$$\boxed{\lambda_n = \frac{2L}{2n+1}}$$ - konačno rj koje zadovoljava OBA rubna uvjeta

Predavanje 26.10.2009 II ciklus

Energija mehaničkog vala

gustoća energije w $[J\,m^{-3}]$ $w = \frac{1}{2}\rho\omega^2 A^2$

energetski tok P $[W]$ $P = \frac{1}{2}\rho\omega^2 A^2 S v$

intenzitet vala I $[W/m^2]$ $I = \frac{1}{2}\rho\omega^2 A^2 v$

za zvučni val:

$$p_{MAX} = A\omega\rho v$$

$$I = \frac{1}{2}\frac{p_{MAX}^2}{\rho v}$$

Jakost i glasnoća zvuka

nivo jakosti zvuka $\boxed{L = 10\log\frac{I}{I_0}\ [dB]}$ $I_0 = 10^{-12}\,W/m^2$

glasnoća zvuka [fon] - referenca na 1000Hz

$$L = 10\log\frac{\frac{1}{2}\frac{p_{MAX}^2}{\rho v}}{\frac{1}{2}\frac{(p_{MAX})^2_{I_0}}{\rho v}} = 10\log\frac{p_{MAX}^2}{(p_{MAX})^2_{I_0}} =$$

$$L = 20\log\frac{p_{MAX}}{(p_{MAX})_0}$$ $(p_{MAX})_0^2 \approx 10^{-5}\,Pa$

$p_0 \approx 10^5\,Pa$

Doplerov efekt

Izvor Detektor

a) ~~Izvor S~~

Detektor se giba prema ili od izvora

I ⟵ (D) ⟶

Detektor prema izvoru

f, λ, v

$$f = \frac{v}{\lambda} \Rightarrow \lambda = \frac{v}{f}$$

rel. brzina $= v + u$

$$f' = \frac{u+v}{\lambda}$$

$$f' = \frac{u+v}{\frac{v}{f}} = f\,\frac{u+v}{v} = f\left(1 + \frac{u}{v}\right)$$

Detektor od izvora

rel brzina $v - u$

$$\boxed{f' = f\,\frac{v-u}{v}}$$

b) Izvor se giba prema ili od izvora

⟵ (I) ⟶ D

Izvor prema detektoru

$$f' = \frac{v}{\lambda'}$$

$$\lambda' = \lambda - ut = \lambda - \frac{u}{f}$$

$$f' = f\,\frac{v}{v-u}$$

Izvor od detektora

$$\boxed{f' = f\,\frac{v}{v+u}}$$

općenito

$$\boxed{f' = f\,\frac{v - \vec{r}_0 \cdot \vec{v}_D}{v - \vec{r}_0 \cdot \vec{v}_I}}$$

MAXWELL-ove jednadžbe

GAUSS-ov zakon (I. Max. jed) - za elek. polje

Faraday - uveo pojam polja

svojstva naboja

1. kvantiziran

$q_0 = 1.6 \cdot 10^{-19}$ C $\qquad \bar{e} = -q_0 \qquad p = +q_0$

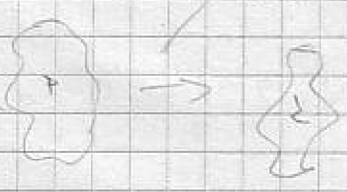
kvarkovi

d s b $\quad -\frac{1}{3} q_0$

u c t $\quad \frac{2}{3} q_0$

2. očuvan

važi procesi



$q_p = q_k$

naboji su jednaki

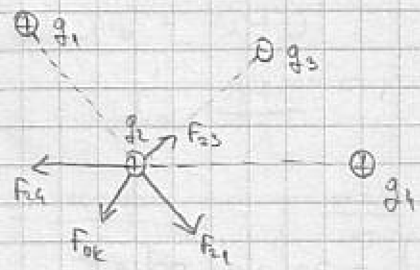
Električno polje

$\vec{E}$

Predavanje 28.10.2009

q_0 $\qquad$ q' $\qquad E = \dfrac{\vec{F}}{q'}$ $\qquad E > 3 \cdot 10^6 \ V/m$ - zrak postaje vodljiv

Princip superpozicije za elek. polje

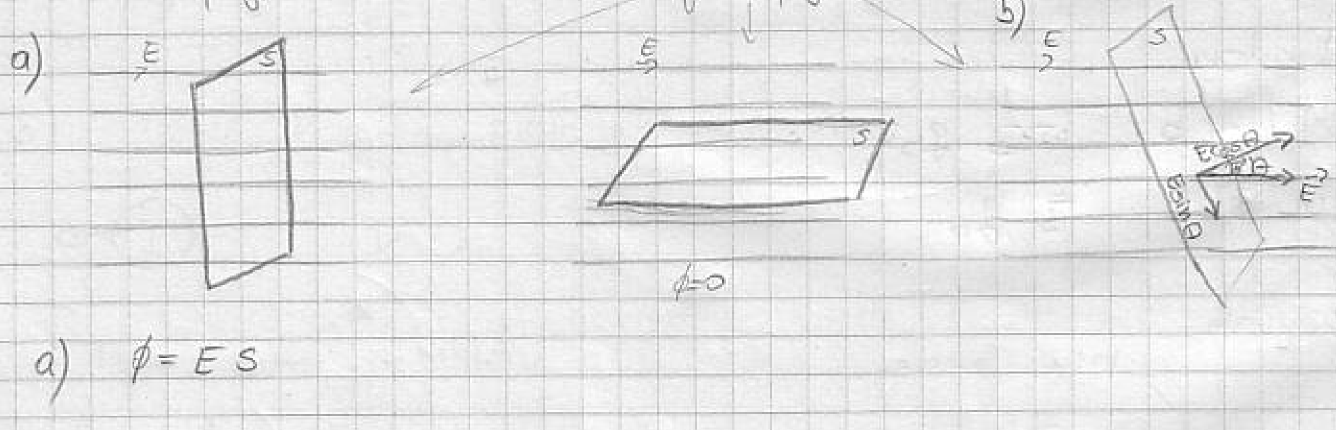


$$\vec{F} = \sum_{i=1}^{N} \vec{F_i} = \sum_{i=1}^{N} q' \vec{E_i} = q' \sum_{i=1}^{N} \vec{E_i} = q' \vec{E}$$

$$\boxed{\vec{E} = \sum_{i=1}^{N} \vec{E_i}}$$

Tok elek. polja

homogeno polje

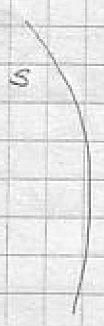


a) $\phi = E S$

b) $\phi = E S \cos\theta$

$\boxed{\phi = \vec{E} \cdot \vec{S}}$ $\qquad \vec{S} = S \cdot \vec{n}$

elek polje nije homogeno



$\phi = ?$

ΔS_i – podjelimo S na puno malih ploha

$\vec{n}_i$

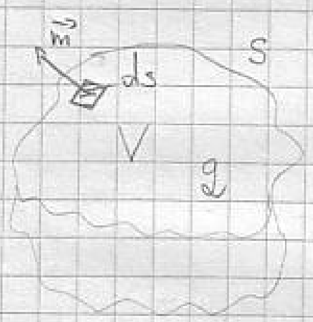
$\vec{n}_i$ – na svakoj plohi polje je homogeno

$$\phi \approx \sum_{i=1}^{N} \vec{E}_i \cdot \Delta \vec{S}_i$$

$$\phi = \lim_{\substack{N \to \infty \\ \Delta S_i \to 0}} \sum_{i=1}^{N} \vec{E}_i \, \Delta S_i = \iint_S \vec{E} \, d\vec{s}$$ – plošni int.

$$\boxed{\phi = \int_s \vec{E} \, d\vec{s}}$$ $\qquad d\vec{s} = \vec{n} \, ds$

zatvorena ploha



$$\boxed{\oint_S \vec{E} \, d\vec{s} = \frac{q}{\varepsilon_0}}$$

gaussov zakon

$\varepsilon_0 = 8{,}85 \cdot 10^{-12} \, C^2 / N m^2$

$\vec{n}$ – gleda iz volumena „prema van"

Tok elek polja kroz bilo koju zatvorenu plohu ~~je~~ proporcionalan je elek. naboju obuhvaćenom tom plohom

Gustoća naboja

$\rho(\vec{r}, t)$

$Q = \int_V \rho \, dV$

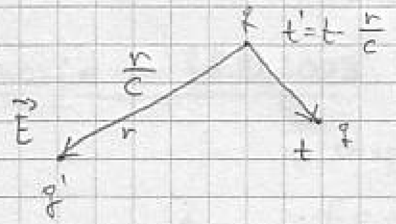
I MAX. JED.

$$\boxed{\oint_S \vec{E} \, ds = \frac{1}{\varepsilon_0} \int_V \rho \, dV}$$

$q = 0 \Rightarrow \phi = 0$

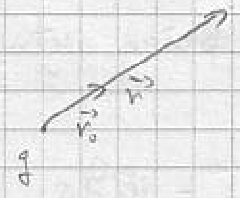
$E(\vec{r})$, $v \ll c$

Gaussov zakon <=> coulombov zakon (za brzine manje od c)



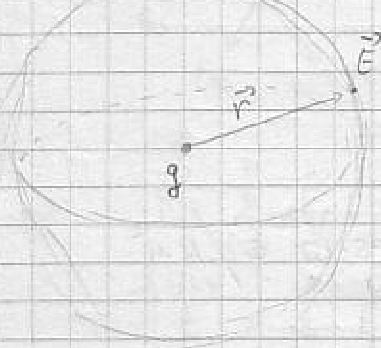
retardacija

za točkasti naboj $\quad \vec{E} = E(r)\,\vec{r_0}$



- sferno simetrična ovisnost

Gauss -> Coulomb



$$d\vec{s} = \vec{r_0}\, dS$$

$$\oint_S \underbrace{E(r)\,\vec{r_0}}_{\vec{E}} \cdot \underbrace{\vec{r_0}\, dS}_{d\vec{s}} =$$

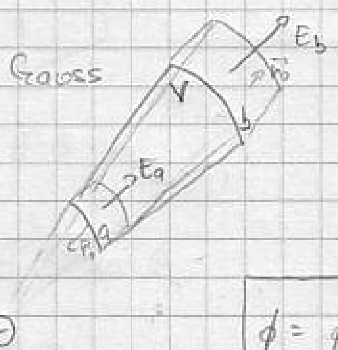
$$E(r)\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E(r) = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}\,\vec{r_0}}$$

- elek polje točkastog naboja

Coulomb -> Gauss



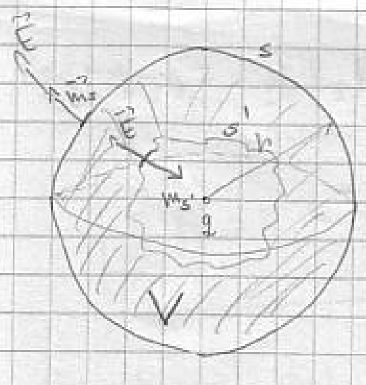
a $\quad \vec{m_a} = -\vec{r_0}$

b $\quad \vec{m_b} = \vec{r_0}$

$$\boxed{\phi = \phi_b - \phi_a = 0}$$

tok polja u zatvorenoj plohi bez naboja je nula

tok pada sa kvadratom udaljenost
površinu raste sa kvadratom udaljenosti
polje u a = polje u b



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\phi = 4\pi r^2 E = 4\pi r^2 \frac{1}{4\pi\varepsilon}\frac{q}{r^2} = \frac{q}{\varepsilon_0}$$

$$\phi = \frac{q}{\varepsilon_0}$$

$$\boxed{\phi_s - \phi_{s'} = 0} \qquad \boxed{\phi_s = \phi_{s'}}$$

Coulombov zakon + superpozicija = Gauss za $v \ll c$

Gaussov zakon u dif. obliku

S, V → S', V' → S'', V'' → $T(x,y,z)$

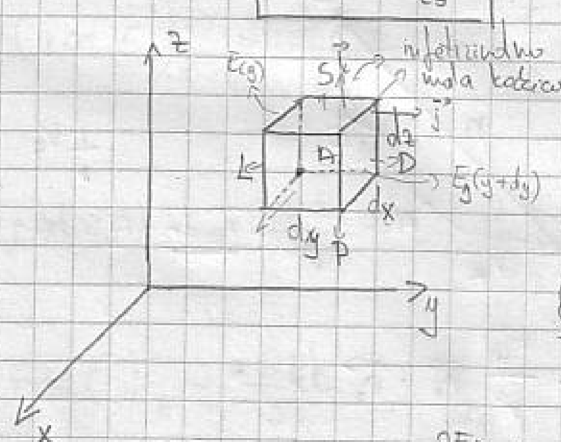
$$\lim_{V\to 0}\frac{\oint_S \vec{E}\,d\vec{s}}{\Delta V} = \operatorname{div}\vec{E}$$

- divergencija elek. polja

– tok vektora elek. polja po jediničnom volumenu u infinitezimalnom malom području promatrane točke

$$\lim_{\Delta V\to 0}\frac{1}{\varepsilon_0}\frac{\int_{\Delta V}\rho\,dV}{\Delta V} = \lim_{\Delta V\to 0}\frac{1}{\varepsilon_0}\frac{\rho\,\Delta V}{\Delta V} = \frac{\rho}{\varepsilon_0}$$

$$\boxed{\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0}}$$



infinitezimalno mala kocka $A(x,y,z)$

dx, dy, dz

$$dV = dx\,dy\,dz$$

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$$

$$\oint \vec{E}\,d\vec{S} = \overbrace{E_y(y+dy)\,dx\,dz - E_y(y)\,dx\,dz}^{D+L} + \overbrace{E_z(z+dz)\,dx\,dy - E_z(z)\,dx\,dy}^{S+P} + E_x(x+dx)\,dy\,dz - E_x(x)\,dy\,dz =$$

$$E_y(y+dy) = E(y) + \frac{\partial E_y}{\partial y}dy + \dots$$

→

$$= \cancel{E_{y(y)}\,dx\,dz} + \frac{\partial E_y}{\partial y} dy\,dx\,dz - \cancel{E_{y(y)}\,dx\,dz} + \cancel{E_{z(z)}\,dx\,dy} + \frac{\partial E_z}{\partial z} dz\,dx\,dy - \cancel{E_{z(y)}\,dx\,dy}$$

$$+ \cancel{E_{x(y)}\,dy\,dz} + \frac{\partial E_x}{\partial x} dx\,dy\,dz - \cancel{E_{x(y)}\,dy\,dz} =$$

$$= \frac{\partial E_y}{\partial y} \underbrace{dx\,dy\,dz}_{dV} + \frac{\partial E_z}{\partial z} \underbrace{dx\,dy\,dz}_{dV} + \frac{\partial E_x}{\partial x} \underbrace{dx\,dy\,dz}_{dV}$$

$$= \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right) dV$$

$$\operatorname{div} \vec{E} = \lim_{\Delta V \to 0} \frac{\oint \vec{E}\, d\vec{s}}{\Delta V} \qquad \frac{\partial}{\partial x} = \partial_x$$

$$\boxed{\operatorname{div} \vec{E} = \partial_x E_x + \partial_y E_y + \partial_z E_z}$$

nabla $\nabla = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}$

$$\boxed{\operatorname{div} \vec{E} = \nabla \cdot \vec{E}}$$

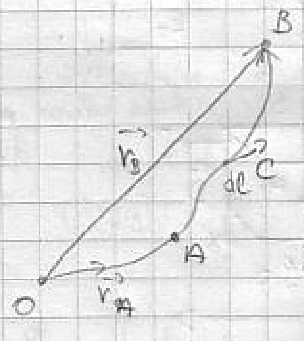
Gaussov zakon:

$$\boxed{\nabla \vec{E} = \frac{\rho}{\varepsilon_0}}$$

Coulombova sila je konzervativna sila

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q q'}{r^2} \vec{r}_0$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{r}_0$$



$\perp \vec{r}_0$ $\quad$ $\perp \vec{r}_0$

$\parallel$ $\quad$ $\parallel$

$d\vec{l} = \underline{\vec{r}_0\, dr} + \vec{\theta}_0\, r\, d\theta + \vec{\varphi}_0\, r \sin\theta\, d\varphi$

$$\int_C \vec{E}\, d\vec{l} = \int_C \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{r}_0 \cdot \vec{r}_0\, dr =$$

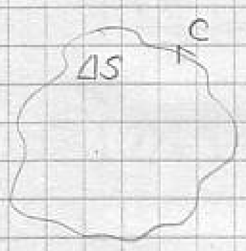
$$= \int_C \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2} dr = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

Predavanje 02. 11. 2009

$$\boxed{\oint_C \vec{E}\, d\vec{l} \to}$$

$d\vec{l} = dl\, \vec{t_0}$

cirkulacija el. polja



$\odot$ - smjer jed. vektora ("van iz biljež.")

$\vec{n}, \vec{E}$

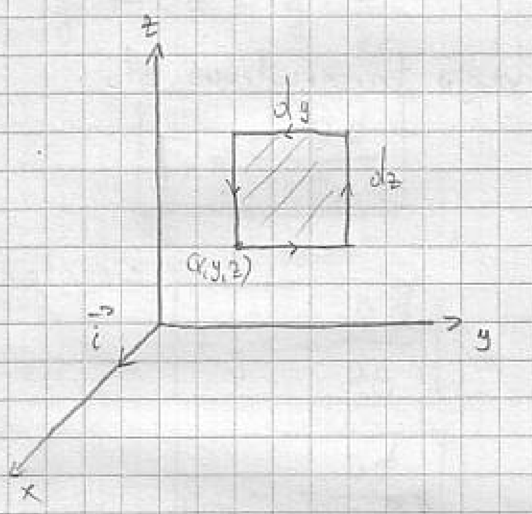
$$\lim_{\Delta S \to 0} \frac{\oint_C \vec{E}\, d\vec{l}}{\Delta S} = \vec{n}\, \mathrm{rot}\, \vec{E}$$

rotacija el. polja

$\mathrm{div}\, \vec{E}$ - skalar

$\mathrm{rot}\, \vec{E}$ - vektor

x y z



$dS = dy\, dz$

$$\boxed{\vec{n} = \vec{i}} \Rightarrow \underbrace{\vec{i}\, \mathrm{rot}\, \vec{E}}_{(\mathrm{rot}\, \vec{E})_x}$$

$$\frac{\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) \cancel{\Delta S}}{\cancel{\Delta S}}$$

$$(\mathrm{rot}\, \vec{E})_x = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$$

$$(\mathrm{rot}\, \vec{E})_y = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$$

$$(\mathrm{rot}\, \vec{E})_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$$

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$\boxed{\mathrm{rot}\,\vec{E} = \nabla \times \vec{E}}$$

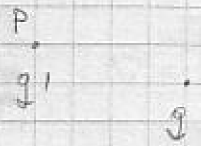
$$\boxed{\mathrm{rot}\,\vec{E} = 0}$$ – rotacija elektrostatskog polja je nula

Električna potencijalna energija

$U_{(P_0)} = U_0$

$U_{(P)} = ?$

$$U_{(P)} = U_{(P_0)} - \int_{P_0}^{P} \vec{F}\, d\vec{l}$$ – vrijedi za bilo koju konzervativnu silu



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{r}_0$$

$U_{(P)} = ?$

$P_0 = \infty$

$U_{(\infty)} = 0$

$$U_{(P)} = -\int_{\infty}^{P(\vec{r})} \frac{1}{4\pi\varepsilon_0} \frac{q \cdot q'}{r^2} \cdot \vec{r}_0\, d\vec{l} = \dots = \frac{1}{4\pi\varepsilon_0} \frac{q q'}{r}$$

u sfernom sustavu

$$\boxed{U = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot q'}{r}}$$ – pot. energija

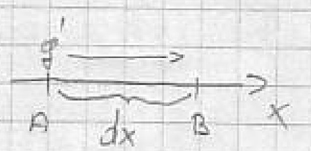
$$U = -\int_{\infty}^{P} q' \vec{E} \cdot d\vec{l} \quad / \, q'$$

$$\frac{U}{q'} = -\int_{\infty}^{P} \vec{E} \cdot d\vec{l}$$

potencijal

$$\boxed{\phi = -\int_{\infty}^{P} \vec{E} \cdot d\vec{l}}$$ – elek. potencijal

kako iz izraza za potencijal izračunati el. polje



$$dW = F_x \cdot dx = \underbrace{q' E_x \, dx}$$

$\boxed{\Delta W = -\Delta U}$ – rad konz. sile jednak je negativnoj promjeni pot. energije

$$dW = -dU = -q' d\phi = \underbrace{-q' \frac{\partial \phi}{\partial x} dx}$$

$$q' E_x = -q' \frac{\partial \phi}{\partial x}$$

$$\boxed{E_x = -\frac{\partial \phi}{\partial x}}$$

analogno:

$$\boxed{E_y = -\frac{\partial \phi}{\partial y}}$$

$$\boxed{E_z = -\frac{\partial \phi}{\partial z}}$$

skraćeno

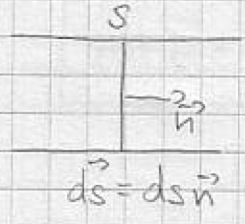
$$\boxed{\vec{E} = -\nabla \phi}$$

$\nabla \phi$ – gradijent ϕ

$$\boxed{E = -\text{grad}\, \phi}$$

Jakost i gustoća električne struje

$\vec{j}$ - vektor gustoće struje



$$I = \int_S \vec{j}\, d\vec{s}$$

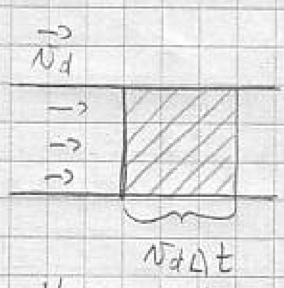
drift velocity

$N_g \quad \vec{v}_i \quad N_i$

$$\boxed{\vec{v}_d = \frac{1}{N_g} \sum_{i=1}^{N_g} N_i \vec{v}_i}$$

elektron u vodiču - oko 10^{-5} m/s

N_0, q



$$\Delta q = N_0 q v_d \Delta t$$

$$j = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = N_0 q v_d$$

$$\boxed{\vec{j} = N_0 q \vec{v}_d}$$ - iznos vektora gustoće struje

$N_0 q = S$

Magnetsko polje

$$\boxed{\vec{F}_m = q \vec{v} \times \vec{B}}$$

$[\vec{B}] = T$

u prostoru 2 polja $\vec{E}, \vec{B}$

$$\boxed{F = \vec{F}_e + \vec{F}_m = q\vec{E} + q\vec{v} \times \vec{B}}$$

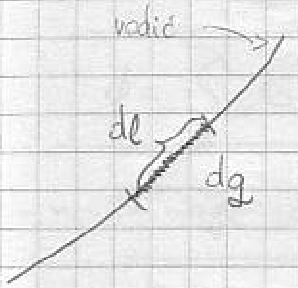
$\vec{F}_m \perp \vec{v}$ - sila ne vrši rad

$\hookrightarrow$ Lorentzova sila

Sila na vodič u magnetskom polju

homogeno mag. polje $\vec{B}$, I, $\vec{v}_d$, q, ρ, S

ρ – gustoća naboja, S – presjek vodiča



$$dF = \vec{v}_d \times B\, dq$$

$$dq = \rho S dl$$

$\vec{dl}$ – vektor $\vec{dl}$ ima iznos dl a smjer vektora $\vec{j}$

$$dF = \underbrace{\rho \vec{v}_d}_{\vec{j}} \times \vec{B} S dl$$

$$d\vec{F} = \vec{j} \times \vec{B} S dl$$

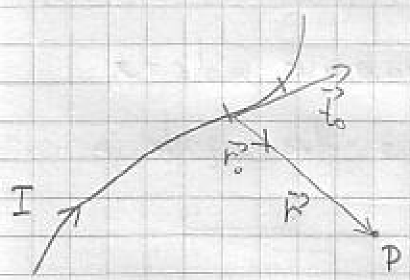
$$\underbrace{\vec{j} S}_{I} dl = I d\vec{l}$$

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

$$\boxed{\vec{F} = \int_A^B I d\vec{l} \times \vec{B}}$$

za stalnu struju $\vec{F} = I \int_A^B d\vec{l} \times \vec{B}$

Biot – Savart -ov zakon



$$d\vec{l} = dl\, \vec{t}_0 \qquad \vec{r} = r \vec{r}_0$$

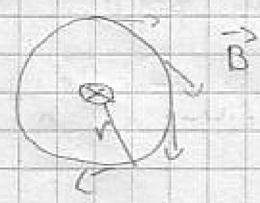
$d\vec{B}$

kroz – ako vodič protječe struja, ona stvara mag. polje

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}_0}{r^2}}$$

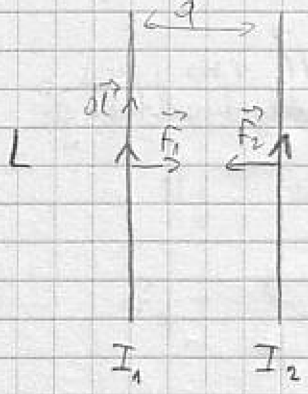
$$\boxed{B = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \vec{r}_0}{r^2}}$$

$$B = \frac{\mu_0 I}{2\pi r}$$



jakost mag. polja beskonačno dugačkog vodiča

Definiranje Ampera



$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

$$F_1 = \int_A^B I_1 B_2 \, dl = I_1 \frac{\mu_0 I_2}{2\pi d} \int_A^B dl =$$

$$= 2 \frac{\mu_0}{4\pi} \frac{I_1 I_2}{d} L$$

$$|\vec{F}_1| = |\vec{F}_2|$$

$$\frac{F_1}{L} = 2 \frac{\mu_0}{4\pi} \frac{I_1 I_2}{d}$$

$d = 1\,m$

$I_1 = I_2 = I$

i ako je sila $\frac{F}{L} = 2 \cdot 10^{-7}\, N m^{-1}$ onda kroz vodič teče struja od 1A

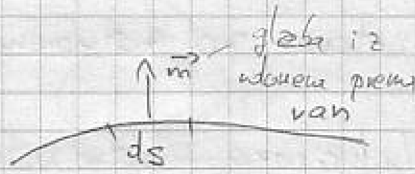
Jedan je amper jakost one stalne struje koja prolazeći kroz 2 ravna usporedna i neizmjerno dugačka vodiča zanemarivo malog kružnog presjeka u vakuumu međusobno udaljena 1m uzrokuje između njih silu od $2 \cdot 10^{-7}$ N/m.

II MAX. jna

Gaussov zakon za mag. polje

$$\boxed{\phi = \int_S \vec{B} \cdot d\vec{s}}$$

$$d\vec{s} = ds\,\vec{n}$$



- nema mag. monopola ⇒

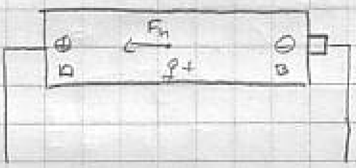
$$\boxed{\oint_S \vec{B}\, d\vec{s} = 0}$$

III MAX. jna

Faraday-ev zakon mag. indukcije

- elektromotorna sila ($\mathcal{E}$)

B2K



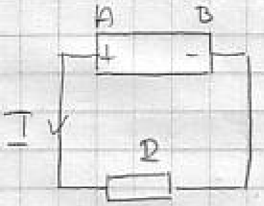
$$F_n = q\,\vec{E}_n$$

$$\frac{\vec{F}_n}{q} = \vec{E}_n$$

$\mathcal{E}$ - unutrašnji napon izvora jednak je radu koji obavi neelektrostatska sila pri prijenosu jediničnoga pozitivnog naboja unutar izvora od ⊖ do ⊕ pola.

$$\mathcal{E} = \int_B^A \frac{\vec{F}_n}{q}\, d\vec{l} = \int_B^A \vec{E}_n\, d\vec{l}$$

zatvoreni strujni krug



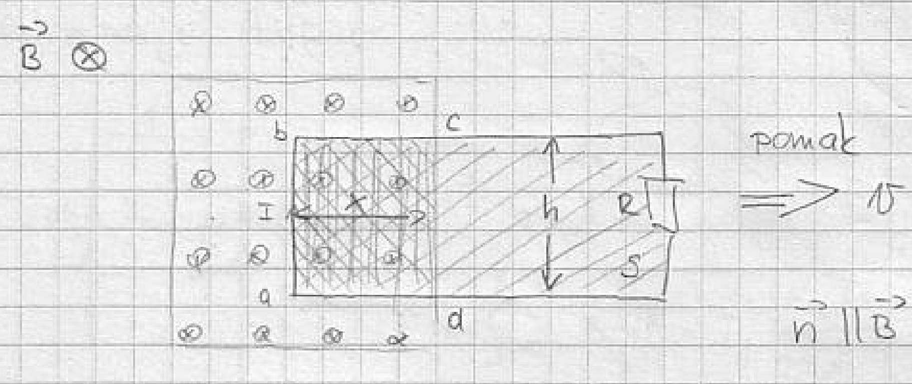
$$\vec{E} = \vec{E}_{EL} + \vec{E}_N$$

$$\oint \vec{E}\, d\vec{l} = \underbrace{\oint \vec{E}_{EL} \cdot d\vec{l}}_{=0} + \oint \vec{E}_N\, d\vec{l} = \int_B^A \vec{E}_N\, d\vec{l} = \mathcal{E}$$

$$\boxed{\mathcal{E} = \oint \vec{E}\, d\vec{l}}$$

EMS u strujnom krugu jednaka je krivuljnom integralu elek. polja uzetom po zatvorenom strujnom krugu

Inducirana elektromotorna sila



$$\mathcal{E} = \oint \vec{E}\, d\vec{l} = \int_A^B vB\, dl = vBh$$

$$\frac{F}{q} = vB$$

$$\mathcal{E} = vBh$$

$$\phi = \int \vec{B}\, d\vec{S} = B \int dS = Bhx$$

$$\frac{d\phi}{dt} = -Bh \underbrace{\frac{dx}{dt}}_{v} = -Bhv$$

$$\boxed{\mathcal{E} = -\frac{d\phi}{dt}}$$ - pravilo toka

Elektromagnetska indukcija - II max jed

1831 - M. Faraday

3 vrste pokusa

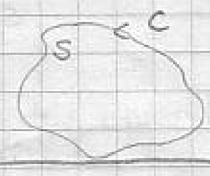
1) - vodič se giba u mag. polju - ind. se struja

2) - zavojnica i magnet, gibamo magnet inducira se struja
magnet stoji, vodič gibamo

3) - 2 zavojnice, kroz 1. protječe struja, u drugoj se inducira struja

- promjenjivo mag. polje inducira el. polje

$$\mathcal{E} = \oint \vec{E}\, d\vec{l} = -\frac{d\phi}{dt}$$



$$\boxed{\oint_C \vec{E}\, d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\, d\vec{s}}$$

$\boxed{\oint_C \vec{E}\, d\vec{l} = 0}$ - okolo nema promjenjivog mag. polja

$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ - dif. oblik $\quad \mathrm{rot}\, \vec{E} = \frac{\partial \vec{B}}{\partial t}$

- univerzalno pravilo toka

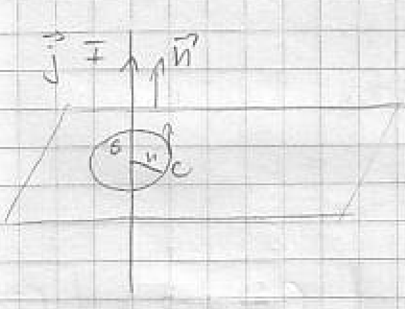
$$\boxed{\mathcal{E} = -\frac{d\phi}{dt}}$$

Lenzovo pravilo

- inducirana el. struja ima takav smjer da proizvodi mag tok koji se suprotstavlja promjeni toka zbog kojega je nastala

IV MAX jed (poopćeni amperov zakon)

amperov zakon



$$B = \frac{\mu_0 I}{2\pi r}$$

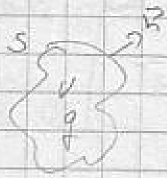
$$\oint_C \vec{B}\, d\vec{l} = \frac{\mu_0 I}{2\pi r} \oint_C dl = \frac{\mu_0 I}{2\pi r} \cdot 2\pi r = \mu_0 I =$$

$$= \mu_0 \int_S \vec{j}\, d\vec{s}$$

$\boxed{\oint_C \vec{B}\, d\vec{l} = \mu_0 \int_S \vec{j}\, d\vec{s}}$ - amperov zakon

$$\oint_c \vec{B}\,d\vec{l} = \mu_0 \int_{S_1} \vec{j}\,d\vec{s} + (?)$$

zakon održanja naboja u int. obliku (Z.O.N.)



$$q = \int_V \rho\, dV$$

naboj se mijenja:

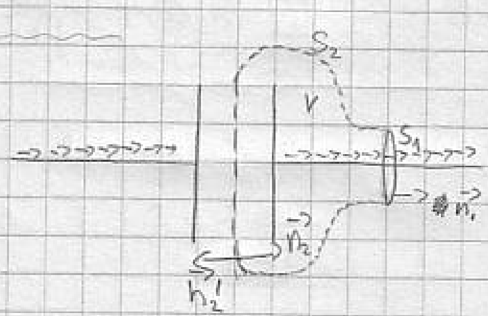
$$I = \int_S \vec{j}\,d\vec{s}$$

$$\frac{dq}{dt} = -I$$

$$\boxed{\frac{d}{dt}\int_V \rho\, dV = -\int_S \vec{j}\,d\vec{s}}$$

$$\boxed{\int_V \frac{\partial \rho}{\partial t}\, dV = -\int_S \vec{j}\,d\vec{s}}$$

$$\boxed{\operatorname{div}\vec{j} = -\frac{\partial \rho}{\partial t}}$$ - zakon očuvanja naboja u dif. obliku



$$\vec{n}_2' = -\vec{n}_2$$

$$S = S_1 \cup S_2$$

$$q = \int_V \rho\, dV = \varepsilon_0 \int_S \vec{E}\,d\vec{s}$$

primjenjujemo I. M. J.

$$\frac{dq}{dt} = \varepsilon_0 \frac{d}{dt}\int_S \vec{E}\,d\vec{s} = \underbrace{\varepsilon_0 \frac{d}{dt}\int_{S_1} \vec{E}\,d\vec{s}}_{0} + \varepsilon_0 \frac{d}{dt}\int_{S_2} \vec{E}\,d\vec{s} = \varepsilon_0 \frac{d}{dt}\int_{S_2} \vec{E}\,\vec{n}_2'\,ds$$

primjenjujemo Z. O. N.

$$\frac{dq}{dt} = -I = -\int_{S_1} \vec{j}\,d\vec{s}$$

$$-\int_{S_1} \vec{j}\, d\vec{s} = -\varepsilon_0 \frac{d}{dt} \int_{S_2} \vec{E}\, \vec{n}_2\, ds$$

$$\boxed{\oint_C \vec{B}\, d\vec{l} = \mu_0 \int_{S_1} \vec{j}\, d\vec{s} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_{S} \vec{E}\, d\vec{S}}$$ – IV Max. jed u int obliku

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$ – dif. oblik

Max jed u dif obliku:

$$\boxed{\begin{aligned} &\operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0} \\ &\operatorname{div} \vec{B} = 0 \\ &\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ &\operatorname{rot} \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

u vakuumu:

$$\boxed{\begin{aligned} &\rho = 0 \quad \vec{j} = 0 \\ &\operatorname{div} \vec{E} = 0 \\ &\operatorname{div} \vec{B} = 0 \\ &\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ &\operatorname{rot} \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

Elektromagnetski valovi

- promjena elek. i mag. polja koju proizvodi promjenjiva struja i giba se naziva SE EMV

$$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Big/ \nabla$$

$$\mathrm{rot}\,\vec{B} = \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \nabla\times\vec{B} = \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t} \quad \Big| \frac{\partial}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\nabla\times\vec{B}\right) = \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

4 MAX. j $\nabla\times\left(\frac{\partial \vec{B}}{\partial t}\right) = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$

3 MAX. j $\nabla\times\nabla\times\vec{E} = -\nabla\times\left(\frac{\partial\vec{B}}{\partial t}\right)$

$$-\Delta\vec{E} = -\nabla\times\left(\frac{\partial \vec{B}}{\partial t}\right) = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\nabla\times\nabla\times\vec{E} = \nabla\left(\nabla\vec{E}\right) - \Delta\vec{E}$$

$$\mathrm{rot}(\mathrm{rot}\,\vec{E}) = \mathrm{grad}\,\mathrm{div}\,\vec{E} - \Delta\vec{E}$$

$$\Delta = \nabla\cdot\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\Delta\vec{E} = \left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right)\vec{i} + \dots$$

$$\boxed{\Delta\vec{E} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0}$$ - valna jed. elek. polja.

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

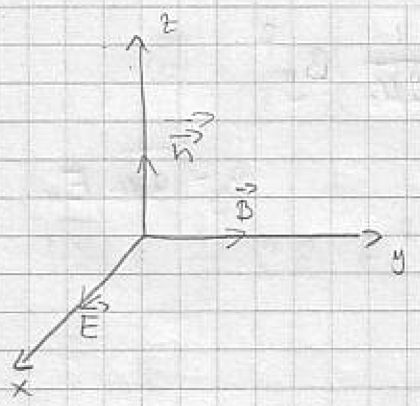
- elek. polje zadovoljava valnu jed.

$$\boxed{v = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 2.998\cdot 10^8\ \mathrm{m/s}}$$ - svjetlost je EM val

analogni postupak: $$\boxed{\Delta\vec{B} - \mu_0\varepsilon_0\frac{\partial\vec{B}}{\partial t} = 0}$$

$$\lambda = \frac{v}{f} = \frac{c}{f} = \frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r\mu_r}}$$

vjsežujc jed. za harmonijski ravni EM val



- monokromatski val

ravni - $\underbrace{\vec{E}(x, y}_{}, z, t) \Rightarrow \vec{E}(z, t)$

keno ovisnost $o\, x, y$
je ino ravni val, a
svije širenja je u smjeru
z osi

$$\vec{E} = E_x(z, t)\,\vec{i}$$

$$E_x(z, t) = E_0 \sin(\omega t - kz)$$

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} - \varepsilon_0 \mu_0 \frac{\partial^2 E_x}{\partial t^2} = 0}$$ - elek polje

$$\omega = \frac{c}{k}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} \Rightarrow \frac{\partial E_x}{\partial z}\vec{j} = -\frac{\partial B_y}{\partial t}\vec{j}$$

$$-k E_0 \cos(\omega t - kz) = -\frac{\partial B_y}{\partial y}$$

$$\Rightarrow B_y = \frac{E_0}{c}\sin(\omega t - kz) \qquad \boxed{B_0 = \frac{E_0}{c}}$$

$$B = B_0 \sin(\omega t - kz)\,\vec{j}$$

$$\boxed{\frac{E_0}{B_0} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

$\vec{E} \times \vec{B}$ kolinearan sa $\vec{n}$

$$\boxed{\vec{B} = \frac{1}{c}\vec{n} \times \vec{E} = \sqrt{\mu_0 \varepsilon_0}\,\vec{n} \times \vec{E}}$$

$$\boxed{\vec{E} = c\,\vec{B} \times \vec{n} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\vec{B} \times \vec{n}}$$

Energija EM vala

- gustoća energije EM vala (w)

$$w = w_E + w_M = \frac{1}{2} \varepsilon_0 E_x^2 + \frac{1}{2\mu_0} B_y^2$$

$$B_y = \sqrt{\varepsilon_0 \mu_0}\, E_x$$

$$w_M = \frac{1}{2\mu_0} B_y^2 = \frac{1}{2\mu_0} \cdot \varepsilon_0 \mu_0 E_x^2$$

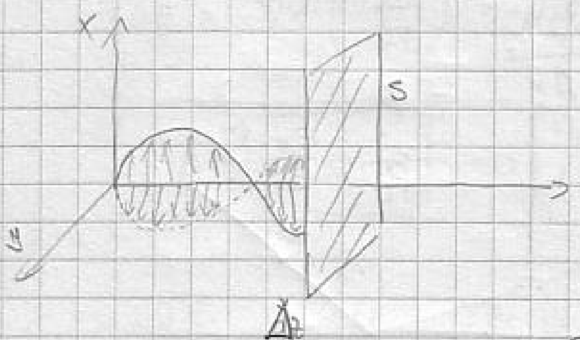
$$= \frac{1}{2} \varepsilon_0 E_x^2 = w_E$$

$$\boxed{w = \varepsilon_0 E_x^2} \qquad \boxed{w = \frac{1}{\mu_0} B_y^2}$$

- intenzitet EM vala (gustoća toka)

pisano S — $\vec{S}$ - poyntingov vektor

$|\vec{S}|$ = intenzitet



$$\Delta z = c \Delta t$$

$$w \cdot \Delta V = \varepsilon_0 E_x^2 \Delta z S = \varepsilon_0 E_x^2 S c \Delta t$$

$$|S| = \lim_{\Delta t \to 0} \frac{\varepsilon_0 E_x^2 S c \Delta t}{S \Delta t} = c \varepsilon_0 E_x^2$$

$$|S| = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \varepsilon_0 E_x \frac{B_y}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\mu_0} E_x B_y$$

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}$$

prosječna gustoća energije

$$\bar{w} = \frac{1}{T}\int_0^T \varepsilon_0 E_x^2 dt = \frac{1}{2}\varepsilon_0 E_0^2$$

$$|\bar{S}| = \frac{1}{2} c\varepsilon_0 E_0^2 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2$$

Geometrijska optika

zakon pravocrtnog širenja svjetlosti

- svjetlost se u homogenom i izotropnom sredstvu širi pravocrtno

zakon neovisnosti svjetlosnih snopova

- 2 snopa se presijecaju. Jedan na drugog ne utječe, šire se kao da onaj drugi ne postoji

zakon refleksije svjetlosti

-

zakon loma svjetlosti (Snellov zakon)

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{n_2}{n_1} \qquad \boxed{n = \frac{c}{v}}$$

$n_1, v_1 \qquad n_2, v_2$

$v_1 = f\lambda_1 \qquad v_2 = f\lambda_2$

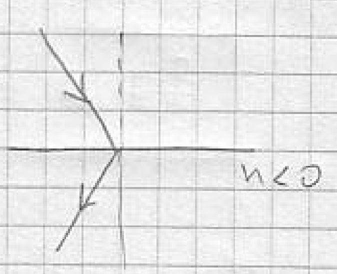
$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\frac{c}{n_1}}{\frac{c}{n_2}} = \frac{n_2}{n_1}$$

$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}}$$

$$n = \frac{\frac{1}{\sqrt{\varepsilon_0 \mu_0}}}{\frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_r \mu_0}}} = \sqrt{\varepsilon_r \mu_r}$$

$$\boxed{n = \sqrt{\varepsilon_r \mu_r}}$$

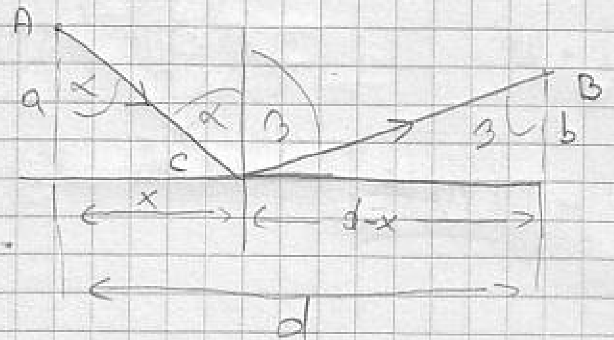
metamaterijali - indeks loma manji od 0



Fermatov princip

- vrijeme t_{AB} potrebno zraci svjetlosti da u sredstvu u kojem se giba brzina $v = \frac{c}{n}$ prijeđe udaljenost $\overline{AB}$ minimalno

$$\boxed{t_{AB} = \int_A^B \frac{n}{c}\, dl}$$



n=konst

$$t_{AB} = t_{AC} + t_{CB} = n\frac{\overline{AC}}{c} + n\frac{\overline{CB}}{c} = \frac{n}{c}\left(\sqrt{a^2+x^2} + \sqrt{b^2+(d-x)^2}\right)$$

$$\frac{d t_{AB}}{dt} = 0$$

$$\frac{n}{c}\left(\frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}\right) = 0$$

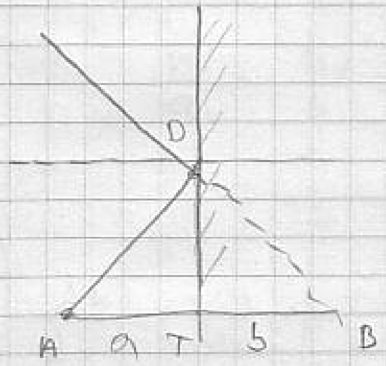
$$\frac{x}{\sqrt{a^2+x^2}} = \frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$\sin\alpha = \sin\beta$$

$\boxed{\alpha=\beta}$ – zakon refleksije svjetlosti

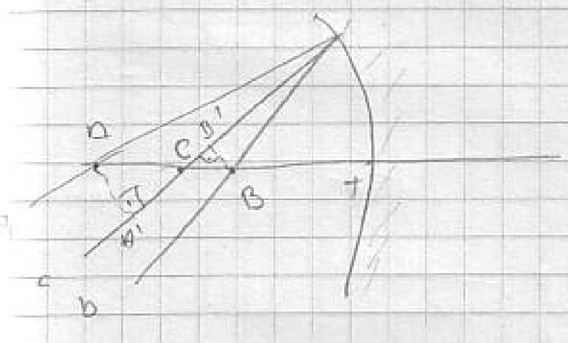
Refleksija na ravnom i sfernom zrcalu

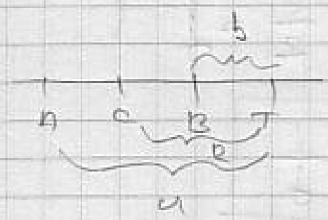
ravno zrcalo



$$\boxed{a=-b}$$

Sferno zrcalo





$\overline{AT}=a$

$\overline{BT}=b$

$\overline{CT}=R$

Gaussove aproksimacije

$$\frac{\overline{AC}}{\overline{AT}} = \frac{\overline{BC}}{\overline{BT}}$$

$$\frac{(a-R)}{a} = \frac{R-b}{b} \Rightarrow$$

$$\boxed{\frac{1}{a}+\frac{1}{b}=\frac{2}{R}}$$

konvencije

$\rightarrow$ $a>0$ $b>0$ | $a<0$ $b<0$

$R_1>0$ | $R_2<0$

$R \to \infty$ - kod ravnog zrcala

$$\frac{1}{a}+\frac{1}{b}=0$$

$$\boxed{a=-b}$$

predmetno zarište F_a

$a=f_a \qquad b=\infty$

$$\boxed{f_a=\frac{R}{2}}$$

slikovno zarište F_b

$a=\infty \qquad b=f_b$

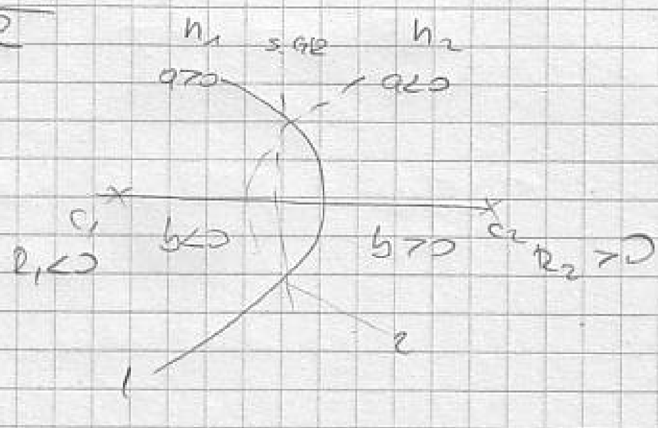
$$\boxed{f_b=\frac{R}{2}}$$

$$f_a=f_b=\frac{R}{2}=f$$

$$\boxed{\frac{1}{a}+\frac{1}{b}=\frac{1}{f}}$$

sferni dioptar

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$



$R \to \infty$ - ravni dioptar

$$\frac{n_1}{a} + \frac{n_2}{b} = 0$$

$$\boxed{f_a = \frac{n_1}{n_2 - n_1} R}$$

$$\boxed{f_b = \frac{n_2}{n_2 - n_1} R}$$

$$\boxed{\frac{f_b}{f_a} = \frac{n_2}{n_1}}$$

$$\boxed{f_b - f_a = R}$$

$$|m| = \frac{y'}{y}$$