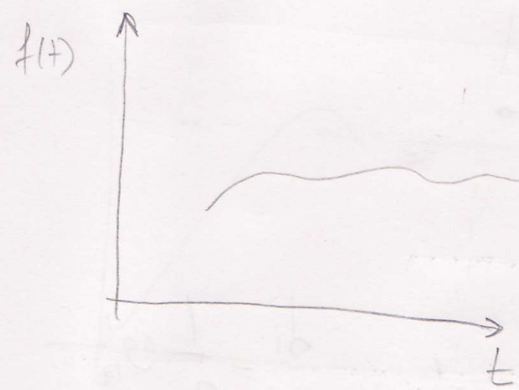
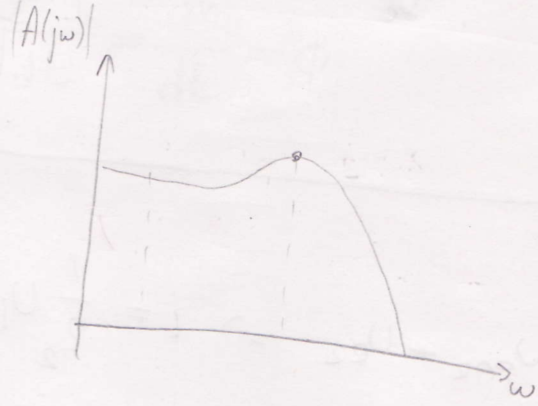
MASS II – 8.10.2009

$\mathcal{L}$ – TRANSFORMACIJA





$f(t) \circ\!\!-\!\!\bullet F(s)$ $\qquad F(s) = \int_{\boxed{-\infty}}^{\infty} f(t) \underbrace{e^{-st}}_{\text{baza transformacije}} dt$ – DVOSTRANA

$F(s) = \int_{\boxed{0}}^{\infty} f(t) e^{-st} dt$ JEDNOSTRANA

zbog kauzalnog singnala $C\,\mu(t)$

LAPLACE TRANSFORMACIJE

$f(t) \circ\!\!-\!\!\bullet F(s)$

$C\,\mu(t) \circ\!\!-\!\!\bullet C \cdot \frac{1}{s}$

$t\mu(t) \circ\!\!-\!\!\bullet \frac{1}{s^2}$

$t^n\mu(t) \circ\!\!-\!\!\bullet \frac{n!}{s^{n+1}}$

$\sin(\omega t)\mu(t) \circ\!\!-\!\!\bullet \frac{\omega}{s^2+\omega^2}$

$\cos(\omega t)\mu(t) \circ\!\!-\!\!\bullet \frac{s}{s^2+\omega^2}$

SVOJSTVA

1) $e^{-at} f(t)\mu(t) \circ\!\!-\!\!\bullet F(s)\Big|_{s+a} = F(s+a)$

$f(t) = \cos(\omega t)^{\mu(t)} \circ\!\!-\!\!\bullet \frac{s}{s^2+\omega^2}$

$g(t) = e^{-3t} \cdot \cos(\omega t)\mu(t)$

$\frac{s}{s^2+\omega^2}\Big|_{s+3} = \frac{s+3}{(s+3)^2+\omega^2}$

1.

2

$$f(t-b)\mu(t-b) \circ\!-\!\bullet\, F(s)e^{-bs}$$

$$f(t) = \cos\left(\frac{\pi}{2}t\right)\mu(t) \circ\!-\!\bullet\, \frac{s}{s^2+\left(\frac{\pi}{2}\right)^2}$$

$$g(t) = \cos\left(\frac{\pi}{2}t\right)\mu(t-2) = \cos\left(\frac{\pi}{2}(t-2+2)\right)\mu(t-2)$$

$$\boxed{\begin{aligned}\sin(x\pm y) &= \sin x\cdot\cos y \pm \cos x \sin y\\ \cos(x\pm y) &= \cos x\cdot\cos y \mp \sin x\cdot\sin y\end{aligned}} \Rightarrow$$

$$= \cos\left(\frac{\pi}{2}(t-2)+\pi\right)\mu(t-2)$$

$$= -\cos\left(\frac{\pi}{2}(t-2)\right)\mu(t-2)$$

$$\frac{-s}{s^2+\left(\frac{\pi}{2}\right)^2}e^{-2s} \quad \bullet\!-\!\circ \quad \boxed{-\cos\left(\frac{\pi}{2}t\right)\mu(t) \circ\!-\!\bullet\, \frac{-s}{s^2+\left(\frac{\pi}{2}\right)^2}}$$

3

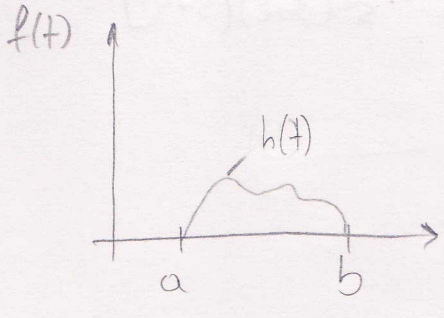
$$f(t) \circ\!-\!\bullet\, F(s)$$

$$f'(t) \circ\!-\!\bullet\, sF(s) - f(0^-)$$

$$f''(t) \circ\!-\!\bullet\, s\left(sF(s)-f(0^-)\right) - f'(0^-) = \underbrace{s^2}_{\text{NAZIVNIK OD } G(s)}F(s) - sf(0^-) - f'(0^-)$$

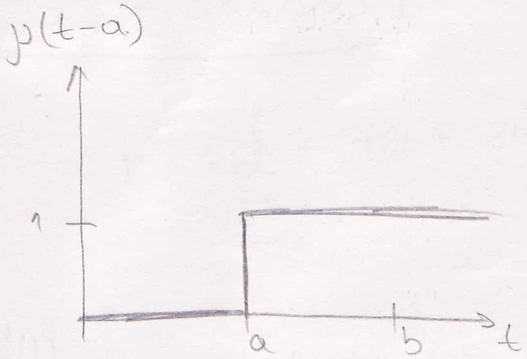
$$Y(s) = \underbrace{U(s)\cdot G(s)}_{\text{ODZIV MIRNOG SUSTAVA}} + \underbrace{A(s)\cdot B(s)}_{\text{ODZIV NEPOBUĐENOG SUSTAVA}}$$

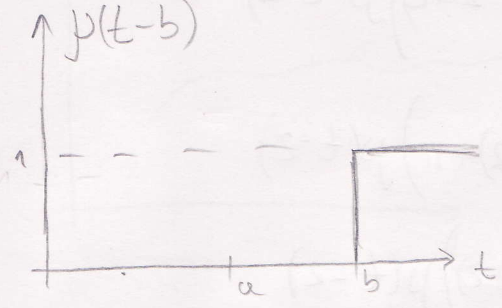
GATE FUNKCIJA (PROZOR, VRATA)

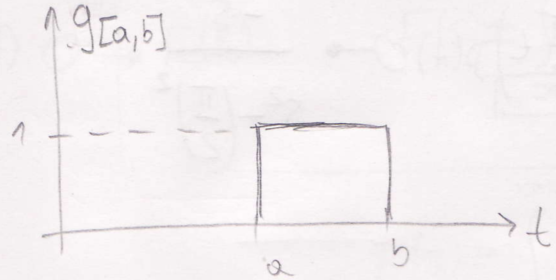


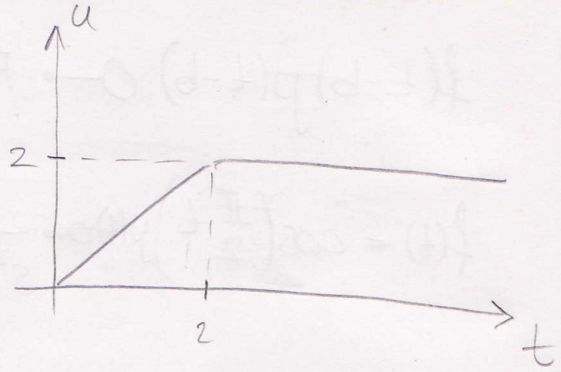
$$f(t) = \begin{cases} 0, & t<a \\ h(t) & a\le t\le b \\ \phi & t>b\end{cases} = h(t)\cdot g_{[a,b]}(t)$$

$$h(t)g_{[a,b]}(t) + z(t)g_{[c,d]}(t) = h(t)\left[\mu(t-a)-\mu(t-b)\right] + z(t)\left[\mu(t-c)-\mu(t-d)\right]$$









$$u(t) = \begin{cases} 0, & t < \emptyset \\ t, & 0 \le t \le 2 \\ 2, & 2 < t < \infty \end{cases}$$

$$= t\left[p(t) - p(t-2)\right] + 2\left[p(t-2) - p(t-\infty)\right]$$

$$= t\,p(t) - (t-2)\,p(t-2)$$

$$\frac{1}{s^2} - \frac{1}{s^2}e^{-2s}$$

$$C_{ij} = \frac{1}{(r-j)!} \lim_{s \to s_0} \left[\frac{d^{r-j}}{ds^{r-j}} \left(A(s) \cdot (s-s_0)^r \right) \right]$$

$r \to$ višestrukost NUL-TOČKE $= j_{max}$

$$A(s) = \frac{s^2+3s+4}{s^2(s+3)^4(s+2)(s+5)} = \frac{C_{11}}{s} + \frac{C_{12}}{s^2} + \frac{C_{21}}{s+3} + \frac{C_{22}}{(s+3)^2} + \frac{C_{23}}{(s+3)^3} + \frac{C_{24}}{(s+3)^4} + \frac{C_{31}}{s+2} + \frac{C_{41}}{s+5}$$

$$C_{23} = \frac{1}{(4-3)!} \lim_{s \to 3} \left[\frac{d^{4-3}}{ds^{4-3}} \frac{s^2+3s+4}{s^2(s+2)(s+5)} \right]$$

$$C_{31} = \lim_{s \to s_3} \left[A(s)(s-s_3) \right]$$

$$C_{41} = \lim_{s \to s_4} \left[A(s) \cdot (s-s_4) \right]$$

$$B(s) = \frac{s+2}{s^2(s+3)\underbrace{(s^2+1)}_{\text{kompleksni } s}} = \frac{C_{11}}{s} + \frac{C_{12}}{s^2} + \frac{C_{21}}{s+3} + \frac{Cs+D}{s^2+1}$$

zbog kompleksnog "s"

PRVO NAĐEMO C_{11}, C_{12}, C_{21} – I SVE RAZMNOŽIMO

$$\cos(x) = \frac{1}{2}\left(e^{jx} - e^{-jx}\right)$$

$$\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

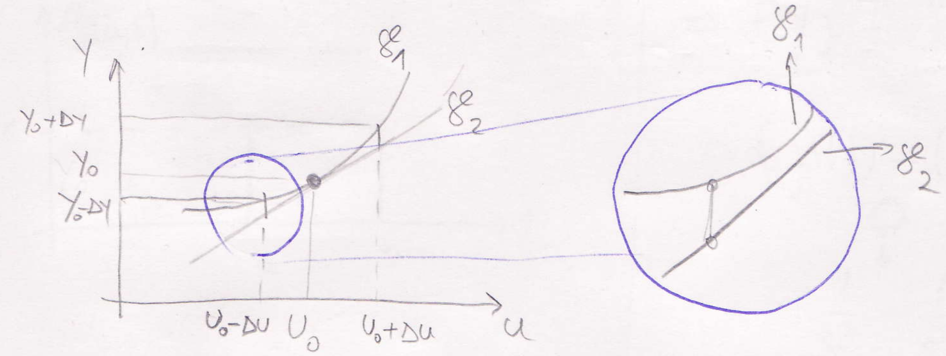
2 LINEARIZACIJA

$$\left.\begin{aligned} f(\alpha u_1) &= \alpha f(u_1) \quad \text{HOMOGENOST} \\ f(\alpha u_1 \pm \beta u_2) &= f(\alpha u_1) \pm f(\beta u_2) \quad \text{ADITIVNOST} \end{aligned}\right\}$$

$$f(\alpha u_1 + \beta u_2) = \alpha f(u_1) + \beta f(u_2) \quad - \text{ SUPERPOZICIJA}$$

$$f(x) = \sqrt{x} \qquad f(\alpha x) = \alpha f(x)$$

$$\sqrt{\alpha x} \neq \alpha \sqrt{x}$$



1) $Y = f(u)$

$$Y - Y_0 = \Delta Y \qquad u - U_0 = \Delta u$$

$$Y = f(U_0) + \left.\frac{df}{du}\right|_{U=U_0} (U-U_0)^1 + \frac{1}{2!} \left.\frac{d^2 f}{du^2}\right|_{U=U_0} (U-U_0)^2 + \dots + \frac{1}{n!} \left.\frac{d^n f}{du^n}\right|_{U=U_0} (U-U_0)^2$$

$$\Delta Y + \cancel{Y_0} = \underbrace{\cancel{f(U_0)}}_{Y_0} + \left.\frac{df}{du}\right|_{U=U_0} \Delta u \rightarrow \boxed{\Delta Y = \left.\frac{df}{du}\right|_{ST} \Delta u}$$

2) ...

3) $y = f(u, w, p)$

$$\underbrace{y}_{\Delta y + \cancel{y_0}} = \underbrace{f(u_0, w_0, p_0)}_{\cancel{y_0}} + \frac{\partial f}{\partial u}\Big|_{ST}(u-u_0)^1 + \frac{\partial f}{\partial w}\Big|_{ST}(w-w_0)^1 + \frac{\partial f}{\partial p}\Big|_{ST}(p-p_0)^1$$

$\boxed{\text{2DZ grupa B}}$

1) $4y'' + 4y' \cdot \ln y + 3y = 3e^u$ $\qquad u_0 = 2$

$\qquad y_0'' = y_0' = \phi$

$$\cancel{4y_0''} + \cancel{4y_0'} + \ln y_0 + 3y_0 = 3e^{u_0}$$

$0 \qquad 0$

$$y_0 = e^{u_0} = 2$$

$y = \Delta y + y_0 \qquad u = \Delta u + u_0$

$y' = \Delta y' + y_0'$

$y'' = \Delta y'' + y_0''$

$$y'' = \frac{3}{4}e^u - y' \ln y - \frac{3}{4}y = f(u, y, y')$$

$$\Delta y'' + \cancel{y_0''} \approx \underbrace{\cancel{f(u_0, y_0, y_0')}}_{y_0'' = \phi} + \frac{\partial f}{\partial u}\Big|_{ST}(u-u_0)^1 + \frac{\partial f}{\partial y}\Big|_{ST}(y-y_0)^1 + \frac{\partial f}{\partial y'}\Big|_{ST}(y'-y_0')^1$$

$$\Delta y'' = \frac{\partial f}{\partial u}\Big|_{ST}\Delta u + \frac{\partial f}{\partial y}\Big|_{ST}\Delta y + \frac{\partial f}{\partial y'}\Big|_{ST}\Delta y' \qquad S.T = \begin{cases} u = u_0 \\ y = y_0 \\ y' = \phi \end{cases}$$

$$\Delta y'' = \left(\frac{3}{4}e^{u_0}\right)\Delta u + \left(-\frac{y_0'}{y_0} - \frac{3}{4}\right)\Delta y + \left(-\ln(y_0)\right)\Delta y_0'$$

$$\Delta y'' + 2\Delta y' + \frac{3}{4}\Delta y = \frac{3}{4}e^{u_0}\Delta u$$ LINEARIZIRANA DIF JEDNADŽBA

$$Y(s)=\mathcal{L}\left\{\Delta y(t)\right\} \qquad U(s)=\mathcal{L}\left\{\Delta u(t)\right\}$$

$$G(s)=\frac{Y(s)}{U(s)}=\frac{\frac{3}{4}e^{2}}{s^{2}+2s+\frac{3}{4}}$$

GRUPA B

(1)

$$U(s)=\frac{1}{s^{2}}-\frac{1}{s^{2}}e^{-2s}$$

$$Y(s)=U(s)\cdot G(s)=\frac{\frac{3}{4}e^{2}}{s^{2}\left(s+\frac{1}{2}\right)\left(s+\frac{3}{2}\right)}-\frac{\frac{3}{4}e^{2}}{s^{2}\left(s+\frac{1}{2}\right)\left(s+\frac{3}{2}\right)}\cdot e^{-2s}$$

$$f(t)\mu(t) \circ\!-\!\bullet\, F(s)$$

$$f(t-a)\mu(t-a) \circ\!-\!\bullet\, F(s)e^{-as}$$

$$\frac{\frac{3}{4}}{s^{2}\left(s+\frac{1}{2}\right)\left(s+\frac{3}{2}\right)}=\frac{C_{11}}{s}+\frac{C_{12}}{s^{2}}+\frac{C_{21}}{s+\frac{1}{2}}+\frac{C_{31}}{s+\frac{3}{2}}=-\frac{8}{3}\frac{1}{s}+\frac{1}{s^{2}}+\frac{3}{s+\frac{1}{2}}-\frac{1}{3}\frac{1}{s+\frac{3}{2}}$$

$$\left(-\frac{8}{3}+t+3e^{-\frac{1}{2}t}-\frac{1}{3}e^{-\frac{3}{2}t}\right)\mu(t)$$

$$C_{21}=\left\{s=-\frac{1}{2}\right\}=3$$

$$C_{31}=-\frac{1}{3}$$

$$C_{12}=1$$

$$C_{11}=-\frac{8}{3}$$

$$Y(s) \circ\!\!-\!\!\bullet\; e^2\left(-\frac{8}{3} + t + 3e^{-\frac{1}{2}t} - \frac{1}{3}e^{-\frac{3}{2}t}\right)\mu(t)$$

$$- e^2\left(-\frac{8}{3} + (t-2) + 3e^{-\frac{1}{2}(t-2)} - \frac{1}{3}e^{-\frac{3}{2}(t-2)}\right)\mu(t-2) \qquad = \Delta y(t)$$

$$y(t) = \Delta y(t) + y_0$$

$$\Delta y_{ss} = \lim_{t\to\infty} e^2 \underbrace{\left(-\frac{8}{3} + t + \cancel{3e^{-\frac{1}{2}t}}^{\,0} - \cancel{\frac{1}{3}e^{-\frac{3}{2}t}}^{\,0}\right)}_{1.\ \text{ČLAN UZ } \mu(t)} - \underbrace{\left(-\frac{8}{3} + (t-2) + \cancel{3e^{-\frac{1}{2}(t-2)}}^{\,0} - \cancel{\frac{1}{3}e^{-\frac{3}{2}(t-2)}}^{\,0}\right)}_{2\ \text{ČLAN UZ } \mu(t-2)}$$

$$= \lim_{t\to\infty} e^2\left(-\frac{8}{3} + t + \frac{8}{3} - t + 2\right) = 2e^2$$

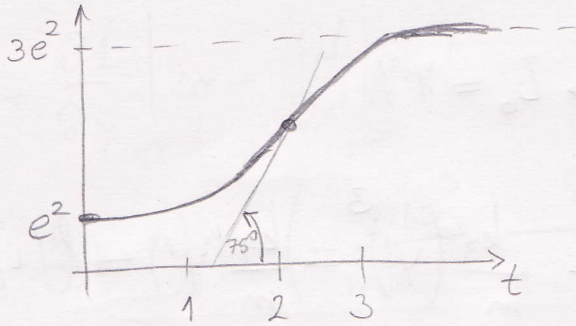
$$y(\infty) = 2e^2 + e^2 = 3e^2$$

$$\Delta y'(t) = e^2\left(1 - \frac{3}{2}e^{-\frac{1}{2}t} + \frac{1}{2}e^{-\frac{3}{2}t}\right)\mu(t) - e^2\left(1 - \frac{3}{2}e^{\frac{1}{2}(t-2)} + \frac{1}{2}e^{-\frac{3}{2}(t-2)}\right)\mu(t-2)$$

$$\alpha = \mathrm{arctg}(k)$$

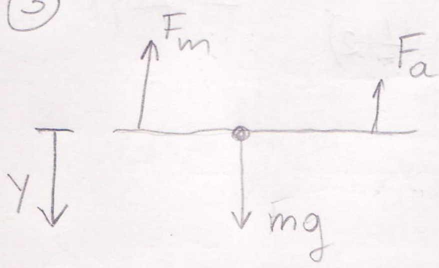
$$\Delta y'(0) = 0$$

$$\Delta y'(2) = 3{,}5$$



1 MI 2007/2008

3



$$F_m = \frac{k_1 i}{(y + k_2)^3}$$

$$F_a = k_3 (y')^3 + k_4 y'$$

UZROK = POSLJEDICA $\underbrace{\quad}_{}$ gibanje tijela

$$mg - F_m - F_a = my''$$

$$my'' + \frac{k_1 i}{(y+k_2)^3} + k_3 (y')^3 + k_4 y' = mg$$

$y_0 = 0{,}01\ m$

$$m\cancel{y_0''}^{\,0} + \frac{k_1 i_0}{(y_0+k_2)^3} + \cancel{k_3 (y_0')^3}^{\,0} + \cancel{k_4 y_0'}^{\,\phi} = mg$$

$$i_0 = \frac{mg}{k_1}(y_0+k_2)^3 \rightarrow i_0 = 1\ A$$

$$y'' = g - \frac{k_1}{m}\frac{i}{(y+k_2)^3} - \frac{k_3}{m}(y')^3 - \frac{k_4}{m}(y') = f(i, y, y_0')$$

$$\Delta y'' = \underbrace{f(i_0, y_0, y_0')}_{\phi} + \frac{\partial f}{\partial i}\bigg|_{ST} \Delta i + \frac{\partial f}{\partial y}\bigg|_{ST} \Delta y + \frac{\partial f}{\partial y'}\bigg|_{ST} \Delta y'$$

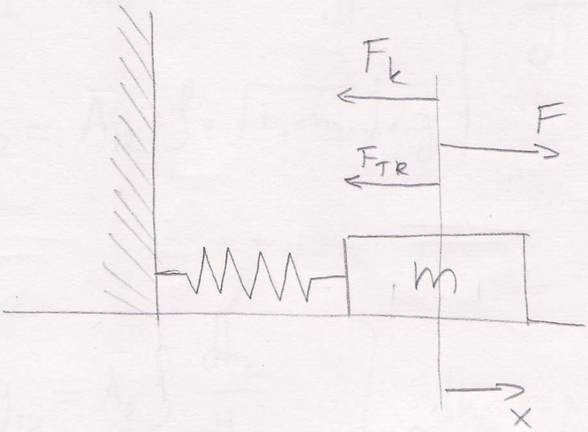
$$\Delta y'' = \frac{-k_1/m}{(y_0+k_2)^3}\Delta i + \left(-\frac{k_1}{m}(-3)\frac{i_0}{(y_0+k_2)^4}\right)\Delta y + \left(-\frac{3k_3}{m}y_0'^2 - \frac{k_4}{m}\right)\Delta y'$$

9

$$\Delta y'' + 20\Delta y' - 490{,}74\,\Delta y = -9{,}81\,\Delta i$$

1MI 08/09

2zd



$$F_{TR} = \frac{2\cdot 10^3}{\pi} \operatorname{arctg}(x')$$

$$F_k = kx^3$$

UZROK = POSLJEDICA

$$F - F_k - T_{TR} = mx''$$

$$mx'' + \frac{2\cdot 10^3}{\pi} \operatorname{arctg}(x') + kx^3 = F$$

$$X_0 = 0{,}2\ m \rightarrow kx_0^3 = F_0 \rightarrow F_0 = 800\ N$$

LINEARIZACIJA

$$x'' = \frac{1}{m} F - \frac{2\cdot 10^3}{m\pi} \operatorname{arctg}(x') - \frac{k}{m} x^3 = f(F, x, x')$$

$$\Delta x'' = \left.\frac{\partial f}{\partial F}\right|_{ST} \Delta F + \left.\frac{\partial f}{\partial x}\right|_{ST} \Delta x + \left.\frac{\partial f}{\partial x'}\right|_{ST} \Delta x'$$

$$\Delta x'' = \left(\frac{1}{m}\right)\Delta F + \left(-\frac{k}{m} 3x_0^2\right)\Delta x + \left(-\frac{2\cdot 10^3}{m\pi} \frac{1}{1 - x_0'^2}\right)\Delta \dot{x}$$

$$\Delta x'' + 127{,}32\,\Delta x' + 2400\,\Delta x = 0{,}2\ \Delta F$$

PRIJENOSNA F-ja

$$G(s) = \frac{X(s)}{F(s)} = \frac{0{,}2}{s^2 + 127{,}32s + 2400}$$

$$X(s) = \mathcal{L}\{\Delta x(t)\}$$

$$F(s) = \mathcal{L}\{\Delta F(t)\}$$

LINEARNI MODEL

$$\Delta F = -500\, \mu(t) \text{ N}$$

$$F(s) = -\frac{500}{s}$$

$$\Delta x(\infty) = \lim_{t\to\infty} \Delta x(t) = \lim_{s\to\phi} sX(s)$$

$$\Delta x(\infty) = \lim_{s\to\phi} s \cdot F(s) \cdot G(s) = \lim s \frac{-500}{\cancel{s}} \frac{0{,}2}{s^2 + 127{,}32s + 2400}$$

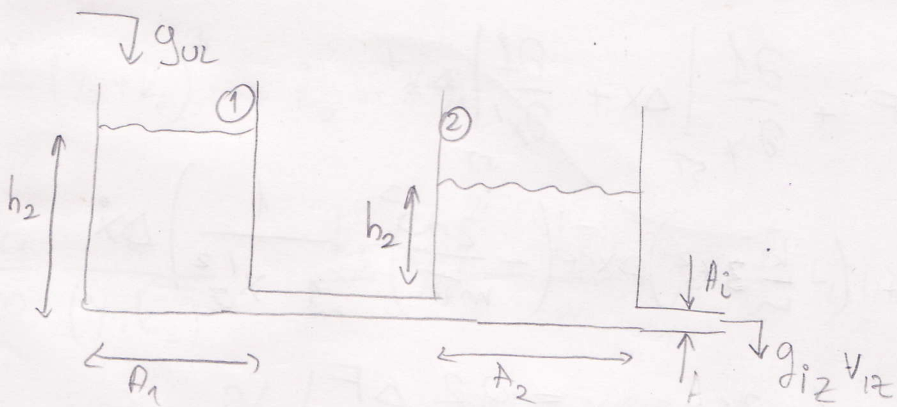
$$\Delta x(\infty) = -0{,}04166 \text{ m} \qquad x(\infty) = 0{,}1583 \text{ m}$$

NELINEARAN MODEL

$$F_0 = 300\text{N} \rightarrow x_0 = \sqrt[3]{\frac{F}{x}} = 0{,}1442$$

$$err = X_{NM} - X_{LM} = 1{,}3 \text{cm}$$

1MI 08/09
3zd



$$g_{UL} = k_{UL}\sqrt{X_{UL}} \quad \left[\frac{kg}{s}\right]$$

11

SPREMNIK 1

$$\left.\begin{aligned} q_{UL} - q_{12} &= A_1 \cdot \rho \frac{dh_1}{dt} \\ q_{12} &= A_{12} \cdot \rho \cdot \sqrt{h_1 - h_2}\sqrt{2g} \end{aligned}\right\} \quad \frac{dh_1}{dt} = \frac{k_{UL}}{A_1 \cdot \rho}\sqrt{x_{UL}} - \frac{A_{12}}{A_1}\sqrt{2g}\sqrt{h_1 - h_2} = f(x, h_1, h_2)$$

SPREMNIK 2

$$\left.\begin{aligned} q_{12} - q_{IZ} &= A_2 \rho \frac{dh_2}{dt} \\ q_{IZ} &= A_i \cdot \rho \sqrt{2g}\sqrt{h_2} \end{aligned}\right\} \quad \frac{dh_2}{dt} = \frac{A_{12}}{A_2}\sqrt{2g}\sqrt{h_1 - h_2} - \frac{A_{iz}}{A_2}\sqrt{2g}\sqrt{h_2} = f(h_1, h_2)$$

3b) RADNA TOČKA

$$\frac{dh_1}{dt} = \frac{dh_2}{dt} = 0$$

$$x_{u0} = 0{,}5$$

$$\frac{A_{12}}{A_2}\sqrt{2g}\sqrt{h_{10} - h_{20}} = \frac{A_i}{A_2}\sqrt{2g}\sqrt{h_2} \quad \Big/^2$$

$$h_{10} = h_{20}\left[1 + \left(\frac{A_i}{A_{12}}\right)^2\right]$$

$$\frac{k_u}{\rho}\sqrt{x_0} = A_{12}\sqrt{2g}\sqrt{h_{10} - h_{20}} \Big/^2 \Rightarrow \quad h_{10} - h_{20} = \left(\frac{k_u}{\rho}\frac{1}{A_{12}\sqrt{2g}}\right)^2 \cdot x_{u0}$$

$$h_{10} = 1{,}274 \text{ m}$$

$$h_{20} = 0{,}637 \text{ m}$$

LINEARIZACIJA

$$\Delta h_1 = \frac{\partial f}{\partial x}\Big|_{ST} \Delta x + \ldots\ldots$$

$$\Delta h_1' = \left(\frac{k_v}{A_1 \rho} \frac{1}{2\sqrt{X_{v\phi}}}\right)\Delta x + \left(-\frac{A_{12}}{A_1}\sqrt{2g}\frac{1}{2\sqrt{h_{1\phi}-h_{2\phi}}}\right)\Delta h_1 + \left(-\frac{A_{12}}{A_1}\sqrt{2g}\frac{-1}{2\sqrt{h_{1\phi}-h_{2\phi}}}\right)\Delta h_2$$

$$\Delta h_2' = \left(\frac{A_{12}}{A_2}\sqrt{2g}\frac{1}{2\sqrt{h_{1\phi}-h_{2\phi}}}\right)\Delta h_1 + \left(\frac{A_{12}}{A_2}\sqrt{2g}\frac{-1}{\sqrt{h_{10}-h_{20}}} - \frac{A_{12}}{A_2}\sqrt{2g}\frac{1}{2\sqrt{h_{2\phi}}}\right)\Delta h_2$$

$$\Delta h_1' = -0{,}005549\ \Delta h_1 + 0{,}005549\ \Delta h_2 + 0{,}007071\ \Delta X_v$$

$$\Delta h_2 = 0{,}005549\ \Delta h_1 - 0{,}011099\ \Delta h_2$$

$$q_{iz} = A_1 \rho \sqrt{2g}\sqrt{h_2} = f(h_2)$$

$$X = \begin{bmatrix}\Delta h_1 \\ \Delta h_2\end{bmatrix} \quad u = [\Delta X_v] \quad Y = [\Delta q_{iz}]$$

$$\Delta q_{iz} = \left(A_1 \rho \sqrt{2g}\frac{1}{2\sqrt{h_{20}}}\right)\Delta h_2$$

$$\begin{bmatrix}\Delta h_1' \\ \Delta h_2'\end{bmatrix} = \begin{bmatrix}-0{,}005549 & 0{,}005549 \\ 0{,}005549 & -0{,}011099\end{bmatrix}\begin{bmatrix}\Delta h_1 \\ \Delta h_2\end{bmatrix} + \begin{bmatrix}0{,}000701 \\ 0\end{bmatrix}[\Delta X_v]$$

$$[\Delta q_{iz}] = [0 \quad 27{,}75]\begin{bmatrix}\Delta h_1 \\ \Delta h_2\end{bmatrix} + [0]\,\Delta X_{vL}$$

$$G(s) = \frac{Q_1(s)}{X_v(s)}$$

1. NAČIN

1) $G(s) = C(sI - A)^{-1}B$

2 NAČIN

$$H_1(s-0{,}005) = 0{,}005 \frac{1}{27{,}75} Q_1(s) + 0{,}007 X_u(s)$$

$$\frac{1}{27{,}15} Q_1(s)\left(s + 0{,}011\right) = 0{,}005 H_1(s)$$

1MI 07/08

$$G(s) = \frac{1-s}{(s+2)^2(3s+9)} e^{-\frac{1}{2}s}$$

LINEARAN: DA! – ima Laplaceovu transformaciju

VR. PROMJENJIV: NE! ⇒

MULTIVARIJABILAN: NE ⇒ morao bi imati više prijenosnih f-ja

KAUZALAN: DA ⇒ po redu brojnika nazivnika