi}{\partial x}\vec{i} + \frac{\partial \varphi}{\partial y}\vec{j} + \frac{\partial \varphi}{\partial z}\vec{k}$

$$\mathrm{grad}\,\varphi = \vec{\nabla}\varphi$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

vektorsko polje

$\vec{A}$

$\vec{A} = \pm\, \mathrm{grad}\,\varphi$

$\varphi \rightarrow$ potencijal polja $\vec{A}$

$\vec{A} \rightarrow$ konzervativno - koje se može prikazati kao gradijent potencijala

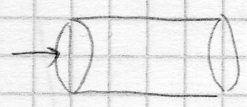
$\vec{\nabla}\vec{A} = \mathrm{div}\,\vec{A}$ divergencija vektorskog polja $\vec{A}$

$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$

$\vec{\nabla}\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ div. vekt. polja je skalar ($\vec{\nabla}\vec{A}$ - skalarni umnožak)

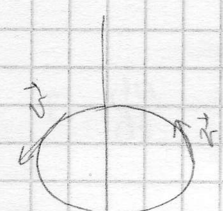
$\vec{\nabla}\vec{A} \to$ tok

fluid



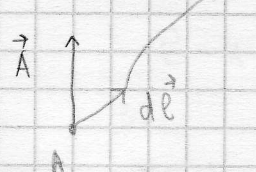
<u>Rotacija vektorskog polja</u>

$$\text{rot}\,\vec{A} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

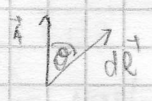


$\text{rot}\,\vec{v} \neq 0$

ako je $\text{rot}\,\vec{v} = 0$ → polje nije vrtložno



$$\int_A^B \vec{A}\,d\vec{l} = \int_A^B A\cos\theta\,dl$$

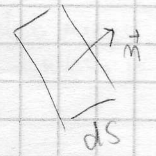


$\oint \vec{A}\,dl$ - integral po zatvorenoj krivulji

Γ ↓

rotacija ili cirkulacija vekt. polja $\vec{A}$ po krivulji

$\oint \vec{A}\,dl = 0$ → ne ovisi o putu



$d\vec{S} = \vec{n}\,dS$

$$\iint_S \vec{A}\,d\vec{S} \to \underbrace{\int_S \vec{A}\,d\vec{S}}$$ → integral po površini

tok vekt. polja $\vec{A}$ kroz površinu

$\oint_S$ → znači da je površina zatvorena

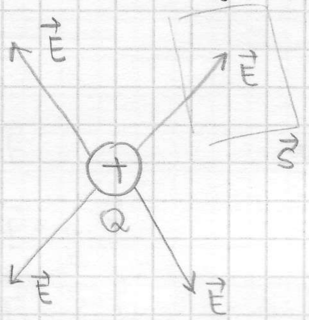
$dV = dx\,dy\,dz$ $\iiint_V \to \int_V$

1. $$\int_A^B \vec{\nabla}\phi \, d\vec{l} = \phi(r_B) - \phi(r_A)$$ teorem o gradijentu

2. $$\int_V \vec{\nabla}\vec{A} \, dV = \oint_S \vec{A} \, d\vec{S}$$ Gaussov teorem

3. $$\int_S (\vec{\nabla} \times \vec{A}) \, d\vec{S} = \oint_l \vec{A} \, d\vec{l}$$ Stokesov teorem

Tok električnog polja

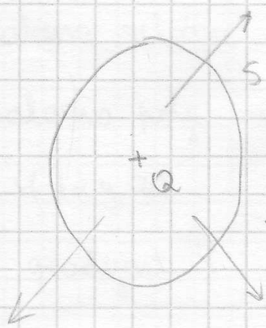


električno polje

$$\Phi_E = \int_S \vec{E} \, d\vec{S}$$

$$\Phi_E = \oint_S \vec{E} \, d\vec{S}$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$



silnice obuhvaćenim plohom — po površini —

$$\boxed{\oint_S \vec{E} \, d\vec{S} = \frac{Q}{\varepsilon_0}}$$

Gaussov zakon za električno polje

1. MAXWELLOVA JEDNADŽBA

na usmenom obavezno

Maxwellove jedn.

nema tih formula na usmenom

Gaussova ploha - uzme obuhvaćeni naboj i tok kroz tu plohu

gustoća naboja

$$\rho = \frac{dQ}{dV}$$

$$dQ = \rho \, dV$$

$$Q = \int_V \rho \, dV$$

G. teorem:

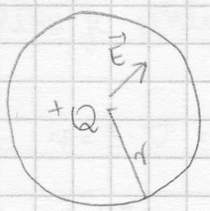
$$\underbrace{\oint_S \vec{E} \, d\vec{S} = \int_V \vec{\nabla}\vec{E} \, dV}$$

$$\frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho \, dV = \int_V \vec{\nabla}\vec{E} \, dV$$

$$\boxed{\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}}$$

divergencija je tok u točki

Električno polje točkastog naboja



$$\oint_S \underbrace{\vec{E}\,d\vec{S}}_{E\,dS} = \frac{Q}{\varepsilon_0}$$

$$E \underbrace{\oint_S dS}_{4r^2\pi} = \frac{Q}{\varepsilon_0}$$

$$E \cdot 4r^2\pi = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \qquad \text{Coulomb}$$

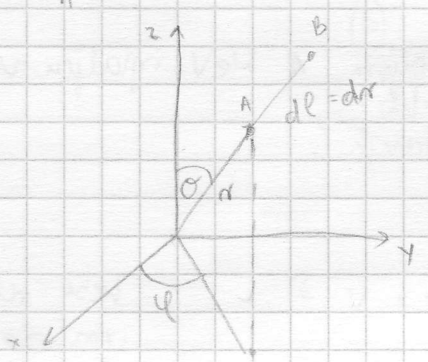
$$I = \frac{dQ}{dt} \qquad 0 \longrightarrow e \qquad S,\ \vec{v},\ n = \frac{N}{V} \qquad I = S \cdot v \cdot e \cdot n$$

Rotacija električnog polja

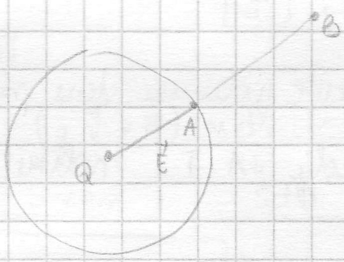
← po zatvorenoj krivulji = 0

$$\vec{E} = k\frac{Q}{r^2}\,\vec{r}_0$$

$$\int_A^B \vec{E}\,d\vec{l}$$



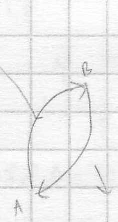
r, θ, φ



$$k\int_A^B \frac{Q}{r^2}\,dr = kQ\left(-\frac{1}{r}\right)\Bigg|_A^B = -kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$V(r_B)$

$$= V(r_B) - V(r_A)$$



$$\boxed{V(r) = k\frac{Q}{r}} \quad \text{potencijal}$$

$$\oint_A \vec{E}\,d\vec{l} = 0 \rightarrow \text{konzervativno el. polje}$$

Stokesov tm: $\oint_K \vec{E}\,d\vec{l} = \int_S (\vec{\nabla} \times \vec{E})\,d\vec{S}$

$$\boxed{\vec{\nabla} \times \vec{E} = 0} \quad \text{3. MAXWELLOVA JEDNADŽBA}$$

polje - grad potencijala

ne postoji krivulja el. polja stalnog

Pr. Odredite rad koji je potrebno obaviti kako bi se dva protona iz beskonačnosti dovela na međusobnu udaljenost 10^{-15} m.

$d = 10^{-15}$ m

$W = \vec{F} \cdot \vec{s}$ $\quad \vec{F} = Q \cdot \vec{E}$

$$W = kQ^2 \int_{\infty}^{a} \frac{dr}{r^2} = k\frac{Q_2}{a} = \frac{3{,}31 \cdot 10^{-13} J}{1{,}6 \cdot 10^{-19}} = 1{,}44 \text{ MeV}$$

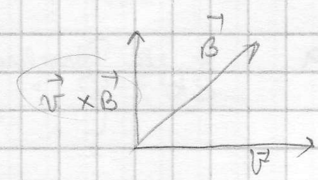
Magnetsko polje

$\vec{B}$

$\vec{F} = q\vec{E}$ električno polje

$q\vec{v} \times \vec{B}$ magnetsko

$$\boxed{\vec{F}_L = q\vec{E} + q\vec{v} \times \vec{B}}$$ LORENTZOVA SILA



pravilo desne ruke

prsti od v prema B

centrifuga veš mašine ima rupe da voda ide van

okomita sila uzrokuje kružno gibanje čestica

znači da će se naboj početi gibati kružno

$\vec{B} \rightarrow [T]$

Zad. Izračunajte polumjer staze protona koji je energijom 1 MeV ispaljen u homogeno mag. polje 1 T. Koliki je polumjer putanje?

$E = 1$ MeV

$B = 1$ T

$r = ?$

$\vec{v} \perp \vec{B}$

$F_m = qvB = F_{cp} = \frac{mv^2}{r}$

$q = e$

$m_p = 1{,}67 \cdot 10^{-27}$ kg

$E_0 = mc^2 = \frac{1{,}67 \cdot 10^{-27}}{1{,}6 \cdot 10^{-19}} \cdot (3 \cdot 10^8)^2 = 939$ MeV

Ako je $v \cong c$ treba relativistički izraz za energiju

$E << E_0$ ne treba rel. pa je

ako je en. puno manja od en. mirovanja ne treba

$E = \frac{1}{2}mv^2 \Rightarrow v = 1{,}38 \cdot 10^7$ m/s

$r = \frac{mv}{qB} = 14{,}4$ cm

$E_0 >> E$

klasična relacija je dobra

primjer, Horvat

α- čestica giba se brzinom $5\cdot10^5$ m/s po kružnoj putanji, $B = 4,5$ mT.

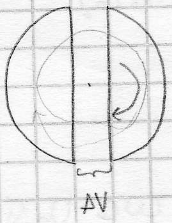
Kolika je ciklotronska frekvencija α-čestice?

$v = 5\cdot 10^5$ m/s

αč $\to$ $q = 2e$

$B = 4,5$ mT

$\omega = ?$



$\vec{v} \perp \vec{B}$

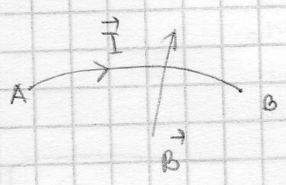
$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

$$2r\pi = v\cdot T \quad ; \quad \omega = 2\pi f \; ; \; f = \frac{1}{T}$$

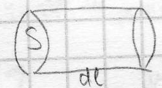
$$\omega = \frac{2\pi}{T} = \frac{\cancel{2\pi}\cdot v}{2r\cancel{\pi}} = \frac{v}{r} = \frac{\cancel{v}}{\frac{m\cancel{v}}{qB}} = \frac{qB}{m}$$

Sila na vodič u $\vec{B}$



$q, \vec{v}$

$F = q\,\vec{v}\times\vec{B}$



$dV = S\cdot dl$

$n\cdot dV$

$$d\vec{F} = (\underbrace{q}_{\to e}\,\vec{v}\times\vec{B})\; n\,\underbrace{dV}_{\downarrow\, S dl}$$

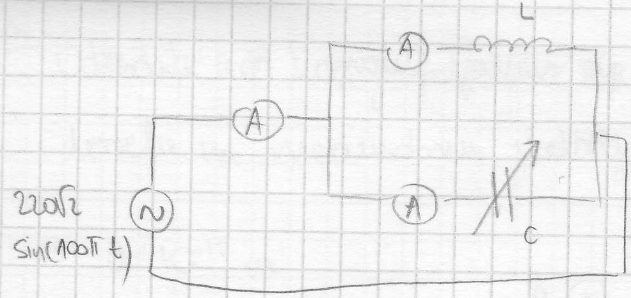
$I = Sven$

$$\boxed{d\vec{F} = I\,d\vec{l}\times\vec{B}}$$ sila na element

$\vec{v} \parallel d\vec{l}$

$$\vec{F} = I\int_A^B d\vec{l}\times\vec{B}$$

11.11.2013.



POKUS

$$i(t) = i_L(t) + i_C(t)$$

$$0 = \frac{U_0}{\omega L} \cdot \sin\left(\omega t - \frac{\pi}{2}\right) + \omega C \cdot U_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

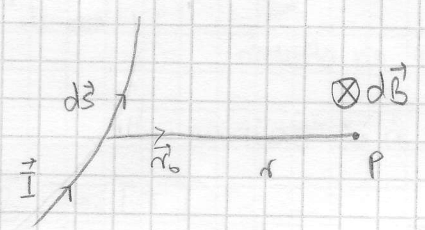
$$0 = U_0 \cdot \frac{1}{\omega L} \cos \omega t + \omega C U_0 \cos \omega t$$

L.T.

$$0 = \underset{\text{nije 0}}{U_0} \underset{\substack{\text{nije 0} \\ \text{(većinom)}}}{\cos \omega t} \cdot \left[\underbrace{\omega C - \frac{1}{\omega L}}_{\text{mora biti 0}}\right]$$

$$\omega^2 = \frac{1}{LC}$$

BIOT - SAVART



$$d\vec{B} = \frac{\mu}{4\pi} I \frac{d\vec{s} \times \vec{r}_0}{r_0}$$

$$\vec{B} = \int_{\text{po vodiču}} d\vec{B}$$

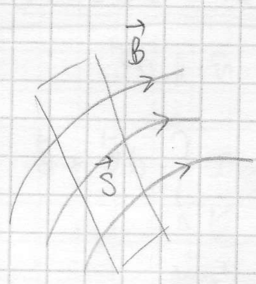
$$\boxed{B = \mu \frac{I}{2\pi r}}$$

od $d\vec{s}$ prema $\vec{r}_0$ → gleda u ploču polje

silnice mag. polja - zatvorene krivulje → ukupan tok na jednom dijelu = 0

Magnetski tok

$\vec{B}$



$\phi_M =$

$$\phi_M = \int_S \vec{B} \, d\vec{S}$$

$$[\phi_M] = Wb$$

$$\oint_S \vec{B} \, d\vec{S} = 0$$

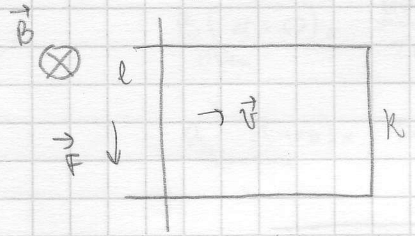
Gaussov tm: $\oint_S \vec{B} d\vec{S} = \int_V \vec{\nabla} \vec{B} dV = 0 \Rightarrow \vec{\nabla}\vec{B} = 0$

divergencija be

$$\boxed{\oint \vec{B} d\vec{S} = 0 \ , \ \vec{\nabla}\vec{B} = 0}$$ Gaussov zakon za $\vec{B}$
(2. MAXWELLOVA JEDNADŽBA)

$$\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$$

Elektromagnetska indukcija



$\vec{v} \rightarrow ds = v dt$

$$\vec{F} = -e\vec{v} \times \vec{B}$$

$$\boxed{\vec{E}_{ind.} = \frac{\vec{F}}{-e}}$$ inducirano el. polje
polje nastalo zbog gibanja žice

ind. elektromotorna sila $\boxed{\mathcal{E} = - \frac{d\phi}{dt}}$ (Lorentz)

$$\phi = \vec{B} \cdot \vec{S}$$

$$\mathcal{E} = \frac{W}{q} \qquad W = F \cdot l = q \underbrace{(\vec{v} \times \vec{B})}_{vB \text{ (okomiti su)}} l = q vBl$$

$$\mathcal{E} = vBl$$

$$dS = l\,ds = l v dt$$

(konst. B, površina se mijenja) $d\phi = B d\vec{S} = Blvdt$

$$\boxed{\frac{d\phi}{dt} = -Blv}$$

jer imamo smanjenje mag. toka

$$\mathcal{E} = \frac{W}{q} = \frac{\vec{F} \cdot d\vec{l}}{q} = \frac{\cancel{q} \vec{E}_{ind} \cdot d\vec{l}}{\cancel{q}}$$

$$\mathcal{E} = \oint_K \vec{E} d\vec{l} = \oint_K (\vec{v} \times \vec{B}) d\vec{l}$$

K
po konturi

St. tm: $\oint_K \vec{E} d\vec{l} = \int_S (\vec{\nabla} \times \vec{E}) d\vec{S}$

inducirano električno polje
ovo nije konzervativno polje

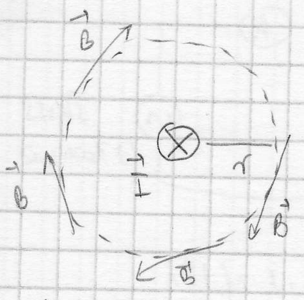
$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(\vec{B} \cdot \vec{S})$$

rotacija ind. el. polja 3. MAXWELL.

$$\mathcal{E} = -\frac{\partial}{\partial t}\int_S B d\vec{S} = \oint_K \vec{E} d\vec{l} = \int_S \vec{\nabla} \times \vec{E} d\vec{S} \Rightarrow \boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

3. Maxwellova (ako nema promjene polja, = 0)

Amperov zakon



palac - smjer struje
prsti - " - polja

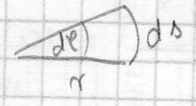
Kolika je cirkulacija polja?

$$\oint_K \vec{B}\, d\vec{s} = ?$$

$$B = \mu_0 \frac{I}{2r\pi}$$

da obiđem cijelu kružnicu

$$\oint_K \mu_0 \frac{I}{2r\pi}\, r\, d\varphi = \frac{\mu_0 I}{2\pi} \underbrace{\int_0^{2\pi} d\varphi}_{\varphi|_0^{2\pi}} = \mu_0 I$$



$$\left.\begin{aligned} \oint_K \vec{B}\, d\vec{s} &= \mu_0 \sum_i I_i \\ I &= \int_S \vec{j}\, d\vec{S} \end{aligned}\right\} \boxed{\oint \vec{B}\, d\vec{s} = \int_S \vec{j}\, d\vec{S}}$$ Amperov zakon

St. tm

$$\oint_K \vec{B}\, d\vec{s} = \int_S (\vec{\nabla} \times \vec{B})\, dS = \mu_0 \int_S \vec{j}\, d\vec{S}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$$

AUDITORNE

Horvat, 1 BIOT-SAVAROT

Zavojnica sa 100 navoja ima $r = 5$ cm. Koliki je iznos mag. polja B u središtu zavojnice ako

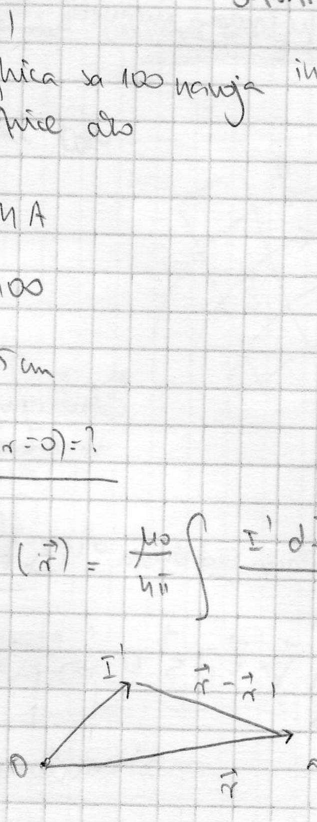
$I = 4$ A

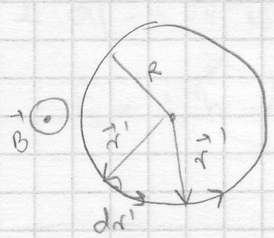
$N = 100$

$R = 5$ cm

$B(r=0) = ?$

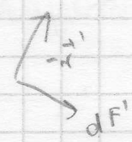
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I' d\vec{r}\,' \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$





$$\vec B(\vec r=0) = -\frac{\mu_0}{4\pi}\int \frac{I'\, d\vec r'\, \vec r'}{|\vec r'|^3} =$$

$$= -\frac{\mu_0}{4\pi}\int \frac{I'\, d\vec r' \times \hat r'}{r'^2}$$



po iznosu

$$B(\vec r=0) = -\frac{\mu_0}{4\pi}\int \frac{I'\, dr'}{r'^2} = -\frac{\mu_0 I}{4\pi R^2}\int_0^{2R\pi} dr' = -\frac{\mu_0 I}{4\pi R^2}\cdot 2R\pi = -\frac{\mu_0 I}{2R}\ (\hat k)$$

$$B_{prsten}(r=0) = \frac{\mu_0 I}{2R}$$

$$B_{zavojnica}(r=0) = N'\frac{\mu_0 I}{2R} = 5{,}03\cdot 10^{-3}\,T$$

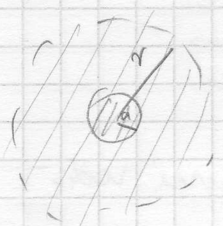
3.3.

$I,\ J = \text{konst.}$ $\qquad I = J a^2 \pi$

$B(r) = ?$

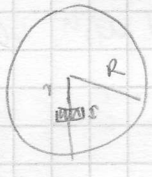
(1) $r > a$ $\qquad \oint_C \vec B\, d\vec r = \mu_0 \int_S \vec J\, d\vec S$

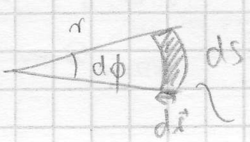
$B(r,\varphi) = B(r)$



$$\int_0^{2\pi} B(r)\, dl = \mu_0 \int_S J(r)\, dS$$

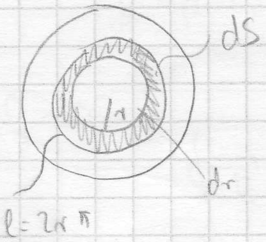
$$J(r) = \begin{cases} J, & r \le a \\ 0, & r > a \ \text{izvan vodiča} \end{cases}$$





$dl = r\cdot d\varphi$

$dS = dr\cdot dl = dr\cdot r\cdot d\varphi$



$$dS = \int_0^{2\pi} dl\, dr\, r = 2\pi r\, dr$$

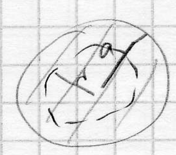
$$\int_0^{2\pi} B(r)\, dl = \mu_0 \int_S J(r)\, dS = \mu_0\, 2\pi \int_0^{r} J(r')\, r'\, dr' =$$

$$= \mu_0 2\pi \int_0^a J(r')\, r' dr' + \overset{=0}{\cancel{\int_a^r J(r')\, r' dr'}} = 2\pi\mu_0 J \frac{a^2}{2} = \mu_0 J a^2 \pi = \mu_0 \cdot I$$

$$B(r)\, 2r\pi = \mu_0 I$$

$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}, \quad r > a$$

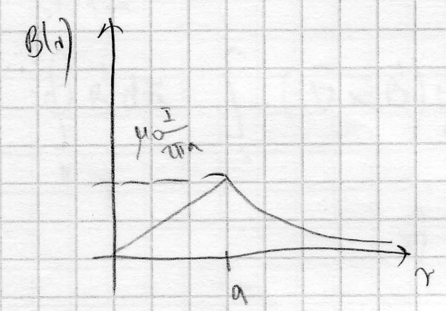
(B) $r \leq a$



$$B(r) \cdot 2r\pi = \mu_0 2\pi \int_0^r J\, r' dr' = \mu_0 2\pi \frac{r^2}{2} J = \mu_0 r^2 \pi J$$

$$B(r)\, 2\cancel{r}\pi = \mu_0 I \frac{r^{\cancel{2}}}{a^2}$$

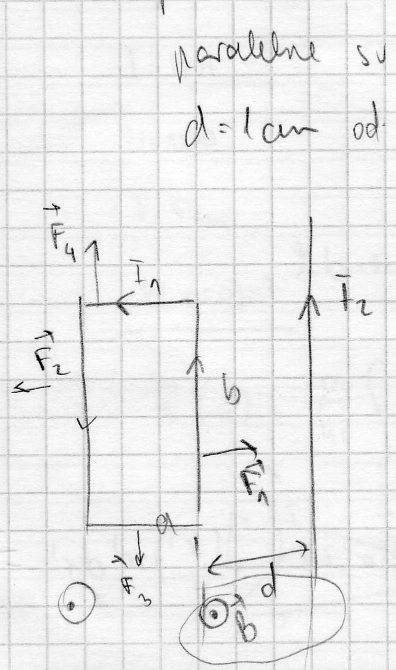
$$\boxed{B(r) = \mu_0 \frac{I}{2\pi a^2} r}, \quad r \leq a$$



Ovo je vrlo, vrlo elementarno!

Horvat, P.5.3.

Pravokutna petlja kojom teče struja $I_1 = 1\,A$ i beskonačni ravni vodič kojim teče struja $I_2 = 2A$ leže u istoj ravnini. Duljine stranica pravokutnika su $a = 2cm$ i $b = 3cm$. Stranice pravok. duljine b paralelne su s beskonačnim vodičem, a bliža od njih udaljena je $d = 1cm$ od.



$$B(r) = \frac{\mu_0 I_2}{2r\pi}$$

$$F = q\, \vec{v} \times \vec{B}$$

$$I = \frac{dq}{dt}$$

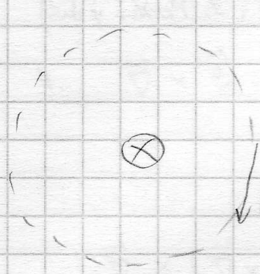
$$\vec{v} = \frac{d\vec{r}}{dt} \quad \Rightarrow \vec{F} = I\, \vec{l} \times \vec{B}$$

$F_1 = I_1 b \frac{\mu_0 I_2}{2\pi d}$ TRIVIĆ

$F_2 = I_1 b \frac{\mu_0 I_2}{2\pi (d+a)}$ $\qquad |\vec{R}| = \frac{b \mu_0 I_1 I_2}{2\pi}\left(\frac{1}{d} - \frac{1}{d+a}\right)$

13.11.2013.

Amperov zakon



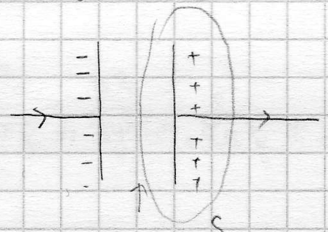
$$\oint_K \vec{B} d\vec{s} = \mu_0 \sum I_i = \mu_0 \int_S \vec{J} d\vec{S}$$

$$I = \int_S \vec{J} d\vec{S}$$

$$\oint_K \vec{B} d\vec{n} \to 0 \Rightarrow \int_S \vec{J} d\vec{S} \to 0$$

sažimam u točku

kondenzator



$$\frac{dq}{dt}$$

nema struje

imamo tok

struje je imamo vremensku promjenu naboja odnosno nap. toka

struja dalje teče kao PROVODNA struja J_p

$$\oint_S \vec{J} d\vec{S} = -\frac{dq}{dt}$$

kad se izlija ili nalija

$$\oint_S \vec{J} d\vec{S} = \int_V \vec{\nabla} \vec{J} dV \qquad q = \int_V \rho dV$$

$$\vec{\nabla} \vec{J} = -\frac{\partial \rho}{\partial t}$$

div. rotacije

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla}(\vec{\nabla} \times \vec{B}) = 0$$

$$\mu_0 \vec{\nabla} \vec{J} = 0$$

$\frac{Q}{\varepsilon_0}$

$$\oint_S \vec{J} d\vec{S} = -\frac{dQ}{dt} \to Q_{unutra}$$

unutar površine

$\vec{J}_p$ - provodna struja

G. zakon:

$$\oint_S \vec{E} d\vec{S} = \frac{Q_{unutra}}{\varepsilon_0} \Rightarrow -\frac{d}{dt}\left(\frac{Q_{unutra}}{\varepsilon_0}\right) = -\frac{d}{dt}\oint_S \vec{E} d\vec{S}$$

$$-\frac{\partial}{\partial t}(Q_{unutra}) = \varepsilon_0 \frac{d}{dt}\oint_S \vec{E} d\vec{S}$$

$$\boxed{\oint_S \vec{B}\, d\vec{s} = \mu_0 \underbrace{\int_S \vec{J}\, d\vec{s}}_{\text{Amper}} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\int_S \vec{E}\, d\vec{s}}$$

Amper - Maxwellov zakon

→ Maxwellov doprinos

↓

kada ima vremenske promjene električnog polja

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

promjenjivo el. će stvoriti kružno magnetsko polje (veza, 3. Maxw. jedn.)

Maxwellove jednadžbe

sigurno na ispitu

1. $\oint_S \vec{E}\, d\vec{s} = \frac{Q}{\varepsilon_0}$ $\qquad \vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$ Gauss

2. $\oint_S \vec{B}\, d\vec{s} = 0$ $\qquad \vec{\nabla}\vec{B} = 0$

slika: silnice zatvaramo na jednom dijelu - koliko ih je ušlo toliko ih je izašlo

3. Faradayev zakon indukcije, linijski integral (vodič)

$$\oint_K \vec{E}\, d\vec{s} = -\frac{d\phi}{dt}, \qquad \phi = \vec{B}\cdot\vec{S}$$

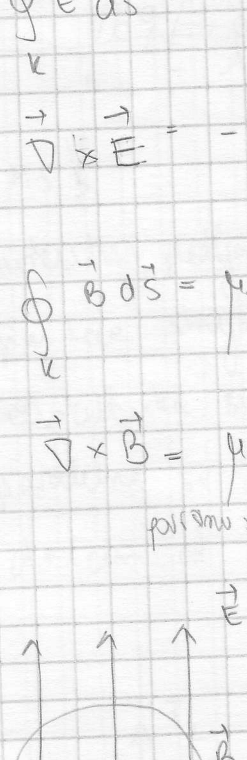
u kojoj se mijenja B (a ne S)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

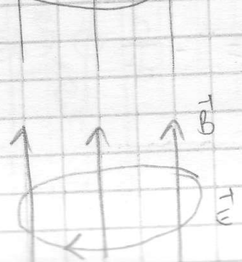
4. $$\oint_K \vec{B}\, d\vec{s} = \mu_0 \int_S \vec{J}\, d\vec{s} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\int_S \vec{E}\, d\vec{s}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

polako sažimam u točku, nema ds



$\frac{\partial \vec{E}}{\partial t} > 0$, onda nastaje kružno magn. polje B



$\frac{\partial \vec{B}}{\partial t} > 0$

suprotan smjer kružnog el. polja zbog minusa

2.12.2013.

Valna jednadžba elektromagnetskih valova u vakuumu

1. $\vec{\nabla}\vec{E} = 0$

2. $\vec{\nabla}\vec{B} = 0$

3. $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

4. $\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$

$$\frac{\partial\vec{E}}{\partial t} = \frac{1}{\varepsilon_0\mu_0}\vec{\nabla}\times\vec{B} \Big|'$$

$$\frac{\partial^2\vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\vec{\nabla}\times\frac{\partial\vec{B}}{\partial t} = -\frac{1}{\varepsilon_0\mu_0}\vec{\nabla}\times(\vec{\nabla}\times\vec{E})$$

Laplace

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\vec{A}) - \Delta\vec{A} \rightarrow \vec{\nabla}\underbrace{(\vec{\nabla}\vec{E})}_{=0} - \Delta\vec{E}$$

$$= \frac{1}{\varepsilon_0\mu_0}\Delta\vec{E}$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\boxed{\frac{\partial^2\vec{E}}{\partial t^2} - \frac{1}{\varepsilon_0\mu_0}\Delta\vec{E} = 0}$$

$$\frac{1}{\varepsilon_0\mu_0} = c^2, \quad c = 3\cdot 10^8 \text{ m/s}$$

$$\frac{\partial^2\vec{B}}{\partial t^2} = c^2\Delta\vec{B}$$

$$\frac{\partial^2 E}{\partial t^2} = c^2\Delta\vec{E}$$

$$\frac{\partial^2 B}{\partial t^2} = c^2\Delta\vec{B}$$

---

amplitude vala

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(\vec{k}\vec{r} - \omega t)}$$

$k = \frac{2\pi}{\lambda}$, $\vec{k}$ - valni vektor u smjeru širenja vala

Pr. smjer širenja vala

$E_x = E_0\sin(kz - \omega t)$, $E_y = E_z = 0$

$\vec{B} = ?$ $\quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial}{\partial t}(B_x\vec{i} + B_y\vec{j} + B_z\vec{k}) =$$

$$= (-1)^3\vec{j}\left(-\frac{\partial E_x}{\partial E_z}\right) = -\frac{\partial}{\partial t}B_y\vec{j}$$

$$\frac{\partial E_x}{\partial z} = \frac{\partial B_y}{\partial t}$$

$$B_y = -\int \frac{\partial E_x}{\partial z} \cdot dt = -\int K E_0 \cos(kz - \omega t)\, dt = \frac{K E_0}{\omega} \sin(kz - \omega t)$$

$$\frac{K}{\omega} = \frac{\frac{\cancel{2\pi}}{\lambda}}{\cancel{2\pi}\nu} = \frac{1}{\lambda \nu} = \frac{1}{c}$$

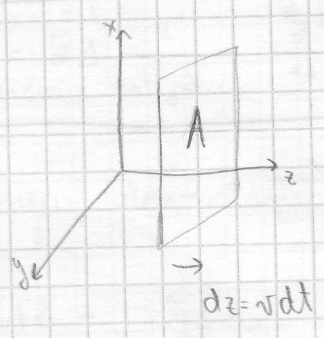
$$B_y = \underbrace{\frac{E_0}{c}}_{B_0} \sin(kz - \omega t) \qquad B_0 = \frac{E_0}{c}, \quad B_x = B_z = 0$$

$$E_x = E_0 \sin(kz - \omega t), \quad E_y = E_z = 0$$

$$\vec{E}_0 = \vec{B}_0 \times \underbrace{\vec{v}}_{c}$$

Intenzitet EM vala = gustoća toka energije

E_x, B_y



LC

Kondenzator:

$$E_{el} = \frac{1}{2} C U^2$$

$$C = \varepsilon \frac{S}{d}, \quad U = Ed$$

$$E_{el} = \frac{1}{2} \varepsilon \frac{S}{d} (Ed)^2 \quad /:V, \quad V = S \cdot d$$

$$\omega_{el} = \frac{E_{el}}{V} = \frac{1}{2} \varepsilon E^2$$

Zavojnica:

energija mag. polja $E_m = \frac{1}{2} L I^2$

$$L = \mu \cdot \frac{N^2 S}{l} \qquad V = S \cdot l$$

$$B = \mu \frac{NI}{l}$$

$$\omega_{el} = \frac{E_m}{V} = \frac{1}{2\mu} B^2$$

$$\boxed{\omega = \frac{1}{2} \varepsilon E_x^2 + \frac{1}{2\mu} B_y^2}$$ ukupna gustoća energije

eu: $\omega \cdot dV = \omega A v dt$

eu: $P \cdot dt = \omega dV = \omega A v dt$

$$\boxed{S = \frac{P}{A} = \omega \cdot v}$$

$E_x = v B_y$

$S = \left[\frac{1}{2}\varepsilon E_x^2 + \frac{1}{2\mu} B_y^2\right] \cdot v \qquad v^2 = \frac{1}{\varepsilon\mu}$

$E_x v B_y \qquad B_y \cdot \frac{E_x}{v}$

$S = \frac{1}{2}\varepsilon v^2 E_x B_y + \frac{1}{2\mu} E_x B_y$

$S = \frac{1}{2}\varepsilon \frac{1}{\varepsilon\mu} E_x B_y + \frac{1}{2\mu} E_x B_y = \frac{1}{\mu} E_x B_y$

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

Poynting

$\overrightarrow{\text{brzina}}$, $\overrightarrow{\text{valni vektor } k}$, $\overrightarrow{\text{Poynting}}$ u smjeru širenja vala

$\vec{E} = \vec{E_0} \sin\left((x+y)\cdot 10^7 - \omega t\right)$ smjer?

$\sin(\vec{k}\vec{r} - \omega t)$

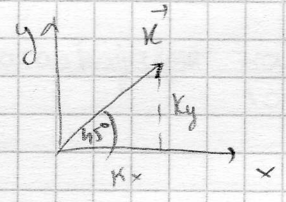
$\vec{k} = k_x\vec{i} + k_y\vec{j} + k_z\vec{k}$

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{k}\vec{r} = k_x x + k_y y + k_z z$

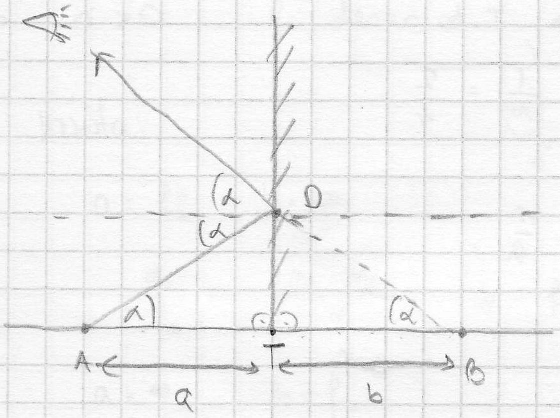
$(x+y)\cdot 10^7 = k_x x + k_y y$

$k_x = k_y = 10^7$



Ravno zrcalo

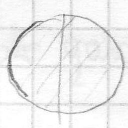
9.12.2013.



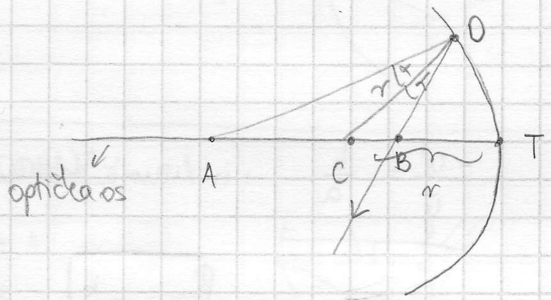
ΔATD i ΔTBD

$a = b$

SFERNA ZRCALA



1. udubljeno - konkavno
2. izbočeno - konvetsno



$\overline{AT} = a$

$\overline{BT} = b$

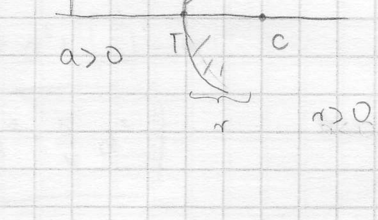
Gausove aproksimacije → paraaksijalne zrake - zrake koje su paralelne s optičkom osi, odnosno kut upada = 0

$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{2}{r}}$$

1. ⟶

$a>0,\ b>0,\ r>0$ lijevo od T

$a<0,\ b<0,\ r<0$ desno od T



1. $b = \infty$

$a = f_a$

$$\frac{1}{f_a} + \underbrace{\frac{1}{\infty}}_{\to 0} = \frac{2}{r}$$

$$f_a = \frac{r}{2}$$

2. $a = \infty$ predmet je beskon. daleko

$b = f_b$

$$\frac{1}{f_b} + \underbrace{\frac{1}{\infty}}_{0} = \frac{2}{r}$$

$$f_b = \frac{r}{2}$$

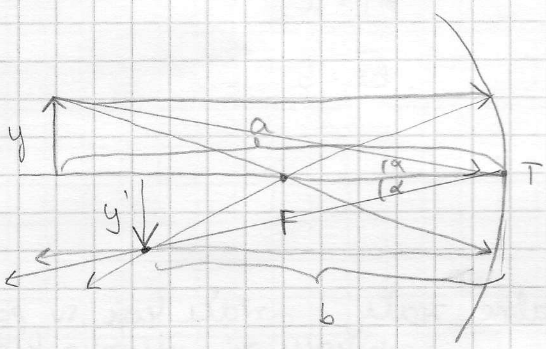
$$\boxed{f_a = f_b = f = \frac{r}{2}}$$

$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{2}{r} = \frac{1}{f}}$$ jednadžba sfernog zrcala

ravno zrcalo: $r \to \infty$

$$\frac{1}{a} = -\frac{1}{b} \Rightarrow \underbrace{a = -b}$$

Poprečno (transverzalno) povećanje - linearno povećanje sfernog zrcala

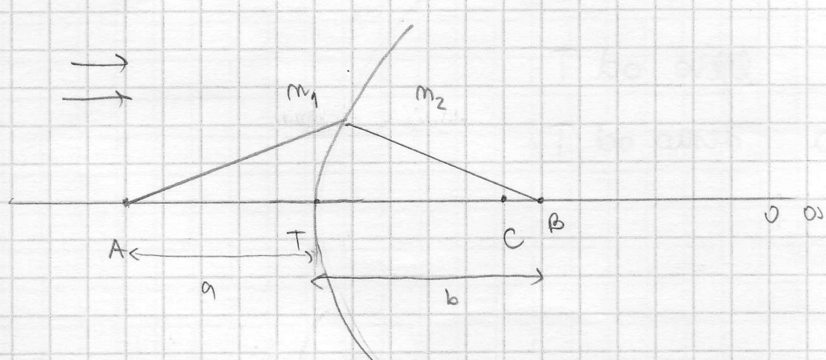


$$\underbrace{-\frac{y'}{y}}_{m} = \frac{b}{a}$$ → iz sličnosti trokuta

$$\boxed{m = -\frac{b}{a}}$$ linearno povećanje zrcala

SFERNI DIOPTAR - granica između dvaju sredstava različitih indeksa loma

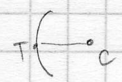
gledamo indeks loma sredstva gdje upadaju zrake svjetlosti, a ne gdje je predmet



upadna zraka

$$\boxed{\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{r}}$$ jdžba sfernog dioptra

1. $a > 0$ predmet lijevo od T
2. $b > 0$ slika desno od T
3. $r > 0$ C desno od T



Žarišta:

$a = f_a \qquad b = \infty$

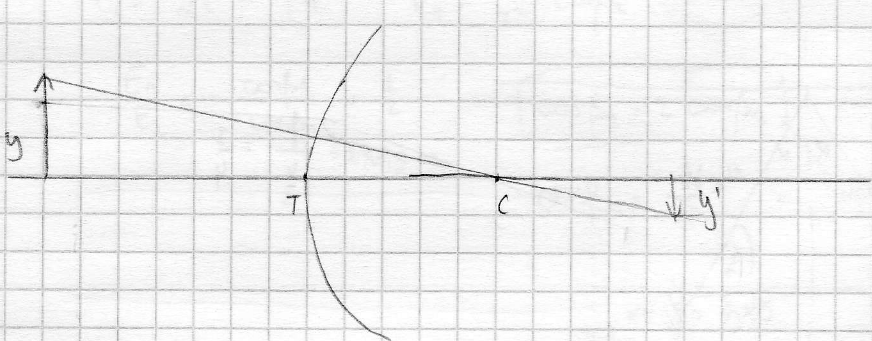
$$\frac{n_1}{f_a} = \frac{n_2 - n_1}{r} \Rightarrow f_a = \frac{n_1}{n_2 - n_1} r$$

$a = \infty \qquad b = f_b$

$$\frac{n_2}{f_b} = \frac{n_2 - n_1}{r} \Rightarrow f_b = \frac{n_2}{n_2 - n_1} r$$

$$\frac{f_b}{f_a} = \frac{n_2}{n_1}$$

$$f_b - f_a = \frac{n_2 - n_1}{n_2 - n_1} r = \underline{\underline{r}}$$



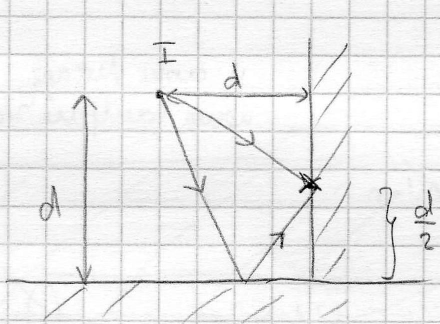
$$\frac{y'}{y} = \frac{b-r}{a+r}$$

$$\frac{y'}{y} = -\frac{n_1}{n_2}\frac{b}{a}$$

m

AUDITORNE

P. 7.1.



$$E = \frac{I}{r^2}\cos\beta$$

direktno

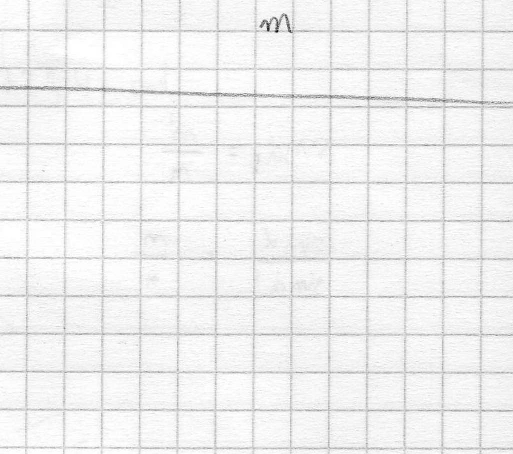
$$r_d^2 = d^2 + \left(\frac{d}{2}\right)^2$$

$$\boxed{r_d^2 = \frac{5}{4}d^2}$$

$$\cos\beta_d = \frac{d}{r_d}$$

$$E_d = \frac{I}{r_d^2}\cos\beta_d = \frac{I d}{r_d^3}$$

reflektirano



$$r_R^2 = d^2 + \left(\frac{3}{2}d\right)^2$$

$$\boxed{r_R^2 = \frac{13}{4}d^2}$$

$$\cos\beta_R = \frac{d}{r_R} \Rightarrow E = \frac{I d}{r_R^3}$$

$$\frac{E_{direktno}}{E_{direktno} + E_{reflektirano}} = ?$$

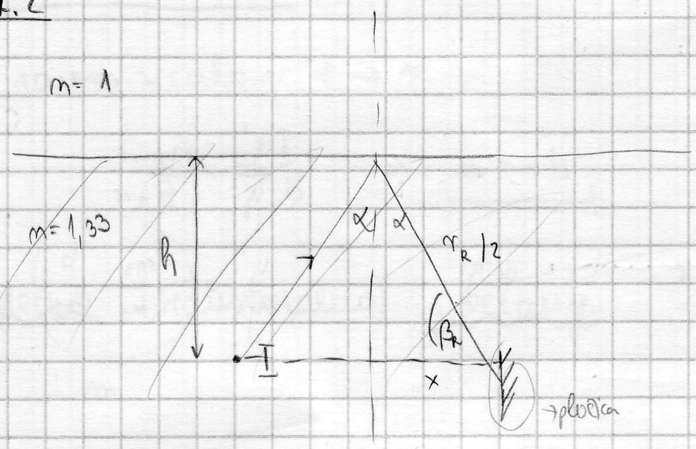
$$\frac{E_d}{E_d + E_R} = \frac{I_d}{r_d^3} \cdot \frac{1}{I_0\left(\frac{1}{r_d^3} + \frac{1}{r_R^3}\right)} = \frac{1}{1 + \left(\frac{r_d}{r_R}\right)^3}$$

$$\left(\frac{r_d}{r_R}\right)^2 = \frac{5}{\cancel{4}} \cancel{d^2} \frac{\cancel{4}}{13 \cancel{d^2}} = \frac{5}{13}$$

$$\frac{E_d}{E_d + E_R} = \frac{1}{1 + \left(\frac{5}{13}\right)^{3/2}} = 0{,}808$$

kad imamo zastor preko, dobivamo 80% osvjetljenja

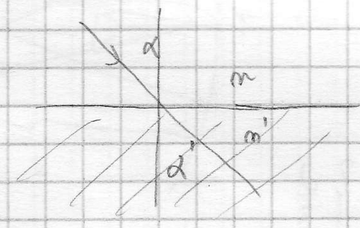
z. 7.2



$$\sin\alpha = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\sin\alpha_g = \frac{n'}{n}$$

$$\frac{\sin\alpha}{\sin\alpha'} = \frac{n'}{n}$$



u ovom slučaju nema lomljene zrake, $\sin\alpha' \to 1$

$$E = \frac{I}{r^2}\cos\beta_R$$

$$\cos\beta_R = \sin\alpha = \frac{3}{4}$$

$$\frac{h}{r_R/2} = \sin\beta_R$$

$$r_R = \frac{2h}{\sin\beta_R} = \frac{2h}{\sqrt{1-\cos^2\beta_R}} = \frac{2h}{\sqrt{1-\frac{9}{16}}} = \frac{2h}{\sqrt{7}} \cdot 4 = \frac{8h}{\sqrt{7}}$$

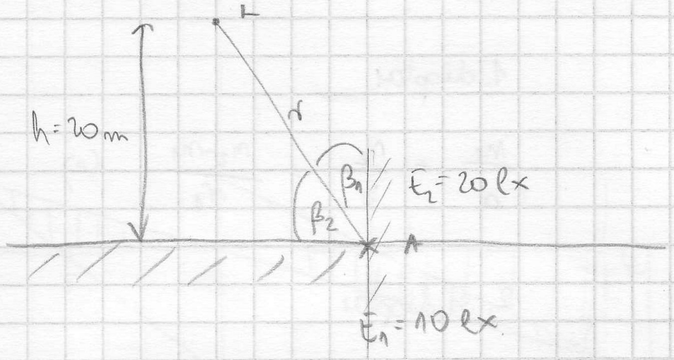
$$\Rightarrow E_R = \frac{I}{64h^2} 7 \cdot \frac{3}{4}$$

$$\cos\beta_R = \frac{r_d}{r_R} \Rightarrow r_d = \frac{3}{4} \cdot \frac{8h}{\sqrt{7}} = \frac{6h}{\sqrt{7}} =$$

$$E_d = \frac{I}{r_d^2} \cdot 1 = \frac{I}{36h^2} \cdot 7$$

$$E_d + E_R = 27{,}65 \ lx$$

Horvat, 7.3. = z.7.3. , razlika što rješavamo s 20 m umjesto 15



$I = ?$

$$E = \frac{I}{r^2}\cos\beta$$

$$I_1 = I_2 = I \quad , \quad r_1 = r_2 = r$$

$$E_1 = \frac{I}{r^2}\cos\beta_1 \quad , \quad E_2 = \frac{I}{r^2}\cos\beta_2$$

$$\frac{E_1}{E_2} = \frac{\cos\beta_1}{\cos\beta_2} = \frac{1}{2} \qquad \boxed{\cos\beta_2 = 2\cos\beta_1}$$

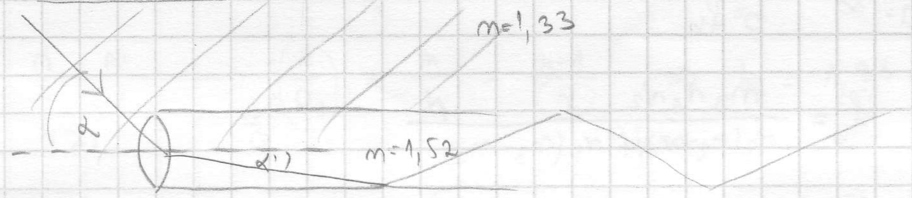
$$\cos\beta_1 = \frac{h}{r} = \sin\beta_2$$

$$\vdots$$

$$\Rightarrow r = h\sqrt{5}$$

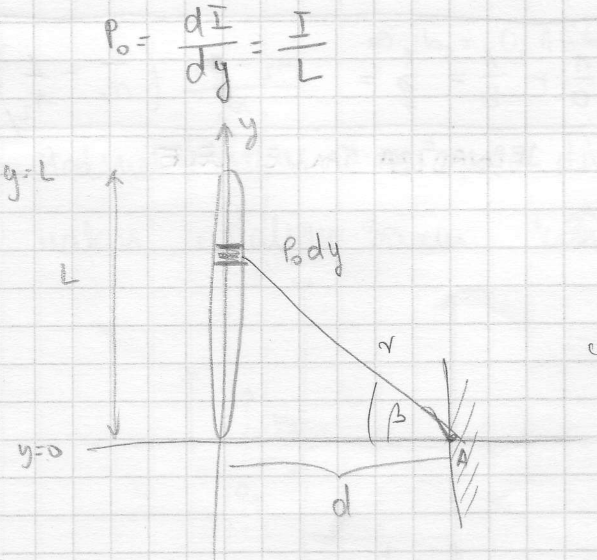
$$I = 44721{,}4 \text{ cd}$$

z 5.3.



z. 7.3.

$$P_0 = \frac{dI}{dy} = \frac{I}{L}$$



$$E = \frac{I}{r^2}\cos\beta \qquad dE_A = \frac{P_0\,dy}{r^2}\,\frac{d}{r} = P_0 d\,\frac{dy}{r^3}$$

$$\cos\beta = \frac{d}{r}$$

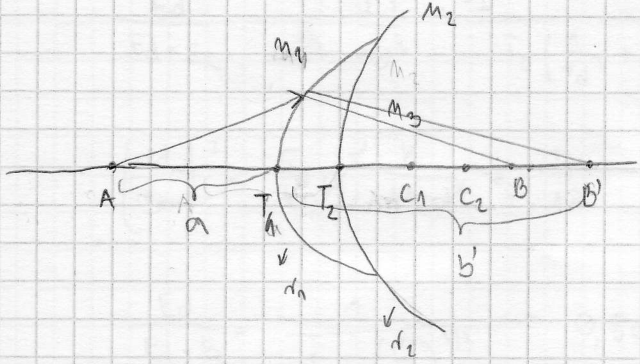
$$r^2 = y^2 + d^2$$

$$dE_A = (P_0 d)\cdot\frac{dy}{(y^2+d^2)^{3/2}}$$

$$E_A = \int_0^L P_0 d\,\frac{dy}{(y^2+d^2)^{3/2}} = \frac{2P_0}{d}$$ - karakteristika izvora

Jednadžba tanke leće

11. 12. 2013.



tanka leća: $\overrightarrow{T_1T_2} \to 0$

$\overline{AT_1} = a$

$\overline{T_1B'} = b'$

$\overline{T_2B'} = a'$, $a' < 0$

$\overline{T_1B} = b = \overline{T_2B}$

$|a'| = |b'|$

1. dioptar

$$\frac{n_1}{a} + \frac{n_2}{b'} = \frac{n_2 - n_1}{r_1} \qquad (1)$$

2. sf. dioptar

$$\frac{n_2}{a'} + \frac{n_3}{b} = \frac{n_3 - n_2}{r_2}$$

$$\frac{n_2}{-b'} + \frac{n_3}{b} = \frac{n_3 - n_2}{r_2} \qquad (2)$$

ne zanima nas [illegible] u kojoj je pala zraka

(1) + (2)

$$\frac{n_1}{a} + \cancel{\frac{n_2}{b'}} + \cancel{\frac{n_2}{-b'}} + \frac{n_3}{b} = \frac{n_2 - n_1}{r_1} + \frac{n_3 - n_2}{r_2}$$

$$\boxed{\frac{n_1}{a} + \frac{n_3}{b} = \frac{n_2 - n_1}{r_1} + \frac{n_3 - n_2}{r_2}}$$

$b = \infty$: $a = f_a$

$$\frac{n_1}{f_a} = \frac{r_2(n_2 - n_1) + r_1(n_3 - n_2)}{r_1 r_2} \Rightarrow \boxed{f_a = \frac{n_1 r_1 r_2}{r_2(n_2 - n_1) + r_1(n_3 - n_2)}}$$

$a = \infty$ $b = f_b$

$$f_b = \frac{n_3 r_1 r_2}{r_2(n_2 - n_1) + r_1(n_3 - n_2)}$$

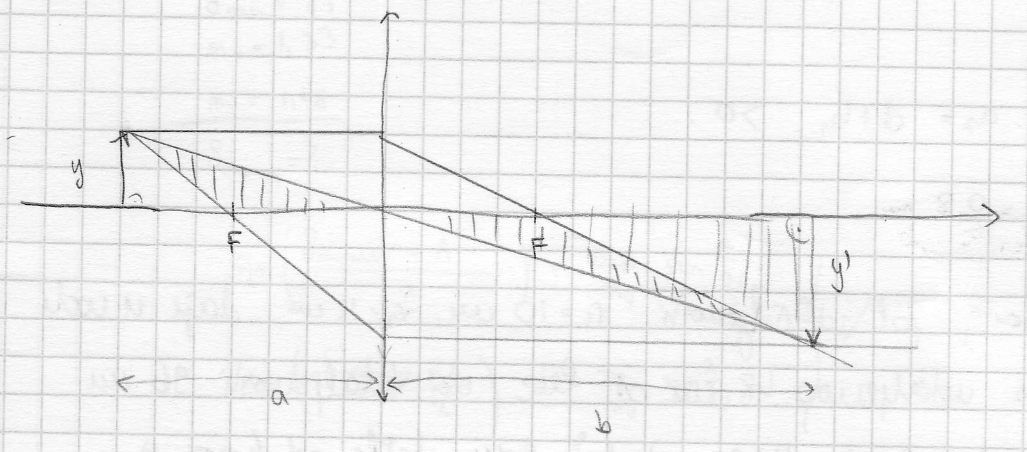
$$\frac{f_b}{f_a} = \frac{n_3}{n_1} \qquad \frac{f_a}{a} + \frac{f_b}{b} = 1$$

$$n_1 = n_3 \Rightarrow f_a = f_b = f \Rightarrow \boxed{\frac{1}{a} + \frac{1}{b} = \frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

JEDNADŽBA TANKE LEĆE

$\frac{1}{f} = j$ → dioptrija

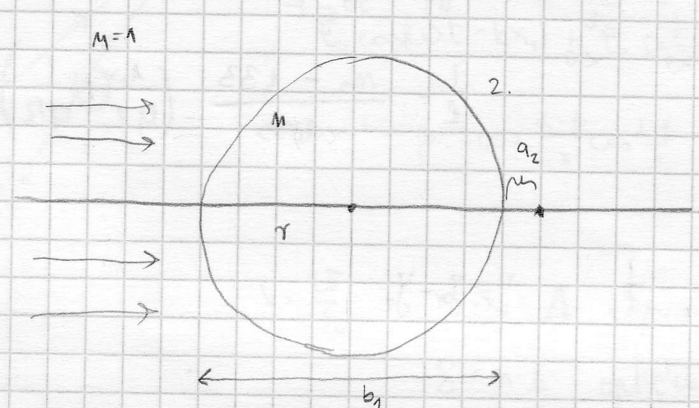
Povećanje kod tanke leće



$$\frac{-y'}{y} = \frac{b}{a}$$

$$m = \frac{y'}{y} = -\frac{b}{a}$$ linearno povećanje

ZI 2012 Sunčeva svjetlost pada na staklenu kuglicu indeksa loma 1,6 i polumjera 1 mm. Odredi udaljenost između tjemena kuglice kroz koje svjetlost iz nje izlazi i točke u kojoj se svjetlost u njoj fokusira. Odredite



$a_1 = \infty$

$r > 0$

$$\frac{1}{a_1} + \frac{n}{b_1} = \frac{n-1}{r} \Rightarrow b_1 = 1{,}33 \text{ mm}$$

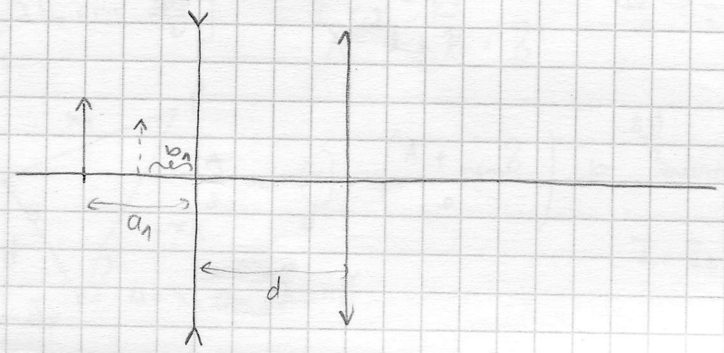
$\frac{1}{a_1} \to 0$, $r \to 0{,}5$ mm

$$\frac{n}{a_2} + \frac{1}{b_2} = \frac{1-n}{r} \qquad a_2 = -(b_1 - 2r)$$

$r < 0$

$$\Rightarrow b_2 = 0{,}166 \text{ mm}$$

Zad Predmet se nalazi 1 m ispred div. leće. Jakost je $-1\ m^{-1}$. Iza div. se nalazi udaljena 30 cm kon. leća žarišne daljine $f_2 = 40$ cm.



$a_1 = 1$ m

$J_1 = \frac{1}{f_1} = -1\ m^{-1}$

$f_2 = 40$ cm

$d = 30$ cm

① $\frac{1}{a_1}+\frac{1}{b_1}=\frac{1}{f_1} \Rightarrow b_1=-0,5$ m

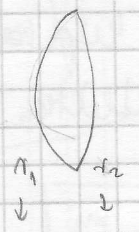
② $\frac{1}{a_2}+\frac{1}{b_2}=\frac{1}{f_2}$, $f_2>0$ $a_2=d+b_1 \quad >0$

$b_2=0,8$ m

5.13? kut. rad.

③ Bikonveksna tanka leća, pol. zakrivljenosti $r_1=10$ cm, $r_2=11$ cm daje u vodi realnu sliku predmeta udaljenog 48 cm od leće na udaljenosti 96 cm od leće. Indeks loma vode je 4/3. Indeks loma stakla od kojeg je napravljena leća?

$r_1=10$ cm
$r_2=12$ cm
$a_1=48$ cm
$b=96$ cm
$n_1=4/3$
$n_2=?$

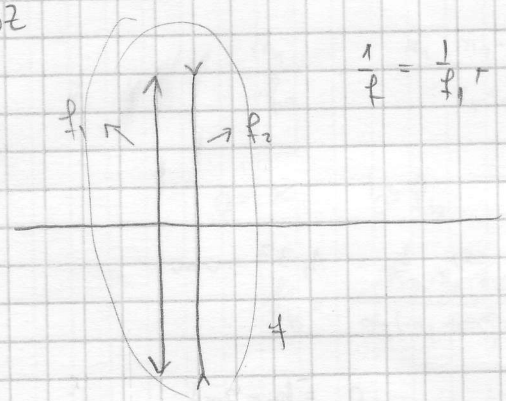


$$\frac{1}{a}+\frac{1}{b}=\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\frac{1}{48}+\frac{1}{96}=\frac{1}{f} \Rightarrow f=32\text{ cm}$$

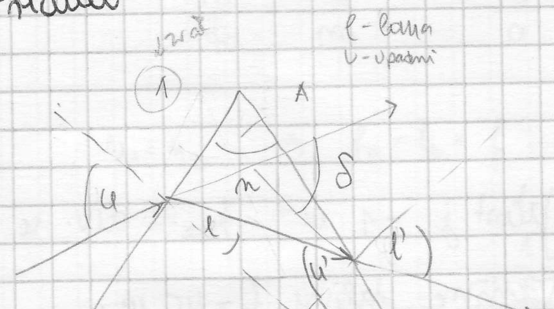
$$\frac{1}{f}=\frac{n_2-1,33}{1,33}\left(\frac{1}{0,1}+\frac{1}{0,12}\right) \Rightarrow n_2=1,1$$

5.15 DZ



$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \qquad j=j_1+j_2$$

Prizma



$$\frac{\sin u}{\sin l}=n$$

$$\frac{\sin u'}{\sin l'}=\frac{1}{n}$$

u ovim formulama

$$\delta = u + l' - A$$

uvjet za minimum DEVIJACIJE $\sin\left(\frac{\delta_{min}+A}{2}\right)=n \sin\frac{A}{2}$

$\frac{n_{prizme}}{n_{sredstva}}$

Zad. 5.4.

$A = 50^\circ$

$\delta_{min} = 12^\circ$

$n_v = 1{,}33$

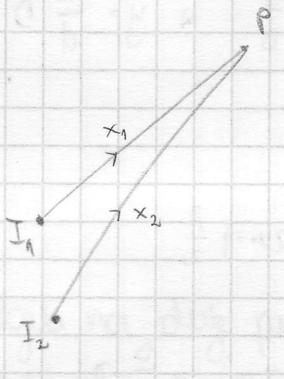
$n_u = 1{,}48$

$\delta_{min,ulj} = ?$

$$\left.\begin{aligned} \sin\left(\frac{\delta_{min,voda} + A}{2}\right) &= \frac{n_p}{n_v}\sin\frac{A}{2} \\ \sin\left(\frac{\delta_{min,ulje} + A}{2}\right) &= \frac{n_p}{n_u}\sin\frac{A}{2} \end{aligned}\right\} / \quad \Rightarrow \delta_{min,ulj} = 5{,}14^\circ$$

16.12.2013.

Interferencija 2 vala:



$$\vec{E}_1(t, x_1) + \vec{E}_2(t, x_2) = \vec{E}(t, x)$$

$$\vec{E}_0\cos(\omega t - kx_1) + \vec{E}_0\cos(\omega t - kx_2) = \vec{E}(t, x)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} \rightarrow \vec{E}(t,x) = \underbrace{2\vec{E}_0\cos\left[\frac{k(x_1-x_2)}{2}\right]}_{\text{amplituda}}\cos\left[\omega t - \frac{k}{2}(x_2-x_1)\right]$$

$k = \frac{2\pi}{\lambda}$ $\quad x_2 - x_1 = \Delta$ geometrijska razlika u hodu

$\delta = n\cdot\Delta$ optička razlika u hodu $\quad n_2x_2 - n_1x_1 = \delta$

$I \sim E^2$ $\quad I_{maks} \rightarrow \cos\frac{\Delta\phi}{2} = \pm 1 \rightarrow \frac{\Delta\phi}{2} = m\pi, \; m = 0, \pm 1, \dots$

$$\frac{k(x_2-x_1)}{2} = \frac{\Delta\phi}{2} \qquad I_{min} \rightarrow \cos\frac{\Delta\phi}{2} = 0$$

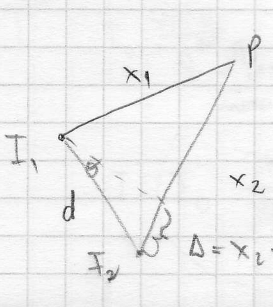
$$\frac{1}{\cancel{2}}\cdot\frac{2\pi}{\lambda}\cdot\delta = m\pi \rightarrow \boxed{\delta = m\lambda}$$

KONSTRUKTIVNA INTERFERENCIJA

$$\frac{\Delta\phi}{2} = \frac{1}{2}\cdot\frac{2\pi}{\lambda}\cdot\delta = (2m+1)\frac{\pi}{2}$$

$$\rightarrow \boxed{\delta = (2m+1)\frac{\lambda}{2}} \quad \text{DESTRUKTIVNA INTERFERENCIJA}$$

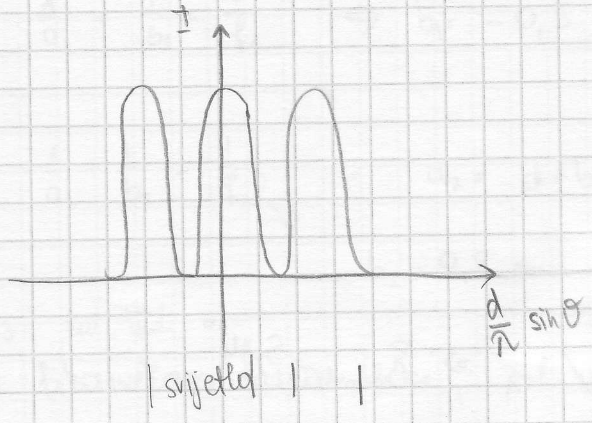
$$I \sim \left[2E_0\cos\frac{\Delta\phi}{2}\right]^2 = I_0\cos^2\frac{\Delta\phi}{2}, \qquad \frac{\Delta\phi}{2} = \frac{1}{2}\cdot\frac{2\pi}{\lambda}\delta = \frac{\pi}{\lambda}\delta$$



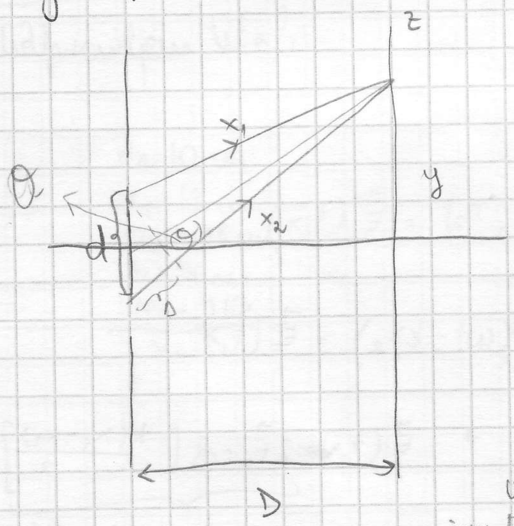
$\Delta = x_2 - x_1 = d\sin\theta$

$$\frac{\Delta\phi}{2} = \frac{\pi}{\lambda}d\sin\theta \qquad (n=1)$$

$$I = I_0\cos^2\left[\frac{\pi}{\lambda}d\sin\theta\right]$$



Youngov pokus



$\sin\theta = \frac{\Delta}{d}$ $\quad \sin\theta \approx \operatorname{tg}\theta$

$\operatorname{tg}\theta = \frac{y}{D}$ $\quad \frac{\Delta}{d} = \frac{y}{D} \Rightarrow y = \frac{\Delta}{d} D$

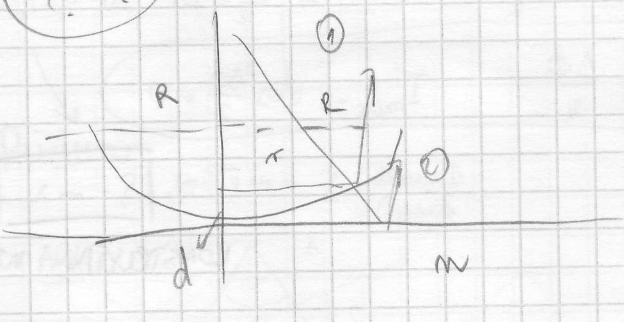
maks $y_{max} = \frac{D}{d} m\lambda$

min $y_{min} = \frac{D}{d}(2m+1)\frac{\lambda}{2}$

udaljenost između dva susjedna maks $y_{m+1} - y_m = \frac{D}{d}\lambda(m+1-m) = \frac{D}{d}\lambda$ = konst.

Newtonovi kolobari

??



$\delta = 2dn + \frac{\lambda}{2}$

$(R-d)^2 + r^2 = R^2$

$R^2 - 2Rd + \underbrace{d^2}_{\approx 0} + r^2 = R^2$

$d = \frac{r^2}{2R}$

$$\boxed{\delta = n \cdot \frac{r^2}{R} + \frac{\lambda}{2}}$$

$m\lambda$ max $\quad (2m+1)\frac{\lambda}{2}$ min

(*) 18.12.2013. fali 1. sat

$$I = \underbrace{E_0^2 N^2}_{I_0} \frac{\sin^2\left[\frac{\pi}{\lambda} b \sin\alpha\right]}{\left[\frac{\pi}{\lambda} b \sin\alpha\right]^2}$$

$$I = I_0 \frac{\sin^2 y}{y^2}$$

$$\sin\left[\frac{\pi}{\lambda} b \sin\alpha\right] = 0 \quad \text{min}$$

$$\frac{\cancel{\pi}}{\lambda} b \sin\alpha = m \cancel{\pi}$$

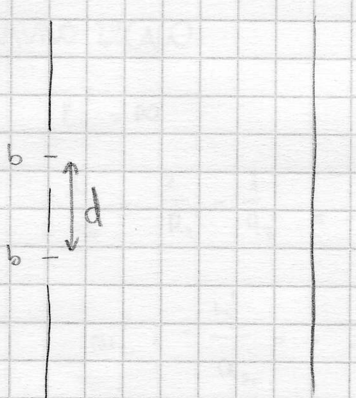
$$b \sin\alpha = m \lambda \quad ; \quad m \in \mathbb{Z}$$

max:
$$\frac{2 \sin y \cos y \cdot y^2 - \sin^2 y \cdot 2y}{y^4} = 0$$

$$y \cos y = \sin y \quad |:\cos y$$

$$\operatorname{tg} y = y \quad \Rightarrow \quad b \sin\alpha = (2m+1)\frac{\lambda}{2}$$

Ogib na dvije pukotine



Interferencija svjetlosti iz 2 izvora

$$I = E_0^2 \frac{\sin^2\left(\frac{\overset{N}{2}\Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)}$$

ogibni

$$I_0 \frac{\sin^2 y}{y^2}$$

ogib na pukotini

interferencija

$$I = I_0 \frac{\sin^2\left[\frac{\pi}{\lambda} b \sin\alpha\right]}{\left[\frac{\pi}{\lambda} b \sin\alpha\right]^2} \cdot \frac{\sin^2\left[\frac{2\pi d}{\lambda} \sin\alpha\right]}{\sin^2\left[\frac{\pi d}{\lambda} \sin\alpha\right]}$$

Ogib

$b\sin\alpha = m\lambda$ min

$b\sin\alpha = (2m+1)\frac{\lambda}{2}$ maks

Interf.

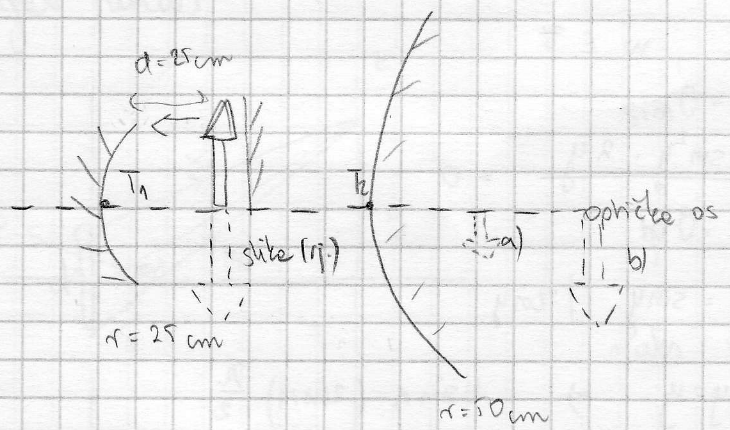
$d\sin\alpha = m'\lambda$ maks

$d\sin\alpha = (2m'+1)\frac{\lambda}{2}$ min

holografija - informativno

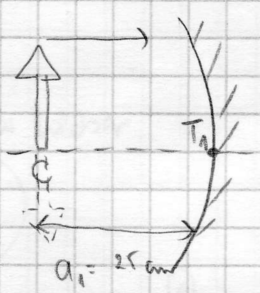
AUDITORNE

Z. 5.11



a)

1



(+) konvencija za zrcala

- zrake →
- C lijevo od T ($R>0$)
- predmet lijevo od T ($a>0$)
- slika lijevo od T ($b>0$)

$R_1 = +25$ cm

$a_1 = +25$ cm

$$\frac{1}{a}+\frac{1}{b}=\frac{2}{R} \quad , m=-\frac{b}{a}$$

$$\frac{1}{a_1}+\frac{1}{b_1}=\frac{2}{R_1} \qquad m_1=-\frac{b_1}{a_1}=-\frac{25\text{ cm}}{25\text{ cm}}=-1$$

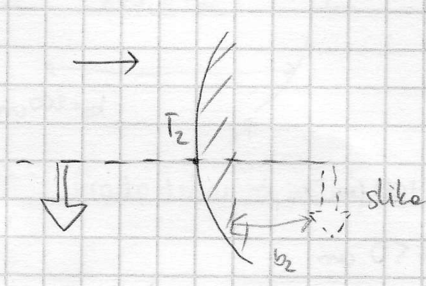
$$\frac{1}{b_1}=\frac{2}{R_1}-\frac{1}{R_1}=\frac{1}{R_1}$$

$$b_1=R_1=+25\text{ cm}$$

$$\underbrace{m_1=-1}$$

(2) $a_2=+25$ cm

$R_2=-50$ cm



$$\frac{1}{a_2}+\frac{1}{b_2}=\frac{2}{R_2}$$

$$\frac{1}{b_2}=\frac{2}{R_2}-\frac{1}{a_2}=\frac{2}{-50\text{ cm}}-\frac{1}{25\text{ cm}}=-\frac{2}{25\text{ cm}} \Longrightarrow \underline{b_2=-12{,}5\text{ cm}}$$

$$m_2=-\frac{b_2}{a_2}=-\frac{-12{,}5\text{ cm}}{25\text{ cm}}=+\frac{1}{2}$$

b) RAVNO ZRCALO

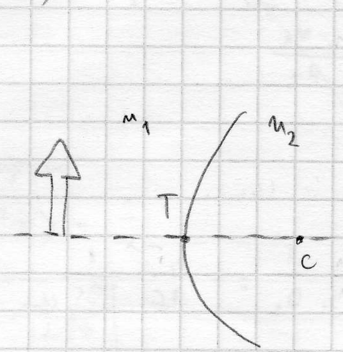
$R_2'=\infty$

$$\frac{1}{b_2'}=\frac{2}{R_2'}-\frac{1}{a_2'}=-\frac{1}{a_2}=-\frac{1}{25\text{ cm}} \Rightarrow b_2'=-25\text{ cm}$$

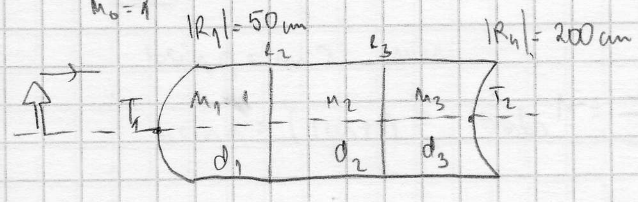
$$m_2'=-\frac{b_2'}{a_2}=-\frac{-25\text{ cm}}{25\text{ cm}}=+1 \quad // \quad \checkmark$$

5.18.

konvencija za dioptar

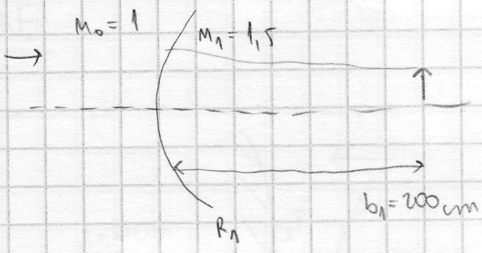


- zrake →
- C desno od T $(R>0)$
- predmet lijevo od T $(a>0)$
- slika desno od T $(b>0)$



treba nešto docrtati dok se rješava ←

①



$a_1 = +400$ cm

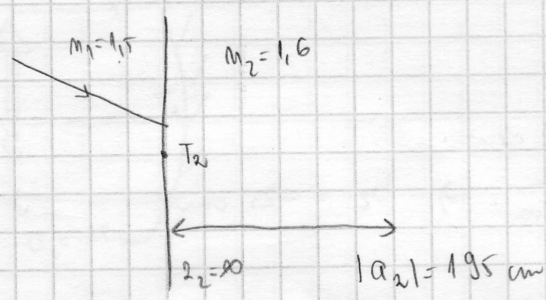
$R = +50$ cm

$$\frac{n_1}{b_1} = \frac{n_1 - n_0}{R_1} - \frac{n_0}{a_1}$$

$$\frac{3}{2b_1} = \frac{1}{2R_1} - \frac{1}{a_1} = \frac{1}{100\,\text{cm}} - \frac{1}{400\,\text{cm}} = \frac{4-1}{400} = \frac{3}{400\,\text{cm}}$$

$$\boxed{b_1 = +200\ \text{cm}}$$

②



$a_2 = -195$ cm

$$\frac{n_1}{a_2} + \frac{n_2}{b_2} = \cancel{\frac{n_2 - n_1}{R_2}} \to 0$$

$$\frac{n_2}{b_2} = -\frac{n_1}{a_1}$$

$$\frac{1}{b_2} = -\frac{n_1}{n_2}\frac{1}{a_1} = -\frac{1,5}{1,6}\frac{1}{195\,\text{cm}} \Rightarrow \boxed{b_2 = +208\ \text{cm}}$$

③

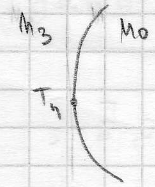
$a_3 = -201$ cm

$R_3 \to \infty$

$$\frac{n_2}{a_3} + \frac{n_3}{b_3} = \cancel{\frac{n_3 - n_2}{R_3}} \to 0$$

$$\frac{1}{b_3} = -\frac{n_2}{n_3}\frac{1}{a_3} = -\frac{1,6}{1,7}\frac{1}{(-201\,\text{cm})} \Rightarrow \boxed{b_3 = +213,6\ \text{cm}}$$

1



$a_4 = -205{,}6$ cm

$R_4 = +200$ cm

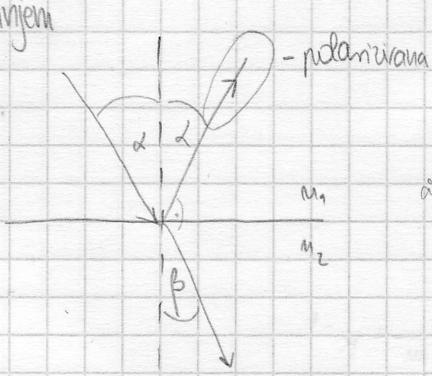
$$\frac{n_3}{a_4} + \frac{n_0}{b_4} = \frac{n_0 - n_3}{R_4} \Rightarrow \frac{1}{b_4} = \frac{-0{,}7}{+200\text{ cm}} - \frac{1{,}7}{-205{,}6\text{ cm}} \Rightarrow \boxed{b_4 = +209{,}71\text{ cm}}$$

Ž. S. 18.

8. 1. 2014.

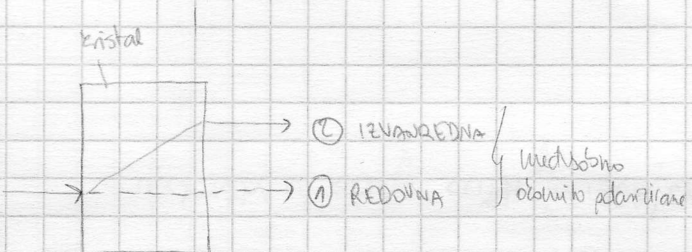
Polarizacija svjetlosti ("rado dolazi na ispitu")

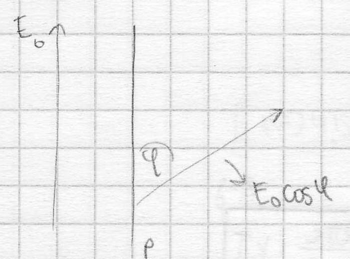
1. odbijanjem



$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$

ako je $\alpha + \beta = 90°$ $\rightarrow$ $\frac{\sin\alpha}{\sin(90° - \alpha)} = \boxed{\operatorname{tg}\alpha = \frac{n_2}{n_1}}$ BREWSTEROV ZAKON

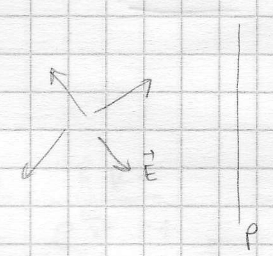




$$\boxed{I = \underbrace{E_0^2}_{I_0} \cos^2\varphi}$$ MALUSOV ZAKON

ako je na ulazu nepol. sv. (pod svim kutevima)

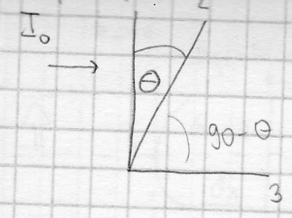
od polaroida dobijemo jednu zraku



srednja vrijednost

$$\langle \cos^2 \pi \rangle = \frac{1}{2}$$

za nepol. sv. $\boxed{I = \frac{1}{2} I_0}$



a) $I(\theta) = ?$
b) $\theta \to I_{max}$

$$\underline{I_3 = \frac{1}{2} I_0 \cos^2\theta \cdot \cos^2(90-\theta) = \frac{1}{2} I_0 \cos^2\theta \sin^2\theta = \frac{I_0}{8} \sin^2}$$

a) $I_1 = \frac{1}{2} I_0$

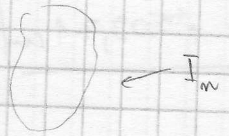
$I_2 = I_1 \cos^2\theta$

$I_3 = I_2 \cos^2(90-\theta)$

b) $\frac{dI_3}{d\theta} = 0 = \frac{I_0}{4} \sin 4\theta \rightarrow \theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$

$\frac{d^2 I_3}{d\theta^2} = I_0 \cos 4\theta < 0$ za $\frac{\pi}{4} \Rightarrow$ maks.

KVANTNA FIZIKA



$a + r + t = 1$

$a = \frac{I_a}{I_n}$, $r = \frac{I_r}{I_n}$, $t = \frac{I_t}{I_n}$, ε - koef. emisije

$$\frac{\varepsilon(\lambda, T)}{a(\lambda, T)} = 1$$ KIRCOFFOV ZAKON

$P = \frac{I}{S}$ $\quad dI = f(\lambda, T)\, d\lambda$

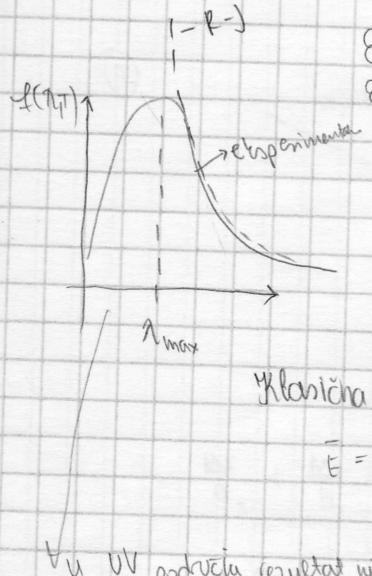
$$I = \int_0^\infty f(\lambda, T)\, d\lambda$$

$\lambda \cdot \nu = c$

Štefan, Boltzmann

$I = \varepsilon \cdot \sigma \cdot T^4$ $\quad \sigma = 5{,}67 \cdot 10^{-8}\ W m^{-2} K^{-1}$

$\varepsilon = 1$ crno
$\varepsilon = 0$ bjelo



$\lambda_{max} \cdot T$ - konst. $= 2{,}898 \cdot 10^{-3}$ Km

Wien

Klasična fizika

$\bar{E} = k \cdot T$ Rayleigh i Jeans su izveli:

$$f(\lambda, T) = \frac{2\pi c}{\lambda^4} \cdot \bar{E} = \boxed{\frac{2\pi c}{\lambda^4} \cdot kT}$$ R-J

u UV području rezultat nije bio dobar
→ neuspjeh klasične fiz. da objasni zračenje crnog tijela

Planck

$E = n \cdot h \cdot \nu$

ISPIT

Maxwell-Boltzmann

$e^{-\frac{h\nu}{kT}} = x$ $\quad \sum_{n=0} x^n = \frac{1}{1-x}$ $\quad \frac{d}{dx}\left(\frac{1}{1-x}\right)$

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E \cdot e^{-\frac{E}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{E}{kT}}} = \frac{h\nu \sum_{n=0}^{\infty} n e^{-\frac{nh\nu}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{nh\nu}{kT}}} = \frac{h\nu \sum_{n=0}^{\infty} n x^n}{\sum_{n=0}^{\infty} x^n} = h\nu x \frac{1 + 2x + 3x^2 + \dots}{1 + x + x^2 + \dots} =$$

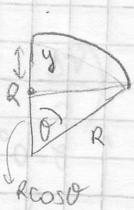
$$= h\nu x \frac{\frac{1}{(1-x)^2}}{\frac{1}{1-x}} = \frac{h\nu x}{1-x} = \frac{h\nu e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

FIZIKA 15. 1. 2014.

$q\vec{v} \times \vec{B}$

$\vec{F} = e\vec{v} \times \vec{B}$

$\vec{v} \perp \vec{B}$



$evB = \frac{mv^2}{R}$

$y = R - R\cos\theta$

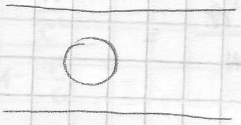
$y = R(1-\cos\theta)$ $\Rightarrow$ R odredimo

$\frac{e}{m} = \frac{Bv}{R}$

J. J. Thompson

$\frac{e}{m} = 1{,}759 \cdot 10^{11}\ \mathrm{As\,kg^{-1}}$

Millikan



$\uparrow \vec{F}_{ot}$

$\bigcirc \uparrow \vec{U} = \rho_0 g V$

$\downarrow \vec{G} = m\vec{g}$

$F_{ot} = 6\pi\eta r v$

v_1 - brzina padanja

$V = \frac{4}{3} r^3 \pi$

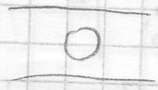
$\rho = \frac{m}{V} \Rightarrow m = \frac{4}{3} r^3 \pi \rho$

$\vec{U} + \vec{F}_{ot} + \vec{G} = 0$

$\rho_0 g V + 6\pi\eta r v_1 - mg = 0$

$6\pi\eta r v_1 = mg - \rho_0 g V =$

$= \frac{4}{3} r^3 \pi g (\rho - \rho_0) \rightarrow v_1$



$\uparrow \vec{F}_e = q \cdot \vec{E}$

$\downarrow \vec{G} \quad \vec{U}$

$F_{ot} \Rightarrow v_2$

$$\underbrace{\vec{G} + \vec{U}}_{6\pi\eta r v_1} + \underbrace{\vec{F}_{ot}}_{6\pi\eta r v_2} + \underbrace{\vec{F}_e}_{qE} = 0$$

$$q = \frac{1}{E}\left[6\pi\eta r (v_1 - v_2)\right]$$

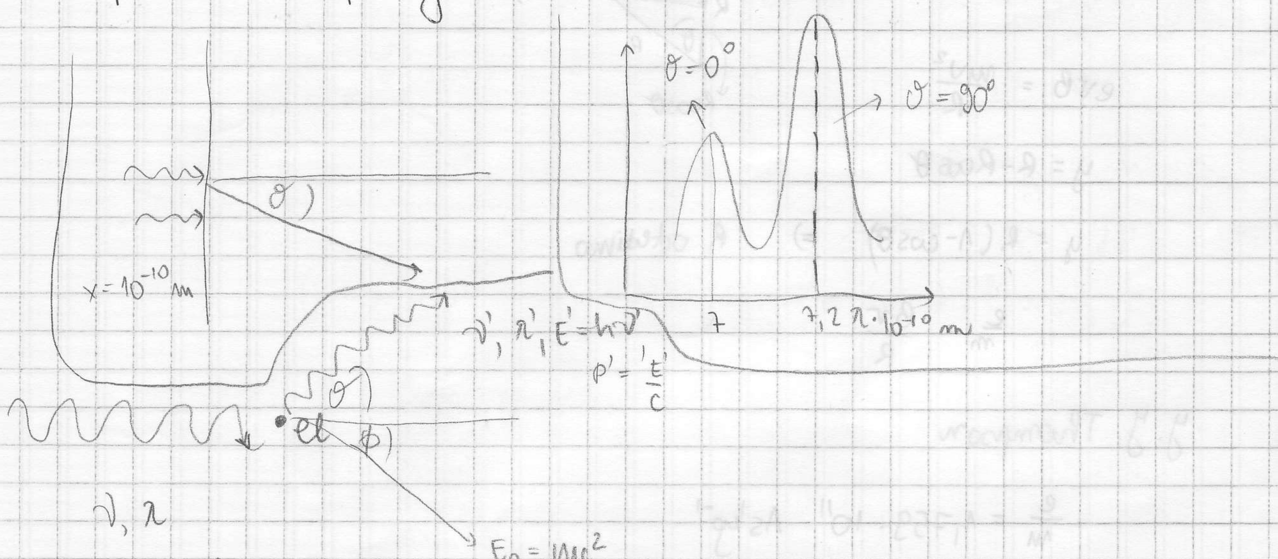
$q = Ve \rightarrow e$

$\frac{e}{m}$ $\qquad e = 1{,}6 \cdot 10^{-19}\ C$

$U_g \cdot e = h\nu - W_i$

$\Rightarrow m = 9{,}1 \cdot 10^{-31}\ kg$

Comptonovo raspršenje



ν, λ

$E = h \cdot \nu$

$p = \frac{E}{c} = \frac{h \cdot \nu}{c}$ $\qquad$ el $\qquad E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$, $\quad E = E_0 + E_k$

1. Količina gibanja

$$\vec{p} = \vec{p}\,' + \vec{p}_e$$

$$\Delta \nu = \nu - \nu'$$

$$\underline{\Delta \lambda = \lambda - \lambda'}$$

$$\vec{p}_c = \vec{p} - \vec{p}\,' \quad \Big|^2$$

$$p_c^2 = p^2 + p'^2 + 2pp' \cos\theta$$

$$pc = \frac{mv^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{m^2 v^2}{1 - \frac{v^2}{c^2}} = \frac{h^2 \nu^2}{c^2} + \frac{h^2 \nu'^2}{c^2} + 2\frac{h\nu \cdot h\nu^2}{c^2} \cos\vartheta$$

$\Delta \nu = \nu - \nu' \quad \Rightarrow \quad \nu' = \nu - \Delta \nu$

① $\underline{\quad ? \quad}$ ν' $/ : m^2c^2$

$$\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = 2 \cdot \frac{h^2 \nu^2}{m^2c^4}\left(1 - \frac{\Delta\nu}{\nu}\right)(1-\cos\theta) + \frac{h^2(\Delta\nu)^2}{m^2c^4} \qquad \text{I}$$

② Očuvanje energije

$$h\nu + E_0 = h\nu' + E$$

$E \rightarrow E_0 + E_k$

$$h(\underbrace{\nu-\nu'}_{\Delta\nu}) = E - E_0 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 \quad / : mc^2$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1 + \frac{h\Delta\nu}{mc^2} \quad /^2$$

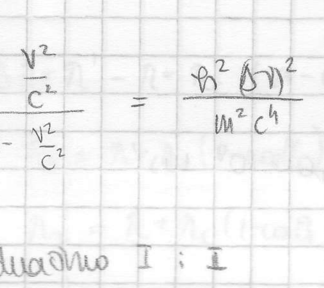
$$\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = \frac{h^2(\Delta\nu)^2}{m^2c^4} + 2\frac{h\Delta\nu}{mc^2} \qquad \text{II}$$

izjednačimo I i II

$$\Delta\nu = \frac{h\nu\nu'}{mc^2}(1-\cos\theta)$$

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

$\lambda_c = 2{,}426 \cdot 10^{-12}$ m



$p = p_e \cos\phi + p\cos\theta$

$0 = p' \sin\theta + p_e \sin\phi$

8.14.

$\nu = 1{,}2 \cdot 10^{18}\ Hz$

$E_k = 0{,}85\ eV$

$\theta = ?$

$\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\theta)$ $\quad \lambda_c = 2{,}426 \cdot 10^{-12}\ m$

$\nu \cdot \lambda = c \rightarrow \lambda = \dots$

$h\nu = E_k + h\nu'$

$h\nu' = h\nu - E_k \Rightarrow \nu', \lambda'$

$\theta = 10{,}72^\circ$

8.15.

$\Delta\nu = 1{,}233 \cdot 10^{20}\ Hz$

$v = ?$

$h\nu = E_k + h\nu'$, $\quad E_k = \dfrac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$

$E_k = h\nu - h\nu' = h\Delta\nu$

$\dfrac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 = h\Delta\nu \Rightarrow v = \dots = 2{,}6 \cdot 10^8\ ms^{-1}$

* 8.16.

Dobro dočitajte pažnju.

$E_{kmax} = 0{,}44\ MeV$

$\lambda = ?$

$\Delta\lambda = \lambda' - \lambda = \dfrac{h}{mc}(1-\cos\theta)$

$E = h\nu = \dfrac{hc}{\lambda}$

$\lambda \cdot \nu = c \rightarrow \lambda = \dfrac{hc}{E}$

$$\frac{hc}{E'} - \frac{hc}{E} = \frac{h}{mc}(1-\cos\theta)$$

$$E' = \frac{E}{1 + \frac{E}{mc^2}(1-\cos\theta)}$$

$E = E_k + E'$

$E' \rightarrow$ 1. $\theta = 0$ $\quad E' = E \quad \rightarrow E_k = 0$

2. $\theta = \pi$ $\quad E' = \dfrac{E}{1 + \frac{2E}{mc^2}} \quad \rightarrow E_{k\,max}$

$\theta = \pi \Rightarrow \Delta\lambda = \dfrac{2h}{mc}$

$E_k = 0{,}44 \text{ MeV} = h\Delta\nu = h(\nu - \nu') = h\left(\dfrac{c}{\lambda} - \dfrac{c}{\lambda'}\right)$

$\dfrac{hc}{\lambda} - \dfrac{hc}{\lambda'} = E_k = 0{,}44 \cdot 10^6 \cdot 1{,}6 \cdot 10^{-19} \rightarrow \lambda = 2 \cdot 10^{-12}\ m$

8.17.

$\theta_1 = 120^\circ \longrightarrow \lambda_1'$

$\theta_2 = 30^\circ \longrightarrow \lambda_2'$

$\dfrac{\lambda_1'}{\lambda_2'} = 3 \qquad \lambda = ?$

$\Delta\lambda = \lambda' - \lambda = \lambda_c(1 - \cos\theta)$

$\lambda_1' = \lambda + \lambda_c(1 - \cos\theta_1)$

$\lambda_2' = \lambda + \lambda_c(1 - \cos\theta_2)$

$\lambda_1' = 3\lambda_2' \Rightarrow \lambda = 1{,}33 \cdot 10^{-12}\ m$

8.18.

$\nu = 3 \cdot 10^{19}\ Hz$

$\theta = 60^\circ$

$\phi, E_k = ?$

$p = p_e\cos\phi + p'\cos\theta$

$0 = p'\sin\theta + p_e\sin\phi$

$$\left.\begin{aligned} &1.\ p'\cos\theta = p - p_e\cos\phi \\ &2.\ p'\sin\theta = p_e\sin\phi \end{aligned}\right\}$$

$\operatorname{tg}\theta = \dfrac{p_e\sin\phi}{p - p_e\cos\phi}$

$\phi = 54{,}40^\circ$

8.1

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$p = mv$$

$$E^2 = p^2c^2 + E_0^2$$

$$p = \frac{1}{c}\sqrt{E^2 - E_0^2}$$

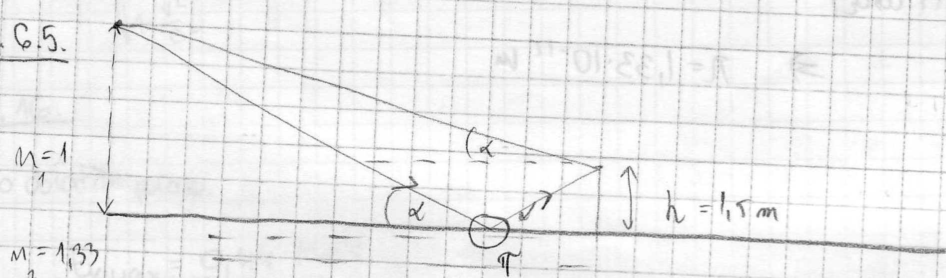
$$\lambda = \frac{h}{p} = \frac{h}{\frac{1}{c}\sqrt{E^2 - E_0^2}}$$

$$\lambda = \frac{hc}{\sqrt{\left(\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 - (mc^2)^2}} = \frac{h\cancel{c}}{mc^{\cancel{2}}\sqrt{\frac{1}{1-\frac{v^2}{c^2}} - 1}}$$

$$\lambda = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{mv}$$

AUDITORNE

pr. 6.5.

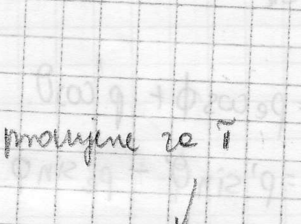


$\lambda = 25$ cm

INTERFERENCIJA:

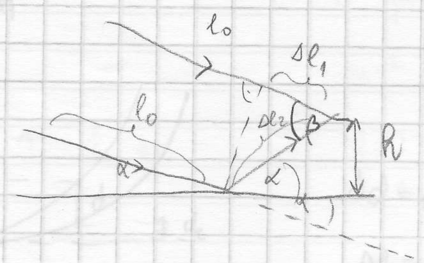
- razlika faza $\Delta\varphi = k \cdot \Delta s$ + eventualne promjene za π
- razlike opt. puteva Δs
- razl. fiz. puteva Δl

samo ako postoji refleksija na optički gušćem sredstvu!



$$\Delta\varphi = k \cdot \Delta s + \pi$$

$$\Delta s = \Delta l$$



$$l_1 = l_0 + \Delta l_1$$

$$l_2 = l_0 + \Delta l_2$$

$$\Delta l = l_2 - l_1 = \Delta l_2 - \Delta l_1$$

$$\beta = 2\alpha$$

$$\Delta l_2 \sin\alpha = h$$

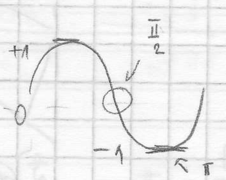
$$\underline{\Delta l_2 \cos\beta = \Delta l_1}$$

$$\Delta l = \Delta l_2 - \Delta l_1 = \Delta l_2 - \Delta l_2 \cos\beta = \Delta l_2 (1 - \cos 2\alpha) =$$

$$= 2 \Delta l_2 \sin^2\alpha = \underline{2h \sin\alpha}$$

$$\Delta\varphi = \frac{2\pi}{\lambda} \cdot 2h \sin\alpha + \pi$$

$$\cos\left(\frac{\Delta\varphi}{2}\right) = \pm 1$$



maximum:

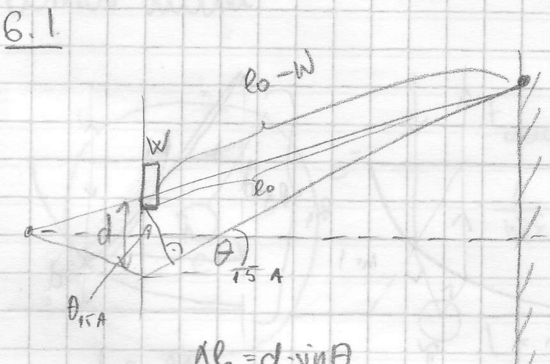
$$\frac{\Delta\varphi}{2} = 0, \pi, 2\pi, \ldots$$

$$\Delta\varphi = 0, 2\pi, 4\pi, \ldots$$

$$\Delta\varphi = \frac{2\pi}{\lambda} \cdot 2h \sin\alpha + \pi = 2\pi$$

$$\frac{4\cancel{\pi}}{\lambda} h \sin\alpha = \cancel{\pi} \Rightarrow \sin\alpha = \frac{\lambda}{4h} \Rightarrow \alpha = 2{,}4^\circ$$

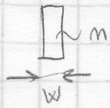
Z. 6.1



$\lambda = 500$ nm

$\Delta l \neq 0$

$\Delta s = 0$



(A) 15-ti maksimum, θ_{15A}

(B) $\theta_{0,B} = \theta_{15,A}$

$\downarrow$

$\Delta\varphi_B = 0 = k\cdot\Delta S_B$

$S_{1B} = (l_0 - W)\cdot 1 + W\cdot n$

$\underline{S_{2B} = l_0 + d\sin\theta_{15A}}$

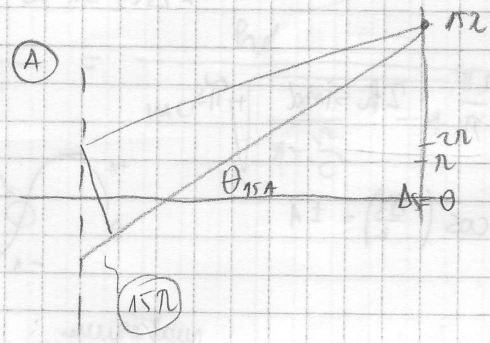
$\Delta S_B = l_0 + d\sin\theta_{15A} - l_0 + W - Wn = d\sin\theta_{15A} - W(n-1)$

$\Delta S_B = 0$

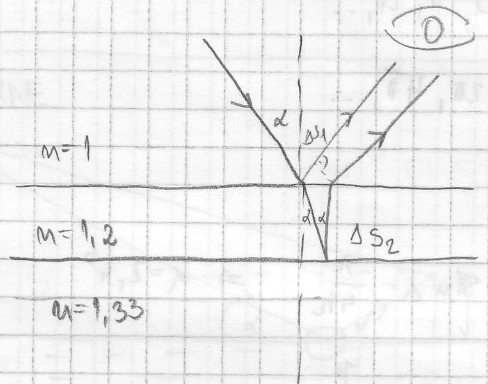
$d\sin\theta_{15A} = W(n-1)$

$$W = \frac{d\sin\theta_{15A}}{n-1}$$

$$W = \frac{15\lambda}{n-1} = 1{,}25\cdot 10^{-5}\ \text{m}$$



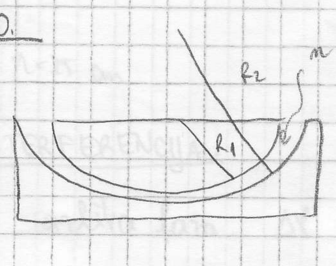
Z.6.6.



plavi

MORPHO leptir

Z.6.10.



Newtonovi kolobarji...

$R_1 = 10$ m

$R_2 = 20$ m

$h = ?$

$\lambda = 0{,}589\ \mu$m

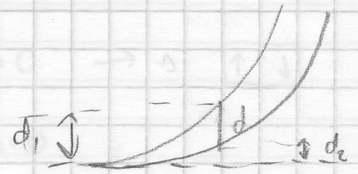
3. temni kolobar, $r_3 = 5{,}1$ mm

$\Delta l = 2d$

$(R_1 - d)^2 + r^2 = R_1^2$

$\cancel{R_1^2} - 2R_1 d + \cancel{d^2} + r^2 = R_1^2$

$r^2 = 2R_1 d_1$



$\Delta d = d_1 - d_2$

$\Delta l = 2\Delta d$

$\Delta S = m \cdot \Delta l = 2m\Delta d$

$r^2 = 2R_2 d_2$

$d_1 - d_2 = \frac{r^2}{2R_1} - \frac{r^2}{2R_2} = \frac{r^2}{2R_1}\left(1 - \frac{1}{2}\right) = \frac{r^2}{4R_1} = \Delta d$

$\Delta S = 2 \cdot m \cdot \frac{r^2}{4R_1}$

20.1.2014.

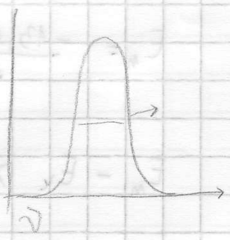
$\Delta x \cdot \Delta p_x \geq \hbar \qquad \hbar = \frac{h}{2\pi}$

RELACIJE NEODREĐENOSTI

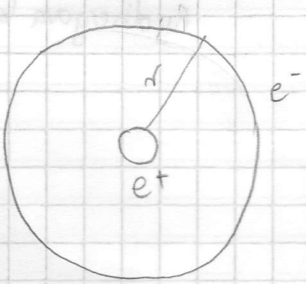
$\Delta E \cdot \Delta t \geq \hbar$

$\downarrow$

$h\Delta\nu \cdot \Delta t \geq \hbar$



Bohrov model



$F_{columb} = F_{cp}$

$\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r}$

$L = r_n \cdot m \cdot v_n = n\hbar \qquad \hbar = \frac{h}{2\pi}$ reducirana Planckova konst.

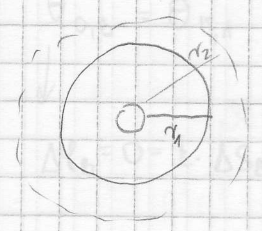
$v_n = \frac{n\hbar}{r_n \cdot m}$

$\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n^{\cancel{2}}} = \frac{m}{\cancel{r_n}} \cdot \left(\frac{n\hbar}{r_n \cdot m}\right)^2$

$\boxed{r_n = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} \cdot n^2}$ $\quad n = 1, 2, 3 \ldots$

$n = 1$

$$r_1 = \frac{4\pi\varepsilon_0 \hbar^2}{m e^2} = 0{,}528 \cdot 10^{-10}\ \text{m}$$



$$\boxed{r_n = r_1 \cdot n^2}$$

staze su kvantizirane

$$v_n = \frac{e^2}{4\pi\varepsilon_0 \hbar} \cdot \frac{1}{n}$$

Ukupna energija

$$E = E_k + E_p = \frac{1}{2} m v_n^2 - \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n}$$

$$E = \frac{1}{2} m \left(\frac{e^2}{4\pi\varepsilon_0 \hbar}\right)^2 \cdot \frac{1}{n^2} - \frac{1 \cdot e^2}{4\pi\varepsilon_0} \cdot \frac{m e^2}{4\pi\varepsilon_0 \hbar^2} \cdot \frac{1}{n^2} = \frac{m e^4}{32\pi^2 \varepsilon_0^2 \hbar^2} \cdot \frac{1}{n^2}$$

$$E_n \sim \frac{1}{n^2} \qquad n = 1, 2, 3, \ldots$$

$n = 1 \quad E_1 = V_1 = -13{,}6\ \text{eV}$ ENERGIJA IONIZACIJE

$$E_n = \frac{-13{,}6\ \text{eV}}{n^2}$$

* $E_n - E_k = h\nu$

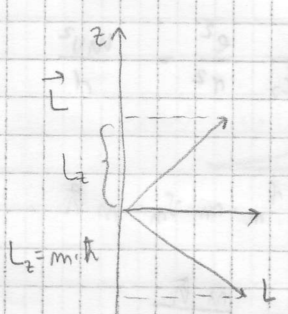
$$h\nu = \frac{m e^4}{32\pi^2 \varepsilon_0^2 \hbar^2}\left(\frac{1}{k^2} - \frac{1}{n^2}\right) \Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right)$$

$R \downarrow$ Rydbergova konst.

$\nu \cdot \lambda = c$

E_n n - glavni

$L \sim l \rightarrow$ orbitalni



$$L = \hbar \sqrt{l(l+1)}$$

m - magnetski kvantni broj

K ljuska

$n = 1$

$l = 0 \longrightarrow s$ $\boxed{\uparrow \downarrow}$

$m = 0$

L ljuska

$n = 2$

$l = 0 \longrightarrow 2s$ $\boxed{\uparrow \downarrow}$

$l = 1$

$m = 0, \pm 1$ $\boxed{\uparrow \downarrow}\boxed{\uparrow \downarrow}\boxed{\uparrow \downarrow}$ p

3 razlicita m-a