2 11.07 - 7 Biramo $y \in [0,2]$, zatim $x \in [y,2]$. Odredi gustoću i očekivanje od x.

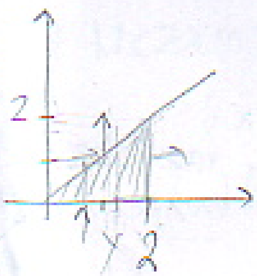
→ razdioba od x ovisi o y pa je to uniformna razdoba

$$f_Y(y) = \underbrace{\frac{1}{2}}_{\text{duljina intervala}}, \quad y \in [0,2] \quad \rightarrow \text{gustoća}$$

$$f_{X|Y=y}(x) = \frac{1}{2-y}$$

$$\left.\begin{matrix} f_Y(y) \\ f_{X|Y=y}(x) \end{matrix}\right\} \quad f(x,y) = \frac{1}{2} \cdot \frac{1}{2-y} \qquad 0 \le y \le x \le 2$$

$$f(x,y) = f_{X|Y=y}(x) \cdot f_Y(y)$$



MARGINALNA GUSTOĆA

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^x \frac{1}{2(2-y)}\,dy = $$

$$= \frac{1}{2}\left[\ln 2 - \ln(2-x)\right], \quad x \in [0,2]$$

1. način

$$E(X) = \int_0^2 x f_X(x)\,dx = \frac{1}{2}\int_0^2 x\left[\ln 2 - \ln(2-x)\right]dx = \ldots =$$

2. način

$$E(X) = \int_{-\infty}^{\infty} E(X|Y=y)\, f_Y(y)\,dy$$

$$E(X|Y=y) = \int_{-\infty}^{\infty} x f_{X|Y=y}(x)\,dx$$

$$E(X|Y=y) = \int_y^2 x \cdot \frac{1}{2-y}\,dx = \frac{1}{2-y} \cdot \frac{x^2}{2}\Big|_y^2 = \frac{1}{2}(y+2)$$

↳ stavljamo granice tamo gdje je definirana uvjetna gustoća

$$E(X) = \int_0^2 \frac{1}{2}(y+2) \cdot \frac{1}{2}\,dy = \frac{1}{4}\left(\frac{y^2}{2} + 2y\right)\Big|_0^2 = \frac{3}{2}$$

↳ stavljamo granice gdje je definiran y

$\boxed{2.MI-11.-6}$ $X \in [1,2]$ Radijus kruga je jednoliko distr. varijabla na 1 do 2.
↳ uniformno

$T > \frac{1}{2}$

$$f_X(x) = \frac{1}{1} = 1$$

$$P(A|X=x) = |\text{geom. vj.}| = \frac{x^2\pi - \frac{1}{4}\pi}{x^2\pi} = 1 - \frac{1}{4}\frac{1}{x^2}$$

$$P(A) = \int_1^2 \left(1 - \frac{1}{4x^2}\right) \cdot 1\,dx = \left(x + \frac{1}{4x}\right)\Big|_1^2 = \frac{7}{8}$$

↳ x definiran od 1 do 2

7.3. UVJETNE RAZDIOBE

Neka je $f(x,y)$ gustoća od vektora X,Y te neka je poznata razdioba od Y. Tada se uvjetna gustoća od X uz uvjet $Y=y$

$$f_{X|Y=y}(x) = \frac{f(x,y)}{f_Y(y)} \quad , \quad f(x,y) = f(x|y) \cdot f_Y(y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x|y) f_Y(y)\,dy$$

$$E(X|Y=y) = \int_{-\infty}^{\infty} x f_{X|Y=y}(x)\,dx$$

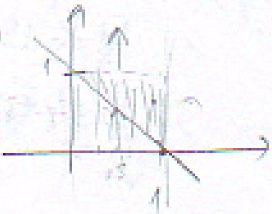
$$E(X) = \int_{-\infty}^{\infty} E(X|Y=y) f_Y(y)\,dy$$

$$P(A) = \int_{-\infty}^{\infty} P(A|X=x) f_X(x)\,dx$$

1-07-6

$$f(x,y) = \begin{cases} Cxy, & x \in [0,1], \ y \in [0,1] \\ 0, & \text{inače} \end{cases}$$

a) $C = ?$



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^1 dx \int_0^1 Cxy\,dy = \frac{C}{4} = 1$$

b) Marginale

$$f_X(x) = \int_0^1 4xy\,dy = 2x, \quad x \in [0,1]$$

$$f_Y(y) = \int_0^1 4xy\,dx = 2y, \quad y \in [0,1]$$

c) Nezavisnost

$$2x \cdot 2y = 4xy$$

$$f(x)f(y) = f(x,y) \rightarrow \text{NEZAVISNI}$$

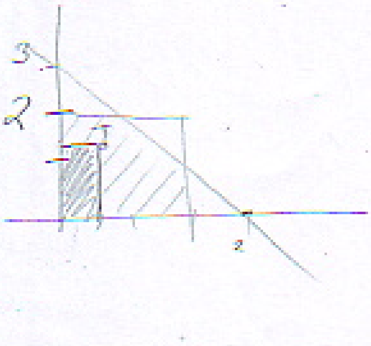
d) $$P(X+Y>1) = \iint_G f(x,y)\,dx\,dy = \int_0^1 dx \int_{1-x}^{1} 4xy\,dy = \frac{5}{6}$$

$y > 1-x$

21-09-10

Biramo točku unutar Ω, $x, y \in [0,2]$, $x+y<3$. Neka je X x-koordinata, a Y y-koordinata. Odredi funkciju gustoće itd.

$y < 3-x$



$$F(x,y) = P(X<x, Y<y) = \frac{x \cdot y}{3+\frac{1}{2}} = \frac{xy}{\frac{7}{2}}$$

$$f(x,y) = \frac{\partial^2 F}{\partial x \partial y} = \frac{2}{7} \rightarrow \text{UNIFORMNA RAZDIOBA}$$

$$f(x,y) = \frac{1}{m(\Omega)}$$

$\boxed{5.DZ.2}$ Unutar polukuga polumjera R. Neka je X udaljenost točke do središta, a Y do polumjera. Odredi funkciju gustoće sluč. vektora X,Y.

P.K. ISJEČKA

$p = r^2\pi \frac{\alpha}{2\pi} = \frac{1}{2} r^2 \alpha$

$\boxed{x \geq y}$

$x \in (0,R)$
$y \in (0,R)$

$$F(x,y) = P(X<x, \overset{\text{presjek}}{,} Y<y)$$

$$= \frac{2\left[\frac{1}{2} \cdot y \cdot \sqrt{x^2-y^2} + \frac{1}{2} x^2 \arcsin \frac{y}{x}\right]}{\frac{1}{2} R^2 \pi}$$

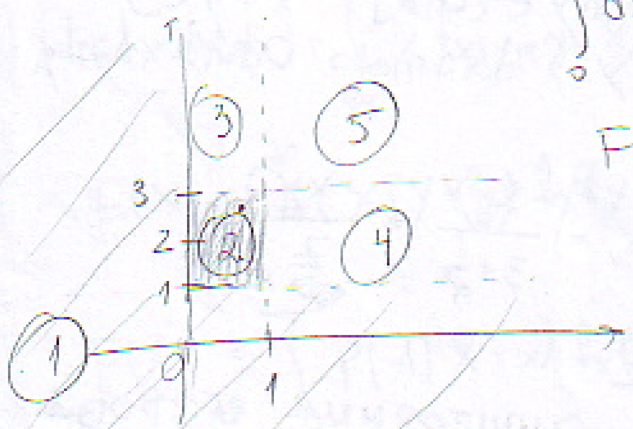
$\varphi = \arcsin \frac{y}{x}$

$$f(x,y) = \frac{\partial^2 F}{\partial x \partial y} = \frac{4x}{R^2 \pi \sqrt{x^2-y^2}} \quad , 0 \leq y \leq x \leq 3$$

$\boxed{5.DZ\,1.}$ Vektor ima gustoću $f(x,y) = C(5-x-y)$

$1<y<3$, $F(x,y)=?$
$0<x<1$

$$\int_0^1 dx \int_1^3 C(5-x-y)\,dy = 5C = 1 \Rightarrow C = \frac{1}{5}$$



$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\,dx\,dy$$

1. $\int_{-\infty}^{x} dx \int_{-\infty}^{y} 0\,dy = 0$ za $x<0$ ili $y<1$

2. $\int_0^x dx \int_1^y \frac{1}{5}(5-x-y)\,dy = \frac{1}{10} x(y-1)(9-x-y)$ $\quad x \in (0,1)$, $y \in (1,3)$

(3) $\int_0^x dx \int_1^3 \frac{1}{5}(5-x-y)dy = \frac{1}{5}x(6-x)$ $\quad x \in (0,1)$

$y > 3$

(4.) $\int_0^1 dx \int_1^y \frac{1}{5}(5-x-y)dy$

↳ od 1 do tego y
jest sno posti [illegible]
[illegible]

(5.) $\int_0^1 dx \int_1^3 \frac{1}{5}(5-x-y)dy = 1$ $\quad x > 1,\ y > 3$

→ jest snoposti [illegible] gdy [illegible] x i y [illegible]

$\boxed{2MI-09-6}$ $X_1, X_2 \in [0,1] \rightarrow X = \max\{X_1, X_2\}$ 10.5.2012. ČETVRTAK

$\rightarrow Y \in [0, X]$

$f_Y(y) = ?$

$E(Y) = ?$

$f_{X_1}(x) = \frac{1}{1}, \; f_{X_2}(x) = 1$

$F_X(x) = P(X < x) = P(\max\{X_1, X_2\} < x)$

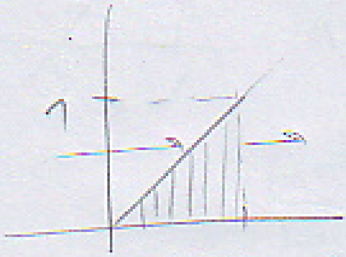
$= P(X_1 < x, X_2 < x) = P(X_1 < x) \cdot P(X_2 < x) =$

$= \int_{-\infty}^{x} f_{X_1}(x)dx \cdot \int_{-\infty}^{x} f_{X_2}(x)dx = \int_0^x dx \int_0^x dx = x^2$

$f_X(x) = F'_X(x) = 2x, \quad x \in (0,1)$

$f_{Y|X=x}(y) = \frac{1}{x} \quad, y \in (0, x) \quad 0 \le y \le x$

$f(x,y) = f_X(x) \cdot f_{Y|X}(y) = 2x \frac{1}{x} = 2$



$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{x=y}^{x} 2dx = 2 - 2y \, , \; y \in (0,1)$

$E(Y) = \int_{-\infty}^{\infty} y f_Y(y)dy = \int_0^1 y(2-2y)dy = \frac{1}{3}$

8. FUNKCIJE SLUČAJNIH VEKTORA

pitanje: ako znamo razdiobu od X i Y, kakva je razdioba od $Z = \psi(X, Y)$

zadatak) X i Y su nezavisne s exp razdiobom s parametrom 2. Naći razdiobu od $Z = X + Y$

$G(z) = ?$

$f_X(x) = \lambda e^{-\lambda x}$

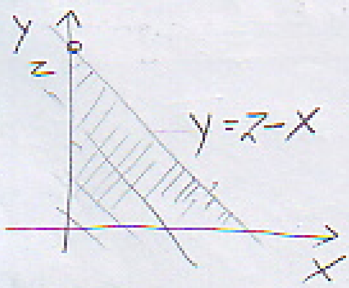
$$\left.\begin{array}{l} f_X(x) = 2e^{-2x}, \quad x > 0 \\ f_Y(y) = 2e^{-2y}, \quad y > 0 \end{array}\right\} \Rightarrow \text{zbog nezavisnosti}$$

$$f(x, y) = 4e^{-2x-2y}$$

$z \in (0, \infty\rangle$

$$G(z) = P(Z < z) = P(X + Y < z) = \iint_G f(x, y)\,dx\,dy$$

$\rightarrow$ ovo nije površina

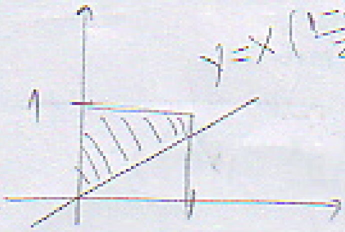


$$= \int_0^z dx \int_0^{z-x} 4e^{-2x-2y}\,dy$$

$$= 1 - (1 + 2z)e^{-2z}, \quad z > 0$$

$$g(z) = G'(z) = 4ze^{-2z}, \quad z > 0$$

5. DZ-15. $[0,1]\times[0,1]$ Odredi gustoću od varijable $Z=\frac{X}{X+Y}$



$y=x\left(\frac{1-z}{z}\right)$ $\quad f(x,y)=\frac{1}{1}=1$ → jer je jedinična

$z\in(0,1)$

$$G(z)=P(Z<z)=P\left(\frac{X}{X+Y}<z\right)$$

$$=P\left(y>x\left(\frac{1-z}{z}\right)\right)$$

$$=\int_0^1 dx\int_{x\left(\frac{1-z}{z}\right)}^1 1\,dy$$ → POVRŠINA zato što je funkcija gustoće 1

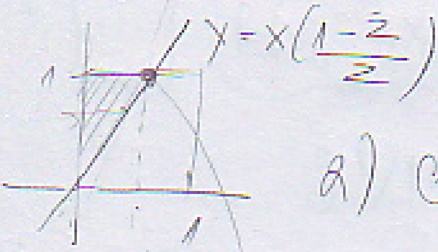
$x<zx+zy$

$zy>x-zx$

$y>\frac{x-zx}{z}$

$y>x\left(\frac{1-z}{z}\right)$ – BROJ! → pravac koji prolazi kroz ishodište

1) $G(z)=\frac{1}{2}\frac{z-1}{z}$, $z\in(0,\frac{1}{2}]$ dijelimo na 2 dijela



nacrtah točku

2) $$G(z)=P\left(y>x\left(\frac{1-z}{z}\right)\right)=\int_0^{z(1-z)} dx\int_{x\left(\frac{1-z}{z}\right)}^1 dy=$$

$$=\frac{1}{2}\frac{z}{1-z},\ z\in\left(\frac{1}{2},1\right]$$

$$g(z)=\begin{cases}\frac{1}{2}\frac{1}{z^2}, & z\in\left(0,\frac{1}{2}\right)\\ \frac{1}{2}\frac{1}{(1-z)^2}, & z\in\left(\frac{1}{2},1\right)\end{cases}$$

↑ zamijenjeni intervali

8.7) $X_1, \dots, X_n$ nezávislé s.v. s exp. rozdelením s parametrami $\lambda_1, \dots, \lambda_n$. Odveď rozdelenie od Y

$= \min\{X_1, \dots, X_n\}$

$G(y) = P(Y < y) = P(\min\{X_1, \dots, X_n\} < y)$

$= 1 - P(\min\{X_1, \dots, X_n\} \geq y)$

$= 1 - P(X_1 \geq y, X_2 \geq y, \dots, X_n \geq y) \overset{\text{ak}}{\underset{NZ}{=}} 1 - P(X_1 \geq y) \cdot P(X_2 \geq y)$

$\cdot P(X_n \geq y) =$

$X_1 \sim E(\lambda_1)$

$P(X_1 \geq y) = 1 - F_1(y)$

$= 1 - (1 - e^{-\lambda_1 y})$

$= e^{-\lambda_1 y}$

$G(y) = 1 - e^{-\lambda_1 y} \dots e^{-\lambda_2 y} \dots e^{-\lambda_n y} = 1 - e^{-(\lambda_1 + \dots + \lambda_n) y}$

$Y \sim E(\lambda_1 + \dots + \lambda_n)$

$Y = \varphi(X)$

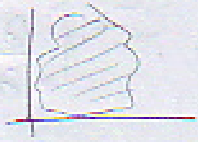
$g(y) = f(x) \left| \frac{dx}{dy} \right|$

$x = \varphi^{-1}(y)$

$(x_1, \dots, x_n) \to f(x_1, \dots, x_n)$ $\qquad \left. \begin{array}{l} y_1 = \psi_1(x_1) \\ y_n = \psi_n(x_n) \end{array} \right\}$

$g(y_1 \dots, y_n) = ?$

$P((x_1 \dots x_n) \in G) = P((y_1 \dots, y_n) \in G')$



$$\int_G \cdots \int f(x_1 \dots, x_n)\, dx_1 \dots dx_n = \left| \begin{array}{l} x_1 = \chi_1(y_1) \\ \vdots \\ x_n = \chi(y_n) \end{array} \right. \qquad J = \frac{\partial(x_1 \dots x_n)}{\partial(y_1 \dots y_n)}$$

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \dots & \frac{\partial x_n}{\partial y_1} \\ \vdots & & \\ \frac{\partial x_n}{\partial y_1} & \dots & \frac{\partial x_n}{\partial y_n} \end{vmatrix} =$$

$$= \int_{G'} \cdots \int \underbrace{f(x_1, \dots, x_n) |J|}_{g(y_1 \dots y_n)} dy_1 \dots dy_n$$

$$g(y_1 \dots, y_n) = f(x_1 \dots y_n) \left| \frac{\partial(x_1 \dots, x_n)}{\partial(y_1 \dots y_n)} \right|$$

$n = 2$

$z = \varphi(x, y)$ $\quad \left.\begin{array}{l} x = x \\ y = \chi(x, z) \end{array}\right\} J = \frac{\partial(x,y)}{\partial(x,z)} = \begin{vmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial z} \end{vmatrix}$

$$= \begin{vmatrix} 1 & 0 \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial z} \end{vmatrix} = \frac{\partial y}{\partial z}$$

$$\Rightarrow g(x, z) = f(x, y) \left| \frac{\partial y}{\partial z} \right|$$

$g(z) = ?$ $\quad$ $\searrow$ i tu

marginalna gustoća $\boxed{g(z) = \int_{-\infty}^{\infty} f(x, y) \left| \frac{\partial y}{\partial z} \right| dx}$

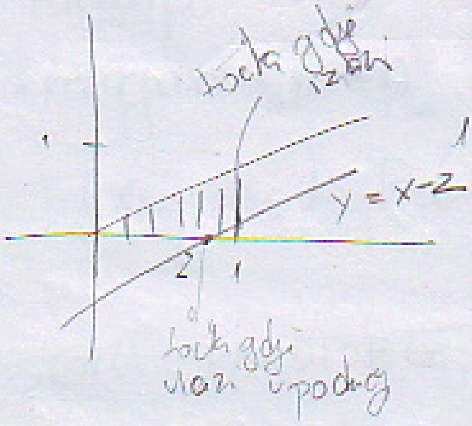
uvrstiti y jer nesmijemo imati y zato što integriramo po x-u

Princip za granice

$\boxed{2MI.-11-7}$ Slučajni vektor x,y ima gustoću

$$f(x,y)=Cx,\quad 0<y\leq x<1\quad,\ z=x-y$$

$$\int_0^1 dx\int_0^x Cx\,dy=\frac{1}{3}C=1,\ C=3$$



1. korak:

$$z=x-y\Rightarrow z\in(0,1)$$

$$\boxed{y=x-z}\rightarrow \text{nacrtati}$$

$$\left|\frac{\partial y}{\partial z}\right|=|-1|=1$$

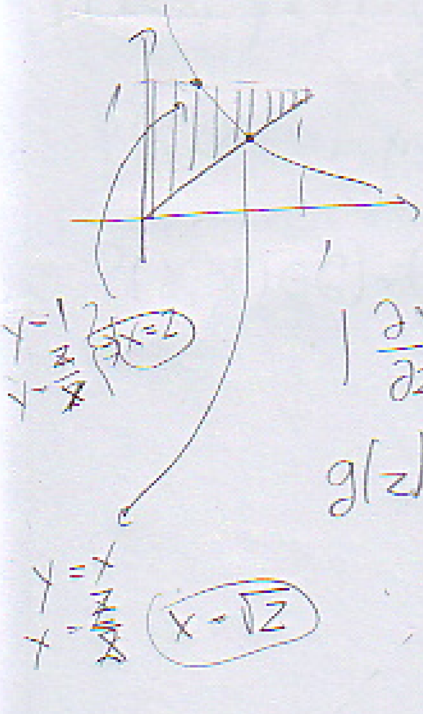
$$g(z)=\int_z^1 3x\cdot 1\,dx=\frac{3x^2}{2}\Big|_z^1=\frac{3}{2}(1-z^2)\ ,\ z\in(0,1)$$

$$0\leq x-z\leq x\leq 1$$

pa su granice od z do 1

$\boxed{5.DZ-19}$ $f(x,y)=C(x+y)\ ,\ 0\leq x\leq y<1\ ,\ z=x\cdot y$

$$\int_0^1 dx\int_x^1 C(x+y)dy=\frac{1}{2}C=1\Rightarrow \boxed{C=2}$$



$$z=x\cdot y\rightarrow z\in(0,1)\Rightarrow y=\frac{z}{x}$$

$$\left|\frac{\partial y}{\partial z}\right|=\frac{1}{x}$$

$$g(z)=\int_z^{\sqrt{z}} 2\cdot\left(x+\frac{z}{x}\right)\frac{1}{x}\,dx$$

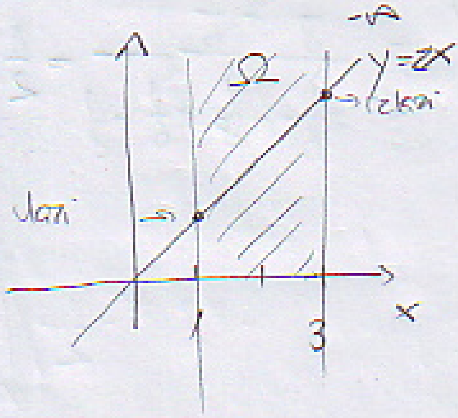
$\boxed{\text{2MI. 08. -7.}}$

$\begin{cases} Y \sim E(1) \\ X \sim U[1,3] \end{cases}$ Odredi razdiobu od varijable $Z = \frac{Y}{X}$, $P\left(\frac{Y}{X} < 1\right) = ?$

$$\left.\begin{array}{l} f_Y(y) = e^{-y}, \ y > 0 \\ f_X(x) = \frac{1}{2}, \ x \in [1,3] \end{array}\right\} \text{nez.} \quad f(x,y) = \frac{1}{2} e^{-y}$$

1. način formula $\Rightarrow y = \chi(x,z)$

$$g(z) = \int_{-\infty}^{\infty} f(x,y) \left| \frac{\partial \chi}{\partial z} \right| dx$$



$$z = \frac{y}{x} \Rightarrow y = z \cdot x \quad \left| \frac{\partial y}{\partial z} \right| = x$$

višak y

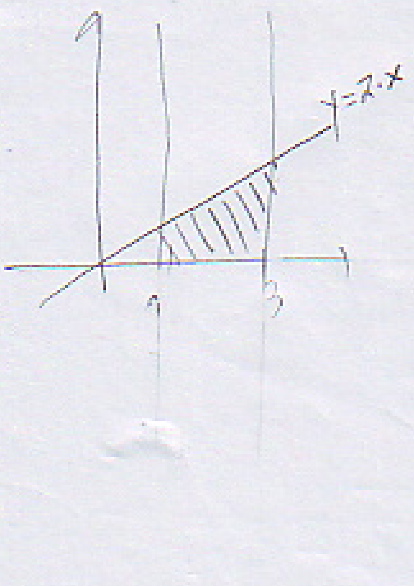
$$g(z) = \int_1^3 \frac{1}{2} e^{-zx} \, x \, dx = \ldots = \frac{1}{2z^2}\left(e^{-z} - e^{-3z}\right) + \frac{1}{2z}\left(e^{-z} - 3e^{-3z}\right), \quad z \in \langle 0, \infty \rangle$$

nejde!

$$P(Z<1) = \int_0^1 g(z)\,dz$$

$G(z) =$

2. način po definiciji



konst

$$G(z) = P(Z<z) = P\left(\frac{Y}{X} < z\right) = P(Y < z \cdot x)$$
$$= \iint f(x,y)\,dx\,dy = \int_1^3 dx \int_0^{zx} \frac{1}{2} e^{-y} dy$$
$$= 1 + \frac{1}{2z}\left(e^{-3z} - e^{-z}\right), \quad z \in \langle 0, +\infty \rangle$$

$g(z) = G'(z) = .$

$P(z<1) = G(1) = 1 + \frac{1}{2}\left(e^{-3} - e^{-1}\right)$

$\boxed{2MI-10-6}$ X i Y su nez. s eksp. razdobom s očekanjem 2

$X \sim E(\frac{1}{2})$ ← parametar $\qquad E(X) = \frac{1}{\lambda}$ → očekanj

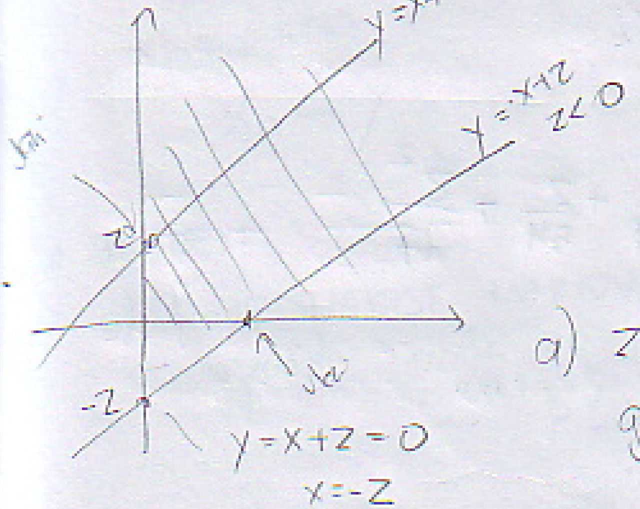
$Y \sim E(\frac{1}{2})$

$f_X(x) = \frac{1}{2} e^{-\frac{1}{2}x}, \; x>0 \qquad f(x,y) = \frac{1}{4} e^{-\frac{1}{2}(x+y)}$

$f_Y(y) = \frac{1}{2} e^{-\frac{1}{2}y}, \; y>0$

$Z = Y - X \; ; \; g(z) = ?$

<u>1. način</u>



$z \in \langle -\infty, +\infty \rangle$

$y = z + x$

$\left|\frac{\partial y}{\partial z}\right| = 1$

$g(z) = \int \frac{1}{4} e^{-\frac{1}{2}(x + z + x)} \cdot 1 \, dx$

a) $z > 0$

$g(z) = \int_0^{\infty} \frac{1}{4} e^{-\frac{1}{2}(x+z+x)} \, 1 \cdot dx = \frac{1}{4} e^{-\frac{1}{2}z}, \; z>0$

po x-u

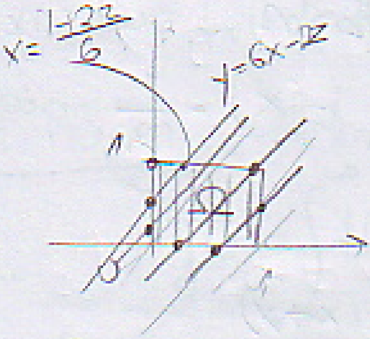
b) $z < 0$

$g(z) = \int_{-z}^{\infty} \frac{1}{4} e^{-\frac{1}{2}(x+z+x)} \, 1\, dx = \frac{1}{4} e^{\frac{1}{2}z}, \; z \le 0$

2 način DZ,

$\boxed{\text{PRMI-07-7}}$ $f(x,y) = x^2 + Cy, \quad 0 \le x,y \le 1$

$Z = 3x - \frac{1}{2}y$ $\quad g(z) = ?$



$$\int_0^1 dx \int_0^1 (x^2 + Cy)\,dy = 1$$

$\boxed{C = \frac{4}{3}}$

$\boxed{y = 6x - 2z}$

$\left|\frac{\partial y}{\partial z}\right| = 2$

$z \in [-\frac{1}{2}, 3]$

$$g(z) = \int \left(x^2 + \frac{4}{3}(6x - 2z)\right) \cdot 2\,dx =$$

1. dio $\int_0^{\frac{1+2z}{6}} \left(x^2 + \frac{4}{3}(6x-2z)\right) \cdot 2\,dx =$

$z \in (-\frac{1}{2}, 0)$

2. interval $z \in (0, \frac{5}{2})$ $\begin{matrix} x=1 \\ y=1 \\ z=\frac{5}{2} \end{matrix}$

$$g(z) = \int_{\frac{1}{3}z}^{\frac{1+2z}{6}} \left(x^2 + \frac{4}{3}(6x - 2z)\right) 2\,dx = \frac{73}{324} + \frac{z}{54} + \frac{z^2}{27}$$

3. interval $z \in (\frac{5}{2}, 3)$

$$g(z) = \int_{\frac{1}{3}z}^{1} \left(x^2 + \frac{4}{3}(6x - 2z)\right) 2\,dx =$$

9. ZAKON VELIKIH BROJEVA I CENTRALNI GRANIČNI TEOREM

→ motivacija: indikatorska varijabla $I_k \sim \begin{pmatrix} 0 & 1 \\ 1-p & p \end{pmatrix} \sim B(\overset{n,p}{1,p})$

→ n puta ponavljamo: $X_n = I_1 + I_2 + \dots + I_n \sim B(n,p)$

→ koliko puta se dogodi neki događaj

- intuitivno: $\frac{X_n}{n} \overset{?}{\rightarrow} p$ —— kako je to limes slučajnih varijabli

9.1. ZAKONI VELIKIH BROJEVA

- def. Niz (X_n) konvergira po vjerojatnosti ka slučajnoj varijabli Y ako za $\forall\, \varepsilon > 0$ vrijedi

$$\lim_{n\to\infty} P(|X_n - Y| > \varepsilon) = 0$$

$$X_n \overset{P}{\rightarrow} Y$$

1. TEOREM

a) NEJEDNAKOST MARKOVA: Ako X poprima nenegativne vrijednosti onda za $\forall\, \varepsilon > 0$ vrijedi

$$P(X \geq \varepsilon) \leq \frac{E(X)}{\varepsilon}$$

b) L_p NEJEDNAKOST: Za svaku slučajnu varijablu X s očekivanjem m_X i svaki $p > 0$ vrijedi

$$P(|X - \underset{\text{očekivanje}}{E(X)}| \geq \varepsilon) \leq \frac{E(|X - E(X)|^p)}{\varepsilon^p}$$

c) NEJEDNAKOST ČEBIŠEVA: Za $p = 2$ vrijedi

$$P(|X - E(X)| \geq \varepsilon) \leq \frac{D(X)}{\varepsilon^2}$$

DOKAZ Markova

$$P(X \geq a) = \int_{x \geq a} dF(x) \leq \int_{x \geq a} \frac{x}{a} dF(x) \leq \frac{1}{a} \int_0^{\infty}$$

$$P(X \geq \varepsilon) = \int_{\varepsilon}^{+\infty} f(x)dx \leq \int_{\varepsilon}^{+\infty} \frac{x}{\varepsilon} f(x)dx \leq \frac{1}{\varepsilon} \underbrace{\int_{0}^{+\infty} x f(x)dx}_{\text{formula za}} = \frac{E(X)}{\varepsilon}$$

$\varepsilon > 0$

b) $P(|X - E(X)| \geq \varepsilon) = P(|X - E(X)|^p \geq \varepsilon^p) \leq \frac{E(|X - E(X)|^p)}{\varepsilon^p}$

$\boxed{6.DZ.1.}$ Broj ~~[illegible]~~ sunčanih dana u g

$E(X) = 75$

Dokažite da je vjerojatnost da će biti u 1. g više od 200 sunčanih dana, veća od $\frac{5}{8}$

$P(X \leq 200) > \frac{5}{8}$ = ?

nepoznate razdiobe

$$P(X \leq 200) = 1 - \underbrace{P(X \geq 200)}_{\leq \frac{E(X)}{\varepsilon}} > 1 - \frac{75}{200} = \frac{5}{8}$$

cijelo minus ispred onda okrećemo znak nejednakosti

$\boxed{6.DZ.-2}$ $E(X) = 25$ km/h

$\sigma(X) = 4.5$ km/h

Koliku brzinu motora očekivati s vjerojatnosti ne manjom od 0.9.?

$$P(|X - E(X)| \geq \varepsilon) \leq \frac{D(X)}{\varepsilon^2}$$

$$P(|X - E(X)| < \varepsilon) > 1 - \frac{D(X)}{\varepsilon^2} > 0.9$$

zato što mora biti veće od 0.9

$$1 - \frac{\sigma^2}{\varepsilon^2} > 0.9 \qquad |X - 25| < 14.23$$

$$\varepsilon = 14.23 \qquad -14.23 < X - 25 < 14.23$$

$$\boxed{10.77 < X < 39.23}$$