(6 bodova) Na raspolaganju su uzorci dvaju razreda za koje se pretpostavlja da slijede višedimenzionalnu normalnu razdiobu. Uzorci iz razreda ω_1 imaju središte u $\vec{m}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ i kovarijacijsku matricu $C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$. Uzorci iz razreda ω_2 imaju središte u $\vec{m}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ i kovarijacijsku matricu $C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Pretpostavite da su vjerojatnosti pojavljivanja uzoraka iz ω_1 i ω_2 jednake. Napišite jednadžbu granice između razreda koju za ovakve uzorke daje Bayesov klasifikator, i to u obliku $a \cdot x_1^2 + b \cdot x_2^2 + c \cdot x_1 \cdot x_2 + d \cdot x_1 + e \cdot x_2 + f = 0$

BAYES - ZAVRŠNI 2008/2009

$|C_1| = 2 \quad |C_2| = 1 \qquad C_1^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} \quad C_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$-\frac{1}{2}\ln 2 + \underbrace{\frac{1}{2}\ln 1}_{0} - \frac{1}{2}\left(\begin{bmatrix} x_1 & x_2 - 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 - 1 \end{bmatrix}\right) + \frac{1}{2}\left(\begin{bmatrix} x_1 - 1 & x_2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 - 1 \\ x_2 \end{bmatrix}\right) = 0 \quad / \cdot \frac{1}{2}$$

$$-\ln 2 - \left(\frac{1}{2}x_1^2 + (x_2 - 1)^2\right) + (x_1 - 1)^2 + x_2^2 = 0$$

$$-\ln 2 - \frac{1}{2}x_1^2 - \cancel{x_2^2} + 2x_2 - \cancel{1} + x_1^2 - 2x_1 + \cancel{1} + \cancel{x_2^2} = 0$$

$$\frac{1}{2}x_1^2 - 2x_1 + 2x_2 - \ln 2 = 0$$

RJEŠENJE U TRAŽENOJ FORMI!

$$\frac{1}{2}x_1^2 + 0x_2^2 + 0x_1x_2 - 2x_1 + 2x_2 + \ln 2 = 0$$

Na raspolaganju su uzorci dvaju razreda za koje se pretpostavlja da slijede višedimenzionalnu normalnu razdiobu. Uzorci iz razreda ω_1 imaju središte u $\vec{m}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ i kovarijacijsku matricu $C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. Uzorci iz razreda ω_2 imaju središte u $\vec{m}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ i kovarijacijsku matricu $C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Pretpostavite da su vjerojatnosti pojavljivanja uzoraka iz ω_1 i ω_2 jednake. Napišite jednadžbu granice između razreda koju za ovakve uzorke daje Bayesov klasifikator.

BAYES - ISPIT 10.7.2006.

$|C_1| = 4 \qquad |C_2| = 1$

$C_1^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \qquad C_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

- $d_1 - d_2 = 0$

$-\cancel{\frac{1}{2}} \ln 4 + \frac{1}{2} \cancel{\ln 1}_{0} - \cancel{\frac{1}{2}} \vec{x}^T \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \vec{x} + \cancel{\frac{1}{2}} \left(\vec{x} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left(\vec{x} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$

$= 0 \quad / \cdot \frac{1}{2}$

$\ln 4 - \frac{1}{2} x_1^2 - \frac{1}{2} x_2^2 + \begin{bmatrix} x_1 - 1 & x_2 \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_2 \end{bmatrix} = 0$

$+ (x_1 - 1)^2 + x_2^2 = 0$

$\ln 4 - \frac{1}{2} x_1^2 - \frac{1}{2} x_2^2 + x_1^2 - 2x_1 + 1 + x_2^2 = 0$

$\frac{1}{2} x_1^2 + \frac{1}{2} x_2^2 - 2x_1 + 1 + \ln 4 = 0 \quad / \cdot 2$

$\boxed{2x_1^2 + 2x_2^2 - 4x_1 + 4.773 = 0}$

Na raspolaganju su uzorci dvaju razreda za koje se pretpostavlja da slijede višedimenzionalnu normalnu razdiobu. Uzorci iz razreda ω_1 imaju središte u $\vec{m}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ i kovarijacijsku matricu $C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$. Uzorci iz razreda ω_2 imaju središte u $\vec{m}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ i kovarijacijsku matricu $C_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$. Pretpostavite da su vjerojatnosti pojavljivanja uzoraka iz ω_1 i ω_2 jednake. Napišite jednadžbu granice između razreda koju za ovakve uzorke daje Bayesov klasifikator.

BAYES - ISPIT 20.6.2006.

$|C_1| = 1 \quad |C_2| = 1$

$C_1^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} \quad C_2^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$

$d_1 - d_2 = \emptyset$

$$-\frac{1}{2} x^T \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} x + \frac{1}{2} [x_1 - 2 \quad x_2] \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 - 2 \\ x_2 \end{bmatrix} - \frac{1}{2} \cancel{\ln 1} + \frac{1}{2} \cancel{\ln 1} = \emptyset$$

$$-\frac{1}{2} x_1^2 - 2 x_2^2 + 2(x_1 - 2)^2 + \frac{1}{2} x_2^2 = \emptyset$$

$$-\frac{1}{2} x_1^2 - \frac{3}{2} x_2^2 + 2 x_1^2 - 4 x_1 + 4 = \emptyset$$

$$\boxed{\frac{3}{2} x_1^2 - \frac{3}{2} x_2^2 - 4 x_1 + 4 = \emptyset}$$

(7 bodova) Zadani su dvodimenzionalni uzorci iz dvaju razreda za koje se pretpostavlja da slijede višedimenzionalnu normalnu razdiobu. Uzorci iz prvoga razreda su

$\omega_1 = \{[1, 3]^T, [2, 0]^T, [2, 6]^T, [3, 3]^T\}$

Uzorci iz ω_2 imaju središte u ishodištu i kovarijacijsku matricu $C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Pretpostavlja se da su vjerojatnosi pojavljivanja uzoraka iz oba razreda jednake. Napišite jednadžbu granice između razreda i to u obliku:

$a \cdot x_1^2 + b \cdot x_2^2 + c \cdot x_1 \cdot x_2 + d \cdot x_1 + e \cdot x_2 + f = 0$

BAYES – ZAVRŠNI 2009/2010

$$\vec{m}_1 = \frac{1}{4}\begin{bmatrix}8\\12\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$$

$$C_1 = \frac{1}{4}\left(\begin{bmatrix}-1 & 0\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix} + \begin{bmatrix}0 & -3\end{bmatrix}\begin{bmatrix}0\\-3\end{bmatrix} + \begin{bmatrix}0 & 3\end{bmatrix}\begin{bmatrix}0\\3\end{bmatrix} + \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\right)$$

$$C_1 = \frac{1}{4}\cdot\left(\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & 9\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & 9\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\right) = \frac{1}{4}\begin{bmatrix}2 & 0\\0 & 18\end{bmatrix} = \begin{bmatrix}1/2 & 0\\0 & 9/2\end{bmatrix}$$

$$|C_1| = \frac{9}{4} \quad C_1^{-1} = \begin{bmatrix}2 & 0\\0 & 2/9\end{bmatrix} \quad C_2^{-1} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \quad |C_2| = 1$$

$$\cancel{\ln P(\omega_1) - \ln P(\omega_2)} \; -\frac{1}{2}\ln|C_1| + \frac{1}{2}\ln|C_2| - \frac{1}{2}\left(\vec{x} - \begin{bmatrix}2\\3\end{bmatrix}\right)^T C_1^{-1}\left(x - \begin{bmatrix}2\\3\end{bmatrix}\right)$$

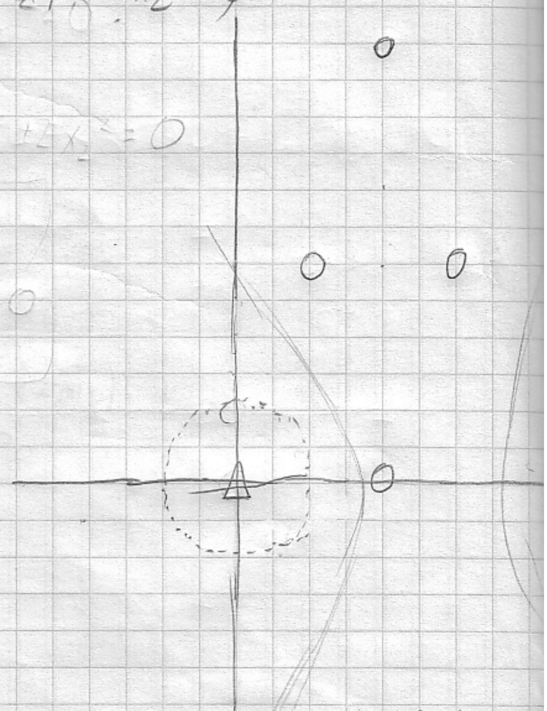
$$+\frac{1}{2}\vec{x}\, C_2^{-1}\vec{x}^{T} = 0$$

$$-\frac{1}{2}\ln\frac{9}{4} + \cancel{\frac{1}{2}\ln 1}_{\emptyset} - \frac{1}{2}\left(\begin{bmatrix}x_1-2 & x_2-3\end{bmatrix}\begin{bmatrix}2 & 0\\0 & 2/9\end{bmatrix}\begin{bmatrix}x_1-2\\x_2-3\end{bmatrix}\right) + \frac{1}{2}\left(\begin{bmatrix}x_1 & x_2\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)$$

$$-\frac{1}{2}\ln\frac{9}{4} - \frac{1}{2}\left(2(x_1-2)^2 + \frac{2}{9}(x_2-3)^2\right) + \frac{1}{2}\left(x_1^2 + x_2^2\right) = \emptyset \;/\cdot\frac{1}{2}$$

$$-\ln\frac{9}{4} - 2x_1^2 + 8x_1 - 8 - \frac{2}{9}x_2^2 + \frac{4}{3}x_2 - 2 + x_1^2 + x_2^2 = \emptyset$$

$$-x_1^2 + \frac{7}{9}x_2^2 + 8x_1 + \frac{4}{3}x_2 - 10.811 = \emptyset$$



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BAYESOV KLASIFIKATOR - AUDITORNE 2010/2012

$\omega_1 = \{[-1\ 0]^T, [0\ -1]^T, [1\ 0]^T, [0\ 1]^T\}$

$\omega_2 = \{[-2\ 0]^T, [0\ -2]^T, [2\ 0]^T, [0\ 2]^T\}$

$P(\omega_1) = \frac{4}{8} = \frac{1}{2}$; $P(\omega_2) = \frac{1}{2}$ // PRVO IZRAČUNAMO VJEROJATNOSTI KLASA

IZRAČUNAMO SREDINE SVAKE KLASE

$$\vec{m}_1 = \vec{m}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

IZRAČUNAMO KOVARIJACIJSKE MATRICE C_i

$$C_1 = \frac{1}{N_1} \sum (\vec{x}_i - \vec{m}_1) \cdot (\vec{x}_i - \vec{m}_1)^T$$

$$C_1 = \frac{1}{4}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$C_2 = \frac{1}{4}\left(\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \right) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

IZRAČUNAMO DETERMINANTE I INVERZE, TREBAĆE POSLIJE

$|C_1| = \frac{1}{4}$ $|C_2| = 4$

$$C_1^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad C_2^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

DECIZIJSKA FUNKCIJA KLASE

$$d_i = \ln P(\omega_i) + \ln P(\vec{x} | \omega_i)$$

$$= \ln P(\omega_i) - \frac{1}{2} \ln |C_i| - \frac{1}{2} (\vec{x} - \vec{m}_i)^T C_i^{-1} (\vec{x} - \vec{m}_i)$$

MARGINA RAZDVAJANJA KLASIFIKATORA:

$$d_1 - d_2 = 0$$

$$\ln P(\omega_1) - \ln P(\omega_2) - \frac{1}{2}\ln|C_1| + \frac{1}{2}\ln|C_2| - \frac{1}{2}(\vec{x}-\vec{m}_1)^T C_1^{-1}(\vec{x}-\vec{m}_1)$$
$$+\frac{1}{2}(\vec{x}-\vec{m}_2)^T C_2^{-1}(\vec{x}-\vec{m}_2)$$

$$-\frac{1}{2}\ln 4^{-1} + \frac{1}{2}\ln 4 \quad -\frac{1}{2}\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\ln 4 \quad = \begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{x_1}{4} & \frac{x_2}{4} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

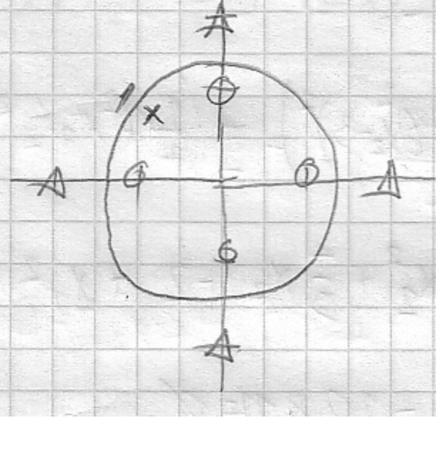
$$\ln 4 \quad - x_1^2 - x_2^2 + \frac{x_1^2}{4} + \frac{x_2^2}{4} = 0 \quad / \cdot 4$$

$$4\ln 4 \quad - 3x_1^2 - 3x_2^2 = 0$$

$$x_1^2 + x_2^2 = \frac{4}{3}\ln 4$$

$$x_1^2 + x_2^2 = 1.848 = 1.359^2$$

OVO GORE JE FUNKCIJA KRUŽNICE U ISHODIŠTU SA RADIJUSOM 1.359



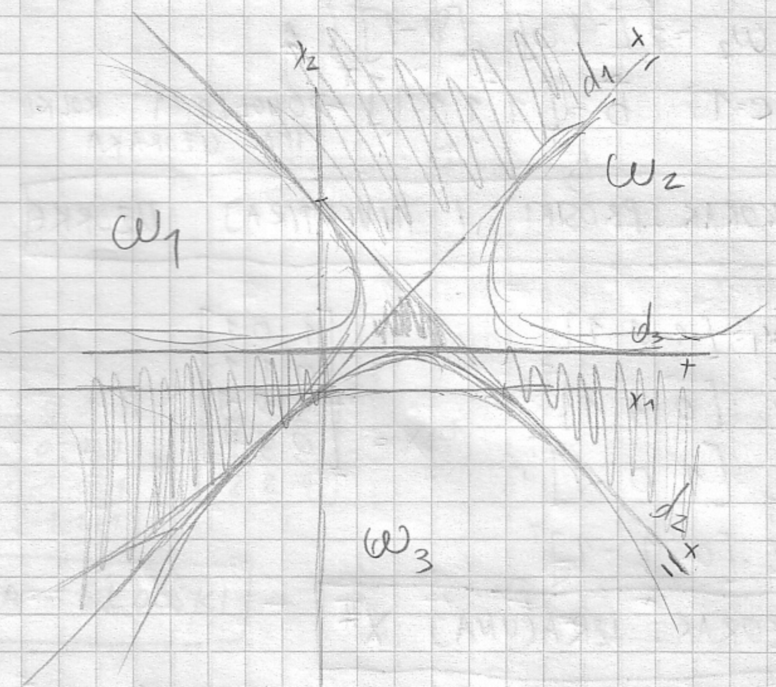
⑥ CRTANJE DECIZIJSKIH FUNKCIJA

1. SLUČAJ FJA ODVAJA SVAKI RAZRED OD OSTALIH

$d_1(\vec{x}) = -x_1 + x_2$

$d_2(\vec{x}) = x_1 + x_2 - 5$

$d_3(\vec{x}) = -x_2 + 1$

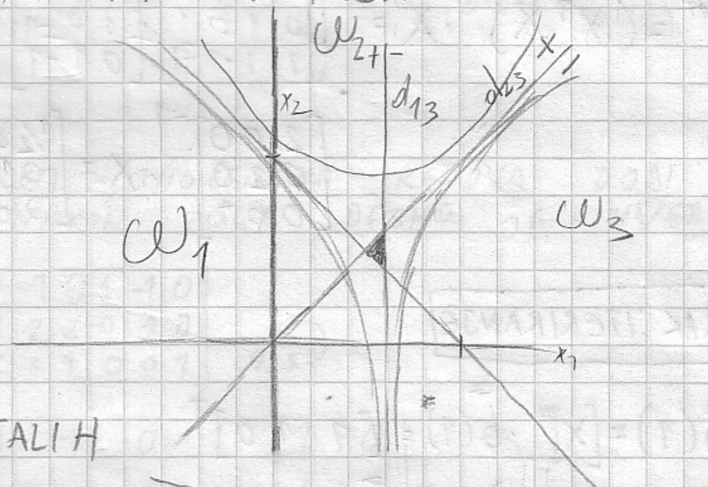


2. SLUČAJ FJA ZA SVAKI PAR RAZREDA

$d_{12}(\vec{x}) = -x_1 - x_2 + 5$

$d_{13}(\vec{x}) = -x_1 + 3$

$d_{23}(\vec{x}) = -x_1 + x_2$



3. SLUČAJ FJA VEĆA OD OSTALIH

$d_1(\vec{x}) = -x_1 + x_2$

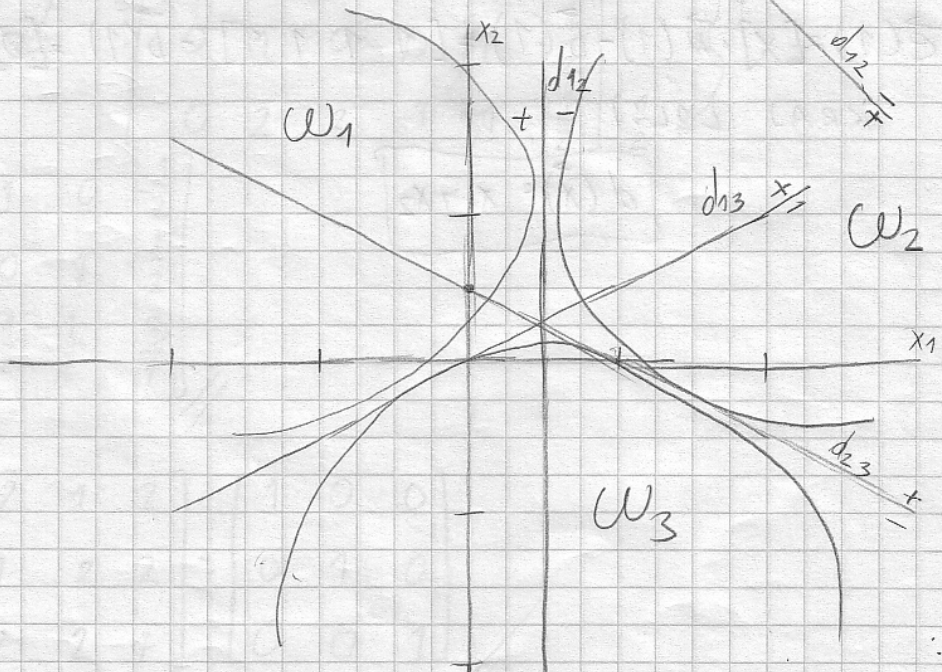
$d_2(\vec{x}) = x_1 + x_2 - 1$

$d_3(\vec{x}) = -x_2$

$d_{12} = -2x_1 + 1$

$d_{13} = -x_1 + 2x_2$

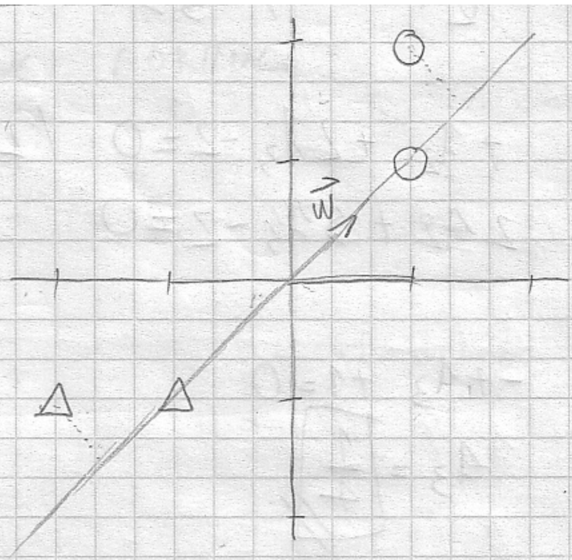
$d_{23} = x_1 + 2x_2 - 1$



1) FLD SA 2 RAZREDA

$\omega_1 = \{ [1,1]^T, [1,2]^T \} \Rightarrow \bigcirc$

$\omega_2 = \{ [-1,-1]^T, [-2,-1]^T \} \Rightarrow \Delta$



1. KORAK: MATRICE RASPRŠENJA

$$\vec{m}_1 = \frac{1}{n_1}\sum_{i=0}^{n_1} \vec{x}_{1i} = \frac{1}{2}\left(\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\1.5\end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{n_2}\sum \vec{x}_{1i} = \frac{1}{2}\left(\begin{bmatrix}-1\\-1\end{bmatrix} + \begin{bmatrix}-2\\-1\end{bmatrix}\right) = \begin{bmatrix}-1.5\\-1\end{bmatrix}$$

$$S_1 = \sum_{x \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$= \left(\begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}1\\1.5\end{bmatrix}\right)\left(\begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}1\\1.5\end{bmatrix}\right)^T + \left(\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}1\\1.5\end{bmatrix}\right)\left(\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}1\\1.5\end{bmatrix}\right)^T =$$

$$= \begin{bmatrix}0\\-0.5\end{bmatrix}\begin{bmatrix}0 & -0.5\end{bmatrix} + \begin{bmatrix}0\\0.5\end{bmatrix}\begin{bmatrix}0 & 0.5\end{bmatrix} =$$

$$= \begin{bmatrix}0 & 0\\0 & 0.25\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & 0.25\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0.5\end{bmatrix}$$

$$S_2 = \sum_{x \in \omega_2} (x_i - m_2)(x - m_2)^T =$$

$$= \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} \right) \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} \right)^T + \left(\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} \right) \left(\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \end{bmatrix} + \begin{bmatrix} -0.5 \\ 0 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix} //$$

$$S_W = S_1 + S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

2.) METODA ZA 2 RAZREDA

$$\vec{w} = S_W^{-1} (\vec{m}_1 - \vec{m}_2)$$

$$S_W^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

3) NORMALIZACIJA RJEŠENJA

$$\vec{w}' = \frac{\vec{w}}{|\vec{w}|} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} //$$

ALT. METODA $\rightarrow$

1. METODA ZA C RAZREDA ZA ZADATAK S 2 RAZREDA

$$\lambda S_W \cdot \vec{w} = S_B \vec{w} \quad / \cdot S_W^{-1}$$

$$\lambda \cdot \vec{w} = S_W^{-1} \cdot S_B \cdot \vec{w}$$

$$(S_W^{-1} \cdot S_B - \lambda I) \cdot \vec{w} = \emptyset$$

$$S_W^{-1} \cdot S_B - \lambda I = \emptyset$$

$$S_B = \sum_{i=1}^{2} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T$$

$$\vec{m} = \frac{1}{\cancel{4}_2}(\cancel{2}\vec{m}_1 + \cancel{2}\vec{m}_2) = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 1.5 \end{bmatrix} + \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -0.25 \\ 0.25 \end{bmatrix}$$

$$S_B = 2 \cdot \left(\begin{bmatrix} 1 \\ 1.5 \end{bmatrix} - \begin{bmatrix} -0.25 \\ 0.25 \end{bmatrix}\right)\left(\begin{bmatrix} 1 \\ 1.5 \end{bmatrix} - \begin{bmatrix} -0.25 \\ 0.25 \end{bmatrix}\right)^T + 2\left(\begin{bmatrix} -1.5 \\ -1 \end{bmatrix} - \begin{bmatrix} -0.25 \\ 0.25 \end{bmatrix}\right)\left(\;\; \right)^T$$

$$= 2 \cdot \begin{bmatrix} 1.25 \\ 1.25 \end{bmatrix}\begin{bmatrix} 1.25 & 1.25 \end{bmatrix} + 2\begin{bmatrix} -1.25 \\ -1.25 \end{bmatrix}\begin{bmatrix} -1.25 & -1.25 \end{bmatrix}$$

$$= 2\begin{bmatrix} 1.25^2 & 1.25^2 \\ 1.25^2 & 1.25^2 \end{bmatrix} + 2\begin{bmatrix} 1.25^2 & 1.25^2 \\ 1.25^2 & 1.25^2 \end{bmatrix}$$

$$S_B = \begin{bmatrix} 6.25 & 6.25 \\ 6.25 & 6.25 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 6.25 & 6.25 \\ 6.25 & 6.25 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 12.5 - \lambda & 12.5 \\ 12.5 & 12.5 - \lambda \end{bmatrix} = 0$$

$$12.5 - \lambda + 12.5 = 0$$

$$12.5 + 12.5 - \lambda = 0 \quad \Rightarrow \boxed{\lambda = 25}$$

$$\left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 6.25 & 6.25 \\ 6.25 & 6.25 \end{bmatrix} - \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}\right)\cdot \vec{w} = 0$$

$$\begin{bmatrix} -12.5 & 12.5 \\ 12.5 & -12.5 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$$-12.5\, w_1 + 12.5\, w_2 = 0$$

$$12.5\, w_1 - 12.5\, w_2 = 0$$

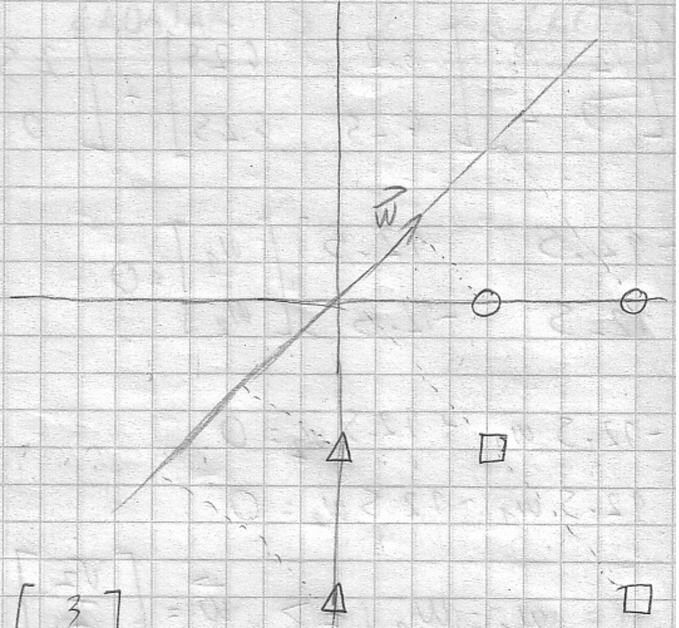
$$w_1 = w_2 \Rightarrow \vec{w} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

② FLD ZA C RAZREDA

$\omega_1 = \{ [2,0]^T, [4,0]^T \}$ ○

$\omega_2 = \{ [0,-2]^T, [0,-4]^T \}$ △

$\omega_3 = \{ [2,-2]^T, [4,-4]^T \}$ □



1. KORAK: MATRICE S_W i S_B

$$\vec{m}_1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \quad \vec{m}_2 = \begin{bmatrix} 0 \\ -3 \end{bmatrix} \quad \vec{m}_3 = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$\vec{m} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$S_W = S_1 + S_2 + S_3 = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} //$$

$$S_B = 2 \cdot \left(\begin{bmatrix} 1 \\ -2 \end{bmatrix} \begin{bmatrix} 1 & -2 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix} \begin{bmatrix} -2 & -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} \right)$$

$$= 2 \cdot \left(\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right) = 2 \cdot \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix} =$$

$$S_B = \begin{bmatrix} 12 & +6 \\ +6 & 12 \end{bmatrix} //$$

$$S_B \vec{w} = \lambda S_W \vec{w}$$

$$(S_B - \lambda S_W)\vec{w} = \emptyset$$

$$\begin{vmatrix} 12 - 4\lambda & 6 + 2\lambda \\ 6 + 2\lambda & 12 - 4\lambda \end{vmatrix} = 0$$

$$144 - 96\lambda + 16\lambda^2 - 36 - 24\lambda + 4\lambda^2 = 0$$

$$12x^2 - 120\lambda + 108 = 0 \implies \lambda_1 = 1, \lambda_2 = 9$$

$$(S_B - 9 S_W)\vec{w} = \emptyset$$

$$\begin{bmatrix} 12 - 36 & 6 + 18 \\ 6 + 18 & 12 - 36 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$$\begin{matrix} -24 w_1 + 24 w_2 = 0 \\ 24 w_1 - 24 w_2 = 0 \end{matrix} = w_1 = w_2 \implies \vec{w} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

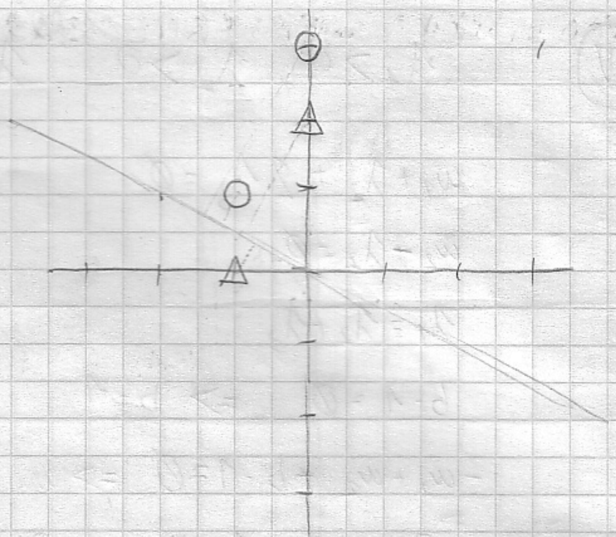
3. (3 boda) Za skup uzoraka $\quad \omega_2 = \{[0,2]^T, [\bar{1},0]^T\}$

$\omega_1 = \{ [0, 3]^T, [-1, 1]^T \}$,

$\omega_2 = \{ [0, 2]^T, [-1, 0]^T \}$,

naći pravac koji daje optimalnu projekciju tih uzoraka u smislu maksimizacije raspršenja između razreda i minimizacije raspršenja unutar razreda. Nacrtati pravac, uzorke i njihove projekcije.

FLD 1.MI 2008/2009



$\omega_1 = \{[0\ 3]^T, [-1\ 1]^T\}$ o

$\omega_2 = \{[0\ 2]^T, [-1\ 0]^T\}$ Δ

1)

$$\vec{m}_1 = \begin{bmatrix} -\frac{1}{2} \\ 2 \end{bmatrix}$$

$$\vec{m}_2 = \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$$

$$S_1 = \sum_{x \in \omega_1} (x_i - m_i)(x_i - m_i)^T = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} \\ -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 \\ 0 & 2 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} \\ -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & 2 \end{bmatrix}$$

$$S_W = S_1 + S_2 = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\vec{w} = S_W^{-1}(\vec{m}_1 - \vec{m}_2)$$

$$S_W^{-1} = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 2 & -1 & 1 \end{array}\right] \cdot \frac{1}{2} =$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]; \quad S_W^{-1} = \begin{bmatrix} 2 & -1 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} 2 & -1 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix} //$$

(7 bodova) Za skup uzoraka

$\omega_1 = \{ [0, 0]^T, [1, 3]^T \}$,

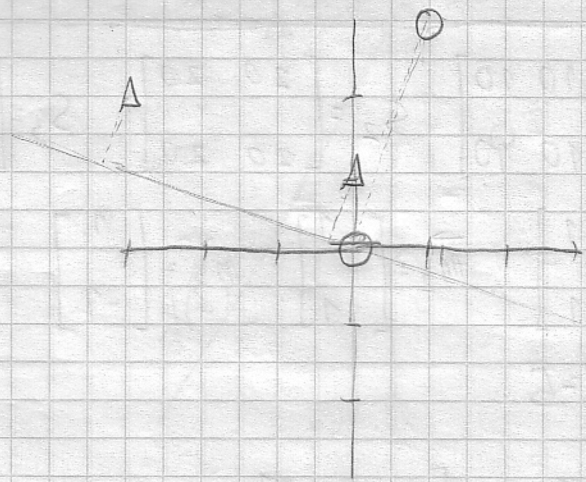
$\omega_2 = \{ [0, 1]^T, [-3, 2]^T \}$,

naći pravac koji daje optimalnu projekciju tih uzoraka u smislu maksimizacije raspršenja između razreda i minimizacije raspršenja unutar razreda. Nacrtati pravac, uzorke i njihove projekcije.

FLD ZA 2 RAZREDA ŽAVRŠNI 2009/2010

$\omega_1 = \{[0 \ \ 0]^T, [1 \ \ 3]^T\}$ o

$\omega_2 = \{[0 \ \ 1]^T, [-3 \ \ 2]^T\}$ Δ



1) $\vec{m}_1 = \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}$

$\vec{m}_2 = \begin{bmatrix} -1.5 \\ 1.5 \end{bmatrix}$

$S_1 = \begin{bmatrix} -0.5 \\ -1.5 \end{bmatrix} [-0.5 \ \ -1.5] + \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix} [0.5 \ \ 1.5] = \begin{bmatrix} 0.25 & 0.75 \\ 0.75 & 2.25 \end{bmatrix} \cdot 2 = \begin{bmatrix} 0.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

$S_2 = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix} [1.5 \ \ -0.5] + \begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix} [-1.5 \ \ 0.5] = \begin{bmatrix} 2.25 & -0.75 \\ -0.75 & 0.25 \end{bmatrix} + \begin{bmatrix} 2.25 & -0.75 \\ -0.75 & 0.25 \end{bmatrix} =$

$S_2 = \begin{bmatrix} 4.5 & -1.5 \\ -1.5 & 0.5 \end{bmatrix}$

$S_W = S_1 + S_2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

2) $\vec{w} = S_W^{-1} (\vec{m}_1 - \vec{m}_2) = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ 0 \end{bmatrix}$

2. METODA

$$\vec{m} = \frac{\vec{m}_1 + \vec{m}_2}{2} = \begin{bmatrix} -0.5 \\ 1.5 \end{bmatrix}$$

$$S_B = 2 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} \right) = 2 \cdot \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right) = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}$$

$$S_w^{-1} \cdot S_B - \lambda I = 0$$

$$\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\cancel{\begin{bmatrix} \frac{4}{5} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

$$\begin{bmatrix} \frac{4}{5} - \lambda & 0 \\ 0 & -\lambda \end{bmatrix} \cdot w = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\frac{4}{5} - \lambda \right) \cdot (-\lambda) = 0$$

$$-\frac{4}{5}\lambda + \lambda^2 = 0$$

$$\lambda \left(\lambda - \frac{4}{5} \right) = 0$$

$$\lambda_1 = 0$$

$$\lambda_2 = \frac{4}{5}$$

$$\frac{4}{5} - \lambda = 0$$

$$-\lambda = 0$$

$$\begin{bmatrix} \frac{4}{5} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{matrix} w_1 \\ w_2 \end{matrix} = 0$$

Četiri razreda dvodimenzionalnih uzoraka zadana su svojim matricama raspršenja, središtima i brojem uzoraka u razredu

$S_1 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$ $m_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$S_2 = \begin{bmatrix} 20 & 20 \\ 20 & 20 \end{bmatrix}$ $m_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$S_3 = \begin{bmatrix} 20 & 20 \\ 20 & 20 \end{bmatrix}$ $m_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

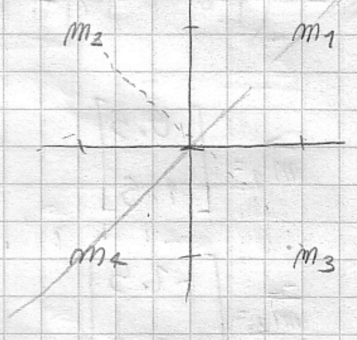
$S_4 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$ $m_4 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Pronaći vektor **w** koji daje optimalnu projekciju ovakvih uzoraka u smislu Fisherovog kriterija.

FLD ZA VIŠE OD 2 RAZREDA | 2. MI 2008/2009

$$S_1=\begin{bmatrix}10 & 10\\ 10 & 10\end{bmatrix} \quad S_2=\begin{bmatrix}20 & 20\\ 20 & 20\end{bmatrix} \quad S_3=\begin{bmatrix}20 & 20\\ 20 & 20\end{bmatrix} \quad S_4=\begin{bmatrix}10 & 10\\ 10 & 10\end{bmatrix}$$

$$\vec{m}_1=\begin{bmatrix}1\\ 1\end{bmatrix} \quad \vec{m}_2=\begin{bmatrix}-1\\ 1\end{bmatrix} \quad \vec{m}_3=\begin{bmatrix}1\\ -1\end{bmatrix} \quad \vec{m}_4=\begin{bmatrix}-1\\ -1\end{bmatrix}$$



$n_{1234}=5$

1)

$$S_W=\sum_{i=1}^{4} S_i=\begin{bmatrix}60 & 60\\ 60 & 60\end{bmatrix}$$

$$\vec{m}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$S_B=\sum_{i=1}^{N} n_i(\vec{m}_i-\vec{m})(\vec{m}_i-\vec{m})^T$$

$$S_B=5\cdot\left(\begin{bmatrix}1\\ 1\end{bmatrix}\begin{bmatrix}1 & 1\end{bmatrix}+\begin{bmatrix}-1\\ 1\end{bmatrix}\begin{bmatrix}-1 & 1\end{bmatrix}+\begin{bmatrix}1\\ -1\end{bmatrix}\begin{bmatrix}1 & -1\end{bmatrix}+\begin{bmatrix}-1\\ -1\end{bmatrix}\begin{bmatrix}-1 & -1\end{bmatrix}\right)$$

$$=5\cdot\left(\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}+\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right)$$

$$=5\cdot\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}=\begin{bmatrix}20 & 0\\ 0 & 20\end{bmatrix}$$

$$(S_B-\lambda S_W)\cdot\vec{w}=\emptyset$$

$$\det(S_B-\lambda S_W)=0$$

$$S_B-\lambda S_W=\begin{bmatrix}20 & 0\\ 0 & 20\end{bmatrix}-\begin{bmatrix}60\lambda & 60\lambda\\ 60\lambda & 60\lambda\end{bmatrix}=\begin{bmatrix}20-60\lambda & -60\lambda\\ -60\lambda & 20-60\lambda\end{bmatrix}$$

$$\det=(20-60\lambda)^2-(-60\lambda)^2=0$$

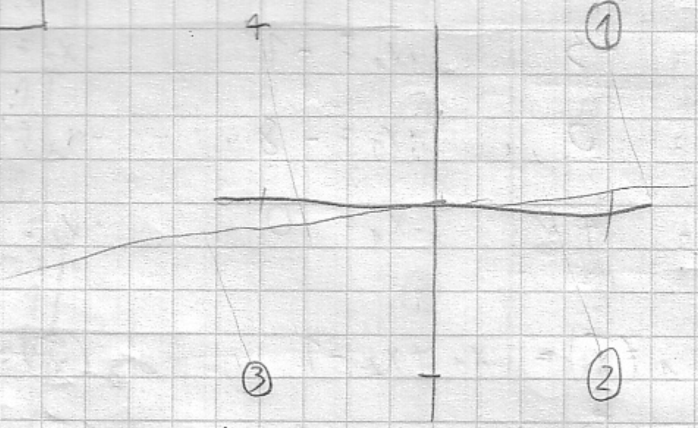
$$400-2400\lambda+\cancel{3600\lambda^2}-\cancel{3600\lambda^2}=0$$

$$2400\lambda=400 \Rightarrow \lambda=\frac{4}{24}=\frac{1}{6}$$

$$\begin{bmatrix}10 & -10\\ -10 & 10\end{bmatrix}\cdot\begin{bmatrix}w_1\\ w_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \qquad w_1=w_2 \Rightarrow \vec{w}=\begin{bmatrix}\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{bmatrix}$$

FLD ZA VIŠE RAZREDA

MOJ ZADATAK



1: [1 1]

2: [1 -1]

3: [-1 -1]

4: [-1 1]

$$S_B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} \qquad S_W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 8-\lambda & -\lambda \\ -\lambda & 8-\lambda \end{bmatrix}$$

$$(8-\lambda)^2 - \lambda^2 = 0$$

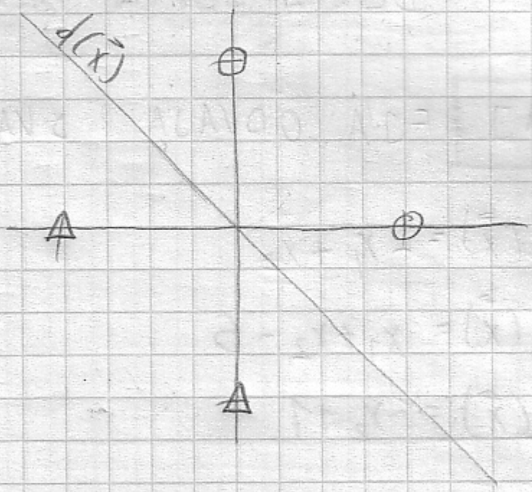
$$8 - 16\lambda = 0 \Rightarrow \lambda = \frac{1}{2}$$

$$\begin{bmatrix} 7.5 & -0.5 \\ -0.5 & 7.5 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$7.5 w_1 = 0.5 w_2$ $\qquad 15 w_1 = w_2$

$$w = \begin{bmatrix} 15 \\ 1 \end{bmatrix}$$

HO-KASHYAP / AUDITORNE



$\omega_1 = \{ [1\ 0]^T, [0\ 1]^T \}$ $\circ$

$\omega_2 = \{ [-1\ 0]^T, [0\ -1]^T \}$ Δ

$c = 1$ $\quad \vec{b} = [1\ 1\ 1\ 1]^T$ → ONOLKO '1' KOLKO IMA UZORAKA

1. KORAK: PROŠIRI I INVERTIRAJ UZORKE

$x_1 = [1\ 0\ 1]^T$

$x_2 = [0\ 1\ 1]^T$

$x_3 = [1\ 0\ -1]^T$

$x_4 = [0\ 1\ -1]^T$

$$X = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

2. KORAK: IZRAČUNAJ $X^{\#}$

$$X^{\#} = (X^T X)^{-1} \cdot X^T = \left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \right)^{-1} \cdot X^T =$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}^{-1} \cdot X = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & -1/4 & -1/4 \end{bmatrix}$$

3. KORAK: ITERIRANJE

$$\vec{w}(1) = [X]^{\#} \cdot \vec{b}(1) = [1\ 1\ 0]^T$$

$$\vec{e}(1) = [X] \cdot \vec{w}(1) - \vec{b}(1) = [1\ 1\ 1\ 1] - \vec{b}(1) = [\emptyset] \checkmark$$

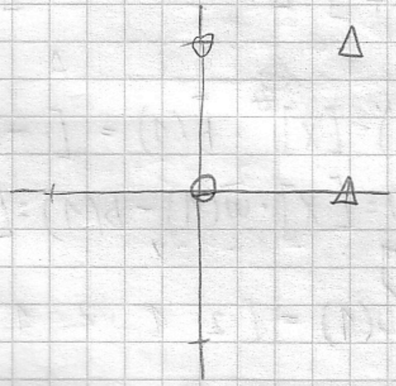
KRAJ LOL!)

$$\boxed{d(\vec{x}) = x_1 + x_2}$$

HO-KASHYAP / PREDAVANJA

$\omega_1 = \{[0\ 0]^T, [0\ 1]^T\}$ ○ $\quad b(1) = [1\ 1\ 1\ 1]^T$

$\omega_2 = \{[1\ 0]^T, [1\ 1]^T\}$ Δ $\quad c = 1$



1)

$x_1 = [0\ 0\ 1]^T$

$x_2 = [0\ 1\ 1]^T \qquad X = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

$x_2 = [-1\ 0\ -1]^T$

$x_3 = [-1\ -1\ -1]^T$

2)

$$X^{\#} = (X^T X)^{-1} X^T = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 4 \end{bmatrix}^{-1} \cdot X^T$$

ALGORITAM INVERTIRANJA - ŠKOLSKI

1. PODJELI 2x ŠIRU MATRICU I ZDESNA DODAJ I

$$X^{-1} = \left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

2) ZBRAJANJEM, ODUZIMANJEM, MNOŽENJEM REDOVA DOĐI DO I S LIJEVE STRANE. NAKON TOGA, DESNO JE INVERZ

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right] - \;=\; \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -1 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right] \cdot 0{,}5 \; -$$

$$= \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 1 & 1 & 2 & 0 & 0 & \frac{1}{2} \end{array}\right] - \;=\; \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 2 & 2 & -1 & 1 & \frac{1}{2} \end{array}\right] \cdot \frac{1}{2} \; + \; - \; =$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4} \end{array}\right] //$$

PROVJERA:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

$$X^{\#}=\begin{bmatrix}1 & 0 & -\frac{1}{2}\\ 0 & 1 & -\frac{1}{2}\\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4}\end{bmatrix}\begin{bmatrix}0 & 0 & -1 & -1\\ 0 & 1 & 0 & -1\\ 1 & 1 & -1 & -1\end{bmatrix}=\begin{bmatrix}-\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} & +\frac{1}{4}\end{bmatrix}\qquad X=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 1\\ -1 & 0 & -1\\ -1 & -1 & -1\end{bmatrix}$$

3) $\vec{w}(1)=[X]^{\#}\cdot\vec{b}(1)=[-2\quad 0\quad 1]^T$

$\vec{e}(1)=[X]\cdot\vec{w}(1)-b(1)=[1\quad 1\quad 1\quad 1]^T-[1\quad 1\quad 1\quad 1]^T=\emptyset$ ✓

ZA VJEŽBU

NPR: $b(1)=[2\quad 1\quad 2\quad 1]$

$\vec{w}(1)=[X]^{\#}\cdot\vec{b}(1)=[-3\quad 0\quad \frac{3}{2}]^T$

$\vec{e}(1)=[X]\cdot\vec{w}(1)-b=[\frac{3}{2}\quad\frac{3}{2}\quad\frac{3}{2}\quad\frac{3}{2}]-[2\quad 1\quad 2\quad 1]^T=[-\frac{1}{2}\quad\frac{1}{2}\quad-\frac{1}{2}\quad\frac{1}{2}]^T$

$\vec{b}(2)=[2\quad 2\quad 2\quad 2]^T$ /// $b_i += e_i$ AKO $e_i>0$

$\vec{w}(2)=[X]^{\#}\cdot\vec{b}(2)=[-4\quad 0\quad 2]^T$

$\vec{e}(2)=[X]\cdot\vec{w}(2)-\vec{b}(2)=[2\quad 2\quad 2\quad 2]-[2\quad 2\quad 2\quad 2]=\emptyset$ ✓

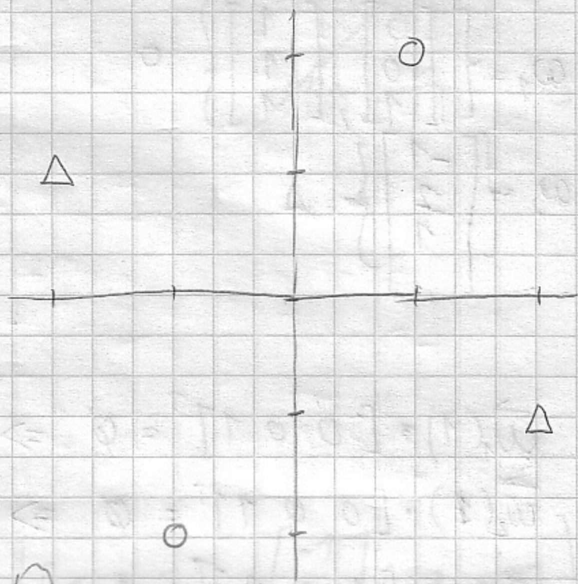
1. **(6 bodova) Postupkom Ho-Kashyapa želimo naći linearnu decizijsku funkciju za skup dovodimenzionalnih uzoraka. Zadana je matrica uzoraka, X, i njezin generalizirani inverz, $X^{\#}$**

$$X = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & -1 \\ 2 & -1 & -1 \\ -1 & -2 & 1 \end{bmatrix} \qquad X^{\#} = \begin{bmatrix} 0.1 & -0.2 & 0.2 & -0.1 \\ 0.2 & 0.1 & -0.1 & -0.2 \\ 0.25 & -0.25 & -0.25 & 0.25 \end{bmatrix}$$

HO-KASHYAP - ZAVRŠNI 2010

$$X = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & -1 \\ 2 & -1 & -1 \\ -1 & -2 & 1 \end{bmatrix} \qquad X^{\#} = \begin{bmatrix} 0,1 & -0,2 & 0,2 & -0,1 \\ 0,2 & 0,1 & -0,1 & -0,2 \\ 0,25 & -0,25 & -0,25 & 0,25 \end{bmatrix}$$

$\vec{b}(1) = [1\ 1\ 1\ 1]^T \qquad c = 1$



ZADATAK, RASPISAT ALGORITAM U 1. KORAKU

a) $\vec{\omega}(1) = [X]^{\#} \cdot \vec{b}(1) = [0\ \ 0\ \ 0]^T$

$\vec{e}(1) = [X] \cdot \vec{\omega}(1) - \vec{b}(1) = [-1\ \ -1\ \ -1]^T$

c) $J(1) = \frac{1}{2} \| [X] \cdot \vec{\omega}(1) - \vec{b}(1) \|^2 = \frac{1}{2} \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} [-1\ -1\ -1] = \frac{3}{2}$

d) POŠTO SU SVI ELEMENTI $\vec{e} < 0 \Rightarrow$ RAZREDI LINEARNO NEODVOJIVI

KL TRANSFORMACIJA - PRIMJER IZ SKRIPTE

$$\omega_1 = \{[0\ 0\ 0]^T, [1\ 0\ 0]^T, [1\ 0\ 1]^T, [1\ 1\ 0]^T\}$$
$$\omega_2 = \{[0\ 0\ 1]^T, [0\ 1\ 0]^T, [0\ 1\ 1]^T, [1\ 1\ 1]^T\}$$

$$P(\omega_1) = \frac{4}{8} \qquad P(\omega_2) = \frac{4}{8}$$

$$R = \sum_{i=1}^{K} P(\omega_i) \cdot E\{x_i \cdot x_i^T\} = \frac{1}{2} E\{\vec{x}_1, \vec{x}_1^T\} + \frac{1}{2} E\{\vec{x}_2, \vec{x}_2^T\}$$

- E JE OČEKIVANJE, IZRAČUNA SE KAO SREDNJA VRIJEDNOST SVIH $\vec{x_i} \cdot \vec{x_i}^T$ UZORAKA

$$R = \frac{1}{2} \cdot \frac{1}{4} \cdot \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot [0\ 0\ 0] + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot [1\ 0\ 0] + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \cdot [1\ 0\ 1] + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot [1\ 1\ 0] \right) +$$

$$\frac{1}{2} \cdot \frac{1}{4} \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot [0\ 0\ 1] + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [0\ 1\ 0] + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} [0\ 1\ 1] + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [1\ 1\ 1] \right)$$

$$= \frac{1}{8} \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) +$$

$$\frac{1}{8} \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

$$= \frac{1}{8} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \frac{1}{8} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} //$$

- RAČUNAMO SADA LAMBDE PO FORMULI $(R - \lambda I)\vec{e} = 0$ TAKO DA PRETPOSTAVIMO DA JE $(R - \lambda I)$ REGULARNA MATRICA ŠTO ZNAČI DA JE DETERMINANTA TOGA $= \emptyset$

$$(R - \lambda I)\vec{e} = \emptyset$$

$$\left(\frac{1}{4} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right) \cdot \vec{e} = \emptyset \Rightarrow \begin{vmatrix} 0.5-\lambda & 0.25 & 0.25 \\ 0.25 & 0.5-\lambda & 0.25 \\ 0.25 & 0.25 & 0.5-\lambda \end{vmatrix} = \emptyset$$

$$(0.5-\lambda)\begin{vmatrix}0.5-\lambda & 0.25\\ 0.25 & 0.5-\lambda\end{vmatrix} - 0.25\begin{vmatrix}0.25 & 0.25\\ 0.25 & 0.5-\lambda\end{vmatrix} + 0.25\begin{vmatrix}0.25 & 0.5-\lambda\\ 0.25 & 0.25\end{vmatrix}$$

$(0.5-\lambda)((0.5-\lambda)^2 - 0.25^2) - 0.25(0.125 - 0.25\lambda - 0.0625)$

$+0.25(0.0625 - 0.125 + 0.25\lambda) = 0 \,/\cdot 4$

$(2-4\lambda)(0.25 - \lambda + \lambda^2 - 0.0625) - 0.0625 + 0.25\lambda$

$-0.0625 + 0.25\lambda = 0$

$0.5 - 2\lambda + 2\lambda^2 - 0.125 - \lambda + 4\lambda^2 - 4\lambda^3 + 0.25\lambda - 0.125 + 0.5\lambda =$

$-4\lambda^3 + 6\lambda^2 - 2.25\lambda + 0.25 = \emptyset \,/\cdot 4$

$-16\lambda^3 + 24\lambda^2 - 9\lambda + 1 = \emptyset$

$-\lambda(16\lambda^2 - 24\lambda + 9) + 1 = 0$

$-\lambda((4\lambda-3)^2) + 1 = 0$

$\lambda(4\lambda-3)^2 = 1$

– INEZNAM DALJE, ALI $\lambda = [1 \ \frac{1}{4} \ \frac{1}{4}]^T$

TRAŽIMO SVOJSTVENE VEKTORE

$$R\vec{e} = \lambda\vec{e}$$

1) $\frac{1}{4}\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix} = \lambda\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix} \,/\cdot 4$

$\frac{1}{2}e_1 + \frac{1}{4}e_2 + \frac{1}{4}e_3 = e_1$

$-2e_1 + e_2 + e_3 = 0 \Rightarrow e_1 = [1 \ 1 \ 1]^T$

2) $e_1 + 2e_2 + e_3 = e_2$

$e_1 + e_2 + e_3 = 0 \Rightarrow e_2 = [-2 \ 1 \ 1]^T$

3) $e_1 + e_2 + 2e_3 = e_3 \quad = [0 \ -1 \ 1]$

TREBA TAKO IZABRAT VEKTORE $\vec{e}$ DA ZADOVOLJAVAJU

$e_i^T \cdot e_j = \emptyset$, I SVAKAKO NORMALIZIRATI $\hat{e}_i = \frac{1}{\|\vec{e}_i\|} e_i$

DALJE SE IZRAČUNA MATRICA A, ODABERU
PRVE 2 DIMENZIJE I POMNOŽE SE PRIMJERI S A

KL TRANSFORMACIJA - AUDITORNE 2010/2011

$$\omega = \{[-4\ -2]^T, [1, 3]^T, [3\ -1]^T\}$$

$$K = \frac{1}{N-1}\sum_{i=1}^{N}(\vec{x}_i - \vec{m})(\vec{x}_i - \vec{m})^T$$

$$m = \frac{1}{3}\left(\begin{bmatrix}-4\\-2\end{bmatrix} + \begin{bmatrix}1\\3\end{bmatrix} + \begin{bmatrix}3\\-1\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$$

$$K = \frac{1}{2}\left(\begin{bmatrix}-4\\-2\end{bmatrix}[-4\ -2] + \begin{bmatrix}1\\3\end{bmatrix}[1\ \ 3] + \begin{bmatrix}3\\-1\end{bmatrix}[3\ -1]\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix}16 & 8\\8 & 4\end{bmatrix} + \begin{bmatrix}1 & 3\\3 & 9\end{bmatrix} + \begin{bmatrix}9 & -3\\-3 & 1\end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix}26 & 8\\8 & 14\end{bmatrix} = \begin{bmatrix}13 & 4\\4 & 7\end{bmatrix}$$

$$(K - \lambda I)\vec{e} = 0$$

$$|K - \lambda I| = 0$$

$$\begin{vmatrix}13-\lambda & 4\\4 & 7-\lambda\end{vmatrix} = 0 \Rightarrow (13-\lambda)(7-\lambda) - 16 = 0$$

$$91 - 13\lambda - 7\lambda + \lambda_2 - 16 = 0$$

$$\lambda_2 - 20\lambda + 75 = 0$$

$$\lambda_{1/2} = \frac{20 \pm \sqrt{20^2 - 4\cdot 75}}{2} = \frac{20 \pm 10}{2} = \begin{bmatrix}15\\5\end{bmatrix}$$

$$(K - \lambda I)\vec{e} = 0$$

$$\begin{bmatrix}-2 & 4\\4 & -8\end{bmatrix}\cdot\begin{bmatrix}e_1\\e_2\end{bmatrix} = 0 \Rightarrow -2e_1 + 4e_2 = 0 \quad e_1 = 2e_2 \quad e = \begin{bmatrix}2\\1\end{bmatrix}$$

$$\boxed{e = \frac{1}{\sqrt{5}}\begin{bmatrix}2\\1\end{bmatrix}}$$

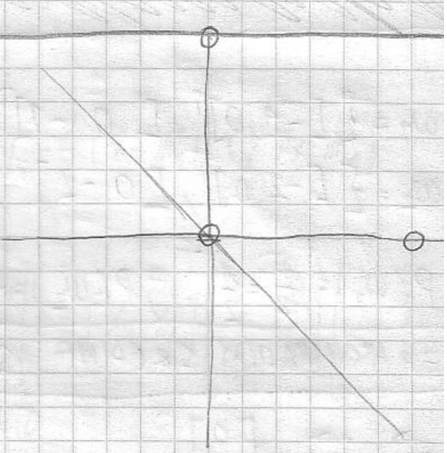
2. (6 bodova) Skup uzorka

$\{ [0, 0]^T, [6, 0]^T [0, 6]^T \}$

transformirajte iz dvodimenzionalnog u jednodimenzionalni prostor uporabom KL transformacije. Uputa: koristiti **kovarijacijsku** matricu.

KL TRANSFORMACIJA – ZAVRŠNI 2010/2011

$\omega = \{ [0 \;\; 0]^T, [6 \;\; 0]^T, [0 \;\; 6]^T \}$



BITNA INFORMACIJA:

KORELACIJSKA MATRICA: R

KOVARIJACIJSKA ILI KOVARIJANTNA JE: K

I ODUZIMA SE SREDNJA VRIJEDNOST!

$$\vec{m} = \frac{1}{3}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$K = \frac{1}{3-1}\left(\begin{bmatrix} -2 \\ -2 \end{bmatrix} [-2 \;\; -2] + \begin{bmatrix} 4 \\ -2 \end{bmatrix} [4 \;\; -2] + \begin{bmatrix} -2 \\ 4 \end{bmatrix} [-2 \;\; 4] \right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 16 & -8 \\ -8 & 4 \end{bmatrix} + \begin{bmatrix} 4 & -8 \\ -8 & 16 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 24 & -12 \\ -12 & 24 \end{bmatrix} = \begin{bmatrix} 12 & -6 \\ -6 & 12 \end{bmatrix}$$

$$|K - \lambda I| = 0$$

$$\begin{vmatrix} 12-\lambda & -6 \\ -6 & 12-\lambda \end{vmatrix} = 0$$

$$(12-\lambda)^2 - 36 = 0$$

$$\lambda^2 + 144 - 24\lambda - 36 = 0$$

$$\lambda^2 - 24\lambda + 108 = 0$$

$$\lambda_{1/2} = \frac{24 \pm \sqrt{576 - 4 \cdot 108}}{2}$$

$$= \frac{24 \pm 12}{2}$$

$$= \begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

a) $(K - 18 I) \cdot \vec{w} = \vec{0}$

$$\begin{bmatrix} -6 & -6 \\ -6 & -6 \end{bmatrix} \cdot \vec{w} = \vec{0}$$

$$-6w_1 - 6w_2 = 0 \Rightarrow w_1 = -w_2 \Rightarrow \vec{w}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

b) $\begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} \vec{w} = \vec{0}$

$$6 \cdot w_1 - 6w_2 = 0 \Rightarrow \vec{w}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$A = \left[\frac{\sqrt{2}}{2} \;\; -\frac{\sqrt{2}}{2} \right]$, $\vec{y}_i = A \cdot \vec{x}_i$

$y = \{ 0, 3\sqrt{2}, -3\sqrt{2} \}$

KL TRANSFORMACIJA - 2. ZADATAK IZ SKRIPTE

$$E = \begin{bmatrix} 1,9 \\ 2 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.3889 & 0.3456 \\ 0,3456 & 0.4044 \end{bmatrix}$$

$$(K - \lambda I)\vec{w} = \vec{\emptyset}$$

$$|K - \lambda I| = \emptyset$$

$$(0.3889 - \lambda)(0.4044 - \lambda) - 0.3456^2 = \emptyset$$

$$0.3889 \cdot 0.4044 - 0.3889\lambda - 0.4044\lambda + \lambda^2 - 0.3456^2 = \emptyset$$

$$0.03783 - 0.7933\lambda + \lambda_2 = 0$$

$$\lambda_{1/2} = \frac{0.7933 \pm \sqrt{0.62932489 - 0.15132}}{2}$$

$$\lambda_{1/2} = \frac{0.7933 \pm 0.691379}{2}$$

$$\lambda_1 = 0.7423 \quad , \quad \lambda_2 = 0.051$$

$$(K - \lambda_1 I)\vec{w} = \emptyset$$

$\vec{w_1}$)
$$\begin{bmatrix} -0.3534 & 0.3456 \\ 0.3456 & -0.3379 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$$

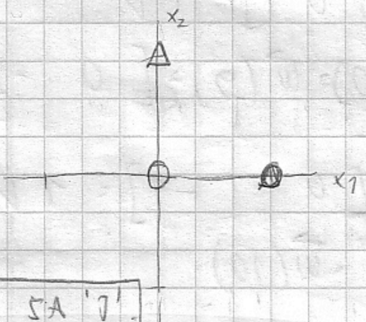
$$w_2 = \frac{0.3534}{0.3456} w_1 \Rightarrow w_2 = 1.02912\, w_1 \qquad \vec{w_1} = \begin{bmatrix} 0.697 \\ 0.717 \end{bmatrix}$$

$\vec{w_2}$)
$$\begin{bmatrix} 0.3379 & 0.3456 \\ 0.3456 & 0.3534 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$$

$$w_1 = \frac{-0.3456}{0.3379} w_2 \Rightarrow w_2 = -1.0227 \qquad w_2 = \begin{bmatrix} -0.717 \\ 0.6991 \end{bmatrix}$$

④ PERCEPTRON ZA 2 RAZREDA

PRIMJER IZ AUDITORNIH



$\omega_1 = \{ [0,0]^T, [1,0]^T \}$ o

$\omega_2 = \{ [0,1]^T \}$ △

$c = 1$

1. KORAK: PROŠIRIT DIMENZIJU UZORAKA SA '1'. I POMNOŽIT ω_2 UZORKE SA -1

$\omega_1 = \{ [0,0,1], [1,0,1] \}$

$\omega_2 = \{ [0,-1,-1] \}$

$\vec{w}^T(1) = [0,0,0]$

2. KORAK: U PETLJI MNOŽI w SA UZORCIMA, I MIJENJAJ w AKO NIJE ISPRAVNO KLASIFICIRAN

$\vec{w}^T(1) \cdot [0\ 0\ 1] = 0$ ✗

KOREKCIJA

$\vec{w}(2) = \vec{w}(1) + c \cdot [0\ 0\ 1] = [0\ 0\ 1]^T$

$\vec{w}_2^T(2) \cdot [1\ 0\ 1] = 1$ ✓

$\vec{w}(3) = \vec{w}(2)$

$\vec{w}(3)^T \cdot [0\ -1\ -1] = -1$ ✗

$\vec{w}(4)^T = \vec{w}(3)^T + c \cdot [0\ -1\ -1] = [0\ -1\ 0]$

$\vec{w}(4)^T \cdot [0\ 0\ 1] = 0$ ✗

$\vec{w}(5) = \vec{w}(4) + c \cdot [0\ 0\ 1] = [0\ -1\ 1]$

$\vec{w}(5)^T \cdot [1\ 0\ 1] = 1$ ✓

$\vec{w}(6) = \vec{w}(5)$

$\vec{w}(6)^T \cdot [0\ -1\ -1] = 0$ ✗

$\vec{w}(7) = \vec{w}(6) + c \cdot [0\ -1\ -1] = [0\ -2\ 0]$

$\vec{w}(7)^T \cdot [0\ 0\ 1] = 0$ ✗

$\vec{w}(8) = \vec{w}(7) + c \cdot [0\ 0\ 1] = [0\ -2\ 1]$

$\vec{w}(9)^T \cdot [1\ \ 0\ \ 1] = 1$ ✓

$\vec{w}(10) = \vec{w}(9) = [0\ \ -2\ \ 1]$

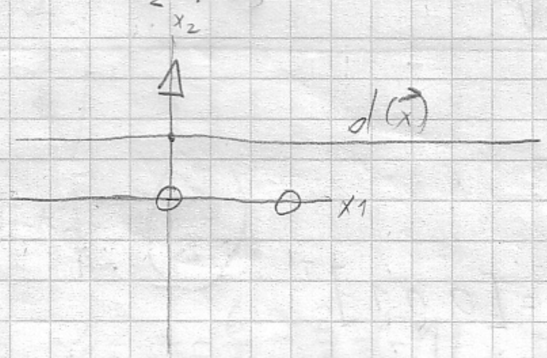
$\vec{w}(10)^T \cdot [0\ \ -1\ \ -1] = 1$ ✓

$\vec{w}(11) = \vec{w}(10)$

$\vec{w}(11) \cdot [0\ \ 0\ \ 1] = 1$ ✓

ALGORITAM JE PROŠAO CIJELU EPOHU BEZ KOREKCIJE, DECIZIJSKA FUNKCIJA JE NAUČENA

$$d(\vec{x}) = -2x_2 + 1$$



⑤ PERCEPTRON ZA VIŠE OD 2 RAZREDA

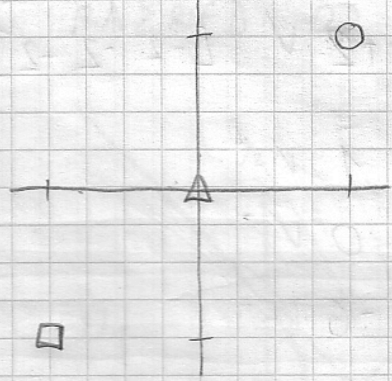
$\omega_1 = \{[1,1]^T\}$ o

$\omega_2 = \{[0,0]^T\}$ △

$\omega_3 = \{[-1\ -1]^T\}$ □

$c = 1$

$\vec{w}_1 = \vec{w}_2 = \vec{w}_3 = \vec{\emptyset}$



1. KORAK, PROŠIRUJEMO UZORKE SA [1]

$\omega_1 = [1\ 1\ 1]^T$

$\omega_2 = [0\ 0\ 1]^T$

$\omega_3 = [-1\ -1\ 1]^T$

1. epoha

- $\vec{w}_1(1)^T \cdot [1\ 1\ 1] = \emptyset \rightarrow \vec{w}_1(2) = \vec{w}_1(1) + c \cdot [1\ 1\ 1] = [1\ 1\ 1$
$\vec{w}_2(1)^T [1\ 1\ 1] = \emptyset \rightarrow \vec{w}_2(2) = \vec{w}_2(1) - c \cdot [1\ 1\ 1] = [-1\ -1\ -$
$\vec{w}_3(1)^T [1\ 1\ 1] = \emptyset \rightarrow \vec{w}_3(2) = \vec{w}_3(1) - c \cdot [1\ 1\ 1] = [-1\ -1$

$\vec{w}_1(2)^T \cdot [0\ 0\ 1] = 1 \rightarrow [1\ 1\ 0]$
- $\vec{w}_2(2)^T \cdot [0\ 0\ 1] = -1 \rightarrow [-1\ -1\ 0]$
$\vec{w}_3(2)^T \cdot [0\ 0\ 1] = -1 \rightarrow [-1\ -1\ -2]$

$\vec{w}_1(3)^T \cdot [-1\ -1\ 1] = -2 \rightarrow [1\ 1\ 0]$
$\vec{w}_2(3)^T \cdot [-1\ -1\ 1] = 2 \rightarrow [0\ 0\ -1]$
- $\vec{w}_3(3)^T \cdot [-1\ -1\ 1] = \emptyset \rightarrow [-2\ -2\ -1]$

- $\vec{w}_1(4) \cdot [1\ 1\ 1] = 2$ ✓
$\vec{w}_2(4) \cdot [1\ 1\ 1] = -1$ ✓
$\vec{w}_3(4) \cdot [1\ 1\ 1] = -5$ ✓

$\vec{w}_1(5) \cdot [0\ 0\ 1] = 0 \rightarrow [1\ 1\ -1]$
- $\vec{w}_2(5) \cdot [0\ 0\ 1] = -1 \rightarrow [0\ 0\ 0]$
$\vec{w}_3(5) \cdot [0\ 0\ 1] = -1 \rightarrow [-2\ -2\ -2]$

$\vec{w}_1(6)^T \cdot [-1\ -1\ 1] = -3$ ✓ $[1\ 1\ -1]$

$\vec{w}_2(6)^T \cdot [-1\ -1\ 1] = 0$ ✓ $[0\ 0\ 0]$

$\vec{w}_3(6)^T [-1\ -1\ 1] = +2$ ✓ $[-2\ -2\ -2]$

$\vec{w}_1(7)^T \cdot [1\ 1\ 1] = 1$ ✓

$\vec{w}_2(7)^T \cdot [1\ 1\ 1] = 0$ ✓

$\vec{w}_3(7)^T \cdot [1\ 1\ 1] = -6$ ✓

$\vec{w}_1(8)^T \cdot [0\ 0\ 1] = -1$ ✓

$\vec{w}_2(8)^T \cdot [0\ 0\ 1] = 0$ ✓

$\vec{w}_3(8)^T \cdot [0\ 0\ 1] = -2$ ✓

$d_1(\vec{x}) = x_1 + x_2 - 1$

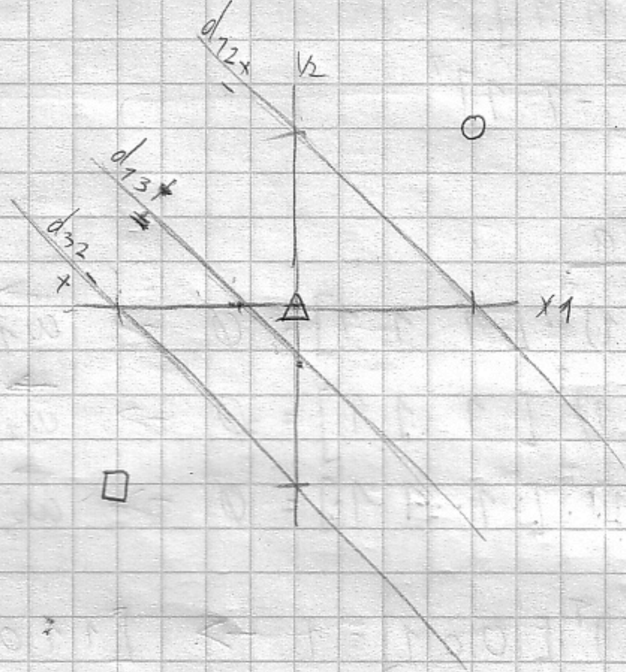
$d_2(\vec{x}) = \emptyset$

$d_3(\vec{x}) = -2x_1 - 2x_2 - 2$

$d_{12}(\vec{x}) = x_1 + x_2 - 1$

$d_{13}(\vec{x}) = 3x_1 + 3x_2 + 1$

$d_{32}(x) = -2x_1 - 2x_2 - 2$



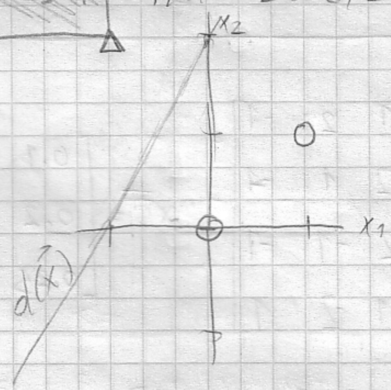
(4 boda) Za skup uzoraka

$\omega_1 = \{ [0, 0]^T, [1, 1]^T \}$,

$\omega_2 = \{ [-1, 2]^T \}$,

naći granice između razreda poopćenim postupkom perceptrona **za više od 2 razreda**. Neka su na početku svi težinski vektori nul-vektori, a konstanta c=1. Biste li isto rješenje dobili postupkom perceptrona za 2 razreda? Komentirajte zašto.

PERCEPTRON ZA VIŠE OD 2 RAZREDA | 1.MI 2008/2009



$\omega_1 = \{ [0\ 0]^T, [1\ 1]^T \}$

$\omega_2 = \{ [-1\ 2]^T \}$

$\vec{w}(0) = [\emptyset]$

$c = 1$

1) $\omega_1 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ o

$\omega_2 = \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$ △

2)

$\vec{w}_1(1) \cdot [0\ 0\ 1]^T = \emptyset \Rightarrow \vec{w}_1(2) = \vec{w}_1(1) + c \cdot [0\ 0\ 1] = [0\ 0\ 1]^T$

$\vec{w}_2(1) \cdot [0\ 0\ 1]^T = \emptyset \Rightarrow \vec{w}_2(2) = \vec{w}_2(1) - c \cdot [0\ 0\ 1] = [0\ 0\ -1]^T$

$\vec{w}_1(2) \cdot [1\ 1\ 1]^T = 1$ ✓

$\vec{w}_2(2) \cdot [1\ 1\ 1]^T = -1$ ✓

$\vec{w}_1(3) \cdot [-1\ 2\ 1] = 1 \Rightarrow \vec{w}_1(4) = [1\ -2\ 0]^T$

$\vec{w}_2(3) \cdot [-1\ 2\ 1] = -1 \Rightarrow \vec{w}_2(4) = [-1\ 2\ 0]^T$

$\vec{w}_1(4) \cdot [0\ 0\ 1] = \emptyset \Rightarrow \vec{w}_1(5) = [1\ -2\ 1]^T$

$\vec{w}_2(4) \cdot [0\ 0\ 1] = \emptyset \Rightarrow \vec{w}_2(5) = [-1\ 2\ -1]^T$

$\vec{w}_1(5) \cdot [1\ 1\ 1] = \emptyset \Rightarrow \vec{w}_1(6) = [2\ -1\ 2]^T$

$w_2(5) \cdot [1\ 1\ 1] = \emptyset \Rightarrow \vec{w}_2(6) = [-2\ 1\ -2]$

$w_1(6) \cdot [-1\ 2\ 1] = -2$ ✓

$\vec{w}_2(6) \cdot [-1\ 2\ 1] = 2$ ✓

$w_1(7) \cdot [0\ 0\ 1] = 2$ ✓

$w_2(7) \cdot [0\ 0\ 1] = -2$ ✓

$w_1(8) \cdot [1\ 1\ 1] = 3$ ✓

$w_2(8) \cdot [1\ 1\ 1] = -3$ ✓

$d(\vec{x}) = 2x_1 - x_2 + 2$

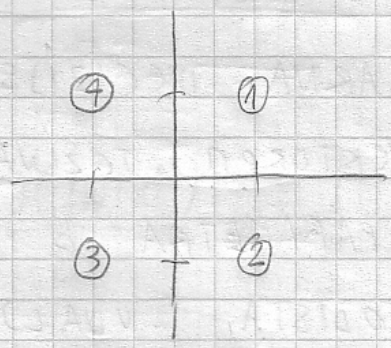
PERCEPTRON ZA VIŠE RAZREDA | MOJ ZADATAK

$w_1 = \{[1\ 1]\}$ $\quad X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $X_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$w_2 = \{[1\ -1]\}$

$w_3 = \{[-1\ -1]\}$ $\quad X_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ $X_4 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$w_4 = \{[-1\ 1]\}$



$w_1(1) = [0\ 0\ 0] \cdot x_1 = 0 \uparrow = [1\ 1\ 1]$

$w_2(1) = [0\ 0\ 0] \cdot x_1 = 0 \downarrow \quad [-1\ -1\ -1]$

$w_3(1) = [0\ 0\ 0] \cdot x_1 = 0 \downarrow \quad [-1\ -1\ -1]$

$w_4(1) = [0\ 0\ 0] \cdot x_1 = 0 \downarrow \quad [-1\ -1\ -1]$

$w_1 \cdot X_2 = 1 \downarrow \quad [0\ 2\ 0]$

$w_2 \cdot X_2 = -1 \uparrow \quad [0\ -2\ 0]$

$w_3 \cdot X_2 = -1 \downarrow \quad [-2\ 0\ -2]$

$w_4 \cdot X_2 = -1 \downarrow \quad [-2\ 0\ -2]$

$w_1 \cdot X_3 = -2 \quad [0\ 2\ 0]$

$w_2 \cdot x_3 = 2 \quad [0\ -2\ 0]$

$w_3 \cdot X_3 = 4 \uparrow \quad [-3\ -1\ -3]$

$w_4 \cdot X_3 = 4 \downarrow \quad [-1\ 1\ -1]$

$w_1 \cdot X_4 = 2 \downarrow \quad [1\ 1\ -1]$

$w_2 \cdot X_4 = -2 \quad [0\ -2\ 0]$

$w_3 \cdot X_4 = -1 \quad [-3\ -1\ -3]$

$w_4 \cdot X_4 = 1 \uparrow \quad [-2\ 2\ 0]$

$w_1 \cdot x_1 = 1 \checkmark \qquad \cdot x_2 = -1 \qquad \cdot x_3 = -3 \quad [1\ 1\ -1]$

$w_2 \cdot x_1 = -2 \qquad \cdot x_2 = 2 \checkmark \qquad \cdot x_3 = 2 \downarrow \quad [1\ -1\ -1]$

$w_3 \cdot x_1 = -7 \qquad \cdot x_2 = -5 \qquad \cdot x_3 = 1 \uparrow \quad [-4\ -2\ -2]$

$w_4 \cdot x_1 = 0 \qquad \cdot x_2 = -4 \qquad \cdot x_3 = 0 \quad [-2\ 2\ 0]$

$\cdot x_4 = -1$ $\quad \cdot x_1 = 1$ ✓ $\quad \cdot x_2 = -1$ $\quad \cdot x_3 = -3$ ✓

$\cdot x_4 = -3$ $\quad \cdot x_1 = -1$ $\quad \cdot x_2 = 1$ ✓ $\quad \cdot x_3 = -1$ ✓

$\cdot x_4 = 0$ $\quad \cdot x_1 = -8$ $\quad \cdot x_2 = -4$ $\quad \cdot x_3 = 4$ ✓

$\cdot x_4 = 4$ ✓ $\quad \cdot x_1 = 0$ $\quad \cdot x_2 = -4$ $\quad \cdot x_3 = 0$ ✓

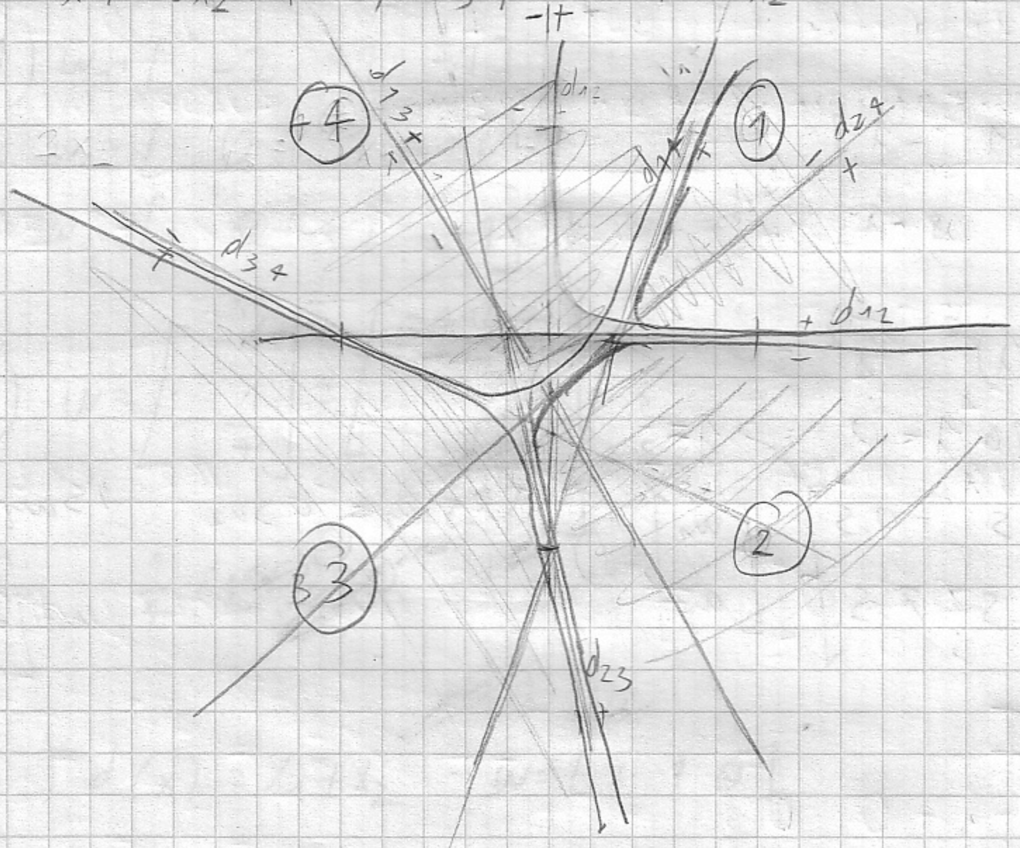
$d_1(\vec{x}) = x_1 + x_2 - 1 \qquad d_3(\vec{x}) = -4x_1 - 2x_2 - 2$

$d_2(\vec{x}) = x_1 - x_2 - 1 \qquad d_4(\vec{x}) = -2x_1 + 2x_2$

$d_{12} = 2x_2 \;;\; d_{13} = 5x_1 + 3x_2 + 1$

$d_{14} = 3x_1 - x_2 - 1 \;;\; d_{23} = 5x_1 + x_2 + 1$

$d_{24} = 3x_1 - 3x_2 - 1 \;;\; d_{34} = -2x_1 - 4x_2 - 2$



POOPĆENE DECIZIJSKE FUNKCIJE - AUDITORNE 2009/2010

○ $\omega_1 = \{ [0\ 2]^T, [-1\ -2]^T, [1, -2]^T \}$

△ $\omega_2 = \{ [0\ 0]^T \}$

$w(1) = [0]\ ;\ c = 1$

- POLINOM DRUGOG STUPNJA JE ZADAN

$$\vec{X}^* = \begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

- PROJEKCIJA ULAZNOG PROSTORA

○ $x_1 = [0\ \ 0\ \ 4\ \ 0\ \ 2\ \ 1]^T$

○ $x_2 = [1\ \ 2\ \ 4\ \ -1\ \ -2\ \ 1]^T$

○ $x_3 = [1\ \ -2\ \ 4\ \ 1\ \ -2\ \ 1]^T$

△ $x_4 = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ -1]^T$ # OVOG SMO POMNOŽILI SA -1 ZBOG ALGORITMA PERCEPTRONA JER PRIPADA KLASI ω_2

$\boxed{\text{VRTIMO ALGORITAM PERCEPTRONA}}$ $\Rightarrow \vec{w}^T \vec{x} > 0$ MORA VRIJEDITI

$\vec{w}(1) = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$

$\vec{w}(1)^T \cdot [0\ \ 0\ \ 0\ \ 4\ \ 0\ \ 2\ \ 1]^T = 0 \Rightarrow$ KOREKCIJA!

$\vec{w}(2) = \vec{w}(1) + c \cdot \vec{x_1} = [0\ \ 0\ \ 0\ \ 4\ \ 0\ \ 2\ \ 1]^T$

$\vec{w}(2)^T \cdot [1\ \ 2\ \ 4\ \ -1\ \ -2\ \ 1]^T = 13 \checkmark \Rightarrow \vec{w}(3) = \vec{w}(2)$

$\vec{w}(3)^T \cdot [1\ \ -2\ \ 4\ \ 1\ \ -2\ \ 1]^T = 13 \checkmark \Rightarrow \vec{w}(4) = \vec{w}(3)$

$\vec{w}(4)^T \cdot [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ -1]^T = -1 \Rightarrow$ KOREKCIJA!

$\vec{w}(5) = \vec{w}(4) + c \cdot x_4 = [0\ \ 0\ \ 4\ \ 0\ \ 2\ \ 0]^T$

$\vec{w}(5) \cdot [0\ \ 0\ \ 4\ \ 0\ \ 2\ \ 1]^T = 20 \checkmark\ \ \vec{w}(6) = \vec{w}(5)$

$\vec{w}(6) \cdot [1\ \ 2\ \ 4\ \ -1\ \ -2\ \ 1]^T = 12 \checkmark\ \ \vec{w}(7) = \vec{w}(6)$

$\vec{w}(7) \cdot [1\ \ -2\ \ 4\ \ 1\ \ -2\ \ 1]^T = 12 \checkmark\ \ \vec{w}(8) = \vec{w}(6)$

$\vec{w}(8) \cdot [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ -1]^T = 0 \Rightarrow$ KOREKCIJA

$\vec{w}(9) = \vec{w}(8) + c \cdot X_4 = [0\ \ 0\ \ 4\ \ 0\ \ 2\ \ -1]$

$\vec{w}(9) \cdot [\ 0\ \ 0\ \ 4\ \ 0\ \ 2\ \ 1\ ] = 19\ \checkmark \qquad \vec{w}(10) = \vec{w}(9)$

$\vec{w}(10) \cdot [\ 1\ \ 2\ \ 4\ \ -1\ \ -2\ \ 1\ ] = 11\ \checkmark \qquad \vec{w}(11) = \vec{w}(10)$

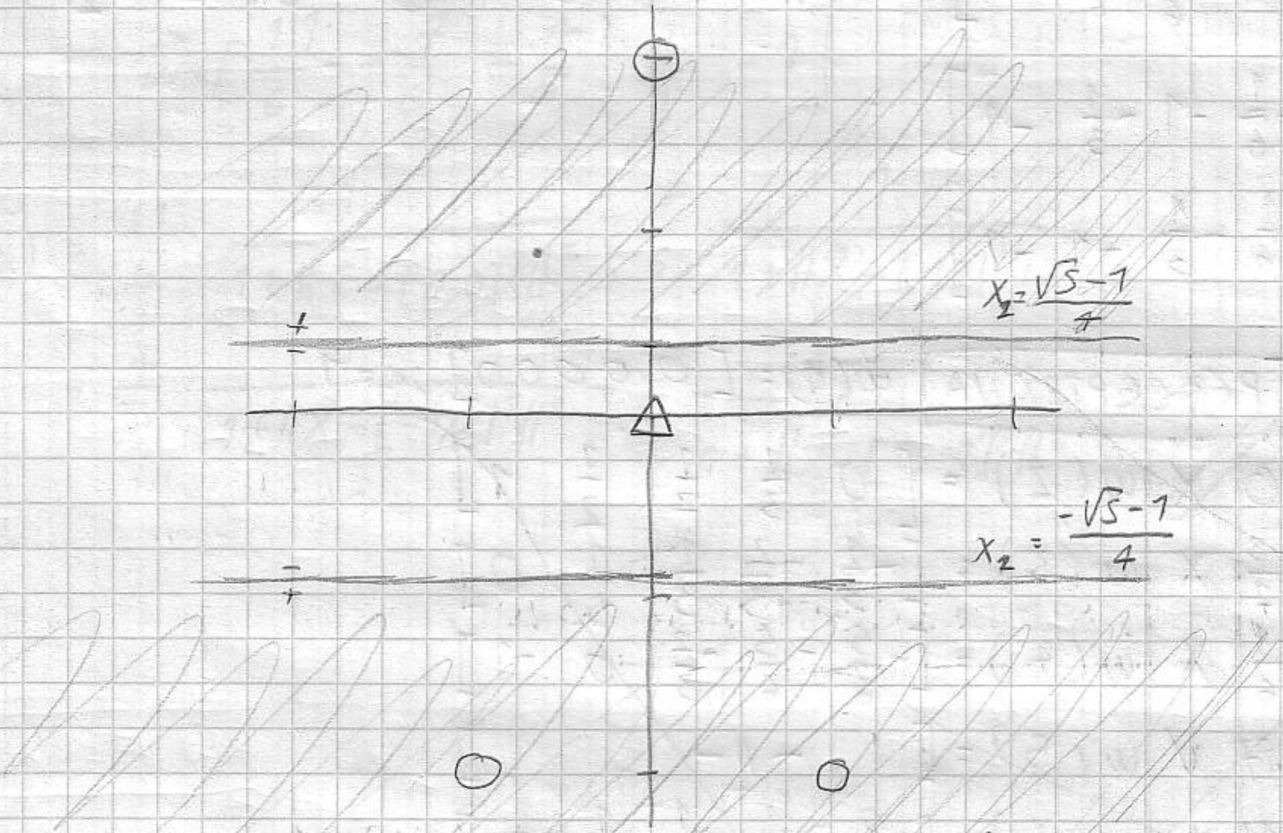
$\vec{w}(11) \cdot [\ 1\ \ -2\ \ 4\ \ 1\ \ -2\ \ 1\ ] = 11\ \checkmark \qquad \vec{w}(12) = \vec{w}(11)$

$\vec{w}(12) \cdot [\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ -1\ ] = 1\ \checkmark$ **- GOTOVO!**

$$d(\vec{x}) = 4x_2^2 + 2x_2 - 1 = 0$$

$$x_{2\,1/2} = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm 2\sqrt{5}}{8}$$

$$x_{21} = \frac{-\sqrt{5}-1}{4}; \quad x_{22} = \frac{\sqrt{5}-1}{4}$$



RADIJALNE BAZNE FUNKCIJE - AUDITORNE 2010/2011

$\omega_1 = \{[0\ 0]\}$ 0

$\omega_2 = \{[1\ 1], [-1\ 0], [0\ -1]\}$ 1

$$f(X) = \frac{1}{1 + \|\vec{x} - \vec{c}_i\|^2}$$

$\vec{c}_i = \vec{x}_i$

- PRVO TRANSFORMIRAMO TOČKE $\vec{x}_i^* = [\ f(x_i, c_1)\ f(x, c_2)\ \dots\ f(x, c_N)$

$\vec{x}_1^* = \left[1\ \ \frac{1}{3}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ 1\right]$ ← ZADNJI ELEMENT JE PROŠIRENJE ZA POSTUPAK PERCEPTRONA.

$\vec{x}_2^* = \left[-\frac{1}{3}\ \ -1\ \ -\frac{1}{6}\ \ -\frac{1}{6}\ \ -1\right]$ ← MNOŽIMO S -1 JER JE $\in \omega_2$

$\vec{x}_3^* = \left[-\frac{1}{2}\ \ -\frac{1}{6}\ \ -1\ \ -\frac{1}{3}\ \ -1\right]$

$\vec{x}_4^* = \left[-\frac{1}{2}\ \ -\frac{1}{6}\ \ -\frac{1}{3}\ \ -1\ \ -1\right]$

POSTUPAK PERCEPTRONA $w(1) = [0\ 0\ 0\ 0\ 0]$; $c = 1$

$w(1) \cdot x_1 = 0$	X	$w(2) = \left[0\ \ \frac{1}{3}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ 1\right]$
$w(2) \cdot x_2 = -\frac{3}{2}$	X	$w(3) = \left[-\frac{1}{3}\ \ -\frac{2}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\right]$
$w(3) \cdot x_3 = -\frac{1}{6}$	X	$w(4) = \left[-\frac{5}{6}\ \ -\frac{5}{6}\ \ -\frac{2}{3}\ \ 0\ \ -1\right]$
$w(4) \cdot x_4 = 1.77$	V	$w(5) = [\ -\|\!\|-\ ]$
$w(5) \cdot x_1 = -1.6$	X	$w(6) = \left[-\frac{5}{6}\ \ -\frac{1}{2}\ \ -\frac{1}{6}\ \ \frac{1}{2}\ \ 0\right]$
$w(6) \cdot x_2 = 0.7$	V	$w(7) = [\ -\|\!\|-\ ]$
$w(7) \cdot x_3 = 0.5$	V	$w(8) = [\ -\|\!\|-\ ]$
$w(8) \cdot x_4 = 0.05$	V	$w(9) = [\ -\|\!\|-\ ]$
$w(9) \cdot x_1 = 0$	X	$w(10) = \left[-\frac{5}{6}\ \ -\frac{1}{6}\ \ \frac{1}{3}\ \ 1\ \ 1\right]$
$w(10) \cdot x_2 = -0.7$	X	$w(11) = \left[-\frac{7}{6}\ \ -\frac{7}{6}\ \ \frac{1}{6}\ \ \frac{5}{6}\ \ 0\right]$
$w(11) \cdot x_3 = 0.3$	V	$w(12) = [\ -\|\!\|-\ ]$
$w(12) \cdot x_4 = -0.1$	X	$w(13) = \left[-\frac{10}{6}\ \ -\frac{4}{3}\ \ -\frac{1}{6}\ \ -\frac{1}{6}\ \ -1\right]$
$w(13) \cdot x_1 = -1.6$	X	$w(14) = \left[-\frac{10}{6}\ \ -1\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\right]$
$w(14) \cdot x_2 = 1.44$	V	$w(15) = [\ -\|\!\|-\ ]$
$w(15) \cdot x_3 = 0.56$	V	$w(16) = [\ -\|\!\|-\ ]$

$w(16) \cdot x_4 = 0.56$	✓	$w(17) = [\quad - 11 - \quad]$
$w(17) \cdot x_1 = -0.3$	✗	$w(18) = \left[-\frac{10}{6} \quad -\frac{2}{3} \quad \frac{5}{6} \quad \frac{5}{6} \quad 1 \right]$
$w(18) \cdot x_2 = -0.06$	✗	$w(19) = \left[-2 \quad -\frac{10}{6} \quad \frac{2}{3} \quad \frac{2}{3} \quad 0 \right]$
$w(19) \cdot x_3 = 0.39$	✓	$w(20) = [\quad - 11 - \quad]$
$w(20) \cdot x_4 = 0.39$	✓	$w(21) = [\quad - 11 - \quad]$
$w(21) \cdot x_1 = 0.11$	✓	$w(22) = [\quad - 11 - \quad]$
$w(22) \cdot x_2 = 2.11$	✓	$w(23) = [\quad - 11 - \quad]$

KRIVO SAM IZRAČUNAO x_1^*, NEDA MI SE RASPISIVAT SVE PONOVO, RJEŠENJE JE (PROVJERENO S PYTHON SKRIPTOM)

$$\vec{w} = \left[\frac{11}{6} \quad -\frac{7}{6} \quad -\frac{4}{3} \quad \frac{2}{3} \quad -1 \right]$$

$$d(\vec{x}) = \frac{11}{6} \cdot \frac{1}{1 + \|\vec{x} - [0,0]^T\|^2} - \frac{7}{6} \cdot \frac{1}{1 + \|\vec{x} - [1,1]^T\|^2}$$

$$- \frac{4}{3} \cdot \frac{1}{1 + \|\vec{x} - [-1,0]^T\|^2} + \frac{2}{3} \frac{1}{1 + \|x - [0,-1]^T\|^2} - 1$$

Za skup uzoraka $[2, 2]^T \in \omega_1$, $[-1, -1]^T \in \omega_1$, $[0, 0]^T \in \omega_2$ naći granicu između razreda, i to u **obliku polinoma drugog stupnja** koja se dobiva postupkom perceptrona sa stalnim prirastom. Neka je na početku **w** nul-vektor, a stopa učenja $c = 1$. Redoslijed pojavljivanja uzoraka neka bude onaj kojime su navedeni u zadatku.

POOPĆENE LINEARNE FJE - ISPIT 10.7.2006.

$\omega_1 = \{[-1\ -1]^T, [0\ 0]^T\}$ O

$\omega_2 = \{[2\ 2]^T\}$ Δ

$X^* = [x_1^2\ x_2^2\ x_1x_2\ x_1\ x_2\ 1]^T$ - POLINOM DRUGOG STUPNJA

-PROJEKCIJE ULAZNOG PROSTORA

$x_1 = [1\ 1\ 1\ -1\ -1\ 1]^T$

$x_2 = [0\ 0\ 0\ 0\ 0\ 1]^T$

$x_3 = [-4\ -4\ -4\ -2\ -2\ -1]^T$ # IMAMO 2 RAZREDA PA MNOŽIMO OVOG SA -1 DA SI OLAKŠAMO ŽIVOT ;)

$w(1) = [0\ 0\ 0\ 0\ 0\ 0]$; $c=1$

$w(1) \cdot x_1 = 0$ [X] $w(2) = [1\ 1\ 1\ -1\ -1\ 1]^T$

$w(2) \cdot x_2 = 1$ [V] $w(3) = w(2)$

$w(3) \cdot x_3 = -9$ [X] $w(4) = [-3\ -3\ -3\ -3\ -3\ 0]^T$

$w(4) \cdot x_1 = -3$ [X] $w(5) = [-2\ -2\ -2\ -4\ -4\ 1]^T$

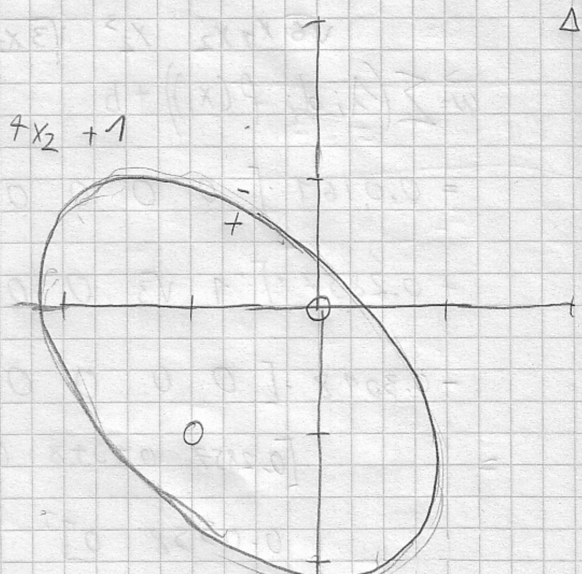
$w(5) \cdot x_2 = 1$ [V] $w(6) = w(5)$

$w(6) \cdot x_3 = 39$ [V] $w(7) = w(6)$

$w(7) \cdot x_1 = 3$ [V] ✓

$d = -2x_1^2 - 2x_2^2 - 2x_1x_2 - 4x_1 - 4x_2 + 1$

EDIT: FAK, SKUŽIO SAM DA SAM FULO KLASE UZORAKA ALI NEMA VEZE.



WOLFRAM ALPHA.COM PLOT

(3 boda) Zadani su uzorci iz tri razreda:

$\omega_1 = \{[0,0]^T\}$

$\omega_1 = \{[2,0]^T, [-1,1]^T\}$

$\omega_3 = \{[1,1]^T\}$

Postupkom preceptrona sa stalnim prirastom potrebno je nači decizijske funkciju za ove ozorke, i to u obliku **polinoma drugog stupnja**. Napišite prvu epohu algoritma (prvi prolaz kroz uzorke) algoritma koji nalazi ovakve decizijske funkcije. Neka su na početku svi težinski vektori nul-vektori, a konstanta $c = 1$.

POOPĆENE LINEARNE FJE - 2. MI - 2009/2010

$\omega_1 = \{[0\ 0]^T\}$ $\omega_2 = \{[2\ 0]^T, [-1\ 1]^T\}$ $\omega_3 = [1\ 1]^T\}$

$\boxed{\text{PROJICIRAMO UZORKE}}$ $X^* = [x_1^2\ x_2^2\ x_1 x_2\ x_1\ x_2\ 1]^T$

$x_1 = [0\ 0\ 0\ 0\ 0\ 1]$

$x_2 = [-4\ 0\ 0\ 2\ 0\ 1]$

$x_3 = [-1\ 1\ -1\ -1\ 1\ 1]$

$x_4 = [1\ 1\ 1\ 1\ 1\ 1]$

$\boxed{\text{ITERIRAMO}}$ $w_1 = w_2 = w_3 = [0]$; $c = 1$

$w_1(1) \cdot x_1 = 0$ X $w_1(2) = [0\ 0\ 0\ 0\ 0\ 1]$
$w_2(1) \cdot x_1 = 0$ X $w_2(2) = [0\ 0\ 0\ 0\ 0\ -1]$
$w_3(1) \cdot x_1 = 0$ X $w_3(2) = [0\ 0\ 0\ 0\ 0\ -1]$

$w_1(2) \cdot x_2 = 1$ X $w_1(3) = [-4\ 0\ 0\ -2\ 0\ 0]$
$w_2(2) \cdot x_2 = -1$ X $w_2(3) = [4\ 0\ 0\ 2\ 0\ 0]$
$w_3(2) \cdot x_2 = -1$ ✓ $w_3(3) = w_2(3)$

$w_1(3) \cdot x_3 = -4$ ✓
$w_2(3) \cdot x_3 = 2$ ✓
$w_3(3) \cdot x_3 = -1$ ✓

$w_1(4) \cdot x_4 = -6$ ✓ $w_1(4) = w_1(3)$
$w_2(4) \cdot x_4 = 6$ X $w_2(4) = [3\ -1\ -1\ 1\ -1\ -1]$
$w_3(4) \cdot x_4 = -1$ X $w_3(4) = [1\ 1\ 1\ 1\ 1\ 0]$

$w_1(5) \cdot x_1 =$
$w_2(5) \cdot x_1 =$
$w_3(5) \cdot x_1 =$

I TAKO DALJE. KONAČNI w-OVI:

$w_1 = [-4\ 0\ 0\ -2\ 0\ 1]$

$w_2 = [3\ -1\ -3\ -1\ -1\ -1]$

$w_3 = [-2\ 2\ 2\ 0\ 2\ -1]$

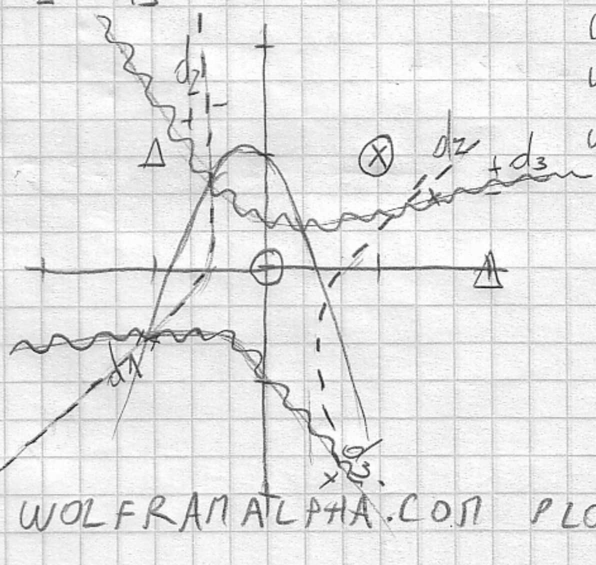
$w_1(6) \cdot x_2 =$
$w_2(6) \cdot x_2 =$
$w_3(6) \cdot x_2 =$

$w_1(7) \cdot x_3 =$
$w_2(7) \cdot x_3 =$
$w_3(7) \cdot x_3 =$

$w_1(8) \cdot x_4 =$
$w_2(8) \cdot x_4 =$
$w_3(8) \cdot x_4 =$

$w_1(9) \cdot x_1 =$
$w_2(9) \cdot x_1 =$
$w_3(9) \cdot x_1 =$

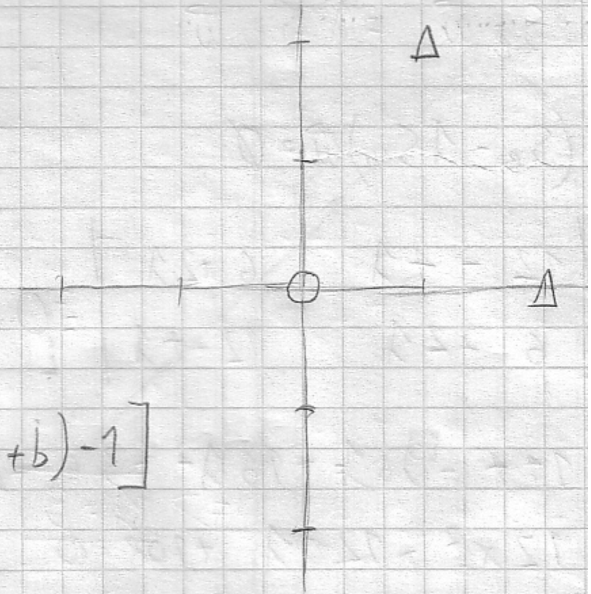
$w_1(10) \cdot x_2 =$
$w_2(10) \cdot x_2 =$
$w_3(10) \cdot x_2 =$



WOLFRAMALPHA.COM PLOT

$w = ($
$w_2 =$
$w_3 = ($

③ SVM - PRIMARNI PROBLEM



$$\omega_1 = \{ [0,0]^T \} \; \circ$$
$$\omega_2 = \{ [2,0]^T, [1,2]^T \} \; \Delta$$

KRITERIJSKA FUNKCIJA

$$J(\vec{w}, b, \vec{\lambda}) = \frac{1}{2}\vec{w}^T\vec{w} - \sum_{i=1}^{n} \lambda_i \left[d_i(\vec{w}^T\vec{x} + b) - 1 \right]$$

UVJETI

a) $\lambda_i \left[d_i(\vec{w}^T\vec{x} + b) - 1 \right] = \emptyset$

b) $\lambda_i \geq \emptyset$

c) $(\vec{w}^T\vec{x} + b) - 1 \geq \emptyset$

1: UVRSTITI UZORKE U KRITERIJSKU FUNKCIJU

$$J(\vec{w}, b, \vec{\lambda}) = \frac{1}{2}\vec{w}^T\vec{w} - \lambda_1(b-1) + $$
$$- \lambda_2(-2w_1 - b - 1)$$
$$- \lambda_3(-w_1 - 2w_2 - b - 1)$$

$$J(\vec{w}, b, \vec{\lambda}) = \frac{w_1^2 + w_2^2}{2} - \lambda_1 b + 1$$
$$+ 2\lambda_2 w_1 + \lambda_2 b + \lambda_2$$
$$+ \lambda_3 w_1 + 2\lambda_3 w_2 + \lambda_3 b + \lambda_3$$

2: PARCIJALNO DERIVIRAJ

$$\frac{\partial J}{\partial w_1} = w_1 + 2\lambda_2 + \lambda_3 = \emptyset \quad \Rightarrow w_1 = -2\lambda_2 - \lambda_3$$

$$\frac{\partial J}{\partial w_2} = w_2 + 2\lambda_3 = \emptyset \quad \Rightarrow w_2 = -2\lambda_3$$

$$\frac{\partial J}{\partial b} = -\lambda_1 + \lambda_2 + \lambda_3 = \emptyset \quad \Rightarrow \lambda_1 = \lambda_2 + \lambda_3$$

3. UVRSTITI UVJETE

$\lambda_1 (b-1) = 0 \Rightarrow \lambda_1 b = \lambda_1$

$\lambda_2(-2w_1 - b - 1) = 0 \qquad 2\lambda_2 w_1 + \lambda_2 b + \lambda_2 = 0$

$\lambda_3(-w_1 - 2w_2 - b - 1) = 0 \qquad \lambda_3 w_1 + 2\lambda_3 w_2 + \lambda_3 b + 1 = 0$

I. $\lambda_1 = \lambda_2 = \lambda_2 = 0 \Rightarrow$ TRIVIJALNO RJEŠENJE, NE GLEDA SE

II. $\lambda_1 > 0 \quad \lambda_2 = 0 \quad \lambda_3 = \lambda_1$

$w_1 = -\lambda_1$

$w_2 = -2\lambda_1$

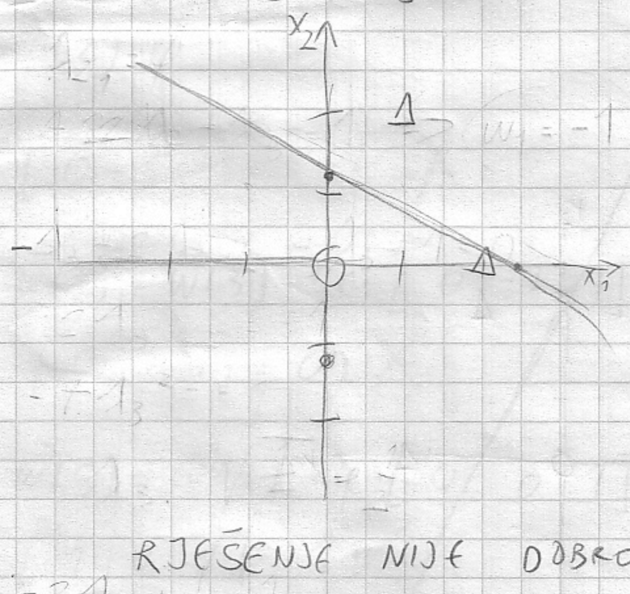
$b - 1 = 0 \Rightarrow b = 1$

$-w_1 - 2w_2 - b - 1 = 0$

$\lambda_1 + 4\lambda_1 - 1 - 1 = 0$

$\lambda_1 = \frac{2}{5} \Rightarrow w = \begin{bmatrix} -\frac{2}{5} \\ -\frac{4}{5} \end{bmatrix} \quad b = 1$

$d(\vec{x}) = -\frac{2}{5}x_1 - \frac{4}{5}x_2 + 1 = 0$



RJEŠENJE NIJE DOBRO JER NE UZIMA JEDAN UZORAK U OBZIR KOD IZGRADNJE SVM-A, VRTIMO DRUGE VARIJANTE

(III) $\lambda_1 \geq 0, \lambda_2 > 0, \lambda_3 > 0$, DOBIT ĆEMO SLIČNU STVAR KO (II)

(IV) $\lambda_1 > 0 \quad \lambda_2 > 0 \quad \lambda_3 > 0$

$b - 1 = 0 \Rightarrow b = 1$

$2w_1 + b + 1 = 0 \Rightarrow -4\lambda_2 - 2\lambda_3 + 2 = 0 \quad /:2$

$w_1 + 2w_2 + b + 1 = 0 \qquad -2\lambda_2 - \lambda_3 - 4\lambda_3 + 2 = 0$

$-4\lambda_3 = -1 \Rightarrow \lambda_3 = \frac{1}{4}$

$-4\lambda_2 - \frac{1}{2} + 2 = 0$

$-4\lambda_2 = -\frac{3}{2} \quad /:-4$

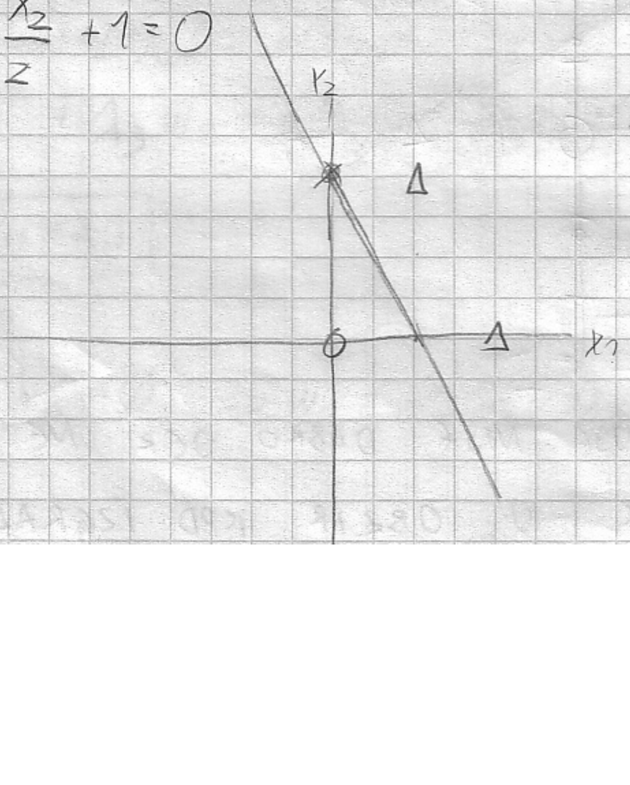
$\boxed{\lambda_2 = \frac{3}{8}}$

$\lambda_1 = \lambda_2 + \lambda_3 = \frac{5}{8}$

$w_1 = -2\lambda_2 - \lambda_3 = -\frac{6}{8} - \frac{2}{8} = -1$

$w_2 = -2\lambda_3 = -\frac{1}{2}$

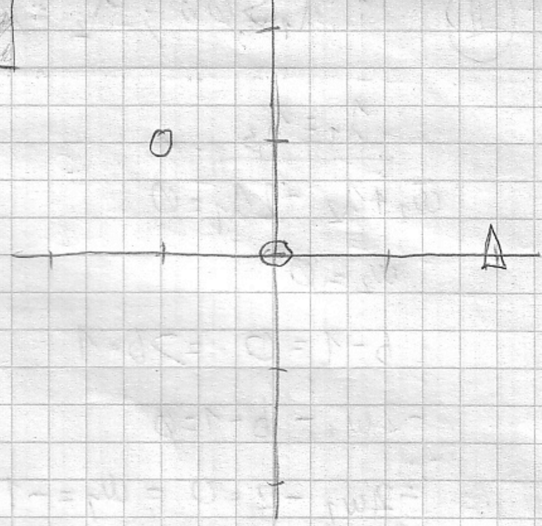
$d(\vec{x}) = -x_1 - \frac{x_2}{2} + 1 = 0$



SVM AUDITORNE fer2.net@mala

$\omega_1 = \{ [0\ 0]^T, [-1\ 1]^T \}$ O

$\omega_2 = \{ [2\ 0]^T \}$ Δ



RJEŠAVAMO KAO PRIMARNI PROBLEM

1)
$$J(\vec{w}, b, \vec{\lambda}) = \frac{1}{2}\vec{w}^T\vec{w} - \lambda_1(b-1) - \lambda_2(-w_1 + w_2 + b - 1) - \lambda_3(-2w_1 - b - 1)$$

$$J(\vec{w}, b, \vec{\lambda}) = \frac{w_1^2 + w_2^2}{2} - \lambda_1 b + \lambda_1 + \lambda_2 w_1 - \lambda_2 w_2 - \lambda_2 b + \lambda_2 + 2\lambda_3 w_1 + \lambda_3 b + \lambda_3$$

2)
$$\frac{\partial J}{\partial w_1} = w_1 + \lambda_2 + 2\lambda_3 = \emptyset$$

$$\frac{\partial J}{\partial w_2} = w_2 - \lambda_2 = \emptyset$$

$$\frac{\partial J}{\partial b} = -\lambda_1 - \lambda_2 + \lambda_3 = \emptyset \Rightarrow \lambda_1 = -\lambda_2 + \lambda_3$$

3)
$$\lambda_1(b-1) = \emptyset$$
$$\lambda_2(-w_1 + w_2 + b - 1) = \emptyset$$
$$\lambda_3(-2w_1 - b - 1) = \emptyset$$

4) VRTIT SLUČAJEVE λ

(I) $\lambda_1 = \lambda_2 = \lambda_3 = \emptyset$ ⇒ TRIVIJALNO RJEŠENJE, OD B ACUJEMO

II $\lambda_1 > 0$, $\lambda_2 = 0$ $\lambda_3 > 0$,

$\lambda_1 = \lambda_3$

$w_1 + \lambda_2 + 2\lambda_1 = 0$

$w_2 = 0$

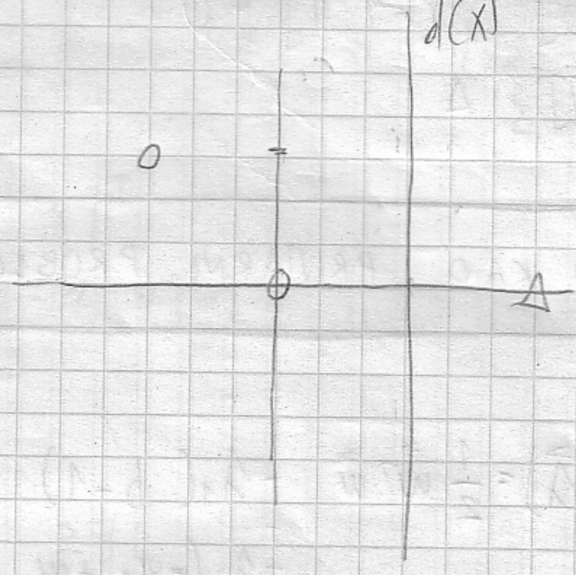
$b - 1 = 0 \Rightarrow b = 1$

$-2w_1 - b - 1 = 0$

$-2w_1 - 2 = 0 \Rightarrow w_1 = -1$

$\vec{w} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $b = 1$

$d(\vec{x}) = -x_1 + 1$



ISPRAVNA GRANICA RAZREDA ALI NE I OPTIMALNA

III $\lambda_1 > 0$ $\lambda_2 > 0$ $\lambda_3 = 0$

$\lambda_1 = \lambda_2$

$w_1 + \lambda_2 = 0$

$w_2 + \lambda_2 = 0$ $\Big\} + \Rightarrow w_1 = -w_2$

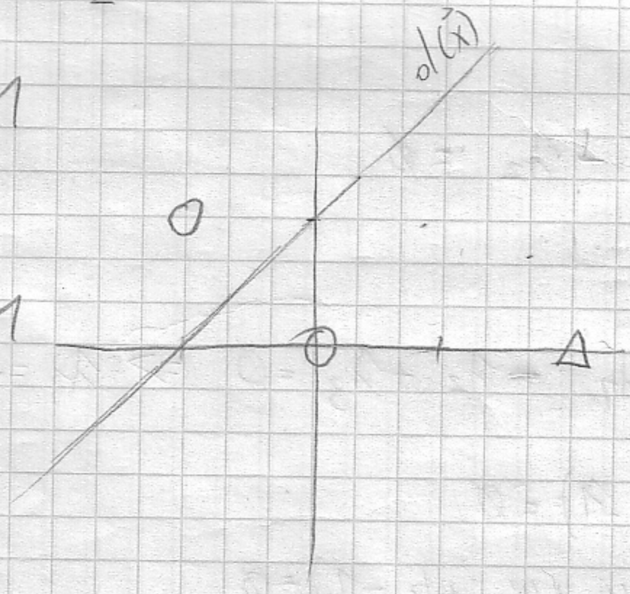
$b - 1 = 0 \Rightarrow b = 1$

$-w_1 + w_2 + b - 1 = 0$

$-w_1 + w_1 - 2 = 0 \Rightarrow w_1 = 1$

$w = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $b = 1$

$d(\vec{x}) = x_1 - x_2 + 1$



NE KLASIFICIRA DOBRO RAZREDE

IV $\lambda_1 > 0, \lambda_2 > 0 \quad \lambda_3 > 0$

$$w_1 + \lambda_2 + 2\lambda_3 = \emptyset$$

$$w_2 - \lambda_2 = \emptyset$$

$$\lambda_1 = \lambda_2 + \lambda_3$$

$$b - 1 = \emptyset \Rightarrow b = 1$$

$$-w_1 + w_2 + b - 1 = \emptyset$$

$$-2w_1 - b - 1 = \emptyset$$

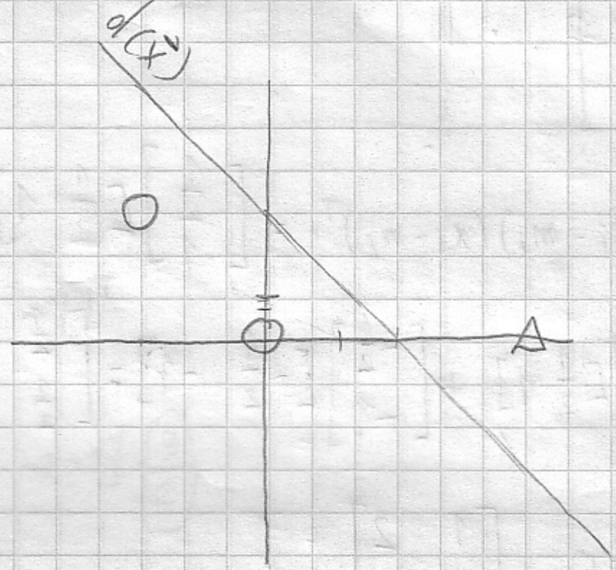
$$-2w_1 - 2 = 0$$

$$w_1 = -1$$

$$-1 + w_2 = 0$$

$$w_2 = -1$$

$$\vec{w} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad b = 1$$



$$d(\vec{x}) = -x_1 - x_2 + 1$$

$\boxed{\text{PROVJERA UVJETA:}}$

$$\vec{w}^T[x] + b \geq 1$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix}[0 \; 0] + 1 \geq 1 \checkmark$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix}[-1 \; -1] + 1 \geq 1 \checkmark$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix}[2 \; 0] + 1 \geq 1 \checkmark$$

DAKLE GRANICA JE OPTIMALNA!

SVM DUALNI PROBLEM

$\omega_1 = \{ [0\ 0]^T, [-1\ 1]^T \}$
$\omega_2 = \{ [2\ 0]^T \}$

$$\boxed{\begin{gathered} \min f(\vec{\lambda}) = \frac{1}{2} \vec{\lambda}^T Q \vec{\lambda} + \vec{c}^T \vec{\lambda} \\ \underline{\text{uz uvjet}} \\ A\vec{\lambda} \leq b\ ,\ E\vec{\lambda} = d \end{gathered}}$$

SAMO IZRAČUNAJ <u>Q, E, d, A, b, c</u> I TO JE TO
MATLAB IZRAČUNA LAMBDE IZ TOGA, TJ. U ISPITU ĆEŠ
DOBITI LAMBDE, NE TREBA IZVESTI KVADRATNO PROGRAMIRANJE

E   LISTA DECIZIJSKIH OZNAKA, 1 ZA C_1, -1 ZA C_2

$E = [1\ \ 1\ \ -1]$

b   SU SVE NULE TRANSPONIRANO (VERTIKALNO)

$b = [0\ \ 0\ \ 0]^T$ # ONOLKO NULA KOLKO IMA PRIMJERA

A $A = -I$

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

d   d JE UVIJEK $[0]$

$d = [0]$

c   c JE $[-1 \dots -1]^T$, N ELEMENATA

$c = [-1\ \ -1\ \ -1]^T$

Q) OVDJE JEDINO IMA POSLA

$$Q = d_i d_j \vec{x}_i^T \vec{x}_j \quad \text{\# MATRICA } N \times N$$

$$Q_{11} = 1 \cdot 1 \cdot [0 \ 0] \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

NPR: $$Q_{22} = 1 \cdot 1 \cdot [-1 \ 1] \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2$$

$$Q_{12} = 1 \cdot 1 \cdot [0 \ 0] \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 0$$

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

SADA MATLAB NA TEMELJU TIH ULAZA U quadprog() FUNKCIJU IZRAČUNA $\vec{\lambda} = [0.5 \ \ 0 \ \ 0.5]^T$

- IZRAČUNA SE $\vec{w}$:

$$\vec{w} = \sum_{i=1}^{3} \lambda_i d_i \vec{x}_i = 0.5 \cdot 1 \cdot [0 \ 0]^T + 0 \cdot 1 \cdot [-1 \ 1]^T + 0.5 \cdot (-1) \cdot [2 \ 0]^T = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

- RAČUNAMO b (ILI w_3 TJ. ODMAK HIPERRAVNINE):
  - UZME SE JEDAN PRIMJER I KLASIFIKACIJOM NAMJESTI b

$$w^T \vec{x}_i + b = \pm 1$$

NPR $$[-1 \ 0] \begin{bmatrix} 0 \\ 0 \end{bmatrix} + b = \pm 1 \Rightarrow b = 1 \quad \text{JER JE } d_1 = 1$$

ILI $$[-1 \ 0] \begin{bmatrix} 2 \\ 0 \end{bmatrix} + b = \pm 1 \Rightarrow -2 + b = \underset{d_3 = -1}{-1} \Rightarrow b = 1$$

$$\tilde{\vec{w}} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow -x_1 + 1 = 0$$

(4 boda) Za općeniti problem kvadratnog programiranja:

$$\min_{x} \frac{1}{2}\vec{x}^T Q\vec{x} + \vec{c}^T\vec{x}$$

uz uvjete

$$A\vec{x} \leq \vec{b}$$

$$E\vec{x} = \vec{d}$$

naći matrice Q, A i E, te vektora c, b i d tako da rješenje gornjeg problema daje rješenje dualnog problema SVM za skup uzoraka

$\omega_1 = \{[1,0]^T, [0,2]^T\}$

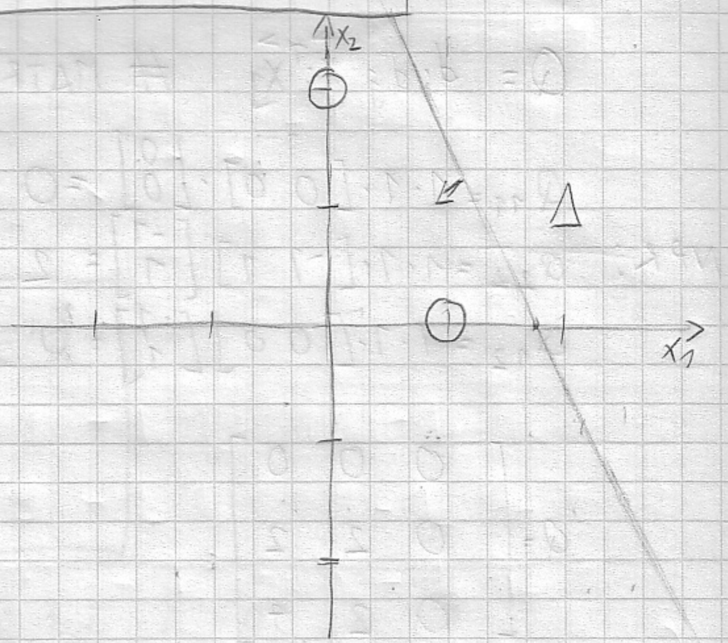
$\omega_2 = \{[2,1]^T, [2,3]^T\}$

Pretpostavite da tražimo linearnu decizijsku funkciju. Ako smo kao rješenje problema dobili vektor $\left[\frac{8}{9}, \frac{2}{9}, \frac{10}{9}, 0\right]^T$ (*moguće je da brojevi nisu dobro prepisani*) , napišite jednadžbe granice između razreda.

SVM DUALNI PROBLEM – 2. MI 2009/2010

a) $\omega_1 = \{[1\ 0]^T, [0\ 2]^T\}$ ○

$\omega_2 = \{[2\ 1]^T, [2\ 3]^T\}$ △



$E = [1\ 1\ -1\ -1]$

$b = [0\ 0\ 0\ 0]^T$

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$d = [0]$

$c = [-1\ -1\ -1\ -1]^T$

$$Q = \begin{bmatrix} 1 & 0 & -2 & -2 \\ 0 & 4 & -2 & -6 \\ -2 & -2 & 5 & 7 \\ -2 & -6 & 7 & 13 \end{bmatrix}$$

b) $\lambda = \left[\frac{8}{9}\ \frac{2}{9}\ \frac{10}{9}\ 0\right]^T$

$$\vec{\omega} = \frac{8}{9} \cdot 1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{2}{9} \cdot 1 \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix} - \frac{10}{9} \cdot 1 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8/9 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4/9 \end{bmatrix} - \begin{bmatrix} 20/9 \\ 10/9 \end{bmatrix}$$

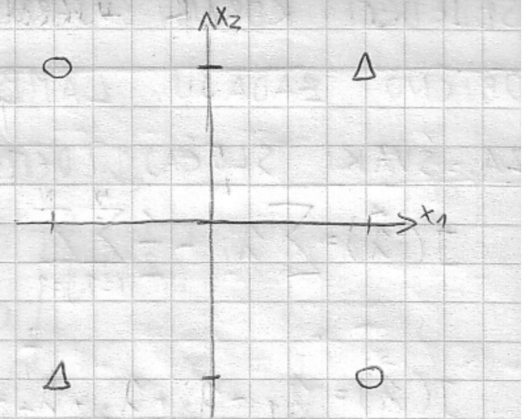
$$= \begin{bmatrix} -\frac{12}{9} \\ -\frac{6}{9} \end{bmatrix} = \begin{bmatrix} -\frac{4}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$$\vec{w} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b = 1$$

$$-\frac{4}{3} + b = 1 \Rightarrow b = \frac{7}{3}$$

$$w = \begin{bmatrix} -\frac{4}{3} \\ -\frac{2}{3} \\ \frac{7}{3} \end{bmatrix} \Rightarrow -\frac{4}{3}x_1 - \frac{2}{3}x_2 + \frac{7}{3} = 0 \quad / \cdot 3 = \boxed{-4x_1 - 2x_2 + 7 = 0}$$

JEZGRENI SVM – XOR PRIMJER



$\omega_1 = \{ [-1 \ \ 1]^T, [1 \ \ -1]^T \}$ O

$\omega_2 = \{ [-1 \ \ -1]^T, [1 \ \ 1]^T \}$ Δ

JEZGRO: $K(\vec{x}, \vec{x}_i) = (1 + \vec{x}^T \vec{x}_i)^2$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \vec{x}_i = \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix}$$

$$K(\vec{x}, \vec{x}_i) = \left(1 + [x_1 \ \ x_2] \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix}\right)^2 = 1 + 2 [x_1 \ x_2] \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix} + \left([x_1 \ x_2] \begin{bmatrix} x \\ x_1 \end{bmatrix} \right.$$

$$= 1 + 2(x_1 x_{i1} + x_2 x_{i2}) + (x_1 x_{i1} + x_2 x_{i2})^2 =$$

$$1 + 2 x_1 x_{i1} + 2 x_2 x_{i2} + x_1^2 x_{i1}^2 + 2 x_1 x_{i1} x_2 x_{i2} + x_2^2 x_{i2}^2 =$$

SLIKA ULAZNOG PROSTORA:

$$\varphi(\vec{x}) = [1, x_1^2, \sqrt{2}\, x_1 x_2, x_2^2, \sqrt{2} x_1, \sqrt{2} x_2]$$

$$\varphi(\vec{x}_i) = [1, x_{i2}^2, \sqrt{2}\, x_{i1} x_{i2}, x_{i2}^2, \sqrt{2} x_{i1}, \sqrt{2}\, x_{i2}]$$

OTKUD TI ELEMENTI? $\Rightarrow \sqrt{2} x_1 \cdot \sqrt{2} x_{i1} = 2 x_1 x_{i1}$

KONSTRUIRA SE **K** MATRICA ŠTO JE ISTO KAO I Q U DUALNOM SVM U, SAMO SU PRIMJERI PROVUČENI KROZ JEZ

$$\varphi(x_1) = \varphi([-1 \ \ 1]^T) = [1 \ \ 1 \ \ -\sqrt{2} \ \ 1 \ \ -\sqrt{2} \ \ \sqrt{2}]$$

$$\varphi(x_2) = \varphi([1 \ \ -1]^T) = [1 \ \ 1 \ \ -\sqrt{2} \ \ 1 \ \ \sqrt{2} \ \ -\sqrt{2}]$$

$$\varphi(x_3) = \varphi([-1 \ \ -1]^T) = [1 \ \ 1 \ \ \sqrt{2} \ \ 1 \ \ -\sqrt{2} \ \ -\sqrt{2}]$$

$$\varphi(x_4) = \varphi([1 \ \ 1]^T) = [1 \ \ 1 \ \ \sqrt{2} \ \ 1 \ \ \sqrt{2} \ \ \sqrt{2}]$$

NPR:

$$K_{12} = \varphi(x_1)\varphi(x_2) = 1 + 1 + 2 + 1 - 2 - 2 = 1$$

$$K_{11} = \varphi(x_1)\varphi(x_2) = 1 + 1 + 2 + 1 + 2 + 2 = 9$$

$$K = \begin{bmatrix} 9 & 1 & 1 & 1 \\ 1 & 9 & 1 & 1 \\ 1 & 1 & 9 & 1 \\ 1 & 1 & 1 & 9 \end{bmatrix}$$

SLJEDEĆI KORAK VJEROJATNO NIJE POTREBAN JER SE NA ISPITU OBIČNO ZADAJU LAMBDE IZRAČUNATE PREKO quadprog() ALI ZA SVAKI SLUČAJ, IDEMO RUČNO NAĆI LAMBDE.

$$J(\vec{\lambda}) = \sum \lambda_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_i \lambda_j d_i d_j \cdot K(\vec{x}_i, \vec{x}_j)$$

$$J(\vec{\lambda}) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 -$$
$$\frac{1}{2} \cdot \big(\lambda_1^2 \cdot 1 \cdot 1 \cdot 9 + \lambda_1 \lambda_2 \cdot 1 \cdot 1 \cdot 1 - \lambda_1 \lambda_3 \cdot 1 \cdot 1 \cdot 1 - \lambda_1 \cdot \lambda_4 \cdot 1 \cdot 1 \cdot 1$$
$$+ \lambda_1 \lambda_2 \cdot 1 \cdot 1 \cdot 1 + \lambda_2^2 \cdot 1 \cdot 1 \cdot 9 - \lambda_2 \lambda_3 \cdot 1 \cdot 1 \cdot 1 - \lambda_2 \lambda_4 \cdot 1 \cdot 1 \cdot 1$$
$$- \lambda_1 \lambda_3 \cdot 1 \cdot 1 \cdot 1 - \lambda_2 \lambda_3 \cdot 1 \cdot 1 \cdot 1 + \lambda_3^2 \cdot 1 \cdot 1 \cdot 9 + \lambda_3 \lambda_4 \cdot 1 \cdot 1 \cdot 1$$
$$- \lambda_1 \lambda_4 \cdot 1 \cdot 1 \cdot 1 - \lambda_2 \lambda_4 \cdot 1 \cdot 1 \cdot 1 + \lambda_3 \lambda_4 \cdot 1 \cdot 1 \cdot 1 + \lambda_4^2 \cdot 1 \cdot 1 \cdot 9 \big)$$

$$= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 -$$
$$- \frac{1}{2} \big(9\lambda_1^2 + 9\lambda_2^2 + 9\lambda_3^2 + 9\lambda_4^2 + 2\lambda_1 \lambda_2 - 2\lambda_1 \lambda_3 - 2\lambda_1 \lambda_4$$
$$- 2\lambda_2 \lambda_3 - 2\lambda_2 \lambda_4 + 2\lambda_3 \lambda_4 \big)$$

PARCIJALNO DERIVIRAJ PO SVIM LAMBDAMA I IZJEDNAČI S NULOM

$$\frac{\partial J}{\partial \lambda_1} = 1 - 9\lambda_1 - \lambda_2 + \lambda_3 + \lambda_4 = 0$$

$$\frac{\partial J}{\partial \lambda_2} = 1 - 9\lambda_2 - \lambda_1 + \lambda_3 + \lambda_4 = 0$$

$$\frac{\partial J}{\partial \lambda_3} = 1 - 9\lambda_3 + \lambda_1 + \lambda_2 - \lambda_4 = 0$$

$$\frac{\partial J}{\partial \lambda_4} = 1 - 9\lambda_4 + \lambda_1 + \lambda_2 - \lambda_3 = 0$$

– RJEŠAVANJEM GORNJEG SUSTAVA JEDNADŽBI DOBIVA SE:

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \frac{1}{8}$$

UVRSTIMO LAMBDE U FORMULU

$$\vec{w}_0 = \sum_{i=1}^{N} \lambda_i d_i \varphi(\vec{x}_i) + b$$

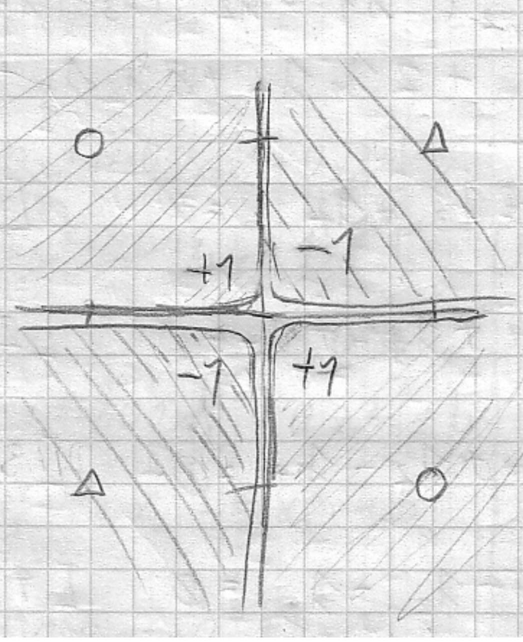
$$w = \frac{1}{8} \cdot \left(\begin{bmatrix} 1 \\ 1 \\ -\sqrt{2} \\ 1 \\ -\sqrt{2} \\ \sqrt{2} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -\sqrt{2} \\ 1 \\ \sqrt{2} \\ -\sqrt{2} \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ \sqrt{2} \\ 1 \\ -\sqrt{2} \\ -\sqrt{2} \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ \sqrt{2} \\ 1 \\ \sqrt{2} \\ \sqrt{2} \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \\ \frac{-4\sqrt{2}}{8} \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftarrow b$$

$$-\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$$

IZRAČUNAMO DECIZIJSKU FUNKCIJU

$$\vec{w}^T \vec{\varphi}(\vec{x}) = \emptyset$$

$$\begin{bmatrix} 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ x_1^2 \\ \sqrt{2} x_1 x_2 \\ x_2^2 \\ \sqrt{2} x_1 \\ \sqrt{2} x_2 \end{bmatrix} = \emptyset \Rightarrow \boxed{-x_1 x_2 = 0}$$



JEZGRENI SVM - AUDITORNE 2009/2010

$$\omega_1 = \{ [0 \ 0]^T, [1 \ 1]^T \}$$
$$\omega_2 = \{ [0 \ 1]^T, [1 \ 0]^T \}$$

$$K(\vec{x}, \vec{x}_i) = (\vec{x}\vec{x}_i + 1)^2$$

$$K = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 9 & 4 & 4 \\ 1 & 4 & 4 & 1 \\ 1 & 4 & 1 & 4 \end{bmatrix}$$

- U OVOM ZADATKU SU ZADANE LAMBDE $\lambda = [\frac{10}{3} \ 2 \ \frac{8}{3} \ \frac{8}{3}]$ ALI ĆU RJEŠIT KLASIČNIM POSTUPKOM RADI VJEŽBE

$$\begin{aligned} J(\vec{\lambda}) = & \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\ & - \frac{1}{2}(\lambda_1^2 + \lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_1\lambda_4 \\ & + \lambda_1\lambda_2 + 9\lambda_2^2 - 4\lambda_2\lambda_3 - 4\lambda_2\lambda_4 \\ & - \lambda_1\lambda_3 - 4\lambda_2\lambda_3 + 4\lambda_3^2 + \lambda_3\lambda_4 \\ & - \lambda_1\lambda_4 - 4\lambda_2\lambda_4 + \lambda_3\lambda_4 + \lambda_4^2) \end{aligned}$$

$$\begin{aligned} = & \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \frac{1}{2}(-\lambda_1^2 - 9\lambda_2^2 - 4\lambda_3^2 - 4\lambda_4^2 - 2\lambda_1\lambda_2 + 2\lambda_1\lambda_3 \\ & + 2\lambda_1\lambda_4 + 8\lambda_2\lambda_3 + 8\lambda_2\lambda_4 - 2\lambda_3\lambda_4) \end{aligned}$$

$$\frac{\partial J}{\partial \lambda_1} = 1 - \lambda_1 - \lambda_2 + \lambda_3 + \lambda_4 = \emptyset \quad [1]$$

$$\frac{\partial J}{\partial \lambda_2} = 1 - 9\lambda_2 - \lambda_1 + 4\lambda_3 + 4\lambda_4 = \emptyset \quad [2]$$

$$\frac{\partial J}{\partial \lambda_3} = 1 - 4\lambda_3 + \lambda_1 + 4\lambda_2 - \lambda_4 = \emptyset \quad [3]$$

$$\frac{\partial J}{\partial \lambda_4} = 1 - 4\lambda_4 + \lambda_1 + 4\lambda_2 - \lambda_3 = \emptyset \quad [4]$$

[3]−[4]: $4\lambda_4 - 4\lambda_3 + \lambda_3 - \lambda_4 = 0 \Rightarrow \boxed{\lambda_3 = \lambda_4}$ [5]

[1]+[3]: $2 - \cancel{\lambda_1} - 4\lambda_3 - \lambda_2 + \cancel{\lambda_1} + \lambda_3 + 4\lambda_2 + \cancel{\lambda_4} - \cancel{\lambda_4} = 0$

$$2 - 3\lambda_3 + 3\lambda_2 = 0 \Rightarrow \boxed{\lambda_2 = \frac{3\lambda_3 - 2}{3}} \quad [6]$$

[1]−[2]: $9\lambda_2 - \cancel{\lambda_1} + \cancel{\lambda_1} - \lambda_2 - 4\lambda_3 + \lambda_3 - 4\lambda_4 + \lambda_4 = 0$

$$8\lambda_2 - 3\lambda_3 - 3\lambda_4 = 0 \Leftarrow [6]$$

$$8\lambda_2 - 6\lambda_3 = 0$$

$$\boxed{\lambda_2 = \frac{6}{8}\lambda_3}$$

$$\frac{6}{8}\lambda_3 = \frac{3\lambda_3 - 2}{3} \;/\cdot 3 \Rightarrow \frac{9}{4}\lambda_3 = 3\lambda_3 - 2 \Rightarrow -\frac{3}{4}\lambda_3 = 2$$

$$\boxed{\lambda_4 = \lambda_3 = \frac{8}{3}} \quad [7]$$

$$\lambda_2 = \frac{6}{\cancel{8}}\cdot\frac{\cancel{8}}{\cancel{3}} = 2 \Rightarrow \boxed{\lambda_2 = 2} \quad [8] \Longrightarrow \lambda_1 + 2 - \frac{8}{3} - \frac{8}{3} = 0$$

$$\lambda_1 = \boxed{\frac{10}{3}}$$

$$1 - \lambda_1 = 2 + 2\cdot\frac{8}{3} = 0 \Rightarrow \boxed{\lambda_1 \neq \frac{13}{3}}$$

OVO BI VRIJEDILO KADA NE BI POSTO
UVIJET $\sum d_i \lambda_i = 0$, ZATO JE $\lambda_1 = 10/3$ JER
JE TO NAJMANJA VRIJEDNOST FUNKCIJE
KOJA MOŽE BITI UZ TAJ UVIJET

ECIZIJSKA FJA

$$d(\vec{x}) = \sum_{i=1}^{N} \lambda_i d_i K(\vec{x}_i, \vec{x}) + \vec{b}$$

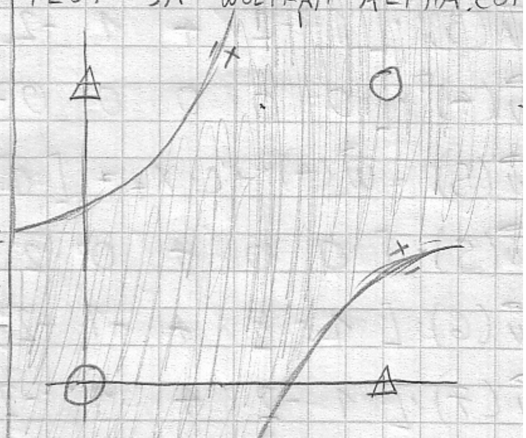
$$d(\vec{x}) = \frac{10}{3}\left[1 + 2\cdot 0x_1 + 2\cdot 0x_2 + 0x_1^2 + 2\cdot 0\cdot 0x_1x_2 + 0x_2^2\right]$$
$$+ 2\left[1 + 2\cdot 1x_1 + 2\cdot 1x_2 + 1x_1^2 + 2\cdot 1\cdot 1x_1x_2 + 1x_2^2\right]$$
$$- \frac{8}{3}\left[1 + 2\cdot 0x_1 + 2\cdot 1x_2 + 0x_1^2 + 2\cdot 0\cdot 1x_1x_2 + 1x_2^2\right]$$
$$- \frac{8}{3}\left[1 + 2\cdot 1x_1 + 2\cdot 0x_2 + 1x_1^2 + 2\cdot 0\cdot 1x_1x_2 + 0x_2^2\right] + b$$
$$= 0 - \frac{4}{3}x_1 - \frac{4}{3}x_2 - \frac{2}{3}x_1^2 + 4x_1x_2 - \frac{2}{3}x_2^2 + b$$

b

$$\frac{4}{3}\cdot 0 - \frac{4}{3}\cdot 0 - \frac{2}{3}\cdot 0^2 + 4\cdot 0\cdot 0 - \frac{2}{3}\cdot 0^2 + b = 1$$

$$\boxed{b = 1}$$

PLOT SA WOLFRAM ALPHA.COM



$$d(\vec{x}) = -\frac{2}{3}x_1^2 - \frac{2}{3}x_2^2 - \frac{4}{3}x_1 - \frac{4}{3}x_2 + 4x_1x_2 + 1$$

(6 bodova) Za skup uzoraka

$\omega_1 = \{[0, 0]^T\}$

$\omega_2 = \{[0, 1]^T, [1, 0]^T, [0, -1]^T\}$

Tražimo granicu između razreda strojem s potpornim vektorima i to u obliku polinoma drugog stupnja. Rješavanjem dualnog problema SVM dobili smo rješenje

$[\lambda_1 \ \ \lambda_2 \ \ \lambda_3 \ \ \lambda_4] = [8/3 \ \ 1 \ \ 2/3 \ \ 1]$

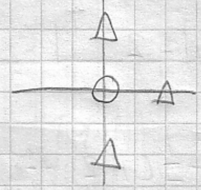
Kako glasi jednadžba granice između razreda u obliku

$ax_1^2 + bx_2^2 + cx_1x_2 + dx_1 + ex_2 + f = 0$

JEZGRENI SVM - ZAVRŠNI 2008/2009

$\omega_1 = \{[0 \ 0]^T\}$ O

$\omega_2 = \{[0 \ 1]^T, [1 \ 0]^T, [0 \ -1]^T\}$ Δ



$\lambda = [\frac{8}{3} \ 1 \ \frac{2}{3} \ 1]^T$

$\varphi(x) = [x_1^2 \ \ x_2^2 \ \ \sqrt{2}x_1x_2 \ \ \sqrt{2}x_1 \ \ \sqrt{2}x_2 \ \ 1]$

$\vec{\omega} = \sum \lambda_i d_i \varphi(x_i)$

$$\vec{\omega} = \frac{8}{3}\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix} - \begin{bmatrix}0\\1\\0\\0\\\sqrt{2}\\1\end{bmatrix} - \frac{2}{3}\begin{bmatrix}1\\0\\0\\\sqrt{2}\\0\\1\end{bmatrix} - \begin{bmatrix}0\\1\\0\\0\\-\sqrt{2}\\1\end{bmatrix} = \begin{bmatrix}-\frac{2}{3}\\-2\\0\\-\frac{2\sqrt{2}}{3}\\0\\0\end{bmatrix}$$

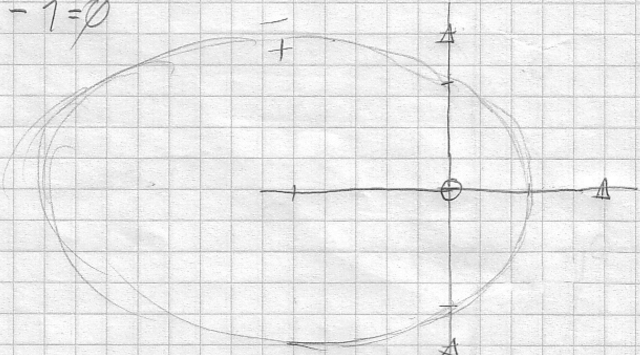
$\omega \ -\frac{2}{3}x_1^2 - 2x_2^2 - \frac{4}{3}x_1 + b = 0$

- UBACIMO [0 0] KOJEM JE UDALJENOST 1

$b = 1$

WOLFRAMALPHA.COM PLOT

$\frac{2}{3}x_1^2 + 2x_2^2 + \frac{4}{3}x_1 - 1 = 0$



(7 bodova) Zadani su dvodimenzionalni uzorci iz dvaju razreda za koje se pretpostavlja da slijede višedimenzionalnu normalnu razdiobu. Uzorci iz prvoga razreda su

$\omega_1 = \{[1, 3]^T, [2, 0]^T, [2, 6]^T, [3, 3]^T\}$

Uzorci iz ω_2 imaju središte u ishodištu i kovarijacijsku matricu $C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Pretpostavlja se da su vjerojatnosi pojavljivanja uzoraka iz oba razreda jednake. Napišite jednadžbu granice između razreda i to u obliku:

$a \cdot x_1^2 + b \cdot x_2^2 + c \cdot x_1 \cdot x_2 + d \cdot x_1 + e \cdot x_2 + f = 0$

JEZGRENI SVM - ZAVRŠNI 2010/2011

ZADATAK JE ISTI KAO I OD 2008/2009, UZ DODATAK IZRAČU

MATRICA:

$$\varphi(\vec{x}) = [x_1^2 \;\; x_2^2 \;\; x_1x_2 \;\; x_1 \;\; x_2 \;\; 1]^T$$

$$K(x, x_i) = \left(1 + [x_{11} \; x_{12}] \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}\right)^2$$

$$K = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 1 & 0 \\ 1 & 1 & 4 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix} \quad Q = d_i d_j K_{ij} = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 4 & 1 & 0 \\ -1 & 1 & 4 & 1 \\ -1 & 0 & 1 & 4 \end{bmatrix}$$

$$c = [-1 \;\; -1 \;\; -1 \;\; -1]^T$$

$\boxed{\text{UVJET} \quad A\vec{x} \leq b}$

$$A = -I = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$b = [0 \;\; 0 \;\; 0 \;\; 0]$$

$\boxed{\text{UVJET} \quad E\vec{x} = d}$

$$E = [1 \;\; -1 \;\; -1 \;\; -1]$$

$$d = [0]$$

U MATLABU PROVJERENE MATRICE, DOBIVAJU SE TOČNE LAMBDE! :)

Za općeniti problem kvadratnog programiranja:

$$\min_x \quad \frac{1}{2} x^T Q x + c^T x$$

uz uvjete

Ax $\leq b$

Ex=d

naći vrijednost matrica Q, A, E te vektora c, b i d tako da rješenje gornjeg problema daje rješenje dualnog problema SVM za skup uzoraka

$\omega_{1=\{[0,0]^T\}}$

$\omega_2 = \{[0,1]^T,\ [1,0]^T,\ [-1,-1]^T\}$

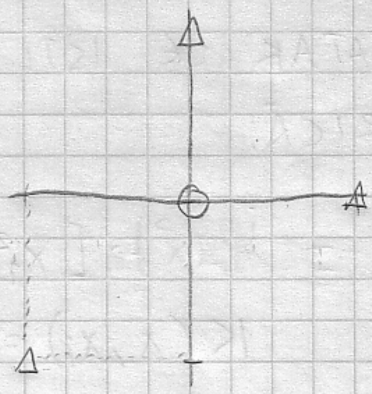
Pretpostavite da tražimo nelinearnu decizijsku funkciju pomoću jezgrene funkcije

K(x, x_i)= $e^{-\frac{1}{2\sigma^2}\|x-x_i\|^2}$ uz $\sigma = 1$.

JEZGRENI SVM - 2.MI 2008/2009

$\omega_1 = \{[0\ 0]^T\} = \circ$

$\omega_2 = \{[0\ 1]^T, [1\ 0]^T, [-1\ -1]^T\} = \Delta$



$$K(x, x_i) = e^{-\frac{1}{2}\|x - x_i\|^2}$$

$$K = \begin{bmatrix} 1 & 0.6 & 0.6 & 0.37 \\ 0.6 & 1 & 0.37 & 0.08 \\ 0.6 & 0.37 & 1 & 0.08 \\ 0.36 & 0.08 & 0.08 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & -0.6 & -0.6 & -0.37 \\ -0.6 & 1 & 0.37 & 0.08 \\ -0.6 & 0.37 & 1 & 0.08 \\ -0.37 & 0.08 & 0.08 & 1 \end{bmatrix}$$

OSTALE MATRICE ISTE KAO I U PROŠLOM ZADATKU
(ZAVRŠNI 2010/2011)

Za općeniti problem kvadratnog programiranja:

$$\min_{\vec{x}} \frac{1}{2} \vec{x}^T Q \vec{x} + \vec{c}^T \vec{x}$$

uz uvjete

$$A\vec{x} \leq \vec{b}$$

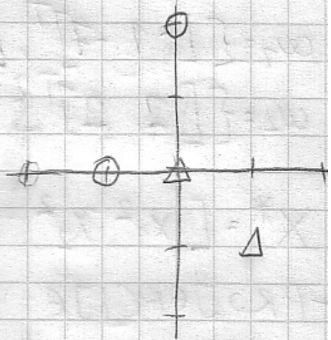
$$E\vec{x} = \vec{d}$$

naći vrijednosti matrica Q, A i E, te vektora c, b i d tako da rješenje gornjeg problema daje rješenje dualnog problema SVM za skup uzoraka $[0, 2]^T \in \omega_1$, $[-1, 0]^T \in \omega_1$, $[0, 0]^T \in \omega_2$, $[1, -1]^T \in \omega_2$. Pretpostavite da tražimo decizijsku funkciju u obliku polinoma trećeg stupnja.

JEZGRENI SVM - POLINOM 3. STUPNJA - ISPIT 29.06.2006.

$\omega_1 = \{[-0 \ 2], [-1 \ 0]\}$ o $\qquad K = (1 + x^T x_i)^3$

$\omega_2 = \{[0 \ 0], [1 \ -1]\}$ Δ



$$K = \begin{bmatrix} 125 & -1 & 1 & -1 \\ -1 & 8 & 1 & 0 \\ +1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 27 \end{bmatrix} \quad Q = \begin{bmatrix} 125 & -1 & -1 & +1 \\ 1 & 8 & -1 & 0 \\ -1 & -1 & 1 & 1 \\ +1 & 0 & 1 & 27 \end{bmatrix}$$

- OSTALE MATRICE ISTE KO U PROŠLOM ZADATKU OSIM $\epsilon = [1 \ 1 \ -1 \ -1]$

- MATLAB DAJE LAMBDE:

$$\lambda = [0.0161 \quad 0.2857 \quad 0.3018 \quad 0]^T$$

$$K(x, x_i) = x_1^3 x_{i1}^3 + 3x_1^2 x_{i1}^2 + 3x_1^2 x_2 x_{i1}^2 x_{i2} + 3x_2^2 x_1 x_{i2}^2 x_{i1} + 3x_1 x_{i1} + 6 x_1 x_2 x_{i1} x_{i2} + x_2^3 x_{i2}^3 + 3x_2^2 x_{i2}^2 + 3x_2 x_{i2} + 1$$

$$\varphi(\vec{x}) = [x_1^3 \ \sqrt{3}x_1^2 \ \sqrt{3}x_1^2 x_2 \ \sqrt{3}x_2^2 x_1 \ \sqrt{3}x_1 \ \sqrt{6}x_1 x_2 \ x_2^3 \ \sqrt{3}x_2^2 \ \sqrt{3}x_2 \ 1]^T$$

$$\vec{w} = \sum(\lambda_i d_i \varphi(x_i)) + b$$

$$= 0.0161 \cdot [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 8 \ 4\sqrt{3} \ 2\sqrt{3} \ 1]^T$$

$$+ 0.2857 \cdot [1 \ \sqrt{3} \ 0 \ 0 \ -\sqrt{3} \ 0 \ 0 \ 0 \ 0 \ 1]^T$$

$$- 0.3018 \cdot [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]^T$$

$$= [0.2857 \ 0.4948 \ 0 \ 0 \ -0.4948 \ 0 \ 0.1288 \ 0.1115 \ 0.0558 \ 0]^T$$

UBACIMO $[0 \ 0] \Rightarrow b = -1$

$$d(\vec{x}) = 0.2857x_1^3 + 0.857x_1^2 - 0.857x_1 + 0.1288x_2^3 + 0.1931x_2^2 + 0.0966x_2 - 1$$