TVORIN MASOVNE

Q $\quad$ λ $\quad$ σ $\quad$ ρ

Q_{uk} $\quad \sum Q_i$ $\quad \int \lambda\, dl$ $\quad \iint \sigma\, dS$ $\quad \iiint \rho\, dV$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r}$$

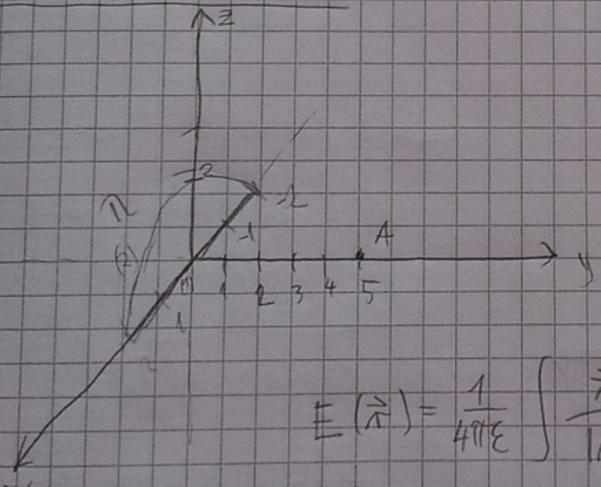
$$\Phi_e = \iint \vec{D}\vec{n}\, dS$$

$$\varphi = -\int_{ref}^{tocka} \vec{E}\, \vec{dl} \Rightarrow \vec{E} = -\nabla\varphi \qquad \oint_c \vec{E}\, \vec{dl} = 0 \qquad \nabla \times \vec{E} = 0$$

$$\Delta \varphi = 0$$

GAUSSOV ZAKON

$$\oiint \vec{D}\, \vec{n}\, dS = \iiint \rho\, dV \Rightarrow \nabla \vec{D} = \rho$$

① 

$$\lambda = \frac{1}{\sqrt{x^2+10}} \quad mC/m$$

$$\vec{E}(A) = ?$$

$$E(\vec{r}) = \frac{1}{4\pi\varepsilon}\int \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\lambda(\vec{r}')\, dl$$

$$\vec{r} = 5\vec{a}_y$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

(1) $\vec{r}\,' = x\,\vec{a}_x$

$dl = dx$

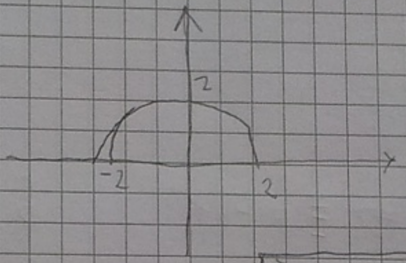
$$E_1(\vec{r}) = \frac{1}{4\pi\varepsilon}\int_{-2}^{2} \frac{-x\,\vec{a}_x + 5\,\vec{a}_y}{\sqrt{x^2+25}^{\,3}}\,\rho\, dx$$

(2)

$$\vec{r}_2\,' = x\,\vec{a}_x + z\,\vec{a}_z$$

$x^2 + z^2 = 4$

$z = \sqrt{4 - x^2}$



$x = 2\cos t$

$y = 0$

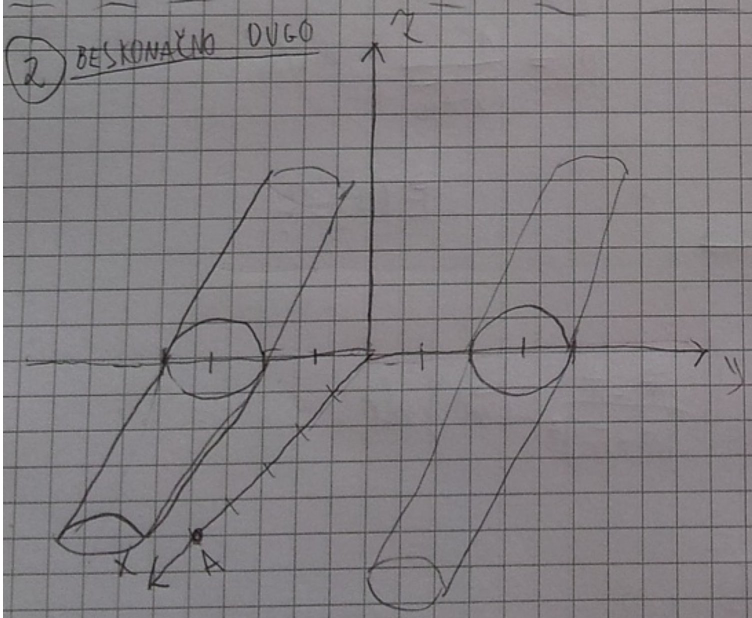
$z = 2\sin t$

$$dl = \sqrt{\left(\frac{\partial x}{\partial t}\right)^2 + \left(\frac{\partial y}{\partial t}\right)^2 + \left(\frac{\partial z}{\partial t}\right)^2}\, dt$$

$t : 0 \to \pi$

$$E_2(\vec{r}) = \frac{1}{4\pi\varepsilon}\int_{0}^{\pi} \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|}\,\rho\, dl$$

(2) BESKONAČNO DUGO



$\to$ 2 CILINDRA $\quad z = 0$

$y_1 = 3\,cm \qquad y_2 = -3\,cm$

$R = 1\,cm$

$\sigma = 20\ nC/m^2$

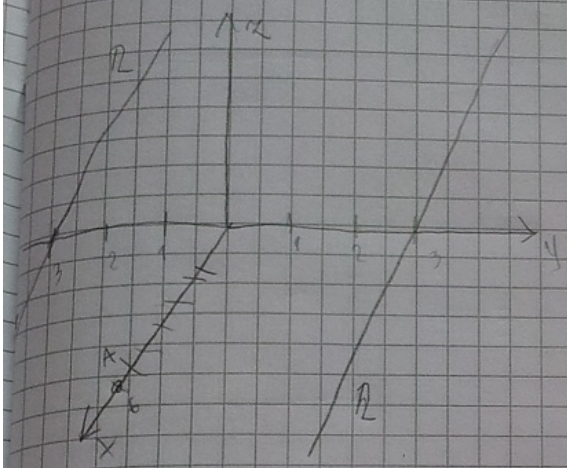
$A(6, 0, 2) \Rightarrow E = ?$

δ

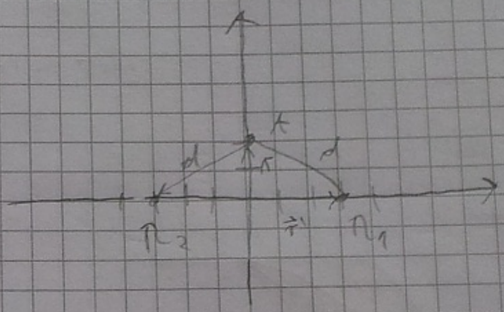
$\Rightarrow$ λ

$Q = \iint \delta \, dS = \delta S$ $\qquad$ $Q = \lambda \cdot l$

$$\lambda = \delta \cdot \frac{S}{l} = \delta \frac{2\pi R \cdot l}{l} = 12{,}86 \ nC/m$$



ZA $\quad x = 6$



$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon d} = E_r \Rightarrow \vec{E} = \frac{\lambda}{2\pi\varepsilon d} \cdot \vec{a}_r$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \qquad \vec{E} = \frac{\lambda}{2\pi\varepsilon} \cdot \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^2}$$

$r = 2\vec{a}_z$

$\vec{r}_1 = 3\vec{a}_y$

$\vec{r}_1 = 3\vec{a}_y$

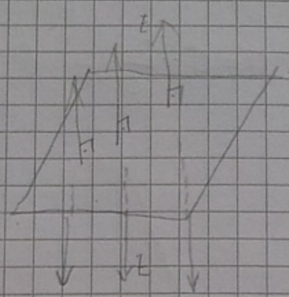
3. RAVNINA $2x + 3y - 4z = 12$

$\sigma = 10\ nC/m^2$

Ele polje duž x-osi = ?

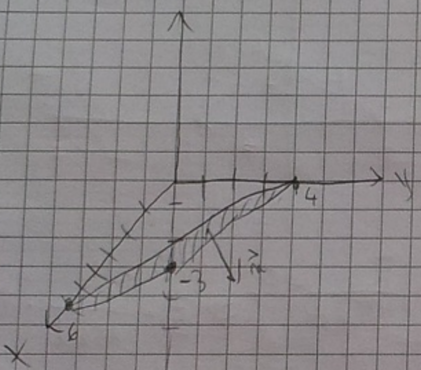
$$|\vec{E}| = \frac{\sigma}{2\varepsilon}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon} \cdot \frac{\pm\vec{n}}{|\vec{n}|}$$

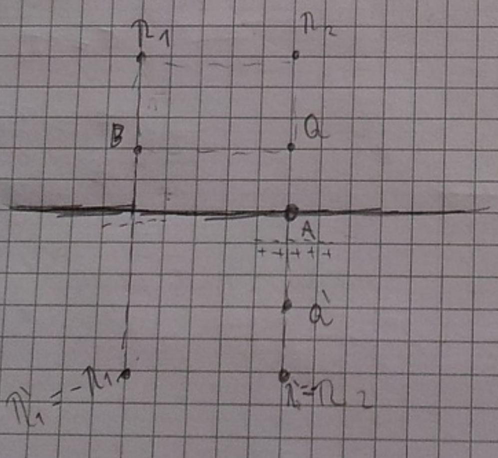


$\vec{n} = 2\vec{a}_x + 3\vec{a}_y - 4\vec{a}_z \quad x \geq 6.$

$\vec{n} = -2\vec{a}_x - \vec{a}_y + 4\vec{a}_z \quad x < 6$



4.



$r_1 Q > 0 \qquad r_2 < 0$

- za točkasti naboj

$$|\vec{E}| = \frac{Q}{4\pi\varepsilon r^2}\vec{a}_r$$

$$|\vec{E}| = \frac{Q}{4\pi\varepsilon}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\varphi = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{1}{r_{ref}}\right)$$

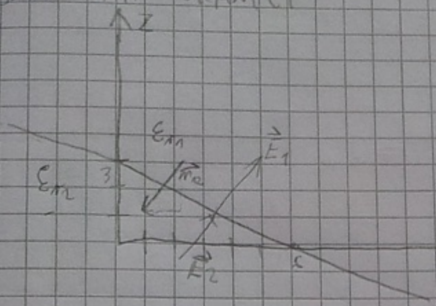
- Za linijski

$$\varphi = \frac{\tau}{2\pi\varepsilon} \ln \frac{r_{ref}}{r}$$

$r_{ref} \Rightarrow$ udaljenost naboja do $\varphi = 0$

$$E =$$

5. UVJETI NA GRANICI



$\varepsilon_{r1} = 2$

$\varepsilon_{r2} = 5$

$\vec{E}_1 = 4\vec{a}_x - 2\vec{a}_y + 3\vec{a}_z$

$\vec{D}_2 = ?$

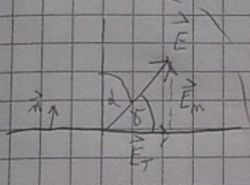
$\vec{n}_{12} \Rightarrow$ jedinična normala iz 1 u 2

$$\vec{n}_{12} = \frac{-3\vec{a}_y - 6\vec{a}_z}{\sqrt{9+36}}$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

$$\vec{n}_{12} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma$$

$$\vec{n}_{12} \times (\vec{E}_2 - \vec{E}_1) = \vec{0}$$



$$E_n = \vec{n} \cdot \vec{E}$$

$$\vec{E}_n = E_n \vec{n}$$

$$\vec{E}_T = \vec{n} \times \vec{E}$$

$$\vec{D} = \vec{\varepsilon}_0 E + \vec{P}$$

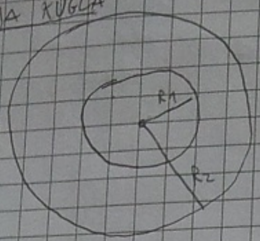
$$Q_{POL} = \iint \vec{P}\, \vec{n}\, dS$$

$$|\vec{n} \cdot \vec{E}| = |\vec{n}| \cdot |\vec{E}| \cos\alpha$$

$$|\vec{n} \times \vec{E}| = |\vec{n}|\,|\vec{E}| \sin\alpha$$

$$U_{AB} = \varphi_A - \varphi_B = -\int_B^A \vec{E}\, d\vec{l} = (\vec{n} \times \vec{E})(\vec{r}_B - \vec{r}_A)$$

(6.) ŠUPLJA KUGLA



$$\rho(r) = \begin{cases} \frac{4}{(r+2)r} & , R_1 < r < R_2 \\ 0 & , \text{inače} \end{cases}$$

$$\oint\!\!\oint_S \vec{D}\,\vec{n}\,dS = \iiint_V \rho\,dV = Q_{uk}$$

$$\vec{D} \oint\!\!\oint dS = Q_{uk}$$

1) $r < R_1$

$$Q_{uk} = \iiint \rho\,dV = 0 \Rightarrow \vec{E} = 0$$

2) $R_1 < r < R_2$

$$\varepsilon_0 \vec{E}_2 \oint\!\!\oint dS = \varepsilon_0 \vec{E}\, 4 r^2 \pi = \iiint \rho\,dV$$

$$\varepsilon_0 E_2 4 r^2 \pi = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta \int_{R_1}^{r} \frac{4 \cdot r^2}{r(r+2)}\,dr$$

$$E_2 = [f(r)]\,\vec{a}_r$$

3) $r < R_2$

$$\varepsilon_0 E 4 r^2 \pi = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta \int_{R_1}^{R_2} \frac{4 \cdot r^2}{r(r+2)}\,dr$$

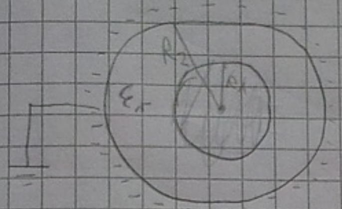
$$\vec{E}_3 = [f_3(r)]\,\vec{a}_r$$

$$Q = 4\pi \int_{R_1}^{R_2} \rho\, r^2\,dr = \varepsilon 4 r^2 \pi \Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\varphi = -\int_{ref}^{r} \vec{E}\, d\vec{l} = -\int_{ref}^{r} E\, dr$$

$$\varphi(r=0) = -\int_{\infty}^{R_2} E_3\, dr - \int_{R_2}^{R_1} E_2\, dr$$

7. CILINDAR



$$\rho = \begin{cases} 4+2r, & r < R_1 \\ 0, & r > R_1 \end{cases}$$

E, $r < R_1$

$$C = \frac{Q}{U}$$

$$\oiint \varepsilon_0 \vec{E}\,\vec{n}\, dS = \iiint_V \rho\, dV$$

$$\varepsilon_0 E\, 2r\pi l = \int_0^l dz \int_0^{2\pi} d\varphi \int_0^r \rho\, r\, dr$$

$R_1 < r < R_2$

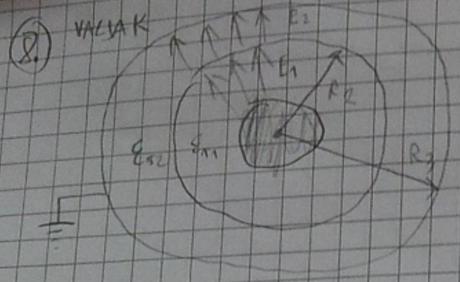
$$Q = 2\pi l \int_0^{R_1} \rho\, r\, dr$$

$$\lambda = 2\pi \int_0^{R_1} \rho\, r\, dr$$

$$\vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{a}_r$$

$$U = -\int_{R_2}^{R_1} E_2\, dr$$

8. VALJAK



$\lambda = 10\ nC/m$

$\varepsilon_{r1} = 2$

$\varepsilon_{r2} = 5$

$C = ?$

$\vec{D}_1 = \vec{D}_2$

$$\oiint \vec{D}\,\vec{n}\,dS = \iiint \rho\,dV$$

$R_1 < r < R_2$

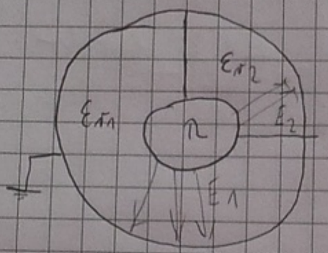
$$E_1 = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r1} r}$$

$R_2 < r < R_3$

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r2} r}$$

$$U = -\int_{R_3}^{R_2} E_2\,dr - \int_{R_2}^{R_1} E_1\,dr$$

9. CILINDRIČNI



$\vec{E}_1 = \vec{E}_2$

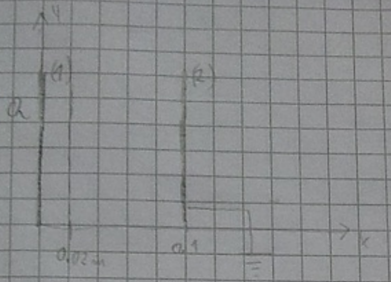
$$\oiint \varepsilon_0 \varepsilon_r \vec{E}\,\vec{n}\,dS = \iiint \rho\,dv$$

$$\varepsilon_0 E \oiint \varepsilon_r\,dS = Q$$

$$\varepsilon_0 E \left[\varepsilon_{r1} \frac{3}{4} 2r\pi l + \frac{1}{4}\varepsilon_{r2} 2r\pi l\right] = Q$$

$$E = \frac{\lambda}{\varepsilon_0 \frac{1}{2} r (3\varepsilon_1 + \varepsilon_{r2})}$$

(10) *



$S = 0.03\ m^2$

$d = 0.1\ m$

(1) $U = 100\ V$

$\varepsilon_r = \frac{4 \cdot d}{x + d}$

$U = 100\ V = -\int_{0.1}^{0} E\, dx$

$100 = \frac{-\delta}{0.4\varepsilon} \int_{0.1}^{0} (x + 0.1)\, dx$

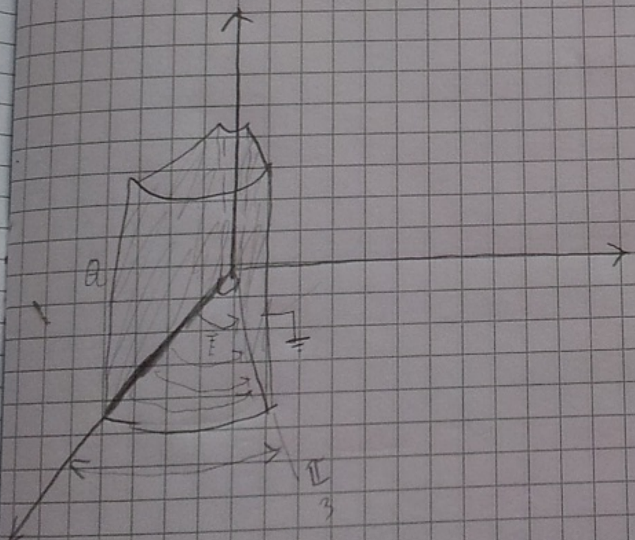
$E = E_1 + E_2 = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$

$E = \frac{\delta}{\varepsilon_0} \frac{(x + 0.1)}{0.4}$

$C = \frac{Q}{U}$

$U = \varphi_1 - \varphi_2$

$\Delta \varphi = 0$



$\vec{E} = \vec{E}\, \vec{a}_\alpha \Rightarrow$ potencijal mijenja u α smjeru

$\underbrace{\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \varphi}{\partial r}\right)}_{=0} + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \alpha^2} + \underbrace{\frac{\partial^2 \varphi}{\partial z^2}}_{=0} = 0$

$\frac{\partial^2 \varphi}{\partial \alpha^2} = 0 \;/ \int d\alpha$

$\frac{\partial \varphi}{\partial \alpha} = C_1$

$\varphi = C_1 \cdot \alpha + C_2$