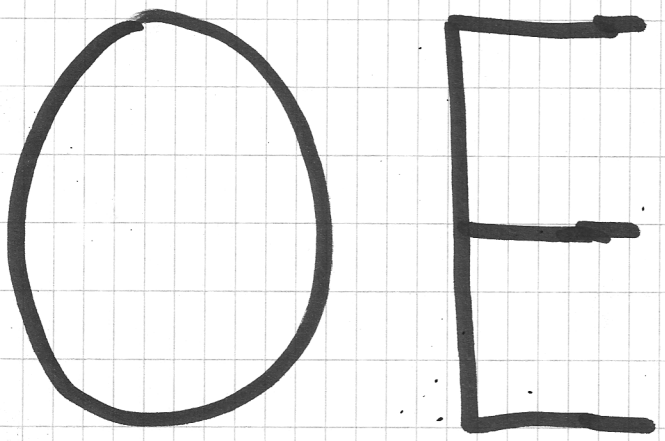


**ZADATAK** Trošilo impedancije $\underline{Z} = 10 + j20\ \Omega$ priklj. je na idealni sinusni naponski izvor, efektivne vrij. napona 220V i frekv. 50 Hz. Odredite iznos kapacitet. kondenzatora koji treba spojiti paralelno trošilu tako da bi se faktor snage trošila povećao na 0.9 (induktivno).

**Rj:**

U našem primjeru imamo induktivno trošilo. Najprije odredimo koliku su radnu i jalovu snagu trošila.

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = 9.8387 \angle -63.4349\ A$$

$$P = |I|^2 R = 968\ W$$

$$Q_L = |I^2| \cdot X = 1936\ VAr$$

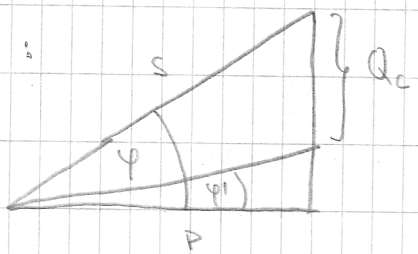
Fazni kut je $\arctg\left(\frac{Q_L}{P}\right) = 63.4349$. Iz toga slijedi da je faktor snage trošila jednak $\cos\varphi = \cos(63.4349) = 0.45$ (ind.).

Potrebno je poboljšati faktor snage na 0.9. Spajanjem kond. paralelno trošilu mora kut snage trebao biti $\arccos(0.9) = 25.84$.

U novom trokutu snage radna snaga je i dalje 968 W, kut je 25.84, stoga je prostorna reaktivna snaga $Q = P \cdot \tg\varphi' = 468.78\ VAr$. Kondenz. mora preuzeti razliku $Q_C = Q_L - Q = 1467.22\ VAr$. Napon na kondenz. je 220 V pa slijedi:

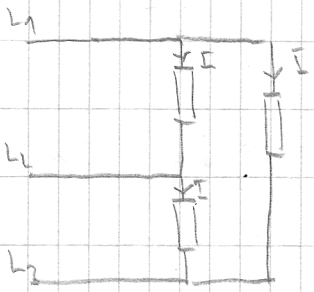
$$C = \frac{Q_C}{\omega U^2} = 96.49\ \mu F$$

-skica:



ZADATAK Kolika je snaga simetričnog trofaznog trošila spojenog u trokut ($Z_1 = Z_2 = Z_3 = 100\angle 30° \Omega$) priklj. na simetričan trofazni izvor linijskog napona $U_L = 380\,V$?

Rj.




$$I = \frac{U_f\angle 0° V - U_f\angle -120° V}{Z} = 3.8\,A$$

$$U_f = \frac{U_L\sqrt{3}}{3} = 219.3531\,V$$

$$P_{UK} = 3P = 3|I|^2 R = 3 \cdot 3.8^2 \cdot 86.60 = 3751.512\,W \checkmark$$

$$Z = 100\angle 30° = 86.60 + 50i$$

ZADATAK Osnovna ćelija izvora napona ima napon praznog hoda $E = 1.5\,V$ i unut. otpor $R_i = 0.005\,\Omega$. Koliko je ćelija potrebno serijski spojiti u bateriju, da bi pri opterećenju strujom $I = 20\,A$ baterija imala napon $U = 112\,V$?

Rj.

$$U = U_0 - IR_i$$

$$112 = x \cdot 1.5 - 20 \cdot x \cdot 0.005 \Rightarrow x = 80$$

ZADATAK Dva otpornika jedn. nazivnog napona $U_n$ serijski su spojeni i priklj. na izvor nazivnog napona. Odredite ukupnu snagu spoja, ako je zadano $P_{n1} = 50\,W$, $P_{n2} = 150\,W$.

Rj.

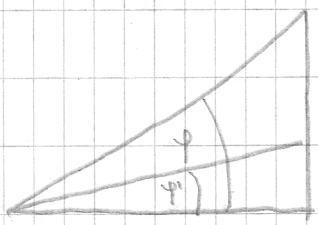
$$P_{UK} = P_1 + P_2 = I^2R_1 + I^2R_2 = I^2(R_1 + R_2) = \frac{U_n^2}{R_1 + R_2} = \frac{U_n^2}{\frac{U_n^2}{P_{n1}} + \frac{U_n^2}{P_{n2}}} =$$

$$I = \frac{U_n}{R_1 + R_2} \qquad P_n = \frac{U_n^2}{R}$$

$$= \frac{1}{\frac{1}{P_{n1}} + \frac{1}{P_{n2}}} = 37.5\,W$$

ZADATAK Induktivnom trošilu koje je priključeno na gradsku mrežu efektivne vrijednosti napona 220V i frekvencije 50 Hz i koje razvija snagu od 240W paralelno je spojen kondenzator kapaciteta 150 μF. Ukupan faktor snage za cijelu kombinaciju iznosi 0.8 (ind.). Koliki je bio faktor snage trošila prije spajanja kondenzatora?

Rj:




$U = 220V$, $f = 50Hz$

$P = 240W$, $C = 150\mu F$

$\cos\varphi' = 0.8$

---

$\cos\varphi = ?$

$Q_L = Q_c + Q$, $\varphi' = \arccos(0.8) = 36.8699 \rightarrow tg\varphi' = \frac{Q}{P} \Rightarrow Q = P\,tg\varphi' = 180VAr$

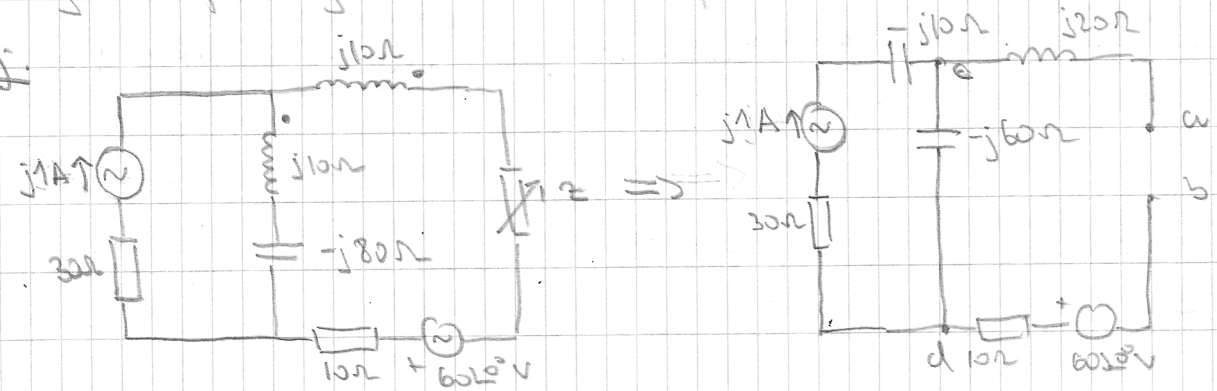
$C = \frac{Q_c}{\omega U^2} \Rightarrow Q_c = C\omega U^2 = 150\mu F \cdot 2\pi f \cdot 220^2 = 2280.75\ VAr$

$Q_L = Q_c + Q = 2460.7963\ VAr$

$tg\varphi = \frac{Q_L}{P} \Rightarrow \cos\varphi = \cos\left[\operatorname{arctg}\left(\frac{Q_L}{P}\right)\right] = 0.097$

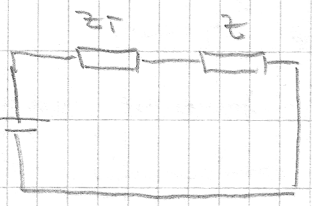
ZADATAK Odredite maks. snagu koja se može razviti na promjenjivoj impedanciji u mreži prema slici.

Rj:




$U_{ab} = U_{cd} + 60\angle 0°V = 120V$

$\Rightarrow$



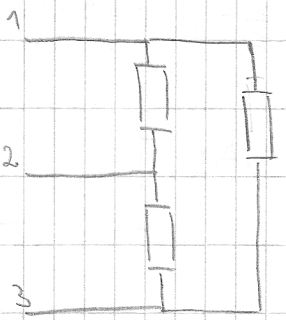

$Z_{TH} = Z_{AB} = 10 - j40\,\Omega$

$Z = Z_{TH}^* = 10 + j40\,\Omega$

$$I = \frac{U}{Z_{TH}+Z} = \frac{120\,V}{20\,\Omega} = 6\,A \qquad P_{max} = I^2\,Re(Z) = 6^2 \cdot 10\,\Omega = 360\,W \;\checkmark$$

**ZADATAK** Tri impedancije spojene su u trokut između čvorišta 1, 2 i 3 te priključene na trofazni izvor linijskog napona $U_L = 400\,V$. Ako impedancije iznose $Z_{12} = 100 + j20\,\Omega$, $Z_{13} = 100 + j150\,\Omega$, $Z_{23} = 100 - j30\,\Omega$, odredite ukupnu jalovu snagu spoja.

**Rj.**




$Z_{12} = 100 + j20\,\Omega$

$Z_{13} = 100 + j150\,\Omega$

$Z_{23} = 100 - j30\,\Omega$

$U_L = 400\,V \Rightarrow U_f = \frac{400\sqrt{3}}{3}$

---

$Q_{uk} = ?$

$$Q_{uk} = Q_{12} + Q_{13} + Q_{23} = |I_{12}|^2\,Im(Z_{12}) + |I_{13}|^2\,Im(Z_{13}) + |I_{23}|^2\,Im(Z_{23})$$

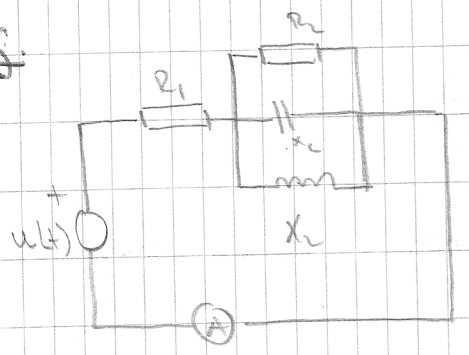
$$I_{12} = \frac{U_f \angle 0^\circ V - U_f \angle -120^\circ V}{Z_{12}} = 3.5223 \angle 18.65\,A \qquad \boxed{Q_{uk} = 102\,VAr}$$

$$I_{23} = \frac{U_f \angle -120^\circ V - U_f \angle -240^\circ V}{Z_{23}} = 3.8313 \angle -73.3\,A$$

$$I_{13} = \frac{U_f \angle 0^\circ V - U_f \angle -240^\circ V}{Z_{13}} = 3.9557 \angle -38.53\,A$$

**ZADATAK** Na trošilo priključeno sličnom prikazanom je izvor nesinusoidalnog napona koji se može opisati jednadžbom $u(t) = 10 + 100\sin(\omega t) + 50\sin(2\omega t)$ V. Odredite struju ampermetra ako je poznato: $R_1 = R_2 = X_C = 10\,\Omega$, $X_L = 5\,\Omega$.

**Rj:**




$u(t) = 10 + 100\sin(\omega t) + 50\sin(2\omega t)$

$R_1 = R_2 = X_C = 10\,\Omega$, $X_L = 5\,\Omega$

$I_A = ?$

1° $u(t) = 10\,V$

$I_1 = \frac{U}{R_1} = 1\,A$

2° $u(t) = 100\sin(\omega t)$

$I_2 = \frac{U_{eff}}{Z} = \frac{\frac{100\sqrt{2}}{2}}{10} = 4.714\,A$

2° $X_C = 5\,\Omega$

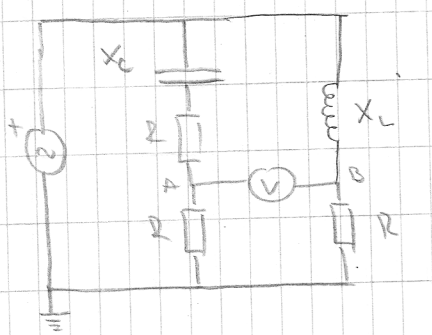
$X_L = 10\,\Omega$

$I_3 = \frac{25\sqrt{2}}{Z} = 2.357\,A$

$I_A = \sqrt{I_1^2 + I_2^2 + I_3^2} = 5.36\,A$

**ZADATAK** Odredite pokazivanje voltmetra u mreži prema slici ako je napon izvora $U = 10\,V$ i vrijedi $R = X_L = \frac{1}{2}X_C = A$.

**Rj:**




$U_V = U_{AB} = \varphi_A - \varphi_B = U - U_C - U_R - U + U_L$

$= U_L - U_C - U_R =$

$\varphi_A = U - U_C - U_R$

$\varphi_B = U - U_L$

$$I_1 = \frac{U}{2R - jX_L} \quad , \quad I_2 = \frac{U}{R + jX_L}$$

$U_L = I_2 \cdot (jX_L)$

$U_C = I_1 (-jX_C)$

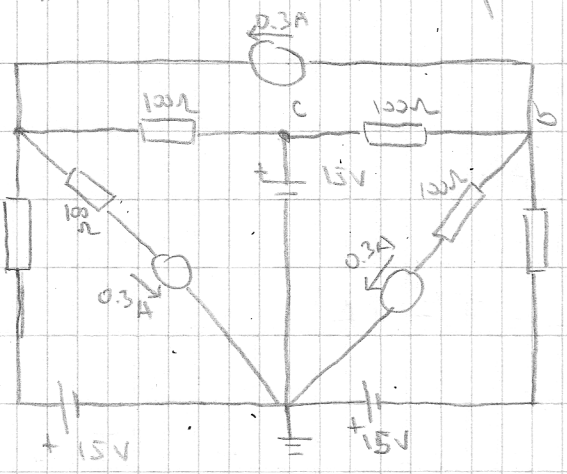
$U_R = I_1 R$

$$U_V = U_L - U_C - U_R = \frac{U}{R+jA}(jA) + j\frac{U}{2A - j\frac{A}{2}}\frac{A}{2} - \frac{U}{2R - j\frac{A}{2}} A$$

$$= j\frac{AU}{R+jA} + j\frac{AU}{4A - jA} - \frac{2AU}{4R - jA} =$$

ZADATAK Odredite napon $U_{ab}$ u mreži prema slici.

Rj.




Primj. metodu napona čvorova:

$$\varphi_x G_{xx} - \sum_{\substack{i=1 \\ i \neq x}}^{n} \varphi_x G_{ix} = \text{alg} \sum_{\substack{i=1 \\ i=x}}^{n} U_{ix} G_{xi}$$

* ako je u grani strujni strujni izvor, njegovu vodlj. zanemarujemo

(čvor a) $\varphi_a \left(\frac{1}{100} + \frac{1}{100}\right) - \varphi_c \cdot \frac{1}{100} = \frac{15}{100} - 0.3 + 0.3$ /·

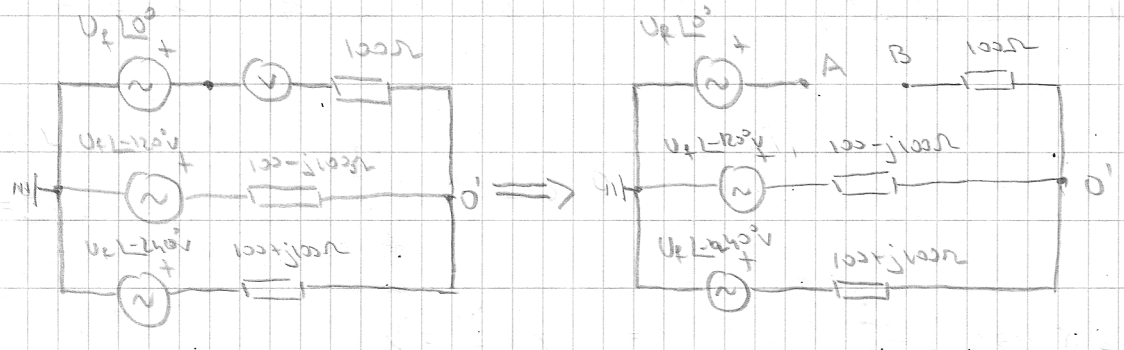
(čvor b) $\varphi_b \left(\frac{1}{100} + \frac{1}{100}\right) - \varphi_c \cdot \frac{1}{100} = -\frac{15}{100} - 0.3 - 0.3$

$$\Rightarrow \quad \frac{2}{100}(\varphi_a - \varphi_b) = \frac{30}{100} + 0.6 = 0.9$$

$$\frac{2}{100} U_{ab} = 0.9 \Rightarrow \boxed{U_{ab} = 45\,V}$$

ZADATAK Koliki napon pokazuje voltmeter u mreži prema slici ako je $U_f = 180\,V$?

Rj:

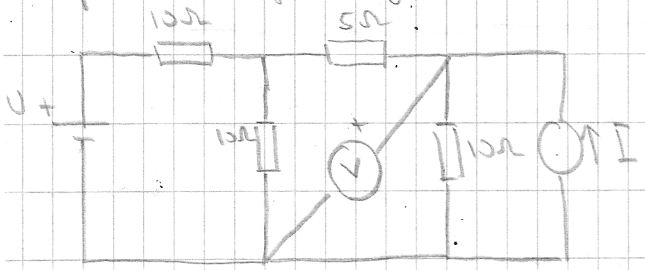



$U_v = U_{ab} = \varphi_a - \varphi_b = 180 \angle 65.88\,V = 114.11\,V$

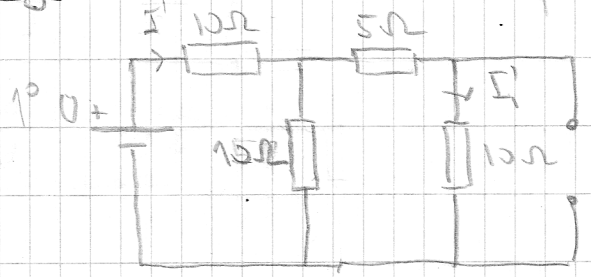
$\varphi_a = 180\,V$ , $\varphi_b = \varphi_{b'}$

$$U_{oo'} = \varphi_{o'} = \varphi_b = \frac{\frac{U_f \angle -120°}{100 - j100\Omega} + \frac{U_f \angle -240°}{100 + j100\Omega}}{\frac{1}{100 - j100\Omega} + \frac{1}{100 + j100\Omega}} = 65.88\,V$$

ZADATAK Voltmetar u spoju na slici pokazuje 10V. Ako se udvostruči jakost struje strujnog izvora I, voltmeter pokazuje 15 V. Koliki je napon naponskog izvora U?



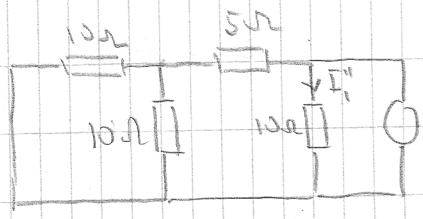

Rj: Koristimo metodu superpozicije.




$R_{ek} = 10 + 10 \,||\, (10 + 5) = 16\,\Omega$

$I' = \frac{U}{16}$ , $I_1' = I' \frac{10}{25} = \frac{U}{40}\,A$

2°




$R_{ek} = 5\,\Omega$

$I_1'' = \frac{I}{2}$

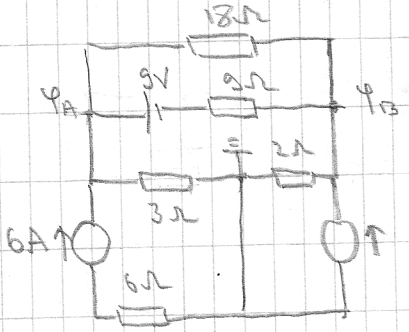
Postavljamo sustav jednadžbi za oba slučaja:

$$\begin{cases} \frac{U}{40} + \frac{I}{2} = 1\,A \\ \frac{U}{40} + \frac{2I}{2} = 1.5\,A \end{cases} \Rightarrow I = 1\,A$$

$$\boxed{U = 20\,V}$$

ZADATAK Odredite iznos struje kroz otpornik $R = 3\,\Omega$ u mreži prema slici.

Rj.




$$\begin{cases} \varphi_A\left(\frac{1}{3}+\frac{1}{5}+\frac{1}{18}\right) - \frac{\varphi_B}{9} - \frac{\varphi_B}{18} = 6 + \frac{9V}{9\Omega} \\ -\varphi_B\left(\frac{1}{9}+\frac{1}{18}+\frac{1}{2}\right) - \frac{\varphi_A}{9} - \frac{\varphi_A}{18} = 0.5A - \frac{9V}{9\Omega} \end{cases}$$

$$\begin{cases} \frac{\varphi_A}{2} - \frac{\varphi_B}{6} = 7 \;/\cdot 6 \\ \frac{2}{3}\varphi_B - \frac{\varphi_A}{6} = -\frac{1}{2} \;/\cdot 6 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 3\varphi_A - \varphi_B = 42 \;/\cdot 4 \\ 4\varphi_B - \varphi_A = -3 \end{cases} / + \quad \Rightarrow \quad 11\varphi_A = 165$$

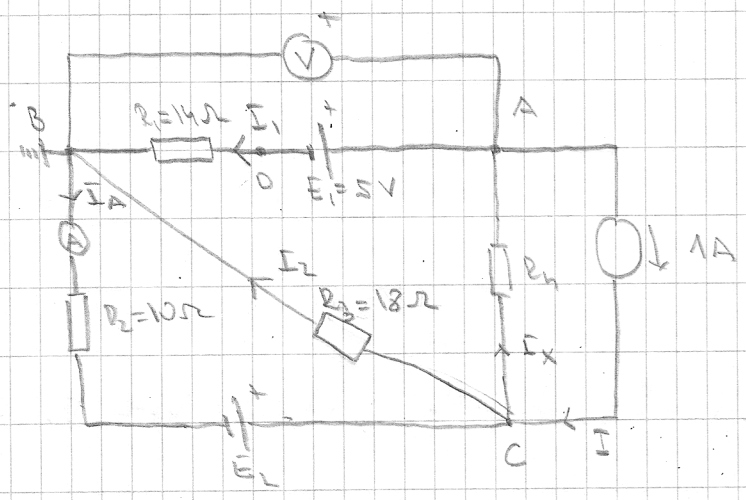
$$\varphi_A = 15\,V$$

$U_{A0} = 15V - 0V = 15V$

$$\boxed{I_3 = \frac{U_{A0}}{R_3} = \frac{15V}{3\Omega} = 5\,A}$$

ZADATAK Ako idealni ampermetar pokazuje $I_A = 1.5\,A$, a idealni voltmetar $U_V = U_{AB} = 12\,V$, odredite iznos otpornika $R_4$.

Rj.:




$U_{AB} = \varphi_A - \varphi_B = 12\,V$

$I_A = 1.5\,A$

$R_4 = ?$

$U_{DB} = \varphi_D - \varphi_B = 7\,V$

$I_1 = \frac{U_{DB}}{R_1} = 0.5\,A$

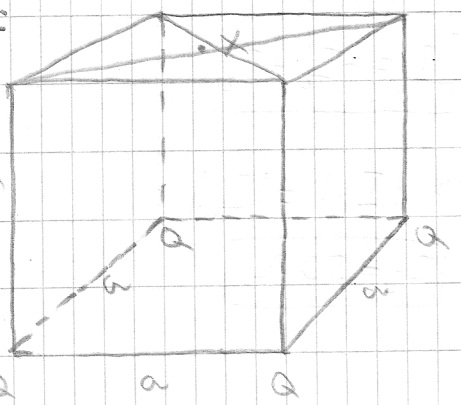
$I_1 + I_2 = I_A \Rightarrow I_2 = I_A - I_1 = 1.5A - 0.5A = 1A \Rightarrow \boxed{I_2 = 1A} \Rightarrow \boxed{\varphi_C = 18\,V}$

$I + I_A = I_x + I_2 \Rightarrow I_x = I + I_A - I_2 = 1A + 1.5A - 1A = 1.5\,A$

$U_{CA} = \varphi_C - \varphi_A = 18V - 12V = 6V$ ; $I_x = \frac{U_{CA}}{R_4} \Rightarrow R_4 = \frac{U_{CA}}{I_x} = \frac{6V}{\frac{3}{2}A} = 4\Omega$ ✓

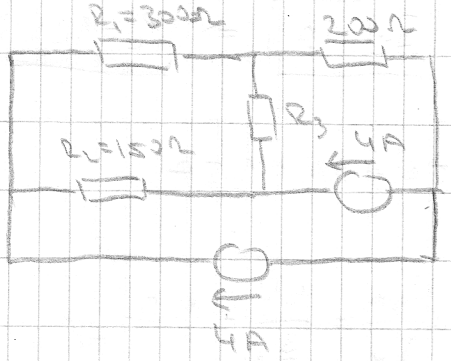
ZADATAK Četiri točkasta naboja smještena su u četiri kuta kocke kao na slici. Ako je $Q = 5\,\mu C$ i $a = 0.2\,m$, kolika je jakost elekt. polja u točki X ?

Rj.:



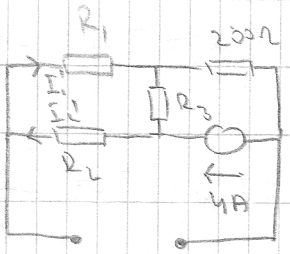
ZADATAK: Koliki treba biti $R_3$ da bi struja kroz $R_1$ i $R_2$ bile jednake?

Rj:




$I = 4\ A$

1°)




suprotni smj.

$$I_1' = I\frac{R_3}{450+R_3}\ (+), \quad I_2' = \frac{R_3}{450+R_3}\ (-)$$

2°)


$$I_1'' = I\frac{150+R_3}{450+R_3}$$

$$I_2'' = I\frac{300}{450+R_3}$$

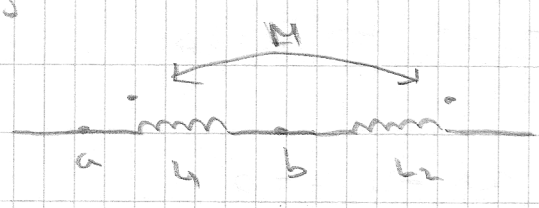
$$\begin{cases} I_1 = I_1' + I_1'' \\ I_2 = -I_2' + I_2'' \end{cases}$$

$$\Rightarrow |I_1| = |I_2|$$

$$\Rightarrow \boxed{R_3 = 50\,\Omega}$$

ZADATAK Dva svitka $L_1 = 3\,mH$ i $L_2 = 12\,mH$ spojena su u seriju s fakt. magn. veze $k = 1$. Odredi napon $U_{ab}$ ako struja raste lin. brz. $10\,A/s$.

Rj.




$$u = L\frac{di}{dt}$$

$$u_a = 30\,mV$$

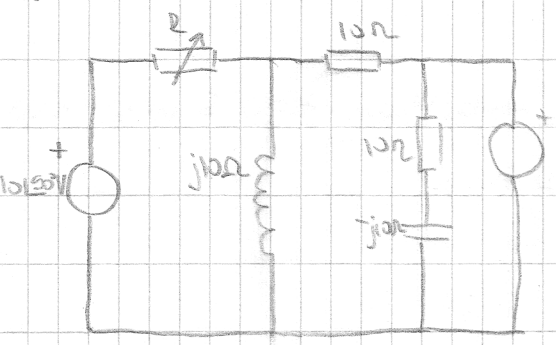
$$u_2 = M\frac{di}{dt}, \quad M = k\sqrt{L_1 L_2}$$

$$= 60\,mV$$

$$U_{ab} = U_a - U_2 = -30\,mV$$

ZADATAK Odredite maks. snagu koja se može ostvariti na otporu u mreži prema slici.

Rj.




$$U_{TH} = 7.536\angle\frac{45}{2}\,V$$

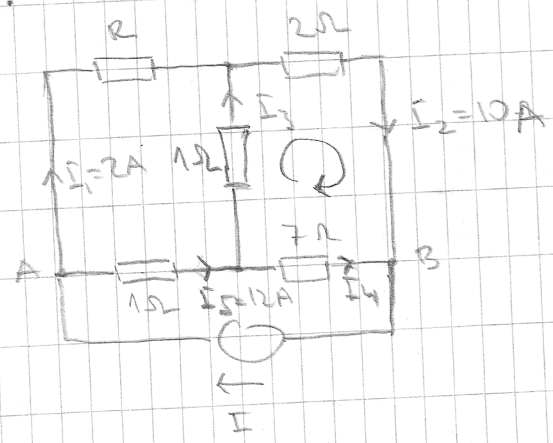
$$Z_{TH} = 5 + 5i \Rightarrow R = 5\,\Omega$$

$$I = \frac{U}{Z_{TH} + R} = 0.6845\angle -4.065\,A$$

$$P = |I|^2 R = 2.34\,W$$

ZADATAK U_z porvnite struje i dopse na slici odredite napon U_ab.

Rj:




$I_1 + I_3 = I_2 \Rightarrow I_3 = 8A$

$-8V - 20V + I_4 \cdot 7 = 0$

$I_4 = 4A$

$I = I_4 + I_2 = 14A$

$I = I_5 + I_1 \Rightarrow \boxed{I_5 = 12A}$

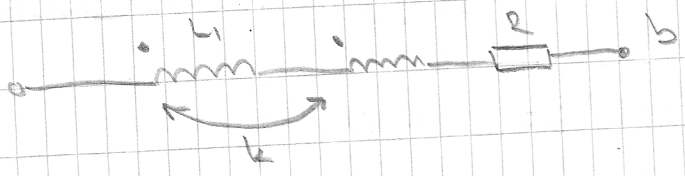
$U_{ab} = \varphi_A - \varphi_B = 12V + 28V = 40V$

$\varphi_A = 12V = I_5 \cdot 1\Omega$

$\varphi_B = -28V = -I_4 \cdot 7\Omega$

ZADATAK Odredite napon $u_{ab}(t)$ u trenutku $t = 0.5$ ms ako je $L_1 = 1$ mH, $L_2 = 4$ mH, $k = 0.5$ i $R = 2\Omega$.

Rj:




$t_1 = 0.5$ ms

$L_1 = 1$ mH, $L_2 = 4$ mH, $R = 2\Omega$

$k = 0.5$

$u_{ab}(t_1) = ?$

$u_{L_1} = (L_1 + M)\frac{di}{dt} = 2V$

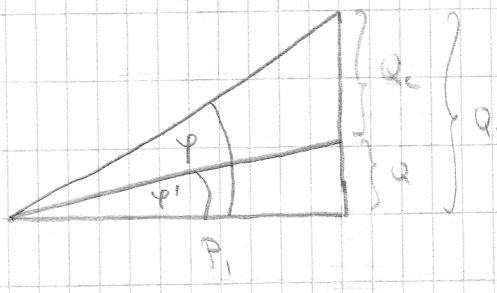
ZADATAK Na trofazni sustav faznog napona $U_f = 220V$ frekv. 50 Hz priključ. je elektromotor snage 6kW ($\cos\varphi = 0.77$). Treba izvršiti kompenzaciju jalove snage s tri kondenzatora odgov. spojena na trofazni sustav. Odredi najmanji kapacitet kojim bismo ukupni faktor snage povećali na $\cos\varphi' = 0.86$.

Rj: $U_f = 220V$, $f = 50Hz$

$P = 6kW$, $\cos\varphi = 0.77$

$\cos\varphi' = 0.86$

$C = ?$




$P_1 = \frac{P}{3} = 2000W$

$\varphi' = \arccos(0.86) = 30.68$

$\varphi = 39.64$

$Q = P_1 \operatorname{tg}\varphi' = 1186.569\ VAr$

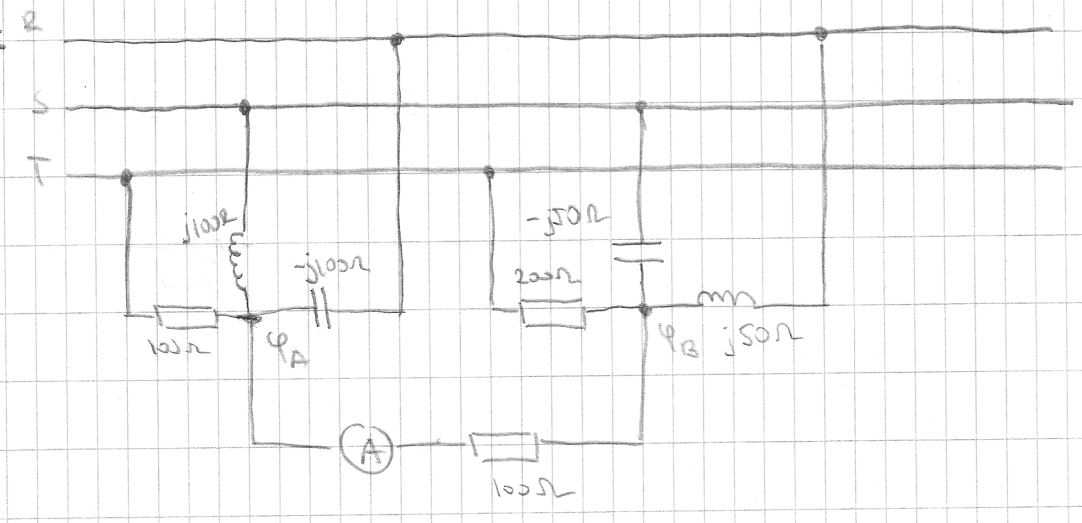
$Q_L = P_1 \operatorname{tg}\varphi = 1656.897\ VAr$

$\Big\} \quad Q_c = Q_L - Q = 470.328 VAr$

$C = \frac{Q_c}{3\omega U^2} = 10.3\mu F$

ZADATAK Trofazna trošila spojena u zvijezdu napajaju se iz trofazne mreže. Odredite pokazivanje ampermetra, ako je zadan fazni napon izvora $U_f = 220\,V$.

Rj.




$$I_A = \left|\frac{U_{AB}}{100\Omega}\right| = \left|\frac{\varphi_A - \varphi_B}{100\Omega}\right| = \left|\frac{476.3\angle 120}{100}\right| = 4.76\,A \checkmark$$

$$\begin{cases} \varphi_A\left(\frac{1}{100} + \frac{1}{j100} + \frac{1}{-j100} + \frac{1}{100}\right) - \frac{\varphi_B}{100} = \frac{U_f\angle 0}{-j100} + \frac{U_f\angle -120}{j100} + \frac{U_f\angle -240}{100} \\ \varphi_B\left(\frac{1}{100} + \frac{1}{200} + \frac{1}{j50} + \frac{1}{-j50}\right) - \frac{\varphi_A}{100} = \frac{U_f\angle 0°\,V}{j50\Omega} + \frac{U_f\angle -120°\,V}{-j50\Omega} + \frac{U_f\angle -240}{200} \end{cases}$$

$$\frac{\varphi_A}{50} = \frac{\varphi_B}{100} + 6.01\angle 120 \quad /:2 \Rightarrow \quad \frac{\varphi_A}{100} = \frac{\varphi_B}{200} + 3.005\angle 120$$

$$\frac{3\varphi_B}{200} = \frac{\varphi_A}{100} + 6.52\,1\angle -60$$

$$\frac{3\varphi_B}{200} = \frac{\varphi_B}{200} + 3.005\angle 120 + 6.52\,1\angle -60$$

$$\Rightarrow \varphi_B = 351.6\angle -60$$

$$\varphi_A = 124.7\angle 120\,V$$

$$\therefore U_{AB} = 476.3\angle 120\,V$$