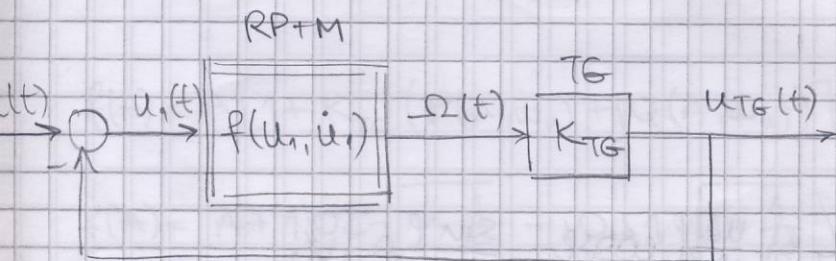


$$|u_1| > b$$

$$I \frac{d\Omega}{dt} + \frac{1}{\beta_0} \cdot \Omega = M_0$$

β_0 - koef. nagiba meh. karakter. motora

$$\beta_0 = \frac{\omega_n}{M_0}$$



u_1 - javlja se zbog inercijskih svojstava u motoru, a sam NE ih nema

$$u_1(t) = u(t) - u_{TG}(t)$$

$$u_{TG}(t) = K_{TG} \cdot \Omega(t) = K_{TG} f(u_1, i_1)$$

Dif. jed. zatvorenog kruga

$$u(t) + K_{TG} f(u_1, i_1) = u(t)$$

$$\Omega = \left[P(A, \omega_u) + \frac{Q(A, \omega_u)}{\omega_u} P \right] u_1(t)$$

↳ opisna f-ja NE koji ima inercijska svojstva (i_1)

$$\left\{ 1 + K_{TG} \left[P(A, \omega_u) + \frac{Q(A, \omega_u)}{\omega_u} P \right] \right\} u_1(t) = u(t)$$

bifaziti
preko u_1

$$u = U_m \sin(\omega_u t)$$

$$= U_m \sin[(\omega_u t + \varphi) - \psi] = U_m \cos \psi \sin(\omega_u t + \varphi) - U_m \sin \psi \cos(\omega_u t + \varphi)$$

$u_1 = A \sin(\omega_u t + \varphi) \rightarrow$ prepostavljeni oblik signala na ulazu

$$i_1 = A \omega_u \cos(\omega_u t + \varphi) \quad u \text{ NE}$$

$$\sin(\omega_n t + \varphi) = \frac{u_i}{A}$$

$$\cos(\omega_n t + \varphi) = \frac{u_r}{A \omega_n}$$

$$u(t) = U_m \cos \varphi \cdot \frac{u_i}{A} - U_m \sin \varphi \cdot \frac{u_r}{A \omega_n}$$

$$u(t) = \frac{U_m}{A} \left(\cos \varphi - \frac{\sin \varphi}{\omega_n} P \right) u_i$$

→ jed. faziv. kružna

$$\left\{ 1 + K_{TG} \left[P + \frac{Q}{\omega_n} P \right] \right\} u_i = \frac{U_m}{A} \left[\cos \varphi - \frac{\sin \varphi}{\omega_n} P \right] u_i$$

→ karakter jed. fazivorelnoj kružna:

$$1 + K_{TG} \left[P + \frac{Q}{\omega_n} P \right] - \frac{U_m}{A} \left(\cos \varphi - \frac{\sin \varphi}{\omega_n} P \right) = 0$$

$$P = j\omega$$

$$1 + K_{TG} \underbrace{\left[P + jQ \right]}_{G_N(A, \omega_n)} - \underbrace{\frac{U_m}{A} \left[\cos \varphi - j \sin \varphi \right]}_{e^{-j\varphi}} = 0$$

$$\underbrace{A \left[1 + K_{TG} G_N(A, \omega_n) \right]}_{f(A)} = U_m e^{-j\varphi}$$

$$G_N(A, \omega_n) = G_{NI}(A) \cdot G(s)$$

\downarrow
opisiva
fja RP

\downarrow
je prijenosna
motorna

$$G_{NI}(A) = \frac{U_m}{U_n} = P_1(A) + jQ_1(A) = P_1(A) = \frac{4C}{\pi A} \sqrt{1 - \left(\frac{b}{A} \right)^2}$$

$$G(S) = \frac{1}{1 + ST_M} = \frac{\Omega}{U_m}$$

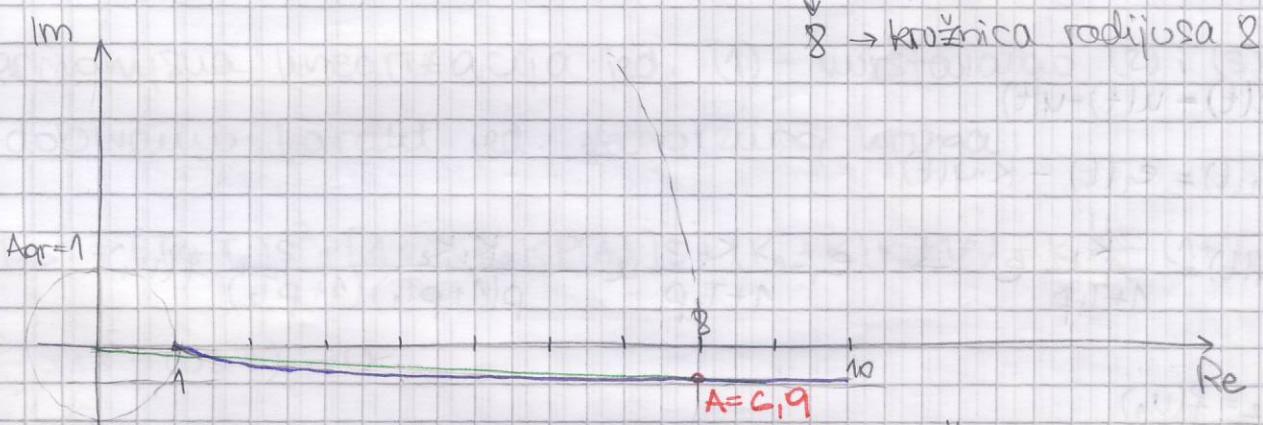
$$T_M = \frac{\Omega_m}{M_0} \cdot T = I \cdot \beta_0$$

$$G_N(A, \omega_n) = \frac{4C}{\pi A} \sqrt{1 - \left(\frac{b}{A}\right)^2} \cdot \frac{1}{1 + \omega_n^2 T_m^2} = P_1(A) \cdot \frac{1 - j\omega_n T_m}{1 + \omega_n^2 T_m^2} =$$

$$= \underbrace{\frac{4C}{\pi A^2} \frac{\sqrt{A^2 - b^2}}{(1 + \omega_n^2 T_m^2)}}_{P(A, \omega_n)} - j \underbrace{\frac{4C \sqrt{A^2 - b^2} \omega_n T_m}{\pi A^2 (1 + \omega_n^2 T_m^2)}}_{Q(A, \omega_n)}$$

$$f(A) = A \left[1 + K_{TG} (P(A, \omega_n) + jQ(A, \omega_n)) \right] = U_m e^{-j\varphi}$$

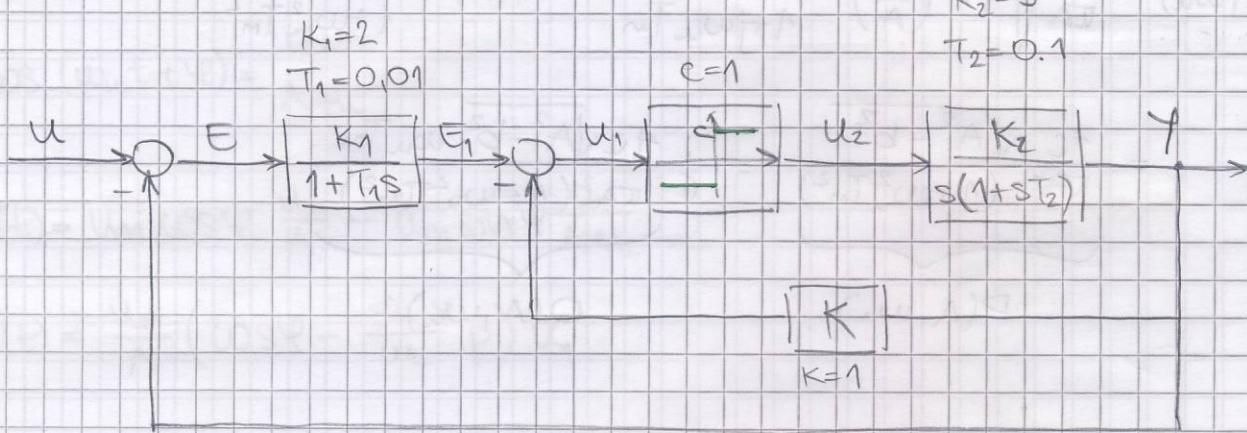
$$f(A) = A + 1,02 \sqrt{1 - \frac{1}{A^2}} - j0,51 \sqrt{1 - \frac{1}{A^2}} = U_m e^{-j\varphi}$$



A	0	1	2	3	5	∞	
f(A)	∞-j∞	1	2,52-j0,16	3,96-j0,48	6-j0,5	∞-j0,5	

$$\Omega(t) = \underbrace{\frac{6,9}{K_{TG}}}_{690} \sin(10t + \varphi)$$

ANALITIČKI PRIMJERI



$$u = U_m \sin(\omega_u t)$$

$$\omega_u = 30 \text{ [s}^{-1}\text{]}, \underline{U_m = ?}$$

$$e(t) = u(t) - y(t)$$

$$u_1(t) = e_1(t) - K \cdot y(t)$$

$$e_1(t) = \frac{K_1}{1+T_1 p} e \rightarrow e_1 = \frac{K_1}{1+T_1 p} u - \frac{K_1 K_2}{p(1+pT_1)(1+pT_2)} f(u_1)$$

$$u_2 = f(u_1)$$

$N_1, N_2 = A$

$$y(t) = \frac{K_2}{p(1+pT_2)} u_2(t)$$

→ zatvoreni kruž

$$u_1(t) + K \frac{K_2}{p(1+pT_2)} f(u_1) = \frac{K}{1+pT_1} u - \frac{K_1 K_2}{p(1+pT_1)(1+pT_2)} f(u_1)$$

$$(1+pT_1)(1+pT_2) pu_1(t) + K \cdot K_2 (1+pT_1) f(u_1) = K \cdot p(1+pT_2) u - K \cdot K_2 f(u_1)$$

$$(1+pT_1)(1+pT_2) pu_1(t) + [K T_1 p + K_1 + K] K_2 f(u_1) = K_1 (1+pT_2) p u_1(t) \quad . \quad (1)$$

↳ dif. jed. zatvorenog kruža

- aliž su se uspostavile prirodne osc. onda je na ulazu u NE:

$$u_1(t) = A \sin(\omega_u t + \varphi)$$

A, φ - nepoznato

ω_u - poznato

$\rightarrow u(t)$ čemo prikazati preko $u_1(t) \rightarrow$ isto kao i u prethodnom primjeru

$$u_1(t) = \frac{U_m}{A} [\cos \varphi - \frac{\sin \varphi}{\omega_n} \cdot p] u_1(t) \quad (2)$$

NE - dvoopol. retež

↳ linearizacija opisnog f-jona

$$f(u_1) = P(A) u_1 \quad (3)$$

$$P(A) = \frac{4C}{IA}$$

Harmonička linearizacija jed. (1) - vršimo (2); (3)

- dobivamo karakterist. jed. zatvoreneg kruža:

$$T_1 T_2 s^3 + (T_1 + T_2)s^2 + [1 + T_1 K_2 K P(A)]s + (K_1 + K)K_2 P(A) + K_1 \frac{U_m}{A} (1 + T_2 s) = 0$$

$$\left(\frac{s}{\omega_n} \sin \varphi - \cos \varphi \right) = 0$$

$$\boxed{s = j\omega_n}$$

$$\text{Re: } (K_1 + K)K_2 P(A) - (T_1 + T_2)\omega_n^2 + \frac{K_1 U_m \omega_n}{A} (T_2 \omega_n \cos \varphi - \sin \varphi) = 0$$

$$\text{Im: } [1 + T_1 K_2 K P(A)]\omega_n - T_1 T_2 \omega_n^3 - \frac{K_1 U_m \omega_n}{A} [T_2 \omega_n \sin \varphi + \cos \varphi] = 0$$

I možimo Re $s = T_2 \omega_n$

$$\text{I) } X = Y = 0 \quad (\text{rješit čemo se slava sa } \sin \varphi \text{ te možemo izr. } \cos \varphi)$$

II možimo Im $s = -T_2 \omega_n$

$$\text{II) } X = Y \quad (\text{rješit čemo se } \cos \varphi \text{ i dobit sin})$$

$$\sin^2 \varphi + \cos^2 \varphi = 1 \rightarrow A$$

I

$$T_2 w_u (K_1 + K) K_2 P(A) - T_2 (T_1 + T_2) w_u^2 + \frac{K_1 K_2 w_0^2 T_2^2}{A} \cos \varphi - \frac{K_1 K_2 w_0^2 T_2}{A} \sin \varphi =$$

$$= [1 + T_1 K_2 K P(A)] w_0 - T_1 T_2 w_u^2 - \frac{K_1 K_2 w_0^2 T_2}{A} \sin \varphi - \frac{K_1 K_2 w_0}{A} \cos \varphi$$

$$P(A) = \frac{4C}{\pi A}$$

$$-T_2 K_2 \frac{4C}{\pi A} (R_1 + K) + 1 + T_1 K_2 K \frac{4C}{\pi A} + T_2 w_u^2 (T_1 + T_2) - T_1 T_2 w_0^2 - \frac{K_1 K_2 (T_2^2 w_0^2 + 1) \cos \varphi}{\pi A}$$

$$-4CT_2 K_2 (K_1 + K) + \pi A + 4CT_1 K_2 K + \pi A T_2 w_u^2 (T_1 + T_2) - \pi A T_1 T_2 w_0^2 - \pi K_1 K_2$$

$$(\pi T_2^2 w_0^2 + 1) \cos \varphi = 0$$

$$-\pi K_1 K_2 (T_2^2 w_0^2 + 1) \cos \varphi = 4CK_2 (T_2 K_1 + T_2 K - T_1 K) - \pi A (T_2^2 w_0^2 + 1)$$

$$-\pi K_1 K_2 \cos \varphi = \frac{4CK_2 (T_2 K_1 + T_2 K - T_1 K)}{\pi T_2^2 w_0^2 + 1} - \pi A$$

$\underbrace{\qquad\qquad\qquad}_{E = 0.58}$

$$-\pi K_1 K_2 \cos \varphi = 0.58 - \pi A$$

(5)

II

$$\pi K_1 K_2 \sin \varphi = \frac{4CK_2 (K_1 + K + T_1 T_2 K w_0^2)}{w_0 (1 + T_2^2 w_0^2)} - T_1 w_0 \pi A$$

$\underbrace{\qquad\qquad\qquad}_{M = 0.3}$

$H = 0.26$

$$\pi K_1 K_2 \sin \varphi = 0.26 - 0.3 \pi A$$

(6)

$$\cos \varphi = - \frac{E - \bar{\mu}A}{\pi K_1 U_m}$$

$$\sin \varphi = \frac{H - M\bar{\mu}A}{\pi K_1 U_m}$$

$$\sin^2 \varphi + \cos^2 \varphi = 1$$

$$(E - \bar{\mu}A)^2 + (H - M\bar{\mu}A)^2 = \pi^2 K_1^2 U_m^2$$

$$(1 + M^2) \pi^2 A^2 - 2(E + HM)\bar{\mu}A + E^2 + H^2 - \pi^2 K_1^2 U_m^2 = 0$$

$$A = \frac{(E + HM) \pm \sqrt{(E + HM)^2 - (1 + M^2)(E^2 + H^2 - \pi^2 K_1^2 U_m^2)}}{\pi(1 + M^2)} \quad (7)$$

Za realni i pozitivni A treba biti zadovljeno

a) $E + HM > 0 \quad 0.658 > 0 \quad \checkmark$

b) $(E + HM)^2 \geq (1 + M^2)(E^2 + H^2 - \pi^2 K_1^2 U_m^2)$

$$\frac{(E + HM)^2}{(1 + M^2)} \geq E^2 + H^2 - \pi^2 K_1^2 U_m^2$$

$$\pi^2 K_1^2 U_m^2 \geq E^2 + H^2 - \frac{(E + HM)^2}{1 + M^2}$$

$$\pi^2 K_1^2 U_m^2 \geq \frac{(H - EM)^2}{1 + M^2}$$

$$U_{\text{kor}} = - \sqrt{\frac{(H - EM)^2}{\pi^2 K_1^2 (1 + M^2)}} = 1,31 \cdot 10^{-2}$$

neka je $U_m = 8.6 \cdot 10^{-2}$

uz takav U_m iz (7) dobivamo:

$$A_1 = 0.362 \quad \begin{cases} \text{SAMO} \\ \text{jedno} \end{cases}$$

$$A_2 = 0.023 \quad \begin{cases} \text{je} \\ \text{stabilno} \end{cases}$$

iz (5) i (6): $\operatorname{tg} \varphi = - \frac{H - M\bar{\mu}A}{E - \bar{\mu}A}$

$$\boxed{\varphi = - \operatorname{arctg} \frac{H - M\bar{\mu}A}{E - \bar{\mu}A}}$$

$$A_1 = 0.362$$

$$A_2 = 0.023$$

Stabilnost:

Homoogena dif. jed. od (1) glasi:

$$(1+T_1p)(1+T_2p)pu_1(t) + (KT_1p + K_1 + K)K_2f(u_1) = 0$$

radni režim (točka): $u_1^*(t) = A \cdot \sin \omega_n t$

$$\downarrow u_1(t) = u_1^*(t) + \Delta u_1(t)$$

mala varijacija oko radne točke

periodički kret.

$$(1+T_1p)(1+T_2p)pu_1(t) + (KT_1p + K_1 + K)K_2 \left(\frac{\partial f}{\partial u_1} \right)^* \Delta u_1 = 0$$

↳ s tom jed. opisana je dinamika zatvorenog

kruga kada sustav radi u radnoj točki te

su dozvoljene male promjene oko radne točke

→ period. kret. je moguće izmjeriti konst. kret.

$$\left(\frac{\partial f}{\partial u_1} \right)^* = \alpha$$

↳ jednaku je srednjoj vrijednosti unutar periode

$$(1+T_1p)(1+T_2p)pu_1(t) + (KT_1p + K_1 + K)K_2 \alpha \Delta u_1 = 0$$

$$\alpha = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{\partial f}{\partial u_1} \right)^* d(\omega_n t)$$

$$\frac{du_1}{d(\omega_n t)} = A \cdot \cos \omega_n t - \sqrt{A^2 - u_1^2}$$

$$d(\omega_n t) = \frac{du_1}{\sqrt{A^2 - u_1^2}}$$

$$\Re = \frac{q}{2\pi} \int_{-\bar{u}_2}^{\bar{u}_2} \left(\frac{df}{du_1} \right)^* \frac{du_1}{\sqrt{A^2 - u_1^2}}$$

$$\Re = \frac{1}{\pi A} \int_{-A}^A \frac{df}{du_1} du_1 = \frac{2c}{\pi A}$$

$$P(A) = \frac{Ac}{\pi A}$$

$$\Re < P(A)$$

↳ relativera uvjet
stabilnosti

Homogenu karakter. jed. se svodi na oblik:

$$a_3 p^3 + a_2 p^2 + a_1 p + a_0 = 0$$

$$a_3, a_2, a_1, a_0 > 0$$

$$a_2 a_1 - a_3 a_0 > 0$$

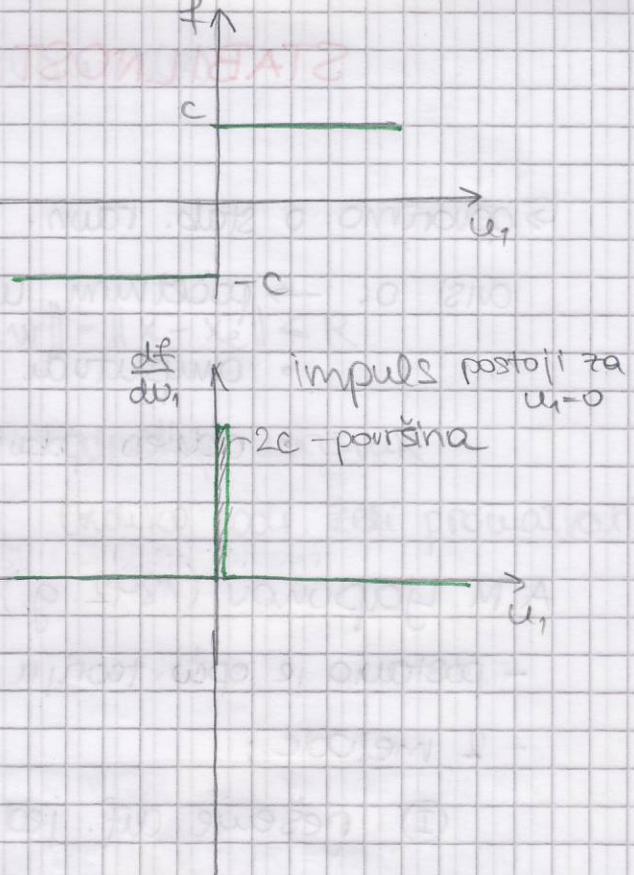
$$-T_1 T_2 (K_1 + K) K_2 \frac{2c}{\pi A} + (T_1 + T_2) \left(1 + K_2 K T_1 \frac{2c}{\pi A} \right) > 0$$

$$A_1 = 0.362 \quad \rightarrow \quad -0.026 + 0.119 > 0$$

stabilno

~~$$A_2 = 0.023 \quad \rightarrow \quad -0.415 + 0.262 < 0$$~~

$$\varphi = -\arctg \frac{H - M \pi A_1}{E - \pi A_1} = -\arctg 0.1456 = -8,28^\circ$$



STABILNOST NELIN. SUSTAVА

RAVNOSTЕŽNIH STANJA

↳ govorimo o stab. ravn. stanja za razliku od lin. sust

avisi o: → početnim uvjetima

- amplitudi pokrete
- obliku pokrete

A.M. Ljapunov (1892. g.)

- postavio je opću teoriju stabilnosti

- 2 metode:

① rješenje dif. jed. i zaključiti o stabilnosti
temeljem trajektorije rješenja

② kvadratna metoda → ne zatvara rješ. dif. jed.

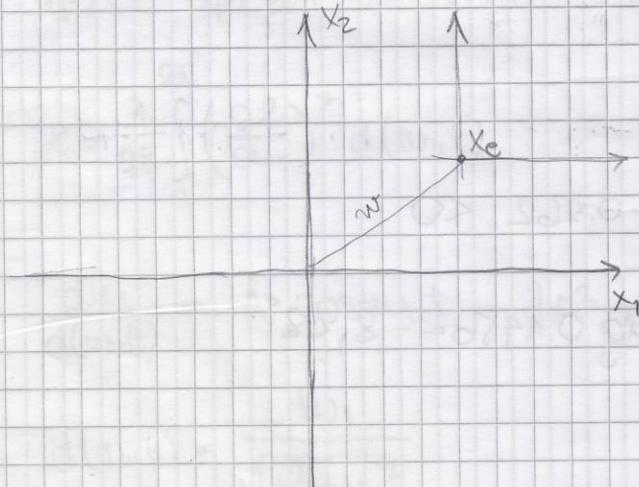
stabilnost u smislu Ljapunova

↳ použanje sustava kada $t \rightarrow \infty$

$$\dot{x} = f(x, u, t)$$

- ravnotežno stanje: $x_e (x_r)$ $\dot{x}=0$ funkc. vrijednost
je nula, neva
promjenjuje stanja

- Ljapunov pretpostavlja da je u - konst.



$$w = x - x_e$$

↳ trans. koord.

VARIJACIJA
PROCESA

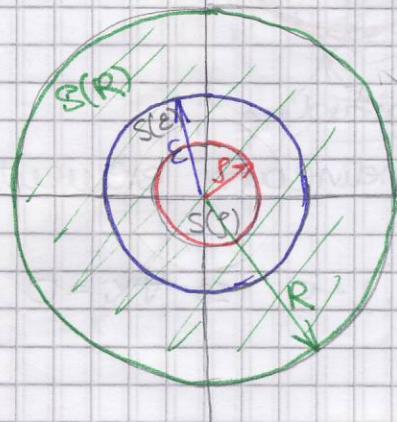
→ stabilnost se provodila unutar područja $S(R)$

- u ravnini $S(R)$ je kružnica

- u prostoru $S(R)$ je kugla

x^2

$$\|w\| = \|x - x_0\| \leq R$$



R - određuje korisnik

(koliko podr. želi provjeravati)

→ područje početnih uvjeta $S(f)$

→ područje koničnih stanja $S(\varepsilon)$

Sustav je stabilan ako za svaki ε , postoji f , tako da trajektorija stanja koja započinje unutar ili na hipersferi $S(f)$ završi ($t \rightarrow \infty$) unutar hiperstere $S(\varepsilon)$

$$f \leq \varepsilon \leq R$$

→ stabilan u malom ako je R mali

→ - - - u velikom - " - R velike

→ globalno stabilan ako je $R \infty$

Asimptotska stabilnost - sustav započinje u nekoj poč. točki i smiri se u ravn. točki (istodioštu)

$$\dot{x}_1(t) = f_1(x_1, x_2, \dots)$$

$$\dot{x}_2(t) = f_2(x_1, x_2, \dots)$$

⋮

$$\dot{x}_n(t) = f_n(x_1, x_2, \dots)$$

↓
funkcionalne ovisnosti su proizvijene i
mogu sadržavati bilo koji neilinearni element

$$f_1(x_1, x_2, \dots) = f_2(x_1, x_2, \dots) = f_n(x_1, x_2, \dots) = 0 \quad \text{za } \forall x_i = 0$$

↳ sustav je u ravnotežnom stanju

$$\dot{x}_1 = \dot{x}_2 = \dot{x}_n = 0$$

FUNKCIJA YAPUNOVA (f-ja po var. stanja)

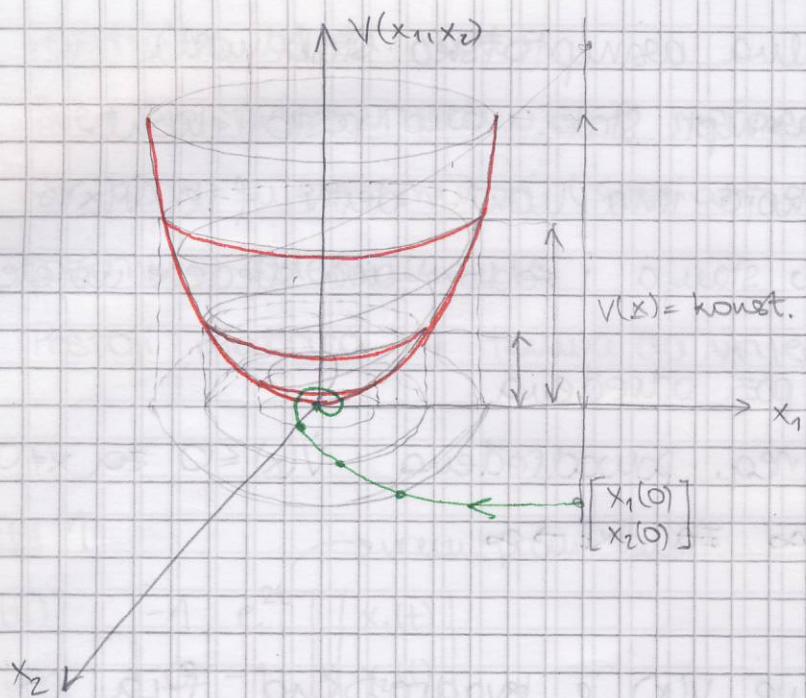
$V(x)$ - skalarna f-ja

1. → svaču f-ju $V(x)$ koja za $x=0 \Rightarrow V(x)=0$

$$\begin{aligned} 2. \quad V(x) &= \frac{dV}{dt} - \frac{\partial V}{\partial x_1} \cdot \frac{dx_1}{dt} + \frac{\partial V}{\partial x_2} \cdot \frac{dx_2}{dt} + \dots + \frac{\partial V}{\partial x_n} \cdot \frac{dx_n}{dt} \\ &= \frac{\partial V}{\partial x_1} \dot{x}_1 + \dots + \frac{\partial V}{\partial x_n} \dot{x}_n \end{aligned}$$

$$\dot{V}(x) < 0 \quad \text{za } x \neq 0 \quad \text{tj. } \dot{V}(x) = 0 \quad \text{za } x = 0$$

↳ opadajuća f-ja (gradjanci su negativni)



↳ Asimptotska stabilnost - kada se zavrsavi u ishodištu

TEOREM 1 (Asimptotske stabiliteti)

Ravn. stanje će biti asimpt. stab. u dužici išodišta ako postoji kont. f-ja $V(\underline{x})$ koja ima kont. prve parne deriv po var. stanja u čitavom okolišu

$$1. \quad V(\underline{x}) \text{ je pozitivno određena} \quad \begin{cases} V(\underline{x}) > 0 & \text{za } \underline{x} \neq \underline{0} \\ V(\underline{x}) = 0 & \text{za } \underline{x} = \underline{0} \end{cases}$$

$$2. \quad \dot{V}(\underline{x}) \text{ je negativno određena} \quad \begin{cases} \dot{V}(\underline{x}) < 0 & \text{za } \underline{x} \neq \underline{0} \\ \dot{V}(\underline{x}) = 0 & \text{za } \underline{x} = \underline{0} \end{cases}$$

↳ Teorem lokalne asimpt. stab. - traži se zadovoljavanje uvjeta u dužini ravn. stanja išodišta

TEOREM 2 (Globalna asimptotska stabilnost)

Sustav je glob. asimpt. stab. ako postoji kont.

skal. fja $V(x)$ koja ima kont. deriv fje $\dot{V}(x)$

u čitavom prostoru stavlja i zadovoljava slijedeće uvjete

1. uvjet - $V(x)$ je poz. određena

2. uvjet - $\dot{V}(x) \leq 0$ za $x \neq 0$

3. uvjet - $V(x) \rightarrow \infty$ za $\|x\| \rightarrow \infty$

Za linearne sustave $V(x)$ je kvadratična fja po var. stavlja

$$\dot{x}(t) = A \cdot x(t)$$

$$V(x) = x^T P x$$

P - poz. definitna matica (simetrična)

$$\dot{V}(x) = \underbrace{\dot{x}^T P x}_{(Ax)^T} + \underbrace{x^T P \dot{x}}_{(Ax)} = x^T (A^T P + PA)x = -x^T Q x \leq 0$$

poz. det.
mat.
(simet.)

JEDNADŽBA YAPUNOVA za linearni sustav

$$-Q = A^T P + PA$$

Q se uobičajeno postavlja kao jedinična mat.

\Rightarrow UVIJETI STABILNOSTI : $P > 0$

Test Positivno odred. mat:

- ① Sve svojstvene vrijednosti P moraju biti > 0
- ② Svi gornji ijni minori (submatrice) P_k imaju pozitivne determinante
- ③ Postoji matrica W tako da vrijedi $P = W^T W$

Primjer 1

→ vrem. prom. sust.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & e^{2t} \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\operatorname{Re}\{\lambda_i\} < 0$$

$$\lambda_{1,2} = -1$$

$$\dot{x}_2(t) = -x_2(t) \rightarrow x_2(t) = x_2(0) \cdot e^{-t} \quad (\text{priješanje})$$

$$\dot{x}_1(t) = -x_1(t) + e^{2t} x_2(t)$$

$$\begin{aligned} \dot{x}_1(t) + x_1(t) &= e^{2t} x_2(t) \\ &= e^t \cdot x_2(0) \quad (\text{respiracija}) \end{aligned}$$

$$V(x) = x^T \dot{x} + \dot{x}^T x = x^T [A(t) + A^T(t)] x \leftarrow \lambda x^T x = -\lambda V(x)$$

$$\forall t \geq 0 : V(x) = x^T x \geq 0$$

$$V(t) \leq V(0) e^{-\lambda t}$$

Fürmjer 2

$$\dot{x}_1(t) = -x_1(t) + 2x_2(t)$$

$$\dot{x}_2(t) = x_1(t) - 4x_2(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

↳ vrewu. nepr. sust.

$$\tilde{A}^T P + P A = -Q$$

$$Q = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad P_{12} = P_{21}$$

$$\left. \begin{array}{l} -2P_{11} + 2P_{12} = -1 \\ -2P_{11} - 5P_{12} + P_{22} = 0 \\ -4P_{12} - 8P_{22} = -1 \end{array} \right\}$$

$$P = \begin{bmatrix} \frac{23}{60} & \frac{-7}{60} \\ \frac{-7}{60} & \frac{11}{60} \end{bmatrix}$$

$$\det P_1 = \left| \frac{23}{60} \right| > 0 \quad \text{w}$$

$$\det P_2 = \det P = \frac{17}{360} > 0 \quad \text{w}$$

} poz.
det.

$\underline{x}_e = 0 \rightarrow$ asymptotiski stabo

$$V(\underline{x}) = \underline{x}^T P \underline{x} = [x_1 \ x_2] \begin{bmatrix} \frac{23}{60} & \frac{-7}{60} \\ \frac{-7}{60} & \frac{11}{60} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{60} (23x_1^2 - 14x_1x_2 + 11x_2^2)$$

$$\dot{V}(\underline{x}) = -\|\underline{x}\|^2 = -(x_1^2 + x_2^2)$$

Primjer 3

$$\dot{x}_1 = x_2 - \alpha x_1 (x_1^2 + x_2^2) \quad \alpha > 0$$

$$\dot{x}_2 = -x_1 - \alpha x_2 (x_1^2 + x_2^2)$$

→ kvadratična forma f -je Liapunova

$$1. V(x) = x_1^2 + x_2^2 > 0 \quad \forall x \neq 0$$

$$2. \ddot{V}(x) = 2x_1 \dot{x}_1 + 2x_2 \dot{x}_2 = -2\alpha(x_1^2 + x_2^2) < 0 \quad \forall x \neq 0$$

$$3. V(x) \rightarrow \infty \quad \text{za} \quad \|x\| \rightarrow \infty$$

↳ ovaj sustav je GUBAUNO asimptotski stabilan

F-ja Liapunova u formi Krasovskog

$$\frac{dx}{dt} = f(x) \quad x_0 = 0$$

↳ nepoželjni velini. Sustav sa ravn. stanjem u ishodištu je linearizirao Jacobijanom

$$A_J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$$V(x) = f^T(x) P f(x)$$

$$\frac{df(x)}{dt} = \frac{\partial f(x)}{\partial x} \cdot \frac{dx}{dt} = A_J \cdot f(x)$$

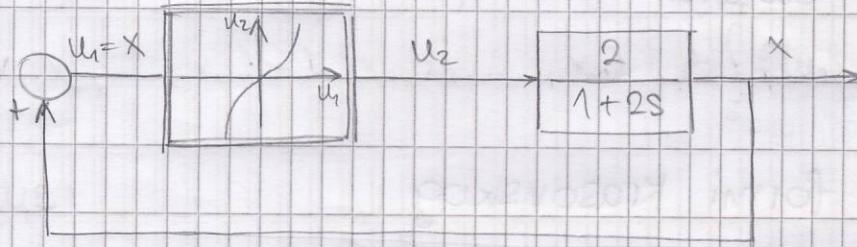
$$\dot{V}(x) = \frac{dV}{dt} = \frac{\partial f^T(x)}{\partial t} \cdot P \cdot f(x) + f^T(x) \cdot P \cdot \frac{\partial f(x)}{\partial t}$$

$$\dot{V}(x) = f^T(x) \underbrace{[A_J^T P + PA_J]}_{-Q} f(x)$$

$$-Q = A_J^T P + PA_J$$

JEDNADŽBA LJPUNOVA

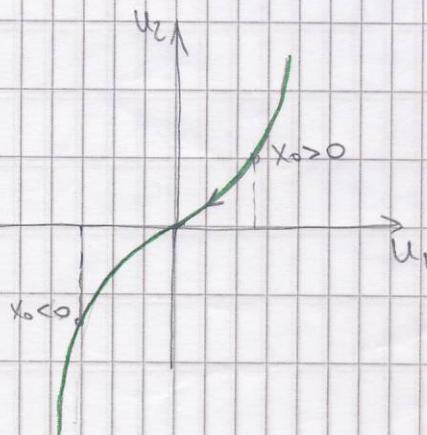
IZQ nelinearni sustav



→ kubna nelin. f-ja : $u_2 = x^3$

$$\frac{dx}{dt} = -0.5x + x^3$$

→ o stabilitetu se može fakultativno i bez f-ja Ljap.
ali rješavanje dif. jed. nije jednostavno



→ ako je $x_0 > 0 \rightarrow$ sust. konvergira prema 0
kada je zad.: $\frac{dx}{dt} < 0$

$$-0.5x + x^3 < 0$$

$$\left| x^2 < \frac{1}{2} \right| \quad (*)$$

\rightarrow ako je $x_0 < 0$

$$\frac{dx}{dt} > 0 \quad \rightarrow \quad -0.5x + x^3 > 0$$

- už $P = I$

$$V(x) = f^T(x) P f(x)$$

$$= (x^3 - 0.5x)^2$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial x} \cdot \frac{dx}{dt} = 2(x^3 - 0.5x)^2 (3x^2 - 0.5) < 0$$

Poz. odr
za x
 > 0

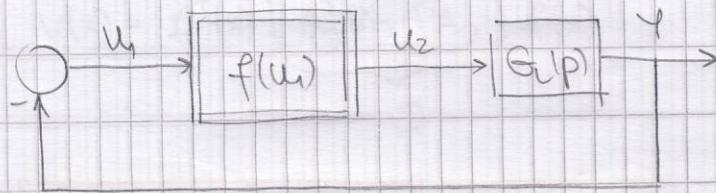
$$3x^2 - 0.5 < 0$$

$$\boxed{x^2 < \frac{1}{6}} \rightarrow \text{stroži ujet za stabilitet se dobije preko Krasovskog od (*)}$$

torjeva

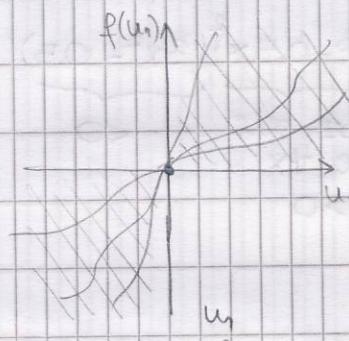
Luré forma f je Ljapunova

Postnikov, Letov (ruska lit.)



NE mora zadovoljavati sljedeće uvjete:

- 1.) $u_1 f(u_1) > 0$ (I i III kvadrant)
- 2.) $f(0) = 0$



$$V(x_1, x_2, \dots, x_n) = L(x_1, x_2, \dots, x_n) + \beta \int_0^x f(u_i) du_i$$

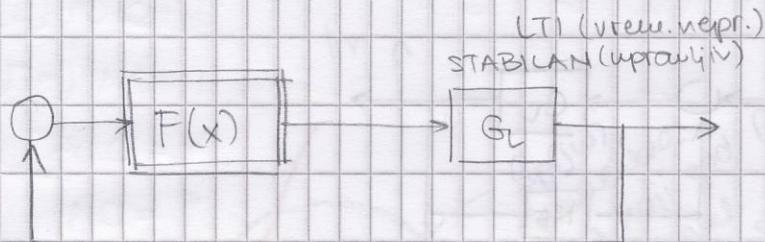
gdje:

$$L(x_1, x_2, \dots, x_n) = \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} x_i x_j$$

α_{ij}, β - konst. skalar

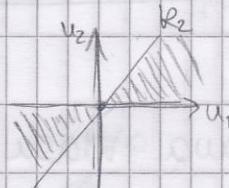
Apsolutne stabilnosti po V.M. Popovu

(Asimptotske stabilnosti u smislu Ljapunova)



$$F(0) = 0$$

$$x \cdot F(x) > 0 ; x \neq 0$$



$$0 < \frac{F(x)}{x} < k_2 ; \text{ za } x \neq 0$$

TEOREM (Popova)

Ravn. stanje nepob. nelin. sust. strukt. kao na slici biti će apsolutno stabilno ako vrijedi:

- 1) lin. dio sustava LTI, upravlјiv, stabilan
- 2) nelin. fja klase $[0, k_2]$
- 3) postoji dva pozitivna realna broja $g > 0$ i proizvodimo mali broj $f > 0$, takvi da za sve poz. frekv. vrijedi:

$$\omega > 0$$

$$\operatorname{Re} \left\{ (1+jgw) G_L(jw) \right\} + \frac{1}{k_2} \geq f > 0$$

odnosno:

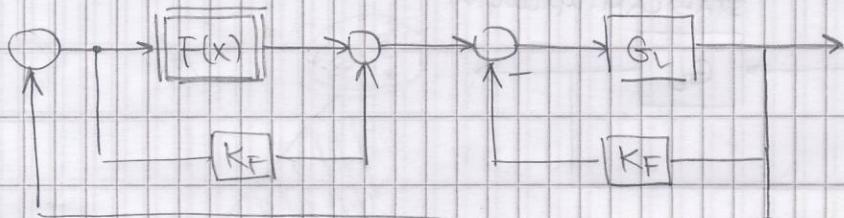
$$\operatorname{Re} \left\{ (1+jgw) G_L(jw) \right\} + \frac{1}{k_2} > 0$$

gdje: $k_2 < \infty$

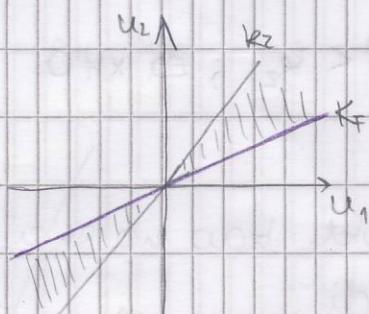
$$\lim_{\omega \rightarrow \infty} G_L(j\omega) = 0$$

↳ stabilnost se može analizirati u frekv. domeni

→ u slučaju da G_L nije stabilan stabiliziramo ga povratnom vrednost.



K_F dodatno ograničava klasu nelin. f-je



→ geometrijska interpretacija teorema Popova

$$\operatorname{Re}\{G_L(j\omega)\} - \frac{1}{2} \operatorname{Im}\{\omega G_L(j\omega)\} + \frac{1}{k_F} > 0$$

$\underbrace{\operatorname{Re}\{G_L\}}_{u(\omega)} \quad \underbrace{\operatorname{Im}\{G_L\}}_{v(\omega)}$

$$G_L(j\omega) = \underbrace{\operatorname{Re}\{G_L\}}_{u(\omega)} + j \underbrace{\operatorname{Im}\{G_L\}}_{v(\omega)} \quad \left. \begin{array}{l} G_L(j\omega) \text{ i } G_p(j\omega) \text{ sijeku} \\ \text{realnu os u istoj točki} \end{array} \right.$$

$$G_p(j\omega) = \underbrace{\operatorname{Re}\{G_p\}}_{u_p(\omega)} + j \omega \underbrace{\operatorname{Im}\{G_p\}}_{v_p(\omega)} \quad \left. \begin{array}{l} \\ \end{array} \right.$$

Popovićeva f-ja (frekv. karakter. Popova)

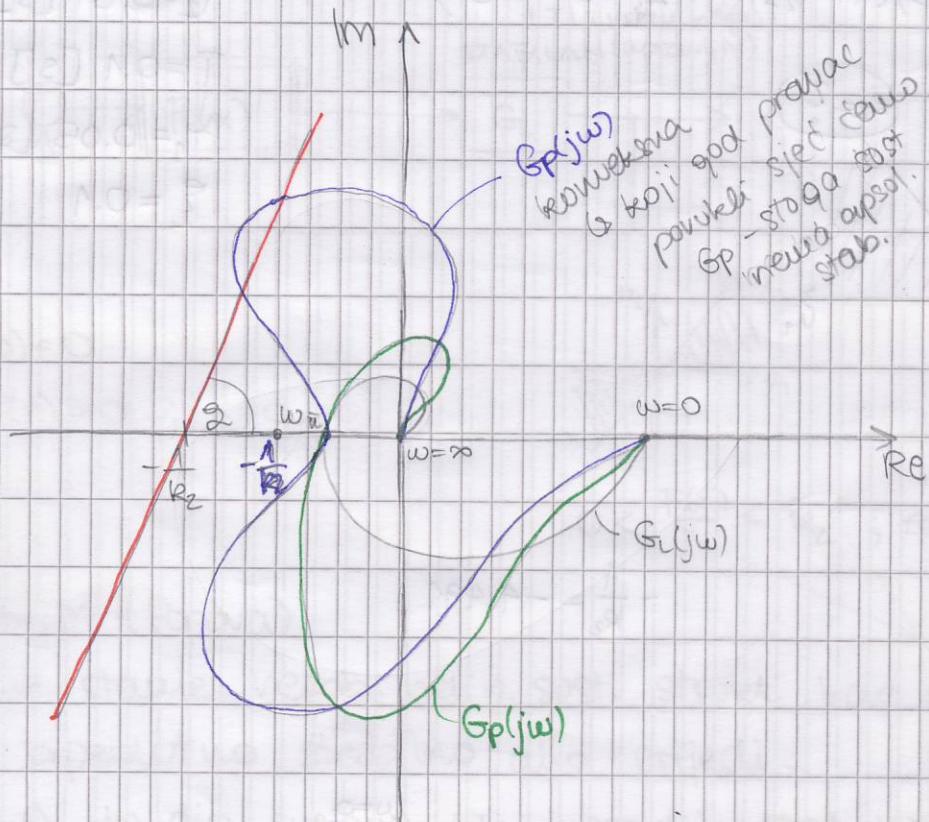
$$\operatorname{Im}\{G_p\} = \omega \cdot V(\omega) = V_p(\omega)$$

↪ w peta uvedenim Im dio
linearne f-je

$V(\omega)$ - neparna f-ja
$V_p(\omega)$ - parna f-ja

$G_p(j\omega)$ polazi s realne osi

$G(j\omega)$ može započeti na imagu. osi



↳ hodograf od G je simetričan obziru na Re os
(za neg. ω bi bio isti samo zrcaljen preko Re)

- ako kroz $-\frac{1}{k_2}$ povučemo pravac s nagibom 2
i taj pravac ne dodiruje karakt. $G_p(j\omega)$ tada
je sustav apsolutno stabilan

Primjer 4

$$G(s) = \frac{K(1 + \gamma_1 s)}{(1 + T_1 s)(1 + T_2 s)(1 + 2\zeta \tau s + \tau^2 s^2)}$$

$$K=10$$

$$T_1=1 [s]$$

$$T_2=0.01 [s]$$

$$\tau=0.1 [s]$$

$$\gamma_1=0.05 [s]$$

$$\zeta=0.1$$

$$u_{21} \quad R_2=?$$

ω_1

Im

$$-\frac{1}{R_2} = -4,491$$

-20

10

-10

20

-30

$\omega=0$

10

Re

$\omega=1$

α, β, γ

ω_0

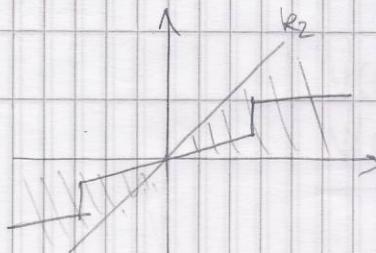
→ tangira popora (sjede G_0)

$$g^{-1} = \tan \gamma \Rightarrow \gamma = 87^\circ \text{ - nagn. pravca}$$

$$R_2 = \frac{1}{4,491} = 0.22$$

→ sektor $[0, 0.22]$ - tu naoru leži nelin. karakter.

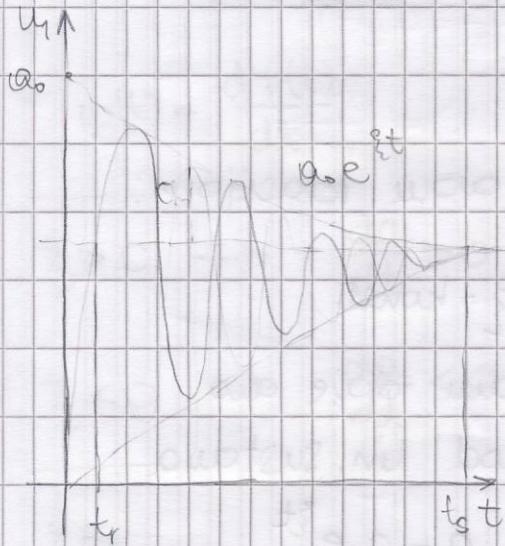
(ako leži u tom sektoru sustav je apsolutno stab.)



Ocjena kvalitete prijelaznog procesa

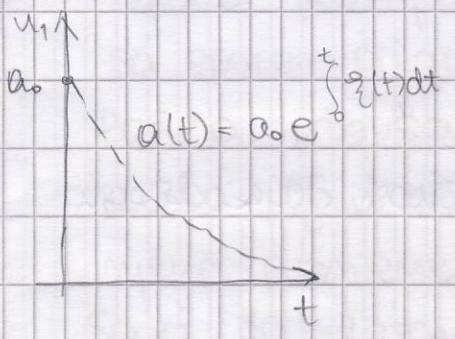
Približna analiza

UN. SUST.

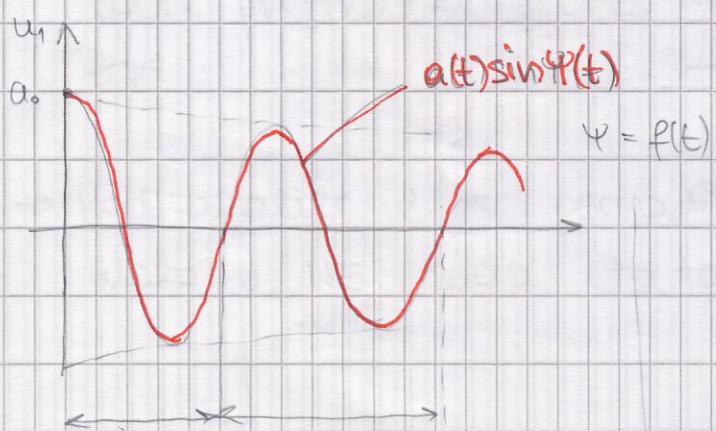


ξ - faktor priqušenja = konst.
 ω - frekv. = konst.

Nelin. sustav



$\xi(t)$ nije konst. } AKKO $\xi(t)$ i
 ω nije konst } $\omega(t)$ sporo
prom. vel.



$\xi(t)$ - POKAZATELJ priqušenja
 $\omega(t)$ - frekv.

→ treba razlikovati od faktora

ξ i ω kod lin. sust.
qdje su konst.

$$\downarrow \varphi = f(t) \Rightarrow \frac{d\varphi}{dt} = \omega(t)$$

$$T_1 \neq T_2 \Rightarrow \omega = f(t)$$

$$\varphi = \int w(t) dt + \varphi_0$$

→ ako je $\omega(t) = \omega = \text{konst.}$

$$\varphi = \int_0^t \omega(t) dt + \varphi_0 = \omega t + \varphi_0$$

↳ početni fazni polmak

$$\sin \varphi = \sin(\omega t + \varphi_0)$$

→ amplitudna svojstva se mogu izraziti ovom relacijom:

$$\boxed{\frac{da}{dt} = a \xi(t)}$$

→ ako je $\xi(t) = \xi = \text{konst.}$

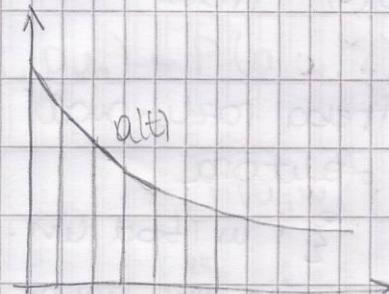
ova relacija naucu daje da
sto i uzmimo kod lin. sustava

$$\frac{da}{a} = \xi dt \rightarrow a = a_0 e^{\xi t}$$

↳ ako $\xi(t) \neq \text{konst.}$

$$\frac{da}{a} = \xi(t) dt \Rightarrow a(t) = a_0 e^{\int_0^t \xi(u) du}$$

→ slika amelope pokazuje da vrem. ciljnost daje drugu
sliku (prom. ξ)



Amplitude



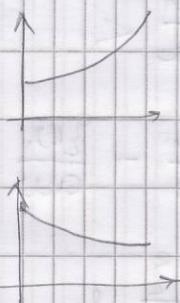
$$u_1(t) = a(t) \sin \psi(t)$$

$$\frac{da}{a} = \xi(t) dt$$

$$w(t) = \frac{d\psi(t)}{dt}$$

$$\xi > 0 \rightarrow \frac{da}{dt} > 0 \text{ respirirajuće}$$

$$\xi < 0 \rightarrow \frac{da}{dt} < 0 \text{ prikušenje}$$



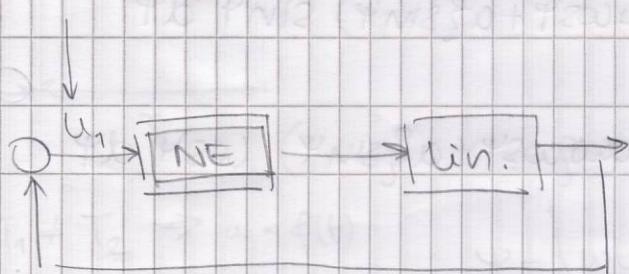
točka mesta u lin. sustavima

- spora promjenjivost w i ξ ne treba se poistovjetiti sa sporom prikušjućim ili raspirujućim procesom
Brzina prikušenja oscilacija određuje se iz iznosa ξ u dijurnom trenutku a ne iz karaktera mijene promjene.

Spori priklazni proces $\rightarrow \xi$ malo

Brzi $\rightarrow \xi$ veliko

- kod analize vl. vsc. periodičko rješenje je traženo na ulazu u NE, sada isto tako



(narm. linear)

Formule za koef. linearne opisne f-je

$$P \stackrel{\text{def}}{=} \frac{d}{dt}$$

$$u_1(t) = a(t) \sin \varphi$$

$$pu_1 = aw \cos \varphi + a\dot{\varphi} \sin \varphi$$

$$\frac{da}{dt} = a\ddot{\varphi}(t)$$

$$\sin \varphi = \frac{u_1}{a} <$$

$$\dot{\varphi}(t) = \frac{d\varphi(t)}{dt}$$

$$\cos \varphi = \frac{pu_1}{aw} - \frac{\ddot{u}_1}{aw}$$

$$\cos \varphi = \frac{P - Q}{aw} u_1$$

$$\begin{aligned} u_2 &= f(u_1, pu_1) = G_n(A, \omega) u_1 \\ &= \underbrace{[P(A, \omega) + jQ(A, \omega)]}_{\text{NE}} u_1 \end{aligned}$$

$$A \sin \omega t = u_1 \rightarrow \boxed{\text{NE}} \parallel u_2$$

$$\text{sada je} \\ u_1 = a \sin \varphi$$

$$u_2 = a [P(a, \omega, \varphi) \sin \varphi + Q(a, \omega, \varphi) \cos \varphi]$$

$$f(u_1, pu_1) = P(a, \omega, \varphi) u_1 + Q(a, \omega, \varphi) \frac{P - Q}{aw} u_1$$

$$= \left[P(a, \omega, \varphi) - \frac{Q}{\omega} Q(a, \omega, \varphi) \right] u_1 + \frac{Q(a, \omega, \varphi)}{\omega} pu_1$$

$$P(a, \omega, \varphi) = \frac{1}{\omega a} \int_0^{2\pi} f(a \sin \varphi, aw \cos \varphi + a\dot{\varphi} \sin \varphi) \sin \varphi d\varphi$$

$$Q(a, \omega, \varphi) = \frac{1}{\omega a} \int_0^{2\pi} f(a \sin \varphi, aw \cos \varphi + a\dot{\varphi} \sin \varphi) \cos \varphi d\varphi$$

→ tipelinearnosti bez inercije

$$f(u_1)$$

$$P(a, \xi) = \frac{1}{\pi a} \int_0^{2\pi} f(a \sin \varphi) \sin \varphi d\varphi$$

$$Q(a, \xi) = \frac{1}{\pi a} \int_0^{2\pi} f(a \sin \varphi) \cos \varphi d\varphi$$

→ odnos $\frac{\xi}{\omega}$ određuje brzinu priqušenja ($\xi < 0$) ili raspiravanja ($\xi > 0$) unutar jedne periode

$|\frac{\xi}{\omega}|$ mali \Rightarrow sporo prom. proc.

$|\frac{\xi}{\omega}|$ veliki \Rightarrow brzo prom. proc.

$|\xi| \ll \omega$ - proc. sa malo prom. amplitudom a u tom slučaju će koef. harm. linearizacije biti isti kao koef. harm. lin. za vr. osc.

$$P(a, \omega) = \frac{1}{\pi a} \int_0^{2\pi} f(a \sin \varphi, \omega \cos \varphi) \sin \varphi d\varphi$$

$$Q(a, \omega) = \frac{1}{\pi a} \int_0^{2\pi} f(a \sin \varphi, \omega \cos \varphi) \cos \varphi d\varphi$$

$$f(u_1) = \mathbb{F}_N(a) u_1$$

kada je $\boxed{\xi = 0} \Rightarrow a(t) = A = \text{konst.}$

nalazimo se na vlastitim osc.

$$w(t) = \omega$$

$$u_1(t) = a(t) \sin \varphi(t)$$

$$\frac{da}{a} = \xi(t) dt \quad ; \quad \omega(t) = \frac{d\varphi(t)}{dt}$$

uz početne uvjete:

$$a(0) = a_0$$

$$\varphi(0) = \varphi_0$$

$$t = \int_{a_0}^a \frac{da}{a\xi(a)}$$

ova rel. omogućava određ.
vremena smirivanja

pri $t=0$

$$\varphi = \int_0^t \omega(t) dt + \varphi_0 = \int_0^t \frac{\omega(a)}{a\xi(a)} da + \varphi_0$$

$$\begin{cases} \xi = \xi(a) \\ \omega = \omega(a) \end{cases} \quad \left. \begin{array}{l} f - je \, dobivena \, rješavanjem \\ jed. harm. ravnoteže \end{array} \right.$$

Jed. harm. ravnoteže - vel. osc.

$$\frac{Y}{U_2} = G_v(s) = \frac{R(s)}{C(s)}$$

$$C(s)u_1 + R(s)G_N(A, \omega)u_1 = 0$$

uz $s = j\omega$

$$C(j\omega)u_1 + R(j\omega)\left[P(A, \omega) + \frac{Q(A, \omega)}{\omega}j\omega\right]u_1 = 0$$

Jed. harm. ravn. - prijelazni proces

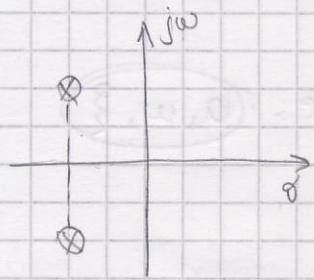
$$C(s)u_1 + R(s)\left[P(a, \omega, \xi) + \frac{s-\xi}{\omega}Q(a, \omega, \xi)\right]u_1 = 0$$

Za jednostavnije tipove NE:

$$C(s)u_1 + R(s)P(a, \xi)u_1 = 0$$

LIN. SUST.

$$u_1 = a_0 e^{st} \sin(\omega t + \varphi)$$



$$s = \xi \pm j\omega$$

↳ po istoj vojici tražimo ri za priel. proc.

(to možemo jer je proc. sporo prom)

$$s = \xi + j\omega$$

$$C(\xi + j\omega) + R(\xi + j\omega)[P + jQ] = 0$$

↳ razvoj u Taylorov red

$$C(\xi + j\omega) = C(\xi) + \left(\frac{dC}{dp}\right)_{\xi} j\omega + \frac{1}{2!} \left(\frac{d^2C}{dp^2}\right)_{\xi} (j\omega)^2 + \dots + \frac{1}{n!} \left(\frac{d^nC}{dp^n}\right)_{\xi} (j\omega)^n$$

n - red karakteristične jednadžbe

$$R(\xi + j\omega) = \dots \quad (\text{isti princip})$$

Kod malih ξ (sporo prom. proc.) prikladnije je unijesto

T. reda, koristiti razvoj potencije ξ
po $j\omega$

$$C(\xi + j\omega) = C(j\omega) + \left(\frac{dC}{dp}\right)_{j\omega} \xi$$

$$R(\xi + j\omega) = \dots$$



→ kada je $Q=0$

$$C(\xi + j\omega) + R(\xi + j\omega) P(a, \xi) = 0$$

razvoj u T. red. ovisno
o iznosu ξ

→ nepoznajice: (a, ω, ξ)

Re: $X = X(a, \omega, \xi)$

} daje nepom. izračunamo

Im: $Y = Y(a, \omega, \xi)$

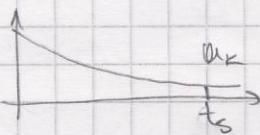
} preko treće

$$\begin{aligned} \xi &= \xi(a) \\ \omega &= \omega(a) \end{aligned}$$

VRIJEME SMIRIVANJA

$$t_s = \left\{ \begin{array}{l} a_k \\ a_0 \end{array} \right.$$

a_k - granična vr. ampl. pri kojoj se amplituda "smiruje"



⇒ alt) se može grafički odrediti

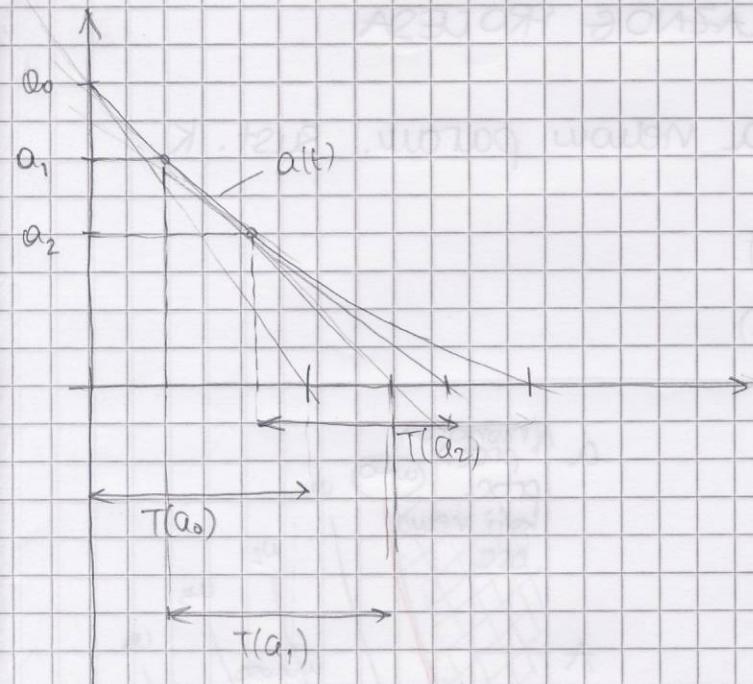
→ Uzodi se tzw. "tekuća" vrem. konst. amplitude prijelaznog procesa na osnovi poznate $\xi(a)$

$$T(a) = \left| \frac{1}{\xi(a)} \right| \quad (\text{kod lin. je konst., a to nije!})$$

, prig. ili rasp.

$$\frac{da}{dt} = \frac{a}{T(a)} \operatorname{sign} \xi$$

↳ nalogi tang. na amplitudu



broj oscilacija:

$$N = \frac{\Psi - \Psi_0}{2\pi} = \frac{1}{2\pi} \int_{\alpha_0}^{\alpha_k} \frac{w(\alpha)}{\alpha\Psi(\alpha)} d\alpha$$

DIJAGRAM KVALITETE PRIJELAZNOG PROCESA

↳ daju on snost w i ξ o a nelow parau. sust. K

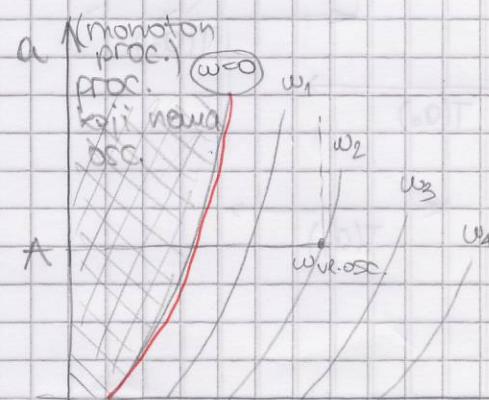
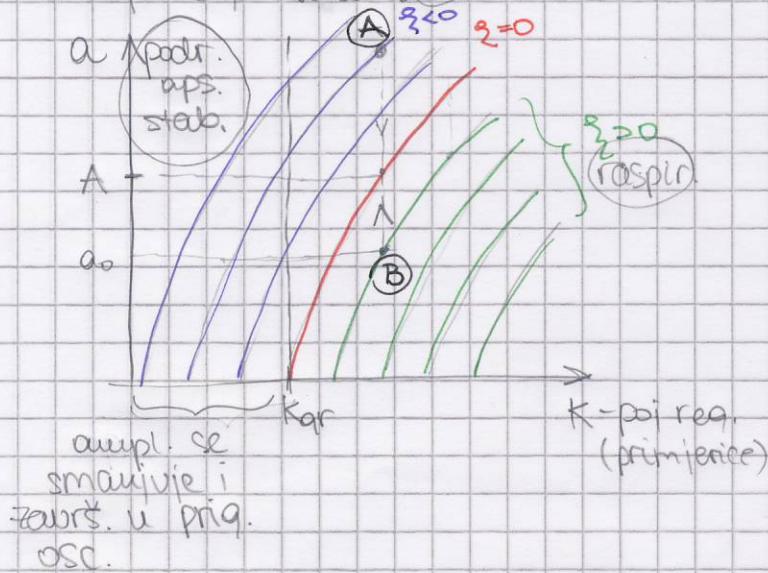
$$\xi = \xi(a)$$

$$w = w(a)$$

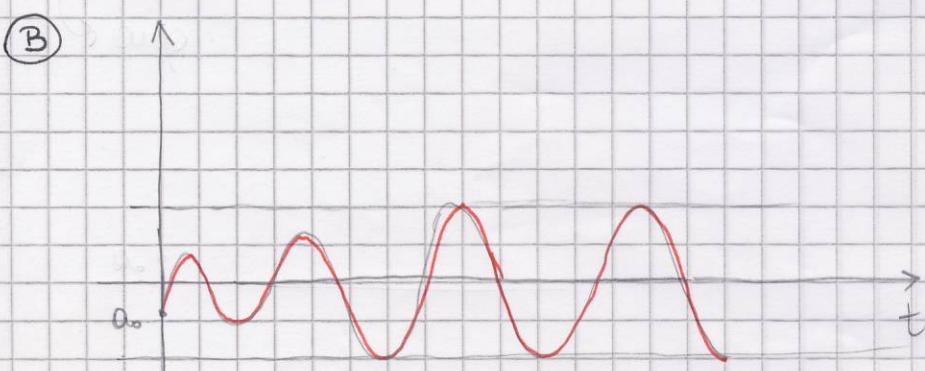
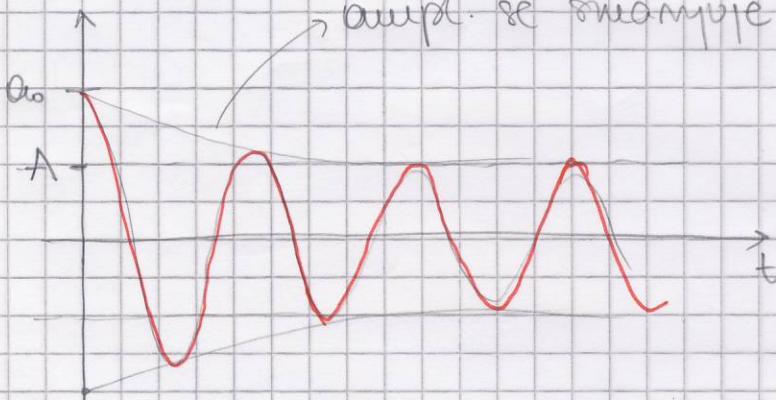
$$\xi = \xi(K)$$

$$w = w(K)$$

crtaju dij. kval. priq.

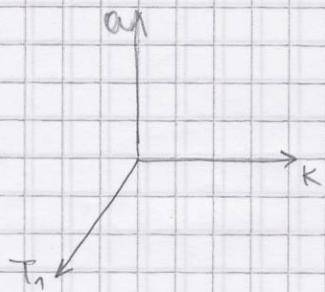


→ ampl. se smanjuje i završ. u vel. osc.

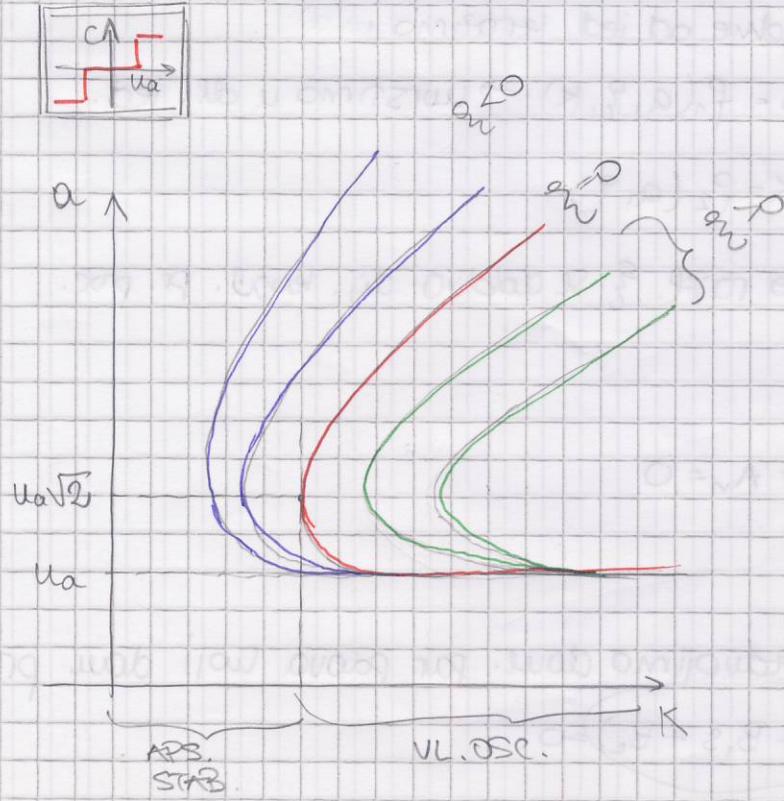
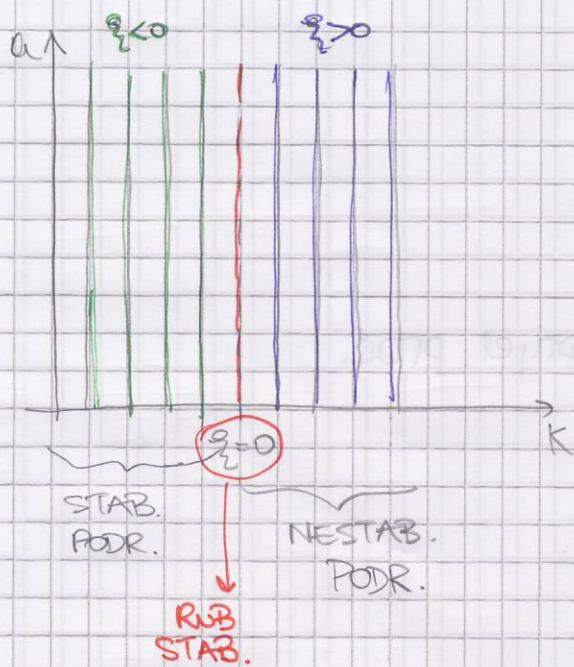


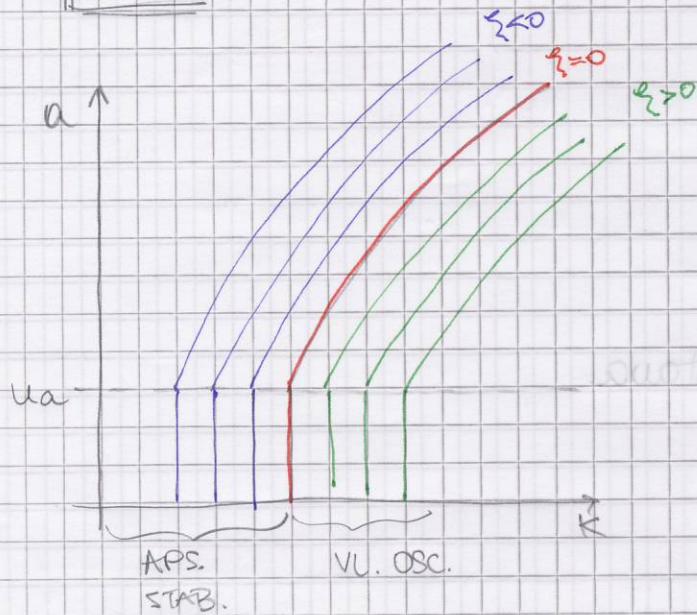
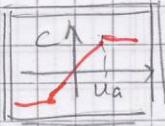
→ moždce je dobiti i pohle:

m



Dijagram. kvalitete lin. sustava





Postupci crtanja dijag. kval. prijel. proc.

I POSTUPAK

$$C(\xi + j\omega) + R(\xi + j\omega)[P + jQ] = 0$$

↓ ↓

razvij u T. red po pot. (jω)

$$\text{Re: } X(a, \omega, \xi) = 0 \quad \} \text{ iz jedne od jed. izrazimo}$$

$$\text{Im: } Y(a, \omega, \xi) = 0 \quad \} \quad \omega = f_1(a, \xi, K) \rightarrow \text{vrstimo u dr. jed.}$$

$$K = f_2(a, \xi)$$

→ Za razl. ξ se dodatno dij. kval. pr. proc.

II POSTUPAK

$$s^n + A_1 s^{n-1} + \dots + A_{n-1} s + A_n = 0$$

A_i - koef. fje ξ, ω i a

→ rastavimo na faktore (izdvajimo dom. par polova koji daju. proc.)

$$(s^{n-2} + C_1 s^{n-3} + C_{n-2})(s^2 + B_1 s + B_2) = 0$$

polovi barem $8 \times (\xi \omega)$ u lijevo

$$\varrho_1 = -\frac{B_1}{2}$$

$$\omega^2 = B_2 - \varrho_1^2$$

$$A_1 = C_1 + B_1$$

$$A_2 = B_2 + C_2 + B_1 C_1$$

$$A_n = C_{n-2} B_2$$

→ karakter. jed. III reda

$$s^3 + A_1 s^2 + A_2 s + A_3 = (s + C_1)(s^2 + B_1 s + B_2)$$

$$A_1 = C_1 + B_1$$

$C_1 > \left| \frac{B_1}{2} \right|$ ako želimo da treći korijen malo utječe na dominantnu dinamu.

$$A_2 = B_2 + B_1 C_1$$

$$A_3 = C_1 B_2$$

$$\varrho_2 = -\frac{A_1 A_2 - A_3}{2(A_2 + (A_1 + 2\varrho_1)^2)}$$

$$\omega^2 = \frac{A_3}{A_1 + 2\varrho_1} - \varrho_2^2$$

→ sustav IV reda

$$s^4 + A_1 s^3 + A_2 s^2 + A_3 s + A_4 = (s^2 + C_1 s + C_2)(s^2 + B_1 s + B_2)$$

$$A_1 = C_1 + B_1$$

$$A_2 = C_2 + B_2 + C_1 B_1$$

$$A_3 = B_1 C_2 + B_2 C_1$$

$$A_4 = C_1 B_2$$

$$\varrho_2 = -\frac{A_3(A_1 A_2 - A_3) - A_1^2 A_4}{2(A_1 + 2\varrho_1)\{A_2 + (A_1 + 2\varrho_1)^2\}^2 - 4A_4 + A_1 A_3}$$

$$\omega^2 = \frac{A_4(A_1 + 4\varrho_1)}{(A_1 + 2\varrho_1)(A_2 + 2\varrho_1) - A_3^2} - \varrho_2^2$$

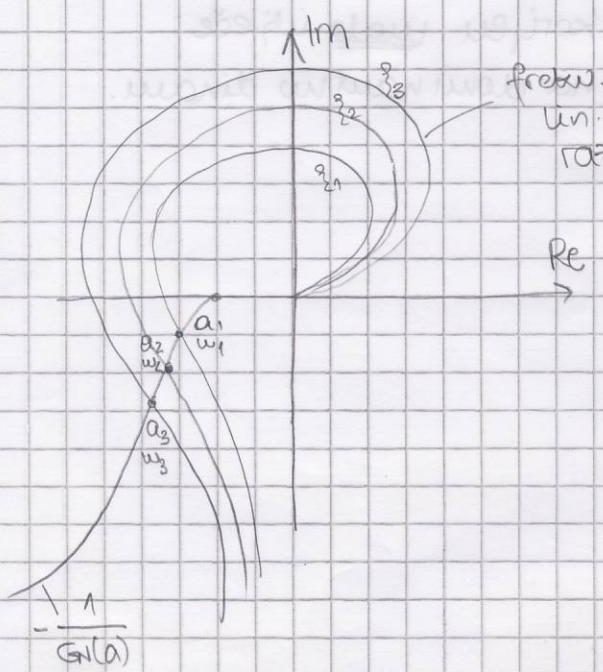
III POSTUPAK

$P(a), Q(a) \rightarrow$ bez inercije (ovise samo o ampl., ne o frekv.)

$$G_L(\xi + j\omega) = -\frac{1}{G_N(a)}$$

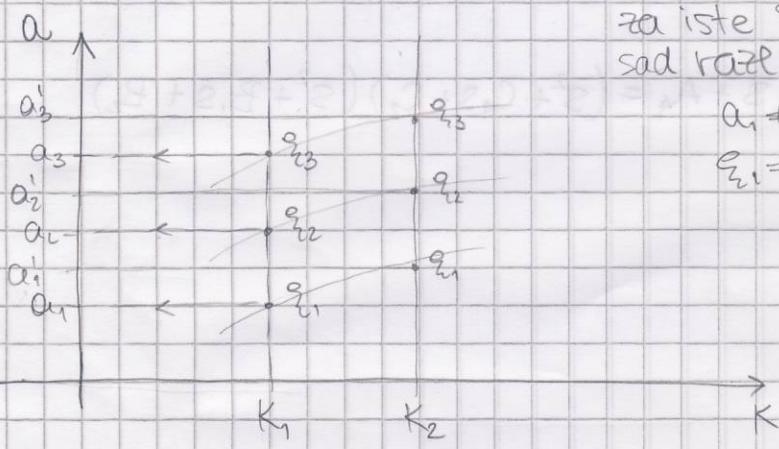
$$G_N(a) = P(a) + jQ(a)$$

$$G_L(\xi + j\omega) = \frac{R(\xi + j\omega)}{C(\xi + j\omega)} \rightarrow \text{T. red po pot. (j\omega)}$$



frekv. karakter.
un. djela $\Rightarrow a$
razl. ξ uz $K = K_1$

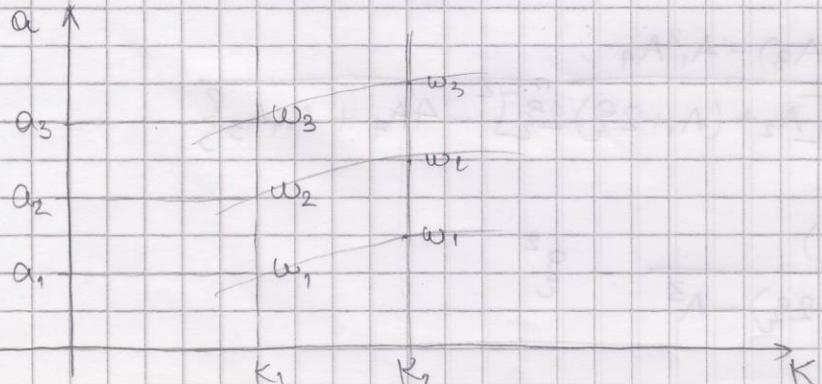
onda se opet
erto familija
krivulja za dr. $K = K_2$



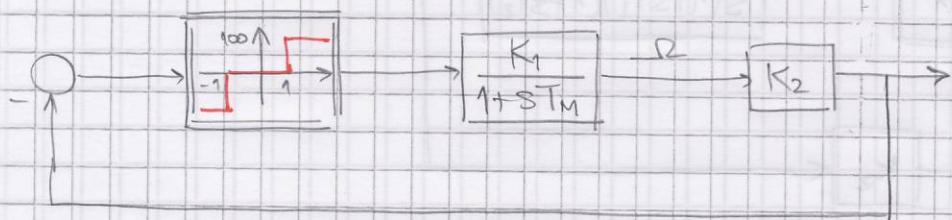
$K = K_2$
za iste ξ dobivaju se
sad razl. a

$$a_1 \neq a'_1$$

$$\xi_1 = \xi_1$$



Treba odrediti dij. kvadr. pr. proc. mehan. sust.



$$G_L(s) G_N(a) + 1 = 0$$

$$G_L(s) = \frac{K_1 K_2}{1 + s T_m}$$

$$G_N(a) = \frac{400}{\pi a} \sqrt{1 - \frac{1}{a^2}}$$

$$K_1 K_2 G_N(a) + T_m s + 1 = 0$$

$$L(s) = T_m s + K_1 K_2 G_N(a) + 1 = 0$$

$$\left. \frac{dL(s)}{ds} \right|_{s=0} = T_m$$

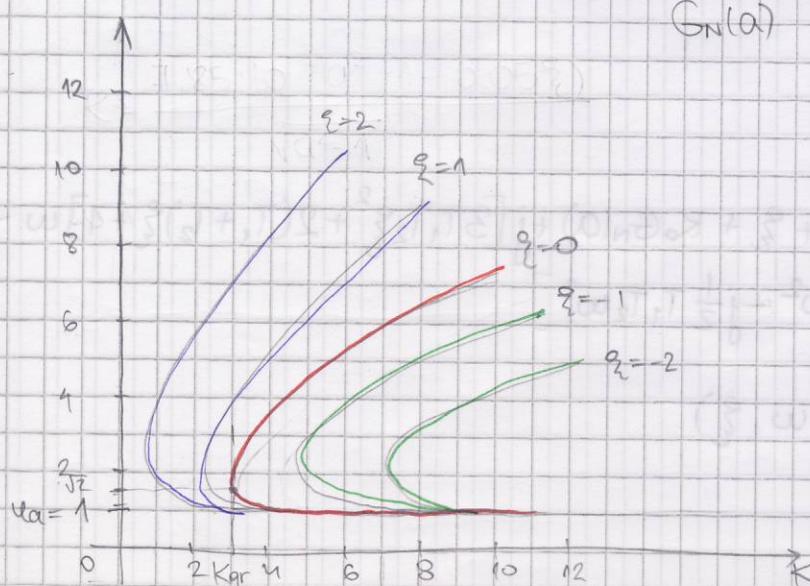
$$L(\xi + j\omega, a) = T_m \xi + K_1 K_2 G_N(a) + 1 + j T_m \omega$$

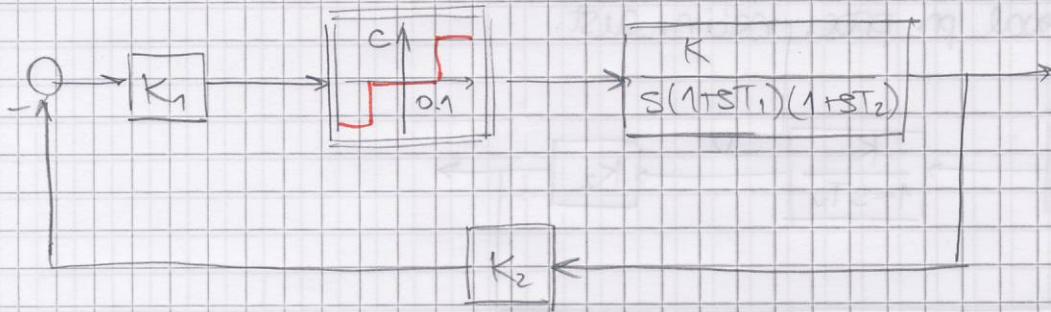
$$\text{Re: } X(a, \omega, \xi) : T_m \xi + K_1 K_2 G_N(a) + 1 = 0$$

$$\text{Im: } Y(a, \omega, \xi) : T_m \omega = 0 \rightarrow \underline{\omega = 0}$$

$$K G_N(a) = 1 - T_m \xi \rightarrow K = \frac{1 - T_m \xi}{G_N(a)} = \frac{\pi a^2 (1 - T_m \xi)}{400 \sqrt{a^2 - 1}}$$

$$K = \frac{7,85 \cdot 10^{-3} a^2 (1 - 0,05 \xi)}{\sqrt{a^2 - 1}}$$





$$K = 1500 \text{ [S]}$$

$$K_1 = 1$$

$$T_1 = 0.2 \text{ [s]}$$

$$K_2 = 7500$$

$$T_2 = 2 \text{ [s]}$$

$$G(s) G_n(a) + 1 = 0$$

$$G_n(s) = \frac{K_1 K_2 K}{s(1+sT_1)(1+sT_2)}$$

$$G_n(a) = P(a) - \frac{4C}{ua} \sqrt{1 - \left(\frac{ua}{a}\right)^2}$$

$$L(s) = T_1 T_2 s^3 + (T_1 + T_2) s^2 + s + K_0 P(a) = 0$$

$$(K_0 = K_1 K_2 K)$$

$L(s)$ se razvija u T. red

$$s = \xi + j\omega$$

$$L(\xi + j\omega) = L(s) \Big|_{s=\xi} + \frac{dL(s)}{ds} \Big|_{s=\xi} \cdot j\omega + \frac{1}{2!} \frac{d^2 L(s)}{ds^2} \Big|_{s=\xi} (\omega)^2 + \frac{1}{3!} \frac{d^3 L(s)}{ds^3} \Big|_{s=\xi} (\omega)^3$$

$$\frac{dL(s)}{ds} \Big|_{s=\xi} = 3T_1 T_2 \xi^2 + 2(T_1 + T_2) \xi + 1$$

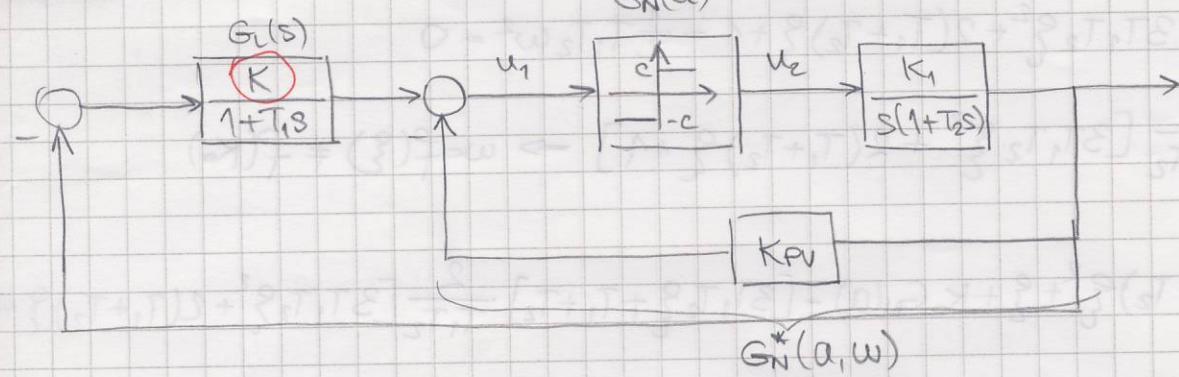
$$\frac{1}{2!} \frac{d^2 L(s)}{ds^2} \Big|_{s=\xi} = 3T_1 T_2 \xi + (T_1 + T_2)$$

$$\frac{1}{3!} \frac{d^3 L(s)}{ds^3} \Big|_{s=\xi} = \frac{T_1 T_2}{2}$$

$$L(\xi + j\omega) = T_1 T_2 \xi^3 + (T_1 + T_2) \xi^2 + \xi + K_0 G_n(a) + j [3T_1 T_2 \xi^2 + 2(T_1 + T_2) \xi + 1] \omega - [3T_1 T_2 \xi + (T_1 + T_2)] \omega^2 - j \frac{1}{2} T_1 T_2 \omega^3$$

$$= X(a, \omega, \xi) + j Y(a, \omega, \xi)$$

Primjer - sami



$$T_1 = T_2 = 0.1 \text{ [s]}$$

Dijagram kvalitete po K

$$K_P = 5$$

$$K_1 = 1$$

$$c = 10$$

$$\gamma(a, \omega, \xi) = T_1 T_2 \xi^3 + (T_1 + T_2) \xi^2 + \xi + K_0 G_N(a) - [3T_1 T_2 \xi + T_1 + T_2] \omega^2 = 0$$

$$\gamma(a, \omega, \xi) = 3T_1 T_2 \xi^2 + 2(T_1 + T_2) \xi + 1 - \frac{1}{2} T_1 T_2 \omega^2 = 0$$

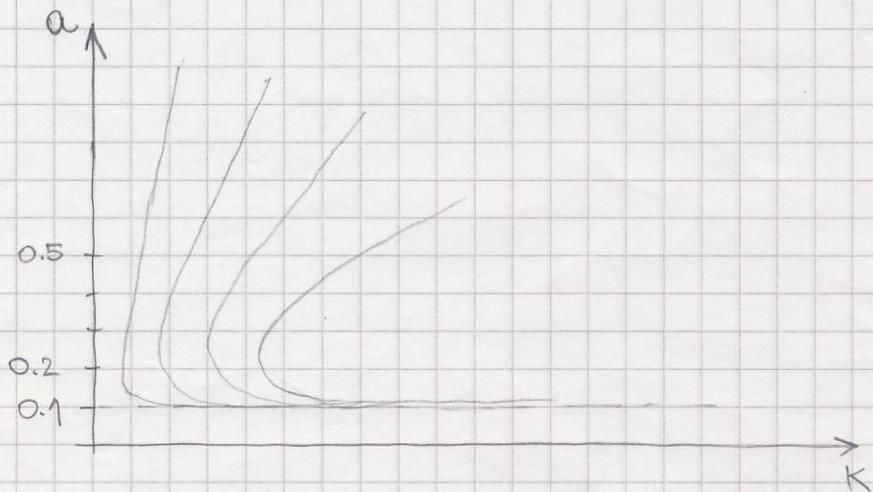
$$\rightarrow \omega^2 = \frac{2}{T_1 T_2} [3T_1 T_2 \xi^2 + 2(T_1 + T_2) \xi + 1] \Rightarrow \omega = f(\xi) \neq f(K_0)$$

$$T_1 T_2 \xi^3 + (T_1 + T_2) \xi^2 + \xi + K_0 G_N(a) - [3T_1 T_2 \xi + T_1 + T_2] \cdot \frac{2}{T_1 T_2} [3T_1 T_2 \xi^2 + 2(T_1 + T_2) \xi + 1] = 0$$

$$0.4 \xi^3 + 2.2 \xi^2 + \xi + K_0 G_N(a) = (3\xi + 5.5)(1.2 \xi^2 + 4.4 \xi + 1)$$

$$K_0 = \frac{1}{G_N(a)} (3.2 \xi^3 + 17.6 \xi^2 + 26.2 \xi + 5.5)$$

$$\hookrightarrow K_0 = f(a) \mid_{\xi \text{-parametar}}$$



Budući da je $\omega = f(\xi)$ to slijedi da su kružnje koje smo načrtali za $K = f(a) \mid_{\xi=\text{konst.}}$ ujedno i kružnje

$$K = f(a) \mid_{\omega=\text{konst.}}$$

KRITERIJI OPTIMALNOSTI (OSU)

(18)

↳ u mat. formi se izražava f(jau) f(je)

$$J = \int_{t_0}^{t_k} F(x, u, f_s) dt$$

↓
 upravljanje
 ↓
 stanje

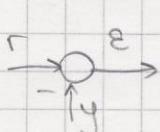
poremećaj

→ izražava meki cilj koji treba postići

Pri. najveća brzina dolaska

$$J = \int_{t_0}^{t_k} dt = t_k - t_0 = \min!!!$$

→ točnost



$$J = \int_{t_0}^{t_k} \varepsilon^2 dt \rightarrow \min$$

ITAE
ISE
:

→ minimum gubitka energije

$$J = \int_{t_0}^{t_k} u(t) \cdot i(t) dt \rightarrow \min = \int_{t_0}^{t_k} \lambda u^2(t) dt$$

	Potencijal	Tok
el.	napon	struja
meh.	brzina	sila
term.	temp.	tok temp.

$$el: \lambda = \frac{1}{R}$$

$$\int_{t_0}^{t_k} (\lambda_1 x^2 + \lambda_2 u^2) dt$$

težinski koef.

(očv. dali manu je bitnija točnost λ_1 ili energija λ_2)

LQR regulator

$$J = \int_{t_0}^{t_k} (x Q x^T + u R u^T) dt$$

→ kvadratični kriteriji se mogu analitički rješiti

Pr. \rightarrow gubitak goriva

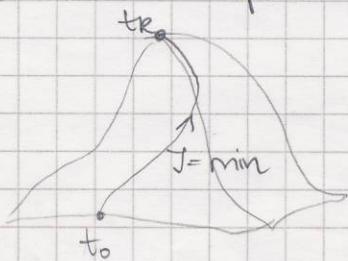
19

$$J = \int_{t_0}^{t_K} \lambda |u(t)| dt$$

\rightarrow vektorski kriterij

$$J = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_n \end{bmatrix} \rightarrow \text{složene kriterije nije moguće rješiti on-line}$$

\rightarrow svi kriteriji s integralima su dinamički kriteriji
(dinam. optimalno upravljanje)



Statičko optimалno upr.

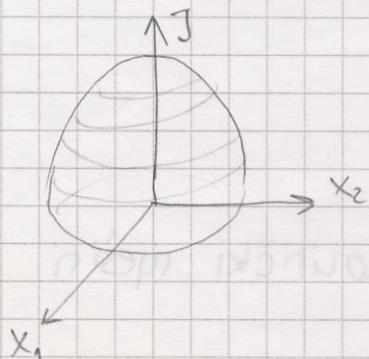
$$J = F(x_s, u_s, f_s) \rightarrow \text{držimo sustav u radnji točki}$$

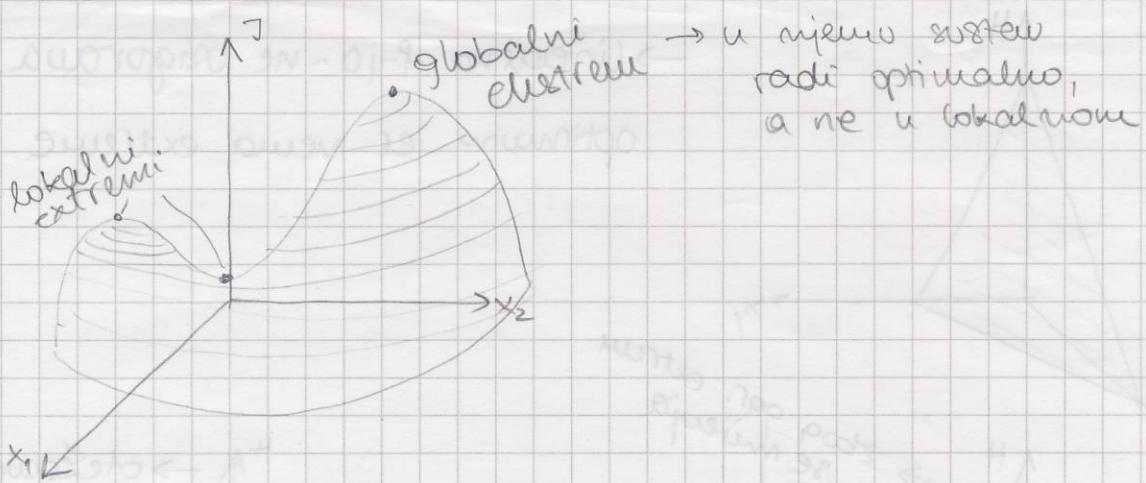
\hookrightarrow kako ostati u ekstremu (pr. min)

\hookrightarrow nije problem
ostati u extremu

\hookrightarrow problem ostati
u extremu

\rightarrow vektorski prikaz (više ravni u ravnini već u prostoru)





Statičko opt. lepr.

Dva osnovna tipa:

- 1.) Nelinearne f-je statičkih karakteristika koje imaju ekstrem !!!

$$F(u, y, f_s)$$

- 2.) Funkcije kriterija kvalitete - opisuju tehnički ili tehnico-ekonomijski pokazateli

Normalni režim rada - ustajeno stanje

$$\rightarrow \dot{x} = f(x, u) \rightarrow f(x_s, u^*) = 0 \rightarrow \underline{u}^* \rightarrow \text{vektor stanja}$$

dif. jed. → aug. jed

→ promjena kriterija u ustajenom stanju se

moe izraziti:

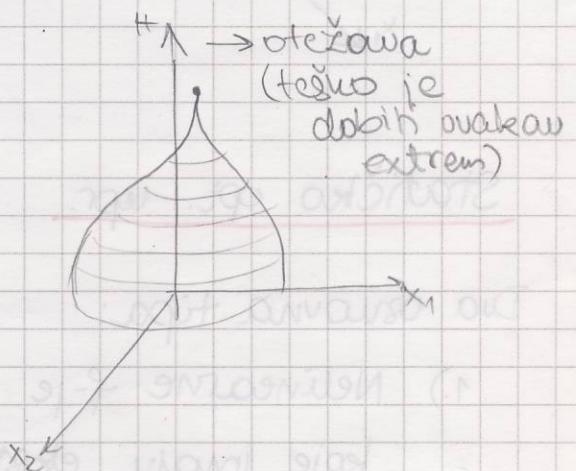
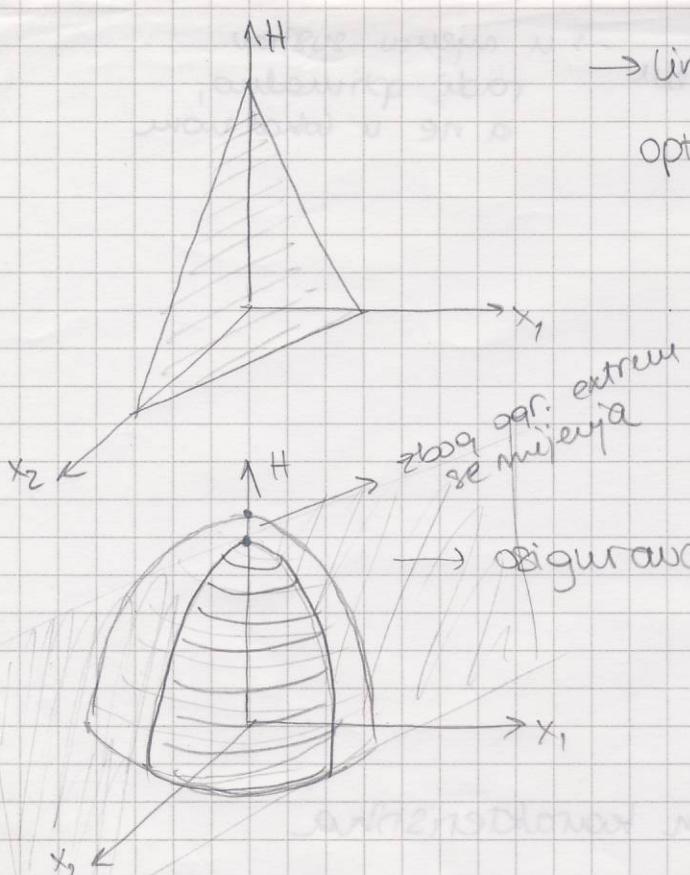
$$\left(\frac{dJ}{dt} \right)_s = F(x_s, u^*) = H(x_s) = H(x_{s1}, x_{s2}, \dots, x_{sn})$$

→ dali možemo odr. extrem f-je ovise o 3 stvare:

- priroda f-je +
- dimenzionalnost (br. var. stanja)
- ogranicenja

→ linearne f-ja - ne osigurava

optimum jer nema extreme



→ priroda f je H

→ kada imamo puno var. stanja teško je odrediti optimum

→ ograničenja

$$h_\gamma(x_1, x_2, \dots, x_p) = 0 ; \quad \gamma = 1, 2, \dots, p < n$$

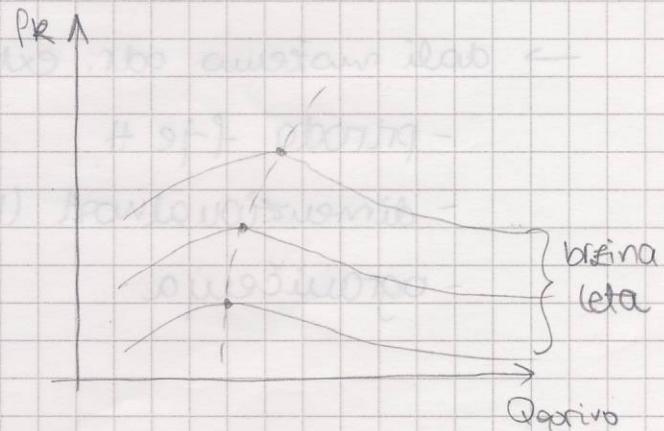
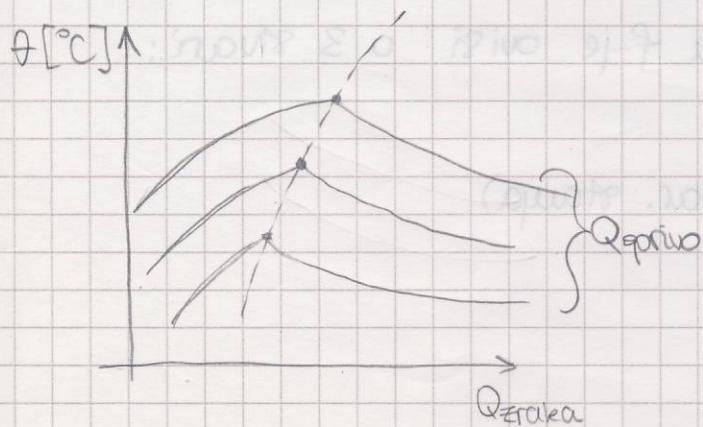
$$h_\gamma(\dots) < 0 ; \quad \dots$$

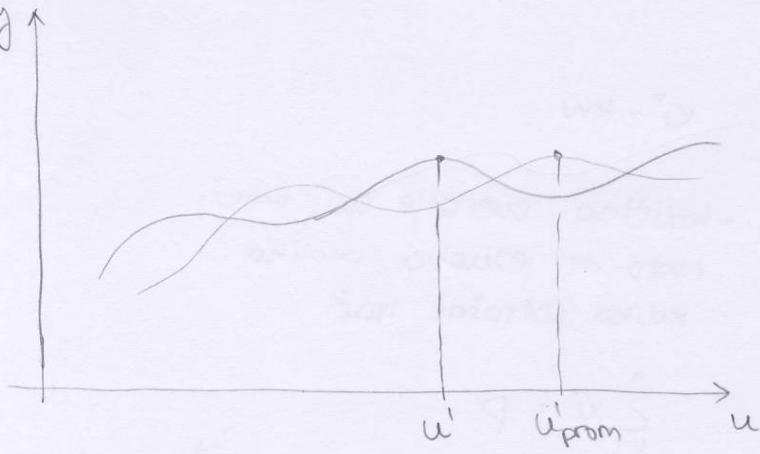
Nelin. sted. karakter. - prikazuju svojstva sustava

→ nema slobode (prostora) jer manji je proces takav

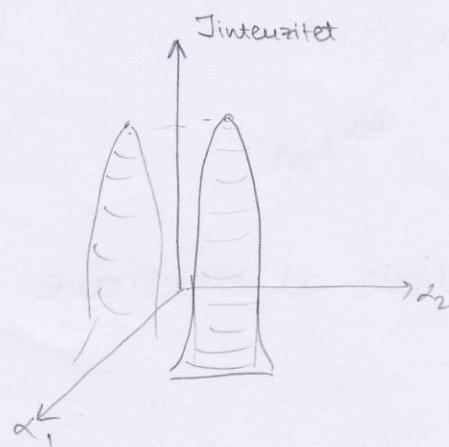
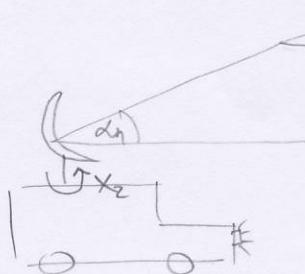
(kakav je)

Pri. vožiste





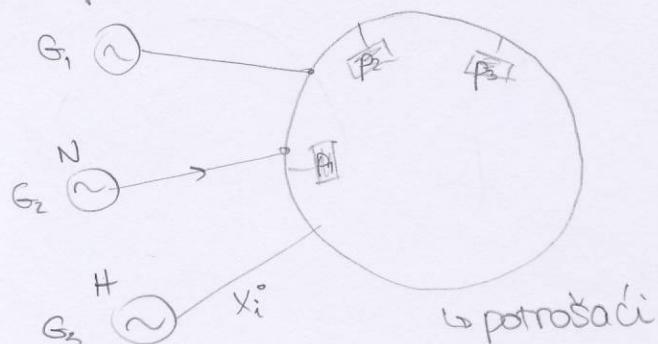
- ↳ s vremenom dolazi do promjena
 → stat. opt. upr. će držati taj extrem
 → pr. - primanje signala



- ograničenja:
- 1) prirodna - vezana uz sam objekt upravljanja (avion-kut propada, brod-kut prevrtaju, pojedalo-zasidrenje...)
 - 2) mjetna - mudi se radi što kvalitetnijeg rada sustava
 (brzina polata, max. utržanje...)
- $|x_{il}| < x_{\max}$

Primjer EES (elektroener. sustav)

(23)



Elektr.

$x_i^\circ - \text{kw}$

- količina energije u mreži
mora biti jednaka onoliko
koliko potrošač traži

$$\sum_{i=1}^n x_i^\circ = P$$

(SDHz označava da je rel. zad.)

$> P$ (50,1 Hz)

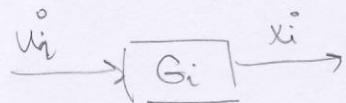
$< P$ (49,9 Hz)

Ufreku. je pokazatelj funkcije

$$\left(\frac{dJ}{dt}\right)^\circ = \sum_{i=1}^n v_i^\circ(x_i^\circ) \text{ kw/sat}$$

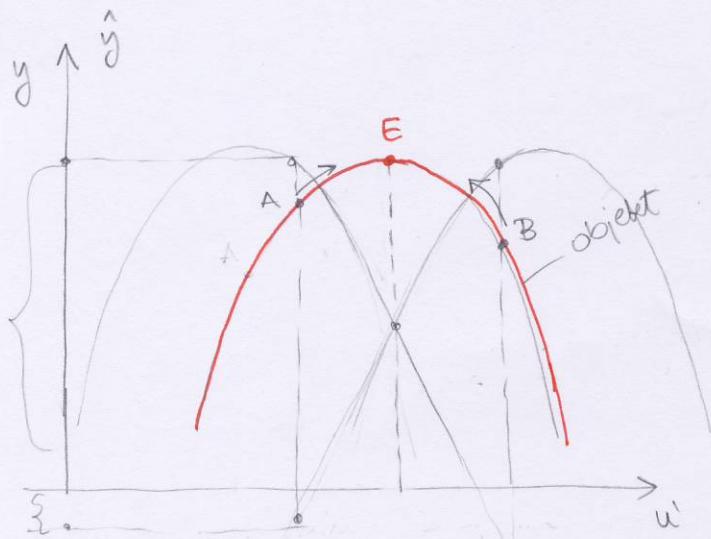
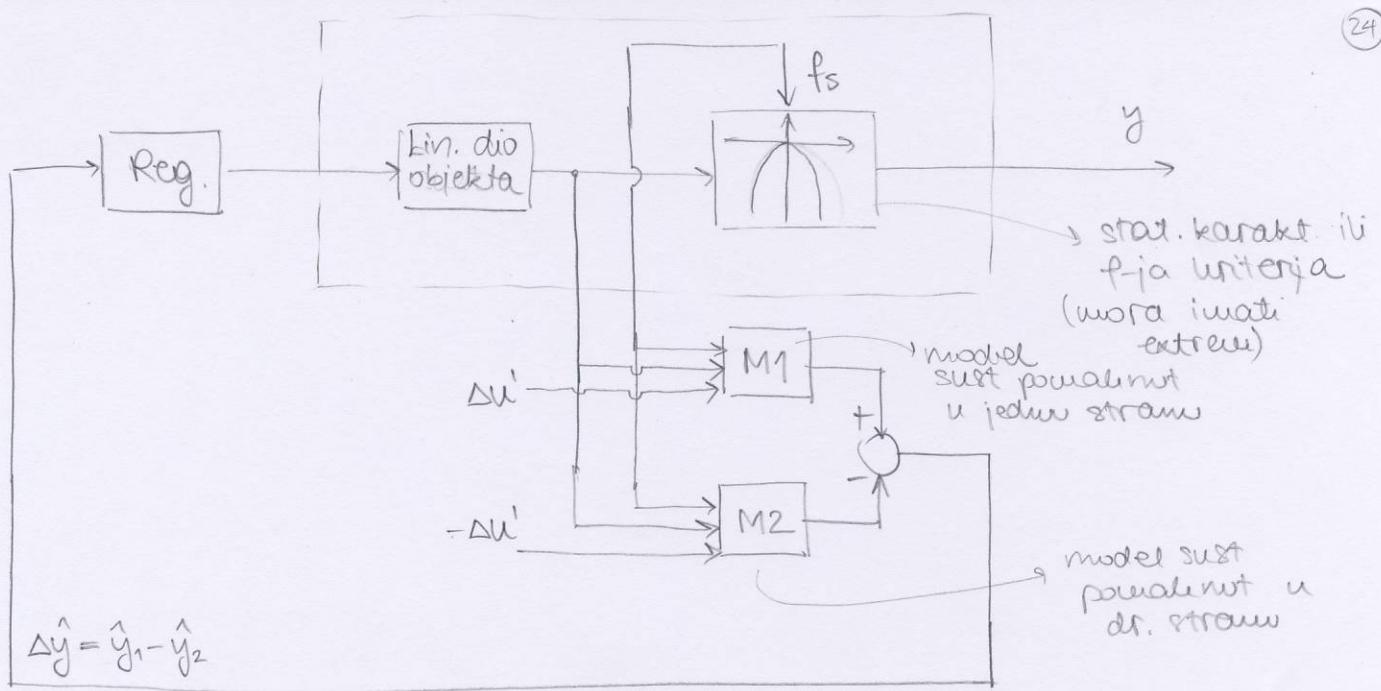
cijena

↳ drugačije su cijene od elektrane do elektrane



Držanje sustava u extremu (metode ekstremalnog upr. jednost. sust.)

- 1.) metode sa traženjem s obzirom na objektivnu (metode s moduliranim test signalom)
- 2.) metode s pomicanjem extrema
- 3.) metode s određivanjem gradijenta
- 4.) metode bez traženja extrema



- nalazimo se u A sa procesom ili u točki B - sust. navi ide
prema extreuu

- posali navi mora davan signal ϕ na extreuu

- u realnom sustav varira dva extrema - oscijacije