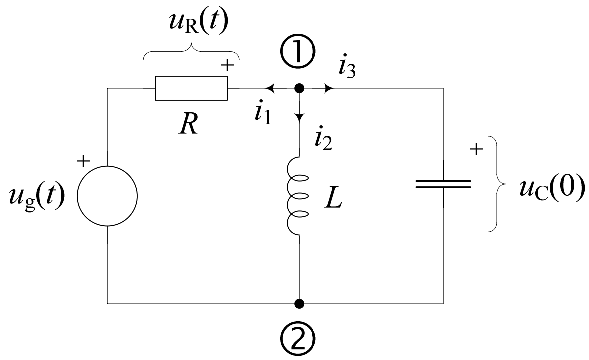
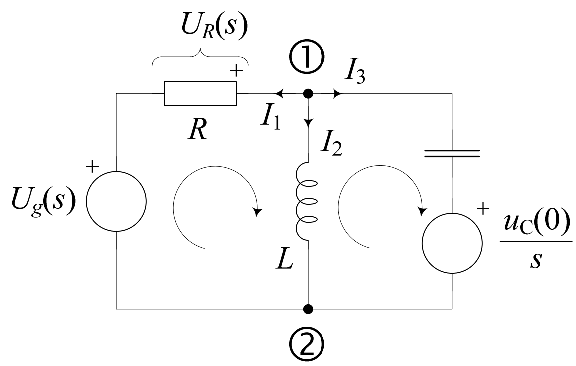
ELEKTRIČNI KRUGOVI SKRIPTA ISPITNIH ZADATAKA

KZN-KZS

3. Za električni krug na slici izračunati napon $U_R(s)$ ako su zadane normalizirane vrijednosti elemenata R =1, L = 1,C = 1, $u_C(0)$ = 2, te $u_g(t)$ = $S(t)$. Koristiti KZS i KZN te oznake grana i čvorova prema slici.



Rješenje: Primjena $\mathcal{L}$-transformacije



$N_b = 3$ (broj grana)

$N_v = 2$ (broj čvorova)

Broj jednadžbi $\text{KZS} = N_v - 1 = 2 - 1 = 1$

Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 3 - 2 + 1 = 2$

Jednadžbe Kirchhoffovih zakona (3 jednadžbe):

1) $I_1 + I_2 + I_3 = 0$ KZS

2) $-U_1 + U_2 = 0$ KZN

3) $-U_2 + U_3 = 0$ KZN

Naponsko – strujne jednadžbe grana (3 jednadžbe):

4) $U_1 = I_1 \cdot R + U_g$

5) $U_2 = I_2 \cdot sL$

6) $U_3 = I_3 \cdot \dfrac{1}{sC} + \dfrac{u_C(0)}{s}$

Sustav ima ukupno $2N_b$=6 jednadžbi i 6 nepoznanica (sve struje i svi naponi grana)

Naponsko – strujne jednadžbe grana uvrstimo u (1) jednadžbu:

$$\left.\begin{array}{l} 4)\ I_1 = \dfrac{U_1}{R} - \dfrac{U_g}{R} \\ 5)\ I_2 = \dfrac{1}{sL} \cdot U_2 \\ 6)\ I_3 = sC \cdot U_3 - C \cdot u_C(0) \end{array}\right\} \rightarrow (1)$$

$$1)'\ \frac{U_1}{R} - \frac{U_g}{R} + \frac{1}{sL} \cdot U_2 + sC \cdot U_3 - C \cdot u_C(0) = 0 \Rightarrow \frac{U_1}{R} + \frac{1}{sL} \cdot U_2 + sC \cdot U_3 = \frac{U_g}{R} + C \cdot u_C(0)$$

$$2)\ U_1 = U_2 \rightarrow (1)$$

$$3)\ U_2 = U_3 \rightarrow (1) \Rightarrow U_1\left(\frac{1}{R} + \frac{1}{sL} + sC\right) = \frac{U_g}{R} + C \cdot u_C(0)$$

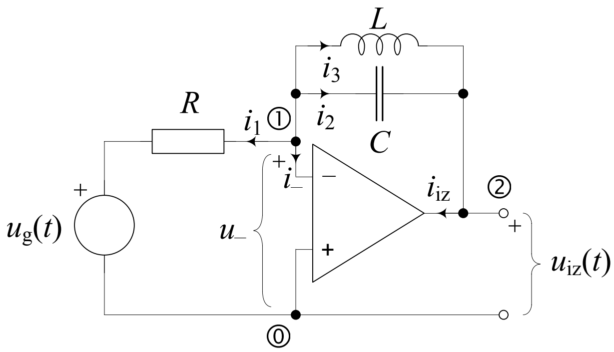
$$\Rightarrow U_1 = \frac{\dfrac{U_g}{R} + C \cdot u_C(0)}{\left(\dfrac{1}{R} + \dfrac{1}{sL} + sC\right)}$$

$$I_1 = \frac{U_1}{R} - \frac{U_g}{R} = \frac{\dfrac{U_g}{R} + C \cdot u_C(0)}{R\left(\dfrac{1}{R} + \dfrac{1}{sL} + sC\right)} - \frac{U_g}{R}$$

$$= \frac{\dfrac{1}{s} + 2}{1 + \dfrac{1}{s} + s} - \frac{1}{s} = \frac{1 + 2s}{1 + s + s^2} - \frac{1}{s} = \frac{s + 2s^2 - 1 - s - s^2}{s\left(1 + s + s^2\right)} = \frac{s^2 - 1}{s\left(1 + s + s^2\right)}$$

$$U_{R1}(s) = I_1 \cdot R = \frac{s^2 - 1}{s\left(s^2 + s + 1\right)}$$

3. Za električni krug na slici izračunati napon $u_{iz}(t)$ ako su zadane normalizirane vrijednosti elemenata $R=1$, $L=1$, $C=1$, uz početne uvjete jednake nuli te $u_g(t)=S(t)$. Koristiti KZS i KZN te oznake grana i čvorova prema slici. Napisati:
a) Broj neovisnih jednadžbi KZS i KZN (mreža ima 5 grana i 3 čvora);
b) Jednadžbe KZS;
c) Jednadžbe KZN;
d) Naponsko-strujne jednadžbe za grane;
e) Napon na izlazu $u_{iz}(t)$.



Rješenje:

$N_b=5$ (broj grana)
$N_v=3$ (broj čvorova)
Broj jednadžbi $\text{KZS} = N_v - 1 = 3 - 1 = 2$
Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 5 - 3 + 1 = 3$ (1 bod)

Jednadžbe Kirchhoffovih zakona (5 jednadžbi):
1) $I_1 + I_2 + I_3 + I_- = 0$ KZS
2) $I_{iz} - I_2 - I_3 = 0$ KZS(1 bod)
3) $U_1 - U_- = 0$ KZN
4) $-U_2 + U_3 = 0$ KZN
5) $-U_- + U_2 + U_{iz} = 0$ KZN(1 bod)

Naponsko–strujne jednadžbe grana (5 jednadžbi):
1) $U_1 = I_1 \cdot R + U_g$
2) $U_2 = I_2 \cdot \frac{1}{sC}$
3) $U_3 = I_3 \cdot sL$ (1 bod)
4) $U_- = 0$
5) $I_- = 0$

Sustav ima ukupno $2N_b=10$ jednadžbi i 10 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe (1)–(5):
1) $\frac{U_1}{R} - \frac{U_g}{R} + sCU_2 + \frac{1}{sL}U_3 = 0$
2) $I_{iz} = sCU_2 + \frac{1}{sL}U_3$

3) $U_1 = U_- = 0$

4) $U_2 = U_3$

5) $U_{iz} = -U_2$

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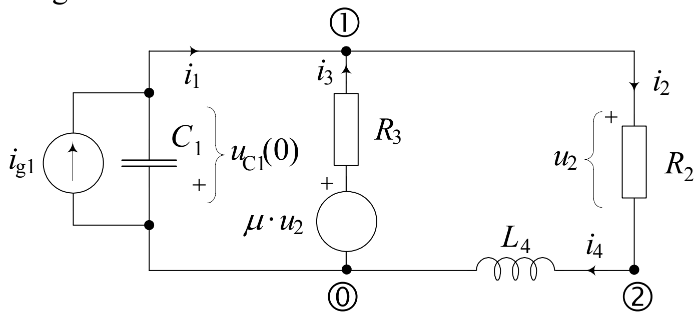
1) $\Rightarrow \left(sC + \frac{1}{sL}\right)U_2 = \frac{U_g}{R} \Rightarrow U_{iz} = -U_2 = -\frac{1}{R}\frac{U_g}{\left(sC + \frac{1}{sL}\right)}$

$$U_{iz}(s) = -\frac{\frac{1}{s}}{s + \frac{1}{s}} = -\frac{1}{s^2 + 1}$$

$u_{iz}(t) = -\sin(t) \cdot S(t)$ (1 bod)

MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug na slici i pridruženim orijentacijama grana te čvorovima zadane su normalizirane vrijednosti elemenata C_1=1, R_2=1, R_3=1, L_4=1 te μ=2, $u_{C1}(0)$=1, $i_{g1}(t)$=$S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati:
a) Jednadžbe KZS i KZN;
b) Naponsko-strujne jednadžbe za grane;
c) Napon na otporu R_2 $U_2(s)$;
d) Napon na otporu R_2 $u_2(t)$;
e) Da li je električni krug stabilan? Zašto?



Rješenje:

a) N_b=4 (broj grana)
N_v=3 (broj čvorova)
Broj jednadžbi $\text{KZS} = N_v - 1 = 3 - 1 = 2$
Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 4 - 3 + 1 = 2$
Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):
1) $-I_1 + I_2 - I_3 = 0$ KZS
2) $-I_2 + I_4 = 0$ KZS
3) $U_1 - U_3 = 0$ KZN
4) $U_2 + U_3 + U_4 = 0$ KZN (1 bod)

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):
1) $U_1 = \dfrac{1}{sC_1} \cdot I_1 - \dfrac{1}{sC_1} \cdot I_{g1} + \dfrac{u_{C1}(0)}{s}$
2) $U_2 = R_2 \cdot I_2$
3) $U_3 = R_3 \cdot I_3 - \mu U_2 = R_3 \cdot I_3 - \mu R_2 I_2$
4) $U_4 = sL_4 \cdot I_4$ (1 bod)

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c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:
1) $I_1 = I_2 - I_3$
2) $I_2 = I_4$
3) $\dfrac{1}{sC_1} \cdot I_1 - \dfrac{1}{sC_1} \cdot I_{g1} + \dfrac{u_{C1}(0)}{s} - R_3 \cdot I_3 + \mu R_2 I_2 = 0$
4) $R_2 \cdot I_2 + R_3 \cdot I_3 - \mu R_2 I_2 + sL_4 \cdot I_4 = 0$

$$1) \rightarrow 3) \Rightarrow \frac{1}{sC_1}\cdot(I_2-I_3)-\frac{1}{sC_1}\cdot I_{g1}+\frac{u_{C1}(0)}{s}-R_3\cdot I_3+\mu R_2 I_2=0$$

$$\left(\frac{1}{sC_1}+\mu R_2\right)\cdot I_2-\left(\frac{1}{sC_1}+R_3\right)\cdot I_3-\frac{1}{sC_1}\cdot I_{g1}+\frac{u_{C1}(0)}{s}=0$$

$$2) \rightarrow 4) \Rightarrow R_2\cdot I_2+R_3\cdot I_3-\mu R_2 I_2+sL_4\cdot I_2=0 \Rightarrow I_3=-\frac{1}{R_3}\left[(1-\mu)R_2+sL_4\right]\cdot I_2$$

$$\Rightarrow\left(\frac{1}{sC_1}+\mu R_2\right)\cdot I_2+\left(\frac{1}{sC_1}+R_3\right)\cdot\frac{1}{R_3}\left[(1-\mu)R_2+sL_4\right]\cdot I_2=\frac{1}{sC_1}\cdot I_{g1}-\frac{u_{C1}(0)}{s}\Big/\cdot sC_1$$

$$\left(1+\mu R_2 sC_1\right)\cdot I_2+\left(\frac{1}{R_3}+sC_1\right)\cdot\left[(1-\mu)R_2+sL_4\right]\cdot I_2=I_{g1}-C_1u_{C1}(0)$$

$$I_2=\frac{I_{g1}-C_1u_{C1}(0)}{1+\mu R_2sC_1+\left(\frac{1}{R_3}+sC_1\right)\cdot\left[(1-\mu)R_2+sL_4\right]};\ U_2=R_2I_2=\frac{R_2\cdot(I_{g1}-C_1u_{C1}(0))}{1+\mu R_2sC_1+\left(\frac{1}{R_3}+sC_1\right)\cdot\left[(1-\mu)R_2+sL_4\right]}$$

$$=\frac{\frac{1}{s}-1}{1+2s+(1+s)\cdot(-1+s)}=\frac{\frac{1}{s}-1}{1+2s+s^2-1}=\frac{\frac{1}{s}-1}{s(s+2)}=\frac{1-s}{s^2(s+2)}\quad \text{(1 bod)}$$

d) Odziv u vremenskoj domeni (rastav na parcijalne razlomke)

$$U_2(s)=\frac{1-s}{s^2(s+2)}=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+2}$$

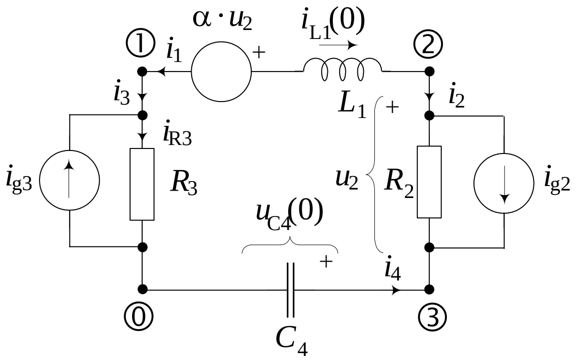
$$A(s+2)+Bs(s+2)+Cs^2=1-s$$

$$(B+C)s^2+(A+2B)s+2A=1-s \Rightarrow A=\frac{1}{2},B=-\frac{1+A}{2}=-\frac{3}{4},C=-B=\frac{3}{4}$$

$$U_2(s)=\frac{1}{4}\left(\frac{2}{s^2}-\frac{3}{s}+\frac{3}{s+2}\right) \Rightarrow \underline{u_2(t)=\frac{1}{4}\left(2t-3+3e^{-2t}\right)\cdot S(t)}\quad \text{(1 bod)}$$

e) Stabilnost:
NE, jer ima dvostruki pol u ishodištu
(odziv na konačnu pobudu teži u ∞ kad $t\rightarrow\infty$.) (1 bod)

4. Za električni krug na slici i pridruženim orijentacijama grana zadane su normalizirane vrijednosti elemenata L_1=1, R_2=1, R_3=2, C_4=1/2, te α=2, $u_{C4}(0)$=2, $i_{L1}(0)$=1, $i_{g2}(t)$=$i_{g3}(t)$=$S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati:
a) Jednadžbe KZS i KZN;
b) Naponsko-strujne jednadžbe za grane;
c) Napon na otporu R_2: $U_2(s)$ i $u_2(t)$;
d) Struju kroz otpor R_3: $I_{R3}(s)$ i $i_{R3}(t)$;



Rješenje:

a) N_b=4 (broj grana)
N_v=4 (broj čvorova)
Broj jednadžbi $\text{KZS} = N_v - 1 = 4 - 1 = 3$
Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 4 - 4 + 1 = 1$
Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):
1) $-I_1 + I_3 = 0$ KZS
2) $I_1 + I_2 = 0$ KZS
3) $-I_2 - I_4 = 0$ KZS
4) $U_1 - U_2 + U_3 + U_4 = 0$ KZN (1 bod)

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):
1) $U_1 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \alpha U_2 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \alpha(I_2 - I_{g2})R_2$
2) $U_2 = (I_2 - I_{g2}) \cdot R_2$
3) $U_3 = (I_3 + I_{g3}) \cdot R_3$
4) $U_4 = \frac{1}{sC_4} \cdot I_4 - \frac{u_{C4}(0)}{s}$ (1 bod)

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c) Sustav ima ukupno 2N_b=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:

1) , 2), 3) $I_1 = -I_2 = I_3 = I_4$

4) $sL_1 I_1 + L_1 i_{L1}(0) + \alpha(I_2 - I_{g2})R_2 - (I_2 - I_{g2})R_2 + (I_3 + I_{g3})R_3 + \frac{1}{sC_4} I_4 - \frac{u_{C4}(0)}{s} = 0$

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1)→4) $\Rightarrow$ $sL_1 I_1 + L_1 i_{L1}(0) - \alpha(I_1 + I_{g2})R_2 + (I_1 + I_{g2})R_2 + (I_1 + I_{g3})R_3 + \frac{1}{sC_4} I_1 - \frac{u_{C4}(0)}{s} = 0$

$$\Rightarrow I_1\left(sL_1 + (1-\alpha)R_2 + R_3 + \frac{1}{sC_4}\right) + L_1 i_{L1}(0) + (1-\alpha)I_{g2}R_2 + I_{g3}R_3 - \frac{u_{C4}(0)}{s} = 0$$

$$\Rightarrow\ I_1(s)=\frac{-L_1 i_{L1}(0)-(1-\alpha)I_{g2}R_2-I_{g3}R_3+\dfrac{u_{C4}(0)}{s}}{sL_1+(1-\alpha)R_2+R_3+\dfrac{1}{sC_4}}$$

$$I_1(s)=\frac{-1+\dfrac{1}{s}-\dfrac{2}{s}+\dfrac{2}{s}}{s-1+2+\dfrac{2}{s}}=\frac{-1+\dfrac{1}{s}}{s+1+\dfrac{2}{s}}=\frac{-s+1}{s^2+s+2}$$ (1 bod)

$$\Rightarrow\ I_2(s)=-I_1(s)=\frac{s-1}{s^2+s+2}\ \Rightarrow$$

$$U_2(s)=\left[I_2(s)-I_{g2}(s)\right]R_2=\frac{s-1}{s^2+s+2}-\frac{1}{s}=\frac{s-1}{s^2+s+\frac{1}{4}+\frac{7}{4}}-\frac{1}{s}=\frac{s+\frac{1}{2}-\frac{3}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{7}{4}}-\frac{1}{s}$$

$$U_2(s)=\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}-\frac{3}{\sqrt{7}}\frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}-\frac{1}{s}$$

$$u_2(t)=e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{7}}{2}t-\frac{3}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right)S(t)-S(t)$$ (1 bod)

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d) Struja kroz R_3

$$\Rightarrow\ I_3(s)=I_1(s)=\frac{-s+1}{s^2+s+2}\ \Rightarrow$$

$$I_{R3}(s)=I_3(s)+I_{g3}(s)=-\frac{s-1}{s^2+s+2}+\frac{1}{s}=-\left(\frac{s-1}{s^2+s+2}-\frac{1}{s}\right)$$

[= minus izraz za $U_2(s)$ gore]

$$i_{R3}(t)=-e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{7}}{2}t-\frac{3}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right)S(t)+S(t)$$ (1 bod)

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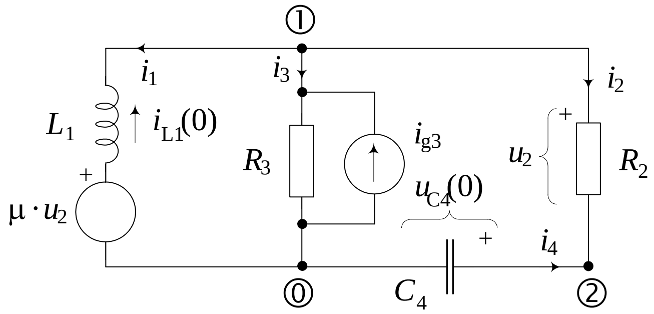
2. Za električni krug na slici i pridruženim orijentacijama grana zadane su normalizirane vrijednosti elemenata L_1=1, R_2=1, R_3=1, C_4=1, te μ=2, $u_{C4}(0)$=1, $i_{L1}(0)$=1, $i_{g3}(t)=S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati: a) Jednadžbe KZS i KZN (odabrati referentne smjerove petlji u smjeru kazaljke na satu); b) Naponsko-strujne jednadžbe za grane; c) Napon na otporu R_2 $U_2(s)$; d) Napon na otporu R_2 $u_2(t)$; e) Da li je električni krug stabilan? Zašto?



Rješenje: Laplaceova transformacija

a)

N_b=4 (broj grana)

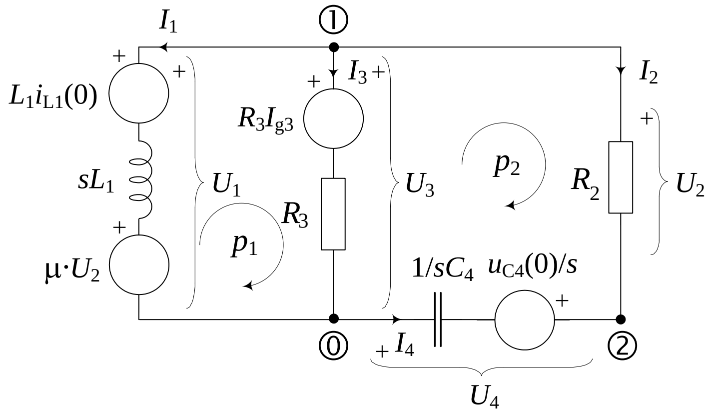
N_v=3 (broj čvorova)

Broj jednadžbi

$\text{KZS} = N_v - 1 = 3 - 1 = 2$

Broj jednadžbi

$\text{KZN} = N_b - N_v + 1 = 4 - 3 + 1 = 2$



Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):

1) $I_1 + I_2 + I_3 = 0$ KZS čvorište (1)

2) $-I_2 - I_4 = 0$ KZS čvorište (2)

3) $-U_1 + U_3 = 0$ KZN petlja p_1

4) $U_2 - U_3 - U_4 = 0$ KZN petlja p_2 **(1 bod)**

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):

1) $U_1 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \mu U_2 = sL_1 \cdot I_1 + \mu R_2 \cdot I_2 + L_1 i_{L1}(0)$

2) $U_2 = R_2 \cdot I_2$

3) $U_3 = R_3 \cdot I_3 + I_{g3} R_3$

4) $U_4 = \dfrac{1}{sC_4} \cdot I_4 - \dfrac{u_{C4}(0)}{s}$ **(1 bod)**

c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana) Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:

1) $I_1 = -I_2 - I_3$

2) $I_2 = -I_4$

3) $-sL_1 \cdot I_1 - \mu R_2 \cdot I_2 - L_1 i_{L1}(0) + R_3 \cdot I_3 + I_{g3} R_3 = 0$

4) $R_2 \cdot I_2 - R_3 \cdot I_3 - I_{g3} R_3 - \dfrac{1}{sC_4} \cdot I_4 + \dfrac{u_{C4}(0)}{s} = 0$

$$1) \to 3) \Rightarrow sL_1 \cdot (I_2 + I_3) - \mu R_2 \cdot I_2 - L_1 i_{L1}(0) + R_3 \cdot I_3 + I_{g3} R_3 = 0$$

$$2) \to 4) \Rightarrow R_2 \cdot I_2 - R_3 \cdot I_3 - I_{g3} R_3 + \frac{1}{sC_4} \cdot I_2 + \frac{u_{C4}(0)}{s} = 0 \Rightarrow$$

$$(1')\ I_2(sL_1 - \mu R_2) + I_3(sL_1 + R_3) = L_1 i_{L1}(0) - I_{g3} R_3$$

$$(2')\ \left(R_2 + \frac{1}{sC_4}\right) \cdot I_2 - R_3 \cdot I_3 = I_{g3} R_3 - \frac{u_{C4}(0)}{s}$$

$\Rightarrow I_2(s),\ I_3(s)$ koristimo metodu determinanti:

$$\begin{bmatrix} sL_1 - \mu R_2 & sL_1 + R_3 \\ R_2 + \frac{1}{sC_4} & -R_3 \end{bmatrix} \cdot \begin{bmatrix} I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} L_1 i_{L1}(0) - I_{g3} R_3 \\ I_{g3} R_3 - \frac{u_{C4}(0)}{s} \end{bmatrix}$$

$$\Delta = \begin{vmatrix} sL_1 - \mu R_2 & sL_1 + R_3 \\ R_2 + \frac{1}{sC_4} & -R_3 \end{vmatrix} = -R_3(sL_1 - \mu R_2) - (sL_1 + R_3)\left(R_2 + \frac{1}{sC_4}\right)$$

$$\Delta = -R_3 sL_1 + \mu R_2 R_3 - sL_1 R_2 - R_3 R_2 - \frac{L_1}{C_4} - R_3 \frac{1}{sC_4} = -s + 2 - s - 1 - 1 - \frac{1}{s} = -2s - \frac{1}{s}$$

$$\Delta_2 = \begin{vmatrix} L_1 i_{L1}(0) - I_{g3} R_3 & sL_1 + R_3 \\ I_{g3} R_3 - \frac{u_{C4}(0)}{s} & -R_3 \end{vmatrix} = -R_3\left(L_1 i_{L1}(0) - I_{g3} R_3\right) - (sL_1 + R_3)\left(I_{g3} R_3 - \frac{u_{C4}(0)}{s}\right)$$

$$\Delta_2 = -R_3 L_1 i_{L1}(0) + R_3^2 I_{g3} - sL_1 I_{g3} R_3 - R_3^2 I_{g3} + sL_1 \frac{u_{C4}(0)}{s} + R_3 \frac{u_{C4}(0)}{s} =$$

$$= -R_3 L_1 i_{L1}(0) - sL_1 I_{g3} R_3 + L_1 u_{C4}(0) + R_3 \frac{u_{C4}(0)}{s} = -1 - s \cdot \frac{1}{s} + 1 + \frac{1}{s} = -1 + \frac{1}{s}$$

$$I_2(s) = \frac{\Delta_2}{\Delta} = \frac{-1 + \frac{1}{s}}{-2s - \frac{1}{s}} = \frac{1 - \frac{1}{s}}{2s + \frac{1}{s}} = \frac{s-1}{2s^2+1};\quad R_2 = 1$$

$$U_2(s) = I_2(s) R_2 = \frac{s-1}{2s^2+1}$$ **(1 bod)**

d) Napon na otporu R_2 $u_2(t)$:

$$U_2(s) = \frac{1}{2} \cdot \frac{s-1}{s^2 + \frac{1}{2}} = \frac{1}{2} \cdot \frac{s}{s^2 + \frac{1}{2}} - \frac{1}{2} \cdot \frac{1}{s^2 + \frac{1}{2}} = \frac{1}{2} \cdot \frac{s}{s^2 + \left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{2}} \cdot \frac{\frac{1}{\sqrt{2}}}{s^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$u_2(t) = \left[\frac{1}{2} \cdot \cos\left(\frac{t}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}} \cdot \sin\left(\frac{t}{\sqrt{2}}\right)\right] S(t)$$ **(1 bod)**

e) Stabilnost:

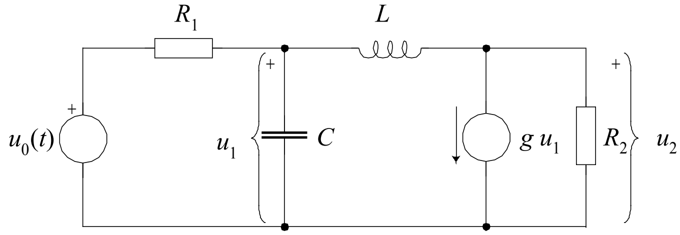
Električni krug je marginalno stabilan (na rubu stabilnosti).

Polovi $s^2 + \frac{1}{2} = 0 \Rightarrow s_{p1,2} = \pm j\frac{\sqrt{2}}{2}$ su jednostruki i nalaze se na imaginarnoj osi.

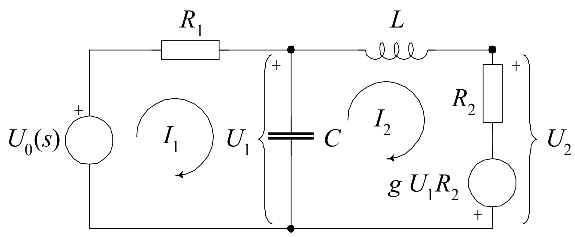
(1 bod)

MATRICE

3. Za krug prikazan slikom napisati jednadžbe petlji. Konačni oblik jednadžbi prikazati u formi matrične jednadžbe. Izračunati napon $U_2(s)$, ako je zadana pobuda $u_0(t)= 3S(t)$, konstanta g=1, a normirane vrijednosti elemenata su: $R_1= R_2=1$, L=1 i C=1. Početni uvjeti su jednaki nuli.



Rješenje: Primjena Laplaceove transformacije



$$(1)\qquad -U_0(s)+I_1(s)R_1+\left(I_1(s)-I_2(s)\right)\frac{1}{sC}=0$$

$$(2)\qquad -\left(I_1(s)-I_2(s)\right)\frac{1}{sC}+sLI_2(s)+R_2I_2(s)-gU_1(s)R_2=0$$

$$U_1(s)=\left(I_1(s)-I_2(s)\right)\frac{1}{sC}$$

$$(1)\qquad U_0=\left(R_1+\frac{1}{sC}\right)I_1-\frac{1}{sC}I_2$$

$$(2)\qquad 0=-\left(1+gR_2\right)\frac{1}{sC}I_1+\left(\left(1+gR_2\right)\frac{1}{sC}+sL+R_2\right)I_2$$

$$\begin{bmatrix} R_1+\frac{1}{sC} & -\frac{1}{sC} \\ -\left(1+gR_2\right)\frac{1}{sC} & \left(1+gR_2\right)\frac{1}{sC}+sL+R_2 \end{bmatrix}\cdot\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} U_0 \\ 0 \end{bmatrix} \qquad \Rightarrow \qquad I_2=\frac{\Delta_2}{\Delta}$$

$$\Delta=\begin{vmatrix} R_1+\frac{1}{sC} & -\frac{1}{sC} \\ -\left(1+gR_2\right)\frac{1}{sC} & \left(1+gR_2\right)\frac{1}{sC}+sL+R_2 \end{vmatrix}=R_1\left(\frac{1+gR_2}{sC}+sL+R_2\right)+\frac{1}{sC}\left(sL+R_2\right)$$

$$\Delta_1=\begin{vmatrix} U_0 & -\frac{1}{sC} \\ 0 & \left(1+gR_2\right)\frac{1}{sC}+sL+R_2 \end{vmatrix}=U_0\left(\frac{1+gR_2}{sC}+sL+R_2\right)$$

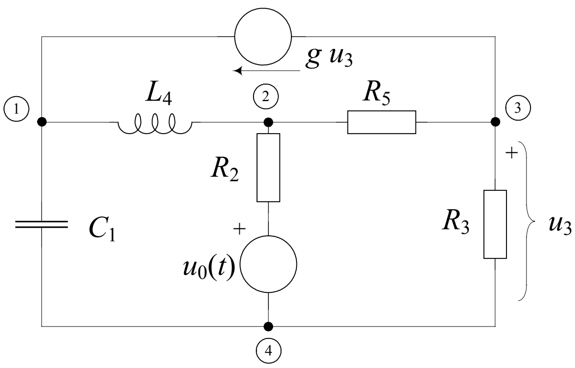
$$\Delta_2 = \begin{vmatrix} R_1 + \dfrac{1}{sC} & U_0 \\ -(1+gR_2)\dfrac{1}{sC} & 0 \end{vmatrix} = \frac{1+gR_2}{sC}U_0$$

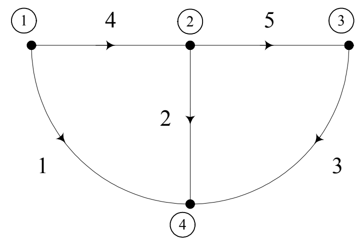
$$U_2 = R_2 I_2 - \frac{gR_2}{sC}(I_1 - I_2) = \frac{-gR_2\Delta_1}{sC\Delta} + \left(R_2 + \frac{gR_2}{sC}\right)\frac{\Delta_2}{\Delta} = U_0 \frac{-\dfrac{gR_2}{sC}sL + \dfrac{R_2}{sC}}{R_1\left(\dfrac{1+gR_2}{sC} + sL + R_2\right) + \dfrac{1}{sC}(sL + R_2)}$$

$$U_2 = U_0 \frac{-gsLR_2 + R_2}{R_1(1 + gR_2 + sC(sL + R_2)) + sL + R_2}$$

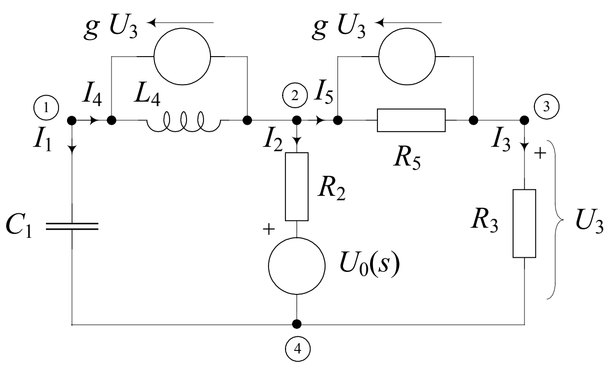
$$U_2 = \frac{3}{s} \cdot \frac{-s+1}{s^2 + 2s + 3}$$

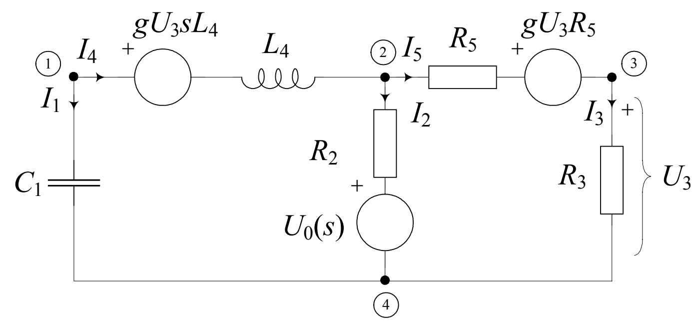
2. Za krug prikazan slikom i pridruženi orijentirani graf napisati matricu incidencija $\mathbf{A}_a$, temeljnu spojnu matricu $\mathbf{S}$, temeljnu rastavnu matricu $\mathbf{Q}$, matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$. Matrica $\mathbf{Z}_b$ mora biti regularna. (Uputa: grane stabla: 1, 2 i 3.)





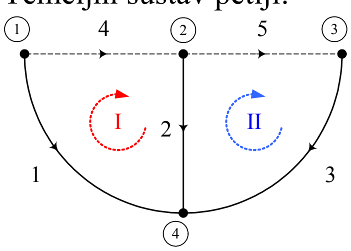
Rješenje: Primjena Laplaceove transformacije i posmicanje strujnog izvora



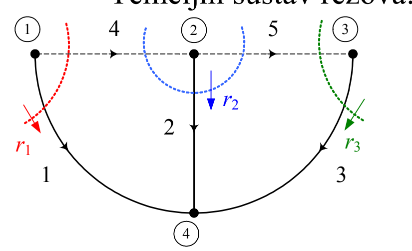


Matrica incidencija: $\mathbf{A}_a = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \end{bmatrix}$

Temeljni sustav petlji:



Temeljni sustav rezova:



Temeljna spojna matrica: $\mathbf{S} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix}$,

Temeljna rastavna matrica: $\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}$

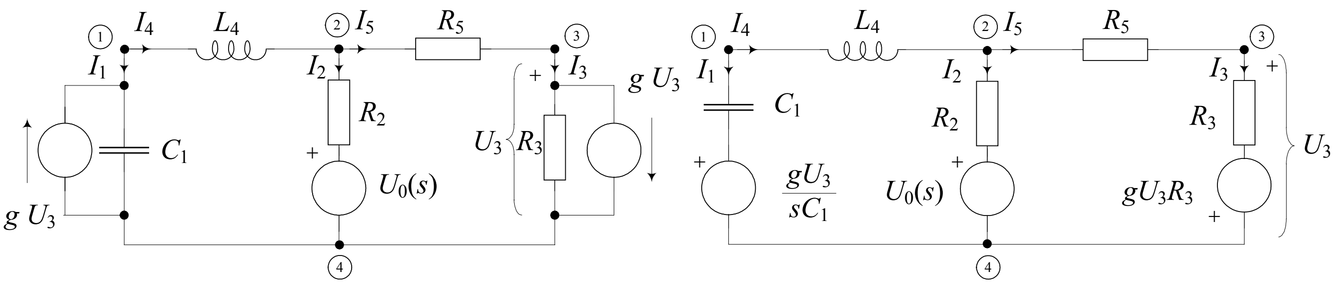
Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

$$U_1 = I_1 \cdot \frac{1}{sC_1}$$
$$U_2 = I_2 \cdot R_2 + U_0(s)$$
$$U_3 = I_3 \cdot R_3$$
$$U_4 = I_4 \cdot sL_4 + g \cdot U_3 \cdot sL_4 = gR_3 sL_4 \cdot I_3 + I_4 \cdot sL_4$$
$$U_5 = gU_3 \cdot R_5 + I_5 \cdot R_5 = gI_3 \cdot R_3 \cdot R_5 + I_5 \cdot R_5$$

$$\mathbf{Z}_b = \begin{bmatrix} \frac{1}{sC_1} & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & gR_3 sL_4 & sL_4 & 0 \\ 0 & 0 & gR_3 R_5 & 0 & R_5 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} 0 \\ U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrica $\mathbf{Z}_b$ je regularna jer nema niti jedan redak niti stupac jednak nuli.

<u>2. način:</u> posmicanje ovisnog strujnog izvora u čvor 4



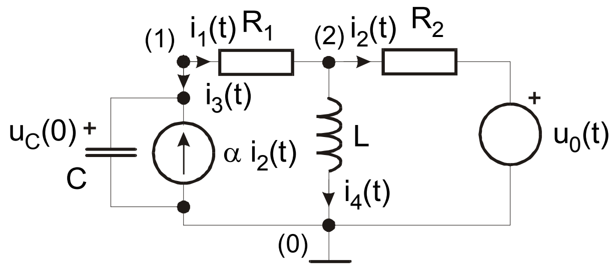
Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

$$U_1 = I_1 \cdot \frac{1}{sC_1} + gU_3 \cdot \frac{1}{sC_1} \Rightarrow U_3 = I_3 \cdot \frac{R_3}{1+gR_3} \Rightarrow U_1 = I_1 \cdot \frac{1}{sC_1} + I_3 \cdot \frac{gR_3}{1+gR_3} \cdot \frac{1}{sC_1}$$
$$U_2 = I_2 \cdot R_2 + U_0(s)$$
$$U_3 = I_3 \cdot R_3 - gU_3 \cdot R_3 \Rightarrow U_3 \cdot (1+gR_3) = I_3 \cdot R_3 \Rightarrow U_3 = I_3 \cdot \frac{R_3}{1+gR_3}$$
$$U_4 = I_4 \cdot sL_4$$
$$U_5 = I_5 \cdot R_5$$

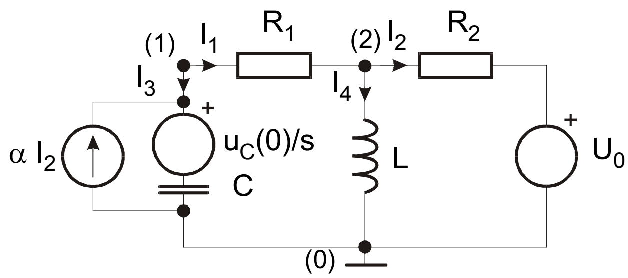
$$\mathbf{Z}_b = \begin{bmatrix} \frac{1}{sC_1} & 0 & \frac{gR_3}{(1+gR_3)sC_1} & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_3}{1+gR_3} & 0 & 0 \\ 0 & 0 & 0 & sL_4 & 0 \\ 0 & 0 & 0 & 0 & R_5 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} 0 \\ U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

ostalo je sve isto

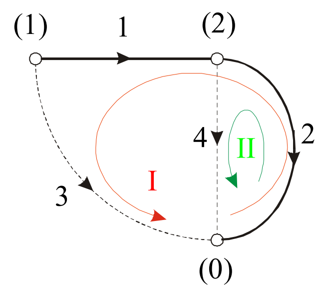
2. Zadan je električni krug prema slici. Poštujući oznake grana i čvorišta nacrtati pripadni orijentirani graf. Napisati naponsko-strujne jednadžbe grana u matričnom obliku te ispisati matricu impedancija grana $\mathbf{Z}_b$ i vektor izvora grana $\mathbf{U}_{0b}$. Napisati temeljni sustav jednadžbi petlji, odnosno odrediti matrice impedancija petlji $\mathbf{Z}_p$ i vektor izvora petlji $\mathbf{U}_{0p}$. (Uputa: grane 1 i 2 su grane stabla.)



Rješenje: Uz primjenu Laplaceove transformacije :



Električni krug



orijentirani graf i temeljni sustav petlji

$$
\text{Spojna matrica:}\qquad S = \begin{array}{c} \\ \mathrm{I} \\ \mathrm{II} \end{array}
\overset{\overbrace{\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}}^{grane}}{\begin{bmatrix} -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}}
$$

$\uparrow$

temeljne petlje

Naponsko strujne relacije grana:

$$
\begin{aligned}
U_1 &= I_1 R_1 \\
U_2 &= I_2 R_2 + U_0(s) \\
U_3 &= (I_3 + \alpha I_2)\frac{1}{sC} + \frac{u_C(0)}{s} \\
U_4 &= I_4 sL
\end{aligned}
\qquad \Rightarrow \qquad
\begin{aligned}
U_1 &= I_1 R_1 \\
U_2 &= I_2 R_2 + U_0(s) \\
U_3 &= \frac{1}{sC} I_3 + \alpha \frac{1}{sC} I_2 + \frac{u_C(0)}{s} \\
U_4 &= I_4 sL
\end{aligned}
$$

Naponsko-strujne relacije grana u matričnom obliku:

$$
\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & \dfrac{\alpha}{sC} & \dfrac{1}{sC} & 0 \\ 0 & 0 & 0 & sL \end{bmatrix}}_{\mathbf{Z}_b} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} 0 \\ U_0 \\ \dfrac{u_C(0)}{s} \\ 0 \end{bmatrix}}_{\mathbf{U}_{0b}}
$$

Temeljni sustav jednadžbi petlji :

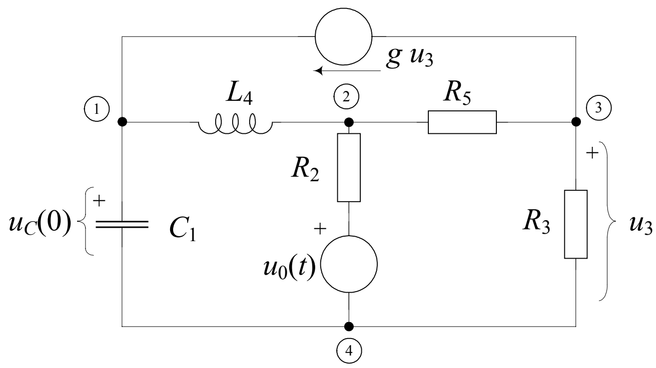
$$\mathbf{Z}_p = \mathbf{S}\mathbf{Z}_b\mathbf{S}^T = \begin{bmatrix} -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & \dfrac{\alpha}{sC} & \dfrac{1}{sC} & 0 \\ 0 & 0 & 0 & sL \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

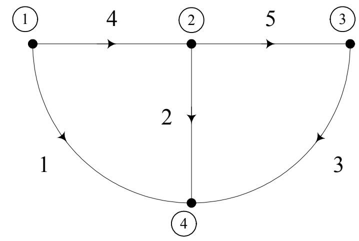
$$= \begin{bmatrix} -R_1 & -R_2 + \dfrac{\alpha}{sC} & \dfrac{1}{sC} & 0 \\ 0 & -R_2 & 0 & sL \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1 + R_2 - \dfrac{\alpha}{sC} + \dfrac{1}{sC} & R_2 - \dfrac{\alpha}{sC} \\ R_2 & R_2 + sL \end{bmatrix} =$$

$$= \begin{bmatrix} R_1 + R_2 + \dfrac{1-\alpha}{sC} & R_2 - \dfrac{\alpha}{sC} \\ R_2 & R_2 + sL \end{bmatrix}$$

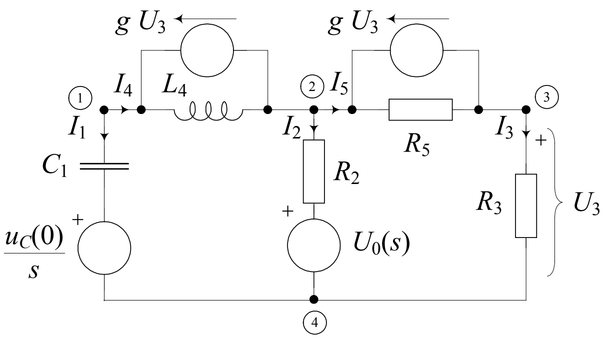
$$\mathbf{U}_{0p} = -\mathbf{S}\mathbf{U}_{0b} = -\begin{bmatrix} -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ U_0 \\ \dfrac{u_C(0)}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} U_0 - \dfrac{u_C(0)}{s} \\ U_0 \end{bmatrix}$$

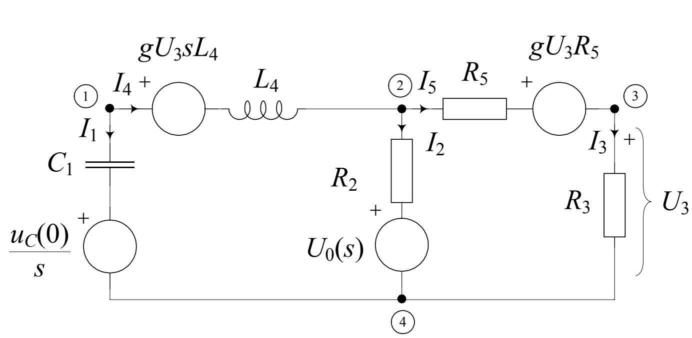
2. Za krug prikazan slikom i pridruženi orijentirani graf napisati temeljni sustav jednadžbi petlji u matričnom obliku (odrediti matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$ preko matrica impedancija grana $\mathbf{Z}_b$ i nezavisnih izvora grana $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna. (Uputa: grane stabla: 1, 2 i 3.)



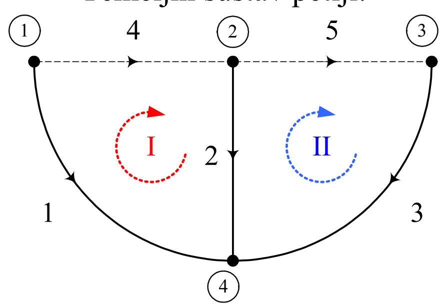


Rješenje: Primjena Laplaceove transformacije i posmicanje strujnog izvora





Temeljni sustav petlji:



Temeljna spojna matrica: $\mathbf{S}=\begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix}$,

Naponsko – strujne relacije grana:

$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$$

$$U_1 = I_1 \cdot \frac{1}{sC_1} + \frac{u_C(0)}{s}$$

$$U_2 = I_2 \cdot R_2 + U_0(s)$$

$$U_3 = I_3 \cdot R_3$$

$$U_4 = I_4 \cdot sL_4 + g \cdot U_3 \cdot sL_4 = gR_3 sL_4 \cdot I_3 + I_4 \cdot sL_4$$

$$U_5 = gU_3 \cdot R_5 + I_5 \cdot R_5 = gI_3 \cdot R_3 \cdot R_5 + I_5 \cdot R_5$$

$$\mathbf{Z}_b = \begin{bmatrix} \frac{1}{sC_1} & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & gR_3 sL_4 & sL_4 & 0 \\ 0 & 0 & gR_3 R_5 & 0 & R_5 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} \frac{u_C(0)}{s} \\ U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrica $\mathbf{Z}_b$ je regularna jer nema niti jedan redak niti stupac jednak nuli.

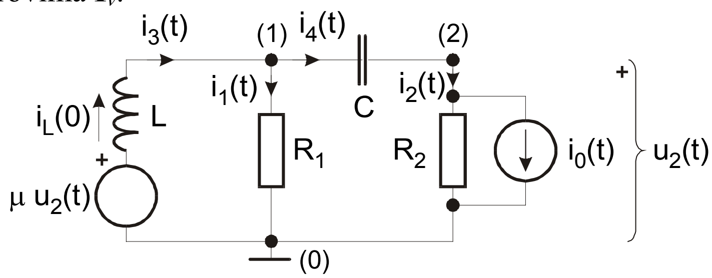
Temeljni sustav jednadžbi petlji: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{sC_1} & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & gR_3 sL_4 & sL_4 & 0 \\ 0 & 0 & gR_3 R_5 & 0 & R_5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

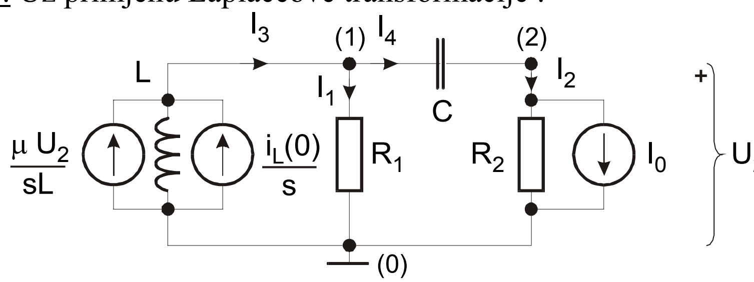
$$= \begin{bmatrix} -\frac{1}{sC_1} & R_2 & gR_3 sL_4 & sL_4 & 0 \\ 0 & -R_2 & R_3 & 0 & R_5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{sC_1} + R_2 + sL_4 & -R_2 + gR_3 sL_4 \\ -R_2 & R_2 + R_3 + R_5 \end{bmatrix}$$

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{u_C(0)}{s} \\ U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{u_C(0)}{s} - U_0(s) \\ U_0(s) \end{bmatrix}$$

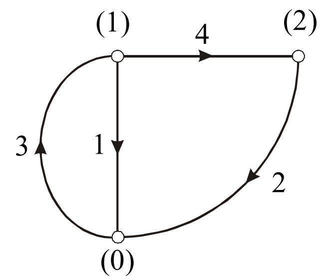
2. Zadan je električni krug prema slici. Poštujući oznake grana i čvorišta nacrtati pripadni orijentirani graf. Napisati reduciranu matricu incidencija **A**. Napisati naponsko-strujne jednadžbe grana u matričnom obliku te ispisati matricu admitancija grana $\mathbf{Y}_b$ i vektor naponskih izvora grana $\mathbf{U}_{0b}$. Napisati sustav jednadžbi čvorova, odnosno odrediti matrice admitancija čvorova $\mathbf{Y}_v$ i vektor strujnih izvora u čvorovima $\mathbf{I}_v$.



Rješenje: Uz primjenu Laplaceove transformacije :



Električni krug



orijentirani graf

Matrica incidencija (reducirana):

$$\mathbf{A} = \begin{array}{c} \\ (1) \\ (2) \end{array} \overset{\overbrace{\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}}^{grane}}{\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}}$$

$$\uparrow$$

cvorovi

Naponsko strujne relacije grana (naponi izraženi pomoću struja):

$$\begin{aligned} U_1 &= I_1 R_1 & \qquad & U_1 = I_1 R_1 \\ U_2 &= (I_2 - I_0) R_2 & & U_2 = I_2 R_2 - I_0 R_2 \\ U_3 &= I_3 sL - Li_L(0) - \mu U_2 & \Rightarrow \quad & U_3 = -\mu I_2 R_2 + I_3 sL - Li_L(0) + \mu I_0 R_2 \\ U_4 &= I_4 \frac{1}{sC} & & U_4 = I_4 \frac{1}{sC} \end{aligned}$$

Naponsko-strujne relacije grana u matričnom obliku:

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & -\mu R_2 & sL & 0 \\ 0 & 0 & 0 & \dfrac{1}{sC} \end{bmatrix}}_{\mathbf{Z}_b} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} 0 \\ -I_0 R_2 \\ \mu I_0 R_2 - Li_L(0) \\ 0 \end{bmatrix}}_{\mathbf{U}_{0b}}$$

Naponsko strujne relacije grana (struje izražene pomoću napona):

$$I_1 = U_1 \frac{1}{R_1}$$

$$I_2 = U_2 \frac{1}{R_2} + I_0$$

$$I_3 = U_2 \frac{\mu}{sL} + U_3 \frac{1}{sL} + \frac{i_L(0)}{s}$$

$$\underline{I_4 = U_4 sC}$$

Naponsko-strujne relacije grana u matričnom obliku:

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} = \underbrace{\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix}}_{\mathbf{Y}_b} \underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} + \underbrace{\begin{bmatrix} 0 \\ I_0 \\ \frac{i_L(0)}{s} \\ 0 \end{bmatrix}}_{\mathbf{I}_{0b}}$$

Sustav jednadžbi čvorova u matričnom obliku:

$$\mathbf{Y}_v = \mathbf{A}\mathbf{Y}_b\mathbf{A}^T = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{R_1} & -\frac{\mu}{sL} & -\frac{1}{sL} & sC \\ 0 & \frac{1}{R_2} & 0 & -sC \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{sL} + sC & -\frac{\mu}{sL} - sC \\ -sC & \frac{1}{R_2} + sC \end{bmatrix}$$

$$\mathbf{I}_v = \mathbf{A}\mathbf{Y}_b\mathbf{U}_{0b} = \begin{bmatrix} \frac{1}{R_1} & -\frac{\mu}{sL} & -\frac{1}{sL} & sC \\ 0 & \frac{1}{R_2} & 0 & -sC \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -I_0 R_2 \\ \mu R_2 I_0 - L i_L(0) \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{i_L(0)}{s} \\ -I_0 \end{bmatrix}$$

sustav jednadžbi čvorova: $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_v$

Drugi način izračunavanja matrice admitancija grana $\mathbf{Y}_b$ invertiranjem matrice impedancija grana $\mathbf{Z}_b$.

$$\mathbf{Z}_b = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & -\mu R_2 & sL & 0 \\ 0 & 0 & 0 & \dfrac{1}{sC} \end{bmatrix}$$

Označimo submatricu (2x2) unutar matrice $\mathbf{Z}_b$ sa

$$\mathbf{Z'}_b = \begin{bmatrix} R_2 & 0 \\ -\mu R_2 & sL \end{bmatrix}$$

sada invertirajmo sumatricu $\mathbf{Z'}_b$

$$\mathbf{Z'}_b^{-1} = \begin{bmatrix} R_2 & 0 \\ -\mu R_2 & sL \end{bmatrix}^{-1} = \frac{1}{R_2 sL}\begin{bmatrix} sL & -(-\mu R_2) \\ 0 & R_2 \end{bmatrix}^T = \frac{1}{R_2 sL}\begin{bmatrix} sL & 0 \\ \mu R_2 & R_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{R_2} & 0 \\ \dfrac{\mu}{sL} & \dfrac{1}{sL} \end{bmatrix},$$

i vratimo je na svoje mjesto unutar matrice $\mathbf{Z}_b^{-1}$. Nadalje, ako invertiramo ostale elemente na dijagonali, konačno dobivamo:

$$\mathbf{Y}_b = \mathbf{Z}_b^{-1} = \begin{bmatrix} \dfrac{1}{R_1} & 0 & 0 & 0 \\ 0 & \dfrac{1}{R_2} & 0 & 0 \\ 0 & \dfrac{\mu}{sL} & \dfrac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix}$$

Naponsko strujne relacije grana (drugi oblik: struje grana su izražene pomoću napona grana):

$$I_1 = \frac{1}{R_1} U_1$$

$$I_2 = \frac{1}{R_2} U_2 + I_0(s)$$

$$I_3 = \frac{1}{sL} U_3 + \frac{\mu}{sL} U_2 + \frac{i_L(0)}{s}$$

$$\underline{I_4 = sCU_4}$$

Naponsko-strujne relacije grana u matričnom obliku:

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} = \underbrace{\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix}}_{\mathbf{Y}_b} \underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} + \underbrace{\begin{bmatrix} 0 \\ I_0 \\ \frac{i_L(0)}{s} \\ 0 \end{bmatrix}}_{\mathbf{I}_{0b}}$$

Sustav jednadžbi čvorova: $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_v$, gdje su:

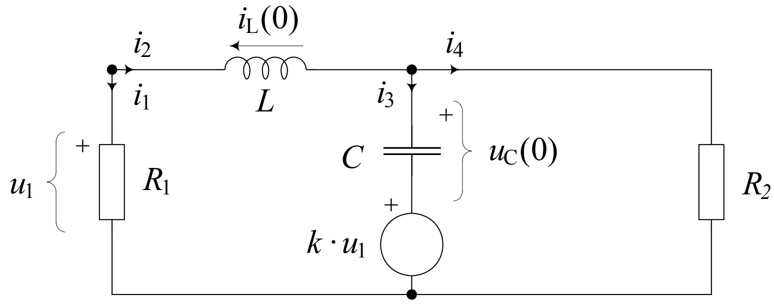
$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{R_1} & -\frac{\mu}{sL} & -\frac{1}{sL} & sC \\ 0 & \frac{1}{R_2} & 0 & -sC \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{sL} + sC & -\frac{\mu}{sL} - sC \\ -sC & \frac{1}{R_2} + sC \end{bmatrix}$$

$$\mathbf{I}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{U}_{0b} = \begin{bmatrix} \frac{1}{R_1} & -\frac{\mu}{sL} & -\frac{1}{sL} & sC \\ 0 & \frac{1}{R_2} & 0 & -sC \end{bmatrix} \begin{bmatrix} 0 \\ -I_0 R_2 \\ \mu I_0(s) R_2 - L i_L(0) \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\mu}{sL} I_0 R_2 - \frac{\mu}{sL} I_0 R_2 + \frac{i_L(0)}{s} \\ -I_0(s) \end{bmatrix} = \begin{bmatrix} \frac{i_L(0)}{s} \\ -I_0(s) \end{bmatrix}$$

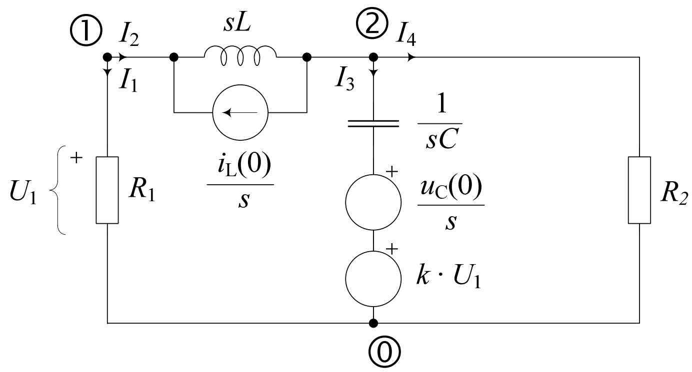
2. Za krug na slici i pridružene oznake čvorova i grana: *a*) nacrtati orijentirani graf (grane stabla: 1, 2). Napisati: *b*) matricu incidencija; *c*) strujno-naponske jednadžbe grana u matričnom obliku (matrice $\mathbf{Y}_b$ i $\mathbf{I}_{0b}$). Matrica $\mathbf{Y}_b$ mora biti regularna. Topološkom analizom odrediti: *d*) matricu admitancija čvorova $\mathbf{Y}_v$; *e*) matricu strujnih doprinosa neovisnih izvora i početnih uvjeta u čvorovima $\mathbf{I}_{0v}$.



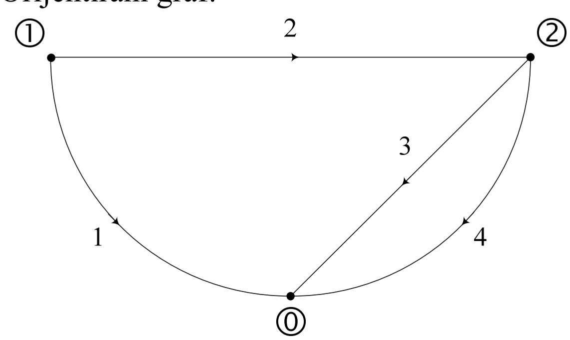
Rješenje:

a) nacrtati orijentirani graf: (1 bod)

Primjena $\mathcal{L}$ − transformacije:



Orijentirani graf:



b) napisati matricu incidencija: (1 bod)

Matrica incidencija:

$$\mathbf{A_a} = \begin{array}{c} \\ 1 \\ 2 \\ 0 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ -1 & 0 & -1 & -1 \end{bmatrix} \end{array} \qquad \mathbf{A} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

c) strujno-naponske jednadžbe grana u matričnom obliku: (1 bod)

I-U jednadžbe grana:

$$I_1 = U_1 \cdot \frac{1}{R_1}$$

$$I_2 = U_2 \cdot \frac{1}{sL} - \frac{i_L(0)}{s}$$

$$I_3 = \left(U_3 - k \cdot U_1 - \frac{U_C(0)}{s} \right) \cdot sC = U_3 \cdot sC - k \cdot U_1 \cdot sC - U_C(0) \cdot C$$

$$I_4 = U_4 \cdot \frac{1}{R_2}$$

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} = \underbrace{\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{sL} & 0 & 0 \\ -k \cdot sC & 0 & sC & 0 \\ 0 & 0 & 0 & \frac{1}{R_2} \end{bmatrix}}_{\mathbf{Y}_b} \left(\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} - \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{u_C(0)}{s} \\ 0 \end{bmatrix}}_{\mathbf{U}_{0b}} \right) + \underbrace{\begin{bmatrix} 0 \\ -\frac{i_L(0)}{s} \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{I}_{0b}}$$

Da bi bila regularna, matrica impedancija grana $\mathbf{Y}_b$ ne smije sadržavati niti redak niti stupac sa svim nulama!

d) topološkom analizom odrediti matricu admitancija čvorova $\mathbf{Y}_v$: (1 bod)

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{sL} & 0 & 0 \\ -k \cdot sC & 0 & sC & 0 \\ 0 & 0 & 0 & \frac{1}{R_2} \end{bmatrix} \cdot \mathbf{A}^T =$$

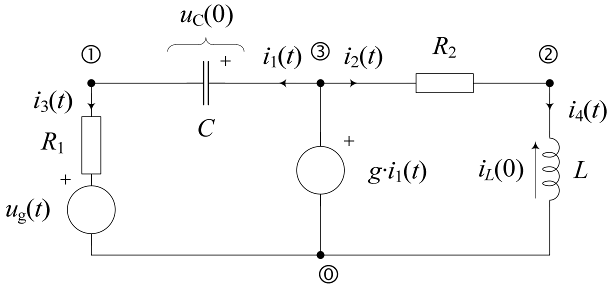
$$= \begin{bmatrix} \frac{1}{R_1} & \frac{1}{sL} & 0 & 0 \\ -k \cdot sC & -\frac{1}{sL} & sC & \frac{1}{R_2} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{sL} & -\frac{1}{sL} \\ -k \cdot sC - \frac{1}{sL} & \frac{1}{sL} + sC + \frac{1}{R_2} \end{bmatrix}$$

e) topološkom analizom odrediti matricu strujnih doprinosa neovisnih izvora i početnih uvjeta u čvorovima $\mathbf{I}_{0v}$: (1 bod)

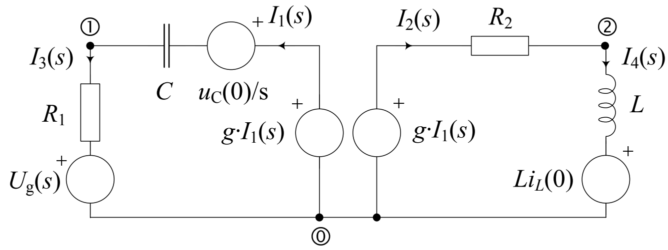
$$\mathbf{I}_{0v} = \mathbf{A} \cdot (-\mathbf{I}_{0b} + \mathbf{Y}_b \cdot \mathbf{U}_{0b}) = \mathbf{A} \cdot \left(-\mathbf{I}_{0b} + \begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{sL} & 0 & 0 \\ -k \cdot sC & 0 & sC & 0 \\ 0 & 0 & 0 & \frac{1}{R_2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \frac{u_C(0)}{s} \\ 0 \end{bmatrix} \right) =$$

$$= \mathbf{A} \cdot \left(-\begin{bmatrix} 0 \\ -\frac{i_L(0)}{s} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_C(0) \cdot C \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \frac{i_L(0)}{s} \\ u_C(0) \cdot C \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{i_L(0)}{s} \\ -\frac{i_L(0)}{s} + u_C(0) \cdot C \end{bmatrix}$$

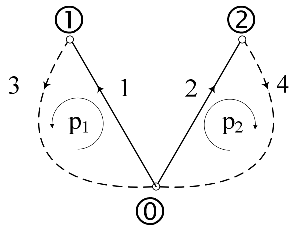
2. Za električni krug prikazan slikom, poštujući oznake čvorišta i grana, odrediti: *a*) orijentirani graf i temeljni sustav petlji; *b*) spojnu matricu **S**; *c*) naponsko-strujne jednadžbe grana u matričnom obliku; *d*) temeljni sustav jednadžbi petlji u matričnom obliku (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$ preko matrica $\mathbf{Z}_b$ i $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna.



Rješenje: Primjena $\mathcal{L}$-transformacije i posmicanje naponskog izvora



a) Orijentirani graf:



b) Spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(1bod)

c) Naponsko-strujne jednadžbe grana:

(2 boda: 1 za jednadžbe i 1 za sliku u $\mathcal{L}$-domeni s posmicanjem izvora)

$$U_1 = I_1 \cdot \frac{1}{sC} + \frac{u_C(0)}{s} - gI_1(s)$$
$$U_2 = I_2 \cdot R_2 - gI_1(s)$$
$$U_3 = I_3 \cdot R_1 + U_g(s)$$
$$U_4 = I_4 \cdot sL + L \cdot i_L(0)$$

U matričnom obliku: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} \frac{1}{sC} - g & 0 & 0 & 0 \\ -g & R_2 & 0 & 0 \\ 0 & 0 & R_1 & 0 \\ 0 & 0 & 0 & sL \end{bmatrix}}_{\mathbf{Z}_b} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} \frac{u_C(0)}{s} \\ 0 \\ U_g(s) \\ Li_L(0) \end{bmatrix}}_{\mathbf{U}_{0b}}$$

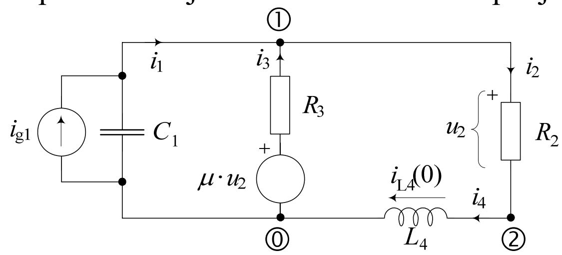
d) Temeljni sustav jednadžbi petlji u matričnom obliku $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$, gdje su:

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{sC} - g & 0 & 0 & 0 \\ -g & R_2 & 0 & 0 \\ 0 & 0 & R_1 & 0 \\ 0 & 0 & 0 & sL \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{sC} - g & 0 & R_1 & 0 \\ -g & R_2 & 0 & sL \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{sC} - g + R_1 & 0 \\ -g & R_2 + sL \end{bmatrix} \quad \text{(1 bod)}$$

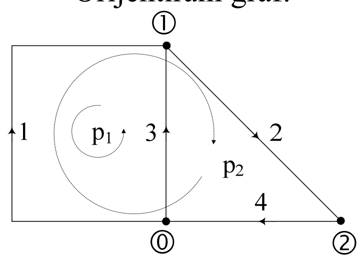
$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{u_C(0)}{s} \\ 0 \\ U_g(s) \\ Li_L(0) \end{bmatrix} = \begin{bmatrix} -\frac{u_C(0)}{s} - U_g(s) \\ -Li_L(0) \end{bmatrix} \quad \text{(1 bod)}$$

2. Za električni krug prikazan slikom i pridruženim orijentacijama grana te čvorovima (grane stabla: 1, 2) treba odrediti temeljni sustav jednadžbi petlji topološkom analizom. Napisati: a) spojnu matricu **S**, b) matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$, c) matricu impedancija petlji $\mathbf{Z}_p$ i d) vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.



Rješenje:

Orijentirani graf:



Spojna matrica:

$$\mathbf{S} = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(1 bod)

Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

(1) $U_1 = \dfrac{1}{sC_1} \cdot I_1 - \dfrac{1}{sC_1} \cdot I_{g1}$

(2) $U_2 = R_2 \cdot I_2$

(3) $U_3 = R_3 \cdot I_3 - \mu U_2 = R_3 \cdot I_3 - \mu R_2 I_2$

(3) $U_4 = sL_4 \cdot I_4 - L_4 i_{L4}(0)$

$$\mathbf{Z}_b = \begin{bmatrix} \dfrac{1}{sC_1} & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & -\mu R_2 & R_3 & 0 \\ 0 & 0 & 0 & sL_4 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} -\dfrac{I_{g1}}{sC_1} \\ 0 \\ 0 \\ -L_4 i_{L4}(0) \end{bmatrix}$$

(1 bod)

Matrica $\mathbf{Z}_b$ je regularna.

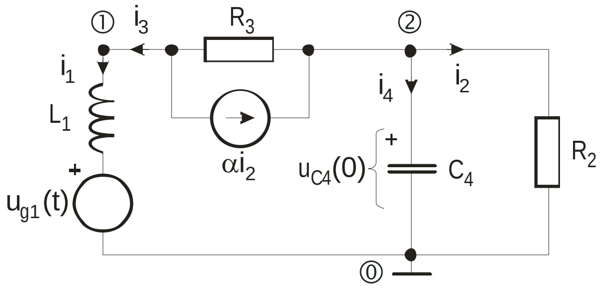
Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

$$\mathbf{S} \cdot \mathbf{Z}_b = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dfrac{1}{sC_1} & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & -\mu R_2 & R_3 & 0 \\ 0 & 0 & 0 & sL_4 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{sC_1} & -\mu R_2 & R_3 & 0 \\ \dfrac{1}{sC_1} & R_2 & 0 & sL_4 \end{bmatrix}$$

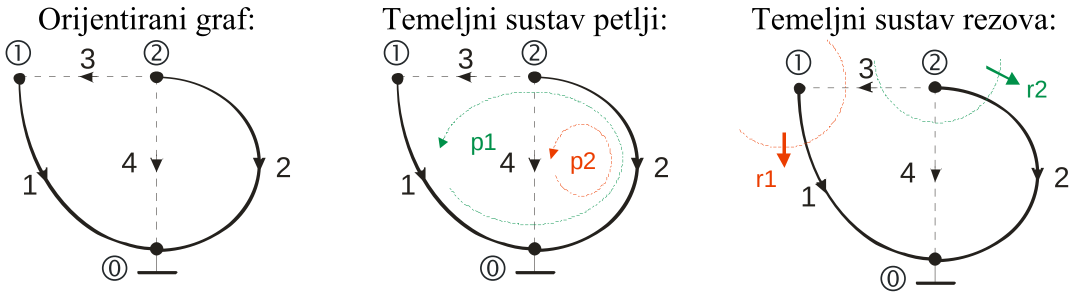
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} -\dfrac{1}{sC_1} & -\mu R_2 & R_3 & 0 \\ \dfrac{1}{sC_1} & R_2 & 0 & sL_4 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{sC_1} + R_3 & -\dfrac{1}{sC_1} - \mu R_2 \\ -\dfrac{1}{sC_1} & \dfrac{1}{sC_1} + R_2 + sL_4 \end{bmatrix}$$ (1 bod)

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\begin{bmatrix} -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\dfrac{I_{g1}}{sC_1} \\ 0 \\ 0 \\ -L_4 \cdot i_{L4}(0) \end{bmatrix} = \begin{bmatrix} -\dfrac{I_{g1}}{sC_1} \\ \dfrac{I_{g1}}{sC_1} + L_4 \cdot i_{L4}(0) \end{bmatrix}$$ (1 bod)

2. Za električni krug prikazan slikom i pridružene orijentacije grana i čvorove napisati: a) matricu incidencija $\mathbf{A}_a$; b) temeljnu spojnu matricu $\mathbf{S}$, temeljnu rastavnu matricu $\mathbf{Q}$; c) matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i neovisnih izvora grana $\mathbf{U}_{0b}$; d) matricu admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i neovisnih strujnih izvora grana $\mathbf{I}_{0b}$; e) pomoću navedenih matrica odrediti sustav jednadžbi čvorova (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$).



Rješenje:



a) matrica incidencija $\mathbf{A}_a$: **(1 bod)**

$$\mathbf{A}_a = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \\ -1 & -1 & 0 & -1 \end{bmatrix}$$

b) temeljna spojna matrica $\mathbf{S}$, temeljna rastavna matrica $\mathbf{Q}$: **(1 bod)**

Temeljna spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Temeljna rastavna matrica:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

c) matrica impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$: **(1 bod)**

Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

(1) $U_1 = sL_1 \cdot I_1 + U_{g1}$

(2) $U_2 = R_2 \cdot I_2$

(3) $U_3 = R_3 \cdot (I_3 + \alpha I_2) = \alpha R_3 \cdot I_2 + R_3 \cdot I_3$

(3) $U_4 = \dfrac{1}{sC_4} \cdot I_4 + \dfrac{u_{C4}(0)}{s}$

$$\mathbf{Z}_b = \begin{bmatrix} sL_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & \alpha R_3 & R_3 & 0 \\ 0 & 0 & 0 & \dfrac{1}{sC_4} \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} U_{g1} \\ 0 \\ 0 \\ \dfrac{u_{C4}(0)}{s} \end{bmatrix}$$

d) matrica admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$: **(1 bod)**

Strujno – naponske relacije grana: $\mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{0b}$

(1) $I_1 = \frac{1}{sL_1} \cdot U_1 - \frac{1}{sL_1} \cdot U_{g1}$

(2) $I_2 = \frac{1}{R_2} \cdot U_2$

(3) $I_3 = \frac{1}{R_3} \cdot U_3 - \alpha I_2 = -\frac{\alpha}{R_2} \cdot U_2 + \frac{1}{R_3} \cdot U_3$

(3) $\underline{I_4 = sC_4 \cdot U_4 - C_4 u_{C4}(0)}$

$$\mathbf{Y}_b = \begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & -\frac{\alpha}{R_2} & \frac{1}{R_3} & 0 \\ 0 & 0 & 0 & sC_4 \end{bmatrix}, \ \mathbf{I}_{0b} = \begin{bmatrix} -\frac{U_{g1}}{sL_1} \\ 0 \\ 0 \\ -C_4 u_{C4}(0) \end{bmatrix}$$

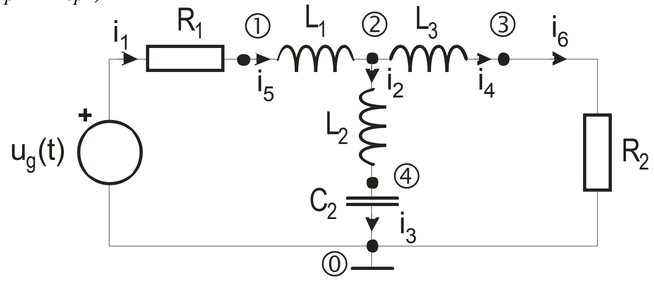
e) sustav jednadžbi čvorova (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$): **(1 bod)**

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & -\frac{\alpha}{R_2} & \frac{1}{R_3} & 0 \\ 0 & 0 & 0 & sC_4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{sL_1} & \frac{\alpha}{R_2} & -\frac{1}{R_3} & 0 \\ 0 & \frac{1}{R_2} - \frac{\alpha}{R_2} & \frac{1}{R_3} & sC_4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{sL_1} + \frac{1}{R_3} & \frac{\alpha}{R_2} - \frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_2} - \frac{\alpha}{R_2} + \frac{1}{R_3} + sC_4 \end{bmatrix}$$

$$\mathbf{I}_{0v} = -\mathbf{A} \cdot \mathbf{I}_{0b} = -\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -\frac{U_{g1}(s)}{sL_1} \\ 0 \\ 0 \\ -C_4 u_{C4}(0) \end{bmatrix} = \begin{bmatrix} \frac{U_{g1}(s)}{sL_1} \\ C_4 u_{C4}(0) \end{bmatrix}$$

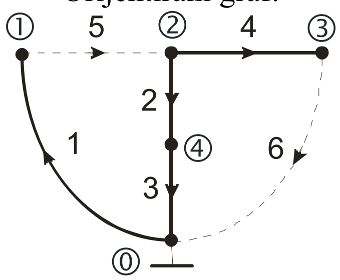
2. Za električni krug prikazan slikom i pridružene orijentacije grana i čvorove nacrtati: a) orijentirani graf, temeljni sustav petlji i temeljni sustav rezova; b) napisati matricu incidencija $\mathbf{A}_a$, temeljnu spojnu matricu $\mathbf{S}$, temeljnu rastavnu matricu $\mathbf{Q}$; c) matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$; d) matricu admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$; e) pomoću navedenih matrica odrediti temeljni sustav jednadžbi petlji (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$).



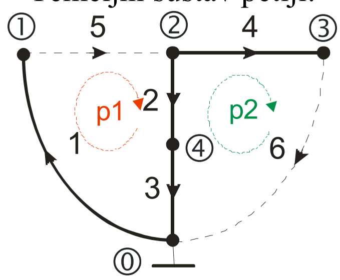
Rješenje:

a) orijentirani graf, temeljni sustav petlji i temeljni sustav rezova: **(1 bod)**

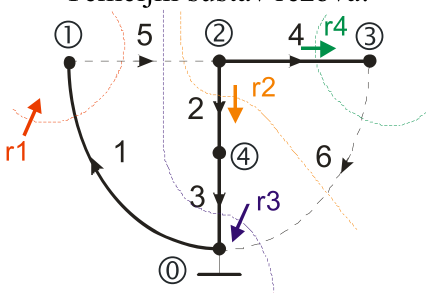
Orijentirani graf:



Temeljni sustav petlji:



Temeljni sustav rezova:



b) matrica incidencija $\mathbf{A}_a$, temeljna spojna matrica $\mathbf{S}$, temeljna rastavna matrica $\mathbf{Q}$: **(1 bod)**

Matrica incidencija:

$$\mathbf{A}_a = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

Temeljna rastavna matrica:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

Temeljna spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{bmatrix}$$

c) matrica impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$: **(1 bod)**

Naponsko – strujne relacije grana:

$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}, \text{ odn. } \mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{0b}$$

$$U_1 = I_1 \cdot R_1 - U_g(s) \quad \Rightarrow \quad I_1 = \frac{1}{R_1} \cdot U_1 + \frac{U_g(s)}{R_1}$$

$$U_2 = I_2 \cdot sL_2 \quad \Rightarrow \quad I_2 = \frac{1}{sL_2} \cdot U_2, \text{ itd.}$$

$$U_3 = I_3 \cdot \frac{1}{sC_2},\ U_4 = I_4 \cdot sL_3,\ U_5 = I_5 \cdot sL_1,\ U_6 = I_6 \cdot R_2$$

Iz gornjeg sustava se mogu pročitati:

$$\mathbf{Z}_b = \begin{bmatrix} R_1 & & & & & \\ & sL_2 & & & & \\ & & \dfrac{1}{sC_2} & & 0 & \\ & & & sL_3 & & \\ & 0 & & & sL_1 & \\ & & & & & R_2 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} -U_g(s) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

d) matrica admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$:
Jedan način je da se gornji sustav napiše tako da su s lijeve strane struje grana, a drugi način je da se invertira matrica $\mathbf{Y}_b = \mathbf{Z}_b^{-1}$. U slučaju dijagonalne matrice to je lako jer elementi na dijagonali inverzne matrice imaju recipročnu vrijednost elemenata originalne matrice. **(1 bod)**

$$\mathbf{Y}_b = \begin{bmatrix} \dfrac{1}{R_1} & & & & & \\ & \dfrac{1}{sL_2} & & & & \\ & & sC_2 & & 0 & \\ & & & \dfrac{1}{sL_3} & & \\ & 0 & & & \dfrac{1}{sL_1} & \\ & & & & & \dfrac{1}{R_2} \end{bmatrix}, \mathbf{I}_{0b} = \begin{bmatrix} U_g(s)/R_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

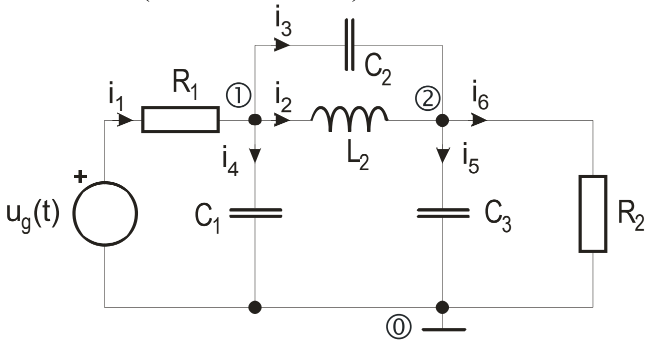
e) sustav jednadžbi petlji (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$): **(1 bod)**

$$\mathbf{Z}_p = \mathbf{S}\cdot\mathbf{Z}_b\cdot\mathbf{S}^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_1 & & & & & \\ & sL_2 & & & & \\ & & \dfrac{1}{sC_2} & & 0 & \\ & & & sL_3 & & \\ & 0 & & & sL_1 & \\ & & & & & R_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} R_1 & sL_2 & \dfrac{1}{sC_2} & 0 & sL_1 & 0 \\ 0 & -sL_2 & -\dfrac{1}{sC_2} & sL_3 & 0 & R_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1 + sL_2 + \dfrac{1}{sC_2} + sL_1 & -sL_2 - \dfrac{1}{sC_2} \\ -sL_2 - \dfrac{1}{sC_2} & sL_2 + \dfrac{1}{sC_2} + sL_3 + R_2 \end{bmatrix}$$

$$\mathbf{U}_{0p} = -\mathbf{S}\cdot\mathbf{U}_{0b} = -\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_g(s) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -U_g(s) \\ 0 \end{bmatrix}$$

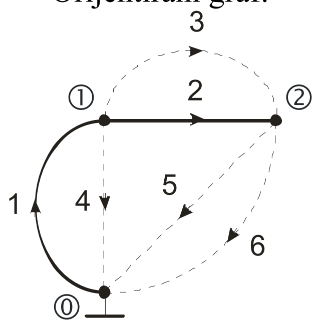
2. Za električni krug prikazan slikom i pridružene orijentacije grana i čvorove nacrtati: a) orijentirani graf, temeljni sustav petlji i temeljni sustav rezova; b) napisati matricu incidencija $\mathbf{A}_a$, temeljnu spojnu matricu **S**, temeljnu rastavnu matricu **Q**; c) matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$; d) matricu admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$; e) pomoću navedenih matrica odrediti sustav jednadžbi čvorova (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$).



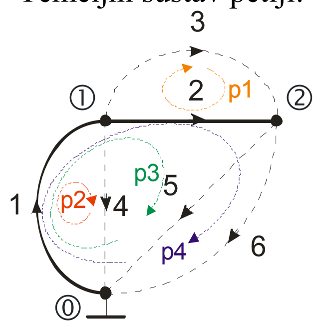
Rješenje:

a) orijentirani graf, temeljni sustav petlji i temeljni sustav rezova: **(1 bod)**

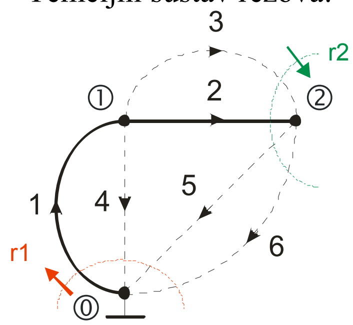
Orijentirani graf:



Temeljni sustav petlji:



Temeljni sustav rezova:



b) matrica incidencija $\mathbf{A}_a$, temeljna spojna matrica **S**, temeljna rastavna matrica **Q**: **(1 bod)**

Matrica incidencija:

$$\mathbf{A}_a = \begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

Temeljna spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Temeljna rastavna matrica:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & 0 & -1 & -1 \end{bmatrix}$$

c) matrica impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$: **(1 bod)**

Naponsko – strujne relacije grana:

$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}, \text{ odn. } \mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{0b}$$

$$U_1 = I_1 \cdot R_1 - U_g(s) \quad \Rightarrow \quad I_1 = \frac{1}{R_1} \cdot U_1 + \frac{U_g(s)}{R_1}$$

$$U_2 = I_2 \cdot sL_2 \quad \Rightarrow \quad I_2 = \frac{1}{sL_2} \cdot U_2, \text{ itd.}$$

$$U_3 = I_3 \cdot \frac{1}{sC_2},\ U_4 = I_4 \cdot \frac{1}{sC_1},\ U_5 = I_5 \cdot \frac{1}{sC_3},\ U_6 = I_6 \cdot R_2$$

Iz gornjeg sustava se mogu pročitati:

$$\mathbf{Z}_b = \begin{bmatrix} R_1 & & & & & \\ & sL_2 & & & 0 & \\ & & \frac{1}{sC_2} & & & \\ & & & \frac{1}{sC_1} & & \\ & 0 & & & \frac{1}{sC_3} & \\ & & & & & R_2 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} -U_g(s) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

d) matrica admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$: **(1 bod)**

Jedan način je da se gornji sustav napiše tako da su s lijeve strane struje grana, a drugi način je da se invertira matrica $\mathbf{Y}_b = \mathbf{Z}_b^{-1}$. U slučaju dijagonalne matrice to je lako jer elementi na dijagonali inverzne matrice imaju recipročnu vrijednost elemenata originalne matrice.

$$\mathbf{Y}_b = \begin{bmatrix} \frac{1}{R_1} & & & & & \\ & \frac{1}{sL_2} & & & & \\ & & sC_2 & & 0 & \\ & & & sC_1 & & \\ & 0 & & & sC_3 & \\ & & & & & \frac{1}{R_2} \end{bmatrix}, \mathbf{I}_{0b} = \begin{bmatrix} U_g(s)/R_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

e) sustav jednadžbi čvorova (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$): **(1 bod)**

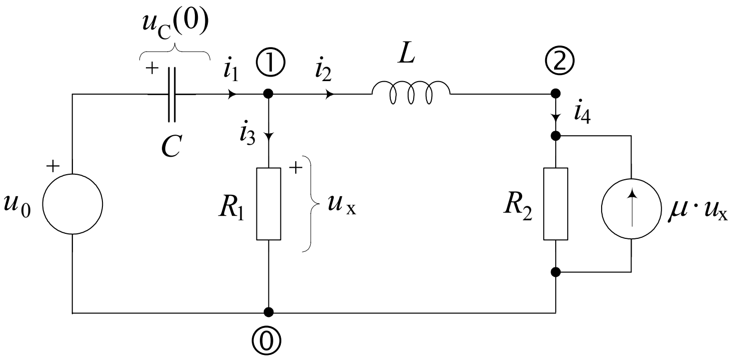
$$\mathbf{Y}_v = \mathbf{A}\cdot\mathbf{Y}_b\cdot\mathbf{A}^T = \begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{R_1} & & & & & \\ & \frac{1}{sL_2} & & & & \\ & & sC_2 & & 0 & \\ & & & sC_1 & & \\ & 0 & & & sC_3 & \\ & & & & & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{1}{R_1} & \frac{1}{sL_2} & sC_2 & sC_1 & 0 & 0 \\ 0 & -\frac{1}{sL_2} & -sC_2 & 0 & sC_3 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1}+\frac{1}{sL_2}+sC_1+sC_2 & -\frac{1}{sL_2}-sC_2 \\ -\frac{1}{sL_2}-sC_2 & \frac{1}{sL_2}+sC_2+sC_3+\frac{1}{R_2} \end{bmatrix}$$

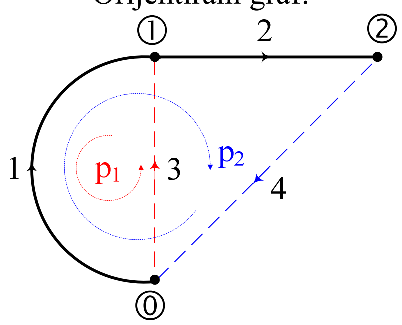
$$\mathbf{I}_{0v} = -\mathbf{A}\cdot\mathbf{I}_{0b} = -\begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} U_g(s)/R_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} U_g(s)/R_1 \\ 0 \end{bmatrix}$$

$$\mathbf{U}_{0p} = -\mathbf{S}\cdot\mathbf{U}_{0b} = -\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} \frac{u_C(0)}{s} - U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} U_0(s) - \frac{u_C(0)}{s} \\ U_0(s) - \frac{u_C(0)}{s} \end{bmatrix}$$ (1 bod)

5. Za električni krug prikazan slikom i pridruženim orijentacijama grana te čvorovima (grane stabla: 1, 2) treba odrediti temeljni sustav jednadžbi petlji primjenom grafova. Napisati: a) spojnu matricu **S**, b) matricu impedancija grana $\mathbf{Z}_b$, c) vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$, d) matricu impedancija petlji $\mathbf{Z}_p$ i e) vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.



Rješenje:



Spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(1 bod)

Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

(1) $U_1 = \dfrac{1}{sC} \cdot I_1 + \dfrac{u_C(0)}{s} - U_0$

(2) $U_2 = sL \cdot I_2$

(3) $U_3 = R_1 \cdot I_3$

(4) $\underline{U_4 = R_2 \cdot I_4 + R_2 \cdot \mu U_x ; U_x = U_3}$

$$\mathbf{Z}_b = \begin{bmatrix} \frac{1}{sC} & 0 & 0 & 0 \\ 0 & sL & 0 & 0 \\ 0 & 0 & R_1 & 0 \\ 0 & 0 & \mu R_1 R_2 & R_2 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} \frac{u_C(0)}{s} - U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(1 bod) (1 bod)

Matrica $\mathbf{Z}_b$ je regularna.

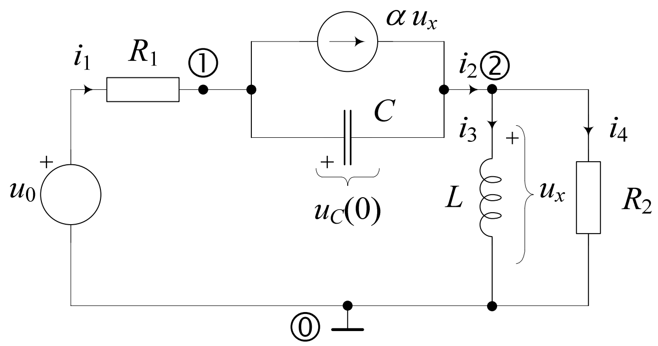
Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

$$\mathbf{S} \cdot \mathbf{Z}_b = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{sC} & 0 & 0 & 0 \\ 0 & sL & 0 & 0 \\ 0 & 0 & R_1 & 0 \\ 0 & 0 & \mu R_1 R_2 & R_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{sC} & 0 & R_1 & 0 \\ \frac{1}{sC} & sL & \mu R_1 R_2 & R_2 \end{bmatrix}$$

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} \frac{1}{sC} & 0 & R_1 & 0 \\ \frac{1}{sC} & sL & \mu R_1 R_2 & R_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{sC} + R_1 & \frac{1}{sC_1} \\ \frac{1}{sC} + \mu R_1 R_2 & \frac{1}{sC} + sL + R_2 \end{bmatrix}$$

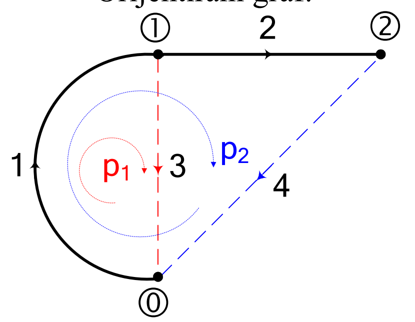
(1 bod)

2. Za električni krug prikazan slikom i pridruženim orijentacijama grana te čvorovima (grane stabla: 1, 2) treba odrediti temeljni sustav jednadžbi petlji primjenom grafova. Napisati: a) spojnu matricu **S**, b) matricu impedancija grana $\mathbf{Z}_b$, c) vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$, d) matricu impedancija petlji $\mathbf{Z}_p$ i e) vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.



Rješenje:

Orijentirani graf:



Spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(1 bod)

Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

(1) $U_1 = R_1 \cdot I_1 - U_0$

(2) $U_2 = \dfrac{1}{sC} \cdot I_2 - \alpha \dfrac{L}{C} I_3 + \dfrac{u_C(0)}{s}$

(3) $U_3 = sL \cdot I_3$

(4) $U_4 = R_2 \cdot I_4$

$$\mathbf{Z}_b = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & \dfrac{1}{sC} & -\alpha \dfrac{L}{C} & 0 \\ 0 & 0 & sL & 0 \\ 0 & 0 & 0 & R_2 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} -U_0(s) \\ \dfrac{u_C(0)}{s} \\ 0 \\ 0 \end{bmatrix}$$

(1 bod) (1 bod)

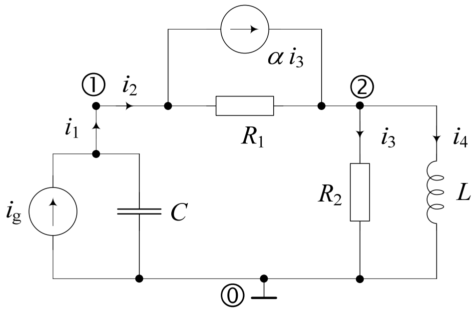
Matrica $\mathbf{Z}_b$ je regularna.

Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

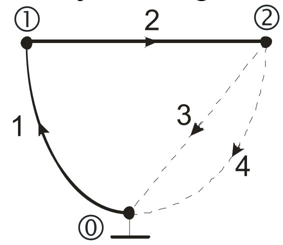
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} R_1 + \dfrac{1}{sC} + sL - \alpha \dfrac{L}{C} & R_1 + \dfrac{1}{sC} \\ R_1 + \dfrac{1}{sC} - \alpha \dfrac{L}{C} & R_1 + \dfrac{1}{sC} + R_2 \end{bmatrix}$$ (1 bod)

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = \begin{bmatrix} U_0(s) - \dfrac{u_C(0)}{s} \\ U_0(s) - \dfrac{u_C(0)}{s} \end{bmatrix}$$ (1 bod)

2. Za električni krug prikazan slikom i pridružene orijentacije grana i čvorove napisati: a) reduciranu matricu incidencija **A**, temeljnu spojnu matricu **S** i temeljnu rastavnu matricu **Q**; b) matricu admitancija grana $\mathbf{Y}_b$; c) vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$; d) matricu admitancija čvorova $\mathbf{Y}_v$; e) vektor početnih uvjeta i nezavisnih strujnih izvora čvorova $\mathbf{I}_{0v}$.



Rješenje:



a) reducirana matrica incidencija **A**, temeljna spojna matrica **S**, temeljna rastavna matrica **Q**: (1 bod)

Reducirana matrica incidencija:

$$\mathbf{A}=\begin{bmatrix}-1 & 1 & 0 & 0\\ 0 & -1 & 1 & 1\end{bmatrix}$$

Temeljna spojna matrica:

$$\mathbf{S}=\begin{bmatrix}1 & 1 & 1 & 0\\ 1 & 1 & 0 & 1\end{bmatrix}$$

Temeljna rastavna matrica:

$$\mathbf{Q}=\begin{bmatrix}0 & 1 & -1 & -1\\ 1 & 0 & -1 & -1\end{bmatrix}$$

Naponsko – strujne relacije grana: $\mathbf{I}_b=\mathbf{Y}_b\cdot\mathbf{U}_b+\mathbf{I}_{0b}$

$$I_1=sC\cdot U_1+I_g(s)$$

$$I_2=\frac{1}{R_1}\cdot U_2+\alpha\cdot I_3=\frac{1}{R_1}\cdot U_2+\frac{\alpha}{R_2}\cdot U_3$$

$$I_3=\frac{1}{R_2}\cdot U_3$$

$$I_4=\frac{1}{sL}\cdot U_4$$

Sustav jednadžbi grana u matričnom obliku:

$$I_b=\begin{bmatrix}sC & 0 & 0 & 0\\ 0 & \frac{1}{R_1} & \frac{\alpha}{R_2} & 0\\ 0 & 0 & \frac{1}{R_2} & 0\\ 0 & 0 & 0 & \frac{1}{sL}\end{bmatrix}\cdot\begin{bmatrix}U_1\\ U_2\\ U_3\\ U_4\end{bmatrix}+\begin{bmatrix}I_g\\ 0\\ 0\\ 0\end{bmatrix}.$$

b) matrica admitancija grana $\mathbf{Y}_b$: (1 bod)

$$\mathbf{Y}_b = \begin{bmatrix} sC & 0 & 0 & 0 \\ 0 & \frac{1}{R_1} & \frac{\alpha}{R_2} & 0 \\ 0 & 0 & \frac{1}{R_2} & 0 \\ 0 & 0 & 0 & \frac{1}{sL} \end{bmatrix}$$

c)vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$: (1 bod)

$$\mathbf{I}_{0b} = \begin{bmatrix} I_g \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Sustav jednadžbi čvorova

d) matrica $\mathbf{Y}_v$: (1 bod)

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} sC + \frac{1}{R_1} & \frac{\alpha}{R_2} - \frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{sL} + \frac{1-\alpha}{R_2} \end{bmatrix}$$

e) matrica $\mathbf{I}_{0v}$: (1 bod)

$$\mathbf{I}_{0v} = -\mathbf{A} \cdot \mathbf{I}_{0b} = \begin{bmatrix} I_g \\ 0 \end{bmatrix}$$

3. Za neki električni krug poznate su slijedeće matrice:

$$\mathbf{Z}_b=\begin{bmatrix} R & 0 & 0\\ \mu R & \dfrac{1}{sC} & 0\\ 0 & 0 & sL\end{bmatrix},\quad \mathbf{U}_{0b}=\begin{bmatrix} 0\\ \dfrac{u_C(0)}{s}\\ -I_0 sL+Li_L(0)\end{bmatrix},\quad \mathbf{A}=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & -1\end{bmatrix}.$$

Odrediti temeljni sustav jednadžbi petlji primjenom grafova i pritom: a) Nacrtati zadanu električnu shemu kruga. b) Nacrtati orijentirani graf i napisati spojnu matricu **S**. Napisati: c) matricu impedancija petlji $\mathbf{Z}_p$ i d) vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.

<u>Rješenje:</u>

Naponsko – strujne relacije grana: $\mathbf{U}_b=\mathbf{Z}_b\cdot\mathbf{I}_b+\mathbf{U}_{0b}$

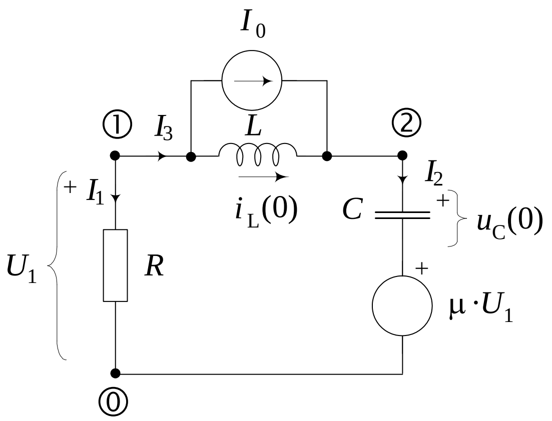
$$\begin{bmatrix}U_1\\U_2\\U_3\end{bmatrix}=\begin{bmatrix} R & 0 & 0\\ \mu R & \dfrac{1}{sC} & 0\\ 0 & 0 & sL\end{bmatrix}\cdot\begin{bmatrix}I_1\\I_2\\I_3\end{bmatrix}+\begin{bmatrix} 0\\ \dfrac{u_C(0)}{s}\\ -I_0 sL+Li_L(0)\end{bmatrix}$$

(1) $U_1=R\cdot I_1$

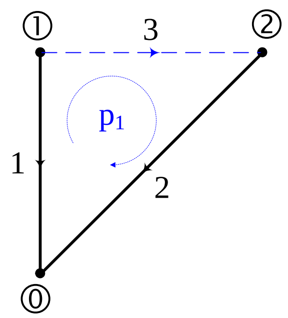
(2) $U_2=\mu R\cdot I_1+\dfrac{1}{sC}\cdot I_2+\dfrac{u_C(0)}{s}=\mu\cdot U_1+\dfrac{1}{sC}\cdot I_2+\dfrac{u_C(0)}{s}$

(3) $U_3=sL\cdot I_3-I_0sL+Li_L(0)=(I_3-I_0)sL+Li_L(0)$

(1 bod)



(1 bod)



(1 bod)

Spojna matrica: $\mathbf{S}=\begin{bmatrix}-1 & 1 & 1\end{bmatrix}$

Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p\cdot\mathbf{I}_p=\mathbf{U}_{0p}$

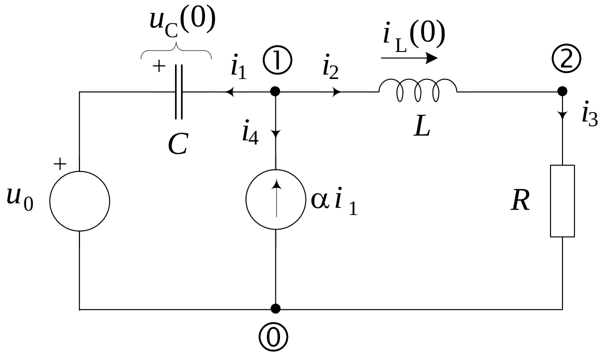
$$\mathbf{Z}_p=\mathbf{S}\cdot\mathbf{Z}_b\cdot\mathbf{S}^T=\begin{bmatrix}-1 & 1 & 1\end{bmatrix}\cdot\begin{bmatrix} R & 0 & 0\\ \mu R & \dfrac{1}{sC} & 0\\ 0 & 0 & sL\end{bmatrix}\cdot\begin{bmatrix}-1\\1\\1\end{bmatrix}=$$

(1 bod)

$$=\begin{bmatrix}-R+\mu R & \dfrac{1}{sC} & sL\end{bmatrix}\cdot\begin{bmatrix}-1\\1\\1\end{bmatrix}=\begin{bmatrix}R(1-\mu)+\dfrac{1}{sC}+sL\end{bmatrix}$$

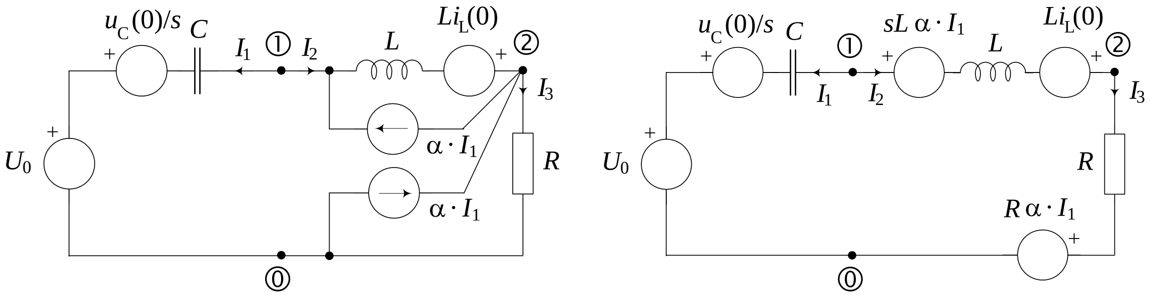
$$\mathbf{U}_{0p}=-\mathbf{S}\cdot\mathbf{U}_{0b}=-\begin{bmatrix}-1 & 1 & 1\end{bmatrix}\cdot\begin{bmatrix} 0\\ \dfrac{u_C(0)}{s}\\ -I_0 sL+Li_L(0)\end{bmatrix}=\begin{bmatrix}-\dfrac{u_C(0)}{s}+I_0sL-Li_L(0)\end{bmatrix}$$ (1 bod)

2. Za električni krug prikazan slikom i pridruženim orijentacijama grana te čvorovima (grane stabla: 1, 2) treba odrediti temeljni sustav jednadžbi petlji primjenom grafova. a) Na usamljeni strujni izvor u grani 4 treba primijeniti postupak posmicanja strujnog izvora (pritom grana 4 nestaje). b) Nacrtati orijentirani graf i napisati spojnu matricu **S**. Napisati: c) matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$, d) matricu impedancija petlji $\mathbf{Z}_p$ i e) vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.



Rješenje:

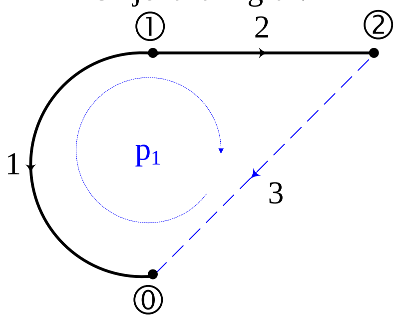
a) Primjena Laplaceove transformacije i posmicanje strujnog izvora:



(1 bod)

b) Orijentirani graf i spojna matrica:

Orijentirani graf:



Spojna matrica:

$$\mathbf{S} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}$$

(1 bod)

c) Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

(1) $U_1 = \frac{1}{sC} \cdot I_1 - \frac{u_C(0)}{s} + U_0$

(2) $U_2 = sL \cdot I_2 + sL\alpha \cdot I_1 - Li_L(0)$

(3) $U_3 = R \cdot I_3 + R \cdot \alpha \cdot I_1$

$$\mathbf{Z}_b = \begin{bmatrix} \frac{1}{sC} & 0 & 0 \\ sL\alpha & sL & 0 \\ R\alpha & 0 & R \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} -\frac{u_C(0)}{s} + U_0(s) \\ -Li_L(0) \\ 0 \end{bmatrix}$$

(1 bod)

Matrica $\mathbf{Z}_b$ je regularna.

d) i e) Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

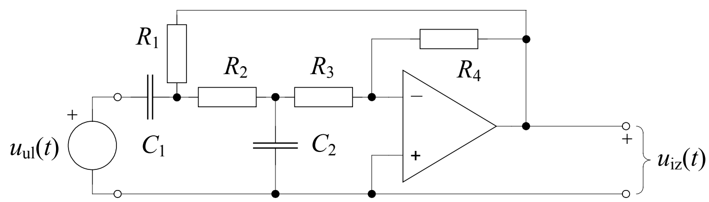
$$\mathbf{S} \cdot \mathbf{Z}_b = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{sC} & 0 & 0 \\ sL\alpha & sL & 0 \\ R\alpha & 0 & R \end{bmatrix} = \begin{bmatrix} -\frac{1}{sC} + sL\alpha + R\alpha & sL & R \end{bmatrix}$$

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} -\frac{1}{sC} + sL\alpha + R\alpha & sL & R \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{sC} - (sL + R)\alpha + sL + R \end{bmatrix} \text{ (1 bod)}$$

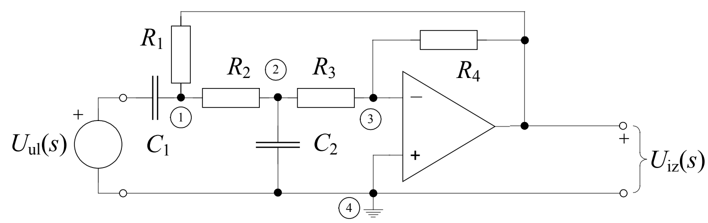
$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{u_C(0)}{s} + U_0(s) \\ -Li_L(0) \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{u_C(0)}{s} + U_0(s) + Li_L(0) \end{bmatrix} \text{ (1 bod)}$$

PRIJENOSNE FUNCIJE

3. Odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom. Nacrtati raspored polova i nula te funkcije u kompleksnoj s-ravnini. Izračunati i nacrtati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$. Zadano je $R_1=R_2=R_3=1$, $R_4=2$, $C_1=C_2=1$.



Rješenje: Primjena Laplaceove transformacije



Metoda napona čvorova (čvorište 4 je referentno) :

(1) $$U_{ul}(s)sC_1+U_{iz}(s)\frac{1}{R_1}=U_1\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)-U_2\frac{1}{R_2}$$

(2) $$0=-U_1\frac{1}{R_2}+U_2\left(sC_2+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

(3) $$U_{iz}\frac{1}{R_4}=-U_2\frac{1}{R_3}+U_3\left(\frac{1}{R_3}+\frac{1}{R_4}\right)$$

$$A\to\infty,\quad U_3=0\ \Rightarrow\ (3)\ \ U_{iz}(s)\frac{1}{R_4}=-U_2\frac{1}{R_3}\Big/\cdot R_4$$

$$(3)\Rightarrow U_{iz}=-\frac{R_4}{R_3}U_2$$

$$(2)\Rightarrow U_1=\left(sR_2C_2+1+\frac{R_2}{R_3}\right)U_2\ \to(1)\Rightarrow U_{ul}sC_1+U_{iz}\frac{1}{R_1}=\left(\left(sR_2C_2+1+\frac{R_2}{R_3}\right)\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{1}{R_2}\right)U_2$$

$$U_{ul}(s)sC_1+U_{iz}(s)\frac{1}{R_1}=-\frac{R_3}{R_4}\left(\left(sR_2C_2+\frac{R_2}{R_3}\right)\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)+sC_1+\frac{1}{R_1}\right)U_{iz}$$

$$\frac{U_{iz}}{U_{ul}}=-\frac{R_4}{R_3}\frac{sC_1}{s^2R_2C_1C_2+sC_1\frac{R_2}{R_3}+sR_2C_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{R_2}{R_3}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+sC_1+\left(1+\frac{R_4}{R_3}\right)\frac{1}{R_1}}$$

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{-\frac{R_4}{R_3}\cdot\frac{s}{R_2C_2}}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}+\frac{1}{R_3C_2}\right)+\frac{R_1+R_2+R_3+R_4}{R_1R_2R_3C_1C_2}}$$

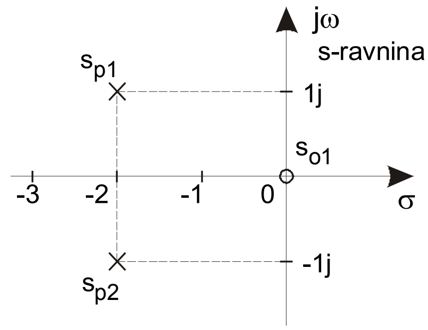
Uvrštenjem vrijednosti elemenata:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{-\frac{2}{1}\frac{1}{1\cdot 1}\cdot s}{s^2+(1+1+1+1)\cdot s+\frac{1+1+1+2}{1\cdot 1\cdot 1\cdot 1}}=\frac{-2\cdot s}{s^2+4\cdot s+5}$$

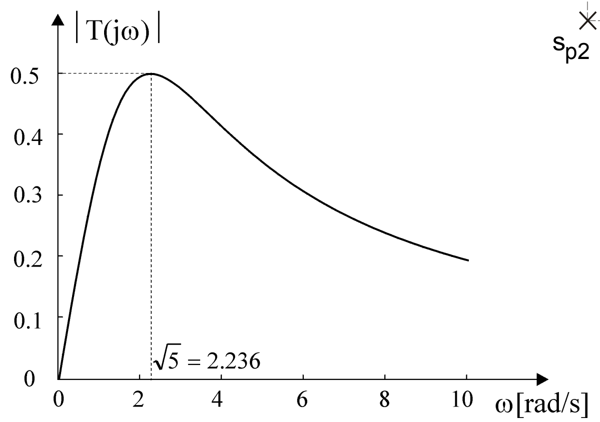
nule: $s_{o1}=0,\quad s_{o2}=\infty$

polovi: $s^2+4\cdot s+5=0 \Rightarrow s_{p1,2}=-2\pm\sqrt{4-5}=-2\pm j$

$$T(j\omega)=\frac{-2\cdot j\omega}{-\omega^2+4\cdot j\omega+5}\Rightarrow |T(j\omega)|=\frac{2\cdot|\omega|}{\sqrt{(5-\omega^2)^2+16\omega^2}}$$



a-f karakteristika:



2. jednostavniji način: odmah uvrstiti vrijednosti elemenata u jednadžbe čvorova :

(1) $\quad U_{ul}s+U_{iz}=U_1(s+2)-U_2$

(2) $\quad 0=-U_1+U_2(s+2)$

(3) $\quad \frac{U_{iz}}{2}=-U_2+U_3\frac{3}{2}$

$A\to\infty,\quad U_3=0 \;\Rightarrow\; (3)\; U_2=-\frac{U_{iz}}{2}$

$(2)\Rightarrow U_1=U_2(s+2)=-U_{iz}\frac{(s+2)}{2}$

$$U_{ul}s+U_{iz}=-U_{iz}\frac{(s+2)}{2}(s+2)+U_{iz}\frac{1}{2}$$

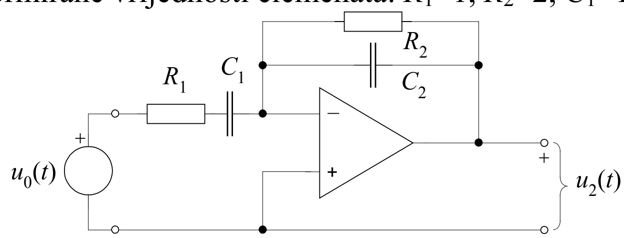
$$2U_{ul}s=-(s^2+4s+5)U_{iz}$$

$$\frac{U_{iz}}{U_{ul}}=-\frac{2s}{s^2+4s+5}$$

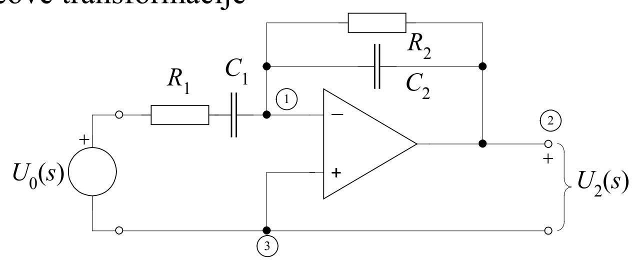
$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{-2\cdot s}{s^2+4\cdot s+5},$$

ostali dio postupka je isti.

4. Za mrežu prikazanu slikom odrediti napon na izlazu operacijskog pojačala $u_2(t)$, ako je pobuda $u_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=2$, $C_1=1$, $C_2=1$, $A\to\infty$.



Rješenje: Primjena Laplaceove transformacije



(1) $$\frac{U_0}{\frac{1}{sC_1}+R_1}=U_1\left(\frac{1}{\frac{1}{sC_1}+R_1}+sC_2+\frac{1}{R_2}\right)-U_2\left(sC_2+\frac{1}{R_2}\right)$$

$$U_1=0,\quad \text{jer}\ \ A\to\infty$$

(1) $$\Rightarrow\qquad U_2=-\frac{U_0}{\left(\frac{1}{sC_1}+R_1\right)\left(sC_2+\frac{1}{R_2}\right)}=-\frac{U_0}{\frac{R_1}{s}\left(s+\frac{1}{R_1C_1}\right)C_2\left(s+\frac{1}{R_2C_2}\right)}$$

$$U_2=-\frac{\frac{1}{R_1C_2}s}{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}U_0=-\frac{1\cdot s\cdot\frac{1}{s}}{\left(s+\frac{1}{2}\right)(s+1)}=\frac{-1}{\left(s+\frac{1}{2}\right)(s+1)}$$

Rastav na parcijalne razlomke:

$$U_2(s)=\frac{-1}{\left(s+\frac{1}{2}\right)(s+1)}=\frac{A}{s+\frac{1}{2}}+\frac{B}{s+1}=\frac{A(s+1)+B\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)(s+1)}$$

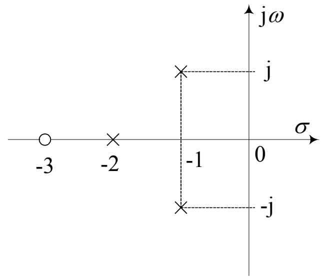
$$A+B=0\ \Rightarrow\ B=-A$$
$$A+\frac{B}{2}=-1\ \Rightarrow\ A-\frac{A}{2}=-1$$

$$\frac{A}{2}=-1\ \Rightarrow\ A=-2$$
$$\underline{B=-A=2}$$

$$U_2(s)=\frac{-2}{s+\frac{1}{2}}+\frac{2}{s+1}$$

Konačno je: $u_2(t)=\left(-2e^{-\frac{t}{2}}+2e^{-t}\right)\cdot S(t)$

4. Zadan je raspored polova i nula prijenosne funkcije $H(s)=U_{iz}(s)/U_{ul}(s)$ nekoga električnog kruga prema slici. Odrediti prijenosnu funkciju ako se traži da bude $|H(\mathrm{j}\omega)|=1$ za $\omega=1$. Odrediti fazor odziva ako je pobuda $u_{ul}(t)=\cos(3t+20^\circ)$.



Rješenje:

Prijenosna funkcija

$$H(s)=\frac{k(s+3)}{(s+1+\mathrm{j})(s+1-\mathrm{j})(s+2)}=\frac{k(s+3)}{(s+2)(s^2+2s+2)}$$

$$H(\mathrm{j}\omega)\Big|_{\omega=1}=\left|\frac{k(\mathrm{j}\omega+3)}{(\mathrm{j}\omega+2)(-\omega^2+2\mathrm{j}\omega+2)}\right|_{\omega=1}=\left.\frac{k\sqrt{\omega^2+9}}{\sqrt{\omega^2+4}\sqrt{(2-\omega^2)^2+4\omega^2}}\right|_{\omega=1}=k\sqrt{\frac{2}{5}}=1 \qquad \Rightarrow \ k=\sqrt{\frac{5}{2}}$$

$$H(s)=\sqrt{\frac{5}{2}}\cdot\frac{(s+3)}{(s+2)(s^2+2s+2)}$$

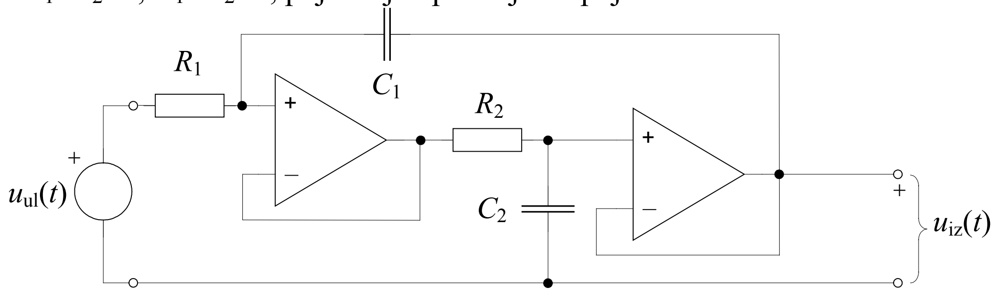
Fazor odziva

$$U_{iz}(\mathrm{j}\omega)=H(\mathrm{j}\omega)\cdot U_{ul}(\mathrm{j}\omega)=\sqrt{\frac{5}{2}}\cdot\frac{(\mathrm{j}\omega+3)\cdot U_{ul}(\mathrm{j}\omega)}{(\mathrm{j}\omega+2)(-\omega^2+2\mathrm{j}\omega+2)}$$

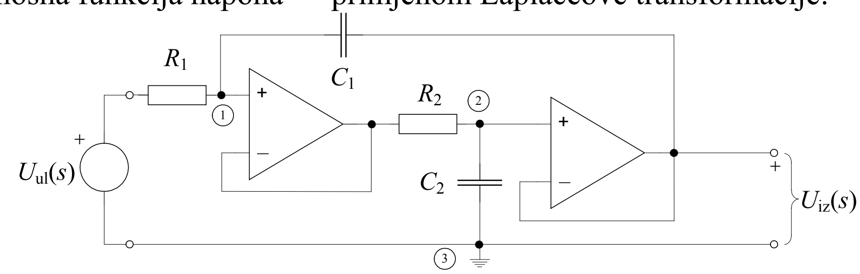
$$U_{ul}(\mathrm{j}\omega)\Big|_{\omega=3}=1\angle 20^\circ=e^{j20^\circ}=\cos(20^\circ)+j\sin(20^\circ)=0{,}9397+j0{,}342$$

$$U_{iz}(\mathrm{j}\omega)\Big|_{\omega=3}=\sqrt{\frac{5}{2}}\cdot\frac{(\mathrm{j}3+3)\cdot U_{ul}(\mathrm{j}\omega)}{(\mathrm{j}3+2)(-7+6\mathrm{j})}=\sqrt{\frac{5}{2}}\cdot\frac{3\sqrt{2}\cdot e^{j45^\circ}\cdot e^{j20^\circ}}{\sqrt{13}\cdot e^{j56{,}3^\circ}\sqrt{85}\cdot e^{j139{,}4^\circ}}=0{,}201\cdot e^{-j130{,}7^\circ}$$

5. Za električni krug prikazan slikom: *a*) Odrediti prijenosnu funkciju napona $H(s)=U_{iz}(s)/U_{ul}(s)$;
b) Izračunati polove i nule prijenosne funkcije i prikazati njihov raspored u *s*-ravnini;
c) Izračunati i skicirati amplitudno-frekvencijsku karakteristiku $|H(j\omega)|$;
d) Izračunati i skicirati logaritamsku mjeru pojačanja $\alpha(\omega)$.
Zadano je: $R_1=R_2=1$, $C_1=C_2=1$, pojačanje operacijskih pojačala $A\to\infty$.



Rješenje: *a*) Prijenosna funkcija napona — primjenom Laplaceove transformacije:



Metoda napona čvorova (čvorište 3 je referentno) :

(1) $$\frac{U_1-U_0}{R_1}+(U_1-U_2)sC_1=0 \quad\Rightarrow\quad U_0=U_1(1+sC_1R_1)-U_2sC_1R_1$$

(2) $$\frac{U_2-U_1}{R_1}+U_2sC_2=0 \quad\Rightarrow\quad U_1=U_2(1+sC_2R_2)$$

$$\Rightarrow\quad U_0=\left[(1+sC_1R_1)(1+sC_2R_2)-sC_1R_1\right]U_2$$

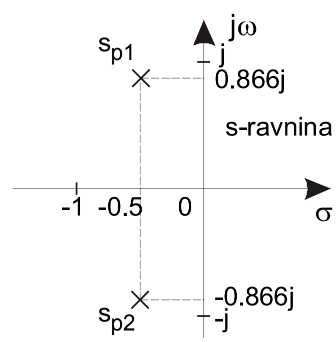
$$\Rightarrow\quad U_2=\frac{U_0}{s^2C_1R_1C_2R_2+sC_2R_2+1}=\frac{\dfrac{U_0}{C_1R_1C_2R_2}}{s^2+s\dfrac{1}{C_1R_1}+\dfrac{1}{C_1R_1C_2R_2}}$$

$$H(s)=\frac{U_{iz}}{U_0}=\frac{\dfrac{1}{C_1R_1C_2R_2}}{s^2+s\dfrac{1}{C_1R_1}+\dfrac{1}{C_1R_1C_2R_2}}=\frac{1}{s^2+s+1}$$

b) Polovi i nule:

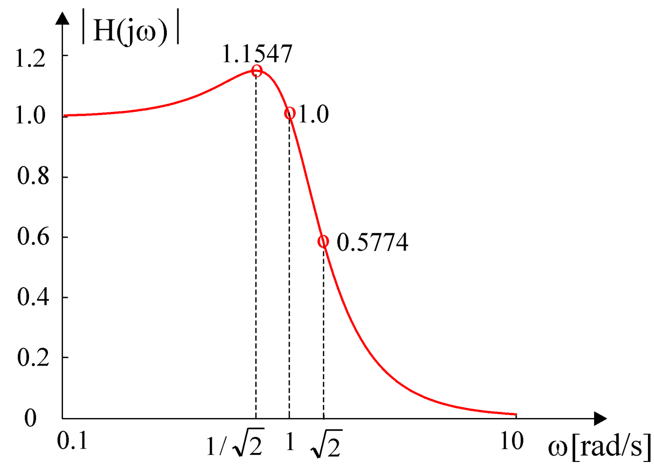
polovi: $s^2+s+1=0\Rightarrow s_{p1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-1}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$

nule: $s_{o1,2}\to\infty$



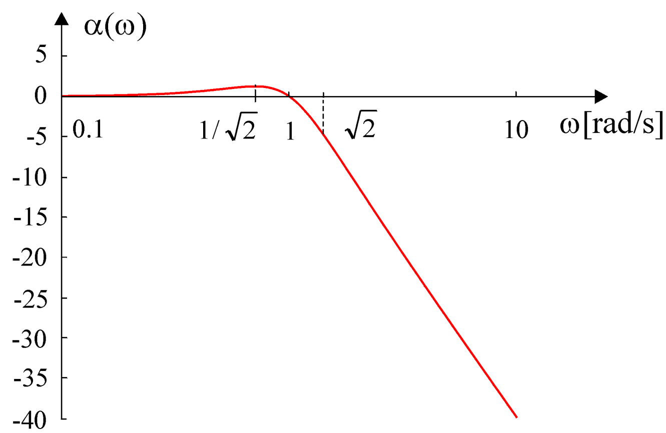
c) A-f karakteristika $|H(j\omega)|$:

$$s = j\omega \Rightarrow \; H(j\omega) = \frac{1}{1-\omega^2 + j\omega} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + \omega^2}} = \frac{1}{\sqrt{1-\omega^2 + \omega^4}}$$



d) Logaritamska mjera pojačanja $\alpha(\omega)$:

$$\alpha(\omega) = 20\log|H(j\omega)| = -10\log(\omega^4 - \omega^2 + 1)$$



c) ukupna naponska prijenosna funkcija $T(s) = \dfrac{U_{iz2}}{U_{ul}} = T_1(s) \cdot T_2(s) = \dfrac{U_{iz1}}{U_{ul}} \cdot \dfrac{U_{iz2}}{U_{iz1}} = \dfrac{2}{sRC} \cdot \dfrac{\frac{1}{RC}}{s + \frac{1}{RC}}$

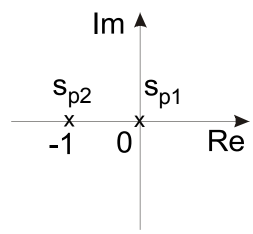
Uz uvrštene normalizirane vrijednosti elemenata R=1 i C=1 :

$$T(s) = \frac{U_{iz2}}{U_{ul}} = \frac{2}{s} \cdot \frac{1}{s+1} = \frac{2}{s^2 + s}$$

Raspored nula i polova u s-ravnini:

Polovi : $s(s+1) = 0 \Rightarrow s_{p1} = 0, s_{p2} = -1$

Nule: $s_{o1,2} = \infty$



Amplitudno-frekvencijska karakteristika:

$$T(j\omega) = \frac{U_{iz2}}{U_{ul}} = \frac{2}{-\omega^2 + j\omega} \quad \Rightarrow \quad \left|T(j\omega)\right| = \frac{U_{iz2}}{U_{ul}} = \frac{2}{\sqrt{\omega^4 + \omega^2}} = \frac{2}{\omega\sqrt{1+\omega^2}}$$

Crtanje a-f karakteristike točku po točku :

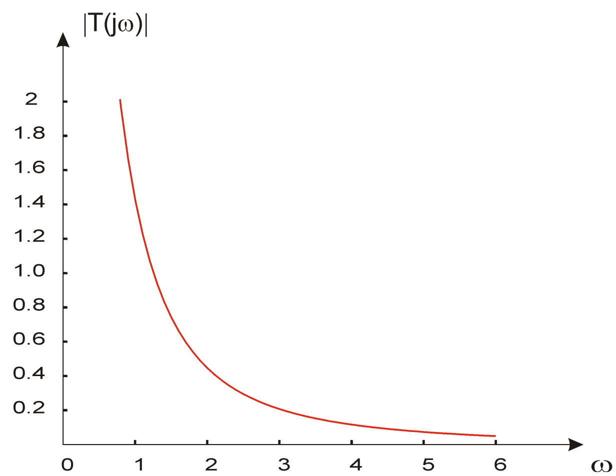
$\omega = 0 \quad \Rightarrow \quad \left|T(j\omega)\right| = \infty$

$\omega = 1 \quad \Rightarrow \quad \left|T(j\omega)\right| = \dfrac{2}{\sqrt{2}} = \sqrt{2} = 1.41$

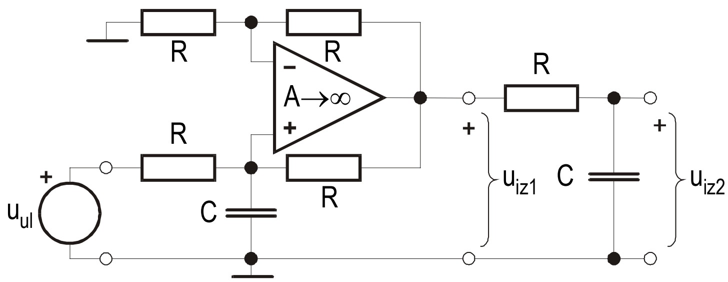
$\omega = 2 \quad \Rightarrow \quad \left|T(j\omega)\right| = \dfrac{2}{2\sqrt{1+4}} = \dfrac{1}{\sqrt{5}} = 0.4472$

$\omega = 3 \quad \Rightarrow \quad \left|T(j\omega)\right| = \dfrac{2}{3\sqrt{10}} = 0.2108$

$\omega = \infty \quad \Rightarrow \quad \left|T(j\omega)\right| = 0$

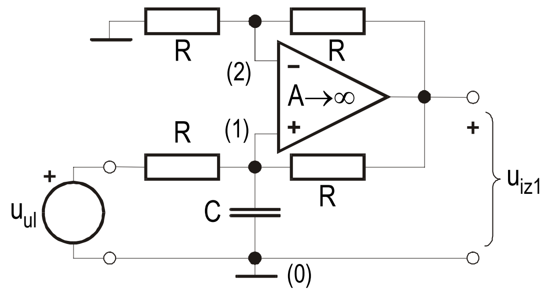


4. Za električni krug prikazan slikom odrediti naponske prijenosne funkcije $T_1(s)=U_{iz1}(s)/U_{ul}(s)$ i $T(s)=U_{iz2}(s)/U_{ul}(s)$. Nacrtati raspored polova u kompleksnoj ravnini te amplitudno-frekvencijsku karakteristiku ukupne prijenosne funkcije $T(s)$. Zadane su normalizirane vrijednosti R=1, C=1.



Rješenje:

a) prva naponska prijenosna funkcija $T_1(s)= U_{iz1}(s)/U_{ul}(s)$:
Jednadžbe čvorova:



(1) $$U_1\left(\frac{1}{R}+\frac{1}{R}+sC\right)-U_{ul}\frac{1}{R}=\frac{U_{iz1}}{R}$$

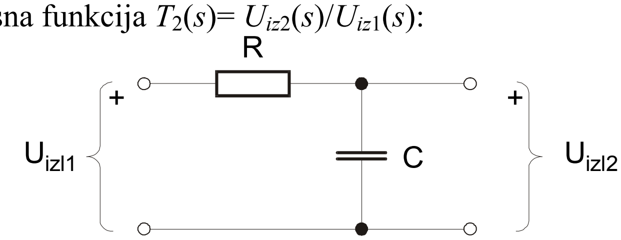
(2) $$U_2\left(\frac{1}{R}+\frac{1}{R}\right)-U_{iz1}\frac{1}{R}=0 \quad\Rightarrow\quad U_2\frac{2}{R}=U_{iz1}\frac{1}{R} \quad\Rightarrow\quad U_2=\frac{U_{iz1}}{2}$$

(3) $$U_{iz1}=A(U_1-U_2)\big/:A,\quad A\to\infty \quad\Rightarrow\quad U_1=U_2$$

(1) $$\frac{U_{iz1}}{2}\left(\frac{2}{R}+sC\right)-U_{ul}\frac{1}{R}=\frac{U_{iz1}}{R}\Big/\cdot R \quad\Rightarrow\quad U_{iz1}+U_{iz1}\frac{sRC}{2}-U_{iz1}=U_{ul}$$

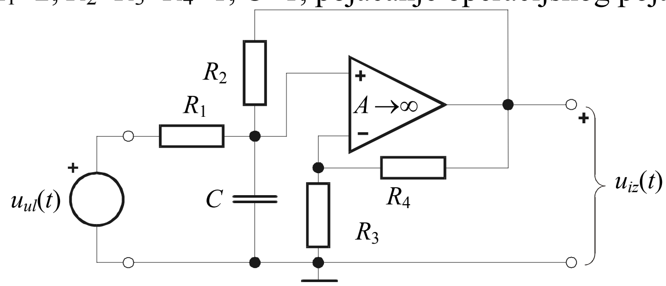
$$\Rightarrow\quad U_{iz1}=\frac{1}{\frac{sRC}{2}}U_{ul} \quad\Rightarrow\quad T_1(s)=\frac{U_{iz1}}{U_{ul}}=\frac{2}{sRC}$$

b) druga naponska prijenosna funkcija $T_2(s)= U_{iz2}(s)/U_{iz1}(s)$:



$$T_2(s)=\frac{U_{iz2}}{U_{iz1}}=\frac{\frac{1}{sC}}{R+\frac{1}{sC}}=\frac{\frac{1}{RC}}{s+\frac{1}{RC}}$$

4. Za električni krug prikazan slikom: *a*) Odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$;
b) Izračunati polove i nule prijenosne funkcije i prikazati njihov raspored u *s*-ravnini;
c) Odrediti odziv mreže $u_{iz}(t)$ na pobudu $u_{ul}(t)=S(t)$. Skicirati odziv. Da li je električni krug stabilan? Zadano je: $R_1=2$, $R_2=R_3=R_4=1$, $C=1$, pojačanje operacijskog pojačala $A\to\infty$.



Rješenje: *a*) Naponska prijenosna funkcija: čvor (1) je na (+) ulazu u operacijsko pojačalo, a čvor (2) na (–) ulazu u operacijsko pojačalo. Slijede jednadžbe čvorova:

$$(1)\qquad \left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)U_1(s)-\frac{1}{R_1}U_{ul}(s)-\frac{1}{R_2}U_{iz}(s)=0$$

$$(2)\qquad \left(\frac{1}{R_3}+\frac{1}{R_4}\right)U_2(s)-\frac{1}{R_4}U_{iz}(s)=0$$

$$(3)\qquad A\left[U_1(s)-U_2(s)\right]=U_{iz}(s)\Rightarrow U_1(s)-U_2(s)=\frac{U_{iz}(s)}{A}$$

$$A\to\infty\Rightarrow U_1(s)=U_2(s)$$

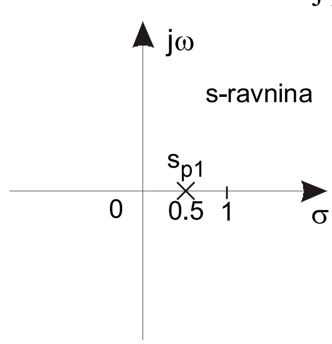
$$(2)\Rightarrow\quad \left(\frac{R_4}{R_3}+1\right)U_2(s)=U_{iz}(s),(3)\Rightarrow U_2(s)=U_1(s)\to(1)$$

$$(1)\qquad \left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)\frac{U_{iz}(s)}{\left(\frac{R_4}{R_3}+1\right)}-\frac{1}{R_1}U_{ul}(s)-\frac{1}{R_2}U_{iz}(s)=0$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+sC-\frac{1}{R_2}\frac{R_4}{R_3}-\frac{1}{R_2}\right)U_{iz}(s)=\frac{1}{R_1}\left(\frac{R_4}{R_3}+1\right)U_{ul}(s)$$

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{\frac{1}{R_1}\left(\frac{R_4}{R_3}+1\right)}{sC+\frac{1}{R_1}-\frac{1}{R_2}\frac{R_4}{R_3}}=\frac{0.5(1+1)}{s+0.5-1}=\frac{1}{s-0.5}$$

b) Polovi i nule:
polovi: $s-0.5=0\Rightarrow s_{p1}=0.5$ (pol je realan i nalazi se u desnoj poluravnini); nule: $s_{o1}\to\infty$



c) Odziv na Step $u_{ul}(t)=S(t)\Rightarrow U_{ul}(s)=\frac{1}{s}$

$$U_{iz}(s)=T(s)U_{ul}(s)=\frac{1}{s-0.5}\frac{1}{s}=\frac{A}{s-0.5}+\frac{B}{s}=\frac{As+Bs-0.5B}{(s-0.5)s}$$

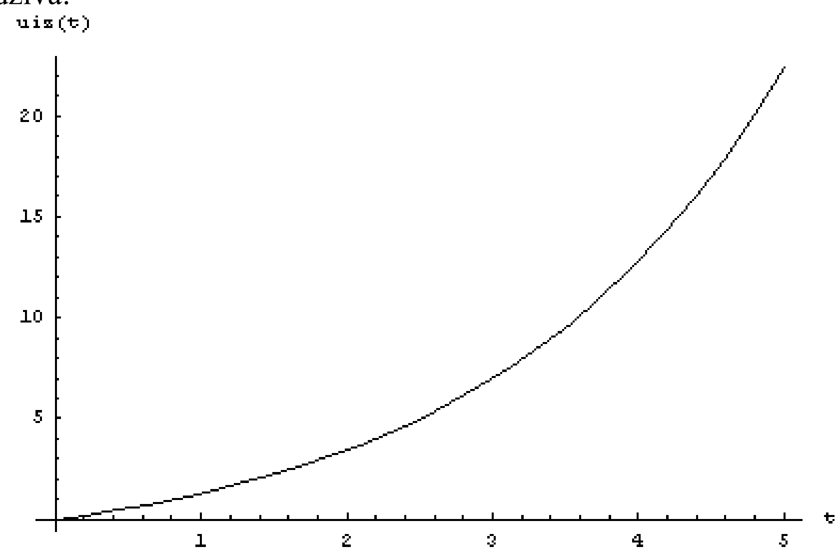
$$A+B=0\Rightarrow A=-B=2$$

$$-0.5B=1\Rightarrow B=-2$$

$$U_{iz}(s)=\frac{2}{s-0.5}-\frac{2}{s}$$

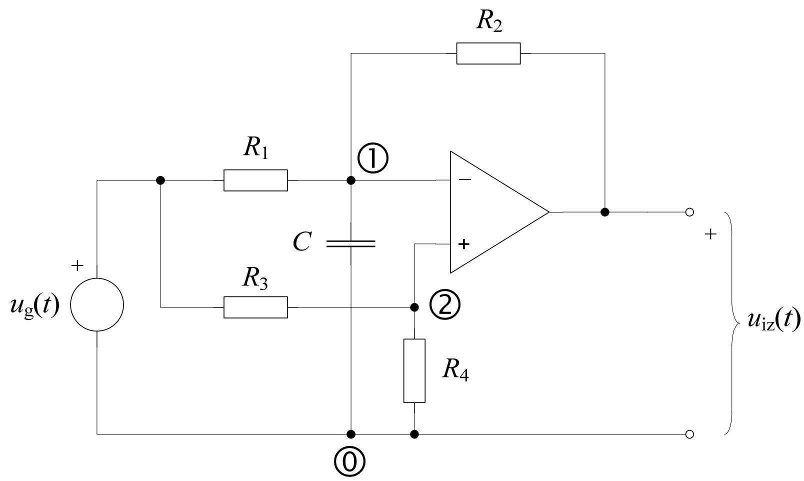
$$\underline{u_{iz}(t)=2e^{0.5t}S(t)-2S(t)=2(e^{0.5t}-1)S(t)}$$

Skica odziva:



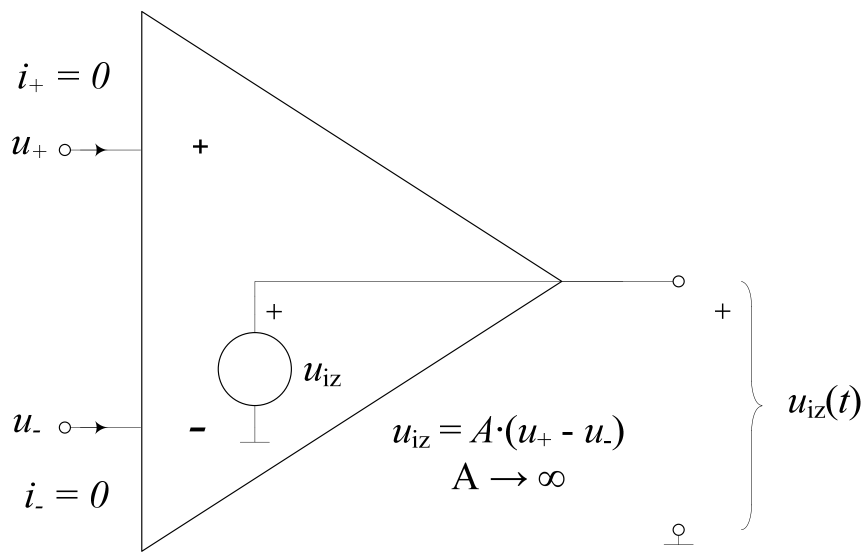
Odgovor: Odziv je raspirujući stoga je električni krug nestabilan.
To je zato jer se pol nalazi u desnoj poluravnini s-ravnine.

5. Zadan je električni krug prema slici. Odrediti napon na izlazu $u_{iz}(t)$ ako je zadano: $R_1 = R_2 = R_3 = 1$, $R_4 = 2$, $C = 1$, $u_g(t) = S(t)$. Početni uvjeti su jednaki nula. Koristiti metodu napona čvorova.



Rješenje:

Nadomjesni sklop operacijskog pojačala:



Jednadžbe čvorova:

$$1)\ U_1\left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)-U_{iz}\frac{1}{R_2}-U_g\frac{1}{R_1}=0$$

$$2)\ U_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)-U_g\frac{1}{R_3}=0$$

$$U_{iz}=A(U_2-U_1)$$

$$\frac{U_{iz}}{A}=U_2-U_1 \qquad A\to\infty$$

$$\Rightarrow U_1=U_2$$

$$1)\ U_1\left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)-U_{iz}\frac{1}{R_2}-U_g\frac{1}{R_1}=0\Rightarrow U_{iz}=U_1R_2\left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)-U_g\frac{R_2}{R_1}$$

$$2)\ U_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)-U_g\frac{1}{R_3}=0\Rightarrow U_2=U_1=U_g\frac{1}{R_3\cdot\frac{R_3+R_4}{R_3R_4}}=U_g\frac{R_4}{R_3+R_4}$$

$\downarrow$

$$1)\ U_{iz}=U_g\frac{R_4}{R_3+R_4}R_2\left(\frac{R_1+R_2+sR_1R_2C}{R_1R_2}\right)-U_g\frac{R_2}{R_1}=U_g\left(\frac{R_4R_2}{R_3+R_4}\left(\frac{R_1+R_2+sR_1R_2C}{R_1R_2}\right)-\frac{R_2}{R_1}\right)$$

$$\Rightarrow U_g\left(\frac{R_4R_2}{R_3+R_4}\frac{R_1+R_2}{R_1R_2}-\frac{R_2}{R_1}+\frac{R_4R_2}{R_3+R_4}\frac{sR_1R_2C}{R_1R_2}\right)=U_g\left(\frac{R_4}{R_3+R_4}\frac{R_1+R_2}{R_1}-\frac{R_2}{R_1}+\frac{R_4R_2}{R_3+R_4}sC\right)=$$

$$U_{iz}=U_g\left(\frac{R_1R_4+R_2R_4}{R_1(R_3+R_4)}-\frac{R_2R_3+R_2R_4}{R_1(R_3+R_4)}+\frac{R_4R_2}{R_3+R_4}sC\right)=U_g\frac{1}{R_3+R_4}\left(\frac{R_1R_4-R_2R_3}{R_1}+sR_4R_2C\right)$$

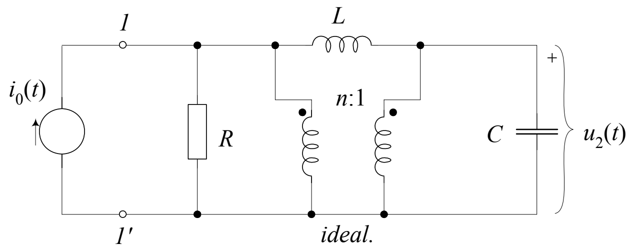
Uvrstimo vrijednosti:

$$\Rightarrow U_{iz}(s)=\frac{1}{3}\left(\frac{2-1}{1}+2s\right)=\frac{1}{3}+\frac{2}{3}s$$

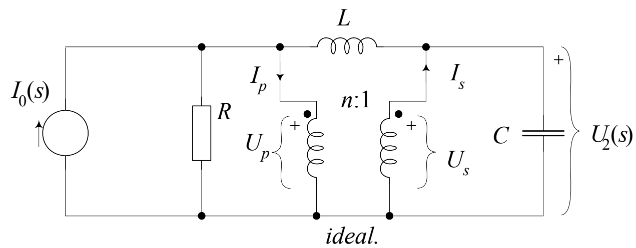
$$\Rightarrow u_{iz}(t)=\frac{1}{3}S(t)+\frac{2}{3}\delta(t)$$

DRUGI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2009

1. Za električni krug na slici s pobudom $i_0(t)$ i normiranim vrijednostima elemenata: R =2, L=1 i C=1, te prijenosnim omjerom transformatora n=2 odrediti: *a*) ulaznu impedanciju $Z_{ul}(s)$ gledanu s priključnica 1-1'; *b*) prijenosnu impedanciju $Z_{21}(s)=U_2(s)/I_0(s)$; *c*) polove i nule funkcije $Z_{21}(s)$i njihov prikaz u s-ravnini; *d*) odziv $u_2(t)$ ako je zadana pobuda $i_0(t)=S(t)$, *e*) odziv $u_2(t)$ ako je $i_0(t)$ stacionarna sinusna pobuda valnoga oblika $i_0(t)$=2 cos(t).



Rješenje: Jednadžbe čvorišta



(1) $$I_0(s)-I_p(s)=\left(\frac{1}{R}+\frac{1}{sL}\right)U_p(s)-\frac{1}{sL}U_s(s)$$ $$U_p(s)=nU_s(s)$$

(2) $$I_s(s)=-\frac{1}{sL}U_p(s)+\left(sC+\frac{1}{sL}\right)U_s(s)$$ $$I_p(s)=\frac{1}{n}I_s(s)$$

---

(1) $$I_0-\frac{I_s}{n}=\left(\frac{1}{R}+\frac{1}{sL}\right)nU_s-\frac{1}{sL}U_s$$

(2) $$I_s(s)=\left(sC+\frac{1-n}{sL}\right)U_s(s)$$ $$I_0=\frac{1}{n}\left(sC+\frac{1-n}{sL}\right)U_s(s)+\left(\frac{1}{R}+\frac{1}{sL}\right)nU_s-\frac{1}{sL}U_s$$

$$U_s(s)=\frac{I_0}{\frac{sC}{n}+\frac{1-n}{nsL}+\frac{n}{R}+\frac{n-1}{sL}}=\frac{nI_0s}{C\left(s^2+s\frac{n^2}{RC}+\frac{(n-1)^2}{LC}\right)}=\frac{2sI_0}{s^2+2s+1}$$

a) $$Z_{ul}(s)=\frac{U_p}{I_0}=\frac{nU_s}{I_0}=\frac{n^2s}{C\left(s^2+s\cdot n^2/RC+(n-1)^2/LC\right)}=\frac{4s}{s^2+2s+1}=\frac{4s}{(s+1)^2}$$ (1 bod)

b) $$Z_{21}(s)=\frac{U_s}{I_0}=\frac{ns}{C\left(s^2+s\cdot n^2/(RC)+(n-1)^2/(LC)\right)}=\frac{2s}{s^2+2s+1}=\frac{2s}{(s+1)^2}$$ (1 bod)

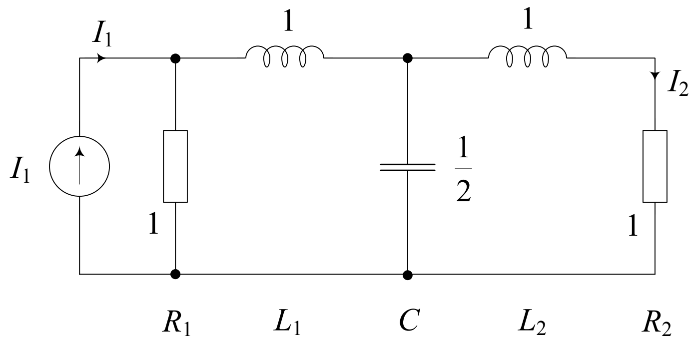
c) nula s=0, polovi: s_{p12}=–1 (1 bod)

d) $u_s(t)=2\cdot t\cdot e^{-t}S(t)$ (1 bod)

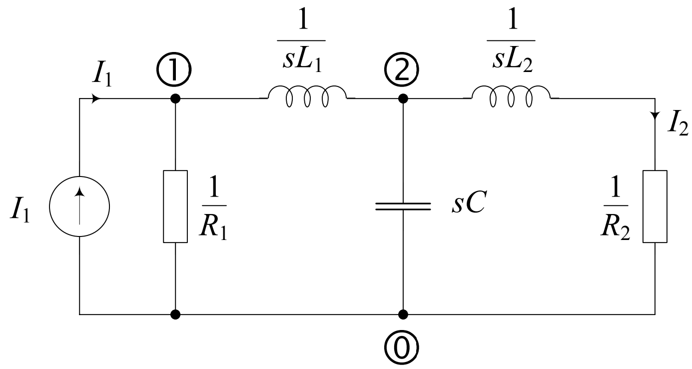
e) Fazori $i_0(t)=2\cdot\cos(t)$ $I_0(j\omega)=2\cdot e^{j0}=2$, $\omega=1$ $U_s(j\omega)=Z_{21}(j\omega)I_0(j\omega)=\frac{2j\omega}{-\omega^2+2j\omega+1}\cdot 2=2$

$u_s(t)=2\cdot\cos(t)$ (1 bod)

3. Za mrežu na slici izračunati prijenosni omjer struja $H_i(s) = I_2(s)/I_1(s)$ ako su zadane normalizirane vrijednosti elemenata: R_1=R_2=1, L_1=L_2=1 i C=1/2. (Koristiti metodu napona čvorova)



Rješenje: Primjena $\mathcal{L}$ − transformacije:



1) $U_1\left(\frac{1}{R_1}+\frac{1}{sL_1}\right)-U_2\left(\frac{1}{sL_1}\right)=I_1$

2) $-U_1\frac{1}{sL_1}+U_2\left(\frac{1}{sL_1}+sC+\frac{1}{sL_2+R_2}\right)=0$

3) $I_2=\frac{U_2}{sL_2+R_2}$ (2 boda)

Postupak: najprije naći $\frac{U_2}{I_1}$ (eliminacijom U_1), pa se uz $\frac{U_2}{sL_2+R_2}=I_2$ lako dobije $H_i(s)=\frac{I_2(s)}{I_1(s)}$.

2) $\Rightarrow U_1=U_2\cdot sL_1\cdot\left(\frac{1}{sL_1}+sC+\frac{1}{sL_2+R_2}\right)\rightarrow 1)$

1) $U_2\cdot sL_1\cdot\left(\frac{1}{sL_1}+sC+\frac{1}{sL_2+R_2}\right)\left(\frac{1}{R_1}+\frac{1}{sL_1}\right)-U_2\left(\frac{1}{sL_1}\right)=I_1$ (odavde odmah slijedi U_2/I_1)

---

3) $\rightarrow$ 1) $I_2\cdot(sL_2+R_2)\cdot\left[sL_1\cdot\left(\frac{1}{sL_1}+sC+\frac{1}{sL_2+R_2}\right)\left(\frac{1}{R_1}+\frac{1}{sL_1}\right)-\left(\frac{1}{sL_1}\right)\right]=I_1$

Nakon malo sređivanja slijedi:

$$I_2 \cdot (sL_2 + R_2) \cdot \left[\left(1 + s^2 CL_1 + \frac{sL_1}{sL_2 + R_2} \right) \left(\frac{1}{R_1} + \frac{1}{sL_1} \right) - \left(\frac{1}{sL_1} \right) \right] = I_1$$

$$I_2 \cdot (sL_2 + R_2) \cdot \left(\frac{1}{R_1} + \frac{s^2 CL_1}{R_1} + \frac{sL_1}{R_1 (sL_2 + R_2)} + \frac{1}{sL_1} + sC + \frac{1}{sL_2 + R_2} - \frac{1}{sL_1} \right) = I_1 \Big/ \cdot R_1$$

$$I_2 \cdot (sL_2 + R_2) \cdot \left(1 + s^2 CL_1 + \frac{sL_1}{sL_2 + R_2} + sCR_1 + \frac{R_1}{sL_2 + R_2} \right) = I_1 \cdot R_1$$

$$I_2 \cdot \left(sL_2 + R_2 + s^2 CL_1 (sL_2 + R_2) + sL_1 + sCR_1 (sL_2 + R_2) + R_1 \right) = I_1 \cdot R_1$$

$$I_2 \cdot \left(sL_2 + R_2 + s^3 CL_1 L_2 + s^2 CL_1 R_2 + sL_1 + s^2 CR_1 L_2 + sCR_1 R_2 + R_1 \right) = I_1 \cdot R_1$$

$$H_i(s) = \frac{I_2}{I_1} = \frac{R_1}{s^3 CL_1 L_2 + s^2 C(L_1 R_2 + R_1 L_2) + s(L_1 + L_2 + CR_1 R_2) + R_1 + R_2}$$

$$H_i(s) = \frac{I_2}{I_1} = \frac{\frac{R_1}{CL_1 L_2}}{s^3 + s^2 \frac{L_1 R_2 + R_1 L_2}{L_1 L_2} + s \frac{L_1 + L_2 + CR_1 R_2}{CL_1 L_2} + \frac{R_1 + R_2}{CL_1 L_2}} \quad \text{(2 boda)}$$

Uz uvrštene vrijednosti elemenata:

$$H_i(s) = \frac{I_2}{I_1} = \frac{2}{s^3 + 2s^2 + 5s + 4} = \frac{2}{(s+1)(s^2 + s + 4)} \quad \text{(1 bod)}$$

5. Za električni krug prikazan slikom:

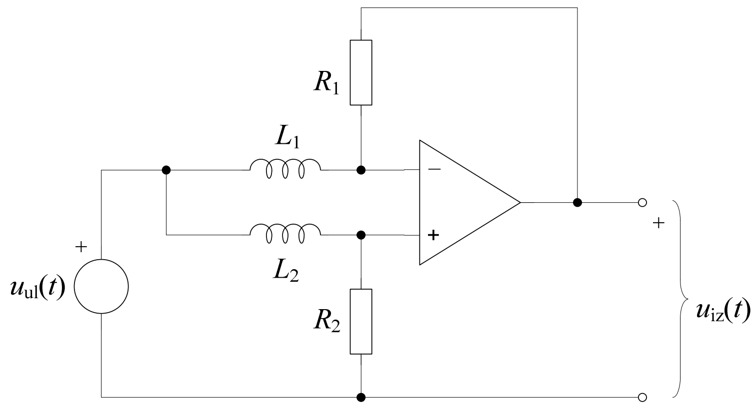
a) Odrediti prijenosnu funkciju napona $H(s)=U_{iz}(s)/U_{ul}(s)$

b) Izračunati polove i nule prijenosne funkcije i prikazati njihov raspored u s-ravnini

c) Izračunati i skicirati A-F karakteristiku $|H(j\omega)|$

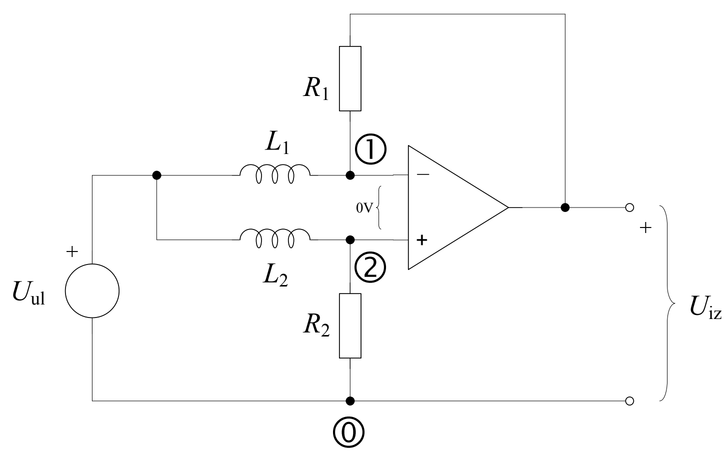
d) Izračunati logaritamsku mjeru pojačanja $\alpha(\omega)$

Zadano je: $R_1 = R_2 = 1$, $L_1 = 2$, $L_2 = 1$, $A \to \infty$ (A je pojačanje operacijskog pojačala).



Rješenje:

a) Određivanje prijenosne funkcije napona $H(s)=U_{iz}(s)/U_{ul}(s)$:



1) $U_1\left(\dfrac{1}{R_1}+\dfrac{1}{sL_1}\right)=U_{ul}\dfrac{1}{sL_1}+U_{iz}\dfrac{1}{R_1}$

2) $U_2\left(\dfrac{1}{R_2}+\dfrac{1}{sL_2}\right)=\dfrac{U_{ul}}{sL_2} \;\Rightarrow\; U_2=\dfrac{U_{ul}}{sL_2\left(\dfrac{1}{R_2}+\dfrac{1}{sL_2}\right)}=\dfrac{R_2}{R_2+sL_2}U_{ul}$

3) $U_1=U_2$

————————————————————————————————(1 bod)

2) $\to$ 1) $\Rightarrow$ $U_{ul}\dfrac{R_2}{R_2+sL_2}\left(\dfrac{1}{R_1}+\dfrac{1}{sL_1}\right)=U_{ul}\dfrac{1}{sL_1}+U_{iz}\dfrac{1}{R_1}$

————————————————————————————————

$$\Rightarrow \qquad U_{ul}\left[\frac{R_2}{R_2+sL_2}\left(\frac{1}{R_1}+\frac{1}{sL_1}\right)-\frac{1}{sL_1}\right]=U_{iz}\frac{1}{R_1}\Big/\cdot R_1$$

$$U_{ul}\left[\frac{R_2}{R_2+sL_2}\left(\frac{R_1}{sL_1}+1\right)-\frac{R_1}{sL_1}\right]=U_{iz}$$

$$H(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{R_2}{R_2+sL_2}\left(\frac{R_1}{sL_1}+1\right)-\frac{R_1}{sL_1}=\left(\frac{R_2}{R_2+sL_2}-1\right)\frac{R_1}{sL_1}+\frac{R_2}{R_2+sL_2}$$

$$=\frac{-sL_2R_1}{(R_2+sL_2)sL_1}+\frac{R_2}{R_2+sL_2}=\frac{sL_1R_2-sL_2R_1}{(R_2+sL_2)sL_1}=\frac{2s-s}{2s(1+s)}=\frac{s}{2s(1+s)}=\frac{1}{2}\cdot\frac{1}{s+1}$$ (1 bod)

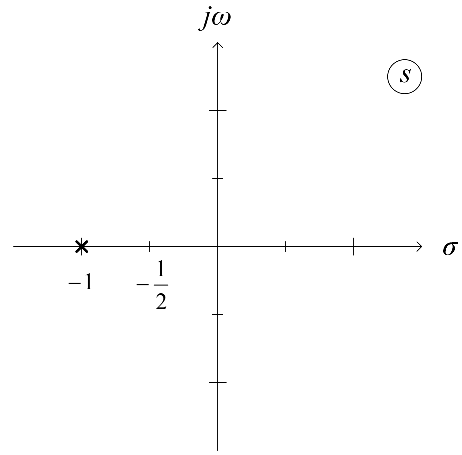
b) Raspored polova i nula: (1 bod)

Polovi: $s+1=0$

$s_{p1}=-1$

Nule:

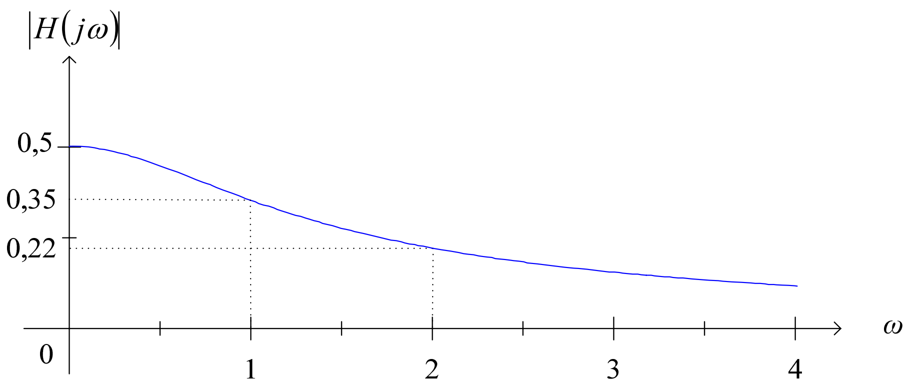
$\lim\limits_{s\to\infty}\frac{1}{2}\cdot\frac{1}{s+1}=0 \Rightarrow s_{01}=\infty$



c) Amplitudno-frekvencijska karakteristika: (1 bod)

Uvrstimo $s = j\omega$ u $H(s)$

$$H(j\omega)=\frac{1}{2}\frac{1}{j\omega+1} \Rightarrow |H(j\omega)|=\frac{1}{2\sqrt{1+\omega^2}}$$



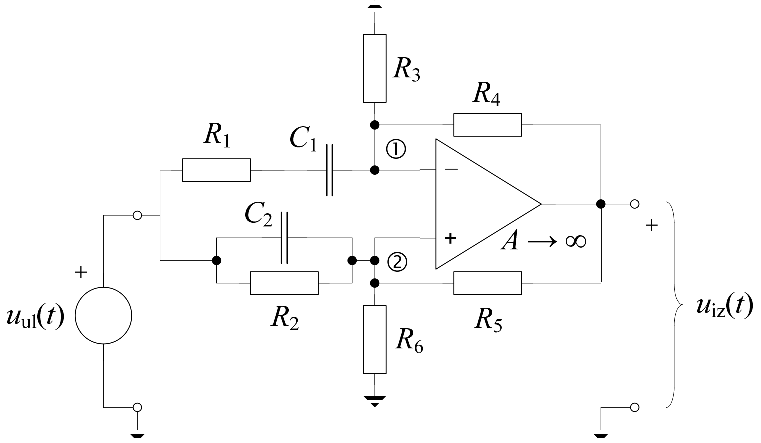
Pomoćne točke za skiciranje a-f karakteristike:

ω	$\lvert H(j\omega)\rvert$
0	0.5
1	$\frac{1}{2\sqrt{2}}=0.3536$
2	$\frac{1}{2\sqrt{5}}=0.2236$
∞	0

d) Logaritamska mjera pojačanja: (1 bod)

$$\alpha(\omega)=20\log|H(j\omega)|=20\log 0.5-10\log(\omega^2+1)=-6.02-10\log(\omega^2+1)[\mathrm{dB}]$$

4. Za električni krug na slici zadane su normalizirane vrijednosti elemenata: $R_1 = R_2 = R_3 = R_4 = R_5 = 1$, $R_6 = 1/2$, $C_1 = C_2 = 1$, $A \to \infty$ i poticaj $u_{ul}(t) = \delta(t)$. Odrediti: a) jednadžbe čvorišta; b) prijenosnu funkciju: $T(s)=U_{iz}(s)/U_{ul}(s)$; c) polove i nule prijenosne funkcije $T(s)$; te d) odziv $U_{iz}(s)$ i odziv $u_{iz}(t)$.



Rješenje:

a) Jednadžbe čvorišta:

Ako za početni račun pojednostavimo:

$$Z_1 = R_1 + \frac{1}{sC_1}$$

$$Z_2 = R_2 \parallel \frac{1}{sC_2} = \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{sR_2C_2 + 1}$$

Tada:

1) $A \to \infty \Rightarrow U_1 = U_2$

2) $U_1\left(\frac{1}{Z_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) - U_{ul}\frac{1}{Z_1} - U_{iz}\frac{1}{R_4} = 0$ (1 bod)

3) $U_2\left(\frac{1}{Z_2} + \frac{1}{R_5} + \frac{1}{R_6}\right) - U_{ul}\frac{1}{Z_2} - U_{iz}\frac{1}{R_5} = 0$ (1 bod)

---

b) Prijenosna funkcija:

$$2) \Rightarrow U_1 = \frac{U_{ul}\frac{1}{Z_1} + U_{iz}\frac{1}{R_4}}{\frac{1}{Z_1} + \frac{1}{R_3} + \frac{1}{R_4}}$$

$$3) \Rightarrow U_2 = \frac{U_{ul}\frac{1}{Z_2} + U_{iz}\frac{1}{R_5}}{\frac{1}{Z_2} + \frac{1}{R_5} + \frac{1}{R_6}}$$

Uz uvrštene $\frac{1}{Z_1} = Y_1$, $\frac{1}{Z_2} = Y_2$, $\frac{1}{R_1} = G_1$ ... $\frac{1}{R_6} = G_6$ slijedi

1) $U_1 = U_2 \Rightarrow \dfrac{U_{ul}Y_1 + U_{iz}G_4}{Y_1 + G_3 + G_4} = \dfrac{U_{ul}Y_2 + U_{iz}G_5}{Y_2 + G_5 + G_6}$

$$\frac{U_{ul}Y_1}{Y_1 + G_3 + G_4} - \frac{U_{ul}Y_2}{Y_2 + G_5 + G_6} = \frac{U_{iz}G_5}{Y_2 + G_5 + G_6} - \frac{U_{iz}G_4}{Y_1 + G_3 + G_4}$$

$$\frac{U_{iz}}{U_{ul}} = \frac{\dfrac{Y_1}{Y_1 + G_3 + G_4} - \dfrac{Y_2}{Y_2 + G_5 + G_6}}{\dfrac{G_5}{Y_2 + G_5 + G_6} - \dfrac{G_4}{Y_1 + G_3 + G_4}}$$

Nakon supstitucije Z_1 i Z_2:

$$Y_1 = \frac{1}{Z_1} = \frac{1}{1 + \frac{1}{s}} = \frac{s}{s+1}, \qquad Y_2 = \frac{1}{Z_2} = s + 1$$

$$\frac{U_{iz}}{U_{ul}} = \frac{\dfrac{s/(s+1)}{s/(s+1)+2} - \dfrac{s+1}{s+1+3}}{\dfrac{1}{s+1+3} - \dfrac{1}{s/(s+1)+2}} = \frac{\dfrac{s}{s+2(s+1)} - \dfrac{s+1}{s+4}}{\dfrac{1}{s+4} - \dfrac{s+1}{s+2(s+1)}} = \frac{\dfrac{s}{3s+2} - \dfrac{s+1}{s+4}}{\dfrac{1}{s+4} - \dfrac{s+1}{3s+2}} =$$

$$= \frac{s(s+4)-(s+1)(3s+2)}{3s+2-(s+1)(s+4)} = \frac{s^2+4s-3s^2-5s-2}{3s+2-s^2-5s-4} = \frac{-2s^2-s-2}{-s^2-2s-2} = \frac{2s^2+s+2}{s^2+2s+2}$$

$$T(s) = \frac{U_{iz}(s)}{U_{ul}(s)} = \frac{2s^2+s+2}{s^2+2s+2}$$ (1 bod)

---

c) Polovi, nule i odziv:

Nule: $2s^2 + s + 2 = 0 \Rightarrow s_{o_{1,2}} = -\dfrac{1}{4} \pm j\dfrac{\sqrt{15}}{4}$

$s_{o_3} = \infty$

Polovi: $s^2 + 2s + 2 = 0 \Rightarrow s_{p_{1,2}} = -1 \pm j$

(1 bod)

---

d) Odziv $u_{iz}(t)$:

Odziv:

$$U_{iz}(s) = T(s)\cdot U_{ul}(s);\ u_{ul}(t) = \delta \Rightarrow U_{ul}(s) = 1 \ \Rightarrow U_{iz}(s) = \frac{2s^2+s+2}{s^2+2s+2}$$

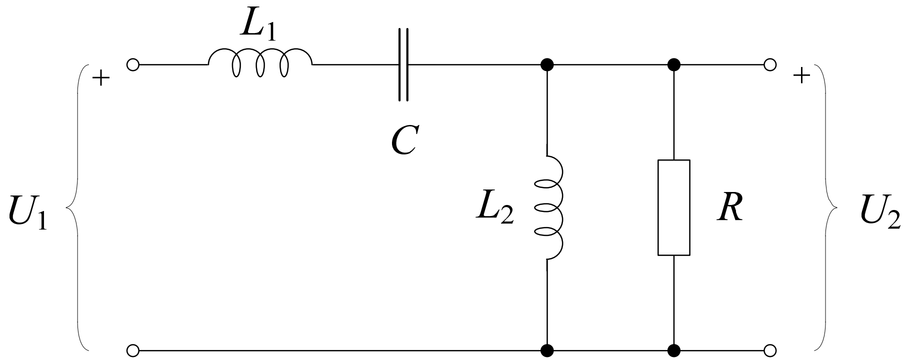
$$(2s^2+s+2):(s^2+2s+2) = 2 - \frac{3s+2}{(s+1)^2+1} = 2 - \frac{3s+3-1}{(s+1)^2+1} = 2 - 3\cdot\frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}$$

$$\underline{-2s^2-4s-4}$$

$$= \quad -3s-2$$

$$\Rightarrow u_{iz}(t) = \left(2\delta - 3e^{-t}\cos(t) + e^{-t}\sin(t)\right)\cdot S(t)$$ (1 bod)

3. Za četveropol na slici zadane su normalizirane vrijednosti elemenata $L_1 = 2/3$, $L_2 = 2$, $R = 1$, $C = 3/4$. a) Odrediti prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$; b) polove i nule $T(s)$; c) Napisati izraz za amplitudno-frekvencijsku karakteristiku i skicirati ju; d) Napisati izraz za fazno-frekvencijsku karakteristiku (skica poželjna, nije neophodna)



Rješenje:

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a) Prijenosna funkcija:

$$U_2 = \frac{Z_2}{Z_1 + Z_2} U_1$$

$$Z_1 = sL_1 + \frac{1}{sC};\; Z_2 = sL_2 \| R = \frac{sL_2 R}{sL_2 + R}$$

$$T(s) = \frac{U_2}{U_1} = \frac{Z_2}{Z_2 + Z_1} = \frac{\dfrac{sL_2R}{sL_2+R}}{\dfrac{sL_2R}{sL_2+R} + sL_1 + \dfrac{1}{sC}} \quad \begin{array}{l} /\cdot(sL_2+R) \\ /\cdot(sL_2+R) \end{array}$$

$$= \frac{sL_2R}{sL_2R + (sL_2+R)(sL_1 + 1/sC)} = \frac{s^2L_2RC}{s^3L_1L_2C + s^2L_2CR + s^2L_1CR + sL_2 + R} =$$

$$= \frac{s^2L_2RC}{s^3L_1L_2C + s^2RC(L_1+L_2) + sL_2 + R}$$

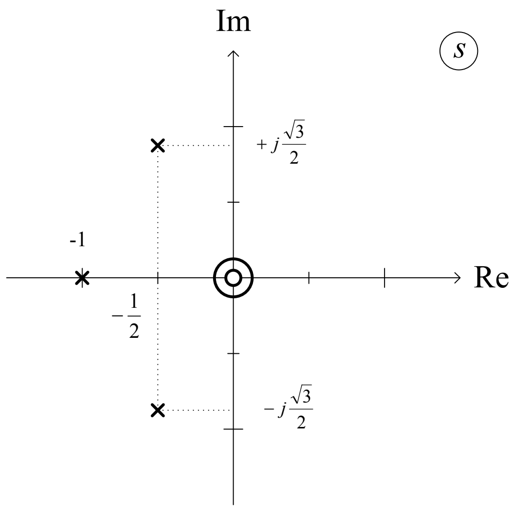
$$T(s) = \frac{\frac{3}{2}s^2}{s^3 + 2s^2 + 2s + 1} = \frac{\frac{3}{2}s^2}{(s+1)(s^2+s+1)}$$ (2 boda)

b) Polovi i nule:

Nule: $s^2 = 0 \quad \Rightarrow \quad s_{o_{1,2}} = 0$

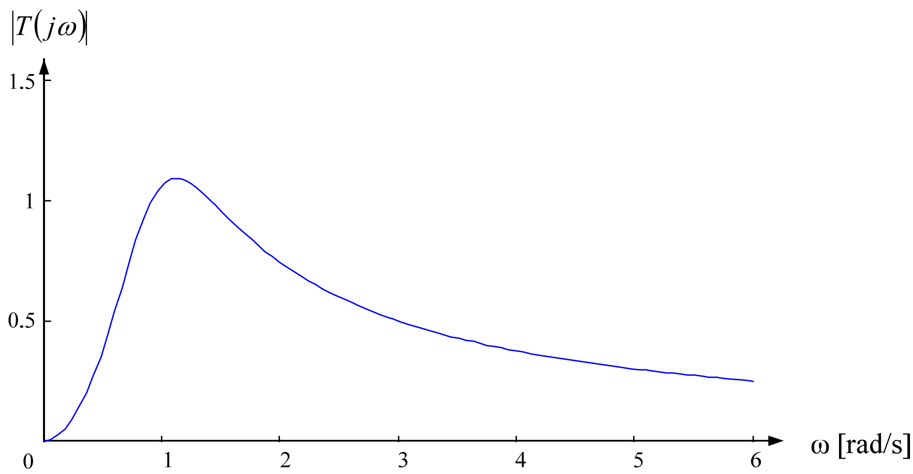
u brojniku s^2 u nazivniku $s^3 \quad \Rightarrow \quad s_{o_3} = \infty$

Polovi: $(s+1)(s^2+s+1) = 0 \quad \Rightarrow \quad s_{p_1} = -1$, $s_{p_{2,3}} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ (1 bod)



c) A-F karakteristika:

$$T(j\omega)=\frac{-\frac{3}{2}\omega^2}{(j\omega+1)(-\omega^2+j\omega+1)} \qquad \Rightarrow \qquad |T(j\omega)|=\frac{\frac{3}{2}\omega^2}{\sqrt{\left((1-\omega^2)^2+\omega^2\right)\cdot(\omega^2+1)}}$$



(1 bod)

d) F-F karakteristika:

$$\varphi(\omega)=\pi-\left[\arctan(\omega)+\arctan\left(\frac{\omega}{1-\omega^2}\right)\right]$$ (1 bod)

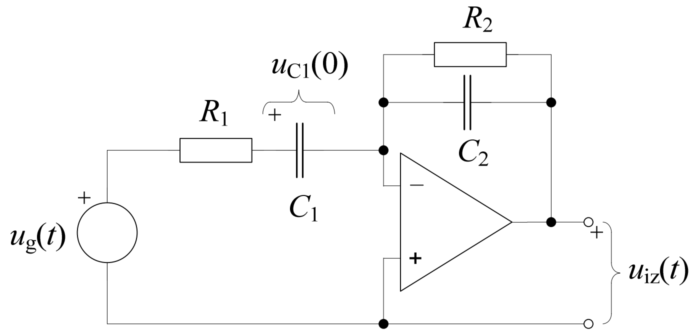
5. Zadan je električni krug prema slici. Odrediti napon na izlazu $u_{iz}(t)$ ako je zadano: $R_1=R_2=1$, $C_1=C_2=1$, $u_g(t)=S(t)$. Početni napon na kapacitetu C_1 je $u_{C1}(0)=2$, a na C_2 je jednak nuli. Koristiti bilo koju metodu u izračunu odziva. Koristeći princip superpozicije, izračunati:

a) Odziv $U_{iz1}(s)$ uslijed naponskog izvora $u_g(t)$;

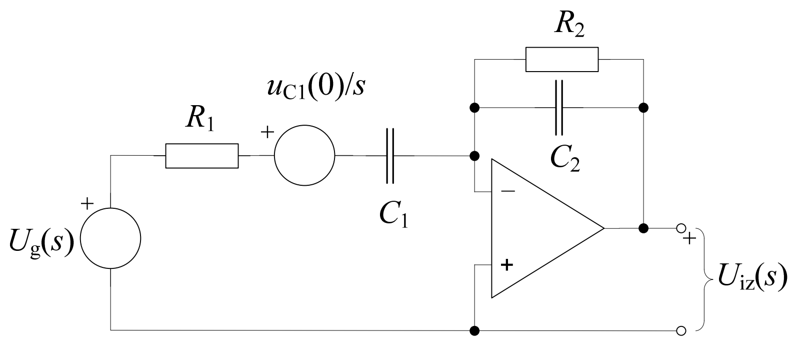
b) Odziv $U_{iz2}(s)$ uslijed početnog uvjeta $u_{C1}(0)$;

c) Ukupni odziv $U_{iz}(s)$;

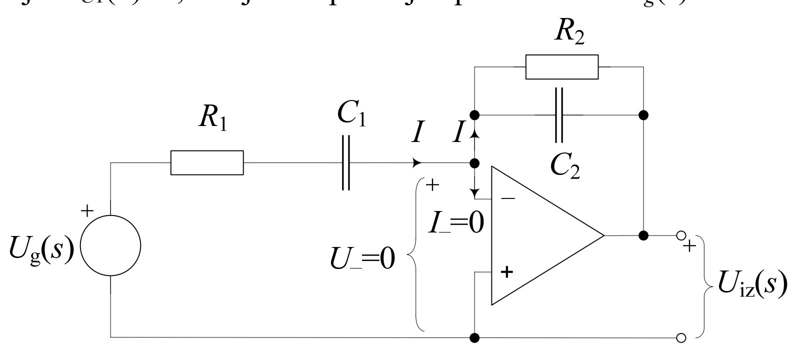
d) Ukupni odziv $u_{iz}(t)$.



Rješenje: Primjena $\mathcal{L}$-transformacije:



Primjena principa superpozicije:
a) Isključen početni uvjet $u_{C1}(0)=0$, ostaje kao poticaj naponski izvor $U_g(s)$:



$$1)\ I = U_g(s)\cdot \frac{1}{R_1 + \frac{1}{sC_1}}$$

$$2)\ I = -U_{iz}(s)\cdot\left(\frac{1}{R_2} + sC_2\right)$$

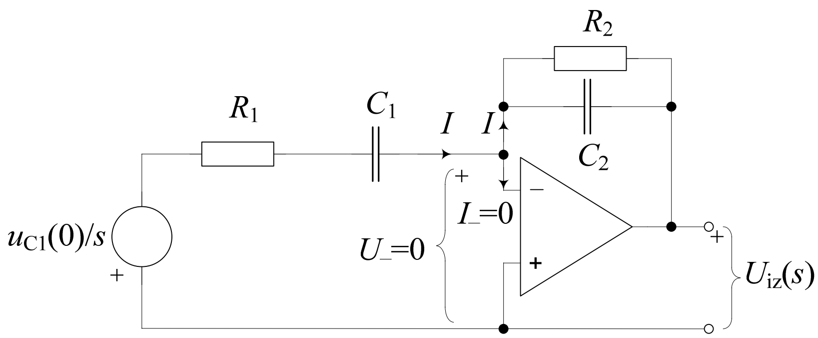
$$\Rightarrow \quad U_{iz}(s) = \frac{-U_g(s)}{\left(\frac{1}{R_2}+sC_2\right)\left(R_1+\frac{1}{sC_1}\right)} = -U_g(s)\cdot\frac{s\frac{1}{R_1C_2}}{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)} =$$

$$= -U_g(s)\cdot\frac{s}{(s+1)(s+1)} = -U_g(s)\cdot\frac{s}{(s+1)^2}$$

Uvrstimo vrijednosti:

$$U_{iz}(s) = -\frac{1}{s}\cdot\frac{s}{(1+s)^2} = -\frac{1}{(1+s)^2} \quad \text{(1 bod)}$$

b) Isključen naponski izvor $U_g(s)$=0, ostaje kao poticaj početni uvjet $u_{C1}(0)$:



$$U_{iz}(s) = \frac{u_{C1}(0)}{s}\cdot\frac{1}{\left(\frac{1}{R_2}+sC_2\right)\left(R_1+\frac{1}{sC_1}\right)}$$

Uvrstimo vrijednosti:

$$U_{iz}(s) = 2\frac{1}{(1+s)^2} \quad \text{(1 bod)}$$

c) Ukupni odziv:

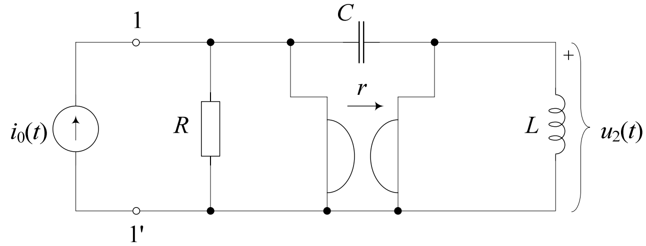
$$U_{iz}(s) = -\frac{1}{(1+s)^2}+\frac{2}{(1+s)^2} = \frac{1}{(1+s)^2} \quad \text{(1 bod)}$$

d) Inverzna Laplaceova transformacija:

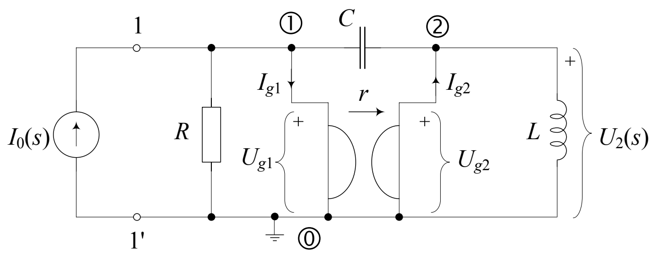
$$u_{iz}(t) = (te^{-t})\cdot S(t) \quad \text{(1 bod)}$$

DRUGI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2010

1. Za električni krug na slici s pobudom $i_0(t)$ i normiranim vrijednostima elemenata: R =2, L=1 i C=1, te konstantom giratora r=2 odrediti: *a*) ulaznu impedanciju $Z_{ul}(s)$ gledanu s priključnica 1-1'; *b*) prijenosnu impedanciju $Z_{21}(s)=U_2(s)/I_0(s)$; *c*) polove i nule funkcije $Z_{21}(s)$ i njihov prikaz u s-ravnini; *d*) odziv $u_2(t)$ ako je zadana pobuda $i_0(t)=S(t)$, *e*) odziv $u_2(t)$ ako je $i_0(t)$ stacionarna sinusna pobuda valnoga oblika $i_0(t)$=5 cos(t).



Rješenje: Jednadžbe čvorišta



$$U_1(s)=U_{g1}(s)$$
$$U_2(s)=U_{g2}(s)$$

(1) $\quad I_0(s)-I_{g1}(s)=\left(\frac{1}{R}+sC\right)U_1(s)-sCU_2(s)$ $\qquad U_{g1}(s)=-rI_{g2}(s)\Rightarrow I_{g2}(s)=-\frac{1}{r}U_1(s)$

(2) $\quad I_{g2}(s)=-sCU_1(s)+\left(sC+\frac{1}{sL}\right)U_2(s)$ $\qquad U_{g2}(s)=-rI_{g1}(s)\Rightarrow I_{g1}(s)=-\frac{1}{r}U_2(s)$

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(1) $\quad I_0(s)+\frac{1}{r}U_2(s)=\left(\frac{1}{R}+sC\right)U_1(s)-sCU_2(s)$

(2) $\quad -\frac{1}{r}U_1(s)=-sCU_1(s)+\left(sC+\frac{1}{sL}\right)U_2(s)$

---

(1) $\quad I_0(s)=\left(\frac{1}{R}+sC\right)U_1(s)-\left(sC+\frac{1}{r}\right)U_2(s)$

(2) $\quad 0=-\left(sC-\frac{1}{r}\right)U_1(s)+\left(sC+\frac{1}{sL}\right)U_2(s) \;\Rightarrow\; U_1(s)=\frac{sC+1/(sL)}{sC-1/r}U_2(s) \;\rightarrow (1)$

$$\Rightarrow I_0(s)=\left(\frac{1}{R}+sC\right)\frac{sC+1/(sL)}{sC-1/r}U_2(s)-\left(sC+\frac{1}{r}\right)U_2(s)$$

$$\Rightarrow I_0(s)\left(sC-\frac{1}{r}\right)=\left(\frac{1}{R}+sC\right)\left(sC+\frac{1}{sL}\right)U_2(s)-\left(sC+\frac{1}{r}\right)\left(sC-\frac{1}{r}\right)U_2(s)$$

$$\Rightarrow I_0(s)\left(sC-\frac{1}{r}\right)=\left(\frac{sC}{R}+\frac{1}{RsL}+\frac{sC}{sL}+\frac{1}{r^2}\right)U_2(s) \;\Rightarrow\; U_2(s)=\frac{\left(sC-\frac{1}{r}\right)I_0(s)}{\frac{sC}{R}+\frac{1}{RsL}+\frac{sC}{sL}+\frac{1}{r^2}}$$

$$U_2(s)=\frac{rRLs(srC-1)}{s^2r^2LC+sR(r^2C+L)+r^2}I_0(s)$$

$$a)\ Z_{ul}(s)=\frac{U_1}{I_0}=\frac{U_1}{U_2}\cdot\frac{U_2}{I_0}=\frac{sC+\frac{1}{sL}}{\left(sC-\frac{1}{r}\right)}\cdot\frac{\left(sC-\frac{1}{r}\right)I_0(s)}{\frac{sC}{R}+\frac{1}{RsL}+\frac{sC}{sL}+\frac{1}{r^2}}=\frac{\left(sC+\frac{1}{sL}\right)I_0(s)}{\frac{sC}{R}+\frac{1}{RsL}+\frac{sC}{sL}+\frac{1}{r^2}}$$

$$Z_{ul}(s)=\frac{U_1}{I_0}=\frac{r^2R(s^2LC+1)}{s^2r^2LC+sR(r^2C+L)+r^2}$$ (uvrstimo R=2, L=1, C=1, r=2)

$$Z_{ul}(s)=\frac{U_1}{I_0}=\frac{8(s^2+1)}{4s^2+10s+4}=2\frac{s^2+1}{s^2+\frac{5}{2}s+1}$$ (1 bod)

$$b)\ Z_{21}(s)=\frac{U_2}{I_0}=\frac{rRLs(srC-1)}{s^2r^2LC+sR(r^2C+L)+r^2}$$ (uvrstimo R=2, L=1, C=1, r=2)

$$Z_{21}(s)=\frac{4s(2s-1)}{4s^2+10s+4}=\frac{2s(2s-1)}{2s^2+5s+2}=\frac{s(2s-1)}{s^2+\frac{5}{2}s+1}=2\frac{s\left(s-\frac{1}{2}\right)}{s^2+\frac{5}{2}s+1}$$ (1 bod)

c) $Z_{21}(s)$: nule: $s(s-1/2)=0 \Rightarrow s_{o1}=0,\ s_{o2}=1/2$,

polovi: $s^2+\frac{5}{2}s+1=0 \Rightarrow s_{p1,2}=-\frac{5}{4}\pm\sqrt{\left(\frac{5}{4}\right)^2-1}=-\frac{5}{4}\pm\sqrt{\frac{25-16}{16}}=-\frac{5}{4}\pm\frac{3}{4}$

$s_{p1}=-1/2,\ s_{p2}=-2$, (1 bod)

$$d)\ U_2(s)=Z_{21}(s)\cdot I_0(s)=2\frac{s\left(s-\frac{1}{2}\right)}{s^2+\frac{5}{2}s+1}\cdot\frac{1}{s}=\frac{2s-1}{s^2+\frac{5}{2}s+1}=\frac{2s-1}{\left(s+\frac{1}{2}\right)(s+2)}$$

Rastav na parcijalne razlomke:

$$U_2(s)=\frac{2s-1}{(s+1/2)(s+2)}=\frac{A}{s+1/2}+\frac{B}{s+2}=\frac{A(s+1)+B(s+1/2)}{(s+1/2)(s+2)}=\frac{(A+B)s+2A+B/2}{(s+1/2)(s+2)}$$

$$\begin{array}{ll} & A=2-B\Rightarrow 2(2-B)+B/2=-1 \\ A+B=2 & \\ \underline{2A+B/2=-1} & \Rightarrow 4-2B+B/2=-1\Rightarrow 2B-B/2=5 \\ & \Rightarrow 3B=10\Rightarrow B=\frac{10}{3},\ A=\frac{6-10}{3}=-\frac{4}{3} \end{array}$$

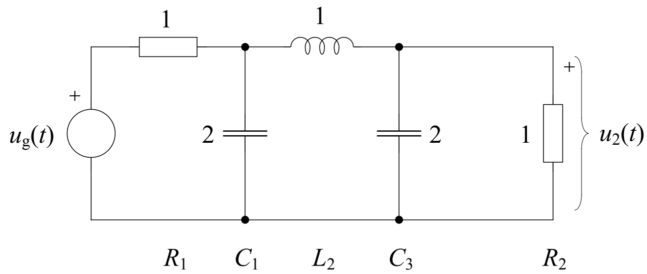
$$U_2(s)=\frac{-4/3}{s+1/2}+\frac{10/3}{s+2}\ \Rightarrow\ u_2(t)=\left(-\frac{4}{3}\cdot e^{-\frac{t}{2}}+\frac{10}{3}\cdot e^{-2t}\right)S(t)$$ (1 bod)

e) Fazori $i_0(t)=5\cdot\cos(t)$ $I_0(j\omega)=5\cdot e^{j0}=5$, $\omega=1$ $U_2(j\omega)=Z_{21}(j\omega)I_0(j\omega)=\frac{-2\omega^2-j\omega}{-\omega^2+(5/2)j\omega+1}\cdot 5$

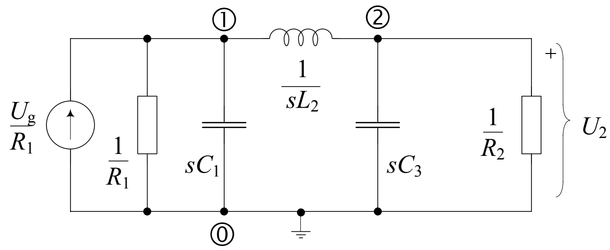
$U_2(j1)=\frac{-2-j}{(5/2)j}\cdot 5=\left(-\frac{2}{j}-1\right)2=(2j-1)2=-2+4j=2\sqrt{5}e^{j\varphi}$ φ=arctan(–2)=116.56° (II kvadrant)

$u_2(t)=2\sqrt{5}\cdot\cos(t+116.56°)$ (1 bod)

3. Za mrežu na slici izračunati naponsku prijenosnu funkciju $H(s)=U_2(s)/U_g(s)$ ako su zadane normalizirane vrijednosti elemenata: $R_1=R_2=1$, $C_1=C_3=2$ i $L_2=1$. (Koristiti metodu napona čvorova) Napisati: a) Jednadžbe za čvorove (1) i (2) za izračun $H(s)$; b) $H(s)$ kao funkciju elemenata; c) $H(s)$ s uvrštenim elementima.



Rješenje:



1) $U_1\left(\frac{1}{R_1}+sC_1+\frac{1}{sL_2}\right)-U_2\left(\frac{1}{sL_2}\right)=\frac{U_g}{R_1}$

2) $-U_1\frac{1}{sL_2}+U_2\left(\frac{1}{sL_2}+sC_3+\frac{1}{R_2}\right)=0$ (1 bod)

---

2) $\Rightarrow U_1=U_2 sL_2\left(\frac{1}{sL_2}+sC_3+\frac{1}{R_2}\right)\Rightarrow U_1=U_2\left(s^2L_2C_3+s\frac{L_2}{R_2}+1\right)$

2) $\rightarrow$ 1) $\Rightarrow\ U_2\left(s^2L_2C_3+s\frac{L_2}{R_2}+1\right)\left(\frac{1}{R_1}+sC_1+\frac{1}{sL_2}\right)-U_2\left(\frac{1}{sL_2}\right)=\frac{U_g}{R_1}$

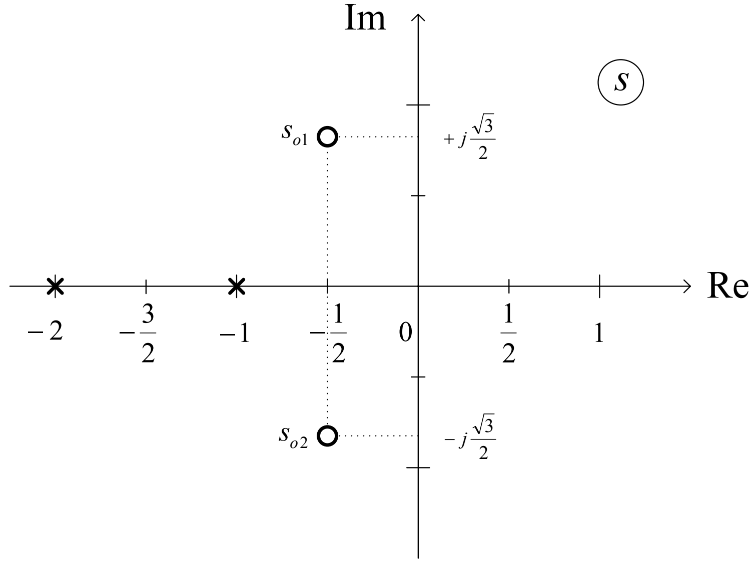
$$U_2\left(s^2\frac{L_2C_3}{R_1}+s\frac{L_2}{R_1R_2}+\frac{1}{R_1}+s^3L_2C_1C_3+s^2\frac{L_2C_1}{R_2}+sC_1+sC_3+\frac{1}{R_2}+\frac{1}{sL_2}-\frac{1}{sL_2}\right)=\frac{U_g}{R_1}\Big/\cdot R_1R_2$$

$$H(s)=\frac{U_2}{U_g}=\frac{R_2}{s^3L_2C_1C_3R_1R_2+s^2(L_2C_3R_2+L_2C_1R_1)+s(L_2+R_1R_2C_1+R_1R_2C_3)+R_1+R_2}$$ (1 bod)

Uz uvrštene vrijednosti elemenata:

$$H(s)=\frac{U_2(s)}{U_g(s)}=\frac{1}{4s^3+4s^2+5s+2}=\frac{\frac{1}{4}}{s^3+s^2+\frac{5}{4}s+\frac{1}{2}}$$ (1 bod)

4. Zadan je raspored polova i nula prema slici prijenosne funkcije $H(s)=U_{iz}(s)/U_{ul}(s)$ nekog električnog kruga. Odrediti: a) prijenosnu funkciju $H(s)$ ako se traži da je $H(0)=1/2$; b) kompleksnu frekvencijsku karakteristiku $H(j\omega)$; c) odziv $u_{iz}(t)$ za pobudu $u_{ul}(t)=\sin(t)$; $\quad -\infty < t < \infty$.



Rješenje:

a) Opći oblik prijenosne funkcije (funkcije mreža) napisan pomoću nula i polova:

$$H(s)=k\cdot\frac{\prod_i (s-s_{0i})}{\prod_j (s-s_{pj})}$$

Nule: $s_{01}=-\frac{1}{2}+j\frac{\sqrt{3}}{2}$; $\quad s_{02}=-\frac{1}{2}-j\frac{\sqrt{3}}{2}$

Polovi: $s_{p1}=-1$; $s_{p2}=-2$

$$H(s)=k\cdot\frac{\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\left(s+\frac{1}{2}+-j\frac{\sqrt{3}}{2}\right)}{(s+1)(s+2)}=k\cdot\frac{\left(s+\frac{1}{2}\right)^2-\left(j\frac{\sqrt{3}}{2}\right)^2}{s^2+3s+2}=$$

$(a-b)(a+b)=a^2-b^2$ (Upotrijebili smo pravilo: 'Razlika kvadrata')

$$=k\cdot\frac{s^2+s+\frac{1}{4}+\frac{3}{4}}{s^2+3s+2}=k\cdot\frac{s^2+s+1}{s^2+3s+2}$$

Konstanta k u općem obliku prijenosne funkcije slijedi iz:

$$H(0)=k\cdot\left.\frac{s^2+s+1}{s^2+3s+2}\right|_{s=0}=k\cdot\frac{1}{2}=\frac{1}{2}\Rightarrow k=1$$

Konačno je: $H(s)=\dfrac{s^2+s+1}{s^2+3s+2}$ (1 bod)

---

b) Kompleksna frekvencijska karakteristika prijenosne funkcije:

$$H(j\omega)=\frac{-\omega^2+j\omega+1}{-\omega^2+3j\omega+2}$$ (1 bod)

c) Odziv na pobudu sve-vremenskom sinus funkcijom frekvencije ω=1:

$$u_{ul}(t)=\sin(t)\Rightarrow U_{ul}(j\omega)=1\angle 0°$$

$$U_{iz}(j\omega)=H(j\omega)\cdot U_{ul}(j\omega)=H(j\omega)\cdot 1\angle 0°$$

$$H(j1)=\frac{-1+j+1}{-1+3j+2}=\frac{j}{1+3j}$$

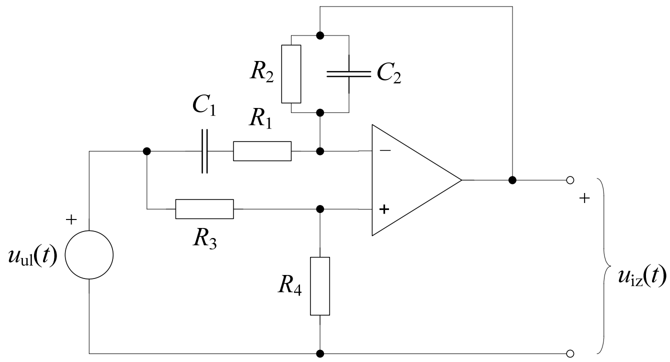
$$|H(j1)|=\frac{1}{\sqrt{3^2+1}}=\frac{1}{\sqrt{10}}=0.3162$$

$$H(j1)=\frac{j}{(1+3j)}\cdot\frac{1-3j}{1-3j}=\frac{3+j}{10}$$

$\angle H(j1)=\arctan(1/3)=18.43°$ (jer je fazor u prvom kvadrantu)

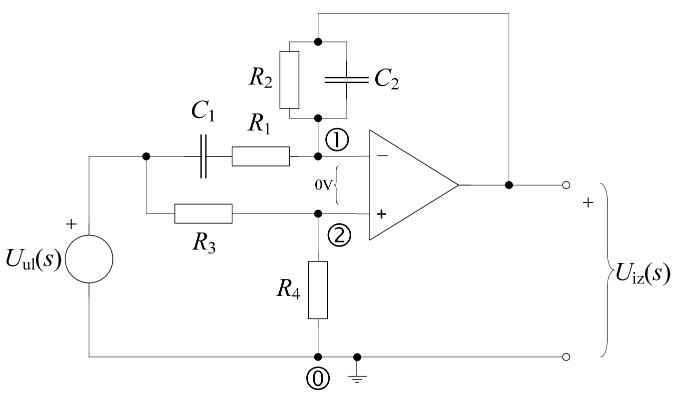
$u_{iz}(t)=0.3162\sin(t+18.43°)$ (1 bod)

5. Za električni krug prikazan slikom: a) Odrediti prijenosnu funkciju napona $H(s)=U_{iz}(s)/U_{ul}(s)$; b) Izračunati polove i nule prijenosne funkcije; c) Izračunati A-F karakteristiku $|H(j\omega)|$; d) Izračunati F-F karakteristiku $\varphi(\omega)=\arg H(j\omega)$; e) Izračunati logaritamsku mjeru pojačanja $\alpha(\omega)$. Zadano je: $R_1=R_2=1$, $C_1=C_2=1$, $R_3=R_4=1$, $A\to\infty$ (A je pojačanje operacijskog pojačala).



Rješenje:

a) Određivanje prijenosne funkcije napona $H(s)=U_{iz}(s)/U_{ul}(s)$:



$$Z_1=\frac{1}{sC_1}+R_1=\frac{1+sR_1C_1}{sC_1}$$

$$Z_2=\frac{1}{1/R_2+sC_2}=\frac{R_2}{1+sR_2C_2}$$

1) $U_1\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)=U_{ul}\frac{1}{Z_1}+U_{iz}\frac{1}{Z_2}$

2) $U_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)=\frac{U_{ul}}{R_3}\;\Rightarrow\;U_2=\frac{U_{ul}}{R_3\left(\frac{1}{R_3}+\frac{1}{R_4}\right)}=\frac{R_4}{R_4+R_3}U_{ul}$

3) $U_1=U_2$

---

2), 3) $\to$ 1) $\Rightarrow\;\frac{R_4}{R_4+R_3}U_{ul}\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)=U_{ul}\frac{1}{Z_1}+U_{iz}\frac{1}{Z_2}\Big/\cdot Z_2$

$$\Rightarrow\;\frac{R_4}{R_4+R_3}U_{ul}\left(\frac{Z_2}{Z_1}+1\right)-U_{ul}\frac{Z_2}{Z_1}=U_{iz}\;\Rightarrow U_{ul}\left[\frac{R_4}{R_4+R_3}\left(\frac{Z_2}{Z_1}+1\right)-\frac{Z_2}{Z_1}\right]=U_{iz}$$

$$H(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{R_4}{R_4+R_3}\left(\frac{Z_2}{Z_1}+1\right)-\frac{Z_2}{Z_1}=\frac{R_4}{R_4+R_3}\frac{Z_2}{Z_1}+\frac{R_4}{R_4+R_3}-\frac{Z_2}{Z_1}=$$

$$=\left(\frac{R_4}{R_4+R_3}-1\right)\frac{Z_2}{Z_1}+\frac{R_4}{R_4+R_3}=\frac{-R_3}{R_4+R_3}\frac{Z_2}{Z_1}+\frac{R_4}{R_4+R_3}=\frac{R_4}{R_4+R_3}\cdot\frac{Z_1-(R_3/R_4)Z_2}{Z_1}=$$

$$=\frac{R_4}{R_4+R_3}\cdot\frac{\frac{1+sR_1C_1}{sC_1}-\frac{R_3}{R_4}\frac{R_2}{1+sR_2C_2}}{\frac{1+sR_1C_1}{sC_1}}=\frac{R_4}{R_4+R_3}\cdot\frac{(1+sR_1C_1)(1+sR_2C_2)-(R_3/R_4)sC_1R_2}{(1+sR_1C_1)(1+sR_2C_2)}$$

Uz uvrštene vrijednosti elemenata:

$$H(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{1}{2}\cdot\frac{(1+s)^2-s}{(1+s)^2}=\frac{1}{2}\cdot\frac{s^2+2s+1-s}{s^2+2s+1}=\frac{1}{2}\cdot\frac{s^2+s+1}{s^2+2s+1}$$

(1 bod)

b) Izračun polova i nula:

Polovi: $(s+1)^2=0$

$s_{p1,2}=-1$

Nule: $s^2+s+1=0$

$$s_{o1,2}=-\frac{1}{2}\pm\sqrt{\left(\frac{1}{2}\right)^2-1}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$

(1 bod)

c) Amplitudno-frekvencijska karakteristika:

Uvrstimo $s=j\omega$ u $H(s)$

$$H(j\omega)=\frac{1}{2}\cdot\frac{-\omega^2+j\omega+1}{(j\omega+1)^2}\Rightarrow|H(j\omega)|=\frac{\sqrt{(1-\omega^2)^2+\omega^2}}{2(1+\omega^2)}$$

(1 bod)

d) Fazno-frekvencijska karakteristika:

$$\varphi(\omega)=\arctan\left(\frac{\omega}{1-\omega^2}\right)-2\arctan(\omega)$$

(1 bod)

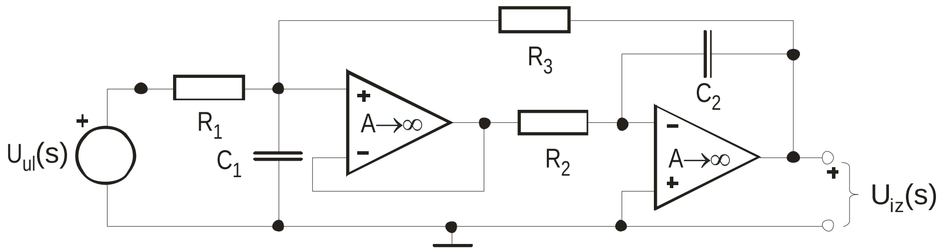
e) Logaritamska mjera pojačanja:

$$\alpha(\omega)=20\log|H(j\omega)|=20\log 0.5+10\log\left(\omega^2+(1-\omega^2)^2\right)-20\log\left(\omega^2+1\right)=$$

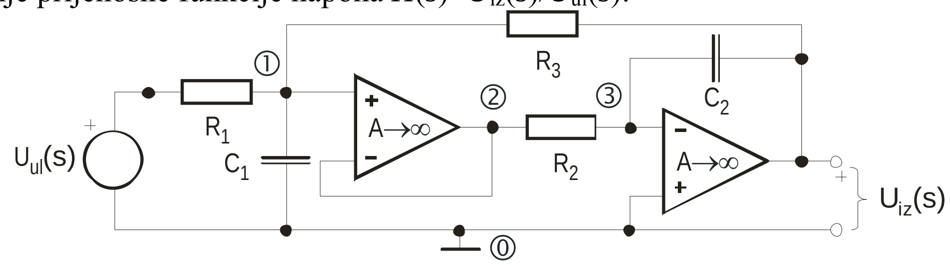
$$=-6.0206+10\log\left(\omega^4-\omega^2+1\right)-20\log\left(\omega^2+1\right)[\mathrm{dB}]$$

(1 bod)

4. Za električni krug prikazan slikom: a) Odrediti prijenosnu funkciju napona $H(s)=U_{iz}(s)/U_{ul}(s)$; b) Izračunati polove i nule prijenosne funkcije; c) Izračunati amplitudno-frekvencijsku karakteristiku $\left|H(j\omega)\right|$; d) Izračunati logaritamsku mjeru pojačanja $\alpha(\omega)$; e) Izračunati fazno-frekvencijsku karakteristiku $\varphi(\omega)$ = arg $H(j\omega)$. Zadano je: R_1=2, R_2=1, R_3=2, C_1=1/2, C_2=1, $A\to\infty$ (A je pojačanje operacijskog pojačala).



Rješenje:
a) Određivanje prijenosne funkcije napona $H(s)=U_{iz}(s)/U_{ul}(s)$:



1) $U_1\left(\frac{1}{R_1}+\frac{1}{R_3}+sC_1\right)=U_{ul}\frac{1}{R_1}+U_{iz}\frac{1}{R_3}$

2) $A\cdot(U_1-U_2)=U_2 \Rightarrow U_1-U_2=\frac{U_2}{A}\Rightarrow U_1-U_2=0;(A\to\infty)\Rightarrow U_1=U_2$

3) $U_3\left(\frac{1}{R_2}+sC_2\right)=U_2\frac{1}{R_2}+U_{iz}sC_2$

4) $A\cdot(-U_3)=U_{iz}\Rightarrow -U_3=U_{iz}/A\Rightarrow U_3=0;(A\to\infty)$

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2) $\to$ 1) $\Rightarrow$ $U_2\left(\frac{1}{R_1}+\frac{1}{R_3}+sC_1\right)=U_{ul}\frac{1}{R_1}+U_{iz}\frac{1}{R_3}$

4) $\to$ 3) $\Rightarrow$ $0=U_2\frac{1}{R_2}+U_{iz}sC_2 \Rightarrow \frac{U_{iz}}{U_2}=-\frac{1}{sR_2C_2}$

$$\Rightarrow \quad -U_{iz}sR_2C_2\cdot\left(\frac{1}{R_1}+\frac{1}{R_3}+sC_1\right)=U_{ul}\frac{1}{R_1}+U_{iz}\frac{1}{R_3}\Big/\cdot(-R_1R_3)$$

$$U_{iz}sR_2C_2\cdot\left(R_3+R_1+sC_1R_1R_3\right)=-U_{ul}R_3-U_{iz}R_1$$

$$U_{iz}\cdot\left(R_1+sR_2C_2(R_3+R_1)+s^2C_1R_1R_3C_2R_2\right)=-U_{ul}R_3$$

$$H(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{-R_3}{R_1+sR_2C_2(R_3+R_1)+s^2C_1R_1R_3C_2R_2}=\frac{-\frac{R_3}{R_1}\cdot\frac{1}{C_1R_3C_2R_2}}{\frac{1}{C_1R_3C_2R_2}+s\frac{(R_3+R_1)}{C_1R_1R_3}+s^2}=$$

$$= -\frac{1}{1+2s+s^2} = -\frac{1}{(1+s)^2} \text{ (1 bod)}$$

b) Raspored polova i nula: (1 bod)

Polovi: $s+1=0$

$s_{p1,2} = -1$

Nule:

$$\lim_{s\to\infty} \frac{1}{(s+1)^2} = 0 \Rightarrow s_{o1,2} = \infty$$

c) Amplitudno-frekvencijska karakteristika:

Uvrstimo $s = j\omega$ u $H(s)$

$$H(j\omega) = -\frac{1}{(j\omega+1)^2} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{(1+\omega^2)^2}} = \frac{1}{1+\omega^2} \text{ (1 bod)}$$

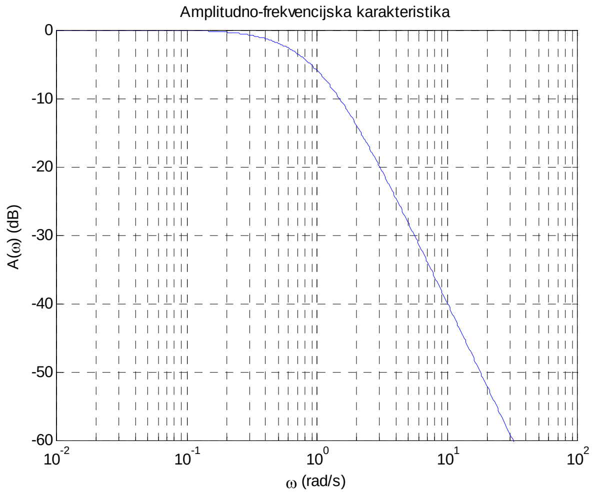
d) Logaritamska mjera pojačanja:

$$\alpha(\omega) = 20\log|H(j\omega)| = 20\log 1 - 20\log(\omega^2+1) = -20\log(\omega^2+1)[\mathrm{dB}] \text{ (1 bod)}$$

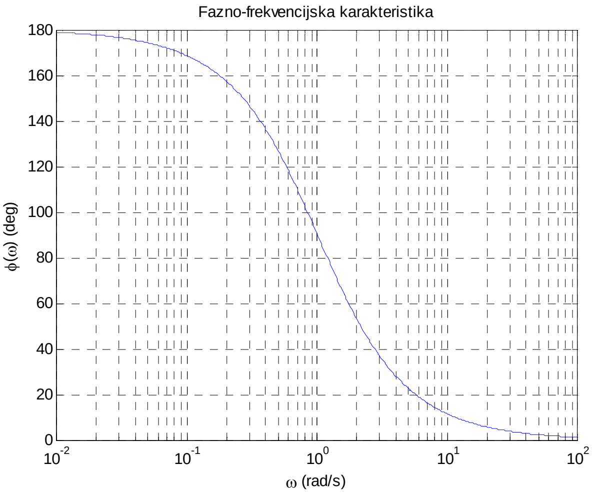
e) Fazno-frekvencijska karakteristika:

$$\varphi(\omega) = \arg H(j\omega) = \pi - 2\arg(j\omega+1) = \pi - 2\arctan(\omega) \text{ (1 bod)}$$

Amplitudno-frekvencijska karakteristika u Matlabu

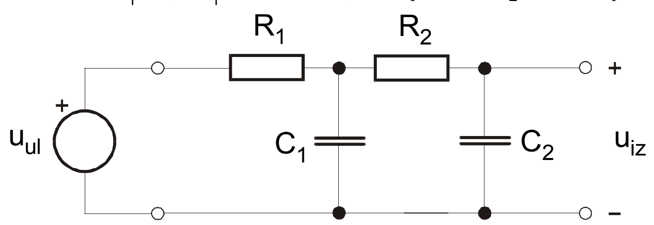


Fazno-frekvencijska karakteristika u Matlabu

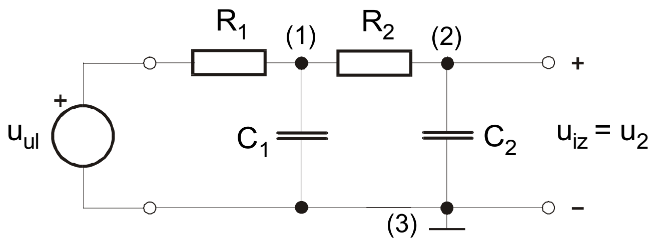


Prijenosne funkcije i a.-f. karakteristika: Zadaci sa rješenjima za vježbu

1. Za električni krug na slici odrediti prijenosnu funkciju napona $T(s)=\dfrac{U_{iz}(s)}{U_{ul}(s)}$. Prikazati frekvencijsku karakteristiku $|T(j\omega)|$. Zadano je: $R_1=1$, $R_2=1$, $C_1=1$, $C_2=1$.

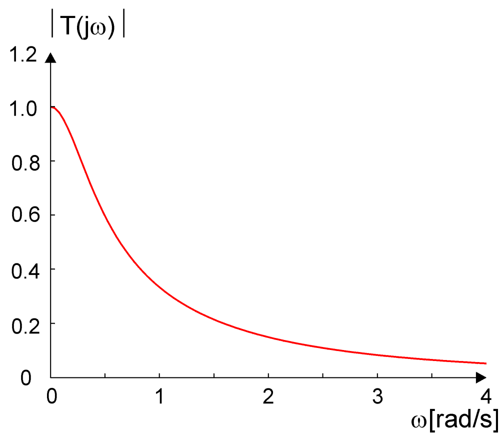


Rješenje:

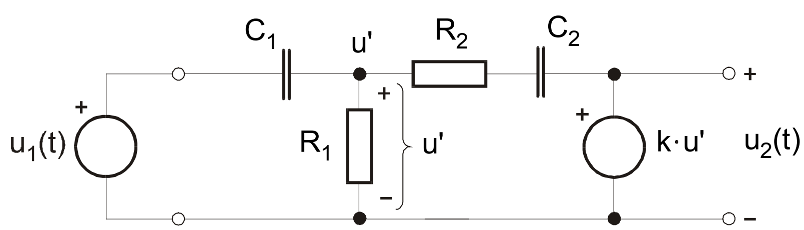


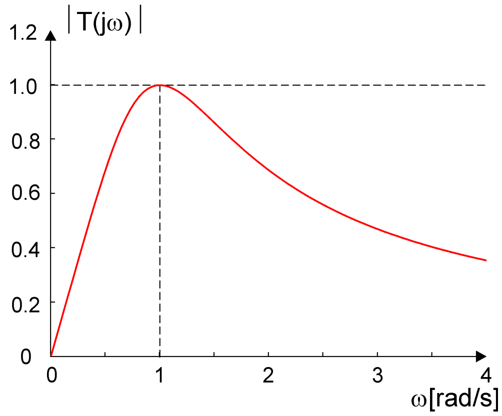
$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{1}{s^2R_1R_2C_1C_2+s(R_1C_1+R_2C_2+R_1C_2)+1}=\frac{1}{s^2+3s+1}$$

$$T(j\omega)=\frac{1}{-\omega^2+3j\omega+1}=\frac{1}{(1-\omega^2)+3j\omega}\qquad\Rightarrow\qquad |T(j\omega)|=\frac{1}{\sqrt{(1-\omega^2)^2+9\omega^2}}$$

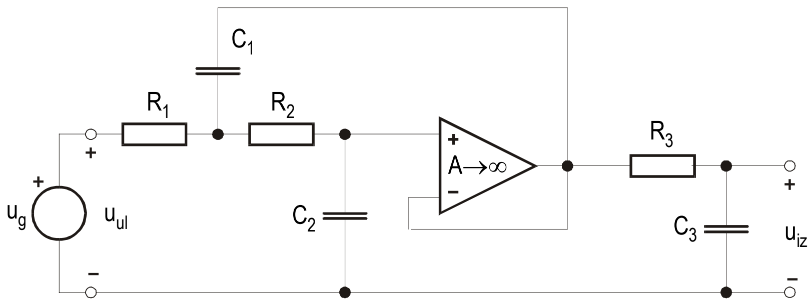


2. Odrediti prijenosnu funkciju $T(s)=\dfrac{U_2(s)}{U_1(s)}$ za električni krug na slici. Zadane su vrijednosti elemenata: $R_1=R_2=10\,K\Omega$, $C_1=C_2=1\,nF$, i parametar k=2. Izvršiti normalizaciju elemenata po frekvenciji $\omega_0=10^5$rad/s i impedanciji $R_0=10^4\Omega$. Odrediti normaliziranu prijenosnu funkciju. Prikazati raspored nula i polova u s ravnini za normaliziranu prijenosnu funkciju.





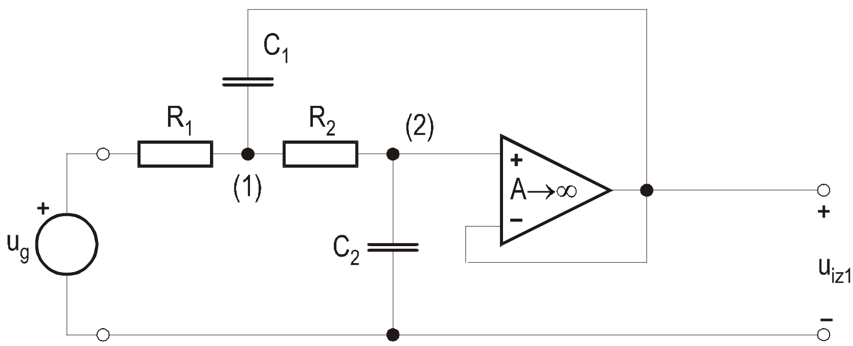
11. Za električni krug prikazan slikom odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s-ravnini te nacrtati amplitudno-frekvencijsku karakteristiku prijenosne funkcije i označiti karakteristične točke. Zadano je R_1=1, R_2=1, R_3=1, C_1=2, C_2=1/2, C_3=1.



Rješenje:

Ukupna prijenosna funkcija sastoji se od dva dijela i ima slijedeći oblik: $T(s)=T_1(s)\cdot T_2(s)$

a) prvi dio:

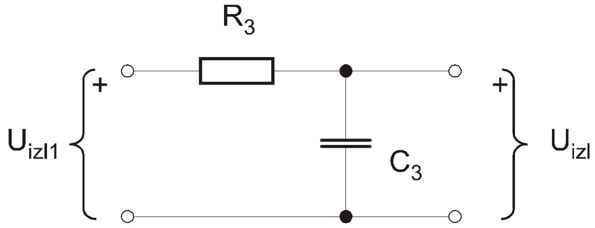


$$T_1(s)=\frac{U_{iz1}}{U_g}=\frac{\dfrac{1}{R_1}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+sC_1+s\dfrac{R_2C_2}{R_1}+sC_2+s^2R_2C_1C_2-\dfrac{1}{R_2}-sC_1}$$

$$=\frac{1}{s^2R_1R_2C_1C_2+s(R_1C_2+R_2C_2)+1}=\frac{1}{s^2+s+1}$$

b) drugi dio: $T_2(s) = \dfrac{U_{iz}}{U_{iz1}}$

$$T_2(s) = \frac{\dfrac{1}{sC_3}}{R_3 + \dfrac{1}{sC_3}} = \frac{\dfrac{1}{R_3C_3}}{s + \dfrac{1}{R_3C_3}} = \frac{1}{s+1}$$



Konačno je ukupna prijenosna funkcija: $T(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

polovi: $s^2 + s + 1 = 0$

$$s_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm j\sqrt{3}}{2}$$

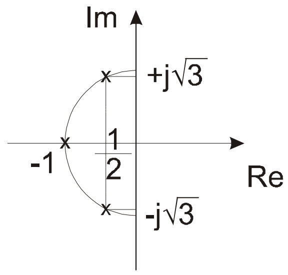
$s + 1 = 0 \Rightarrow \qquad s_3 = -1$

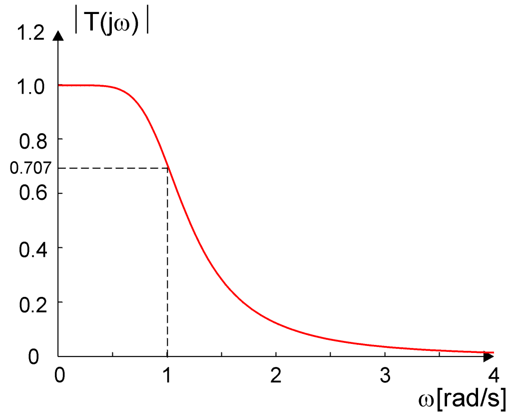
$s_1 = s_2^*$

nule: postoje tri nule u beskonačnosti

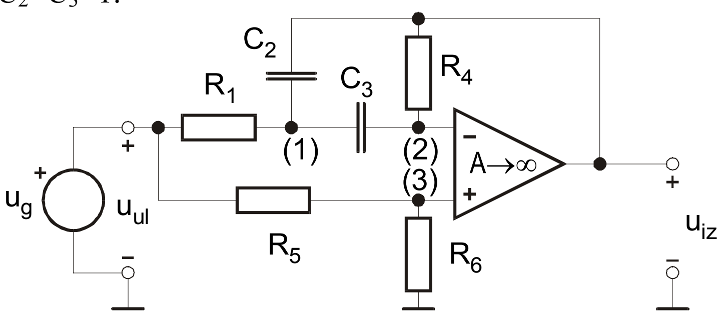
a-f karakteristika:

$$T(j\omega) = \frac{1}{(j\omega+1)(-\omega^2 + j\omega + 1)} \quad \Rightarrow \quad |T(j\omega)| = \frac{1}{\sqrt{\omega^2+1}\sqrt{(1-\omega^2)^2+\omega^2}}$$





12. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su normalizirane vrijednosti elemenata: $R_1=1/2$, $R_4=R_6=2$, $R_5=1$, $C_2=C_3=1$.



Rješenje:

$$T(s)=\frac{U_{iz}}{U_{ul}}=\alpha\cdot\frac{s^2C_2C_3R_4R_1+s\left[\left(1-\frac{1}{\alpha}\right)C_3R_4+C_2R_1+C_3R_1\right]+1}{s^2C_2C_3R_4R_1+s(C_2R_1+C_3R_1)+1},\ \alpha=\frac{R_6}{R_5+R_6}\ (0<\alpha<1);$$

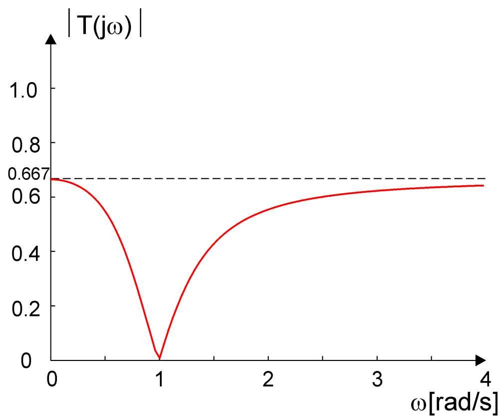
Uvrstimo vrijednosti, konačno je prijenosna funkcija:

$$\boxed{T(s)=\frac{2}{3}\cdot\frac{s^2+1}{s^2+s+1}}$$

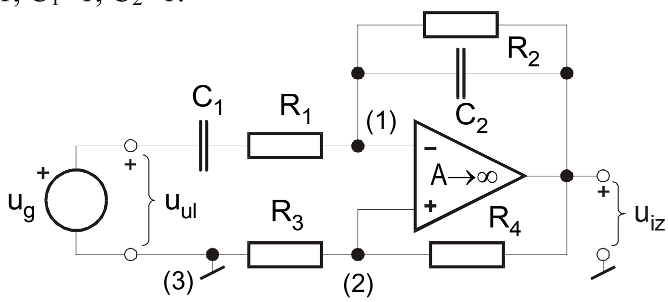
$$\text{nule}: s_{o1,2}=\pm j$$

$$\text{polovi}: s_{p1,2}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$

$$T(j\omega)=\frac{2}{3}\frac{\left|1-\omega^2\right|}{-\omega^2+j\omega+1}\Rightarrow\left|T(j\omega)\right|=\frac{2}{3}\frac{\left|1-\omega^2\right|}{\sqrt{(1-\omega^2)^2+\omega^2}}$$



13. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su normalizirane vrijednosti elemenata: $R_1=1/2$, $R_2=1$, $R_3=1$, $R_4=1$, $C_1=1$, $C_2=1$.



Rješenje:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{-\left(1+\frac{R_3}{R_4}\right)s\frac{1}{R_1C_2}}{s^2+s\frac{R_1C_1+R_2C_2-R_2C_1R_3/R_4}{R_1C_1R_2C_2}+\frac{1}{R_1C_1R_2C_2}}$$

uz uvrštene vrijednosti elemenata je prijenosna funkcija:

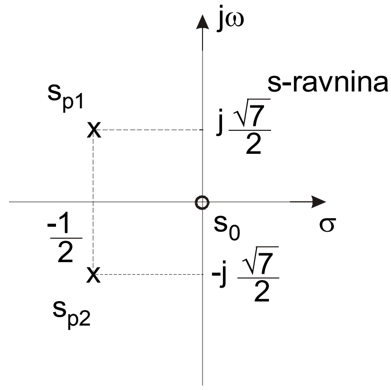
$$T(s)=\frac{-4\cdot s}{s^2+s+2}$$

Polovi: $s^2+s+2=0$

$$s_{p1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-2}=-\frac{1}{2}\pm j\frac{\sqrt{7}}{2}$$

Nule: s=0

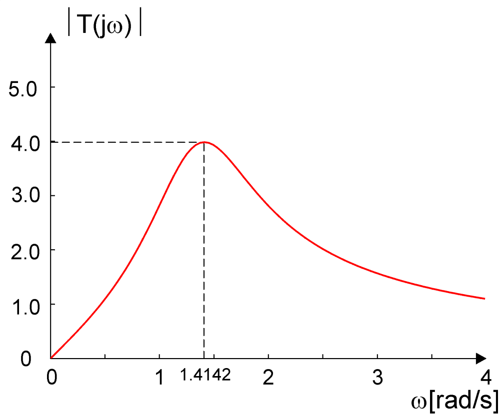
$s_{o1}=0,\quad s_{o2}=\infty$



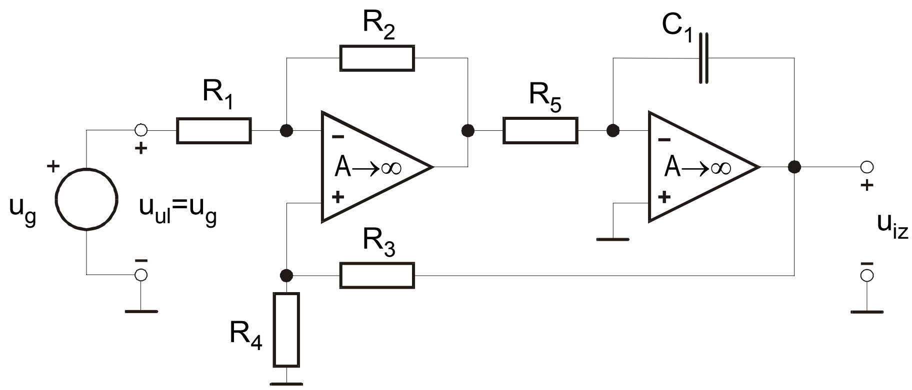
a-f karakteristika : $T(j\omega)=\dfrac{-4\cdot j\omega}{-\omega^2+j\omega+2}\quad\Rightarrow$

$$|T(j\omega)|=\frac{4\cdot\omega}{\sqrt{(2-\omega^2)^2+\omega^2}}$$

Graf:



14. Odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom.

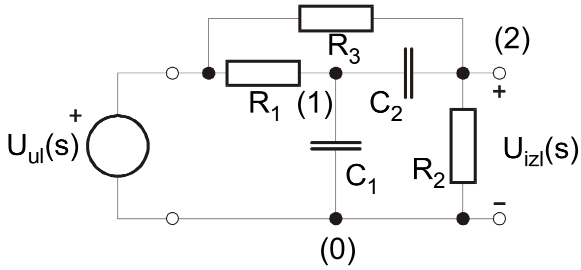


Rješenje:

$$T(s)=\frac{U_6}{U_1}=\frac{1}{s\dfrac{R_1}{R_2}R_5C_1+\dfrac{R_4}{R_3+R_4}\cdot\dfrac{R_1+R_2}{R_2}}=\frac{\dfrac{R_2}{R_1}\cdot\dfrac{1}{R_5C_1}}{s+\dfrac{R_4}{R_3+R_4}\cdot\dfrac{R_1+R_2}{R_1}\cdot\dfrac{1}{R_5\cdot C_1}}=$$

$$=\frac{R_2(R_3+R_4)}{sC_1R_1R_5(R_3+R_4)+R_4(R_1+R_2)}$$

15. Naći naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$ električnog kruga na slici. Nacrtati položaj nula i polova u s-ravnini i konstruirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$, ako su zadane normalizirane vrijednosti elemenata $R_1=R_2=R_3=1$, $C_1=C_2=1$.



Rješenje:

$$U_1\left(\frac{1}{R_1}+sC_1+sC_2\right)-U_2\cdot sC_2=U_{ul}\cdot\frac{1}{R_1}$$

$$-U_1\cdot sC_2+U_2\left(\frac{1}{R_2}+\frac{1}{R_3}+sC_2\right)=U_{ul}\cdot\frac{1}{R_3}$$

$$U_2=\frac{\Delta_2}{\Delta};\qquad T(s)=\frac{U_{iz}}{U_{ul}};\qquad U_{iz}=U_2$$

$$\Delta=\begin{vmatrix}\frac{1}{R_1}+sC_1+sC_2 & -sC_2\\ -sC_2 & \frac{1}{R_2}+\frac{1}{R_3}+sC_2\end{vmatrix}=\left(\frac{1}{R_1}+sC_1+sC_2\right)\cdot\left(\frac{1}{R_2}+\frac{1}{R_3}+sC_2\right)-(sC_2)^2$$

$$\Delta_2=\begin{vmatrix}\frac{1}{R_1}+sC_1+sC_2 & U_{ul}\cdot\frac{1}{R_1}\\ -sC_2 & U_{ul}\cdot\frac{1}{R_3}\end{vmatrix}=U_{ul}\cdot\left[\frac{1}{R_3}\left(\frac{1}{R_1}+sC_1+sC_2\right)+\frac{sC_2}{R_1}\right]$$

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{\Delta_2/\Delta}{U_{ul}}=\frac{\frac{1}{C_1C_2R_1R_2}+s\left[\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3}+\frac{1}{R_1C_1}\right]}{\frac{1}{R_1C_1C_2}\left(\frac{1}{R_2}+\frac{1}{R_3}\right)+s\left[\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\cdot\left(\frac{1}{R_2}+\frac{1}{R_3}\right)+\frac{1}{R_1C_1}\right]+s^2}$$

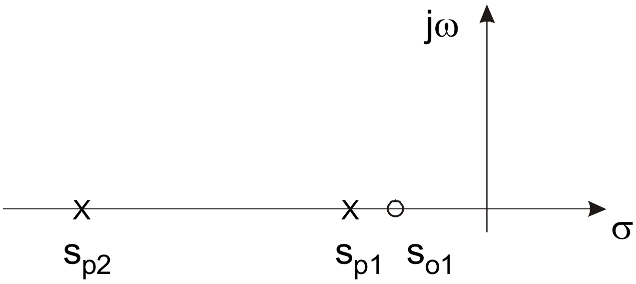
Uz uvrštene vrijednosti elemenata $R_1=R_2=R_3=1,\ C_1=C_2=1$, prijenosna funkcija glasi:

$$T(s)=\frac{1+3s}{2+5s+s^2}$$

$$\text{nule}:\quad 1+3s=0\ \Rightarrow\ s_{o1}=-\frac{1}{3},\qquad s_{o2}=\infty$$

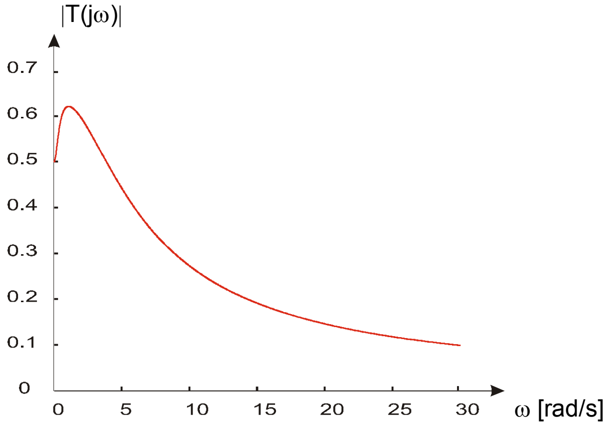
$$\text{polovi}: s^2+5s+2=0\ \Rightarrow\quad s_{p_{1,2}}=\frac{-s\pm\sqrt{25-8}}{2}=\frac{-5\pm\sqrt{17}}{2}$$

$$s_{p_1}=-0.4385\,,\quad s_{p_2}=-4.561$$

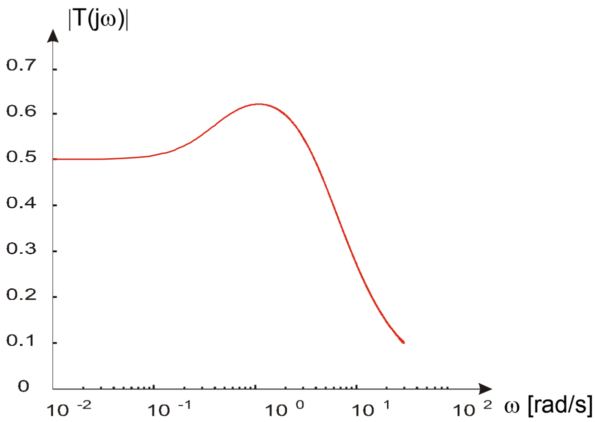


a-f karakteristika:

$$T(j\omega)=\frac{1+3j\omega}{2+5j\omega-\omega^2} \quad \Rightarrow \quad |T(j\omega)|=\frac{\sqrt{1+(3\omega)^2}}{\sqrt{(2-\omega^2)^2+(5\omega)^2}}$$

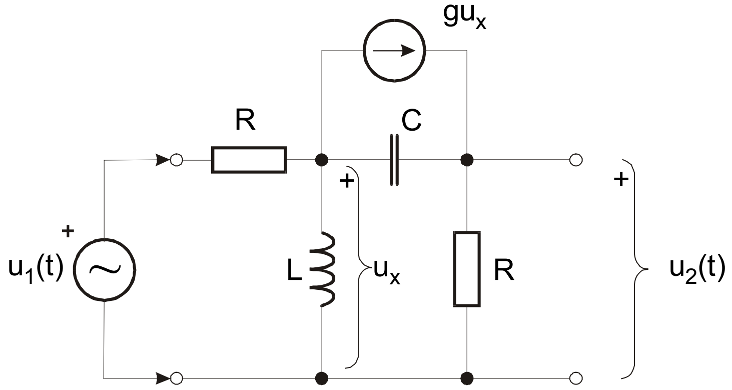


frekvencijska os je u linearnom mjerilu



frekvencijska os je u logaritamskom mjerilu

16. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s)=\dfrac{U_2(s)}{U_1(s)}$ ako su zadane normalizirane vrijednosti elemenata: *R*=1, *C*=1, *L*=2 i parametar *g*=2. Za pobudu $u_1(t)=10\cos t$ (stacionarni sinusni valni oblik) naći valni oblik izlaznog napona $u_2(t)$.



Rješenje:

Prijenosna funkcija napona glasi: $T(s)=\dfrac{U_2(s)}{U_1(s)}=\dfrac{2s^2+4s}{4s^2+7s+1}$

Pošto je pobuda sinusoidalnog stacionarnog valnog oblika u prijenosnu funkciju $T(s)$ supstituiramo $s = j\omega$ i dobivamo:

$$T(j\omega) = \frac{-2\omega^2 + 4j\omega}{(1-4\omega^2) + 7j\omega} \quad (*)$$

Posljednji izraz se može napisati tako da se može razdvojiti realni i imaginarni dio :

$$T(j\omega) = \frac{-2\omega^2 + 4j\omega}{(1-4\omega^2) + 7j\omega} \cdot \frac{(1-4\omega^2) - 7j\omega}{(1-4\omega^2) - 7j\omega} = \frac{2\omega^2(13+4\omega^2) + 2j\omega(2-\omega^2)}{(1-4\omega^2)^2 + (7\omega)^2} \quad (**)$$

Frekvencija pobudnog signala je jednaka 1 stoga uvrštavamo $\omega = 1$ u gornje izraze za $T(j\omega)$.

$$T(1j) = \frac{-2+4j}{-3+7j} \quad (*);\ \text{ili} \qquad T(1j) = 2\frac{17+j}{9+49} = \frac{17+j}{29} \quad (**)$$

Fazor pobudnog signala je : $U_1(j\omega)=10\angle 0°$

Fazor odzivnog signala se izračuna iz : $U_2(j\omega)= T(j\omega)\, U_1(j\omega)$, odnosno:

Amplituda izlaznog signala je: $|U_2(j\omega)| = |T(j\omega)|_{\omega=1} \cdot |U_1(j\omega)|$

Fazni kut izlaznog signala je: $\arg U_2(j\omega) = \arg T(j\omega)_{\omega=1} + \arg U_1(j\omega)$

Amplituda izlaznog signala:

(*) $\Rightarrow$

$$|U_2(j\omega)| = 10 \cdot \left.\frac{\sqrt{4\omega^4 + 16\omega^2}}{\sqrt{(1-4\omega^2)^2 + (7\omega)^2}}\right|_{\omega=1} = 10\frac{\sqrt{2^2+4^2}}{\sqrt{3^2+7^2}} = 10\frac{\sqrt{20}}{\sqrt{58}} = 5.8722 \text{ (jednostavniji način)}$$

(**) $\Rightarrow$

$$|U_2(j\omega)| = 10 \cdot \left.\frac{\sqrt{4\omega^4(13+4\omega^2)^2 + 4\omega^2(2-\omega^2)^2}}{(1-4\omega^2)^2 + (7\omega)^2}\right|_{\omega=1} = 10\frac{\sqrt{17^2+1^2}}{29} = 5.8722$$

Fazni kut izlaznog signala:

(*) $\Rightarrow$

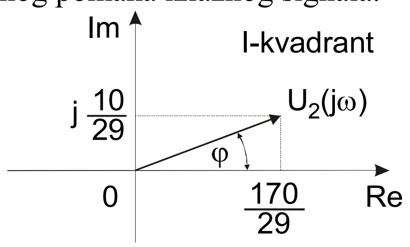
$$\varphi = \varphi_{Brojnika} - \varphi_{Nazivnika} = \left.\text{arctg}\frac{4\omega}{-2\omega^2}\right|_{\omega=1} - \left.\text{arctg}\frac{7\omega}{1-4\omega^2}\right|_{\omega=1} =$$

$$= \text{arctg}\frac{4}{-2} - \text{arctg}\frac{7}{-3} = -63.43 - (-66.8) = 3.37° \text{ (jednostavniji način)}$$

(**) $\Rightarrow$

$$\varphi = \left.\text{arctg}\frac{2\omega(2-\omega^2)}{2\omega^2(13+4\omega^2)}\right|_{\omega=1} = \text{arctg}\frac{1}{17} = 3.37°$$

Uputno je nacrtati fazor izlaznog signala $U_2(j\omega)$ kako bi se vidjelo u kojem kvadrantu se fazor nalazi i odredio točan iznos faznog pomaka izlaznog signala.



Konačno rješenje tj. valni oblik odziva je: $u_2(t)$=5.87 cos (t + 3.37°).

17. Odziv nekog električnog kruga na pobudu $x(t)=S(t)$ glasi: $y(t)=e^{-3t}\mathrm{ch}(2t)S(t)$. Odrediti funkciju mreže i fazor odziva na pobudu $x_1(t)=2\cos(3t+45^\circ)$ (pobuda je sinusoidalni stacionarni signal).

Rješenje:

$$X(s)=\frac{1}{s}$$

$$Y(s)=\frac{s+3}{(s+3)^2-4}=\frac{s+3}{s^2+6s+5}$$

$$H(s)=\frac{s(s+3)}{(s+3)^2-4}=\frac{s(s+3)}{s^2+6s+5}$$

Fazori:

$$H(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}$$

$$X_1(\mathrm{j}\omega)=2e^{\mathrm{j}\pi/4}$$

$$Y_1(\mathrm{j}\omega)=H(\mathrm{j}\omega)X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}2e^{\mathrm{j}\pi/4}$$

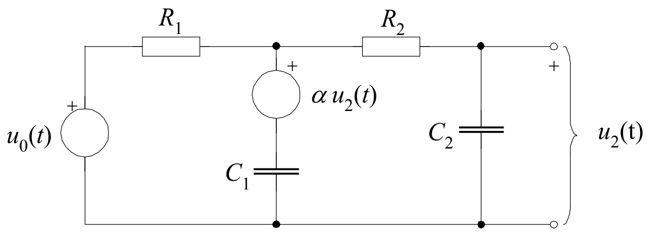
$$\omega=3$$

$$Y_1(\mathrm{j}3)=\frac{\mathrm{j}3\cdot(\mathrm{j}3+3)}{(\mathrm{j}3)^2+18\mathrm{j}+5}\cdot 2\cdot\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)$$

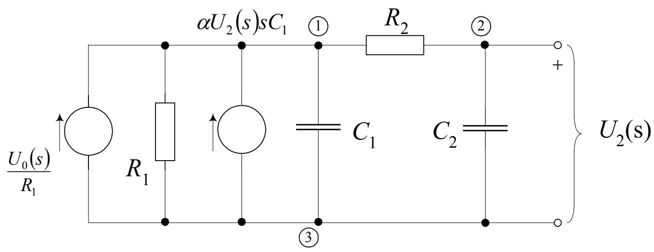
$$Y_1(\mathrm{j}3)=\frac{9\sqrt{2}}{85}\cdot(2+j9)\quad \text{(prvi kvadrant: } 0<\varphi<90^\circ)$$

$$y_1(t)=\frac{9\sqrt{2}\sqrt{85}}{85}\cos(3t+77.47^\circ)$$

18. Odrediti prijenosnu funkciju $H(s)=U_2(s)/U_0(s)$ električnog kruga prema slici. U kojim se granicama mora kretati iznos konstante α da bi polovi funkcije $H(s)$ bili realni? Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $C_1=1$, $C_2=1$.



Rješenje:



$$H(s)=\frac{U_2}{U_0}=\frac{1}{s^2C_1C_2R_1R_2+s\left(C_2R_1+C_2R_2+(1-\alpha)C_1R_1\right)+1}$$

$$H(s)=\frac{U_2}{U_0}=\frac{1}{s^2+s(3-\alpha)+1} \quad\Rightarrow\quad s_{p1,2}=-\frac{3-\alpha}{2}\pm\sqrt{\left(\frac{3-\alpha}{2}\right)^2-1}$$

1. uvjet $\left(\frac{3-\alpha}{2}\right)^2-1\geq 0 \quad\Rightarrow\quad \left(\frac{3-\alpha}{2}\right)^2\geq 1 \;\Rightarrow\; 9-6\alpha+\alpha^2\geq 4 \;\Rightarrow\; \alpha^2-6\alpha+5\geq 0$

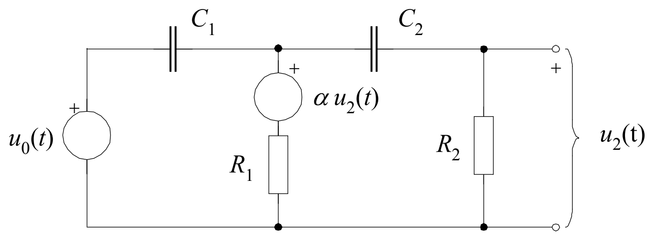
$\Rightarrow\; \alpha_{1,2}=3\pm\sqrt{9-5}=3\pm 2 \;\Rightarrow\; \alpha\leq 1 \,\&\, \alpha\geq 5$

2. uvjet stabilnost: polovi moraju biti u lijevoj poluravnini:

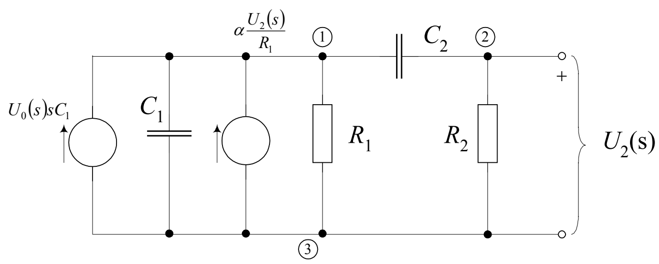
$$p_{1,2}=-\frac{3-\alpha}{2}\pm\sqrt{\left(\frac{3-\alpha}{2}\right)^2-1}\leq 0 \;\Rightarrow\; \alpha\leq 1$$

konačno α mora biti $\alpha\leq 1$.

19. Odrediti prijenosnu funkciju $H(s)=U_2(s)/U_0(s)$ električnog kruga prema slici. U kojim se granicama mora kretati iznos konstante α da bi polovi funkcije $H(s)$ bili kompleksni? Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $C_1=1$, $C_2=1$.



Rješenje:



$$H(s)=\frac{U_2}{U_0}=\frac{s^2C_1C_2R_2R_1}{s^2C_1C_2R_2R_1+(1-\alpha)sR_2C_2+sC_1R_1+sC_2R_1+1}$$

$$H(s)=\frac{U_2}{U_0}=\frac{s^2}{s^2+s(3-\alpha)+1} \quad\Rightarrow\quad s_{p1,2}=-\frac{3-\alpha}{2}\pm\sqrt{\left(\frac{3-\alpha}{2}\right)^2-1}$$

1. uvjet $\left(\frac{3-\alpha}{2}\right)^2-1<0 \quad\Rightarrow\quad \left(\frac{3-\alpha}{2}\right)^2<1 \;\Rightarrow\; 9-6\alpha+\alpha^2<4 \;\Rightarrow\; \alpha^2-6\alpha+5<0$

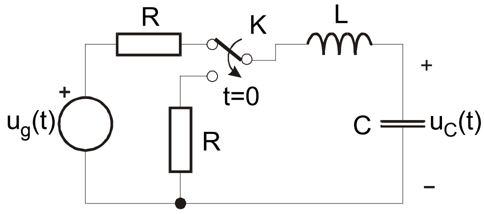
$\Rightarrow\; \alpha_{1,2}=3\pm\sqrt{9-5}=3\pm 2 \;\Rightarrow\; 1<\alpha<5$

2. uvjet stabilnost : $\mathrm{Re}[p_{1,2}]=-\frac{3-\alpha}{2}\leq 0 \;\Rightarrow\; \alpha\leq 3$

konačno α mora biti u granicama $1<\alpha\leq 3$.

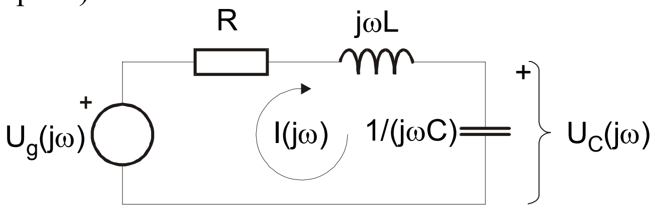
Prijelazne pojave

20. Za mrežu na slici odrediti napon na kapacitetu $u_C(t)$ ako se u trenutku t=0 prebaci sklopka K. Zadano je: R=4, C=1/2, L=2, $u_g(t)$=10sin(2t); $-\infty<t<\infty$ (sinusoidalno stacionarno stanje).



Rješenje:
Zadatak se rješava u dva koraka: u prvom a) koraku se pomoću fazora za t<0 izračuna utjecaj pobude tako da se nađu početni uvjeti: napon na kapacitetu i početna struja kroz induktivitet. U drugom b) koraku se za $t\geq0$ uz poznate početne uvjete pomoću Laplaceove transformacije izračuna traženi napon na kapacitetu $u_C(t)$.

a) t<0 (fazori stuje i napona)

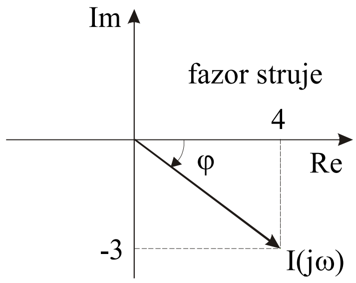


$$U_g(j\omega)=10\angle 0°,\quad \omega=2$$

Fazor struje u električnom krugu:

$$I(j\omega)=\frac{U_g(j\omega)}{R+j\omega L+\dfrac{1}{j\omega C}}=\frac{U_g(j\omega)}{R+j\left(\omega L-\dfrac{1}{\omega C}\right)}=\frac{10}{4+j\left(2\cdot 2-\dfrac{1}{2\cdot(1/2)}\right)}=$$

$$=\frac{10}{4+j3}\cdot\frac{4-j3}{4-j3}=\frac{10\cdot(4-j3)}{25}=\frac{2}{5}\cdot(4-j3)$$



$$|I(j\omega)|=\frac{10}{\sqrt{4^2+3^2}}=\frac{10}{\sqrt{25}}=\frac{10}{5}=2$$

$$\varphi=arctg\frac{\mathrm{Im}}{\mathrm{Re}}=arctg\frac{-3}{4}=-36.87°$$

Iz fazora slijede podaci o struji u električnom krugu u vremenskoj domeni:

$$i(t)=2\cdot\sin(2t-36.87°)$$

U trenutku t=0 se tada može izračunati početna struja u el. krugu koja je ujedno i početna struja kroz induktivitet.

$$i_L(0)=i(t)_{t=0}=2\cdot\sin(-36.87°)=2\cdot(-0.6)=-1.2\,A$$

Rješenje:

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{k\cdot s\cdot\left(s+\dfrac{1}{R_2C_2}\right)}{s^2+s\dfrac{R_1C_1+R_2C_2+R_1C_2(1-k)}{R_1R_2C_1C_2}+\dfrac{1}{R_1R_2C_1C_2}}$$

uz uvrštene vrijednosti elemenata:

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{2\cdot s\cdot\left(s+10^5\right)}{s^2+10^5s+10^{10}}$$

---

normalizacija na: $R_0=10^4\,\Omega$, $\omega_0=10^5\ rad/s$:

$$R_n=\frac{R}{R_0}=\frac{10^4}{10^4}=1 \qquad \Rightarrow \qquad R_1=R_2=1$$

$$C_n=\omega_0\cdot R_0\cdot C=10^5\cdot 10^4\cdot 10^{-9} \qquad \Rightarrow \qquad C_1=C_2=1$$

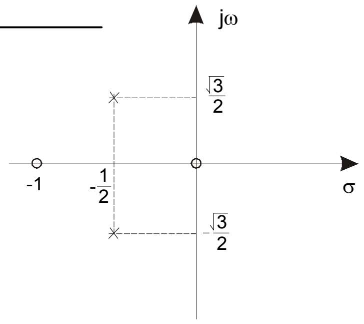
---

uz uvrštene normalizirane vrijednosti elemenata:

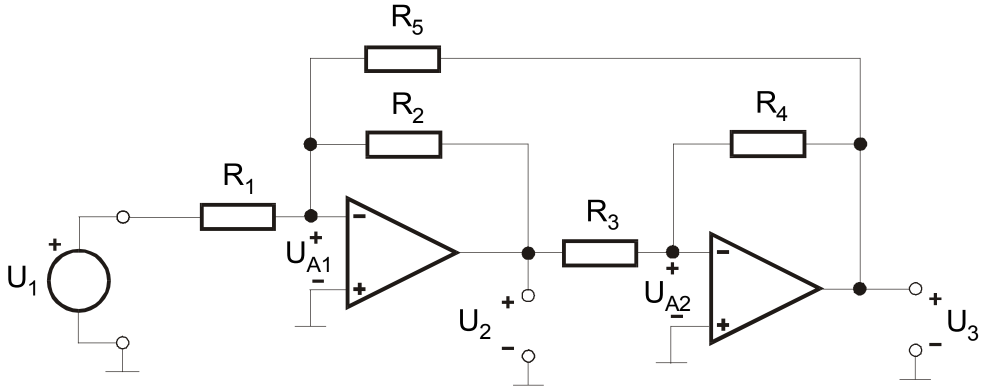
$$T(s)=\frac{U_2}{U_1}=\frac{2s(s+1)}{s^2+s+1}$$

nule: $s_{01}=0,\quad s_{02}=-1$

polovi: $s_{p1,2}=\dfrac{-1\pm\sqrt{1-4}}{2}=-\dfrac{1}{2}\pm j\dfrac{\sqrt{3}}{2}$



3. Za električni krug na slici odrediti naponsku prijenosnu funkciju $T(s)=\dfrac{U_3(s)}{U_1(s)}$. Zadane su normalizirane vrijednosti elemenata: $R_1=R_2=R_3=R_4=1$, $R_5=2$.

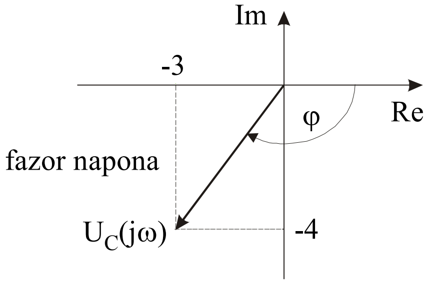


Rješenje:

$$T(s)=\frac{U_3}{U_1}=\frac{R_2R_4R_5}{R_1(R_3R_5-R_2R_4)}=2$$

Fazor napon na kapacitetu C u električnom krugu je:

$$U_C(j\omega) = I(j\omega)\cdot\frac{1}{j\omega C} = \frac{10}{4+j3}\cdot\frac{1}{j2\frac{1}{2}} = \frac{10}{4+j3}\cdot\frac{1}{j} = \frac{10}{-3+j4} = \frac{2}{5}(-3-j4)$$



$$\left|U_C(j\omega)\right| = \frac{10}{\sqrt{3^2+4^2}} = \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$$

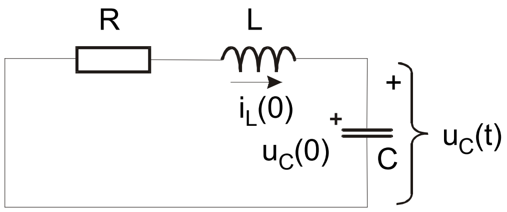
$$\varphi = arctg\frac{\mathrm{Im}}{\mathrm{Re}} = arctg\frac{-3}{-4} = -126.87^\circ$$

$$u_C(t) = 2\cdot\sin(2t-126.87^\circ)$$

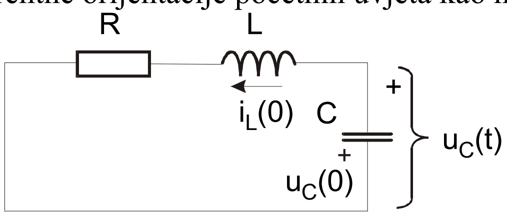
$$u_C(0) = 2\cdot\sin(-126.87^\circ) = 2\cdot(-0.8) = -1.6V$$

b) $t\geq 0$ (Laplaceova transformacija)

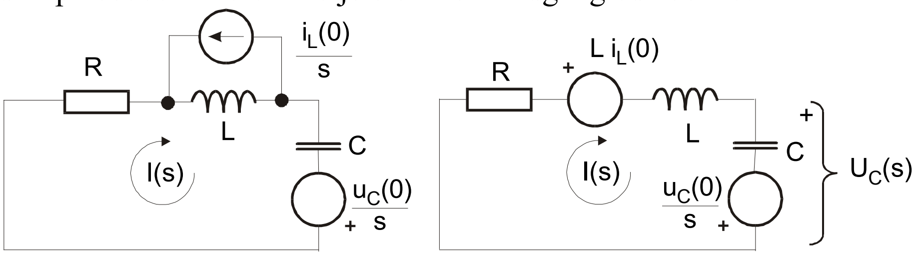
Uz poznate početne uvjete $i_L(0) = -1.2A$ i $u_C(0) = -1.6V$, te pobudu $u_g(t) = 0$, (za $t\geq 0$) električni krug izgleda ovako:



Ako bismo htjeli uvrstiti pozitivne vrijednosti početnih uvjeta $i_L(0) = 1.2A$ i $u_C(0) = 1.6V$, tada trebamo izmjeniti referentne orijentacije početnih uvjeta kao na slijedećoj slici:



Uz primjenu Laplaceove transformacije električni krug izgleda ovako:



Jednadžba za struju za električni krug

$$I(s)\cdot\left(R+\frac{1}{sC}+sL\right)+L\cdot i_L(0)-\frac{u_C(0)}{s} = 0$$

uz uvrštene vrijednosti:

$$I(s)\left(4+\frac{2}{s}+2s\right)+2\cdot 1.2-\frac{1.6}{s}=0$$

$$I(s)=\frac{\frac{1.6}{s}-2.4}{4+\frac{2}{s}+2s}=\frac{1.6-2.4s}{2s^2+4s+2}=\frac{0.8-1.2s}{s^2+2s+1}$$

Traženi napon na kapacitetu je:

$$U_C(s)=I(s)\cdot\frac{1}{sC}-\frac{u_C(0)}{s}$$

uz uvrštene vrijednosti:

$$U_C(s)=\underbrace{\frac{0.8-1.2s}{s^2+2s+1}\cdot\frac{2}{s}}_{(*)}-\frac{1.6}{s}$$

Rastav na parcijalne razlomke izraza (*):

$$(*)=\frac{1.6-2.4s}{s^2+2s+1}\cdot\frac{1}{s}=\frac{As+B}{s^2+2s+1}+\frac{C}{s}=\frac{As^2+Bs+Cs^2+2Cs+C}{(s^2+2s+1)\cdot s}=\frac{(A+C)s^2+(B+2C)s+C}{(s^2+2s+1)\cdot s}$$

$$A+C=0$$
$$B+2C=-2.4$$
$$C=1.6$$

A=–C=–1.6;
B=–2C–2.4=–3.2–2.4=–5.6.

$$(*)=\frac{-1.6s-5.6}{s^2+2s+1}+\frac{1.6}{s}$$

Konačno je:

$$U_C(s)=\underbrace{-1.6\cdot\frac{s}{(s+1)^2}-5.6\frac{1}{(s+1)^2}+\frac{1.6}{s}}_{(*)}-\frac{1.6}{s}=-1.6\cdot\frac{s}{(s+1)^2}-5.6\frac{1}{(s+1)^2}=$$

$$=-1.6\left[\frac{s+1}{(s+1)^2}-\frac{1}{(s+1)^2}\right]-5.6\frac{1}{(s+1)^2}=$$

$$=-1.6\frac{1}{s+1}+\frac{1.6}{(s+1)^2}-5.6\frac{1}{(s+1)^2}=-\frac{1.6}{s+1}-\frac{4}{(s+1)^2}$$

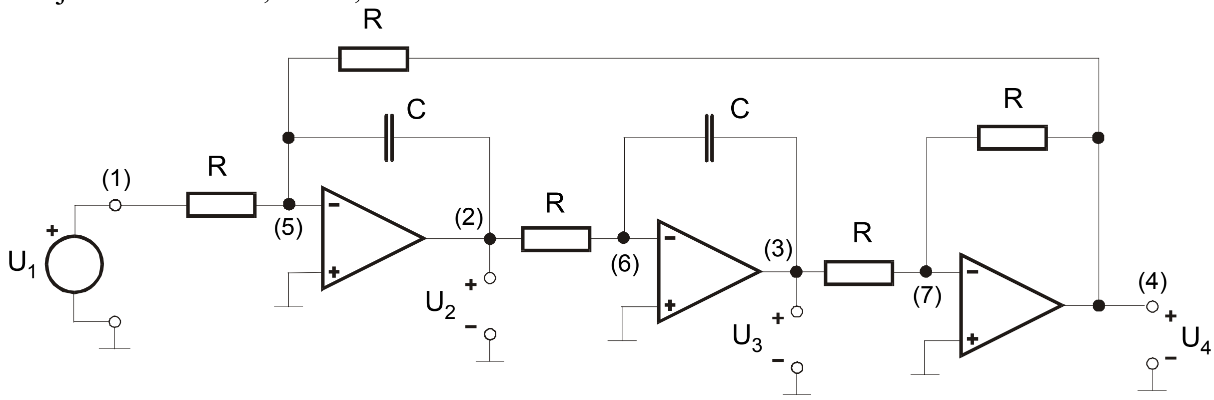
$$u_C(t)=(-1.6\cdot e^{-t}-4t\cdot e^{-t})\cdot S(t)$$

4. Za električni krug prikazan slikom naći prijenosnu funkciju napona:

A) $T(s)=\dfrac{U_4(s)}{U_1(s)}$

B) $T(s)=\dfrac{U_3(s)}{U_1(s)}$

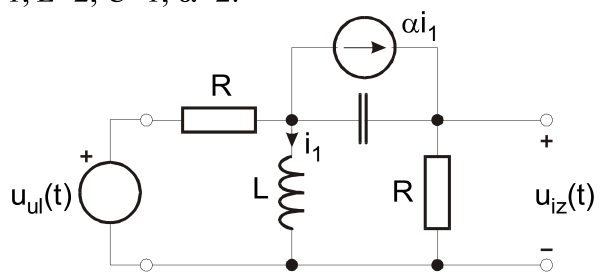
ako je zadano $R=1$, $C=1$, $A\to\infty$.



Rješenje:

A) $T(s)=\dfrac{U_4}{U_1}=-\dfrac{1}{1+s^2}$ B) $T(s)=\dfrac{U_3}{U_1}=\dfrac{1}{1+s^2}$

5. Odrediti prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom. Nacrtati raspored polova i nula te funkcije u kompleksnoj s-ravnini. Odrediti i nacrtati funkciju $|T(j\omega)|$. Zadano je $R=1$, $L=2$, $C=1$, $\alpha=2$.



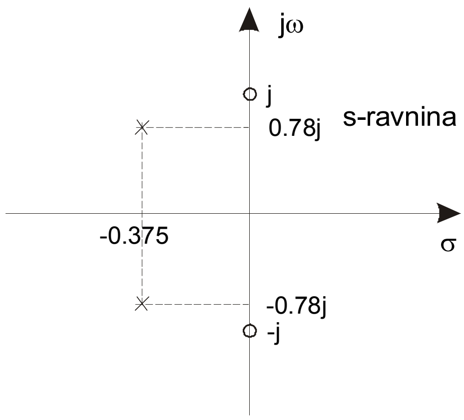
Rješenje:

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{2(s^2+1)}{(s+1)(2s^2+2s+3)-2s(s^2+1)}=\frac{2(s^2+1)}{4s^2+3s+3}=\frac{1}{2}\cdot\frac{s^2+1}{s^2+\frac{3}{4}s+\frac{3}{4}}$$

nule: $s^2+1=0 \Rightarrow s^2=-1 \;/\sqrt{\;} \Rightarrow s_{o1,2}=\pm j$

polovi: $s^2+\frac{3}{4}s+\frac{3}{4}=0 \Rightarrow s_{p1,2}=\frac{-3}{8}\pm\sqrt{\frac{9}{64}-\frac{3}{4}}=\frac{-3}{8}\pm\sqrt{\frac{9-48}{64}}=\frac{-3}{8}\pm j\frac{\sqrt{39}}{8}$

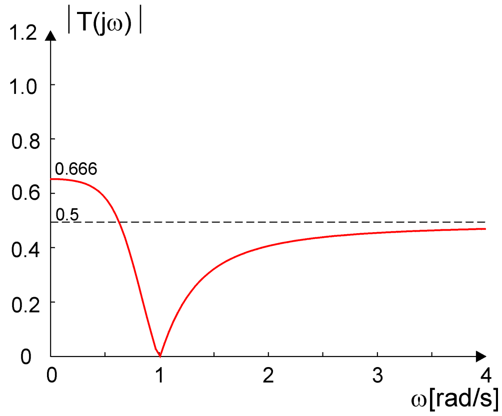
$\Rightarrow s_{p1,2}=-0.375\pm j0.78065$



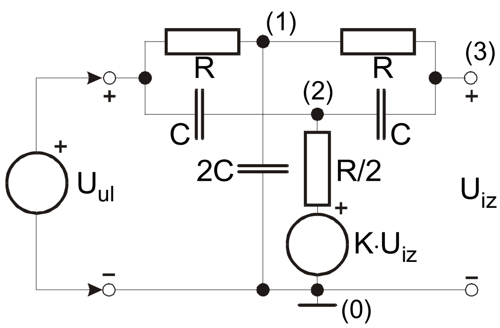
$|T(j\omega)|$:

$$T(j\omega)=\frac{1}{2}\cdot\frac{-\omega^2+1}{-\omega^2+\frac{3}{4}j\omega+\frac{3}{4}} \Rightarrow |T(j\omega)|=\frac{1}{2}\cdot\frac{|1-\omega^2|}{\sqrt{\left(\frac{3}{4}-\omega^2\right)^2+\left(\frac{3}{4}\omega\right)^2}}$$

karakteristične točke: $T(0)=\frac{1}{2}\cdot\frac{1}{\frac{3}{4}}=\frac{1}{2}\cdot\frac{4}{3}=\frac{2}{3}=0.666$, $T(\infty)=\lim\limits_{s\to\infty}\frac{1}{2}\cdot\frac{1+\frac{1}{s^2}}{1+\frac{3}{4}\frac{1}{s}+\frac{3}{4}\frac{1}{s^2}}=\frac{1}{2}=0.5$



6. Odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom, ako je zadano: R=1, C=1, k=1.5. Nacrtati raspored polova i nula u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$.



Rješenje:

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{s^2+1}{s^2+s+1}$$

Polovi:

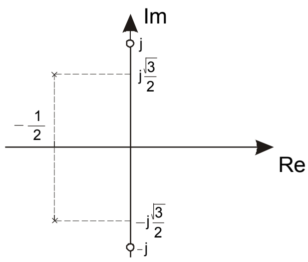
$$s^2 + s + 1 = 0$$

$$s_{P1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

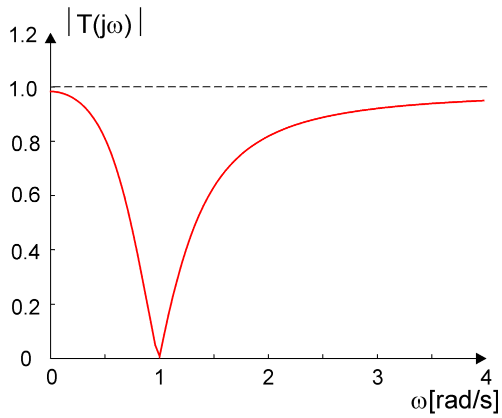
Nule:

$$s^2 + 1 = 0$$

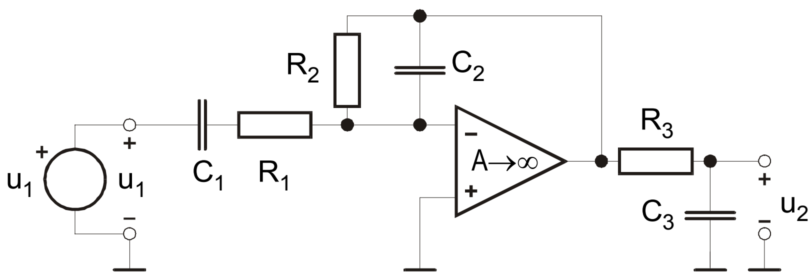
$$s_{o1,2} = \pm j$$



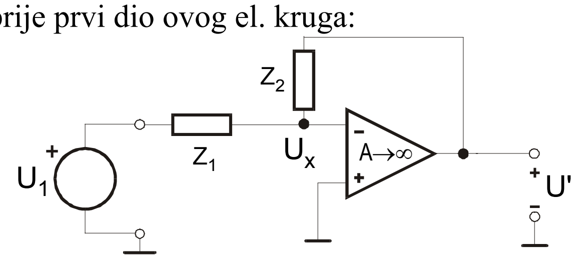
a-f karakteristika:



7. Naći naponsku prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$ električnog kruga na slici. Nacrtati raspored nula i polova u s-ravnini i konstruirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$, ako su zadane normalizirane vrijednosti elemenata $R_1=R_2=R_3=1$, $C_1=1$, $C_2=1/2$, $C_3=2$.



Rješenje: Proučimo najprije prvi dio ovog el. kruga:



$$\boxed{\frac{U'}{U_1}=-\frac{\dfrac{1}{Z_1}}{\dfrac{1}{A}\left(\dfrac{1}{Z_1}+\dfrac{1}{Z_2}\right)+\dfrac{1}{Z_2}}}$$

$$A\to\infty\ \Rightarrow\ \frac{U'}{U_1}=-\frac{\dfrac{1}{Z_1}}{\dfrac{1}{Z_2}}=-\frac{Z_2}{Z_1},\quad Z_1(s)=R_1+\frac{1}{sC_1}=\frac{1+sR_1C_1}{sC_1},\ Z_2(s)=\frac{R_2\dfrac{1}{sC_2}}{R_2+\dfrac{1}{sC_2}}=\frac{R_2}{1+sR_2C_2}$$

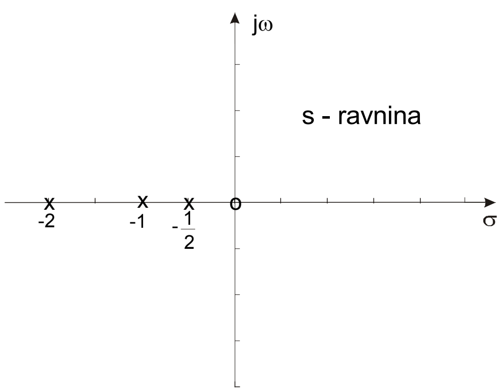
$$\frac{U'}{U_1}=-\frac{sC_1R_2}{(1+sR_1C_1)(1+sR_2C_2)}=\frac{-s\dfrac{1}{R_1C_2}}{\left(s+\dfrac{1}{R_1C_1}\right)\left(s+\dfrac{1}{R_2C_2}\right)}$$

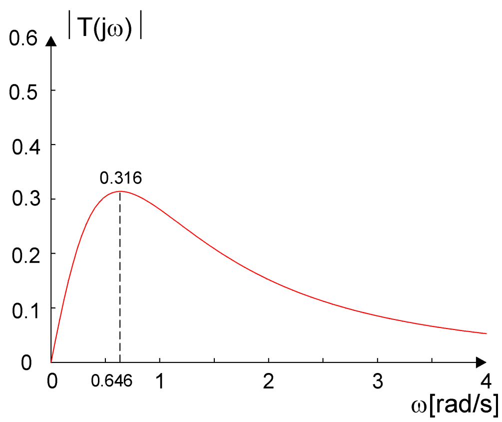
$$\frac{U_2}{U'}=\frac{\dfrac{1}{sC_3}}{R_3+\dfrac{1}{sC_3}}=\frac{1}{sR_3C_3+1}=\frac{\dfrac{1}{R_3C_3}}{s+\dfrac{1}{R_3C_3}}$$

$$T(s)=\frac{U_2}{U_1}=\frac{U'}{U_1}\cdot\frac{U_2}{U'}=\frac{-s\dfrac{1}{R_1C_2}}{\left(s+\dfrac{1}{R_1C_1}\right)\left(s+\dfrac{1}{R_2C_2}\right)}\cdot\frac{\dfrac{1}{R_3C_3}}{\left(s+\dfrac{1}{R_3C_3}\right)}$$

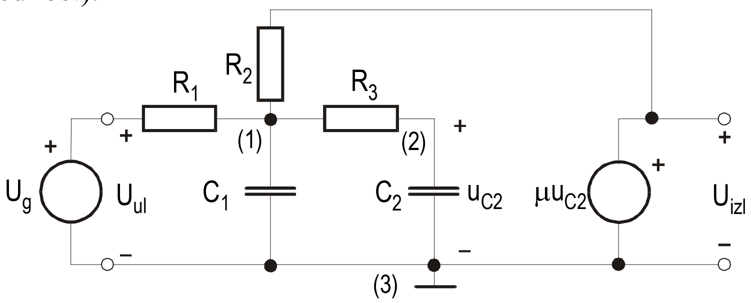
$$R_1=R_2=R_3=1;\quad C_1=1;\quad C_2=\frac{1}{2};\quad C_3=2$$

$$T(s)=\frac{U_2}{U_1}=\frac{-s}{(s+1)\left(s+\dfrac{1}{2}\right)(s+2)}$$





8. Za električni krug prikazan slikom naći prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati raspored nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su vrijednosti elemenata: $R_1=R_2=0.5$, $R_3=1$, $C_1=C_2=1$, $\mu=-3$, (pažnja: naponsko pojačanje μ naponski upravljanog naponskog izvora u mreži ima negativnu vrijednost).



Rješenje:

$$T(s)=\frac{U_2}{U_1}=\frac{-\mu\cdot R_2}{s^2(R_1R_2R_3C_1C_2)+s(R_1R_2C_1+R_2R_3C_2+R_1R_3C_2+R_1R_2C_2)+R_1\cdot(1+\mu)+R_2}$$

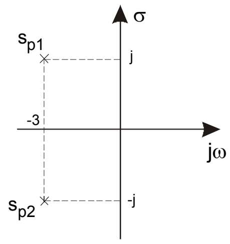
$$T(s)=\frac{-3\cdot 0.5}{0.25s^2+s(0.25+0.5+0.5+0.25)+0.5+4\cdot 0.5}=\frac{-1.5 \quad /\cdot 4}{0.25s^2+1.5s+2.5 \quad /\cdot 4}=$$

$$=\frac{-6}{s^2+6s+10}=\frac{-\frac{3}{5}\cdot 10}{s^2+6s+10}$$

Na frekvenciji $\omega=0$ pojačanje prijenosne funkcije je $T(0)=-3/5$.

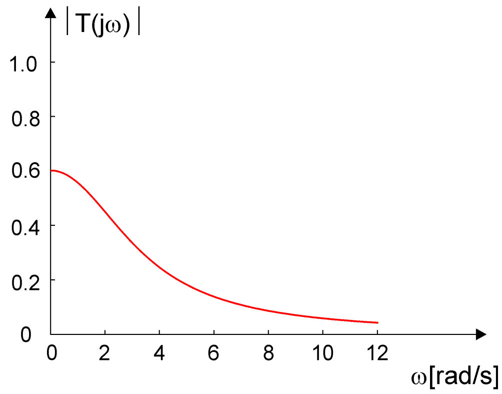
polovi: $s^2+6s+10=0$; $\quad \Delta=b^2-4ac=36-4\cdot 10=-4$

$$s_{p1,2}=\frac{-6\pm j2}{2}=-3\pm j$$

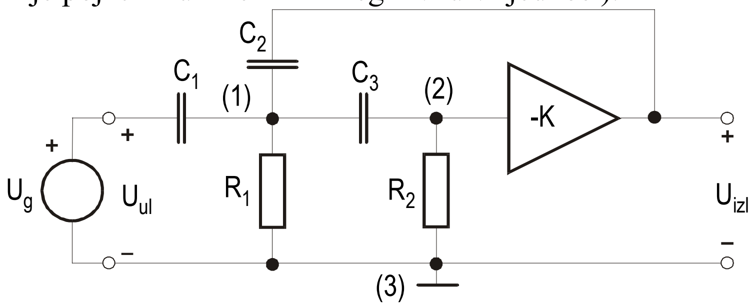


nule: dvije nule u beskonačnosti

$$|T(j\omega)|=\left|\frac{-6}{-\omega^2+6j\omega+10}\right|=\frac{6}{\sqrt{(10-\omega^2)^2+36\omega^2}}$$



9. Za električni krug prikazanu slikom naći prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati raspored nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su vrijednosti elemenata: $R_1=R_2=1$, $C_1=C_2=0.5$, $C_3=1$, $|K|=3$, (pažnja: pojačanje pojačala u mreži ima negativnu vrijednost).



Rješenje:

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{-Ks^2R_1R_2C_1C_3}{s^2R_1R_2C_3\left(C_1+(1+K)C_2\right)+s\left[R_2C_3+R_1(C_1+C_2)+R_1C_3\right]+1}$$

$$T(s)=-\frac{1.5s^2}{2.5s^2+s\cdot[2+1]+1}=-\frac{0.6s^2}{s^2+1.2s+0.4}$$

Na frekvenciji ω=∞ pojačanje prijenosne funkcije je T(0)=–0.6.

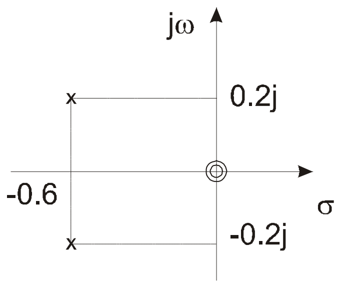
polovi: $s^2+1.2s+0.4=0$

$$s_{p1,2}=\frac{-1.2\pm\sqrt{1.2^2-4\cdot 0.4}}{2}=-0.6\pm 0.2j$$

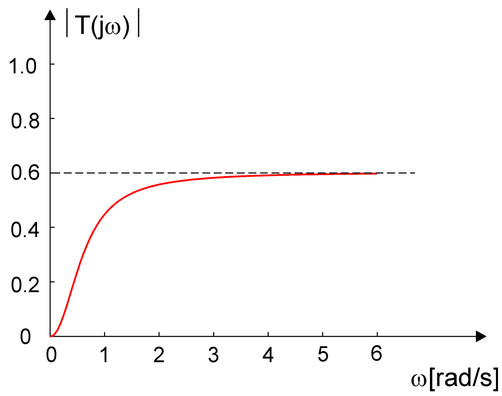
nule: $s^2=0$

$s_{01,2}=0$ dvostruka nula u ishodištu

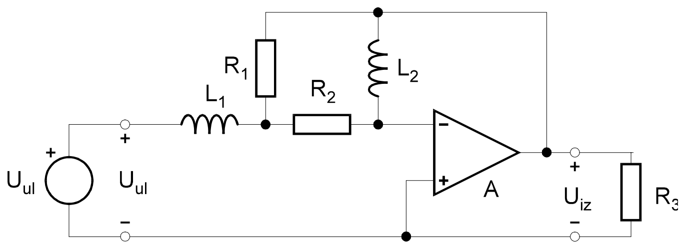
raspored nula i polova u s-ravnini:



$$|T(j\omega)|=\left|\frac{-0.6\omega^2}{-\omega^2+1.2j\omega+0.4}\right|=\frac{0.6\omega^2}{\sqrt{(0.4-\omega^2)^2+(1.2\omega)^2}}$$



10. Za električni krug prikazan slikom treba naći prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$, položaj polova i nula u kompleksnoj s-ravnini i konstruirati amplitudno-frekvencijsku karakteristiku. Zadane su normalizirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $R_3=1$, $L_1=1/\sqrt{2}$, $L_2=\sqrt{2}$ te $A\to\infty$.



Rješenje:

$$T(s)=\frac{U_{izl}}{U_{ul}}=\frac{-s\cdot\dfrac{R_1}{L_1}}{s^2+s\dfrac{R_1+R_2}{L_2}+\dfrac{R_1R_2}{L_1L_2}}$$

uz uvrštene vrijednosti dobivamo:

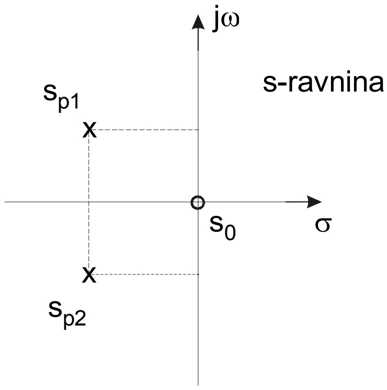
$$T(s)=\frac{U_{iz}}{U_{ul}}=-\frac{s\cdot\sqrt{2}}{s^2+\dfrac{2}{\sqrt{2}}s+\dfrac{1}{\dfrac{1}{\sqrt{2}}\cdot\sqrt{2}}}=-\frac{s\cdot\sqrt{2}}{s^2+\sqrt{2}\cdot s+1}$$

polovi: $s^2+\sqrt{2}\cdot s+1=0$

$$\Rightarrow\ s_{p1,2}=\frac{-\sqrt{2}\pm\sqrt{2-4}}{2}=\frac{-\sqrt{2}\pm j\sqrt{2}}{2}$$

$$s_{p1,2}=-\frac{\sqrt{2}}{2}\pm j\frac{\sqrt{2}}{2}$$

nule: $s_{01}=0$ jedna nula u ishodištu, a druga u beskonačnosti



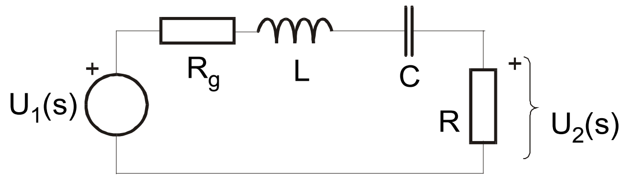
a-f karakteristika (uvrstimo $s=j\omega$ u $T(s)$):

$$T(j\omega)=\frac{U_{iz}(j\omega)}{U_{ul}(j\omega)}=-\frac{-j\omega\cdot\sqrt{2}}{-\omega^2+\sqrt{2}\cdot j\omega+1}$$

$$\left|T(j\omega)\right|=\frac{\omega\sqrt{2}}{\sqrt{\left(1-\omega^2\right)^2+2\omega^2}}=\frac{\omega\sqrt{2}}{\sqrt{1-2\omega^2+\omega^4+2\omega^2}}=\sqrt{2}\cdot\frac{\omega}{\sqrt{1+\omega^4}}$$

FILTRI

4. Za pojasno propusni električni filtar prikazan slikom izračunati naponsku prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$, ako su zadane vrijednosti elemenata L=1mH, R_g=50Ω, C=100nF i R=150Ω. Izračunati i skicirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$. Kolika je širina pojasa propuštanja B, Q-faktor i centralna frekvencija ω_0 , pojačanje u području propuštanja k te gornja i donja granična frekvencija ω_g i ω_d ?



Rješenje: Naponska prijenosna funkcija:

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{Z_2}{Z_1+Z_2}$$

$$Z_1(s)=sL+\frac{1}{sC}+R_g,\quad Z_2(s)=R$$

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{R}{sL+\frac{1}{sC}+R_g+R}=\frac{\frac{R}{L}\cdot s}{s^2+\frac{R+R_g}{L}\cdot s+\frac{1}{LC}}=\frac{\frac{R}{R+R_g}\cdot\frac{R+R_g}{L}\cdot s}{s^2+\frac{R+R_g}{L}\cdot s+\frac{1}{LC}}$$

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{k\cdot\frac{\omega_p}{q_p}\cdot s}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2}\qquad\Rightarrow\qquad \omega_p=\frac{1}{\sqrt{LC}},\qquad k=\frac{R}{R+R_g}$$

$$\frac{\omega_p}{q_p}=\frac{R+R_g}{L}\qquad\Rightarrow\qquad q_p=\omega_p\cdot\frac{L}{R+R_g}=\frac{1}{\sqrt{LC}}\cdot\frac{L}{R+R_g}=\frac{1}{R+R_g}\cdot\sqrt{\frac{L}{C}}$$

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{1.5\cdot10^5\cdot s}{s^2+2\cdot10^5\cdot s+10^{10}}$$

$$\Rightarrow\ T(j\omega)=\frac{1.5\cdot10^5\cdot j\omega}{-\omega^2+2\cdot10^5\cdot j\omega+10^{10}}$$

$$\Rightarrow\ |T(j\omega)|=\frac{1.5\cdot10^5\cdot\omega}{\sqrt{(10^{10}-\omega^2)^2+(2\cdot10^5\cdot\omega)^2}}$$

$$\omega_p=\sqrt{10^{10}}=10^5\ \text{rad/s},\qquad \frac{\omega_p}{q_p}=2\cdot10^5\qquad\Rightarrow\qquad q_p=\frac{10^5}{2\cdot10^5}=\frac{1}{2},\ \ k=\frac{3}{4}$$

Širina pojasa propuštanja $B=\dfrac{\omega_p}{q_p}=2\cdot10^5$ [rad/s]

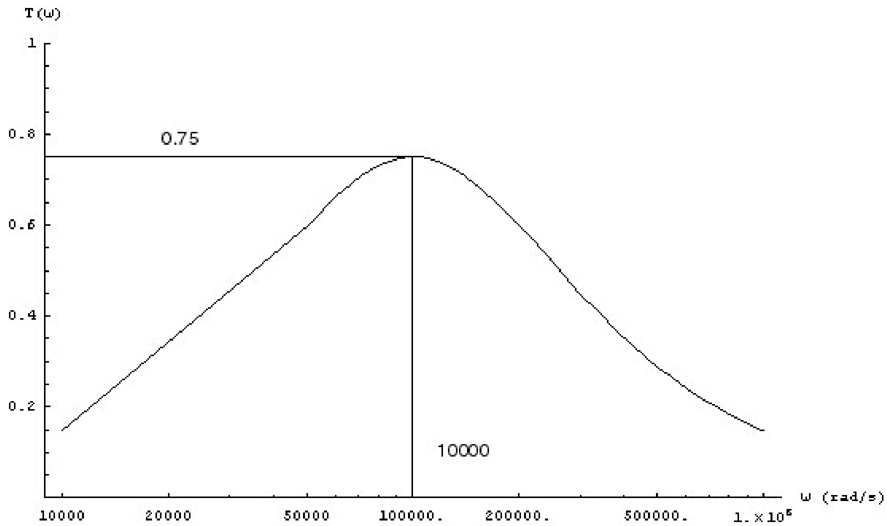
Gornja i donja granična frekvencija su:

$$\omega_{g,d} = \omega_p \sqrt{1 + \frac{1}{4q_p^2}} \pm \frac{\omega_p}{2q_p} = 10^5 \sqrt{1 + \frac{1}{4 \cdot 0.25}} \pm \frac{2 \cdot 10^5}{2} = 10^5 (\sqrt{2} \pm 1) \text{ [rad/s]}$$

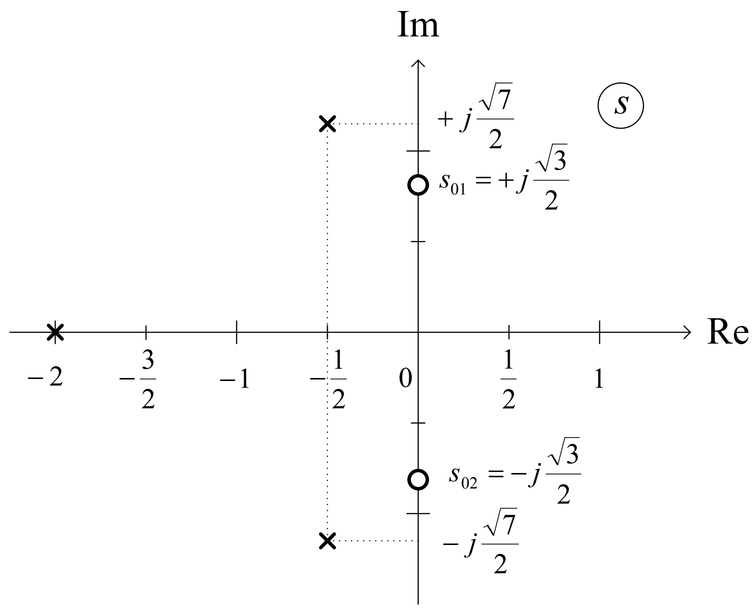
ω_g=241421 [rad/s], ω_d=41421 [rad/s]

B=ω_g–ω_d=241421–41421=20 000= $2 \cdot 10^5$ [rad/s]

Amplitudno-frekvencijska karakteristika:



4. Zadan je raspored polova i nula prema slici prijenosne funkcije $H(s)=U_{iz}(s)/U_{ul}(s)$ nekog električnog kruga. Odrediti: *a*) prijenosnu funkciju $H(s)$; *b*) konstantu k u prijenosnoj funkciji ako se traži da je $H(1)=7/12$; *c*) frekvencijsku karakteristiku $H(j\omega)$; *d*) odziv $u_{iz}(t)$ za pobudu $u_{ul}(t)=\sin(t)$; $\quad -\infty < t < \infty$.



Rješenje:

a) Opći oblik prijenosne funkcije (funkcije mreža) napisan pomoću nula i polova:

$$H(s)=k\cdot\frac{\prod_i (s-s_{0i})}{\prod_j (s-s_{pj})}$$

Nule: $s_{01}=j\frac{\sqrt{3}}{2}$; $\quad s_{02}=-j\frac{\sqrt{3}}{2}$

Polovi: $s_{p1}=-2$; $s_{p2}=-\frac{1}{2}+j\frac{\sqrt{7}}{2}$; $s_{p3}=-\frac{1}{2}-j\frac{\sqrt{7}}{2}$

$$H(s)=k\cdot\frac{\left(s+j\frac{\sqrt{3}}{2}\right)\left(s-j\frac{\sqrt{3}}{2}\right)}{(s+2)\left(s+\frac{1}{2}-j\frac{\sqrt{7}}{2}\right)\left(s+\frac{1}{2}+j\frac{\sqrt{7}}{2}\right)}=k\cdot\frac{(s)^2-\left(j\frac{\sqrt{3}}{2}\right)^2}{(s+2)\left(\left(s+\frac{1}{2}\right)^2-\left(j\frac{\sqrt{7}}{2}\right)^2\right)}=$$

$(a-b)(a+b)=a^2-b^2$ (Upotrijebili smo pravilo: 'Razlika kvadrata')

$$=k\cdot\frac{s^2+\frac{3}{4}}{(s+2)\left(s^2+s+\frac{1}{4}+\frac{7}{4}\right)}=k\cdot\frac{s^2+\frac{3}{4}}{(s+2)(s^2+s+2)}$$ (1 bod)

b) Konstanta k u općem obliku prijenosne funkcije:

$$H(1) = k \cdot \frac{1+3/4}{(1+2)(1+1+2)} = k \cdot \frac{7/4}{3 \cdot 4} = k \cdot \frac{7/4}{12} = k \cdot \frac{7}{4} \cdot \frac{1}{12} = k \cdot \frac{7}{48} = \frac{7}{12} \Rightarrow k = \frac{7}{12} \cdot \frac{48}{7} = 4$$

Konačno je: $H(s) = 4 \cdot \dfrac{s^2 + \dfrac{3}{4}}{(s+2)(s^2+s+2)} = \dfrac{4s^2+3}{(s+2)(s^2+s+2)}$ (1 bod)

---

c) Frekvencijska karakteristika prijenosne funkcije:

$$H(j\omega) = \frac{-4\omega^2+3}{(j\omega+2)(-\omega^2+j\omega+2)}$$ (1 bod)

d) Odziv na pobudu sve-vremenskom sinus funkcijom frekvencije ω=1:

$u_{ul}(t) = \sin(t) \Rightarrow U_{ul}(j\omega) = 1\angle 0°$

$U_{iz}(j\omega) = H(j\omega) \cdot U_{ul}(j\omega) = H(j\omega) \cdot 1\angle 0°$

$$H(j1) = \frac{-4+3}{(j+2)(-1+j+2)} = \frac{-1}{(2+j)(1+j)}$$

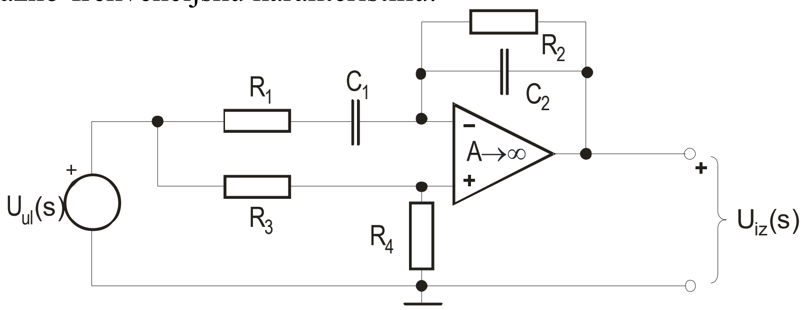
$$|H(j1)| = \frac{1}{\sqrt{(2^2+1)}\sqrt{(1+1)}} = \frac{1}{\sqrt{5}\sqrt{2}} = \frac{1}{\sqrt{10}} = 0.3162$$ (1 bod)

$$H(j1) = \frac{-1}{(2+j)(1+j)} = \frac{-1}{2+2j+j-1} = \frac{-1}{1+3j} \cdot \frac{1-3j}{1-3j} = \frac{-1+3j}{10}$$

$\angle H(j1) = \arctan(-3) = 180° - 71.56° = 108.43°$ (jer je fazor u drugom kvadrantu) (1 bod)

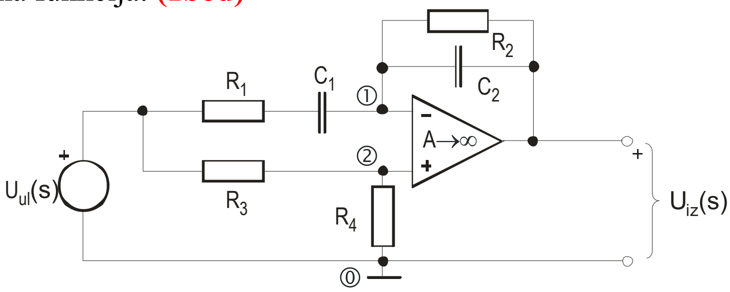
$u_{iz}(t) = 0.3162\sin(t + 108.43°)$

4. Za električni filtar (široko-pojasnu) pojasnu branu prikazanu slikom zadane su vrijednosti elemenata R_1=R_2=2kΩ, C_1=C_2=100nF i R_3=1kΩ. Izračunati: a) naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$; b) otpor R_4 iz uvjeta za pojasnu branu, Q-faktor polova q_p, centralnu frekvenciju ω_0 te pojačanje u području propuštanja k; c) Kolika je širina pojasa gušenja B, te gornja i donja granična frekvencija ω_g i ω_d? Izračunati i skicirati: d) amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$ u dB; e) fazno-frekvencijsku karakteristiku.



Rješenje:

a) Naponska prijenosna funkcija: **(1bod)**



Naponske jednadžbe za čvorove (1) i (2) glase:

$$(1)\quad U_1(s)\left(\frac{1}{R_1+1/sC_1}+sC_2+\frac{1}{R_2}\right)=\frac{U_{ul}(s)}{R_1+1/sC_1}+U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)$$

$$(2)\quad U_2(s)\left(\frac{1}{R_3}+\frac{1}{R_4}\right)=\frac{U_{ul}(s)}{R_3}\quad\Rightarrow\quad U_2(s)=\frac{R_4}{R_3+R_4}U_{ul}(s)$$

Zbog virtualnog kratkog spoja je $U_1(s)=U_2(s)$ pa vrijedi:

$$U_{ul}(s)\frac{R_4}{R_3+R_4}\left(\frac{1}{R_1+\frac{1}{sC_1}}+sC_2+\frac{1}{R_2}\right)=\frac{U_{ul}(s)}{R_1+\frac{1}{sC_1}}+U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)\Bigg/\cdot\left(R_1+\frac{1}{sC_1}\right)$$

$$U_{ul}(s)\frac{R_4}{R_3+R_4}\left[1+\left(sC_2+\frac{1}{R_2}\right)\left(R_1+\frac{1}{sC_1}\right)\right]=U_{ul}(s)+U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)\left(R_1+\frac{1}{sC_1}\right)$$

Nakon kraćeg računanja dobivamo:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{R_4}{R_3+R_4}\cdot\frac{s^2C_1R_1C_2R_2+s\left(C_1R_1+C_2R_2-\frac{R_3}{R_4}C_1R_2\right)+1}{s^2C_1R_1C_2R_2+s\left(C_1R_1+C_2R_2\right)+1}$$

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{R_4}{R_3+R_4}\cdot\frac{s^2+s\frac{C_1R_1+C_2R_2-(R_3/R_4)C_1R_2}{C_1R_1C_2R_2}+\frac{1}{C_1R_1C_2R_2}}{s^2+s\frac{C_1R_1+C_2R_2}{C_1R_1C_2R_2}+\frac{1}{C_1R_1C_2R_2}}$$

b) Parametri prijenosne funkcije: q_p, ω_p i k: **(1bod)**

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=k\cdot\frac{s^2+\omega_p^2}{s^2+(\omega_p/q_p)\cdot s+\omega_p^2}\quad\Rightarrow\quad\omega_p=\frac{1}{\sqrt{C_1R_1C_2R_2}},\qquad k=\frac{R_4}{R_3+R_4},$$

$$\frac{\omega_p}{q_p}=\frac{C_1R_1+C_2R_2}{C_1R_1C_2R_2}\quad\Rightarrow\quad q_p=\omega_p\cdot\frac{C_1R_1C_2R_2}{C_1R_1+C_2R_2}=\frac{\sqrt{C_1R_1C_2R_2}}{C_1R_1+C_2R_2}$$

Uvjet za pojasnu branu je da srednji član u brojniku koji množi s bude jednak nuli (slijedi R_4):

$$C_1R_1+C_2R_2-(R_3/R_4)C_1R_2=0\;\Rightarrow\;C_1R_1+C_2R_2=(R_3/R_4)C_1R_2\;\Rightarrow\;R_4=R_3\cdot\frac{C_1R_2}{C_1R_1+C_2R_2}$$

Uz zadane vrijednosti: $C_1=C_2=C=100\text{nF}$, $R_1=R_2=R=1500\Omega$ i $R_3=1\text{k}\Omega$ slijedi:

$$\omega_p=\frac{1}{RC}=\frac{1}{100\cdot10^{-9}\cdot2000}=5\cdot10^3\,\text{rad/s},\;q_p=\frac{1}{2},\;R_4=R_3\cdot\frac{1}{2}=500\Omega,\;k=\frac{R_3/2}{R_3+R_3/2}=\frac{1/2}{3/2}=\frac{1}{3}$$

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{1}{3}\cdot\frac{s^2+2.5\cdot10^7}{s^2+10^4\cdot s+2.5\cdot10^7}$$

Napomena: za ovaj električni krug te zadanu centralnu frekvenciju $\omega_0=5\cdot10^3$ rad/s, širina pojasa B ne može biti manja od 10^4 rad/s, odn. normirana širina pojasa (omjer) B/ω_0 ne može biti manja od 2. Zato se ovaj električni filtar zove širokopojasni jer se njime ne može realizirati uži pojas gušenja B od navedenoga (i time mu je ograničeno područje primjene).

c) Širina pojasa gušenja B, te gornja i donja granična frekvencija ω_g i ω_d (koriste se isti izrazi kao za pojasno-propusni filtar): **(1bod)**

Širina pojasa gušenja $B=\dfrac{\omega_p}{q_p}=\dfrac{5\cdot10^3}{1/2}=10^4$ [rad/s]

Gornja i donja granična frekvencija pojasa gušenja su:

$$\omega_{g,d}=\omega_p\sqrt{1+\frac{1}{4q_p^2}}\pm\frac{\omega_p}{2q_p}=5\cdot10^3\sqrt{1+\frac{1}{4\cdot0.25}}\pm\frac{5\cdot10^3}{1}=5\cdot10^3(\sqrt{2}\pm1)\;[\text{rad/s}]$$

ω_g=12071 [rad/s], ω_d=2071 [rad/s], $B=\omega_g-\omega_d$=12071−2017=10 000= 10^4 [rad/s]

d) Amplitudno-frekvencijska karakteristika: **(1bod)**

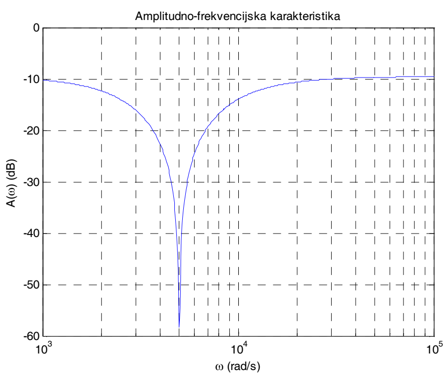
$$\Rightarrow\;T(j\omega)=\frac{N(j\omega)}{D(j\omega)}=\frac{1}{3}\cdot\frac{2.5\cdot10^7-\omega^2}{-\omega^2+10^4\cdot j\omega+2.5\cdot10^7}\;\Rightarrow\;|T(j\omega)|=\frac{1}{3}\cdot\frac{|2.5\cdot10^7-\omega^2|}{\sqrt{(2.5\cdot10^7-\omega^2)^2+(10^4\cdot\omega)^2}}$$

$$\Rightarrow\;A(\omega)[\text{dB}]=20\log|T(j\omega)|=20\log\frac{1}{3}\cdot\frac{|2.5\cdot10^7-\omega^2|}{\sqrt{(2.5\cdot10^7-\omega^2)^2+(10^4\cdot\omega)^2}}$$

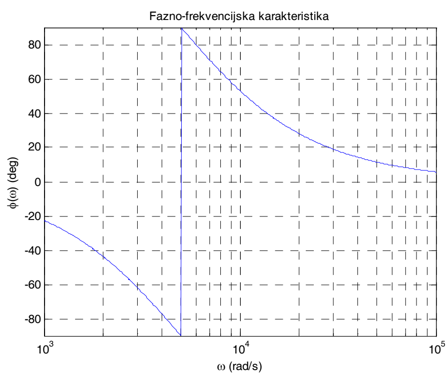
e) Fazno-frekvencijska karakteristika: **(1bod)**

$$\varphi(\omega)=\arctan\frac{\text{Im}[T(j\omega)]}{\text{Re}[T(j\omega)]}=\arctan\frac{\text{Im}[N(j\omega)]}{\text{Re}[N(j\omega)]}-\arctan\frac{\text{Im}[D(j\omega)]}{\text{Re}[D(j\omega)]}=\pi\cdot S(\omega-5\cdot10^3)-\arctan\frac{10^4\cdot\omega}{2.5\cdot10^7-\omega^2}$$

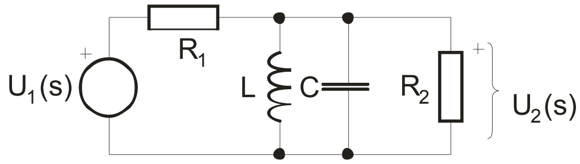
Amplitudno-frekvencijska karakteristika u Matlabu



Fazno-frekvencijska karakteristika u Matlabu



4. Za pojasno propusni električni filtar prikazan slikom zadane su vrijednosti elemenata L=1mH, R_1=50Ω, C=100nF i R_2=50Ω. Izračunati: a) naponsku prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$; b) Q-faktor polova q_p, centralnu frekvenciju ω_0 te pojačanje u području propuštanja k; c) Kolika je širina pojasa propuštanja B, te gornja i donja granična frekvencija ω_g i ω_d? Izračunati i skicirati: d) amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$ u dB; e) fazno-frekvencijsku karakteristiku.



Rješenje:

a) Naponska prijenosna funkcija: **(1bod)**

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{Y_1}{Y_1+Y_2}$$

$$Y_1(s)=\frac{1}{R_1},\quad Y_2(s)=\frac{1}{sL}+\frac{1}{R_2}+sC$$

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{sL}+\frac{1}{R_2}+sC}\frac{/\cdot R_1R_2sL}{/\cdot R_1R_2sL}=\frac{sLR_2}{s^2R_1R_2LC+sL(R_1+R_2)+R_1R_2}$$

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{s\frac{LR_2}{R_1R_2LC}}{s^2+s\frac{L(R_1+R_2)}{R_1R_2LC}+\frac{R_1R_2}{R_1R_2LC}}=\frac{s\frac{1}{R_1C}}{s^2+s\frac{1}{R_pC}+\frac{1}{LC}}=\frac{R_p}{R_1}\cdot\frac{s\frac{1}{R_pC}}{s^2+s\frac{1}{R_pC}+\frac{1}{LC}}$$

Uz zadane vrijednosti:

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{2\cdot10^5\cdot s}{s^2+4\cdot10^5\cdot s+10^{10}}$$

b) Parametri prijenosne funkcije: q_p, ω_p i k: **(1bod)**

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{k\cdot\frac{\omega_p}{q_p}\cdot s}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2}\quad\Rightarrow$$

$$\omega_p=\frac{1}{\sqrt{LC}},\qquad k=\frac{R_p}{R_1}=\frac{R_2}{R_1+R_2},\qquad R_p=\frac{R_1R_2}{R_1+R_2},$$

$$\frac{\omega_p}{q_p}=\frac{1}{R_pC}\quad\Rightarrow\quad q_p=\omega_p\cdot R_pC=\frac{1}{\sqrt{LC}}\cdot R_pC=R_p\cdot\sqrt{\frac{C}{L}}$$

Uz zadane vrijednosti:

$$\omega_p=\sqrt{10^{10}}=10^5\,\text{rad/s},\qquad\frac{\omega_p}{q_p}=4\cdot10^5\quad\Rightarrow\quad q_p=\frac{10^5}{4\cdot10^5}=\frac{1}{4},\quad k=\frac{1}{2}$$

c) Širina pojasa propuštanja B, te gornja i donja granična frekvencija ω_g i ω_d: **(1bod)**

Širina pojasa propuštanja $B=\dfrac{\omega_p}{q_p}=4\cdot10^5$ [rad/s]

Gornja i donja granična frekvencija su:

$$\omega_{g,d}=\omega_p\sqrt{1+\frac{1}{4q_p^2}}\pm\frac{\omega_p}{2q_p}=10^5\sqrt{1+\frac{1}{4\cdot 0.0625}}\pm\frac{4\cdot 10^5}{2}=10^5(\sqrt{5}\pm 2)\ \text{[rad/s]}$$

ω_g=423606,8 [rad/s], ω_d=23606,8 [rad/s]

$B=\omega_g-\omega_d$=423606,8–23606,8=400 000= $4\cdot 10^5$ [rad/s]

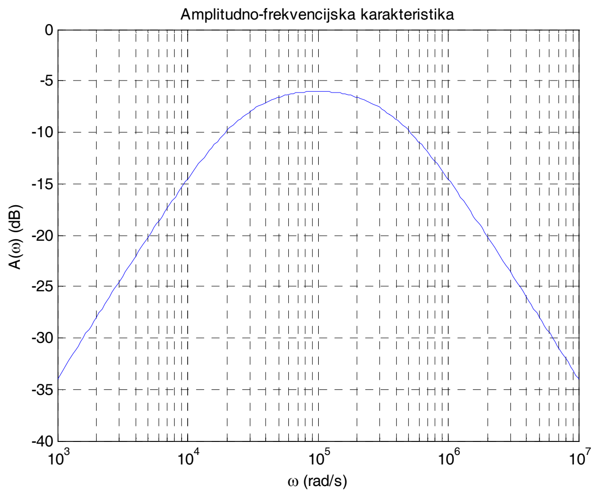
d) Amplitudno-frekvencijska karakteristika: **(1bod)**

$$\Rightarrow\ T(j\omega)=\frac{N(j\omega)}{D(j\omega)}=\frac{2\cdot 10^5\cdot j\omega}{-\omega^2+4\cdot 10^5\cdot j\omega+10^{10}}\ \Rightarrow$$

$$|T(j\omega)|=\frac{2\cdot 10^5\cdot\omega}{\sqrt{(10^{10}-\omega^2)^2+(4\cdot 10^5\cdot\omega)^2}}$$

$$\Rightarrow\ A(\omega)[\text{dB}]=20\log|T(j\omega)|=20\log\frac{2\cdot 10^5\cdot\omega}{\sqrt{(10^{10}-\omega^2)^2+(4\cdot 10^5\cdot\omega)^2}}$$

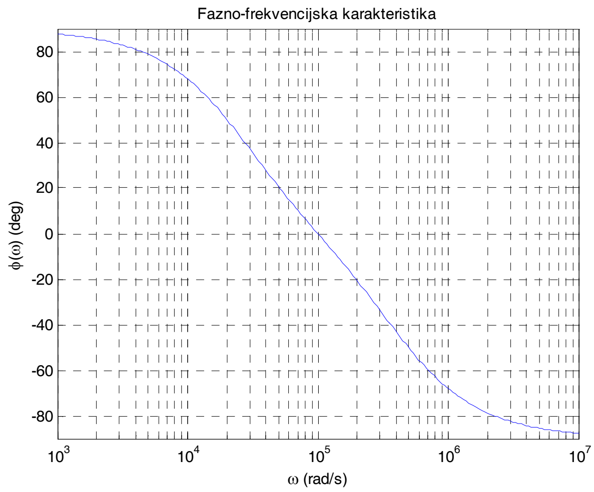
Amplitudno-frekvencijska karakteristika u Matlabu:



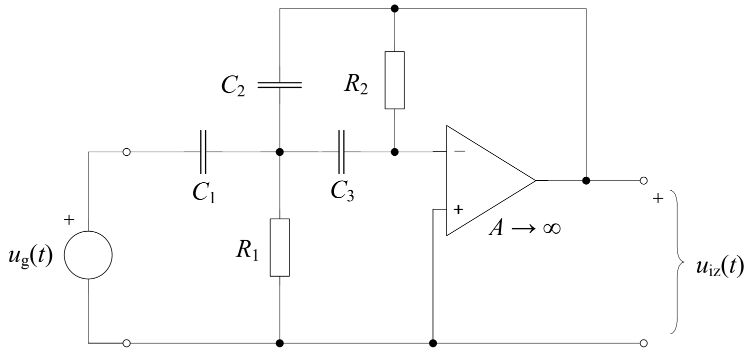
e) Fazno-frekvencijska karakteristika: **(1bod)**

$$\varphi(\omega)=\arctan\frac{\text{Im}[T(j\omega)]}{\text{Re}[T(j\omega)]}=\arctan\frac{\text{Im}[N(j\omega)]}{\text{Re}[N(j\omega)]}-\arctan\frac{\text{Im}[D(j\omega)]}{\text{Re}[D(j\omega)]}=\frac{\pi}{2}-\arctan\frac{4\cdot 10^5\cdot\omega}{10^{10}-\omega^2}$$

Fazno-frekvencijska karakteristika u Matlabu:



4. Zadan je aktivni filtar prikazan slikom i njegova prijenosna funkcija $T(s)=U_{iz}(s)/U_g(s)$. *a*) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja odrediti parametre k, ω_0, Q. O kojem se tipu filtra radi (NP, VP, PP ili PB)? *b*) Ako su zadane normalizirane vrijednosti parametara $\omega_0=1$, $Q=5$ i $|k|=1$ te ako je $C_2=4C_3=1$, izračunati normalizirane vrijednosti kapaciteta C_1 i otpora R_1 i R_2. *c*) Prikazati raspored polova i nula u kompleksnoj ravnini. *d*) Nacrtati amplitudno-frekvencijsku karakteristiku.



$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=-\frac{s^2R_1R_2C_1C_3}{s^2R_1R_2C_2C_3+sR_1(C_1+C_2+C_3)+1}$$

<u>Rješenje:</u>

a) $T(s)=\dfrac{k\cdot s^2}{s^2+\dfrac{\omega_0}{Q}\cdot s+\omega_0{}^2}$ Opći oblik VP (visoki propust)

(uobičajeno je kod el. filtara da je pojačanje k zadano s apsolutnom vrijednosti)
prepišimo $T(s)$ tako da najvišu potenciju od s u nazivniku množi jedinica

$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=\frac{-\dfrac{C_1}{C_2}\cdot s^2}{s^2+s\cdot\dfrac{R_1(C_1+C_2+C_3)}{R_1R_2C_2C_3}+\dfrac{1}{R_1R_2C_2C_3}}$$

-o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ NP (niski propust)
-parametri k, ω_0, Q: (1 bod)

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_2C_3}}\qquad \frac{\omega_0}{Q}=\frac{R_1(C_1+C_2+C_3)}{R_1R_2C_2C_3}=\frac{C_1+C_2+C_3}{R_2C_2C_3}$$

$$\Rightarrow\qquad Q=\frac{R_1R_2C_2C_3}{R_1(C_1+C_2+C_3)}\omega_0=\frac{\sqrt{R_1R_2C_2C_3}}{R_1(C_1+C_2+C_3)},\qquad k=\frac{C_1}{C_2}$$

b) ako su zadane vrijednosti parametara $\omega_0=1$, $Q=5$ i $|k|=1$ te ako je $C_2=4C_3=1$ (tj. $C_2=1$, $C_3=1/4$), izračunati normalizirane vrijednosti kapaciteta C_1 i otpora R_1 i R_2. (2 boda)

uz $C_2=C_3/4=1 \Rightarrow \omega_0=\dfrac{2}{\sqrt{R_1R_2}}=1; \Rightarrow R_1R_2=4 \Rightarrow R_1=\dfrac{4}{R_2}$

$$Q=\frac{\sqrt{R_1R_2C_2C_3}}{R_1(C_1+C_2+C_3)}=\frac{\frac{1}{2}\sqrt{R_1R_2}}{R_1(C_1+1+1/4)}=\frac{2}{9}\cdot\sqrt{\frac{R_2}{R_1}};\ Q=\frac{2}{9}\cdot\sqrt{\frac{R_2}{R_1}}\Rightarrow\frac{R_2}{R_1}=\left(\frac{9}{2}\right)^2Q^2$$

$$k=\frac{C_1}{C_2}=1\Rightarrow C_1=C_2=1;\Rightarrow\frac{R_2^2}{4}=\left(\frac{9}{2}\right)^2Q^2\ \Rightarrow\ R_2^2=4\left(\frac{9}{2}\right)^2Q^2\Rightarrow\ R_2=2\frac{9}{2}Q=9Q$$

$$R_2=9Q=9\cdot5=45,\qquad R_1=4/R_2=4/45=0.08888$$

$\boxed{R_1 = 4/45 = 0.088888,\ R_2 = 45;\ C_1 = C_2 = 1;\ C_3 = 1/4 = 0.25.}$

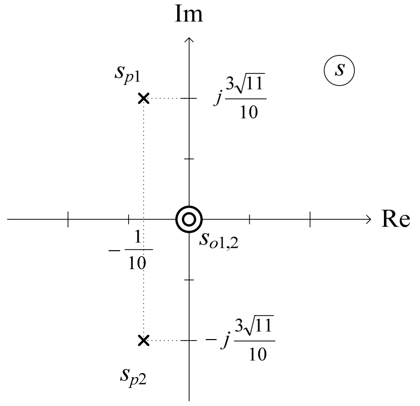
c) raspored polova i nula u kompleksnoj ravnini: (1bod)

$$T(s) = \frac{s^2}{s^2 + \frac{1}{5}\cdot s + 1} = \frac{s^2}{s^2 + 0.2\cdot s + 1}$$

nule $\quad s_{o1,2} = 0$ (dvije nule su u ishodištu)

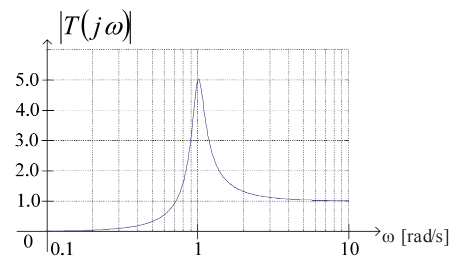
polovi $\quad s^2 + 0.2\cdot s + 1 = 0 \quad \Rightarrow$

$$s_{p1,2} = \frac{-0.2 \pm \sqrt{0.2^2 - 4}}{2} = -\frac{1}{10} \pm j\frac{3\sqrt{11}}{10} = -0.1 \pm j0.994978$$

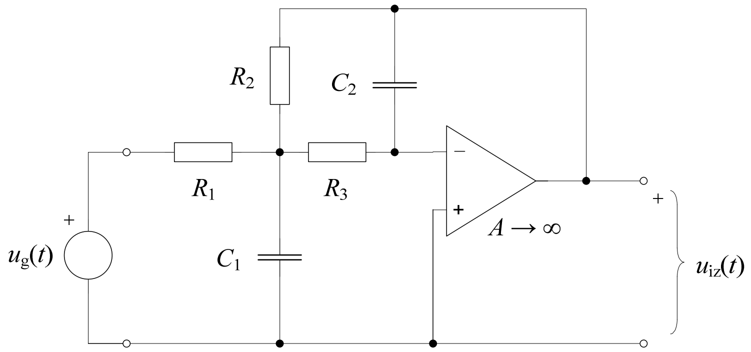


d) amplitudno-frekvencijska karakteristika: (1bod)

$$T(j\omega) = \frac{-\omega^2}{-\omega^2 + j\omega\cdot 0.2 + 1} \quad \Rightarrow \quad |T(j\omega)| = \frac{\omega^2}{\sqrt{(1-\omega^2)^2 + (\omega\cdot 0.2)^2}}$$



4. Zadan je aktivni filtar prikazan slikom i njegova prijenosna funkcija $T(s)=U_{iz}(s)/U_g(s)$. *a*) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja odrediti parametre k, ω_0, Q. O kojem se tipu filtra radi (NP, VP, PP ili PB)? *b*) Ako su zadane normalizirane vrijednosti parametara $\omega_0=1$, $Q=1/\sqrt{2}$ i $|k|=1$ te ako je $R_2=R_3=1$, izračunati normalizirane vrijednosti otpora R_1 i kapaciteta C_1 i C_2. *c*) Prikazati raspored polova i nula u kompleksnoj ravnini. *d*) Nacrtati amplitudno-frekvencijsku karakteristiku.



$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=-\frac{R_2}{s^2R_1R_2R_3C_1C_2+s\left[C_2R_1(R_2+R_3)+C_2R_2R_3\right]+R_1}$$

<u>Rješenje:</u>

a) $T(s)=\dfrac{k\cdot\omega_0^{\,2}}{s^2+\dfrac{\omega_0}{Q}\cdot s+\omega_0^{\,2}}$ Opći oblik NP

(uobičajeno je kod el. filtara da je pojačanje k zadano s apsolutnom vrijednosti)
prepišimo $T(s)$ tako da najvišu potenciju od s u nazivniku množi jedinica

$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=\frac{-\dfrac{R_2}{R_1}\cdot\dfrac{R_1}{R_1R_2R_3C_1C_2}}{s^2+s\cdot\dfrac{C_2R_1(R_2+R_3)+C_2R_2R_3}{R_1R_2R_3C_1C_2}+\dfrac{R_1}{R_1R_2R_3C_1C_2}}$$

-o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ NP (niski propust)
-parametri k, ω_0, Q: (1 bod)

$$\omega_0=\frac{1}{\sqrt{R_2R_3C_1C_2}}\qquad \frac{\omega_0}{Q}=\frac{C_2R_1(R_2+R_3)+C_2R_2R_3}{R_1R_2R_3C_1C_2}=\frac{R_1R_2+R_1R_3+R_2R_3}{R_1R_2R_3C_1}$$

$$\Rightarrow\qquad Q=\frac{R_2R_3C_1C_2}{C_2(R_2+R_3)+C_2R_2R_3/R_1}\omega_0=\frac{\sqrt{R_2R_3C_1C_2}}{C_2(R_2+R_3)+C_2R_2R_3/R_1},\qquad k=\frac{R_2}{R_1}$$

b) ako su zadane vrijednosti parametara $\omega_0=1$, $Q=1/\sqrt{2}$ i $k=1$ te ako je $R_2=R_3=1$, izračunati normalizirane vrijednosti otpora R_1 i kapaciteta C_1 i C_2. (2 boda)

uz $R_2=R_3=1 \Rightarrow \omega_0=\dfrac{1}{\sqrt{C_1C_2}}=1; \Rightarrow C_1=\dfrac{1}{C_2}$ (*)

$$Q=\frac{\sqrt{R_2R_3C_1C_2}}{C_2(R_2+R_3)+k\cdot C_2R_3}=\frac{\sqrt{C_1C_2}}{2C_2+k\cdot C_2}=\sqrt{\frac{C_1}{C_2}}\cdot\frac{1}{2+k};\ Q=\frac{1}{3}\cdot\sqrt{\frac{C_1}{C_2}}\Rightarrow\frac{C_1}{C_2}=9Q^2=\frac{9}{2}\qquad (**)$$

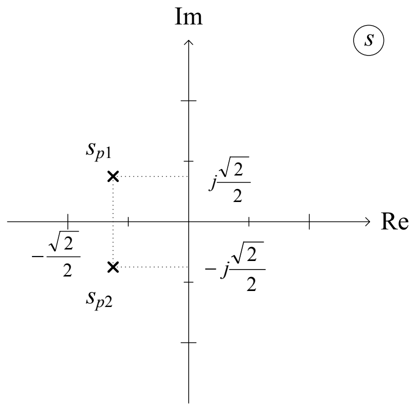
$$k=\frac{R_2}{R_1}=1\Rightarrow R_1=R_2=1;\ (*),\ (**)\Rightarrow\ C_1^2=\frac{9}{2}\quad\Rightarrow\ \boxed{C_1=\frac{3}{\sqrt{2}},\ C_2=\frac{\sqrt{2}}{3};\ R_1=R_2=R_3=1.}$$

c) raspored polova i nula u kompleksnoj ravnini: (1bod)

$$T(s)=\frac{1}{s^2+\sqrt{2}\cdot s+1}$$

nule $\quad s_{o1,2}=\infty$ (dvije nule su u neizmjerno)

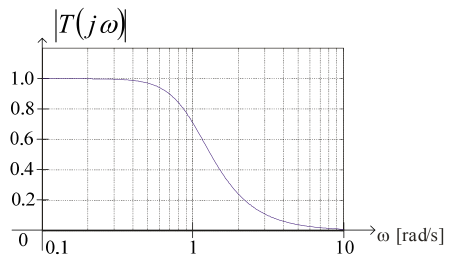
polovi $\quad s^2+\sqrt{2}\cdot s+1=0 \qquad \Rightarrow \qquad s_{p1,2}=-\frac{\sqrt{2}\pm\sqrt{2-4}}{2}=-\frac{\sqrt{2}}{2}\pm j\frac{\sqrt{2}}{2}$



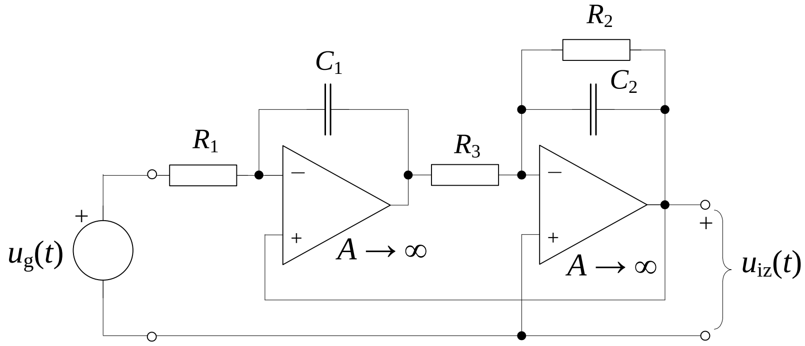
d) amplitudno-frekvencijska karakteristika: (1bod)

$$T(j\omega)=\frac{1}{-\omega^2+j\omega\cdot\sqrt{2}+1} \qquad \Rightarrow$$

$$|T(j\omega)|=\frac{1}{\sqrt{\left(1-\omega^2\right)^2+\left(\omega\cdot\sqrt{2}\right)^2}}=\frac{1}{\sqrt{1-2\omega^2+\omega^4+2\omega^2}}=\frac{1}{\sqrt{1+\omega^4}}$$

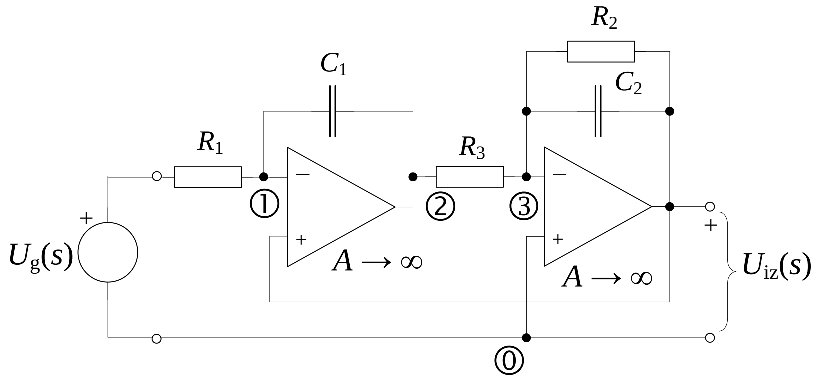


4. Zadan je aktivni-RC električni filtar prikazan slikom. a) Izračunati njegovu naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_g(s)$. b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja odrediti parametre k, ω_0, Q. O kojem se tipu filtra radi (NP, VP, PP ili PB)? c) Ako su zadane normalizirane vrijednosti elemenata C_1=1/2, C_2=2, te R_1=1, R_2=R_3=4, izračunati parametre ω_0, Q i pojačanje k. d) Prikazati raspored polova i nula u kompleksnoj ravnini. e) Nacrtati amplitudno-frekvencijsku karakteristiku.



Rješenje:

a) Metoda čvorišta:



$$(1)\quad U_1\left(sC_1+\frac{1}{R_1}\right)-U_2 sC_1=U_g\frac{1}{R_1}$$

$$(2)\quad -U_2\frac{1}{R_3}+U_3\left(sC_2+\frac{1}{R_2}+\frac{1}{R_3}\right)=U_{iz}\left(sC_2+\frac{1}{R_2}\right)$$

$$(3)\quad A(U_{iz}-U_1)=U_2,\, A\to\infty \Rightarrow U_1=U_{iz}$$

$$(4)\quad A(-U_3)=U_{iz},\, A\to\infty \Rightarrow U_3=0$$

$$(1)\quad U_{iz}\left(sC_1+\frac{1}{R_1}\right)-U_2 sC_1=U_g\frac{1}{R_1}$$

$$(2)\quad -U_2\frac{1}{R_3}=U_{iz}\left(sC_2+\frac{1}{R_2}\right)$$

$$(2)\Rightarrow U_2=-U_{iz}\left(sC_2R_3+\frac{R_3}{R_2}\right)\to(1)\Rightarrow$$

$$U_{iz}\left(sC_1+\frac{1}{R_1}\right)+U_{iz}\left(sC_2R_3+\frac{R_3}{R_2}\right)sC_1=U_g\frac{1}{R_1}\Big/\cdot R_1$$

$$U_{iz}\left(sR_1C_1+1\right)+U_{iz}\left(sC_2R_3+\frac{R_3}{R_2}\right)sR_1C_1=U_g$$

$$U_{iz}\left(s^2R_1C_1C_2R_3+sR_1C_1\frac{R_3}{R_2}+sR_1C_1+1\right)=U_g\,;T(s)=\frac{U_{iz}}{U_g}=\frac{1}{s^2R_1C_1C_2R_3+sR_1C_1\frac{R_3}{R_2}+sR_1C_1+1}$$

$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=\frac{\frac{1}{R_1C_1R_3C_2}}{s^2+s\frac{R_3/R_2+1}{C_2R_3}+\frac{1}{R_1C_1R_3C_2}}\quad\Leftrightarrow\quad T(s)=\frac{k\cdot\omega_0^{\;2}}{s^2+\frac{\omega_0}{Q}\cdot s+\omega_0^{\;2}}\ \text{(Opći oblik NP)}$$

(1 bod)

b) -o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ NP (niski propust)

-parametri k, ω_0, Q:

$$\underline{\omega_0=\frac{1}{\sqrt{R_1R_3C_1C_2}}}\,;\ \underline{k=1};\qquad \frac{\omega_0}{Q}=\frac{R_3/R_2+1}{R_3C_2}\Rightarrow$$

$$Q=\frac{\omega_0}{\frac{R_3/R_2+1}{R_3C_2}}=\frac{R_3C_2}{R_3/R_2+1}\cdot\frac{1}{\sqrt{R_1R_3C_1C_2}}=\underline{\frac{1}{R_3/R_2+1}\cdot\sqrt{\frac{R_3C_2}{R_1C_1}}}\ \textbf{(1 bod)}$$

c) Ako su zadane normalizirane vrijednosti elemenata C_1=0.5, C_2=2, te R_1=1, R_2=R_3=4, izračunati parametre ω_0, Q i pojačanje k. **(1 bod)**

$$\omega_0=\frac{1}{\sqrt{4\cdot0.5\cdot2}}=\frac{1}{2},\ Q=\frac{1}{1+1}\cdot\sqrt{\frac{4\cdot2}{1\cdot0.5}}=\frac{\sqrt{16}}{2}=\frac{4}{2}=2,\ k=1$$

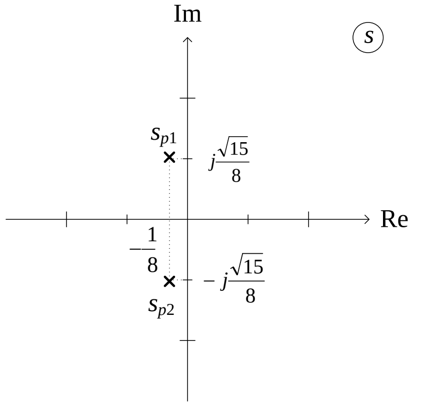
d) raspored polova i nula u kompleksnoj ravnini: **(1 bod)**

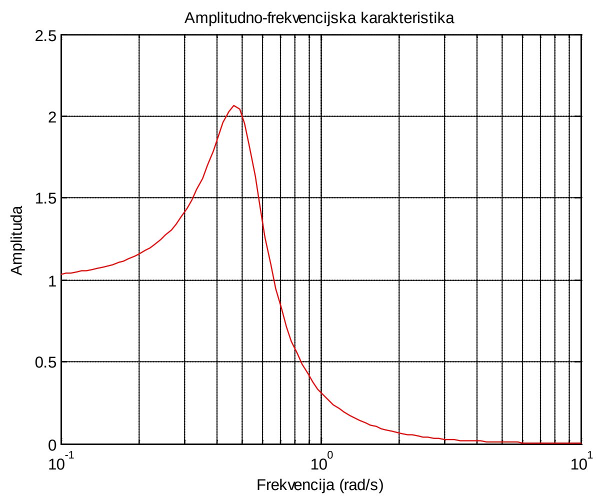
$$T(s)=\frac{\frac{1}{4}}{s^2+\frac{1/2}{2}s+\frac{1}{4}}=\frac{\frac{1}{4}}{s^2+\frac{1}{4}s+\frac{1}{4}}\qquad \text{nule}\quad s_{o1}=\infty,s_{o2}=\infty$$

$$\text{polovi}\quad s^2+\frac{1}{4}s+\frac{1}{4}=0\Rightarrow\qquad s_{p1,2}=-\frac{1}{8}\pm\sqrt{\frac{1}{64}-\frac{1}{4}}=-\frac{1}{8}\pm\sqrt{\frac{1-16}{64}}=-\frac{1}{8}\pm j\frac{\sqrt{15}}{8}$$

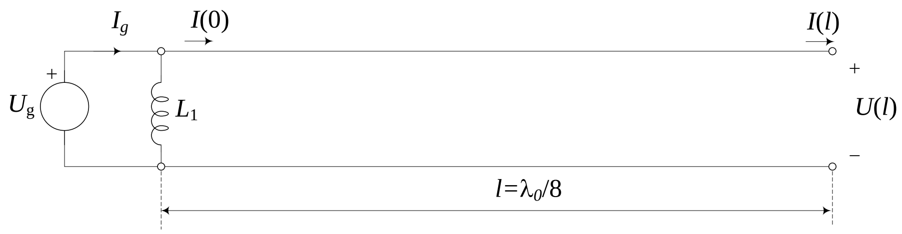
e) amplitudno-frekvencijska karakteristika: **(1 bod)**

$$s{=}j\omega\ \Rightarrow\ T(j\omega)=\frac{\frac{1}{4}}{-\omega^2+j\frac{1}{4}\omega+\frac{1}{4}}\quad\Rightarrow\quad |T(j\omega)|=\frac{\frac{1}{4}}{\sqrt{\left(\frac{1}{4}-\omega^2\right)^2+\left(\frac{1}{4}\omega\right)^2}}$$





5. Zadana je linija bez gubitaka s L=0,2 mH/km i C=80 nF/km. Na ulaz linije je priključen naponski izvor $u_0(t)$ = 20 cos(ω_0t) paralelno s induktivitetom L_1=0,5 mH. Duljina linije je $l=\lambda_0/8$, gdje je λ_0 valna duljina signala pri frekvenciji ω_0. Izlaz linije je otvoren. Odrediti: a) karakterističnu impedanciju Z_0; b) ulaznu impedanciju Z_{ul} linije i frekvenciju ω_0 na kojoj je struja I_g jednaka nuli; c) koeficijent prijenosa γ linije, duljinu linije l u km, valnu duljinu λ_0 signala frekvencije ω_0 i brzinu širenja vala na liniji v; d) struju $I(0)$ na ulazu u liniju; e) napon $U(l)$ i struju $I(l)$ na kraju linije.



Rješenje:

a) Linija bez gubitaka $\rightarrow$ $R=0,\ G=0$ $\quad\Rightarrow\quad Z_0=\sqrt{L/C},\ \gamma=s\sqrt{LC}$

Stac. sinusna pobuda $\rightarrow$ $s=j\omega$ $\quad\Rightarrow\quad \gamma=j\omega\sqrt{LC}=j\beta$

$$Z_0=\sqrt{L/C}=\sqrt{2\cdot10^{-4}/8\cdot10^{-8}}=\sqrt{0{,}25\cdot10^{4}}=50\Omega$$ **(1 bod)**

b) $I(l)=0$

$$U(0)=U(l)\cdot ch(\gamma l)+I(l)Z_0 sh(\gamma l)=U(l)\cdot\cos(\beta l)+jI(l)Z_0\sin(\beta l)=U(l)\cdot\cos(\beta l)$$

$$I(0)=\frac{U(l)}{Z_0}sh(\gamma l)+I(l)ch(\gamma l)=\frac{U(l)}{Z_0}j\sin(\beta l)+I(l)\cos(\beta l)=\frac{U(l)}{Z_0}j\sin(\beta l)$$

$$Z_{ul}=\frac{U(0)}{I(0)}=-jZ_0\mathrm{ctg}(\beta l)\quad\Rightarrow\quad Y_{ul}=j\frac{\mathrm{tg}(\beta l)}{Z_0}=j\frac{\mathrm{tg}(\pi/4)}{Z_0}=j\frac{1}{Z_0}$$

$$Y_{ul}+\frac{1}{j\omega L_1}=j\frac{1}{Z_0}-j\frac{1}{\omega L_1}=0\Rightarrow\quad \omega=\omega_0=\frac{Z_0}{L_1}=\frac{50}{0{,}5\cdot10^{-3}}=10^5[\mathrm{rad/s}]$$ **(1 bod)**

c) $\gamma=j\omega_0\sqrt{LC}=j10^5\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}=j4\cdot10^{-1}=j0{,}4/\mathrm{km}$

$$l=\lambda_0/8=\frac{2\pi}{8\beta}=\frac{\pi}{4\omega_0\sqrt{LC}}=\pi\cdot0{,}625\ \mathrm{km}=1{,}9635\mathrm{km}$$

$$\lambda_0=8l=\pi\cdot5\ \mathrm{km}=15{,}708\mathrm{km}$$

$$v=\frac{\omega_0}{\beta}=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}}=2{,}5\cdot10^5\mathrm{km/s}$$ **(1 bod)**

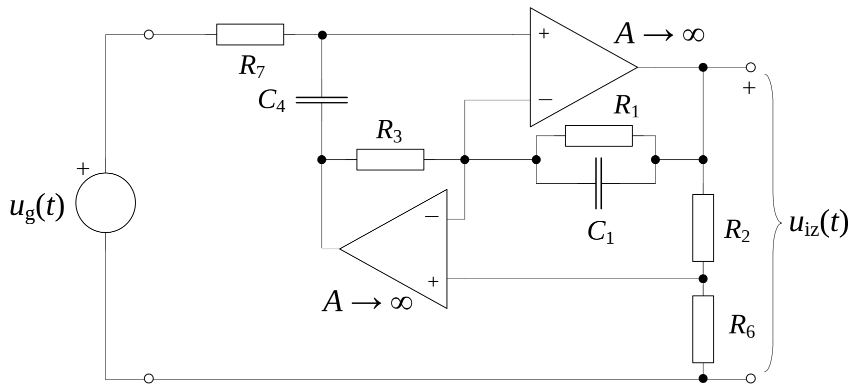
d) $U_g=20\angle0^\circ$

$$I(0)=\frac{U(0)}{Z_{ul}}=\frac{U_g}{-jZ_0}=j\frac{U_g}{Z_0}=j\frac{20}{50}=0{,}4\angle90^\circ\ \mathrm{A}$$ **(1 bod)**

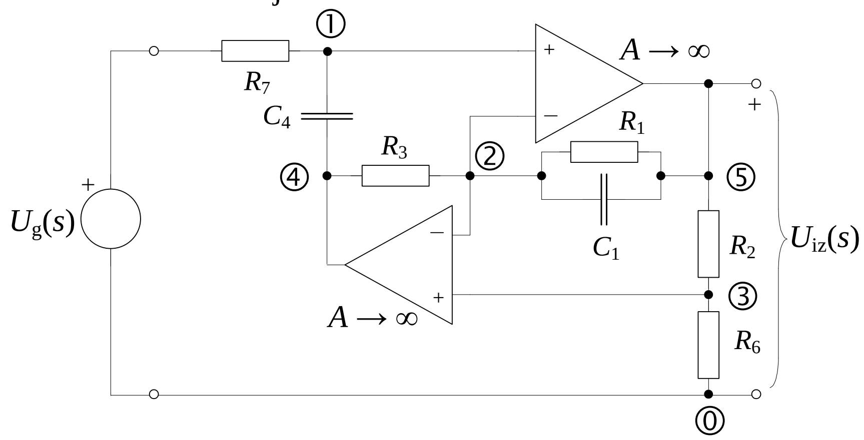
e) $U(0)=U(l)\cdot\cos(\beta\cdot l)=U(l)\cdot\frac{\sqrt{2}}{2}$

$$U(l)=U(0)\cdot\sqrt{2}=U_g\cdot\sqrt{2}=20\cdot\sqrt{2}\angle0^\circ\ \mathrm{V}$$ **(1 bod)**

4. Zadan je aktivni-RC električni filtar prikazan slikom s normaliziranim vrijednostima elemenata C_1=1/2, C_4=2, te R_1=3, R_2=R_3=R_6=R_7=1. a) Izračunati njegovu naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_g(s)$. O kojem se tipu filtra radi (NP, VP, PP ili PB)? b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati parametre k, ω_0, Q. c) Prikazati raspored polova i nula u kompleksnoj ravnini. d) Nacrtati amplitudno-frekvencijsku karakteristiku.



Rješenje: Laplaceova transformacija:



a) Metoda čvorišta:

(1) $U_1\left(\frac{1}{R_7}+sC_4\right)-U_4sC_4=U_g\frac{1}{R_7}$

(2) $-U_4\frac{1}{R_3}+U_2\left(\frac{1}{R_1}+\frac{1}{R_3}+sC_1\right)-U_5\left(\frac{1}{R_1}+sC_1\right)=0$

(3) $-U_5\frac{1}{R_2}+U_3\left(\frac{1}{R_2}+\frac{1}{R_6}\right)=0$

(4) $A\to\infty\Rightarrow U_1=U_2=U_3$

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(1) $U_1\left(\frac{1}{R_7}+sC_4\right)-U_4sC_4=U_g\frac{1}{R_7}$

(2) $-U_4\frac{1}{R_3}+U_1\left(\frac{1}{R_1}+\frac{1}{R_3}+sC_1\right)-U_5\left(\frac{1}{R_1}+sC_1\right)=0$

(3) $-U_5\frac{1}{R_2}+U_1\left(\frac{1}{R_2}+\frac{1}{R_6}\right)=0$

$$(1) \Rightarrow U_4 = -U_g \frac{1}{sR_7C_4} + U_1\left(\frac{1}{sR_7C_4}+1\right)$$

$$(2) \Rightarrow U_4 = U_1\left(\frac{R_3}{R_1}+1+sR_3C_1\right) - U_5\left(\frac{R_3}{R_1}+R_3sC_1\right)$$

$$(3) \Rightarrow U_1 = \frac{R_6}{R_2+R_6}U_5; \quad U_5 = U_{iz} = \left(\frac{R_2}{R_6}+1\right)U_1$$

$$(1), (3) \rightarrow (2) \Rightarrow$$

$$-U_g \frac{1}{sR_7C_4} + U_1\left(\frac{1}{sR_7C_4}+1\right) = U_1\left(\frac{R_3}{R_1}+sR_3C_1+1\right) - U_1\left(\frac{R_2}{R_6}+1\right)\left(\frac{R_3}{R_1}+sR_3C_1\right)$$

$$-U_g \frac{1}{sR_7C_4} + U_1\frac{1}{sR_7C_4} + U_1 = U_1\left(\frac{R_3}{R_1}+sR_3C_1\right) + U_1 - U_1\frac{R_2}{R_6}\left(\frac{R_3}{R_1}+sR_3C_1\right) - U_1\left(\frac{R_3}{R_1}+sR_3C_1\right)$$

$$U_g \frac{1}{sR_7C_4} = U_1\frac{1}{sR_7C_4} + U_1\frac{R_2}{R_6}\left(\frac{R_3}{R_1}+sR_3C_1\right) \Big/ \cdot sR_7C_4$$

$$U_g = U_1\left(1+\frac{R_3}{R_1}\frac{R_2}{R_6}sR_7C_4 + s\frac{R_2}{R_6}R_3C_1sR_7C_4\right)$$

$$U_g = U_1\left(1+s\frac{R_2R_3R_7C_4}{R_1R_6}+s^2\frac{R_2R_3R_7C_1C_4}{R_6}\right)$$

$$U_g = \frac{R_6}{R_2+R_6}U_{iz}\left(1+s\frac{R_2R_3R_7C_4}{R_1R_6}+s^2\frac{R_2R_3R_7C_1C_4}{R_6}\right)$$

$$T(s) = \frac{U_{iz}(s)}{U_g(s)} = \frac{\dfrac{R_2+R_6}{R_6}}{1+s\dfrac{R_2R_3R_7C_4}{R_1R_6}+s^2\dfrac{R_2R_3R_7C_1C_4}{R_6}} = \frac{\left(\dfrac{R_2}{R_6}+1\right)\dfrac{R_6}{R_2R_3R_7C_1C_4}}{s^2+s\dfrac{1}{R_1C_1}+\dfrac{R_6}{R_2R_3R_7C_1C_4}} = \frac{2}{s^2+\dfrac{2}{3}s+1}$$

(2 boda)

-o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ NP

b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati parametre k, ω_0, Q.

$$T(s) = \frac{k\cdot\omega_0^{\;2}}{s^2+\dfrac{\omega_0}{Q}\cdot s+\omega_0^{\;2}}$$ Opći oblik NP (niski propust)

-parametri k, ω_0, Q:

$$k = \frac{R_2}{R_6}+1 = \left(\frac{1}{1}+1\right) = 2$$

$$\omega_0 = \sqrt{\frac{R_6}{R_2R_3R_7C_1C_4}} = \sqrt{\frac{1}{1\cdot1\cdot1\cdot\frac{1}{2}\cdot2}} = 1$$

$$\frac{\omega_0}{Q} = \frac{1}{R_1C_1} \Rightarrow Q = R_1C_1\omega_0 = R_1C_1\sqrt{\frac{R_6}{R_2R_3R_7C_1C_4}} = R_1C_1\cdot\omega_0 = \frac{3}{2}\cdot1 = 1.5.$$ (1 bod)

c) raspored polova i nula u kompleksnoj ravnini: (1 bod)

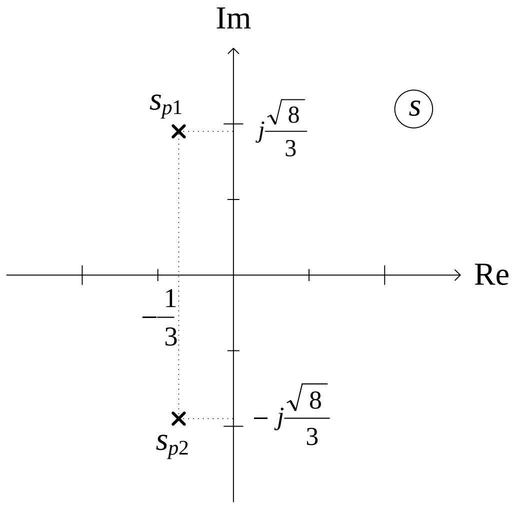
$$T(s)=\frac{2}{s^2+\frac{2}{3}s+1} \qquad \text{nule} \qquad s_{o1}=\infty, s_{o2}=\infty$$

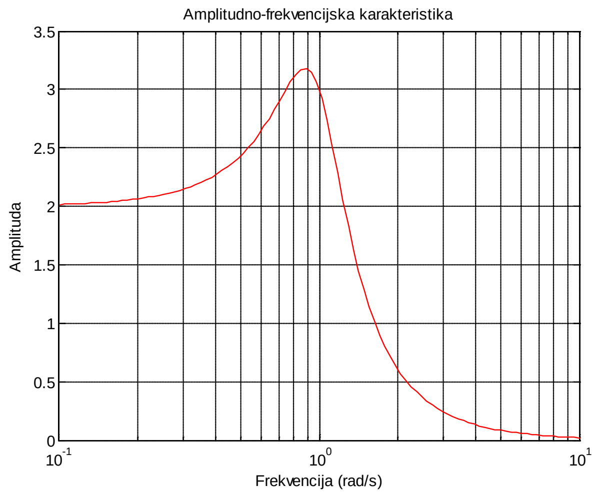
$$\text{polovi} \quad s^2+\frac{2}{3}s+1=0 \qquad \Rightarrow \qquad s_{p1,2}=-\frac{1}{3}\pm\sqrt{\frac{1}{9}-1}=-\frac{1}{3}\pm j\frac{\sqrt{8}}{3}=-\frac{1}{3}\pm j\frac{2\sqrt{2}}{3}$$

d) amplitudno-frekvencijska karakteristika: (1 bod)

$s=j\omega \Rightarrow$

$$T(j\omega)=\frac{2}{-\omega^2+j\frac{2}{3}\omega+1} \qquad \Rightarrow \qquad |T(j\omega)|=\frac{2}{\sqrt{(1-\omega^2)^2+\left(\frac{2}{3}\omega\right)^2}}$$





5. Zadana je linija bez gubitaka s L=2 mH/km i C=6 nF/km, duljine $l=\lambda/4$+40km. Na kraj linije priključen je dvopol impedancije $Z=Z_0$, a na udaljenosti $l=\lambda/4$ od početka linije priključen je otpor R=100 Ω. Napon na ulazu linije je $u(0, t)=4\cdot\cos(10^4\, t)$.

Odrediti:

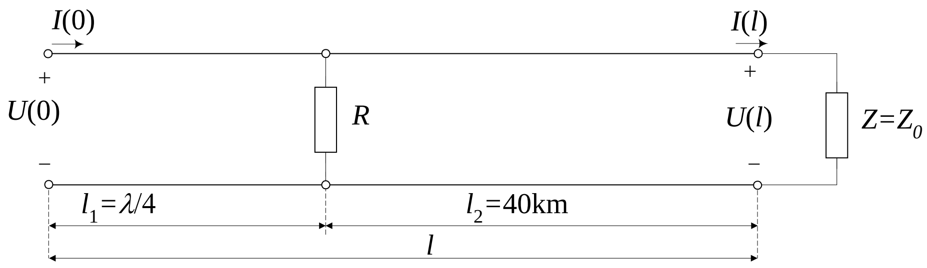
a) karakterističnu impedanciju Z_0, koeficijent prijenosa γ i duljinu linije u km;

b) ulaznu impedanciju prve linije: Z_{ul1};

c) napon na kraju prve linije: $u(l_1, t)$;

d) napon na kraju linije: $u(l, t)$;

e) struju na kraju linije: $i(l, t)$.



ELEKTRIČNI KRUGOVI — Zadaci sa rješenjima za vježbu

Električni filtri

1. Zadana je prijenosna funkcija $T(s)=U_{iz}(s)/U_{ul}(s)$ električnog filtra. Nacrtati raspored nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Izračunati vrijednost faktora dobrote q_p, frekvencije ω_p i pojačanja u području propuštanja K. O kakvoj se vrsti filtra radi (NP, VP, PP ili PB)?

$$T(s)=\frac{0.6s^2}{s^2+1.2s+0.4}$$

Rješenje: izjednačimo zadanu prijenosnu funkciju sa općim oblikom prijenosne funkcije:

$$T(s)=\frac{0.6s^2}{s^2+1.2s+0.4}=\frac{K\cdot s^2}{s^2+\dfrac{\omega_p}{q_p}s+\omega_p^2}$$

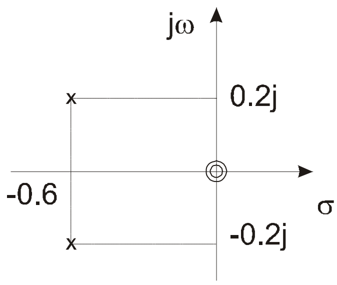
Odatle slijede parametri filtra:

$$\omega_p=\sqrt{0.4}\cong 0.63,\quad q_p=\frac{\omega_p}{2\sigma_p}=\frac{\sqrt{0.4}}{1.2}\cong 0.53,\quad K=0.6$$

Zatim slijede polovi: $s^2+1.2s+0.4=0 \Rightarrow s_{p1,2}=\dfrac{-1.2\pm\sqrt{1.2^2-4\cdot 0.4}}{2}=-0.6\pm 0.2j$

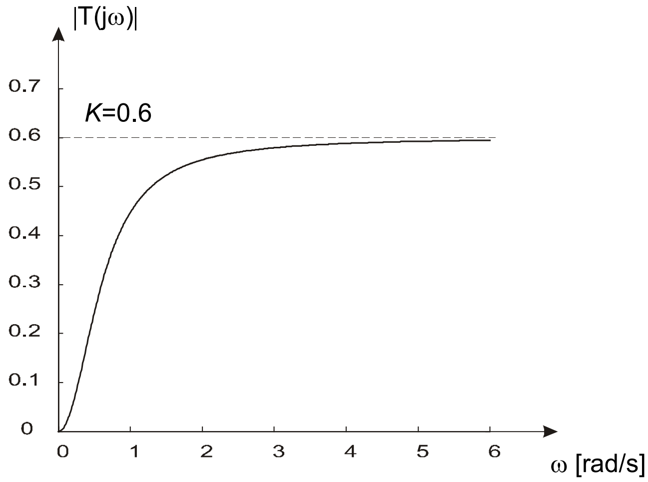
i nule: $s^2=0 \Rightarrow s_{01,2}=0$ dvostruka nula u ishodištu

raspored nula i polova u s-ravnini:



amplitudno-frekvencijska karakteristika slijedi ako uvrstimo s=jω

$$|T(j\omega)|=\left|\frac{0.6\omega^2}{-\omega^2+1.2j\omega+0.4}\right|=\frac{0.6\omega^2}{\sqrt{(0.4-\omega^2)^2+(1.2\omega)^2}}$$



ovo je visoki propust (VP)

2. Zadana je prijenosna funkcija $T(s)=U_{iz}(s)/U_{ul}(s)$ električnog filtra. Izračunati vrijednost faktora dobrote q_p, frekvencije ω_p i pojačanja u području propuštanja K. O kakvoj se vrsti filtra radi (NP, VP, PP ili PB)? Koliko iznosi širina pojasa propuštanja filtra B te gornja i donja granična frekvencija f_g i f_d ? U kakvoj vezi su granične frekvencije sa centralnom frekvencijom ? Nacrtati raspored nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$.

$$T(s)=\frac{31.4159\cdot s}{s^2+31.4159\cdot s+98696}$$

Rješenje: izjednačimo zadanu prijenosnu funkciju sa općim oblikom prijenosne funkcije:

$$T(s)=\frac{31.4159\cdot s}{s^2+31.4159\cdot s+98696}=\frac{K\cdot\frac{\omega_p}{q_p}\cdot s}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2}$$

Odatle slijede parametri filtra:

$$\omega_p=\sqrt{98696}=314.159=2\pi\cdot 50\,[\text{rad/s}]\Rightarrow f_p=50[\text{Hz}]$$

$$q_p=\frac{\omega_p}{2\sigma_p}=\frac{314.159}{31.4159}=10$$

$$K=1$$

(gdje je $2\sigma_p$ član koji množi s u nazivniku prijenosne funkcije)

Ovo je pojasni propust (PP)

Širina pojasa propuštanja : $B=\frac{\omega_p}{q_p}=\frac{314.159}{10}=31.4159\ [\text{rad/s}]$

Gornja i donja granična frekvencija su :

$$\omega_{g,d}=\omega_p\sqrt{1+\frac{1}{4q_p^2}}\pm\frac{\omega_p}{2q_p}=314.159\sqrt{1+\frac{1}{4\cdot 100}}\pm\frac{314.159}{2\cdot 10}=314.551\pm 15.708\ [\text{rad/s}]$$

ω_g=330.259 [rad/s], ω_d=298.844 [rad/s] ili

$f_g=\omega_g/2\pi$=330.259/2π=52.5624 [Hz], $f_d=\omega_d/2\pi$=298.844 /2π=47.5625 [Hz],

$B=\omega_g-\omega_d$=330.259–298.844=31.415 [rad/s] ili

f_g-f_d=31.415/2π=5 [Hz]

centralna frekvencija $\omega_0=\omega_p=314.159=2\pi\cdot 50\,[\text{rad/s}]$ ili

f_0=50 [Hz]

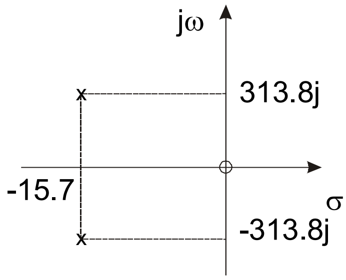
$\omega_0^2=\omega_d\cdot\omega_g\rightarrow$ ω_0 je geometrijska sredina od ω_d i ω_g ili

$f_0^2=f_d\cdot f_g\rightarrow$ f_0 je geometrijska sredina od f_d i f_g

Zatim slijede polovi: $s^2+31.4159\cdot s+98696=0\ \Rightarrow$

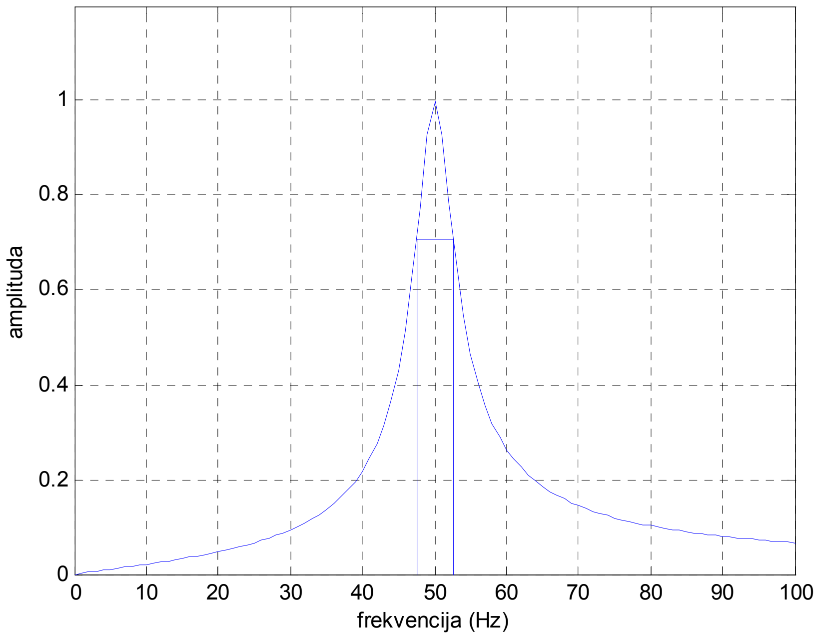
$$s_{p1,2}=\frac{-31.4159\pm\sqrt{31.4159^2-4\cdot 98696}}{2}=-15.708\pm 313.766j$$

i nule: $s=0\ \Rightarrow\ s_{01}=0,\ s_{02}=\infty$ jedna nula u ishodištu, druga u beskonačnosti raspored nula i polova u s-ravnini:

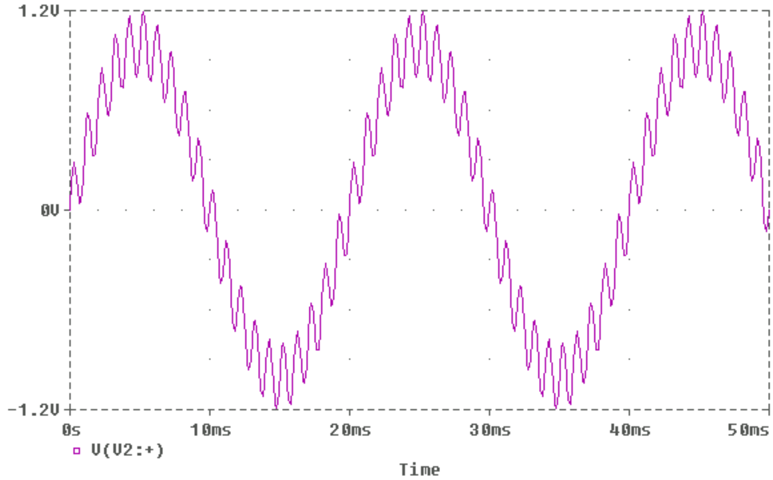


amplitudno-frekvencijska karakteristika slijedi ako uvrstimo $s=j\omega$

$$\left|T(j\omega)\right| = \frac{\left|31.4159\cdot\omega\right|}{\sqrt{(98696-\omega^2)^2+(31.4159\cdot\omega)^2}}\ ,\ \omega=2\pi f$$



3. Telefonski prijenosni sistem sadrži osim korisnoga signala i smetnju od gradske mreže frekvencije 50 Hz. Za ilustraciju neka je ulazni napon sastavljen od sinusnoga signala frekvencije 1000 Hz i smetnje frekvencije 50 Hz : $u_{ul}(t) = \sin(2\pi \cdot 50t) + 0.2 \cdot \sin(2\pi \cdot 1000t)$ i prikazan je na slijedećoj slici.



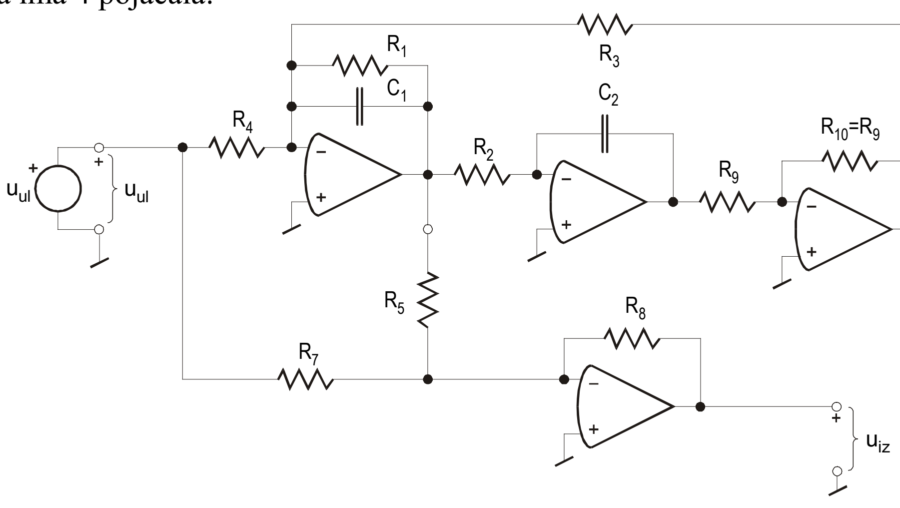
Treba projektirati filtar koji eliminira smetnju. Takav filtar bi trebao biti pojasna brana s frekvencijom nule f_z=50Hz, i sa velikim Q-faktorom polova q_p=10 koji osigurava visoku selektivnost filtra (odn. uski pojas gušenja).

Napomena: navedeni primjer je prvenstveno edukativnog karaktera mada dijelovi ovog dosta opširnog zadatka mogu biti kao primjer ispitnog zadatka. Na predavanjima je prikazan isti zadatak koji je riješen pomoću primjera pasivnog RLC filtra.

Ciljevi:

1. Upoznavanje sa osnovnim principima rada filtara,
2. korištenje metode superpozicije u izračunavanju odziva i fazorskog računa u predstavljanju signala pobude,
3. dobivanje predodžbe o prijenosnoj funkciji, značenju nula prijenosne funkcije,
4. dobivanje predodžbe o a-f i f-f karakteristici filtra,
5. postupak proračuna naočigled kompliciranog filtarskog električnog kruga,
6. uočavanje da navedeni električni krug sadrži operacijska pojačala, te otpore i kapacitete, a ne sadrži induktivitete (kažemo da je to aktivni-RC filtar).

Rješenje: U ovom zadatku ćemo za realizaciju koristiti slijedeću aktivnu RC filtarsku sekciju 2. reda koja ima 4 pojačala:



Prijenosna funkcija se da lako izračunati, npr. pomoću metode napona čvorova, i ona za navedeni električni krug (filtar) glasi:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=-\frac{R_8}{R_7}\cdot\frac{s^2+s\frac{\omega_p}{q_p}\left(1-\frac{R_7}{R_5}\cdot\frac{R_1}{R_4}\right)+\omega_p^2}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2}=-k\cdot\frac{s^2+s\frac{\omega_z}{q_z}+\omega_z^2}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2},$$

gdje su:

$$\omega_p=\frac{1}{\sqrt{R_2R_3C_1C_2}},\ q_p=R_1\sqrt{\frac{C_1}{R_2R_3C_2}},\ \omega_z=\omega_p,\ k=\frac{R_8}{R_7}.$$

Srednji član u brojniku je: $\frac{\omega_z}{q_z}=\frac{\omega_p}{q_p}\left(1-\frac{R_7}{R_5}\cdot\frac{R_1}{R_4}\right)$ i on mora biti jednak nuli da bismo ostvarili prijenosnu karakteristiku pojasne brane na frekvenciji $\omega_z=\omega_p=2\pi\cdot 50=100\pi$.

Da bismo izračunali elemente filtra provedimo slijedeći postupak proračuna:

1. Odaberimo vrijednosti elemenata $R_2=R_3=R_4=R_7=R_9=R_{10}=R$, $C_1=C_2=C$. Tada gornje jednadžbe poprimaju jednostavan oblik:

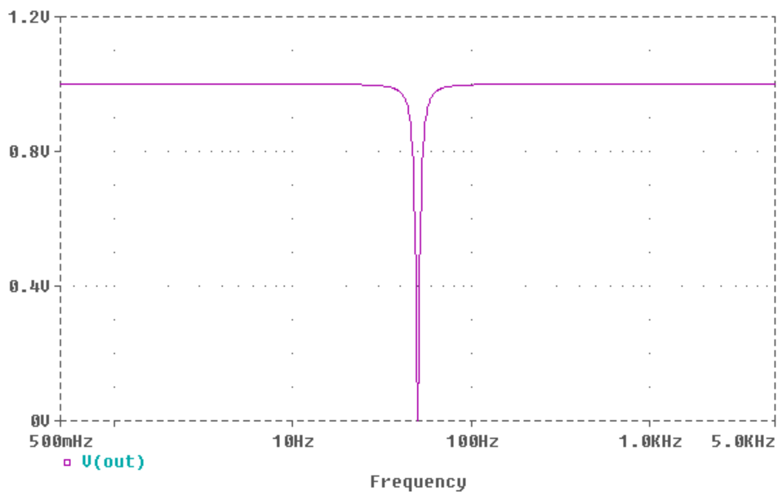
$$\omega_p=\frac{1}{RC},\ q_p=\frac{R_1}{R},\ \frac{q_p}{\omega_p}=\frac{1}{R_1C},\ \omega_z=\omega_p,\ k=\frac{R_8}{R}$$

2. Ako odaberemo kapacitete vrijednosti C=1μF izračunajmo $R=1/(C\ \omega_p)=1/(10^{-6}\ 100\pi)=$ 3183.1 Ω=3.183 kΩ
3. Iz $\omega_p/q_p=(R_1C)^{-1}$ izračunajmo $R_1=q_p/(\omega_pC)=10/(10^{-6}\ 100\pi)=31831$ Ω=31.831 kΩ
4. Izjednačimo $R_5=R_1$ =31.831 kΩ, da bismo ostvarili $q_z=\infty$,
5. Izračunajmo $R_8=k\cdot R=1\cdot 3.183$ kΩ=3.183 kΩ da bismo realizirali jedinično pojačanje u području propuštanja.

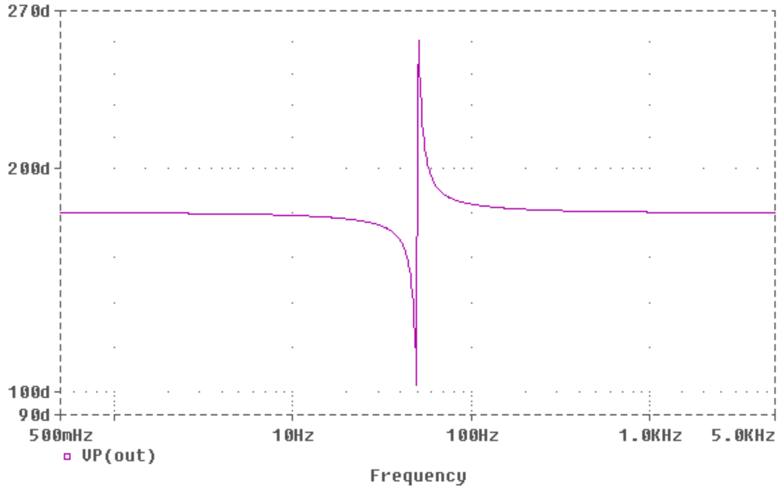
Naponska prijenosna funkcija glasi: $T(s)=-\frac{s^2+98696}{s^2+31.4159\cdot s+98696}$

Za sinusoidalno stacionarno stanje (uvrstimo $s=j\omega$): $T(j\omega)=\frac{98696-\omega^2}{-\omega^2+31.4159\cdot j\omega+98696}$

a-f karakteristika : $|T(j\omega)|=\frac{|98696-\omega^2|}{\sqrt{(98696-\omega^2)^2+(31.4159\cdot\omega)^2}}$



f-f karakteristika : $\varphi(\omega) = \pi \cdot S(\omega - 100\pi) - \arctan \dfrac{31.4159 \cdot \omega}{98696 - \omega^2}$



Koliki je izlazni napon ?

Ulazni napon je $u_{ul}(t) = \sin(2\pi \cdot 50t) + 0.2 \cdot \sin(2\pi \cdot 1000t)$, dakle ima dvije komponente : $u_{ul}(t) = u_{ul1}(t) + u_{ul2}(t)$, gdje su

$$u_{ul1}(t) = \sin(2\pi \cdot 50t);$$
$$u_{ul2}(t) = 0.2 \cdot \sin(2\pi \cdot 1000t).$$

Tim komponentama su pridruženi fazori :

$$U_{ul1}(j\omega) = 1\angle 0°,\ \text{uz frekvenciju } \omega = 2\pi \cdot 50 \text{ rad/s};$$
$$U_{ul2}(j\omega) = 0.2\angle 0°,\ \text{uz frekvenciju } \omega = 2\pi \cdot 1000 \text{ rad/s}.$$

<u>Metodom superpozicije</u> možemo izračunati valni oblik izlaznog signala:

$$u_{iz}(t) = u_{iz1}(t) + u_{iz2}(t).$$

Pritom je svaka komponenta u izlaznom signalu :

i) Odziv $u_{iz1}(t)$ uslijed prvog signala (poticaja) $u_{ul1}(t)$:

Amplituda signala $u_{iz1}(t)$:

$$|U_{iz1}(j\omega)| = |T(j\omega)| \cdot |U_{ul1}(j\omega)| = \frac{|98696 - \omega^2|}{\sqrt{(98696 - \omega^2)^2 + (31.4159 \cdot \omega)^2}} \cdot 1,$$ te uz uvrštenu

frekvenciju ω=2π·50 rad/s dobivamo da je $|U_{iz1}(j\omega)| = 4.45924 \cdot 10^{-6} \approx 0$.

Fazni kut signala $u_{iz1}(t)$: $\varphi_{iz1}(\omega) = \varphi(\omega) + \varphi_{ul1}(\omega)$, gdje je :

$$\varphi(\omega) = \pi \cdot S(\omega - 100\pi) - \arctan \frac{31.4159 \cdot \omega}{98696 - \omega^2}.$$

Fazni kut signala na izlazu jednak je kutu prijenosne funkcije jer je kut signala na ulazu jednak nula.

Dakle, za frekvenciju ω=2π·50 rad/s dobivamo dva rješenja odnosno da je fazni kut $\varphi_{iz1}(\omega)$=180°–89.9997°≈90° ili $\varphi_{iz1}(\omega)$=–89.9997°≈–90° (odnosno +270°). Ili, jednostavnije, možemo prikazati kut izlaznog signala kao $\varphi_{iz1}(\omega)$=±90°. No taj signal ima amplitudu približno jednaku nuli pa njegov doprinos možemo zanemariti, stoga niti njegov fazni pomak nije od velikog interesa.

ii) Odziv $u_{iz2}(t)$ uslijed drugog signala $u_{ul2}(t)$:

Amplituda signala $u_{iz2}(t)$:

$$|U_{iz2}(j\omega)| = |T(j\omega)| \cdot |U_{ul2}(j\omega)| = \frac{|98696 - \omega^2|}{\sqrt{(98696 - \omega^2)^2 + (31.4159 \cdot \omega)^2}} \cdot 0.2,$$

te uz uvrštenu frekvenciju ω=2π·1000 rad/s dobivamo da je

$|U_{iz2}(j\omega)| = 0.999987 \cdot 0.2 = 0.199997 \approx 0.2.$

Fazni kut signala $u_{iz2}(t)$:

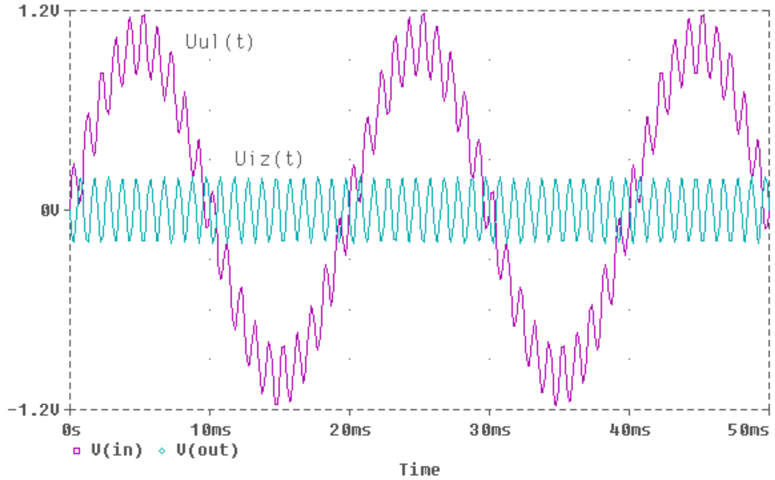
Za frekvenciju ω=2π·1000 rad/s dobivamo koristeći iste izraze za fazni kut kao u prvom slučaju $\varphi_{iz2}(\omega)$= 180°–0.287194°≈180°.

iii) Ukupni odziv $u_{iz}(t)$:

Konačno, izlazni napon ne sadrži više komponentu od 50 Hz pa je:

$$u_{iz}(t) = 0.2 \cdot \sin(2\pi \cdot 1000t + 180°) = -0.2 \cdot \sin(2\pi \cdot 1000t)$$

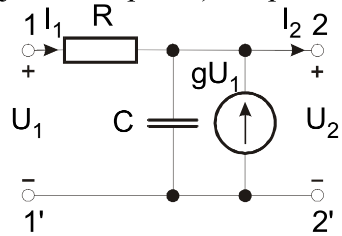
Dakle, cijeli odziv je jednak izlaznom naponu jedino uslijed poticaja signala $u_{ul2}(t)$. Poticaj i odziv su prikazani na slijedećoj slici:



Sve slike su realizirane pomoću programa PSpice.

ČETVEROPOLI

3. Za četveropol prikazan slikom izračunati z-parametre i napisati matricu z-parametara. Pomoću poznatih z-parametara izračunati naponsku prijenosnu funkciju četveropola $T(s)=U_2(s)/U_1(s)$ ako je izlazni prilaz (2–2') otvoren. Da li je četveropol: *a*) recipročan, *b*) simetričan ? Obrazložiti odgovor.

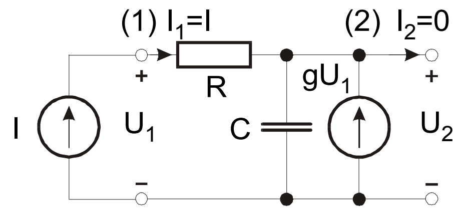


Rješenje:

[z]-parametri:

$$U_1 = z_{11}I_1 - z_{12}I_2$$
$$U_2 = z_{21}I_1 - z_{22}I_2$$

a) I_2=0



(1) $$U_1\frac{1}{R} - U_2\frac{1}{R} = I$$

(2) $$-U_1\frac{1}{R} + U_2\left(\frac{1}{R} + sC\right) = gU_1$$

$$(1) \Rightarrow U_1 = U_2 + I\cdot R,\quad U_2 = U_1 - I\cdot R$$

$$(2) \Rightarrow -U_1\frac{1}{R} + (U_1 - I\cdot R)\left(\frac{1}{R} + sC\right) = gU_1$$

$$-U_1\frac{1}{R} + U_1\left(\frac{1}{R} + sC\right) - gU_1 = I\cdot R\left(\frac{1}{R} + sC\right)$$

$$U_1(sC - g) = I(1 + sRC),\quad I = I_1$$

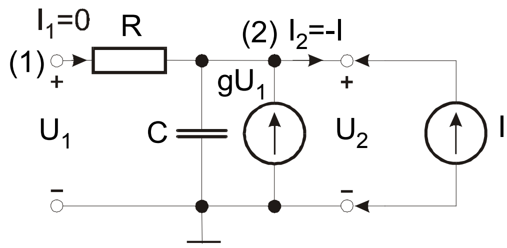
$$z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = \frac{1 + sRC}{sC - g} = \frac{R + \frac{1}{sC}}{1 - \frac{g}{sC}}$$

$$(2) \Rightarrow -U_1\left(\frac{1}{R} + g\right) + U_2\left(\frac{1}{R} + sC\right) = 0 \;\Rightarrow -(U_2 + I\cdot R)\left(\frac{1}{R} + g\right) + U_2\left(\frac{1}{R} + sC\right) = 0$$

$$-U_2\frac{1}{R} - U_2 g - I - I\cdot R\cdot g + U_2\frac{1}{R} + sCU_2 = 0 \Rightarrow U_2(sC - g) = I(1 + Rg)$$

$$z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0} = \frac{1 + Rg}{sC - g} = \frac{1}{sC}\left(1 + g\frac{R + \frac{1}{sC}}{1 - \frac{g}{sC}}\right) = g\frac{sRC + 1}{sC - g} - R \quad \text{i slično}$$

b) I_1=0



$$U_2 sC = gU_1 + I$$

$$\underline{I_1 = 0 \rightarrow U_1 = U_2, \qquad I = -I_2}$$

$$U_2 sC = gU_2 - I_2$$

$$U_2(sC - g) = -I_2, \;\; U_1 = U_2$$

$$z_{22} = -\left.\frac{U_2}{I_2}\right|_{I_1=0} = \frac{1}{sC_2 - g}$$

$$z_{12} = -\left.\frac{U_1}{I_2}\right|_{I_1=0} = \frac{1}{sC_2 - g}$$

$$[z] = \begin{bmatrix} \dfrac{1+sRC}{sC_2 - g} & -\dfrac{1}{sC_2 - g} \\ \dfrac{1+Rg}{sC_2 - g} & -\dfrac{1}{sC_2 - g} \end{bmatrix}$$

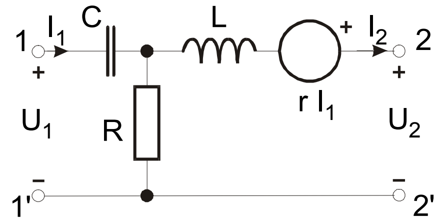
$$[z] = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix}$$

Prijenosna funkcija napona:

$$T(s) = \frac{z_{21}}{z_{11}} = \left.\frac{\dfrac{U_2}{I_1}}{\dfrac{U_1}{I_1}}\right|_{I_2=0} = \left.\frac{U_2}{U_1}\right|_{I_2=0} = \frac{1+Rg}{1+sRC}$$

Odgovori na pitanja:
a) Četveropol nije recipročan jer sadrži naponsko ovisni strujni izvor i parametri z_{12} i z_{21} su stoga različiti;
b) Četveropol nije električki simetričan jer se parametri z_{11} i z_{22} razlikuju.

3. Za četveropol prikazan slikom izračunati *y*-parametre i napisati matricu *y*-parametara. Zadane su normalizirane vrijednosti elemenata R=1, L=2, C=1 i r=2. Da li je četveropol: *a*) recipročan, *b*) simetričan ? Obrazložiti odgovor.



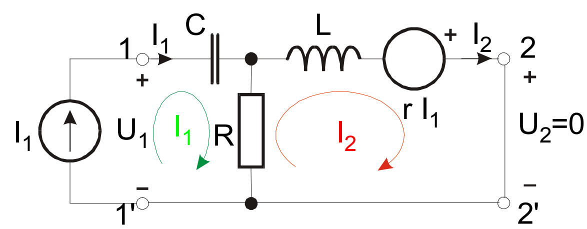
Rješenje:

[*y*]-parametri:

$I_1 = y_{11}U_1 - y_{12}U_2$

$I_2 = y_{21}U_1 - y_{22}U_2$

a) U_2=0



(1) $$I_1\left(R+\frac{1}{sC}\right)-I_2R=U_1$$

(2) $$-I_1R+I_2\left(R+sL\right)=rI_1$$

$$(2)\Rightarrow I_1\left(R+r\right)=I_2\left(R+sL\right)$$

$$(1)\Rightarrow I_1\left(R+\frac{1}{sC}\right)-I_1\frac{R+r}{R+sL}R=U_1$$

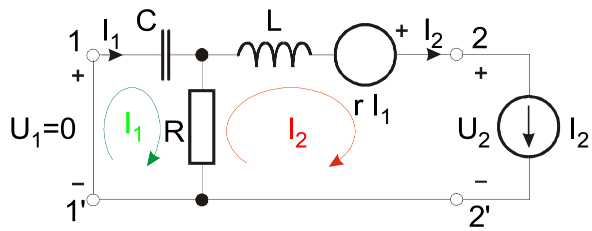
$$y_{11}=\left.\frac{I_1}{U_1}\right|_{U_2=0}=\frac{1}{\left(R+\frac{1}{sC}\right)-\frac{R+r}{R+sL}R}=\frac{R+sL}{\left(R+\frac{1}{sC}\right)\left(R+sL\right)-\left(R+r\right)R}=$$

$$=\frac{1+2s}{\left(1+\frac{1}{s}\right)\left(1+2s\right)-\left(1+2\right)1}=\frac{1+2s}{1+2s+\frac{1}{s}+2-3}=\frac{1+2s}{2s+\frac{1}{s}}=\frac{s(1+2s)}{2s^2+1}$$

$$(1)\Rightarrow I_2\frac{R+sL}{R+r}\left(R+\frac{1}{sC}\right)-I_2R=U_1$$

$$y_{21}=\left.\frac{I_2}{U_1}\right|_{U_2=0}=\frac{1}{\frac{R+sL}{R+r}\left(R+\frac{1}{sC}\right)-R}=\frac{R+r}{\left(R+\frac{1}{sC}\right)\left(R+sL\right)-\left(R+r\right)R}=\frac{1+2}{2s+\frac{1}{s}}=\frac{3s}{2s^2+1}$$

b) U_1=0



(1) $\quad I_1\left(R+\frac{1}{sC}\right)-I_2R=0$

(2) $\quad -I_1(R+r)+I_2(R+sL)=-U_2$

$(1)\Rightarrow I_1\left(R+\frac{1}{sC}\right)=I_2R$

$$(2)\Rightarrow -I_1(R+r)+I_1\frac{R+\frac{1}{sC}}{R}(R+sL)=-U_2$$

$$y_{12}=-\frac{I_1}{U_2}\bigg|_{U_1=0}=\frac{1}{\frac{R+\frac{1}{sC}}{R}(R+sL)-(R+r)}=\frac{R}{\left(R+\frac{1}{sC}\right)(R+sL)-(R+r)R}=$$

$$=\frac{1}{2s+\frac{1}{s}}=\frac{s}{2s^2+1}$$

$$(2)\Rightarrow -I_2\frac{R}{R+\frac{1}{sC}}(R+r)+I_2(R+sL)=-U_2$$

$$y_{22}=-\frac{I_2}{U_2}\bigg|_{U_1=0}=\frac{1}{(R+sL)-\frac{R}{R+\frac{1}{sC}}(R+r)}=\frac{\left(R+\frac{1}{sC}\right)}{\left(R+\frac{1}{sC}\right)(R+sL)-(R+r)R}=$$

$$=\frac{1+\frac{1}{s}}{2s+\frac{1}{s}}=\frac{s+1}{2s^2+1}$$

Konačno rješenje glasi: $[y]=\begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22}\end{bmatrix}$

$$[y]=\frac{1}{\left(R+\frac{1}{sC}\right)(R+sL)-(R+r)R}\begin{bmatrix} R+sL & -R \\ R+r & -\left(R+\frac{1}{sC}\right)\end{bmatrix}=$$

$$=\frac{1}{sLR+\frac{R}{sC}+\frac{L}{C}-rR}\cdot\begin{bmatrix} R+sL & -R \\ R+r & -\left(R+\frac{1}{sC}\right)\end{bmatrix}=\frac{1}{2s+\frac{1}{s}}\cdot\begin{bmatrix} 1+2s & -1 \\ 3 & -\left(1+\frac{1}{s}\right)\end{bmatrix}=$$

$$=\frac{s}{2s^2+1}\cdot\begin{bmatrix} 1+2s & -1 \\ 3 & -\frac{s+1}{s}\end{bmatrix}.$$

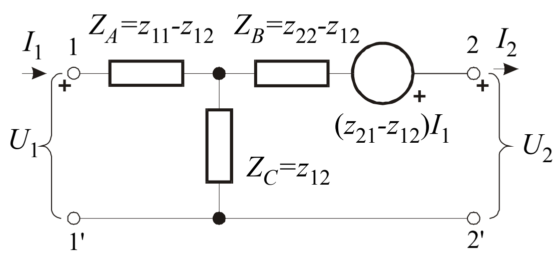
Odgovori na pitanja:

a) Četveropol nije recipročan jer sadrži strujno ovisni naponski izvor i parametri y_{12} i y_{21} su stoga različiti;

b) Četveropol nije električki simetričan jer se parametri y_{11} i y_{22} razlikuju.

Drugi jednostavniji način rješavanja zadatka:

Na slajdovima 14. predavanja "Četveropoli" nalazi se ekvivalentni T-spoj nerecipročnog četveropola (za koji vrijedi $z_{12} \neq z_{21}$):



Opisan je jednadžbama: $z_{11}=Z_A+Z_C$; $z_{12}=Z_C$; $z_{22}=Z_B+Z_C$; $z_{21}-z_{12}=r$; $z_{21}=z_{12}+r$; gdje su z_{11}, z_{12}, z_{21}, z_{22}, z-parametri, a r je parametar strujno-ovisnog naponskog izvora zadanog u zadatku.

Vidljivo je da je: $Z_A=1/(sC)$, $Z_B=sL$, $Z_C=R$, odavde slijede z-parametri: $z_{11}=1/(sC)+R$; $z_{12}=R$; $z_{22}=sL+R$; $z_{21}=R+r$. Odnosno u matričnom obliku:

$$[z]=\begin{bmatrix} R+\dfrac{1}{sC} & -R \\ R+r & -(R+sL) \end{bmatrix}$$

Ako invertiramo matricu sa z-parametrima lako izračunamo y-parametre:

$\Delta z=-\left(R+\dfrac{1}{sC}\right)(R+sL)+R(R+r)$ je determinanta matrice sa z-parametrima

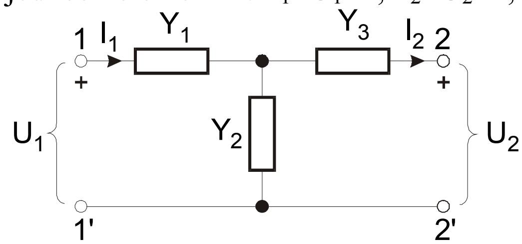
$$[y]=[z]^{-1}=\frac{1}{\Delta z}\begin{bmatrix} -(R+sL) & -(R+r) \\ -(-R) & R+\dfrac{1}{sC} \end{bmatrix}^T=\frac{1}{\Delta z}\begin{bmatrix} -(R+sL) & R \\ -(R+r) & R+\dfrac{1}{sC} \end{bmatrix}$$
$$=\frac{1}{-\Delta z}\begin{bmatrix} R+sL & -R \\ R+r & -\left(R+\dfrac{1}{sC}\right) \end{bmatrix}$$

$$[y]=\frac{1}{\left(R+\dfrac{1}{sC}\right)(R+sL)-R(R+r)}\begin{bmatrix} R+sL & -R \\ R+r & -\left(R+\dfrac{1}{sC}\right) \end{bmatrix}$$

Odnosno uz uvrštene vrijednosti elemenata:

$$[y]=\frac{s}{2s^2+1}\cdot\begin{bmatrix} 1+2s & -1 \\ 3 & -\dfrac{s+1}{s} \end{bmatrix}=\begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

3. Za T-četveropol prikazan slikom izračunati *y*-parametre i napisati matricu *y*-parametara. (Izraziti *y*-parametre pomoću admitancija elemenata Y_1, Y_2 i Y_3.) Ako je izlazni prilaz (2–2') zaključen admitancijom Y_L pomoću poznatih *y*-parametara izračunati: *a*) naponsku prijenosnu funkciju četveropola $T(s)=U_2(s)/U_1(s)$; *b*) ulaznu admitanciju u četveropol $Y_{ul1}(s)=I_1(s)/U_1(s)$. Najprije izraziti $T(s)$ i $Y_{ul1}(s)$ pomoću poznatih *y*-parametara izraženih admitancijama elemenata Y_1, Y_2 i Y_3, a zatim uvrstiti slijedeće vrijednosti elemenata: $Y_1=G_1=1$, $Y_2=G_2=2$, $Y_3=G_3=1$ i $Y_L=G_L=1$.

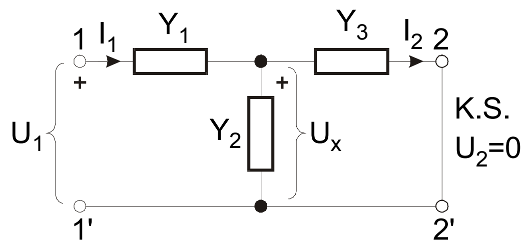


Rješenje:

[*y*]-parametri:

$$I_1 = y_{11}U_1 - y_{12}U_2$$
$$I_2 = y_{21}U_1 - y_{22}U_2$$

a) $U_2=0$



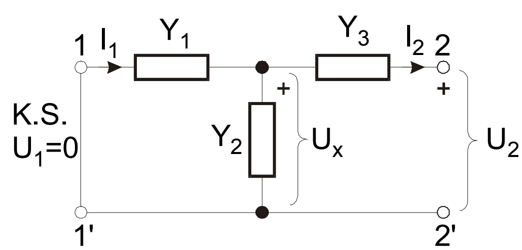
$$U_1 = I_1\left(\frac{1}{Y_1}+\frac{1}{Y_2+Y_3}\right) = I_1\frac{Y_1+Y_2+Y_3}{Y_1(Y_2+Y_3)}$$

$$U_x = I_2\frac{1}{Y_3} = I_1\frac{1}{Y_2+Y_3} \Rightarrow I_2 = I_1\frac{Y_3}{Y_2+Y_3} \Rightarrow I_1 = I_2\frac{Y_2+Y_3}{Y_3}$$

$$\Rightarrow U_1 = I_1\frac{Y_1+Y_2+Y_3}{Y_1(Y_2+Y_3)} = I_2\frac{Y_2+Y_3}{Y_3}\frac{Y_1+Y_2+Y_3}{Y_1(Y_2+Y_3)} = I_2\frac{Y_1+Y_2+Y_3}{Y_1Y_3}$$

$$y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0} = \frac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3}; \quad y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0} = \frac{Y_1Y_3}{Y_1+Y_2+Y_3}$$

b) $U_1=0$



$$U_2 = -I_2\left(\frac{1}{Y_3}+\frac{1}{Y_1+Y_2}\right) = -I_2\frac{Y_1+Y_2+Y_3}{Y_3(Y_1+Y_2)}$$

$$U_x = -I_2\frac{1}{Y_1+Y_2} = -I_1\frac{1}{Y_1} \Rightarrow I_1 = I_2\frac{Y_1}{Y_1+Y_2} \Rightarrow I_2 = I_1\frac{Y_1+Y_2}{Y_1}$$

$$\Rightarrow U_2 = -I_2 \frac{Y_1+Y_2+Y_3}{Y_3(Y_1+Y_2)} = -I_1 \frac{Y_1+Y_2}{Y_1} \frac{Y_1+Y_2+Y_3}{Y_3(Y_1+Y_2)} = -I_1 \frac{Y_1+Y_2+Y_3}{Y_1 Y_3}$$

$$y_{12} = -\left.\frac{I_1}{U_2}\right|_{U_1=0} = \frac{Y_1 Y_3}{Y_1+Y_2+Y_3}\ ; \quad y_{22} = -\left.\frac{I_2}{U_2}\right|_{U_1=0} = \frac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3}$$

$$[y] = \begin{bmatrix} \frac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3} & -\frac{Y_1Y_3}{Y_1+Y_2+Y_3} \\ \frac{Y_1Y_3}{Y_1+Y_2+Y_3} & -\frac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3} \end{bmatrix}, \quad [y] = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

Prijenosna funkcija napona:

$$T(s) = \frac{U_2}{U_1} = \frac{y_{21}}{Y_L + y_{22}} = \frac{\frac{Y_1Y_3}{Y_1+Y_2+Y_3}}{Y_L + \frac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3}} = \frac{Y_1Y_3}{Y_L(Y_1+Y_2+Y_3)+Y_3(Y_1+Y_2)}$$

uz uvrštene vrijednosti:

$$T(s) = \frac{U_2}{U_1} = \frac{G_1G_3}{G_L(G_1+G_2+G_3)+G_3(G_1+G_2)} = \frac{1}{1(1+2+1)+1(1+2)} = \frac{1}{7}$$

Ulazna admitancija u četveropol:

$$Y_{ul1}(s) = \frac{I_1}{U_1} = y_{11} - \frac{y_{12}y_{21}}{Y_L + y_{22}} = \frac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3} - \frac{\left(\frac{Y_1Y_3}{Y_1+Y_2+Y_3}\right)^2}{Y_L + \frac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3}} =$$

$$= \frac{1}{(Y_1+Y_2+Y_3)}\left[Y_1(Y_2+Y_3) - \frac{(Y_1Y_3)^2}{Y_L(Y_1+Y_2+Y_3)+Y_3(Y_1+Y_2)}\right]$$

uz uvrštene vrijednosti:

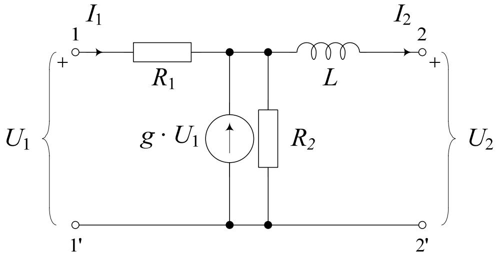
$$Y_{ul1} = \frac{1}{(G_1+G_2+G_3)}\left[G_1(G_2+G_3) - \frac{(G_1G_3)^2}{G_L(G_1+G_2+G_3)+G_3(G_1+G_2)}\right] =$$

$$= \frac{1}{(1+2+1)}\left[1(2+1) - \frac{(1)^2}{1(1+2+1)+1(1+2)}\right] = \frac{1}{4}\left[3-\frac{1}{7}\right] = \frac{1}{4}\cdot\frac{20}{7} = \frac{5}{7}$$

ZAVRŠNI ISPIT IZ PREDMETA ELEKTRIČNI KRUGOVI 2009/10

Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za četveropol na slici izračunati: *a*) [**z**]-parametre i napisati ih u matričnom obliku. Da li je četveropol: *b*) recipročan; *c*) simetričan ? Obrazložiti odgovore.



Rješenje:

a) izračun [**z**] parametara

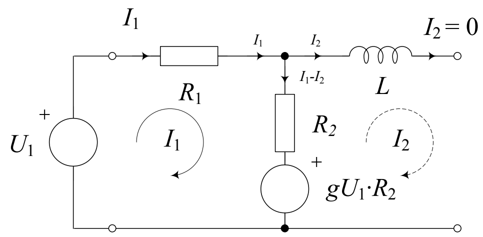
$$U_1 = z_{11} \cdot I_1 - z_{12} \cdot I_2$$
$$U_2 = z_{21} \cdot I_1 - z_{22} \cdot I_2$$

Najjednostavnije je izračunati [**z**]-parametre pomoću jednadžbi petlji (pretpostavimo izvore U_1 i U_2):

$$(1)\ I_1 \cdot (R_1 + R_2) - I_2 \cdot R_2 = U_1 - gU_1 \cdot R_2$$
$$(2)\ -I_1 \cdot R_2 + I_2 (sL + R_2) = gU_1 R_2 - U_2$$

$I_2 = 0$ (na prilazu 2-2' prazni hod)

$$U_1 = z_{11} \cdot I_1 \Rightarrow z_{11} = U_1 / I_1$$
$$U_2 = z_{21} \cdot I_1 \Rightarrow z_{21} = U_2 / I_1$$



$$(1) \Rightarrow I_1 \cdot (R_1 + R_2) = U_1 (1 - g \cdot R_2) \Rightarrow U_1 = \frac{R_1 + R_2}{1 - g \cdot R_2} \cdot I_1 \Rightarrow z_{11} = \frac{U_1}{I_1} = \frac{R_1 + R_2}{1 - g \cdot R_2}$$

$$(2) \Rightarrow -I_1 \cdot R_2 = gU_1 R_2 - U_2 \Rightarrow U_2 = gU_1 R_2 + I_1 \cdot R_2$$

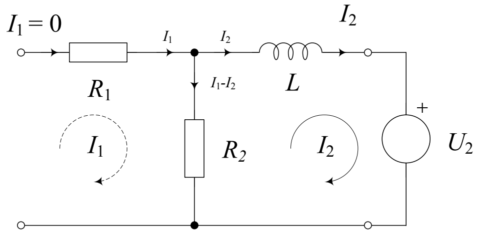
$$(1) \rightarrow (2) \Rightarrow U_2 = gR_2 \frac{R_1 + R_2}{1 - g \cdot R_2} \cdot I_1 + R_2 \cdot I_1 \Rightarrow z_{21} = \frac{U_2}{I_1} = gR_2 \frac{R_1 + R_2}{1 - g \cdot R_2} + R_2$$

Sređivanje z_{21}

$$z_{21} = \frac{U_2}{I_1} = \frac{gR_1 R_2 + gR_2^2 + R_2 - gR_2^2}{1 - g \cdot R_2} = \frac{(1 + gR_1) R_2}{1 - g \cdot R_2}$$

$\underline{I_1 = 0}$ (na prilazu 1-1' prazni hod)

$$U_1 = -z_{12} \cdot I_2 \Rightarrow z_{12} = -U_1 / I_2$$
$$U_2 = -z_{22} \cdot I_2 \Rightarrow z_{22} = -U_2 / I_2$$



$$(1) \Rightarrow -I_2 \cdot R_2 = U_1 - gU_1 \cdot R_2 \Rightarrow U_1 = -\frac{R_2}{1-gR_2} \cdot I_2 \Rightarrow z_{12} = -\frac{U_1}{I_2} = \frac{R_2}{1-gR_2}$$

$$(2) \Rightarrow I_2(sL + R_2) = gU_1R_2 - U_2 \Rightarrow U_2 = gU_1R_2 - I_2(sL + R_2)$$

$$(1) \to (2) \Rightarrow U_2 = -gR_2\frac{R_2}{1-gR_2} \cdot I_2 - (sL + R_2) \cdot I_2 \Rightarrow z_{22} = -\frac{U_2}{I_2} = \frac{gR_2^2}{1-g \cdot R_2} + sL + R_2$$

Slijedi sređivanje z_{22}

$$z_{22} = -\frac{U_2}{I_2} = \frac{gR_2^2 + sL + R_2 - g \cdot R_2 sL - gR_2^2}{1-g \cdot R_2} = \frac{sL + R_2(1 - g \cdot sL)}{1-g \cdot R_2}$$

Matrica [**z**]-parametara:

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{R_1 + R_2}{1-g \cdot R_2} & -\dfrac{R_2}{1-g \cdot R_2} \\ \dfrac{(1+gR_1)R_2}{1-g \cdot R_2} & -\dfrac{R_2(1-g \cdot sL)+sL}{1-g \cdot R_2} \end{bmatrix}$$

(do sada: maksimum 4 boda – ako su sva 4 parametra točna)

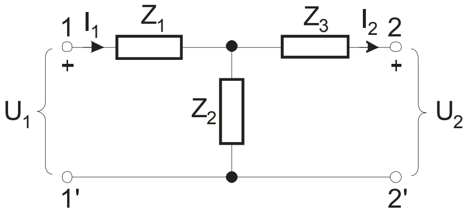
b) Da li je četveropol recipročan ?

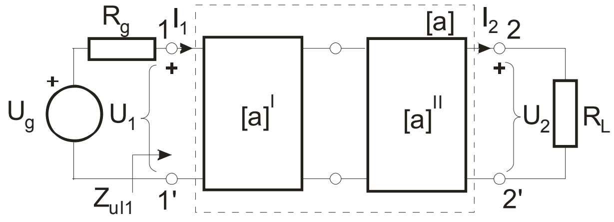
Ne, jer za recipročnost mora vrijediti z_{12}=z_{21}. To očigledno ne vrijedi, a razlog tomu je zavisni izvor.

c) Da li je četveropol simetričan ?

Ne, jer za simetričnost mora vrijediti z_{11}=z_{22}. (1 bod)

3. Za T-četveropol prikazan lijevom slikom izračunati prijenosne a-parametre. a) Napisati parametre A, B, C i D pomoću Z_1, Z_2 i Z_3 te uvrstiti slijedeće vrijednosti elemenata: $Z_1=sL_1=s$, $Z_2=R_2=1$, $Z_3=R_3=1$. b) Ako su dva ista četveropola iz prethodne točke spojena u kaskadu kao na desnoj slici, izračunati ukupne a-parametre kaskade uz uvrštene vrijednosti elemenata. c) Da li je ukupni četveropol (kaskada) recipročan, simetričan? Ako je izlazni prilaz (2–2') zaključen otporom $R_L=1$ pomoću a-parametara izračunati za kaskadu: d) ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$; e) ako je uz to na ulaz priključen generator ulaznog otpora $R_g=1$ izračunati prijenosnu funkciju napona kaskade $H(s)=U_2(s)/U_g(s)$.



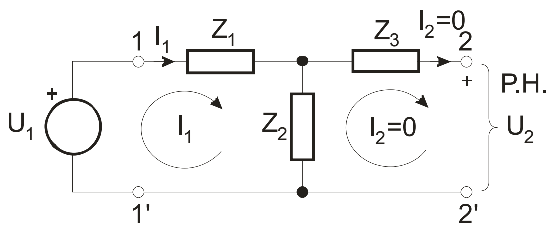


Rješenje:

a) [a]-parametri: **(1 bod)**

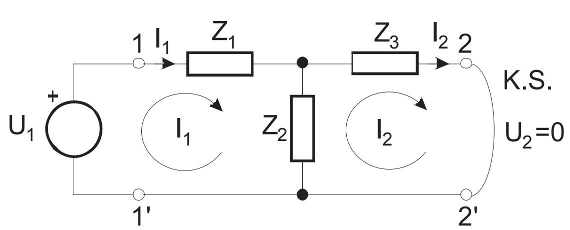
$$U_1 = A \cdot U_2 + B \cdot I_2$$
$$I_1 = C \cdot U_2 + D \cdot I_2$$

$$I_2 = 0 \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} \;;\quad C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$$



$$\begin{aligned}&(1)\;\; U_1 = I_1(Z_1+Z_2)\\ &(2)\;\; U_2 = I_1 Z_2\end{aligned} \quad \Rightarrow \quad A = \frac{U_1}{U_2} = \frac{Z_1+Z_2}{Z_2} = 1 + \frac{Z_1}{Z_2};\quad C = \frac{I_1}{U_2} = \frac{1}{Z_2};$$

$$U_2 = 0 \qquad B = \left.\frac{U_1}{I_2}\right|_{U_2=0} \;;\quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$



$$\begin{aligned}&(1)\;\; U_1 = I_1(Z_1+Z_2) - I_2 Z_2\\ &(2)\;\; 0 = -I_1 Z_2 + I_2(Z_2+Z_3)\end{aligned} \quad \Rightarrow I_1 = I_2 \frac{Z_2+Z_3}{Z_2};$$

$$U_1 = I_2 \frac{Z_2+Z_3}{Z_2}(Z_1+Z_2) - I_2 Z_2 = I_2 \frac{(Z_2+Z_3)(Z_1+Z_2) - Z_2^2}{Z_2} \Rightarrow U_1 = I_2 \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_2}$$

$$\Rightarrow \;\; B = \frac{U_1}{I_2} = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_2} = Z_1 + Z_3 + \frac{Z_1Z_3}{Z_2};\quad D = \frac{I_1}{I_2} = \frac{Z_2+Z_3}{Z_2} = 1 + \frac{Z_3}{Z_2};$$

$$[a]=\begin{bmatrix} A & B \\ C & D \end{bmatrix};\quad [a]=\begin{bmatrix} 1+\dfrac{Z_1}{Z_2} & Z_1+Z_3+\dfrac{Z_1Z_3}{Z_2} \\ \dfrac{1}{Z_2} & 1+\dfrac{Z_3}{Z_2} \end{bmatrix}=\begin{bmatrix} 1+s & 1+2s \\ 1 & 2 \end{bmatrix}$$

(ovo su parametri jednog četveropola)

b) prijenosni a-parametri kaskade dva četveropola **(1bod)**

$$[a]=[a]^I\cdot[a]^{II}=\begin{bmatrix} 1+s & 1+2s \\ 1 & 2 \end{bmatrix}\cdot\begin{bmatrix} 1+s & 1+2s \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} (1+s)^2+1+2s & (1+s)(1+2s)+2(1+2s) \\ 1+s+2 & 1+2s+4 \end{bmatrix}$$

$$[a]=\begin{bmatrix} s^2+4s+2 & 2s^2+7s+3 \\ s+3 & 2s+5 \end{bmatrix}$$

(ovo su parametri dva četveropola spojena u kaskadu)

c) Da li je kaskada recipročna, simetrična? **(1bod)**

Za recipročnost vrijedi: $\Delta = AD - BC = 1$

$$\Delta=(s^2+4s+2)(2s+5)-(2s^2+7s+3)(s+3)=$$
$$=2s^3+8s^2+4s+5s^2+20s+10-(2s^3+7s^2+3s+6s^2+21s+9)=1$$

$\Rightarrow$ Dobiveni čeveropol je recipročan.

Za simetričnost vrijedi: A=D $\Rightarrow$ $s^2+4s+2\neq 5+2s$

$\Rightarrow$ Dobiveni četveropol nije simetričan

Konačno iz jednadžbi

$$\underline{\begin{matrix} U_1=A\cdot U_2+B\cdot I_2 \\ I_1=C\cdot U_2+D\cdot I_2 \end{matrix}},\quad R_L=\frac{U_2}{I_2},\quad U_g=I_1R_g+U_1$$

slijede:

d) Ulazna impedancija u četveropol: **(1 bod)**

$$Z_{ul1}(s)=\frac{U_1}{I_1}=\frac{AU_2+BI_2}{CU_2+DI_2}=\frac{A\dfrac{U_2}{I_2}+B}{C\dfrac{U_2}{I_2}+D}=\frac{AR_L+B}{CR_L+D}$$

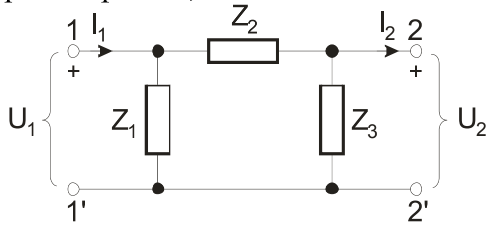
$$Z_{ul1}(s)=\frac{(s^2+4s+2)\cdot 1+(2s^2+7s+3)}{(s+3)\cdot 1+(2s+5)}=\frac{3s^2+11s+5}{3s+8}$$

e) Prijenosna funkcija napona: **(1 bod)**

$$U_g=I_1R_g+U_1=\left(CU_2+D\frac{U_2}{R_L}\right)R_g+AU_2+B\frac{U_2}{R_L}\;\Rightarrow\; H(s)=\frac{U_2}{U_g}=\frac{R_L}{AR_L+B+R_g(CR_L+D)}$$

$$H(s)=\frac{1}{(s^2+4s+2)\cdot 1+2s^2+7s+3+1\cdot[(s+3)\cdot 1+2s+5]}=\frac{1}{3s^2+14s+13}$$

3. Za Π-četveropol prikazan slikom izračunati z-parametre. Napisati: a) parametre z_{11}, z_{21}, z_{12} i z_{22} (izraziti z-parametre pomoću Z_1, Z_2 i Z_3). Ako je izlazni prilaz (2–2') zaključen impedancijom Z_L pomoću z-parametara izračunati: b) strujnu prijenosnu funkciju četveropola $H_i(s)=I_2(s)/I_1(s)$; c) ulaznu impedanciju u četveropol $Z_{ul1}(s)=U_1(s)/I_1(s)$ (izraziti $H_i(s)$ i $Z_{ul1}(s)$ pomoću gore izračunatih z-parametara izraženih općim impedancijama elemenata Z_1, Z_2 i Z_3). d) Izračunati $H_i(s)$ i $Z_{ul1}(s)$ uz slijedeće vrijednosti elemenata: $Z_1=R_1=1$, $Z_2=sL_2=s$, $Z_3=R_3=1$ i $Z_L=R_L=1$. e) Da li je četveropol recipročan, simetričan?



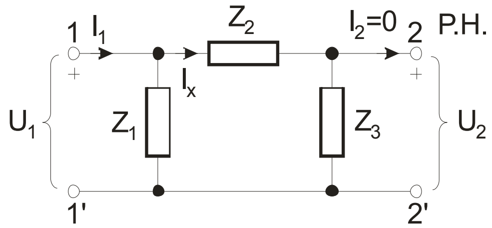
Rješenje:

a) [z]-parametri: **(1 bod)**

$$U_1 = z_{11}I_1 - z_{12}I_2$$

$$U_2 = z_{21}I_1 - z_{22}I_2$$

$I_2=0$ parametri z_{11} i z_{21}:



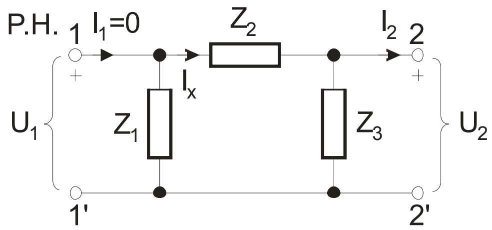
$$I_1 = U_1\left(\frac{1}{Z_1}+\frac{1}{Z_2+Z_3}\right) = U_1\frac{Z_1+Z_2+Z_3}{Z_1(Z_2+Z_3)}$$

$$I_x = U_2\frac{1}{Z_3} = U_1\frac{1}{Z_2+Z_3} \quad\Rightarrow\quad U_2 = U_1\frac{Z_3}{Z_2+Z_3} \Rightarrow U_1 = U_2\frac{Z_2+Z_3}{Z_3}$$

$$\Rightarrow\ I_1 = U_1\frac{Z_1+Z_2+Z_3}{Z_1(Z_2+Z_3)} = U_2\frac{Z_2+Z_3}{Z_3}\frac{Z_1+Z_2+Z_3}{Z_1(Z_2+Z_3)} = U_2\frac{Z_1+Z_2+Z_3}{Z_1Z_3}$$

$$z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = \frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3}\ ;\quad z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0} = \frac{Z_1Z_3}{Z_1+Z_2+Z_3}$$

$I_1=0$ parametri z_{12} i z_{22}:



$$I_2 = -U_2\left(\frac{1}{Z_3}+\frac{1}{Z_1+Z_2}\right) = -U_2\frac{Z_1+Z_2+Z_3}{Z_3(Z_1+Z_2)}$$

$$I_x = -U_2\frac{1}{Z_1+Z_2} = -U_1\frac{1}{Z_1} \quad\Rightarrow\quad U_1 = U_2\frac{Z_1}{Z_1+Z_2} \Rightarrow U_2 = U_1\frac{Z_1+Z_2}{Z_1}$$

$$\Rightarrow\ I_2 = -U_2\frac{Z_1+Z_2+Z_3}{Z_3(Z_1+Z_2)} = -U_1\frac{Z_1+Z_2}{Z_1}\frac{Z_1+Z_2+Z_3}{Z_3(Z_1+Z_2)} = -U_1\frac{Z_1+Z_2+Z_3}{Z_1Z_3}$$

$$z_{12}=-\frac{U_1}{I_2}\bigg|_{I_1=0}=\frac{Z_1Z_3}{Z_1+Z_2+Z_3}\;;\quad z_{22}=-\frac{U_2}{I_2}\bigg|_{I_1=0}=\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}$$

$$[z]=\begin{bmatrix}\frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & -\frac{Z_1Z_3}{Z_1+Z_2+Z_3}\\ \frac{Z_1Z_3}{Z_1+Z_2+Z_3} & -\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}\end{bmatrix},\;[z]=\begin{bmatrix}z_{11} & -z_{12}\\ z_{21} & -z_{22}\end{bmatrix}$$

b) Prijenosna funkcija struje: **(1 bod)**

$$H_i(s)=\frac{I_2}{I_1}=\frac{z_{21}}{Z_L+z_{22}}=\frac{\frac{Z_1Z_3}{Z_1+Z_2+Z_3}}{Z_L+\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}}=\frac{Z_1Z_3}{Z_L(Z_1+Z_2+Z_3)+Z_3(Z_1+Z_2)}$$

c) Ulazna impedancija u četveropol: **(1 bod)**

$$Z_{ul1}(s)=\frac{U_1}{I_1}=z_{11}-\frac{z_{12}z_{21}}{Z_L+z_{22}}=\frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3}-\frac{\left(\frac{Z_1Z_3}{Z_1+Z_2+Z_3}\right)^2}{Z_L+\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}}=$$

$$=\frac{1}{(Z_1+Z_2+Z_3)}\left[Z_1(Z_2+Z_3)-\frac{(Z_1Z_3)^2}{Z_L(Z_1+Z_2+Z_3)+Z_3(Z_1+Z_2)}\right]$$

d) Uz uvrštene vrijednosti: **(1 bod)**

$$H_i(s)=\frac{I_2}{I_1}=\frac{R_1R_3}{R_L(R_1+R_2+R_3)+R_3(R_1+R_2)}=\frac{1}{1(1+s+1)+1(1+s)}=\frac{1}{3+2s}$$

$$Z_{ul1}=\frac{1}{(R_1+R_2+R_3)}\left[R_1(R_2+R_3)-\frac{(R_1R_3)^2}{R_L(R_1+R_2+R_3)+R_3(R_1+R_2)}\right]=$$

$$=\frac{1}{(1+s+1)}\left[1(s+1)-\frac{(1)^2}{1(1+s+1)+1(1+s)}\right]=\frac{1}{2+s}\left[s+1-\frac{1}{3+2s}\right]=$$

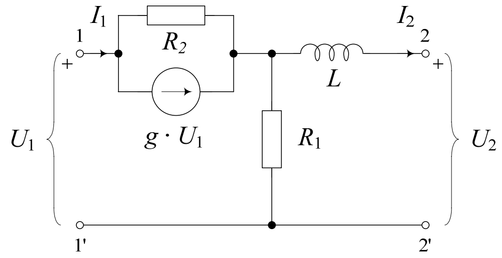
$$=\frac{1}{2+s}\left[\frac{3s+2s^2+3+2s-1}{3+2s}\right]=\frac{2s^2+5s+2}{(2+s)(3+2s)}=\frac{2s^2+5s+2}{2s^2+7s+6}$$

e) Recipročnost i simetričnost četveropola: **(1 bod)**

$$[z]=\begin{bmatrix}\frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & -\frac{Z_1Z_3}{Z_1+Z_2+Z_3}\\ \frac{Z_1Z_3}{Z_1+Z_2+Z_3} & -\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}\end{bmatrix}=\begin{bmatrix}\frac{s+1}{s+2} & -\frac{1}{s+2}\\ \frac{1}{s+2} & -\frac{s+1}{s+2}\end{bmatrix}$$

Četveropol je recipročan jer vrijedi $z_{21}=z_{12}$ i simetričan jer je $z_{11}=z_{22}$.

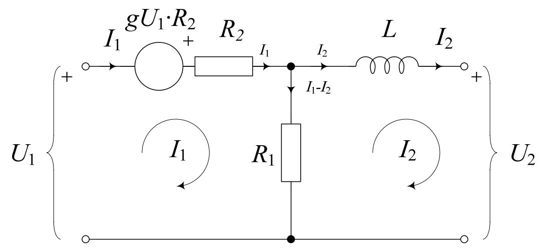
3. Za četveropol na slici: *a*) izračunati [**z**]-parametre i napisati ih u matričnom obliku. *b*) Iz [**z**]-parametara izračunati [**y**]-parametre. Ako je na na ulaz 1–1' spojen naponski izvor $u_1(t)=S(t)$ te na izlaz 2–2' spojena impedancija $Z_2=1/sC$ izračunati: *c*) prijenosnu funkciju napona $H_u(s)=U_2(s)/U_1(s)$ i izlazni napon $U_2(s)$, *d*) ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$ i ulaznu struju $I_1(s)$. Zadano je L=1, C=1, R_1=1, R_2=2, g=3.



Rješenje:

a) izračun [**z**] parametara pomoću jednadžbi petlji:

$$U_1 = z_{11} \cdot I_1 - z_{12} \cdot I_2$$
$$U_2 = z_{21} \cdot I_1 - z_{22} \cdot I_2$$



$$(1)\ I_1 \cdot (R_1 + R_2) - I_2 \cdot R_1 = U_1 + gU_1 \cdot R_2$$
$$(2)\ -I_1 \cdot R_1 + I_2(sL + R_1) = -U_2$$

$$(1)\ I_1 \cdot (R_1 + R_2) - I_2 \cdot R_1 = U_1(1 + g \cdot R_2) / : (1 + g \cdot R_2)$$
$$(2)\ -I_1 \cdot R_1 + I_2(sL + R_1) = -U_2 / \cdot (-1)$$

$$(1)\ I_1 \cdot \frac{R_1 + R_2}{1 + g \cdot R_2} - I_2 \cdot \frac{R_1}{1 + g \cdot R_2} = U_1$$
$$(2)\ I_1 \cdot R_1 - I_2(sL + R_1) = U_2$$

Matrica [**z**]-parametara (uz uvrštene vrijednosti elemenata L=1, C=1, R_1=1, R_2=2, g=3):

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{R_1 + R_2}{1 + g \cdot R_2} & -\dfrac{R_1}{1 + g \cdot R_2} \\ R_1 & -(sL + R_1) \end{bmatrix} = \begin{bmatrix} \dfrac{3}{7} & -\dfrac{1}{7} \\ 1 & -(s+1) \end{bmatrix}$$

(do sada: maksimum 2 boda – ako su svi parametri točni)

b) Matrica [**y**]-parametara: $[\mathbf{y}] = [\mathbf{z}]^{-1}$ (1 bod)

$$\det[\mathbf{z}] = \begin{vmatrix} \dfrac{3}{7} & -\dfrac{1}{7} \\ 1 & -(s+1) \end{vmatrix} = -\frac{3}{7} \cdot (s+1) + \frac{1}{7} = \frac{-3s - 3 + 1}{7} = \frac{-3s - 2}{7} = -\frac{1}{7}(3s + 2)$$

$$[\mathbf{y}]=\begin{bmatrix}\frac{3}{7} & -\frac{1}{7}\\ 1 & -(s+1)\end{bmatrix}^{-1}=\frac{-7}{3s+2}\begin{bmatrix}-(s+1) & -1\\ \frac{1}{7} & \frac{3}{7}\end{bmatrix}^{T}=\frac{-7}{3s+2}\begin{bmatrix}-(s+1) & \frac{1}{7}\\ -1 & \frac{3}{7}\end{bmatrix}=\begin{bmatrix}\frac{7(s+1)}{3s+2} & -\frac{1}{3s+2}\\ \frac{7}{3s+2} & -\frac{3}{3s+2}\end{bmatrix}$$

c) Prijenosna funkcija napona i odziv na Step: (1 bod)

$$H_u(s)=\frac{U_2(s)}{U_1(s)}=\frac{Z_L z_{21}}{z_{11}(z_{22}+Z_L)-z_{12}z_{21}}=\frac{\frac{1}{s}\cdot 1}{\frac{3}{7}\left(s+1+\frac{1}{s}\right)-\frac{1}{7}}=\frac{7}{3\left(s^2+s+1\right)-s}$$

$$=\frac{7}{3\left(s^2+s+1\right)-s}=\frac{7}{3s^2+2s+3}$$

ili

$$H_u(s)=\frac{U_2(s)}{U_1(s)}=\frac{y_{21}}{y_{22}+Y_L}=\frac{\frac{7}{3s+2}}{\frac{3}{3s+2}+s}=\frac{7}{3+s(3s+2)}=\frac{7}{3s^2+2s+3}$$

$$U_1(s)=\frac{1}{s}\Rightarrow U_2(s)=H_u(s)U_1(s)=\frac{1}{s}\cdot\frac{7}{3s^2+2s+3}=\frac{7}{s(3s^2+2s+3)}$$

d) Izračunati ulaznu impedanciju $Z_{ul1}(s)$ i ulaznu struju $I_1(s)$: (1 bod)

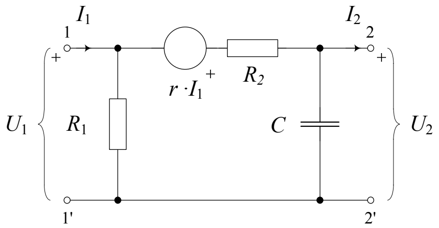
$$Z_{ul1}(s)=\frac{U_1(s)}{I_1(s)}=z_{11}-\frac{z_{12}z_{21}}{z_{22}+Z_L}=\frac{3}{7}-\frac{\frac{1}{7}}{s+1+\frac{1}{s}}=\frac{3}{7}-\frac{s}{7(s^2+s+1)}=\frac{3(s^2+s+1)-s}{7(s^2+s+1)}$$

$$\frac{3(s^2+s+1)-s}{7(s^2+s+1)}=\frac{3s^2+2s+3}{7(s^2+s+1)}\quad\Rightarrow\quad I_1(s)=\frac{U_1(s)}{Z_{ul1}(s)}=\frac{1}{s}\cdot\frac{7(s^2+s+1)}{3s^2+2s+3}$$

ili

$$Y_{ul1}(s)=\frac{I_1(s)}{U_1(s)}=y_{11}-\frac{y_{12}y_{21}}{y_{22}+Y_L}\quad\Rightarrow\quad I_1(s)=U_1(s)\cdot Y_{ul1}(s)=\frac{1}{s}\cdot\frac{7(s^2+s+1)}{3s^2+2s+3}$$

3. Za četveropol na slici izračunati: *a*) prijenosne [*a*]-parametre i napisati ih u matričnom obliku. *b*) Ako je na izlaz četveropola spojena impedancija zaključenja $Z_2=sL$ izračunati ulaznu impedanciju četveropola $Z_{ul1}(s)$. *c*) Da li je četveropol: recipročan, simetričan? Obrazložiti odgovore. Zadane su normalizirane vrijednosti $R_1=R_2=1$, $C=1$, $L=1$, $r=2$.



Rješenje:

Napomena: ako se odmah uvrste numeričke vrijednosti (što je ovdje učinjeno i jednako se priznaje za točno rješenje) tada je postupak znatno jednostavniji i kraći.

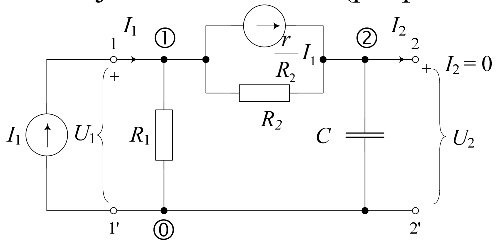
a) izračun [*a*]-parametara

$$U_1 = A\cdot U_2 + B\cdot I_2$$
$$I_1 = C\cdot U_2 + D\cdot I_2$$
prijenosne jednadžbe četveropola

$I_2 = 0$ (na prilazu 2-2' prazni hod)

$$A=\left.\frac{U_1}{U_2}\right|_{I_2=0} \quad ; \quad C=\left.\frac{I_1}{U_2}\right|_{I_2=0}$$

Izračunajmo[*a*]-parametre pomoću jednadžbi čvorova (pretpostavimo izvor I_1):



$$(1)\ U_1\cdot\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-U_2\cdot\frac{1}{R_2}=I_1-I_1\cdot\frac{r}{R_2}$$

$$(2)\ -U_1\cdot\frac{1}{R_2}+U_2\cdot\left(sC+\frac{1}{R_2}\right)=I_1\cdot\frac{r}{R_2}$$

uvrstimo vrijednosti elemenata:

$$(1)\ \ U_1\cdot(1+1)-U_2\cdot 1 = I_1 - I_1\cdot 2 \Rightarrow U_1\cdot 2 - U_2 = -I_1 \Rightarrow U_1 = \frac{U_2}{2}-\frac{I_1}{2}$$

$$(2)\ -U_1\cdot 1 + U_2\cdot(s+1) = I_1\cdot 2 \Rightarrow -U_1+U_2\cdot(s+1)=I_1\cdot 2 \Rightarrow I_1 = -\frac{U_1}{2}+U_2\frac{s+1}{2}$$

Supstitucija I_1: (2) $\rightarrow$ (1) $\Rightarrow U_1 = \frac{U_2}{2}-\frac{1}{2}\left(-\frac{U_1}{2}+U_2\frac{s+1}{2}\right)\Big/\cdot 4$

$$4U_1 = 2U_2 - \left[-U_1 + U_2(s+1)\right]$$

$$3U_1 = 2U_2 - U_2 s - U_2 \ \ \Rightarrow A = \left.\frac{U_1}{U_2}\right|_{I_2=0} = \frac{1-s}{3}$$

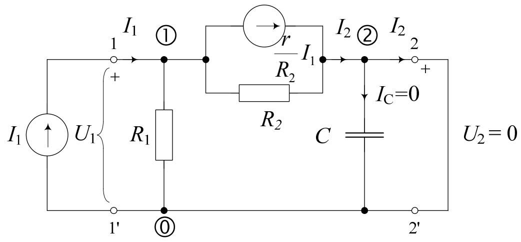
Supstitucija U_1: (1) $\rightarrow$ (2) $\Rightarrow$ $I_1 = -\frac{1}{2}\left(\frac{U_2}{2} - \frac{I_1}{2}\right) + U_2 \frac{s+1}{2} \Big/ \cdot 4$

$$4I_1 = -(U_2 - I_1) + U_2(2s+2)$$

$$3I_1 = -U_2 + 2U_2 + 2sU_2 \;\Rightarrow\; C = \left.\frac{I_1}{U_2}\right|_{I_2=0} = \frac{1+2s}{3}$$

$U_2 = 0$ (na prilazu 2-2' kratki spoj)

$$B = \left.\frac{U_1}{I_2}\right|_{U_2=0} \quad ; \quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$



(1) $U_1 \cdot \frac{1}{R_1} = I_1 - I_2$;

(2) $I_2 = \frac{r}{R_2} I_1 + \frac{U_1}{R_2} \;\Rightarrow\; U_1 = R_2 I_2 - rI_1$

uvrstimo vrijednosti elemenata:

(1) $U_1 = I_1 - I_2$;

(2) $U_1 = I_2 - 2I_1$

$$(2) \rightarrow (1) \Rightarrow I_2 - 2I_1 = I_1 - I_2 \;\Rightarrow\; 2I_2 = 3I_1 \;\Rightarrow\; D = \left.\frac{I_1}{I_2}\right|_{U_2=0} = \frac{2}{3}$$

$$\rightarrow (2) \Rightarrow U_1 = I_2 - 2I_1 = I_2 - 2\frac{2}{3}I_2 = -\frac{1}{3}I_2 \;\Rightarrow\; B = \left.\frac{U_1}{I_2}\right|_{U_2=0} = -\frac{1}{3}$$

Matrica prijenosnih [a]-parametara glasi: $[a] = \frac{1}{3}\begin{bmatrix} 1-s & -1 \\ 2s+1 & 2 \end{bmatrix}$

(do sada: maksimum 3 boda, svaki neispravan parametar 1 bod manje)

b) $Z_{ul1}(s) = \frac{U_1}{I_1} = \frac{AU_2 + BI_2}{CU_2 + DI_2} = \frac{AU_2/I_2 + B}{CU_2/I_2 + D} = \frac{AZ_2 + B}{CZ_2 + D}$; $Z_2 = \frac{U_2}{I_2} = sL$

$$Z_{ul1}(s) = \frac{(1-s)s - 1}{(2s+1)s + 2} = \frac{-s^2 + s - 1}{2s^2 + s + 2}$$ (1 bod)

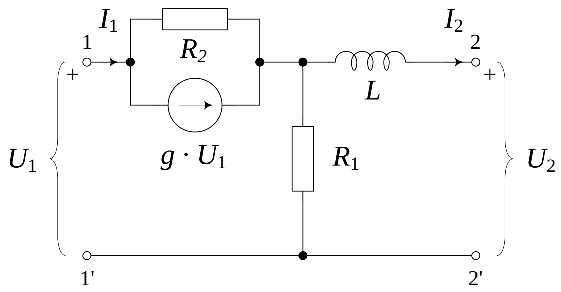
c) Da li je četveropol recipročan ? Ne, jer za recipročnost mora vrijediti det[a]=1.

$$\det[a] = \begin{vmatrix} \frac{1-s}{3} & -\frac{1}{3} \\ \frac{2s+1}{3} & \frac{2}{3} \end{vmatrix} = \frac{2(1-s)}{9} + \frac{2s+1}{9} = \frac{3}{9} = \frac{1}{3} \neq 1$$

To očigledno ne vrijedi, a razlog tomu je zavisni izvor.

Da li je četveropol simetričan ? Ne, jer za simetričnost mora vrijediti A=D. (1 bod)

3. Za četveropol na slici izračunati: a) [*a*]-parametre. Zadano je R_1=1/2, R_2=1/2, L=1/2, g=2. b) Da li je četveropol: recipročan, simetričan ? Obrazložiti odgovore. Ako je izlazni prilaz (2–2') zaključen otporom R_L=1 pomoću [*a*]-parametara izračunati: c) ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$; d) ako je uz to na ulaz priključen generator ulaznog otpora R_g=1 izračunati prijenosnu funkciju napona $H(s)=U_2(s)/U_g(s)$.

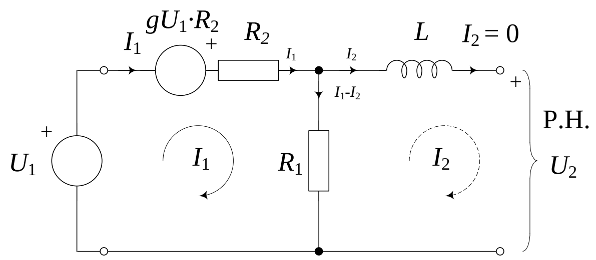


Rješenje:

a) [*a*]-parametri:

$$U_1 = A\cdot U_2 + B\cdot I_2$$
$$I_1 = C\cdot U_2 + D\cdot I_2$$

$$I_2 = 0 \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} \;;\quad C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$$

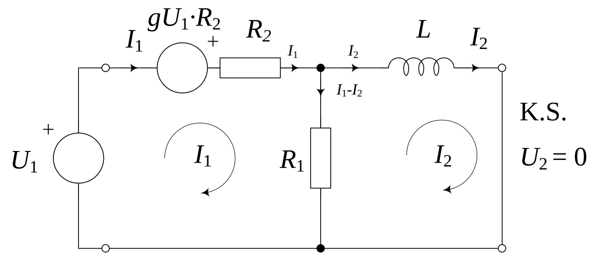


$$I_1\cdot(R_1+R_2) - g\cdot U_1\cdot R_2 = U_1 \Rightarrow I_1 = \frac{g\cdot U_1\cdot R_2 + U_1}{R_1+R_2}$$
$$U_2 = I_1\cdot R_1$$

$$\Rightarrow U_2 = U_1\cdot(1+gR_2)\cdot\frac{R_1}{R_1+R_2} \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} = \frac{\dfrac{R_1+R_2}{1+gR_2}}{R_1} = \frac{\dfrac{1/2+1/2}{1+1}}{1/2} = \frac{1/2}{1/2} = 1$$

$$\Rightarrow U_2 = I_1\cdot R_1, \qquad C = \left.\frac{I_1}{U_2}\right|_{I_2=0} = \frac{1}{R_1} = \frac{1}{1/2} = 2$$

$$U_2 = 0 \qquad B = \left.\frac{U_1}{I_2}\right|_{U_2=0} \;;\quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$



$$I_1 \cdot (R_1 + R_2) - I_2 \cdot R_1 = U_1 + g \cdot U_1 \cdot R_2$$
$$-I_1 \cdot R_1 + I_2 \cdot (R_1 + sL) = 0$$

$$(2) \Rightarrow \; I_1 = I_2 \cdot \frac{R_1 + sL}{R_1}$$

$$(2) \rightarrow (1) \Rightarrow \; I_2 \cdot \frac{R_1 + sL}{R_1} \cdot (R_1 + R_2) - I_2 \cdot R_1 = U_1 \cdot (1 + gR_2)$$

$$I_2 \cdot \left[\left(1 + \frac{sL}{R_1}\right) \cdot (R_1 + R_2) - R_1 \right] = U_1 \cdot (1 + gR_2)$$

$$I_2 \cdot \left[R_1 + R_2 + sL + R_2 \frac{sL}{R_1} - R_1 \right] = U_1 \cdot (1 + gR_2)$$

$$B = \left. \frac{U_1}{I_2} \right|_{U_2 = 0} = \frac{R_2 + sL + R_2 \dfrac{sL}{R_1}}{1 + gR_2} = \frac{\dfrac{sLR_1 + R_2(R_1 + sL)}{1 + gR_2}}{R_1}$$

$$= \frac{\dfrac{s \cdot 1/4 + (1/2) \cdot (1/2 + s \cdot 1/2)}{1+1}}{1/2} = \frac{s \cdot 1/4 + (1/4) \cdot (1 + s)}{2/2} = \frac{s + (1+s)}{4} = \frac{2s+1}{4}$$

$$I_1 = I_2 \cdot \frac{R_1 + sL}{R_1} \qquad D = \left. \frac{I_1}{I_2} \right|_{U_2 = 0} = \frac{R_1 + sL}{R_1} = \frac{1/2 + s \cdot 1/2}{1/2} = s + 1$$

Uvrstimo vrijednosti elemenata R_1=1/2, R_2=1/2, L=1/2, g=2:

$$[a] = \begin{bmatrix} 1 & (2s+1)/4 \\ 2 & s+1 \end{bmatrix}$$ **(2 boda)**

b) Da li je četveropol recipročan, simetričan? **(1bod)**

Za recipročnost vrijedi: $\Delta = AD - BC = 1$

$\Delta = s + 1 - s - 1/2 = 1/2 \quad \Rightarrow$ Čeveropol nije recipročan.

Za simetričnost vrijedi: A=D $\Rightarrow$ $1 \neq s+1 \quad \Rightarrow$ Četveropol nije simetričan

Konačno iz jednadžbi $\begin{array}{l} U_1 = A \cdot U_2 + B \cdot I_2 \\ I_1 = C \cdot U_2 + D \cdot I_2 \end{array}$, $R_L = \dfrac{U_2}{I_2}$, $U_g = I_1 R_g + U_1$ slijede:

c) Ulazna impedancija u četveropol:

$$Z_{ul1}(s) = \frac{U_1}{I_1} = \frac{AU_2 + BI_2}{CU_2 + DI_2} = \frac{A\dfrac{U_2}{I_2} + B}{C\dfrac{U_2}{I_2} + D} = \frac{AR_L + B}{CR_L + D}$$

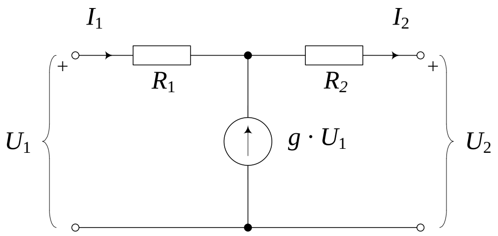
$$\Rightarrow Z_{ul1}(s) = \frac{1 \cdot 1 + s/2 + 1/4}{2 \cdot 1 + s + 1} = \frac{s/2 + 5/4}{s+3} = \frac{1}{4} \cdot \frac{2s+5}{s+3}$$ **(1 bod)**

d) Prijenosna funkcija napona:

$$U_g = I_1 R_g + U_1 = \left(CU_2 + D\frac{U_2}{R_L} \right) R_g + AU_2 + B\frac{U_2}{R_L} \Rightarrow H(s) = \frac{U_2}{U_g} = \frac{R_L}{AR_L + B + R_g(CR_L + D)} \Rightarrow$$

$$H(s) = \frac{1}{1 \cdot 1 + s/2 + 1/4 + 1 \cdot [2 + s + 1]} = \frac{1}{3s/2 + 5/4 + 12/4} = \frac{1}{3s/2 + 17/4} = \frac{4}{6s+17}$$ **(1 bod)**

3. Za četveropol na slici izračunati: a) [a]-parametre. Zadano je R_1=2, R_2=1, g=1/2. b) Da li je četveropol: recipročan, simetričan ? Obrazložiti odgovore. Ako je izlazni prilaz (2–2') zaključen otporom R_L=1 pomoću [a]-parametara izračunati: c) ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$; d) ako je uz to na ulaz priključen generator ulaznog otpora R_g=1 izračunati prijenosnu funkciju napona $H(s)=U_2(s)/U_g(s)$.

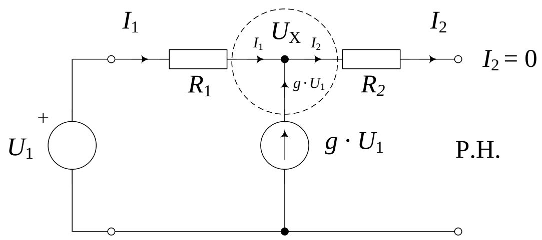


Rješenje:

a) [a]-parametri: (2 boda)

$$U_1 = A\cdot U_2 + B\cdot I_2$$
$$I_1 = C\cdot U_2 + D\cdot I_2$$

$$I_2 = 0 \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} \; ; \quad C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$$

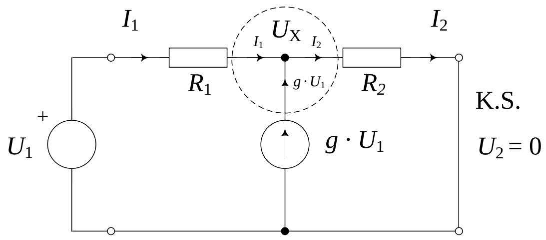


$$I_1 + g\cdot U_1 = 0 \Rightarrow I_1 = -g\cdot U_1$$
$$U_X = U_1 - I_1\cdot R_1$$

$$\Rightarrow U_2 = U_X = U_1 + g\cdot U_1\cdot R_1 = (1+gR_1)\cdot U_1 \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} = \frac{1}{1+gR_1} = \frac{1}{1+\frac{1}{2}2} = \frac{1}{2}$$

$$\Rightarrow U_1 = -\frac{I_1}{g},\; U_2 = (1+gR_1)\cdot U_1 = -\frac{1+gR_1}{g}\cdot I_1 \qquad C = \left.\frac{I_1}{U_2}\right|_{I_2=0} = -\frac{g}{1+gR_1} = \frac{-\frac{1}{2}}{1+1} = -\frac{1}{4}$$

$$U_2 = 0 \qquad B = \left.\frac{U_1}{I_2}\right|_{U_2=0} \; ; \quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$



(1) $I_1 + g\cdot U_1 = I_2$

(2) $U_X = I_2\cdot R_2 = U_1 - I_1\cdot R_1$

$$(1) \Rightarrow U_1 = \frac{I_2}{g} - \frac{I_1}{g}$$

$$(2) \Rightarrow I_1 = \frac{1}{R_1} \cdot U_1 - I_2 \frac{R_2}{R_1}, \; I_2 = \frac{U_1}{R_2} - I_1 \frac{R_1}{R_2}$$

$$\frac{1}{R_1} \cdot U_1 - I_2 \frac{R_2}{R_1} + g \cdot U_1 = I_2$$

$$U_1\left(\frac{1}{R_1} + g\right) = I_2\left(1 + \frac{R_2}{R_1}\right) \qquad B = \left.\frac{U_1}{I_2}\right|_{U_2=0} = \frac{1 + R_2 / R_1}{1/R_1 + g} = \frac{R_1 + R_2}{1 + gR_1} = \frac{2+1}{1+(1/2)\cdot 2} = \frac{3}{2}$$

$$(1) \rightarrow (2) \Rightarrow I_2 = I_2 \frac{1}{gR_2} - I_1 \frac{1}{gR_2} - I_1 \frac{R_1}{R_2}$$

$$I_2\left(\frac{1}{gR_2} - 1\right) = I_1\left(\frac{1}{gR_2} + \frac{R_1}{R_2}\right) \quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0} = \frac{\frac{1}{gR_2} - 1}{\frac{1}{gR_2} + \frac{R_1}{R_2}} = \frac{1 - gR_2}{1 + gR_1} = \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

Uvrstimo vrijednosti elemenata R_1=2, R_2=1, g=1/2:

$$[a] = \begin{bmatrix} 1/2 & 3/2 \\ -1/4 & 1/4 \end{bmatrix}$$

b) Da li je četveropol recipročan, simetričan? (1bod)

Za recipročnost vrijedi: $\Delta = AD - BC = 1$

$$\Delta = \frac{1}{2} \cdot \frac{1}{4} + \frac{3}{2} \cdot \frac{1}{4} = \frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2} \qquad \Rightarrow \text{Čeveropol nije recipročan.}$$

Za simetričnost vrijedi: A=D $\Rightarrow \frac{1}{2} \neq \frac{1}{4} \qquad \Rightarrow$ Četveropol nije simetričan

Konačno iz jednadžbi $\underline{\begin{matrix} U_1 = A \cdot U_2 + B \cdot I_2 \\ I_1 = C \cdot U_2 + D \cdot I_2 \end{matrix}}$, $R_L = \frac{U_2}{I_2}$, $U_g = I_1 R_g + U_1$ slijede:

c) Ulazna impedancija u četveropol:

$$Z_{ul1}(s) = \frac{U_1}{I_1} = \frac{AU_2 + BI_2}{CU_2 + DI_2} = \frac{A\frac{U_2}{I_2} + B}{C\frac{U_2}{I_2} + D} = \frac{AR_L + B}{CR_L + D} \Rightarrow Z_{ul1}(s) = \frac{\frac{1}{2} \cdot 1 + \frac{3}{2}}{-\frac{1}{4} \cdot 1 + \frac{1}{4}} = \frac{2}{0} = \infty$$ (1 bod)

d) Prijenosna funkcija napona:

$$U_g = I_1 R_g + U_1 = \left(CU_2 + D\frac{U_2}{R_L}\right) R_g + AU_2 + B\frac{U_2}{R_L}$$

$$\Rightarrow H(s) = \frac{U_2}{U_g} = \frac{R_L}{AR_L + B + R_g(CR_L + D)} \Rightarrow H(s) = \frac{1}{\frac{1}{2} \cdot 1 + \frac{3}{2} + 1 \cdot \left[-\frac{1}{4} \cdot 1 + \frac{1}{4}\right]} = \frac{1}{2}$$ (1 bod)

Četveropoli

4. Koliki moraju biti α, β i γ (α, β, γ>0) da bi četveropol zadan prijenosnim parametrima bio recipročan i simetričan. Naći ekvivalentni *T*-spoj.

$$[a]=\begin{bmatrix}\frac{1+\alpha s}{1-2s} & \frac{4}{1-2s}\\ \frac{\beta s}{1-2s} & \frac{\gamma+\alpha s}{1-2s}\end{bmatrix}$$

Rješenje:

Uvjet simetrije: $A=D$

$A=D\Rightarrow \gamma=1$

Uvjet recipročnosti: $\Delta=AD-BC=1$ $(\Delta=\det[a])$

$$AD-BC=1 \quad\Rightarrow\quad \frac{(1+\alpha s)^2}{(1-2s)^2}-\frac{4\beta s}{(1-2s)^2}=1$$

$$(1+\alpha s)^2-4\beta s=(1-2s)^2$$

$$1+2\alpha s+\alpha^2 s^2-4\beta s=1-4s+4s^2$$

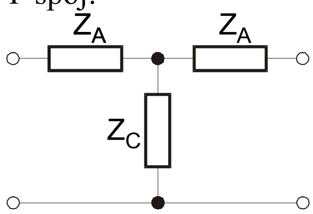
$$1+(2\alpha-4\beta)s+\alpha^2 s^2=1-4s+4s^2$$

$$\alpha^2=4 \quad\Rightarrow\quad \alpha=2$$

$$\underline{2\alpha-4\beta=-4} \quad\Rightarrow\quad 4-4\beta=-4 \quad\Rightarrow\quad \beta=\frac{8}{4}=2$$

$$[a]=\begin{bmatrix}\frac{1+2s}{1-2s} & \frac{4}{1-2s}\\ \frac{2s}{1-2s} & \frac{1+2s}{1-2s}\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}$$

T-spoj:



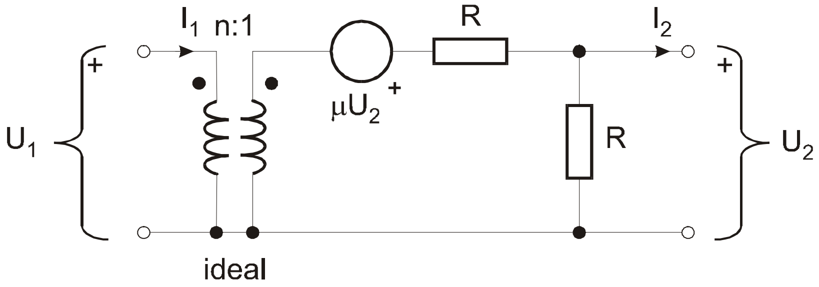
$$z_{11}=\frac{A}{C}=\frac{1+2s}{2s}$$

$$z_{12}=\frac{AD}{C}-B=\frac{(1+2s)^2}{2s(1-2s)}-\frac{4}{1-2s}=\frac{1-2s}{2s}$$

$$Z_A=z_{11}-z_{12}=2$$

$$Z_C=z_{12}=\frac{1-2s}{2s}=\frac{1}{2s}-1$$

5. Za četveropol prikazan slikom izračunati $[y]$ parametre. Odrediti vrijednost za n kako bi četveropol bio simetričan.



Rješenje:

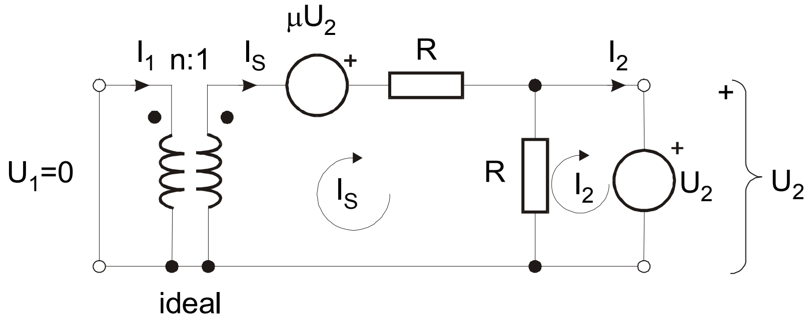
$$I_1 = y_{11} \cdot U_1 - y_{12} \cdot U_2$$
$$I_2 = y_{21} \cdot U_1 - y_{22} \cdot U_2$$

$$y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0} \quad y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0} \quad y_{12} = -\left.\frac{I_1}{U_2}\right|_{U_1=0} \quad y_{22} = -\left.\frac{I_2}{U_2}\right|_{U_1=0}$$

jednadžbe transformatora :

$$U_S = \frac{1}{n} \cdot U_1$$
$$I_S = n \cdot I_1$$

a) $U_1 = 0 \qquad \Rightarrow \qquad U_S = 0$

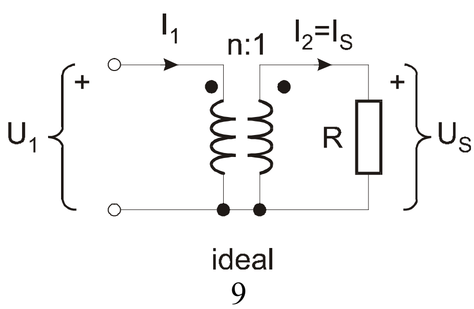


$$I_S \cdot R + U_2 = \mu \cdot U_2 \quad \Rightarrow \quad I_S \cdot R = (\mu - 1) \cdot U_2 \quad \Rightarrow \quad nI_1 \cdot R = (\mu - 1) \cdot U_2 \quad \Rightarrow \quad I_1 = -\frac{1-\mu}{n \cdot R} \cdot U_2$$

$$y_{12} = -\left.\frac{I_1}{U_2}\right|_{U_1=0} = \frac{1-\mu}{n \cdot R}$$

$$U_2 = I_S \cdot R - I_2 \cdot R \quad \Rightarrow \quad U_2 = \frac{\mu - 1}{R} \cdot U_2 \cdot R - I_2 \cdot R \quad \Rightarrow \quad y_{22} = -\left.\frac{I_2}{U_2}\right|_{U_1=0} = \frac{2-\mu}{R}$$

$U_2 = 0$



iz jednadžbi transformatora slijedi $\Rightarrow I_1 = \frac{1}{n} \cdot I_2 \quad I_2 = n \cdot I_1$

$$U_S = I_2 \cdot R \quad \Rightarrow \frac{1}{n} \cdot U_1 = n \cdot I_1 \cdot R \quad \Rightarrow \quad y_{11} = \left. \frac{I_1}{U_1} \right|_{U_2=0} = \frac{1}{n^2 R}$$

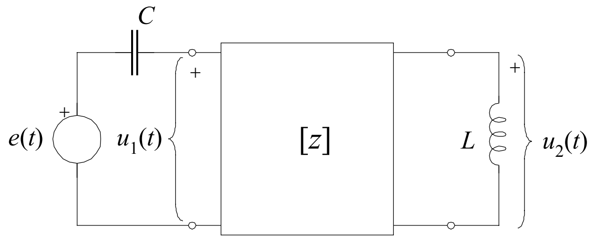
$$y_{21} = \left. \frac{I_2}{U_1} \right|_{U_2=0} = \frac{n \cdot I_1}{U_1} = n \cdot \frac{1}{n^2 R} = \frac{1}{nR}$$

$$[y] = \begin{bmatrix} \frac{1}{n^2 R} & -\frac{1-\mu}{nR} \\ \frac{1}{nR} & -\frac{2-\mu}{R} \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

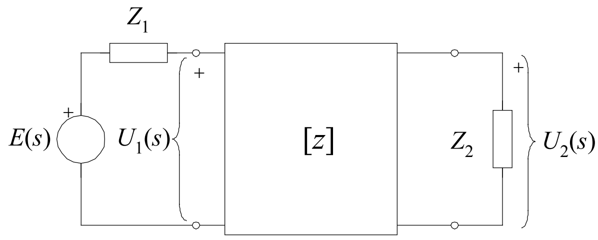
$\frac{1-\mu}{nR} = \frac{1}{nR} \Rightarrow \qquad \mu = 0$ za recipročnost

$\frac{1}{n^2 R} = \frac{2-\mu}{R} \Rightarrow \qquad n = \frac{1}{\sqrt{2-\mu}}$ za simetričnost

6. Naći naponsku prijenosnu funkciju $T(s)=U_2(s)/E(s)$ četveropola na slici. Odrediti napon $u_2(t)$ na izlazu četveropola ako je zadano $e(t)=S(t)$, $C=1$, $L=2$.



$$[\mathbf{z}]=\begin{bmatrix} 2s+1 & -2s \\ 2s & -2s \end{bmatrix}$$



Rješenje:

Prijenosna funkcija: $T(s)=\dfrac{U_2(s)}{E(s)}$

Jednadžbe četveropola

$$U_1 = z_{11}I_1 - z_{12}I_2$$

$$U_2 = z_{11}I_1 - z_{12}I_2$$

$$I_1(s)=\frac{E(s)-U_1(s)}{Z_1} \quad ; \quad I_2(s)=\frac{U_2(s)}{Z_2}$$

$$T(s)=\frac{U_2(s)}{E(s)}=\frac{Z_2\, z_{21}}{(Z_1+z_{11})(Z_2+z_{22})-(z_{12}\, z_{21})}$$

$$T(s)=\frac{s^2}{s^2+s+1}$$

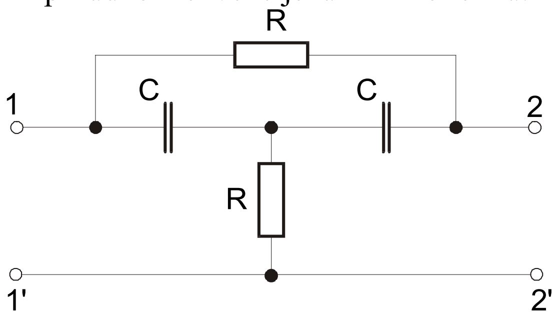
$$U_2(s)=T(s)\cdot E(s)=\frac{s^2}{s^2+s+1}\cdot\frac{1}{s}=\frac{s}{s^2+s+1}$$

$$U_2(s)=\frac{s}{s^2+s+1} \qquad s_{p1,2}=\frac{-1\pm\sqrt{1-4}}{2}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$

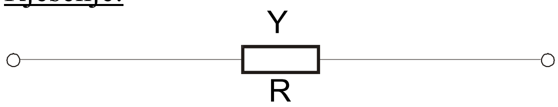
$$U_2(s)=\frac{s+\frac{1}{2}-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}-\frac{1}{2}\frac{2}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

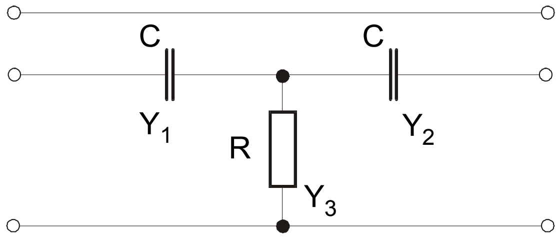
$$u_2(t)=e^{-\frac{1}{2}t}\left(\cos\frac{\sqrt{3}}{2}t-\frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right)\cdot S(t)$$

7. Za četveropol prikazan slikom odrediti matricu [**y**]-parametara i prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$, za prazni hod na izlazu. Nacrtati raspored polova i nula u kompleksnoj s-ravnini i konstruirati amplitudno-frekvencijsku karakteristiku. Zadano je R=1, C=1.



Rješenje:





$$[\mathbf{y}]^{I}=\begin{bmatrix}\frac{1}{R} & -\frac{1}{R}\\ \frac{1}{R} & -\frac{1}{R}\end{bmatrix}$$

$$[\mathbf{y}]^{I}=\begin{bmatrix}Y & -Y\\ Y & -Y\end{bmatrix}$$

$$[\mathbf{y}]^{II}=\begin{bmatrix}\frac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3} & -\frac{Y_1Y_2}{Y_1+Y_2+Y_3}\\ \frac{Y_1Y_2}{Y_1+Y_2+Y_3} & -\frac{Y_2(Y_1+Y_3)}{Y_1+Y_2+Y_3}\end{bmatrix}$$

$$Y_1+Y_2+Y_3=2sC+\frac{1}{R}$$

$$Y_1(Y_2+Y_3)=sC\left(sC+\frac{1}{R}\right)=s^2C^2+\frac{sC}{R}$$

$$Y_1Y_2=s^2C^2$$

$$[\mathbf{y}]^{II}=\begin{bmatrix}\frac{sC(sRC+1)}{2sRC+1} & -\frac{s^2C^2R}{2sRC+1}\\ \frac{s^2C^2R}{2sRC+1} & -\frac{sC(sRC+1)}{2sRC+1}\end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{y}]^{I} + [\mathbf{y}]^{II} = \begin{bmatrix} \dfrac{sC(sRC+1)}{2sRC+1} + \dfrac{1}{R} & -\dfrac{s^2C^2R}{2sRC+1} - \dfrac{1}{R} \\ \dfrac{s^2C^2R}{2sRC+1} + \dfrac{1}{R} & -\left[\dfrac{sC(sRC+1)}{2sRC+1} + \dfrac{1}{R}\right] \end{bmatrix} =$$

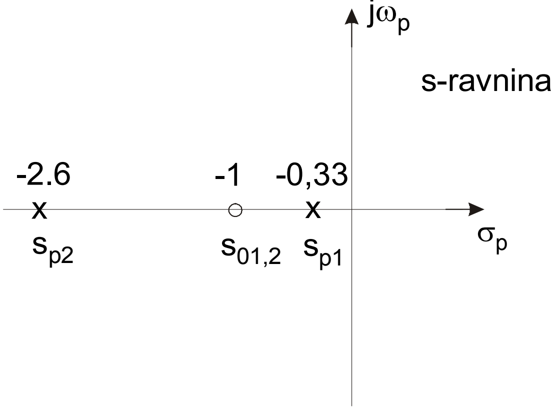
$$= \frac{1}{2s+1}\begin{bmatrix} s^2+3s+1 & -(s^2+2s)+1 \\ s^2+2s+1 & -(s^2+3s+1) \end{bmatrix}$$

$$T(s) = \frac{y_{21}}{y_{22}} = \frac{U_2}{U_1} = \frac{s^2+2s+1}{s^2+3s+1}$$

$$\text{nule}: \quad (s+1)^2 = 0 \quad S_{0_{1,2}} = -1$$

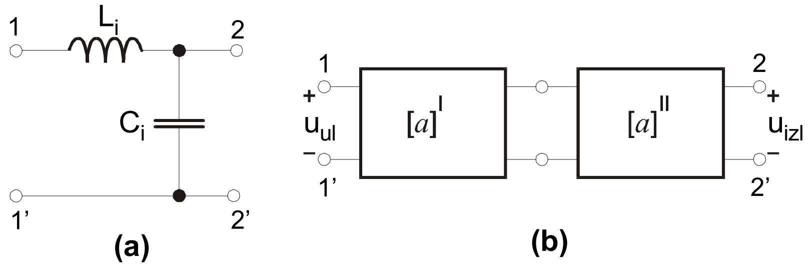
$$\text{polovi}: \quad s^2+3s+1 = 0$$

$$S_{P_{12}} = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2} = \begin{cases} -2.618 \\ -0.382 \end{cases}$$



$$|T(j\omega)| = \frac{\sqrt{4\omega^2 - (1-\omega^2)^2}}{\sqrt{9\omega^2 + (1-\omega^2)^2}}$$

8. Za četveropol na slici **(a)** naći matricu prijenosnih parametara [a]. Za kaskadu dva takva četveropola, koja je prikazana na slici **(b),** naći ukupnu matricu prijenosnih parametara [a]. Pomoću matrice prijenosnih parametara izračunati naponsku prijenosnu funkciju kaskade $T(s)=U_{iz}(s)/U_{ul}(s)$, ako je na izlazu otpornik R=1. Zadano je $C_i=L_i=1$ (i=1, 2).



Rješenje:

prijenosne jednadžbe četeveropola

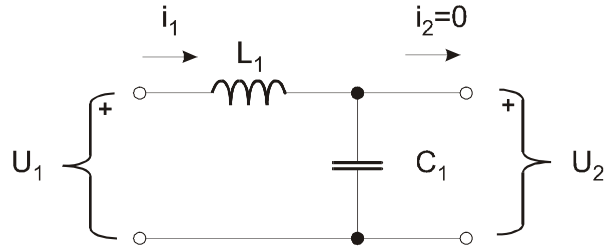
(1) $U_1 = A \cdot U_2 + B \cdot I_2$

(2) $I_1 = C \cdot U_2 + D \cdot I_2$

iz njih slijede prijenosni parametri

$$A = \frac{U_1}{U_2}\Big|_{I_2=0};\quad B = \frac{U_1}{I_2}\Big|_{U_2=0};\quad C = \frac{I_1}{U_2}\Big|_{I_2=0};\quad D = \frac{I_1}{I_2}\Big|_{U_2=0}$$

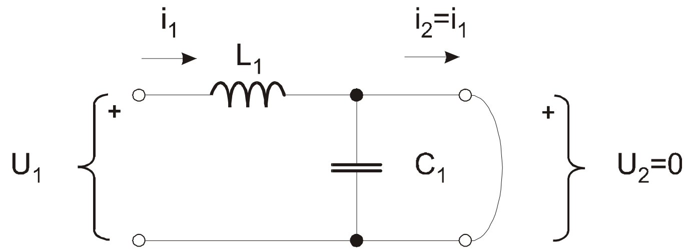
$I_2 = 0$



$$\frac{U_2}{U_1} = \frac{\frac{1}{sC_1}}{sL_1 + \frac{1}{sC_1}} = \frac{1}{s^2 L_1 C_1 + 1} \Rightarrow \qquad A = \frac{U_1}{U_2} = s^2 L_1 C_1 + 1$$

$$U_2 = I_1 \cdot \frac{1}{sC_1} \Rightarrow \qquad C = \frac{I_1}{U_2} = sC_1$$

$U_2 = 0$



$$U_1 = I_1 \cdot sL_1 = I_2 \cdot sL_1 \Rightarrow \qquad B = \frac{U_1}{I_2} = sL_1$$

$$I_1 = I_2 \Rightarrow \qquad D = \frac{I_1}{I_2} = 1$$

Konačno matrica prijenosnih parametara $[a]$ glasi

$[a]^I = \begin{bmatrix} 1+s^2L_1C_1 & sL_1 \\ sC_1 & 1 \end{bmatrix}$ analogno tome slijedi i $[a]^{II} = \begin{bmatrix} 1+s^2L_2C_2 & sL_2 \\ sC_2 & 1 \end{bmatrix}$

Spoj u kaskadu ili lanac:

$$[a]=[a]^I \cdot [a]^{II} = \begin{bmatrix} 1+s^2L_1C_1 & sL_1 \\ sC_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1+s^2L_2C_2 & sL_2 \\ sC_2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+s^2L_1C_1)(1+s^2L_2C_2)+sL_1sC_2 & (1+s^2L_1C_1)sL_2+sL_1 \\ sC_1(1+s^2L_2C_2)+sC_2 & sC_1sL_2+1 \end{bmatrix}$$

$$[a]= \begin{bmatrix} s^4L_1L_2C_1C_2+s^2(L_1C_1+L_2C_2+L_1C_2)+1 & s^3L_1C_1L_2+s(L_1+L_2) \\ s^3L_2C_2C_1+s(C_1+C_2) & s^2L_2C_1+1 \end{bmatrix}$$

uz uvrštene vrijednosti:

$$[a]= \begin{bmatrix} s^4+3s^2+1 & s^3+2s \\ s^3+2s & s^2+1 \end{bmatrix}$$

zaključenje $Z_2 = \dfrac{U_2}{I_2}$

$$(1) \qquad U_1 = A\cdot U_2 + \frac{B}{Z_2}\cdot U_2 = \left(A+\frac{B}{Z_2}\right)\cdot U_2$$

naponska prijenosna funkcija glasi:

$T(s) = \dfrac{U_2}{U_1} = \dfrac{1}{A+\dfrac{B}{Z_2}}$; $Z_2 = R$, odnosno:

$$T(s) = \frac{1}{s^4L_1L_2C_1C_2+s^2(L_1C_1+L_2C_2+L_1C_2)+1+s^3\dfrac{L_1C_1L_2}{R}+s\dfrac{L_1+L_2}{R}}$$

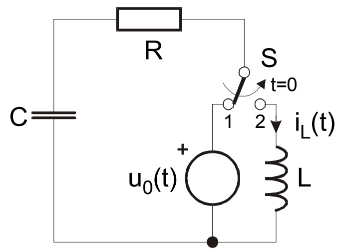
uz uvrštene vrijednosti:

$$T(s) = \frac{1}{s^4+s^3+3s^2+2s+1}$$

PRIJELAZNE POJAVE

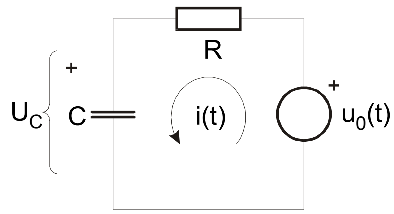
ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA

1. U trenutku t=0 sklopka S se prebaci iz položaja 1 u položaj 2. Izračunati odziv $i_L(t)$. Zadana je pobuda $u_0(t)$=2cos(2t) ($-\infty<t<+\infty$ stacionarna sinusna pobuda) i normirane vrijednosti elemenata: R=2, L=1, C=1/2.



Rješenje:

a) Za $t\leq 0$ stacionarna sinusna pobuda $\rightarrow$ fazori



$$U_0(j\omega)=I(j\omega)\cdot\left(R+\frac{1}{j\omega C}\right),\ U_C(j\omega)=I(j\omega)\cdot\frac{1}{j\omega C}$$

$$U_C(j\omega)=U_0(j\omega)\cdot\frac{1}{1+j\omega RC};\quad U_0(j\omega)=2\angle 0°$$

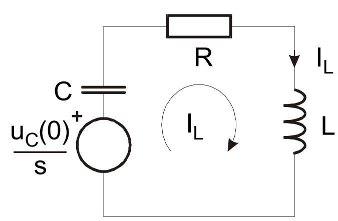
$$U_C(j\omega)=\frac{2}{1+j2\cdot 2\cdot\frac{1}{2}}=\frac{2}{1+j2}\cdot\frac{1-j2}{1-j2}=\frac{2}{5}(1-j2)=\frac{2}{\sqrt{5}}e^{-j\arctan(2)}$$

$$u_C(t)=\frac{2}{\sqrt{5}}\cos(2t-\arctan 2)\quad\Rightarrow\quad u_C(0)=\frac{2}{\sqrt{5}}\cos(\arctan 2)$$

$$\tan x=2,\quad \cos x=?\ \tan x=\frac{\sin x}{\cos x}=\frac{\sqrt{1-\cos^2 x}}{\cos x}\Rightarrow \tan^2 x=\frac{1-\cos^2 x}{\cos^2 x}\Rightarrow \cos^2 x(1+\tan^2 x)=1$$

$$\cos x=\frac{1}{\sqrt{1+\tan^2 x}},\ \cos x=\frac{1}{\sqrt{1+4}}=\frac{1}{\sqrt{5}},\Rightarrow\ u_C(0)=\frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}=\frac{2}{5}=0.4$$

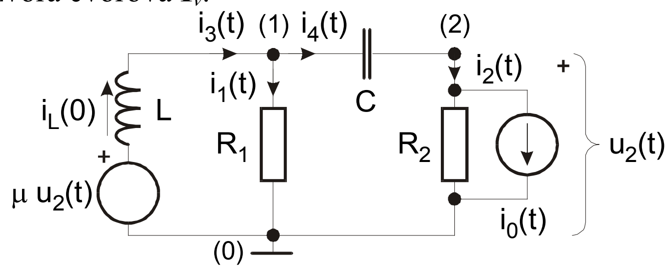
b) Za t>0 Laplaceova transformacija



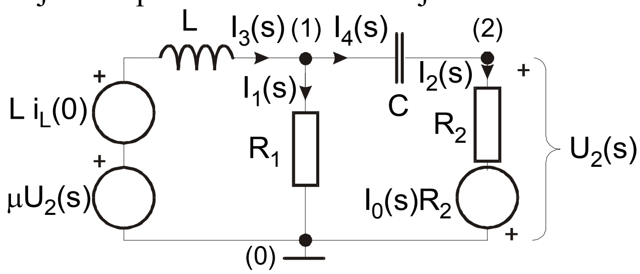
$$I_L(s)=\frac{\frac{u_C(0)}{s}}{R+sL+\frac{1}{sC}}=\frac{Cu_C(0)}{s^2LC+sRC+1}\quad\Rightarrow\quad I_L(s)=\frac{\frac{1}{2}\cdot 0.4}{\frac{s^2}{2}+s+1}=\frac{0.4}{s^2+2s+2}$$

$$I_L(s)=0.4\cdot\frac{1}{(s+1)^2+1}\quad\Rightarrow\quad \underline{i_L(t)=0.4\cdot e^{-t}\sin t\cdot S(t)}$$

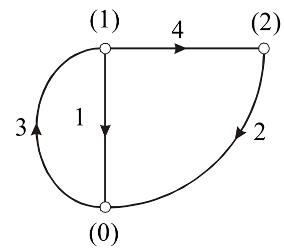
2. Zadan je električni krug prema slici. Poštujući oznake grana i čvorišta nacrtati pripadni orijentirani graf te ispisati reduciranu matricu incidencija **A**. Napisati naponsko-strujne jednadžbe grana u matričnom obliku te ispisati matricu admitancija grana $\mathbf{Y}_b$ i vektor strujnih izvora grana $\mathbf{I}_{0b}$. Topološkom analizom napisati sustav jednadžbi čvorova, odnosno odrediti matrice admitancija čvorova $\mathbf{Y}_v$ i vektor izvora čvorova $\mathbf{I}_v$.



Rješenje: Uz primjenu Laplaceove transformacije :



Električni krug



orijentirani graf

Matrica incidencija: $\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

Naponsko strujne relacije grana (prvi oblik: naponi grana su izraženi pomoću struja grana):

$$\begin{aligned} U_1 &= I_1 R_1 \\ U_2 &= I_2 R_2 - I_0(s) R_2 \\ U_3 &= I_3 sL - \mu U_2 - L i_L(0) \\ U_4 &= I_4 \frac{1}{sC} \end{aligned} \quad \Rightarrow \quad \begin{aligned} U_1 &= I_1 R_1 \\ U_2 &= I_2 R_2 - I_0(s) R_2 \\ U_3 &= -\mu I_2 R_2 + I_3 sL + \mu I_0(s) R_2 - L i_L(0) \\ U_4 &= I_4 \frac{1}{sC} \end{aligned}$$

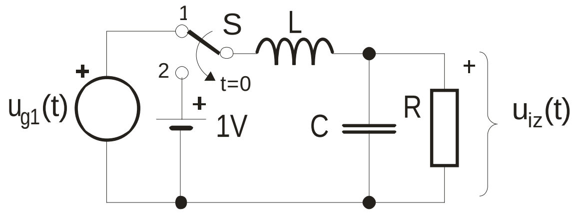
Naponsko-strujne relacije grana u matričnom obliku:

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & -\mu R_2 & sL & 0 \\ 0 & 0 & 0 & \frac{1}{sC} \end{bmatrix}}_{\mathbf{Z}_b} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} 0 \\ -I_0 R_2 \\ \mu I_0(s) R_2 - L i_L(0) \\ 0 \end{bmatrix}}_{\mathbf{U}_{0b}}$$

PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA 2010-2011

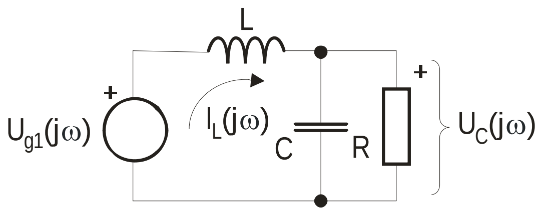
Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za električni krug prikazan slikom se u trenutku t=0 prebaci sklopka S iz položaja 1 u 2. Zadane su normalizirane vrijednosti elemenata: R=1, L=1, C=2, $u_{g1}(t)$=2sin(t); $-\infty<t<\infty$ (sinusno stacionarno stanje) i napon baterije $u_{g2}(t)$=1V (istosmjerni izvor). Odrediti za t<0: a) fazore napona na kapacitetu C i struje kroz induktivitet L; b) valne oblike napona na kapacitetu $u_C(t)$ i struje kroz induktivitet $i_L(t)$; c) početne uvjete $u_C(0)$ i $i_L(0)$. Odrediti za $t\geq 0$: d) napon na izlazu $U_{iz}(s)$; e) valni oblik napona $u_{iz}(t)$.



Rješenje:

a) za t<0 fazori struja i napona



$$I(j\omega)=\frac{U_{g1}(j\omega)}{j\omega L+\cfrac{1}{j\omega C+\cfrac{1}{R}}};\quad U_C(j\omega)=I(j\omega)\cdot\frac{1}{j\omega C+\cfrac{1}{R}}=\frac{U_{g1}(j\omega)}{j\omega L+\cfrac{1}{j\omega C+\cfrac{1}{R}}}\cdot\frac{1}{j\omega C+\cfrac{1}{R}}$$

$$U_C(j\omega)=\frac{U_{g1}(j\omega)}{j\omega L\left(j\omega C+\cfrac{1}{R}\right)+1}=\frac{U_{g1}(j\omega)}{-\omega^2LC+j\omega\cfrac{L}{R}+1}$$

Uz uvrštene vrijednosti elemenata:

$$I(j\omega)=\frac{\left(j\omega C+\cfrac{1}{R}\right)U_{g1}(j\omega)}{j\omega L\left(j\omega C+\cfrac{1}{R}\right)+1}=\frac{\left(j\omega C+\cfrac{1}{R}\right)U_{g1}(j\omega)}{\left(1-\omega^2LC\right)+j\omega\cfrac{L}{R}}=\frac{(1+2j)\cdot 2\angle 0^\circ}{-1+j};$$

$$I(j\omega)=\frac{2(1+2j)}{-1+j}\cdot\frac{-1-j}{-1-j}=2\frac{-1-j-2j+2}{1+1}=1-3j=\underline{\sqrt{10}\cdot e^{-j\arctan(3)^\circ}}$$

$$U_C(j\omega)=\frac{U_{g1}(j\omega)}{\left(1-\omega^2LC\right)+j\omega\cfrac{L}{R}}=\frac{2\angle 0^\circ}{-1+j};$$

$$U_C(j\omega)=\frac{2}{-1+j}\cdot\frac{-1-j}{-1-j}=2\frac{-1-j1}{1+1}=-(1+j)=\underline{-\sqrt{2}\cdot e^{j\arctan(1)^\circ}}$$ **(1 bod)**

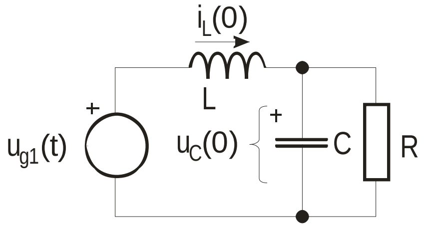
b) za t<0 valni oblici napona na kapacitetu $u_C(t)$ i struje kroz induktivitet $i_L(t)$:

$$i_L(t)=\sqrt{10}\cdot\sin(t-\arctan(3))=3.16228\cdot\sin(t-71.56^\circ)$$

$$u_C(t)=-\sqrt{2}\sin(t+\arctan(1))=-\sqrt{2}\sin(t+45^\circ)=1.41421\cdot\sin(t-135^\circ)$$ **(1 bod)**

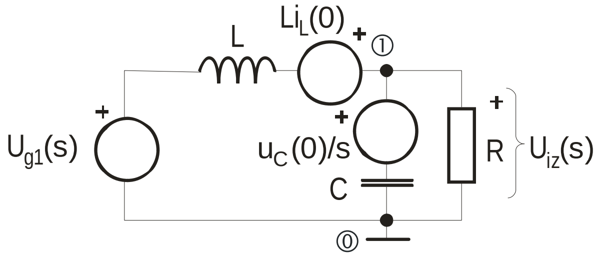
c) za t=0 početni naponi i struje $u_C(0)$ i $i_L(0)$:

$$i_L(0)=\sqrt{10}\cdot\sin(-71.56^\circ)=-3[\mathrm{A}];\qquad u_C(0)=-\sqrt{2}\sin(45^\circ)=-1[\mathrm{V}]$$ **(1 bod)**



d) za $t \geq 0$ Laplaceova transformacija

Uz poznate početne uvjete $i_L(0)=-3$ i $u_C(0)=-1$, te pobudu $u_{g1}(t)=S(t)$ (baterija), (za $t \geq 0$)



Jednadžba za čvor (1): $U_{iz}(s)\left(\frac{1}{sL}+sC+\frac{1}{R}\right)=\frac{U_{g1}(s)}{sL}+\frac{i_L(0)}{s}+Cu_C(0)$

Uz uvrštene vrijednosti elemenata:

$$U_{iz}(s)=\frac{\frac{U_{g1}(s)}{sL}+\frac{i_L(0)}{s}+Cu_C(0)}{\left(\frac{1}{sL}+sC+\frac{1}{R}\right)}=\frac{\frac{1}{s^2}-\frac{3}{s}-2}{\left(\frac{1}{s}+2s+1\right)}=\frac{\frac{1}{s}-3-2s}{s\left(\frac{1}{s}+2s+1\right)}=\frac{\frac{1}{s}-3-2s}{\left(1+s+2s^2\right)}$$ **(1 bod)**

e) valni oblika napona $u_{iz}(t)$:

$$\Rightarrow U_{iz}(s)=\frac{1}{2}\cdot\frac{1/s-3-2s}{\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}=\frac{1}{2}\cdot\frac{1}{s\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}-\frac{1}{2}\cdot\frac{3+2s}{\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}$$

Rastav na parcijalne razlomke prvog pribrojnika:

$$\frac{1}{s\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}=\frac{A}{s}+\frac{Bs+C}{\frac{1}{2}+\frac{s}{2}+s^2}=\frac{\frac{A}{2}+\frac{A}{2}s+As^2+Bs^2+Cs}{s\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}=\frac{\frac{A}{2}+\left(\frac{A}{2}+C\right)s+(A+B)s^2}{s\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}$$

(1) $A+B=0 \Rightarrow B=-A=-2$

(2) $A/2+C=0 \Rightarrow C=-A/2=-1$

(3) $A/2=1 \Rightarrow A=2$

Sređivanje: $U_{iz}(s)=\frac{1}{2}\cdot\frac{2}{s}+\frac{1}{2}\cdot\frac{-2s-1}{\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}+\frac{1}{2}\cdot\frac{-3-2s}{\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}=\frac{1}{s}-2\cdot\frac{s+1}{\left(\frac{1}{2}+\frac{s}{2}+s^2\right)}$

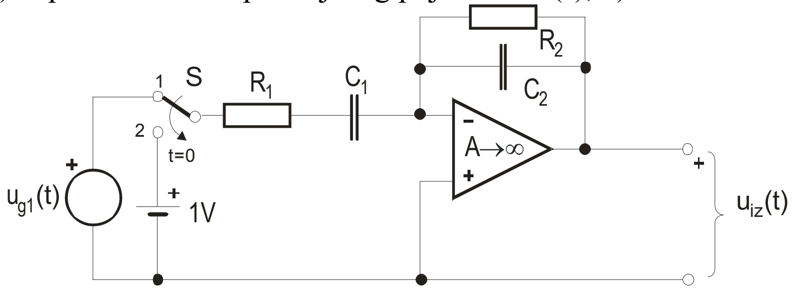
$$U_{iz}(s)=\frac{1}{s}-2\cdot\frac{s+\frac{1}{4}+\frac{3}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}=\frac{1}{s}-2\cdot\frac{s+\frac{1}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}-2\cdot\frac{3}{4}\cdot\frac{1}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$

$$\Rightarrow u_{iz}(t)=S(t)-2e^{-\frac{t}{4}}\left[\cos\left(\frac{\sqrt{7}}{4}\right)+\frac{3}{\sqrt{7}}\sin\left(\frac{\sqrt{7}}{4}\right)\right]\cdot S(t)$$ **(1 bod)**

ZAVRŠNI ISPIT IZ PREDMETA ELEKTRIČNI KRUGOVI 2010/11

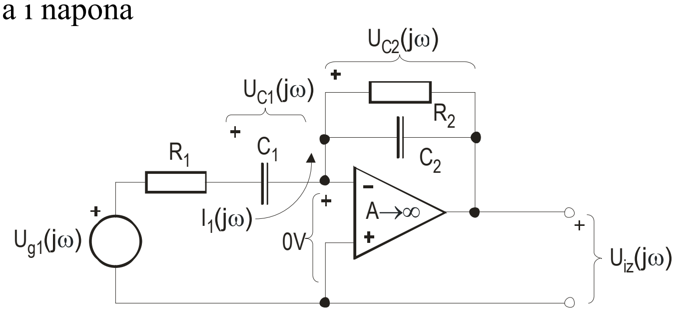
Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za električni krug prikazan slikom se u trenutku t=0 prebaci sklopka S iz položaja 1 u 2 uzrokujući prijelaznu pojavu. Zadane su normalizirane vrijednosti elemenata: R_1=1, R_2=1, C_1=1/2, C_2=1, $u_{g1}(t)$=10sin(2t); –∞<t<∞ (sinusoidalno stacionarno stanje) i napon baterije $u_{g2}(t)$=1V (istosmjerni izvor). Odrediti za t<0: a) fazore napona na kapacitetima C_1 i C_2; b) valne oblike napona na kapacitetima $u_{C1}(t)$ i $u_{C2}(t)$; c) početne napone $u_{C1}(0)$ i $u_{C2}(0)$. Odrediti za t≥0: d) napon na izlazu operacijskog pojačala $U_{iz}(s)$; e) valni oblika napona $u_{iz}(t)$.



Rješenje:

a) za t<0 fazori struja i napona



$$I_1(j\omega)=\frac{U_{g1}(j\omega)}{R_1+\dfrac{1}{j\omega C_1}};\quad U_{C1}(j\omega)=I_1(j\omega)\cdot\frac{1}{j\omega C_1}=\frac{U_{g1}(j\omega)}{1+j\omega R_1C_1}$$

$$U_{C2}(j\omega)=-U_{iz}(j\omega)=\frac{I_1(j\omega)}{j\omega C_2+\dfrac{1}{R_2}}=\frac{U_{g1}(j\omega)}{\left(R_1+\dfrac{1}{j\omega C_1}\right)\left(j\omega C_2+\dfrac{1}{R_2}\right)}$$

Uz uvrštene vrijednosti elemenata:

$$U_{C1}(j\omega)=\frac{U_{g1}(j\omega)}{1+j\omega R_1C_1}=\frac{10\angle 0^\circ}{1+j2\cdot 1\cdot 1/2}=\frac{10}{1+j}=\frac{10}{1+j}\cdot\frac{1-j}{1-j}=\frac{10}{2}(1-j)=\underline{5(1-j)=5\sqrt{2}e^{-j45^\circ}}$$

$$U_{C2}(j\omega)=\frac{U_{g1}(j\omega)}{\left(R_1+\dfrac{1}{j\omega C_1}\right)\left(j\omega C_2+\dfrac{1}{R_2}\right)}=\frac{10\angle 0^\circ}{\left(1+\dfrac{1}{j2\cdot 1/2}\right)(j2\cdot 1+1)}=\frac{10}{(1-j)(1+2j)}$$

$$U_{C2}(j\omega)=\frac{10}{2\cdot 5}(1+j)(1-2j)=\underline{(1+j)(1-2j)=3-j}=\sqrt{2}e^{j45^\circ}\sqrt{5}e^{-j\arctan(2)}=$$

(1 bod)

$$=\sqrt{10}e^{j(45^\circ-63.435^\circ)}=\underline{\sqrt{10}e^{-j18.435^\circ}}\sqrt{2}e^{j45^\circ}\sqrt{5}e^{-j\arctan(2)}=\sqrt{10}e^{j(45^\circ-63.435^\circ)}=\underline{\sqrt{10}e^{-j18.435^\circ}}$$

b) za t<0 valni oblici napona na kapacitetima $u_{C1}(t)$ i $u_{C2}(t)$:

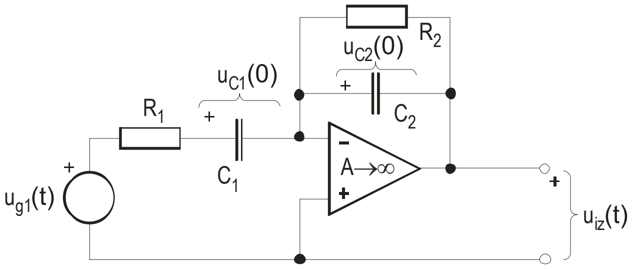
$$\underline{u_{C1}(t)=5\sqrt{2}\sin(2t-45^\circ)=7.071068\sin(2t-45^\circ)}$$

$$\underline{u_{C2}(t)=\sqrt{10}\sin(2t-18.435^\circ)=3.162278\sin(2t-18.435^\circ)}$$ **(1 bod)**

c) za t=0 početni naponi $u_{C1}(0)$ i $u_{C2}(0)$:

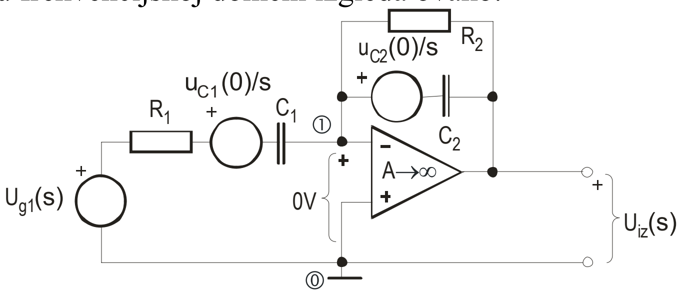
$$u_{C1}(0)=5\sqrt{2}\sin(-45°)=-5\sqrt{2}\frac{\sqrt{2}}{2}=\underline{-5[\mathrm{V}]}$$

$$u_{C2}(0)=\sqrt{10}\sin(-18.435°)=\underline{-1[\mathrm{V}]}$$ **(1 bod)**



d) za t≥0 Laplaceova transformacija

Uz poznate početne uvjete $u_{C1}(0)=-5$ i $u_{C2}(0)=-1$, te pobudu $u_{g1}(t)=S(t)$ (baterija), (za t≥0) električni krug u frekvencijskoj domeni izgleda ovako:



Jednadžba za čvor (1) glasi:

$$(1)\quad U_1(s)\left(\frac{1}{R_1+\frac{1}{sC_1}}+sC_2+\frac{1}{R_2}\right)=\frac{U_{g1}(s)-\frac{u_{C1}(0)}{s}}{R_1+\frac{1}{sC_1}}+\frac{\frac{u_{C2}(0)}{s}}{\frac{1}{sC_2}}+U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)$$

Zbog virtualnog kratkog spoja je $U_1(s)$=0 pa vrijedi:

$$U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)=-\frac{U_{g1}(s)-\frac{u_{C1}(0)}{s}}{R_1+\frac{1}{sC_1}}-C_2u_{C2}(0)\,;\; U_{iz}(s)=-\frac{U_{g1}(s)-\frac{u_{C1}(0)}{s}}{\left(R_1+\frac{1}{sC_1}\right)\left(sC_2+\frac{1}{R_2}\right)}-\frac{C_2u_{C2}(0)}{\left(sC_2+\frac{1}{R_2}\right)}$$

Uz uvrštene vrijednosti elemenata:

$$U_{iz}(s)=-\frac{\frac{1}{s}-\frac{-5}{s}}{(1+2/s)(s+1)}-\frac{1\cdot(-1)}{(s+1)}=-\frac{6}{(s+2)(s+1)}+\frac{1}{s+1}\Rightarrow \underline{U_{iz}(s)=\frac{s-4}{(s+2)(s+1)}}$$ **(1 bod)**

e) valni oblika napona $u_{iz}(t)$: Rastav na parcijalne razlomke:

$$\frac{-6}{(s+2)(s+1)}=\frac{A}{s+2}+\frac{B}{s+1}=\frac{As+A+Bs+2B}{(s+2)(s+1)}=\frac{(A+B)s+(A+2B)}{(s+2)(s+1)}=\frac{6}{s+2}+\frac{-6}{s+1}$$

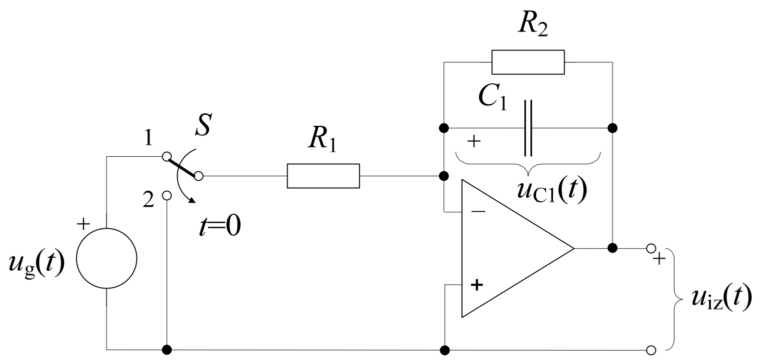
$$(1)\quad A+B=0\Rightarrow A=-B=6$$

$$\underline{(2)\quad A+2B=-6\Rightarrow -B+2B=B=-6}$$

$$U_{iz}(s)=-\frac{6}{(s+2)(s+1)}+\frac{1}{s+1}=\frac{6}{s+2}-\frac{6}{s+1}+\frac{1}{s+1}=\frac{6}{s+2}-\frac{5}{s+1}$$

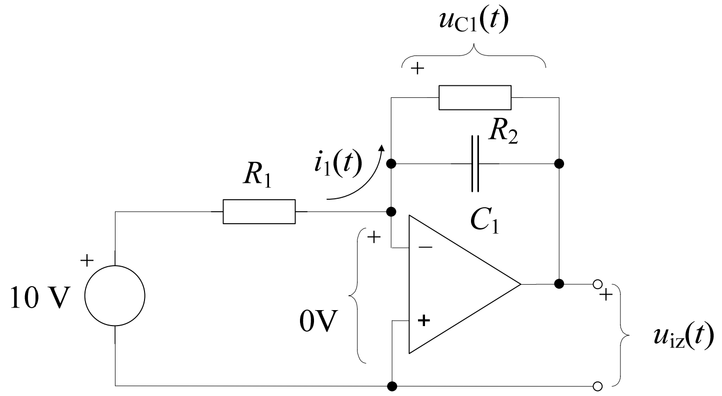
$$\Rightarrow \underline{u_{iz}(t)=(6e^{-2t}-5e^{-t})S(t)}$$ **(1 bod)**

3. Za električni krug prikazan slikom se u trenutku t=0 prebaci sklopka S iz položaja 1 u 2 uzrokujući prijelaznu pojavu. Zadane su normalizirane vrijednosti elemenata: R_1=1, R_2=1/2, C_1=1, $u_g(t)$=10 V; $-\infty<t<\infty$ (istosmjerni napon-baterija). Odrediti za t<0: a) valni oblik napona na kapacitetu $u_{C1}(t)$; b) početni napon $u_{C1}(0)$. Odrediti za $t\geq 0$: c) napon na izlazu operacijskog pojačala $U_{iz}(s)$; d) valni oblika napona $u_{iz}(t)$. e) Ako je R_2=2 odrediti valni oblik napona $u_{iz}(t)$ za $-\infty<t<\infty$.



Rješenje:

a) za t<0 struja i napon su istosmjerne veličine (konstante). Frekvencija signala naponskog izvora $u_g(t)$=10V je nula (istosmjerni signal -> konstanta). Stoga kapacitet C_1 ima neizmjernu impedanciju, tj. kao da ne postoji.



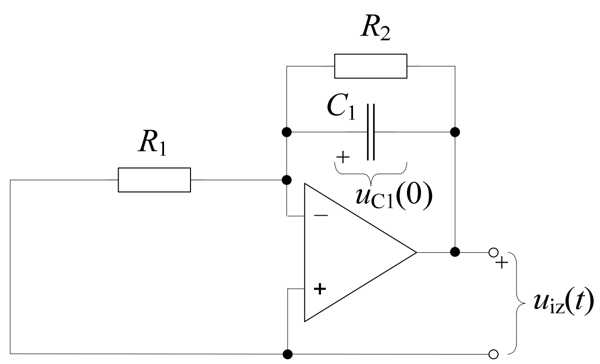
$$i_1(t)=\frac{u_g(t)}{R_1};\quad u_{C1}(t)=i_1(t)\cdot R_2=u_g(t)\cdot\frac{R_2}{R_1}$$

Uz uvrštene vrijednosti elemenata:

$$\underline{u_{C1}(t)=10\,\mathrm{V}\cdot 1\cdot\frac{1}{2}=5\,\mathrm{V};\ t<0}\ \text{(1 bod)}$$

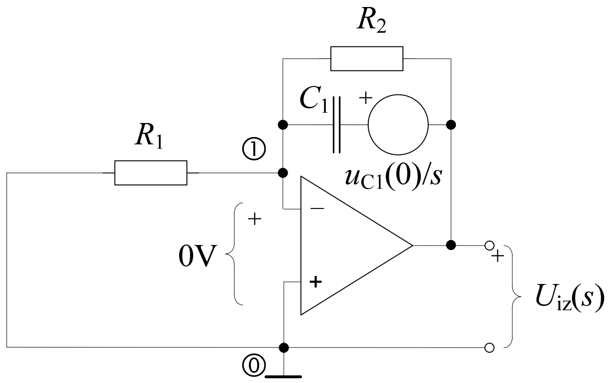
b) za t=0 početni napon $u_{C1}(0)$:

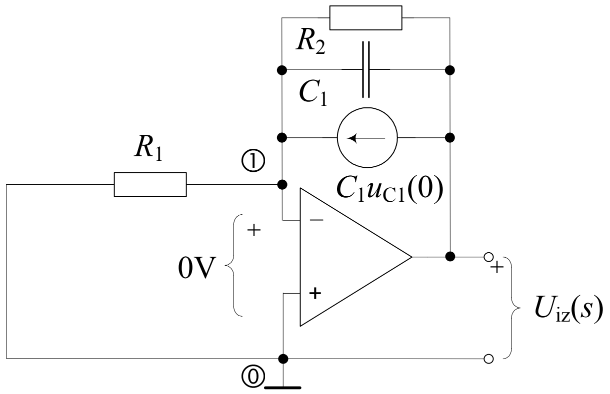
$$\underline{u_{C1}(0)=5\,\mathrm{V}}\ \text{(1 bod)}$$



c) <u>za $t \ge 0$</u> Laplaceova transformacija

Uz poznate početne uvjete $u_{C1}(0)=5$, te isključenu pobudu $u_g(t)=0$, za $t\ge 0$ električni krug u frekvencijskoj domeni izgleda ovako:





Jednadžba za čvor (1) glasi:

$$(1)\quad U_1(s)\left(\frac{1}{R_1}+sC_1+\frac{1}{R_2}\right)=C_1u_{C1}(0)+U_{iz}(s)\left(sC_1+\frac{1}{R_2}\right).$$

Zbog virtualnog kratkog spoja je $U_1(s)=0$ pa vrijedi:

$$U_{iz}(s)=-\frac{C_1u_{C1}(0)}{sC_1+\dfrac{1}{R_2}}.$$

Uz uvrštene vrijednosti elemenata:

$$\underline{U_{iz}(s)=-\frac{1\cdot 5}{s+2}=-\frac{5}{s+2}}.\quad \text{(1 bod)}$$

d) valni oblika napona $u_{iz}(t)$:

$$\Rightarrow \underline{u_{iz}(t)=-5e^{-2t}\cdot S(t)}.\quad \text{(1 bod)}$$

e) valni oblika napona $u_{iz}(t)$ za $-\infty<t<\infty$ ako je $R_2=2$:

<u>za $t<0$</u> $u_{iz}(t)=-u_{C1}(t)=-\dfrac{R_2}{R_1}\cdot u_g(t)=-\dfrac{2}{1}\cdot 10=-20\text{V}$

<u>za $t\ge 0$</u>

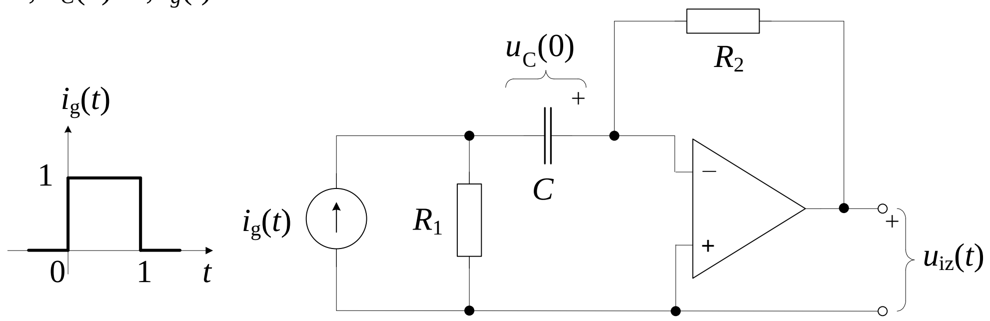
$$U_{iz}(s)=-\frac{C_1u_{C1}(0)}{sC_1+\dfrac{1}{R_2}}=-\frac{C_1u_g(0)\dfrac{R_2}{R_1}}{sC_1+\dfrac{1}{R_2}}=-\frac{\dfrac{R_2}{R_1}}{s+\dfrac{1}{C_1R_2}}\cdot u_g(0)=-\frac{\dfrac{2}{1}}{s+\dfrac{1}{2}}\cdot 10=-\frac{20}{s+1/2}$$

$$u_{iz}(t)=-20e^{-\frac{1}{2}t}\cdot S(t)$$

$$\underline{u_{iz}(t)[\text{V}]=\begin{cases}-20, & t<0\\ -20e^{-\frac{1}{2}t}, & t\ge 0\end{cases}}\quad \text{(1 bod)}$$

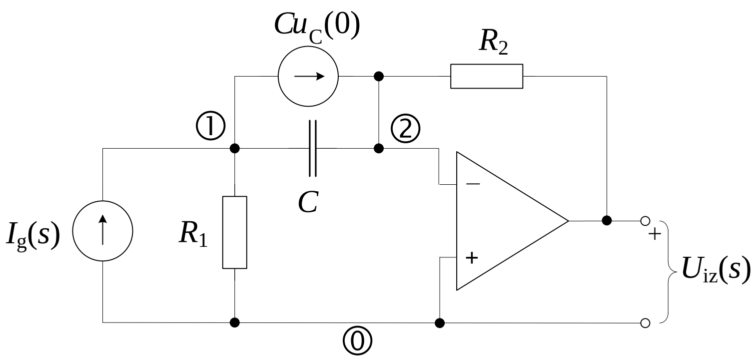
MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2012-2013 – Rješenja

1. Za električni krug prikazan slikom izračunati: a) napon na izlazu operacijskog pojačala $U_{iz}(s)$; b) valni oblik napona $u_{iz}(t)$. Zadane su normalizirane vrijednosti elemenata: R_1=1, R_2=1/2, C=2, $u_C(0)$=1, $i_g(t)$ zadan slikom.



Rješenje:

a) Jednadžbe čvorišta:



$$1)\ U_1\left(sC+\frac{1}{R_1}\right)-U_2sC=I_g(s)-Cu_C(0)$$

$$2)\ -U_1sC+U_2\left(sC+\frac{1}{R_2}\right)=\frac{U_{iz}(s)}{R_2}+Cu_C(0)$$

$$U_2=0$$

$$1)\ U_1\left(sC+\frac{1}{R_1}\right)=I_g(s)-Cu_C(0)$$

$$2)\ -U_1sC=\frac{U_{iz}(s)}{R_2}+Cu_C(0)\ \Rightarrow\ U_1=-\frac{U_{iz}(s)}{sCR_2}-\frac{u_C(0)}{s}$$ (1 bod)

$$\Rightarrow\ -\left(\frac{U_{iz}(s)}{sCR_2}+\frac{u_C(0)}{s}\right)\left(sC+\frac{1}{R_1}\right)=I_g(s)-Cu_C(0)$$

$$\Rightarrow\ U_{iz}(s)=-\frac{I_g(s)-Cu_C(0)}{sC+\frac{1}{R_1}}sCR_2-Cu_C(0)R_2$$ (1 bod)

$$i_g(t)=S(t)-S(t-1)\Rightarrow I_g(s)=\frac{1}{s}-\frac{1}{s}e^{-s}=\frac{1}{s}(1-e^{-s})$$ (1 bod)

$$U_{iz}(s) = -\frac{\frac{1}{s}(1-e^{-s})-2}{2s+1}s - 1 = \frac{2s-1}{2s+1} + \frac{e^{-s}}{2s+1} - 1 =$$

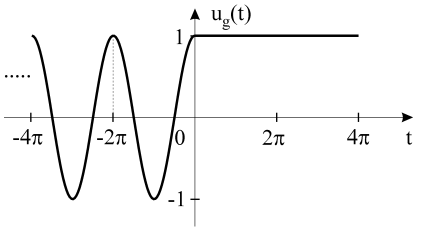
$$= \frac{2s+1-2}{2s+1} + \frac{e^{-s}}{2s+1} - 1 = 1 + \frac{-2}{2s+1} + \frac{e^{-s}}{2s+1} - 1 = \frac{-2}{2s+1} + \frac{e^{-s}}{2s+1} = -\frac{1}{s+\frac{1}{2}} + \frac{\frac{1}{2}}{s+\frac{1}{2}} \cdot e^{-s}$$

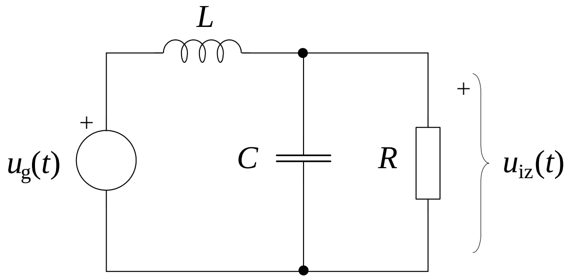
(1 bod)

$\Rightarrow \quad u_{iz}(t) = -e^{-t/2}S(t) + \frac{1}{2}e^{-(t-1)/2}S(t-1)$ (1 bod)

PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2012-2013

1. Za električni krug na slici zadane su normalizirane vrijednosti elemenata $R=1/\sqrt{2}$, $C=1$, $L=1$, te valni oblik pobude $u_g(t)$ prikazan slikom. Odrediti: a) valni oblik napona na kapacitetu $u_C(t)$ i struje kroz induktivitet $i_L(t)$ za $t<0$; b) početni napon $u_C(0)$ i struju $i_L(0)$; c) napon $U_{iz}(s)$; d) valni oblika napona $u_{iz}(t)$ za $t \geq 0$.





<u>Rješenje:</u>

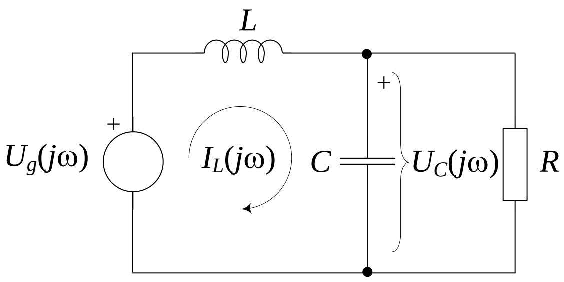
a) <u>za $t<0$</u> napon generatora je svevremenska sinusoida koja se može opisati izrazom: $u_g(t)=\cos(\omega_0 t)$; $-\infty<t<\infty$, gdje se vidi iz slike: $T=2\pi$, $f_0=1/T=1/(2\pi)$ pa je $\omega_0=2\pi f_0=1$ [rad/s]. Stoga se mogu izračunati napon $u_C(t)$ i struja $i_L(t)$ korištenjem fazorskog računa.

$$I_L(j\omega)=\frac{U_g(j\omega)}{j\omega L+\dfrac{1}{j\omega C+1/R}};\quad U_C(j\omega)=U_{iz}(j\omega)=I_L(j\omega)\cdot\frac{1}{j\omega C+\dfrac{1}{R}};$$

$$U_g(j\omega)=1\angle 0^\circ \quad (\omega_0=1\text{rad/s})$$

$$U_C(j\omega)=\frac{U_g(j\omega)}{j\omega L+\dfrac{1}{j\omega C+1/R}}\cdot\frac{1}{j\omega C+\dfrac{1}{R}}=$$

$$=\frac{U_g(j\omega)}{j\omega L\left(j\omega C+\dfrac{1}{R}\right)+1}=\frac{U_g(j\omega)}{1-\omega^2 LC+j\omega\dfrac{L}{R}}$$



$$I_L(j\omega)=\frac{U_g(j\omega)(j\omega C+1/R)}{j\omega L(j\omega C+1/R)+1}=\frac{U_g(j\omega)(j\omega C+1/R)}{1-\omega^2 LC+j\omega L/R}$$

Uz uvrštene vrijednosti elemenata na frekvenciji signala ω_0:

$$I_L(j\omega)=\frac{1\angle 0^\circ\cdot\left(j+\sqrt{2}\right)}{1-1+j\sqrt{2}}=\frac{j+\sqrt{2}}{j\sqrt{2}}=\frac{1}{\sqrt{2}}-j;\qquad U_C(j\omega)=\frac{1}{j\sqrt{2}}=-j\frac{\sqrt{2}}{2}\ \textbf{(1 bod)}$$

Za $t<0$ valni oblici struje kroz induktivitet $i_L(t)$ i napona na kapacitetu $u_C(t)$:

$$|I_L(j\omega)|=\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+1^2}=\sqrt{\frac{1}{2}+1}=\sqrt{\frac{3}{2}},$$

$$\angle I_L(j\omega)=\arctan\left(-\sqrt{2}\right)=-0.955317\text{rad}=-54.7356^\circ$$

$$\Rightarrow |U_C(j\omega)|=\frac{\sqrt{2}}{2},\quad \angle U_C(j\omega)=-\frac{\pi}{2}\text{rad}=-90^\circ$$

$$\underline{i_L(t)=\sqrt{3/2}\cos(\omega_0 t-54.7356^\circ)[A]}$$

$$\underline{u_C(t)=\sqrt{2}/2\cos(\omega_0 t-90^\circ)[V]}\ \textbf{(1 bod)}$$

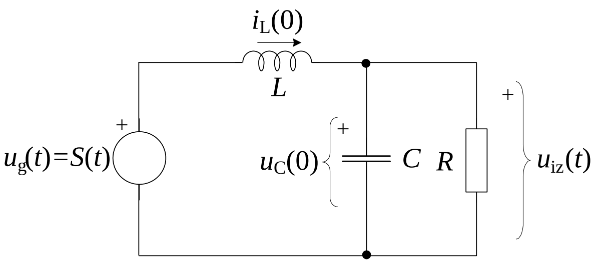
b) početni napon $u_C(0)$ i struja $i_L(0)$: u prethodne izraze uvrstimo t=0.

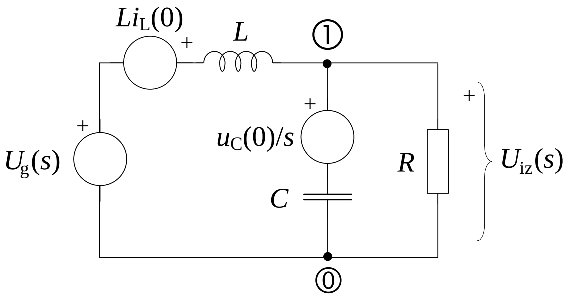
$$\underline{i_L(0) = \sqrt{3/2}\cos(-54.7356°) = 1/\sqrt{2} = \sqrt{2}/2 = 0.707[A]}$$

$$\underline{u_C(0) = \sqrt{2}/2\cos(-90°) = 0[V]}$$ **(1 bod)**

c) za $t \geq 0$ Laplaceova transformacija.

Uz poznate početne uvjete $u_C(0)$ i $i_L(0)$, te pobude $u_g(t)=S(t)$, za $t \geq 0$ električni krug u frekvencijskoj domeni izgleda ovako:





Napon čvorišta (1): $U_{iz}(s)\left(\frac{1}{sL} + sC + \frac{1}{R}\right) = \frac{U_g(s) + Li_L(0)}{sL} + Cu_C(0)$

$$\Rightarrow U_{iz}(s) = \frac{\frac{U_g(s) + Li_L(0)}{sL} + Cu_C(0)}{\left(\frac{1}{sL} + sC + \frac{1}{R}\right)} = \frac{U_g(s) + Li_L(0) + sLCu_C(0)}{s^2LC + s\frac{L}{R} + 1} = \frac{U_g(s)\frac{1}{LC} + \frac{i_L(0)}{C} + su_C(0)}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

$$U_{iz}(s) = \frac{U_g(s)\frac{1}{LC} + \frac{i_L(0)}{C}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}} = \frac{\frac{1}{s} + \frac{\sqrt{2}}{2}}{s^2 + s\sqrt{2} + 1} = \frac{s\frac{\sqrt{2}}{2} + 1}{s\left(s^2 + s\sqrt{2} + 1\right)}$$ **(1 bod)**

d) Povratak u vremensku domenu (rastav na parcijalne razlomke):

$$U_{iz}(s) = \frac{s \cdot \sqrt{2}/2 + 1}{s \cdot \left(s^2 + s\sqrt{2} + 1\right)} = \frac{A}{s} + \frac{Bs + C}{s^2 + s\sqrt{2} + 1} = \frac{As^2 + As\sqrt{2} + A + Bs^2 + Cs}{s\left(s^2 + s\sqrt{2} + 1\right)}$$

$$(A + B)s^2 + (A\sqrt{2} + C)s + A = s \cdot \sqrt{2}/2 + 1 \Rightarrow$$

$$A = 1,$$

$$A + B = 0 \Rightarrow B = -A = -1,$$

$$\underline{C + A\sqrt{2} = \sqrt{2}/2 \Rightarrow C = -\sqrt{2} + \sqrt{2}/2 = -\sqrt{2}/2}$$

Polovi:

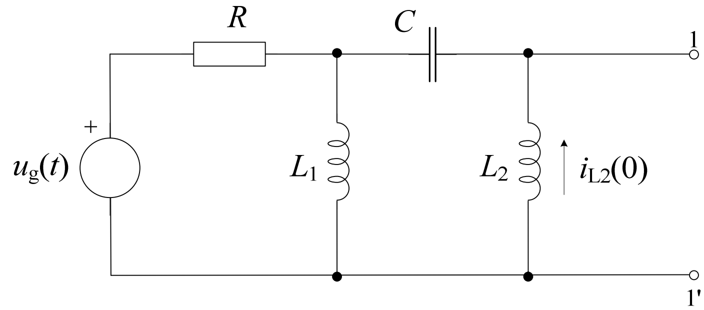
$$s^2 + s\sqrt{2} + 1 = 0 \Rightarrow s_{p1,2} = -\frac{\sqrt{2}}{2} \pm \sqrt{\frac{1}{2} - 1} = -\frac{\sqrt{2}}{2} \pm j\sqrt{\frac{1}{2}} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$$

$$s = 0 \Rightarrow s_{p3} = 0$$

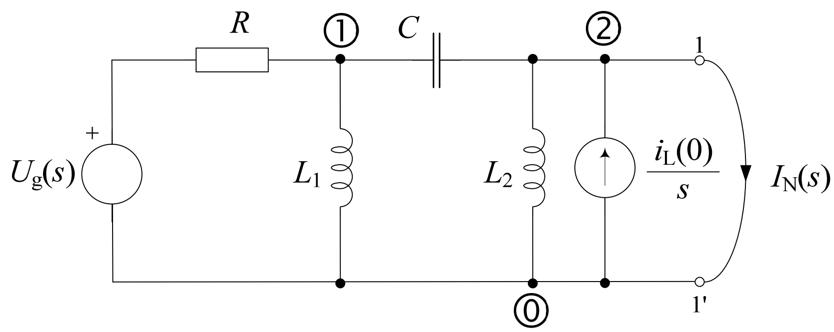
$$U_{iz}(s) = \frac{1}{s} - \frac{\left(s + \frac{\sqrt{2}}{2}\right)}{\left(s + \frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} \quad \Rightarrow \quad \underline{u_{iz}(t) = \left[1 - e^{-\frac{\sqrt{2}}{2}t}\cos\left(\frac{\sqrt{2}}{2}t\right)\right] \cdot S(t)}$$ **(1 bod)**

NORTON

2. Za električni krug na slici pomoću teorema superpozicije odrediti parametre nadomjesnog kruga po Nortonu s obzirom na priključnice 1-1': a) I_N(s); b) $Y_N(s)$. Zadane su normalizirane vrijednosti elemenata: $R = 1$, $L_1 = L_2 = 1$, $C = 2$ te $i_{L2}(0) = 1$, $u_g(t) = S(t)$. Koristiti jednadžbe čvorova.

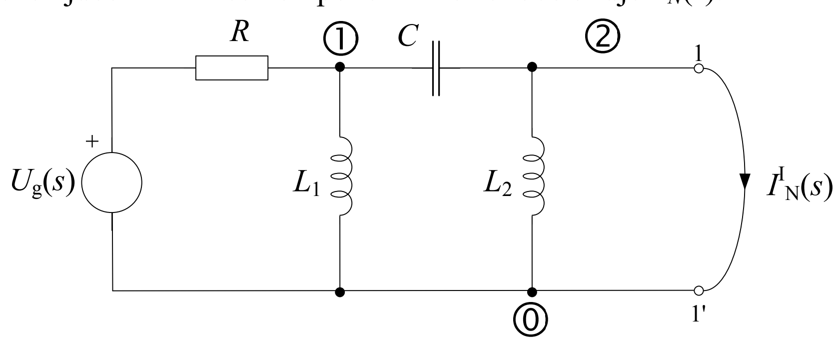


Rješenje: Primjena $\mathscr{L}$-transformacije na električni krug:



a) Nortonova struja $I_N(s)$:

a.1) početni uvjet je isključen i traži se komponenta Nortonove struje $I^I_N(s)$.



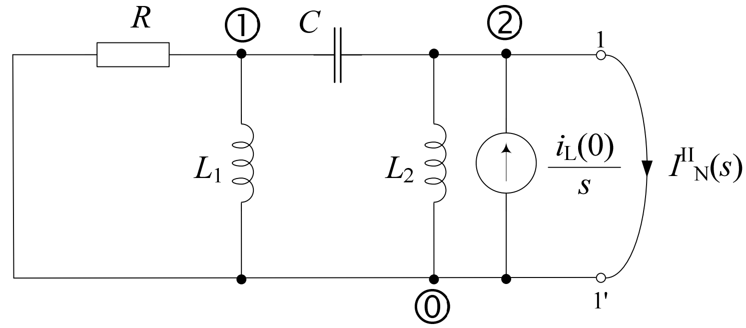
(1) $\dfrac{U_g(s)}{R} = U_1\left(\dfrac{1}{R} + \dfrac{1}{sL_1} + sC\right) - U_2 sC$

(2) $-I_N^I(s) = -U_1 sC + U_2\left(\dfrac{1}{sL_2} + sC\right)$

$U_2 = 0$

(1) $\Rightarrow U_1 = \dfrac{U_g(s)}{R\left(\dfrac{1}{R} + \dfrac{1}{sL_1} + sC\right)}$; (2) $\Rightarrow I_N^I(s) = \dfrac{U_g(s)}{R\left(\dfrac{1}{R} + \dfrac{1}{sL_1} + sC\right)} sC$

a.2) naponski izvor je isključen i traži se komponenta Nortonove struje $I^{II}{}_N(s)$.



(1) $\quad 0 = U_1\left(\frac{1}{R} + \frac{1}{sL_1} + sC\right) - U_2 sC$

(2) $\quad \frac{i_{L2}(0)}{s} - I_N^{II}(s) = -U_1 sC + U_2\left(\frac{1}{sL_2} + sC\right)$

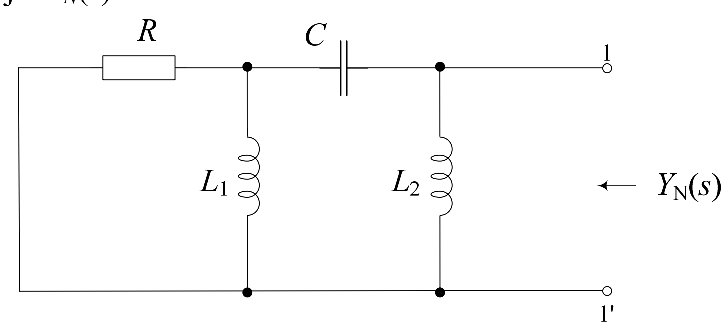
$$U_2 = 0 \Rightarrow U_1 = 0$$

(2) $\Rightarrow I_N^{II}(s) = \frac{i_{L2}(0)}{s}$

Ukupna Nortonova struja je:

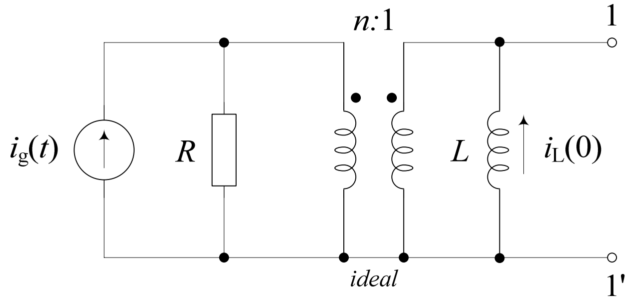
$$I_N(s) = I_N^I(s) + I_N^{II}(s) = \frac{U_g(s)}{R\left(\frac{1}{R} + \frac{1}{sL_1} + sC\right)} sC + \frac{i_{L2}(0)}{s} = \frac{\frac{1}{s}2s}{1\left(1 + \frac{1}{s} + 2s\right)} + \frac{1}{s} = \frac{4s^2 + s + 1}{s\left(2s^2 + s + 1\right)}$$

b) Nortonova admitancija $Y_N(s)$:

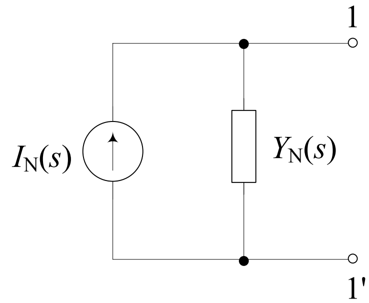


$$Y_N(s) = \frac{1}{sL_2} + \frac{1}{\frac{1}{sC} + \frac{RsL_1}{R + sL_1}} = \frac{2s^3 + 4s^2 + s + 1}{s(2s^2 + s + 1)}$$

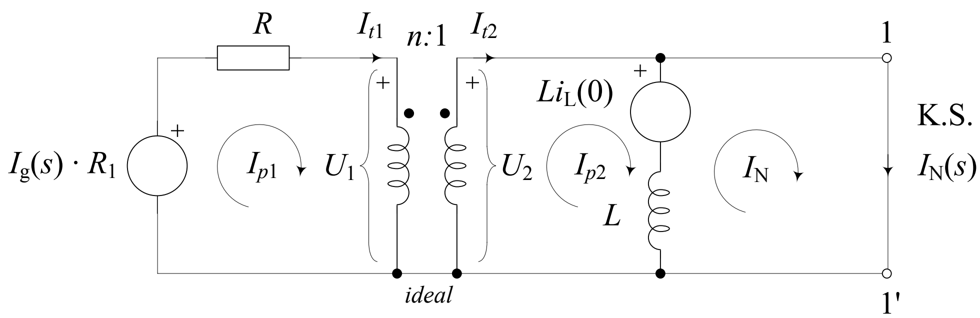
4. Za električni krug na slici izračunati parametre nadomjesnog kruga po Nortonu s obzirom na polove 1–1': $I_N(s)$ i $Y_N(s)$. Zadane su normalizirane vrijednosti elemenata: R=1, L=1, $i_L(0)$=1, n=2, $i_g(t)$=$S(t)$. (Koristiti bilo koju metodu u izračunu; preporučuje se metoda petlji.) Napisati:
a) Nortonovu struju $I_N(s)$ uz uvrštene vrijednosti elemenata;
b) Nortonovu admitanciju $Y_N(s)$ uz uvrštene vrijednosti elemenata;
c) Struju kroz otpor R.



Rješenje:



a) Nortonova struja $I_N(s)$ primjenom $\mathcal{L}$-transformacije na električni krug:



Jednadžbe idealnog transformatora:

$$U_1 = n \cdot U_2 \Rightarrow U_2 = \frac{U_1}{n}$$

$$I_{t1} = \frac{1}{n} \cdot I_{t2} \Rightarrow I_{t2} = n \cdot I_{t1}$$

1) $I_{p1}R = -U_1 + I_g R$ $\qquad I_{p1} = I_{t1}$

2) $(I_{p2} - I_N)sL + Li_L(0) = U_2$ $\qquad I_{p2} = I_{t2}$

Nakon sređivanja jednadžbe glase:

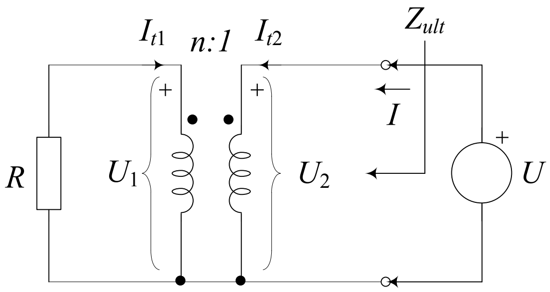
$$\left.\begin{aligned} &1)\ I_{p1}R + nU_2 = I_g R \\ &2)\ I_N = \frac{1}{sL}(I_{p2}sL + Li_L(0) - U_2) \end{aligned}\right\} \quad \left.\begin{aligned} &1)\ I_{p1}R + nU_2 = I_g R \\ &2)\ I_N = \frac{1}{sL}\left[nI_{p1}sL + Li_L(0) - U_2\right] \end{aligned}\right\}$$

$U_2=0$

$$\left.\begin{aligned} &1)\ I_{p1}R = I_g R \\ &\underline{2)\ I_N = \frac{1}{sL}\left[nI_{p1}sL + Li_L(0)\right]} \end{aligned}\right\} \quad I_N = \frac{1}{sL}\left[nI_g sL + Li_L(0)\right]$$

$$I_N = \frac{1}{s}\left[2\frac{1}{s}s + 1\right] = \frac{3}{s}$$ (1 bod)

b) Nortonova admitancija $Y_N(s)$:
Najjednostavnije je izračunati ulaznu impedanciju u transformator zaključen s R. Označimo je s Z_{ult}.



$$Z_{ult} = \frac{1}{n^2}R$$

$$I_{t1} = -\frac{U_1}{R_1} \Rightarrow \frac{U_1}{I_{t1}} = -R$$

$$U = U_2$$

$$I = I_{t2}$$

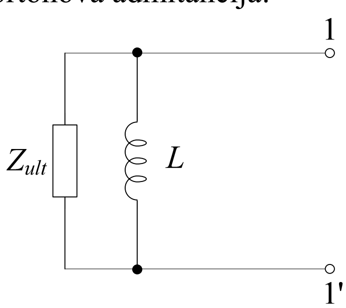
Jednadžbe transformatora su:

$$U_1 = n \cdot U_2 \Rightarrow U_2 = \frac{U_1}{n}$$

$$\underline{I_{t1} = -\frac{1}{n} \cdot I_{t2} \Rightarrow I_{t2} = -n \cdot I_{t1}}$$

$$Z_{ult} = \frac{U}{I} = \frac{U_2}{I_{t2}} = \frac{\frac{U_1}{n}}{-n \cdot I_{t1}} = -\frac{\frac{U_1}{I_{t1}}}{n^2} = -\frac{-R}{n^2} = \frac{R}{n^2}$$

Tada je Nortonova admitancija:

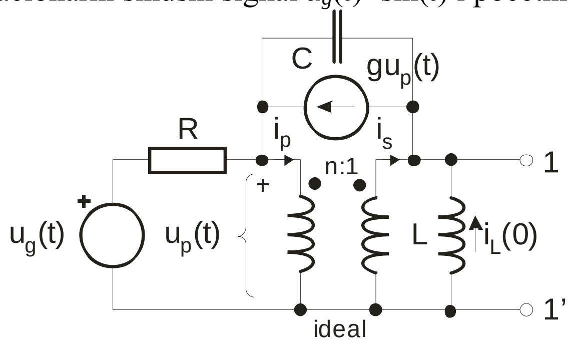


$$\leftarrow Y_N(s) = \frac{1}{Z_{ult}} + \frac{1}{sL}$$

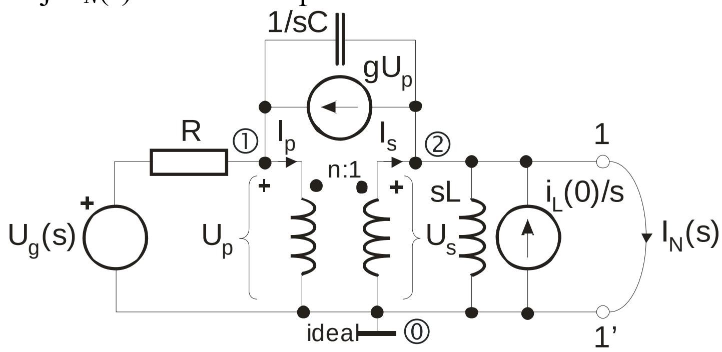
$$Y_N(s) = \frac{n^2}{R} + \frac{1}{sL} = 4 + \frac{1}{s}$$ (1 bod)

c) Za izračun Nortonove struje $I_N(s)$, polovi dvopola 1–1' trebaju biti kratko spojeni.
Struja kroz R je nula jer sva struja iz izvora I_g teče kroz primar idealnog transformatora, odn. $I_g=I_{t1}$. To je zato jer kratki spoj na sekundaru transformatora uzrokuje kratki spoj na primaru transformatora, a cjelokupna struja uvijek teče kroz kratki spoj, a ništa ne teče kroz R. (1 bod)

3. Za mrežu prikazanu slikom odrediti nadomjesne parametre mreže po Nortonu $I_N(s)$ i $Y_N(s)$ s obzirom na polove 1–1'. Koristiti metodu napona čvorova u proračunu. Zadane su normalizirane vrijednosti elemenata: L=1, C=1, R=1, g=2, n=2, $i_L(0)$=1, $u_C(0)$=0 te izvor $u_g(t)=S(t)$. Napisati: a) Jednadžbe čvorova za izračun struje $I_N(s)$; b) Jednadžbe čvorova za izračun $Y_N(s)$. Uz uvrštene vrijednosti elemenata: c) Nortonovu struju $I_N(s)$; d) Nortonovu admitanciju $Y_N(s)$; e) Nortonovu struju $i_N(t)$ ako je pobuda stacionarni sinusni signal $u_g(t)$=sin(t) i početni uvjeti jednaki nula.



Rješenje: Nortonova struja $I_N(s)$ metodom napona čvorova:



$U_1=U_p$, $U_2=U_s$

(1) $$U_1\left(\frac{1}{R}+sC\right)-U_2 sC = gU_1+\frac{U_g}{R}-I_p \qquad U_s=(1/n)\cdot U_p$$

(2) $$-U_1 sC+U_2\left(sC+\frac{1}{sL}\right)=-gU_1+I_s+\frac{i_L(0)}{s}-I_N \qquad I_s=n\cdot I_p$$

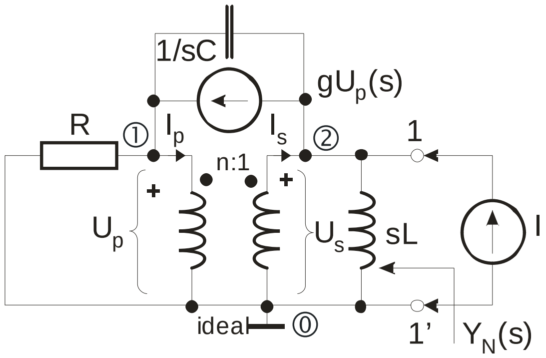
$U_2=U_s=0 \Rightarrow U_1=U_p=0$

(1) $$I_p=\frac{U_g}{R} \Rightarrow I_s=nI_p=n\frac{U_g}{R}$$

(2) $$I_N=I_s+\frac{i_L(0)}{s}=n\frac{U_g}{R}+\frac{i_L(0)}{s}$$ **(1 bod)**

$$\Rightarrow I_N(s)=n\frac{U_g}{R}+\frac{i_L(0)}{s}=\frac{2}{s}+\frac{1}{s}=\frac{3}{s}$$ **(1 bod)**

Nortonova admitancija $Y_N(s)$ (isključeni su početni uvjeti i neovisni izvori):



$Y_N(s)=\dfrac{I}{U_2}$, $U_1=U_p$, $U_2=U_s$

(1) $\quad U_1\left(\dfrac{1}{R}+sC\right)-U_2sC=gU_1-I_p \qquad U_s=(1/n)\cdot U_p$

(2) $\quad -U_1sC+U_2\left(sC+\dfrac{1}{sL}\right)=-gU_1+I_s+I \qquad I_s=n\cdot I_p$ **(1 bod)**

---

(1) $\quad U_1\left(\dfrac{1}{R}+sC-\dfrac{1}{n}sC-g\right)=-I_p$

(2) $\quad U_1\left[-sC+g+\dfrac{1}{n}\left(sC+\dfrac{1}{sL}\right)\right]=nI_p+I$

---

(1) $\quad \Rightarrow nU_2\left(-\dfrac{1}{R}+\dfrac{1}{n}sC-sC+g\right)=I_p$

(2) $\quad \Rightarrow nU_2\left(-sC+g+\dfrac{1}{n}sC+\dfrac{1}{nsL}\right)-nI_p=I$

---

(1) $\rightarrow$ (2) $\Rightarrow$

$$I=nU_2\left(-sC+g+\frac{1}{n}sC+\frac{1}{nsL}\right)-n^2U_2\left(-\frac{1}{R}+\frac{1}{n}sC-sC+g\right)$$

$$Y_N(s)=\frac{I}{U_2}=-nsC+ng+sC+\frac{1}{sL}+n^2\frac{1}{R}-nsC+n^2sC-n^2g=$$

$$=-2s+4+s+\frac{1}{s}+4-2s+4s-8=s+\frac{1}{s}$$ **(1 bod)**

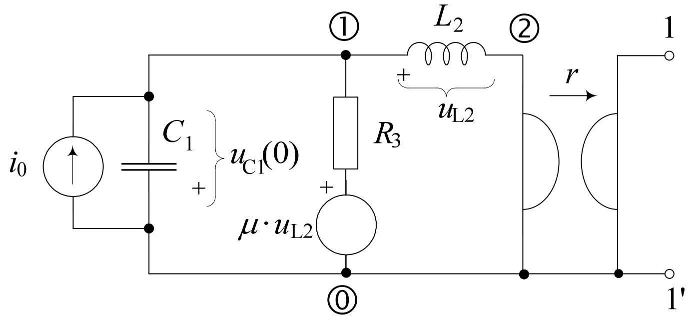
e) Nortonova struja $i_N(t)$ ako je pobuda stacionarni sinusni signal $u_g(t)=\sin(t)$ i početni uvjeti jednaki nula.

$$I_N(j\omega)=\frac{n}{R}U_g(j\omega)=2\cdot 1\angle 0^\circ=2\angle 0^\circ$$

$$\Rightarrow i_N(t)=2\sin(t)$$ **(1 bod)**

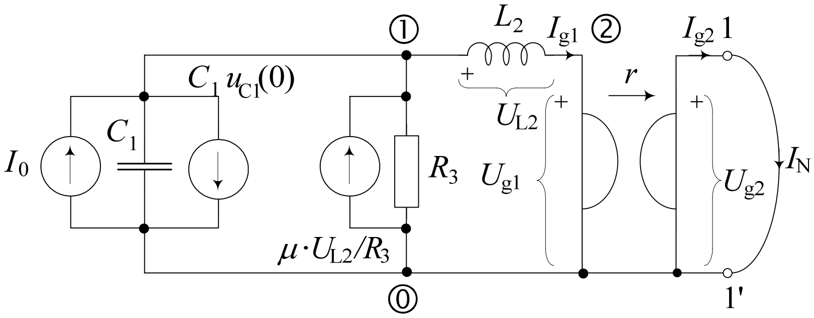
2. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C_1=1, L_2=1, R_3=1 te μ=2, r=1, $u_{C1}(0)$=1, $i_0(t)$=$S(t)$. Odrediti nadomjesne parametre mreže po Nortonu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta (čvorište ⓪ je referentno). U zadatku je potrebno:

a) Nacrtati sklop za izračunavanje Nortonove struje, postaviti jednadžbe napona za čvorišta ① i ②;
b) Odrediti Nortonovu struju $I_N(s)$;
c) Nacrtati sklop za izračunavanje Nortonove admitancije, postaviti jednadžbe napona za čvorišta ① i ②;
d) Odrediti Nortonovu admitanciju $Y_N(s)$.
e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



1) $U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=I_0(s)-C_1u_{C1}(0)+\frac{\mu U_{L2}}{R_3}$

2) $U_{L2}\frac{1}{sL_2}=I_{g1}(s)\,,\ U_{L2}(s)=U_1(s)-U_2(s)$

3) $I_{g2}=-\frac{1}{r}U_{g1}\,,\ U_{g1}=U_2\,,\ I_{g2}=I_N \Rightarrow I_N=-\frac{1}{r}U_2$

4) $I_{g1}=-\frac{1}{r}U_{g2}\,,\ U_{g2}=0 \ \Rightarrow I_{g1}=0 \Rightarrow U_{L2}=0$

---

1) $U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=I_0(s)-C_1u_{C1}(0)$

2) $-U_1\frac{1}{sL_2}+U_2\frac{1}{sL_2}=0 \Rightarrow U_1=U_2 \Rightarrow I_N=-\frac{1}{r}U_1$ (1 bod)

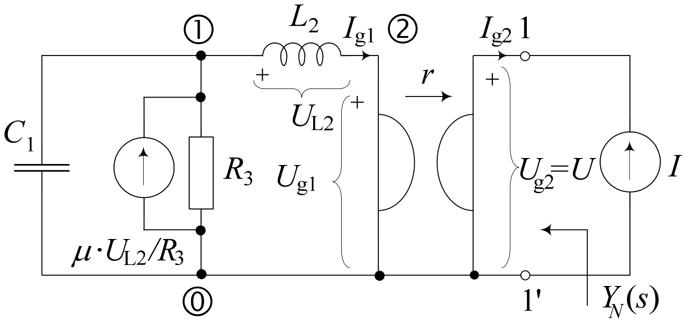
---

b) Nortonova struja $I_N(s)$:

$$U_1\left(sC_1+\frac{1}{R_3}\right)=I_0(s)-C_1u_{C1}(0)\ \Rightarrow U_1=\frac{I_0(s)-C_1u_{C1}(0)}{sC_1+1/R_3}$$

$$I_N(s)=-\frac{1}{r}U_1(s)=-\frac{1}{r}\frac{I_0(s)-C_1u_{C1}(0)}{sC_1+1/R_3}\ \Rightarrow I_N(s)=-\frac{1/s-1}{s+1}=\frac{1-1/s}{s+1}=\frac{s-1}{s(s+1)}$$ (1 bod)

c) Izračunavanje Nortonove admitancije pomoću jednadžbi napona čvorišta ① i ②



$$\Rightarrow Y_N(s)=\frac{I(s)}{U(s)}$$

1) $U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=\frac{\mu U_{L2}}{R_3}$

2) $U_{L2}\frac{1}{sL_2}=I_{g1}(s)$, $U_{L2}(s)=U_1(s)-U_2(s)$

3) $I_{g2}=-\frac{1}{r}U_{g1}$, $U_{g1}=U_2$, $I_{g2}=-I$

4) $I_{g1}=-\frac{1}{r}U_{g2}$, $U_{g2}=U$

---

1) $U_1\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)-U_2\left(\frac{1}{sL_2}-\frac{\mu}{R_3}\right)=0$

2) $-U_1\frac{1}{sL_2}+U_2\frac{1}{sL_2}=-I_{g1}=\frac{1}{r}U$

3) $I=\frac{1}{r}U_2\ \Rightarrow U_2=r\cdot I$ (1 bod)

---

d) Nortonova admitancija $Y_N(s)$:

$$2)\Rightarrow\ -U_1+U_2=\frac{sL_2}{r}U\ \rightarrow 1)\ \left(U_2-\frac{sL_2}{r}U\right)\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)-U_2\left(\frac{1}{sL_2}-\frac{\mu}{R_3}\right)=0$$

$$U_2\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}-\frac{1}{sL_2}+\frac{\mu}{R_3}\right)-\frac{sL_2}{r}U\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)=0$$

$$I\cdot r\cdot\left(sC_1+\frac{1}{R_3}\right)=\frac{sL_2}{r}U\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)\Rightarrow$$

$$Y_N(s)=\frac{I(s)}{U(s)}=\frac{\frac{sL_2}{r}\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)}{r\left(sC_1+1/R_3\right)}$$

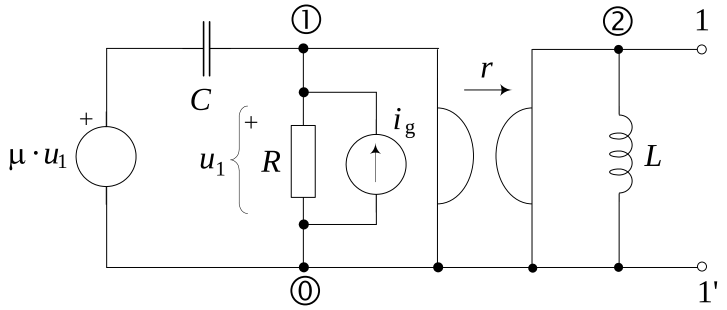
$$Y_N(s)=\frac{I(s)}{U(s)}=\frac{s\left(s+1/s-1\right)}{s+1}=\frac{s^2-s+1}{s+1}=\frac{(s+1)^2-3s}{s+1}=s+1-\frac{3s}{s+1}$$ (1 bod)

e) Da li je električni krug recipročan? Zašto?
NE, električni krug nije recipročan jer sadrži ovisni izvor i girator. (1 bod)

ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2012-2013

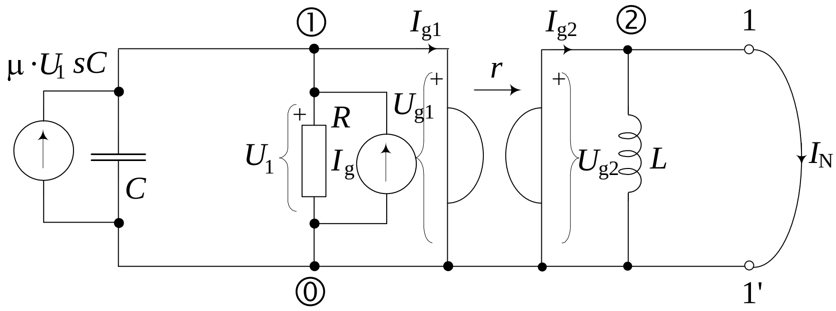
1. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C=1, L=1, R=1 te μ=2, r=1, $i_g(t)=S(t)$. Odrediti nadomjesne parametre mreže po Northonu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta. U zadatku je potrebno:

   a) Nacrtati sklop za izračunavanje Nortonove struje, postaviti jednadžbe napona za čvorišta ① i ②;
   b) Odrediti Nortonovu struju $I_N(s)$;
   c) Nacrtati sklop za izračunavanje Nortonove admitancije, postaviti jednadžbe napona za čvorišta ① i ②;
   d) Odrediti Nortonovu admitanciju $Y_N(s)$.
   e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



$$1)\ U_1\left(sC+\frac{1}{R}\right)=I_g(s)+\mu U_1 sC-I_{g1}$$

$$2)\ U_2\frac{1}{sL}=I_{g2}(s)-I_N(s)$$

$$3)\ I_{g2}=-\frac{1}{r}U_1$$

$$4)\ I_{g1}=-\frac{1}{r}U_2$$

$$U_2=0,\ I_{g1}=0$$

$$1)\ U_1\left(sC-\mu sC+\frac{1}{R}\right)=I_g(s)\ \Rightarrow\ U_1=\frac{I_g(s)}{sC(1-\mu)+1/R}$$

$$2)\ I_N(s)=I_{g2}(s)=-\frac{1}{r}U_1(s)\qquad \text{(1 bod)}$$

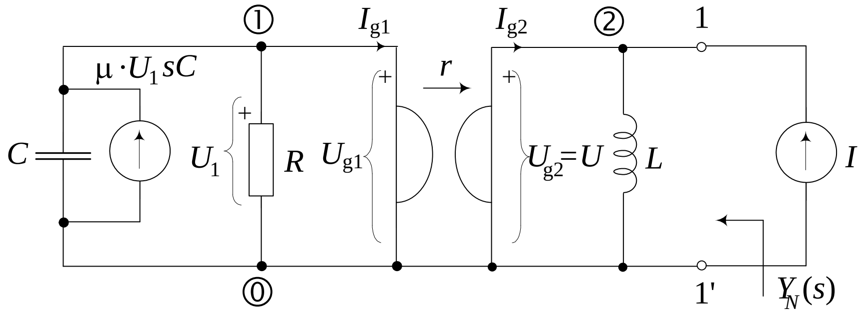
---

b) Nortonova struja $I_N(s)$:

$$1)\rightarrow 2)\Rightarrow\; I_N(s)=-\frac{1}{r}\cdot\frac{I_g(s)}{sC(1-\mu)+1/R}$$

$$I_N(s)=-\frac{1}{1}\cdot\frac{\frac{1}{s}}{-s+1}=-\frac{1}{s(1-s)}=\frac{1}{s(s-1)}\;\Rightarrow\; I_N(s)=\frac{1}{s(s-1)}$$ (1 bod)

c) Izračunavanje Nortonove admitancije pomoću jednadžbi napona čvorišta ① i ②



$$\Rightarrow Y_N(s)=\frac{I(s)}{U(s)}$$

$$1)\; U_1\left(sC+\frac{1}{R}\right)=\mu U_1 sC-I_{g1} \qquad 3)\; I_{g2}=-\frac{1}{r}U_1$$

$$2)\; U_2\frac{1}{sL}=I_{g2}(s)+I(s) \qquad 4)\; I_{g1}=-\frac{1}{r}U_2,\; U_2=U$$

$$1)\; U_1\left[sC(1-\mu)+\frac{1}{R}\right]=\frac{1}{r}U_2\;\Rightarrow\; U_1=\frac{1}{r}\cdot\frac{U_2}{sC(1-\mu)+1/R}$$

$$2)\; U_2\frac{1}{sL}=-\frac{1}{r}U_1+I(s)\;\Rightarrow\; U_2\frac{1}{sL}=-\frac{1}{r^2}\cdot\frac{U_2}{sC(1-\mu)+1/R}+I(s)$$ (1 bod)

d) Nortonova admitancija $Y_N(s)$:

$$I(s)=U(s)\left[\frac{1}{sL}+\frac{1}{r^2}\cdot\frac{1}{sC(1-\mu)+1/R}\right]\Rightarrow Y_N(s)=\frac{I(s)}{U(s)}=\frac{1}{sL}+\frac{1}{r^2}\cdot\frac{1}{sC(1-\mu)+1/R}$$

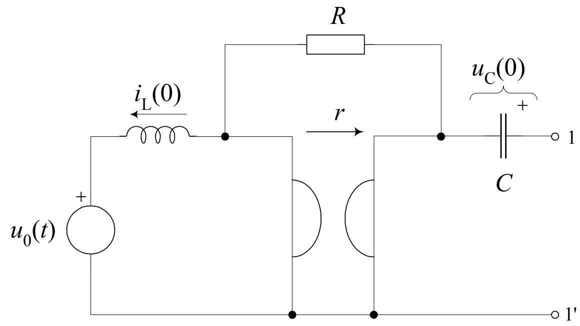
$$Y_N(s)=\frac{I(s)}{U(s)}=\frac{1}{s}+\frac{1}{-s+1}=\frac{-s+1+s}{s(-s+1)}=\frac{1}{s(1-s)}$$ (1 bod)

---

e) Da li je električni krug recipročan? Zašto?
NE, električni krug nije recipročan jer sadrži ovisni izvor i girator. (1 bod)

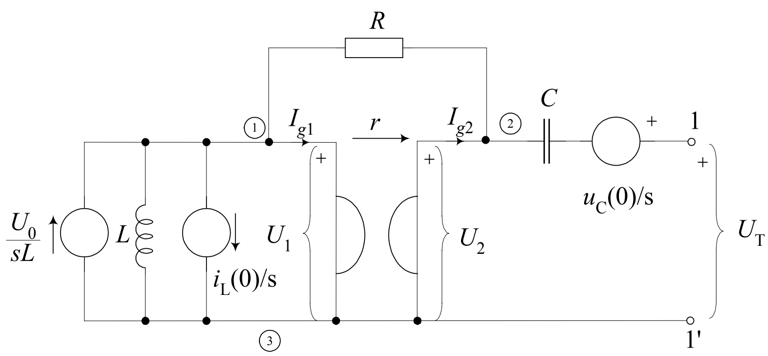
THEVENIN

5. Za mrežu prikazanu slikom odrediti nadomjesnu shemu po Theveninu obzirom na priključnice 1-1', koristeći postupak jednadžbi čvorišta, ako je pobuda $u_0(t)=\delta(t)$. Zadane su normirane vrijednosti elemenata: R=0.5, L=1, C=1, r=1 i početni uvjeti $u_C(0)=1$, $i_L(0)=0.5$.



Rješenje: Primjena Laplaceove transformacije

a) Teveninov napon $U_T(s)$:



(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}-I_{g1}=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\frac{1}{R}$$ $$I_{g1}=-U_2\frac{1}{r}$$

(2) $$I_{g2}=-U_1\frac{1}{R}+U_2\frac{1}{R}$$ $$I_{g2}=-U_1\frac{1}{r}$$

---

(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)$$

(2) $$0=-U_1\left(\frac{1}{R}-\frac{1}{r}\right)+U_2\frac{1}{R}$$

---

(2) $$\Rightarrow \qquad U_1=\frac{U_2\frac{1}{R}}{\frac{1}{R}-\frac{1}{r}}$$

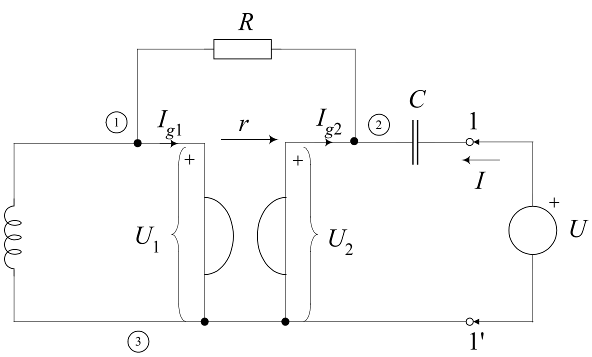
(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}=U_2\frac{\frac{1}{R}}{\frac{1}{R}-\frac{1}{r}}\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right) \qquad \Big/\cdot\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$U_2=\frac{\left(\frac{1}{R}-\frac{1}{r}\right)\left(\frac{U_0}{sL}-\frac{i_L(0)}{s}\right)}{\frac{1}{sRL}+\frac{1}{r^2}}=\frac{\left(r^2-Rr\right)\left(U_0-Li_L(0)\right)}{r^2+sRL}$$

$$U_T(s)=U_2(s)+\frac{u_C(0)}{s}=\frac{\left(r^2-Rr\right)\left(U_0-Li_L(0)\right)}{r^2+sRL}+\frac{u_C(0)}{s}=\frac{(1-0.5)(1-0.5)}{1+0.5s}+\frac{1}{s}$$

$$U_T(s)=\frac{0.5}{s+2}+\frac{1}{s}=\frac{1.5s+2}{s(s+2)}$$

a) Teveninova impedancija $Z_T(s)$:



$$Z_T(s)=\frac{U}{I},\qquad I=(U-U_2)sC\qquad\Rightarrow\qquad Z_T(s)=\frac{U_2}{I}+\frac{1}{sC}$$

(1) $\quad -I_{g1}=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\frac{1}{R}\qquad\qquad I_{g1}=-U_2\frac{1}{r}$

(2) $\quad I_{g2}+I=-U_1\frac{1}{R}+U_2\frac{1}{R}\qquad\qquad I_{g2}=-U_1\frac{1}{r}$

---

(1) $\quad 0=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)\qquad\Rightarrow\qquad U_1=\frac{\frac{1}{R}+\frac{1}{r}}{\frac{1}{sL}+\frac{1}{R}}U_2$

(2) $\quad I=-U_1\left(\frac{1}{R}-\frac{1}{r}\right)+U_2\frac{1}{R}$

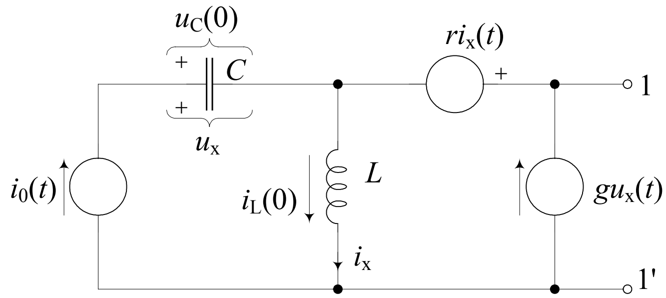
---

(2) $\quad I=-\frac{\frac{1}{R}+\frac{1}{r}}{\frac{1}{sL}+\frac{1}{R}}\left(\frac{1}{R}-\frac{1}{r}\right)U_2+U_2\frac{1}{R}=\frac{\frac{1}{sLR}+\frac{1}{r^2}}{\frac{1}{sL}+\frac{1}{R}}U_2=\frac{r^2+sLR}{r^2(sL+R)}U_2$

$$Z_T(s)=\frac{U_2}{I}+\frac{1}{sC}=\frac{r^2(sL+R)}{r^2+sLR}+\frac{1}{sC}=\frac{1(s+0.5)}{1+0.5s}+\frac{1}{s}=\frac{2s+1}{s+2}+\frac{1}{s}$$

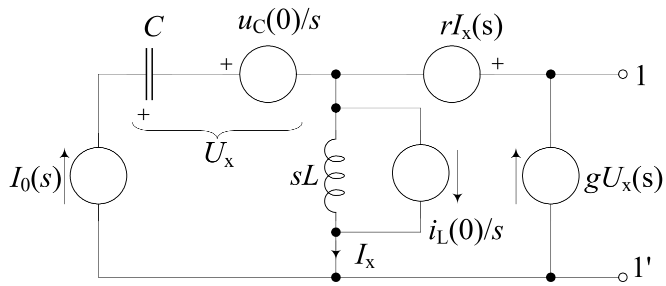
$$Z_T(s)=\frac{2(s^2+s+1)}{s(s+2)}$$

3. Za krug prikazan slikom odrediti nadomjesnu shemu po Theveninu obzirom na priključnice 1-1', ako je pobuda $i_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: R=1, L=1, C=1, r=0.5, g=0.5, a početni uvjeti su $u_C(0)=1$ i $i_L(0)=1$.



Rješenje: Primjena Laplaceove transformacije

a) Theveninov napon $U_T(s)$



$$U_x(s)=I_0\frac{1}{sC}+\frac{u_C(0)}{s}$$

$$I_x(s)=I_0(s)+gU_x(s)=I_0(s)+g\left(I_0\frac{1}{sC}+\frac{u_C(0)}{s}\right)$$

$$U_T(s)=U_L(s)+rI_x(s)=sLI_x(s)-Li_L(0)+rI_x(s)=(sL+r)I_x(s)-Li_L(0)$$

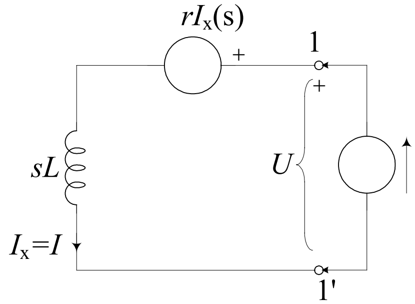
$$U_T(s)=(sL+r)\left[I_0(s)+g\left(I_0\frac{1}{sC}+\frac{u_C(0)}{s}\right)\right]-Li_L(0)$$

$$U_T(s)=(sL+r)\left(1+\frac{g}{sC}\right)I_0(s)+(sL+r)g\frac{u_C(0)}{s}-Li_L(0)$$

$$U_T(s)=\left(s+\frac{1}{2}\right)\left(1+\frac{1}{2s}\right)\frac{1}{s}+\left(s+\frac{1}{2}\right)\frac{1}{2s}-1=\frac{2s^2+5s+1}{4s^2}=\frac{1}{2}+\frac{5}{4s}+\frac{1}{4s^2}$$

$$u_T(t)=\frac{1}{2}\delta(t)+\frac{5}{4}S(t)+\frac{1}{4}tS(t)$$

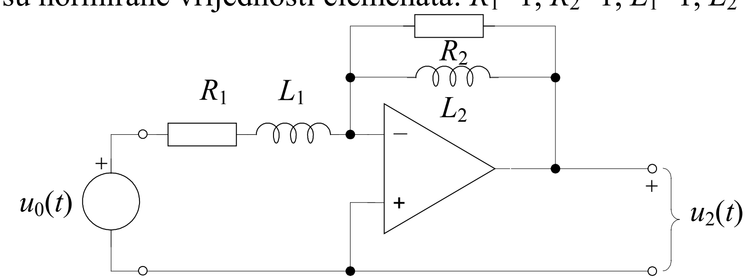
b) Theveninova impedancija $Z_T(s)$



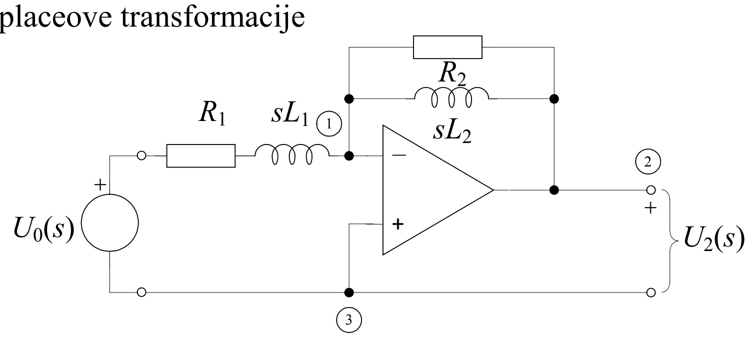
$$U = I_x sL + r \cdot I_x, I_x = I \Rightarrow U = I(sL + r)$$

$$Z_T(s) = \frac{U}{I} = sL + r = s + 1/2$$

4. Za krug prikazan slikom odrediti napon na izlazu operacijskog pojačala $u_2(t)$, ako je pobuda $u_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $L_1=1$, $L_2=1/2$, $A\rightarrow\infty$.



Rješenje: Primjena Laplaceove transformacije



(1) $$\frac{U_0}{sL_1 + R_1} = U_1\left(\frac{1}{sL_1 + R_1} + \frac{1}{sL_2} + \frac{1}{R_2}\right) - U_2\left(\frac{1}{sL_2} + \frac{1}{R_2}\right)$$

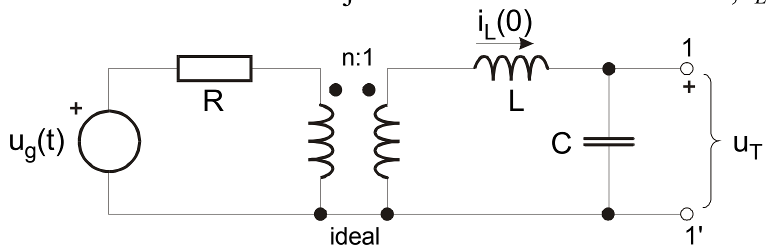
$U_1 = 0$, jer $A \to \infty$

(1) $\Rightarrow$ $$U_2 = -\frac{U_0}{(sL_1 + R_1)\left(\frac{1}{sL_2} + \frac{1}{R_2}\right)} = -\frac{U_0}{\frac{L_1}{sR_2}\left(s + \frac{R_1}{L_1}\right)\left(s + \frac{R_2}{L_2}\right)}$$

$$U_2 = -\frac{\frac{R_2}{L_1}s}{\left(s + \frac{R_1}{L_1}\right)\left(s + \frac{R_2}{L_2}\right)}U_0 = -\frac{1 \cdot s \cdot \frac{1}{s}}{(s+1)(s+2)} = \frac{-1}{(s+1)(s+2)}$$

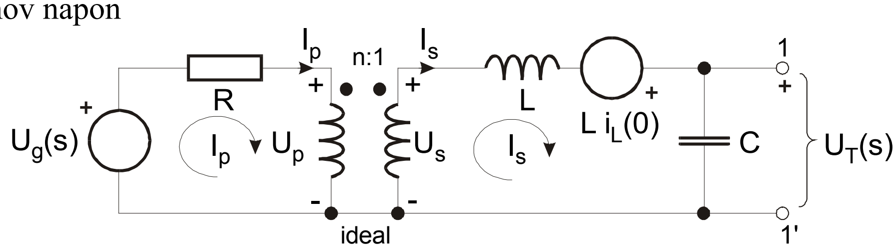
PONOVLJENI ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za električni krug prikazan slikom odrediti $U_T(s)$ i $Z_T(s)$ nadomjesne sheme po Teveninu s obzirom na stezaljke 1-1'. Zadane su normalizirane vrijednosti elemenata: $L=C=R=1$, $i_L(0)=1$, $n=2$, $u_g(t)=S(t)$.



Rješenje:

a) Theveninov napon



Jednadžbe transformatora:

$$U_p = n \cdot U_s \quad \Rightarrow \quad U_s = \frac{U_p}{n}$$

$$I_p = \frac{1}{n} \cdot I_s \quad \Rightarrow \quad I_s = nI_p$$

Jednadžbe petlji :

(1) $\quad U_g(s) = I_p \cdot R + U_p(s)$

(2) $\quad U_s = I_s sL + I_s \dfrac{1}{sC} - Li_L(0)$

(3) $\quad U_T(s) = I_s \cdot \dfrac{1}{sC}$

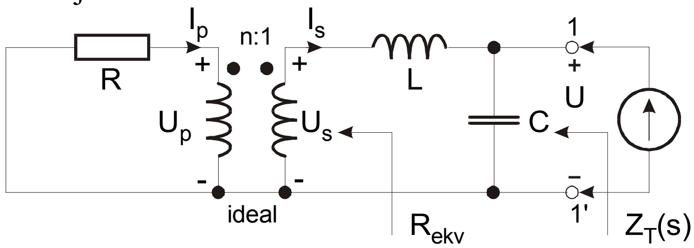
$$(1) \Rightarrow \quad U_g(s) = \frac{1}{n} I_s R + nU_s; \quad (2) \to (1) \Rightarrow \quad U_g(s) = \frac{1}{n} I_s R + n\left(I_s sL + I_s \frac{1}{sC} - Li_L(0)\right) \Big/ : n$$

$$\Rightarrow \quad \frac{U_g(s)}{n} + Li_L(0) = I_s\left(\frac{R}{n^2} + sL + \frac{1}{sC}\right); \quad \Rightarrow \quad I_s = \frac{\dfrac{U_g(s)}{n} + Li_L(0)}{\dfrac{R}{n^2} + sL + \dfrac{1}{sC}}$$

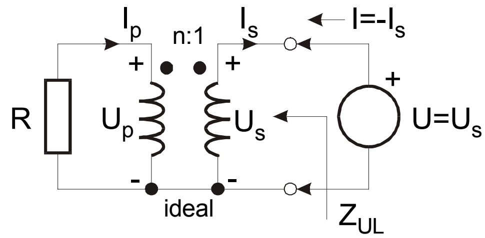
$$\Rightarrow \quad U_T(s) = I_s \cdot \frac{1}{sC} = \frac{\dfrac{U_g(s)}{n} + Li_L(0)}{\dfrac{R}{n^2} + sL + \dfrac{1}{sC}} \cdot \frac{1}{sC} = \frac{\dfrac{U_g(s)}{n} + Li_L(0)}{s^2LC + sC\dfrac{R}{n^2} + 1}$$

$$U_T(s) = \frac{\dfrac{1}{2s} + 1}{s^2 + \dfrac{1}{4}s + 1} = \frac{\dfrac{2}{s} + 4}{4s^2 + s + 4} = \frac{2(2s+1)}{s(4s^2 + s + 4)}$$

b) Teveninova impedancija

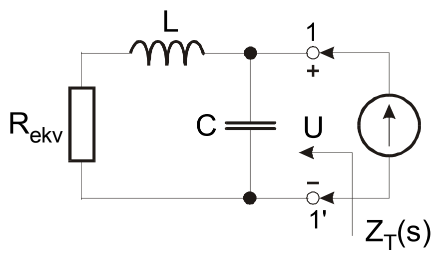


Izračunajmo najprije R_{ekv} :



$$Z_{ul} = \frac{U}{I} = \frac{U_s}{-I_s} = -\frac{\frac{U_p}{n}}{nI_p} = \frac{1}{n^2} \cdot \left(-\frac{U_p}{I_p}\right); \qquad -\frac{U_p}{I_p} = R; \quad \Rightarrow \quad Z_{ul} = \frac{R}{n^2} = R_{ekv}; \quad R_{ekv} = \frac{R}{n^2} = \frac{1}{4}$$

Teveninova impedancija :

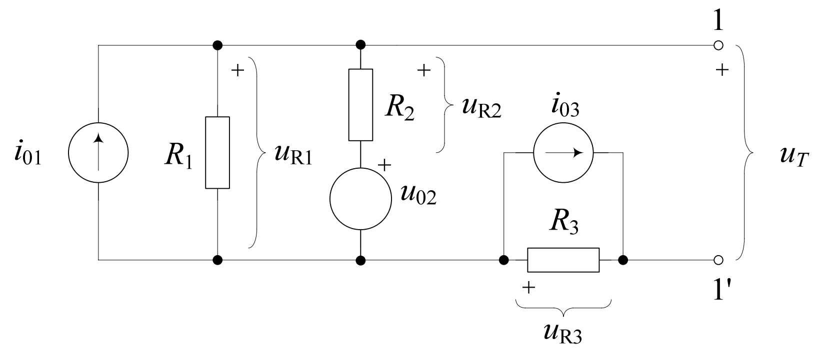


$$Z_T = \frac{U}{I} = \frac{\frac{1}{sC}\left(R_{ekv} + sL\right)}{\frac{1}{sC} + R_{ekv} + sL} = \frac{R_{ekv} + sL}{1 + sCR_{ekv} + s^2CL} = \frac{s + \frac{1}{4}}{s^2 + \frac{1}{4}s + 1} = \frac{4s+1}{4s^2 + s + 4}$$

$$Z_T(s) = \frac{4s+1}{4s^2 + s + 4}$$

PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2009

1. Za električni krug na slici odrediti: a) Theveninov napon $u_T(t)$; b) Theveninov otpor R_T; c) napon na otporu R_3; d) napon na otporu R_2 i e) napon na otporu R_1. Zadano je: R_1 = 5kΩ, R_2 = 10kΩ, R_3 = 10kΩ, i_{01} = 3mA, u_{02} = 3V, i_{03} = 1mA.



Rješenje:

$$u_T = \frac{i_{01}R_2 + u_{02}}{R_1 + R_2} R_1 - i_{03}R_3 = i_{01} \cdot 3{,}333 \cdot 10^{-3} + u_{02} \cdot 3{,}333 - i_{03} \cdot 10^4$$

$$u_T = 10\text{V} + 1\text{V} - 10\text{V} = 1\text{V}$$

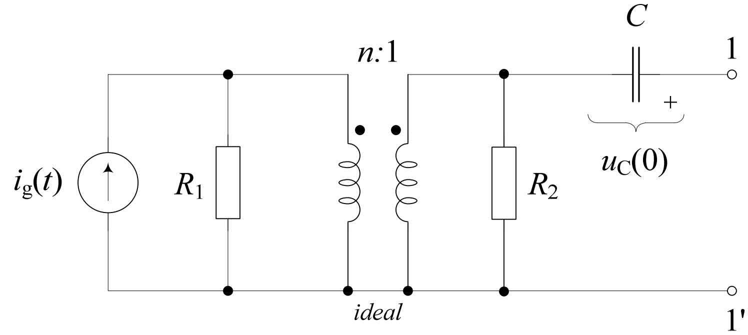
$$R_T = \frac{R_1 R_2}{R_1 + R_2} + R_3 = \frac{5 \cdot 10}{5 + 10} + 10 = 13{,}333\text{k}\Omega$$

$$u_{R3} = -i_{03} \cdot R_3 = -10^{-3} \cdot 10 \cdot 10^3 = -10\text{V}$$

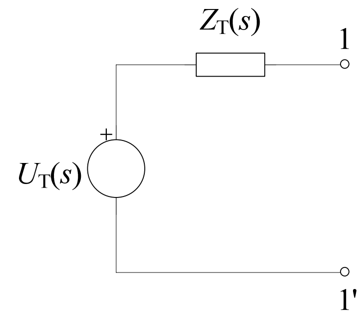
$$u_{R1} = u_T - u_{R3} = 1 - (-10) = 11\text{V}$$

$$u_{R2} = u_T - u_{R3} - u_{02} = 11 - 3 = 8\text{V}$$

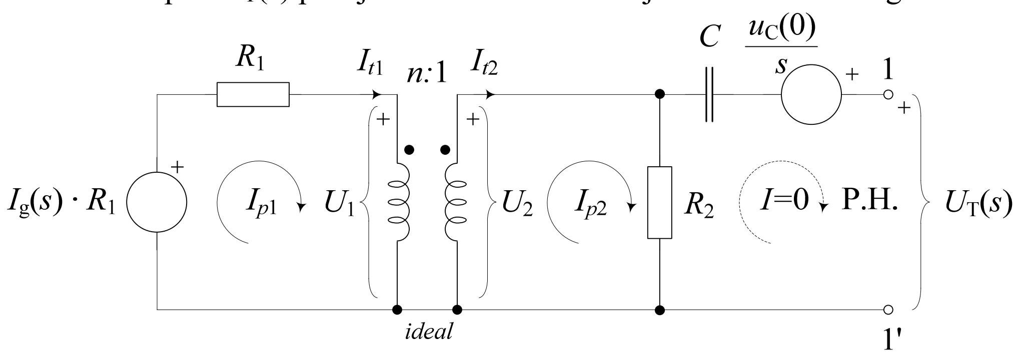
4. Za električni krug na slici izračunati nadomjesne parametre nadomjesnog kruga po Theveninu s obzirom na stezaljke 1 – 1': a) $U_T(s)$ i b) $Z_T(s)$. Zadane su normalizirane vrijednosti elemenata: $R_1 = R_2 = 1$, $C = 1$, $u_C(0) = 1$, $n = 2$, $i_g(t) = S(t)$. Koristiti metodu petlji.



Rješenje:



a) Theveninov napon $U_T(s)$ primjenom $\mathcal{L}$-transformacije na električni krug:



Jednadžbe idealnog transformatora:

$$U_1 = n \cdot U_2 \Rightarrow U_2 = \frac{U_1}{n}$$

$$I_{t1} = \frac{1}{n} \cdot I_{t2} \Rightarrow I_{t2} = n \cdot I_{t1}$$

1) $I_{p1}R_1 = -U_1 + I_g R_1$ $\qquad I_{p1} = I_{t1}$

2) $I_{p2}R_2 = U_2$ $\qquad I_{p2} = I_{t2}$

Nakon sređivanja jednadžbe glase:

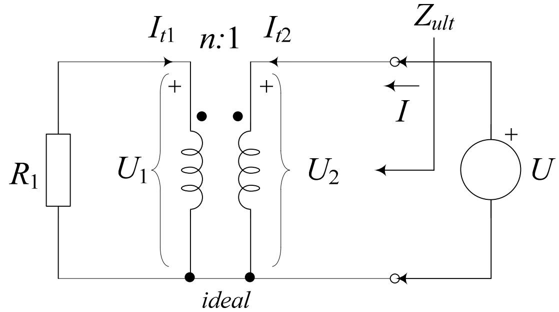
$$\left.\begin{aligned} &1)\ I_{p1}R_1 + nU_2 = I_g R_1 \\ &2)\ -nI_{p1}R_2 + U_2 = 0 \end{aligned}\right\} \qquad \left.\begin{aligned} &I_{p1}R_1 + nU_2 = I_g R_1 \\ &I_{p1} = \frac{U_2}{nR_2} \end{aligned}\right\} \qquad \left.\frac{U_2}{nR_2}R_1 + nU_2 = I_g R_1\right\}$$

$$\left.U_2\left(\frac{1}{nR_2}R_1+n\right)=I_gR_1\quad\right\}\qquad U_2=\frac{I_gR_1}{\dfrac{n^2R_2+R_1}{nR_2}}=I_g\cdot\frac{nR_1R_2}{n^2R_2+R_1}$$

$$U_T(s)=U_2(s)+\frac{u_C(0)}{s}=I_g\cdot\frac{nR_1R_2}{n^2R_2+R_1}+\frac{u_C(0)}{s}=\frac{1}{s}\cdot\frac{2}{5}+\frac{1}{s}=\frac{1}{s}\cdot\frac{7}{5}$$

b) Theveninova impedancija $Z_T(s)$:

1. način (pojednostavljen): Najprije izračunati ulaznu impedanciju u transformator zaključen s R_1. Označimo je s Z_{ult}.



$$Z_{ult}=\frac{1}{n^2}R_1$$

$$I_{t1}=-\frac{U_1}{R_1}\Rightarrow\frac{U_1}{I_{t1}}=-R_1$$

$$U=U_2$$

$$I=I_{t2}$$

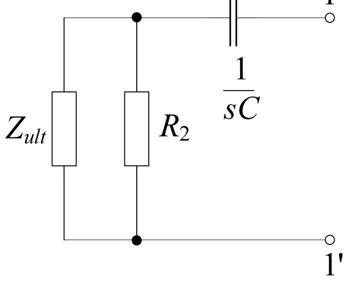
Jednadžbe od transformatora su (obratiti pažnju na referentno usmjerenje I_{t2}):

$$U_1=n\cdot U_2\Rightarrow U_2=\frac{U_1}{n}$$

$$I_{t1}=-\frac{1}{n}\cdot I_{t2}\Rightarrow I_{t2}=-n\cdot I_{t1}$$

$$Z_{ult}=\frac{U}{I}=\frac{U_2}{I_{t2}}=\frac{\dfrac{U_1}{n}}{-n\cdot I_{t1}}=-\frac{\dfrac{U_1}{I_{t1}}}{n^2}=-\frac{-R_1}{n^2}=\frac{R_1}{n^2}$$

Tada je Teveninova impedancija:



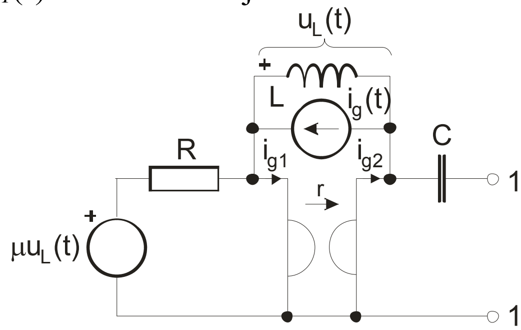
$$\leftarrow Z_T(s)=\frac{1}{sC}+R_2\parallel Z_{ult}$$

$$Z_T(s)=\frac{1}{sC}+\frac{\dfrac{R_1}{n^2}\cdot R_2}{\dfrac{R_1}{n^2}+R_2}=\frac{1}{s}+\frac{\dfrac{1}{4}}{\dfrac{1}{4}+1}=\frac{1}{s}+\frac{\dfrac{1}{4}}{\dfrac{5}{4}}=\frac{1}{s}+\frac{1}{5}$$

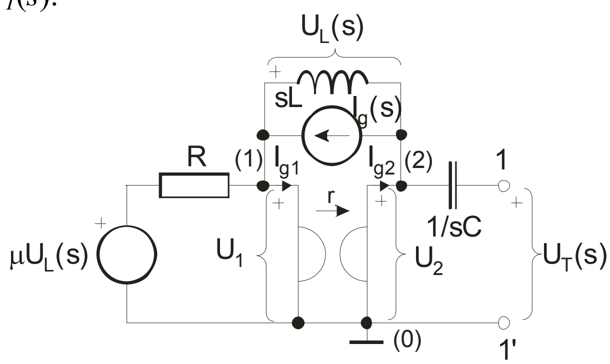
2. način: metodom petlji (nije ovdje prikazan, ali se priznaje u potpunosti).

2. Za mrežu prikazanu slikom odrediti nadomjesne parametre mreže po Theveninu $U_T(s)$ i $Z_T(s)$ s obzirom na polove 1–1'. Koristiti metodu napona čvorova u proračunu. Zadane su normalizirane vrijednosti elemenata: L=1, C=1, R=1, μ=2, r=2 te izvor $i_g(t)$=$S(t)$. Napisati:

a) Jednadžbu za čvor (1) za izračun $U_T(s)$;
b) Jednadžbu za čvor (2) za izračun $U_T(s)$;
c) Theveninov napon $U_T(s)$ uz uvrštene vrijednosti elemenata;
d) Theveninovu impedanciju $Z_T(s)$ uz uvrštene vrijednosti elemenata.



Rješenje: Metodom napona čvorova:
Theveninov napon $U_T(s)$:



$$U_T(s) = U_2(s)$$

(1) $$U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\mu\frac{U_L(s)}{R}+I_g(s)-I_{g1}(s) \quad \text{(1 bod)}$$

(2) $$-U_1\frac{1}{sL}+U_2\frac{1}{sL}=-I_g(s)+I_{g2}(s) \quad \text{(1 bod)}$$

$$U_2=-r\cdot I_{g1}$$
$$U_1=-r\cdot I_{g2} \qquad U_L(s)=U_1(s)-U_2(s)$$

---

(1) $$U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\mu\frac{U_1(s)-U_2(s)}{R}+I_g(s)+\frac{U_2}{r}$$

(2) $$-U_1\frac{1}{sL}+U_2\frac{1}{sL}=-I_g(s)-\frac{U_1}{r}$$

$$(2)\Rightarrow U_2\frac{1}{sL}+I_g(s)=U_1\left(\frac{1}{sL}-\frac{1}{r}\right) \;\Rightarrow\; U_1=\frac{U_2\frac{1}{sL}+I_g(s)}{\frac{1}{sL}-\frac{1}{r}} \rightarrow (1)$$

$$(1) \Rightarrow U_1\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)=U_2\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)+I_g$$

$$(1),\,(2) \Rightarrow \frac{U_2\frac{1}{sL}+I_g(s)}{\frac{1}{sL}-\frac{1}{r}}\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)=U_2\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)+I_g$$

$$\left[U_2\frac{1}{sL}+I_g(s)\right]\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)=U_2\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)\left(\frac{1}{sL}-\frac{1}{r}\right)+I_g\left(\frac{1}{sL}-\frac{1}{r}\right)$$

$$U_2\frac{1}{sL}\left(\frac{1}{R}-\frac{\mu}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)+U_2\frac{1}{r}\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)=$$

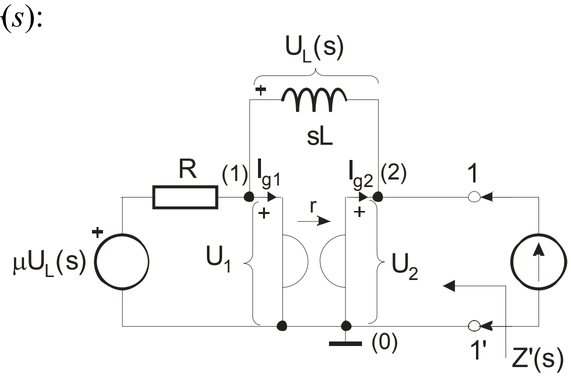
$$=I_g\left(\frac{1}{sL}-\frac{1}{r}\right)-I_g(s)\left(\frac{1}{R}-\frac{\mu}{R}+\frac{1}{sL}\right)$$

$$U_2\left(\frac{1}{sL}\frac{1}{R}+\frac{1}{r^2}-\frac{\mu}{rR}\right)=-I_g(s)\left(\frac{1}{r}+\frac{1-\mu}{R}\right)$$

$$U_2\left(\frac{1}{s}+\frac{1}{4}-1\right)=-I_g(s)\left(\frac{1}{2}-1\right)$$

$$U_2\left(\frac{1}{s}-\frac{3}{4}\right)=\frac{1}{2}I_g(s) \;\Rightarrow\; U_T(s)=U_2=\frac{\frac{1}{2}}{\frac{1}{s}-\frac{3}{4}}I_g(s)=\frac{\frac{1}{2}}{\frac{1}{s}-\frac{3}{4}}\cdot\frac{1}{s}=\frac{2}{4-3s}$$ (1 bod)

Theveninova impedancija $Z_T(s)$:



$$(1) \qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\mu\frac{U_L(s)}{R}-I_{g1}(s)$$

$$(2) \qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=I(s)+I_{g2}(s)$$

$$U_2=-r\cdot I_{g1}$$

$$U_1=-r\cdot I_{g2} \qquad U_L(s)=U_1(s)-U_2(s)$$

$$(1) \qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\mu\frac{U_1(s)-U_2(s)}{R}+\frac{U_2(s)}{r}$$

$$(2) \qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=I(s)-\frac{U_1(s)}{r}$$

$$(1) \qquad U_1\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)-U_2\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)=0$$

$$(2) \qquad -U_1\left(\frac{1}{sL}-\frac{1}{r}\right)+U_2\frac{1}{sL}=I(s)$$

$$(1) \Rightarrow U_1(s)=\frac{\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}}{\frac{1-\mu}{R}+\frac{1}{sL}}U_2(s)$$

$$(1),\,(2) \Rightarrow -\frac{\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}}{\frac{1-\mu}{R}+\frac{1}{sL}}U_2\left(\frac{1}{sL}-\frac{1}{r}\right)+U_2\frac{1}{sL}=I(s)$$

$$Z'(s)=\frac{U_2(s)}{I(s)}=\frac{\frac{1-\mu}{R}+\frac{1}{sL}}{\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)\left(\frac{1}{r}-\frac{1}{sL}\right)+\frac{1}{sL}\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)}=\frac{-1+\frac{1}{s}}{\left(\frac{1}{s}-2+\frac{1}{2}\right)\left(\frac{1}{2}-\frac{1}{s}\right)+\frac{1}{s}\left(-1+\frac{1}{s}\right)}=$$

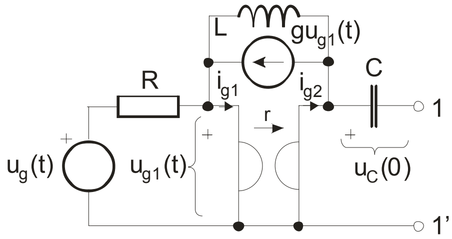
$$=\frac{-1+\frac{1}{s}}{\frac{1}{2s}-\frac{1}{s^2}-1+\frac{2}{s}+\frac{1}{4}-\frac{1}{2s}-\frac{1}{s}+\frac{1}{s^2}}=\frac{-1+\frac{1}{s}}{-\frac{3}{4}+\frac{1}{s}}=\frac{4s-4}{3s-4} \quad \Rightarrow$$

$$Z_T(s)=Z'(s)+\frac{1}{sC}=\frac{4s-4}{3s-4}+\frac{1}{s}=\frac{4s^2-s-4}{s(3s-4)}$$ (1 bod)

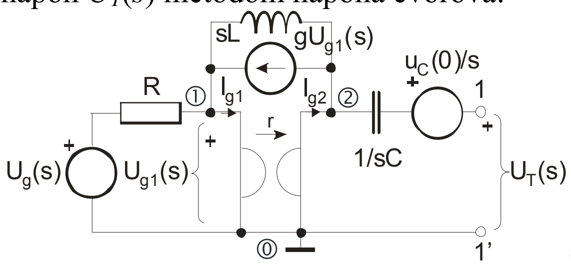
PONOVLJENI ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA 2010/11

Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za mrežu prikazanu slikom odrediti nadomjesne parametre mreže po Theveninu $U_T(s)$ i $Z_T(s)$ s obzirom na polove 1–1'. Koristiti metodu napona čvorova u proračunu. Zadane su normalizirane vrijednosti elemenata: L=1, C=1, R=1, g=2, r=2, $u_C(0)$=1 te izvor $u_g(t)$=$S(t)$. Nacrtati i napisati: a) Slike za izračun $U_T(s)$ i $Z_T(s)$; b) Jednadžbe čvorova za izračun $U_T(s)$; c) Jednadžbe čvorova za izračun $Z_T(s)$. Uz uvrštene vrijednosti elemenata: d) Theveninov napon $U_T(s)$; e) Theveninovu impedanciju $Z_T(s)$.



Rješenje: Theveninov napon $U_T(s)$ metodom napona čvorova:



$$U_T(s)=U_2(s)-\frac{u_C(0)}{s}$$

(0,5 boda)

$$(1)\qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1+\frac{U_g(s)}{R}-I_{g1}(s) \qquad U_2=-r\cdot I_{g1}\Rightarrow I_{g1}=-\frac{U_2}{r}$$

$$(2)\qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)+I_{g2}(s)\ \textbf{(1 bod)} \qquad U_1=-r\cdot I_{g2}\Rightarrow I_{g2}=-\frac{U_1}{r}$$

$$(1)\qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1+\frac{U_g(s)}{R}+\frac{U_2}{r}\ \Rightarrow U_1\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=\frac{U_g(s)}{R}$$

$$(2)\qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)-\frac{U_1(s)}{r}$$

$$(2)\Rightarrow -U_1\left(\frac{1}{sL}-g-\frac{1}{r}\right)+U_2\frac{1}{sL}=0\ \Rightarrow\ U_1=\frac{1/sL}{1/sL-g-1/r}\cdot U_2\ \rightarrow (1)$$

$$(1),(2)\Rightarrow U_2\frac{\frac{1}{sL}}{\frac{1}{sL}-g-\frac{1}{r}}\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=\frac{U_g}{R}$$

$$U_2\frac{1}{sL}\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)=\frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

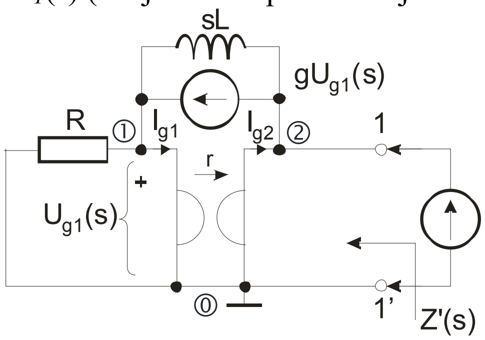
$$U_2\left(\frac{1}{sLR}+\frac{1}{(sL)^2}-\frac{g}{sL}-\frac{1}{(sL)^2}+\frac{g}{sL}+\frac{1}{rsL}-\frac{1}{rsL}+\frac{g}{r}+\frac{1}{r^2}\right)=\frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2\left(\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}\right)=\frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2(s)=\frac{\frac{1}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)}{\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}}U_g(s)==\frac{\frac{1}{s}-2-\frac{1}{2}}{\frac{1}{s}+1+\frac{1}{4}}\cdot\frac{1}{s}=\frac{\frac{1}{s}-\frac{5}{2}}{\frac{1}{s}+\frac{5}{4}}\cdot\frac{1}{s}=\frac{\frac{1}{s}-\frac{5}{2}}{1+\frac{5}{4}s}=\frac{4\frac{1}{s}-10}{4+5s}=\frac{-10s+4}{s(5s+4)}$$

$$\Rightarrow\ U_T(s)=U_2(s)-\frac{u_C(0)}{s}=\frac{-10s+4}{s(5s+4)}-\frac{1}{s}=\frac{-10s+4-(5s+4)}{s(5s+4)}=\frac{-15}{5s+4}$$ **(1 bod)**

Theveninova impedancija $Z_T(s)$ (isključeni su početni uvjeti i neovisni izvori):



(0,5 boda)

$$Z_T(s)=Z'(s)+\frac{1}{sC}$$

$$(1)\qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1-I_{g1}(s)$$

$$(2)\qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)+I_{g2}(s)+I(s)$$ **(1 bod)**

$$(1)\qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1+\frac{U_2}{r}\ \Rightarrow\ U_1\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=0$$

$$(2)\qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)-\frac{U_1(s)}{r}+I(s)$$

$$(2)\Rightarrow\ -U_1\left(\frac{1}{sL}-g-\frac{1}{r}\right)+U_2\frac{1}{sL}=I(s)\ \Rightarrow\ U_1=\frac{\frac{1}{sL}}{\frac{1}{sL}-g-\frac{1}{r}}\cdot U_2-\frac{I(s)}{\frac{1}{sL}-g-\frac{1}{r}}\ \rightarrow(1)$$

$$(1),(2)\Rightarrow\left[\frac{\frac{1}{sL}}{\frac{1}{sL}-g-\frac{1}{r}}\cdot U_2-\frac{I(s)}{\frac{1}{sL}-g-\frac{1}{r}}\right]\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=0$$

$$\left[\frac{1}{sL}\cdot U_2-I(s)\right]\left(\frac{1}{R}+\frac{1}{sL}-g\right)=U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

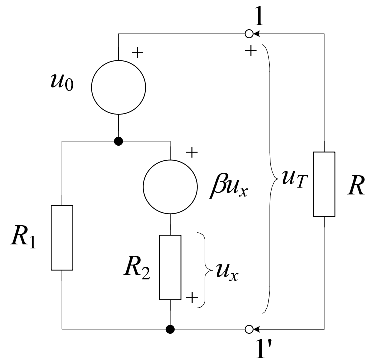
$$U_2\frac{1}{sL}\cdot\left(\frac{1}{R}+\frac{1}{sL}-g\right)-I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)=U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2\frac{1}{sL}\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)=I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)$$

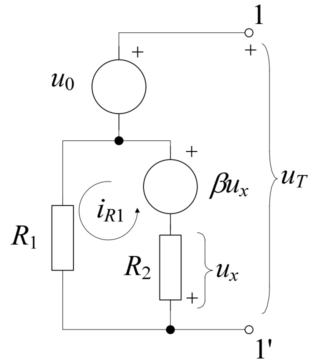
$$U_2\left(\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}\right)=I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)\Rightarrow Z'(s)=\frac{U_2(s)}{I(s)}=\frac{\frac{1}{R}+\frac{1}{sL}-g}{\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}}=\frac{1+\frac{1}{s}-2}{\frac{1}{s}+1+\frac{1}{4}}=\frac{4-4s}{5s+4}$$

$$Z_T(s)=Z'(s)+\frac{1}{sC}=\frac{4-4s}{5s+4}+\frac{1}{s}=\frac{-4s^2+9s+4}{s(5s+4)}$$ **(1 bod)**

4. Za krug prikazan slikom isključiti otpor R i obzirom na priključnice 1–1' odrediti:
   a) Theveninov napon u_T;
   b) Theveninov otpor R_T;
   c) iznos konstante β za koji je $R_T=R$;
   d) napon u_x uz uključen otpor R [β iz zadatka c)].
   e) Za koji iznos konstante β je $R_T=\infty$?
   Zadano je: pobuda u_0=2 V i vrijednosti elemenata $R_1=R_2$=2 Ω, R=4 Ω.



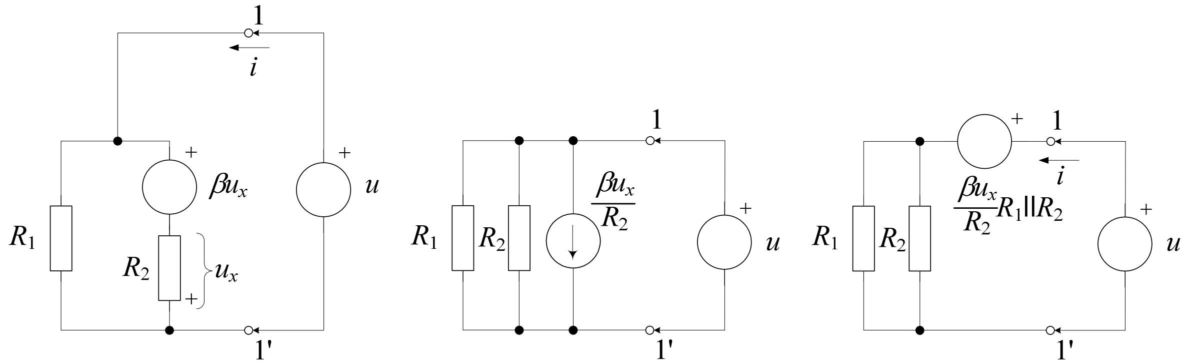
Rješenje:
Isključiti otpor R i odrediti nadomjesni spoj po Theveninu obzirom na priključnice 1–1';
a) Theveninov napon u_T:



$$u_T = u_0 + i_{R1}\cdot R_1; \qquad i_{R1} = \frac{\beta u_x}{R_1+R_2}; \qquad u_x = i_{R1}\cdot R_2$$

$$i_{R1} = \frac{\beta\cdot i_{R1}R_2}{R_1+R_2} \Rightarrow i_{R1}\left(1-\frac{\beta\cdot R_2}{R_1+R_2}\right)=0 \Rightarrow i_{R1}=0 \Rightarrow u_T = u_0 = 2\text{V}$$ (1 bod)

b) Theveninov otpor R_T:



$$-u_x = u - \beta\cdot u_x \Rightarrow u_x(1-\beta) = -u \Rightarrow u_x = \frac{u}{\beta-1}$$

$$i=\frac{u}{R_1}+\frac{u-\beta\cdot u_x}{R_2}=\frac{u}{R_1}+\frac{u-\frac{\beta}{\beta-1}u}{R_2}\quad\Rightarrow\quad R_T=\frac{u}{i}=\frac{1}{\frac{1}{R_1}+\frac{\beta-1-\beta}{R_2(\beta-1)}}=\frac{R_1R_2}{R_2-R_1\frac{1}{\beta-1}}$$

$$R_T=\frac{R_1R_2(\beta-1)}{R_2(\beta-1)-R_1}=\frac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1}$$ (1 bod)

c) odrediti iznos konstante β za koji je $R_T=R$.

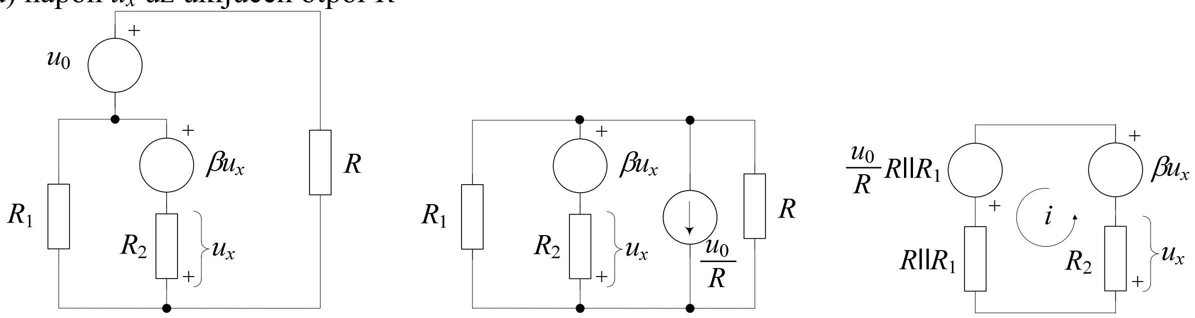
$$[R_2(1-\beta)+R_1]R_T=R_1R_2(1-\beta)\quad\Rightarrow\quad(1-\beta)(R_T-R_1)R_2=-R_1R_T$$

$$1-\beta=\frac{R_1R_T}{(R_1-R_T)R_2}\quad\Rightarrow\quad\beta=1-\frac{R_1R_T}{(R_1-R_T)R_2}$$

$$\beta=1-\frac{2\cdot 4}{(2-4)\cdot 2}=1+\frac{4}{2}=3$$ (1 bod)

Provjera: $R_T=\frac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1}=\frac{2\cdot 2\cdot(1-3)}{2\cdot(1-3)+2}=\frac{2\cdot(-2)}{-2+1}=4\,\Omega$

d) napon u_x uz uključen otpor R



$$\frac{u_0}{R}=\frac{2}{4}=\frac{1}{2}\text{A}$$

$$\frac{u_0}{R}\cdot\frac{RR_1}{R+R_1}+\beta\cdot u_x=i\cdot\left(R_2+\frac{RR_1}{R+R_1}\right)$$

$$u_0\cdot R_1+\beta\cdot u_x(R+R_1)=i\cdot[R_2(R+R_1)+RR_1]$$

$$u_x=i\cdot R_2$$

$$u_0\cdot R_1+\beta\cdot u_x(R+R_1)=\frac{u_x}{R_2}\cdot[R_2(R+R_1)+RR_1]$$

$$u_0\cdot R_1=(1-\beta)\cdot u_x(R+R_1)+u_x\frac{RR_1}{R_2}$$

$$u_x=\frac{u_0\cdot R_1}{(1-\beta)(R+R_1)+\frac{RR_1}{R_2}}=\frac{2\cdot 2}{(1-3)\cdot(4+2)+\frac{4\cdot 2}{2}}=-\frac{1}{2}V$$ (1 bod)

e) Za koji iznos konstante β je $R_T=\infty$? Za $\beta=2$. (1 bod)

$$R_T=\frac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1}=\frac{2\cdot 2\cdot(1-2)}{2\cdot(1-2)+2}=\frac{-4}{-2+2}=\frac{-4}{0}=\infty$$

$$1)\ U_1\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=-\frac{rI_s(s)}{sL}-I_p(s)$$

$$2)\ -U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rI_s(s)}{sL}+I_s(s)+I(s)$$

$$3)\ U_2=\frac{1}{n}U_1$$

$$4)\ I_s=nI_p$$

$$1)\ nU_2\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=-\frac{rnI_p(s)}{sL}-I_p(s)$$

$$2)\ -nU_2\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rnI_p(s)}{sL}+nI_p(s)+I(s)$$ (1 bod)

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d) Theveninova impedancija $Z_T(s)=U_2(s)/I(s)$:
Uvrstimo vrijednosti: $C=1/2$, $L=2$, $R_1=R_2=1$ te $n=2$, $r=4$, $u_C(0)=1$, $i_L(0)=1$, $u_g(t)=S(t)$.

$$1)\ 2U_2\left(\frac{s}{2}+1+\frac{1}{2s}\right)-U_2\frac{1}{2s}=-\frac{4I_p(s)}{s}-I_p(s)\ \Rightarrow$$

$$U_2\left(s+2+\frac{1}{2s}\right)=-I_p(s)\left(\frac{4}{s}+1\right)\ \Rightarrow\ I_p(s)=\frac{-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}$$

$$2)\ -U_2\frac{1}{s}+U_2\left(1+\frac{1}{2s}\right)=\frac{4I_p(s)}{s}+2I_p(s)+I(s)$$

$$\underline{U_2\left(1-\frac{1}{2s}\right)=2I_p(s)\left(1+\frac{2}{s}\right)+I(s)}$$

$$1),2)\Rightarrow U_2\left(1-\frac{1}{2s}\right)=2\frac{-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}\left(1+\frac{2}{s}\right)+I(s)=\frac{-U_2 2[s+2+1/(2s)]}{(s+4)}(s+2)+I(s)$$

$$\Rightarrow U_2(2s-1)=2s\frac{-U_2 2(s+2+1/(2s))}{(s+4)}(s+2)+2sI(s)$$

$$\Rightarrow U_2(2s-1)(s+4)=-U_2 4s\left[(s+2)^2+(s+2)/(2s)\right]+2s(s+4)I(s)$$

$$\Rightarrow U_2\left[(2s-1)(s+4)+4s(s+2)^2+4s(1/2+1/s)\right]=2s(s+4)I(s)$$

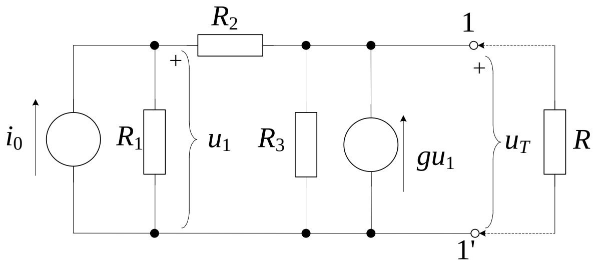
$$\Rightarrow U_2\left[2s^2-s+8s-4+4s^3+16s^2+16s+2s+4\right]=\left[2s^2+8s\right]I(s)$$

$$Z_T(s)=\frac{U_2(s)}{I(s)}=\frac{2s^2+8s}{4s^3+18s^2+25s}=\frac{2(s+4)}{4s^2+18s+25}$$ (1 bod)

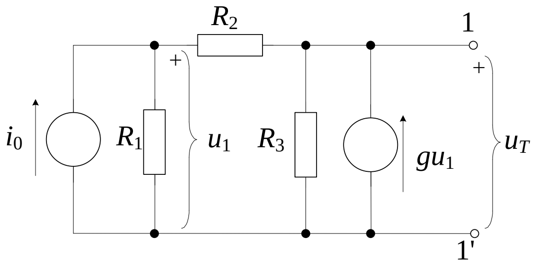
e) Da li je električni krug recipročan? Zašto?
NE jer ima strujno ovisni naponski izvor. (1 bod)

2. Za krug na slici obzirom na priključnice 1–1' i isključen otpor R odrediti:
a) Theveninov napon u_T; b) Theveninov otpor R_T; c) iznos konstante g za koji je $R_T=R$;
d) napon u_1 uz uključen otpor R [g iz zadatka c)]; e) iznos konstante g za koji je $R_T=\infty$.

Zadano je: i_0=2 A i $R_1 = 1\Omega$, $R_1 = \frac{1}{2}\Omega$ i $R_3 = R = \frac{1}{3}\Omega$.



Rješenje: a) Theveninov napon u_T:

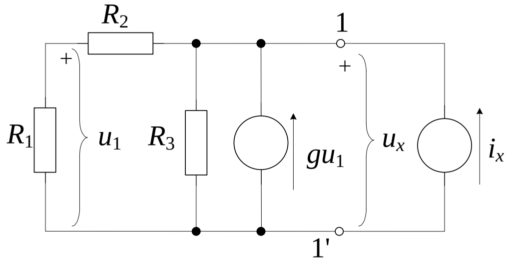


$$u_1(G_1+G_2)-u_2G_2=i_0$$

$$-u_1G_2+u_2(G_2+G_3)=gu_1 \quad \Rightarrow \quad u_1=u_2\frac{G_2+G_3}{g+G_2}$$

$$u_T=u_2=\frac{g+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}i_0=\frac{2g+4}{11-2g}$$ (1 bod)

b) Theveninov otpor R_T:



$$u_1=u_x\frac{G_2}{G_1+G_2}$$

$$i_x+gu_1=u_x\left(G_3+\frac{G_1G_2}{G_1+G_2}\right) \quad \Rightarrow \quad i_x=u_x\left(-g\frac{G_2}{G_1+G_2}+G_3+\frac{G_1G_2}{G_1+G_2}\right)$$

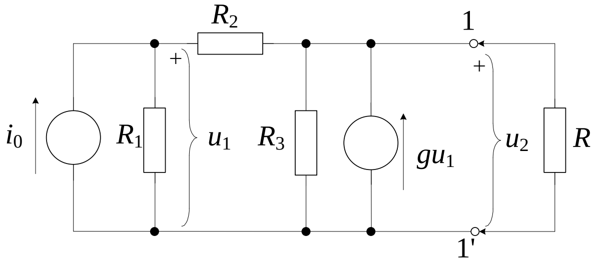
$$R_T=\frac{u_x}{i_x}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}=\frac{3}{11-2g}$$ (1 bod)

c) odrediti iznos konstante g za koji je $R_T=R$.

$$R_T=R=\frac{1}{3}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2} \quad \Rightarrow \quad \frac{1}{3}=\frac{1+2}{3(1+2)+2-2g} \quad \Rightarrow \quad g=1\ \Omega^{-1}$$

(1 bod)

d) napon u_1 uz uključen otpor R



$$-u_1G_2+u_2(G_2+G_3+G)=gu_1$$

$$u_1=u_2\frac{G_2+G_3+G}{g+G_2}=\frac{u_T}{2}\cdot\frac{G_2+G_3+G}{g+G_2}=\frac{1}{2}\cdot\frac{G_2+G_3+G}{g+G_2}\cdot\frac{g+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}i_0=\frac{8}{9}\,\text{V}$$

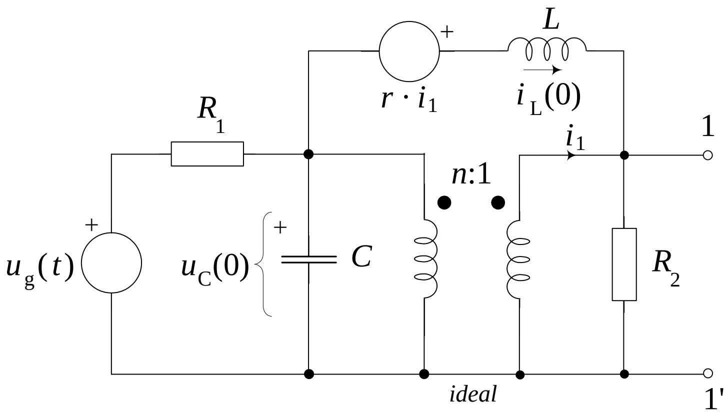
(1 bod)

e) iznos konstante g za koji je R_T=∞?

$$R_T=\frac{u_x}{i_x}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}\Omega$$

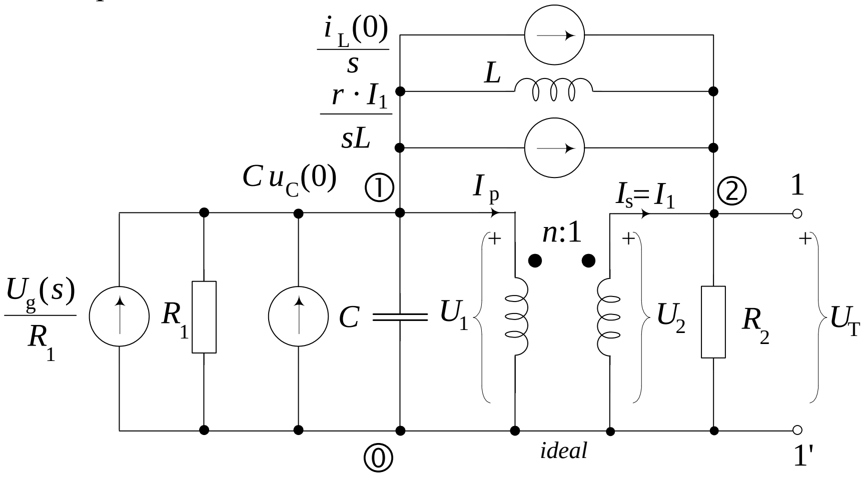
$$G_3(G_1+G_2)+G_1G_2-gG_2=0\Rightarrow \qquad g=\frac{G_3(G_1+G_2)}{G_2}+G_1=\frac{11}{2}\ \Omega^{-1}$$ (1 bod)

5. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C=1/2, L=2, R_1=R_2=1 te n=2, r=4, $u_C(0)$=1, $i_L(0)$=1, $u_g(t)$=$S(t)$. Odrediti nadomjesne parametre mreže po Theveninu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta. U zadatku je potrebno: a) Nacrtati sklop za izračunavanje Theveninovog napona i postaviti jednadžbe napona za čvorišta ① i ②; b) Odrediti Theveninov napon $U_T(s)$; c) Nacrtati sklop za izračunavanje Theveninove impedancije i postaviti jednadžbe napona za čvorišta ① i ②; d) Odrediti Theveninovu impedanciju $Z_T(s)$. e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



$$1)\ U_1\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_g(s)}{R_1}+Cu_C(0)-\frac{rI_s(s)}{sL}-\frac{i_L(0)}{s}-I_p(s)$$

$$2)\ -U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rI_s(s)}{sL}+\frac{i_L(0)}{s}+I_s(s)$$

$$3)\ U_2=\frac{1}{n}U_1$$

$$4)\ I_s=nI_p$$

$$1)\ nU_2\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_g(s)}{R_1}+Cu_C(0)-\frac{rnI_p(s)}{sL}-\frac{i_L(0)}{s}-I_p(s)$$

$$2)\ -nU_2\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rnI_p(s)}{sL}+\frac{i_L(0)}{s}+nI_p(s)$$ (1 bod)

---

b) Theveninov napon $U_T(s)=U_2(s)$:

Uvrstimo vrijednosti: $C=1/2$, $L=2$, $R_1=R_2=1$ te $n=2$, $r=4$, $u_C(0)=1$, $i_L(0)=1$, $u_g(t)=S(t)$.

$$1)\ 2U_2\left(\frac{s}{2}+1+\frac{1}{2s}\right)-U_2\frac{1}{2s}=\frac{1}{s}+\frac{1}{2}-\frac{4I_p(s)}{s}-\frac{1}{s}-I_p(s)\ \Rightarrow$$

$$U_2\left(s+2+\frac{1}{2s}\right)=\frac{1}{2}-I_p(s)\left(\frac{4}{s}+1\right)\ \Rightarrow\ I_p(s)=\frac{\frac{1}{2}-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}$$

$$2)\ -U_2\frac{1}{s}+U_2\left(1+\frac{1}{2s}\right)=\frac{4I_p(s)}{s}+\frac{1}{s}+2I_p(s)$$

$$U_2\left(1-\frac{1}{2s}\right)=\frac{1}{s}+2I_p(s)\left(1+\frac{2}{s}\right)$$

1), 2) $\Rightarrow$

$$U_2\left(1-\frac{1}{2s}\right)=\frac{1}{s}+2\frac{\frac{1}{2}-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}\left(1+\frac{2}{s}\right)=\frac{1}{s}+\frac{1-U_2 2\left(s+2+\frac{1}{2s}\right)}{(s+4)}(s+2)$$

$$\Rightarrow U_2(2s-1)=2+2s\frac{1-U_2 2(s+2+1/(2s))}{(s+4)}(s+2)$$

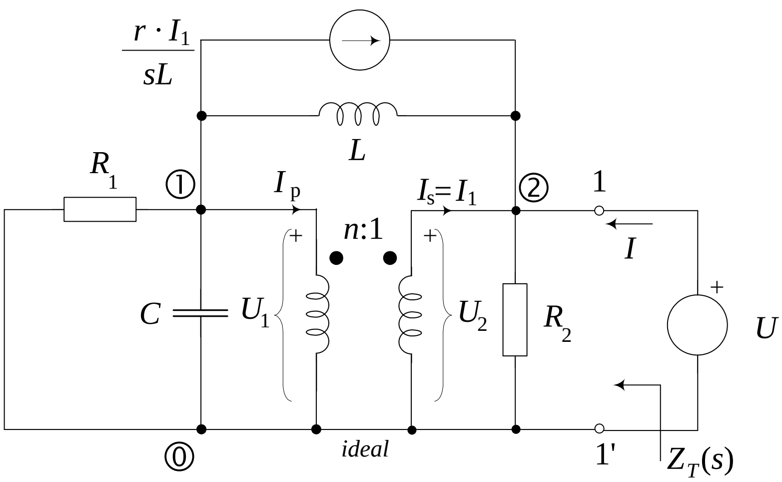
$$\Rightarrow U_2(2s-1)(s+4)=2(s+4)+2s(s+2)-U_2 4s\left[(s+2)^2+\frac{1}{2s}(s+2)\right]$$

$$\Rightarrow U_2\left[(2s-1)(s+4)+4s\left(s^2+4s+4+\frac{1}{2}+\frac{1}{s}\right)\right]=2(s+4)+2s(s+2)$$

$$\Rightarrow U_2\left[2s^2-s+8s-4+4s^3+16s^2+16s+2s+4\right]=2s+8+2s^2+4s$$

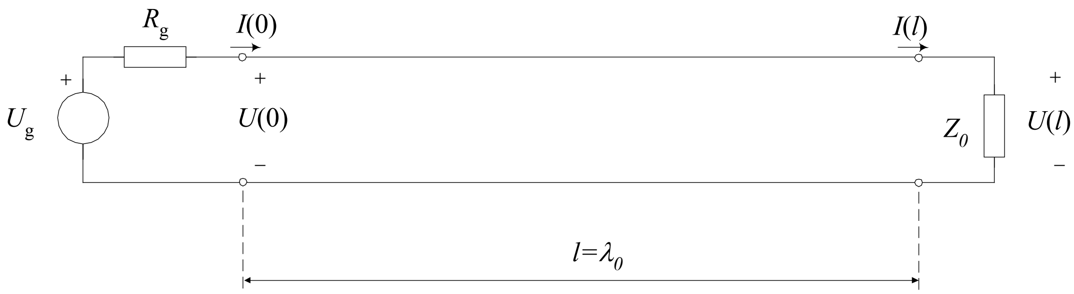
$$U_T(s)=U_2(s)=\frac{2s^2+6s+8}{4s^3+18s^2+25s}=\frac{2(s^2+3s+4)}{s(4s^2+18s+25)}$$ (1 bod)

c) Izračunavanje Theveninove impedancije pomoću jednadžbi napona čvorišta ① i ②



LINIJE

5. Zadana je linija bez gubitaka s L=10 μH/km i C=100 nF/km. Na ulaz linije je priključen naponski izvor $u_g(t)$ = 2 cos(ω_0t) u seriji s otporom R_g=50Ω. Duljina linije je $l=\lambda_0$, gdje je λ_0 valna duljina signala pri frekvenciji $\omega_0=10^6$. Izlaz linije je zaključen karakterističnom impedancijom.
   a) Odrediti karakterističnu impedanciju Z_0 i koeficijent prijenosa γ linije.
   b) Kolika je duljina linije u km?
   c) Odrediti napon i struju na polovici linije.
   d) Odrediti napon i struju na kraju linije.



Rješenje:

a) Linija bez gubitaka $\rightarrow$ $R=0$, $G=0$ $\Rightarrow$ $Z_0=\sqrt{L/C}$, $\gamma=s\sqrt{LC}$

Stac. sinusna pobuda $\rightarrow$ $s=j\omega$ $\Rightarrow$ $\gamma=j\omega\sqrt{LC}=j\beta$

$$Z_0=\sqrt{L/C}=\sqrt{10^{-5}/10^{-7}}=10\Omega$$

$$\gamma=j\omega\sqrt{LC}=j10^6\sqrt{10^{-5}\cdot 10^{-7}}=j$$

b) $$l=\lambda_0=\frac{2\pi}{\beta}=\frac{2\pi}{\omega\sqrt{LC}}=2\pi=6{,}28\text{km}$$

c) $$U(x)=U(0)\cdot ch\gamma x-I(0)Z_0 sh\gamma x$$

$$\underline{I(x)=-\frac{U(0)}{Z_0}sh\gamma x+I(0)ch\gamma x}$$

$$Z_{ul}=Z_0 \Rightarrow U(0)=Z_0I(0) \quad U(0)=\frac{Z_0}{Z_0+R_g}U_g(s)=\frac{10\Omega}{10\Omega+50\Omega}U_g(s)=\frac{1}{6}U_g(s)$$

$$U(x)=U(0)\cdot(ch\gamma x-sh\gamma x)=U(0)\cdot e^{-\gamma x}$$

$$\underline{I(x)=\frac{U(0)}{Z_0}(-sh\gamma x+ch\gamma x)=\frac{U(0)}{Z_0}e^{-\gamma x}}$$

$$U(l/2)=U(0)\cdot\left(ch(\gamma l/2)-sh(\gamma l/2)\right)=U(0)\cdot e^{-\gamma l/2}=U(0)\cdot e^{-j\pi}=-U(0)$$

$$\underline{I(l/2)=\frac{U(0)}{Z_0}\left(-sh(\gamma l/2)+ch(\gamma l/2)\right)=\frac{U(0)}{Z_0}e^{-\gamma l/2}=\frac{U(0)}{Z_0}e^{-j\pi}=-\frac{U(0)}{Z_0}}$$

$$U(0)=\frac{1}{6}U_g(s) \quad \underline{u(l/2,t)=-\frac{1}{3}\cos(\omega_0 t)}, \ \underline{i(l/2,t)=-\frac{1}{30}\cos(\omega_0 t)}$$

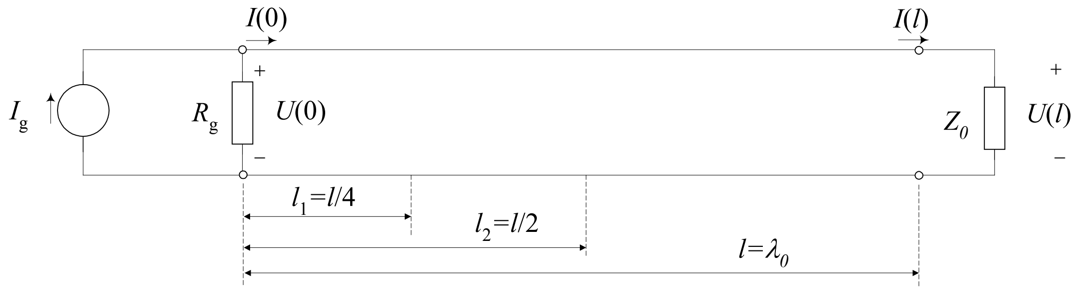
d) $$U(l)=U(0)\cdot\left(ch(\gamma l)-sh(\gamma l)\right)=U(0)\cdot e^{-\gamma l}=U(0)\cdot e^{-j2\pi}=U(0)$$

$$\underline{I(l)=\frac{U(0)}{Z_0}\left(-sh(\gamma l)+ch(\gamma l)\right)=\frac{U(0)}{Z_0}e^{-\gamma l}=\frac{U(0)}{Z_0}e^{-j2\pi}=\frac{U(0)}{Z_0}}$$

$$\underline{u(l,t)=\frac{1}{3}\cos(\omega_0 t)}, \ \underline{i(l,t)=\frac{1}{30}\cos(\omega_0 t)}$$

5. Zadana je linija bez gubitaka s L=20 μH/km i C=8 nF/km. Na ulaz linije je priključen strujni izvor $i_g(t)$ = 0,2 cos(ω_0t) paralelno s otporom R_g=10Ω. Duljina linije je $l=\lambda_0$, gdje je λ_0 valna duljina signala pri frekvenciji $\omega_0 = 5\cdot 10^6$ rad/s. Izlaz linije je zaključen karakterističnom impedancijom.

a) Odrediti karakterističnu impedanciju Z_0 i koeficijent prijenosa γ linije.
b) Kolika je duljina linije u km?
c) Odrediti napon i struju na 1/4 linije.
d) Odrediti napon i struju na 1/2 linije.



Rješenje:

a) Linija bez gubitaka $\rightarrow\ R=0,\ G=0 \quad\Rightarrow\quad Z_0=\sqrt{L/C},\ \gamma=s\sqrt{LC}\ \ \Omega$

Stac. sinusna pobuda $\rightarrow\ s=j\omega_0 \quad\Rightarrow\quad \gamma=j\omega_0\sqrt{LC}=j\beta$

$$Z_0=\sqrt{L/C}=\sqrt{2\cdot 10^{-5}/8\cdot 10^{-9}}=\sqrt{10^4/4}=50\Omega$$

$$\gamma=j\omega_0\sqrt{LC}=j5\cdot 10^6\sqrt{20\cdot 10^{-6}\cdot 8\cdot 10^{-9}}=j5\cdot 10^6\sqrt{16\cdot 10^{-14}}=j5\cdot 10^6\cdot 4\cdot 10^{-7}=j2\ \text{km}^{-1}$$

b) $l=\lambda_0=\dfrac{2\pi}{\beta}=\dfrac{2\pi}{2}=\pi=3{,}14\text{km}\ ; \qquad l_1=\lambda_0/4=\dfrac{\pi}{4}\text{km}\ ; \qquad l_2=\lambda_0/2=\dfrac{\pi}{2}\text{km}$

c)
$$U(x)=U(0)\cdot ch\gamma x-I(0)Z_0 sh\gamma x$$
$$I(x)=-\frac{U(0)}{Z_0}sh\gamma x+I(0)ch\gamma x$$

$$U(0)=I_g\cdot(Z_0\parallel R_g)=I_g\cdot\left(Z_0\cdot R_g/(Z_0+R_g)\right)=\frac{10}{6}=1{,}666$$
$$I(0)=U(0)/Z_0=U(0)/Z_0=0{,}0333$$
$$U(x)=U(0)\cdot(ch\gamma x-sh\gamma x)=U(0)\cdot e^{-\gamma x}$$
$$I(x)=\frac{U(0)}{Z_0}(-sh\gamma x+ch\gamma x)=\frac{U(0)}{Z_0}e^{-\gamma x}$$

$$U(l/4)=U(0)\cdot\left(ch(\gamma l/4)-sh(\gamma l/4)\right)=U(0)\cdot e^{-\gamma l/4}=U(0)\cdot e^{-j\pi/2}$$
$$I(l/4)=\frac{U(0)}{Z_0}\left(-sh(\gamma l/4)+ch(\gamma l/4)\right)=\frac{U(0)}{Z_0}e^{-\gamma l/4}=\frac{U(0)}{Z_0}e^{-j\pi/2}$$

$$u(l/4,t)=1{,}6667\cdot\cos(\omega_0 t-90^\circ)$$
$$i(l/4,t)=0{,}0333\cdot\cos(\omega_0 t-90^\circ)$$

d)
$$U(l/2)=U(0)\cdot\left(ch(\gamma l/2)-sh(\gamma l/2)\right)=U(0)\cdot e^{-\gamma l/2}=U(0)\cdot e^{-j\pi}$$
$$I(l/2)=\frac{U(0)}{Z_0}\left(-sh(\gamma l/2)+ch(\gamma l/2)\right)=\frac{U(0)}{Z_0}e^{-\gamma l/2}=\frac{U(0)}{Z_0}e^{-j\pi}$$

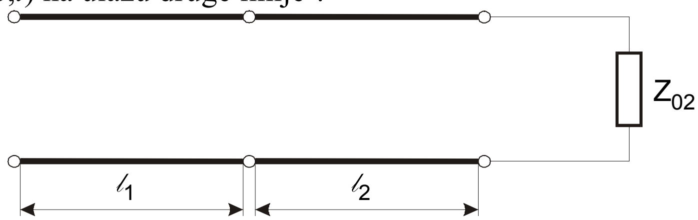
$$u(l/2,t)=-1{,}6667\cdot\cos(\omega_0 t),\qquad i(l/2,t)=-0{,}0333\cdot\cos(\omega_0 t)$$

5. Na liniju bez gubitaka duljine $l_1=\lambda_1/2$, s primarnim parametrima L_1=2 mH/km i C_1=6 nF/km, priključena je linija bez gubitaka zadana sa L_2=0,6 mH/km i C_2=40 nF/km. Druga linija je zaključena svojom karakterističnom impedancijom.

a) Koliki je faktor refleksije prve linije na spojnom mjestu ?

b) Kolika je amplituda polaznog, a kolika reflektiranog vala na spojnom mjestu, ako je napon na ulazu prve linije $u_{\mathrm{I}}(0,t)=2\cos 10^4 t$?

c) Koliki je napon $u_{\mathrm{II}}(0,t)$ na ulazu druge linije ?



Rješenje:

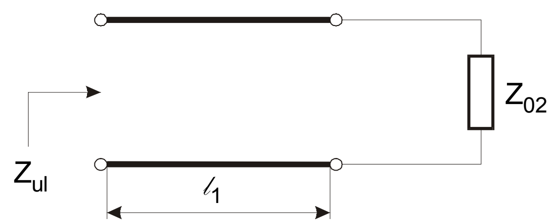
Linija bez gubitaka $\rightarrow\ R=0,\ G=0 \qquad \Rightarrow \qquad Z_0=\sqrt{L/C},\ \gamma=s\sqrt{LC}$

$$Z_{01}=\sqrt{L_1/C_1}=1/\sqrt{3}\cdot 10^3\,\Omega$$

$$Z_{02}=\sqrt{L_2/C_2}=\sqrt{3/2}\cdot 10^2\,\Omega$$

a)
$$\Gamma=\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}=\frac{\sqrt{3/2}\cdot 10^2-1/\sqrt{3}\cdot 10^3}{\sqrt{3/2}\cdot 10^2+1/\sqrt{3}\cdot 10^3}=\frac{3-10\sqrt{2}}{3+10\sqrt{2}}=-\frac{11.1}{17.1}=-0.65$$

Za $l=\lambda/2 \Rightarrow Z_{ul}=Z_2$



$Z_{ul}=Z_{02}$

$$U_p\left(\frac{\lambda_1}{2}\right)=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\beta_1\frac{\lambda_1}{2}}=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\pi}=$$

b)
$$=\frac{U(0)+Z_{01}\dfrac{U(0)}{Z_{ul}}}{2}e^{-j\pi}=\frac{U(0)}{2}\left(1+\frac{1}{\sqrt{3}}10^3\sqrt{\frac{2}{3}}10^{-2}\right)e^{-j\pi}=\frac{2}{2}\left(1+\frac{10\sqrt{2}}{3}\right)e^{-j\pi}=5.71405e^{-j\pi}$$

$u_p(l_1,t)$= 5.71405 (cos 10^4 t– 180°)= – 5.71405 (cos 10^4 t)

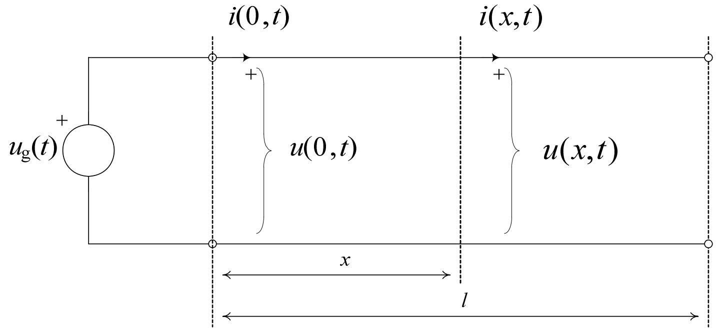
$U_r=U_p\cdot(-0.65)$=5.71405·(–0.65)= – 3.71413

$u_r(l_1,t)$= 3.71413 (cos 10^4 t)

c) $u_{\mathrm{II}}(0,t)=u_p(l_1,t)+u_r(l_1,t)=U_p\,(1+\Gamma)$ (cos 10^4 t– 180°)= – 5.71405·0.35 (cos 10^4 t)=

$u_{\mathrm{II}}(0,t)$= – 2.0 (cos 10^4 t)

5. Na ulazu linije bez gubitaka s L=4mH/km, C=400nF/km, duljine $l = 2.5\lambda$ km, djeluje stacionarni sinusni izvor napona $u_g = 5\cos(2\pi \cdot f_0 \cdot t)$ uz f_0=6,25kHz. Izlaz linije je u praznom hodu. Odrediti: *a*) valnu duljinu λ signala na liniji; *b*) duljinu l linije, *c*) karakterističnu impedanciju Z_0, faktor prijenosa γ te brzinu širenja vala po liniji v; *d*) ulaznu impedanciju Z_{ul}; *e*) napon i struju na sredini linije ($x=l/2$)?



Rješenje:

a) $\omega_0 = 2\pi f_0 = 39{,}2699$ rad/s

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0\sqrt{LC}} = \frac{2\pi}{2\pi \cdot 6{,}25 \cdot 10^3 \sqrt{4 \cdot 10^{-3} \cdot 400 \cdot 10^{-9}}} = 4\text{km} \quad \text{(1bod)}$$

b) $l = 2.5\lambda = 10\text{km}$ (1bod)

c) Za liniju bez gubitaka vrijedi:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{4 \cdot 10^{-3}}{400 \cdot 10^{-9}}} = \frac{1}{10} \cdot 10^3 \Omega = 100\Omega,$$

$$\gamma = j\beta;\ \alpha = 0, \qquad \beta = \frac{2\pi}{\lambda} = \frac{\pi}{2}\left[\frac{\text{rad}}{\text{km}}\right] \qquad \beta l = \beta \cdot 10 = 5\pi$$

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \cdot 10^{-3} \cdot 400 \cdot 10^{-9}}} = \frac{1}{40 \cdot 10^{-6}} = 25 \cdot 10^3 \text{km/s} \quad \text{(1bod)}$$

d) $$Z_{ul} = Z_0 \frac{Z_2 \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{Z_2 \sinh(\gamma l) + Z_0 \cosh(\gamma l)} = Z_0 \,\text{cth}(\gamma l) = Z_0 j\text{ctg}(\beta l)$$

$$\text{ctg}(\beta l) = \text{ctg}(5\pi) = \infty \quad \Rightarrow \quad Z_{ul} = Z_0 j\text{ctg}(\beta l) = \infty \quad \text{(1bod)}$$

e) Napon i struja na mjestu x=2.5 km

$$\beta x = \frac{5\pi}{2}, \qquad U(0) = 5\angle 0°, \qquad I(0) = \frac{U(0)}{Z_{ul}} = 0$$

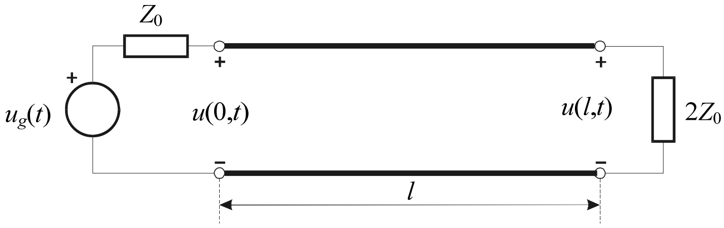
Prijenosne jednadžbe linije:

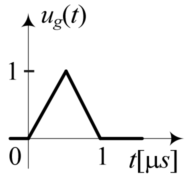
$$U(x) = U(0) \cdot \text{ch}(\gamma x) - I(0) \cdot Z_0 \cdot \text{sh}(\gamma x) = U(0) \cdot \cos(\beta x) = 5 \cdot \cos\left(\frac{\pi}{2}\right) = 0$$

$$I(x) = -\frac{U(0)}{Z_0} \cdot \text{sh}(\gamma x) + I(0) \cdot \text{ch}(\gamma x) = -j\frac{U(0)}{Z_0} \cdot \sin(\beta x) = -j\frac{5}{100} \cdot \sin\left(\frac{\pi}{2}\right) = -j0{,}05 = 0{,}05\angle -90°$$

$i(x,t) = 5\cos(2\pi f_0 t - 90°)V$ (1bod)

5. Zadana je linija s primarnim parametrima R=5Ω/km, L=0.25mH/km, G=2mS/km, C=100nF/km, duljine l=5km. Na liniju je spojen naponski izvor $u_g(t)$=$S(t)$ i serijski otpor jednak valnoj impedanciji linije Z_0. Linija je zaključena s $2Z_0$. Odrediti: a) valnu impedanciju Z_0 i koeficijent prijenosa γ; b) brzinu i vrijeme propagacije signala na liniji; c) faktor refleksije signala na ulazu Γ_1 i na izlazu Γ_2 linije; d) izraz za polazni val na mjestu x linije; e) valni oblik napona na izlazu linije $u(l, t)$.





Rješenje:

$$\frac{R}{L}=\frac{G}{C} \qquad \text{Linija bez distorzije:}$$

a) $$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{250\cdot10^{-6}}{100\cdot10^{-9}}}=50\Omega$$ **(1 bod)**

$$\gamma=\sqrt{RG}+s\sqrt{LC}=\sqrt{5\cdot2\cdot10^{-3}}+s\sqrt{0.25\cdot10^{-3}\cdot1\cdot10^{-7}}=0.1+s\cdot5\cdot10^{-6}$$

b) $$v=\frac{1}{\sqrt{LC}}=\frac{1}{5\cdot10^{-6}}=200\cdot10^{3}\,km/s$$

$$v=\frac{l}{T}\Rightarrow T=\frac{l}{v}=\frac{5}{200\cdot10^{3}}=25\mu s$$ **(1 bod)**

c) $\Gamma_1=0$ $$\Gamma_2=\frac{Z_2-Z_0}{Z_2+Z_0}=\frac{1}{3}$$ **(1 bod)**

d) Polazni val: $$U_p(x,s)=\underbrace{U(0,s)}_{\frac{U_g}{2}}\cdot e^{-\gamma\cdot x}=\frac{U_g(s)}{2}\cdot e^{-\gamma\cdot x}=\frac{U_g(s)}{2}\cdot e^{-\left(0.1+5\cdot10^{-6}s\right)x}$$

$$u_p(x,t)=e^{-0.1\cdot x}\cdot\frac{u_g\left(t-5\cdot10^{-6}\cdot x\right)}{2}$$ **(1 bod)**

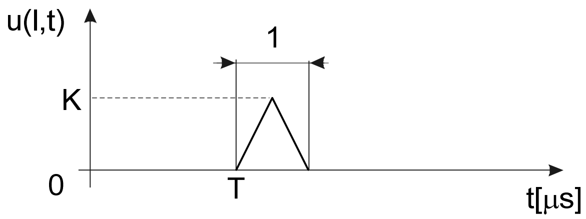
e) Polazni val: $$U_p(l,s)=\frac{U_g(s)}{2}\cdot e^{-\gamma\cdot l}=\frac{U_g(s)}{2}\cdot e^{-\left(0.1+5\cdot10^{-6}s\right)l}=\frac{U_g(s)}{2}\cdot e^{-\left(0.5+25\cdot10^{-6}s\right)}$$

Reflektirani val: $$U_r(l,s)=\Gamma_2\cdot U_p(l,s)=\Gamma_2\cdot U(0,s)\cdot e^{-\gamma\cdot l}=\Gamma_2\cdot\frac{U_g(s)}{2}\cdot e^{-\gamma\cdot l}$$

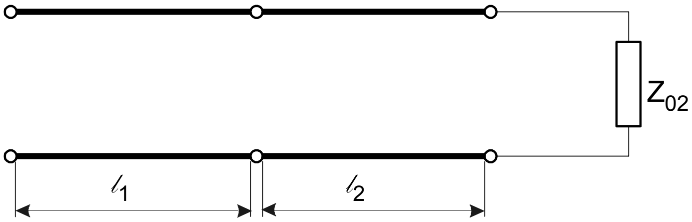
Ukupni napon na izlazu:

$$U(l,s)=U_p(l,s)+U_r(l,s)=(1+\Gamma_2)\cdot U_p(l,s)=\frac{4}{3}\cdot\frac{U_g(s)}{2}\cdot e^{-(25\cdot10^{-6}\cdot s)}\cdot e^{-0.5}\Rightarrow$$

$$u(l,t)=\frac{2}{3\sqrt{e}}u_g(t-25\cdot10^{-6})=K\cdot u_g(t-T)\Rightarrow K=\frac{2}{3\sqrt{e}}$$ **(1 bod)**



5. Na ulazu linije bez gubitaka duljine $l_1=\lambda_1/2$, s primarnim parametrima L_1=1mH/km i C_1=400nF/km, djeluje napon $u_I(0,t)=2\cos(10^4 t)$, a na izlaz je priključena linija bez gubitaka zadana sa L_2=2,25 mH/km i C_2=400 nF/km, koja je zaključena svojom karakterističnom impedancijom. Izračunati: a) valne impedancije obiju linija Z_{01} i Z_{02}; b) faktor refleksije prve linije na spojnom mjestu; c) polazni i reflektirani val struje na spojnom mjestu; d) struju $i_{II}(0,t)$ na ulazu druge linije; e) vrijednost otpora R, kojeg treba spojiti paralelno ulazu druge linije, da bi prva linija bila prilagođena.



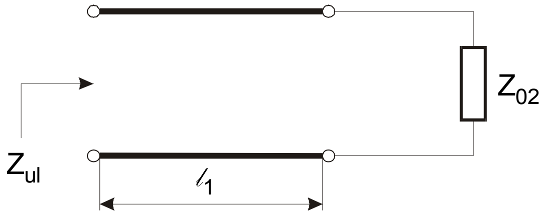
Rješenje:

a) $$Z_{01}=\sqrt{L_1/C_1}=\sqrt{10^{-3}/4\cdot10^{-7}}=50\Omega$$ **(1 bod)**

$$Z_{02}=\sqrt{L_2/C_2}=\sqrt{2{,}25\cdot10^{-3}/4\cdot10^{-7}}=75\Omega$$

b) $$\Gamma_2=\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}=\frac{25}{125}=\frac{1}{5}=0.2$$ **(1 bod)**

Za $l=\lambda/2 \Rightarrow Z_{ul}=Z_2$



$Z_{ul}=Z_{02}$

c) $$U_p\left(\frac{\lambda_1}{2}\right)=A_1e^{-j\beta_1\frac{\lambda_1}{2}}=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\beta_1\frac{\lambda_1}{2}}=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\pi}=$$

$$=\frac{U(0)+Z_{01}\dfrac{U(0)}{Z_{ul}}}{2}e^{-j\pi}=\frac{U(0)}{2}\left(1+\frac{50}{75}\right)e^{-j\pi}=-1.667\text{V}$$

$$I_p(\lambda_1/2)=\frac{U_p(\lambda_1/2)}{Z_{01}}=\frac{-1{,}667}{50}\text{A}=-0{,}03333\text{A}$$

$i_p(l_1,t)$= 0.03333(cos 10^4 t– 180°)= – 0.03333 (cos 10^4 t)

$$U_r=\Gamma_2\cdot U_p=\Gamma_2\cdot A_1e^{-j\beta_1\frac{\lambda_1}{2}}=-0.2\cdot1.667=\underline{-0.3333\text{ V}}$$

$$I_r=-\frac{U_r}{Z_{01}}=-\frac{\Gamma_2\cdot U_p}{Z_{01}}=-\frac{\Gamma_2\cdot A_1e^{-j\beta_1\frac{\lambda_1}{2}}}{Z_{01}}=-\frac{\Gamma_2\cdot A_1e^{-j\beta_1\frac{\lambda_1}{2}}}{Z_{01}}=0.006667A$$

$i_r(l_1,t)$= 0.006667 (cos 10^4 t) **(1 bod)**

d) $i_{II}(0,t)=i_p(l_1,t)+i_r(l_1,t)$= – 0.026667(cos 10^4 t) **(1 bod)**

e) $$Z_{01}=\frac{Z_{02}\cdot R}{Z_{02}+R}\quad\Rightarrow\quad R=\frac{Z_{02}\cdot Z_{01}}{Z_{02}-Z_{01}}=150\Omega$$ **(1 bod)**

5. Zadana je linija bez gubitaka s L=400 μH/km i C=40 nF/km. Na ulaz linije priključen je naponski izvor $u_g(t) = 2\sin(\omega_0 t)$, u seriju s otporom R=50Ω, a na izlazu je karakteristična impedancija Z_0. Duljina linije l jednaka je njenoj valnoj duljini na frekvenciji $\omega_0 = 2\pi 10^5$ rad/s. Odrediti:

a) karakterističnu impedanciju Z_0;
b) faktor prijenosa γ;
c) duljinu linije l u km i brzinu širenja vala po liniji v;
d) ulaznu impedanciju Z_{ul} i ulazni napon linije u(0,t);
e) napon i struju na polovini linije.

<u>Rješenje:</u>

a) $Z_0 = \sqrt{L/C} = \sqrt{4\cdot 10^{-4}/4\cdot 10^{-8}} = 100\Omega$; **(1 bod)**

b) $\gamma = j\omega_0\sqrt{LC} = j2\pi\cdot 10^5\cdot\sqrt{4\cdot 10^{-4}\cdot 4\cdot 10^{-8}} = j2\pi\cdot 0{,}4$ **(1 bod)**

c) $l = \lambda_0 = \dfrac{2\pi}{\omega_0\sqrt{LC}} = \dfrac{2\pi}{2\pi\cdot 10^5\cdot 4\cdot 10^{-6}} = 2{,}5\text{km}$;

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\cdot 10^{-4}\cdot 4\cdot 10^{-8}}} = \frac{10^6}{4} = 250\,000\,\text{km/s}$$ **(1 bod)**

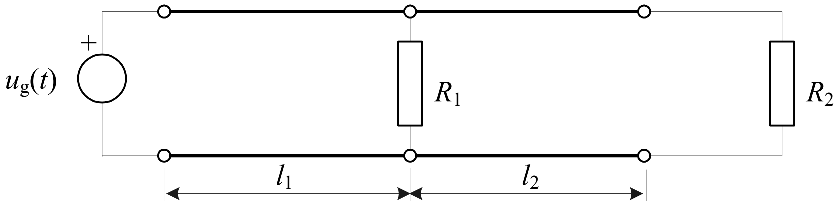
d) $Z_{ul} = \dfrac{U(0)}{I(0)} = Z_0$; $\qquad U(0) = U_g\dfrac{Z_{ul}}{R+Z_{ul}} = 2\dfrac{100}{50+100} = 1{,}3333$ **(1 bod)**

e) $U(x) = U(0)\cdot ch\,\gamma x - I(0)Z_0 sh\,\gamma x = U(0)\cdot e^{-\gamma x} = U(0)\cdot e^{-j\pi} = -U(0)$

$u(x,t) = -1{,}333\sin(\omega_0 t)$

$i(x,t) = \dfrac{u(x,t)}{Z_0} = -\dfrac{1{,}3333}{Z_0}\sin(\omega_0 t) = -0{,}013333\sin(\omega_0 t)$ **(1 bod)**

5. Na ulazu linije bez gubitaka, duljine $l_1=\lambda_1/2$, s primarnim parametrima L_1=4,5 mH/km i C_1=0.8μF/km, djeluje napon $u_g(t)=2\cos(\omega_0 t)$. Na izlaz je priključen je otpor R_1 paralelno s linijom bez gubitaka duljine $l_2=\pi/8$ km, zadanom s L_2=200 μH/km i C_2=0.08 μF/km, zaključenom otporom R_2=25Ω. Izračunati: a) valne impedancije obiju linija Z_{01} i Z_{02}, te frekvenciju ω_0 signala ako je $l_2=\lambda_2/4$; b) duljinu prve linije l_1, te koeficijente prijenosa γ_1 i γ_2; c) ulaznu impedanciju Z_{ul2} druge linije i vrijednost otpora R_1 da bi prva linija bila prilagođena na svome izlazu; d) napon $u_{II}(0,t)$ na ulazu druge linije; e) faktor refleksije i napon $u_{II}(l_2,t)$ na izlazu druge linije.



Rješenje:

a) $Z_{01}=\sqrt{L_1/C_1}=\sqrt{4{,}5\cdot10^{-3}/8\cdot10^{-7}}=75\Omega$, $Z_{02}=\sqrt{L_2/C_2}=\sqrt{2\cdot10^{-4}/8\cdot10^{-8}}=50\Omega$

$$l_2=\frac{\lambda_2}{4}=\frac{2\pi}{4\beta_2}=\frac{\pi}{2\omega_0\sqrt{L_2C_2}}=\frac{\pi}{8}km \qquad \omega_0=\frac{\pi}{2l_2\sqrt{L_2C_2}}=\frac{4}{\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}}=10^6\,\text{rad/s}$$

(1 bod)

b) $$l_1=\frac{\lambda_1}{2}=\frac{2\pi}{2\beta_1}=\frac{\pi}{\omega\sqrt{L_1C_1}}=\frac{\pi}{10^6\sqrt{4{,}5\cdot10^{-3}\cdot8\cdot10^{-7}}}=\frac{\pi}{60}\text{km}$$

$$\gamma_1=j\beta_1=j\omega\sqrt{L_1\cdot C_1}=j10^6\sqrt{4{,}5\cdot10^{-3}\cdot8\cdot10^{-7}}=j60\frac{\text{rad}}{\text{km}} \quad\Rightarrow\quad \beta_1l_1=\beta_1\frac{\lambda_1}{2}=\beta_1\frac{2\pi}{2\beta_1}=\pi$$

$$\gamma_2=j\beta_2=j\omega\sqrt{L_2\cdot C_2}=j10^6\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}=4\frac{\text{rad}}{\text{km}} \quad\Rightarrow\quad \beta_2l_2=\beta_2\frac{\lambda_2}{4}=\beta_2\frac{2\pi}{4\beta_2}=\frac{\pi}{2}$$

(1 bod)

c) $$Z_{ul2}=\frac{R_2ch(\gamma_2l_2)+Z_{02}sh(\gamma_2l_2)}{\frac{R_2}{Z_{02}}sh(\gamma_2l_2)+ch(\gamma_2l_2)}=Z_{02}\frac{R_2\cos(\beta_2l_2)+jZ_{02}\sin(\beta_2l_2)}{R_2j\sin(\beta_2l_2)+Z_{02}\cos(\beta_2l_2)}=\frac{Z_{02}^2}{R_2}=\frac{2500}{25}=100\Omega$$

$$Z_{ul2}=Z_{01}=\frac{Z_{02}^2}{R_2} \qquad \frac{1}{Z_{01}}=\frac{1}{Z_{ul2}}+\frac{1}{R_1}=\frac{R_2}{Z_{02}^2}+\frac{1}{R_1}\Rightarrow\frac{1}{R_1}=\frac{1}{Z_{01}}-\frac{1}{Z_{ul2}}=\frac{1}{75}-\frac{1}{100}=\frac{1}{300}$$

$$R_1=300\Omega$$

(1 bod)

d) $$U_2(0)=U_1(0)e^{-j\beta_1l_1}=U_1(0)e^{-j\pi}=-2 \qquad u_2(0,t)=-2\cos(\omega t)$$

(1 bod)

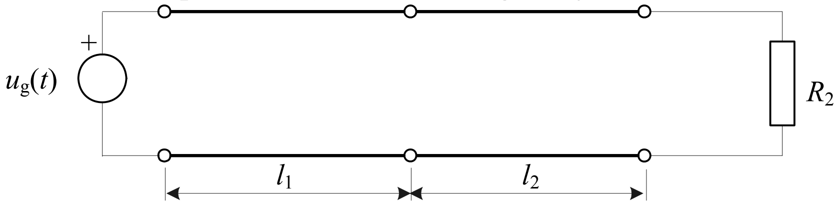
e) $$\Gamma_2=\frac{R_2-Z_{02}}{R_2+Z_{02}}=\frac{25-50}{25+50}=-\frac{25}{75}=-0{,}3333 \qquad I_2(0)=\frac{U_2(0)}{Z_{ul2}}=\frac{2}{100}=0{,}02A$$

$$U_2(l)=U_2(0)ch(j\beta_2l_2)-I_2(0)Z_{02}sh(j\beta_2l_2)=-U_1(0)\left(\cos\left(\frac{\pi}{2}\right)-j\frac{R_2}{Z_{02}}\sin\left(\frac{\pi}{2}\right)\right)=0{,}5jU_1(0)$$

$$u_2(l,t)=\cos\left(\omega t+\frac{\pi}{2}\right)$$

(1 bod)

5. Na ulazu linije bez gubitaka, duljine $l_1=\lambda_1/2$, s primarnim parametrima L_1=0.2 mH/km i C_1=80nF/km, djeluje napon $u_g(t)=2\cos(10^6t)$. Na izlaz je priključena linija bez gubitaka duljine $l_2=\lambda_2/4$, zadana sa L_2=4,5 mH/km i C_2=800 nF/km, zaključena otporom R_2. Izračunati: a) valne impedancije obiju linija Z_{01} i Z_{02}, te koeficijente prijenosa γ_1 i γ_2; b) duljine obiju linija l_1 i l_2; c) vrijednost otpora R_2 da bi prva linija bila prilagođena na svome izlazu; d) napon $u_{II}(0,t)$ na ulazu; e) napon $u_{II}(l_2,t)$ na izlazu druge linije.



Rješenje:

a) $$Z_{01}=\sqrt{L_1/C_1}=\sqrt{2\cdot10^{-4}/8\cdot10^{-8}}=50\Omega,\ Z_{02}=\sqrt{L_2/C_2}=\sqrt{4{,}5\cdot10^{-3}/8\cdot10^{-7}}=75\Omega$$

$$\gamma_{01}=j\omega\sqrt{L_1\cdot C_1}=j10^6\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}=j4\text{ rad/s/km}\Rightarrow\beta_1=4\text{ rad/s/km}$$

$$\gamma_{02}=j\omega\sqrt{L_2\cdot C_2}=j10^6\sqrt{4{,}5\cdot10^{-3}\cdot8\cdot10^{-7}}=j60\text{ rad/s/km}\Rightarrow\beta_2=60\text{ rad/s/km}$$

(1 bod)

b) $$l_1=\frac{\lambda_1}{2}=\frac{2\pi}{2\beta_1}=\frac{\pi}{\omega\sqrt{L_1C_1}}=\frac{\pi}{10^6\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}}=\frac{\pi}{4}km$$

$$l_2=\frac{\lambda_2}{4}=\frac{2\pi}{4\beta_2}=\frac{\pi}{2\omega\sqrt{L_2C_2}}=\frac{\pi}{2\cdot10^6\sqrt{4{,}5\cdot10^{-3}\cdot8\cdot10^{-7}}}=\frac{\pi}{120}km\ (1\text{ bod})$$

c) $$\beta_1l_1=\beta_1\frac{\lambda_1}{2}=\beta_1\frac{2\pi}{2\beta_1}=\pi\qquad\qquad\beta_2l_2=\beta_2\frac{\lambda_2}{4}=\beta_2\frac{2\pi}{4\beta_2}=\frac{\pi}{2}$$

$$Z_{ul2}=\frac{R_2ch(\gamma_2l_2)+Z_{02}sh(\gamma_2l_2)}{\frac{R_2}{Z_{02}}sh(\gamma_2l_2)+ch(\gamma_2l_2)}=Z_{02}\frac{R_2\cos(\beta_2l_2)+jZ_{02}\sin(\beta_2l_2)}{R_2j\sin(\beta_2l_2)+Z_{02}\cos(\beta_2l_2)}=\frac{Z_{02}^2}{R_2}$$

$$Z_{ul2}=Z_{01}=\frac{Z_{02}^2}{R_2}\qquad\Rightarrow\qquad R_2=\frac{Z_{02}^2}{Z_{01}}=\frac{75^2}{50}=112{,}5\Omega\ (1\text{ bod})$$

d) $$U_2(0)=U_1(0)e^{-j\beta_1l_1}=U_1(0)e^{-j\pi}=-U_1(0)\qquad\Rightarrow\qquad u_2(0,t)=-2\cos(\omega t)(1\text{ bod})$$

$$I_2(0)=\frac{U_2(0)}{Z_{ul2}}=\frac{U_2(0)}{Z_{02}^2}R_2$$

e) $$U_2(l)=U_2(0)ch(j\beta_2l_2)-I_2(0)Z_{02}sh(j\beta_2l_2)=-U_1(0)\left(\cos\left(\frac{\pi}{2}\right)-j\frac{R_2}{Z_{02}}\sin\left(\frac{\pi}{2}\right)\right)=1{,}5jU_1(0)$$

$$u_2(l,t)=3\cos\left(\omega t+\frac{\pi}{2}\right)(1\text{ bod})$$

Rješenje: Liniju ćemo analizirati kao dvije linije s istim primarnim parametrima, spojene u kaskadu

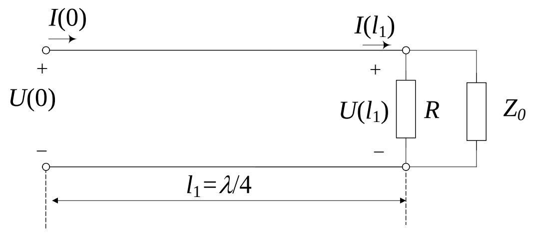
a) Linija bez gubitaka $\rightarrow Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2\cdot 10^{-3}}{6\cdot 10^{-9}}} = \frac{10^3}{\sqrt{3}} = 577{,}35\Omega$

$$\gamma = j\beta = j\omega\sqrt{LC} = j2\cdot\sqrt{3}\cdot 10^{-2} = j0{,}0346/\text{km}$$

Duljina linije: $l = \frac{\lambda}{4} + 40 = \frac{2\pi}{4\beta} + 40 = \frac{2\pi}{4\cdot 2\cdot\sqrt{3}\cdot 10^{-2}} + 40 = 45{,}345 + 40 = 85{,}345\text{km}$ (1bod)

b) Ulazna impedancija linije.

Ulazna impedancija druge linije: $Z_{ul2} = Z_0$



Ukupna impedancija na kraju prve linije: $Z_2 = \frac{R\cdot Z_{ul2}}{R + Z_{ul2}} = \frac{R\cdot Z_0}{R+Z_0} = 85{,}25\Omega$

Ulazna impedancija prve linije: $Z_{ul1} = \frac{U(0)}{I(0)} = \frac{U(l_1)ch(\gamma l_1) + I(l_1)Z_0 sh(\gamma l_1)}{\frac{U(l_1)}{Z_0}sh(\gamma l_1) + I(l_1)Z_0 ch(\gamma l_1)}$

$$\gamma\cdot l_1 = j\beta\cdot l_1 = j\beta\frac{\lambda}{4} = j\frac{\pi}{2}$$

$$Z_{ul1} = \frac{U(l_1)\cos(\beta l_1) + jI(l_1)Z_0\sin(\beta l_1)}{j\frac{U(l_1)}{Z_0}\sin(\beta l_1) + I(l_1)Z_0\cos(\beta l_1)} = Z_0^2\frac{I(l_1)}{U(l_1)} = \frac{Z_0^2}{Z_2} = 3910\Omega$$

(1bod)

c) Napon na kraju prve linije (prva linija je zaključena sa $Z_2 \neq Z_0$):

$$U(l_1) = U(0)\cos(\beta l_1) - jI(0)Z_0\sin(\beta l_1) = -j\frac{U(0)}{Z_{ul1}}Z_0 = -j0.59$$

$$U(l_1) = 0{,}59\cdot e^{-j\pi/2}$$

$$u(l_1,t) = 0{,}59\cdot\cos(10^4 t - 1{,}57) = 0{,}59\cdot\cos(10^4 t - \pi/2)$$

(1bod)

d) Napon na kraju druge linije (druga linija je zaključena sa Z_0):

$$U(l) = U(l_2) = U(l_1)e^{-\gamma l_2} = U(l_1)e^{-j\beta\cdot 40} = U(l_1)e^{-j1{,}386} = 0{,}59\cdot e^{-j\pi/2}\cdot e^{-j1{,}386}$$

$$U(l) = 0{,}59\cdot e^{-j2{,}95}$$

$$u(l,t) = 0{,}59\cdot\cos(10^4 t - 2{,}95) = 0{,}59\cdot\cos(10^4 t - 0{,}94\pi)$$

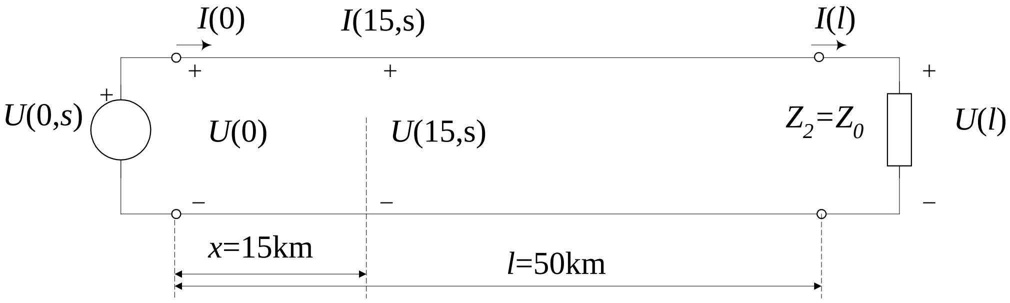
(1bod)

e) Struja na kraju druge linije:

$$i(l,t) = \frac{u(l,t)}{Z_0} = 1{,}02\cdot 10^{-3}\cos(10^4 t - 2{,}95) = 1{,}02\cdot 10^{-3}\cdot\cos(10^4 t - 0{,}94\pi)$$

(1bod)

1. Zadana je linija duljine l=50 km. Primarni parametri linije su R=5,4 Ω/km, L=2mH/km, G=1μS/km i C=6nF/km. Odrediti iznos napona i struje na 15 km od početka linije ako je linija zaključena impedancijom Z_2=Z_0, a napon na ulazu linije je $u(0,t)$= 10 cos(5 10^3t).



Rješenje: Linija je prilagođena $\rightarrow$ $Z_2 = Z_0$ $\Rightarrow$ $Z_{ul1} = Z_0$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{5{,}4 + j10}{10^{-6}(1 + j30)}} = \sqrt{10^5(3{,}39 - j1{,}69)} = 10^2(5{,}99 - j1{,}41)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = 10^{-3}\sqrt{(5{,}4 + j10)(1 + j30)} = 0{,}0048 + j0{,}0178$$

$$U(0) = U(x)\cdot ch\gamma x + I(x)Z_0 sh\gamma x$$

$$I(0) = \frac{U(x)}{Z_0} sh\gamma x + I(x) ch\gamma x$$

$$U(x) = U(0)\cdot ch\gamma x - I(0)Z_0 sh\gamma x$$

$$I(x) = -\frac{U(0)}{Z_0} sh\gamma x + I(0) ch\gamma x$$

$$U(0) = Z_0 I(0)$$

$$U(x) = U(0)\cdot(ch\gamma x - sh\gamma x) = U(0)\cdot e^{-\gamma x}$$

$$I(x) = \frac{U(0)}{Z_0}(-sh\gamma x + ch\gamma x) = \frac{U(0)}{Z_0} e^{-\gamma x}$$

$$U(15) = 10\cdot e^{-\gamma 15} = 8{,}071 - j2{,}458 = 9{,}3\cdot e^{-j15^\circ}\,[V]$$

$$I(15) = \frac{10}{Z_0} e^{-\gamma 15} = 0{,}015 - j0{,}00055 = 15{,}1\cdot e^{-j2^\circ}\,[mA]$$

$$u(15,t) = 9{,}3\cdot\cos(\omega t - 15^\circ)[V]$$
$$i(15,t) = 15{,}1\cdot\cos(\omega t - 2^\circ)[mA]$$

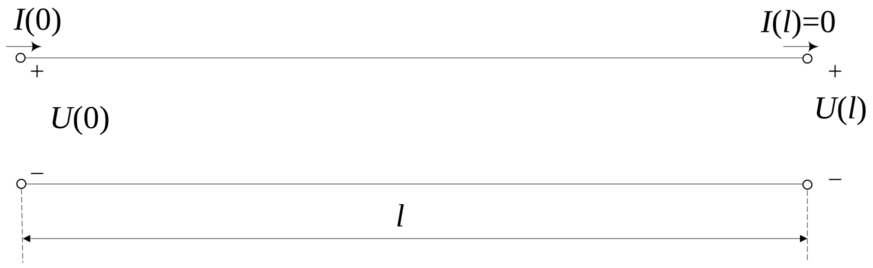
2. Zadana je linija bez gubitaka s L=4 mH/km i C=8 nF/km.

a. Koliko najmanje mora biti duga ova linija da kod ω=10^6 rad/s ulazna impedancija bude jednaka nuli, kad je suprotni kraj otvoren?

b. Koliki su $u(0,t)$, $u(l,t)$ i $i(l,t)$ na toj liniji ako je $i(0,t) = 5\cos(10^6\, t)$?

c. Odrediti brzinu širenja signala po liniji,

d. Odrediti valnu duljinu signala.



<u>Rješenje:</u> Linija bez gubitaka $\rightarrow$ $R = 0$, $G = 0$ $\quad\Rightarrow\quad$ $Z_0 = \sqrt{L/C}$, $\gamma = s\sqrt{LC}$

Stac. sinusna pobuda $\rightarrow$ $s = j\omega$ $\quad\Rightarrow\quad$ $\gamma = j\omega\sqrt{LC} = j\beta$

a) $I(l) = 0$

$$U(0) = U(l)\cdot ch\gamma\, l + I(l)Z_0 sh\gamma\, l = U(l)\cdot ch\gamma\, l$$

$$I(0) = \frac{U(l)}{Z_0} sh\gamma\, l + I(l) ch\gamma\, l = \frac{U(l)}{Z_0} sh\gamma\, l$$

$$Z_{ul1} = \frac{U(0)}{I(0)} = Z_0 \frac{\mathrm{ch}(\gamma\, l)}{\mathrm{sh}(\gamma\, l)} = 0 \qquad \Rightarrow \qquad \mathrm{ch}(\gamma\cdot l) = 0$$

$$\mathrm{ch}(\gamma\cdot l) = \mathrm{ch}(j\beta\cdot l) = \cos(\beta\cdot l) = \cos\left(\omega\sqrt{LC}\cdot l\right) = 0 \;\Rightarrow\; \omega\sqrt{LC}\, l = \pi/2 \;\Rightarrow\; l = \frac{\pi}{2\omega\sqrt{LC}} = \frac{\pi}{8\sqrt{2}}\,\mathrm{km}$$

b)

$$u(0,t) = 0$$

$$i(0,t) = 5\cos(10^6 t)$$

$$i(l,t) = 0$$

$$U(l) = U(0)\cdot ch(\gamma\, l) - I(0)Z_0 sh(\gamma\, l) = -I(0)Z_0 \mathrm{sh}(\,j\beta l) = -I(0)Z_0\, j\sin(\pi/2)$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{10^3}{\sqrt{2}}$$

$$U(l) = -jI(0)Z_0 = -j\frac{5\cdot 10^3}{\sqrt{2}} = \frac{5\cdot 10^3}{\sqrt{2}} e^{-j\pi/2}$$

$$u(l,t) = \frac{5\cdot 10^3}{\sqrt{2}} \cos\left(10^6 t - \frac{\pi}{2}\right)$$

c)
$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\cdot 10^{-3}\cdot 8\cdot 10^{-9}}} = \frac{10^6}{\sqrt{32}} = 176.776{,}69\ \mathrm{km/s}$$

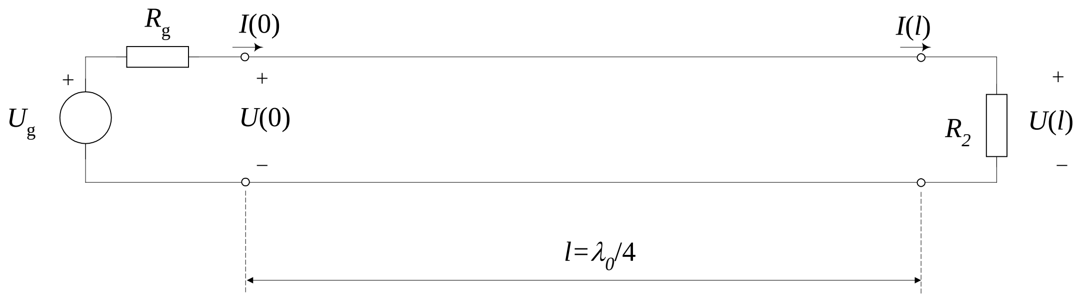
d)
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{2\pi}{10^6\cdot\sqrt{4\cdot 10^{-3}\cdot 8\cdot 10^{-9}}} = \frac{2\pi}{\sqrt{32}} = 1{,}1107\ \mathrm{km}$$

3. Zadana je linija bez gubitaka s L=0,8 mH/km, C=80 nF/km i $l=\lambda_0/4$ kod ω_0. Na ulaz linije priključen je generator napona $u_g(t)$ s unutarnjim otporom R_g, a na kraju linije je otpor R_2=1kΩ. Na frekvenciji $\omega_0=10^5$ rad/s ulazna impedancija je prilagođena na Rg.

a. Koliki je Rg?

b. Koliko je duga linija?

c. Odrediti $u(l,t)$ i $i(l,t)$ na toj liniji ako je $u_g(t) = 4\cos(\omega_0 t)$.



Rješenje:

Linija bez gubitaka $\rightarrow$ $R = 0$, $G = 0$ $\Rightarrow$ $Z_0 = \sqrt{L/C}$

$\gamma = s\sqrt{LC}$ Stac. sinusna pobuda $\rightarrow$ $s = j\omega$ $\Rightarrow$ $\gamma = j\omega\sqrt{LC} = j\beta$

a) $U(l) = R_2 I(l)$

$$U(0) = U(l)\cdot ch(\gamma l) + I(l) Z_0 sh(\gamma l) = I(l)\cdot\left(R_2 ch(\gamma l) + Z_0 sh(\gamma l)\right)$$

$$I(0) = \frac{U(l)}{Z_0} sh(\gamma l) + I(l) ch(\gamma l) = I(l)\left(\frac{R_2}{Z_0} sh(\gamma l) + ch(\gamma l)\right)$$

$$Z_{ul1} = \frac{R_2 ch(\gamma l) + Z_0 sh(\gamma l)}{\frac{R_2}{Z_0} sh(\gamma l) + ch(\gamma l)}$$

$$l = \frac{\lambda_0}{4} = \frac{2\pi}{\beta}\cdot\frac{1}{4} = \frac{\pi}{2\beta}$$

$$\gamma\cdot l = j\beta\cdot l = j\frac{\pi}{2}$$

$$sh(\gamma l) = sh\left(j\frac{\pi}{2}\right) = j\sin\left(\frac{\pi}{2}\right) = j$$

$$ch(\gamma l) = ch\left(j\frac{\pi}{2}\right) = j\cos\left(\frac{\pi}{2}\right) = 0$$

$$R_g = Z_{ul1} = \frac{R_2\cos(\pi/2) + Z_0 j\sin(\pi/2)}{\frac{R_2}{Z_0} j\sin(\pi/2) + \cos(\pi/2)} = \frac{Z_0^2}{R_2} = \frac{L}{CR_2} = \frac{8\cdot 10^{-4}}{8\cdot 10^{-8}\cdot 10^{3}} = 10\,\Omega$$

b) $$l = \frac{\lambda_0}{4} = \frac{2\pi}{\beta}\cdot\frac{1}{4} = \frac{\pi}{2\omega_0\sqrt{LC}} = \frac{\pi}{2\cdot 10^5\sqrt{8\cdot 10^{-4}\cdot 8\cdot 10^{-8}}} = \frac{\pi}{1{,}6} = 1{,}9634\text{ km}$$

c)

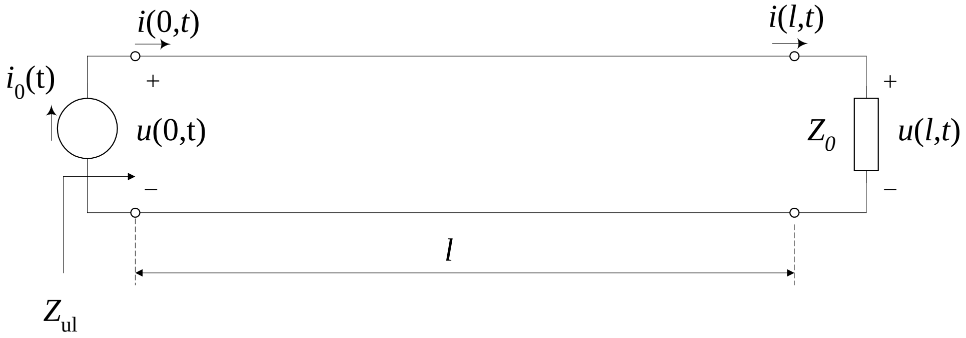
$$U(l) = U(0)\cdot ch(\gamma l) - I(0) Z_0 sh(\gamma l) = -jI(0)\cdot Z_0 = -j\frac{U_g}{R_g + Z_{ul1}}\cdot Z_0 = -j\frac{U_g}{2R_g}\cdot Z_0 = -j20 = 20e^{-j\frac{\pi}{2}}$$

$$I(l) = -\frac{U(0)}{Z_0}\cdot sh(\gamma l) - I(0) ch(\gamma l) = -j\frac{U(0)}{Z_0} = -j\frac{U_g}{Z_0}\cdot\frac{Z_{ul1}}{R_g + Z_{ul1}} = -j\frac{U_g}{2Z_0} = -j0{,}02 = 0{,}02e^{-j\frac{\pi}{2}}$$

$$u(l,t) = 20\cos\left(\omega t - 90^\circ\right)\text{V}$$

$$i(l,t) = 0{,}02\cos\left(\omega t - 90^\circ\right)\text{A}$$

4. Zadana je linija duljine l=100 km. Primarni parametri linije su R=1 Ω/km, L=3mH/km, G=3μS/km i C=9nF/km. Odrediti izraz za napon na izlazu linije ako je linija zaključena svojom karakterističnom impedancijom Z_0, a na ulazu linije je strujni izvor $i(t)=\delta(t)$.



Rješenje:

Linija bez distorzije: $\rightarrow\ R/L = G/C \ \Rightarrow \quad Z_0=\sqrt{L/C}=578\,\Omega,$

$$\gamma=\sqrt{RG}+s\sqrt{LC}=\sqrt{3}\cdot 10^{-3}+s3\sqrt{3}\cdot 10^{-6}$$

Linija je prilagođena $\rightarrow\ Z_2=Z_0$

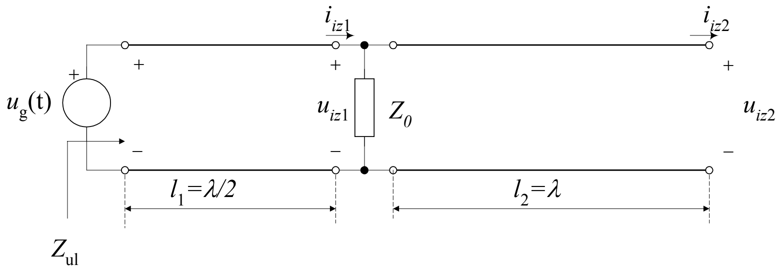
$$U(l,s)=U(0,s)\cdot ch(\gamma l)-I(0,s)Z_0 sh(\gamma l)=I(0,s)\cdot Z_0\big(ch(\gamma l)-sh(\gamma l)\big)=I(0,s)\cdot Z_0 e^{-\gamma l}$$

$$U(l,s)=I(0,s)\cdot Z_0 e^{-\gamma l}=\frac{10^3}{\sqrt{3}}e^{-\left(0,1\sqrt{3}+s3\sqrt{3}\cdot 10^{-4}\right)}=\frac{10^3}{\sqrt{3}}e^{-0,1\sqrt{3}}\cdot e^{-s3\sqrt{3}\cdot 10^{-4}}$$

$$u(l,t)=\frac{10^3}{\sqrt{3}}e^{-0,1\sqrt{3}}\delta\left(t-3\sqrt{3}\cdot 10^{-4}\right)=485\cdot\delta\left(t-3\sqrt{3}\cdot 10^{-4}\right)$$

Linije

9. Zadan je spoj dviju linija bez gubitaka s primarnim parametrima C=1nF/km, L=1mH/km, duljina $\ell_1=\lambda/2$ i $\ell_2=\lambda$ prema slici. Odrediti valne oblike napona i struje na krajevima linija u_{izli} i i_{izli}; (i=1,2) ako je zadano $u_g(t)=\cos(2\pi\ 10^3 t)$; $-\infty<t<\infty$.



$$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{1\cdot 10^{-3}}{1\cdot 10^{-9}}}=\sqrt{10^6}=10^3\,\Omega \qquad \gamma=\alpha+j\beta;\quad \alpha=0,\ \beta=\omega_0\sqrt{LC}$$

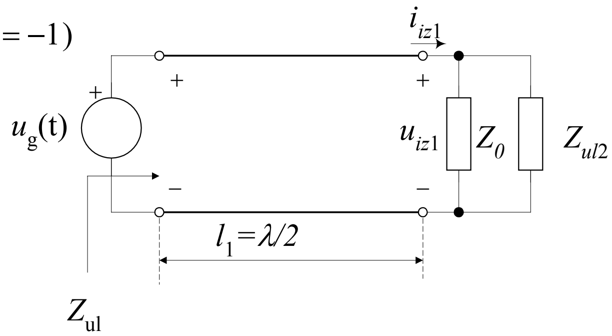
ω_0 - frekvencija sinusne pobude; $\lambda=2\pi/\beta$ - valna duljina signala

Prijenosne jednadžbe linije :

$$U(0)=U(l)\mathrm{ch}(\gamma l)+Z_0 I(l)\mathrm{sh}(\gamma l)=U(l)\cos(\beta l)+Z_0 I(l)j\sin(\beta l)$$

$$I(0)=\frac{U(l)}{Z_0}j\mathrm{sh}(\gamma l)+I(l)\mathrm{ch}(\gamma l)=\frac{U(l)}{Z_0}j\sin(\beta l)+I(l)\cos(\beta l)$$

($\mathrm{ch}\,jx=\cos x$, $\mathrm{sh}\,jx=j\sin x$, $\sin\pi=0$, $\cos\pi=-1$)



a) prva linija: $l_1=\dfrac{\lambda}{2}=\dfrac{\pi}{\beta}\ \Rightarrow\ \beta\cdot l_1=\pi$

$$U^I(0)=-U^I(l_1)$$
$$\underline{I^I(0)=-I^I(l_1)}$$
$$U^I(l_1)=(Z_0\,\|\,Z_{ul2})I^I(l_1)$$

b) druga linija: $l_2=\lambda=\dfrac{2\pi}{\beta}\ \Rightarrow\ \beta\cdot l_2=2\pi$

$$Z_{ul2}=\frac{U^{II}(0)}{I^{II}(0)}=\frac{U^{II}(l)\mathrm{ch}(\gamma l_2)+Z_0 I^{II}(l)\mathrm{sh}(\gamma l_2)}{\frac{U^{II}(l)}{Z_0}\mathrm{sh}(\gamma l_2)+I^{II}(l)\mathrm{ch}(\gamma l_2)}=\frac{Z_2\cos(\beta l_2)+jZ_0\sin(\beta l_2)}{j\frac{Z_2}{Z_0}\sin(\beta l_2)+\cos(\beta l_2)}=Z_2$$

($\mathrm{ch}\,jx=\cos x$, $\mathrm{sh}\,jx=j\sin x$, $\sin 2\pi=0$, $\cos 2\pi=1$)

$Z_2=\infty$ pa je $Z_{ul2}=\infty$

$$U^I(l_1)=Z_0 I^I(l_1)$$

$$U^I(l_1)=-U^I(0)=-1=1\cdot e^{j\pi}\ \Rightarrow\ \underline{u_l^I(t)=u_{izl1}(t)=-\cos(2\pi\cdot 10^3 t)}$$

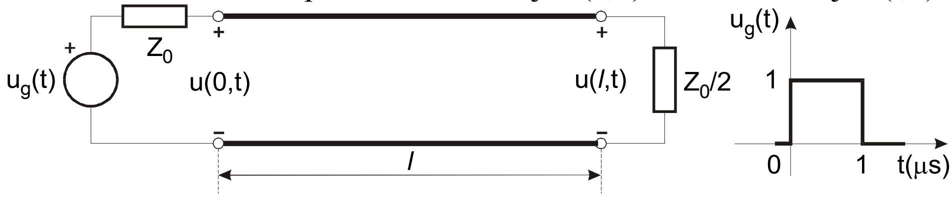
$$\underline{I^I(l_1)=\frac{U^I(l_1)}{Z_0}}=-10^{-3}=10^{-3}\cdot e^{j\pi}\ \Rightarrow\ \underline{i_l^I(t)=i_{izl1}(t)=-10^{-3}\cos(2\pi\cdot 10^3 t)}$$

$$Z_{ul}^I=\frac{U^I(0)}{I^I(0)}=Z_0$$

$$U^{II}(l_2)=U^{II}(0)\cdot\underbrace{\cos 2\pi}_{1}-I^{II}(0)\cdot Z_0 j\underbrace{\sin 2\pi}_{0}=U^{II}(0)=U^I(l_1)$$

$$\underline{u_{izl2}(t)=u_{izl1}(t)=-\cos(2\pi\cdot 10^3 t)} \qquad \underline{i_{izl2}(t)=0}$$

10. Zadana je linija s primarnim parametrima R=0.5Ω/km, L=10μH/km, G=0,2mS/km, C=4nF/km, duljine l=1km. Na liniju je spojen generator $u_g(t)$ s unutarnjim otporom jednakim zrcalnoj impedanciji linije Z_0 i valnim oblikom prema slici, a linija je zaključena s $Z_0/2$. Odrediti i nacrtati valne oblike napona na ulazu linije $u(0, t)$ i na izlazu linije $u(l, t)$.



Rješenje:

$$\frac{R}{L}=\frac{0.5\Omega}{10\mu H}=\frac{0.5}{10\cdot10^{-6}}=5\cdot10^{4} \qquad \frac{G}{C}=\frac{0{,}2mS}{4nF}=5\cdot10^{4} \Rightarrow \frac{R}{L}=\frac{G}{C}$$

Linija bez distorzije:

$$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{10\cdot10^{-6}}{4\cdot10^{-9}}}=50\Omega \qquad \gamma=s\sqrt{LC}+\sqrt{RG}=20\cdot10^{-6}s+0{,}01$$

Polazni val na izlazu: $U_p(l)=\underbrace{U(0)}_{\frac{U_g}{2}}\cdot e^{-\gamma\cdot l}=\underbrace{U(0)}_{\frac{U_g}{2}}\cdot e^{-(20\cdot10^{-6}\cdot s)}\cdot e^{-0{,}01}$

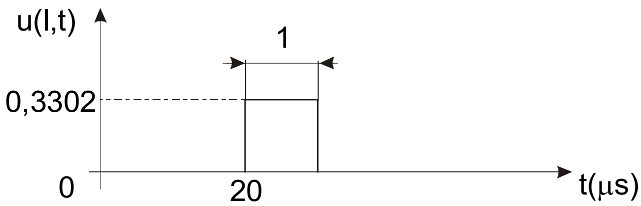
faktor refleksije na izlazu: $\Gamma_2=\frac{Z_2-Z_0}{Z_2+Z_0}=\frac{\frac{Z_0}{2}-Z_0}{\frac{Z_0}{2}+Z_0}=-\frac{1}{3}$

reflektirani val na izlazu: $U_r(l)=\Gamma_2\cdot U_p(l)=\Gamma_2\cdot\frac{U_g(s)}{2}\cdot e^{-(20\cdot10^{-6}\cdot s)}\cdot e^{-0{,}01}$

ukupni napon na izlazu:

$$U_{izl}=U_p(l)+U_r(l)=(1+\Gamma_2)\cdot U_p(l)=(1+\Gamma_2)\cdot\frac{U_g(s)}{2}\cdot e^{-(20\cdot10^{-6}\cdot s)}\cdot e^{-0{,}01}=\frac{2}{3}\cdot\frac{U_g(s)}{2}\cdot e^{-(20\cdot10^{-6}\cdot s)}\cdot e^{-0{,}01}$$

$$\Rightarrow u_{izl}(t)=\frac{1}{3e^{0{,}01}}\cdot u_g(t-20\cdot10^{-6}), \qquad e^{0{,}01}=1{,}01005$$



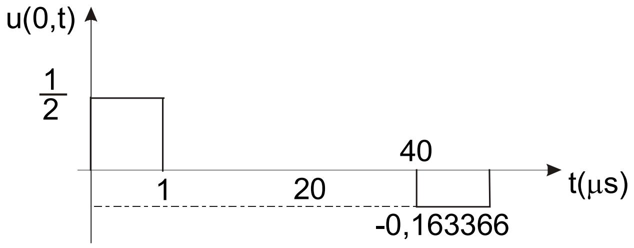
Rreflektirani val na ulazu:

$$U_r(0)=U_r(l)\cdot e^{-(20\cdot10^{-6}\cdot s+0{,}01)}=\Gamma_2\cdot U_p(l)\cdot e^{-(20\cdot10^{-6}\cdot s+0{,}01)}=-\frac{1}{3}\cdot\frac{U_g(s)}{2e^{0{,}01}}\cdot e^{-(20\cdot10^{-6}\cdot s)}\cdot e^{-(20\cdot10^{-6}\cdot s)}\cdot e^{-0{,}01}$$

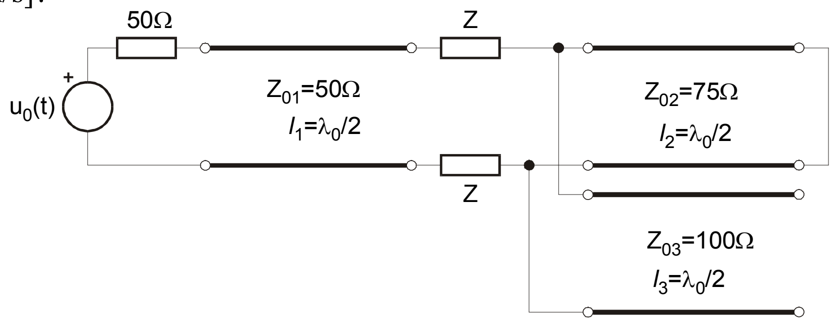
$$U_r(0)=-\frac{1}{6e^{0{,}02}}\cdot U_g(s)\cdot e^{-(40\cdot10^{-6}\cdot s)} \Rightarrow u_r(0,t)=-\frac{1}{6e^{0{,}02}}\cdot u_g(t-40\cdot10^{-6})$$

Ukupni napon na ulazu :

$$u(0,t)=\frac{u_g(t)}{2}-\frac{1}{6e^{0{,}02}}\cdot u_g(t-40\cdot10^{-6}), \qquad e^{0{,}02}=1{,}0202$$



11. Zadan je sustav linija bez gubitaka prikazan slikom. Odrediti impedanciju Z da bi prva linija bila prilagođena po zrcalnim impedancijama. Odrediti napone na kraju svake linije ako je u_0=sin ($4\pi\ 10^5\ t$)[mV]. Koliko su duge linije ako je brzina širenja vala na linijama $v=4\cdot10^5$[km/s]?



Rješenje:

linija bez gubitaka uz sinusoidalnu pobudu

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \gamma = \alpha + j\beta \Rightarrow \alpha = 0\,,\ \beta = \omega_0\sqrt{LC} \qquad \lambda = \frac{v}{f_0} = \frac{2\cdot\pi\cdot v}{\omega_0}$$

$f_0 = \dfrac{\omega_0}{2\pi}$ frekvencija signala

$v = \dfrac{\omega}{\beta}$ brzina širenja vala duž linije

$\lambda = \dfrac{2\pi}{\beta}$ valna dužina

$$\Rightarrow \lambda_0 = \frac{2\pi}{\omega_0\sqrt{LC}} \Rightarrow \lambda_0 = \frac{2\pi\cdot(4\cdot10^{-5})}{4\pi\cdot10^5} = 2km$$

$$l_1 = l_2 = l_3 = \frac{\lambda_0}{2} = 1km$$

prijenosne jednadžbe linije:

$$U(0) = U(x)\cdot ch(\gamma x) + I(x)Z_0 sh(\gamma x) = U(x)\cdot\cos(\beta x) + jI(x)Z_0\sin(\beta x)$$

$$I(0) = \frac{U(x)}{Z_0}sh(\gamma x) + I(x)ch(\gamma x) = j\frac{U(x)}{Z_0}\sin(\beta x) + I(x)\cos(\beta x)$$

---

ako je $x = l \Rightarrow \gamma x = \gamma l = g$ i ako je linija zaključena impedancijom $Z_2 = \dfrac{U(l)}{I(l)}$ tada je ulazna impedancija $Z_{ul} = \dfrac{U(0)}{I(0)} = \dfrac{Z_2 ch\,g + Z_0 sh\,g}{\dfrac{Z_2}{Z_0}sh\,g + ch\,g}$

općenito vrijedi: $ch\, jx = \cos x$

$sh\, jx = j \sin x$

ako je: $l = \dfrac{\lambda_0}{2} \Rightarrow g = \gamma \cdot l = j\beta \cdot \dfrac{1}{2} \cdot \dfrac{2\pi}{\beta} = j\dfrac{1}{2} \cdot 2\pi = j\pi$

$ch\, g = ch\, j\pi = \cos \pi = -1$

$sh\, g = sh\, j\pi = j \sin \pi = 0$

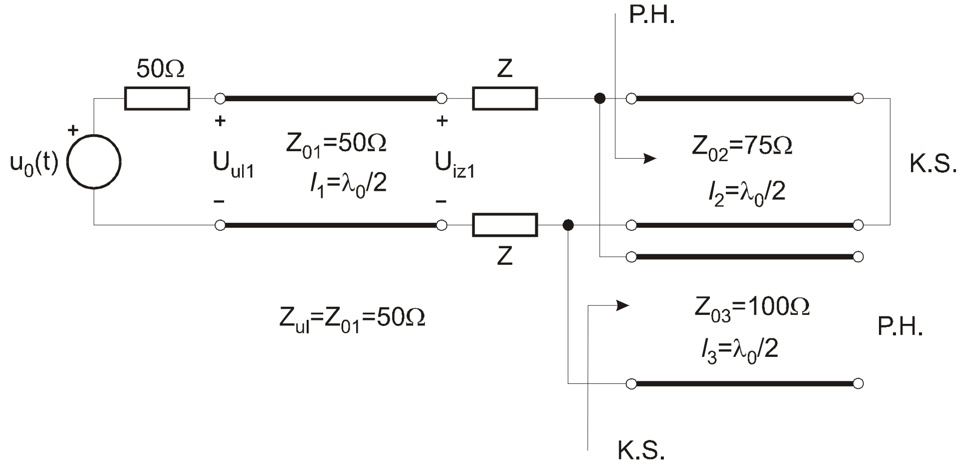
$$Z_{ul} = \frac{Z_2 \cdot (-1) + Z_0 \cdot 0}{\dfrac{Z_2}{Z_0} \cdot 0 + (-1)} = Z_2$$

imamo slijedeće slučajeve:

a) $Z_2 = \infty \Rightarrow \quad Z_{ul} = \infty$

b) $Z_2 = 0 \Rightarrow \quad Z_{ul} = 0$

c) $Z_2 = Z_0 \Rightarrow \quad Z_{ul} = Z_0$



$$Z = \frac{Z_0}{2} = 25\Omega$$

prijenosna funkcija na prvoj liniji uz zaključenje $Z_2 = Z_0 = \dfrac{U_{iz1}}{I_{iz1}} = 50\Omega$

$$U(0) = U(l) ch\gamma l + \underbrace{I(l) \cdot Z_0}_{U(l)} \cdot sh\gamma l = U(l) \cdot (ch\gamma l + sh\gamma l) = U(l) \cdot e^{g}$$

$T(s) = \dfrac{U(l)}{U(0)} = e^{-g}; \qquad g = j\pi$, dolazi do zakreta faze za $-\pi$, a gušenja nema

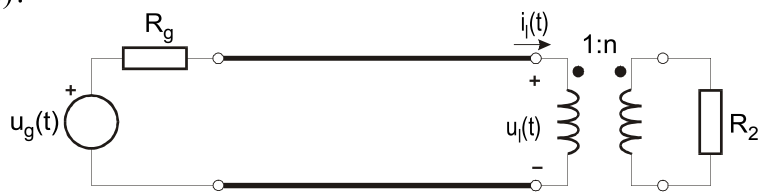
$$sh\, x = \frac{e^{-x} - e^{-x}}{2}; \qquad ch\, x = \frac{e^{x} + e^{-x}}{2} \Rightarrow \qquad sh\, x + ch\, x = e^{x}$$

$$U_{ul1} = \frac{U_0}{2}$$

$$U_{iz1} = U_{ul1} \cdot e^{-j\pi} = \frac{U_0}{2} \cdot e^{-j\pi}$$

$$u_{iz1}(t) = \frac{1}{2} \sin(4\pi \cdot 10^5 t - \pi) [mV] \qquad u_{iz2}(t) = u_{iz3}(t) = 0 [V]$$

12. Zadana je linija bez gubitaka s primarnim parametrima L=1mH/km i C=400nF/km duljine l=314m prema slici. Na ulaz je spojen naponski izvor $u_g(t)=10\sin 10^6 t$ unutrašnjeg otpora jednakog karakterističnoj impedanciji linije. Koliki mora biti omjer transformatora n da bi na izlazu bilo postignuto prilagođenje ako je R_2=800Ω. Koliki su napon i struja na izlazu linije $u_l(t)$ i $i_l(t)$?



Rješenje:

Linija bez gubitaka: $R = 0;\quad G = 0$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{1\cdot 10^{-3}}{400\cdot 10^{-9}}} = 50\Omega$$

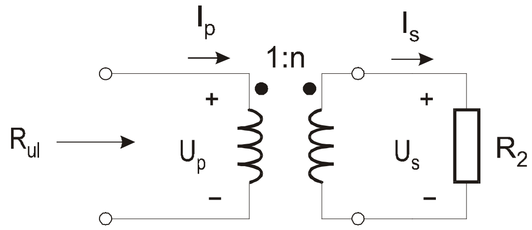
$$\gamma = s\cdot\sqrt{LC} = s\cdot\sqrt{10^{-3}\cdot 400\cdot 10^{-9}} = s\cdot 2\cdot 10^{-5}\,/\,km$$

$$\gamma = \alpha + j\beta = 0 + j\omega_0\sqrt{LC}$$

$$\nu = \frac{\omega_0}{\beta}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0\sqrt{LC}} = \frac{2\pi}{20}\cdot km = 0.314\,km$$

$$\beta = \omega_0\cdot\sqrt{LC} = 10^6\cdot 2\cdot 10^{-5} = 20\,/\,km$$



$$U_s = n\cdot U_p$$

$$I_s = \frac{1}{n}\cdot I_p$$

$$U_s = I_s\cdot R_2$$

$$R_{ul} = \frac{U_p}{I_p} = \frac{\frac{U_s}{n}}{I_s\cdot n} = \frac{\frac{U_s}{I_s}}{n^2} = \frac{R_2}{n^2}$$

$$R_2 = n^2\cdot R_{ul}$$

$$n^2 = \frac{R_2}{R_{ul}} = \frac{800}{50} = 16 \Rightarrow \qquad n = 4$$

$g = j\omega\sqrt{LC} \cdot l = j \cdot 10^6 \cdot 2 \cdot 10^{-5} \cdot 0.314 = j \cdot 2\pi$

prijenosne jednadžbe linije:

$$U(x) = U(0) \cdot ch\gamma x - I(0) \cdot Z(0) \cdot sh\gamma x$$
$$I(x) = -\frac{U(0)}{Z_0} \cdot sh\gamma x + I(0) \cdot ch\gamma x$$

---

na mjestu $x = l$

$$U(l) = U(0) \cdot ch\, g - I(0) \cdot Z(0) \cdot sh\, g$$

na ulazu linije $I(0) = \dfrac{U(0)}{Z_0}$ jer je $Z_{ul} = Z_0 = \dfrac{R_2}{n^2} = 50\Omega$ (prilagođenje)

$$U(l) = U(0) \cdot (ch\, g - sh\, g) = U(0) \cdot e^{-g}$$
$$U(0) = \frac{U_g}{2} = 5\angle 0°$$
$$U(l) = 5 \cdot e^{-j2\pi} = 5 \cdot (\cos 2\pi - j\sin 2\pi) = -j5$$

$$I(l) = -\frac{U(0)}{Z_0} sh\, g + I(0) ch\, g$$
$$I(0) = \frac{U(0)}{Z_0}$$

$$I(l) = -\frac{U_0}{Z_0} sh\, g + \frac{U_0}{Z_0} ch\, g = \frac{U(0)}{Z_0}(ch\, g - sh\, g) = \frac{U(0)}{Z_0} \cdot e^{-g}$$
$$I(l) = \frac{U(l)}{Z_0} = \frac{-j5}{50} = -0.1j$$

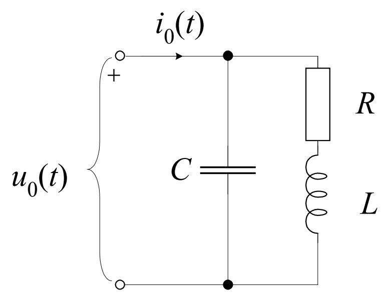
$$u(l,t) = 5\sin(10^6 t - 90°)$$
$$i(l,t) = 0.1\sin(10^6 t - 90°)$$

OSTALO

PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Na priključnice dvopola sastavljenog od paralelnoga spoja kapaciteta C=1 nF i serijske kombinacije otpora R=1000 Ω i induktiviteta L=1 mH, djeluje strujni izvor $i_0(t)=\delta(t)$. Normirati elemente dvopola na frekvenciju $\omega_0=10^6$ rad/s i na otpor R_0=1000 Ω. Odrediti napon $u(t)$ na priključnicama tog dvopola.

Rješenje: Normiranje elemenata zatim primjena Laplaceove transformacije



$$R_n = \frac{R}{R_0} = \frac{1000}{1000} = 1$$

$$Z_{C_n} = \frac{1}{sCR_0} = \frac{1}{\frac{s}{\omega_0}\underbrace{\omega_0 CR_0}_{C_n}} \qquad \Rightarrow \qquad C_n = \omega_0 CR_0 = 10^6 \cdot 10^{-9} \cdot 10^3 = 1$$

$$Z_{L_n} = \frac{sL}{R_0} = \frac{s}{\omega_0} \cdot \underbrace{\frac{\omega_0 L}{R_0}}_{L_n} \qquad \Rightarrow \qquad L_n = \frac{\omega_0 L}{R_0} = \frac{10^6 \cdot 10^{-3}}{10^3} = 1$$

$$Z_n(s) = \frac{\frac{1}{sC}(R+sL)}{\frac{1}{sC}+R+sL} = \frac{R+sL}{1+sCR+s^2LC}$$

Uz uvrštene normirane vrijednosti elemenata impedancija dvopola glasi:

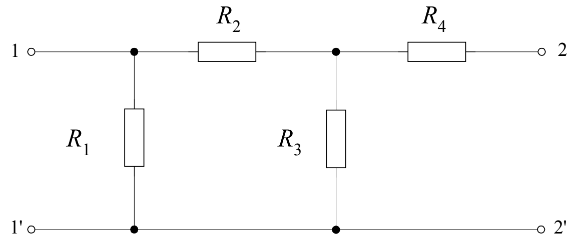
$$Z_n(s) = \frac{1+s}{s^2+s+1} = \frac{s+\frac{1}{2}+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}$$

Odnosno:

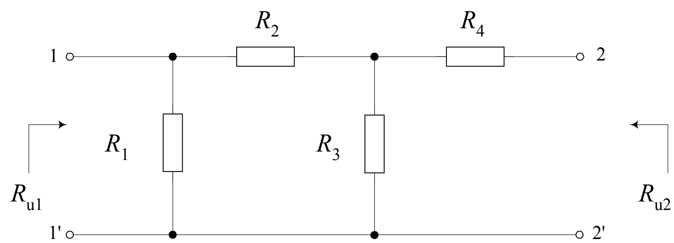
$$U_0(s) = I_0(s)\cdot Z_n(s) = \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} + \frac{\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{2}{\sqrt{3}}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} = \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} + \frac{1}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}$$

pa je $u_0(t) = \mathscr{L}^{-1}\left[U_0(s)\right] = e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right)S(t)$

2. Za krug prikazan slikom odrediti otpor R_1 tako da ukupni otpor gledan sa priključnica 1-1' bude jednak otporu gledanome s priključnica 2-2'. Zadano je: R_2= 40 Ω, R_3=30 Ω i R_4=20 Ω.



Rješenje:



$$R_{u1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = \frac{R_1 \cdot 70}{R_1 + 70}$$

$$R_{u2} = R_4 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_1 + R_2}} = R_4 + \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} = 20 + \frac{30(R_1 + 40)}{R_1 + 70}$$

$$R_{u1} = R_{u2}$$

$$\frac{R_1 \cdot 70}{R_1 + 70} = 20 + \frac{30(R_1 + 40)}{R_1 + 70}$$

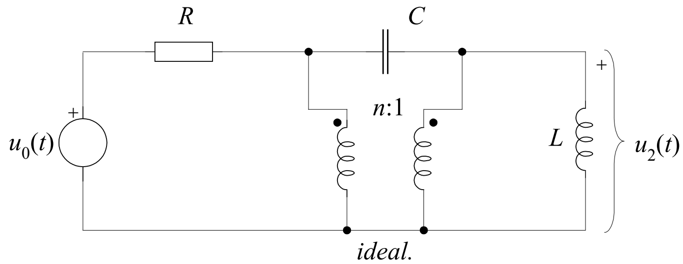
$$\frac{R_1 \cdot 70}{R_1 + 70} = \frac{20(R_1 + 70) + 30(R_1 + 40)}{R_1 + 70}$$

$$R_1 \cdot 70 - R_1 \cdot 50 = 20 \cdot 70 + 30 \cdot 40$$

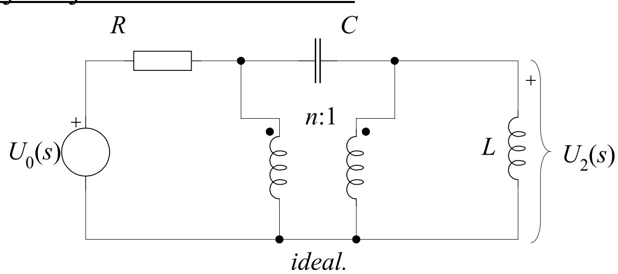
$$R_1 = \frac{20 \cdot 70 + 30 \cdot 40}{20} = 130\,\Omega$$

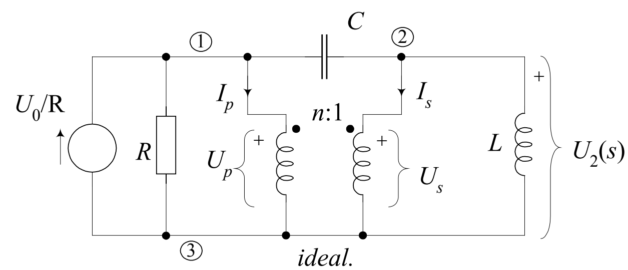
DRUGI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za krug prikazan slikom izračunati odziv $u_2(t)$ ako je zadana pobuda $u_0(t)=S(t)$, prijenosni omjer $n=1/2$, a normirane vrijednosti elemenata su: $R=2$, $L=1$ i $C=1$. Početni uvjeti su jednaki nuli.



<u>Rješenje: Jednadžbe čvorišta</u>





$$(1)\qquad \frac{U_0(s)}{R}-I_p(s)=\left(\frac{1}{R}+sC\right)U_1(s)-sCU_2(s) \qquad\qquad U_p(s)=nU_s(s)\Rightarrow U_1(s)=nU_2(s)$$

$$(2)\qquad -I_s(s)=-sCU_1(s)+\left(sC+\frac{1}{sL}\right)U_2(s) \qquad\qquad I_p(s)=-\frac{1}{n}I_s(s)$$

$$(1)\qquad \frac{U_0}{R}+\frac{I_s}{n}=\left(\frac{1}{R}+sC\right)nU_2-sCU_2$$

$$(2)\qquad -I_s(s)=-sCnU_2+\left(sC+\frac{1}{sL}\right)U_2(s) \qquad \Rightarrow \qquad I_s(s)=\left(sCn-sC-\frac{1}{sL}\right)U_2(s)$$

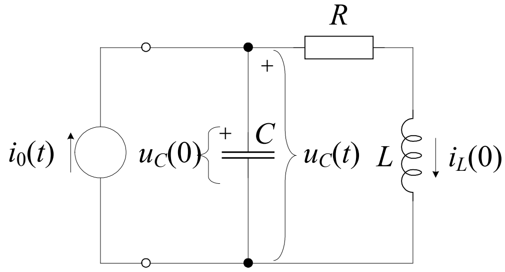
$$\frac{U_0}{R}=\left(-sC+\frac{1}{n}\left(sC+\frac{1}{sL}\right)\right)U_2+\left(\frac{1}{R}+sC\right)nU_2-sCU_2$$

$$\frac{U_0}{R}=\left(\frac{1}{n}\left(sC+\frac{1}{sL}\right)+\left(\frac{1}{R}+sC\right)n-2sC\right)U_2$$

$$U_2(s)=\frac{nU_0}{R\left(\left(sC+\frac{1}{sL}\right)+\left(\frac{1}{R}+sC\right)n^2-2nsC\right)}=\frac{snU_0}{RC\left(s^2\left(1+n^2-2n\right)+s\frac{n^2}{RC}+\frac{1}{LC}\right)}=\frac{2}{2s^2+s+8}$$

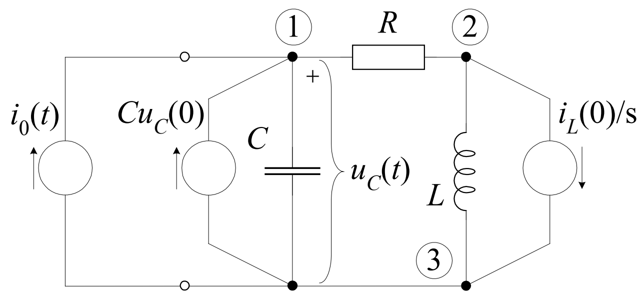
$$U_2(s)=\frac{1}{s^2+\frac{s}{2}+4}=\frac{1}{\left(s+\frac{1}{4}\right)^2+\frac{63}{16}}=\frac{4}{\sqrt{63}}\cdot\frac{\frac{\sqrt{63}}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{63}}{4}\right)^2} \qquad u_2(t)=\frac{4}{3\sqrt{7}}\cdot e^{-t/4}\sin\left(\frac{3\sqrt{7}}{4}\right)S(t)$$

3. Odrediti odziv $u_C(t)$ mreže prema slici za $t \geq 0$, ako su zadane normirane vrijednosti elemenata: $R=1$, $C=2$ i $L=1$, početni uvjeti u mreži $u_C(0)=1/5$ i $i_L(0)=-1/5$, a pobuda je $i_0(t)=S(t)$.



Rješenje:

Za $t \geq 0$: Primjena Laplaceove transformacije → Jednadžbe čvorišta



$$I_0(s)+Cu_C(0)=\left(sC+\frac{1}{R}\right)U_1(s)-\frac{1}{R}U_2(s)$$

$$\frac{-i_L(0)}{s}=-\frac{1}{R}U_1(s)+\left(\frac{1}{sL}+\frac{1}{R}\right)U_2(s) \qquad \Rightarrow \qquad U_2(s)=\frac{sL}{(R+sL)}U_1(s)-i_L(0)\frac{RL}{(R+sL)}$$

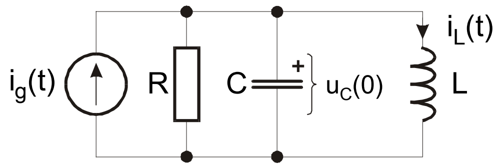
$$U_1(s)=\frac{(R+sL)I_0(s)+C(R+sL)u_C(0)-Li_L(0)}{s^2LC+sRC+1}=\frac{(1+s)+2s(1+s)u_C(0)-si_L(0)}{s(2s^2+2s+1)}$$

$$U_1(s)=\frac{1}{5s}\cdot\frac{5(1+s)+2s(1+s)+s}{2s^2+2s+1}=\frac{1}{5s}\cdot\frac{2s^2+8s+5}{2s^2+2s+1}=\frac{1}{5}\left(\frac{5}{s}-4\frac{s+0{,}5}{(s+0{,}5)^2+0{,}25}+2\frac{0{,}5}{(s+0{,}5)^2+0{,}25}\right)$$

$$u_1(t)=\frac{1}{5}\left(5-4e^{-0{,}5t}\cos(0{,}5t)+2e^{-0{,}5t}\sin(0{,}5t)\right)S(t)$$

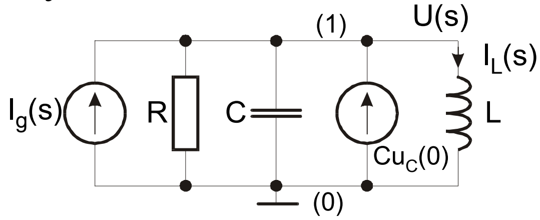
ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA

1. U električnom krugu na slici odrediti odziv struje kroz induktivitet $i_L(t)$ ako su zadane normalizirane vrijednosti elemenata: C=2, R=1/2, L=1/4, početni uvjeti $i_L(0)$=0, $u_C(0)$=1 te struja pobude $i_g(t)$=$e^{-2t}\cdot S(t)$.



Rješenje:

Primjena Laplaceove transformacije:



$$U(s)\cdot\left(sC+\frac{1}{R}+\frac{1}{sL}\right)-C\cdot u_C(0)-I_g(s)=0$$

$$U(s)=\frac{I_g(s)+C\cdot u_C(0)}{sC+\frac{1}{R}+\frac{1}{sL}}$$

$$I_L(s)=U(s)\cdot\frac{1}{sL}=\frac{I_g(s)+C\cdot u_C(0)}{s^2LC+s\frac{L}{R}+1}$$

$$i_g(t)=e^{-2t}\cdot S(t)\ \Rightarrow\ I_g(s)=\frac{1}{s+2}$$

$$I_L(s)=\frac{\frac{1}{s+2}+2}{\frac{s^2}{2}+\frac{s}{2}+1}=\frac{2(1+2s+4)}{(s+2)(s^2+s+2)}=\frac{4s+10}{(s+2)(s^2+s+2)}$$

Slijedi rastav na parcijalne razlomke izraza $\frac{4s+10}{(s+2)(s^2+s+2)}=\frac{A}{s+2}+\frac{Bs+C}{s^2+s+2}$

$$As^2+As+2A+Bs^2+2Bs+Cs+2C=4s+10$$

$$(A+B)s^2+(A+2B+C)s+(2A+2C)=4s+10$$

(1) $\quad A+B=0$

(2) $\quad A+2B+C=4$

(3) $\quad 2(A+C)=10$

(1)$\Rightarrow\ B=-A$, (3)$\Rightarrow\ C=5-A$, (2) $\Rightarrow\ A+2B+5-A=4$

$$\Rightarrow \quad 2B = -1 \;\Rightarrow\quad B = -\frac{1}{2},\; A = \frac{1}{2},\; C = 5 - A = 5 - \frac{1}{2} = \frac{10}{2} - \frac{1}{2} = \frac{9}{2}$$

$$\Rightarrow I_L(s) = \frac{4s+10}{(s+2)(s^2+s+2)} = \frac{1}{2}\cdot\frac{1}{s+2} - \frac{1}{2}\cdot\frac{s}{s^2+s+2} + \frac{9}{2}\cdot\frac{1}{s^2+s+2} =$$

$$= \frac{1}{2}\cdot\frac{1}{s+2} - \frac{1}{2}\cdot\frac{s+\frac{1}{2}-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2} + \frac{9}{2}\cdot\frac{1}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2} =$$

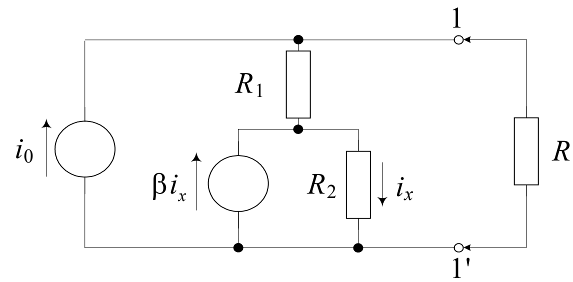
$$= \frac{1}{2}\cdot\frac{1}{s+2} - \frac{1}{2}\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2} + \frac{19}{4}\cdot\frac{\frac{2}{\sqrt{7}}\cdot\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2} =$$

$$= \frac{1}{2}\cdot\frac{1}{s+2} - \frac{1}{2}\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2} + \frac{19}{2\sqrt{7}}\cdot\frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2} =$$

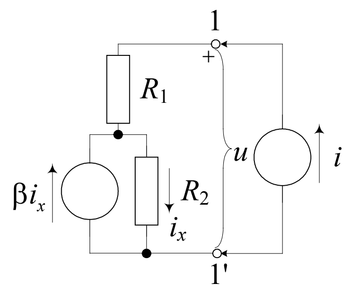
$$i_L(t) = \mathscr{L}\{I_L(s)\} \quad\Rightarrow\quad \underline{i_L(t) = \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-\frac{t}{2}}\cos\frac{\sqrt{7}}{2}t + \frac{19}{2\sqrt{7}}e^{-\frac{t}{2}}\sin\frac{\sqrt{7}}{2}t\right)\cdot S(t)}$$

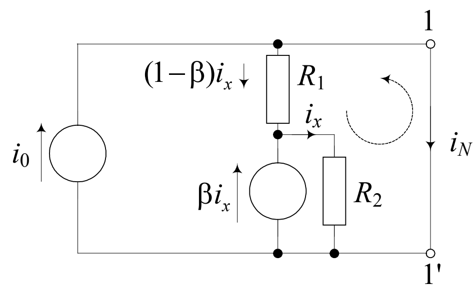
PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za krug prikazan slikom:
   a) isključiti otpor R i odrediti nadomjesni spoj po Nortonu obzirom na priključnice 1–1';
   b) odrediti iznos konstante β za koji je $Y_N(s)=1/R$.
   c) uz uključen otpor R primjenom transformacija izvora i Kirchhoffovih zakona odrediti struju i_x;
   Zadana je pobuda $i_0=2$ A, i vrijednosti elemenata $R_1=R_2=4\ \Omega$, $R=16\ \Omega$.



Rješenje:
a) isključiti otpor R i odrediti nadomjesni spoj po Nortonu obzirom na priključnice 1–1';





Nortonova admitancija: $i_x=\beta i_x+i \Rightarrow i_x(1-\beta)=i \Rightarrow i_x=\dfrac{i}{1-\beta}$, $u=iR_1+\dfrac{i}{1-\beta}R_2 \Rightarrow Z_T=\dfrac{u}{i}=R_1+\dfrac{R_2}{1-\beta}$,

$$Y_N=\frac{1}{Z_T}=\frac{1}{R_1+\dfrac{R_2}{1-\beta}}=\frac{1-\beta}{R_1(1-\beta)+R_2}$$

Nortonova struja: $i_N=(\beta-1)i_x+i_0$,

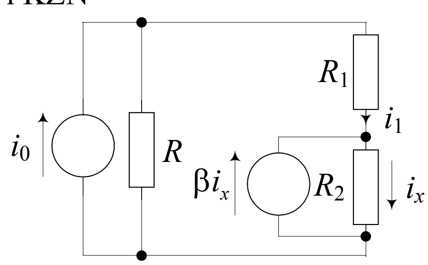
$$(1-\beta)i_xR_1+i_xR_2=0 \;\Rightarrow\; (\beta-1)i_xR_1=i_xR_2 \Rightarrow i_x=0 \Rightarrow i_N=i_0=2A$$

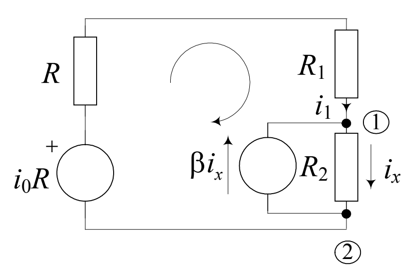
b) odrediti iznos konstante β za koji je $Y_N(s)=1/R$.

$$Z_T=R_1+\frac{R_2}{1-\beta}=R \Rightarrow 4+\frac{4}{1-\beta}=16 \Rightarrow \frac{4}{1-\beta}=12 \Rightarrow \frac{1}{1-\beta}=3 \Rightarrow 1-\beta=\frac{1}{3} \Rightarrow \beta=1-\frac{1}{3}=\frac{2}{3},$$

konačno je: $Y_N=1/Z_T=1/16$.

c) Jednadžbe KZS i KZN





KZS za čvor (1): $0=-i_1-\beta i_x+i_x \quad \Rightarrow \quad i_1=-(\beta-1)i_x=(1-\beta)i_x$

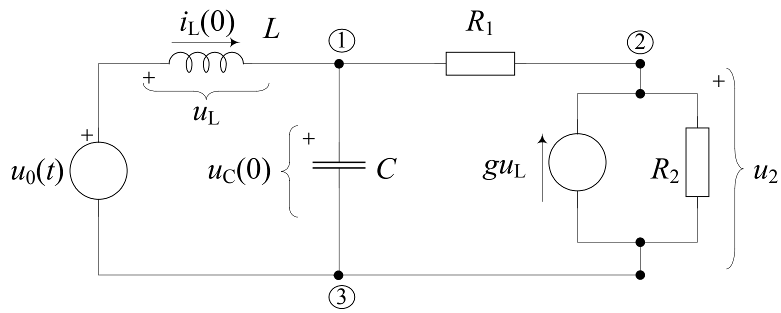
KZN za petlju: $i_1(R+R_1)+i_xR_2=i_0R$

$$\Rightarrow \quad (1-\beta)i_x(R+R_1)+i_xR_2=i_0R \quad \Rightarrow$$

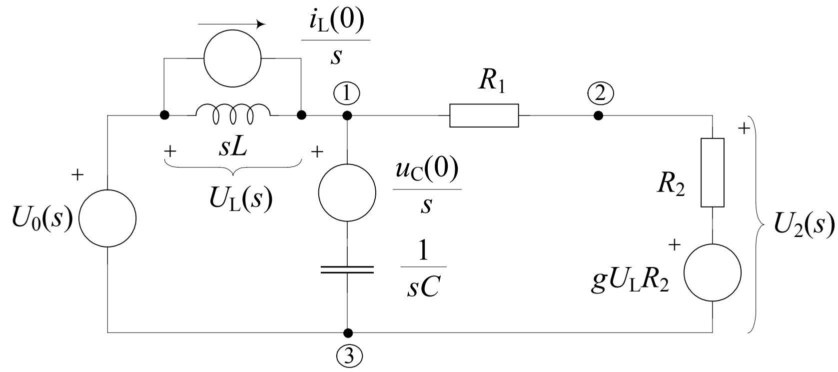
$$i_x=\frac{i_0R}{(1-\beta)(R+R_1)+R_2}$$

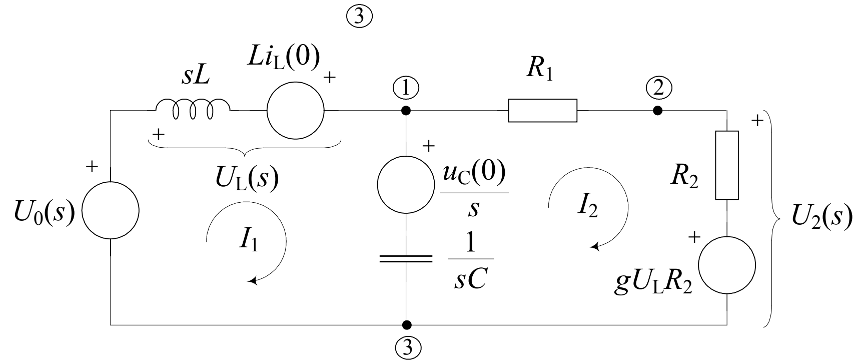
$$i_x=\frac{2\cdot16}{(1-2/3)(16+4)+4}=\frac{32}{(1/3)\cdot20+4}=\frac{3\cdot32}{20+12}=\frac{96}{32}=3A$$

2. Za krug prikazan slikom napisati jednadžbe petlji. Izračunati napon $U_2(s)$, ako je zadana pobuda $u_0(t)$= 2S(t), konstanta g=1, normirane vrijednosti elemenata su: R_1=R_2=1, L=1 i C=1, a početni uvjeti su $u_C(0)$=1, $i_L(0)$=1.



Rješenje: Primjena Laplaceove transformacije





Jednadžbe petlji:

(1) $$I_1(s)sL+[I_1(s)-I_2(s)]\frac{1}{sC}=U_0(s)+Li_L(0)-\frac{u_C(0)}{s}$$

(2) $$-[I_1(s)-I_2(s)]\frac{1}{sC}+I_2(s)R_1+I_2(s)R_2=-gU_L(s)R_2+\frac{u_C(0)}{s}$$

$$U_L(s) = I_1(s)sL - Li_L(0)$$
$$U_2(s) = I_2(s)R_2 + gU_L(s)R_2 = I_2(s)R_2 + g\left[I_1(s)sL - Li_L(0)\right]R_2$$

$$(1) \qquad I_1(s)sL + \left[I_1(s) - I_2(s)\right]\frac{1}{sC} = U_0(s) + Li_L(0) - \frac{u_C(0)}{s}$$

$$(2) \qquad -\left[I_1(s) - I_2(s)\right]\frac{1}{sC} + I_2(s)R_1 + I_2(s)R_2 = -g\left[I_1(s)sL - Li_L(0)\right]R_2 + \frac{u_C(0)}{s}$$

$$(1) \qquad I_1(s)\left(sL + \frac{1}{sC}\right) - I_2(s)\frac{1}{sC} = U_0(s) + Li_L(0) - \frac{u_C(0)}{s}$$

$$(2) \qquad -I_1(s)\left(\frac{1}{sC} - gR_2sL\right) + I_2(s)\left(\frac{1}{sC} + R_1 + R_2\right) = gR_2Li_L(0) + \frac{u_C(0)}{s}$$

Uvrstimo vrijednosti elemenata:

$$(1) \qquad I_1(s)\left(s + \frac{1}{s}\right) - I_2(s)\frac{1}{s} = \frac{2}{s} + 1 - \frac{1}{s} = 1 + \frac{1}{s} \Big/ \cdot s$$

$$(2) \qquad -I_1(s)\left(\frac{1}{s} - s\right) + I_2(s)\left(\frac{1}{s} + 2\right) = 1 + \frac{1}{s} \Big/ \cdot s$$

$$(1) \qquad I_1(s)\left(s^2 + 1\right) - I_2(s) = s + 1$$

$$(2) \qquad I_1(s)\left(s^2 - 1\right) + I_2(s)(2s + 1) = s + 1$$

$$\Delta = \begin{vmatrix} s^2+1 & -1 \\ s^2-1 & 2s+1 \end{vmatrix} = (s^2+1)(2s+1) + s^2 - 1 = 2s^3 + 2s + s^2 + 1 + s^2 - 1 = 2s^3 + 2s^2 + 2s$$

$$\Delta_1 = \begin{vmatrix} s+1 & -1 \\ s+1 & 2s+1 \end{vmatrix} = (s+1)(2s+1) + s + 1 = 2s^2 + 2s + s + 1 + s + 1 = 2s^2 + 4s + 2$$

$$\Delta_2 = \begin{vmatrix} s^2+1 & s+1 \\ s^2-1 & s+1 \end{vmatrix} = (s^2+1)(s+1) - (s^2-1)(s+1) = (s+1)(s^2+1-s^2+1) = 2(s+1)$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2s^2+4s+2}{2s^3+2s^2+2s} = \frac{2(s^2+2s+1)}{2(s^3+s^2+s)} = \frac{s^2+2s+1}{s^3+s^2+s}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{2(s+1)}{2(s^3+s^2+s)} = \frac{s+1}{s^3+s^2+s}$$

$$U_2(s) = I_2(s)R_2 + g\left[I_1(s)sL - Li_L(0)\right]R_2 = \frac{s+1}{s^3+s^2+s} + \frac{s^2+2s+1}{s^3+s^2+s}s - 1$$

$$U_2(s) = \frac{s+1+s^3+2s^2+s-s^3-s^2-s}{s^3+s^2+s} = \frac{s^2+s+1}{s^3+s^2+s} = \frac{s^2+s+1}{s(s^2+s+1)} = \frac{1}{s}$$

Rastav na parcijalne razlomke:

$$U_2(s)=\frac{-1}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}=\frac{A(s+2)+B(s+1)}{(s+1)(s+2)}=\frac{(A+B)s+(2A+B)}{(s+1)(s+2)}$$

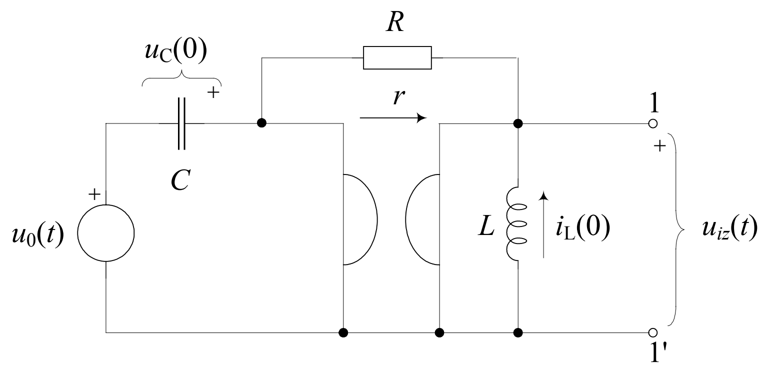
$$A+B=0\ \Rightarrow\ B=-A$$

$$2A+B=-1\ \Rightarrow\ 2A-A=-1\ \Rightarrow\ A=-1\Rightarrow\ B=1$$

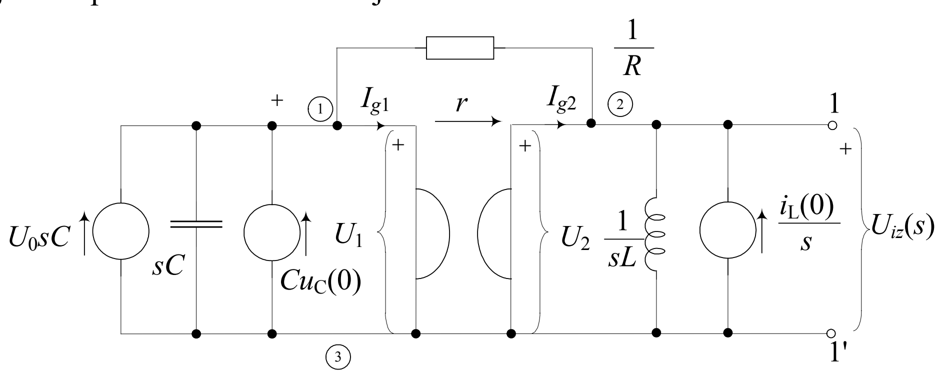
$$U_2(s)=\frac{-1}{s+1}+\frac{1}{s+2}$$

Konačno je: $u_2(t)=\left(-e^{-t}+e^{-2t}\right)\cdot S(t)$

5. Za krug prikazan slikom odrediti napon $u_{iz}(t)$ na priključnicama 1-1', koristeći postupak jednadžbi čvorišta, ako je pobuda $u_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: R=0.5, L=1, C=1, r=1 i početni uvjeti $u_C(0)$=2, $i_L(0)$=1.



Rjешenje: Primjena Laplaceove transformacije



(1) $\quad U_1\left(sC+\frac{1}{R}\right)-U_2\frac{1}{R}=U_0(s)sC+Cu_C(0)-I_{g1} \qquad I_{g1}=-U_2\frac{1}{r}$

(2) $\quad -U_1\frac{1}{R}+U_2\left(\frac{1}{R}+\frac{1}{sL}\right)=\frac{i_L(0)}{s}+I_{g2} \qquad I_{g2}=-U_1\frac{1}{r}$

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(1) $\quad U_1\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)=U_0(s)sC+Cu_C(0)$

(2) $$-U_1\left(\frac{1}{R}-\frac{1}{r}\right)+U_2\left(\frac{1}{R}+\frac{1}{sL}\right)=\frac{i_L(0)}{s}$$

---

(2) $$\Rightarrow \qquad U_1=\frac{U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}}{\frac{1}{R}-\frac{1}{r}}$$

(1) $$\frac{U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}}{\frac{1}{R}-\frac{1}{r}}\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)=U_0sC+Cu_C(0) \qquad \Big/\cdot\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$\left[U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}\right]\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)\left(\frac{1}{R}-\frac{1}{r}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$U_2\left(\frac{1}{R}+\frac{1}{sL}\right)\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R^2}-\frac{1}{r^2}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)$$

$$U_2\left(\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{R^2}+\frac{1}{RsL}-\frac{1}{R^2}+\frac{1}{r^2}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)$$

$$U_2(s)=\frac{\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)}{\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{RsL}+\frac{1}{r^2}}$$

$$U_2(s)=\frac{\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)}{\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{RsL}+\frac{1}{r^2}}$$

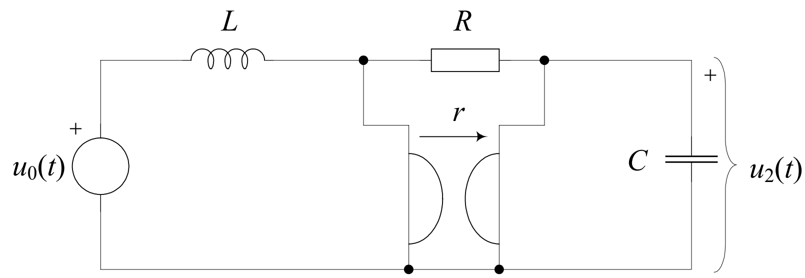
$$U_2(s)=\frac{\left[\frac{1}{s}s+1\cdot 2\right](2-1)+\frac{1}{s}(s+2)}{2s+1+\frac{2}{s}+1}=\frac{3+\frac{2}{s}+1}{2s+2+\frac{2}{s}}=\frac{4+\frac{2}{s}}{2s+2+\frac{2}{s}}=\frac{2+\frac{1}{s}}{s+1+\frac{1}{s}}=\frac{2s+1}{s^2+s+1}$$

$$U_2(s)=\frac{2s+1}{s^2+s+1}=2\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

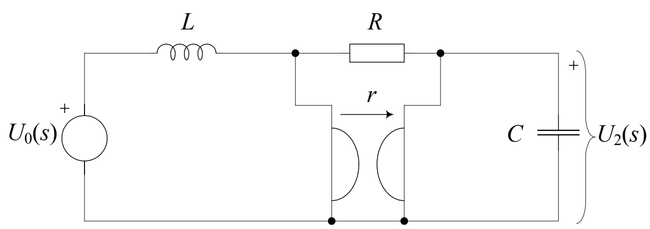
$$u_2(t)=2e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}\right)\cdot S(t)$$

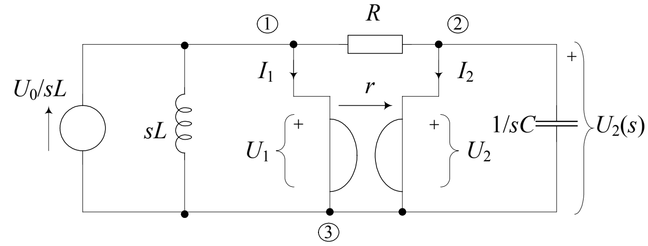
DRUGI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za krug prikazan slikom izračunati odziv $u_2(t)$ ako je zadana pobuda $u_0(t)=2\delta(t)$, konstanta giratora r=2, a normirane vrijednosti elemenata su: R=1, L=2 i C=1/2. Početni uvjeti su jednaki nuli.



<u>Rješenje: Jednadžbe čvorišta</u>





(1) $\quad \dfrac{U_0(s)}{sL} - I_1(s) = \left(\dfrac{1}{sL}+\dfrac{1}{R}\right)U_1(s) - \dfrac{1}{R}U_2(s)$ $\qquad U_1(s) = rI_2(s) \quad \Rightarrow \quad I_2(s) = \dfrac{1}{r}U_1(s)$

(2) $\quad -I_2(s) = -\dfrac{1}{R}U_1(s) + \left(sC+\dfrac{1}{R}\right)U_2(s)$ $\qquad U_2(s) = -rI_1(s) \quad \Rightarrow \quad I_1(s) = -\dfrac{1}{r}U_2(s)$

---

(1) $\quad \dfrac{U_0(s)}{sL} + \dfrac{U_2(s)}{r} = \left(\dfrac{1}{sL}+\dfrac{1}{R}\right)U_1(s) - \dfrac{1}{R}U_2(s)$

(2) $\quad -\dfrac{U_1(s)}{r} = -\dfrac{1}{R}U_1(s) + \left(sC+\dfrac{1}{R}\right)U_2(s)$

---

(1) $\quad \dfrac{U_0(s)}{sL} = \left(\dfrac{1}{sL}+\dfrac{1}{R}\right)U_1(s) - \left(\dfrac{1}{R}+\dfrac{1}{r}\right)U_2(s)$

(2) $\quad 0 = -\left(\dfrac{1}{R}-\dfrac{1}{r}\right)U_1(s) + \left(sC+\dfrac{1}{R}\right)U_2(s)$

$$\Delta = \begin{vmatrix} \dfrac{1}{sL}+\dfrac{1}{R} & -\left(\dfrac{1}{R}+\dfrac{1}{r}\right) \\ -\left(\dfrac{1}{R}-\dfrac{1}{r}\right) & sC+\dfrac{1}{R} \end{vmatrix} = \left(\frac{1}{sL}+\frac{1}{R}\right)\left(sC+\frac{1}{R}\right) - \left(\frac{1}{R}+\frac{1}{r}\right)\left(\frac{1}{R}-\frac{1}{r}\right) = \frac{sC}{sL}+\frac{sC}{R}+\frac{1}{RsL}+\frac{1}{R^2}-\frac{1}{R^2}+\frac{1}{r^2}$$

$$= \frac{sC}{sL}+\frac{sC}{R}+\frac{1}{RsL}+\frac{1}{r^2}$$

$$\Delta_2 = \begin{vmatrix} \frac{1}{sL}+\frac{1}{R} & \frac{U_0}{sL} \\ -\left(\frac{1}{R}-\frac{1}{r}\right) & 0 \end{vmatrix} = \frac{U_0}{sL}\left(\frac{1}{R}-\frac{1}{r}\right)$$

Uvrstimo vrijednosti: r=2, R=1, L=2 i C=1/2, $U_0(s)$=2.

$$\Delta = \frac{1}{4}+\frac{1}{2}s+\frac{1}{2}\cdot\frac{1}{s}+\frac{1}{4} = \frac{1}{2}+\frac{1}{2}s+\frac{1}{2s}$$

$$\Delta_2 = \frac{2}{2s}\left(1-\frac{1}{2}\right) = \frac{1}{2s}$$

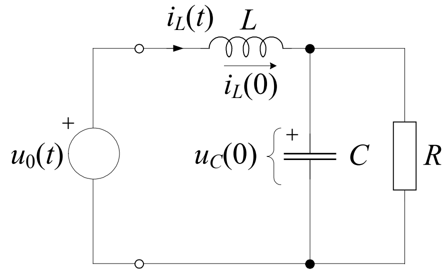
$$U_2(s) = \frac{\Delta_2}{\Delta} = \frac{\frac{1}{2s}}{\frac{1}{2}+\frac{1}{2}s+\frac{1}{2s}} = \frac{1}{s^2+s+1}$$

$$s^2+s+1=0 \Rightarrow s_{1,2} = \frac{-1\pm\sqrt{1-4}}{2} = -\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$

$$U_2(s) = \frac{1}{s^2+s+\frac{1}{4}+\frac{3}{4}} = \frac{2}{\sqrt{3}}\cdot\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

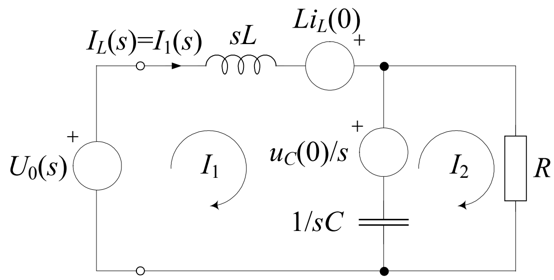
$$u_2(t) = \mathscr{L}^{-1}[U_2(s)] = \frac{2}{\sqrt{3}}\cdot e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)\cdot S(t)$$

3. Odrediti odziv $i_L(t)$ mreže prema slici za $t \geq 0$, ako su zadane normirane vrijednosti elemenata: $R=1$, $L=2$ i $C=1$, početni uvjeti u mreži $u_C(0)=-1/5$ i $i_L(0)=1/5$, a pobuda je $u_0(t)=S(t)$.



Rješenje:

Za $t \geq 0$: Primjena Laplaceove transformacije → Jednadžbe petlji



(1) $$U_0(s)+Li_L(0)-\frac{u_C(0)}{s}=\left(sL+\frac{1}{sC}\right)I_1(s)-\frac{1}{sC}I_2(s)$$

(2) $$\frac{u_C(0)}{s}=-\frac{1}{sC}I_1(s)+\left(\frac{1}{sC}+R\right)I_2(s)$$

---

(2) $\Rightarrow$ $$I_2(s)\frac{1+sRC}{sC}=\frac{1}{sC}I_1(s)+\frac{u_C(0)}{s} \Rightarrow I_2(s)=\frac{1}{1+sRC}I_1(s)+\frac{Cu_C(0)}{1+sRC} \rightarrow (1) \Rightarrow$$

$$U_0(s)+Li_L(0)-\frac{u_C(0)}{s}=\frac{s^2LC+1}{sC}I_1(s)-\frac{1}{sC}\cdot\frac{I_1(s)+Cu_C(0)}{1+sRC}\Big/\cdot sC(1+sRC)$$

$$Cu_C(0)+(1+sRC)(sCU_0(s)+sCLi_L(0)-Cu_C(0))=I_1(s)\left[(s^2LC+1)(1+sRC)-1\right]$$

$$I_1(s)=\frac{(1+sRC)[U_0(s)+Li_L(0)]-RCu_C(0)}{s^2RLC+sL+R}=\frac{U_0(s)\left(\frac{1}{R}+sC\right)+Li_L(0)\left(\frac{1}{R}+sC\right)-Cu_C(0)}{s^2LC+s\frac{L}{R}+1}$$

$$I_1(s)=\frac{\frac{1}{s}(1+s)+\frac{2}{5}(1+s)+\frac{1}{5}}{2s^2+2s+1}=\frac{1}{5s}\cdot\frac{5(1+s)+2s(1+s)+s}{2s^2+2s+1}$$

$$I_1(s)=\frac{1}{5s}\cdot\frac{2s^2+8s+5}{2s^2+2s+1}=\frac{1}{5}\left(\frac{5}{s}-4\frac{s+0{,}5}{(s+0{,}5)^2+0{,}25}+2\frac{0{,}5}{(s+0{,}5)^2+0{,}25}\right)$$

$$i_L(t)=i_1(t)=\frac{1}{5}\left(5-4e^{-0{,}5t}\cos(0{,}5t)+2e^{-0{,}5t}\sin(0{,}5t)\right)\cdot S(t)$$

4. Odziv neke mreže na pobudu $x(t)$=S(t) glasi: $y(t)$=e^{-3t}ch(2t)·S(t). Odrediti funkciju mreže i fazor odziva na pobudu $x(t)$=2 cos(3t+45°).

Rješenje:

$$X(s)=\frac{1}{s}$$

$$Y(s)=\frac{s+3}{(s+3)^2-4}=\frac{s+3}{s^2+6s+5}$$

$$H(s)=\frac{s(s+3)}{(s+3)^2-4}=\frac{s(s+3)}{s^2+6s+5}$$

Fazori:

$$H(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}$$

$$X_1(\mathrm{j}\omega)=2e^{\mathrm{j}\pi/4}$$

$$Y_1(\mathrm{j}\omega)=H(\mathrm{j}\omega)X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}2e^{\mathrm{j}\pi/4}$$

$$\omega=3$$

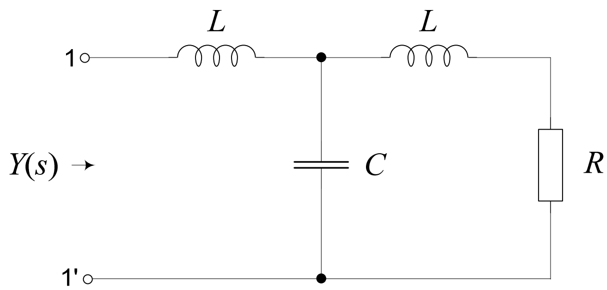
$$Y_1(\mathrm{j}3)=\frac{\mathrm{j}3\cdot(\mathrm{j}3+3)}{(\mathrm{j}3)^2+18\mathrm{j}+5}\cdot 2\cdot\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)$$

$$Y_1(\mathrm{j}3)=\frac{9\sqrt{2}}{85}\cdot(2+j9)$$

$$y_1(t)=\frac{9\sqrt{2}}{\sqrt{85}}\cos(3t+77.47°)$$

PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2010

1. Zadan je dvopol sastavljen od normiranih elemenata R=1, C=2, L=1.
   a) Izračunati ulaznu admitanciju $Y(s)$ na priključnicama 1-1' tog dvopola;
   b) Denormirati elemente dvopola na frekvenciju ω_0=10^6 rad/s i na otpor R_0=1000 Ω;
   c) Odrediti denormiranu ulaznu admitanciju $Y(s)$;
   d) Koliki je iznos denormirane ulazne admitancije $Y(s)$ na frekvenciji nula $Y(0)$?



<u>Rješenje:</u>

a) Ulazna admitancija:

$$Y_n(s)=\frac{1}{sL+\dfrac{1}{sC+\dfrac{1}{sL+R}}}=\frac{1}{sL+\dfrac{sL+R}{sC(sL+R)+1}}=\frac{sC(sL+R)+1}{sL\left[sC(sL+R)+1\right]+sL+R}=$$

$$=\frac{s^2LC+sCR+1}{sL\left[s^2LC+sCR+1\right]+sL+R}=\frac{s^2LC+sCR+1}{s^3L^2C+s^2LCR+s2L+R}=\frac{2s^2+2s+1}{2s^3+2s^2+2s+1}$$ (1 bod)

b) Denormiranje elemenata: (1 bod)

$$R=R_0\cdot R_n=1000\cdot 1=1000\Omega=1k\Omega$$

$$Z_{C_n}=\frac{1}{sCR_0}=\frac{1}{\dfrac{s}{\omega_0}\underbrace{\omega_0 CR_0}_{C_n}}\qquad\Rightarrow\qquad C=\frac{C_n}{\omega_0 R_0}=\frac{2}{10^6\cdot 10^3}=2\cdot 10^{-9}F=2nF$$

$$Z_{L_n}=\frac{sL}{R_0}=\frac{s}{\omega_0}\cdot\underbrace{\frac{\omega_0 L}{R_0}}_{L_n}\qquad\Rightarrow\qquad L=\frac{L_n R_0}{\omega_0}=\frac{1\cdot 10^3}{10^6}=1\cdot 10^{-3}H=1mH$$

c) Uz uvrštene denormirane vrijednosti elemenata admitancija dvopola glasi:

$$Y(s)=\frac{s^2LC+sCR+1}{s^3L^2C+s^2LCR+s2L+R}=\frac{s^2\cdot 10^{-3}\cdot 2\cdot 10^{-9}+s\cdot 2\cdot 10^{-9}\cdot 10^3+1}{s^3\cdot(10^{-3})^2\cdot 2\cdot 10^{-9}+s^2\cdot 10^{-3}\cdot 2\cdot 10^{-9}\cdot 10^3+s\cdot 2\cdot 10^{-3}+10^3}=$$

$$=\frac{s^2\cdot 2\cdot 10^{-12}+s\cdot 2\cdot 10^{-6}+1}{s^3\cdot 2\cdot 10^{-15}+s^2\cdot 2\cdot 10^{-9}+s\cdot 2\cdot 10^{-3}+10^3}=\frac{s^2\cdot\dfrac{2\cdot 10^{-12}}{2\cdot 10^{-15}}+s\cdot\dfrac{2\cdot 10^{-6}}{2\cdot 10^{-15}}+\dfrac{1}{2\cdot 10^{-15}}}{s^3+s^2\cdot\dfrac{2\cdot 10^{-9}}{2\cdot 10^{-15}}+s\cdot\dfrac{2\cdot 10^{-3}}{2\cdot 10^{-15}}+\dfrac{10^3}{2\cdot 10^{-15}}}=$$

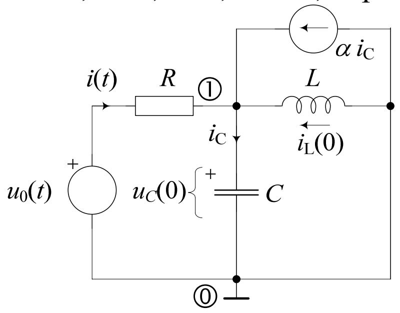
$$=\frac{s^2\cdot 10^3+s\cdot 10^9+5\cdot 10^{14}}{s^3+s^2\cdot 10^6+s\cdot 10^{12}+5\cdot 10^{17}}$$ (1 bod)

d) Uz uvrštene denormirane vrijednosti elemenata admitancija dvopola na frekvenciji nula glasi:

$$Y(0)=\frac{5\cdot 10^{14}}{5\cdot 10^{17}}=10^{-3}$$ (1 bod)

PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug prikazan slikom odrediti odziv $i(t)$ ako je zadan poticaj $u_0(t)=\cos(t)S(t)$. Zadani su normalizirani elementi R=1, C=1, L=2, α=1/2, te početni uvjeti $u_C(0)$=1, $i_L(0)$=1/2.



Rješenje:

Metoda napona čvorova:

$$(1)\ U_1 \cdot \left(\frac{1}{R} + sC + \frac{1}{sL}\right) = \frac{U_0}{R} + \frac{i_L(0)}{s} + \alpha I_C + Cu_C(0) \quad \text{(1 bod)}$$

$$U_1 = I_C \frac{1}{sC} + \frac{u_C(0)}{s} \Rightarrow I_C = sC\left(U_1 - \frac{u_C(0)}{s}\right) \quad \text{(1 bod)}$$

Uz uvrštene vrijednosti elemenata: $I_C = s\left(U_1 - \frac{1}{s}\right) = sU_1 - 1$

$$U_1 \cdot \left(1 + s + \frac{1}{2s}\right) = \frac{s}{s^2+1} + \frac{1}{2s} + \frac{s}{2}U_1 - \frac{1}{2} + 1$$

$$U_1 \cdot \left(1 + s + \frac{1}{2s} - \frac{s}{2}\right) = \frac{s}{s^2+1} + \frac{1}{2s} + \frac{1}{2}$$

$$U_1 \cdot \left(1 + \frac{s}{2} + \frac{1}{2s}\right) = \frac{2s^2 + s^2 + 1 + s^3 + s}{2s(s^2+1)}$$

$$U_1 \frac{1+2s+s^2}{2s} = \frac{s^3+3s^2+s+1}{2s(s^2+1)} \quad \Rightarrow \quad U_1 = \frac{s^3+3s^2+s+1}{(s+1)^2(s^2+1)} \quad \text{(1 bod)}$$

$$U_1 = U_0 - IR \Rightarrow I = U_0 - U_1$$

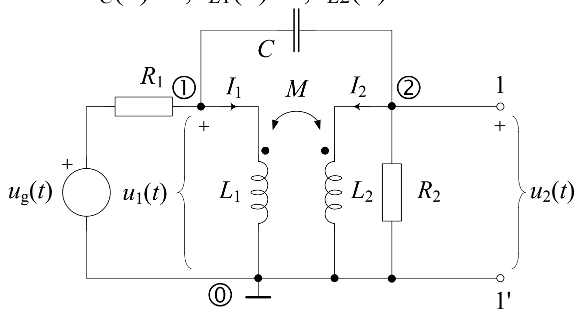
$$I = \frac{s}{s^2+1} - \frac{s^3+3s^2+s+1}{(s+1)^2(s^2+1)} = \frac{s(s^2+2s+1)-(s^3+3s^2+s+1)}{(s+1)^2(s^2+1)}$$

$$= \frac{-(s^2+1)}{(s+1)^2(s^2+1)} = \frac{-1}{(s+1)^2} \quad \text{(1 bod)}$$

$$I(s) = \frac{-1}{(s+1)^2} \Rightarrow i(t) = -te^{-t}S(t) \quad \text{(1 bod)}$$

ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug na slici izračunati odziv $u_2(t)$ na prilazu 1–1', ako je zadan poticaj $u_g(t)=e^{-t}\cdot S(t)$. Zadane su normalizirane vrijednosti elemenata: $R_1=R_2=1$, $L_1=L_2=1$, $M=1$, $C=1$. Početni uvjeti su jednaki nula: $u_C(0)=0$, $i_{L1}(0)=0$, $i_{L2}(0)=0$.



Rješenje:

Napomena: ako se odmah uvrste numeričke vrijednosti (što se jednako priznaje za točno rješenje) tada je postupak znatno jednostavniji i kraći.

Postavimo jednadžbe čvorova:

$$(1)\ U_1\cdot\left(\frac{1}{R_1}+sC\right)-U_2\cdot sC=\frac{U_g}{R_1}-I_1$$

$$(2)\ -U_1\cdot sC+U_2\cdot\left(\frac{1}{R_2}+sC\right)=-I_2$$

$$\begin{aligned}U_1&=sL_1\cdot I_1+sM\cdot I_2\\U_2&=sM\cdot I_1+sL_2\cdot I_2\end{aligned}\quad \text{(jednadžbe vezanih induktiviteta)}$$

(2 boda)

Uvrstimo vrijednosti elemenata:

$$(1)\ U_1\cdot(1+s)-U_2\cdot s=\frac{1}{s+1}-I_1$$

$$(2)\ -U_1\cdot s+U_2\cdot(1+s)=-I_2$$

$$(3)\ U_1=s\cdot I_1+s\cdot I_2$$

$$(4)\ U_2=s\cdot I_1+s\cdot I_2$$

(1 bod)

$$(3),(4)\Rightarrow U_1=U_2$$

$$(1)\ I_1=\frac{1}{s+1}-U_1\cdot(1+s)+U_1\cdot s=\frac{1}{s+1}-U_1$$

$$(2)\ I_2=-U_2$$

$$(4)\ U_2=s\cdot\overbrace{\left(\frac{1}{s+1}-U_1\right)}^{I_1}+s\cdot I_2=s\cdot\left(\frac{1}{s+1}-U_2\right)-s\cdot U_2=\frac{s}{s+1}-2s\cdot U_2$$

$$U_2(1+2s)=\frac{s}{s+1}$$

$$U_2(s)=\frac{s}{(s+1)(2s+1)}=\frac{s}{2(s+1)(s+1/2)}=\frac{A}{s+1}+\frac{B}{s+1/2}\Rightarrow A=1,\quad B=-\frac{1}{2}$$ (1 bod)

$$\Rightarrow u_2(t)=\left(e^{-t}-\frac{1}{2}e^{-\frac{t}{2}}\right)\cdot S(t)$$ (1 bod)