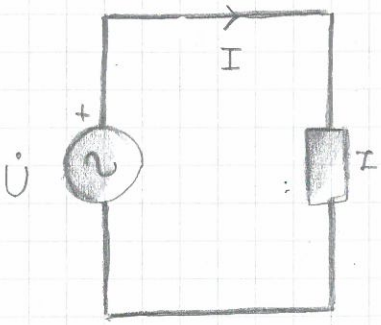
SNAGA IZMJENIČNE STRUJE

LED - dioda	10 mW
E BIKE	250 W
MOTOR AUTOMOBILA	60 kW
ŽARULJA	100 W

↙ 90%



$\dot{I} = |\dot{I}| \angle \varphi$

$\dot{U} = |\dot{U}| \angle \alpha_u$

$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{|U|}{|Z|} \angle \alpha_u - \varphi$

$\alpha_i = \alpha_u - \varphi$

$u(t) = U_m \sin(\omega t + \alpha_u)$

$i(t) = I_m \sin(\omega t + \alpha_i)$

TRENUTNA SNAGA

$$p(t) = u(t) \cdot i(t) = U_m \sin(\omega t + \alpha_u) \cdot I_m \sin(\omega t + \alpha_i)$$

$$\left[\sin\alpha \sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))\right]$$

$$= \underbrace{\frac{1}{2} U_m I_m}_{U_{ef} \cdot I_{ef}} \left[\underbrace{\cos(\alpha_u - \alpha_i)}_{\text{KONSTANTNI ČLAN}} - \underbrace{\cos(2\omega t + \alpha_u + \alpha_i)}_{\text{ČLAN DVOSTRUKE FREKVENCIJE}}\right]$$

konst. član : $p(t) = U_{ef} \cdot I_{ef} \cdot \cos\varphi$

član 2f : $p(t) = -U_{ef} \cdot I_{ef} \cdot \cos(2\omega t + \alpha_u + \alpha_i)$

$\dot{U} = 2 \angle 0°$

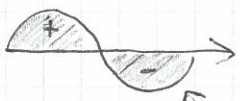
$\dot{I} = \frac{4}{3} \angle 60°$

$\dot{I} = 1.5 \angle -60°$

$p(t) > 0$ → izvor predaje snagu trošilu

$p(t) < 0$ → energija se vraća prema izvoru

SREDNJA VRIJEDNOST SNAGE



$$P_{sred.} = \frac{1}{T} \int_0^T p(t)\,dt = \frac{1}{T} U_{ef} I_{ef} \int_0^T (\cos\varphi - \underbrace{\cos(2\omega t + \alpha_u + \alpha_i)}_{=0}) dt$$

$$= \frac{1}{T} U_{ef} I_{ef} \cdot \cos\varphi \cdot T \rightarrow \text{RADNA SNAGA}$$

FAKTOR SNAGE

$$\cos\varphi = \frac{P}{U \cdot I}$$

$S = UI$ [VA] PRIVIDNA SNAGA

$P = UI\cos\varphi$ [W] RADNA SNAGA $\Rightarrow$ $S = \sqrt{P^2 + Q^2}$ (trokut)

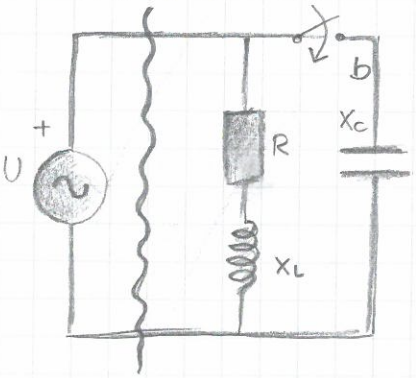
$Q = UI\sin\varphi$ [VAr] JALOVA SNAGA

$\varphi > 0$, $Q > 0$ induktivno ponašanje

$\varphi < 0$, $Q < 0$ kapacitivno ponašanje

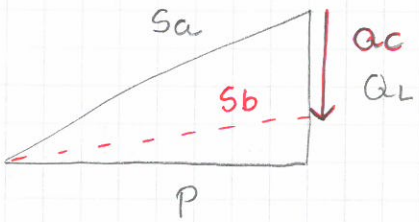
L = trošilo jalove snage C = izvor jalove snage

- KOMPENZACIJA a



izvor nije opterećen jer kondenzator daje jalovu snagu koju troši induktivitet

Trokut snage za induk. trošilo



smanji se struja → gubitci na mreži se smanje

Primjer: Odredi C kojeg dodajemo paralelno $Z = 20\angle 30°$ da bi faktor snage bio 0.95.

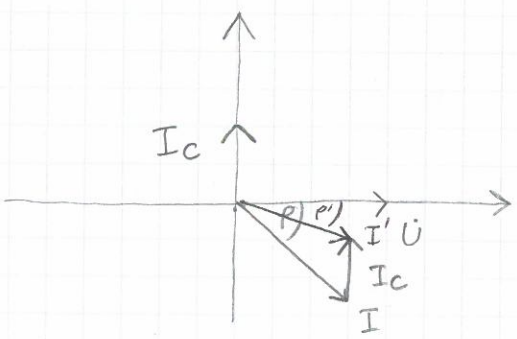
$C = ?$
$Z = 20\angle 30°$
$\cos\varphi = 0.95$
$U = 120$ V

$Z = |Z|\angle\varphi$

$I = \frac{U}{Z} = \frac{120}{20} = 6$ A

$\varphi = 30°$

$\cos\varphi = 0.95 \rightarrow \varphi = 18.19°$



$$tg\,\varphi' = \frac{I\sin\varphi - I_C}{I\cos\varphi}$$

$I_C = I\sin\varphi - I\cos\varphi\, tg\,\varphi'$

$I_C = 1{,}292$ A

$X_C = \frac{U}{I_C} = \frac{1}{\omega C} \Rightarrow$

$C = 34{,}29\,\mu$F

IZRAČUN SNAGE U KOMPLEKSNOM PODRUČJU

- $\dot{U} = U \angle \alpha_u$
  $\dot{I} = I \angle \alpha_i$; $\varphi = \alpha_u - \alpha_i$

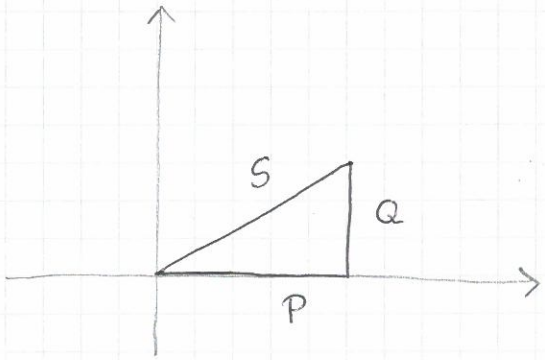
  $S = \dot{U} \cdot \dot{I}^*$
  ↑ konjugirano kompleksno

  $S = U \cdot I (\cos\varphi + j \sin\varphi)$

  $P = U I \cos\varphi$

  $Q = U I \sin\varphi$

  $S = P + jQ$



- $Z = |Z| \angle \varphi$

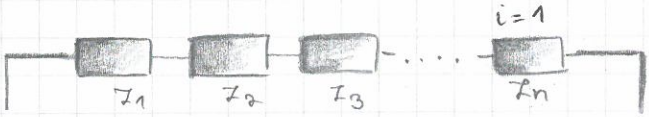
  $\dot{U} = \dot{I} \cdot Z$

  $S = Z \cdot \dot{I} \cdot \dot{I}^* = Z \cdot |I|^2 = |I|^2 (R + jX_L) =$

  $= \underbrace{|I|^2 \cdot R}_{P} + j \underbrace{|I|^2 X}_{Q}$

Serijski spoj n impedancija

$\dot{U} = \dot{U}_1 + \dot{U}_2 \dots + \dot{U}_m = \sum_{i=1}^{m} \dot{U}_i$



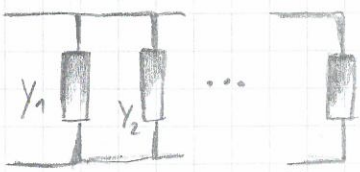
- $S = \dot{U} \cdot \dot{I}^* = \sum_{i=1}^{m} S_i = \sum_{i=1}^{m} (P_i + jQ_i) = \sum_{i=1}^{m} P_i + \sum_{i=1}^{m} jQ_i$

$|S| = \sqrt{(\sum_{i=1}^{m} P_i)^2 + (\sum_{i=1}^{m} Q_i)^2}$

Paralelni spoj $\dot{I} = \dot{U} \cdot Y$ $S = \dot{U} \cdot \dot{I}^* = \dot{U} \cdot \dot{U}^* \cdot Y^* = |U|^2 \cdot Y^*$

$= |U|^2 \cdot (G - jB)$

$S = P + jQ$; $P = |U|^2 G$; $Q = -|U|^2 B$



$\dot{I} = \dot{I}_1 + \dot{I}_2 \dots + \dot{I}_m$

$\dot{U} = \dot{U}_1 = \dot{U}_2 \dots + \dot{U}_m$

- $S = \dot{U} \cdot \dot{I}^* = \sum_{i=1}^{m} \dot{U} \cdot \dot{I}_i^* = \sum_{i=1}^{m} S_i = \sum_{i=1}^{m} P_i + \sum_{i=1}^{m} jQ_i$

$|S| = \sqrt{(\sum_{i=1}^{m} P_i)^2 + (\sum_{i=1}^{m} Q_i)^2}$ ← isto je zbog očuvanja energije

Primjer: Na izvor je priključena impedancija Z. Odredite S, P, Q

$\dot{U} = 100 \angle 30° \ V$

$Z = 3 + j4 \ \Omega = \sqrt{3^2+4^2} \angle \operatorname{arctg} \frac{4}{3} = 5 \angle 53{,}13°$

$\dot{I} = \frac{\dot{U}}{Z} = \frac{100 \angle 30°}{5 \angle 53{,}13°} = 20 \angle -23{,}13°$

a) $P = I^2 \cdot R = 20^2 \cdot 3 = 1200 \ W$
$Q = I^2 \cdot X = 20^2 \cdot 4 = 1600 \ VAr$
$S = \sqrt{P^2+Q^2} = 2000 \ VA$

b) $P = UI \cos\varphi$
$Q = UI \sin\varphi$
$S = UI$

Primjer. Paralela Z_1 i Z_2, snaga na otporniku $\overset{I_1}{V}$ je 20 W.

$Z_1 = 2 - j5 \ \Omega$

$Z_2 = 1 + j \ \Omega$

$P(Z_1) = 20W$

$P = I_1^2 \cdot R$

$I_1^2 = \frac{P}{R} = \frac{20}{2} = 10$

$I_1 = \sqrt{10} \ A$

$Q(Z_1) = |I_2|^2 \cdot (-5) = -50 \ VAr$

$|U| = |I_1| \cdot |Z_1| = \sqrt{10} \cdot \sqrt{2^2+3^2} = \sqrt{290}$

$I_2 = \frac{U}{|Z_2|} = \frac{\sqrt{290}}{\sqrt{2}} = \sqrt{145}$

$Q_2 = I_2^2 \cdot X_2 = 145 \ VAr$

$Q_{uk} = Q_1 + Q_2 = 145 - 50 = 95 \ VAr$

U i I metog dvopola su $u(t) = 100 \sin(\omega t)$, $i(t) = 5 \sin(\omega t - \frac{\pi}{3})$
Odredite Z, S, P, Q

$u(t) = 100 \sin(\omega t)$
$i(t) = 5 \sin(\omega t - \frac{\pi}{3})$

$U = 50\sqrt{2} \angle 0°$
$I = \frac{5\sqrt{2}}{2} \angle 60°$

$Z = \frac{\dot{U}}{\dot{I}} = \frac{50\sqrt{2} \angle 0°}{\frac{5\sqrt{2}}{2} \angle 60°} = 20 \angle -60°$

$P = U \cdot I \cdot \cos\varphi = 125 \ W$
$Q = U \cdot I \cdot \sin\varphi = 216.5 \ VAr$
$S = \sqrt{P^2+Q^2} = 250 \ VA$

Na izvor su spojena paralelno 3 trošila

1) 250 VA, $\cos\varphi = 0.5$ (ind)
2) 180 W, $\cos\varphi = 0{,}8$ (kap)
3) 300 VA, $\cos\varphi = ?$, 100 Var (ind)

$P, Q, S, \cos\varphi$

moram zbrajati P i Q, ne S

1) $S_1 = 250\ VA$

$P_1 = S_1 \cos\varphi_1 = 125\ W$

$Q_1 = S_1 \sin\varphi_1 = 216,5\ VAr$

2) $P_2 = 180\ W$

$S_2 = P_2 / \cos\varphi_2 = 225\ VA$

$Q_2 = P_2 / \sin\varphi_2 = (-)135\ VAr$

kapacitivan

3) $S_3 = 300\ VA$

$Q_3 = 100\ VAr$

$P_3 = \sqrt{S_3^2 - Q_3^2} = 282,84\ W$

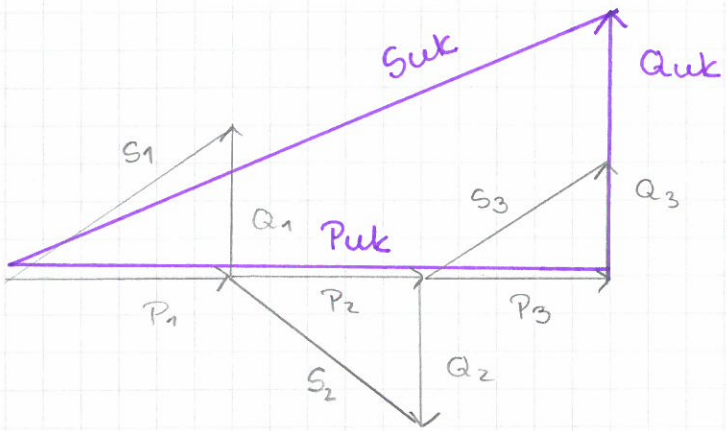
$P_{uk} = P_1 + P_2 + P_3 = 587,84\ W$

$Q_{uk} = Q_1 + Q_2 + Q_3 = 181,5\ VAr$ (ind)

$S = \sqrt{P_{uk}^2 + Q_{uk}^2} = 615,22\ VA$ ($S = \cancel{S_1 + S_2 + S_3}$)

NE SMIJEM !

$\cos\varphi = \frac{P_{uk}}{S_{uk}} = 0.955$



Zbirka SNAGA U KRUGOVIMA IZHJENIČNE STRUJE

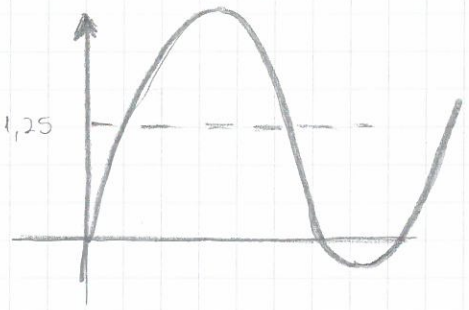
P1. $u(t) = 5 \sin(314t)$ V
$i(t) = 1 \sin(\omega t + \pi/3)$ A

struja prethodi naponu → kapacitivan karakter

$p(t) = U \cdot I \cos\varphi - UI \cos(2\omega - \varphi)$ $\quad \varphi = \varphi_u - \varphi_i$

ef. vrij. ↓ ↓ harmonijska funkcija dvostruke frekvencije

ISTOSMJERNA KOMPONENTA (srednja vrijednost)



KONDENZATOR → prilikom nabijanja skuplja energiju, a kod izbijanja ju vraća u krug.

Ako je dvopol samo L ili C tada je srednja vrijednost jednaka nuli.
Ako je samo otpornik u krugu snaga nema neg. vrijednosti jer on ne može vraćati E u krug.

P2. $U = 3{,}5$ V
$I = 0{,}7$ A
$\varphi = -60°$

radna snaga (srednja vrijednost trenutačne) $P = UI\cos\varphi = 1{,}225$ W

$P = I^2R \Rightarrow R = 2{,}5\,\Omega$

$S = UI = 2{,}5$ VA → prividna snaga

jalova, reaktivna snaga (ma L i C) $Q = UI\sin\varphi = -2{,}12$ VAr

$Q = I^2X_C \Rightarrow X_C = 4{,}33\,\Omega$

induktivna reaktivna snaga - pozitivna

kapacitivna reaktivna snaga - negativna

P3. maks. trenutačna snaga 300 VA
min. -11- -100 VA

„od vrha do vrha" = 400 VA ⇒ 200 je amplituda ⇒ PRIVIDNA SNAGA

$P = 100\,W$

$Q = \sqrt{S^2 - P^2} = 173\ VAr$

$\cos\varphi = \frac{P}{S} = 0{,}5$

P4. $R = 10\,\Omega$
$X_L = 5\,\Omega$
$X_C = 15\,\Omega$
$I = 1\,A$

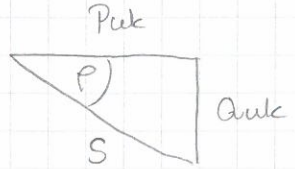
$P = I^2 \cdot R = 10\,W$ $\quad P_{uk} = 10\,W$

$Q_L = I^2 \cdot X_L = 5\,VAr$
$Q_C = -I^2 \cdot X_C = -15\,VAr$ $\Big\}$ $Q_{uk} = -10\,VAr$ $\quad S_{uk} = 14{,}1\,VA$

$S_{uk} = \sqrt{P_{uk}^2 + Q_{uk}^2}$

$\cos\varphi = \frac{P}{S} \Rightarrow \varphi = -45°$ kapacitivan spoj

$\varphi_u - \varphi_i \Rightarrow \varphi_i > \varphi_u$



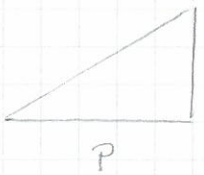
Ako je spoj u rezonanciji reaktivna se energija u cijeosti razmjenjuje između reaktivnih elemenata unutar spoja.

P6. $U = 120\,V$
$f = 50\,Hz$
$Z = 20 + 30j\,\Omega$

kompenzacija reaktivne snage - zbog suprotnih karaktera induktivne i kapacitivne snage.
⇒ postiže se da se razmjena reaktivne snage odigrava na strani trošila, a ne između trošila i izvora

$Z = 36\,\Omega$
$I = U/Z = 3{,}33\,A$

$P = I^2 R = 3{,}33^2 \cdot 20 = 221{,}778\,W$
$Q_L = I^2 \cdot X_L = 3{,}33^2 \cdot 30 = 332{,}7\,VAr$



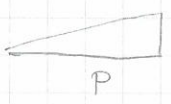
$\varphi = \operatorname{arctg}\frac{Q_L}{P} = 56{,}3°$

$\cos\varphi = 0.55$ (ind.) ⇒ paralelno treba spojiti kondenzator

$\arccos(0{,}95) = 18{,}2°$

$Q = \operatorname{tg} 18{,}2° \cdot P = 72{,}89\ VAr$ $\quad Q_C = \frac{U^2}{X_C}$

$Q_C = 332{,}7 - 72{,}89 = 259{,}8$ $\quad C = 57\,\mu F$



P7. $U = 100\ V$
$R_i = 10\ \Omega$
$X_i = 5\ \Omega$ (kap)
$R_{tc} = 20\ \Omega$
$X_{tp} = 20\ \Omega$ (kap)
$X_{tc} = 20\ \Omega$ (ind)

maksimalna snaga $\Rightarrow$ $R_t = R_i$
$X_t = -X_i$ (istog iznosa ali suprotnog predznaka)

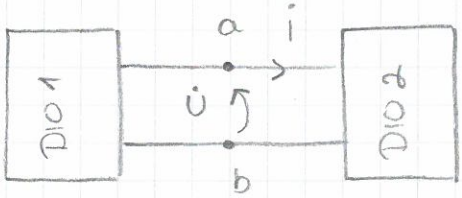
SERIJSKA REZONANCIJA

Napomena: Ako se može mijenjati samo ohmsko radnog dijela trošila tada je uvjet za maks. snagu

$R_t = \sqrt{X_i^2 + R_i^2}$ reaktivni dio impedancije trošila dodaje se umnožak jednoj impedanciji izvora

ZADACI:

1.



$U = 220V$
$I = 10A$
$\varphi_u = 0°$
$\varphi_i = 45°$

$U = 220\ (\cos 0° + j \sin 0°) = 220\ V$
$I = 10\ (\cos 45° + j \sin 45°) = 7{,}07 + 7{,}07j\ A$

$S = U \cdot I$

2. $P_{max} = 1600\ VA$
$P_{min} = -400\ VA$

Amplituda = S

"od vrha do vrha" = $2000\ VA \Rightarrow S = 1000\ VA$

$P = 600\ W$ $\quad Q = \sqrt{S^2 - P^2} = 800\ VA$

$\varphi = \operatorname{arctg} \frac{Q}{P} \Rightarrow \cos\varphi = 0{,}6$

3. $U_m = 220\ V$
$f = 50\ Hz$
$I_m = 4{,}5\ A$
$\cos\varphi = 0{,}866$
$C = ?$

$S = U \cdot I = 990\ VA$
$P = U \cdot I \cdot \cos\varphi = 857{,}34\ W$
$Q = U \cdot I \cdot \sin\varphi = 495{,}04\ VAr$

$\cos\varphi = 0{,}95 \Rightarrow \varphi = 18{,}2°$ $\quad tg\ 18{,}2° = \frac{Q}{P} \Rightarrow Q = 281{,}379$

$Q_c = 213{,}16$

$C = \frac{Q_c}{\omega U^2} = 14\ \mu F$

4. ① $S_1 = 250\ VA$; $\cos\varphi_1 = 0{,}5$ (ind.)

② $P_2 = 180\ W$; $\cos\varphi_2 = 0{,}8$ (kap.)

③ $S_3 = 300\ VA$; $Q_3 = 100\ VAr$

① $P_1 = S_1 \cdot \cos\varphi_1 = 125\ W$
$Q_1 = S_1 \cdot \sin\varphi_1 = 216{,}5\ VAr$

② $S_2 = 225\ VA$
$Q_2 = S_2 \cdot \sin\varphi = 135\ VAr$

③ $Q_3 = S_3 \cdot \sin\varphi \Rightarrow \sin\varphi = \frac{1}{3}$
$P_3 = S_3 \cdot \cos\varphi = 282{,}84\ W$

$P_{uk} = P_1 + P_2 + P_3 = 587{,}84\ W$

$Q_{uk} = Q_1 - Q_2 + Q_3 = 181{,}5\ VAr$ (ind)

$S = \sqrt{P^2 + Q^2}$

$= 615\ VA$

5. $\cos\varphi = 0{,}5 \Rightarrow$ vzporedno serijski ?

$$Z_p = \frac{R \cdot (-jX_C)}{R - jX_C} \cdot \frac{R + jX_C}{R + jX_C} = \frac{R^2(-jX_C) + R(X_C)^2}{R^2 + X_C^2}$$

$$S_p = I_p^2 \cdot Z_p = \underbrace{I_p^2 \cdot \frac{RX_C^2}{R^2 + X_C^2}}_{P} - \underbrace{I_p^2 \cdot \frac{R^2 X_C j}{R^2 + X_C^2}}_{Q}$$

$$\cos\varphi = \frac{P}{S}$$

$$|S| = I_p^2 \sqrt{\frac{R^2X_C^4}{(R^2+X_C^2)^2} + \frac{R^4X_C^2}{(R^2+X_C^2)^2}}$$

$$\frac{1}{2} = \frac{\frac{RX_C^2}{R^2+X_C^2}}{\sqrt{R^2X_C^4 + R^4X_C^2}} = \frac{RX_C^2}{RX_C\sqrt{X_C^2+R^2}} = \frac{1}{2} \quad /^2$$

$$\frac{X_C^2}{X_C^2 + R^2} = \frac{1}{4}$$

$$4X_C^2 = X_C^2 + R^2 \Rightarrow R^2 = 3X_C^2$$

$$Z = R - jX_C \qquad S = I_S^2 \cdot Z = \underbrace{I_S^2 R}_{P} - I_S^2 jX_C$$

$$\cos\varphi = \frac{P}{S} = \frac{I_S R}{I_S \sqrt{R^2+X_C^2}} = \frac{R}{\sqrt{R^2+X_C^2}} \quad /^2$$

$$\cos^2\varphi = \frac{R^2}{R^2+X_C^2} = \frac{3X_C^2}{4X_C^2} \Rightarrow \cos\varphi = \frac{\sqrt{3}}{2}$$

6. $U = 110\,V$

$Z_i = 4\sqrt{2}\,\angle 45° = 4 + 4j$

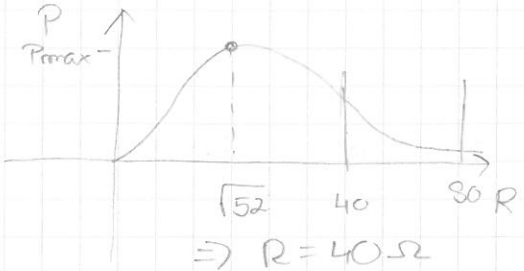
$X_C = -j\,10\,\Omega = 10\,\Omega$ ↗ ne morem ga vgraditi pa ga dodajem izvoru

$R_1 = 40\,\Omega$

$R_2 = 80\,\Omega$ $\Big\}\ R = [40, 80]\,\Omega$

$Z_i' = Z_i - jX_C = 4 + 4j - j10 = 4 - 6j$

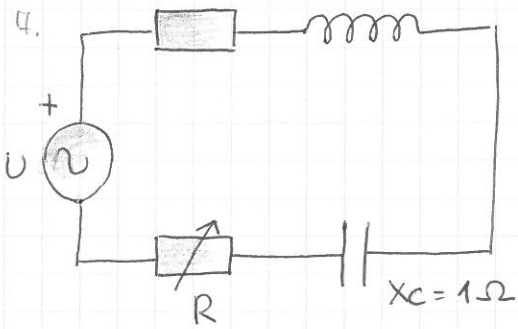
$P = I^2 \cdot R = \left(\frac{U}{(R_i + R)^2 + 4}\right)^2 \cdot R$



$R(P_{max}) = \sqrt{R_i^2 + X_i^2} = \sqrt{52}$

$P = \left(\frac{U}{\sqrt{(R_i + R)^2 + 4(X_i - X_C)^2}}\right)^2 \cdot R = 245{,}4\,W$

4.



$U = 9{,}48\,V$ $\Rightarrow R = 40\,\Omega$

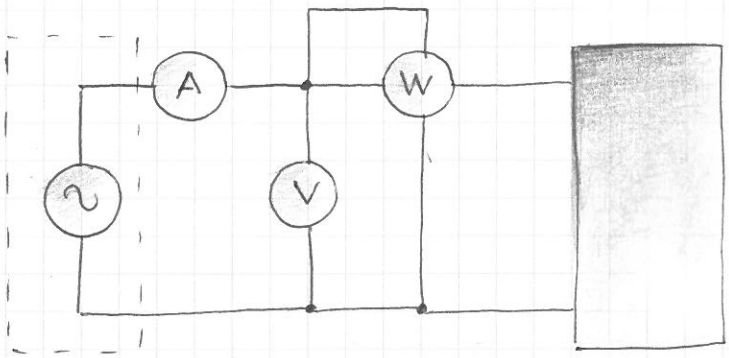
$R = \sqrt{R_i^2 + (X_L - X_C)^2} = \sqrt{16 + 9} = 5\,\Omega$

$Z = 4 + 4j - j + 5 = 9 + 3j = 3\sqrt{10}\,\angle 16{,}7°$

$I = \frac{U}{Z} = 0.99\,\angle -16{,}7°$

$P = I^2 \cdot R = 5\,W\,\angle -16{,}7°$

8.



$P = 30\,W$

$U = 220\,V$

$I = 300\,mA$

$f = 50\,Hz$

$P = U \cdot I \cdot \cos\varphi$

$30 = 220 \cdot 0{,}3 \cdot \cos\varphi$

$\varphi = 82{,}96°$

$Z = \frac{U}{I} = 733{,}33$

$S = I^2 Z = \underbrace{I^2 R}_{P} + \underbrace{I^2 X_L}_{Q}$

$30 = 0{,}3^2 \cdot R$

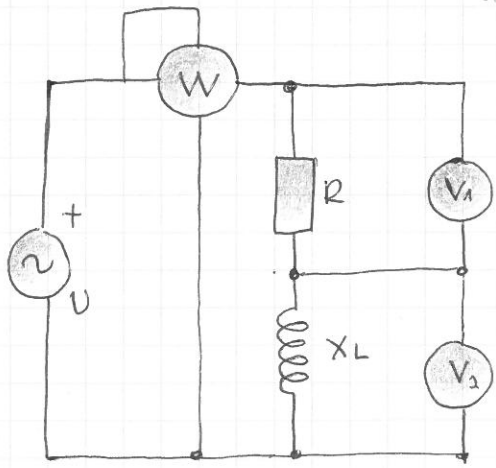
$R = 333\,\Omega$

$Q = S \sin\varphi = 58{,}785 = 0{,}3^2 \cdot X_L$

$X_L = 653{,}17 = 2\pi f \cdot L$

$L = 2{,}079\,H$

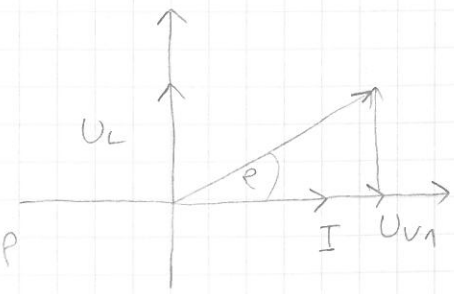
9.



$U_{V1} = 30\,V$

$U_{V2} = 50\,V$

$P_W = 30\,W$



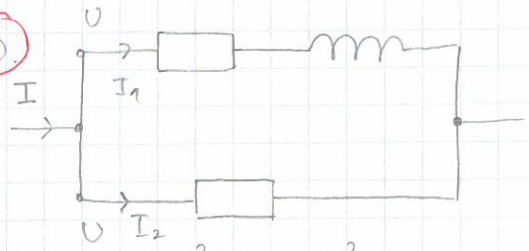
$U = 58{,}3\,V$

$P = U \cdot I \cdot \cos\varphi$

$I = 1\,A$

$P = I^2 R \Rightarrow R = 30\,\Omega$

10.



$R_1 = 10\,\Omega$
$R_2 = 3\,\Omega$
$X_L = 4\,\Omega$
$P_{uk} = 1100\,W$

$I = I_1 + I_2$

$P_{uk} = I_1^2 \cdot R_1 + I_2^2 \cdot R_2$
$1100 = 10I_1^2 + 3I_2^2$
$1100 = 40I_2^2 + 3I_2$
$I_2 = 5{,}06\,A$

$Z_1 = R_2 + jX_L = 3 + 4j = 5\,\angle 53{,}13°$

$Z_1 = R_1 = 10\,\angle 0°$

SLJEDEĆA STRANICA

11. $R = 5\,\Omega$
$Z = 3 + 4j\,\Omega$

$S = \underbrace{I^2R}_{P} + j\underbrace{I^2X}_{Q}$

$Q = 4 \cdot I^2$
$I = 5\,A$

$P = I^2R = 5^2 \cdot 8 = 200\,W$

$Q = 100\,VAr$

$Z_{uk} = 8 + 4j\,\Omega$

12. $R = 5\,\Omega$; $U_R = 31{,}6\,V \Rightarrow I = 6{,}32\,A$
$X_L = 15\,\Omega$

$S = I^2R + jI^2 \cdot X = 200 + 600j$

13. $Z_1 = 2 - 4j$ $\quad |Z_1| = 2\sqrt{5}$
$Z_2 = 4 + 2j$ $\quad P = 20\,W$ $\quad |Z_2| = 2\sqrt{5}$ } jednaka struja

$20 = 4 \cdot I^2$
$I = \sqrt{5}$

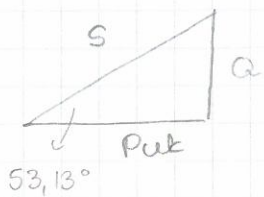
upola manji R, jednaka struja

$P_{R_2} = 20\,W$; $P_{R_1} = 10\,W$ $\quad P_{uk} = 30\,W$

$Q_1 = 4 \cdot I^2 = -20\,VAr$ $\quad Q_2 = 10\,VAr$ $\quad Q_{uk} = 10\,VAr$ (kap)

$S_{uk} = 31{,}63\,VA$

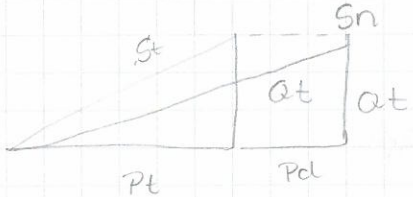
14. $S = 25\,kVA = U \cdot I$
$P_t = 12\,kW$
$\cos\varphi = 0{,}6$



$S = \frac{P_t}{\cos\varphi} = \frac{12\,kW}{0{,}6} = 20\,kVA$

$P_{uk} = 15\,kW$

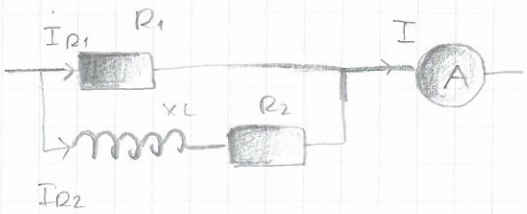
$n = \frac{12}{15} = 0{,}8$



$(P_t + P_{cl})^2 + Q_t^2 = S_n^2$
$P_{cl} = 7{,}2\,kW$

Primjer: Odredite struju ampermetra.

$P_{uk} = 1100\ W$
$R_1 = 10\ \Omega$
$R_2 = 3\ \Omega$
$X_L = 4\ \Omega$



$Z_1 = 10\ \Omega$
$Z_2 = 3 + 4j = 5\ \angle 53{,}13°$

$$Z_{uk} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{10 \cdot (3+4j)}{13+4j} = \frac{30+40j}{13+4j} \cdot \frac{13-4j}{13-4j}$$

$$= \frac{390 - 120j + 520j + 160}{169 + 16} = \frac{550}{185} + \frac{400j}{185} = 2{,}97 + 2{,}16j$$

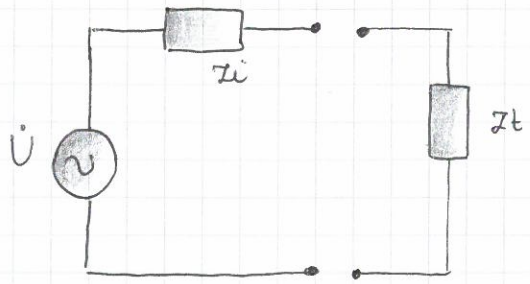
$$= 3{,}67\ \angle 36°$$

$$P = U \cdot I \cdot \cos f = I^2 \cdot I \cdot \cos f$$

$$1100\ W = I^2 \cdot 3{,}67 \cdot \cos 36°$$

$$I = 19{,}25\ A$$

TEOREM PRIJENOSA MAKSIMALNE SNAGE



$$Z_i = R_i + jX_i$$

$$Z_t = R_t + jX_t$$

$$\dot{I} = \frac{\dot{U}}{Z_i + Z_t} = \frac{\dot{U}}{(R_i + R_t) + j(X_i + X_t)}$$

$$\dot{I} = \frac{U}{\sqrt{(R_i + R_t)^2 + (X_i + X_t)^2}}$$

$$P_t = I^2 \cdot R_t = U^2 \frac{R_t}{(R_i + R_t)^2 + (X_i + X_t)^2} = f(R_t, X_t)$$

(1) $$\frac{dP_t}{dR_t} = U^2 \frac{(R_i + R_t)^2 + (X_i + X_t)^2 - R_t \cdot 2 \cdot (R_i + R_t)}{((R_i + R_t)^2 + (X_i + X_t)^2)^2} = 0$$

(2) $$\frac{dP_t}{dX_t} = U^2 \frac{-R_t \cdot 2(X_i + X_t)}{((R_i + R_t)^2 + (X_i + X_t)^2)^2} = 0$$

$$\frac{dP_t}{dX_t} = 0 \ ; \quad X_t = -X_i$$

u (1) $$\frac{dP_t}{dR_t} = 0 \qquad (R_i + R_t)(R_i + R_t - 2R_t) + (X_i + X_t)^2 = 0$$

$$R_i = R_t$$

$$Z_t = Z_i^* \qquad *$$

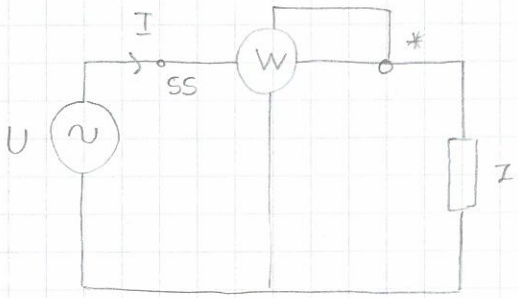
2. Ako je u trošilu samo radni otpor Rt

$$Rt = \sqrt{Ri^2 + Xi^2}$$

3. Ako imam samo Xt

$$Xt = -Xi$$

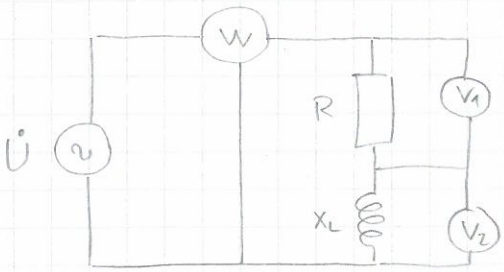
Mjerenje snage izmjenične struje

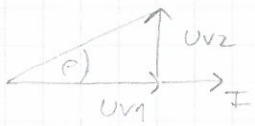


$P_w = U_w I_w \cos \angle U_w I_w$

$U_{V1} = 30\ V$
$U_{V2} = 50\ V$
$P_w = 30\ W$

$R, X_L, U = ?$





$U = 58,3$
$\varphi = 59°$

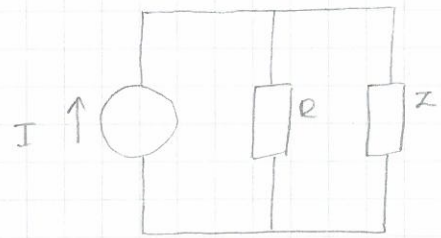
$P_w = U \cdot I \cdot \cos\varphi$
$30 = 58,3 \cdot I \cdot \cos 59°$
$I = 1\ A$

$Q = I \cdot U \cdot \sin\varphi = I^2 X_L$
$X_L = 49,97\ \Omega$

$P = \frac{U_e^2}{R} = \frac{30^2}{30} = 30\ \Omega$

Zbirka:

15. $R = 10\ \Omega$
$I = 8\ \angle -30°$
$I = 5A$



$\underline{I} = 4\sqrt{3} - j4$

$R_p = R \| \underline{I} = \frac{R \cdot \underline{I}}{R + \underline{I}} = 4,4 - j1.3$

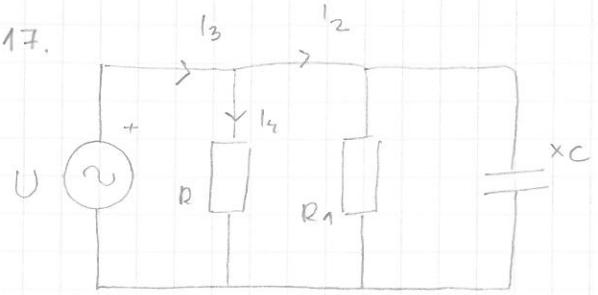
$\underline{U} = \underline{I} \cdot \underline{I}_p = 5 \cdot (4,4 - j1.3) = 23\ V$

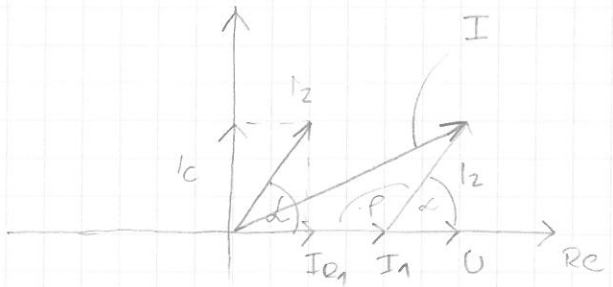
$P = I^2 R_p = 110\ W$

$P_R = \frac{U^2}{R} = 52,9\ W$

$Q = I^2 \cdot X_p = -33\ VAr$

17.





$I^2 = I_1^2 + I_2^2 - 2I_1 I_2 \cos\beta$

$\cos\beta = \frac{-1}{8}$

$\beta = 97,18°$

$\alpha = 180 - \beta = 82,82°$

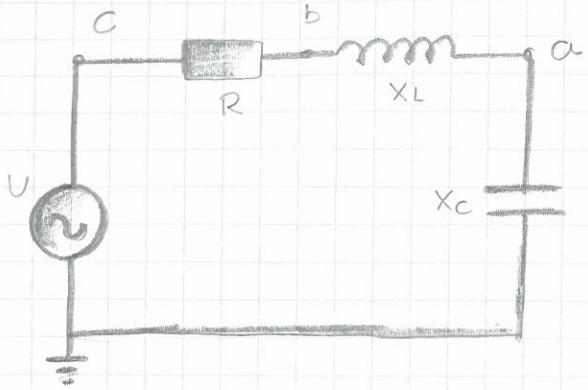
$\cos\alpha = \frac{I_{R1}}{I_2}$ $\quad \sin\alpha = \frac{I_c}{I_2}$ $\quad tg\alpha = \frac{I_c}{I_{R1}} = \frac{\frac{U}{X_C}}{\frac{U}{R}} = \frac{R_1}{X_C}$

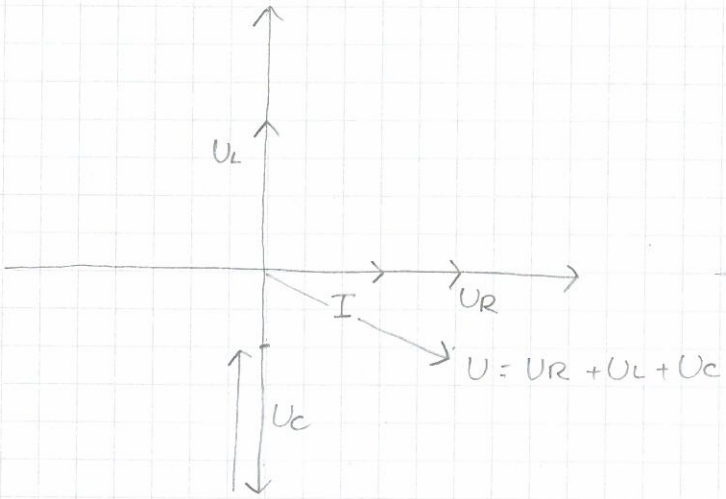
$I_{R1} = I_2 \cos\alpha$ $\quad I_c = I_2 \sin\alpha$ $\quad X_C = \frac{R_1}{tg\alpha}$

$Q = I_c^2 X_C = (I_2 \sin\alpha)^2 \cdot \frac{R_1}{tg\alpha} = 4 \cdot \sin 82,82° \cdot \frac{4}{tg\,82,82°}$

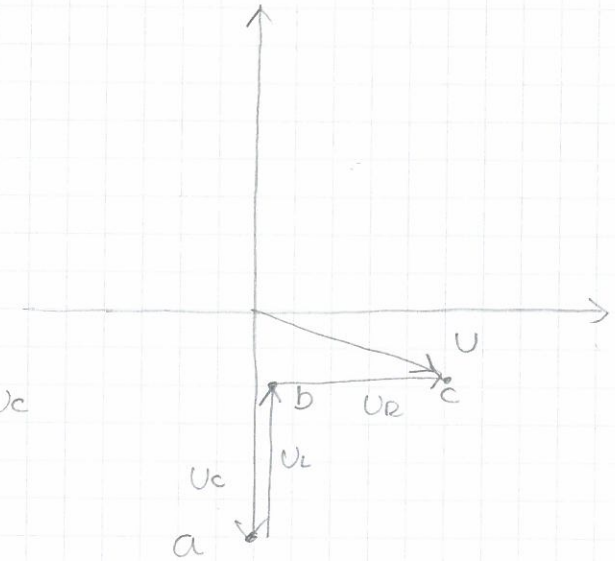
$= 1,98$ VAr

TOPOGRAFSKI DIJAGRAM

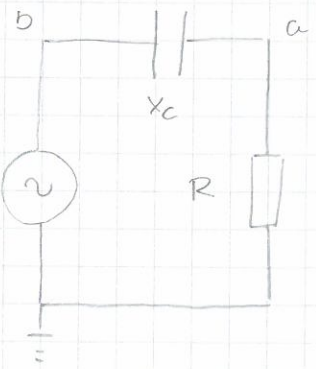


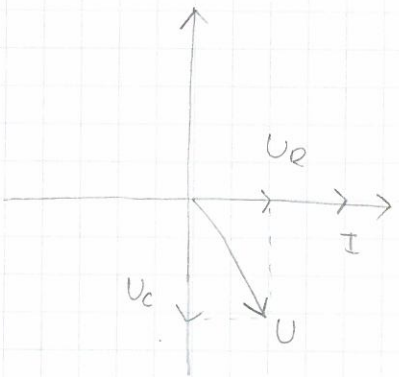


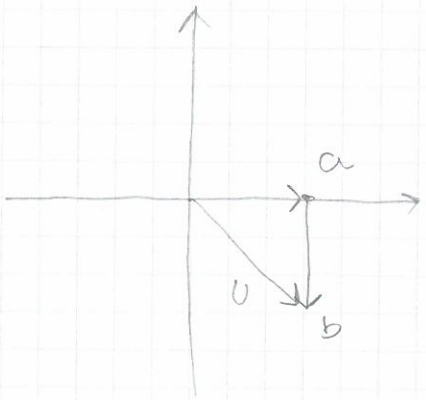
FAZORSKI DIJAGRAM



TOPOGRAFSKI DIJAGRAM

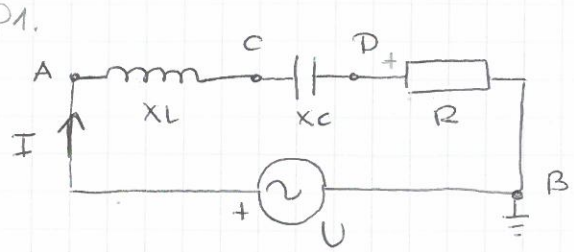






Topografski dijagram - zbirka

P1.



$R = 20\Omega$
$X_L = 20\Omega$
$X_C = 40\Omega$
$U = 20V$

$\underline{Z} = 20 - 20j\,\Omega = 20\sqrt{2}\ \angle -45°$

$\underline{U} = 20\ \angle 0°$

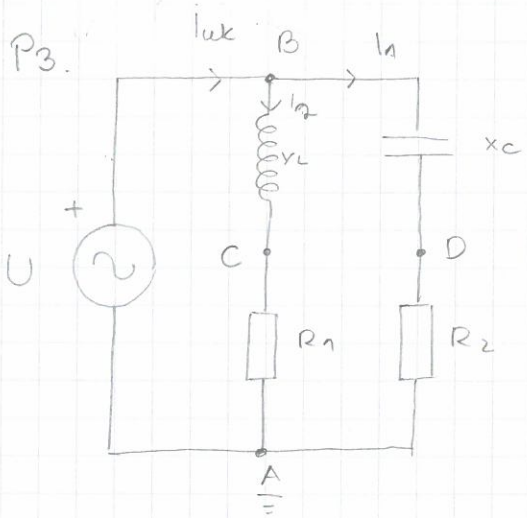
$\underline{I} = \frac{\underline{U}}{\underline{Z}} = 0{,}7\ \angle 45°$

$\underline{P_D} = \underline{I} \cdot R = 0{,}5\ \angle 45° \cdot 20 = 14{,}1\ \angle 45° = 10 + 10j\ V$

$\underline{P_C} = \underline{P_D} + \underline{I}\,X_C = 10 + 10j + 0{,}707\ \angle 45° \cdot 40\ \angle -90° = 30\ \angle -45° + 10 + 10j$

$= 30 - 10j\ V$

P3.



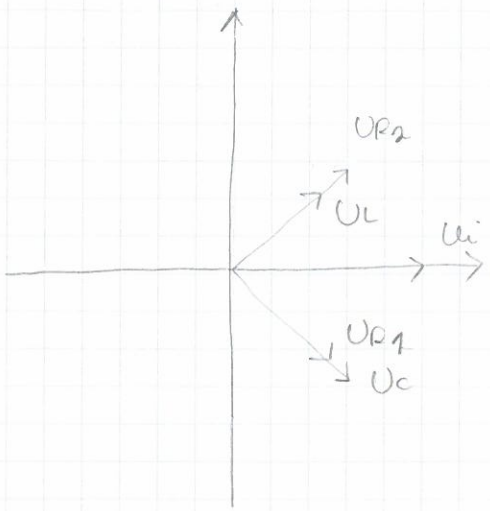
$R_1 = R_2 = X_C = X_L = 24\Omega$; $U = 24V$

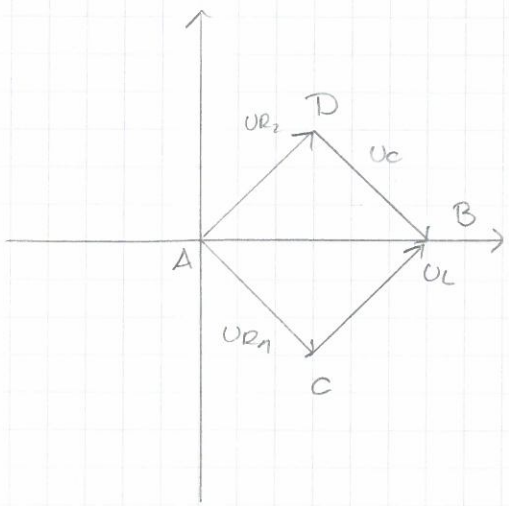
$\underline{Z_1} = 24 - 24j$

$\underline{Z_2} = 24 + 24j$

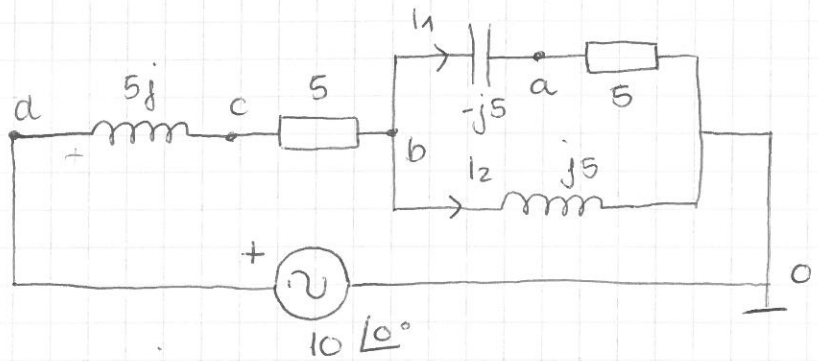
$\underline{I_1} = \frac{\underline{U}}{\underline{Z_1}} = 0{,}707\ \angle 45°$

$\underline{I_2} = 0{,}707\ \angle -45°$





P5.



$Y = 10 + 10j$

$= 10\sqrt{2}\ \angle 45°$

$I = \frac{U}{Z} = \frac{10\ \angle 0°}{10\sqrt{2}\ \angle 45°} = \frac{\sqrt{2}}{2}\ \angle -45 = 0{,}707\ \angle -45°$

$U_{dc} = I \cdot 5j = 0{,}707\ \angle -45° \cdot 5\ \angle 90° = 3{,}535\ \angle 45°\ V$

$U_{cb} = I \cdot 5 = 3{,}535\ \angle -45°\ V$

$U_p = 5\ V$

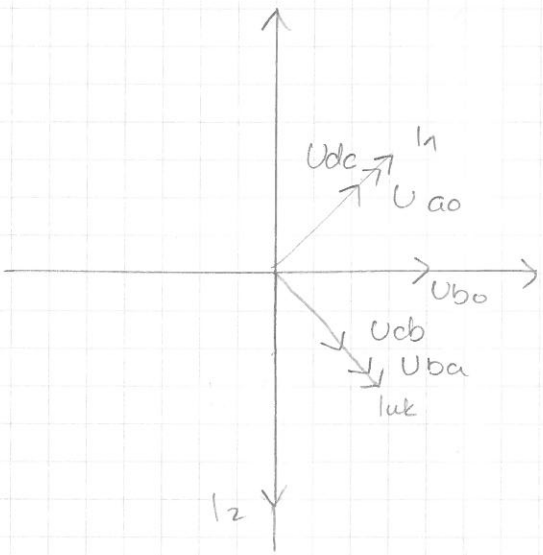
$I_1 = \frac{U_p}{5-5j} = 0{,}707\ \angle 45°\ A$

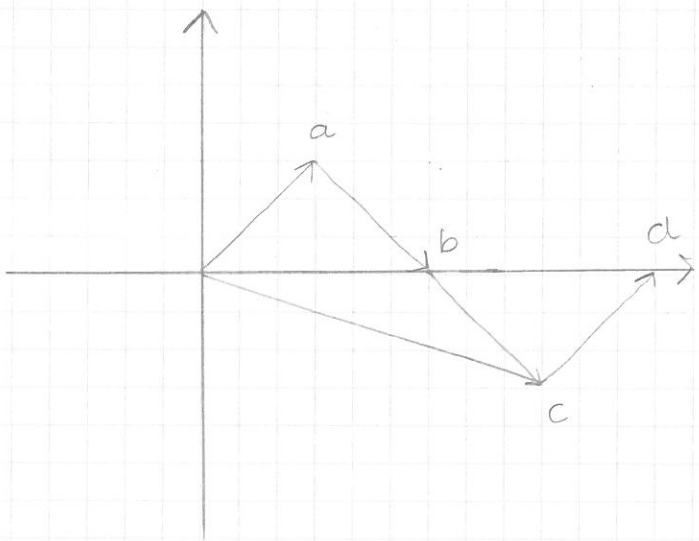
$I_2 = 1\ \angle -90°\ A$

$U_{ba} = 3{,}535\ \angle -45°$

$U_{ao} = 3{,}535\ \angle 45°$

$U_{bo} = I_2 \cdot 5j = 5$

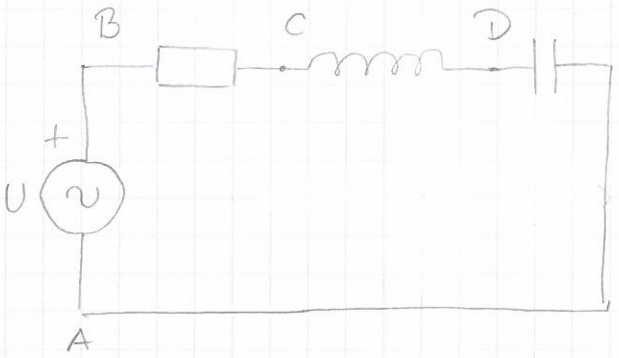




TOPOGRAFSKI DIJAGRAM

→ vektorski dijagram napona u krugu

- vektori napona su nadovezani jedan na drugi kako su elementi spojeni u shemi
- odaberemo 1 čvor kao referentni $\varphi = 0$
- potencijale ostalih čvorova u odnosu na čvor ref. pot.

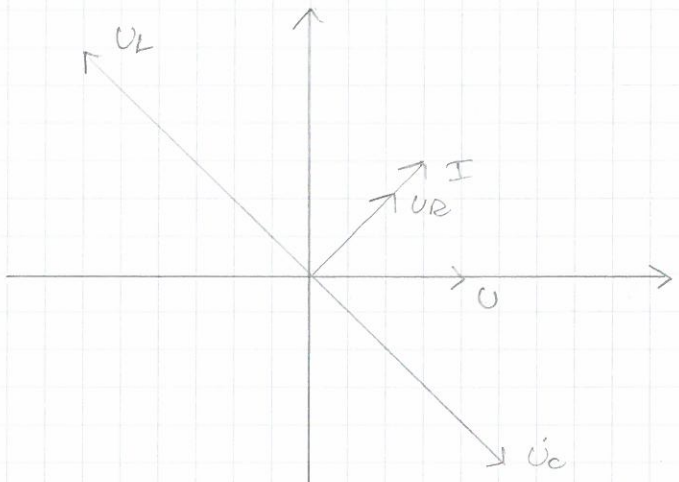


$U = 6 \angle 0° \; V$
$R = 2\,\Omega$
$X_L = 8\,\Omega$
$X_C = 10\,\Omega$

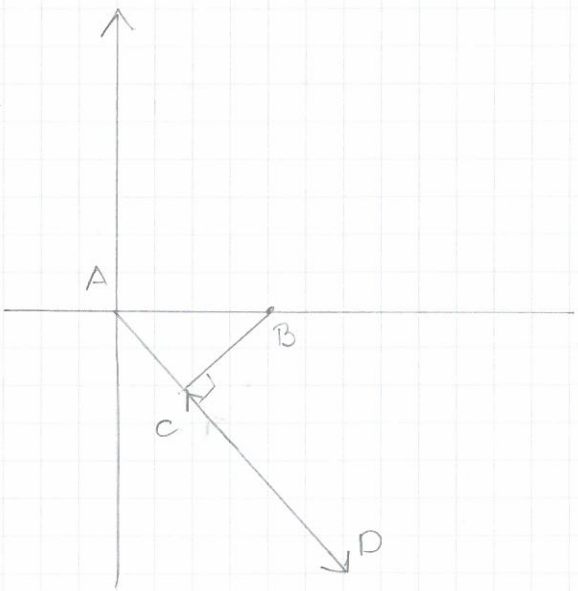
$$Z = R + j(X_L - X_C) = 2\sqrt{2} \angle -45°$$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{3}{\sqrt{2}} \angle 45°$$

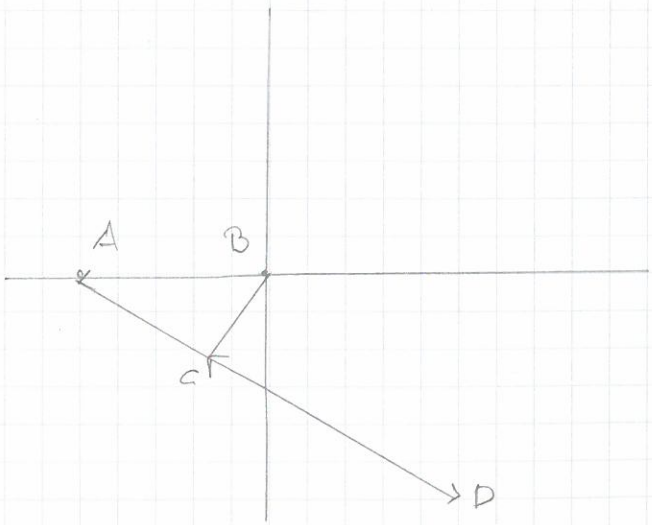
$\dot{U}_R = \dot{I} \cdot R = 3 + 3j \; V$
$\dot{U}_L = \dot{I} \, jX_L = -12 + j12 \; V$
$\dot{U}_C = 15 - 15j \; V$



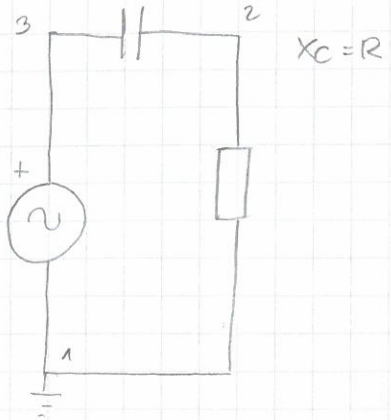
V1

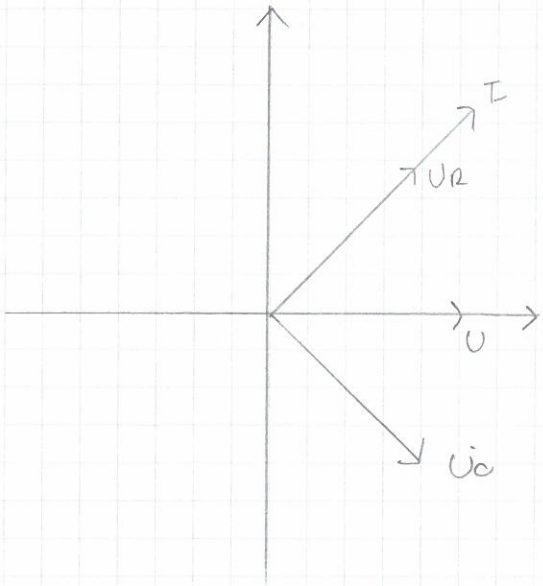


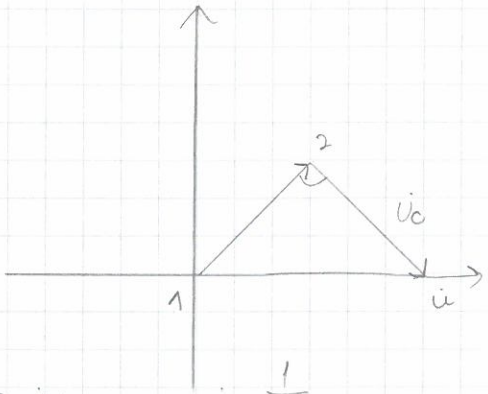
V2. $\varphi_B = 0$



MJESNI DIJAGRAM

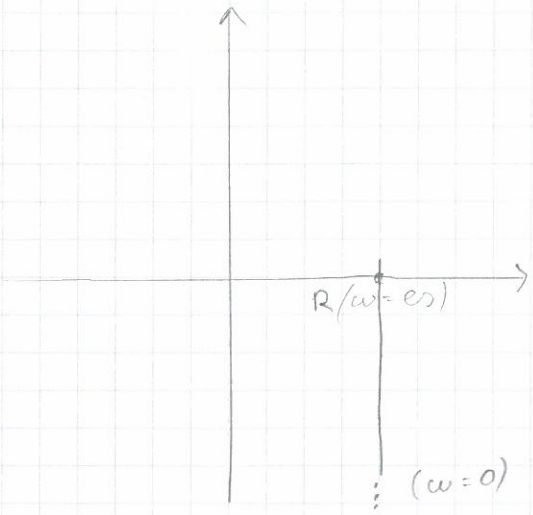


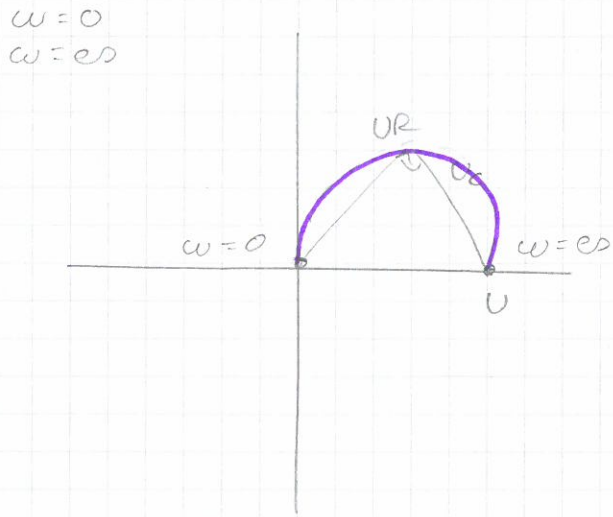




Mjesni dijagram je prikaz u kompleksnoj ravnini koje neka veličina poprima pri promjeni nekog parametra.

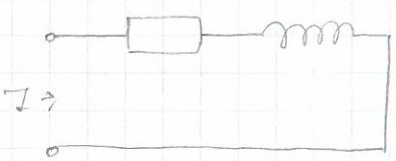
$$Z = R - jX_C = R - j \cdot \frac{1}{\omega C}$$

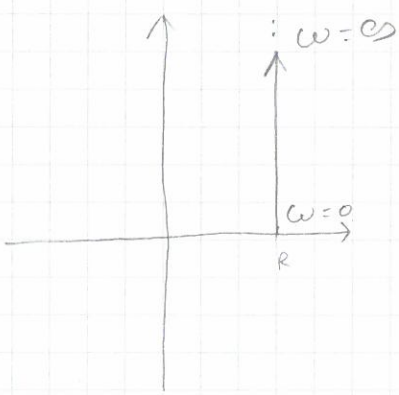


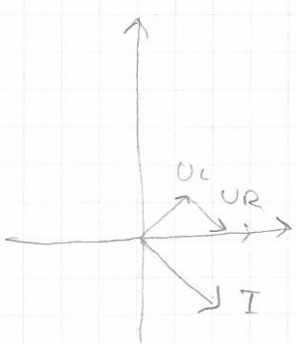


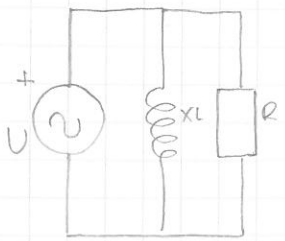
$\omega = \infty \quad U_R = U \; ; \; U_C = 0$

$\omega = 0 \quad U_R = 0$ (jer je $I = 0$) ; $U_C = U_R$





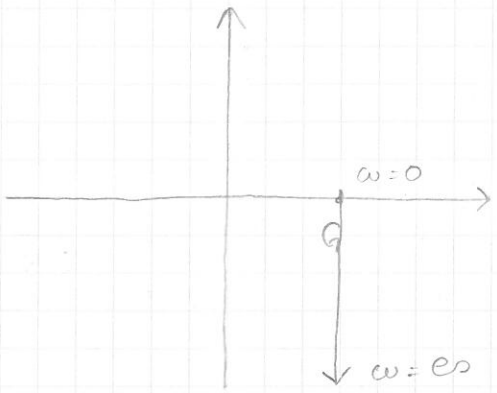




$$Z = \frac{R \cdot jXL}{R + jXL} \cdot \frac{R - jXL}{R - jXL} = \frac{R^2 jXL + RXL^2}{R^2 + XL^2}$$

$\omega = 0$

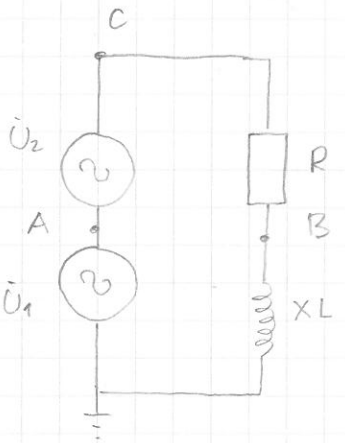
$\omega = \infty$ 0

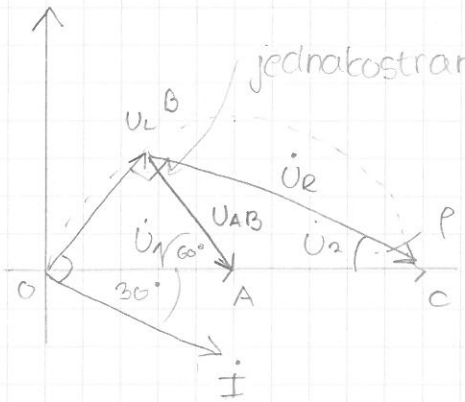


$$G = \frac{1}{R}$$

$$Be = \frac{1}{\omega L}$$

Zadatak: Odredite L pri kojem je kut između napona U_1 i U_{ab} jednak 60° pri $\omega = 1{,}3 \cdot 10^3$ rad/s. $U_1 = U_2 = U$. $R = 100\,\Omega$





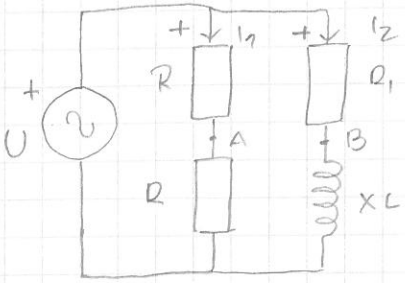
$|\dot{U}_{AB}| = |\dot{U}|$

→ $\triangle OAB$ jednakostraničan $|\dot{U}_L| = |\dot{U}|$

$\varphi = 90° - 60° = 30°$

$$\operatorname{tg}\varphi = \frac{U_L}{U_R} = \frac{XL \cdot I}{R \cdot I} = \frac{XL}{R} \Rightarrow L = \frac{R \operatorname{tg}\varphi}{\omega} = 39{,}3\,\text{mH}$$

U krugu prema slici mijenja se L, kako se pri tome mijenja U_{AB}. Nacrtajte dijagram promjene struje I_2



$$I_1 = \frac{U}{2R}$$

$$I_2 = \frac{U}{R_1 + jX_L}$$

$$\dot{U}_{AB} = I_2 \cdot R_1 - I_1 \cdot R$$

$$= \frac{\dot{U}}{R_1 + jX_L} \cdot R_1 - \frac{\dot{U}}{2R} \cdot R$$

$$\dot{U}_{AB} = \dot{U}\left[\frac{-1}{2} + \frac{R_1}{R_1 + jX_L} \cdot \frac{R - jX_L}{R - jX_L}\right]$$

$$= \dot{U}\left[\underbrace{\frac{-1}{2} + \frac{R_1^2}{R_1^2 + X_L^2}}_{a}\right] - j\left[\underbrace{\frac{R_1 X_L}{R_1^2 + X_L^2}}_{b}\right]$$

$$\dot{U}_{AB} = \dot{U} \cdot (a + jb) = \dot{U} \cdot c \cdot e^{j\varphi}$$

$$I_2 = R_1 + jX_L \quad |Z_2| \angle \varphi_2$$

$$tg\,\varphi_2 = \frac{X_L}{R_1}$$

$$c = \sqrt{a^2 + b^2} = \frac{1}{2}$$

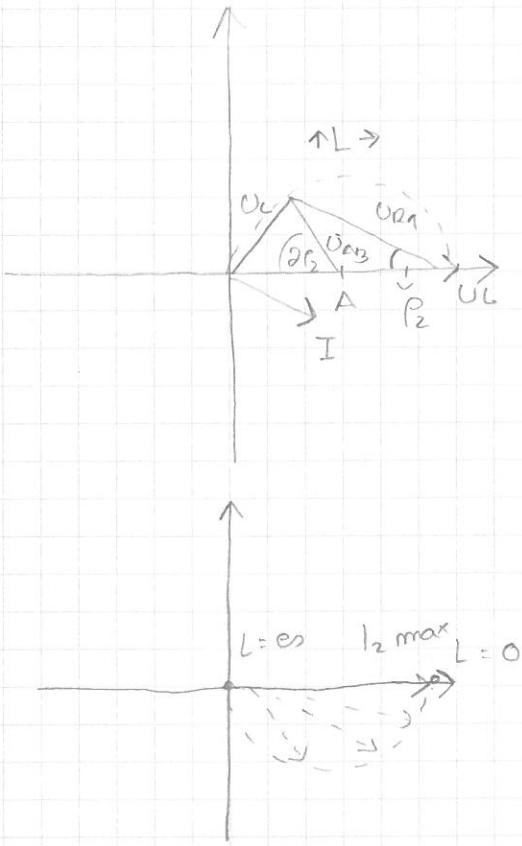
$$tg\,\varphi = \frac{b}{a} = \frac{2R_1 X_L}{R_1^2 - X_L^2} \cdot \frac{\frac{1}{R_1^2}}{\frac{1}{R_1^2}}$$

$$= \frac{-2\frac{X_L}{R_1}}{1 - \left(\frac{X_L}{R_1}\right)^2} = \frac{-2\,tg\,\varphi_2}{1 - tg^2\,\varphi_2}$$

$$\dot{U}_{AB} = \dot{U} \cdot c \cdot e^{j\varphi} = \frac{1}{2} U e^{-j2\varphi_2}$$

$$tg\,\varphi = -tg(2\varphi_2)$$

$$\varphi = -2\varphi_2$$



$I_2(L)$

$$I_2 = \frac{\dot{U}}{R_1 + jX_L}$$

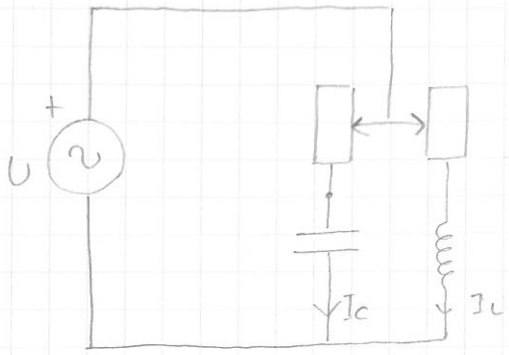
$$|I_2| = \frac{U}{\sqrt{R_1^2 + X_L^2}} = \frac{U}{R_1\sqrt{1 + \left(\frac{X_L}{R}\right)^2}} =$$

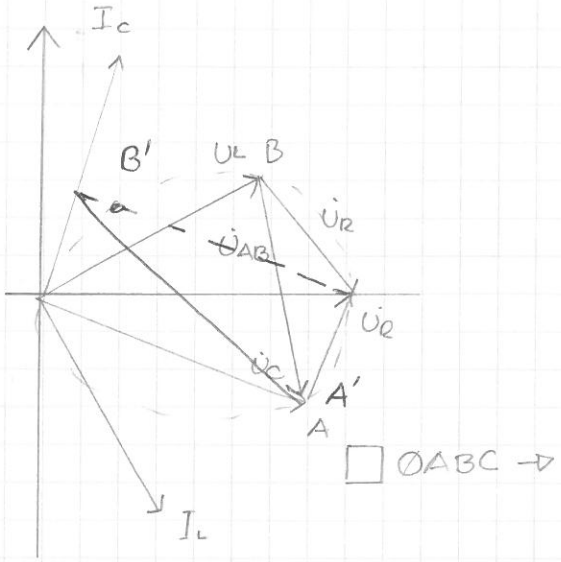
$$\frac{U}{R_1\sqrt{1 + tg^2\,\varphi_2}}$$

$L = 0 \Rightarrow I_2 = \frac{U}{R_1}$

$L = \infty \Rightarrow I_2 = 0$

U krugu prema sici kazaljkom se kontinuirano mijenja R $[0, R_P]$
Odredite R pri kojem je R ($U_{AB} = U_{AB}$ max)





$\overline{OB'} = \overline{A'C}$
$\overline{OA'} = \overline{B'C}$

$I_L \omega L = I_C \cdot R \quad (1)$

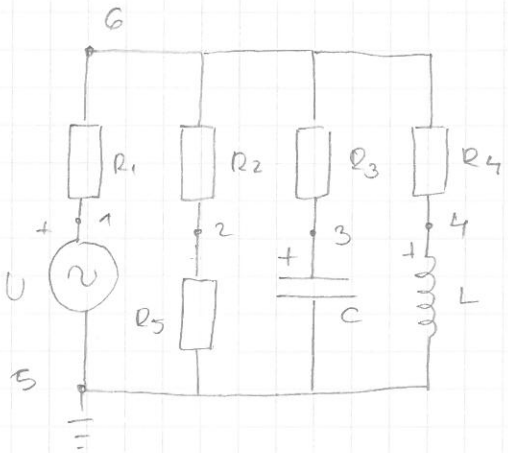
$I_C \cdot \frac{1}{\omega C} = I_L \cdot R \quad (2)$

$I_L \cdot R = I_C \cdot \frac{1}{\omega C} \quad (3)$

1 : 3 $\quad \frac{\omega L}{R} = R \omega C \quad R^2 = \frac{L}{C} \Rightarrow R = \sqrt{\frac{L}{C}}$

Krug rezonira na svim frekvencijama

Nacrtajte topografski dijagram za mrežu prema sici



$U_{65} = 100 \angle 0° V$
$R_1 = 10, \; R_2 = 50 \quad R_3 = 50 \quad R_4 = R_5 = 50 \Omega$
$L = 160 mH$
$C = 64 \mu F$
$\omega = 312{,}5 \; s^{-1}$

$Z_{uk} = Z_{pm} + R_1 = \frac{130}{3} \Omega$

$U_{65} = U \cdot I R_1$

$I = \frac{U_{65}}{Z_{pm}} = 3 \angle 0°$

$U = U_{65} + I R_1$
$= 130 \angle 0° V$

$Y_2 = \frac{1}{R_2 + R_5} = 10 \; mS$

$Y_3 = \frac{1}{R_3 - jXc} = \frac{1}{50 - j \cdot \frac{1}{312{,}564 \cdot 10^{-6}}} = 10 + j10 \; mS$

$Y_4 = \frac{1}{R_4 + jX_L} = 10 - j10 \; mS$

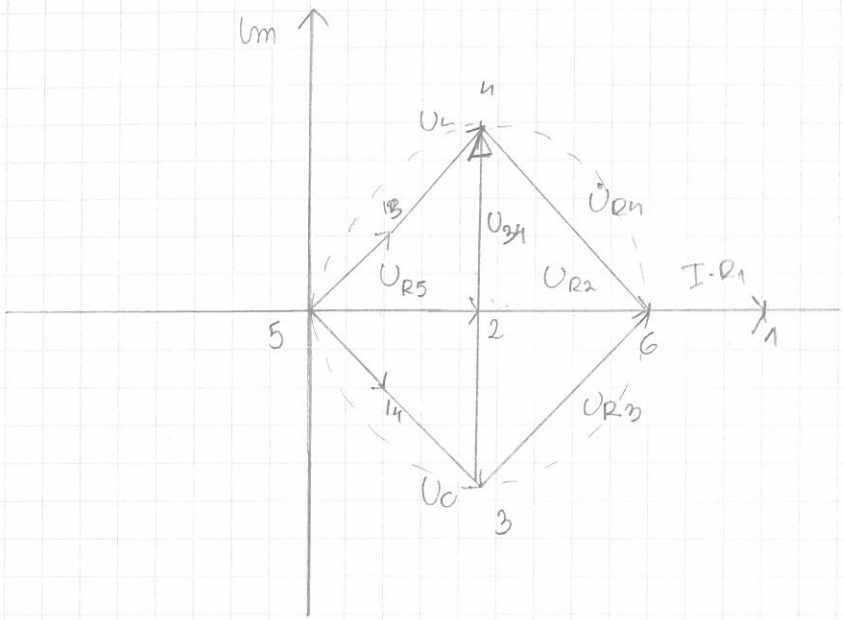
$Y_{pm} = Y_2 + Y_3 + Y_4 = 30 mS$

$Z_{pm} = \frac{1}{Y_{pm}} = \frac{100}{3} \Omega$

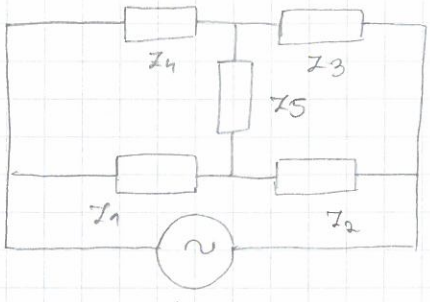
$I_4 = U_{65} \cdot Y_4 = \sqrt{2} \angle 45°$

$I_3 = U_{65} \cdot Y_3 = \sqrt{2} \angle 45°$

$U_{43} = I_4 jX_L - I_3 \cdot (-jXc) = j100$

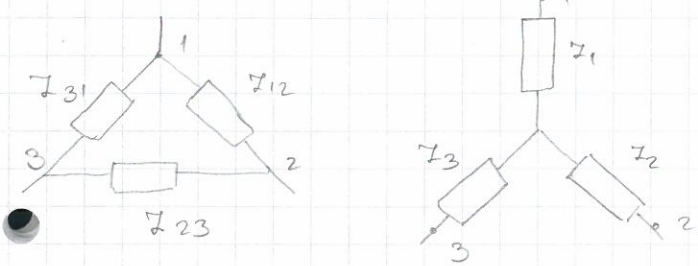


MOSNI SPOJ



$Z_1 Z_3 = Z_2 Z_4$

Pretvorba $\Delta \rightarrow \curlyvee$

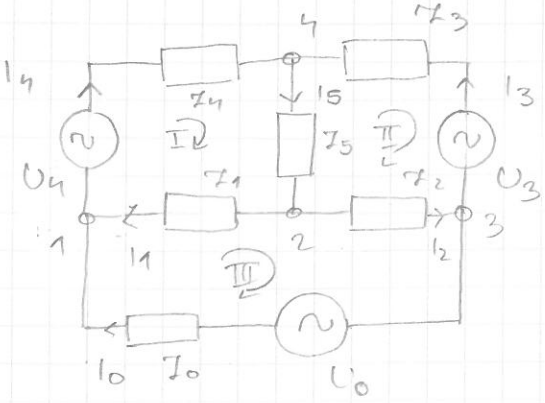


$\Delta \rightarrow \curlyvee \quad Z_\Delta = Z_{12} + Z_{23} + Z_{31}$

$$Z_1 = \frac{Z_{12} Z_{31}}{Z_\Delta} \qquad Z_2 = \frac{Z_{12} \cdot Z_{23}}{Z_\Delta} \qquad Z_3 = \frac{Z_{31} \cdot Z_{23}}{Z_\Delta}$$

$\curlyvee \rightarrow \Delta$

$$Z_{12} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

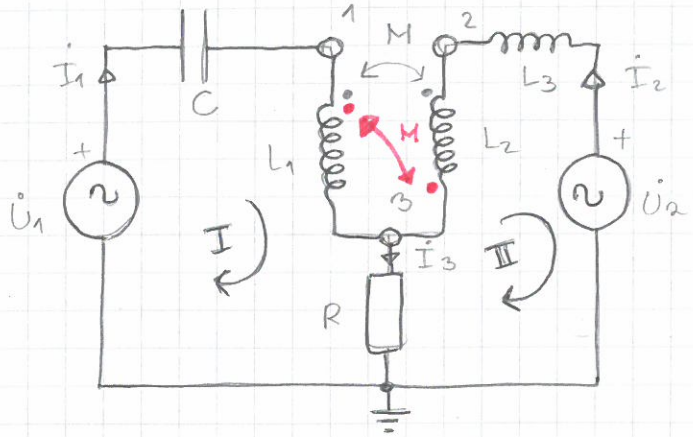


1. $\dot{I}_0 + \dot{I}_1 - \dot{I}_4 = 0$
2. $-\dot{I}_1 - \dot{I}_2 + \dot{I}_5 = 0$
3. $\dot{I}_3 + \dot{I}_4 - \dot{I}_5 = 0$

I. $\dot{U}_4 = I_1 Z_1 + \dot{I}_4 Z_4 + \dot{I}_5 Z_5$
II. $-U_3 = -I_3 Z_3 - I_5 Z_5 - I_2 Z_2$
III. $U_0 = I_0 Z_0 - I_1 Z_1 + I_2 Z_2$

$$\begin{array}{c} \begin{matrix} I_0 & I_1 & I_2 & I_3 & I_4 & I_5 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & I_1 & 0 & 0 & Z_4 & Z_5 \\ 0 & 0 & -Z_2 & -Z_3 & 0 & -Z_5 \\ Z_0 & -Z_1 & Z_2 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ U_4 \\ -U_3 \\ U_0 \end{bmatrix}$$

TRETMAN MEĐUINDUKTIVITETA



u suprotan smjer m.veze

I.K1 $\dot{I}_1 + \dot{I}_2 = \dot{I}_3$

II K2.① $I_1 \cdot \frac{-j}{\omega C} + I_1 \cdot j\omega L_1 \pm I_2 j\omega M + I_3 \cdot R = U_1$

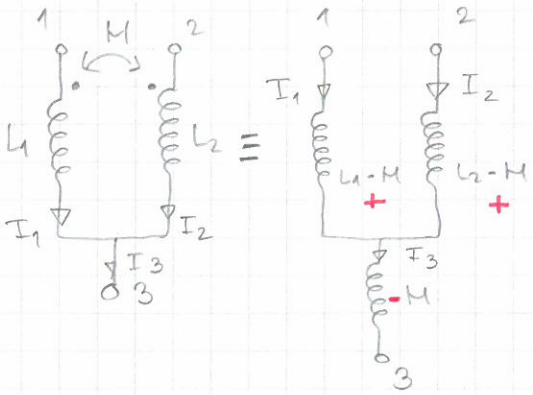
II K1.② $+ I_1 \cdot j\omega M - I_2 \cdot j\omega L_2 - I_2 j\omega L_3 - I_3 \cdot R = -U_2$

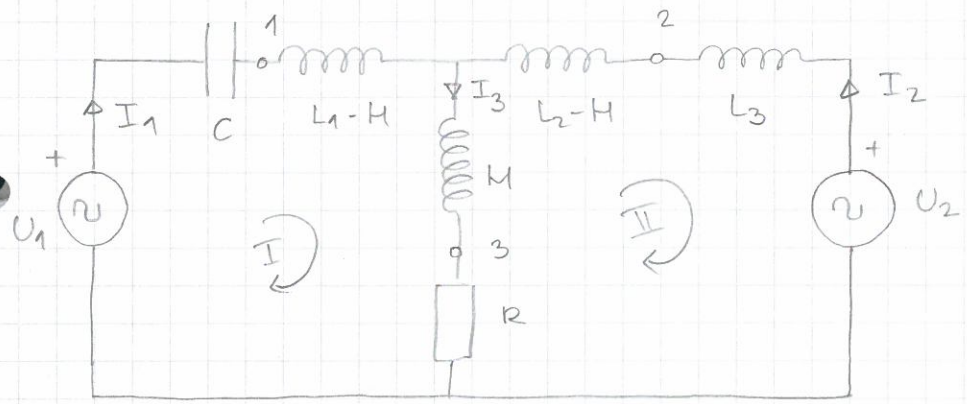
$U_{13} = I_1 \cdot j\omega L_1 \pm I_2 \cdot j\omega M$ $\quad \left[I_2 = I_3 - I_1 \right.$

$U_{23} = I_2 \cdot j\omega L_2 \pm I_1 j\omega M$ $\quad \left[I_1 = I_3 - I_2 \right.$

$U_{13} = I_1 j\omega (L_1 - M) + I_3 \cdot j\omega M$

$U_{23} = I_2 j\omega (L_2 - M) + I_3 \cdot j\omega M$



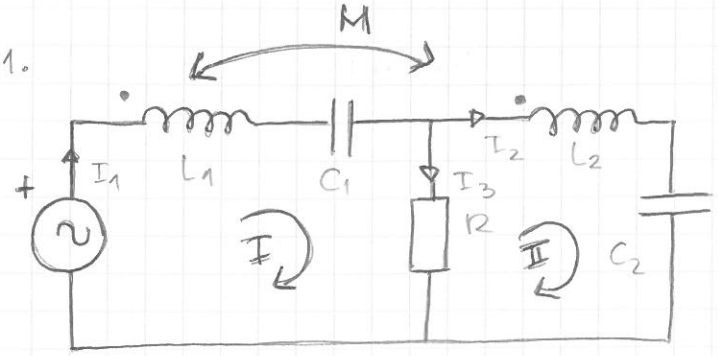


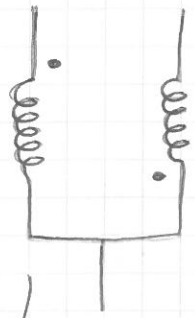
$\dot{I}_1 + \dot{I}_2 = \dot{I}_3$

$U_1 = I_1 \left(\frac{-j}{\omega C} \right) + I_1 \omega (L_1 - M) + I_3 j\omega M + I_3 \cdot R$

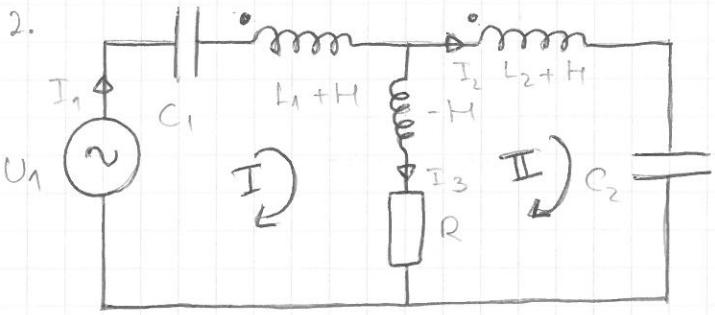
$-U_2 = -I_2 \cdot j\omega L_3 - I_2 (L_2 - M) j\omega - I_3 j\omega M - I_3 \cdot R$

1.





2.



1. $I_1 = I_3 + I_2$ smjer koja ju uzrokuje ulazi u točkicu

$$U = I_1 \cdot j\omega L_1 + I_2 \cdot j\omega H + I_1 \cdot \left(\frac{-j}{\omega C_1}\right) + I_3 \cdot R$$

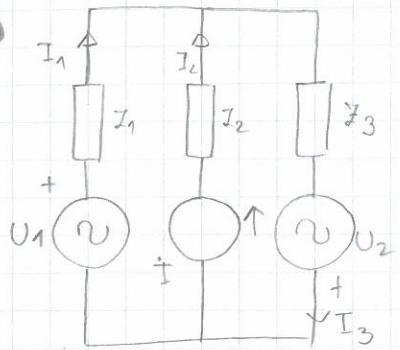
$$0 = -I_3 \cdot R + I_2 \cdot L_2 + I_1 \cdot j\omega H + I_2 \cdot \left(\frac{-j}{\omega C_2}\right)$$

2. $I_1 = I_3 + I_2$

$$U = I_1 \cdot \left(\frac{-j}{\omega C_1}\right) + I_1 \cdot j\omega (L_1 + H) + I_3 \cdot j\omega (-H) + I_3 \cdot R$$

$$0 = -I_3 \cdot R - I_3 j\omega (-H) + I_2 \cdot j\omega (L_2 + H) + I_2 \cdot \left(\frac{-j}{\omega C_2}\right)$$

SUPERPOZICIJA



AKTIVAN U_1

$\dot{I}_1' = \frac{U_1}{Z_1 + Z_3}$; $I_2 = 0$; $\dot{I}_3' = -\dot{I}_1'$

AKTIVAN $\dot{I}$

$\dot{I}_2'' = -I$; $I_1'' = I \frac{Z_3}{Z_1 + Z_3}$; $I_3'' = I \cdot \frac{Z_1}{Z_1 + Z_3}$

AKTIVAN U_2

$\dot{I}_2''' = 0$; $I_1''' = \frac{U_2}{Z_1 + Z_3}$; $I_3''' = -I_1'''$

$$I_1 = I_1' + I_1'' + I_1''' = \frac{U_1 + U_2 + I \cdot Z_3}{Z_1 + Z_3}$$

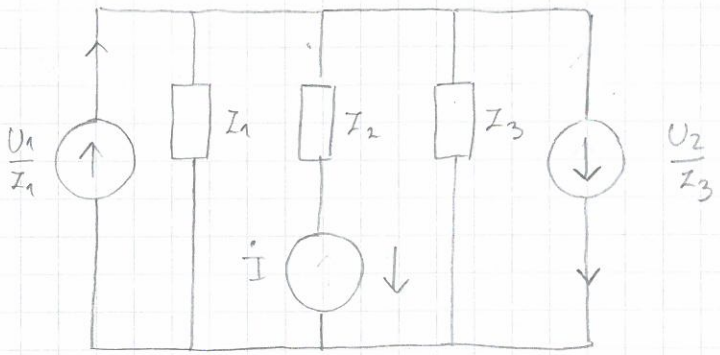
$Z_1 = -jX_C$
$Z_3 = jX_L$
$X_L = X_C$

PROBLEM !

$I_2 = -I$

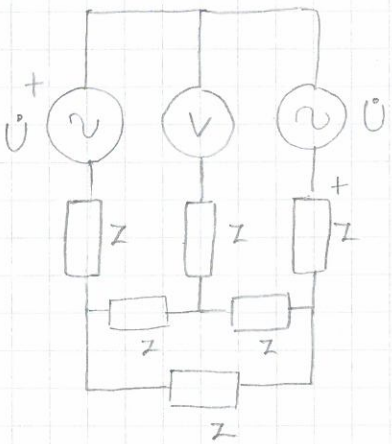
$$I_3 = \frac{-U_1 - U_2 + I \cdot Z_1}{Z_1 + Z_3}$$

Paralelna rezonanca



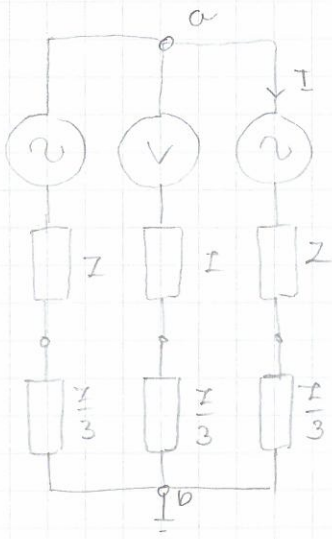
$$Z_{13} = \frac{Z_1 \cdot Z_3}{Z_1 + Z_3} = \infty$$

- Odredite rezim voltmetra



$\Delta \Rightarrow Y$

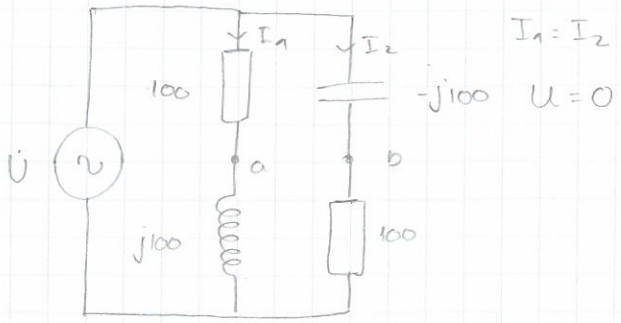
$$Z = \frac{Z_{12} \cdot Z_{31}}{Z_\Delta} = \frac{Z^2}{3Z} = \frac{Z}{3}$$



$$I = \frac{20}{2 \cdot \frac{4}{3} Z}$$

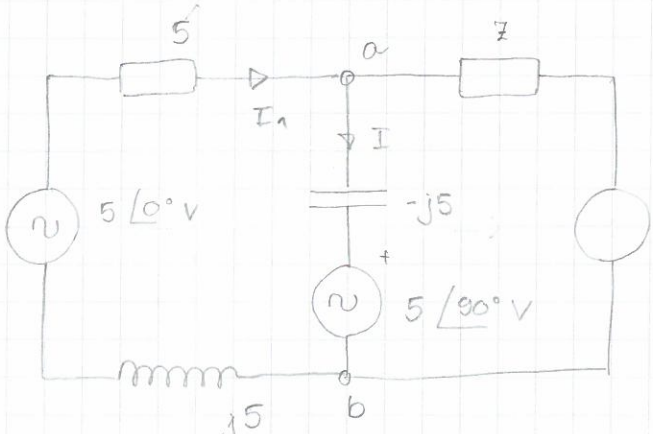
$$U_1 = I \cdot \frac{4}{3} Z = 0$$

$$U_V = U - U_1 = 0$$



$I_1 = I_2$

$U = 0$



$I = 2\angle 0°$ A

$\dot{I} = I_1 + I_2$

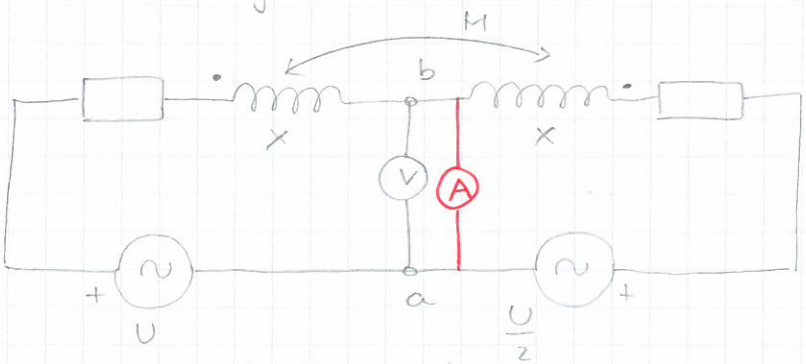
$U_{ab} = 5\angle 90° + I \cdot j5$

$= j5 - j10 = -j5$

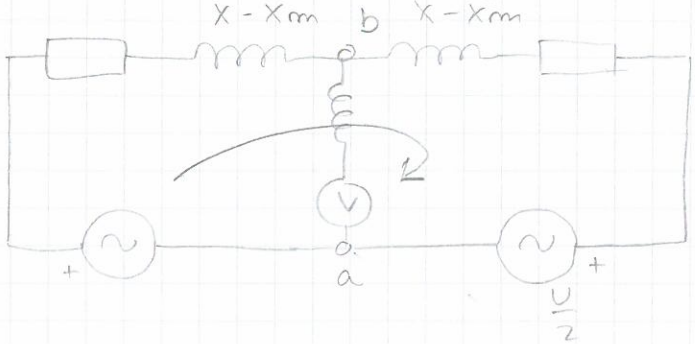
$I_1 = \frac{5\angle 0° - U_{ab}}{5 + j5} = \frac{5 + j5}{5 + j5} = 1\angle 0°$

$I_2 = I - I_1 = 2\angle 0° - 1\angle 0° = 1\angle 0°$ A

1. Odredite U_V
b) Ako V zamjenim sa A odredite I_A



$R = 50\Omega$
$X = 100\Omega$
$X_m = 50\Omega$
$U = 200\angle 0°$ V



$U - \frac{U}{2} = I\,(2R + 2j\,(X - X_m))$

$\frac{U}{2} = 2I\,(R + j\,(X - X_m))$

$I = \frac{U}{4\,(R + j\,(X - X_m))}$

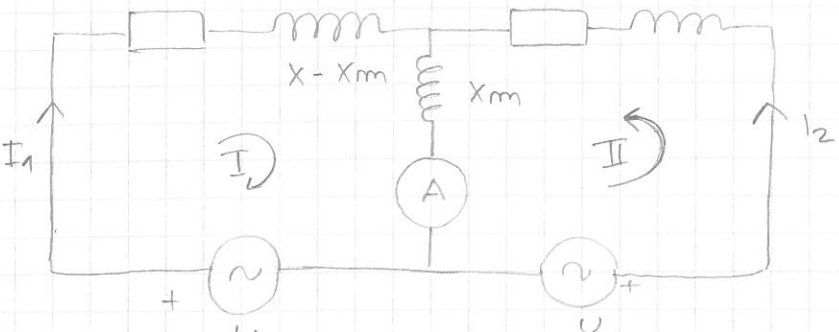
$= \frac{200\angle 0°}{4\,(50 + j50)} = \frac{\sqrt{2}}{2}\angle -45°$

$P_a + U - I \cdot R - I \cdot j\,(X - X_m) = P_B$

$P_A - P_B = U_{ab} = -200\angle 0° + \frac{\sqrt{2}}{2}\angle -45° \cdot 50\sqrt{2}\angle 45° = -150\angle 0°$

$U_V = 150$ V

b)



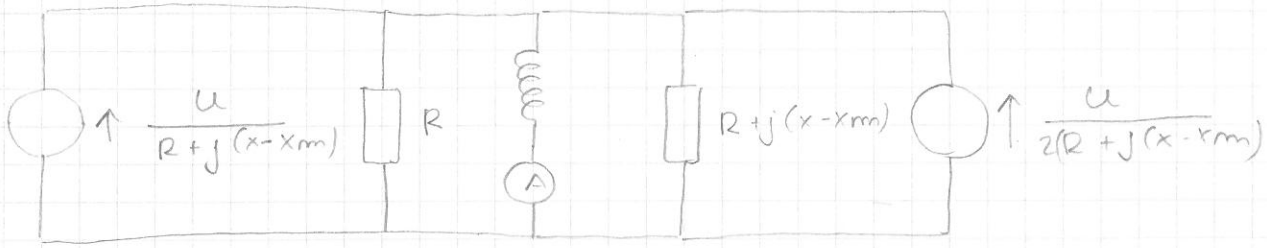
$$U = I_1 \cdot (R + j(X - X_m)) + (2I_1 + I_2) \cdot jX_m$$

$$\frac{U}{2} = I_2 (R + j(X - X_m)) + (I_1 + I_2) \cdot jX_m$$

$$200 \angle 0° = I_1 \cdot (50 + j100) + I_2 \cdot j50$$

$$100 \angle 0° = I_1 \cdot j50 + I_2 \cdot (50 + j100)$$

→ pretvorili naponske u strujne



$$U_{10} = I/Y$$

↑ struje koje ulaze u čvor 1

$$U_{10} = \frac{\frac{U}{R + j(X - X_m)} + \frac{U}{2(R + j(X - X_m))}}{\frac{1}{R + j(X - X_m)} + \frac{1}{jX_m} + \frac{1}{R + j(X - X_m)}}$$

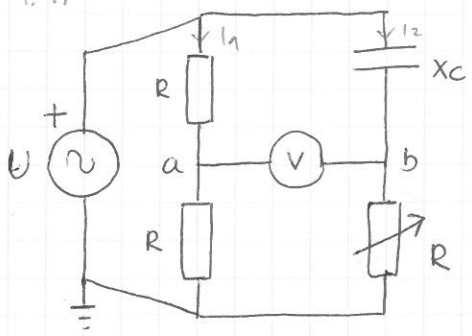
$$= \frac{\frac{3}{2} \frac{U}{R + j(X - X_m)}}{\frac{jX_m + R + j(X - X_m) + jX_m}{jX_m (R + j(X - X_m))}}$$

$$= \frac{3}{2} \frac{U \cdot jX_m}{R + j(X + X_m)}$$

$$I_A = \frac{U_{10}}{jX_m} = \frac{3}{2} \cdot \frac{200 \angle 0°}{50 + j(50 + j150)} = \frac{3}{5}(1 - j3)$$

Zbirka: topografski dijagram

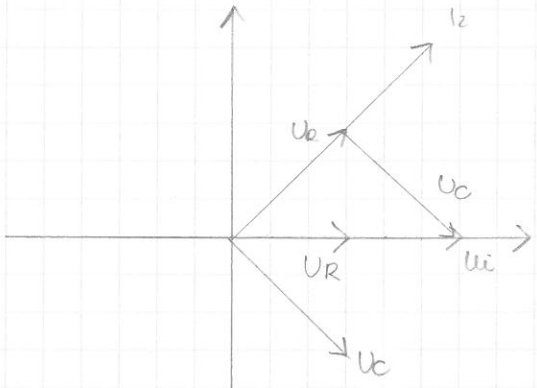
1.1.



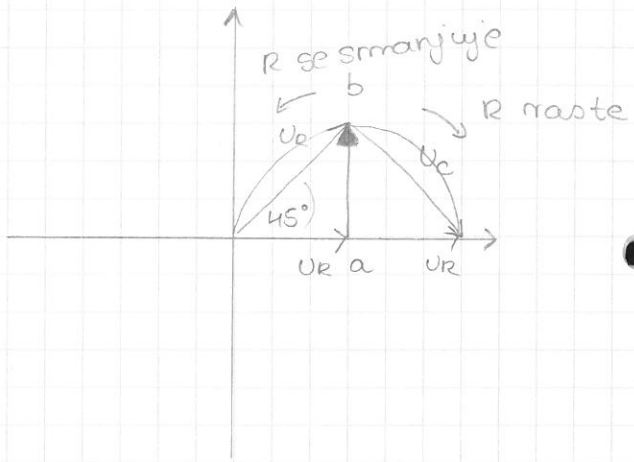
$R = X_C$; $U = 80\,V$

$U_{ab} = U_R \cdot tg45° = 40\,V$

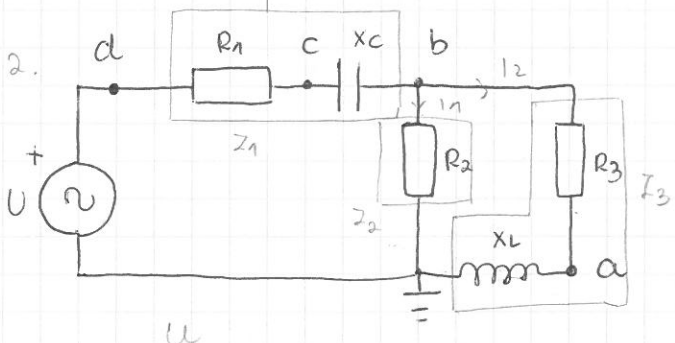
vektorski dijagram



topografski dijagram



1.2.



$U = 8V$
$R_1 = 5\,\Omega$
$X_C = 1\,\Omega$
$R_2 = 5\,\Omega$
$R_3 = 5\,\Omega$
$X_L = 5\,\Omega$

$Z_1 = 5 - j$
$Z_2 = 5$
$Z_3 = 5 + 5j$
$Z_p = \frac{Z_2 \cdot Z_3}{Z_2 + Z_3} = 3 + j$
$Z_{uk} = Z_1 + Z_p = 8$

$I_{uk} = \frac{U}{Z_{uk}} = 1\,A$

$U_{R1} = R \cdot I = 5 \cdot 1 \angle 0° = 5V$

$\frac{I_1}{I_2} = \frac{Z_3}{Z_2} = \frac{5+5j}{5}$

$\frac{I_1}{1 - I_1} = 1 + j$

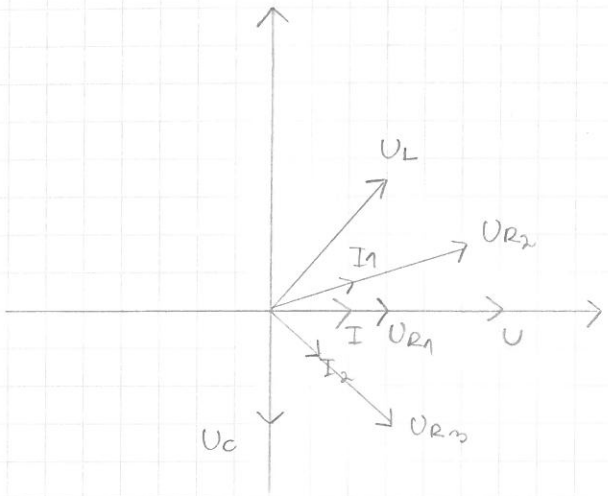
$I_1 = 1 - I_1 + j - I_1 \cdot j$

$I_1 (1 + 1 + j) = 1 + j$

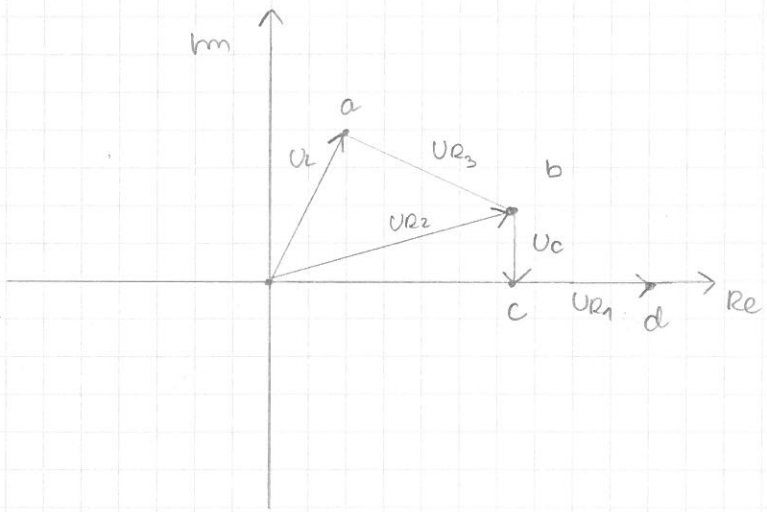
$I_1 = 0,632 \angle 18,43°\,A$

$I_2 = 1 \angle 0° - I_1 = 0.447 \angle -26,57°\,A$

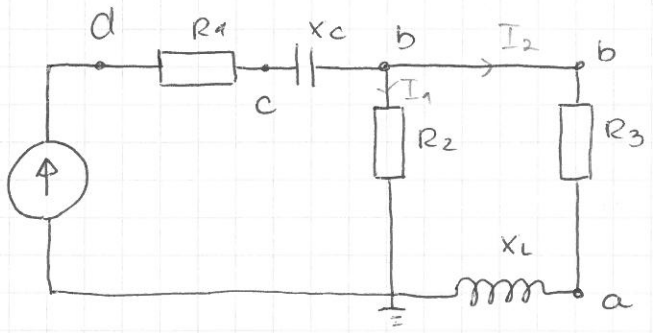
vektorski dijagram



topografski dijagram



1.3.



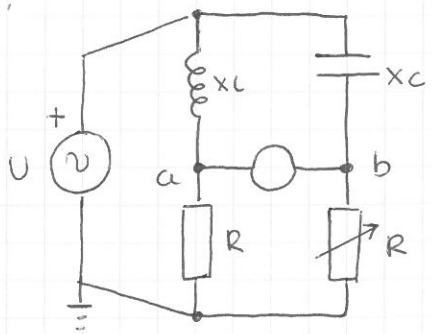
$I = 2\,A$

$Z_{uk} = 8\,\Omega\,\angle 0°$

$U = I \cdot Z_{uk} = 16\,V\,\angle 0°$

$U_1 = 2 \cdot 5 = 10\,V\,\angle 0°$

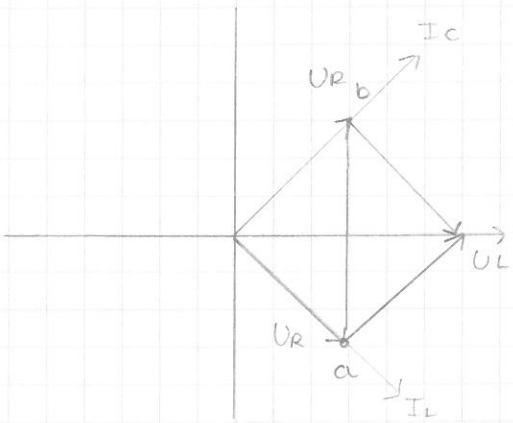
1.4.



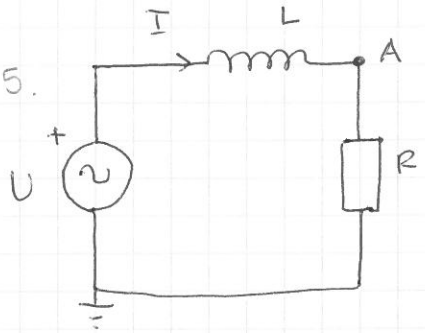
$U_V = 24\,V$

$X_L = X_C = R$

$U_i = 24\,V$



1.5.

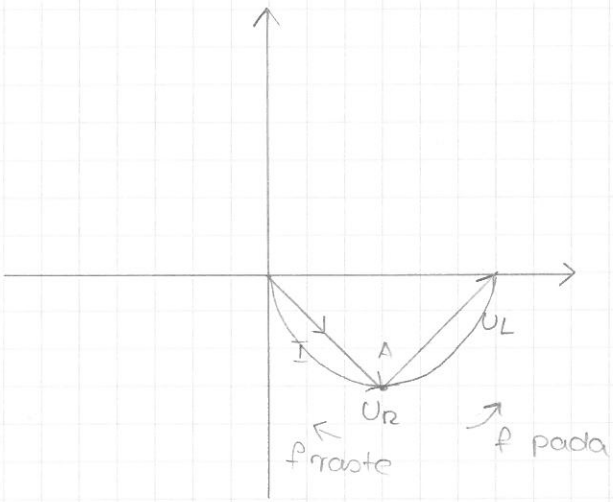


$R = 1\,k\Omega$

$L = 1\,H$

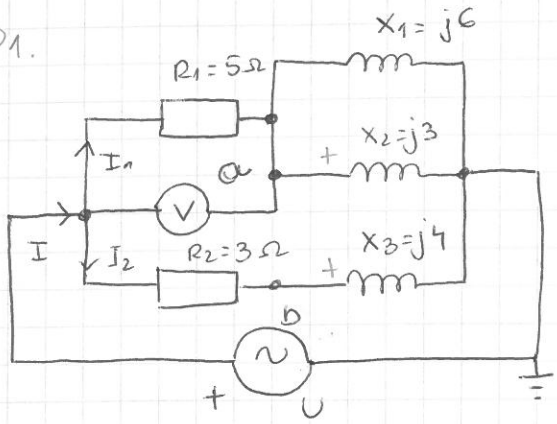
$R = 2\pi f \cdot L$

$f = 160\,Hz$



ZBIRKA: SLOŽENI KRUGOVI IZMJENIČNE STRUJE

P1.



$I_1 = \frac{45}{5} = 9A \angle 0° \quad Z_p = 2j\,\Omega$

$U = 45 \angle 0° \qquad U_p = 2j \cdot 9A = 18j$

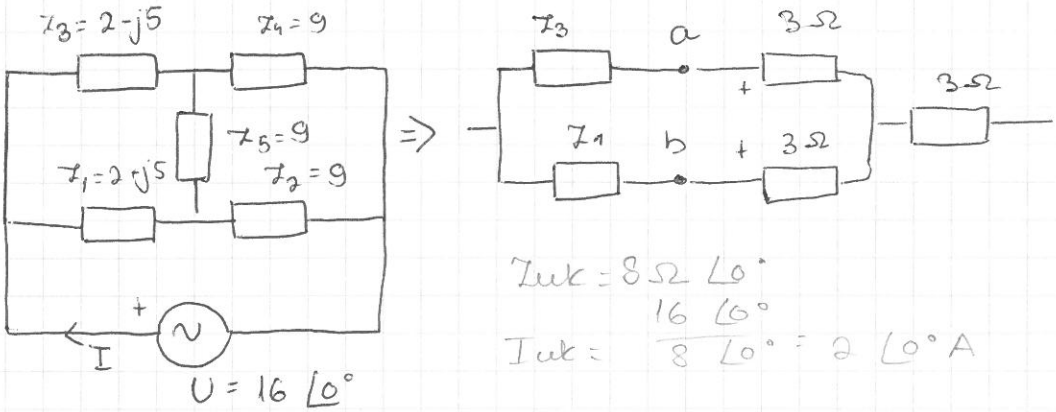
$U_{uk} = 43,47 \angle 21,8°$

$I_2 = \frac{U_{uk}}{3+4j} = 9,67 \angle -31,13°$

$I_{uk} = 13 \angle -16° A$

$U_{ab} = -I_2 \cdot X_3 + U_p =$

P2.



$Z_{uk} = 8\,\Omega \angle 0°$

$I_{uk} = \frac{16 \angle 0°}{8 \angle 0°} = 2 \angle 0° A$

$\frac{I_1}{I_2} = \frac{5+5i}{5-5i}$

$\frac{I_1}{1-I_1} = i$

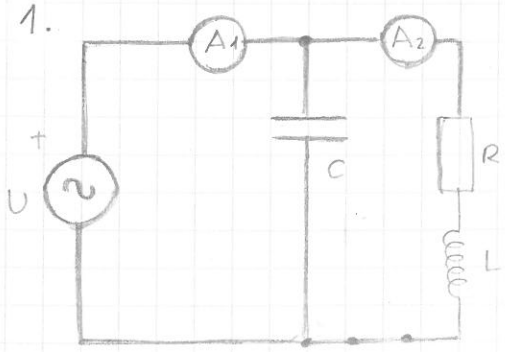
$I_1 = i - I_1 \cdot i$

$I_1(1+i) = i$

$I_1 = \frac{\sqrt{2}}{2} \angle 45° \qquad I_2 = \frac{\sqrt{2}}{2} \angle -45°$

$U_{ab} = -I_2 \cdot 3 + I_1 \cdot 3 = 6j\,V$

1.



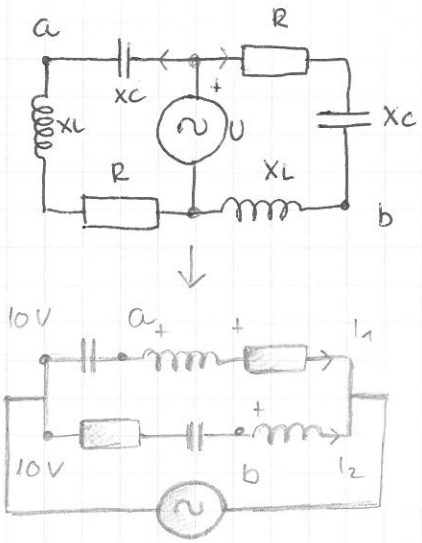
$I_A = 0{,}5\,A$
$f = 60\,Hz$
$U = 110\,V$

$\frac{1}{220} = \frac{1}{220} + \frac{1}{R+L}$

$I = \frac{U}{I} = 220\,\Omega$

$X_C = 220\,\Omega$

2.



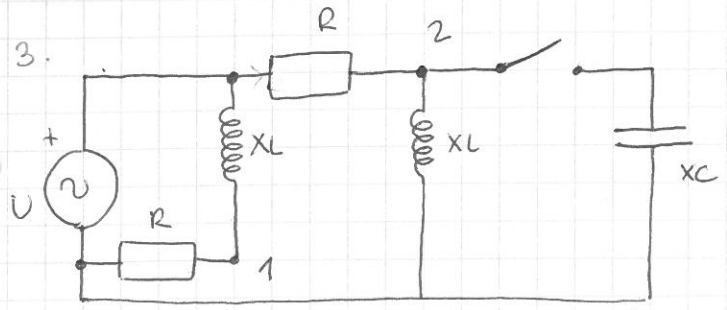
$U = 10\,V \qquad R = X_L = X_C = 10\,\Omega$

$I_1 = \frac{10}{10} = 1\,A$

$I_2 = 1\,A$

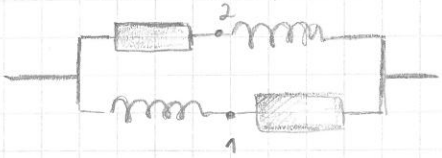
$U_{ab} = -1 \cdot jX_L + 1 \cdot R + 1 + jX_L = 10\,V$

3.

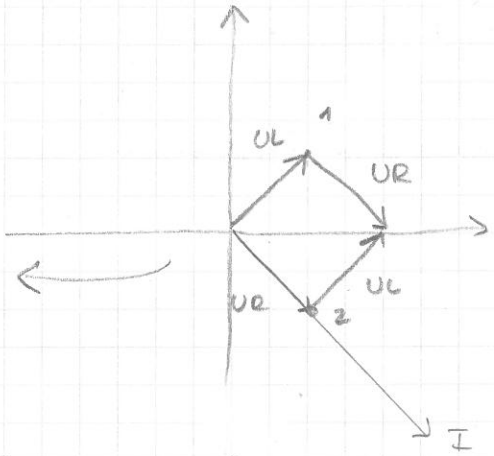


$R = X_L = X_C$

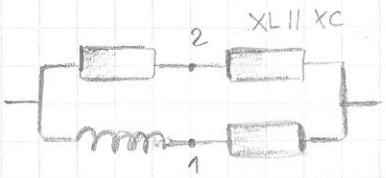
otwarta sklepka $U_{12} = 100 \angle 0°$



$U_i = 100 \angle 0°$

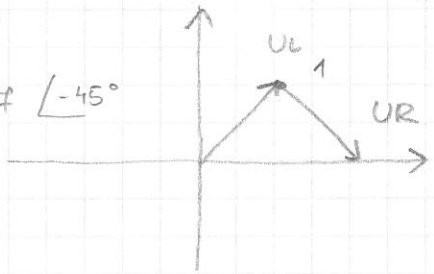


zatwarta sklepka

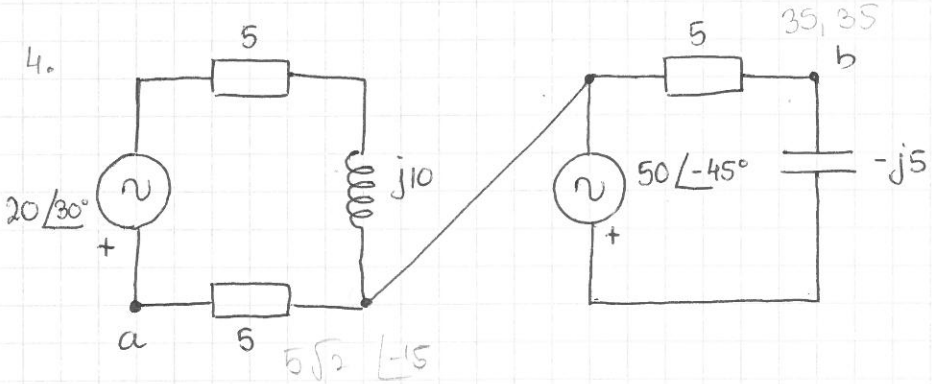


$Z = \frac{jX_L \cdot (-jX_C)}{jX_L - jX_C} = \infty$

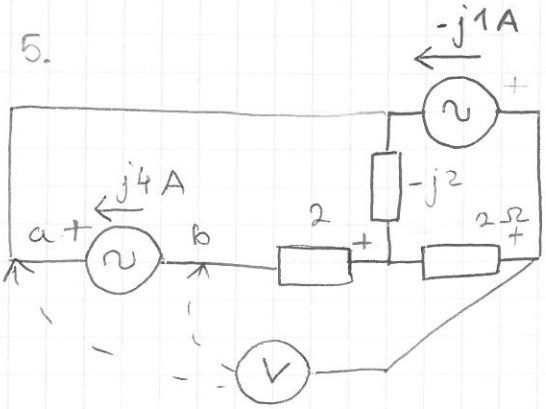
$U = \sqrt{50^2 + 50^2} = 70,7 \angle -45°$



4.



5.



a) $Z' = 2 - 2j\ \Omega$

$U = I_1 \cdot Z$

$U' = -2 - 2i$

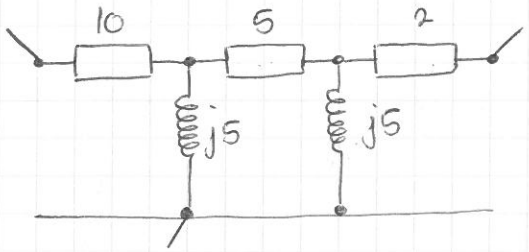
$Z'' = 2 - 2i\ \Omega$

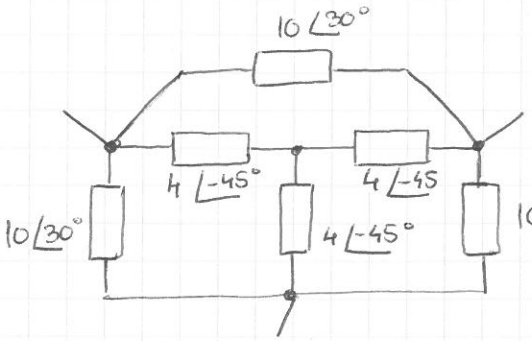
$U'' = 8 + 8i$

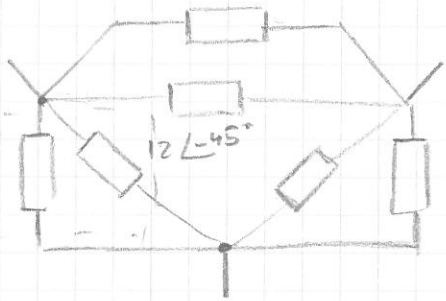
$U =$

$U_b = 2i + 4 = 10 \angle 90° V$

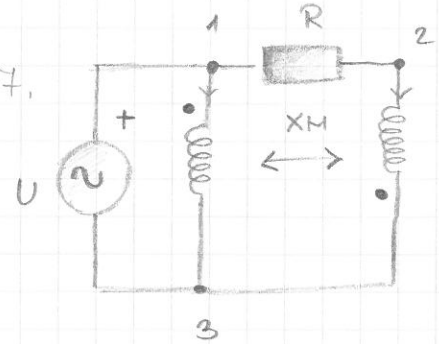
6.





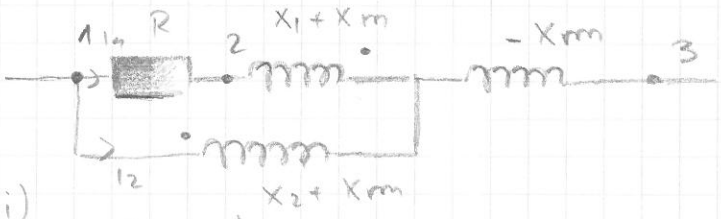


7.



$X_1 = X_2 = 25\ \Omega$ $X_M = 20\ \Omega$ $U = 25$ V

nesuglasna veza



$Z_{ek} = \dfrac{45i \cdot (25 + 45i)}{25 + 90i} + (-20i)$

$= 7,11 \angle 35°32\ \Omega$

$I = \dfrac{U}{Z} = 3,516 \angle -35°53'$ A

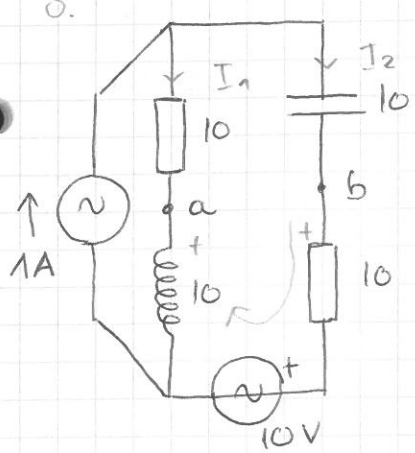
$\dfrac{I_1}{I_2} = \dfrac{45i}{25 + 45i}$

$\dfrac{I_1}{3,516 \angle -35,53} = I_1 = 0,874 \angle 29°$

$I_1 = 1,69 \angle -20,03°$ A

$P = I^2 R = 71,4$ W

8.



aktivan U $U_{ab} = -5 - 5j$

$Z_{uk} = 20\,\Omega$

$I' = 0{,}5\ \angle 0°$

aktivan I

$Z_{uk} = 10\,\Omega$

$\frac{I_2}{I_1} = \frac{10+10j}{10-10j}$

$\frac{I_2}{1-I_2} = i$

$I_2 = i - I_2 \cdot i$

$I_2 (1+i) = i$

$I_2 = 0{,}707\ \angle 45°$

$I_1 = 0{,}707\ \angle -45°$

$U_R = 7{,}07\ \angle 45°$

$U_L = 7{,}07\ \angle 45°$

$U_{ab} = -7{,}07\ \angle 45° + 7{,}07\ \angle 45° = 0$

$U_{ab} = -5 - 5j = 7{,}07\ \angle 45°$

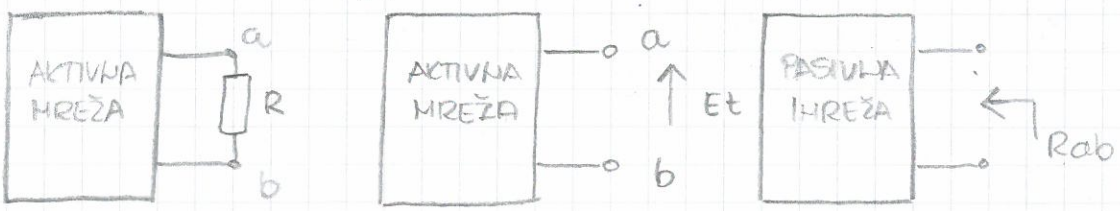
THEVENINOV TEOREM

→ zanima nas pojava na samo jednom elementu ili dijelu mreže

DIO AKTIVNE MREŽE SA STEZALJKI A i B MOŽE SE NADOMJESTITI REALNIM NAPONSKIM IZVOROM S PARAMETRIMA Et i Rt.

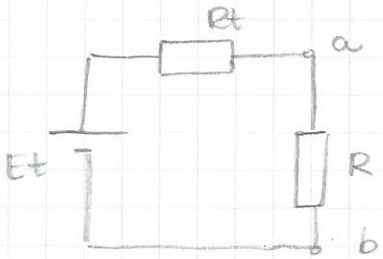
THEVENINOV NAPON Et - NAPON OTVORENIH PRIKLJUČNICA A i B

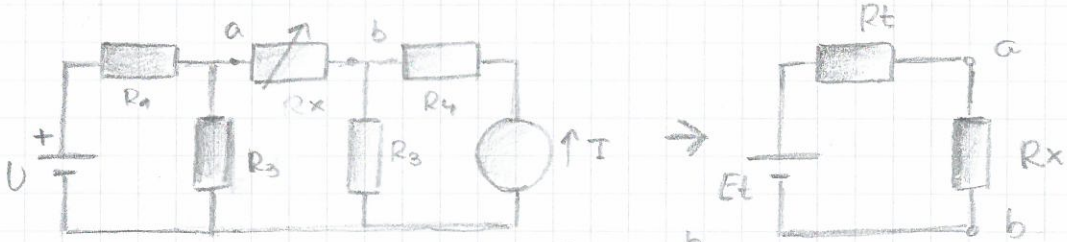
THEVENINOV OTPOR Rt - OTPOR PASIVNE MREŽE S OTVORENIH PRIKLJUČNICA A i B



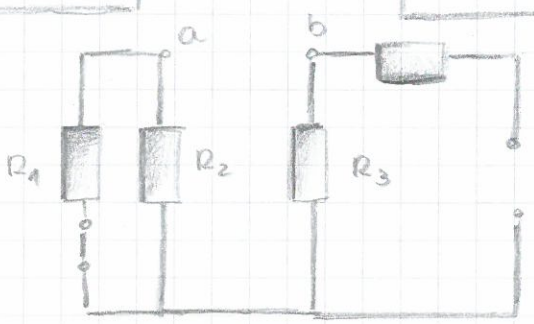
pasivna mreža
→ ugašeni izvori

NAPONSKI - KS
STRUJNI - PH





$R_1 = R_2 = R_3 = 1\,\Omega$
$R_4 = 2\,\Omega$
$U = 30\,V$
$I = 10\,A$
$R_x (P_{max}) = ?$

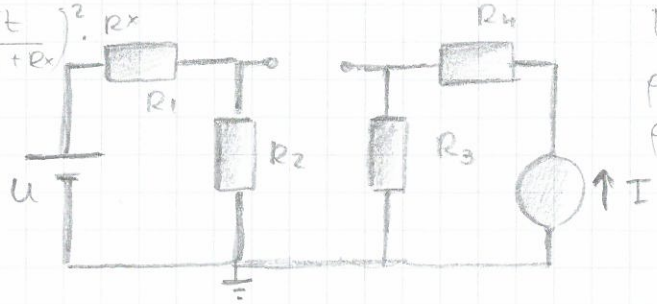


parametre tražim s otvorenih prikl. ali kasnije nova shema pa nije više Et !

$R_t = R_1 \| R_2 + R_3 = 1.5\,\Omega$

$R_x (P_{max}) = 1,5\,\Omega$

$P_{max} = I^2 R_x = \left(\frac{E_t}{R_t + R_x}\right)^2 \cdot R_x$
$= 4,17\,W$



$E_t = U_{ab} = P_a - P_b$

$P_A = I_1 \cdot R_2 = \frac{U}{R_1 + R_2} \cdot R_2 = 15\,V$

$P_B = I R_3 = 10\,V$

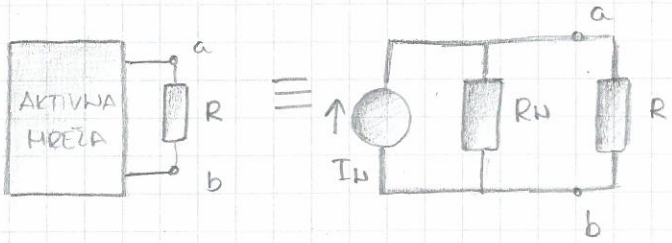
NORTONOV TEOREM

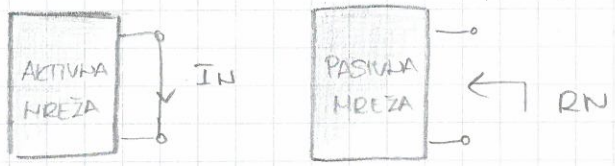
→ zanima nas pojava na samo jednom elementu

Dio aktivne mreže može se nadomjestiti s priključnica a i b realnim strujnim izvorom s parametrima I_N i R_N.

R_N - otpor pasivne mreže s otvorenih priključnica a i b (isto kao Rt)

I_N - struja kroz kratkospojene stezaljke a i b.



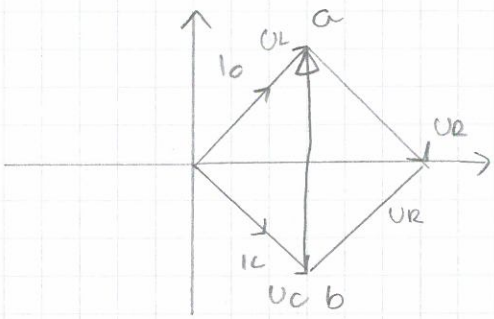


SPECIJALNI SLUČAJEVI:

$R_t = R_n = 0 \Rightarrow$ Theveninov teorem

$R_t = R_n = \infty \Rightarrow$ Nortonov teorem

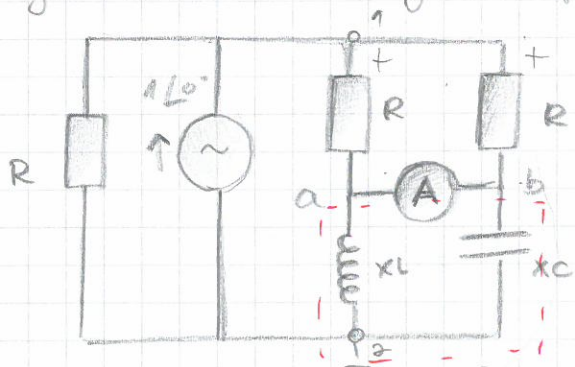
može se dobiti paralelnom rezonancijom



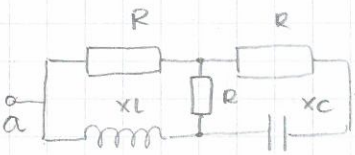
Ekvivalencija T. i N. teorema

$$R_N = R_T$$
$$I_N R_N = E_T$$
$$I_N = \frac{E_T}{R_T}$$

Primjer: Odredite struju ampermetra.



$R = X$



most nije u ravnoteži

$$U_{ab} = U_{12} \angle 90°$$

$$\dot{I}_{12} = \frac{(R+jX)\cdot(R-jX)}{2R} = \frac{R^2+X^2}{2R} = R$$

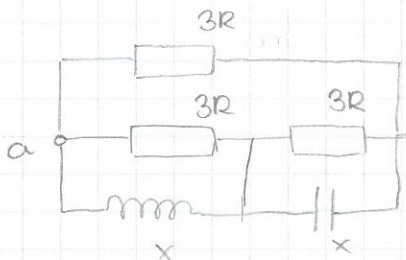
$$U_{12} = \frac{\dot{I}}{2}\cdot R = 0.5R \angle 0°$$

$$U_{ab} = \dot{E}_t = U_{12}\angle 90° = \frac{R}{2}\angle 90°$$

$$I_N = \frac{E_t}{Z_t} = \frac{\frac{R}{2}\angle 90°}{\frac{R}{2}} = 1\angle 90°$$

$\Delta \rightarrow Y$

$$Z_\Delta = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3} = \frac{3R^2}{R} = 3R$$



$$Y_{ab} = \frac{1}{3R} + \frac{1}{Z_P}$$

$$Y_{ab} = \frac{G}{3R}$$

$$Z_{ab} = \frac{1}{2}R$$

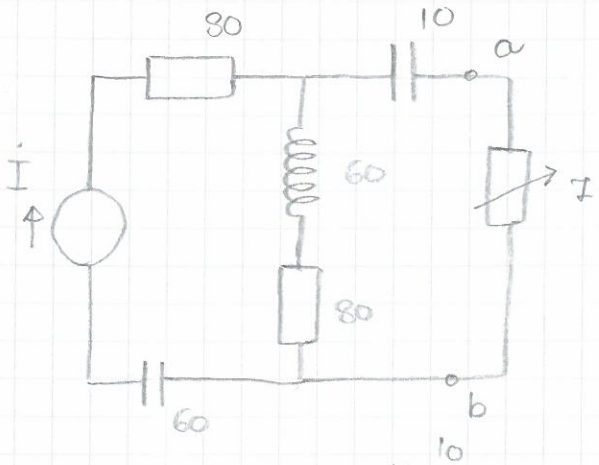
$$Z_{RXL} = 3R \parallel jX_L$$

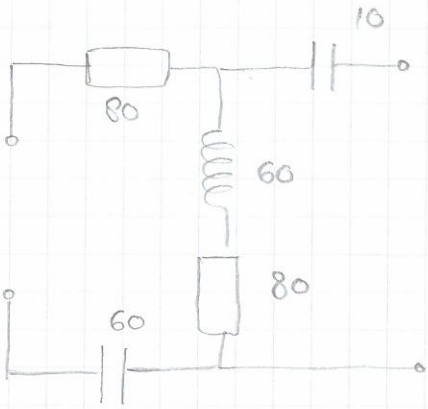
$$Z_{RXC} = 3R \parallel (-jX_C)$$

$$Z_P = Z_{RXL} + Z_{RXC} = \frac{3}{5}R$$

$$Y_P = \frac{1}{Z_P}$$

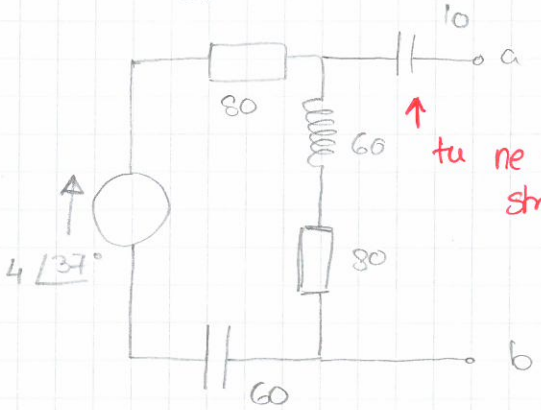
moraci smo kombinirati teoreme jer bi kao posljedica brtko spojnika bila paralelna rez.





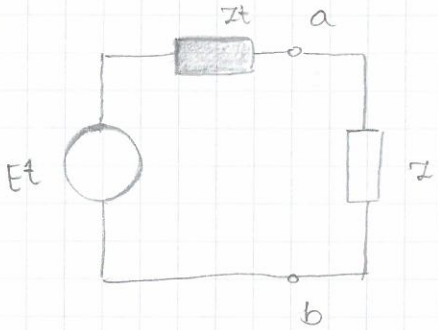
Z_a

$Z_{ab} = 80 + j60 - j10 = 80 + j50 = Z_t$



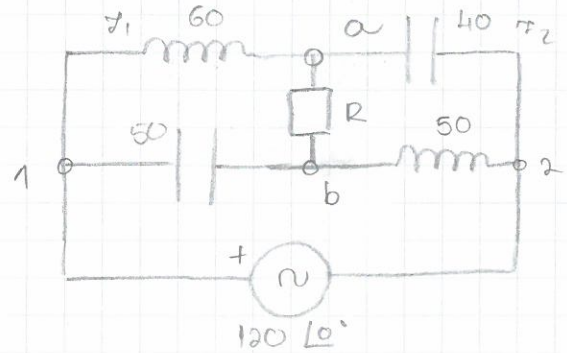
tu ne teče struja

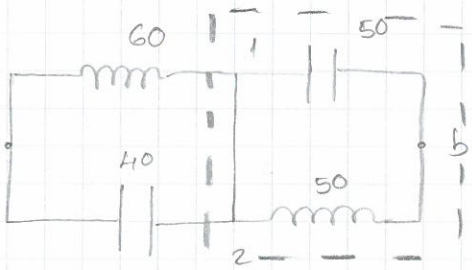
$E_t = I \cdot (80 + 60j) = 400 \angle 74°$



$P_{max} = \left(\frac{E_t}{Z_R + Z}\right)^2 \cdot Re(Z) = \frac{400}{(2 \cdot 80)^2} \cdot 80 = 500\ W$

Odredite P na otporu $R = 10\ \Omega$



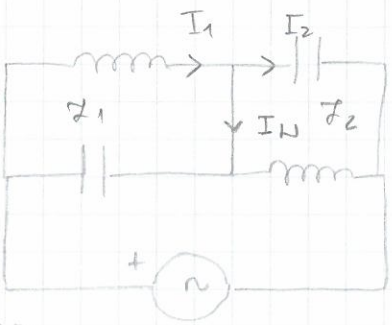


$j50 \parallel -j50 = \infty$

$Z_{ab} = \infty \Rightarrow$ NORTON

$Z_1 = j60 \parallel (-j50) = -j300$

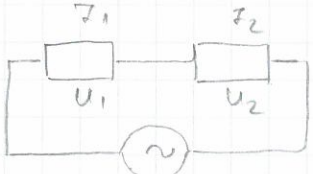
$Z_2 = -j40 \parallel j50 = -j200$



$I_1 = \frac{U_1}{j60} = -j1{,}2A$

$I_2 = \frac{U_2}{-j40} = j1{,}2A$

$I_N = I_1 - I_2 = -j2{,}4A$

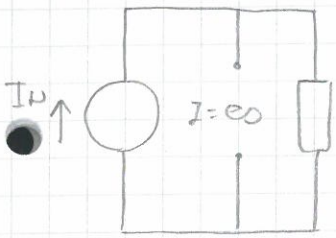


$U_1 = \frac{U}{Z_1 + Z_2} \cdot Z_1$

$= \frac{120 \angle 0°}{-j500} \cdot -j300$

$= 72 \angle 0°\ V$

$U_2 = 48 \angle 0°\ V$



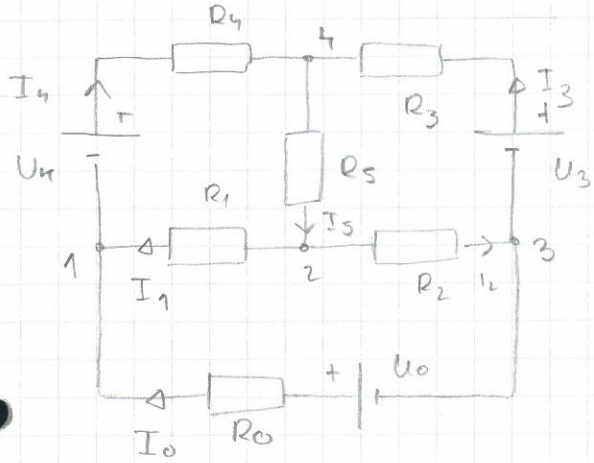
$P(R) = I_N^2 \cdot R = 2{,}4^2 \cdot 10 = 57{,}6\ W$

METODA NAPONA ČVOROVA (Jednadžbe potencijala čvorova)

↓

nastaju uvrštenjem II KZ u I KZ

→ strujne jednadžbe



$\varphi_4 = 0$

1. $I_0 + I_1 - I_4 = 0$
2. $I_5 - I_1 - I_2 = 0$
3. $I_2 - I_3 - I_0 = 0$

$$I_0 = \frac{\varphi_3 - \varphi_1 + U_0}{R_0}$$

$$I_1 = \frac{\varphi_2 - \varphi_1}{R_1}$$

$$I_2 = \frac{\varphi_2 - \varphi_3}{R_2}$$

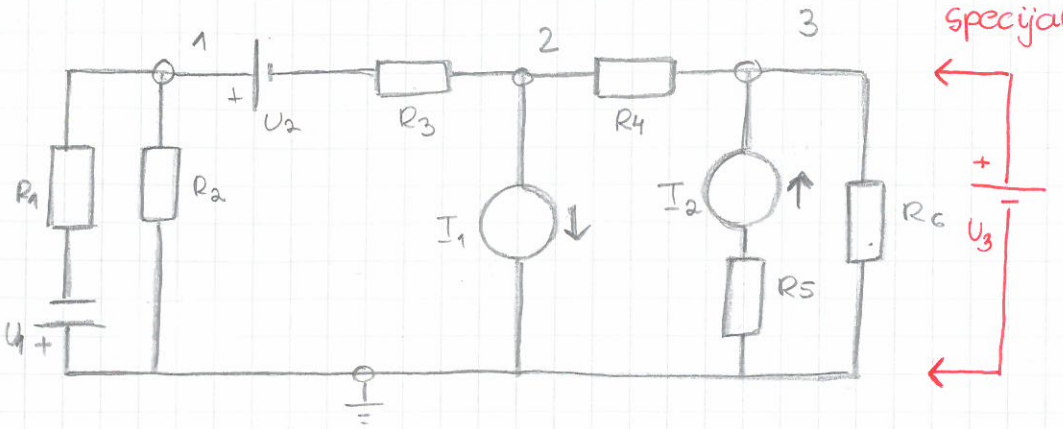
$$I_3 = \frac{\varphi_3 - \varphi_4 + U_3}{R_3}$$

$$I_4 = \frac{\varphi_1 - \varphi_4 + U_4}{R_4}$$

$$I_5 = \frac{(\varphi_4 - \varphi_2)}{R_5}$$

1. $\frac{\varphi_3 - \varphi_1 + U_0}{R_0} + \frac{\varphi_2 - \varphi_1}{R_1} - \frac{\varphi_1 + U_4}{R_5} = 0$
2. $\frac{-\varphi_2}{R_5} - \frac{\varphi_2 - \varphi_1}{R_1} - \frac{\varphi_2 - \varphi_3}{R_2} = 0$
3. $\frac{\varphi_2 - \varphi_3}{R_2} - \frac{\varphi_3 + U_3}{R_3} - \frac{\varphi_3 - \varphi_1 + U_0}{R_0} = 0$

1. $\varphi_1 \left[\frac{1}{R_0} + \frac{1}{R_1} + \frac{1}{R_4}\right] - \varphi_2 \frac{1}{R_1} - \varphi_3 \cdot \frac{1}{R_0} = \frac{U_0}{R_0} - \frac{U_4}{R_4}$
2. $-\varphi_2 \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right] - \varphi_3 \frac{1}{R_2} = 0$
3. $-\varphi_1 \frac{1}{R_0} - \varphi_2 \cdot \frac{1}{R_2} + \varphi_3 \left[\frac{1}{R_0} + \frac{1}{R_2} + \frac{1}{R_3}\right] = -\frac{U_0}{R_0} - \frac{U_3}{R_3}$



specijalni slučaj

1. $\varphi_1 \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \varphi_2 \frac{1}{R_3} = \frac{U_2}{R_3} - \frac{U_1}{R_1}$

2. $\varphi_2 \cdot \left(\frac{1}{R_3} + \frac{1}{R_4}\right) - \varphi_1 \frac{1}{R_3} - \varphi_3 \cdot \frac{1}{R_4} = -\frac{U_2}{R_3} - I_1$

3. $\varphi_3 \cdot \left(\frac{1}{R_4} + \cancel{\frac{1}{R_5}} + \frac{1}{R_6}\right) - \varphi_2 \cdot \frac{1}{R_4} = I_2$

vodljivost grane sa stranim izvorom u seriji je 0

$\varphi_B = 0$

X.2. $\varphi_A \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{U_1}{R_1} + \frac{U_2}{R_2}$

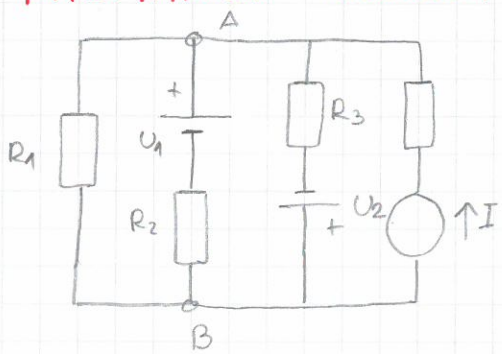
$$\varphi_A = \frac{\frac{U_1}{R_1} + \frac{U_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{12}{5}$$

$$I_1 = \frac{U_1 - U_{ab}}{R_1} = \frac{U_1 - \varphi_A}{R_1} = \frac{24 - \frac{12}{5}}{3} = 3{,}2\ A$$

$$I_2 = \frac{U_2 - \varphi_A}{R_2} = -0{,}8\ A$$

$$I_3 = \frac{\varphi_A}{R_3} = 2{,}4\ A$$

MILMANOV TEOREM



$$U_{AB} = \frac{\sum_{alg} \frac{E_i}{R_i}}{\sum G_i} = \frac{\frac{U_1}{R_2} - \frac{U_2}{R_3} + I}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cancel{\frac{1}{R_4}}}$$

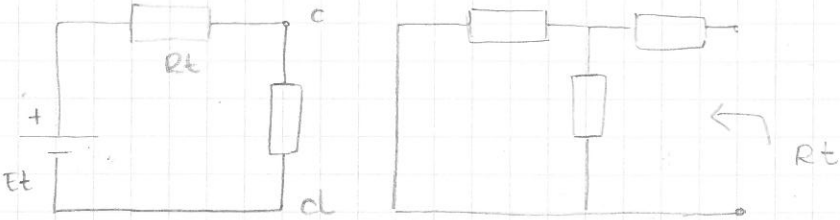
X.1. 1. $\varphi_1 \left(\frac{1}{R_3} + \frac{1}{R_1}\right) - \varphi_2 \cdot \left(\frac{1}{R_3}\right) = I_1$ $\Rightarrow$ $\varphi_1 = 6\,V$

2. $-\varphi_1 \frac{1}{R_3} + \varphi_2 \left(\frac{1}{R_3} + \frac{1}{R_2}\right) = -I_2$ $\varphi_2 = -6\,V$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}$$

$I_3 = \frac{\varphi_1 - \varphi_2}{R_3} = 1\,A$

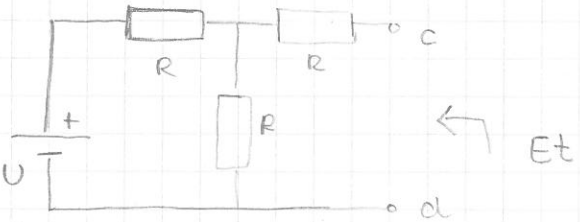
X.3. $U = 12\,V$
$R_t = 100\,\Omega$
$R = 57{,}735\,\Omega$
$U_{cd} / U_{ab} = ?$



$R_T = R + R \parallel R = \frac{3}{2}R$

$E_t = \frac{U}{R+R} \cdot R = \frac{U}{2}$

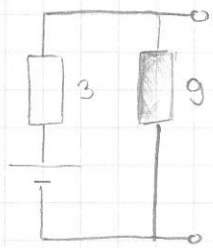
$U_{cd} = \frac{E_t}{R_T + R_t} \cdot R_t$



$\frac{U_{cd}}{U_{ab}} = \frac{U_{cd}}{U} = \frac{R_t}{3R + 2R_t} = 0{,}268$

X.6. Dva Theveninova izvora

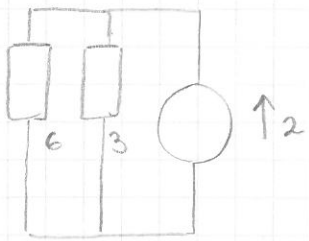
lijevo



$R_t = \frac{9}{4}$

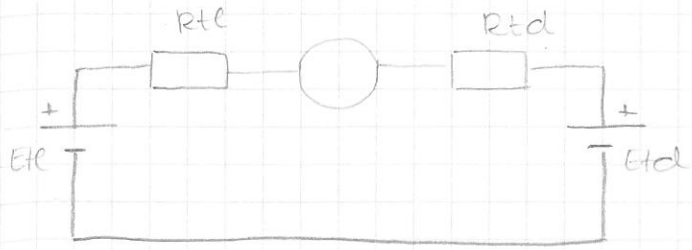
$E_t = \frac{12}{3+9} \cdot 9 = 9\,V$

desno



$R_{td} = \frac{6 \cdot 3}{6+3} = 2\,\Omega$

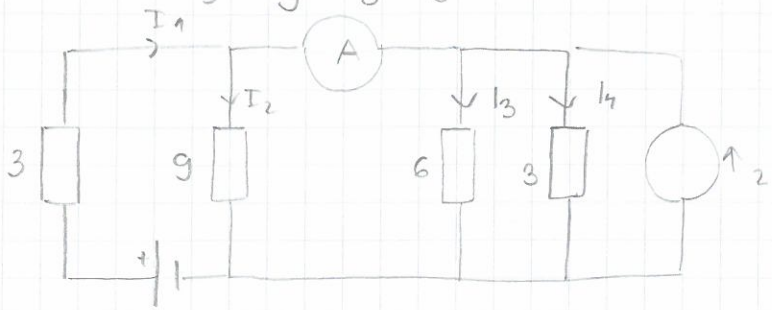
$E_{td} = 2 \cdot 2 = 4\,V$



$I_A = \frac{E_{tc} - E_{td}}{R_{tc} + R_{td}} = 1{,}762\,A$

b) Milmanov teorem

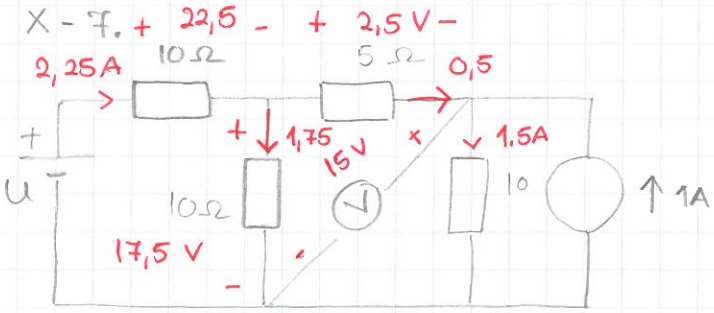
$$\varphi_A = \frac{\frac{12}{3} + 2}{\frac{1}{3} + \frac{1}{9} + \frac{1}{3} + \frac{1}{6}} = 6{,}353\ V$$



$$I_A = I_1 - I_2 = \frac{U - \varphi_A}{3} - \frac{\varphi_A}{9}$$

$$I_A = I_3 + I_4 - 2 = \frac{\varphi_A}{6} + \frac{\varphi_A}{3} - 2$$

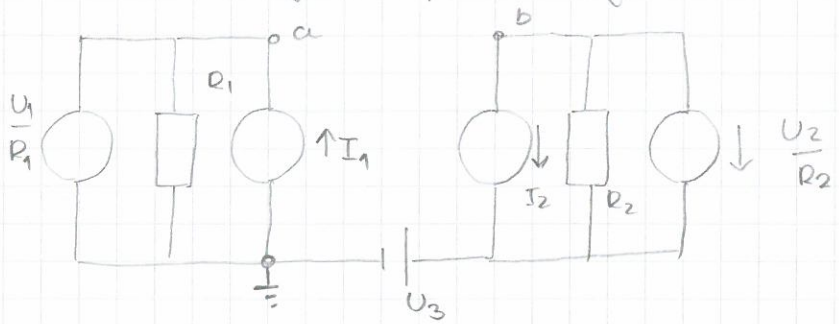
X - 7.



X - 9.

$R_t = R_1 + R_2 = 30\ \Omega$

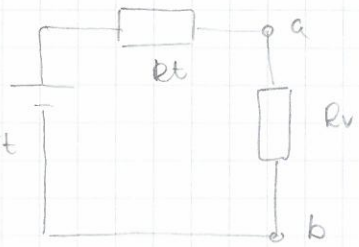
. transformacija map. u strujne



$$\varphi_A = \left(\frac{U_1}{R_1} + I_1\right) \cdot R_1$$

$$\varphi_B = U_3 + \left[-\frac{U_2}{R_2} - I_2\right] \cdot R_2$$

$$Et = Uab = \varphi_A - \varphi_B = 60\ V$$



$$P = \left(\frac{Et}{Rv + Rt}\right)^2 \cdot Rv$$

$$= 30\ W$$

X - 11. $P_1 = 20\,V$

$P_3 = 10\,V$

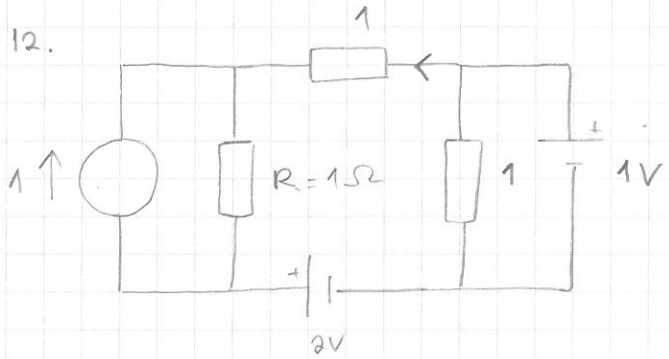
$$P_2 \cdot \left(\frac{1}{5} + \frac{1}{4} + \frac{1}{2}\right) - P_1 \cdot \frac{1}{5} - P_3 \cdot \frac{1}{2} = 0$$

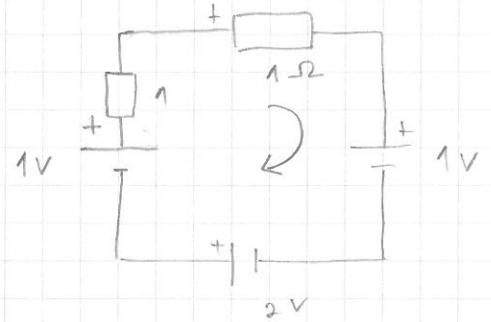
$$P_2 = \frac{4+5}{\frac{19}{20}} = \frac{180}{19} = 9{,}47\ V$$

$$P_4 \cdot \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right) - P_1 \cdot \frac{1}{5} - P_3 \cdot \frac{1}{4} = 0$$

$$P_4 = 8{,}298\ V$$

X - 12.





$$I \cdot 2 = 1 - 2 - 1 = -2$$

$$I = -1\ A$$

X - 15. $P_3 = U_4$

$$P_1 \cdot \left(\frac{1}{I_1} + \frac{1}{I_3} + \frac{1}{I_5}\right) - P_2 \cdot \frac{1}{I_1} = \frac{-U_1}{I_1} + \frac{U_4}{I_5}$$

$$-P_1 \cdot \frac{1}{I_1} + P_2 \cdot \left(\frac{1}{I_1} + \frac{1}{I_2} + \frac{1}{I_6}\right) = \frac{U_1}{I_1} + \frac{U_2}{I_2} + \frac{U_4}{I_2}$$

$P_1 = -1 - j3 \qquad P_2 = 1 - j$

$$I_1 = \frac{P_1 - P_2 + U_1}{I_1} = -j2$$

X-17. $Zt = 6 \| j2 + 2 \| -j2 = \frac{21j+49}{29}$

$P_1 = \frac{50}{5+j2} \cdot j2$

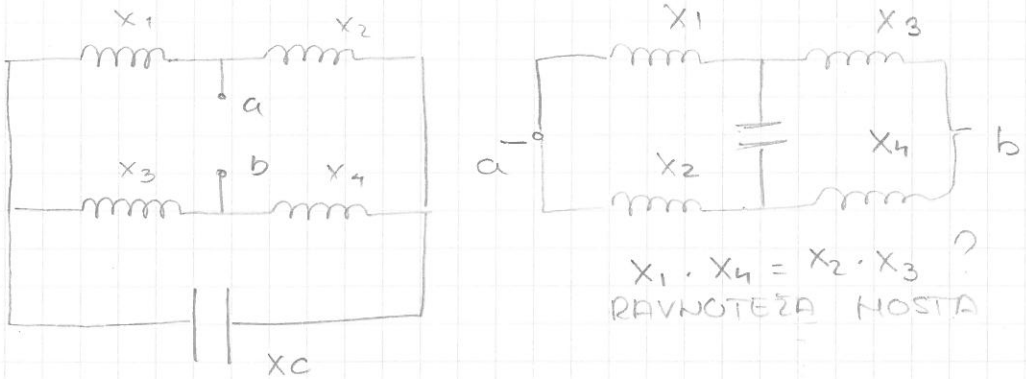
$P_2 = \frac{j50}{2-j2} \cdot (-j2)$

$Et = P_1 - P_2 =$

$|Et| = 19,69$

$|I| = \frac{|Et|}{|Zt+R|} = 3,43A$

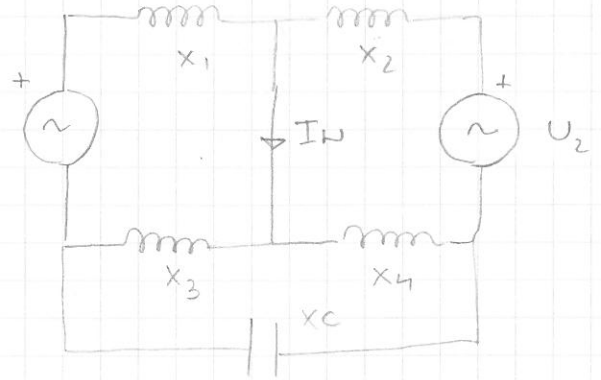
X-18.



$X_1 \cdot X_4 = X_2 \cdot X_3$?

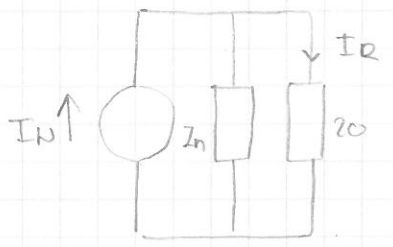
RAVNOTEŽA MOSTA

$Z_N = \frac{j(X_1+X_3) \cdot j(X_2+X_4)}{j(X_1+X_2+X_3+X_4)} = j20\,\Omega$



$I_N = \frac{U_1}{j(X_1+X_3)} + \frac{U_2}{j(X_2+X_4)}$

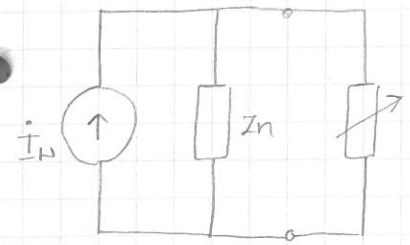
$= \frac{1}{6}(2-j1)$



$\dot{I}_R = \dot{I}_N \cdot \frac{I_N}{I_N+20}$

$I_R = 0,261\ A$

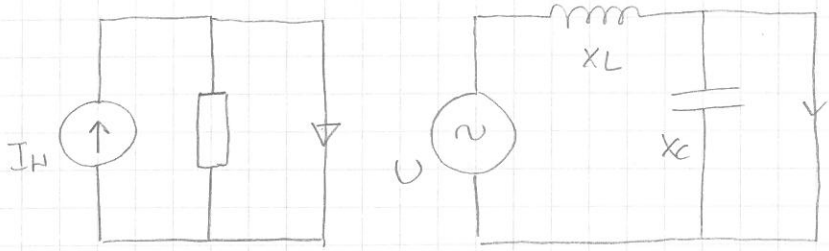
X.19.



$$Z_u = \frac{jX_L(-jX_C)}{jX_L - jX_C} = \infty$$

(1) $X_L = X_C$

$$\omega L = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega^2 L} = 2{,}65\,\mu F$$



$$I_N = \frac{\dot{U}}{jX_L} \Rightarrow X_L = \frac{U}{I_N} = \frac{U}{I_Z}$$

$$= \frac{120}{0{,}1} = 1200\,\Omega$$

$$L = 3{,}82\,H$$

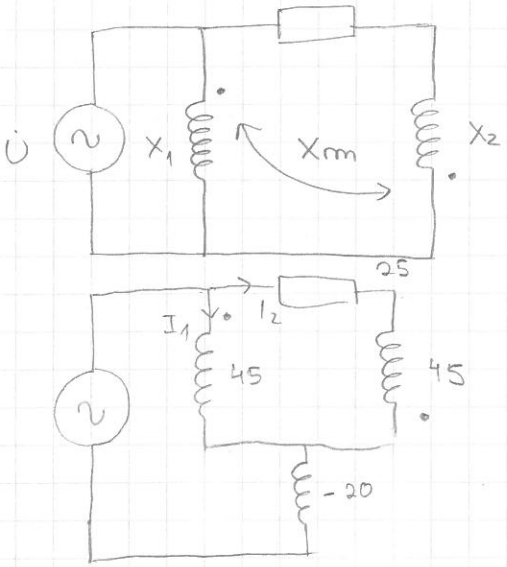
P(R)?

$R = 25\,\Omega$

$X_1 = X_2 = 25\,\Omega$

$X_m = 20\,\Omega$

$U = 25\,V$



$$Z = \frac{(25+45j)\cdot 45j}{25+45j+45j} - 20j$$

$$Z = 7{,}11\ \angle 35{,}32°$$

$$I = \frac{U}{Z} = 3{,}515\ \angle -35{,}32°$$

$$\frac{I_1}{I_2} = \frac{25+45j}{45j}$$

$$\frac{3{,}515\ \angle -35{,}32°}{I_2} = 1{,}144\ \angle -29$$

$$3{,}515\ \angle -35{,}32 - I_2 = 1{,}144\ \angle -29\ I_2$$

$$3{,}515\ \angle -35{,}32 = 2{,}076\ \angle -15{,}52\ I_2$$

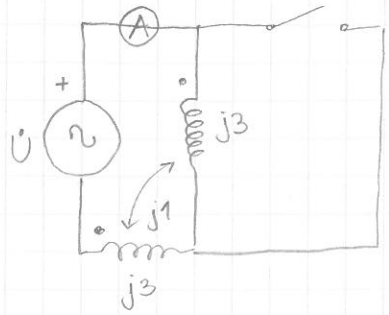
$$I_2 = 1{,}69\ \angle -19{,}8\ A$$

$$P = I^2 \cdot R = 71{,}67$$

II KZ.

$$I_1 jX_1 - I_2 jX_m = \dot{U}$$

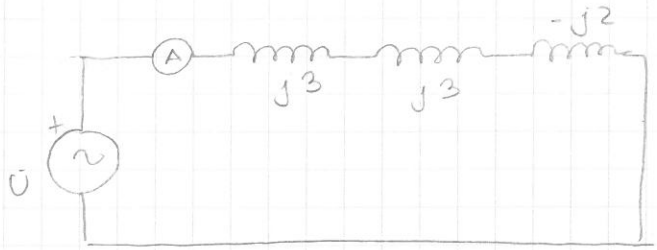
$$-I_1 jX_m + I_2(R + jX_2) = \dot{U}$$



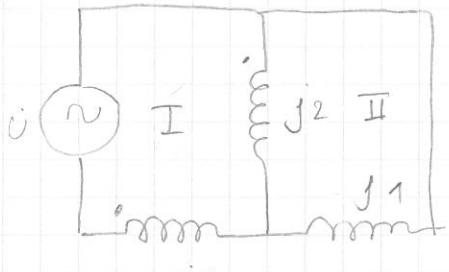
I_A otvoreno / I_A zatvoreno

$$\frac{\frac{U}{4j}}{\frac{30}{3j}} = \frac{8j}{12j} = \frac{2}{3}$$

Skeapka otvorena



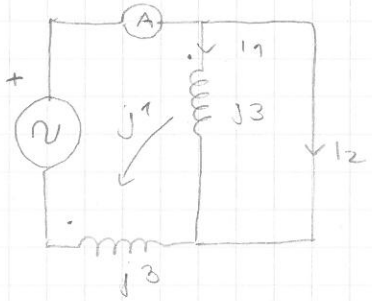
$$I_{Aot} = \frac{U}{j4}$$



$$I_{Azatvoreno} = \frac{U}{\frac{8}{3j}}$$

$$\dot{U} = (I_1 + I_2) j3 - I_1 j1$$

$$-I_1 j3 + (I_1 + I_2) \cdot j1 = 0$$



TROFAZNI SUSTAVI

2 fazni sustav

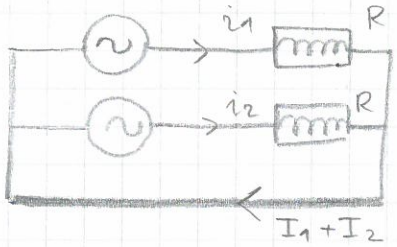
$i_1 = I_m \sin(\omega t)$
$i_2 = I_m \cos(\omega t)$

$p_{uk} = p_1 + p_2 = i_1^2 R + i_2 R = I_m^2 \cdot R \,[\sin^2(\omega t) + \cos^2 \omega t] = I_m^2 \cdot R$

$\dot{I}_1 = \frac{I_m}{\sqrt{2}} \angle 0°$

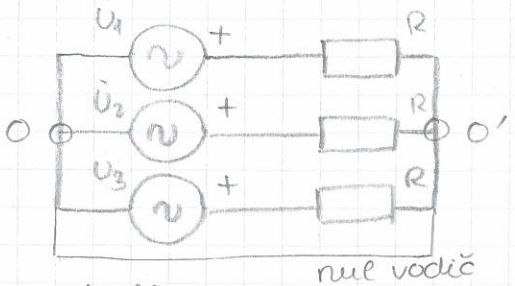
$\dot{I}_2 = \frac{I_m}{\sqrt{2}} \angle 90°$

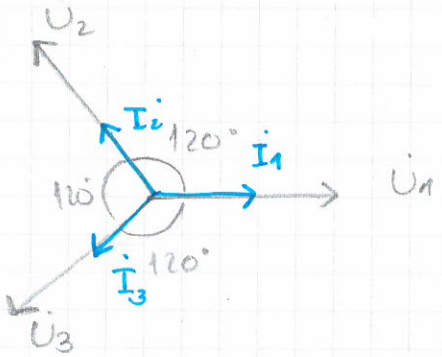
$\dot{I}_1 + \dot{I}_2 = \frac{I_m}{\sqrt{2}} \angle 0° + \frac{I_m}{\sqrt{2}} \angle 90° = I_m \angle 45°$



← mora biti debeji asimetično

3 fazni sustav





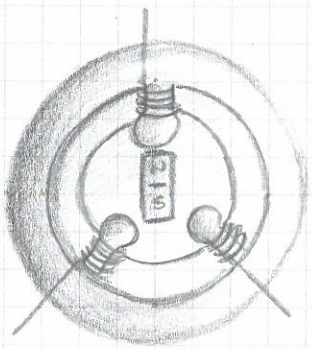
$U_1 = U \angle 0°$
$U_2 = U \angle -120°$
$U_3 = U \angle -240°$

$U_1 + U_2 + U_3 = 0$

Millman

$$U_{O'O} = \frac{\frac{U_1}{R} + \frac{U_2}{R} + \frac{U_3}{R}}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = 0$$

Ako je $U_{O'O} = 0$ ⇒ nema smije kroz nul vodič pa on nije potreban



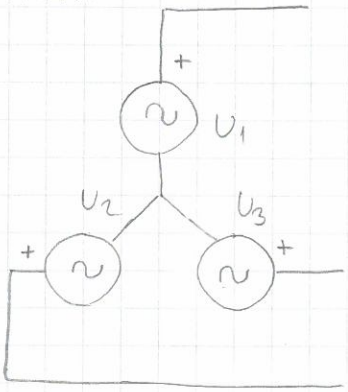
$\Phi = \Phi_m \sin(\omega t)$

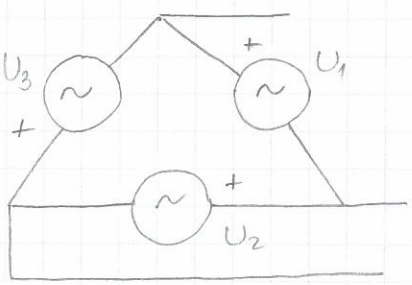
$u = -\frac{d\Phi}{dt}$ ⇒ sinusne funkcije

pomak u Φ za 120° ujetavam geometrijski

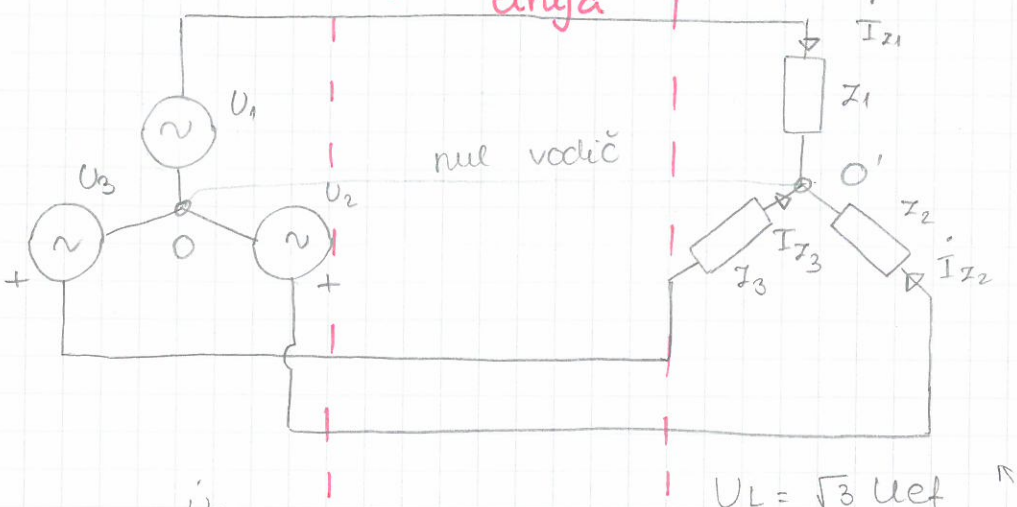
→ vrijedi i za napone

2 SPOJA





generator | prijenosni vod (linija) | trošilo



Y spoj

linijske struje = faznim strujama

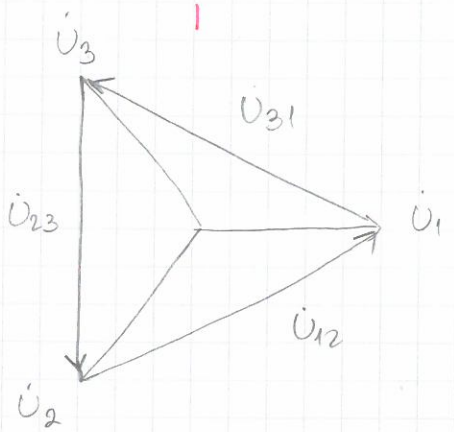
Vanjski napon

$\dot{U}_{12} = \dot{U}_1 - \dot{U}_2$

$\dot{U}_{23} = \dot{U}_2 - \dot{U}_3$

$\dot{U}_{31} = \dot{U}_3 - \dot{U}_1$

$U_L = \sqrt{3}\, U_{ef}$ ↖ fazni napon



$\dot{U}_1 = U \angle 0°$

$\dot{U}_2 = U \angle -120°$

$\dot{U}_3 = U \angle -240°$

$\dot{U}_{12} = \dot{U}_1 - \dot{U}_2 = U\angle 0° - U\angle 120°$

$= U\left[1 - \left[-0{,}5 - j\frac{\sqrt{3}}{2}\right]\right]$

$= U \cdot \frac{1}{2}\left[3 + j\sqrt{3}\right]$

$|\dot{U}_{12}| = |U| \cdot \frac{1}{2} \cdot \sqrt{3^2 + 3} = |U| \cdot \frac{1}{2} \cdot 2\sqrt{3}$

$= |U| \cdot \sqrt{3}$

$I_N = \frac{\dot{U}_1}{Z_1} = \frac{\dot{U}_2}{Z_2} + \frac{\dot{U}_3}{Z_3} = 0$

↑ $\{Z_1 = Z_2 = Z_3\}$

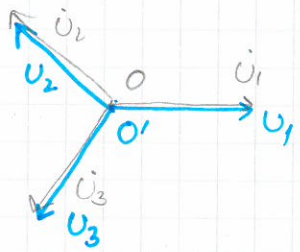
struja kroz nul vodič

ŠTO AKO $Z_1 \neq Z_2 \neq Z_3$ I NEMA NUL VODIČA?

Millerman

$$U_{O'O} = \frac{\frac{U}{Z_1} + \frac{U_2}{Z_2} + \frac{U_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} \neq 0$$

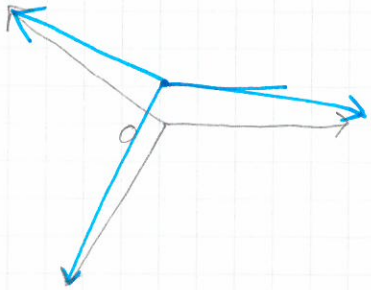
simetrično trošilo + nul vodič



- trošilo
- generator

L_1	R	U
L_2	S	V
L_3	T	W

asimetrično trošilo

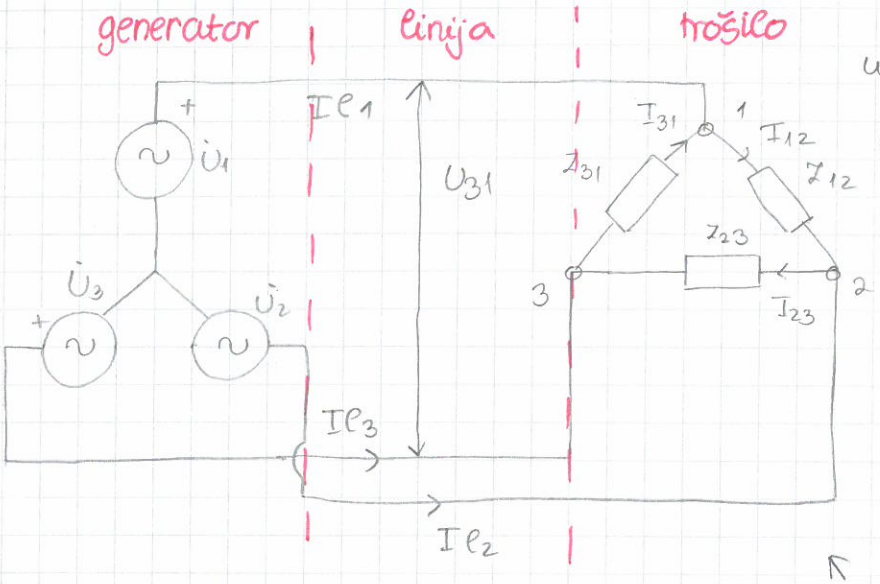


ako postoji nul vodič kroz njega teče struja

$\dot{I}_O = I_{f1} + I_{f2} + I_{f3} =$

$\frac{U_f \angle 0°}{Z_1} + \frac{U_f \angle -120°}{Z_2} + \frac{U_f \angle -240°}{Z_3}$

$I = \frac{U_{O'O} - U_1}{Z_1}$



u Δ mjerna nul vodiča

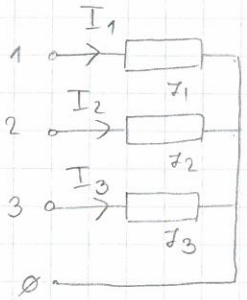
napon faze trošila i linijski napon

$U_f = U_l$

↖ linijski napon

$$\left.\begin{aligned} \dot{I}_{l1} &= \dot{I}_{12} - \dot{I}_{31} \\ \dot{I}_{l2} &= \dot{I}_{23} - \dot{I}_{12} \\ \dot{I}_{l3} &= \dot{I}_{31} - \dot{I}_{23} \end{aligned}\right\} \quad I_l = \sqrt{3}\, I_f$$

Zadatak:



$U_f = 110\ V$

$Z_1 = Z_2 = 110\ \angle 0°\ \Omega$

$Z_3 = 110\ \angle -90°\ \Omega$

$I_1 = 1\ A$

$I_2 = 1\ \angle -120°$

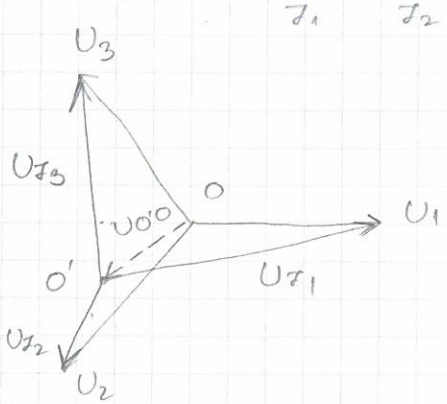
$I_3 = 1\ \angle -150°$

Odredite I_0

$I_N = \sqrt{2}\ \angle -105°$.

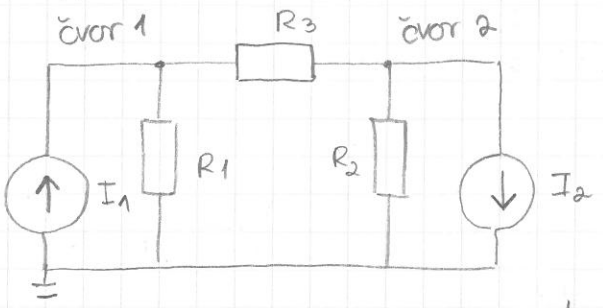
b) prekid nul vodiča $U_{o'o}$

$$U_{o'o} = \frac{\frac{U_1}{Z_1} + \frac{U_2}{Z_2} + \frac{U_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{\sqrt{2}\ \angle 105°}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = -46{,}15 - j52$$



ZBIRKA: POSTUPCI RJEŠAVANJA EL. MREŽA

X-1.



$R_1 = 2\,\Omega$
$R_2 = 6\,\Omega$
$R_3 = 12\,\Omega$
$I_1 = 4\,A$
$I_2 = 2\,A$

$$\varphi_1\left(\frac{1}{R_1}+\frac{1}{R_3}\right) - \varphi_2 \cdot \frac{1}{R_3} = I_1$$

$$-\varphi_1 \cdot \frac{1}{R_3} + \varphi_2 \cdot \left(\frac{1}{R_3}+\frac{1}{R_2}\right) = -I_2$$

$$\varphi_1 \cdot \frac{7}{12} - \varphi_2 \cdot \frac{1}{12} = 4 \quad / \cdot 12$$

$$-\varphi_1 \cdot \frac{1}{12} + \varphi_2 \cdot \frac{1}{4} = -2 \quad / \cdot 12$$

$$7\varphi_1 - \varphi_2 = 48 \quad / \cdot 3$$

$$-\varphi_1 + 3\varphi_2 = -24$$

$$21\varphi_1 - 3\varphi_2 = 144$$

$$+\quad -\varphi_1 + 3\varphi_2 = -24$$

$$20\varphi_1 = 120$$

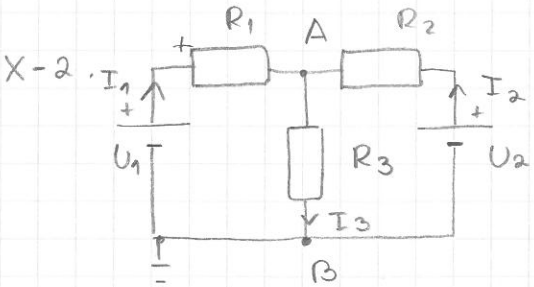
$$\varphi_1 = 6\,V$$

$$\varphi_2 = -6\,V$$

$$U_{12} = \varphi_1 - \varphi_2 = 12\,V$$

$$I_{R3} = \frac{12\,V}{12\,\Omega} = 1\,A \quad (1 \to 3)$$

X-2.



$R_1 = 3\,\Omega$
$R_2 = 3\,\Omega$
$R_3 = 6\,\Omega$
$U_1 = 24\,V$
$U_2 = 12\,V$

Millman

$$U_{AB} = \frac{\frac{U_1}{R_1}+\frac{U_2}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = 14{,}4\,V$$

$$I_3 = \frac{U_{AB}}{R_3} = 2{,}4\,A$$

$$\varphi_A = U_1 - I_1 \cdot R_1$$

$$14{,}4 = 24 - I_1 \cdot 3$$

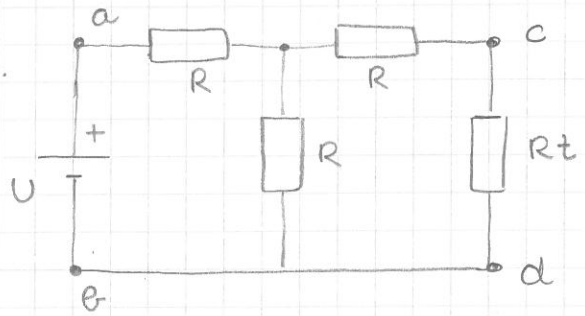
$$I_1 = 3{,}2\,A$$

$$I_1 + I_2 = I_3$$

$$3{,}2 + I_2 = 2{,}4$$

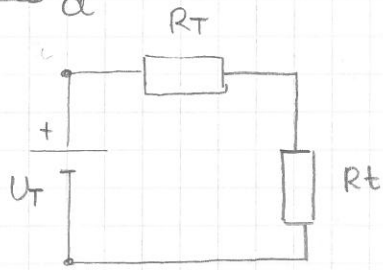
$$I_2 = -0{,}8\,A$$

X-3.



Ucd / Uab = ?

U = 12 V
Rt = 100 Ω
R = 57,735 Ω



$U_T = 6\ V$
$R_T = 86,6025\ \Omega$

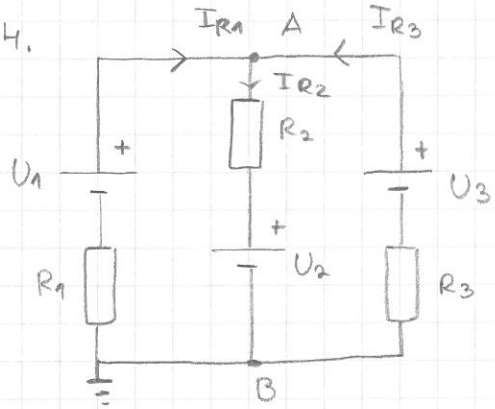
$$\frac{U_{cd}}{U_{ab}} = \frac{3,2154}{6} = 0,268$$

$$\frac{U_{cd}}{6 - U_{cd}} = \frac{100}{86,6025}$$

$$U_{cd} - 86,6025 = 600 - 100\,U_{cd}$$

$$U_{cd} = 3,2154$$

X-4.



$R_1 = 16\ \Omega$
$R_2 = 4\ \Omega$
$R_3 = 24\ \Omega$
$U_1 = 72\ V$
$U_2 = 8\ V$
$U_3 = 48\ V$

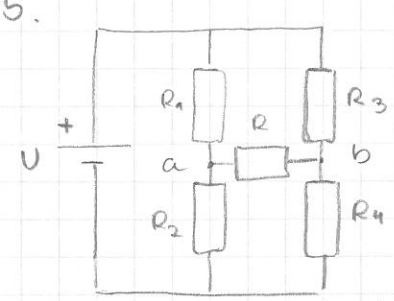
$$U_{AB} = \frac{\frac{U_1}{R_1} + \frac{U_3}{R_3} + \frac{U_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = 24\ V$$

$U_2 + I_{R_2} \cdot R_2 = 24$
$8 + I_{R_2} \cdot 4 = 24$
$I_{R_2} = 4\ A$

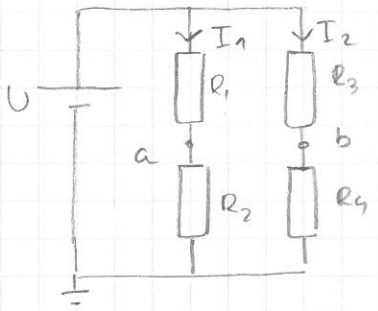
$-I_1 R_1 + U_1 = 24$
$-I_1 \cdot 16 + 72 = 24$
$I_1 = 3\ A$

$I_2 = 1\ A$

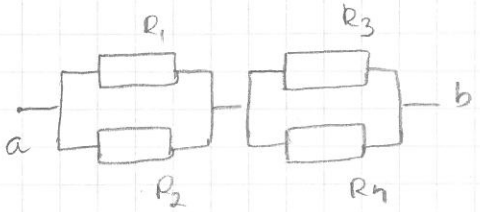
X-5.



$U = 72\ V$
$R = 5\ \Omega$
$R_1 = 6\ \Omega$
$R_2 = 3\ \Omega$
$R_4 = 4\ \Omega$
$R_3 = 12\ \Omega$



$I_1 = 8A$
$I_2 = 4,5A$



$R_T = 5\ \Omega$
$U_T = 6V$

$P_a = I_1 \cdot R_2 = 24V$
$P_b = I_2 \cdot R_4 = 18\ V$

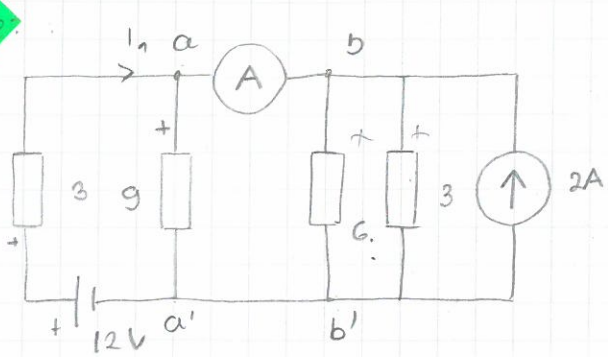
$U_{ab} = P_a - P_b = 6V$

$$I = \frac{6}{5+5} = 0,6A$$

$I_0 = 1,2\ A$

$R_1 = 9\ \Omega$ (most u racunaru)

X-6:



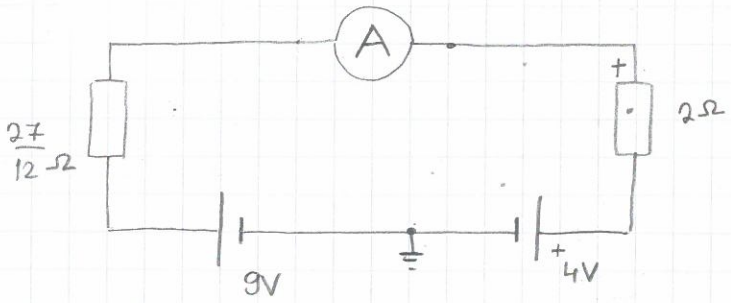
$R_{T1} = 3 \| 9 = \frac{27}{12}\Omega$

$I_1 = \frac{12V}{3+9\Omega} = 1A$

$U_{T1} = 1A \cdot 9\Omega = 9V$

$R_{T2} = 3 \| 6 = 2\Omega$

$U_{T2} = 2\Omega \cdot 2A = 4V$



$I_1 = \frac{9}{\frac{27}{12}+2} = 2,11764$

$I_2 = \frac{4}{\frac{27}{2}+2} = 0,9412$

$I = I_1 - I_2 = 1,17646\ A$

$\varphi_A = \varphi_B = 4 + 2 \cdot 1,176 = 6,352\ V$

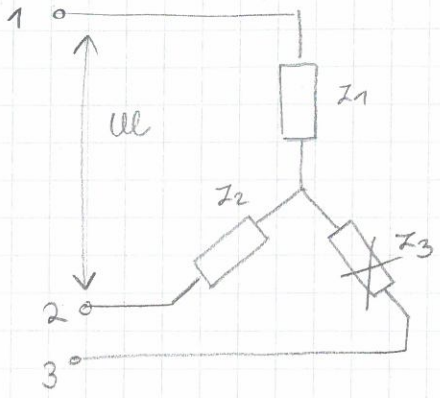
Millerman

$U_{aa'} = \frac{\frac{12}{3} + 2}{\frac{1}{3} + \frac{1}{9} + \frac{1}{6} + \frac{1}{3}} = 6,353\ V$

$I_1 = I_A + I_2 \qquad I_2 = \frac{U_{aa'}}{9} = 0,705\ A$

$12 + I_1 \cdot 3 = 6,353$

$I_1 = 1,882\ A \qquad I_A = 1,177\ A$



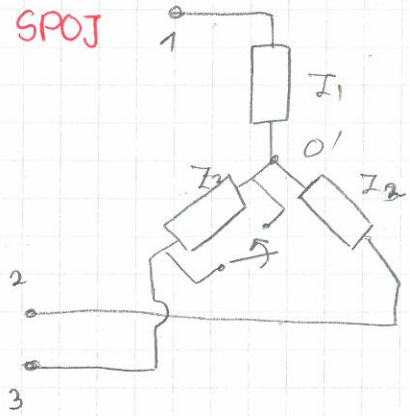
• Prekid faze $Z_1 = Z_2 = Z_3$

$$I_f = \frac{U_f}{Z}$$

$$I_f = \frac{U_l}{2Z} = \frac{\sqrt{3}U_f}{2Z}$$

Struja padne

KRATKI SPOJ

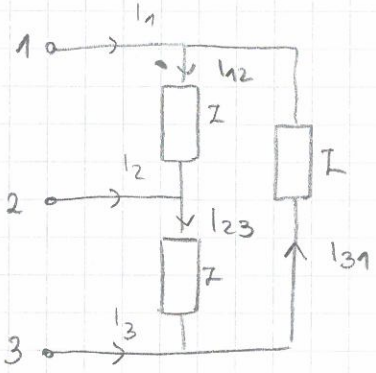


$O' = 3$ ⇒ linijski napon na impedanciji.

$$I_1 = \frac{U_{31}}{Z_1} \qquad I_2 = \frac{U_{23}}{Z_2}$$

SITUACIJE ZA TROKUT

⇒ trošilo u spoju trokuta



fazne struje I_{12}, I_{23}, I_{31}

linijske struje I_1, I_2, I_3

$I_1 = I_{12} - I_{31}$

$I_2 = I_{23} - I_{12}$

$I_3 = I_{31} - I_{23}$

$U_l = 380V$
$Z = 76 \angle 30°$

Izračunaj linijske i fazne struje.

$$I_{12} = \frac{U_{12}}{Z} = \frac{380 \angle 0°}{76 \angle 30°} = 5 \angle -30°$$

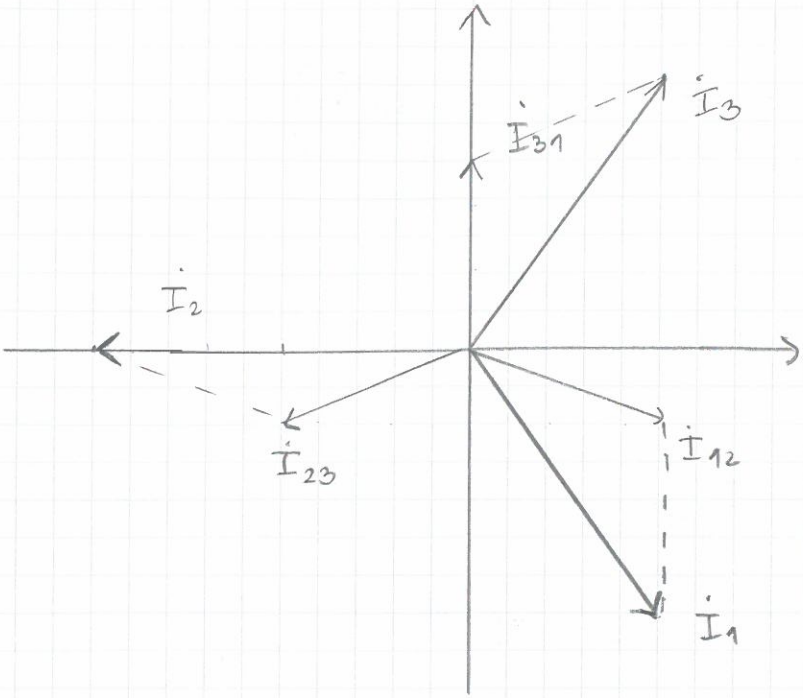
$$I_{23} = \frac{U_{23}}{Z} = \frac{380 \angle -120°}{76 \angle 30°} = 5 \angle -150°$$

$$I_{31} = \frac{U_{31}}{Z} = \frac{380 \angle -240}{76 \angle 30°} = 5 \angle 90°$$

$$I_1 = 5\sqrt{3} \angle -60°$$

$$I_2 = 5\sqrt{3} \angle -150°$$

$$I_3 = 5\sqrt{3} \angle 60°$$



SNAGA U TROFAZNIM SUSTAVIMA - simetrično trofazno trošilo

Snaga 1 faze $P_1 = U_f \cdot I_f \cdot \cos\varphi$

3 faze $P = 3P_1 = 3 U_f I_f \cos\varphi$

LINIJSKE VELIČINE $P = \sqrt{3}\, U_l I_l \cos\varphi$

$Q = \sqrt{3}\, U_l I_l \sin\varphi$

$S = \sqrt{P^2 + Q^2} = \sqrt{3}\, U_l I_l$

Neovisno o spoju

- nesimetrično trošilo

$P = P_1 + P_2 + P_3$ zbrajanje po fazama

$Q = Q_1 + Q_2 + Q_3$

$S = \sqrt{P^2 + Q^2}$

Trošilo s impedancijama spojenim u $\curlywedge$ priključeno je na mrežu linijskog napona 381 V s nul vodičem

$P, Q, S = ?$

$Z_1 = 44 \angle 60°$

$Z_2 = 44 \angle -60°$

$Z_3 = 44 \angle 0°$

$U_f = \frac{U_l}{\sqrt{3}} = \frac{381}{\sqrt{3}} = 220\,V$

$I_1 = \frac{U_1}{Z_1} = \frac{220 \angle 0°}{44 \angle 60°} = 5 \angle -60°$

$I_2 = \frac{U_2}{Z_2} = \frac{220 \angle -120°}{44 \angle -60°} = 5 \angle -60°$

$I_3 = \frac{U_3}{Z_3} = \frac{220 \angle -240}{44 \angle 0°} = 5 \angle -240°$

$I_0 = I_1 + I_2 + I_3 = 5 \angle -60°$

$I_1 = 44 \angle 60° = 44(\cos 60° + j \sin 60°) = 44(0,5 + \frac{j\sqrt{3}}{2})$

$I_2 = 44 \angle -60° = 44\ (0,5 - \frac{j\sqrt{3}}{2})$

$I_3 = 44$

$P_1 = I^2 \cdot R_1 = 5^2 \cdot 44 \cdot 0,5 = 550\ W$

$P_2 = I_2^2 \cdot R_2 = 5^2 \cdot 44 \cdot 0,5 = 550\ W$

$P_3 = I_3^2 R_3 = 5^2 \cdot 44 = 1100\ W$

$P_{uk} = P_1 + P_2 + P_3 = 2200\ W$

$Q_1 = I_1^2 \cdot X_1 = 5^2 \cdot 44 \cdot \frac{\sqrt{3}}{2} = 952,63\ VAr$

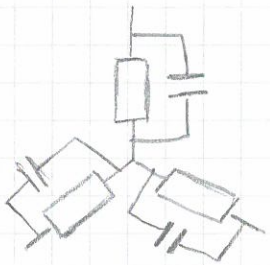
$Q_2 = I_2^2 \cdot X_2 = 5^2 \cdot 44 \cdot \frac{-\sqrt{3}}{2} = -952,63\ Var$

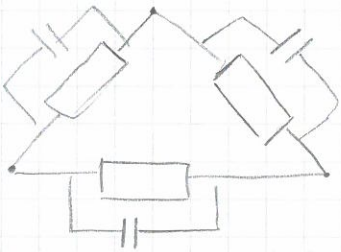
$Q_3 = I_3^2 \cdot X_3 = 0$

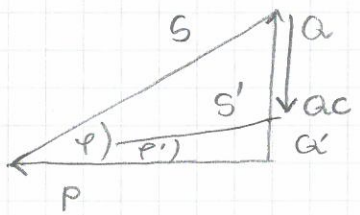
$Q_{uk} = Q_1 + Q_2 + Q_3 = 0$

$S = \sqrt{P_{uk}^2 + Q_{uk}^2} = 2200\ VA$

KOMPENZACIJA JALOVE SNAGE ZA SIMETRIČNO TROŠILO







$tg\ \varphi = \frac{Q}{P}$

$tg\ \varphi' = \frac{Q - Q'}{P}$?

$Q_c = Q - Q' = P(tg\ \varphi - tg\ \varphi')$

$Q_c = \frac{U^2}{X_c} = U^2 \omega C$

$C = \frac{Q_c}{U^2 \omega}$ $\begin{cases} U = U_f & \text{za } \lambda \\ U = U_l & \text{za } \Delta \end{cases}$

Kondenzator $C = 150\ \mu F$ priključuje se prvo između L_1 i L_2 (linijskih vodiča) a zatim između L_1 i nul vodiča mreže 50 Hz. Ako se pritom izmjerene struje kroz kondenzator razlikuju za 7,59 A.

a) U kojem je spoju veća struja?
b) Kolika je linijski napon

$$I_1 = \frac{U_{12}}{X_C} = \frac{\sqrt{3}U_f}{X_C}$$
$$I_2 = \frac{U_{10}}{X_C} = \frac{U_f}{X_C}$$
} veća je struja I_1

$$I_1 - I_2 = 7{,}59 = \frac{U_f}{X_C}\left[\sqrt{3} - 1\right]$$

$$U_f = \frac{(I_1 - I_2)\,X_C}{\sqrt{3} - 1} = 220\ V$$

$$U_l = 380\ V$$

Kako se promjene I_l simetričnog trošila spojenog u Δ ako faze prespojimo u zvijezdu.
Spoj ne mijenja U i Z.

$$I_{l\Delta} = \sqrt{3} I_{f\Delta} = \sqrt{3} \cdot \frac{U_l}{|Z|}$$

$$I_{lY} = I_{fY} = \frac{U_{fY}}{|Z|} = \frac{U_l}{\sqrt{3}|Z|}$$

$$\frac{I_{lY}}{I_{l\Delta}} = \frac{\frac{U_l}{\sqrt{3}|Z|}}{\frac{\sqrt{3}U_l}{|Z|}} = \frac{1}{3}$$

Na trofazni napon gradske mreže priključen je elektromotor, odredi najmanji C kojima bismo faktor snage povećali ma 0,86. Kolika bi trebao biti nazivni napon kondenzatora, i način spajanja

$P = 6\ kW$
$\cos\varphi = 0{,}77$
$\cos\varphi' = 0{,}86$
$C = ?$

spoj $\Delta \Rightarrow U_l$ nazivni $= 400\ V$

$\cos\varphi = 0{,}77 \Rightarrow \varphi = 39{,}64°$

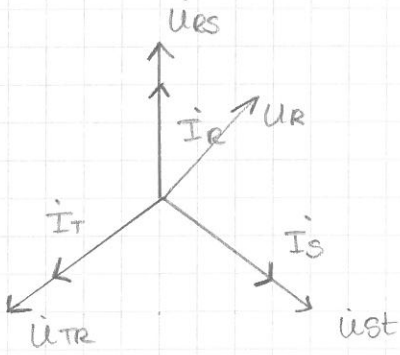
$\cos\varphi' = 0{,}86 \Rightarrow \varphi' = 30{,}68°$

$$Q_C = P(\operatorname{tg}\varphi - \operatorname{tg}\varphi') = 1411{,}58\ VAr$$

$$C = \frac{Q_C}{3\omega U^2} = 10{,}4\ \mu F$$

Vektorski dijagram efinijskih struja i napona simetričnog 3-faznog trošila spojen je u ⅄ prikaza je slikom.

$I_f = 10A$
$U_l = 380V$ odredite Z, P, Q trošila.

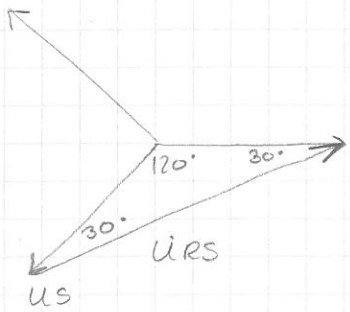


$$Z = \frac{U_R}{I_R} = \frac{\frac{380}{\sqrt{3}}\angle 0°}{10\angle 30°} = 22\angle -30°$$

$$P_{uk} = \sqrt{3}\, U_l I_f \cos\varphi$$
$$= \sqrt{3} \cdot 380 \cdot 10 \cdot \cos(-30°)$$
$$= 5700\ W$$

$$Q_{uk} = U_l \cdot I_f \cdot \sin\varphi \cdot \sqrt{3}$$
$$= -3290.9\ VAr$$

$$S_{uk} = \sqrt{P^2 + Q^2}$$
$$= 6589,8\ VA$$

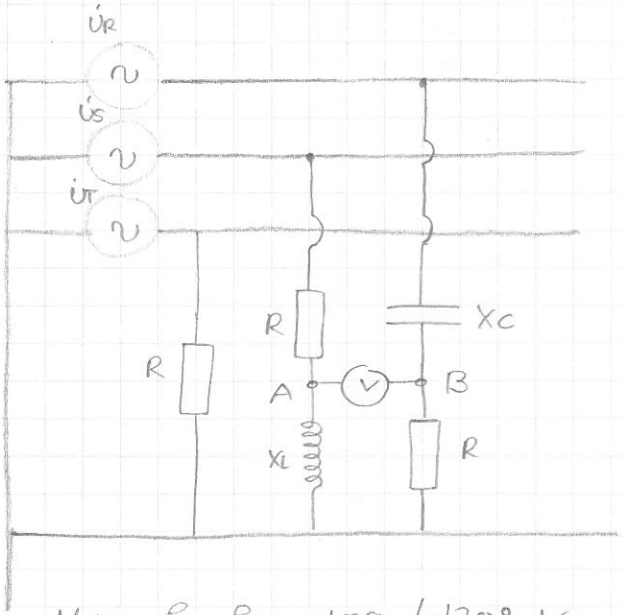


Struja kroz nul vod iznosi 1 A.

a) Odredit U voltmetra, P_{uk}, Q_{uk}
b) Ako se umjesto Ⓥ stavi Ⓐ odredite struju.

$I_\phi = 1A$

$R = \sqrt{3} X_L = \sqrt{3} X_C = 100\Omega$



$$Z_R = 100 - j\frac{100}{\sqrt{3}} = \frac{200\sqrt{3}}{3}\angle -30°$$

$$Z_S = R + jX_L = \frac{200\sqrt{3}}{3}\angle 30°$$

$$Z_T = 100\Omega$$

$$I_0 = I_R + I_S + I_T$$

$$I_R = \frac{U_R}{Z_R} = \frac{U_f\angle 0°}{\frac{200\sqrt{3}}{3}\angle -30°} = \frac{U_f\sqrt{3}}{200}\angle 30°$$

$$I_S = \frac{U_S}{Z_S} = \frac{U_f\angle -120°}{\frac{200\sqrt{3}}{3}\angle 30°} = \frac{U_f\sqrt{3}}{200}\angle -150°$$

$$I_T = \frac{U_T}{Z_T} = \frac{U_f\angle -240°}{100}$$

$I_R + I_S = 0$

$$I_0 = \frac{U_f\angle -240°}{100} \Rightarrow U_f = 100\ V$$

$$U_{AB} = P_A - P_B = 100\angle -120°\ V$$

$$P_A = I_S \cdot jX_L = 50\angle -60°\ V$$

$$P_B = I_R \cdot R = 50\sqrt{3}\angle 30°\ V$$

$P_R = U_R I_R \cos\varphi_R = 100 \frac{\sqrt{3}}{2} \cos(-30°) = 75\ W$

$Q_R = U_R I_R \sin\varphi_R = 100 \frac{\sqrt{3}}{2} \sin(-30°) = -25\sqrt{3}\ VAr$

$P_S = 100 \cdot \frac{\sqrt{3}}{2} \cos 30° = 75\ W$

$Q_S = 100 \cdot \frac{\sqrt{3}}{2} \sin 30° = 25\sqrt{3}\ Var$

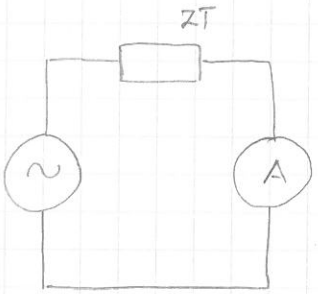
$P_T = 100 \cdot 1 \cdot \cos 0° = 100\ W$

$Q_T = 100 \cdot 1 \cdot \sin 0° = 0\ VAr$

$P_{uk} = P_R + P_S + P_T = 250\ W$

$Q_{uk} = Q_1 + Q_2 + Q_3 = 0$

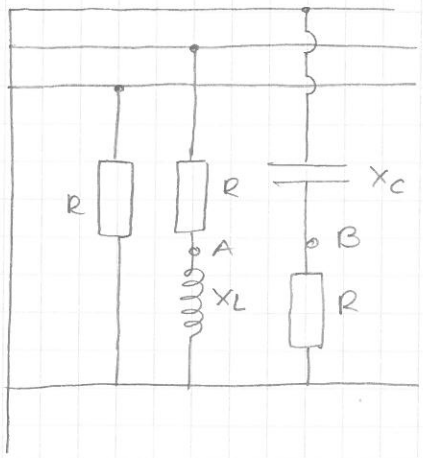
b)

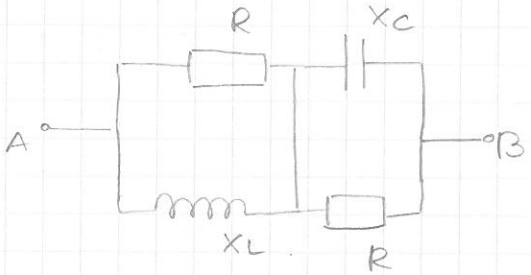


$U_T = U_V$

$I_A = \frac{E_T}{Z_T} = \frac{100\ \angle -120°}{50} = 2\ \angle -120°$

$|I_A| = 2\ A$



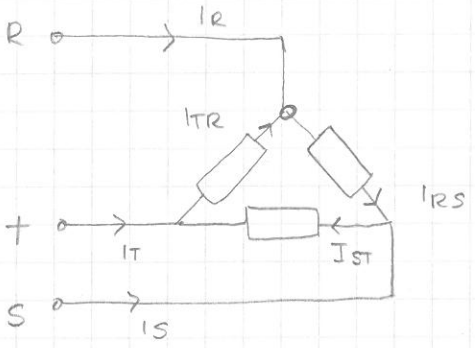


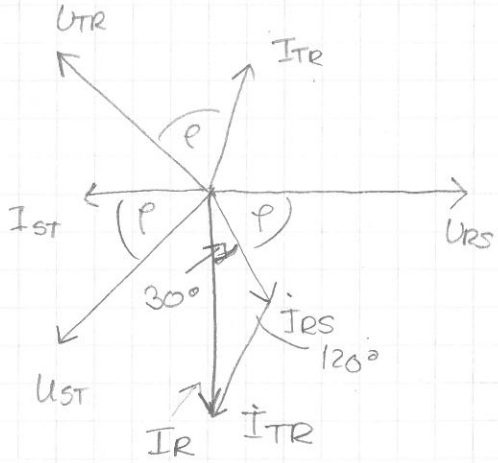
$Z_T = R \,||\, jX_L + R \,||\, (-jX_C)$

$Z_T = 100 \cdot \left(\frac{j}{\sqrt{3}+j} + \frac{-j}{\sqrt{3}-j} \right)$

$Z_T = 50\ \Omega$

U simetričnom trofaznom trošilu spojenom u Δ struja linije R je $100 \angle -60°$ A. Ako je $U_l = 200$ V, odredite I jedne faze trošila te ukupnu radnu i jalovu snagu.





$$\dot{I}_R = \dot{I}_{RS} - \dot{I}_{TR}$$

$$I_R = I_e \angle -30° - \varphi$$

$$\varphi = 30°$$

$$I_e = \sqrt{3} \cdot I_f$$

$$I_f = \frac{I_e}{\sqrt{3}} = \frac{100}{\sqrt{3}}$$

$$Z = \frac{U_L}{I_f} = \frac{200}{\frac{100}{\sqrt{3}}} = 2\sqrt{3}$$

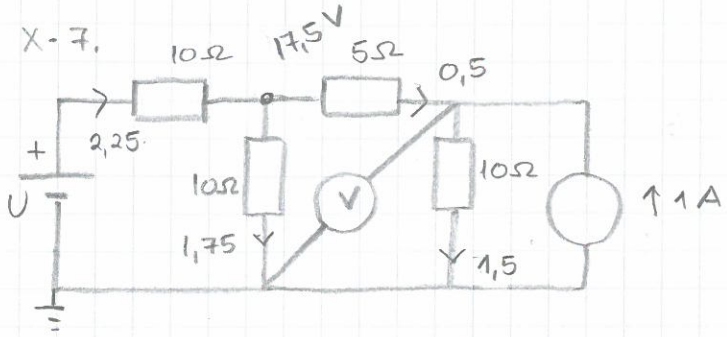
$$Z = 2\sqrt{3} \angle 30°$$

$$P = \sqrt{3}\, U_l \cdot I_e \cdot \cos\varphi$$
$$= \sqrt{3} \cdot 200 \cdot 100 \cdot \cos 30° = 30000 \text{ W}$$

$$Q = \sqrt{3}\, U_l \cdot I_e \cdot \sin\varphi$$
$$= \sqrt{3} \cdot 200 \cdot 100 \cdot \sin 30° = 10000\sqrt{3} \text{ VAr}$$

ZBIRKA : POSTUPCI RJEŠAVANJA EL. MREŽA

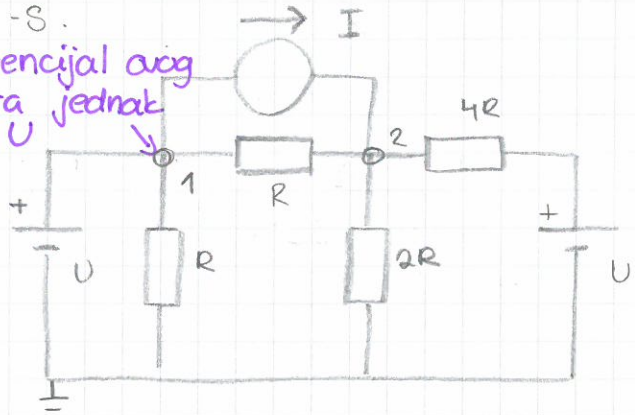
X-7.



$U = 40\ V$

X-8.

potencijal ovog čvora jednak U



$R = 5\Omega$
$I = 2A$
$U = 20\ V$

$$P_2 \cdot \left(\frac{1}{R} + \frac{1}{2R} + \frac{1}{4R}\right) - P_1 \cdot \left(\frac{1}{R}\right) = I + \frac{U}{4R}$$

$$P_2 \cdot \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{20}\right) - 20 \cdot \left(\frac{1}{5}\right) = 2 + \frac{20}{20}$$

$$\frac{7}{20} P_2 - 4 = 3$$

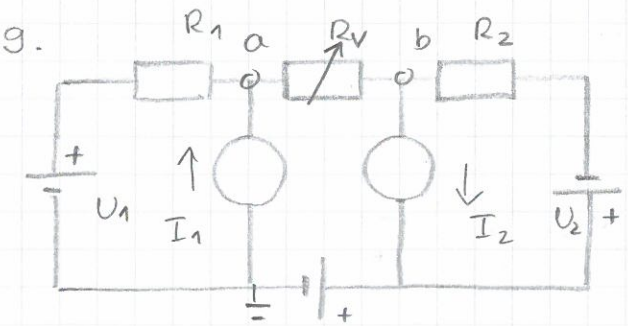
$P_2 = 20 \qquad U_{23} = 20$

promjena smjera struje

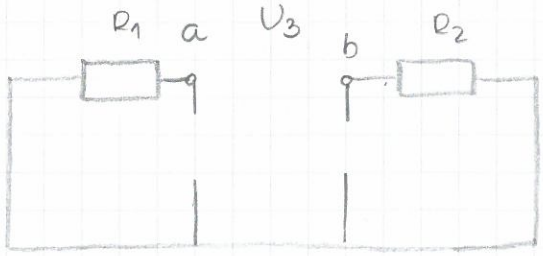
$$P_2 \cdot \frac{7}{20} - 4 = -2 + 1$$

$P_2 = 8{,}57\ V$

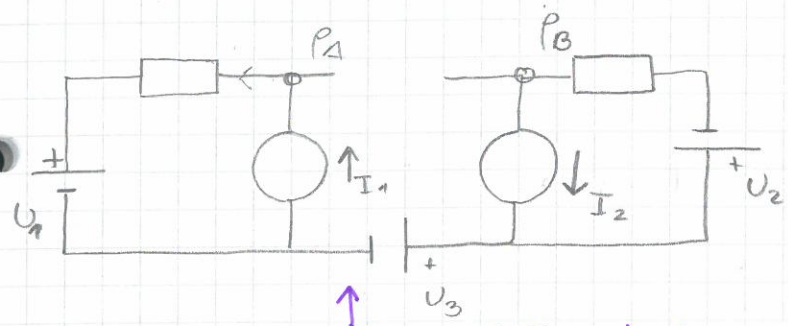
X-9.



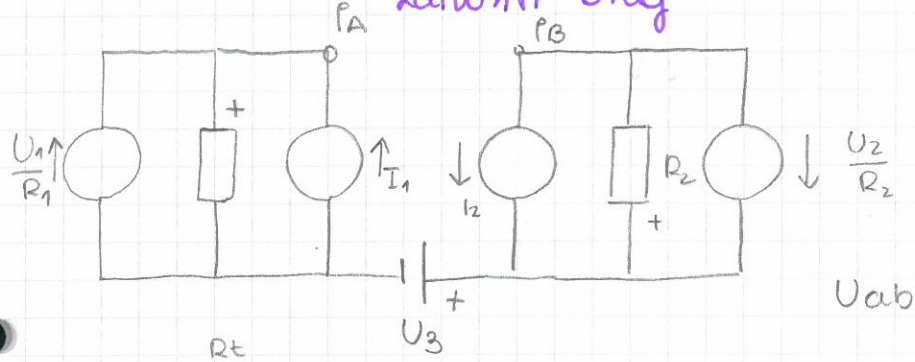
$R_1 = 20\Omega$
$R_2 = 10\Omega$
$U_1 = U_3 = 20V$
$U_2 = 30V$
$I_1 = I_2 = 1\ A$



$R_t = R_1 + R_2 = 30\Omega$



↑ tu ne teče struja ne može se zatvoriti krug

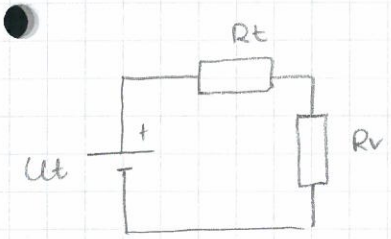


$$P_A = \left(\frac{U_1}{R_1} + I_1\right) \cdot R_1 = 40\,V$$

$$P_B = U_3 - \left(\frac{U_2}{R_2} + I_2\right) R_2$$

$$P_B = -20\,V$$

$$U_{ab} = P_a - P_b = 60\,V$$



P max ⇒ Rt = Rv

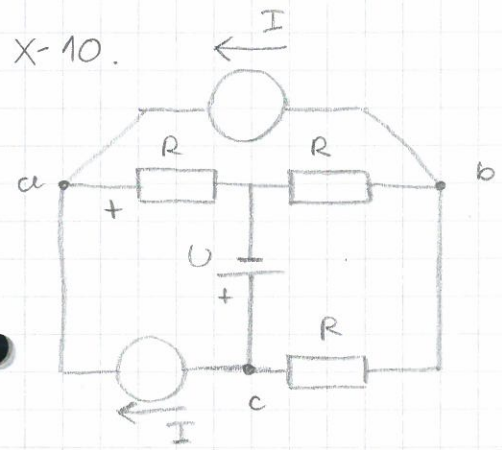
Rv = 30Ω $I = \frac{Ut}{2Rt} = 1A$

P =

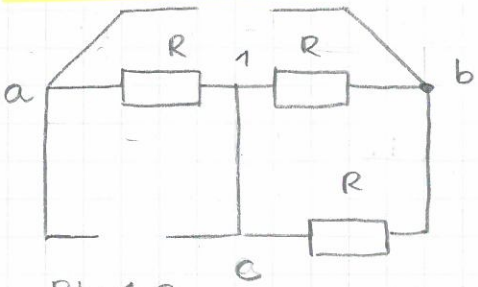
ZAŠTO SU ONI KRATKO SPOJENI ?

↓

X-10.



U = 1 V
R = 1Ω
I = 1 A



Rt = 1Ω

$U_T = -U + 2 \cdot R = 1$

$I_N = 1A = \frac{U_T}{R_T}$

S točaka b i c

Rt = 0,5Ω

Ut = -2 · R

$P_1 = -1\,V$

$P_C =$

$$P_B\left(\frac{1}{r} + \frac{1}{R}\right) - P_1\left(\frac{1}{R}\right) = -I$$

$$P_B \cdot 2 + 1 = -1$$

$$P_B = -1\,V$$

$$I_N = \frac{U_T}{R_T} = 2A$$

$$U_{bc} = -1V - 0V = -1V$$

+ na c

X-11.

$\varphi_1 = 20\,V$

$\varphi_3 = 10\,V$

$$\varphi_2 \cdot \left(\frac{1}{5} + \frac{1}{4} + \frac{1}{2}\right) - \varphi_1 \cdot \left(\frac{1}{5}\right) - \varphi_3 \cdot \left(\frac{1}{2}\right) = 0$$

$$\varphi_2 \cdot \frac{19}{20} - 4 - 5 = 0$$

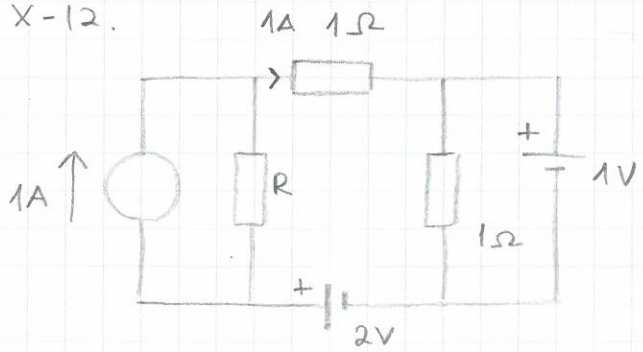
$$\varphi_2 = 9{,}47\,V$$

$$\varphi_4 \cdot \left(\frac{1}{5} + \frac{1}{4} + \frac{1}{3}\right) - \varphi_1 \cdot \frac{1}{5} - \varphi_3 \cdot \frac{1}{4} = 0$$

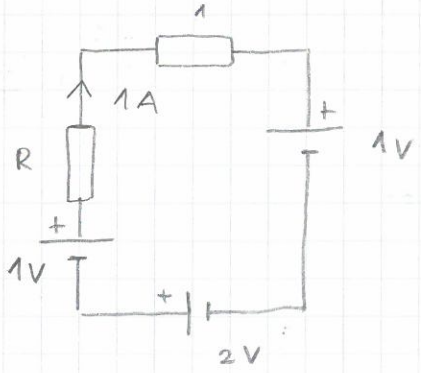
$$\varphi_4 \cdot \frac{47}{60} - 4 - 2{,}5 = 0$$

$$\varphi_4 = 8{,}3\,V$$

X-12.



zanemarimo otpornik spojen paraleelno naponskom izvoru



$R = 2\,\Omega$

$U = 2\,V$

$I = 1A \Rightarrow I_R = 0$

Strujni izvor miješanja smjer: $I_R = 1\,A$

X-13.

$$U_{oa} = \frac{\frac{U_1}{R_1} + \frac{U_2}{R_2} + \frac{U_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = 10\,V \qquad \varphi_a = 10\,V$$

$$U'_{ao} = \frac{\frac{U_1}{R_1} + \frac{U_2}{R_2} + \frac{U_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{1}} = 5\,V \qquad \varphi_a = 5\,V$$

X-14.

$R_t = 1{,}6\ \Omega$

$$P_1 = \frac{\frac{-10}{3{,}2} - \frac{10}{4} - \frac{10}{16}}{\frac{1}{3{,}2} + \frac{1}{4} + \frac{1}{16}} = -10$$

$U_t = -10 + 6{,}8 = -3{,}2$

$$I = \frac{U_t}{2R_t} = 1\,A$$

$P = I^2 \cdot R = 1{,}6\ W$

X-15. $U_1 = 4\,V$ $\quad P_0 = 0\,V$

$U_2 = j8\,V$ $\quad P_3 = 4\sqrt{2}\ \angle -45°\ V$

$U_4 = 4 - j4\,V$

$U_6 = -j4\,V$

$Z_1 = 1 + j\,\Omega$

$Z_2 = -j2\,\Omega$

$Z_3 = 1\,\Omega$

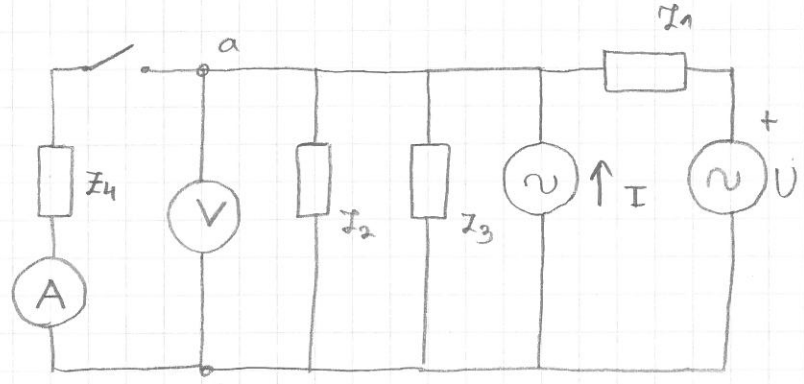
$Z_5 = j\,\Omega$

$Z_6 = j2\,\Omega$

$$P_1 \cdot \left(\frac{1}{Z_3} + \frac{1}{Z_5} + \frac{1}{Z_1}\right) - P_3 \cdot \left(\frac{1}{Z_5}\right) - P_2 \cdot \left(\frac{1}{Z_1}\right) = \frac{-U_1}{Z_1}$$

$$P_2 \cdot \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_6}\right) - P_1 \cdot \left(\frac{1}{Z_1}\right) - P_3 \cdot \left(\frac{1}{Z_2}\right) = \frac{U_2}{Z_2} + \frac{U_1}{Z_1} - \frac{U_6}{Z_6}$$

X-16



$U_w = 10\,V$

$Z_1 = 2 + j2\,\Omega$

$Z_2 = 2 - j2\,\Omega$

$Z_3 = 2\,\Omega$

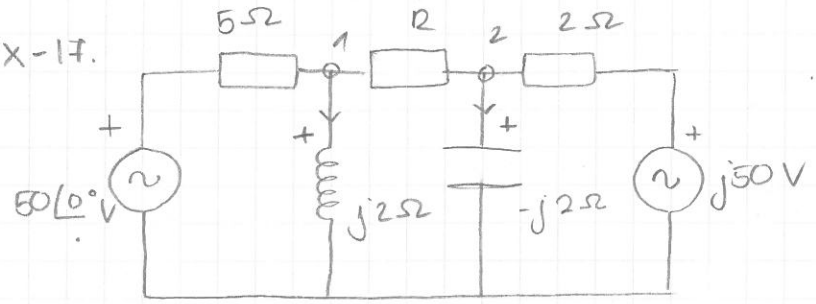
$Z_4 = 1 + j2\,\Omega$

$$U_w = U_{ab} = \frac{I + \frac{U}{Z_1}}{\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_1}} \qquad I = \frac{U}{Z_4} = 3{,}54\ \angle -45°$$

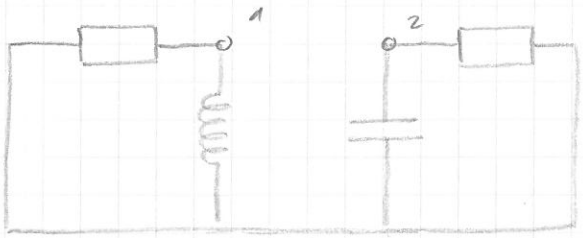
$$10 \cdot 1 = I + \frac{U}{Z_1}$$

$$U_w' = \frac{I + \frac{U}{Z_1}}{\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_1} + \frac{1}{Z_4}} = 7{,}9\ \angle 18{,}43°$$

X-17.



R = 4Ω



$R_T = 5 \| j2 + 2 \| -j2 = 1{,}838 \angle 23{,}2°$

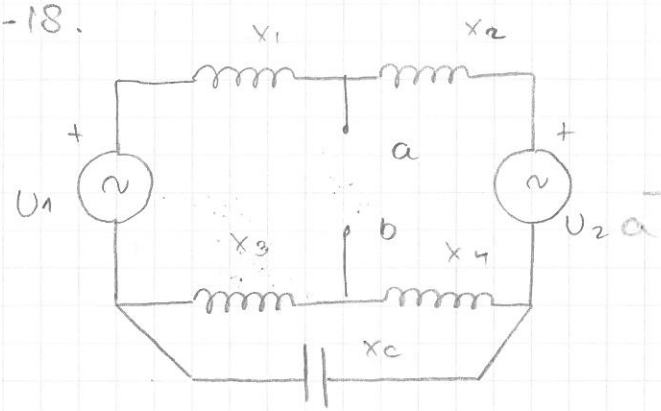
$I_1 = 9{,}28 \angle -21{,}8°$ $\quad I_2 = 17{,}68 \angle 135°$

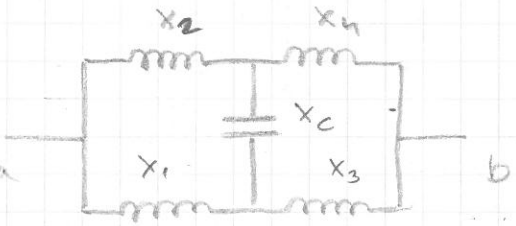
$P_1 = 18{,}57 \angle 68{,}2°$ $\quad P_2 = 35{,}36 \angle 45°$

$U_T = P_1 - P_2 = 19{,}7 \angle -156{,}798$ V

$I = \frac{Ut}{1{,}838 \angle 23{,}2 + 4} = 3{,}43 \angle -164{,}1°$ A

X-18.





↓

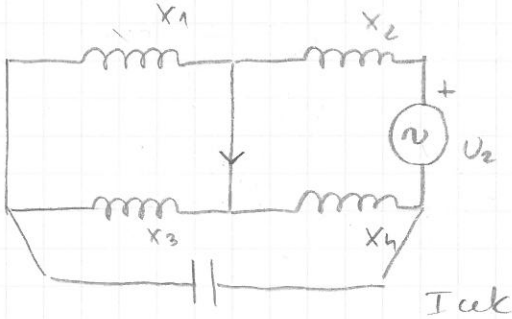
most ve rovnováze

$X_1 = X_4 = 20$
$X_3 = 40$
$X_2 = 10$

$X_C = 30\Omega$
$U_1 = 10V \angle 0°$
$U_2 = 10 \angle 90°$

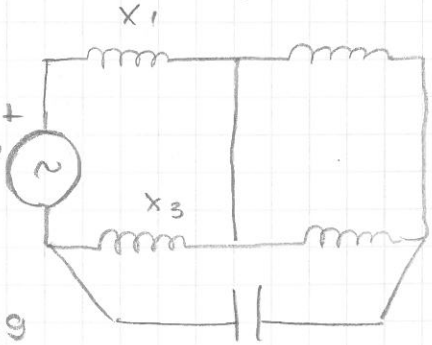
$Z_T = 20j$

$Ut =$

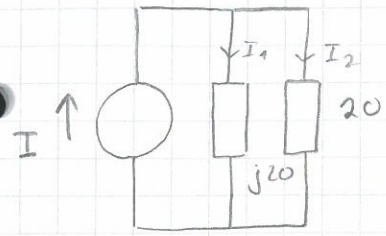


$I = 0{,}33A$

$I = 0{,}167 \angle 90$



$I_{uk} = 0{,}372 \angle -26{,}5879$



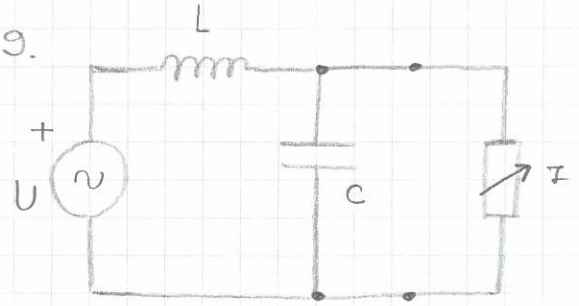
$$\frac{j20}{20} = \frac{I_2}{I_1}$$

$$\frac{j20}{20} = \frac{I_2}{0,37\angle -26,5 - I_2}$$

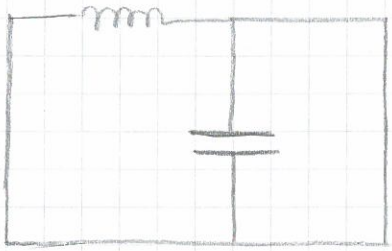
$$5i \cdot 0,37\angle -26,5° - 5i \cdot I_2 = I_2$$

$$I_2 = 0,26\angle 18,5$$

X-19.

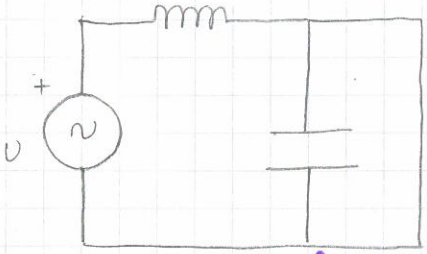


$U = 120V$
$f = 50\ Hz$
$I_N = 0,1\ A$



$$Z_n = \frac{jX_C \cdot (-jX_C)}{jX_L + (-jX_C)} \Rightarrow \text{paralelna rezonancija}$$

$X_L = X_C \Rightarrow Z_n = \infty$



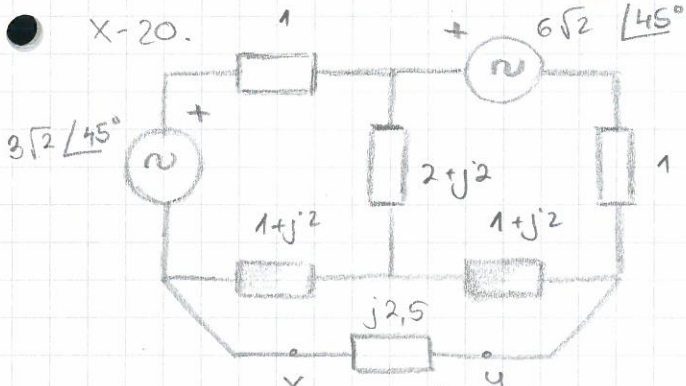
$$I_N = \frac{U}{X_L} \Rightarrow X_L = \frac{U}{I_n} = 120\ \Omega$$

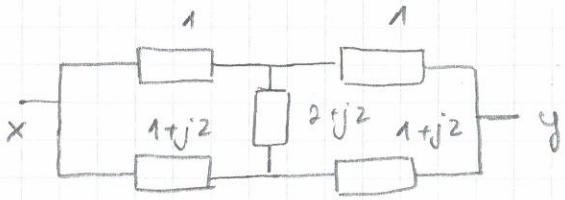
$$\omega L = X_L \Rightarrow L = 3,829\ H$$

$$C = 2,65\ nF$$

kratko spojen, sva struja teče kroz XL

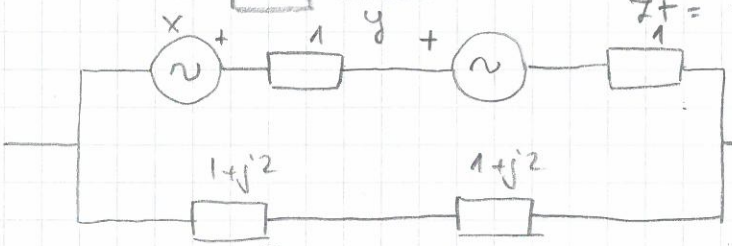
X-20.





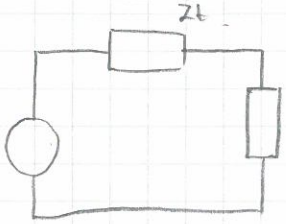
most u ravnoteži

$$Z_t = 1,58\angle 18,43°\ \Omega$$



Millman

$$U_t = \frac{-\frac{3\sqrt{2}\angle 45°}{2} + \frac{6\sqrt{2}\angle 45°}{2}}{\frac{1}{2} + \frac{1}{2(1+j2)}}$$



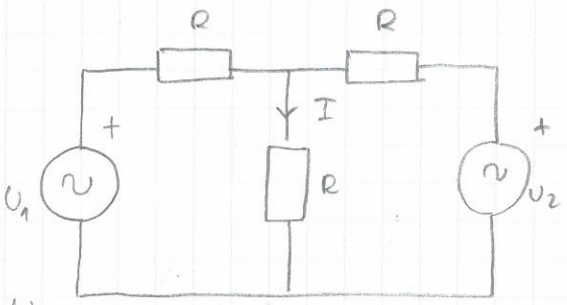
$$I = 2,12\angle 45°$$

$$U_{xy} = 5,3\angle 135° = \frac{\frac{3}{2} + \frac{3}{2}i}{\frac{3}{5} - \frac{1}{5}i} = 3,35\angle 63°$$

X-21. $U_1 = U_2$

$I = 1\,A$

$U_1 \rightarrow \frac{U_1}{2}$



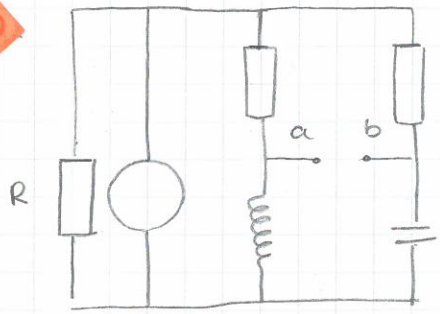
$$Ua = \frac{\frac{U_1}{R} + \frac{U_2}{R}}{\frac{3}{R}} = \frac{2U}{3}$$

$$I = \frac{U}{R} = \frac{2U}{3R} \Rightarrow \frac{U}{R} = \frac{3}{2}$$

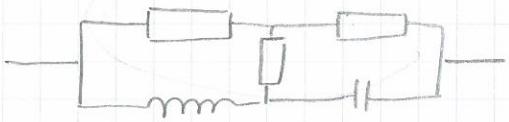
$$Ua' = \frac{\frac{Ui}{R} + \frac{\frac{Ui}{2}}{R}}{\frac{3}{R}} = \frac{\frac{3Ui}{2R}}{\frac{3}{R}} = \frac{Ui}{2}$$

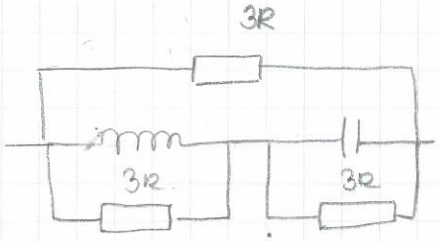
$$I = \frac{\frac{Ui}{2}}{R} = \frac{Ui}{2R} = \frac{3}{2} \cdot \frac{1}{4} = 0{,}75\,A$$

X-22



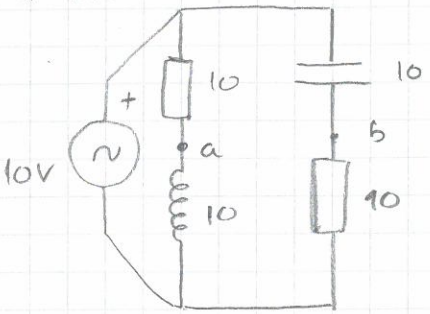
Y → Δ

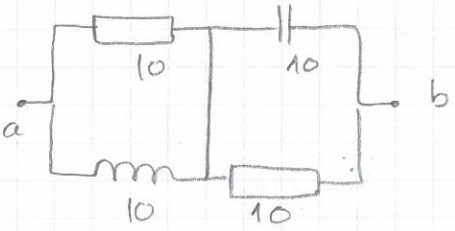




$$\frac{3R \cdot jX_L}{3R + jX_L} + \frac{-3R \cdot jX_C}{3R - jX_C}$$

X-23.



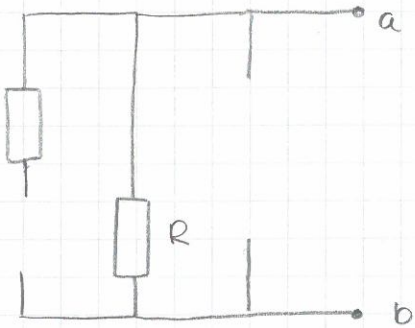


$Z = 10 \| 10j + 10 \| -10j$

$= 10\,\Omega$

most je u ravnoteži $Et = 0\ V$

X-24.



$Zt = 1\,\Omega$

vodljivost grani sa stvarnim izvorom je 0.

$$Uab = \frac{I + \frac{U}{R} - I}{\frac{1}{R}} = 1\ V$$

$$Uab = \frac{I + \frac{U}{R} + I}{\frac{1}{R}} = 3\ V$$

X-25.

$Z = (15 + 5j) \| (5 + 5j) = 4 + 3j$

$$\frac{I_1}{I_2} = \frac{(5 + 5j)}{(15 + 5j)}$$

$$\frac{5\angle 30° - I_2}{I_2} = 0{,}447\angle 26{,}57°$$

$5\angle 30° - I_2 = 0{,}447\angle 26{,}57° \cdot I_2$

$5\angle 30° = \sqrt{2}\angle 8{,}13° \cdot I_2$

$I_2 = 3{,}535\angle 21{,}87°$

$I_1 = 1{,}53\angle 48{,}43°$

$Uab = I_1 \cdot (5 + j5) = 11{,}18\angle 93°$

X-26. $Z = \frac{(10+20i)\cdot 10}{30+20i} + j5 = 10,6 \angle 45° \ \Omega$

$U_{uk} = 10\sqrt{2} \angle 45° \ V$

$Z_{uk} = 20 + 20j = 20\sqrt{2} \angle 45° \ \Omega$

$I = \frac{1}{2} A$

$U_T = -10 \angle 90° + 5 = 11,18 \angle -63,43 \ V$

$I_N = \frac{U_T}{R_t} = 1,05 \angle -108,43° \ A$

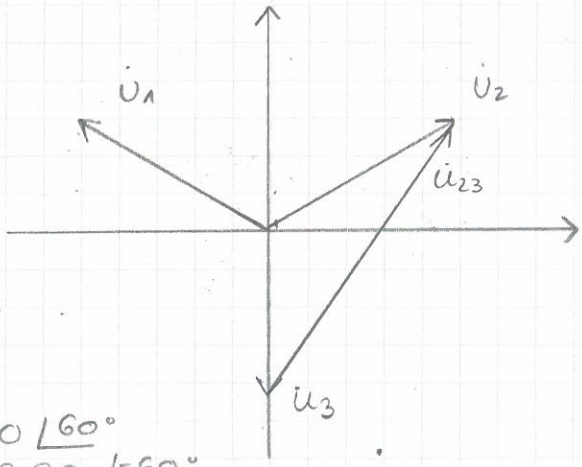
POLIFAZNI SUSTAVI

→ primjeri

X1-P2. $\underline{U}_2 = 190 + j110$

$\underline{U}_{23} = ?$

$|\underline{U}_2| = 220$



$\underline{U}_{23} = \underline{U}_2 - \underline{U}_3$

$= 220 \angle 0° - 220 \angle -90°$

$= 381 \angle 60°$

X1-P3. $\underline{U}_{12} = 190 + j330 = 380 \angle 60°$

$\underline{U}_{23} = 190 - j330 = 380 \angle -60°$

$\underline{U}_{31} = 380 \angle 180°$

X1-P4. $U_l = 380$

$U_f = U_l$

$I_f = \frac{380}{22} = 17{,}27$ A

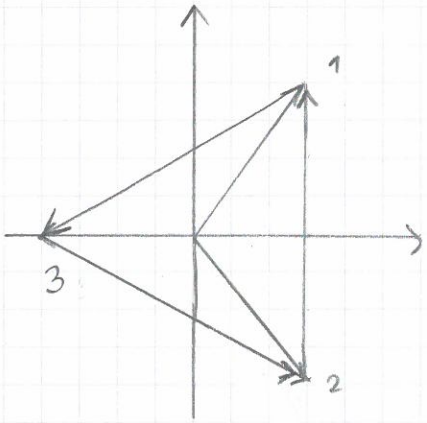
$I_l = \sqrt{3} \cdot I_f = 30$ A

X1-P5. $X_C = 53\,\Omega$

trokut

$I_{23} = ?$ $I_3 = ?$

$U_{12} = 190$ V



$\underline{U}_{23} = 190 \angle -30°$

$\underline{I}_{23} = \frac{\underline{U}_{23}}{-jX_C} = 3{,}58 \angle 60°$

$\underline{U}_{31} = 190° \angle 210°$

$\underline{I}_{31} = \frac{\underline{U}_{31}}{-jX_C} = 3{,}58 \angle -60°$

$\underline{I}_3 + \underline{I}_{23} = \underline{I}_{31}$

$\underline{I}_3 = -6{,}2j$ A

X1-P4.

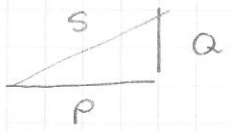
$$U_{OO'} = \frac{\frac{220 \angle -120}{R} + \frac{220 \angle +120°}{R}}{\frac{2}{R}} = -110 \text{ V}$$

X+P11. $I_f = 1A$

$$U_{0'0} = \frac{\frac{U_f}{Z} + \frac{U_f}{Z}}{\frac{2}{Z}} = \frac{U_f}{2} \Rightarrow U_f = 100$$

$$I_f = \frac{U_f}{Z} \Rightarrow Z = 100\Omega$$

$U_l = 380\ V$
$f = 50\ Hz$
$P = 3,5\ kW$
$\cos\varphi = 0,6 \Rightarrow \varphi = 53°$
$\cos\varphi' = 0,8 \quad \varphi' = 36,9°$



$\operatorname{arctg}\varphi = \frac{Q}{P} \Rightarrow Q_1 = 4,66\ kVAr$

$Q_2 = 2,63\ kVAr$

$\Delta Q = 2,03\ kVAr$

$$Q_c = \frac{U_f^2}{X_c} \qquad X_c = \frac{1}{\omega C}$$

$$\frac{1}{\omega C} \cdot Q_c = U_f^2$$

$$Q_c = \omega C U_f^2$$

$$\frac{Q_c}{\omega U_f^2} = C \qquad Q_c = \frac{\Delta Q}{3} =$$

1. trofazno simetrično trošilo - zvijezda

$U_l = 12\ V$

$Z = 100\ \angle 0°\ \Omega$

$U_f = \frac{12}{\sqrt{3}} = 6{,}93\ V$

$I_f = \frac{U_f}{Z} = 0{,}069\ A$

$I_l = I_f = 0{,}069\ A$

$I_0 = 0\ A$

$P = \sqrt{3}\, U_l I_l \cos\varphi = 1{,}43\ W$

2. nesimetrično trošilo u zvijezdi

100 (R)
200 (S)
300 (T)

$U_l = 400\ V \Rightarrow U_f = 231\ V$

$I_R = \frac{U_f}{R} = 2{,}3\ A\ \angle 0°$

$I_S = \frac{U_f\ \angle -120}{200} = 1{,}155\ A\ \angle -120°$

$I_T = \frac{U_f\ \angle -240}{300} = 0{,}77\ A\ \angle -240$

$I_0 = I_R + I_S + I_T$

$= 1{,}37\ \angle -14°\ A$

$P = I_f U_f \cos\varphi = 531.3\ W$

$P_1 = I_{f2} U_f \cos\varphi = 266{,}8\ W$

$P_3 = I_{f3} U_l \cos\varphi = 177{,}87\ W$

$P_{uk} = 977{,}7\ W$

3. $R_1 = 100\ \Omega$
$R_2 = 200\ \Omega$
$R_3 = 300\ \Omega$
$U_l = 400\ V$

$I_1 = \frac{400}{100} = 4\ \angle 0°\ A$

$I_2 = 2\ \angle -120\ A$

$I_3 = 1{,}33\ \angle -240°\ A$

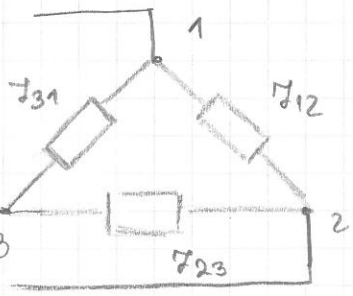
$U_f = 400\ V$

$P_1 = 1600\ W \quad Q_1 = 0$
$P_2 = 800\ W \quad Q_2 = 0$
$P_3 = 532\ W \quad Q_3 = 0$

$P_{uk} = 2932\ W$

$S = 2932\ W$

4. $U_l = 120\ V$
$Z_{12} = Z_{31} = 6 + j8$
$I_{23} = 10\ \Omega$
$U_f = U_l$



$I_{12} = 12\ \angle -53°\ A$

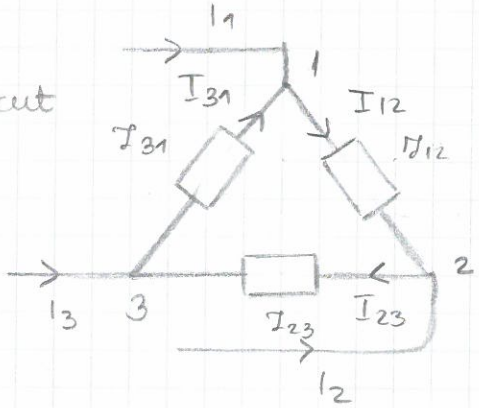
$I_{23} = 12\ \angle -173°\ A$

$I_{31} = 12\ \angle 120°\ A$

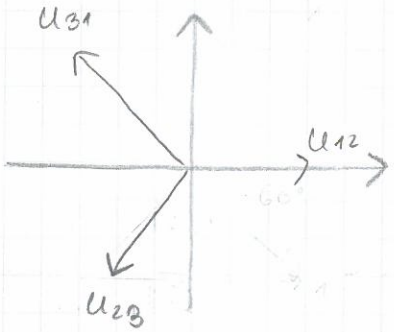
$P_1 = 864 \quad Q_1 = 1152\ VAr$
$P_2 = 864 \quad Q_2 = 1152\ VAr$
$P_3 = 1440 \quad Q_3 = 0$

$P_{uk} = 3168 \quad Q_{uk} = 2304\ VAr$

5. $Z_{12} = 20 \angle 60° \ \Omega$
$Z_{31} = 20 \angle +60° \ \Omega$
$Z_{23} = 10 \angle 0° \ \Omega$
} trokut



$U_{12} = 190 \angle 0° \ V$
$U_{23} = 190 \angle -120° \ V$
$U_{31} = 190 \angle -240 \ V$



$I_{12} = 9{,}5 \angle -60° \ A$

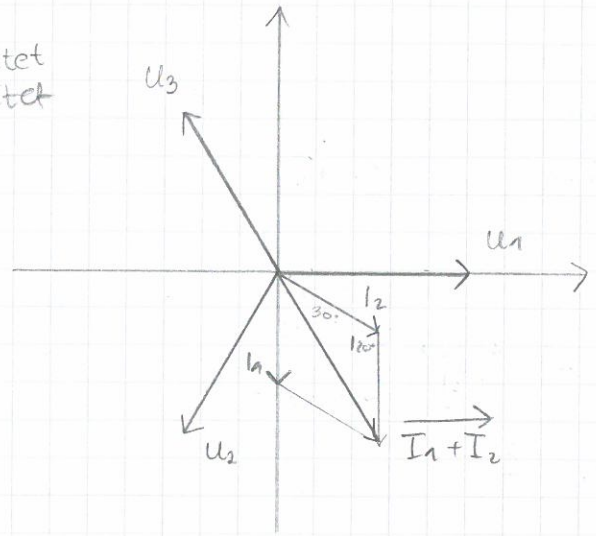
$I_{31} = 9{,}5 \angle 60° \ A$

$I_{23} = 19 \angle -120° \ A$

$I_1 + I_{31} - I_{12} = 0$
$I_1 = I_{12} - I_{31} = 16{,}45$

$I_2 + I_{12} - I_{23} = 0$
$I_2 = I_{23} - I_{12}$

6. $Z_1 = j150 \ \Omega$ induktivitet
$Z_2 = -j150 \ \Omega$ kapacitet
$I_0 = 0$



$(I_1+I_2)^2 = I_1^2 + I_2^2 - 2 I_1 I_2 \cdot \cos 120°$

$= 2I^2 + I^2$
$= 3I^2$
$(I_1+I_2) = \sqrt{3} I \Rightarrow R \ \sqrt{3} \times$ manji

$(I_1+I_2) = \frac{U_3}{R}$ $\quad R = 56{,}6$

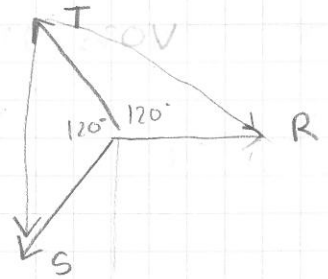
7. $Z_1 = 20 \angle 30° \Omega$
$Z_2 = 40 \angle 0° \Omega$
$Z_3 = 60 \angle -90° \Omega$

$U_f = 230 V$ $\quad U_1 = 230 \angle 0°$
$I_0 = ?$ $\quad U_2 = 230 \angle -120°$
$U_3 = 230 \angle -240°$

$I_1 = 11.5 \angle -30° A$

$I_2 = 5.75 \angle -120° A$ $\quad I_0 = 13,19 \angle -73,4° A$

$I_3 = 3,83 \angle -150° A$

8. 

$U_{RT} = 380 \angle -30°$ $\quad \angle u - \angle i = -30 + 150 = 120°$

$I_S = \frac{380 \angle -120°}{\sqrt{3} \angle 30°} = 44 \angle -150°$

$P = U_{RT} \cdot I_S \cdot \cos 120° = -3360 W$

b) $U_{ST} = 380 \angle -90°$
$I_S = 44 \angle -150$ $\quad \varphi = \angle u - \angle i = 60°$

$P = U_{ST} \cdot I_S \cdot \cos\varphi = 3360 W$

9. $C = 150 \mu F$
$\Delta I = 7,59 A$

između R i S je spojen na linijski napon, a između R i O na fazni napon

$I_1 = \frac{U_l}{X_C} = \frac{\sqrt{3} U_f}{X_C}$

$I_2 = \frac{U_f}{X_C} = \frac{U_l}{X_C}$ $\quad \frac{\sqrt{3} U_f}{X_C} - \frac{U_f}{X_C} = 7,59$

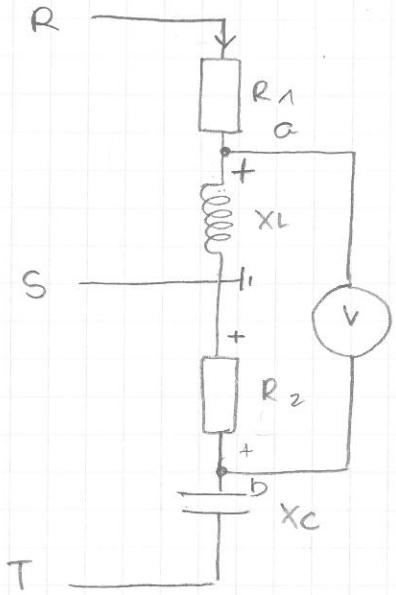
$X_C = \frac{1}{\omega C} = 21,23 \Omega$ $\quad \sqrt{3} U_f - U_f = 7,59 \cdot 21,23$

$U_f = 220 V$

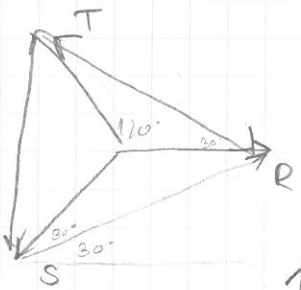
$U_l = 381 V$

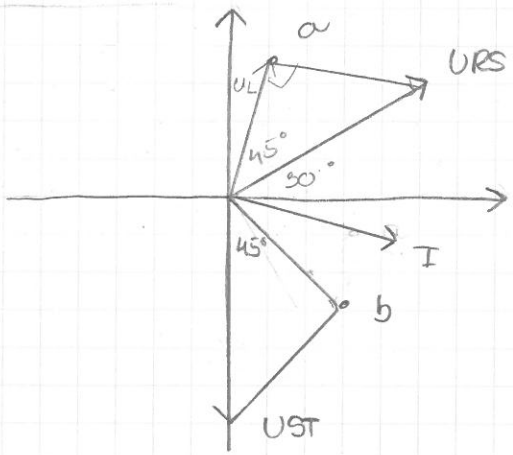
10. $u = 100\ V$

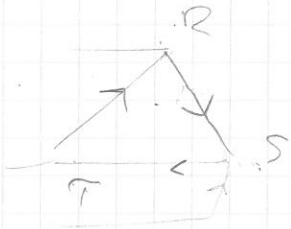
prije zatvaranja sklopke



$U_{RS} = 100 \angle 30°$ $U_{ST} = 100 \angle -90°$







$2U^2 = 100^2$

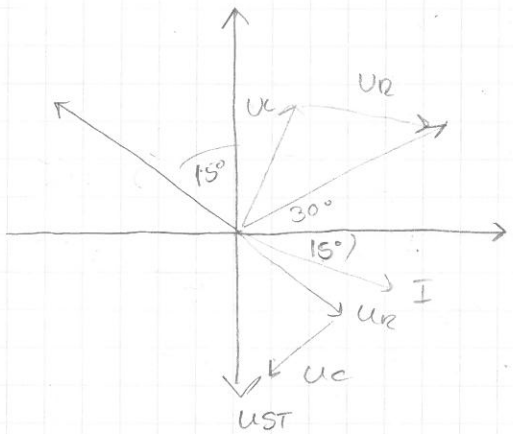
$U = 70,7$

$U_{RS}^2 = U_R^2 + U_l^2$

$100^2 = 2U_R^2$

$U_R = 50\sqrt{2}$

$\frac{U_R}{U_l} = \frac{R_1}{X_L} = \frac{1}{1}$



$U_l = 50\sqrt{2} \angle 75°$ $U_{r2} = 50\sqrt{2} \angle -45°$

$U_r = 50\sqrt{2} \angle -15°$ $U_c = 50\sqrt{2} \angle -135°$

$U_v = U_l + U_{r2} = 70,71 \angle 15°$

$U_{R_2} = 100 \angle 150°$

$P_1 = \frac{50\sqrt{2}^2}{10} \cdot 2 = 1000\ W$

$Q_L = Q_C \Rightarrow Q_{uk} = 0$

$P_2 = \left(\frac{50\sqrt{2}}{10}\right)^2 \cdot 2 + \left(\frac{100}{10}\right)^2$

$= 2000\ W$

$Q_{uk} = 0$

II. $U_u = U_l + U_c = 36,6 \angle 150°$

$P = 1)$

$U_f \cdot I_f \cdot 1 \Rightarrow I_{f1} = 1,73 \angle 0°$

$U_f \cdot I_{f2} \Rightarrow I_{f2} = 3,46 \angle -120°$

$U_f \cdot I_{f3} \Rightarrow I_{f3} = 5,2 \angle -240°$

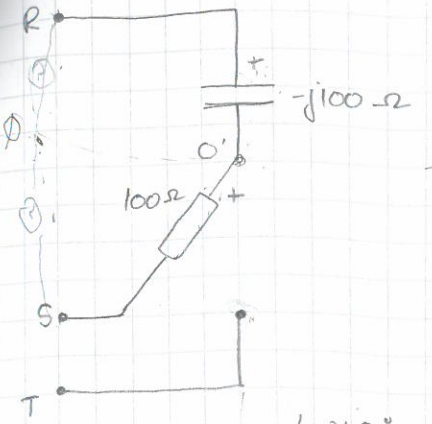
$I_0 = 3,005 \angle 149,9°$

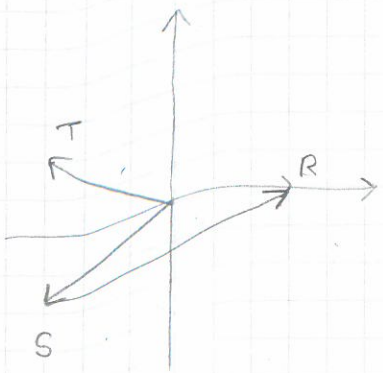
12. $R = X_C = X_L = 10\,\Omega$
$U_f = 100\ V$
$U_l = 173,2\ V$

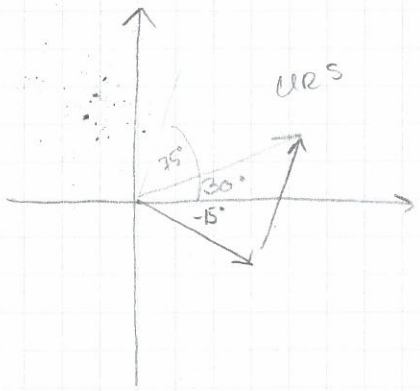
a) trokut
$U_f : U_l = 173,2$
$P = 3000\ W$

b) zvijezda
$U_f = 100$
$P = 1000\ W$

13. $U_{fi} = 220\ V$
$U_l = 381\ V$



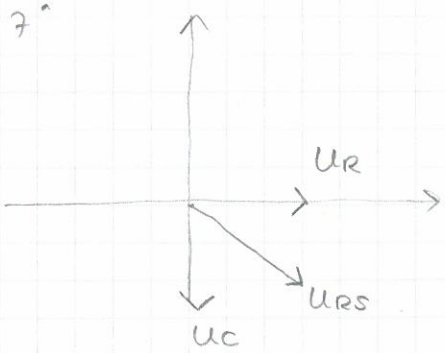




$381^2 = 2U^2$
$U = 269,4$

$P_T = U_{fi} \angle -240° = 220 \angle -240°\ V$

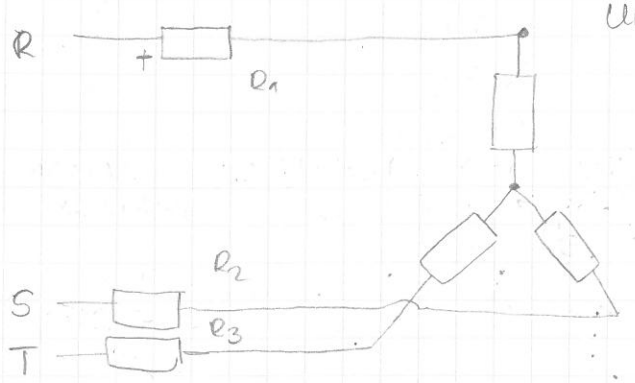
$P_0 = U_R \angle 0° - U_C \angle -90° = 347 \angle 50,7°$



$P_T - P_0 =$

$$U_{OO'} = \frac{\frac{U_{SR}}{-j100\,\Omega} + \frac{U_S}{100\,\Omega}}{\frac{1}{-}}$$

14. $R = 10\,\Omega$



$U_R - U_{R_1} =$

$I =$

PRIJELAZNE POJAVE

Stacionarno stanje - stanje u kojem se uspostave ustaljeni oblici struje i napona

- kod L i C, zbog nemogućnosti trenutačne promjene struje, ne nastupa odmah stacionarno stanje

Prijelazno stanje → nastaje nakon uključenja ili isključenja kruga, nisu uspostavljeni ustaljeni oblici

Prijelazne pojave → vremenske promjene struje i napona tijekom prijelaznog stanja

Vremenska konstanta τ je parametar kruga koji određuje trajanje prijelaznog stanja, ovisi o elementima kruga

P.1. nenabijen

$C = 220\ nF$
$U = 100\ V$
$R = 100\ \Omega$ - serijski spoj

$\tau = RC = 0{,}022$
$63\% = 0{,}11$
$W = \frac{CU^2}{2} = 1{,}1\ J$

P.2. $C = 220 \cdot 10^{-12}\ F$
$U_0 = 100\ V$
$R = 10\ k\Omega$ paralelno

$\tau = RC = 2{,}2 \cdot 10^{-6}$

$37 = 100\, e^{-\frac{t}{\tau}}$
$0{,}37 = e^{-\frac{t}{\tau}}$
$e^{0{,}37} = -\frac{t}{\tau}$

P.3. $L = 10 \cdot 10^{-3}\ H$
$t = 0$
$U = 100\ V$
$R = 100\ \Omega$

$\tau = 1 \cdot 10^{-4}$
$i = \left(\frac{U}{R}\right)\left(1 - e^{-\frac{t}{\tau}}\right) =$

1. $C = 10\,nF$
$W_c = 5\,J$
$U_i = ?$
$W_c = \frac{CU^2}{2} \Rightarrow U = 1000\,V$

2. $C = 100\,nF$
$U = 100\,V$
$R = 10000\,\Omega$
$U_c = 0{,}63\,U$
$U_c = U\,(1 - e^{\frac{-t}{\tau}})$
$0{,}63 = 1 - e^{-t}$
$0{,}37 = \frac{1}{e^t}$
$\tau = RC = 1$
$0{,}37\,e^t = 1$
$e^t = \frac{1}{0{,}37}$ / ln
$t = 1$

3. $R = 1000\,\Omega$
$t = 11\,s$
$\tau = 2{,}2$
$\tau = RC \Rightarrow C = 2{,}2 \cdot 10^{-3}\,F$

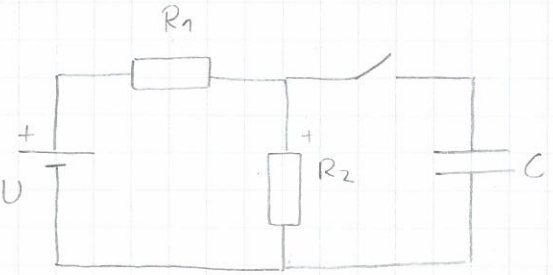
4. $R_1 = 600\,\Omega$
$R_2 = 2{,}4\,k\Omega$
$U = 12\,V$
$C = 5\,nF$
$\tau_2 = 0.012$
$U_c = U_0\,e^{\frac{-t}{\tau}}$
$= 3{,}82\,V$
$\tau = RC = 0.015 \Rightarrow 15\,ms$
$U_c = U\,(1 - e^{\frac{-t}{\tau}})$
$U_c = 12 \cdot (1 - e^{-\frac{15}{30}})$
$U_c = 10{,}376\,V$

5. $U_0 = 20\,V$
$U_1 = 40\,V$
$R = 100\,\Omega$
U_0^+
$U_c = 20\,V$
$U_R = 40\,V$
?

6. $U = 10\,V$
$R_i = 1000\,\Omega$
$t_1 = 20\,ms$
$C_1 = 30\,nF$
$C_2 = 60\,nF$
$C_{uk} = 20 \cdot 10^{-6}$
$\tau = RC = 0{,}02\,s$
$U_{C_2} = 2{,}22\,V$

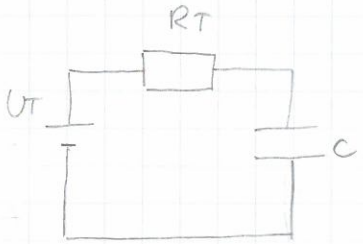
7. $I = \frac{U}{R} = -2{,}5\,A$
$U_c = 0\,V$
$U_R = 5\,V$

8. $R_1 = 4k\,\Omega$
$R_2 = 12k\,\Omega$
$C = 25 \cdot 10^{-9}\,F$
$U = 120\,V$



$R_T = 3k\,\Omega$
$U_T = 90$

$U_c = U(1 - e^{\frac{-t}{\tau}}) = 77{,}82\,V$
$Q = CU = 1{,}945\,\mu C$



$\tau = RC = 7{,}5 \cdot 10^{-5}$
$t = 150 \cdot 10^{-6}$

$I_{R_2} = \frac{U_c}{R_2} = 6{,}485 \cdot 10^{-3}$

$U_{R_1} = 42{,}18\,V \Rightarrow I_{R_1} = 0{,}01$

$i_C = 0{,}0$

9. $R = 2\,\Omega$
$L = 0{,}01\,H$
$U = 10\,V$
$t_1 = 5\,ms$

$\tau = \frac{L}{R} = 5 \cdot 10^{-3}$

$u_L = U e^{-\frac{t}{\tau}} = 3{,}68 \Rightarrow U_R = 6{,}32\,V$

11. $U = 10\,V$
$t = 0{,}2\,s$
$R = 5\,\Omega$
$L = 1\,H$

$u_L = 3{,}68\,V$
$U_R = 6{,}32\,V$
$i_L = \left(\frac{10}{5}\right)\left(1 - e^{-1}\right) = 1{,}264$

$\tau = 0{,}2\,s$

$P = UI = 12{,}64\,W$

12. $U = 12\,V$
$R_1 = 20\,\Omega$
$R_2 = 20\,k\Omega$

$R_T = 19.98$
$I = 5{,}99 \cdot 10^{-4}$
$U_T = 11{,}988$

$i_L = 0{,}6\,A$
$U_{R_2} = 11\,991\,V$

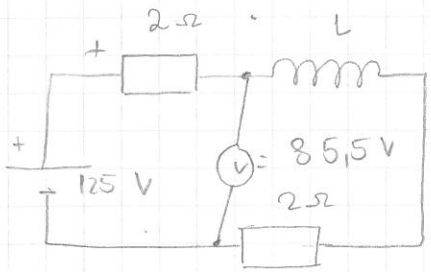
13. $U = 100\,V$
$R_i = R = 10\,\Omega$
$t_1 = 40 \cdot 10^{-3}\,s$
$U_L = 60\,V$
$i_L =$
$P = \frac{W}{t}$

$W = P \cdot 5\tau$

$60 = U \cdot e^{-\frac{t}{\tau}}$
$0{,}6 = e^{-\frac{t}{\tau}}$ /ln
$\ln 0{,}6\,\tau = 0{,}0783$

$P = U^2\left(1 - e^{-\frac{t}{\tau}}\right)e^{-\frac{t}{\tau}} / R = 3{,}346$

14. $U = 125\,V$
$R_i = 2\,\Omega$
$R_S = 2\,\Omega$
$t_1 = 40\,ms$
$U_S = 85{,}5\,V$



$U_{R_i} = 39{,}5\,V$
$U_{R_S} = 39{,}5\,V$

$u_L = 46 = U \cdot e^{-\frac{t}{\tau}}$
$46 = 125 \cdot e^{-\frac{40 \cdot 10^{-3}}{\tau}}$
$0.368 = e^{\cdots}$ /ln
$\ln 0{,}368 = \frac{-40 \cdot 10^{-3}}{\tau}$

$\tau = \frac{L}{R}$

$L = 0.16\,H$

$\tau = 0.04\,H$

$R_1 = 103\,\Omega$
$R_2 = 104\,\Omega$
$R_3 = 13\,\Omega$
$U_f = 212\,V$

$U_R = U_f \angle 0°\ V = 212 \angle 0°$
$U_S = U_f \angle -120°\ V = 212 \angle -120°$
$U_T = U_f \angle -240°\ V = 212 \angle -240°$

$$U_{oo'} = \frac{\frac{U_R}{R_1} + \frac{U_S}{R_2} + \frac{U_T}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = 148.153356 \angle 119.93\ V$$

$U_{RI} = U_R - U_{oo'} = 212 \angle 0° - 148.153356 \angle 119.93 = 313.427 \angle -24,18$
$U_{SI} = U_S - U_{oo'} = 313.6373 \angle -95,87°$
$U_{TI} = U_T - U_{oo'} = 63,847 \angle 120,162°$

$I_R = 3,04298 \angle -24,18$
$I_S = 3,01574 \angle -95,87$
$I_T = 4,9113 \angle 120.162°$

$P_{uk} = 953,7519 + 945,3475 + 313,57128 = 2213,17$
2,21 nakon

$I_R = 2,0582 \angle 0°$
$I_S = 2,03846 \angle -120°$
$I_T = 16,30769 \angle 120°$

$P_{uk} = 436,327 + 432,1532 + 3457,2297 = 4325,7099$
4,33 prije

prije

NESINUSOIDNI VALNI OBLICI

IZMJENIČNE EL. VELIČINE - periodički promjenjiv naponom ili strujom u vremenu jedne periode promjeni smjer (poprimi pozitivne i negativne vrijednosti)

jednaki poz. i neg. dijelovi - čista izmjenična

karakterizacija
- -> maksimalna ili tjemena vrijednost
- -> srednja vrijednost (istosmjerna komponenta)
- -> vrijednost od vrha do dna
- -> efektivna vrijednost
- -> omjerni faktor

SREDNJA VRIJEDNOST

$$I_{SR} = \frac{1}{T}\int_0^T i(t)dt$$ matematička srednja vrijednost u intervalu T

VRIJEDNOST OD VRHA DO DNA Y_{PP}

-> razlika max i min vrijednosti $Y_{PP} = Y_{m1} - Y_{m2}$
-> ne mijenja se s promjenom srednje vrijednosti

EFEKTIVNA VRIJEDNOST

$$I_{ef} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

OMJERNI FAKTOR

Tjemeni faktor $\sigma = \frac{I_m}{I_{ef}}$ -> izobličenje u odnosu na sinusni oblik

Faktor oblika $\xi = \frac{I_{ef}}{I_{sr}}$ -> odredivanje efektivnih vrijednosti kod instrumenata sa srednjim otklonom

OSNOVNI VALNI OBLICI

- sinusni
- pilasti
- vremenski nepromjenjivi (konstantni)

PARAMETRI SINUSNOG VALNOG OBLIKA

$U_{sr} = 0$

$U_{ef} = \frac{U_m}{\sqrt{2}}$ $\quad \sigma = \sqrt{2}$

PARAMETRI PILASTOG VALNOG OBLIKA

$U_{sr} = \frac{U_m}{2}$ izmjenična verzija $U_{sr} = 0$

$\xi = \frac{2}{\sqrt{3}}$

$U_{ef} = \frac{U_m}{\sqrt{3}}$

$\sigma = \sqrt{3}$

PARAMETRI KONST. VRIJEDNOG OBLIKA

$U = U_{sr} = U_{ef}$

Poluvalno ispravljanje sinusoide

$T = \frac{2\pi}{\omega}$ $\quad I_{sr} = \frac{I_m}{\pi}$ $\quad I_{ef} = \frac{I_m}{2}$

Punovalno ispravljanje sinusoide

$I_{sr} = \frac{2I_m}{\pi}$ $\quad I = \frac{I_m}{\sqrt{2}}$

POSEBNI SLUČAJEVI VALNIH OBLIKA

- periodički niz impulsa
- složeni valni oblik → zbroj više valnih oblika

↓

$Y_{ef} = \sqrt{Y_{1ef}^2 + Y_{2ef}^2 \dots}$

PERIODIČKI NIZ IMPULSA

- period ponavljanja T
- vrijeme trajanja τ

$Y_{ef} = Y_{ef\ osnovno} \cdot \sqrt{\frac{\tau}{T}}$

$Y_{sr} = Y_{sr\ osnovno} \cdot \frac{\tau}{T}$

1. $T = 2$
$Um = 50\,V$

$u(t) = \frac{Um}{T} \cdot t$ $\qquad$ $Usr = \frac{Um}{2} = 25\ V$

$Uef = \frac{Um}{\sqrt{3}} = 28{,}868\ V$

2. $Ti = 10\,ms$
$T = 20\,ms$
$Um = 5$

$f = \frac{1}{20 \cdot 10^{-3}} = 50\,Hz$

$Usr = Um \cdot \frac{Ti}{T} = 2{,}5\ V$

$Uef = Um \cdot \sqrt{\frac{Ti}{T}} = 3{,}536\ V$

3. $R = 100\,\Omega$
$Um = 1$ $\qquad$ $Ief = \frac{Im}{2} = \frac{1}{2}$

$P = Ief^2 \cdot R = 25\,W$

2.1. $Upp = 8\,V$ $\qquad$ $Ti = T/2$
$Usr = 0$

$Uef = \frac{4}{\sqrt{3}} \sqrt{\frac{1}{2}}$ $\qquad$ $Ief = \frac{4}{\sqrt{3}}$

2,2. $P = 400\,W$
$R = 25$

$Ief = 4\,A$ $\qquad$ $Ief = \frac{I}{\sqrt{2}} \Rightarrow I = 5{,}66\,A$

$Ief = \frac{I}{\sqrt{3}} \Rightarrow 6{,}93\,A = I$

$\frac{I}{I} = 6{,}92$

$Ief = \frac{I}{2} \Rightarrow I = 8$

3. $3{,}536 = 5 \cdot \sqrt{\frac{10}{x}} \Rightarrow x = 20 \cdot 10^{-3} \Rightarrow f = 50\,Hz$

ZAVRŠNI ISPIT 2016.

1. $\cos\varphi = 0,707$
$R = 22\,\Omega$
$\cos\varphi' = 0,866$

$P = \frac{U^2}{R} = \frac{220^2}{22} = 2200\,W$

$\cos\varphi = \frac{P}{S}$ $\quad \varphi' = 30°$ $\quad tg\,\varphi = \frac{Q}{P}$

$0,707 S = P$
$S = 3111.74$

$Q' = 1270,32$

$3111.74^2 = 2200^2 + Q^2$
$Q = 2200.66$

$\Delta Q = Q_c = 930.34 = \frac{U^2}{X}$

$X_c = 52$

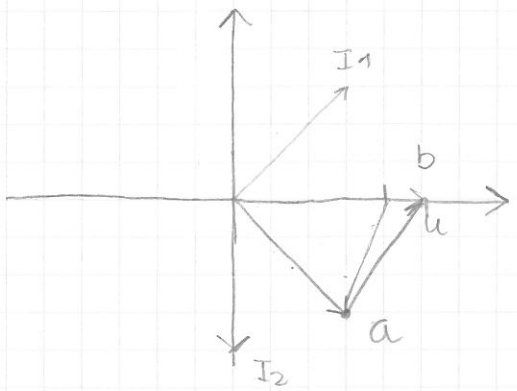
$52 = \frac{1}{2\pi \cdot 50 \cdot C}$

$C = \frac{1}{100\pi \cdot 52} = 6,12 \cdot 10^{-5}\,F$

2. $U_{ab} = 6,403\,V$ $\quad R = X_c = X_L = 10\,\Omega$

$I_1 = R - jX_c = 10 - 10j$ kapacitivna smjja

$I_2 = 10j$ čisto induktivna smjja



$\frac{I_1}{I_2} = \frac{I_2}{I_1}$

$\frac{I_1}{I_2} = \frac{10j}{10 - 10j}$

$\frac{I_1}{I_2} = \frac{\sqrt{2}}{2} \angle 135°$

$I_1 = 0,71 \angle 173,659$

$U_c - U_l =$

$I_1 \cdot -jX_c - I_2 \cdot 0.9j X_L = 6,403$

$\frac{\sqrt{2}}{2} \angle 135° \cdot I_2 \cdot 10 \angle -90 - I_2 \cdot 0.9 \cdot 10 \angle 90° = 6,403$

$I_2 \cdot (5 - 4i) = 6,403$

$I_2 = 1 \angle 38,66°$

$I_{uk} = I_1 + I_2 = 0,71 \angle 83,66°$

3. $\dot{U} = 8 \angle 0° V$
$R = 1\ \Omega$

$$P_1 \cdot \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right) - P_2 \cdot \frac{1}{R} - P_3 \cdot \frac{1}{R} = -\frac{\dot{U}}{R}$$

$$P_1 \cdot 3 - P_2 - P_3 = -8 \angle 0° \cdot$$

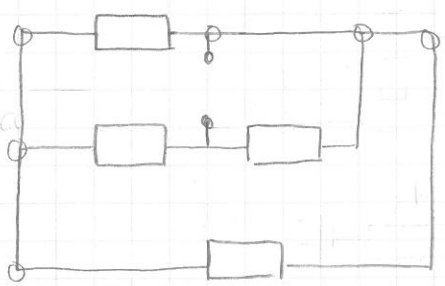
$$P_2 \cdot 3 - P_3 - P_1 = 8 \angle 0°$$

$$\underline{P_3 = \dot{U} = 8 \angle 0°}$$

$$P_1 \cdot 3 - P_2 - 8 \angle 0° = -8 \angle 0°$$
$$3P_1 - P_2 = 0$$
$$P_2 = 3P_1$$

$$P_2 \cdot 3 - 8 \angle 0° - P_1 = 8 \angle 0°$$
$$8P_1 = 16$$
$$P_1 = 2\ V$$

4. $R = 10\ \Omega$
$U = 25\ V$



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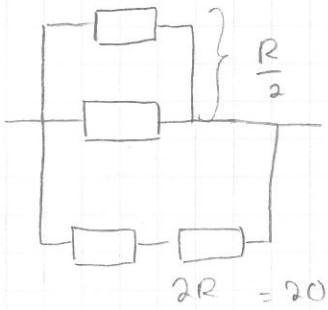


$RT = 4\ \Omega$
$UT = -\frac{50}{3}$

5.

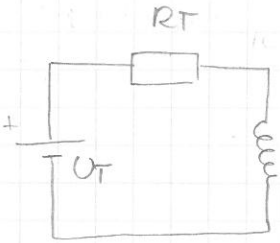
a —

7. $$U_{o'o} = \frac{2\sqrt{2}}{\frac{1}{100} + \frac{1}{100} + \frac{1}{-100j}} = 126,5 \angle -26,56°$$

8. $E = 20\,V$
$R_1 = 10\,\Omega$
$R_2 = 5\,\Omega$
$L = 0,01\,H$

$$\tau = \frac{L}{R} = \frac{0,01}{10} = 1 \cdot 10^{-3}\,s = 1\,ms$$

$U_T = 10\,V$
$R_T = 10\,\Omega$



$$i = 1 \cdot (1 - e^{-1}) = 0,632$$

9. $$I_1 = I_m \cdot \sqrt{\frac{T_1}{T}}$$
$$I_2 = \frac{I_m}{\sqrt{3}} \cdot \sqrt{\frac{T - T_1}{T}}$$

$$I_m^2 \cdot \frac{T_1}{T} + \frac{I_m^2}{3} \cdot \frac{T - T_1}{T} = \frac{I_m^2}{2} \quad / \cdot 6T$$

$$6T_1 I_m^2 + 2 I_m^2 (T - T_1) = 3T I_m^2$$

$$6T_1 + 2(T - T_1) = 3T$$

$$6T_1 + 2T - 2T_1 = 3T$$

$$4T_1 = T$$

$$T_1 = \frac{T}{4}$$

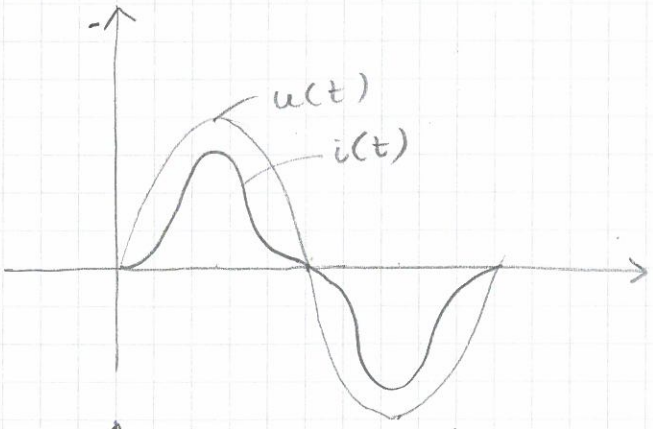
10. $U_{ef} = 65,31$

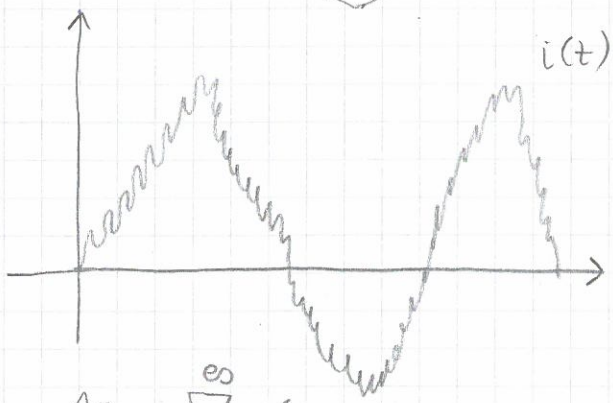
$$Z = \frac{17}{3} \angle 28,1°$$

$I =$

HARMONIČKI SLOŽENI VALNI OBLICI

- transformator (nelinearan zbog željezne jezgre)

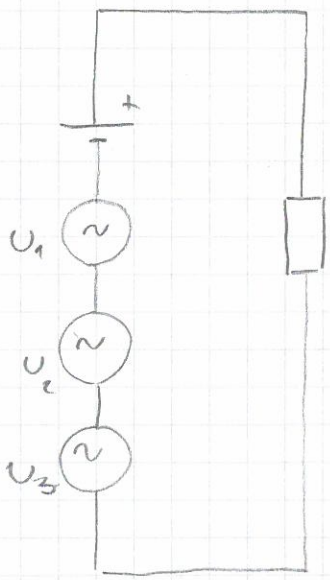




i(t) - električna vuča

$$y(t) = \frac{A_0}{2} + \sum_{i=1}^{\infty} (A_n \cos(n\omega t) + B_n \sin(n\omega t))$$

↳ bilo koji signal koji je periodički - FOURIEOVA ANALIZA



Harmonička analiza → superpozicija

(U_0, U_1, U_2, f_1, f_2 zadani)

X_L, X_C - funkcije frekvencije

↳ NEMOJ ZABORAVIT PROMIJENIT

1. Nesinusoidni napon efektivne vrijednosti 200 V može se prikazati u obliku ... Ako je

$$u(t) = U_0 + U_{m1}\sin(\omega t) + U_{m3}\sin(3\omega t)$$

$U_{m1} = 0{,}8\,U_0$

$U_{m3} = 0{,}5\,U_0$

$$U = \sqrt{U_0^2 + \left(\frac{U_{m1}}{\sqrt{2}}\right)^2 + \left(\frac{U_{m3}}{\sqrt{2}}\right)^2}$$

$$U = U_0 \cdot \sqrt{\left(\frac{0{,}8}{\sqrt{2}}\right)^2 + \left(\frac{0{,}5}{\sqrt{2}}\right)^2}$$

$U_0 = 166{,}38$ V

Za mrežu prema slici odredite

a) Ulazna ef. –

b) Ef. vrijednosti I_R, I_C, I_L

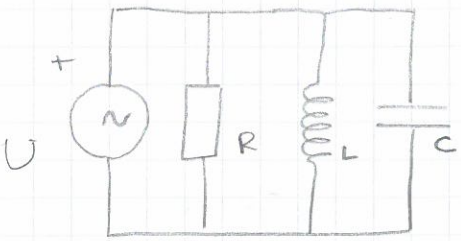
$\omega = 1000\ s^{-1}$

$R = 100\ \Omega$

$L = 0{,}1$ H

$C = 10$ nF

$$u(t) = 100\sin\left(\omega t + \frac{\pi}{6}\right) + 30\sin(3\omega t) + 10\sin\left(5\omega t - \frac{3\pi}{4}\right)$$



$$U_{ef} = \sqrt{\left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{30}{\sqrt{2}}\right)^2 + \left(\frac{10}{\sqrt{2}}\right)^2}$$

$$I_R = \sqrt{I_{R_1}^2 + I_{R_3}^2 + I_{R_5}^2}$$

$$= \sqrt{\left(\frac{100}{\sqrt{2}\cdot 100}\right)^2 + \left(\frac{30}{\sqrt{2}\,100}\right)^2 + \left(\frac{10}{\sqrt{2}\,100}\right)^2}$$

$$= 0{,}741$$

$X_L = m\omega L$

$$X_C = \frac{1}{m\omega C}$$

$$I_L = \sqrt{I_{L_1}^2 + I_{L_3}^2 + I_{L_5}^2}$$

$$= \sqrt{\left(\frac{100}{\sqrt{2}\cdot 1000\cdot 0{,}1}\right)^2 + \left(\frac{30}{\sqrt{2}\cdot 1000\cdot 3\cdot 0{,}1}\right)^2 + \left(\frac{10}{\sqrt{2}\cdot 5\cdot 1000\cdot 0{,}1}\right)^2} = 0{,}711\ \text{A}$$

$$I_C = \sqrt{I_{C_1}^2 + I_{C_3}^2 + I_{C_5}^2}$$

$$= \sqrt{\left(\frac{100\cdot 1000\cdot 10\cdot 10^{-6}}{\sqrt{2}}\right)^2 + \left(\frac{30\cdot 3\cdot 1000\cdot 10\cdot 10^{-6}}{\sqrt{2}}\right)^2 + \left(\frac{10\cdot 5\cdot 1000\cdot 10\cdot 10}{\sqrt{2}}\right)}$$

$= 1{,}015$ A

PROVJERI REZONANCIJE !

1. $R = 1\,\Omega$

$i(t) = 1 + 2\sin \omega t$

$P(t) = i^2 \cdot R = \left(1 + 4\sin \omega t + 4\sin^2 \omega t\right) \cdot R =$

$\left(1 + 4\sin \omega t - \frac{1 - \cos 2\omega t}{2}\right) \cdot R$

$i = \sqrt{1^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = \sqrt{3}$

$P = i^2 \cdot R = 3\ W$

$P = \left(\frac{2}{\sqrt{2}}\right)^2 \cdot 1 = 2\ W$

4. $u(t) = 4\sin(\omega t) + \sin 2\omega t$

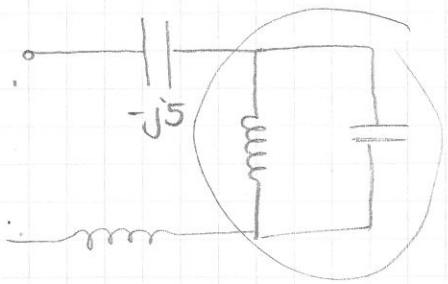
$\omega_1 \quad U_{ef1} = \frac{4}{\sqrt{2}}$

$Z_p = -j10 + j10 + \frac{j5 \cdot (-j20)}{j5 - j20} = j\frac{20}{3}$

$I_{ef1} = \left|\frac{U_{ef1}}{Z_p}\right| = \left|\frac{\frac{4}{\sqrt{2}}}{j\frac{20}{3}}\right| = \left|-j\frac{3}{5\sqrt{2}}\right| = \frac{3}{5\sqrt{2}}$

$\omega_2 \cdot U_{ef_2} = \frac{1}{\sqrt{2}}$

↳ paralelna rezonancija



5. $u(t) = 50 + 50\sin(5\cdot10^3 t) + 30\sin(10^4 t) + 20\sin(2\cdot10^4 t)$

$i(t) = 11{,}2\sin(5\cdot10^3 t + 63{,}4) + 10{,}6\sin(10^4 t + 45°) + 8{,}97\sin(2\cdot10^4 t + 26{,}6°)\ A$

$\omega_0 = 0 \quad U = 50, \quad I = 0, \quad P = 0$

$\omega = 5\cdot10^{-3} \quad U_{ef} = \frac{50}{\sqrt{2}}, \quad I_{ef} = \frac{11{,}2}{\sqrt{2}}, \quad \varphi = 63{,}4°, \quad P = U_{ef} \cdot I_{ef} \cos\varphi = 125{,}37\ W$

$\omega_1 = 10^4 \quad U_{ef} = \frac{30}{\sqrt{2}}, \quad I_{ef} = \frac{10{,}6}{\sqrt{2}}, \quad \varphi = 45° \quad P = U_{ef} I_{ef} \cos\varphi = 112{,}43\ W$

$\omega_2 = 2\cdot10^4 \quad U_{ef} = \frac{20}{\sqrt{2}}, \quad I_{ef} = \frac{8{,}97}{\sqrt{2}}, \quad \varphi = 26{,}6°, \quad P = U_{ef} \cdot I_{ef} \cdot \cos\varphi = 80{,}2\ W$

$P_{uk} = P_1 + P_2 + P_3 = 318\ W$

6. $u(t) = 100 + 141 \sin \omega t + 14{,}1 \sin 3\omega t$

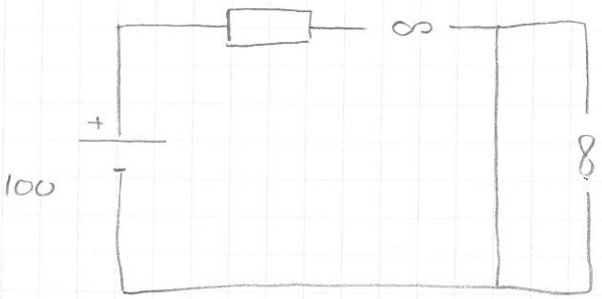
$R = 2{,}5\ \Omega$

$\frac{1}{\omega C_1} = 1{,}125\ \Omega$ } ... pri osnovnoj f

$\omega L = 9\ \Omega$

istosmjerna

ω_0

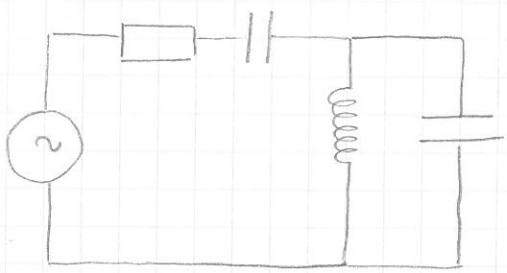


$I_0 = 0$

$U_{RC} = U_0 = 100\ V$

$U_{LC} = 0$

ω_1



$Z = R - jX_{C1} + \frac{jX_L(-jX_C)}{jX_L - jX_{C2}}$

$= 2{,}5\ \Omega$

$I_1 = \frac{100}{2{,}5} = 40\angle 0°\ A$

$U_{RC1} = I_1 \cdot (R - jX_{C1}) = 40\angle 0° \cdot (2{,}5 - j1{,}125) = 100 - j45$

$U_{LC_1} = I_1 \cdot (j125) = j45$

$\omega_3 = 3\omega$

$X_L = 3\ \Omega$ $\Rightarrow$ paralelna rez.
$X_C = 3\ \Omega$

$I_3 = 0\ A$
$U_{RC_3} = 0\ V$
$U_{LC_3} = 10\ V$

$I_{wk} = 40\angle 0°\ A$

$U_{RC} = \sqrt{(U_{RC_0})^2 + (U_{RC_1})^2} = \sqrt{100^2 + 100^2 + 45^2} = 148{,}4\ V$

$U_{LC} = \sqrt{U_{LC_1}^2 + U_{LC_3}^2} = \sqrt{45^2 + 10^2} = 46{,}09\ V$

7 $I_{C1} = 2A$
$I_{L6} = 0,1A$

za ω) $R = X_L = X_C = 5\Omega$
$I_{C1} = 2 \angle 0° \; A$

$U_p = I_{C1} \cdot \sqrt{R^2 + X_C^2} = 2 \cdot \sqrt{50} = 10\sqrt{2}$

$U_p = 10\sqrt{2} \angle -45°$

$I_{L1} = \frac{U_p}{jX_L} = \frac{10\sqrt{2} \angle -45°}{5 \angle 90°} = 2\sqrt{2} \angle -135° \; A$

$\dot{I}_{i1} = \dot{I}_{C1} + \dot{I}_{L1} = -j2$

$U_{RXL1} = I_{i1} \cdot (R + jX_L) = -j2(5 + j5) = 10\sqrt{2} \angle -45° \; V$

$\dot{U}_i = U_p + U_{RXL} = 10\sqrt{2} \angle -45° + 10\sqrt{2} \angle -45° = 20\sqrt{2} \angle -45°$

za 5ω)

$I_{L5} = 0,1A$
$R = 5\Omega$
$X_L = 25\Omega$
$X_C = 1\Omega$

$\dot{U}_{p5} = I_{L5} \cdot jX_L = 0.1 \angle 0° \cdot 25 \angle 90° = 2,5 \angle 90°$

$I_{C5} = \frac{U_{p5}}{R - jX_{C5}} = \frac{2.5 \angle 90°}{5 - j} = \frac{-2,5 + j12.5}{26}$

$I_{i5} = I_{L5} + I_{C5} = 0.1 - \frac{2.5 + j12.15}{26} = \frac{0.1}{26} + j\frac{12.5}{26}$

$\dot{U}_{RXL} = I_{i5} \cdot (R + jX_L) = \frac{-312 + j65}{26}$

$\dot{U}_i = \dot{U}_{RXL} + U_{P5} = 2,5 \angle 90° + \frac{-312 + j65}{26} = -12 + j5$

$U_{ef}^2 = 800 + 12^2 + 5^2 \Rightarrow U_{ef} = 31.13$

1. $f = 50\ Hz$
$Z_1 = 2 + j4\ \Omega$
$Z_2 = 4 - j2\ \Omega$
$P_1 = 18\ W$
$\cos\varphi = 1$

$I_1 = \sqrt{\frac{P_1}{R_1}} = 3A \Rightarrow Q_1 = I^2 \cdot X_1 = 36\ VAr$

$U = |I_1||Z_1| = 3 \cdot \sqrt{2^2+4^2} = 3\sqrt{20}$

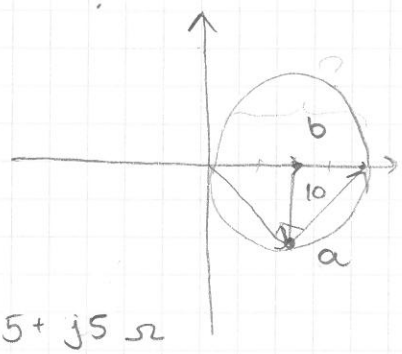
$I_2 = \frac{|U|}{|Z_2|} = \frac{3\sqrt{20}}{\sqrt{4^2+2^2}} = 3\ A$

$Q_2 = I^2 \cdot (-2) = -18\ VAr$

$Q_{uk} = 18\ VAr = \quad Q_c = \frac{U^2}{X_c} = U^2 \omega C$

$C = \frac{Q_c}{U^2 \omega} = 318{,}13\ \mu F$

2. $X_L = X_C = R = 10\ \Omega$
$U_w = 10\ V$



3. $Z = 5 + j5\ \Omega$
$P_A = 10 \angle 0°$

$U_T = P_A - P_B = 10\sqrt{2} \angle -45°$

$\frac{10\sqrt{2}\angle 45°}{10 + j10} \cdot j10$

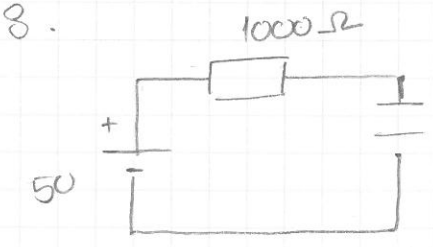
$R = \sqrt{5^2+5^2} = 5\sqrt{2}$

$P = \left(\frac{10\sqrt{2}}{|5 + 5\sqrt{2} + 5j|}\right)^2 5\sqrt{2} = 8{,}28\ W$

6. $U_l = 380\ V$
$U_f = 220 \angle 0°$

$I = \frac{220\angle 0°}{5} + \frac{220 \cdot \angle 120°}{5 - j5} + \frac{220\angle -240°}{5 + j5} = 60{,}1 \angle 0°$

8.



$\tau = RC = 2 \cdot 10^{-5}$

$U_c = 50 \cdot \left(1 - e^{-\frac{5 \cdot 10^{-6}}{2 \cdot 10^{-5}}}\right)$

$= 45{,}89\ V$

istosmjerna

$I_{ef} = 0$ → serijski spojen kapacitet

ω

$Z = 10 - j10 + (j2.11 - j8) = 10 - j\frac{22}{3}$

$I_{ef\omega} = \frac{\sqrt{2}}{2} : |Z| = \frac{1}{2\sqrt{\frac{34}{3}}} = 80,6\ mA$

$\omega \Rightarrow 2\omega$

u paraleli $j2 \rightarrow j4$, $-j8 \rightarrow -j4$ } paralelna rezonancija

$I_{ef} = 0$

5. $u(t) = U_0 + U_{m_1} \sin(\omega t - \pi/6) - U_{m_2} \sin(3\omega t - \pi/4)$

istosmjerna komponenta

↳ nema doprinosa, kondenzator u seriji $\left(\frac{1}{\omega C} - \frac{1}{0} = \infty\right)$

$\omega \Rightarrow \omega_1$

$I = \frac{U_{m_1}/\sqrt{2}}{R + jX_L - jX_C} = \frac{20}{6 - j8} = 2\ A$

$\omega \Rightarrow \omega_3$

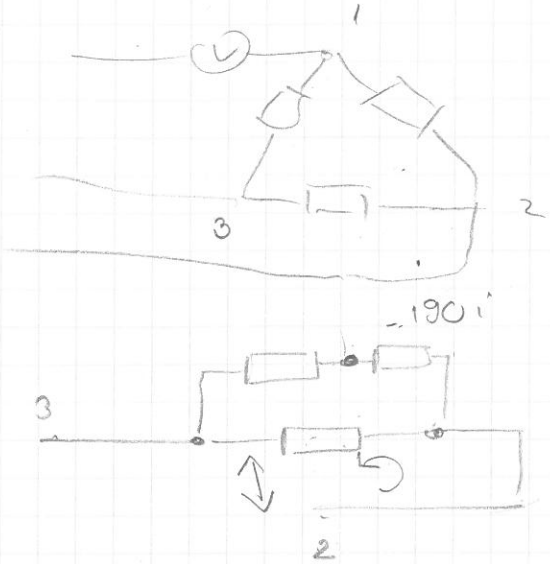
$X_L = 3\ \Omega$, $X_C = 3\ \Omega$ } serijska rezonancija

$I = \frac{U}{R} = \frac{12}{6} =$

$Z = R + jX_\ell - jX_C = 6\ \Omega = 2\ A$

$I_{ef} = \sqrt{I_{ef}^2 + I_{ef}^2} = 2,8\ A$

$U_L = 380$



$$U_2 - U_3 = \frac{380}{\sqrt{3}} \angle -120° - \frac{380}{\sqrt{3}} \angle -240° = -380\,i$$

$$P = \frac{380}{\sqrt{3}} \angle -120° + 190i = -109{,}7$$

$U_w = 330\ V$

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2. $u(t) = 10 + 50 \sin \omega t + 25 \sin 3\omega t$; $\omega = 314$ $R = 5$, $L = 0{,}02\,H$

istosmjerna

$I = \frac{10}{5} = 2\,A$

$\omega \Rightarrow \omega$

$X_L = 6{,}28j$

$R = 5\,\Omega$

$\underline{I} = \frac{50}{\sqrt{20}} : (6{,}28j + 5) = 4{,}4 \angle -51{,}5°$

$\omega \Rightarrow 3\omega$

$X_L = 18{,}84j$

$R = 5\,\Omega$

$\underline{I} = \frac{25}{\sqrt{2}} : (5 + 18{,}84j) = 0.91 \angle -75{,}14°$

$I_{ef} = \sqrt{I_{ef_1}^2 + I_{ef_2}^2 + I_{ef_3}} =$

$I_{ef} = 4{,}92$

$P = U_0 I_0 + U_{ef_1} I_{ef_1} \cos(+51{,}5°) + U_{ef_2} I_{ef_2} (\cos(+75{,}14°)$

$P = 20 + 88{,}11 + 4{,}13 = 112{,}24\,W$

4. $u(t) = 30 \sin \omega t + 15\sqrt{2} \sin 2\omega t$

$R = 3\,\Omega$

$L = 1 \cdot 10^{-3}\,H$

$C = 0{,}25 \cdot 10^{-3}\,F$

$\omega = 1000$

$R = 3\Omega$

$X_L = 1j$

$X_C = -4j$

$\underline{I}_{ef} = \frac{U_{ef}}{\sqrt{2}} : (3 - 3j) = 5 \angle 45°$

$\omega = 2000$

$R = 3\,\Omega$

$X_L = 2j\,\Omega$

$X_C = -2j\,\Omega$

} serijska rez

$I = \frac{U}{R} = \frac{15}{3} = 5\,A$

$P = I_{ef}^2 \cdot R = (5^2 \cdot 5^2) \cdot R = 150\,W$

1. $R = 1\ \Omega$
$i(t) = 1 + 2\sin\omega t$
$P = ?$
$I_{ef} = \sqrt{1 + \left(\frac{2}{\sqrt{2}}\right)^2} = \sqrt{3}$
$P = I^2 R = 3\ W$
$P = 2\ W$

2. $u(t) = 1 + U_{1m}\sin\omega t$
$U_{ef} = 1{,}73\ V$
$U_{ef} = \sqrt{1^2 + \left(\frac{U_{1m}}{\sqrt{2}}\right)^2}$
$3 = 1^2 + \frac{U_{1m}^2}{2}$
$4 = U_{1m}^2$
$U_{1m} = 2$

3. $u(t) = 10\sin 100t + \sin\left(300t + \frac{\pi}{2}\right)\ V$

4. $u(t) = 4\sin\omega t + \sin 2\omega t$
ω
$I = \frac{4}{\sqrt{2}} : Z_{uk} = 1{,}49\ \angle -4{,}79°$
$Z_1 = 10 + \frac{50}{3}i$
$Z_{uk} = 1{,}89\ \angle 4{,}79°$
$I = 1{,}49\ \angle -4{,}79° \cdot \frac{2}{10 + \frac{50}{3}i} = 0.15\ \angle -63{,}8°$
$\omega \rightarrow 2\omega$
paralelna rez.

5. $u(t) = 50 + 50\sin(5\cdot10^3 t) + 30\sin(10^4 t) + 20\sin(2\cdot10^4)$
$i(t) = 11{,}2\sin(5\,10^3 t + 63{,}4°) + 10{,}6\sin(10^4 t + 45°) + 8{,}97\sin(2\cdot10^4 t + 26{,}6°)$

$P_1 = 125{,}37$
$P_2 = 112{,}43$
$P_3 = 80{,}21$
$P_{uk} = 318\ W$

6. $u(t) = 100 + 141 \sin \omega t + 14,1 \sin 3\omega t$

$R = 2,5\ \Omega$

$\frac{1}{\omega C_1} = 1,125$

$\omega L = 1\ \Omega$

$\frac{1}{\omega C_2} = 9\ \Omega$

istosmjerna → prekid zbog kondenzatora $U_{RC} = 100\ V$

$I = 0$

b) $U_{RC_1} = 40 \cdot (2,5 - 1,125j) = 109,66 \angle -24,23°$

ω

c) $U_{LC_2} = 45 \angle 90°$

$Z_{uk} = \frac{5}{2}$

$I_{uk} = 40\ A$

$\omega \rightarrow 3\omega$

paralelna rezonancija

$U_{RC_1} = 0$

$U_{LC_2} = 10$

a) $I_{uk} = 40\ A$

b) $U_{RC_1} = 148,4\ V$

c) $U_{LC_1} = 46,1\ V$

7. $I_{C_1} = 2\ A$ $\quad I_{L_5} = 0,1\ A$

$Z_{uk} = 10 + 10j$ $\quad 0,1 = I_{uk} \cdot \frac{5j}{25j}$

$U_{ef_1} = 20 + 20j$ $\quad I_{uk} = 0.49 \angle 101,3$

$U_{ef_2} = 13,25 \angle 169,14°$

$U = \sqrt{U_{ef_1}^2 + U_{ef_2}^2} = 37,23$

8. $R = 8\ \Omega$ a) I_{ef}
$C = 31{,}8 \cdot 10^{-6}\ F$ b)
$f_1 = 500\ Hz$
$U_{ef} = 120\ V$
$U_3 = 0{,}6\ U$
$U_5 = 0{,}25 U_1$
$U_7 = 0.1321 U_1$

$120^2 = U_1^2 + 0{,}36 U_1^2 + 0{,}0625 U_1^2 + 0.017424 U_1^2$
$U_1 = 100$

ω_1
$R = 8\Omega$
$C = 31{,}8 \cdot 10^{-6} \Rightarrow X_C = \frac{1}{\omega C} = 10j\Omega$

$Z_{uk} = (8 - j10)\ \Omega$
$I_{ef} = \frac{100}{8 - j10} = 7{,}8\ \angle 51{,}34$

ω_3
$R = 8$
$X_C = 3{,}34\ \Omega$
$I_{ef} = \frac{60}{8 - 3{,}34j} = 6{,}92\ \angle 22{,}66°$

ω_5
$R = 8$
$X_C = 2\ \Omega$
$I_{ef} = \frac{25}{8 - 2j} = 3\ \angle 14°$

ω_7
$R = 8$
$X_C = 1{,}43$
$I_{ef} = \frac{13{,}2}{8 - 1{,}43j} = 1{,}62\ \angle 10{,}13°$

$I_{efuk}^2 = I_{ef1}^2 + I_{ef3}^2 + I_{ef5}^2 + I_{ef7}^2 = 10{,}97\ A$

$P_{uk} = 964\ W$

9. $u(t) = 150 \sin(1000t) + 100\sin(2000t) + 75 \sin(3000t)$

ω_1
$U_{ef} = 75\sqrt{2}$ V
$X_C = -3{,}183\,j$
$X_L = 31{,}41\,j$
$R = 5\,\Omega$

$I_1 = 3{,}7 \angle -79{,}96°$

$P_1 = 69{,}1$ W

ω_2
$U_{ef} = 50\sqrt{2}$
$X_C = -1{,}59\,j$
$X_L = 62{,}83\,j$

$I_2 = 1{,}15 \angle -85{,}33°$

$P_2 = 6{,}62$ W

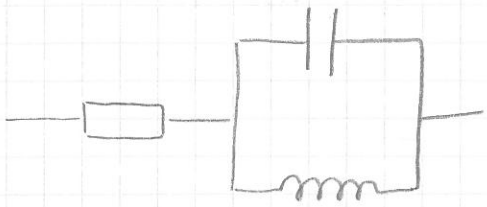
ω_3
$U_{ef} = 53{,}03$ V
$X_C = -1{,}06\,j$
$X_L = 94{,}25\,j$

$I_3 = 0{,}57 \angle -86{,}92°$

$P_3 = 1{,}63$

$P = 77{,}35$

10. $R = 5\,\Omega$



$u(t) = 50 + 20 \sin 500t + 10 \sin 1000t$
$X_L(\omega) = 2\,\Omega$
$X_C = 8\,\Omega$

istosmjerna

$Z_{uk} = R = 5\,\Omega \Rightarrow I = \frac{50}{5} = 10$ A

$\omega = 500t$
$X_L = 2\,\Omega$
$X_C = 8\,\Omega$
$R = 5\,\Omega$
$\Big\}$ $Z_{uk} = 5 + \frac{8}{3}i\ \Omega \Rightarrow I = 2{,}5 \angle -28{,}1°$

$\omega = 1000\,t$
$X_L = 4\,\Omega$
$X_C = 4\,\Omega$
$\Big\}$ paralelna rezonancija $\Rightarrow$ ne teče struja

$\rightarrow$ $i(t) = 10 + 3{,}54 \sin(500t - 28{,}1°)$ A

$P = 531{,}25$ W

11. 1 i 3 harmonik

$I_{m_3} = \frac{I_{m_1}}{2}$

$i(t) = I_{m_1} \sin(\omega t) + \frac{I_{m_1}}{2} \sin(3\omega t)$

$i_1 \angle 0°$

$U_{C_1} = 90 \angle -90°$
$U_{R_1} = 20 \angle 0°$
$U_L = 10 \angle 90°$
$U_{R_2} = 40 \angle 0°$

$U_{uk} = 60 - 80i = 100 \angle -53{,}13$

$R_1 = \frac{20}{I_{m_1}\sqrt{2}}$

$R_2 = \frac{40}{I_{m_1}\sqrt{2}}$

$\left|\frac{X_L}{X_C}\right| = \frac{10}{90} = \frac{1}{9}$

3ω

$\frac{X_L}{X_C} = \frac{3}{3} \Rightarrow$ serijska rezonancija

$Z = R_1 + R_2 = \frac{20}{I_{m_1}} + \frac{40}{I_{m_1}} = \frac{60}{I_{m_1}\sqrt{2}}$

$U_2 = I \cdot Z = \frac{I_{m_1}}{2\sqrt{2}} \cdot \frac{60}{I_{m_1}\sqrt{2}} = \frac{60}{4} = 20 \angle 0°$

$115^2 = \left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{20}{\sqrt{2}}\right)^2 + X$

2. $i(t) = I_{m_1} \cdot \sin \omega t + \frac{I_{m_1}}{2} \sin 3\omega t$

$\omega = \omega_1$

$U_C = 90 \angle -90$
$U_{R_1} = 20 \angle 0°$
$U_L = 10 \angle 90$
$U_{R_2} = 40 \angle 0°$

$U_{ef} = 60 - 80i = 100 \angle -53,13°$

$I = \frac{U}{I} = \frac{100}{\frac{I_m}{\sqrt{2}}} = \frac{100\sqrt{2}}{I_m}$

$\frac{X_L}{X_C} = \frac{1}{9}$ $\qquad \frac{(R_1 + R_2)}{(X_L + X_C)} = \frac{60}{80}$

$\omega \Rightarrow 3\omega$

$\frac{X_L}{X_C} = \frac{3}{3}$ rezonancija

$U = I \cdot I = \frac{I_{m_1}}{2 \cdot \sqrt{2}} \cdot \frac{60\sqrt{2}}{I_m} = 30$

$115^2 = 100^2 + 30^2 + X^2$

$X = 48,29$

12. $u(t) = 40 \sin \omega t + 16 \sin 2\omega t$

$R = 1,33\ \Omega$
$X_L = 1\ \Omega$
$X_C = 4\ \Omega$
I_{ef}.

(ω)

$U_{ef} = 20\sqrt{2}$

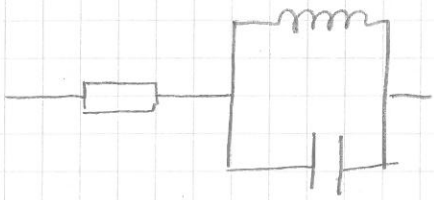
$Z_{ek} = 2,1 \angle 18,42°$ $\Rightarrow I_{ef}$ $13,47 \angle -18,42°$

(2ω)

$U_{ef} = 8\sqrt{2}$

$X_L = 2$
$X_C = 2$ } rez.

13. $R = 5\,\Omega$
$\omega = 500$
$X_L = 2\,\Omega$
$X_C = 3\,\Omega$



$u(t) = 50 + 66{,}57 \sin(500t) + 50{,}5 \sin(1000t)\ V$

istosmjerno

$Z = 5\,\Omega$

$I = 10\ A$

$\omega = 500$
$Z = 5 + \frac{5}{3}i$

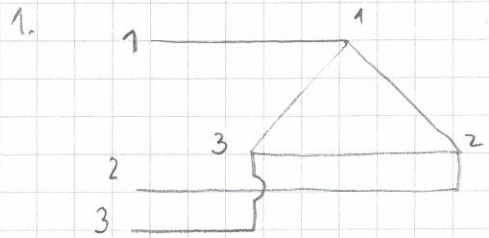
$I = 8{,}31\ \angle 0{,}8$

$\omega = 1000$
$X_L = 4$ } paralelna rez
$X_C = 4$ } ne teče struja
∞ otpor

$I_{ef} = 13\ A$

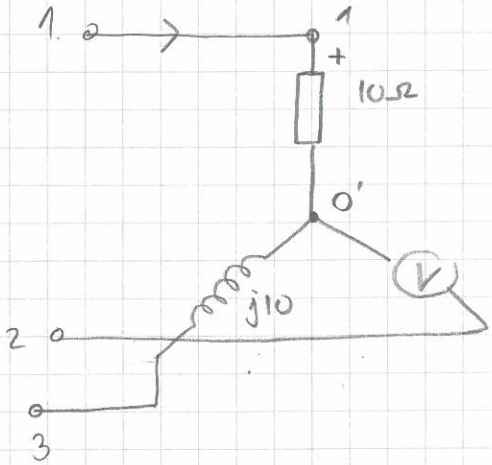
• MASOVNE

1.



$$I_{12} = \frac{U_1 - U_2}{Z} = \frac{380\angle 30°}{30 - j60} = 3{,}8\angle 66{,}36 \qquad P = U \cdot I \cdot \cos(\varphi_u - \varphi_i) = 1155{,}35$$

$$I_{23} = \frac{U_2 - U_3}{Z} = 3{,}8\angle -53{,}13 \qquad P = U \cdot I \cdot \cos(\varphi_u - \varphi_i) = 1155{,}2$$



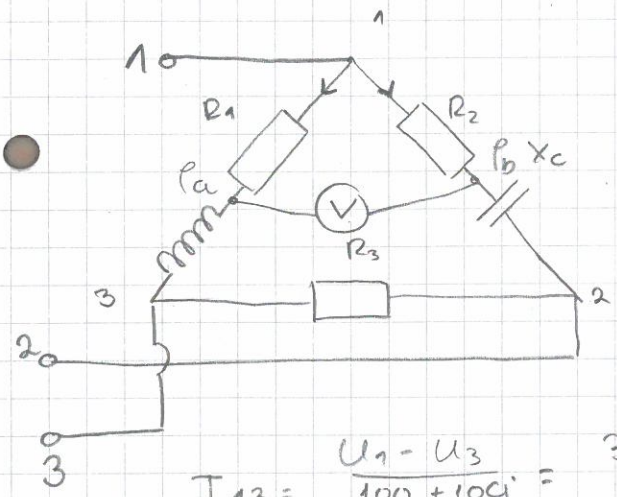
$$U_{13} = U_1 - U_3 = 400\angle -30° \text{ V}$$

$$I_{13} = \frac{U_{13}}{10 + j10} = 20\sqrt{2}\angle -75° \text{ A}$$

$$P_{O'} = P_1 - I_{13} \cdot 10 = \frac{400}{\sqrt{3}}\angle 0° - 20\sqrt{2}\angle -75° \cdot 10$$

$$= 315.47\angle 60° \text{ V}$$

$$U_V = P_{O'} - P_2 = 546{,}41 \text{ V}$$



$$U_{lin} = 381 \text{ V}$$

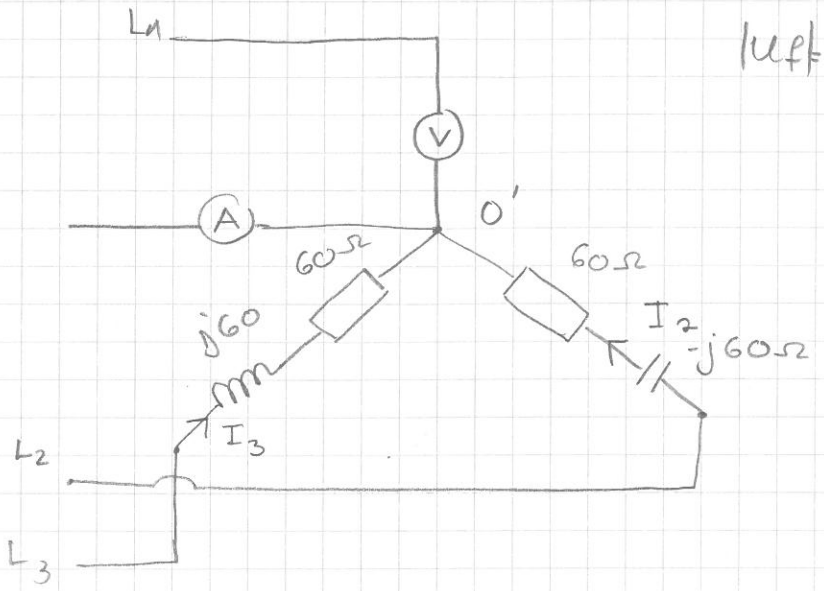
$$I_{12} = \frac{U_1 - U_2}{R_2 - jX_C} = \frac{381\angle 30°}{200 - 200j} = 1.347\angle 75° \text{ A}$$

$$P_b = P_1 - I_{12} \cdot R_2 = 300{,}2\angle -60°$$

$$I_{13} = \frac{U_1 - U_3}{100 + 100j} = \frac{380\angle -30°}{100 + 100j} = 2{,}687\angle -75°$$

$$P_A = P_1 - I_{13} \cdot R_1 = 300\angle 60°$$

$$U_V = P_A - P_B = 519{,}62 \text{ V}$$



$|U_f| = 230\,V$

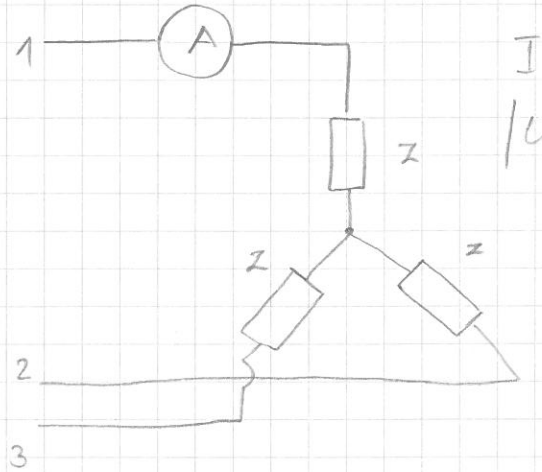
$U_{L_3} - U_0 = 230 \angle -240°$

$I_3 = \frac{U_{L_3}}{60 + j60} = 2{,}71 \angle 75°$

$U_{L_2} - U_0 = 230 \angle -120°$

$I_2 = \frac{U_{L_2}}{60 - j60} = 2{,}71 \angle -75$

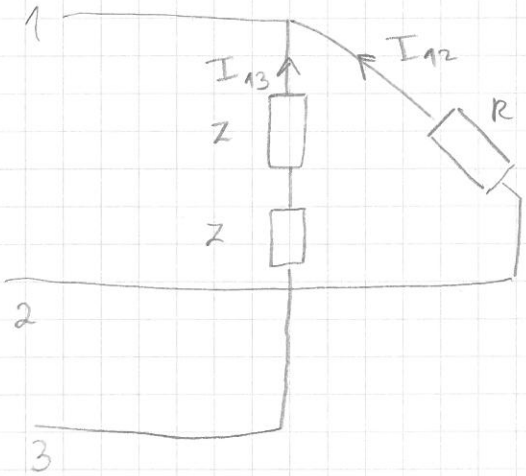
$I_0 = 1{,}4\ A$



$I_e = I_f = 2\ A$

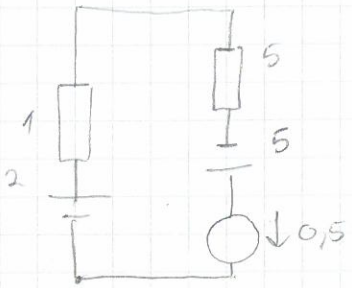
$|U_f| = 200$

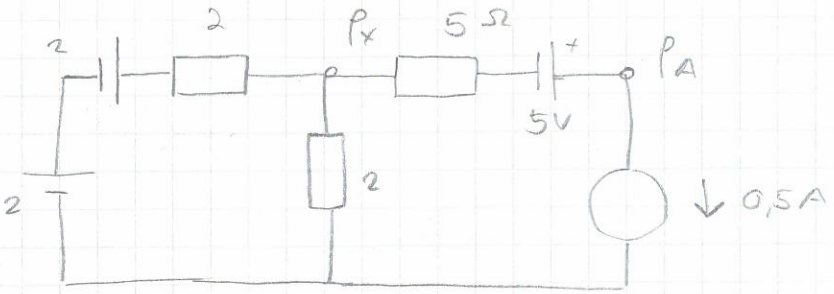
$I_{12} = \frac{U_1 - U_2}{R} = \frac{346 \angle 30°}{100} = 2\sqrt{3} \angle 30°$



$I_{13} = \frac{U_a - U_3}{Z + Z} = \frac{346 \angle 30°}{Z + Z} = \sqrt{3} \angle -90$

$I_{12} + I_{13} = 3\ A$





$$P_X = \frac{\frac{4}{2} - 0,5}{\frac{1}{2} + \frac{1}{2}} : \frac{2-0,5}{1} = 1,5 \text{ V}$$

$$P_A = P_X + 0,5 \cdot 5 = 1,5 + 2,5 = 4 \text{ V}$$

$$U_{AB} = P_A - P_B = 4 - 0 = 4 \text{ V}$$