$$E_k = \sqrt{\frac{p^2}{m^2c^2}\cdot m^2c^4 + m^2c^4} - mc^2 = mc^2\sqrt{\frac{p^2}{m^2c^2}+1} - mc^2$$

$$E_k = 0{,}511\cdot\sqrt{1{,}17^2+1} - 0{,}511 = 0{,}275\ MeV$$

$$E_\gamma = (87{,}6 + 275)\ keV = 363\ keV$$

47. $E_{\gamma 1} = 0{,}51\ MeV$

$E_{\gamma 2} = 80\ keV$

$\varphi = 60°$

$\left(\frac{d\sigma}{d\Omega}\right)_{E_{\gamma 1}} = 10\ \frac{barn}{ster}$

$\left(\frac{d\sigma}{d\Omega}\right)_{E_{\gamma 2}} = ?$

$$\left(\frac{d\sigma}{d\Omega}\right) = k\,\frac{\sin^2\varphi}{\left(1-\frac{v}{c}\cos\varphi\right)} \qquad \beta = \frac{v}{c}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{E_{\gamma 1}} = k\cdot\frac{\sin^2 60°}{(1-\beta_1\cos 60°)}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{E_{\gamma 2}} = k\cdot\frac{\sin^2 60°}{(1-\beta_2\cos 60°)}$$

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)_{E_{\gamma 1}}}{\left(\frac{d\sigma}{d\Omega}\right)_{E_{\gamma 2}}} = \frac{\frac{1}{1-\beta_1\cdot\frac{1}{2}}}{\frac{1}{1-\beta_2\cdot\frac{1}{2}}} = \frac{\frac{2}{2-\beta_1}}{\frac{2}{2-\beta_2}} = \frac{2-\beta_2}{2-\beta_1}$$

$$E = mc^2 = \frac{m_0}{\sqrt{1-\beta^2}}c^2 = m_0c^2 + E_k \;/: m_0c^2$$

$$\frac{1}{\sqrt{1-\beta^2}} = 1 + \frac{E_k}{m_0c^2} \Rightarrow \beta^2 = 1 - \frac{1}{\left(1+\frac{E_k}{m_0c^2}\right)^2} \Rightarrow \beta = \sqrt{1-\frac{1}{\left(1+\frac{E_k}{m_0c^2}\right)^2}}$$

$$\beta_1 = \sqrt{1-\frac{1}{\left(1+\frac{0{,}51}{0{,}51}\right)^2}} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} = 0{,}866$$

$$\beta_2 = \sqrt{1-\frac{1}{\left(1+\frac{80}{510}\right)^2}} = \sqrt{1-\frac{1}{1{,}34}} = \sqrt{1-0{,}747} = 0{,}503$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{E_{\gamma 2}} = 10\ \frac{barn}{steradian}\cdot\frac{2-0{,}866}{2-0{,}503} = 10\cdot\frac{1{,}134}{1{,}497} = 7{,}57\ \frac{barn}{steradian}$$

48. $\sigma_{i,Pb} = 28$ barna, Pb, $Z = 82$
volfram, $Z = 74$, $\sigma_W = ?$
aluminij, Al, $Z = 13$, $\sigma_{Al} = ?$

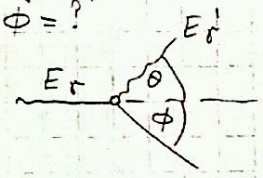
$$\sigma_{i,6} \approx \frac{Z^5}{E^{1/2}}$$

$$E^{1/2} = \frac{Z^5}{\sigma} = \frac{82^5}{28} = 132 \cdot 10^6$$

$$\sigma_W = \frac{74^5}{132 \cdot 10^6} = 17 \text{ barna}$$

$$\sigma_{Al} = \frac{13^5}{132 \cdot 10^6} = 2{,}8 \text{ m-barna}$$

49. $\theta = 90°$
$E_\gamma = 1{,}33$ MeV
$\Phi = ?$




$E_\gamma$ ... kin. ener. elektrona

$E_\gamma = E_\gamma' + E_{ke}$

Comptonovo raspršenje

$$\frac{h \cdot f}{c} = \frac{h \cdot f'}{c} \cdot \cos\theta + p \cos\phi$$

$$0 = \frac{E_\gamma'}{c} \cdot \sin\theta - p \sin\phi$$

$$E_\gamma' = \frac{E_\gamma}{1 + \frac{E_\gamma}{mc^2}(1 - \cos\theta)} = \frac{1{,}33}{1 + \frac{1{,}33}{0{,}511}(1 - 0)} = \frac{1{,}33}{3{,}6} = 0{,}369 \text{ MeV}$$

$$p \sin\phi = \frac{E_\gamma'}{c} \cdot \sin\theta$$

$$\sin\phi = \frac{E_\gamma'}{pc} \cdot \sin\theta$$

$E_{ke} = E_\gamma - E_\gamma' = 0{,}96$ MeV

$E_{ke} = \sqrt{p^2c^2 + m^2c^4} - mc^2$

$E_{ke} + mc^2 = \sqrt{p^2c^2 + m^2c^4}$

$E_{ke}^2 + 2E_{ke}mc^2 + m^2c^4 = p^2c^2 + m^2c^4$

$pc = \sqrt{E_{ke}^2 + 2E_{ke}mc^2} = \sqrt{0{,}36^2 + 2 \cdot 0{,}36 \cdot 0{,}51} = 1{,}38$ MeV

$$\sin\phi = \frac{0{,}369}{1{,}38} \cdot \sin 90° = 0{,}267 \Rightarrow \phi = 15{,}5°$$

50. $xf = 0{,}4\ g/cm^2$

$\varphi = 0 \Rightarrow \theta$ može biti samo $0°$ ili $180°$

$\Rightarrow$ kut $\theta \neq 0$ jer bi onda kinetička energija elektrona bila 0

$\Rightarrow \theta = 180°$

$R_0\,(g/cm^2) = 0{,}52 \cdot E\,(MeV) - 0{,}09$

$0{,}4 = 0{,}52 \cdot E(MeV) - 0{,}09$

$0{,}49 = 0{,}52 \cdot E\,(MeV)$

$E_{kc} = 0{,}94\ MeV$

$E_\gamma = E_\gamma' + E_{kc} = E'_\gamma + 0{,}94$

$\frac{E_\gamma}{c} = \frac{E'_\gamma}{c}\cos\theta + p\cos\varphi \quad / \cdot c$

$E_\gamma = E'_\gamma \cos\theta + pc\cos\varphi$

$E_\gamma = -E'_\gamma + pc$

$E_{kc} = \sqrt{p^2c^2 + m^2c^4} - mc^2$

$pc = \sqrt{E_{kc}^2 + 2E_{kc}mc^2} = \sqrt{0{,}94^2 + 2 \cdot 0{,}94 \cdot 0{,}51} = 1{,}36\ MeV$

$E_\gamma = 1{,}36 - E'_\gamma$

$E'_\gamma + 0{,}94 = 1{,}36 - E'_\gamma$

$2E'_\gamma = 2{,}3 \Rightarrow E'_\gamma = 1{,}15\ MeV$

$E_\gamma = 1{,}36 - 1{,}15 = 0{,}21\ MeV$

51. $\lambda = 0{,}03\ A = 3 \cdot 10^{-12}\ m \qquad \lambda = 10^{-10}\ m$

$\theta = 60°, 90°, 180°$

$E'_\gamma = ?$

$E_\gamma = h \cdot f = h \cdot \frac{c}{\lambda} = 4{,}14 \cdot 10^{-15}\ eV\cdot s \cdot \frac{3 \cdot 10^8\ m/s}{3 \cdot 10^{-12}\ m} = 4{,}14 \cdot 10^5 = 414\ keV$

$h = 6{,}626 \cdot 10^{-34}\ J\,s = \frac{6{,}626 \cdot 10^{-21}\ MeV \cdot s}{1{,}602 \cdot 10^{-13}} = 4{,}14 \cdot 10^{-15}\ eV\cdot s$

$E'_{\gamma,60°} = \frac{414}{1 + \frac{414}{510}(1 - 0{,}5)} = \frac{414}{1{,}406} = 294{,}5\ keV$

$E'_{\gamma,90°} = \frac{414}{1{,}81} = 228\ keV, \quad E'_{\gamma,180°} = \frac{414}{2{,}62} = 157{,}8\ keV$

52. $E_\gamma = 1\ MeV$
$\lambda$ , 25 %
$E_{kc}$ , $\phi = ?$

$$E_\gamma = h \cdot f = h \cdot \frac{c}{\lambda} \Rightarrow \lambda_0 = \frac{h \cdot c}{E} = \frac{4,14 \cdot 10^{-13}\ eVs \cdot 3 \cdot 10^8\ m/s}{10^6\ eV} = 1242\ fm$$

$$\lambda = 1,25 \cdot \lambda_0 = 1,25 \cdot 1242 = 1552\ fm = \lambda'$$

$$E'_\gamma = h \cdot f' = h \cdot \frac{c}{\lambda'} = \frac{4,14 \cdot 10^{-15}\ eVs \cdot 3 \cdot 10^8\ m/s}{1552 \cdot 10^{-15}\ m} = 0,8\ MeV$$

$$E_\gamma = E'_\gamma + E_{kc}$$

$$E_{kc} = E_\gamma - E'_\gamma = 1 - 0,8 = 0,2\ MeV$$

$$\lambda - \lambda_0 = \frac{h \cdot c}{mc \cdot c}(1 - \cos\theta)$$

$$310 \cdot 10^{-15} = \frac{4,14 \cdot 10^{-15}\ eVs \cdot 3 \cdot 10^8\ m/s}{0,51 \cdot 10^6\ eV}(1 - \cos\theta)$$

$$310 \cdot 10^{-15} = 2435 \cdot 10^{-15}(1 - \cos\theta)$$

$$0,127 = 1 - \cos\theta$$
$$-0,8727 = -\cos\theta \Rightarrow \theta = 29,2°$$

$$\sin\phi = \frac{E'_\gamma}{pc} \cdot \sin\theta$$

$$pc = \sqrt{E_{kc}^2 + 2E_{kc}mc^2} = \sqrt{0,2^2 + 2 \cdot 0,2 \cdot 0,511} = 0,49\ MeV$$

$$\sin\phi = \frac{0,8}{0,49} \cdot \sin 29,2° = 0,794 \Rightarrow \phi = 52,8°$$

53. $E_\gamma = 1,2\ MeV$
$\theta = 0°, 30°, 180°$ $\qquad E_0 = E_\gamma$
$\left(\frac{d\sigma}{d\Omega}\right)_\theta = ?$

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\left(\frac{E}{E_0}\right)^2\left(\frac{E_0}{E} + \frac{E}{E_0} - \sin^2\theta\right)$$

$$E = \frac{E_\gamma}{1 + \frac{E_\gamma}{mc^2}(1 - \cos\theta)} = \frac{1,2}{1 + \frac{1,2}{0,511}(1 - 1)} = 1,2\ MeV$$

$$\Rightarrow E = E_0$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{0°} = \frac{r_0^2}{2}(1 + 1 - 0) = \frac{r_0^2}{2} \cdot 2 = r_0^2$$

$$E = \frac{1,2}{1 + \frac{1,2}{0,51}(1-0)} = \frac{1,2}{3,35} = 0,36\,MeV$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{90^\circ} = \frac{r_0^2}{2} \cdot 0,03 \left(3,33 + 0,3 - 1\right) = 0,1 \cdot r_0^2$$

54. $E_\gamma = 3\,MeV$
$E_{kc} = E_{kp}$
$B = 0,1\,T$
$r = ?$

$$E_\gamma = 2mc^2 + E_{kc} + E_{kp}$$

$$E_\gamma = 2mc^2 + 2E_{kp}$$

$$2E_{kp} = E_\gamma - 2mc^2$$

$$E_{kp} = \frac{E_\gamma - 2mc^2}{2} = \frac{3 - 2 \cdot 0,51}{2} = 0,99\,MeV$$

$$\frac{m \cdot v^2}{r} = e \cdot B \cdot v \;/\cdot r$$

$$m \cdot v = e \cdot B \cdot r \Rightarrow p = e \cdot B \cdot r$$

$$r = \frac{p \cdot c}{e \cdot B \cdot c} = \frac{pc}{e \cdot B \cdot c}$$

$$pc = \sqrt{E_{kp}^2 + 2E_{kp}mc^2} = \sqrt{0,99^2 + 2 \cdot 0,99 \cdot 0,51} = 1,41\,MeV$$

$$r = \frac{1,41 \cdot 1,602 \cdot 10^{-13} \cdot 10^6}{1,602 \cdot 10^{-19} \cdot 0,1 \cdot 3 \cdot 10^8} = 4,7\,cm$$

55. $E_{kp} = 2\,MeV$
$E_B = 78,5\,keV$
$E_\gamma = ?$

$$2E_\gamma = E_{kp} + 2mc^2 - E_B$$

$$2E_\gamma = 2 + 2 \cdot 0,51 - 0,0785 = 2 + 1,02 - 0,0785 = 2,94\,MeV$$

$$E_\gamma = 1,47\,MeV$$

## - APSORPCIJA ELEKTROMAGNETSKOG ZRAČENJA U MATERIJI

56. $Eu^{156}$ → $E_{\beta 1} = 0,5$ MeV (60%), $E_{\beta 2} = 2,5$ MeV (40%)

$x_1 = 8$ cm
20 otkucaja/min
$x' = 1$ cm, brzina brojanja = ?
bakar, Cu, $\rho_{Cu} = 8,96$ g/cm³

$N_1 = N_{10} \cdot e^{-\mu x}$

$\frac{\mu_1}{\rho} = 0,0853$ cm²/g $\Rightarrow \mu_1 = 0,764$ cm⁻¹

$\frac{\mu_2}{\rho} = 0,0388$ cm²/g $\Rightarrow \mu_1 = 0,348$ cm⁻¹

$N_1 = N_{10} \cdot e^{-0,764 \cdot 8} = N_{10} \cdot 0,002216$

$450 \cdot N_1 = N_{10}$

$\frac{N_{10}}{N_1} = 451$ (toliko je oslabljeno zračenje nakon prolaska kroz filtar)

$\Rightarrow N_1 = \frac{N_{10}}{451}$

$N_2 = N_{20} \cdot e^{-\mu x}$

$N_2 = N_{20} \cdot e^{-0,348 \cdot 8} = N_{20} \cdot 0,0618$

$N_2 \cdot 16,2 = N_{20}$

$\frac{N_{20}}{N_2} = 16,2 \Rightarrow N_2 = \frac{N_{20}}{16,2}$

$N_{10} = 0,6 \cdot N_0$ $N_{20} = 0,4 \cdot N_0$

$N_2 : N_1 = \frac{0,4 \cdot N_0}{16,2} : \frac{0,6 \cdot N_0}{451} = 0,0245 : 0,0013 = 19 : 1$

- ukupno 20 otkucaja $\Rightarrow$ 19 od zraka 2, a 1 od zraka 1

$\Rightarrow$ promjena filtra, $x' = 1$ cm

$\frac{N_1'}{N_1} = \frac{N_{10} \cdot e^{-\mu x'}}{N_{10} \cdot e^{-\mu x}} = e^{-\mu(x'-x)} = 210$

$N_1' = 210 \cdot N_1 = 210 \cdot 1 = 210$ otkucaja od zraka 1

$\frac{N_2'}{N_2} = \frac{N_{20} \cdot e^{-\mu x'}}{N_{20} \cdot e^{-\mu x}} = e^{-\mu(x'-x)} = 11,4 \Rightarrow N_2' = 11,4 \cdot 19 = 217$ otkucaja

- ukupno brzina: $N' = N_1' + N_2' = 210 + 217 = 427$ otkucaja/min

57. $x\rho = 1,4\ g/cm^2$

$E_{\gamma 1} = 0,4\ MeV$

$E_{\gamma 2} = 1,5\ MeV$

$\frac{\mu_1}{\rho} = 0,0953\ cm^2/g$ $\qquad N = N_0 \cdot e^{-\frac{\mu}{\rho} x\rho} \Rightarrow \frac{N}{N_0} = e^{-\frac{\mu}{\rho} x\rho}$

$\frac{\mu_2}{\rho} = 0,0516\ cm^2/g$

$\eta = \frac{N_0 - N}{N_0} = 1 - e^{-\frac{\mu}{\rho} x\rho}$

$\eta_1 = 1 - 0,875 = 0,124$

$\eta_2 = 1 - 0,93 = 0,07$

$\eta_1 : \eta_2 = 0,124 : 0,07 = 1,77 : 1$

58. $Pb$ , $\rho_{Pb} = 11,3\ g/cm^3$

$I = 10^{-6} \cdot I_0$ , $x = ?$

$R = 3\ m$

$\frac{\Phi_R}{\Phi_x} = ?$

$\frac{\mu}{\rho} = 0,042\ cm^2/g$

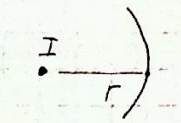
$\frac{I}{I_0} = 10^{-6} = e^{-\mu x} = e^{-\frac{\mu}{\rho} \cdot x\rho}$

$10^{-6} = e^{-0,042 \cdot x \cdot 11,3}$

$10^{-6} = e^{-0,4746 \cdot x}$

$\ln 10^{-6} = -0,4746 \cdot x$

$x = 29\ cm = 0,29$




$\Phi = \frac{I}{4\pi r^2}$

$\frac{\Phi_R}{\Phi_x} = \frac{\frac{I}{4\pi R^2}}{\frac{I}{4\pi x^2}} = \frac{x^2}{R^2} = \frac{0,29^2}{3^2} = 9,3 \cdot 10^{-3}$

59. $E_\gamma = 1,5\,MeV$
Pb , $\rho_{Pb} = 11,3\,g/cm^3$
$\frac{\mu}{\rho} = 0,05\,cm^2/g$
$x = ?$

$\mu = \frac{\mu}{\rho} \cdot \rho = 0,565\,cm^{-1}$

$I_2 = 4 \cdot I_1$

$\frac{I_1}{I_2} = \frac{I_1}{4 \cdot I_1} = \frac{1}{4} = e^{-\mu x}$

$\frac{1}{4} = e^{-0,565 \cdot x}$

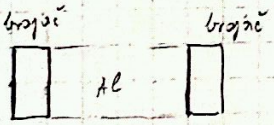
$\ln \frac{1}{4} = -0,565 \cdot x$

$x = 2,45\,cm$

## - PROLAZ ELEKTRONA KROZ MATERIJU

60. C
$x\rho = 10\,mg/cm^2$
Al , $x\rho = 150\,mg/cm^2$
$E_\beta = ?$

$E_e = E_\beta - W$




$R_0 = 150\,mg/cm^2 + 3 \cdot 10\,mg/cm^2 = 180\,mg/cm^2 = 180 \cdot 10^{-3}\,g/cm^2$

$R_0 = 0,52 \cdot E_e - 0,09$

$E_e = \frac{R_0 + 0,09}{0,52} = \frac{180 \cdot 10^{-3} + 0,09}{0,52} = \frac{0,27}{0,52} = 0,519\,MeV$

$W = 0 \Rightarrow E_e = E_\beta = 0,519\,MeV$

61. $x\rho = 0,1\,g/cm^2$
$E_{max} = 0,773\,MeV$
2% - nosprime zrake

$E_{max} = 0,54 \cdot Z^{1/5}$

$0,773 = 0,54 \cdot Z^{1/5}$

$1,43148 = Z^{1/5} \Rightarrow Z = 5,37 \approx 6 \Rightarrow$ ugljik

$$\sigma_T = \frac{8\pi}{3} r_0^2 = \frac{8\pi}{3} \cdot (2,818 \cdot 10^{-15})^2 = 67 \cdot 10^{-30}\ m^2 = 0,67\ barn$$

$$M = A \cdot \rho \cdot z \cdot \sigma_T \cdot \frac{1}{\frac{\Delta N}{N}} = 12,04 \Rightarrow {}^{12}_{6}C$$

62. $Al$, $x\rho = 0,436\ g/cm^2 = R_0$

$$E = \frac{R_0 + 0,09}{0,52} = \frac{0,436 + 0,09}{0,52} = 1,012\ MeV$$

$$E_{max} = 0,54 \cdot Z^{1,15}$$

$$1,012 = 0,54 \cdot Z^{1,15}$$

$$Z^{1,15} = 1,874 \Rightarrow Z = 23$$

63. $l_{rad} = ?$
$N$, $Al$, $Pb$

$\div N_{7}$, $Al_{13}$, $Pb_{82}$

$$E = E_0 \cdot e^{-\frac{x}{l_{rad}}}$$

$$\frac{dE}{dx} = E_0 \cdot \frac{-1}{l_{rad}} \cdot e^{-\frac{x}{l_{rad}}} \quad -E$$

$$\frac{dE}{dx} = -\frac{E}{l_{rad}}$$

$$-\frac{dE}{dx} = \frac{E}{l_{rad}}$$

$$l_{rad} = \frac{E}{\left(-\frac{dE}{dx}\right)} = \frac{E}{\frac{4r_0^2}{137} \cdot E \cdot N \cdot Z^2 \ln \frac{183}{Z^{1/3}}} = \frac{137}{4r_0^2 \cdot N \cdot Z^2 \cdot \ln \frac{183}{Z^{1/3}}}$$

$r_0 = 2,818 \cdot 10^{-15}\ m$ $\quad 22,4\ l$

$N_2$, $Z = 7 \Rightarrow N = \frac{2 \cdot 1000 \cdot N_A}{22,4} = \frac{2 \cdot 1000 \cdot 6,02 \cdot 10^{23}}{22,4} = 5,375 \cdot 10$

$$l_{rad} = \frac{137}{4 \cdot (2,818)^2 \cdot 10^{-30} \cdot 5,375 \cdot 10^{25} \cdot 49 \cdot \ln \frac{183}{1,913}} = 0,00359 \cdot 10^5 = 359\ m$$

43. $E_k = 10\,MeV$ $\quad {}^{27}_{13}Al$ $\quad \rho_{Al} = 2{,}7\,g/cm^3$

$\rho\Delta x = 1\,mg/cm^2$

$\Delta x = \frac{10^{-3}\,g/cm^2}{2{,}7\,g/cm^3} = 3{,}7\cdot 10^{-4}\,cm$

$n = 7{,}8\cdot 10^{23}\,cm^{-3}$

$$\Delta E = 2\pi\cdot(2{,}82\cdot 10^{-15})^2\cdot\frac{0{,}511\,MeV\cdot 2(938+939)\,MeV}{10\,MeV}\cdot 7{,}8\cdot 10^{23}\,cm^{-3}\ln\left(\frac{4\pi 10^3\,eV\cdot 0{,}511\,MeV}{e(938+939)\,MeV\cdot 10^{-4}\,eV}\right)\Delta x$$

$\Delta E = 2\pi\cdot 7{,}95\cdot 10^{-30}\cdot 1496269\cdot 4{,}74\cdot 3{,}7\cdot 10^{-4}\,MeV$

$\Delta E = 320\,keV$

## - PROLAZAK ELEKTROMAGNETSKOG ZRAČENJA KROZ MATERIJU

44. zakon o očuvanju energije $\Rightarrow E_\gamma = E_k$

$$E_k = \sqrt{p^2c^2+m^2c^4} - mc^2 = E_\gamma$$

$$E_\gamma + mc^2 = \sqrt{p^2c^2+m^2c^4}\ /^2$$

$$E_\gamma^2 + 2E_\gamma mc^2 + m^2c^4 = p^2c^2 + m^2c^4$$

$$p^2 = \frac{E_\gamma^2}{c^2} + \frac{2E_\gamma mc^2}{c^2}$$

$$p = \sqrt{\frac{E_\gamma^2}{c^2} + 2E_\gamma m}$$

$p = \frac{E_\gamma}{c}$ (samo za $E_\gamma = 0$)

46. $r = 10\,cm$
$B = 2\cdot 10^{-2}\,T$
$W_1 = 27{,}6\,keV$
$E_\gamma = ?$

$$\frac{m\cdot v^2}{r} = e\cdot v\cdot B\ /:v\cdot r$$

$$m\cdot v = e\cdot B\cdot r$$

$$p = e\cdot B\cdot r$$

$$\frac{p}{mc} = \frac{e\cdot B\cdot r}{mc} = \frac{1{,}6\cdot 10^{-19}\cdot 2\cdot 10^{-2}\cdot 0{,}1}{9{,}1\cdot 10^{-31}\cdot 3\cdot 10^8\cdot 10^{-2}} = 1{,}17 > 1 \Rightarrow \text{RELATIVISTIČKI}$$

$E_\gamma = W_1 + E_k$

$E_k = \sqrt{p^2c^2+m^2c^4} - mc^2$

$\frac{p^2}{m^2c^2} = 1{,}1^2$