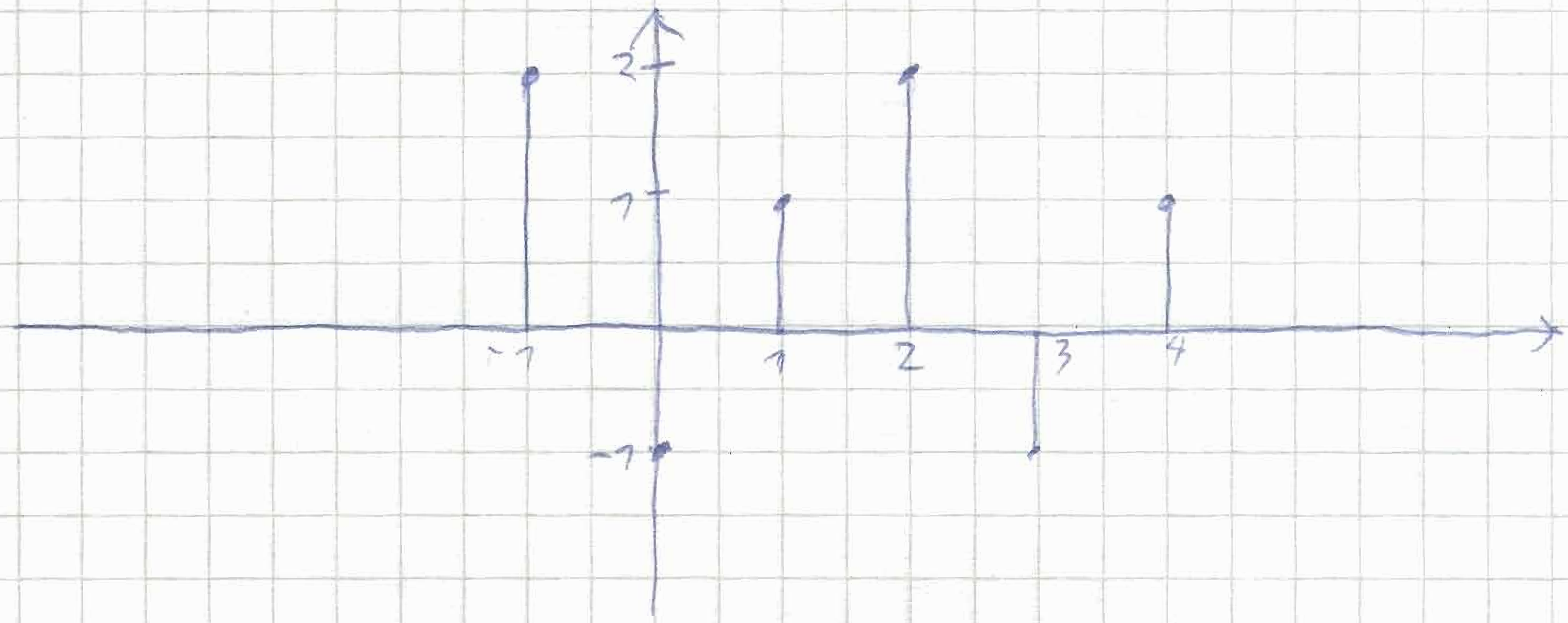
Z TRANSFORMACIJA

$$Z\{h(n)\} = \sum_{k=-\infty}^{\infty} h(k)\, z^{-k} = H(z)$$



$$H(z) = 2\cdot \cancel{z^{-(-1)}} - 1\cdot z^{0} + 1\cdot z^{-1} + 2\cdot z^{-2} - 1\cdot z^{-3} + 1\cdot z^{-4}$$

ako maknemo onda je signal kauzalan

$$u(n) = \mu(n)$$

$$U(z) = Z\{u(n)\} = \underbrace{1}_{z^{0}} + z^{-1} + z^{-2} + z^{-3} + \dots = \frac{1}{1-z^{-1}}$$

$$\sum_{k=0}^{\infty} x^{n} = \frac{1}{1-x} \qquad |x| < 1$$

$$x = z^{-1}$$

$$= \frac{1}{1-z^{-1}} \cdot \frac{z}{z} = \frac{z}{z-1} \qquad |z^{-1}| < 1$$

$$|z| > 1$$

$u(z)$	$U(z)$
$\delta(n)$	1
$\delta(n-m)$	z^{-m}
$\mu(n)$	$\frac{z}{z-1}$
$n \cdot \mu(n)$	$\frac{z}{(z-1)^2}$
$a^n \mu(n)$	$\frac{z}{z-a}$

$\boxed{\text{ZAD}}$

$$6y(n) + y(n-1) - y(n-2) = u(n) - u(n-2)$$

$$E^{-1}y(n) = y(n-1)$$

$$6y(n) + E^{-1}y(n) - E^{-2}y(n) = u(n) - E^{-2}u(n)$$

$$y(n)\Big[\underbrace{6 + E^{-1} - E^{-2}}_{A(E)}\Big) = u(n)\underbrace{\Big[1 - E^{-2}\Big]}_{B(E)} \quad / : A(E)$$

$$y(n) = \underbrace{\frac{B(E)}{A(E)}}_{H(E)} u(n)$$

PRIJENOSNA FUNK.

$H(z)$

$$y(n) = \frac{1 - z^{-2}}{6 + z^{-1} - z^{-2}} \begin{matrix} / \cdot z^2 \\ / \cdot z^2 \end{matrix} u(n) =$$

$$H(z) = \frac{z^2 - 1}{6z^2 + z - 1} = \frac{1}{6} \frac{z^2 - 1}{z^2 + \frac{1}{6}z - \frac{1}{6}}$$

$$u(n) = C \cdot z^n \mu(n)$$

$$z^2 - 1 = 0$$

$$z_{1,2} = \pm 1$$

$$p^2 + \frac{1}{6} p - \frac{1}{6} = 0$$

$$p_{1,2} = \frac{-\frac{1}{6} \pm \sqrt{\frac{1}{36} + \frac{4}{6}}}{2} = -\frac{1}{12} \pm \frac{5}{12}$$

$$p_1 = \frac{1}{3} \qquad p_2 = -\frac{1}{2}$$

$$H(z) = \frac{1}{6} \frac{(z-1)(z+1)}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)}$$ sustav je stabilan

STABILAN SUSTAV $|p| < 1$

$$p = \alpha + j\beta$$

$$|p| = \sqrt{\alpha^2 + \beta^2}$$

$$Z\{h(n)\} = H(z)$$

$$\frac{s+2}{s^2+3s+4} = \frac{A}{(s-s_1)} + \frac{B}{(s-s_2)}$$

$$H(z) = \frac{1}{6}\,\frac{z^2-1}{z^2+\frac{1}{6}z-\frac{1}{6}} = \frac{1}{6}\,\frac{z^2-1}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)}$$

$$= C_1 + \frac{C_2\,z}{\left(z-\frac{1}{3}\right)} + \frac{C_3\,z}{\left(z+\frac{1}{2}\right)}$$

$$C_1 = H(0) = \frac{1}{6}\cdot\frac{-1}{-\frac{1}{6}} = 1$$

$$C_2 = \lim_{z\to\frac{1}{3}}\left[\frac{z-\frac{1}{3}}{z}\cdot H(z)\right] = \lim_{z\to\frac{1}{3}}\left[\frac{\cancel{\left(z-\frac{1}{3}\right)}}{z}\cdot\frac{1}{6}\cdot\frac{z^2-1}{\cancel{\left(z-\frac{1}{3}\right)}\left(z+\frac{1}{2}\right)}\right]$$

$$= 3\cdot\frac{1}{6}\,\frac{\frac{1}{9}-1}{\frac{1}{3}+\frac{1}{2}} = \frac{1}{2}\cdot\frac{-\frac{8}{9}}{} = -\frac{8}{15}$$

$$C_3 = \lim_{z\to-\frac{1}{2}}\left[\frac{\cancel{z+\frac{1}{2}}}{z}\cdot\frac{1}{6}\cdot\frac{z^2-1}{\left(z-\frac{1}{3}\right)\cancel{\left(z+\frac{1}{2}\right)}}\right]$$

$$= -2\cdot\frac{1}{6}\cdot\frac{\frac{1}{4}-1}{-\frac{1}{2}-\frac{1}{3}} = -\frac{3}{10}$$

$$H(z) = 1 - \frac{8}{15}\,\frac{z}{\left(z-\frac{1}{3}\right)} - \frac{3}{10}\,\frac{z}{\left(z+\frac{1}{2}\right)}$$

$$h(n) = Z^{-1}\{H(z)\} = \delta(n) - \frac{8}{15}\left(\frac{1}{3}\right)^n\mu(n) - \frac{3}{10}\left(-\frac{1}{2}\right)^n\mu(n)$$

$$y(n) = h(n) * u(n)$$

$$Y(z) = H(z) \cdot U(z)$$

u zadatku je $U(z) = 1$

$\boxed{ZAD}$

$$F(z) = \frac{6z^2 - 14z + 6}{(z-1)(z-2)}$$

$$= C_1 + \frac{C_2 z}{(z-1)} + \frac{C_3 z}{(z-2)}$$

$$C_1 = F(z)\Big|_{z=0} = 3$$

$$C_2 = \lim_{z \to 1} \left[\frac{\cancel{z-1}}{z} \frac{6z^2 - 14z + 6}{\cancel{(z-1)}(z-2)} \right] = 2$$

$$C_3 = \lim_{z \to 2} \left[\frac{\cancel{z-2}}{z} \frac{6z^2 - 14z + 6}{(z-1)\cancel{(z-2)}} \right]$$

$$= \frac{1}{2} \frac{24 - 28 + 6}{1} = 1$$

$$F(z) = 3 + \frac{2z}{z-1} + \frac{z}{z-2}$$

$$f(n) = 3\,\delta(n) + 2\,1^n \mu(n) + 2^n \mu(n)$$

$\boxed{ZAD}$ $$F(z) = \frac{4z^3 - 18z^2 + 21z}{(z-2)(z-3)^2}$$

$$= c_1 + \frac{c_2 z}{z-2} + \frac{c_3 z}{z-3} + \frac{c_4 z^2}{(z-3)^2}$$

$$c_1 = F(z)\Big|_{z=0} = \frac{0}{6} = 0$$

$$c_2 = \lim_{z\to 2}\left[\frac{z-2}{z}\,\frac{4z^3-18z^2+21z}{\cancel{(z-2)}(z-3)^2}\right]$$

$$= \frac{1}{2}\,\frac{32-72+42}{1} = 1$$

$$c_{3(2)} = c_4 = \lim_{z\to 3}\left[\frac{\cancel{(z-3)^2}}{z^2}\,\frac{4z^3-18z^2+21z}{(z-2)(z-3)^2}\right] = 1$$

c_{nk} k = KRATNOS

$$c_{3(1)} = \lim_{z\to 3}\left[\frac{(z-3)^2}{z^2}\,\frac{(4z^3-18z^2+21z)}{(z-2)(z-3)^2}\right]^{(2-1)}$$

$$c_{nj} = \frac{1}{(k-j)!}\lim_{z\to p}\left[\frac{(z-p)^k}{z}H(z)\right]^{(k-j)}$$ derivacija

$j=1$ prva derivacija nultočke

$$c_3 = \frac{1}{(2-1)!}\lim_{z\to 3}\left[\left[\frac{\cancel{(z-3)}^2}{z}\cdot\frac{4z^3-18z^2+21z}{(z-2)\cancel{(z-3)^2}}\right]^{(2-1)}\right]$$

$$\frac{4z^3-18z^2+21z}{z^3-2z^2} = \frac{(12z^2-36z+21)(z^3-2z^2)-(4z^3-18z^2+21z)(3z^2-4z)}{(z^3-2z^2)^2}$$

$\boxed{ZAD}$

$$F(z) = \frac{10z^3 - 37z^2 + 24z + 18}{z(z-3)^2}$$

$$= c_1 + \frac{c_2}{z} + \frac{c_{3(1)}\, z}{z-3} + \frac{c_{3(2)}\, z^2}{(z-3)^2}$$

$$c_2 = \lim_{z\to 0}\left(\cancel{z}\cdot\frac{10z^3-37z^2+24z+18}{\cancel{z}(z-3)^2}\right) = \frac{18}{9} = 2$$

$$c_{3(1)} = \frac{1}{(2-1)!}\lim_{z\to 3}\left[\frac{(z-3)^2}{z}\cdot\frac{10z^3-37z^2+24z+18}{z(z-3)^2}\right]^{(2-1)} = 5$$

$$c_{3(2)} = \frac{1}{(2-2)!}\lim_{z\to 3}\left[\frac{(z-3)^2}{z}\quad H(z)\right] = 1$$

$$F(z) = c_1 + \frac{2}{z} + \frac{5z}{z-3} + \frac{z^2}{(z-3)^2}$$

$$= \frac{c_1 z(z-3)^2 + 2(z-3)^2 + 5z^2(z-3) + z^3}{z(z-3)^2}$$

$$c_1 z(z^2-6z+9) = c_1 z^3 - 6c_1 z^2 + 9c_1 z$$

$$= \frac{c_1 z(z-3)^2 + 2z^2 - 12z + 18 + 5z^3 - 15z^2 + z^3}{z(z-3)^2}$$

$$= z^3(c_1 + 6) + zz(6c_1 + 2 - 15) + z(9c_1 - 12)$$

$$10 = c_1 + 6$$

$$c_1 = 4$$

$$z\cdot\frac{z}{(z-3)^2} \Rightarrow \delta(n-1) * n3^n\mu(n) = (n+1)3^n\mu(n+1)$$

množenje u z-domeni = konvolucija u n-domeni

$$Z: \quad \frac{z}{(z-a)^2} = \delta(n+1) * \left[n (a)^n \mu(n) \right]$$

$$= (n+1)\, a^n\, \mu(n+1)$$

$\boxed{\text{ZAD}}$ $\quad 6y(n) + y(n-1) - y(n-2) = u(n) - u(n-2)$

$$H(z) = \frac{1}{6} \, \frac{(z-1)(z+1)}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)}$$

$$u(n) = (n+1)\,\mu(n)$$

$$y(-1) = 1 \qquad\qquad y(-2) = -1$$

$$u(n) = n\,\mu(n) + \mu(n)$$

$$U(z) = Z\{u(n)\} = \frac{z}{(z-1)^2} + \frac{z}{z-1}$$

$$\boxed{\begin{aligned} Z\{y(n-1)\} &= z^{-1} Y(z) + y(-1) \\ Z\{y(n-2)\} &= z^{-2} Y(z) + z^{-1} y(-1) + y(-2) \end{aligned}}$$

$$6Y(z) + z^{-1}Y(z) + y(-1) - z^{-2}Y(z) - z^{-1}y(-1) - y(-2)$$

$$= U(z) - z^{-2}U(z) \underbrace{+ z^{-1}u(-1) + u(-2)}_{\text{jer je pobuda kauzalna}}$$

$$Y(z)\left(6 + z^{-1} - z^{-2}\right) = U(z)\left(1 - z^{-2}\right) + \left[-y(-1) + z^{-1}y(-1) + y(-2)\right]$$

$$Y(z) = \underbrace{\frac{1 - z^{-2}}{6 + z^{-2} - z^{-2}} U(z)}_{\substack{\text{ODZIV MIRNOG} \\ \text{SUSTAVA}}} + \underbrace{\frac{-y(-1) + z^{-1} y(-1) + y(-2)}{6 + z^{-1} - z^{-2}}}_{\substack{Y_0 \\ \text{ODZIV NEPOBUĐENOG} \\ \text{SUSTAVA}}}$$

(nema poč. uvjeta)

ne djelujemo pobudom a ima zadane poč uvjete

$$Y(z) = Y_m(z) + Y_0(z)$$

$$Y_m(z) = H(z) \cdot U(z)$$

$$U(z) = Z\{u(n)\} = \frac{z}{(z-1)^2} + \frac{z}{z-1} = \frac{z^2}{(z-1)^2}$$

$$Y_m(z) = \frac{1}{6} \frac{\cancel{(z-1)}(z+1)}{(z-\frac{1}{3})(z+\frac{1}{2})} \cdot \frac{z^2}{(z-1)^{\cancel{2}}}$$

$$= \frac{1}{6} \frac{z^2 (z+1)}{(z-\frac{1}{3})(z+\frac{1}{2})(z-1)}$$

$$Y_0(z) = \frac{-y(-1) + z^{-1} y(-1) + y(-2)}{6 + z^{-1} - z^{-2}} = \frac{-2 + z^{-1}}{6 + z^{-1} - z^{-2}} \begin{matrix} / \cdot z^2 \\ / \cdot z^2 \end{matrix}$$

$$= \frac{-2z^2 + z}{6z^2 + z - 1} = \frac{-2(z^2 - \frac{1}{2}z)}{6(z^2 + \frac{1}{6}z - \frac{1}{6})}$$

$$= -\frac{1}{3} \frac{z(z - \frac{1}{2})}{(z-\frac{1}{3})(z+\frac{1}{2})}$$

$$Y_m(z) = \frac{1}{6} \frac{z^2(z+1)}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)(z-1)}$$

$$= C_1 + \frac{C_2 z}{z-\frac{1}{3}} + \frac{C_3 z}{z+\frac{1}{2}} + \frac{C_4 z}{z-1}$$

$$C_1 = Y_m(0) = 0$$

$$C_2 = \lim_{z \to \frac{1}{3}} \left[\frac{\cancel{z-\frac{1}{3}}}{\cancel{z}} \frac{1}{6} \frac{z^{\cancel{2}}(z+1)}{\cancel{\left(z-\frac{1}{3}\right)}\left(z+\frac{1}{2}\right)(z-1)} \right]$$

$$= \frac{1}{\cancel{6}} \frac{\frac{1}{\cancel{3}} \cdot \frac{4}{3}}{\frac{5}{\cancel{6}} \cdot \left(-\frac{2}{3}\right)} = -\frac{2}{15}$$

$\frac{3}{6}$ -

$$C_3 = \lim_{z \to -\frac{1}{2}} \left[\frac{\cancel{z+\frac{1}{2}}}{\cancel{z}} \cdot \frac{1}{6} \frac{z^{\cancel{2}}(z+1)}{\left(z-\frac{1}{3}\right)\cancel{\left(z+\frac{1}{2}\right)}(z-1)} \right]$$

$$= \frac{1}{6} \frac{-\frac{1}{2} \cdot \frac{3}{2}}{\frac{5}{6}\left(+\frac{3}{2}\right)} = -\frac{1}{30}$$

$$C_4 = \lim_{z \to 1} \left[\frac{\cancel{z-1}}{\cancel{z}} \cdot \frac{1}{6} \cdot \frac{z^{\cancel{2}}(z+1)}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)\cancel{(z-1)}} \right] = \frac{1}{3}$$

$$Y_m(z) = -\frac{2}{15} \frac{z}{\left(z-\frac{1}{3}\right)} - \frac{1}{30} \frac{z}{\left(z+\frac{1}{2}\right)} + \frac{1}{3} \frac{z}{(z-1)}$$

$$y_m(n) = Z^{-1}\{Y_m(z)\} = -\frac{2}{15}\left(\frac{1}{3}\right)^n \mu(n) - \frac{1}{30}\left(-\frac{1}{2}\right)^n \mu(n) + \frac{1}{3} 1^n \mu(n)$$

$$Y_0(z) = -\frac{1}{3}\,\frac{z\left(z-\frac{1}{2}\right)}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)}$$

$$Y_{01}(z) = \frac{Y_0(z)}{z} = -\frac{1}{3}\,\frac{\left(z-\frac{1}{2}\right)}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)}$$

$$F(z) = \frac{A}{z-r_1} + \frac{B}{z-r_2}$$

$$Y_{01}(z) = -\frac{1}{3}\left[\frac{c_1}{\left(z-\frac{1}{3}\right)} + \frac{c_2}{z+\frac{1}{2}}\right]$$

$$= -\frac{1}{3}\left[\frac{c_1 z + \frac{1}{2}c_1 + c_2 z - \frac{1}{3}c_2}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)}\right]$$

$$= -\frac{1}{3}\,\frac{z(c_1+c_2) + \left(\frac{1}{2}c_1 - \frac{1}{3}c_2\right)}{\left(z-\frac{1}{3}\right)\left(z+\frac{1}{2}\right)}$$

$$c_1 + c_2 = 1 \qquad \frac{1}{2}c_1 - \frac{1}{3}c_2 = -\frac{1}{2}$$

$$c_1 = 1 - c_2$$

$$3c_1 - 2c_2 = -3$$

$$3 - 3c_2 - 2c_2 = -3$$

$$-5c_2 = -6$$

$$c_2 = \frac{6}{5}$$

$$c_2 = \frac{6}{5} \qquad c_1 = -\frac{1}{5}$$

$$Y_0(z) = z\,Y_{01}(z) = -\frac{1}{5}\cdot\frac{z}{\left(z-\frac{1}{3}\right)} + \frac{6}{5}\,\frac{z}{\left(z+\frac{1}{2}\right)}$$

$$y_0(n) = Z^{-1}\{Y_0(z)\} = \frac{1}{15}\left(\frac{1}{3}\right)^n \mu(n) - \frac{2}{15}\left(-\frac{1}{2}\right)^n \mu(n)$$

$$y(n) = y_m(n) + y_0(n)$$

$$= \underbrace{-\frac{1}{15}\left(\frac{1}{3}\right)^n \mu(n) - \frac{1}{6}\left(-\frac{1}{2}\right)^n \mu(n)}_{\text{PRIRODNI ODZIV}} + \underbrace{\frac{1}{3}\mu(n)}_{\text{PRISILNI}}$$

$$y_{STAC}(n) = \lim_{n \to \infty} y(n) = \frac{1}{3}\mu(n) = \frac{1}{3}$$

STACIONARNI ODZIV

KONTINUIRANI SUSTAVI

$$y''(t) + a_1 y'(t) + a_2 y(t) = b_0 u''(t) + b_1 u'(t) + b_2 u(t)$$

D - operator deriviranja

$$D y(t) = y'(t)$$

$$D^2 y(t) + a_1 D y(t) + a_2 y(t) = b_0 D^2 u(t) + b_1 D u(t) + b_2 u(t)$$

$$y(t)\underbrace{[D^2 + a_1 D + a_2]}_{A(D)} = u(t)\underbrace{[D^2 b_0 + b_1 D + b_2]}_{B(D)}$$

$$y(t) = \underbrace{\frac{B(D)}{A(D)}}_{H(D)} u(t)$$

$$H(D) \Rightarrow H(s)$$

$$u(t) = c e^{st}$$

$$D u(t) = s \underbrace{c e^{st}}_{u(t)}$$

$$H(s) = \underbrace{b_0}_{K} \frac{s^2 + b_1 s + b_2}{s^2 + a_1 s + a_2}$$

$$\mathcal{L}\{u(t)\} = \int_0^\infty e^{-st} u(t)\, dt$$

LAPLACEOVA TRANSFORMACIJA

$u(t)$	$U(s)$
$\delta(t)$	1
$\delta(t-t_0)$	e^{-st_0}
$\mu(t)$	$\frac{1}{s}$
$t\,\mu(t)$	$\frac{1}{s^2}$
$t^k\,\mu(t)$	$\frac{k!}{s^{k+1}}$
$e^{at}\,\mu(t)$	$\frac{1}{s-a}$
$te^{at}\,\mu(t)$	$\frac{1}{(s-a)^2}$

ZAD

$$F(s) = \frac{7s^2 - 5s - 6}{(s+1)(s-2)(s-1)}$$

$$F(s) = C_1 + \frac{C_2}{s+1} + \frac{C_3}{s-2} + \frac{C_4}{s-1}$$

$$C_1 = \lim_{s\to\infty} F(s) = \lim_{s} \frac{7s^2}{s^3} = 0$$

$$C_2 = \lim_{s\to -1}\left[(s+1)\,F(s)\right] = \lim_{s\to -1}\left[\frac{\cancel{(s+1)}(7s^2-5s-6)}{\cancel{(s+1)}(s-2)(s-1)}\right]$$

$$= \frac{7+5-6}{+3\cdot 2} = \frac{+6}{6} = 1$$

$$C_3 = \lim_{s\to 2}\left[\frac{\cancel{(s-2)}(7s^2-5s-6)}{(s+1)\cancel{(s-2)}(s-1)}\right] = \frac{28-10-6}{3\cdot 1} = 4$$

$$C_4 = \lim_{s\to 1}\left[\frac{7s^2-5s-6}{(s+1)(s-2)}\right] = \frac{7-5-6}{-2\cdot 1} = 2$$

$$F(s) = \frac{1}{s+1} + \frac{4}{s-2} + \frac{2}{s-1}$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = e^{-t}\mu(t) + 4e^{2t}\mu(t) + 2e^{t}\mu(t)$$

$\boxed{\text{ZAD}}$ $$F(s) = \frac{s+1}{(s-1)^2(s+2)}$$

$$F(s) = c_1 + \frac{c_2}{s+2} + \frac{c_{31}}{s-1} + \frac{c_{32}}{(s-1)^2}$$

$$c_1 = \lim_{s\to\infty} F(s) = \frac{s}{s^3} \dots = 0$$

$$c_2 = \lim_{s\to -2}\left[\frac{s+1}{(s-1)^2\cancel{(s+2)}}\cancel{(s+2)}\right] = \frac{-2+1}{9} = -\frac{1}{9}$$

$$c_{kj} = \frac{1}{(k-j)!}\lim_{s\to p}\left[\left[(s-p)^k F(s)\right]^{(k-j)}\right]$$

k - kratnost

p - pol

$$c_{31} = \frac{1}{(2-1)!}\lim_{s\to 1}\left[\cancel{(s-1)^2}\frac{s+1}{\cancel{(s-1)^2}\, s+2}\right]^{(1)}$$

$$= \lim_{s\to 1}\frac{\cancel{s}+2-\cancel{s}-1}{(s+2)^2} = \frac{1}{9}$$

$$c_{32} = \frac{1}{(2-2)!}\lim_{s\to 1}\left[\cancel{(s-1)^2}\frac{s+1}{\cancel{(s-1)^2}(s+2)}\right]$$

$$= \lim_{s\to 1}\frac{s+1}{s+2} = \frac{2}{3}$$

$$F(s) = -\frac{1}{9}\frac{1}{s+2} + \frac{1}{9}\frac{1}{s-1} + \frac{2}{3}\frac{1}{(s-1)^2}$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = -\frac{1}{9}e^{-2t}\mu(t) + \frac{1}{9}e^{t}\mu(t) + \frac{2}{3}te^{t}\mu(t)$$

$\boxed{\text{ZAD}}$

$$y''(t) + 7y'(t) + 10y(t) = x(t) \quad / \mathcal{L}$$

$y(0) = 2$

$y'(0) = -1$

$$\mathcal{L}\{y'(t)\} = sY(s) - y(0^-)$$

$$\mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0^-) - y'(0^-)$$

$$s^2Y(s) - sy(0^-) - y'(0^-) + 7[sY(s) - y(0^-)] + 10Y(s) = U(s)$$

$$Y(s)[s^2 + 7s + 10] = U(s) + sy(0^-) + \underbrace{y'(0^-)}_{-1} + \underbrace{7y(0^-)}_{14}$$

$$Y(s) = \underbrace{\underbrace{\frac{1}{s^2+7s+10}}_{H(s)} U(s)}_{\substack{Y_m(s) \\ \text{ODZIV MIRNOG} \\ \text{SUSTAVA}}} + \underbrace{\frac{2s+13}{s^2+7s+10}}_{\substack{\text{ODZIV NEPOBUĐENOG} \\ \text{SUSTAVA} \\ Y_0(s)}}$$

$$Y_0(s) = \frac{2s+13}{s^2+7s+10} = \frac{2s+13}{(s+2)(s+5)}$$

$s_1 = -2 \qquad s_2 = -5$

$$r_{1,2} = \alpha \pm j\omega$$

$$\mathrm{Re}\{r\} < 0 \quad \Rightarrow \quad \alpha < 0$$

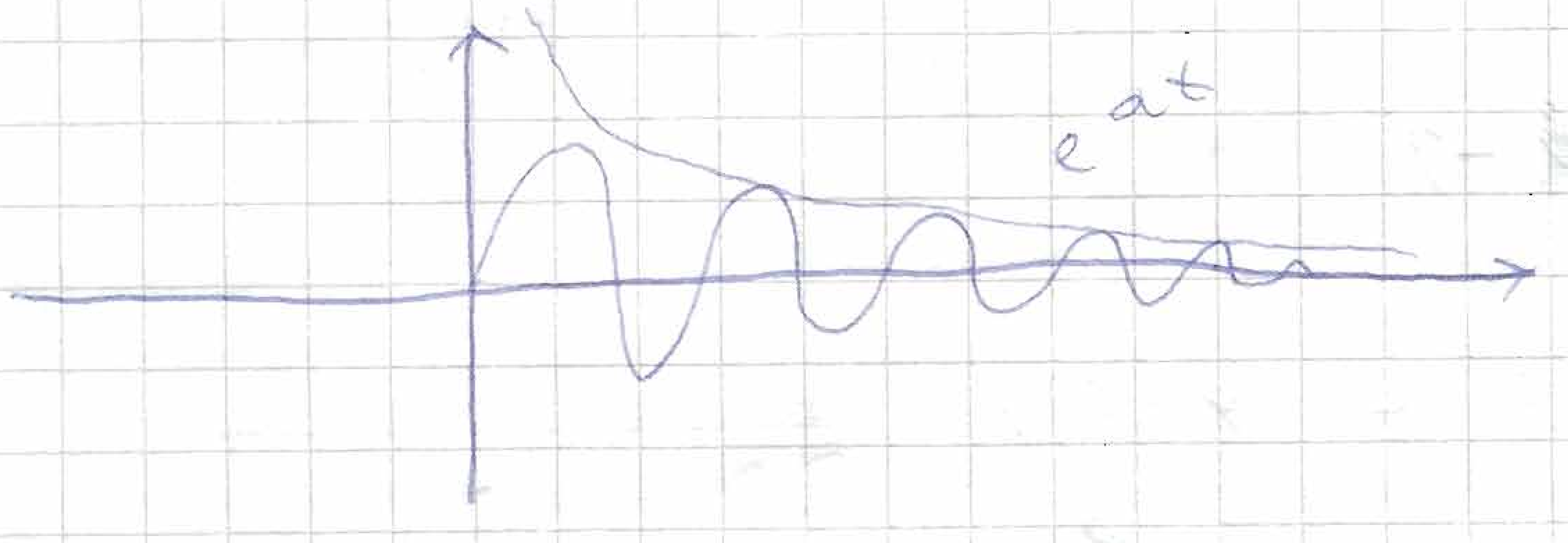
$\alpha = 0$ RUBNO STABILAN

$$\mu_{1,2} = \alpha \pm j\beta$$

$$y(n) = c_1 e^{(\alpha + j\beta)t}\mu(t) + c_2 e^{(\alpha - j\beta)t}\mu(t)$$

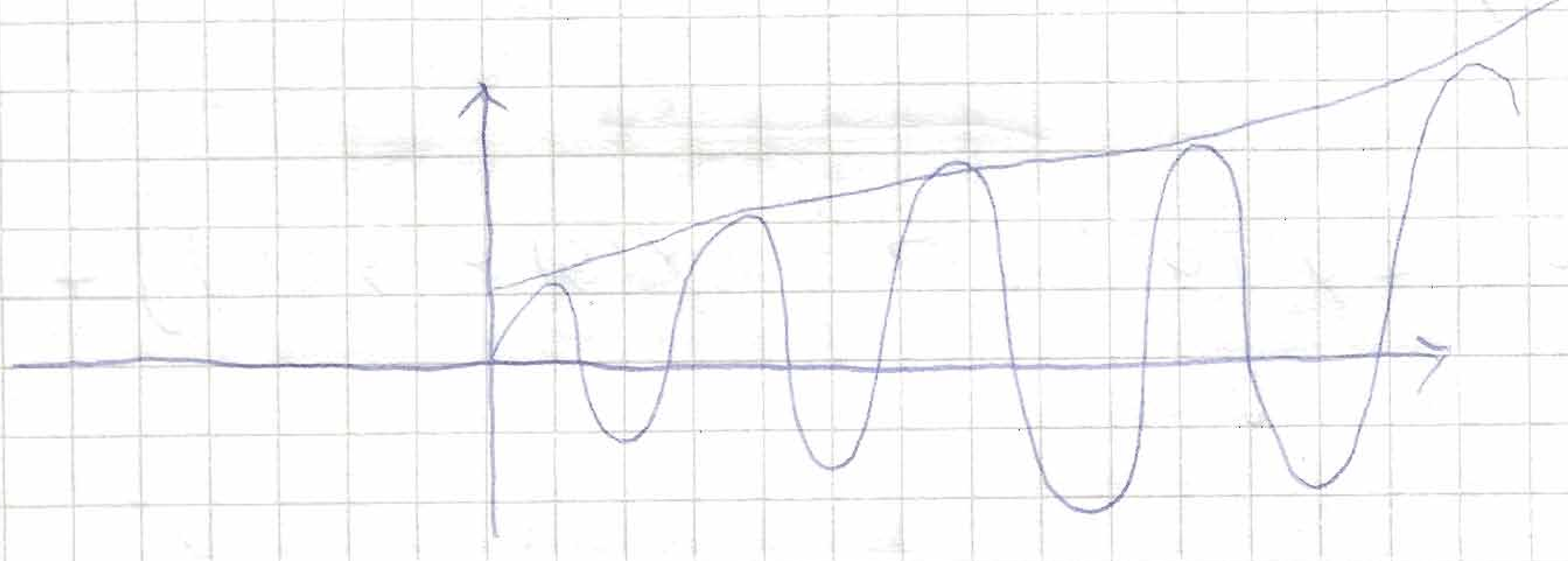
$$= e^{\alpha t}\left[c_1 e^{j\beta t} + c_2 e^{-j\beta t}\right]\mu(t)$$

$\alpha < 0$



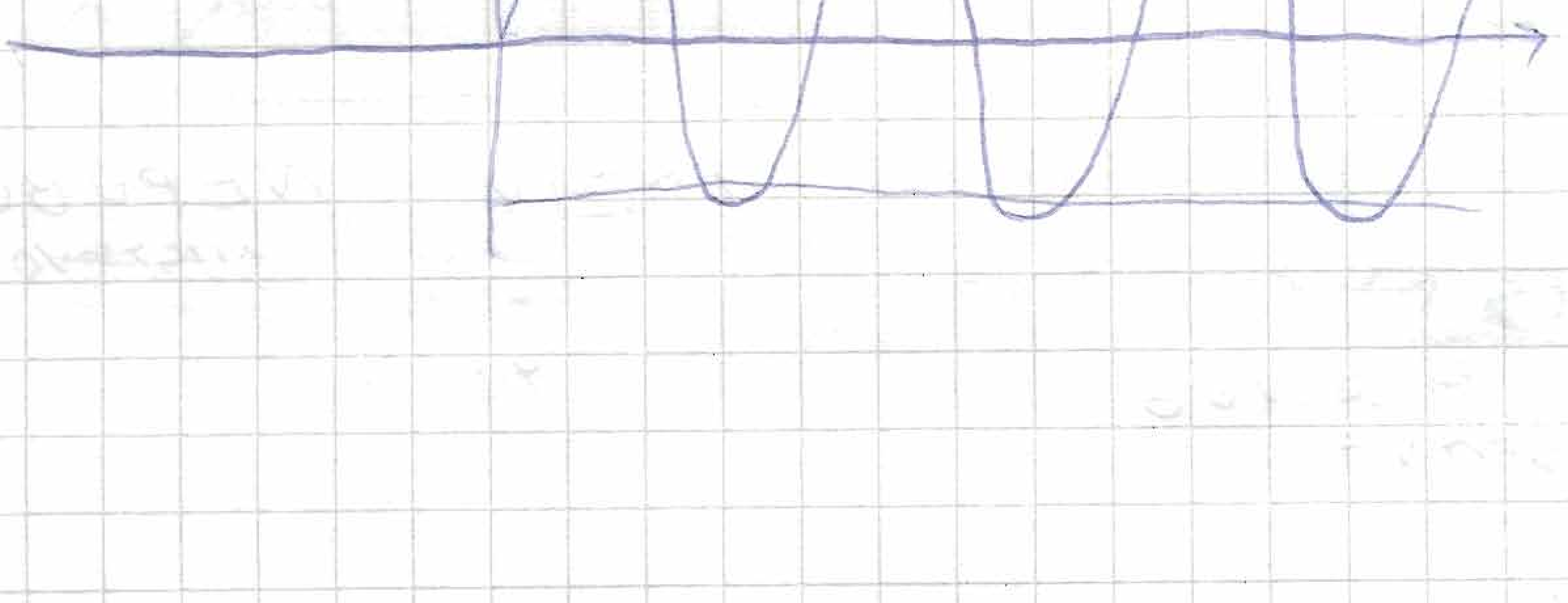
STABILAN

$\alpha > 0$



NESTABILAN

$\alpha = 0$



MARGINALNO STABILAN

$$Y_0(s) = C_1 + \frac{C_2}{s+2} + \frac{C_3}{s+5}$$

$$C_1 = \lim_{s\to\infty} Y_0(s) = 0$$

$$C_2 = \lim_{s\to -2}\left[\cancel{(s+2)}\frac{2s+13}{\cancel{(s+2)}(s+5)}\right] = \frac{9}{3} = 3$$

$$C_3 = \lim_{s\to -5}\left[\cancel{(s+5)}\frac{2s+13}{(s+2)\cancel{(s+5)}}\right] = \frac{3}{-3} = -1$$

$$Y_0(s) = \frac{3}{s+2} - \frac{1}{s+5}$$

$$y_0(t) = \mathcal{L}^{-1}\{Y_0(s)\} = 3e^{-2t}\mu(t) - e^{-5t}\mu(t)$$

$$u(t) = 5e^{-3t}\mu(t) \quad \circ\!\!-\!\!\bullet \quad \frac{5}{s+3} = U(s)$$

$$Y_m(s) = \frac{1}{(s+2)(s+5)} \cdot \frac{5}{s+3} = \frac{5}{(s+2)(s+3)(s+5)}$$

$$= C_1 + \frac{C_2}{s+2} + \frac{C_3}{s+3} + \frac{C_4}{s+5}$$

$$C_1 = \lim_{s\to\infty} Y_m(s) = 0$$

$$C_2 = \lim_{s\to -2}\left[(s+2)\cdot\frac{5}{(s+2)(s+5)(s+3)}\right] = -\frac{5}{3}$$

$$C_3 = \lim_{s\to -3} \qquad = -\frac{5}{2}$$

$$C_4 = \lim_{s\to -5} \qquad = \frac{5}{6}$$

$$y_m(t) = \mathcal{L}^{-1}\{Y_m(t)\} = \frac{5}{3}e^{-2t}\mu(t) - \frac{5}{2}e^{-3t}\mu(t) + \frac{5}{6}e^{-5t}\mu(t)$$

$$y(t) = y_m(t) + y_0(t) = \underbrace{\frac{5}{3}e^{-2t}\mu(t) - \frac{5}{2}e^{-3t}\mu(t)}_{\text{PRISILNI ODZIV zbog pobude}} + \underbrace{\frac{5}{6}e^{-5t}\mu(t) + 3e^{-2t}\mu(t) - e^{-5t}\mu(t)}_{\text{ostalo PRIRODNI}}$$

$\boxed{\text{ZAD}}$ $u(t) - 0{,}1\, y(t) - 0.2\, y'(t) = y''(t)$

$y''(t) + \underset{0.2}{\quad} y'(t) + \underset{0.1}{\quad} y(t) = u(t)$

$u(t) = 3 \cos(1{,}8\, t)$

$y(0^-) = 10$

$y'(0^-) = -5$

$$H(s) = \frac{1}{s^2 + 0.2 s + 0.1}$$

$$p_{1,2} = \qquad = -0.1 \pm j0.3$$

$$H(s) = \frac{1}{(s - (-0.1 + j0.3))(s - (-0.1 - j0.3))}$$

$$s^2 Y(s) - s\,y(0^-) - y'(0^-) + 0.2\left[sY(s) - y(0^-)\right] + 0.1\,Y(s) = U(s)$$

$$Y(s) = \underbrace{\frac{1}{s^2 + 0.2s + 0.1} U(s)}_{Y_m(s)} + \underbrace{\frac{s\,y(0^-) + y'(0^-) + 0.2\,y(0^-)}{s^2 + 0.2s + 0.1}}_{Y_o(s)}$$

$$Y_o(s) = \frac{-10s - 5 + 2}{(s - p_1)(s - p_2)} = \frac{-10s - 7}{(s - p_1)(s - p_2)}$$

$$= c_1 + \frac{c_2}{s - p_1} + \frac{c_3}{s - p_2}$$

$c_1 = 0$

$$c_2 = \lim_{s \to p_1} \left[\cancel{(s - p_1)} \frac{-10s - 7}{\cancel{(s - p_1)}(s - p_2)} \right] = \frac{1 - j3 - 7}{-\cancel{0.1} + j0.3 + \cancel{0.1} + j0.3}$$

$$c_2 = \frac{-6-j3}{j0.6} \qquad \frac{1}{j} = -1$$

$$= j\,\frac{2-j1}{0.2} = 10j + 5$$

$$c_3 = \lim_{s \to r_2} \left[\cancel{(s-r_2)} \frac{-10s-7}{(s-r_1)\cancel{(s-r_2)}} \right] = \frac{1+j3-7}{-\cancel{0.1}-j0.3+\cancel{0.1}-j0.3}$$

$$= \frac{-6+j3}{-j0.6} = 5-j10$$

$$c_3 = c_2^*$$

$$Y_0(s) = (5+j10)\frac{1}{s-r_1} + (5-j10)\frac{1}{s-r_2}$$

$$y_0(t) = \mathcal{L}^{-1}\{Y_0(s)\} = (5+j10)\, e^{(-0.1+j0.3)t} \mu(t)$$

$$+ (5-j10)\, e^{(-0.1-j0.3)t} \mu(t)$$

$$= 5e^{-0.1t}\left[\frac{c_2}{5} e^{j0.3t} + \frac{c_2^*}{5} e^{-j0.3t} \right] \mu(t)$$

$$c = 1+j2 \qquad c_2 = 5+j10$$

$$e^{j\varphi t} = \cos(\varphi t) + j\sin(\varphi t)$$

$$y_0(t) = 5e^{-0.1t}\left[c\left[\cos(0.3t) + j\sin(0.3t)\right] + c^*\left[\underset{\cos(0.3t)}{\overset{\shortparallel}{\cos(-0.3t)}} - j\sin(0.3t)\right]\right.$$

$$y_0(t) = 5e^{-0,1t}\left[(c+c^*)\cos(0{,}3t) + j(c-c^*)\sin(0{,}3t)\right]\mu(t)$$

$$c = 1 + j2$$
$$c^* = 1 - j2$$

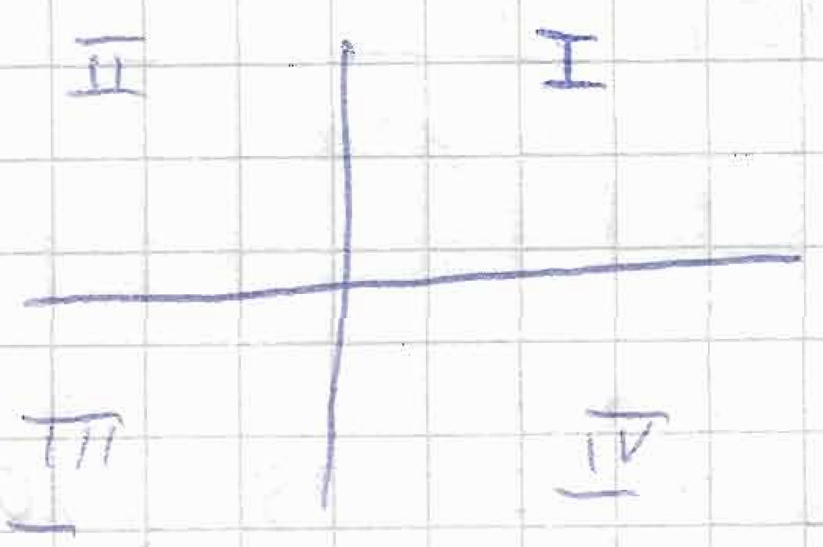
$$y_0(t) = 5e^{-0,1t}\left[2\cos(0{,}3t) - 4\sin(0{,}3t)\right]\mu(t)$$

$$A\cos\omega t + B\sin\omega t = C\sin(\omega t + \varphi)$$

$$C = \sqrt{A^2+B^2}$$

$$\varphi = \operatorname{atan}_2 \frac{B}{A}$$

vodi računa o kvadrantima



$$y_0(t) = 5e^{-0,1t}\sqrt{4+16}\,\sin\left(0{,}3t + \operatorname{atan}_2 \frac{-4}{2}\right)\mu(t)$$

$$= 10\sqrt{5}\,e^{-0,1t}\sin(0.3t + 2{,}034)\,\mu(t)$$

$\arctan -2 = -1{,}1$

$+\pi = 2{,}034$

$$= 10\sqrt{5}\,e^{-0,1t}\cos(0{,}3t \qquad)\,\mu(t)$$

$$-\sin\omega t \circ\!\!-\!\!\bullet \frac{\omega}{s^2+\omega^2}$$

$$\cos\omega t \circ\!\!-\!\!\bullet \frac{s}{s^2+\omega^2}$$

$$Y_m(s) = H(s) \cdot U(s)$$

$$u(t) = 3\cos(1{,}8\,t)\ 1(t)$$

$$U(s) = 3\,\frac{s}{s^2 + 0{,}324}$$

$$H(s) = \frac{1}{s^2 + 0{,}2s + 0{,}1}$$

$$\frac{As + B}{s^2 + 1}$$

$$Y_m(s) = \frac{3s}{(s^2 + 0{,}2s + 0{,}1)(s^2 + 0.324)}$$

$$e^{at}\cos\omega t \;\to\; \frac{s+a}{(s+a)^2 + \omega^2}$$

$$= \frac{3s}{\left((s+0{,}1)^2 + (0{,}3)^2\right)\left(s^2 + 1{,}8^2\right)}$$

$$Y_m(s) = \frac{c_{11}s + c_{12}}{(s+0{,}1)^2 + (0{,}3)^2} + \frac{c_{21}s + c_{22}}{s^2 + 1{,}8^2}$$