MI - 2014/2015

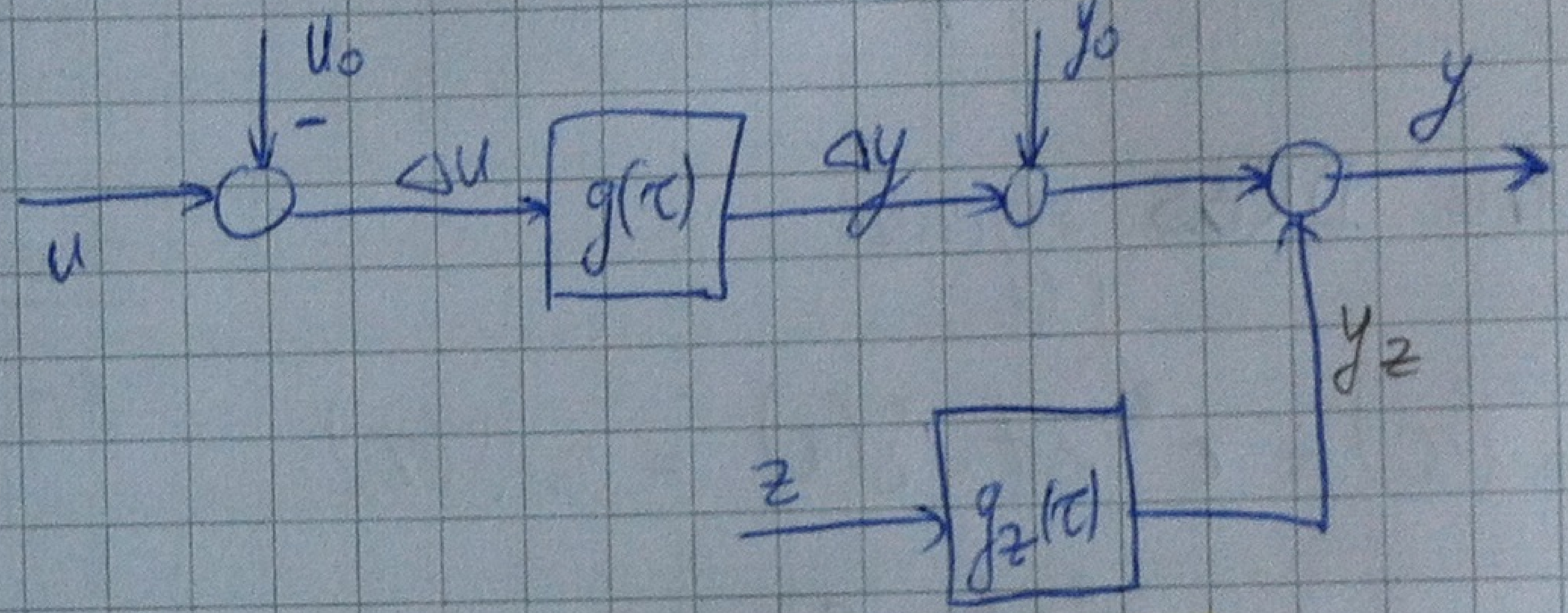
1. ?

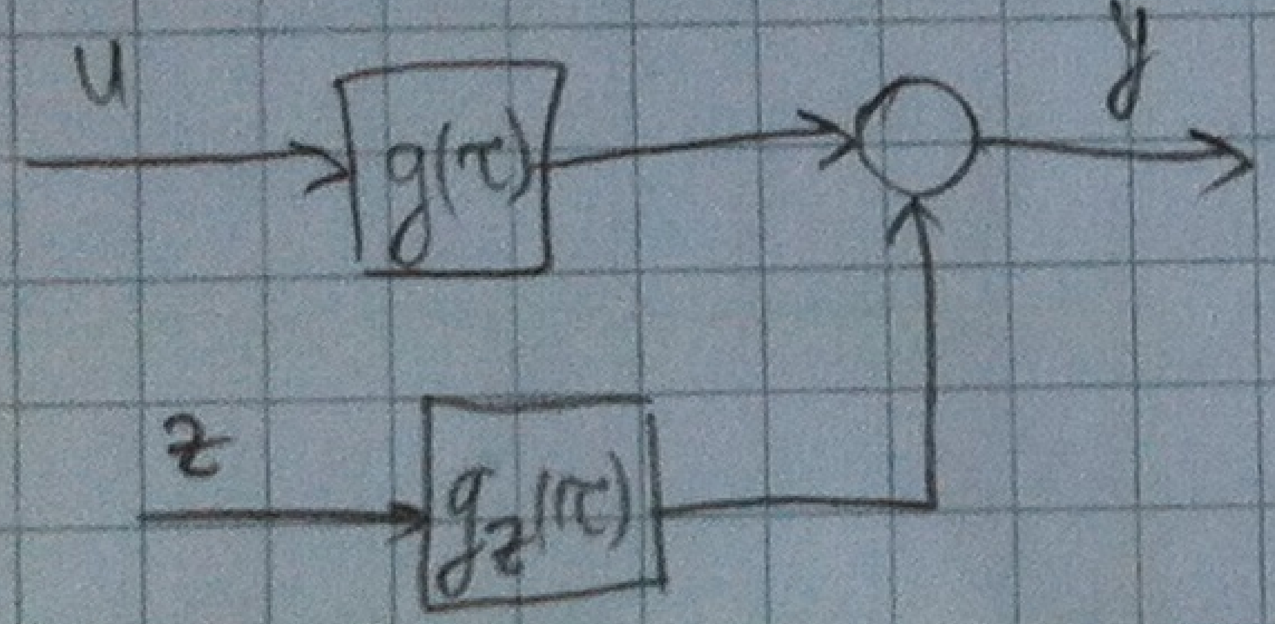
2. $g(\tau) = ?$ u U_0 okolini

$R_{\Delta u z}(\tau) = 0, \quad \forall \tau \in \mathbb{R}$

$S_{\Delta u \Delta u}(\omega) = 4, \quad U_0 y_0 = -4$

$R_{uy}(\tau) = 2e^{-\tau}\cos(\tau)\, S(\tau) - 4$





$$\boxed{R_{uy}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} u(\tau)\, y(t-\tau)\, dt}$$

$$R_{uy}(\tau) = R_{\Delta u + U_0,\, \Delta y + y_0 + y_z}(\tau) = R_{\Delta u \Delta y}(\tau) + R_{\Delta u y_0}(\tau) + \cancelto{0}{R_{\Delta u y_z}(\tau)} + R_{U_0 \Delta y}(\tau) + R_{U_0 y_0}(\tau) + R_{U_0 y_z}(\tau)$$

1°: $R_{\Delta u \Delta y}(\tau) = \int_0^\infty R_{\Delta u \Delta u}(\tau - \sigma)\cdot g(\sigma)\, d\sigma = \int_0^\infty S_{\Delta u \Delta u}(\tau)\,\delta(\tau-\sigma)\cdot g(\sigma)\, d\sigma = \int_0^\infty 4\, g(\sigma)\,\delta(\tau-\sigma)\, d\sigma =$

$$\boxed{R_{uu}(\tau) = S_{uu}(\tau)\,\delta(\tau-\sigma)}$$

$= 4g(\tau)$

2°: $R_{\Delta u y_0}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} \Delta u(t) \cdot y_0(t+\tau)\, dt = y_0 \cdot \lim_{T\to\infty} \frac{1}{2T} \underbrace{\int_{-\infty}^{T} \Delta u(t)\, dt}_{\overline{\Delta u}=0} = 0$

3°: $R_{\Delta u y_z}(\tau) = 0$

4°: $R_{u_0 \Delta y}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} u_0(t) \cdot \Delta y(t+\tau)\, dt = u_0 \cdot \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} \Delta y(t+\tau)\, dt = 0$

5°: $R_{u_0 y_0}(\tau) = -4$

6°: $R_{u_0 y_z}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} u_0(t) \cdot y_z(t+\tau)\, dt = u_0 \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} y_z(t+\tau)\, dt = 0$

$$2e^{-\tau}\cos(\tau)\, S(\tau) \cancel{-4} = 4g(\tau) \cancel{-4}$$

$$g(\tau) = \frac{1}{2} e^{-\tau} \cos(\tau)\, S(\tau)$$

(3.) $R_{zz}(\tau) = 3 \cdot \delta(\tau)$, $\Delta t = 0{,}1$

a) parametre DRBS signala

?

$$R_{zz}(\tau) \doteq c^2 \Delta t\, \delta(\tau) = 3 \cdot \delta(\tau)$$

$$c^2 \Delta t = 3$$

$$c = \sqrt{\frac{3}{\Delta t}} = \sqrt{\frac{3}{0{,}1}} = 5{,}477$$

b) $S_{uu}(0) = ?$

?

$$S_{uu}(0) = c = 5{,}477$$

4. $S_{uu}(\omega) = \frac{1}{2} \frac{9\omega^2+1}{\omega^2+4}$, $S_{uy}(j\omega) = \frac{\dots}{\omega^2+4} \quad \frac{\dots}{3j\omega+1}$

a) $S_{yy} = ?$

$S_{uy}(j\omega) = S_{uu}(\omega) \cdot G(j\omega)$

$G(j\omega) = \frac{S_{uy}(j\omega)}{S_{uu}(\omega)} = \frac{2}{3j\omega + 1}$

$|G(j\omega)|^2 = \frac{S_{yy}(\omega)}{S_{uu}(\omega)}$

$S_{yy} = S_{uu}(\omega) \cdot |G(j\omega)|^2$

$= \frac{1}{2} \cdot \frac{\cancel{9\omega^2+1}}{\omega^2+4} \cdot \frac{4}{\cancel{9\omega^2+1}} = \frac{2}{\omega^2+4}$

b) $\overline{P}_y = ?$

$$\boxed{\overline{P}_y = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} y^2(t)\,dt = R_{yy}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{yy}(\omega)\, e^{j\omega 0}\, d\omega} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{\omega^2+4}\, d\omega =$$

$$= \frac{1}{2\pi} \left(\operatorname{atan} \frac{\omega}{2} \right) \Bigg|_{-\infty}^{\infty} = \frac{1}{2\pi} \left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right) = \frac{1}{2\cancel{\pi}} \cdot \cancel{\pi} = \frac{1}{2}$$

$$\boxed{\int \frac{a}{\omega^2 + a^2}\, d\omega = \operatorname{atan} \frac{1}{a}\omega}$$

5. $G_p(z) = \frac{b_1 + b_2 z^{-1}}{z^2 + a_1 z - a_2} = \frac{b_1 z^{-2} + b_2 z^{-3}}{1 + a_1 z^{-1} - a_2 z^{-2}}$

$T_p = 30\,s$, $T_s = 150\,ms$

a) ρ, ASL (limitiran)

$\rho = ?$

$$\boxed{ASL = \frac{1}{1-\rho} = \frac{T_p}{T_s}}$$

$ASL = \frac{30}{150 \cdot 10^{-3}} = 200$

$1 - \rho = \frac{1}{ASL}$

$\rho = 1 - \frac{1}{ASL} = 0{,}995$

b) $\rho, \rho^2, \rho^3, \dots$

c) $\Phi = ?$, $Y = ?$, $\theta = ?$, $u(0), \dots, u(7)$, $y(0), \dots, y(7)$

$$G_p(z) = \frac{b_1 z^{-2} + b_2 z^{-3}}{1 + a_1 z^{-1} - a_2 z^{-2}} = \frac{Y(z)}{U(z)}$$

$$y(k) = -a_1 y(k-1) + a_2 y(k-2) + b_1 u(k-2) + b_2 u(k-3)$$

$$y(3) = -a_1 y(2) + a_2 y(1) + b_1 u(1) + b_2 u(0)$$
$$y(4) = -a_1 y(3) + a_2 y(2) + b_1 u(2) + b_2 u(1)$$
$$y(5) = -a_1 y(4) + a_2 y(3) + b_1 u(3) + b_2 u(2)$$
$$y(6) = -a_1 y(5) + a_2 y(4) + b_1 u(4) + b_2 u(3)$$
$$y(7) = -a_1 y(6) + a_2 y(5) + b_1 u(5) + b_2 u(4)$$

$$Y = \Phi \cdot \theta - \varepsilon$$

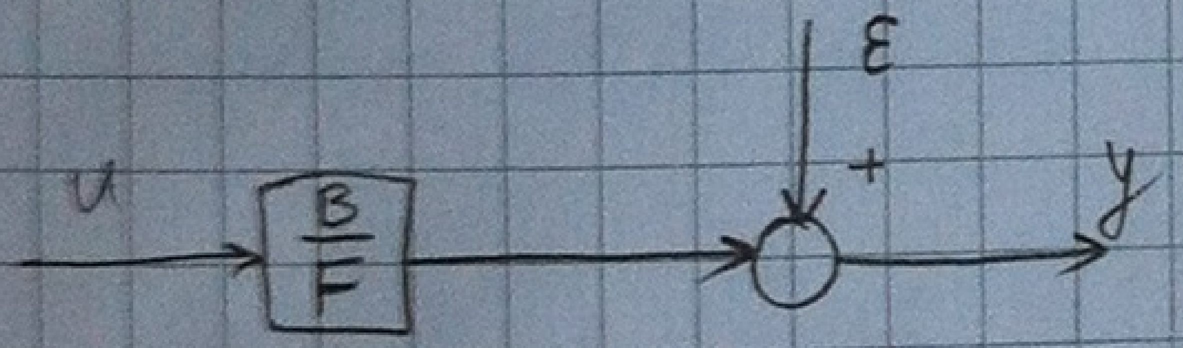
$$Y = \begin{bmatrix} y(3) \\ y(4) \\ y(5) \\ y(6) \\ y(7) \end{bmatrix}, \quad \Phi = \begin{bmatrix} -y(2) & y(1) & u(1) & u(0) \\ -y(3) & y(2) & u(2) & u(1) \\ -y(4) & y(3) & u(3) & u(2) \\ -y(5) & y(4) & u(4) & u(3) \\ -y(6) & y(5) & u(5) & u(4) \end{bmatrix}, \quad \theta = \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix}$$

6.

$$B(z^{-1}) = 1{,}5 z^{-1} + z^{-2}$$
$$F(z^{-1}) = 1 - 1{,}8 z^{-1} + 0{,}81 z^{-2}$$

$$A y = \frac{B}{F} u + \frac{C}{D} \varepsilon$$

a)



b)

$$y(z) = \frac{B(z^{-1})}{F(z^{-1})} \cdot u(z) + \varepsilon(z) = \frac{1{,}5 z^{-1} + z^{-2}}{1 - 1{,}8 z^{-1} + 0{,}81 z^{-2}} \cdot u(z) + \varepsilon(z)$$

$$y(z) = \frac{(1{,}5 z^{-1} + z^{-2}) \cdot u(z) + (1 - 1{,}8 z^{-1} + 0{,}81 z^{-2}) \cdot \varepsilon(z)}{1 - 1{,}8 z^{-1} + 0{,}81 z^{-2}}$$

$$y(k) - 1{,}8 y(k-1) + 0{,}81 y(k-2) = 1{,}5 u(k-1) + u(k-2) + \varepsilon(k) - 1{,}8 \varepsilon(k-1) + 0{,}81 \varepsilon(k-2)$$

c) $y(3) = ?$

$u(k) = \text{Step}$

k	ε(k)
1	0,5
2	0,2
3	-0,2
4	0,3

$$y(3) = 1{,}8\,y(2) - 0{,}81\,y(1) + 1{,}5\,u(2) + u(1) + \varepsilon(3) - 1{,}8\,\varepsilon(2) + 0{,}81\,\varepsilon(1)$$

$$y(2) = 1{,}8\,y(1) \cancel{- 0{,}81\,y(0)} + 1{,}5\,u(1) + u(0) + \varepsilon(2) - 1{,}8\,\varepsilon(1) \cancel{+ 0{,}81\,\varepsilon(0)}$$

$$y(1) = \cancel{1{,}8\,y(0)} \cancel{- 0{,}81\,y(-1)} + 1{,}5\,u(0) + \cancel{u(-1)}^{0} + \varepsilon(1) \cancel{- 1{,}8\,\varepsilon(0)} \cancel{+ 0{,}81\,\varepsilon(-1)}$$

$$y(0) = \cancel{1{,}8\,y(-1)} \cancel{- 0{,}81\,y(-2)} \cancel{+ 1{,}5\,u(-1)}^{0} \cancel{+ u(-2)}^{0} + \varepsilon(0) \cancel{- 1{,}8\,\varepsilon(-1)} \cancel{+ 0{,}81\,\varepsilon(-2)}$$

$$y(0) = \varepsilon(0)$$

$$y(1) = 1{,}5\,u(0) + \varepsilon(1)$$

$$y(2) = 2{,}7\,u(0) + 1{,}8\,\varepsilon(1) + 1{,}5\,u(1) + u(0) + \varepsilon(2) - 1{,}8\,\varepsilon(1)$$

$$y(3) = 4{,}86\,u(0) + \cancel{3{,}24\,\varepsilon(1)} + 2{,}7\,u(1) + 1{,}8\,u(0) + \cancel{1{,}8\,\varepsilon(2)} \cancel{- 3{,}24\,\varepsilon(1)} - 1{,}215\,u(0) \cancel{- 0{,}81\,\varepsilon(1)} +$$
$$+ 1{,}5\,u(2) + u(1) + \varepsilon(3) \cancel{- 1{,}8\,\varepsilon(2)} \cancel{+ 0{,}81\,\varepsilon(1)}$$

$$y(3) = 5{,}445\,u(0) + 3{,}7\,u(1) + 1{,}5\,u(2) + \varepsilon(3) = 5{,}45 + 3{,}7 + 1{,}5 + (-0{,}2) = \underline{\underline{10{,}45}}\ !$$

7.

$$A(z^{-1})\,y(z) = B(z^{-1})\,u(z) + \cancel{\frac{1}{D(z^{-1})}\varepsilon(z)} + v(z)$$

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 u(k-1) + b_2 u(k-2) + v(k)$$

$$v(z) = \frac{\varepsilon(z)}{D(z^{-1})} \Rightarrow v(k) + d_1 v(k-1) + d_2 v(k-2) = \varepsilon(k)$$

$$y(8) = -a_1 y(7) - a_2 y(6) + b_1 u(7) + b_2 u(6) + v(8)$$

$$E[y(8)] = E[-a_1 y(7) - a_2 y(6) + b_1 u(7) + b_2 u(6) + v(8)]$$

$$E[v(8)] = E[-d_1 v(7) - d_2 v(6) + \cancel{\varepsilon(8)}^{0}]$$

$$E[y(7)] = E[-a_1 y(6) - a_2 y(5) + b_1 u(6) + b_2 u(5) + v(7)]$$

$$E[y(6)] = E[-a_1 y(5) - a_2 y(4) + b_1 u(5) + b_2 u(4) + v(6)]$$

$$E[v(6)] = E[y(6)] + E[a_1 y(5) + a_2 y(4) - b_1 u(5) - b_2 u(4)]$$

$$E[v(7)] = E[y(7)] + E[a_1 y(6) + a_2 y(5) - b_1 u(6) - b_2 u(5)]$$

$$E[y(8)] = -a_1 y(7) - a_2 y(6) + b_1 u(7) + b_2 u(6) - d_1 E[v(7)] - d_2 E[v(6)]$$

$$= -a_1 y(7) - a_2 y(6) + b_1 u(7) + b_2 u(6) - d_1 y(7) - a_1 d_1 y(6) - a_2 d_1 y(5) + b_1 d_1 u(6) + b_2 d_1 u(5)$$

$$- d_2 y(6) - a_1 d_2 y(5) - a_2 d_1 y(4) + b_1 d_2 u(5) + b_2 d_2 u(4)$$