pr.

stožerni element — sve smo podijelili s njim ispod

1. r. mat. A = 1. red mat U — elem. koji se više neće mijenjati

LU
$$\begin{bmatrix} 2 & 3 & 1 & 5 \\ 6 & 13 & 5 & 19 \\ 2 & 19 & 10 & 23 \\ 4 & 10 & 11 & 31 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 3 & 1 & 5 \\ 3 & 4 & 2 & 4 \\ 1 & 16 & 9 & 18 \\ 2 & 4 & 9 & 21 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 3 & 1 & 5 \\ 3 & 4 & 2 & 4 \\ 1 & 4 & 1 & 2 \\ 2 & 1 & 7 & A \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 3 & 1 & 5 \\ 3 & 4 & 2 & 4 \\ 1 & 4 & 1 & 2 \\ 2 & 1 & 7 & 3 \end{bmatrix}$$

$(10 - (2)\cdot 3) = 4$

$13 - 3\cdot 3$

$:4$

$:1$

$17 - 7\cdot 2$

$$\underline{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 2 & 1 & 7 & 1 \end{bmatrix} \qquad \underline{U} = \begin{bmatrix} 2 & 3 & 1 & 5 \\ 0 & 4 & 2 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

2. RSLAJ

2.1 LU Dekompozicija

$\underline{A}\,\underline{x} = \underline{b}$

$\underline{A} = \underline{L}\cdot\underline{U}$

$$\underline{L} = \begin{bmatrix} 1 & & & 0 \\ l & 1 & & \\ l & l & 1 & \\ l & 0 & l & 1 \end{bmatrix} \qquad \underline{U} = \begin{bmatrix} u_{11} & & u & \\ & u_{22} & u & u \\ & \emptyset & u & u \\ & & & u_{nn} \end{bmatrix}$$

$\underline{A}\,\underline{x} = \underline{L}\,\underline{U}\,\underline{x} = \underline{b}$, $\underline{U}\,\underline{x} \to \underline{y}$

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1) Supstitucija unaprijed

$\underline{L}\,\underline{y} = \underline{b}$

$y_1 = b_1$

$y_2 = b_2 - l_{21}\cdot y_1$

$\vdots$

$$\boxed{y_i = b_i - \sum_{j=1}^{i-1} l_{ij}\cdot y_j \qquad i = 1 \ldots n}$$

.) Supstitucija unatrag

$\underline{U}\,\underline{x} = \underline{y}$

$x_n = \dfrac{y_n}{u_{nn}}$

$$\left[x_i = \frac{1}{u_{ii}} \left(y_i - \sum_{j=i+1}^{n} u_{ij} \cdot x_j \right), \quad i = n, \ldots, 1 \right]$$

Nem. prostor matrice A može se iskoristiti za smještaj matrica L i U

Nem. prostor vektora b koristi se prvo za vektor y, a potom za vektor x

LU algoritam

```
za i = 1 DO n-1                                   } LU
   za j = i+1 DO n
        A[j,i] /= A[i,i];
        za k = i+1 DO n
            A[j,k] -= A[j,i] * A[i,k];

za i = 1 DO n-1                                   } Supst. unaprijed
   za j = i+1 DO n
        b[j] -= A[j,i] * b[i];

za i = n DO 1                                     } Supst unatrag
   b[i] /= A[i,i];
   za j = 1 DO i-1
        b[j] -= A[j,i] * b[i];
```

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Zadatak.

LU dekompozicija

$$\begin{bmatrix} 4 & 3 & 2 & 1 \\ 4 & 6 & 1 & -1 \\ -8 & 3 & -5 & -6 \\ 12 & 12 & 7 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 3 & -1 & -2 \\ -2 & 9 & -1 & -4 \\ 3 & 3 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 3 & -1 & -2 \\ -2 & 3 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 3 & -1 & -2 \\ -2 & 3 & 2 & 2 \\ 3 & 1 & 1 & 1 \end{bmatrix}$$

Zadatak

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 0 \\ 2 & 4 & 2 \end{bmatrix} \underline{x} = \begin{bmatrix} 25 \\ 10 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & -3 \\ 0 & 4 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & -3 \\ 0 & 4 & 14 \end{bmatrix} = \underline{L} \backslash \underline{U}$$

$$\underline{L}\,\underline{y} = \underline{b} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} \cdot \underline{y} = \begin{bmatrix} 25 \\ 10 \\ 10 \end{bmatrix}$$

$$\Rightarrow y_1 = 25, \quad y_2 = -15, \quad y_3 = 70$$

$$\underline{U}\,\underline{x} = \underline{y} \Rightarrow \begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 14 \end{bmatrix} \cdot \underline{x} = \begin{bmatrix} 25 \\ -15 \\ 70 \end{bmatrix}$$

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$$\Rightarrow x_3 = \frac{70}{14} = 5, \quad x_2 = 0, \quad x_1 = 5 \Rightarrow \underline{x} = \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$$

Zadatak

$$\begin{bmatrix} 6 & 2 & 10 \\ 2 & 3 & 0 \\ 0 & 4 & 2 \end{bmatrix} \underline{x} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

trivijalno rješenje

$$\begin{bmatrix} 6 & 2 & 10 \\ 1/3 & 7/3 & -10/3 \\ 0 & 4 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 2 & 10 \\ 1/3 & 7/3 & -10/3 \\ 0 & 12/7 & 54/7 \end{bmatrix} = \underline{L} \backslash \underline{U}$$

$$\underline{L}\,\underline{y} = \underline{b} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 0 & 12/7 & 1 \end{bmatrix} \underline{y} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \qquad \underline{y} = \begin{bmatrix} 2 \\ 7/3 \\ 0 \end{bmatrix}$$

$$\underline{U}\,\underline{x} = \underline{y} \Rightarrow \begin{bmatrix} 6 & 2 & 10 \\ 0 & 7/3 & -10/3 \\ 0 & 0 & 54/7 \end{bmatrix} \cdot \underline{x} = \begin{bmatrix} 2 \\ 7/3 \\ 0 \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

2.2 Stožerni razvoj (Pivotiranje)

pr. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 1 & 3 & 3 \end{bmatrix} \underline{x} = \begin{bmatrix} 0.5 \\ 2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} \boxed{1} & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boxed{0} & 2 \\ 1 & ? & \end{bmatrix}$ s LU postupkom nemožemo doći do rješenja

- Razlozi

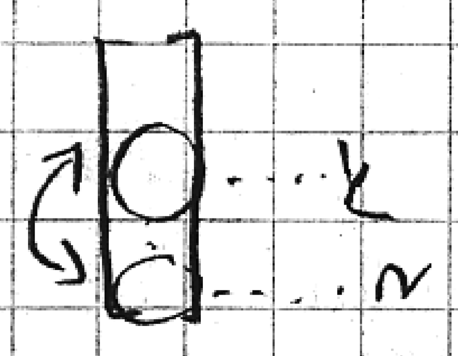
1) Tijekom dekompozicije može se dogoditi da stožerni element postane jednak ∅

2) Ako je stožerni element mali može doći do pogreške zbog zaokruživanja

⇒ PIVOTIRANJE (stož. razvoj) – mijenjamo (zamijenjujemo) stupce ili retke matrice kako bi za stožerni element došli što veći broj po aps. vrijednosti

a) Djelomično (ili po recima ili po stupcima)

- PO STUPCIMA



unutar stupca tražimo |max| vrijednost

mijenjamo 2 retka

$|a_{rk}| = \max |a_{ik}|, \quad i = k, \ldots, n$

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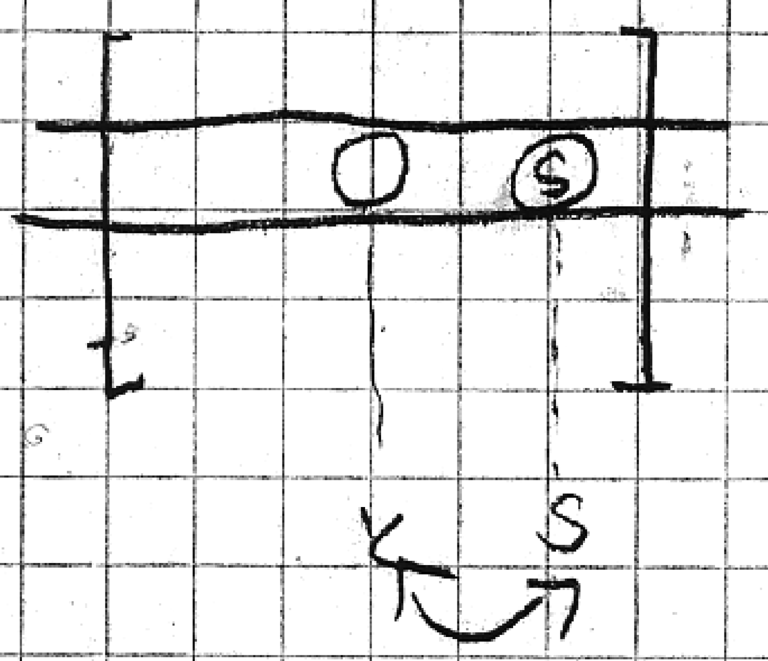
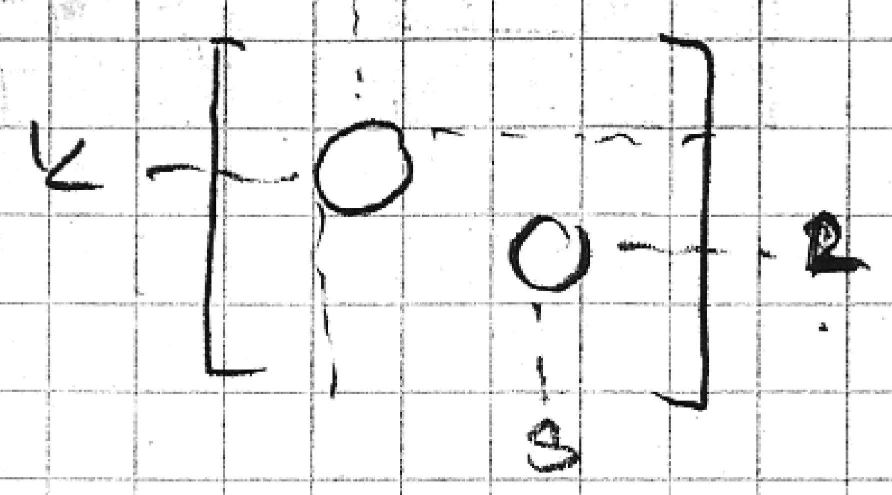
- PO RETCIMA



- mijenjamo 2 stupca
- mijenjamo redoslijed varijabli

b) Potpuno pivotiranje

$$|a_{rs}| = \max_{k} |a_{ij}| \qquad i,j = k, \dots, n$$



pretražujemo cijelu podmatricu

2.3 LUP Dekompozicija

LUP dekompozicija je zapravo LU dekomp. uz djelomično pivotiranje po stupcima

Napomena $A[k,i] == \emptyset$

$$\Rightarrow |A[k,i]| < \varepsilon, \quad \varepsilon \ll$$

- svaka nesingularna matrica može se svesti na L i U uz barem jednu permutaciju redaka ili stupaca matrice (ako matrica nije singularna tada postoji barem 1 permutacija redaka za koju će se matrica moći rastaviti na L i U)

Zadatak

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3_{max} & 2 & 1 & 4 \\ 1 & 3 & 2 & 4 \\ 1 & 4 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 3 & 2 & 1 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \\ 1 & 4 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 3 & 2 & 1 & 4 \\ 1/3 & 4/3 & 8/3 & 8/3 \\ 1/3 & 7/3 & 5/3 & 8/3 \\ 1/3 & \underset{max}{10/3} & 5/3 & 5/3 \end{bmatrix} \to \begin{bmatrix} 3 & 2 & 1 & 4 \\ 1/3 & 10/3 & 5/3 & 5/3 \\ 1/3 & 7/3 & 5/3 & 8/3 \\ 1/3 & 4/3 & 8/3 & 8/3 \end{bmatrix} \to$$

$$\begin{bmatrix} 3 & 2 & 1 & 4 \\ 1/3 & 10/3 & 5/3 & 5/3 \\ 1/3 & 7/10 & 1/2 & 3/2 \\ 1/3 & 4/10 & \underset{max}{2} & 2 \end{bmatrix} \to \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1/3 & 4/10 & 2 & 2 \\ 1/3 & 7/10 & 1/2 & 3/2 \end{bmatrix} \to \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 2 & 2 \\ \cdot & \cdot & 1/4 & 1 \end{bmatrix}$$

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adatak

$$\begin{bmatrix}4 & 10 & 5 & -9\\ -4 & 6 & 6 & 3\\ -8 & 4 & 2 & 6\\ -4 & 8 & 1 & -3\end{bmatrix} \rightarrow \begin{bmatrix}-8 & 4 & 2 & 6\\ -4 & 6 & 6 & 3\\ 4 & 10 & 5 & -9\\ -4 & 8 & 1 & -3\end{bmatrix} \rightarrow \begin{bmatrix}\boxed{-8} & 4 & 2 & 6\\ 1/2 & 4 & 5 & 0\\ -1/2 & 12 & 6 & -6\\ 1/2 & 6 & 0 & -6\end{bmatrix} \rightarrow \begin{bmatrix}-8 & 4 & 2 & 6\\ -1/2 & \boxed{12} & 6 & -6\\ 1/2 & 4 & 5 & 0\\ 1/2 & 6 & 0 & -6\end{bmatrix} \rightarrow \begin{bmatrix}-8 & 4 & 2 & 6\\ 1/2 & 12 & 6 & -6\\ 1/2 & 1/3 & 3 & 2\\ 1/2 & 1/2 & -3 & 3\end{bmatrix} \rightarrow \begin{bmatrix}-8 & 4 & 2 & 6\\ -1/2 & 12 & 6 & -6\\ 1/2 & 1/3 & 3 & 2\\ 1/2 & 1/2 & -1 & -1\end{bmatrix}$$

adatak

Sustav riješiti LUP dekompozicijom

$$\begin{bmatrix}0 & 1 & 2\\ 2 & 0 & 3\\ 3 & 5 & 1\end{bmatrix} \underline{x} = \begin{bmatrix}6\\ 9\\ 3\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}0 & 1 & 2 & 6\\ 2 & 0 & 3 & 9\\ 3 & 5 & 1 & 3\end{array}\right] \rightarrow \left[\begin{array}{ccc|c}3 & 5 & 1 & 3\\ 2 & 0 & 3 & 9\\ 0 & 1 & 2 & 6\end{array}\right] \rightarrow \left[\begin{array}{ccc|c}3 & 5 & 1 & 3\\ 2/3 & -10/3 & 7/3 & 9\\ 0 & 1 & 2 & 6\end{array}\right] \rightarrow \left[\begin{array}{ccc|c}3 & 5 & 1 & 3\\ 2/3 & -10/3 & 7/3 & 9\\ 0 & -3/10 & 27/10 & 6\end{array}\right]$$

$$\underline{L} \cdot \underline{y} = \underline{P} \cdot \underline{b} \Rightarrow \begin{bmatrix}1 & 0 & 0\\ 2/3 & 1 & 0\\ 0 & 3/10 & 1\end{bmatrix} \underline{y} = \begin{bmatrix}3\\ 9\\ 6\end{bmatrix} \Rightarrow \begin{aligned} y_1 &= 3\\ y_2 &= 7\\ y_3 &= 81/10\end{aligned}$$

$$\underline{U} \cdot \underline{x} = \underline{y} \Rightarrow \begin{bmatrix}3 & 5 & 1\\ 0 & -10/3 & 7/3\\ 0 & 0 & 27/10\end{bmatrix} \cdot \underline{x} = \begin{bmatrix}3\\ 7\\ 81/10\end{bmatrix} \qquad \begin{aligned} x_3 &= 0\\ x_2 &= 0\end{aligned} \qquad x_1 = 0$$

adatak

LUP

$$\begin{bmatrix}1 & 5/2 & 1/2\\ 2 & 2 & 0\\ 4 & 2 & -2\end{bmatrix} \cdot \underline{x} = \begin{bmatrix}41/2\\ 22\\ 22\end{bmatrix}$$

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$$\left[\begin{array}{ccc|c}4 & 2 & -2 & 22\\ 2 & 2 & 0 & 22\\ 1 & 5/2 & 1/2 & 41/2\end{array}\right] \rightarrow \left[\begin{array}{ccc|c}\boxed{4} & 2 & -2 & 22\\ 1/2 & 1 & 1 & 22\\ 1/4 & 2 & 1 & 41/2\end{array}\right] \rightarrow \left[\begin{array}{ccc|c}4 & 2 & -2 & 22\\ 1/4 & 2 & 1 & 41/2\\ 1/2 & 1 & 1 & 22\end{array}\right] \rightarrow \left[\begin{array}{ccc|c}4 & 2 & -2 & 22\\ 1/4 & \boxed{2} & 1 & 41/2\\ 1/2 & 1/2 & 1/2 & 22\end{array}\right]$$

$$\underline{L} \cdot \underline{y} = \underline{P} \cdot \underline{b}$$

$$\underline{P} = \begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ 1/2 & 1/2 & 1 \end{bmatrix} \qquad y_1 = 22 \quad y_2 = 15 \quad y_3 = 7/2$$

$$\underline{U} \cdot \underline{x} = \underline{y}$$

$$\underline{U} = \begin{bmatrix} 4 & 2 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & 1/2 \end{bmatrix} \qquad \begin{aligned} x_3 &= 7 \\ x_2 &= 4 \\ x_1 &= 7 \end{aligned}$$

Zadatak

Možemo li rastaviti LU?

$$\begin{bmatrix} 2 & 0 & -2 & 6 \\ 2 & 3 & 1 & 5 \\ -2 & 0 & 5 & -5 \\ 4 & 2 & -6 & 8 \end{bmatrix} \to \begin{bmatrix} \boxed{2} & 0 & -2 & 6 \\ 1 & 3 & 3 & -1 \\ -1 & 0 & 3 & 1 \\ 2 & 2 & -2 & -1 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & -2 & 6 \\ 1 & 3 & 3 & -1 \\ -1 & 0 & 3 & \\ 2 & 2/3 & & \end{bmatrix} //$$

Može ?

2.4 Inverzija matrica

$$\underline{A} \cdot \underline{A}^{-1} = ? = \underline{X}$$

$$\underline{A} \cdot \underline{X} = \underline{E}$$

$$\underline{X} = [\underline{x_1}, \underline{x_2}, \dots, \underline{x_n}]$$

$$A \cdot [\underline{x_1}, \underline{x_2}, \dots, x_n] = [\underline{e_1}, \underline{e_2}, \dots, \underline{e_n}]$$

$$\begin{cases} \underline{A} \cdot \underline{x_1} = \underline{e_1} \\ \underline{A} \cdot \underline{x_2} = \underline{e_2} \\ \vdots \\ \underline{A} \cdot \underline{x_n} = \underline{e_n} \end{cases}$$

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Rješavanje

LUP dekompozicija nad A

n puta supst. unatrag

n puta supst. unaprijed

LUP : $O(n^3)$

SUP : $O(n^2)$

$\left.\right\}$ $O(n^3) + n \cdot O(n^2) = O(n^3)$

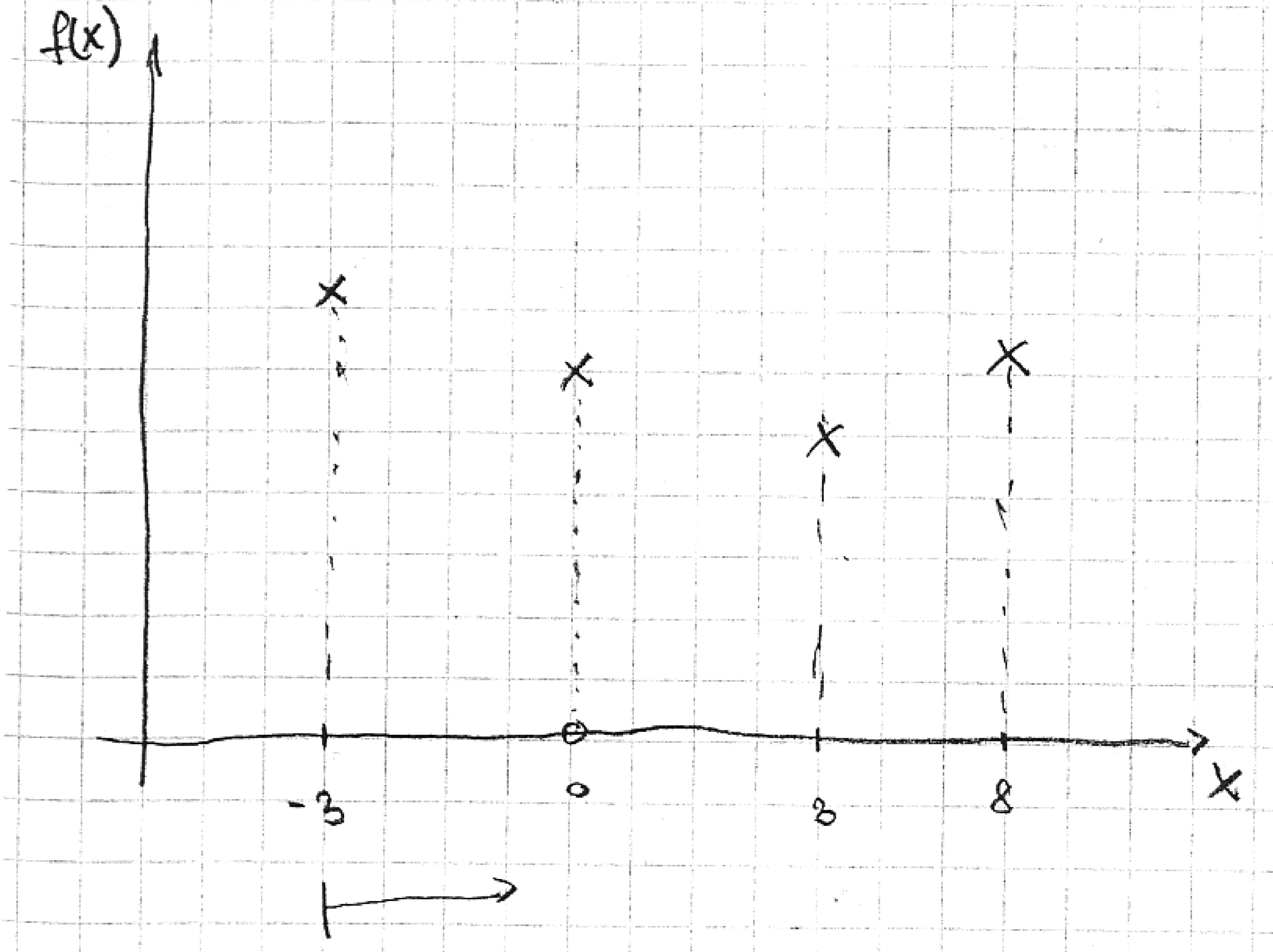
→ zapamtiti

$\underline{P} \cdot \underline{A} = \underline{L} \cdot \underline{U}$

$(\underline{L} \cdot \underline{y} = \underline{P} \cdot \underline{b})$ $\qquad \underline{L} \cdot \underline{y} = \underline{P} \cdot e_1$

$(\underline{U} \cdot \underline{x} = \underline{y})$ $\qquad \underbrace{\underline{U} \cdot \underline{x}_1 = \underline{U}_1}_{\text{invertiranje}}$

3. PNO



ako f-ja ima samo 1 minimum

min. između 0 i 8

3.1 Pronalaženje unimod. intervala

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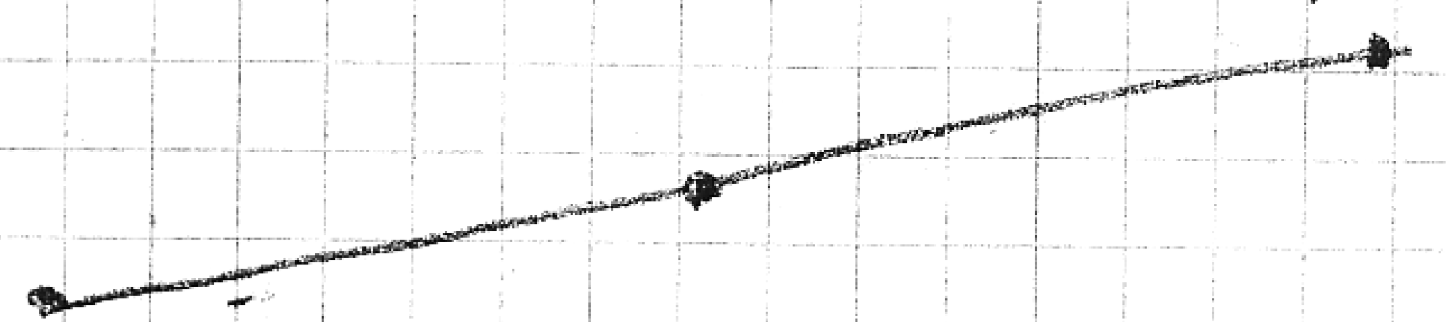
x_0 - početna točka

h - početni pomak (mali)

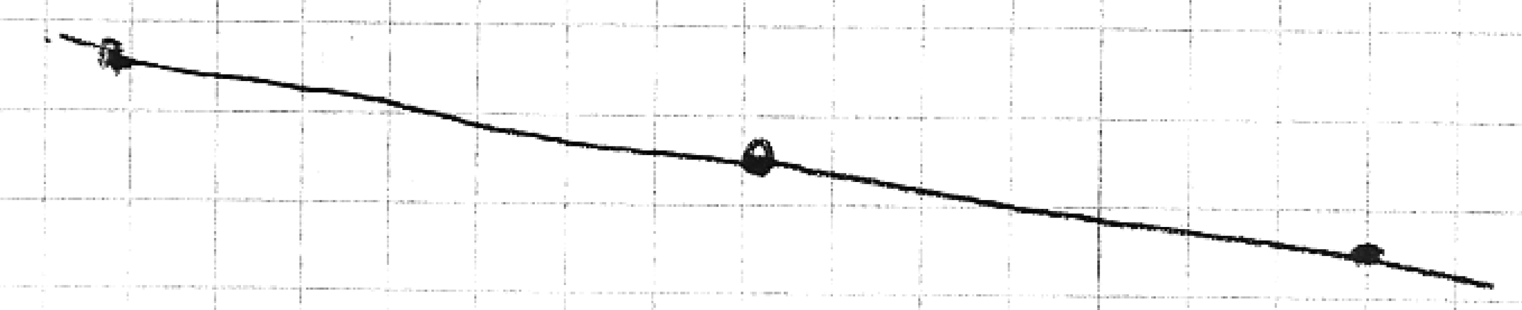
$f(x_0-h)$ $\qquad f(x_0)$ $\qquad f(x_0+h)$



iteracija

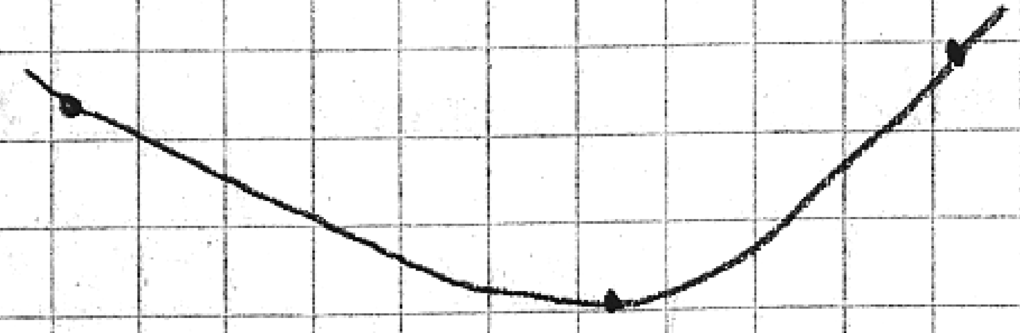
$\Rightarrow f(x_0 - 2^i \cdot h)$, sve do $f(x_0 - 2^k \cdot h) > f(x_0 - 2^{k-1} \cdot h)$

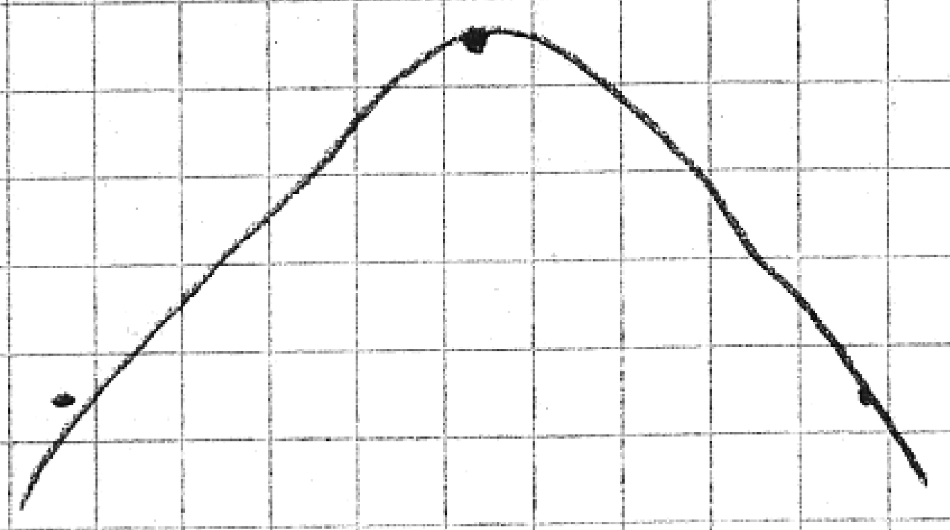
$\Rightarrow$ Interval : $[x_0 - 2^k \cdot h, \; x_0 - 2^{k-2} \cdot h]$



$f(x_0 + 2^i \cdot h)$, sve dok $f(x_0 + 2^k h) > f(x_0 - 2^{k-1} h)$

$\Rightarrow$ Int $[x_0 + 2^{k-2} \cdot h, \; x_0 + 2^k \cdot h]$

3)  $\Rightarrow$ int $[x_0-h,\ x_0+h]$

4)  $\Rightarrow$ Nije unimodalna fja (ima više minimuma) (svejedno na koju stranu odemo)

Zadatak

Zadana je fja $f(x)=x^2-2$, $x_0=100$, $h=1$. Koje se granice intervala dobivaju provođenjem algoritma unimod. int.

$$\underset{f(99)}{f(x_0-h)} < \underset{f(100)}{f(x_0)} < \underset{f(101)}{f(x_0+h)}$$

	x_0-h	x_0-2h	x_0-4h	...	x_0-64h	x_0-128h	x_0-256h
x	99	98	96		32	-28	-156
f(x)							
	>	>	>	>	>	<	

$-28 < -156$

INT $[-156, 32]$

Zad.

Na nepoznatom unimod. fjom postupak pronalaženja unimod. int uz $x_0=0$, $h=2$ daje interval $[-32, -8]$. Za svaku od sljed. tvrdnji odrediti jeli istinita ili lažna ili se istinitost nemože odrediti.

a) $g(2) < g(-2)$ $\longrightarrow$ NE

b) $g(-5) > g(-10)$ $\longrightarrow$ DA $g(-5) > g(-8) > g(-10)$

c) $g(-10) > g(10)$ $\longrightarrow$ NE

d) $g(0) < g(-30)$ $\longrightarrow$? Nemožemo odrediti

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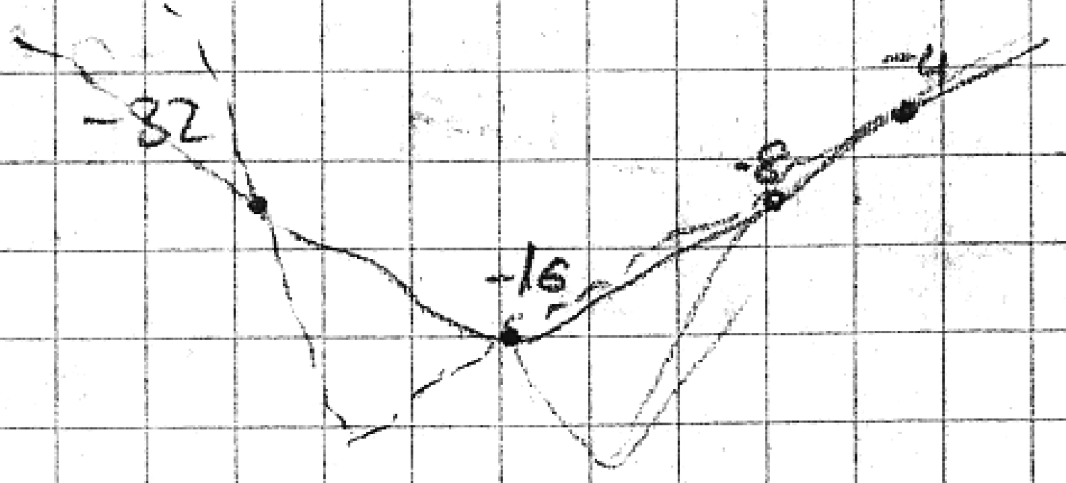
x	-32	-16	-8	-4	-2	0	2
g(x)							
		>	<	<	<	<	<



Zadatak

$g(x)$, $x_0 = 2$, $h = 1$, $[6, 18]$

x	1	2	3	4	5	10	18
$f(x)$							

$>$ $>$ $>$ $>$ $>$ $<$

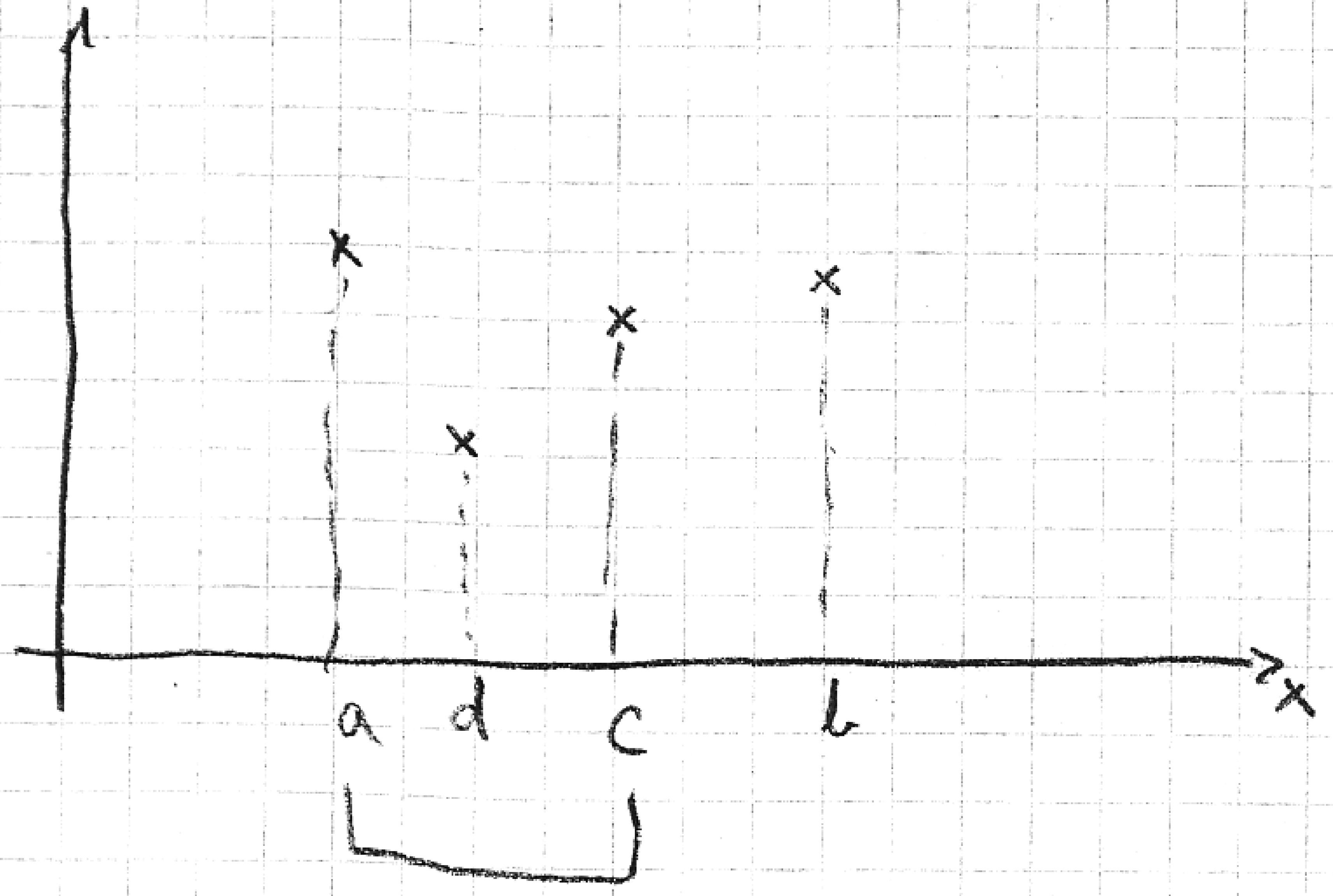
a) $g(1) < g(2) \rightarrow$ NE

b) $g(5) > g(10) \rightarrow$ DA

c) $g(6) > g(18) \rightarrow ?$

d) $g(10) < g(18) \rightarrow$ DA

Pr.



Kad reduciramo interval trebamo dvije nove točke unutar većeg intervala da bi jedne mogli izbaciti

Zadatak: Provedite metodu zlatnog reza do širine intervala $\varepsilon \leq 0.5$

$f(x) = (x+1)^2$, $x \in [-2, 2]$

$c = b - k(b-a)$ (širina intervala)

$d = a + k(b-a)$

	a	c	d	b
0	-2	-0.472	+0.472	2
1	-2	-1.056	-0.472	0.472
2	-2	-1.416	-1.056	-0.472
3	-1.416	-1.056	-0.833	-0.472
4	-1.416	-1.193	-1.056	-0.833
5	-1.193			-0.833

$(a-b) < 0.5$ STOP

$\Rightarrow [-1.193, -0.833]$

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zadatak: Pronaći unimod. int. te ga prez. metodom zlatnog reza

$f(x) = (x-4)$, $x_0 = 0$, $h = 1$, $\varepsilon \leq 1$

x	-1	0	1	2	4	8
f(x)						

$>$ $>$ $>$ $>$ $<$

$\Rightarrow [2, 8]$

	a	c	d	b
0	2	4.292	5.708	8
1	2	3.416	4.292	5.708
2	3.416	4.292	4.832	5.708
	3.416	3.987	4.292	4.832
	3.416			4.292

$\Rightarrow [3.416, 4292]$

Ono što je bliže minimumu je tolje

4 ovdje

zadatak: (širina) Početni unimodalni interval $[-100, +100]$
koliko iteracija postupka zlatnog reza je potrebno
kako bi se int. smanjilo do širine $\varepsilon \leq 0.001$

$I = 200$ šir. poč. int.

$I \cdot k^n \leq \varepsilon$
$\rightarrow$ smanjuje se za faktor k u svakoj iteraciji

$$k^n \leq \frac{\varepsilon}{I} \quad / \log$$

$$n \log(k) \leq \log\left(\frac{\varepsilon}{I}\right) \quad / \log k$$

$$n \geq \frac{\log(\varepsilon / I)}{\log k} = 25.36$$

$$\boxed{n = 26}$$

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Zad.
$f(x) = (x-4)^2$, $[-2, 6]$ Provesti Fibonaccijev postupak do širine intervala $\varepsilon \leq 1$

Broj koraka:

Niz: $1, 1, 2, 3, 5, \boxed{8}, 13, \ldots$

$$F_N \geq \frac{b-a}{\varepsilon} = 8$$

$$c = b - \frac{F_{N-1}}{F_N}(b-a)$$

$$d = a + \frac{F_{N-1}}{F_N}(b-a)$$

$c_1 = 6 - \frac{5}{8}(6+2) = 1$

$d_1 = -2 + \frac{5}{8}(8) = 3$

	a	c	d	b
0	-2	1	3	6
1	1	3	4	6
2	3	4	5	6
3	3	4	4	5 $\Leftarrow$ STOP

u zadnjem koraku ispitujemo vrijednost funkcije (min) sredine cilja u točki bliža intervala

Zadatak (Fibonacci) (prošli)

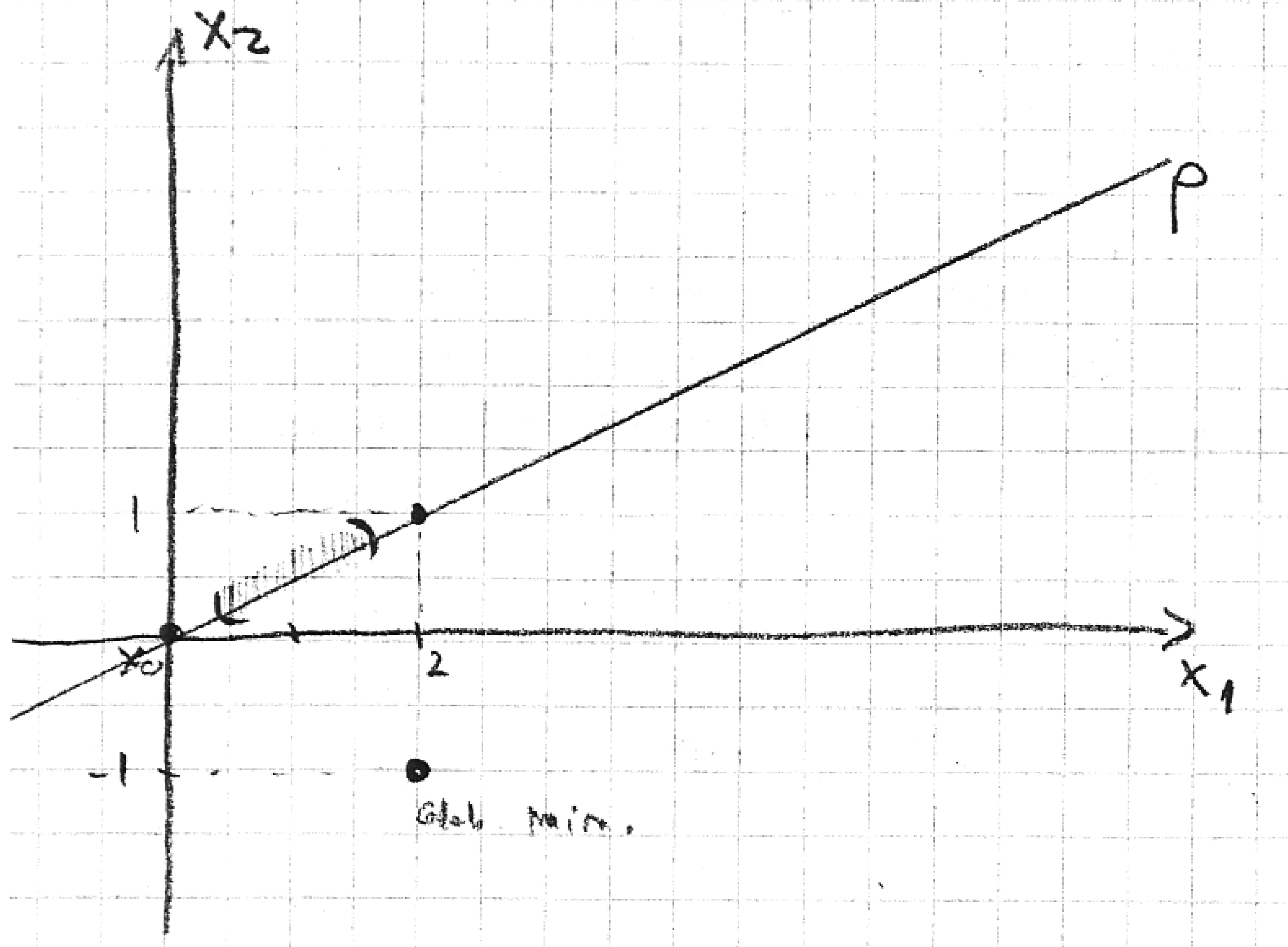
dodatna točka $\rightarrow 4.01$

$$f(x) = (x-4)^2 \Rightarrow [3, 4.01]$$

$$f(x) = (x-4.2)^2 \Rightarrow [4, 5]$$

Zadatak: zadana je f-ja, pronaći minimum zadane f-je na pravcu zadanom točkom $\underline{x_0}$ i smjerom $\underline{v}$ postupkom zlatnog (korak) reza, $\varepsilon \le 0.5$

$$F(\underline{x}) = (x_1-2)^2 + (x_2+1)^2, \quad \underline{x_0} = \begin{bmatrix}0\\0\end{bmatrix}, \quad \underline{v} = \begin{bmatrix}2\\1\end{bmatrix}; \quad h = 1$$



$$\underline{x} = \underline{x_0} + \lambda \cdot v = \begin{bmatrix}0\\0\end{bmatrix} + \lambda\begin{bmatrix}2\\1\end{bmatrix} = \begin{array}{c|c} \begin{bmatrix}2\lambda\\ \lambda\end{bmatrix} & \begin{matrix}x_1\\x_2\end{matrix}\end{array}$$

$F(\underline{x}) = (2\lambda-2)^2 + (\lambda+1)^2 = f(\lambda)$ sve točke koje dobijemo iz f-je F su na pravcu

Unimodalni interval:

λ	-1	0	1	2
$f(\lambda)$	16	5	4	13
		>	>	<

$\Rightarrow \lambda \in [0, 2]$

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zlatni rez:

a	c	d	b
0	0.764	1.236	2
0	0.472	0.764	1.236
0	0.292	0.472	0.764
0.292	0.472		0.764

$f(c) = 3.3$
$f(d) = 5.22$

$f(c) = 3.28 <$

$f(c) = 3.67 >$

$$\lambda \in [0.292, 0.764]$$
λ_1 λ_2

$$x \in [\underline{x}_0 + \eta_1 \cdot \underline{v}, \; \underline{x}_0 + \eta_2 \underline{v}]$$

$$\left[\begin{bmatrix} 0.584 \\ 0.292 \end{bmatrix}, \begin{bmatrix} 1.528 \\ 0.764 \end{bmatrix} \right]$$

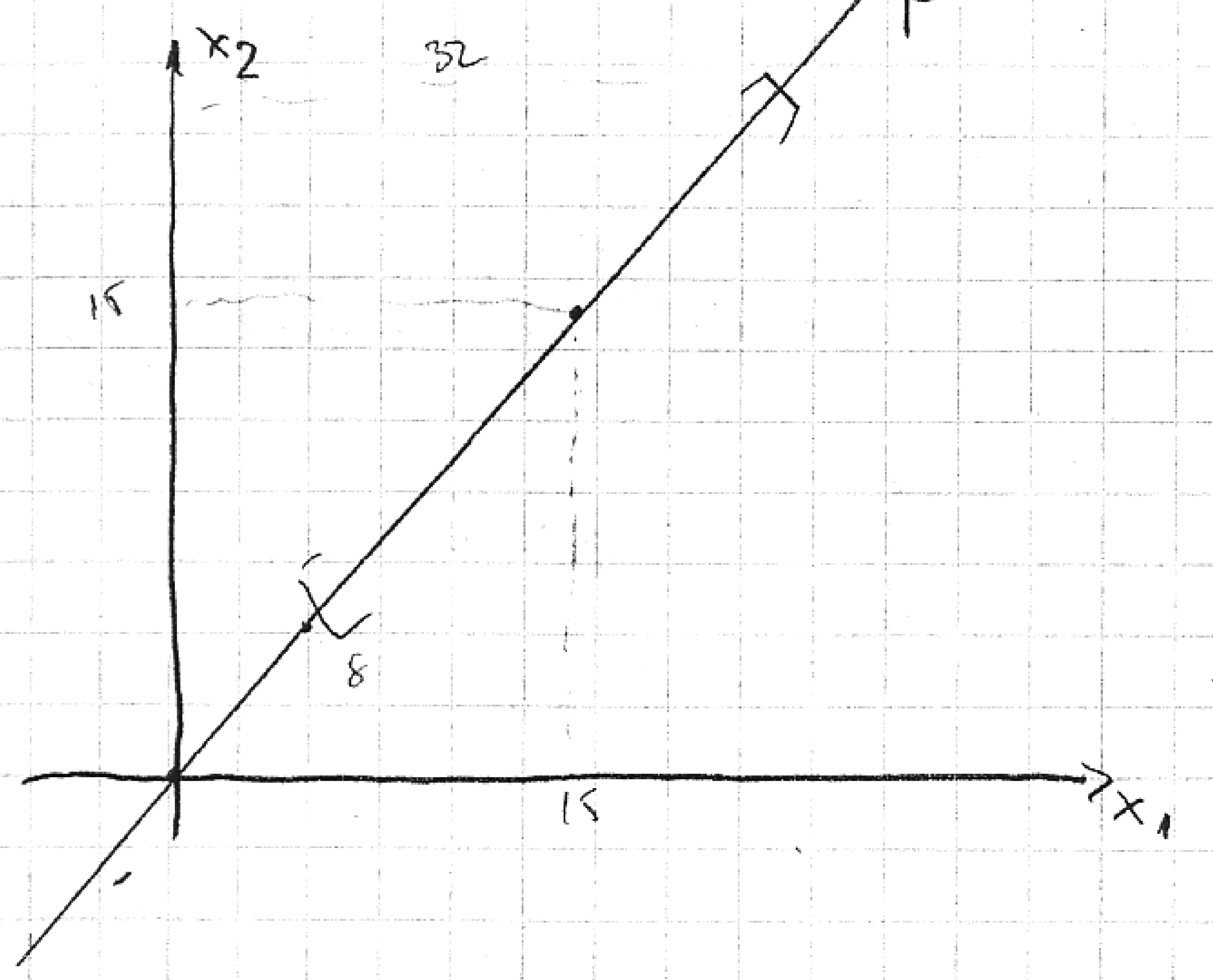
Zadatak: Pronaći minimum f-je na pravcu Fib. postupkom do $\varepsilon \leq 3$

$\underline{x}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $h = 1$

$F(\underline{x}) = (x_1 - 15)^2 + (x_2 - 15)^2$, $\underline{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ → min(15, 15)

$\underline{x}_0 + \eta \cdot \underline{v} = \begin{bmatrix} \eta \\ \eta \end{bmatrix}$

$F(\underline{x}) = 2(\eta - 15)^2 = f(\eta)$



Unimod. int

η	-1	0	1	2	4	8	16	32
$f(\eta)$								
	>	>	>	>	>	>	<	

$\eta \in [8, 32]$

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$F_N \geq \frac{b-a}{\varepsilon} = \frac{32-8}{3} = 8$; $1, 1, 2, 3, 5, \boxed{8}$

$c = b - \frac{F_{N-1}}{F_N}(b-a)$

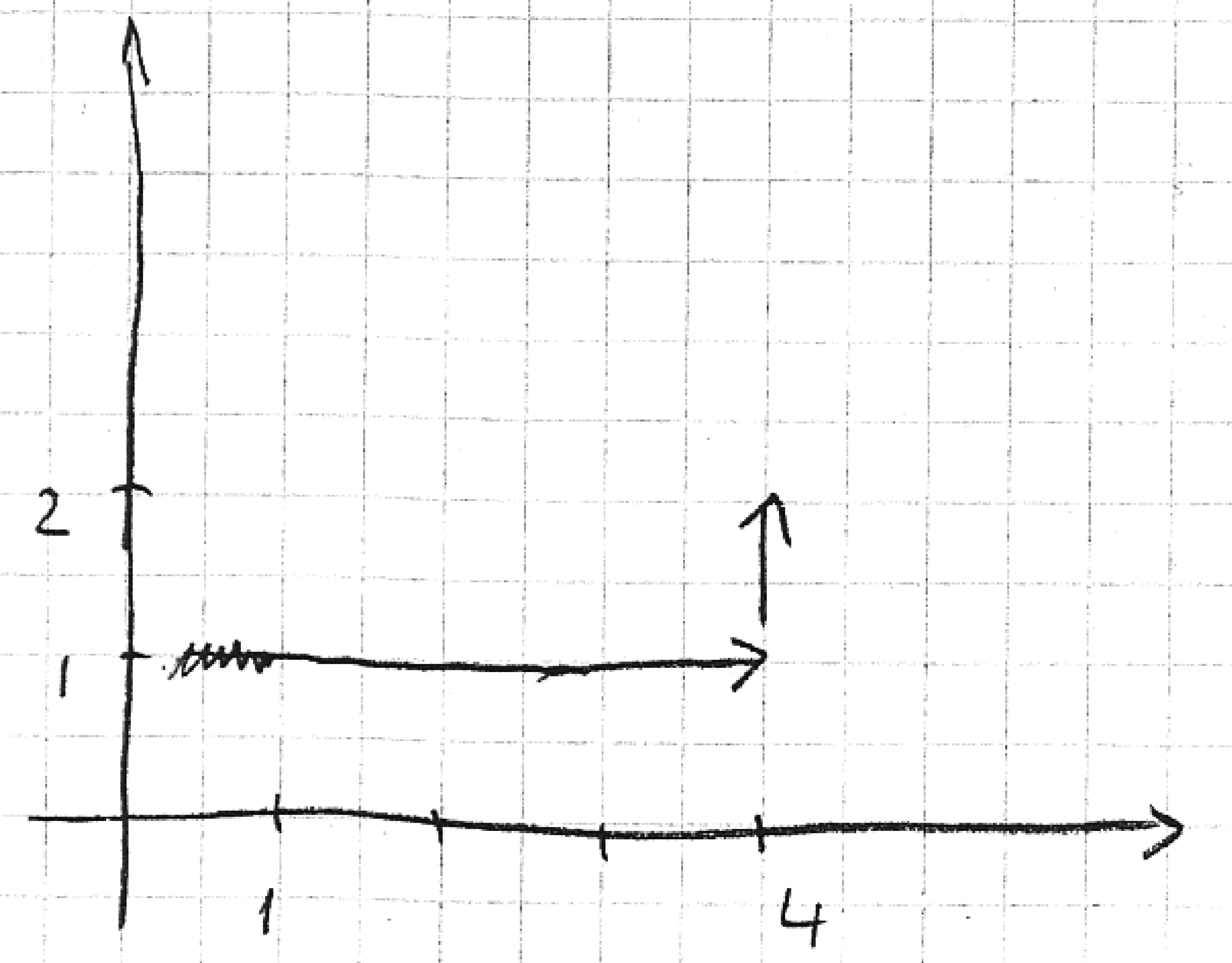
a	c	d	b
8	17	23	32
8	14	17	23
8	11	14	17
11	14	$\boxed{14.01}$	17

$\Rightarrow \eta \in [14, 17]$

$$\underline{x} \in [\underline{x}_0 + \eta_1 \underline{v}, \; \underline{x}_0 + \eta_2 \underline{v}] = \left[\begin{bmatrix} 14 \\ 14 \end{bmatrix}, \begin{bmatrix} 17 \\ 17 \end{bmatrix} \right]$$

Zadatak Postupkom koordinatnog traženja skicirati minimum.

$F(\underline{x}) = (x_1-4)^2 + (x_2-2)^2 \quad x_0 \begin{bmatrix}1\\1\end{bmatrix}$

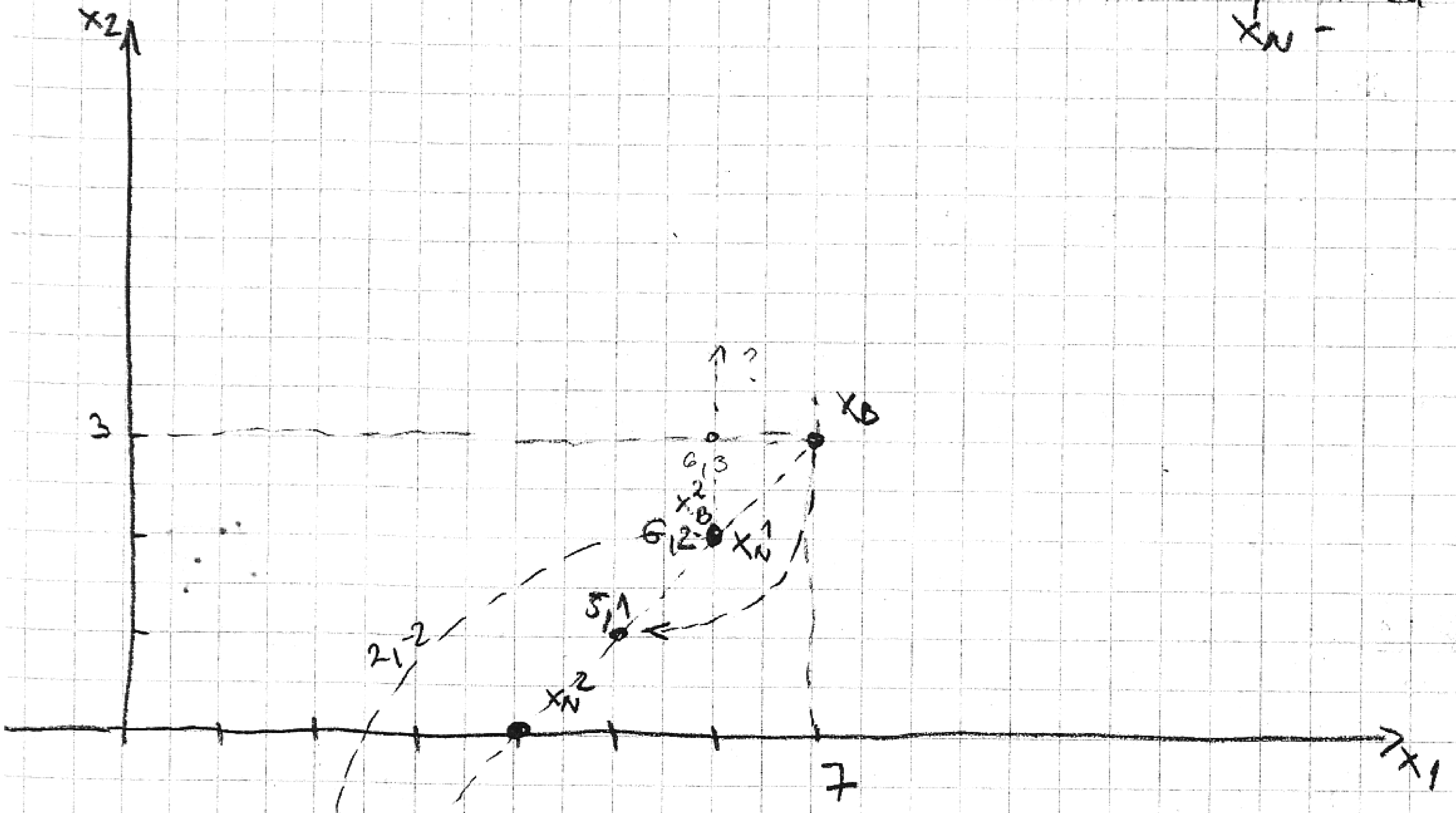


Zadatak:

$f(x) = x_1^2 + 4x_2^2$, $\Delta x = 1$; provesti H.i J. postupak do pomaka $\varepsilon \le 0.25$

$\underline{x}_0 = (7,3)'$

X_B – bazna točka
X_P – točka pretraživanja
X_N –



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X_B	X_P	X_N	$f(X_N) < f(X_B)$	
7,3	7,3	6,2	✓	
6,2	5,1	4,0	✓	
4,0	2,-2	1,-1	✓	(5<16)
1,-1	-2,-2	-1,-1	✗	$\Rightarrow \Delta x = \frac{\Delta x}{2} = 0.5$

zanemarimo ove dvije točke

samo zadržimo baznu točku i krenemo ispočetka

1,-1	1,-1	0.5,-0.5	✓	
0.5,-0.5	0,0	0,0	✓	
0,0	-0.5,-0.5	0,0	✗	$\Delta x = \frac{\Delta x}{2} = 0.25$!

Rješenje je zadnja bazna točka $\underline{x_B} = (0,0)$

zadatak:

$f(\underline{x}) = x_1^2 + x_2^2 + x_3^2$, $\underline{x_0} = (2,3,4)$, $\Delta x = 1$, $\varepsilon = 0.25$

- provesti Hooke-Jeeves postupak do $\varepsilon = 0.25$

$\underline{x_B}$	$\underline{x_P}$	$\underline{x_N}$	$f(\underline{x_N}) < f(\underline{x_B})$
2,3,4	2,3,4	1,2,3	✓
1,2,3	0,1,2	0,0,1	✓
0,0,1	-1,-2,-1	0,-1,0	NE $\Delta x = 0.5$ nismo napredovali u odnosu na x_B
0,0,1	0,0,1	0,0,0.5	✓
0,0,0.5	0,0,0	0,0,0	✓
0,0,0	0,0,-0.5	0,0,0	NE, $\Delta x = 0.25$

Rj: $(0,0,0) = \underline{x_B}$

pred. 4.5.3 Pon po Powellu (4-4² - dokaz ne treba ...

SMJEROVI (4-45)

$\underline{e_1}$	$\underline{e_2}$	$\underline{e_2}, \underline{e_1}, \underline{e_2}$
$\underline{e_2}$	$\underline{v_1}$	$\underline{v_1}, \underline{e_2}, \underline{v_1}$
$\underline{v_1}$	$\underline{v_2}$	$\underline{v_2}, \underline{v_1}, \underline{v_2}$

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zadatak:

$f(\underline{x}) = (x_1-2)^2 + (x_2+1)^2$, $\underline{v} = [0,1]^T$

- za zadanu kvadr. f-ju f provedite analitički smjer smjeru v. Postupak!

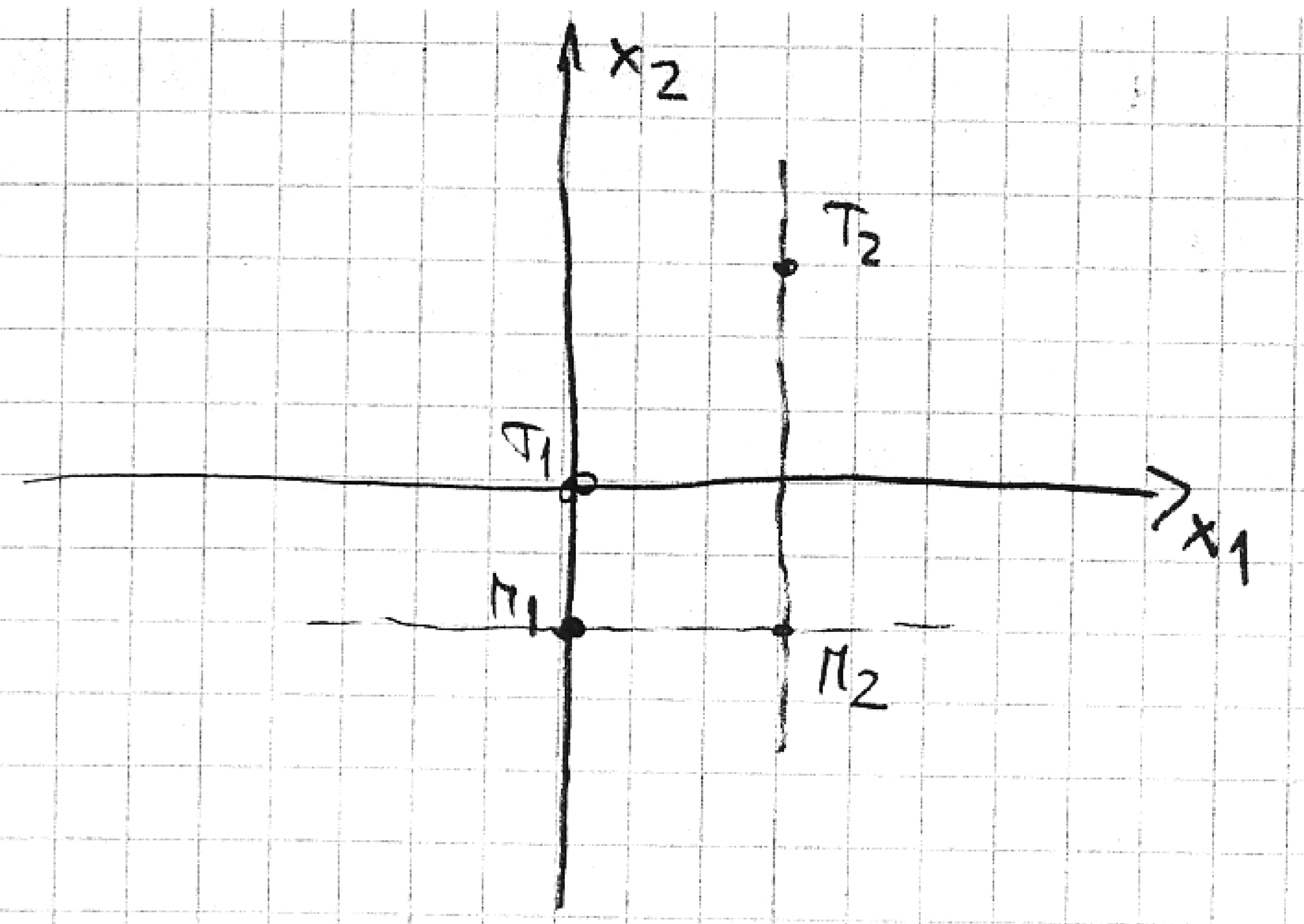
npr. $\underline{T_1} = [0,0]$, $\underline{T_2} = [1,1]$

za T_1: $f(\underline{T_1} + \lambda \underline{v}) = f(0 + \lambda \cdot 0, 0 + \lambda \cdot 1) = f(0, \lambda) = (0-2)^2 + (\lambda+1)^2 = \lambda^2 + 2\lambda + 5$

$\frac{\partial f}{\partial \lambda} = 2\lambda + 2 = 0 \Rightarrow \lambda_1 = -1$, $\underline{M_1} = \underline{T_1} + \lambda_1 \cdot \underline{v} = [0,-1]$

za T_2: $f(\underline{T_2} + \lambda_2 \cdot \underline{v}) = f(1, 1+\lambda) = (1-2)^2 + (1+\lambda+1)^2 = \lambda^2 + 4\lambda + 5$

$\frac{\partial f}{\partial \lambda} = 2\lambda + 4 = 0 \Rightarrow \lambda_2 = -2$ $M_2 = \underline{T_2} + \lambda_2 \cdot \underline{v} = [1,-1]$



konj. f. smjer $= \underline{m}_2 - \underline{m}_1$ (ili $\underline{m}_1 - \underline{m}_2$) $= (-1, 0)$

pred.: 4.5.4. simplex postupak po Nelderu i Meadu

Simplex postupak

podaci:

- $\underline{x}[\phi], \ldots, \underline{x}[N] \to$?
- $\underline{x}[l] \to$ točka s najboljom (najmanjom) vrijednošću f-je cilja
- $\underline{x}[h] \to$ točka s najlošijom (najvećom) –"– –"–
- $x_c \to$ aritmetička sredina svih točaka (osim najlošije)
  ↳ CENTROID

→ REFLEKSIJA:

$$\underline{x}_R = (1+\alpha)\,\underline{x}_c - \alpha\,\underline{x}[h]$$

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→ KONTRAKCIJA:

$$\underline{x}_K = (1-\beta)\,\underline{x}_c + \beta\,\underline{x}[h]$$

→ EKSPANZIJA:

$$\underline{x}_e = (1-\gamma)\,\underline{x}_c + \gamma\,\underline{x}_R$$

→ POMAK PREMA $\underline{x}[l]$

$$\underline{x}[i] = \frac{1}{2}\left(\underline{x}[i] + \underline{x}[l]\right), \quad i = 0, \ldots, n$$

Zad.

$f(\underline{x}) = x_1^2 + x_2^2 + x_3^2$ – naći centroid točke po simplex postupku

$\underline{x}_1 = (1, 2, 3)$

$\underline{x}_2 = (9, 2, 3)$

$\underline{x}_3 = (-2, 0, 3)$

$\underline{x}_4 = (-4, 0, 1)$

$f(\underline{x}_1) = 14$

$f(\underline{x}_2) = 20$ — BEZ — $\underline{x}_2 = \underline{x}[h]$

$f(\underline{x}_3) = 13$

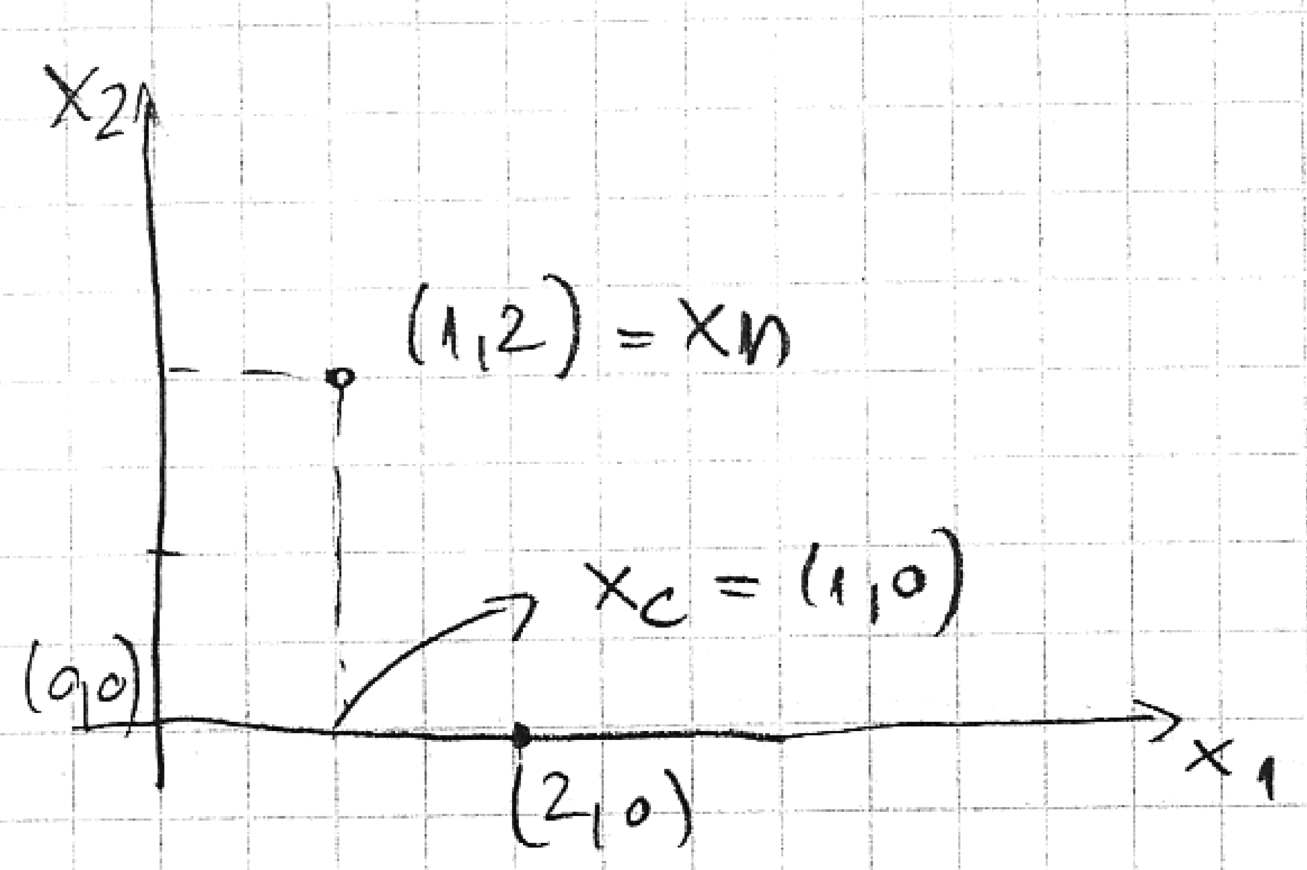
$f(\underline{x}_4) = 17$

$$\underline{x}_c = \frac{1}{3}\begin{bmatrix} 1 & -2 & -4 \\ 2 & +0 & +0 \\ 3 & +3 & +1 \end{bmatrix} = \begin{bmatrix} -5/3 \\ 2/3 \\ 7/3 \end{bmatrix}$$

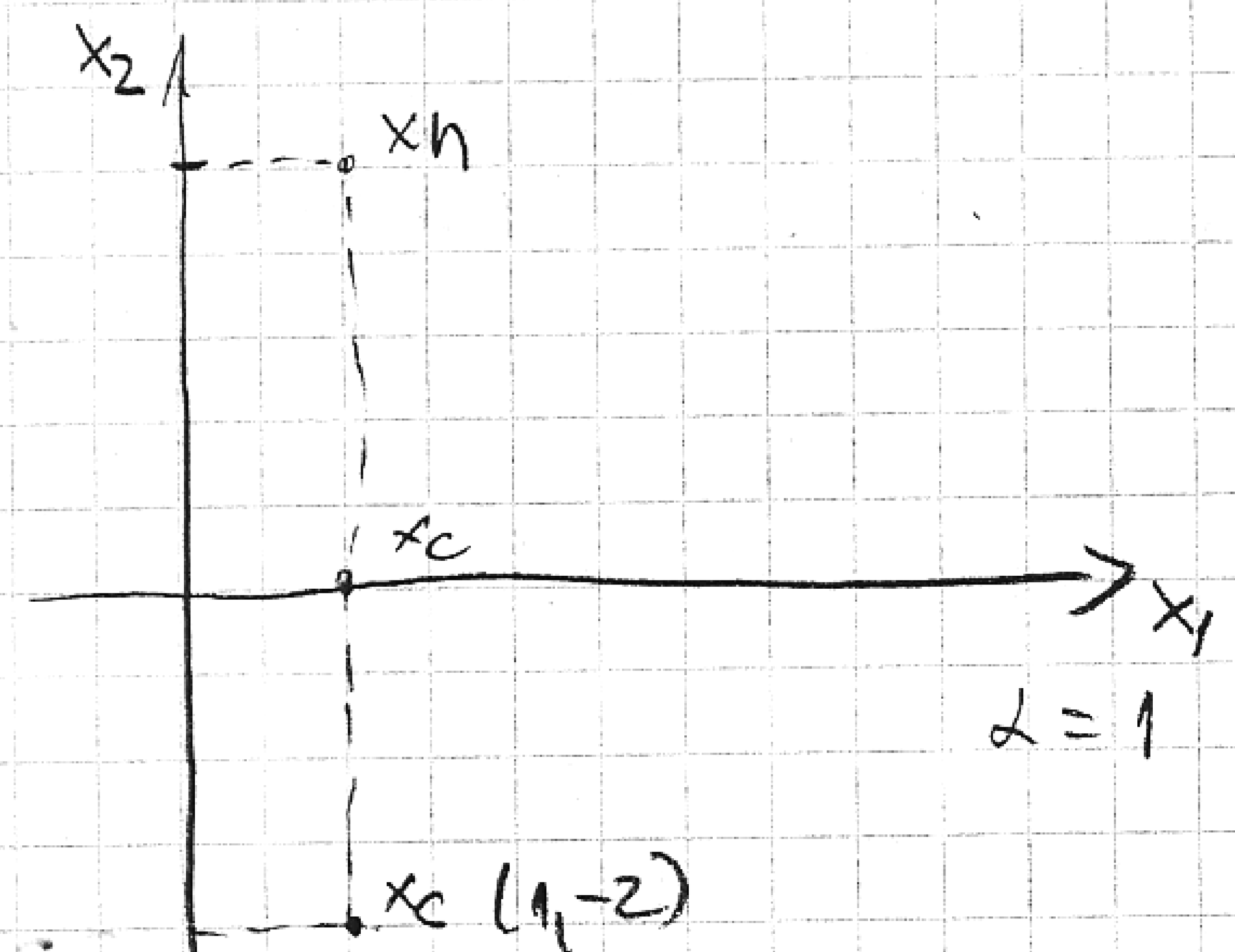
zadatak

$(0,0)$, $(1,2)$, $(2,0)$

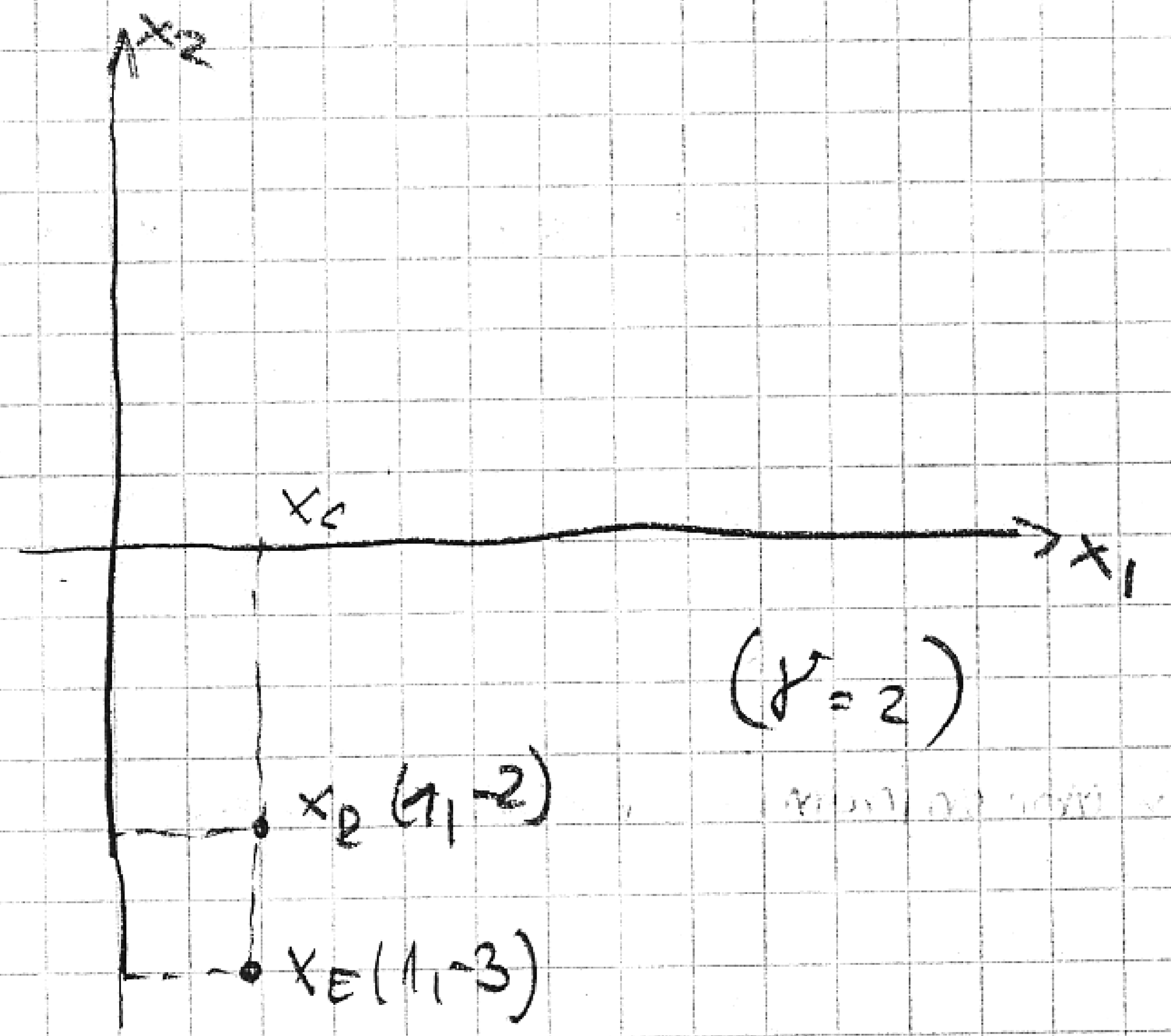
$f(\underline{x}) = x_1^2 + x_2^2$



REFLEKSIJA :



EKSPANZIJA :



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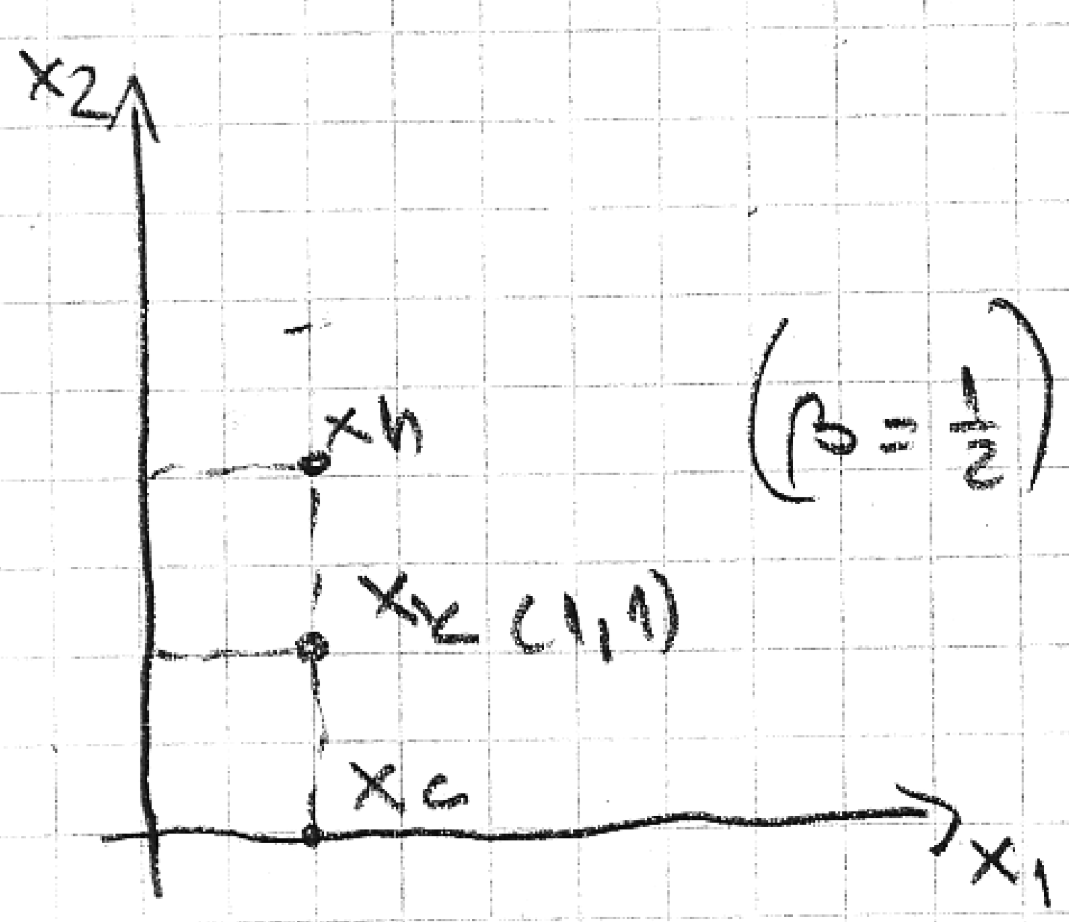
KONTRAKCIJA



Zadatak

$$F(\underline{x}) = (x_1-4)^2 + 4(x_2-2)^2$$

$$x_0 = (0,0)$$

Provesti jednu iteraciju metode najbržeg spusta i min fje na pravcu odrediti analitički

$$\frac{\partial F}{\partial x_1} = 2(x_1-4) = 2x_1 - 8$$

$$\frac{\partial F}{\partial x_2} = 8(x_2-2) = 8x_2 - 16$$

$$\nabla F = \begin{bmatrix} 2x_1 - 8 \\ 8x_2 - 16 \end{bmatrix}$$

$$\nabla \underline{F}(x_0) = \begin{bmatrix} -8 \\ -16 \end{bmatrix} \Rightarrow v_0 = -\frac{\nabla f(x_0)}{\|\nabla f(\underline{x_0})\|} \quad ; \quad \| \quad \| = \sqrt{320}$$

$$v_0 = \frac{1}{\sqrt{320}} \begin{bmatrix} 8 \\ 16 \end{bmatrix} \longrightarrow \text{jedinični vektor}$$

Nenormirani vektor smjera

$$\underline{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ (npr.)}$$

$$F(\underline{x_0} + \lambda v) = (\lambda-4)^2 + 4(2\lambda-2)^2 = 17\lambda^2 - 40\lambda + 32$$

$$\frac{\partial F}{\partial \lambda} = 34\lambda - 40 = 0 \implies \lambda = \frac{40}{34} = 1,176$$

$$\underline{x}_1 = x_0 + \lambda v = \begin{bmatrix} 1.175 \\ 2.353 \end{bmatrix} //$$

te je gotova 1 iteracija

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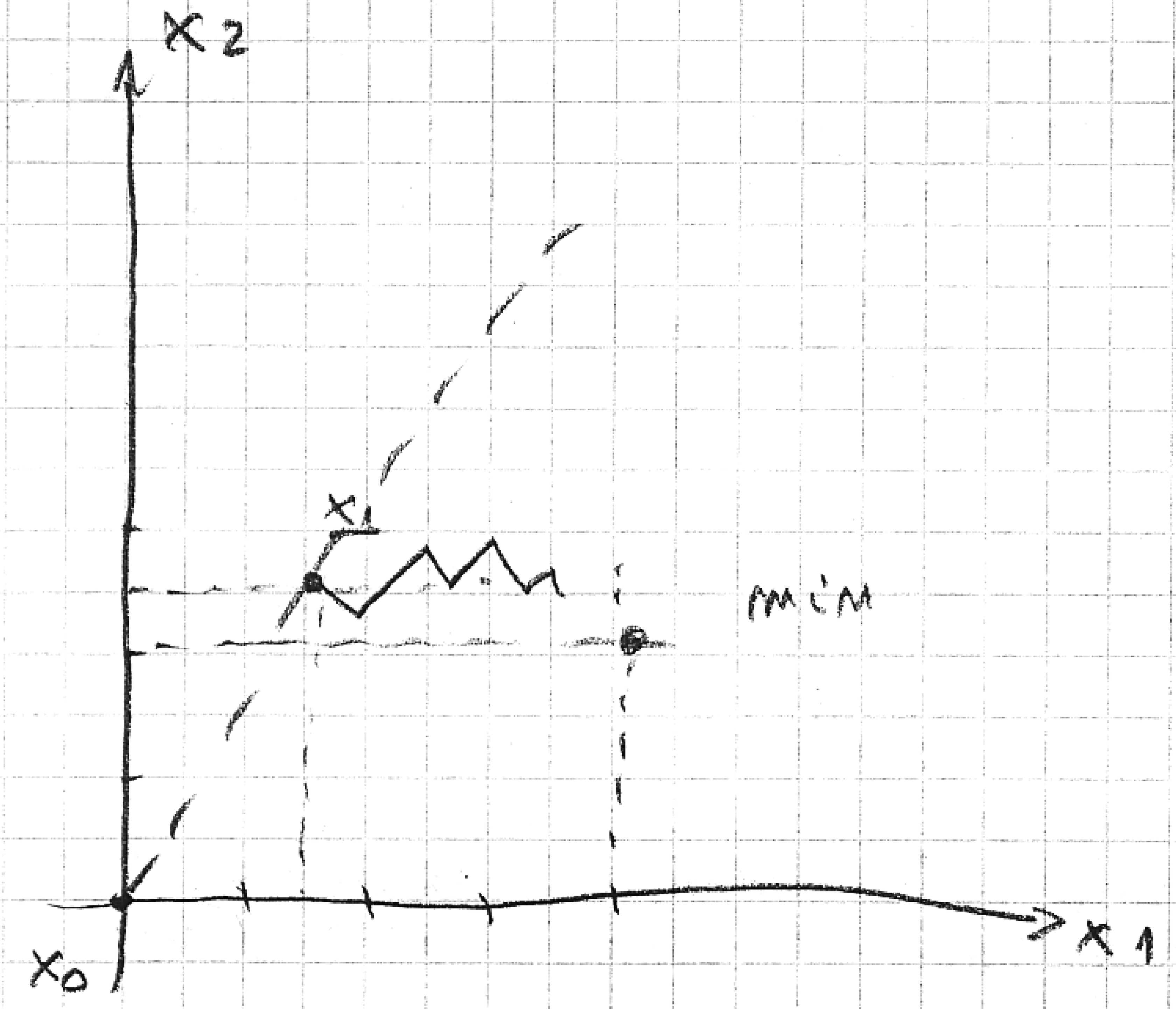

Pokazuje li smjer pretraživanje prema min. f-je ?

NE !

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4.6 2a   N-R postupak za sustav nelinearnih jedn.

Primjer)

$$\left.\begin{aligned} x_1^2+x_2^2-1&=0 \\ x_1-2x_2&=0 \end{aligned}\right\} \text{ Sustav nelinearan}$$

Sustav : $\underline{G}(\underline{x})=\underline{0}$

$$G=\begin{bmatrix} g_1(\underline{x}) \\ \vdots \\ g_n(\underline{x}) \end{bmatrix}$$

Dva pristupa

a) se prevlačimo u 1 f-ju cilja, tj. uzmemo pojedine f-je kvadriramo im i zbrojimo te onda od nove f-je tražimo minimum.

Definiramo pomoćnu f-ju $F(\underline{x})=\sum (g_i(\underline{x}))^2$ (NPR.)

b) koristiti neku od metoda za rješ. nelin. jedn.
npr.

N-R post. za sust. nelin. jdn.

$$\underline{x}^{k+1}=\underline{x}^k-\underbrace{\underline{J}^{-1}(\underline{x}^k)}_{\substack{\text{matrica} \\ \text{prvih der.}}}\cdot\underbrace{\underline{G}(\underline{x}^k)}_{\text{vektor}}$$

svih f-ja po svim varijablama

i

$$\underline{J}(\underline{x}^k)=\begin{bmatrix} \left.\frac{\partial g_1}{\partial x_1}\right|_{x_k} & \cdots & \left.\frac{\partial g_1}{\partial x_n}\right|_{x_k} \\ \vdots & & \vdots \\ \left.\frac{\partial g_n}{\partial x_1}\right|_{x_k} & \cdots & \left.\frac{\partial g_n}{\partial x_n}\right|_{x_k} \end{bmatrix}$$

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$$\underline{x}^{k+1}-\underline{x}^k=\Delta\underline{x}=-\underline{J}^{-1}\underline{G}$$ za pojedinu točku probl. možemo svesti na rješ. sustava linearnih jdn

Rješ kao

$$\underline{J}(\underline{x}^k)\cdot\underline{\Delta x}^k=-\underline{G}(\underline{x}^k)$$

$$\underline{x}^{k+1}=\underline{x}^k+\underline{\Delta x}^k$$

(N-R za sustav jednad.)

$$\underline{J}(\underline{X}_k) \cdot \underline{\Delta X}_k = -\underline{G}(\underline{X}_k)$$

Algoritam:

POČETAK $(\underline{X}_0, \underline{\varepsilon})$

$\underline{X} = \underline{X}_0$;

PONAVLJAJ

IZRAČUNAJ $\underline{G}(\underline{X})$;
IZRAČUNAJ $\underline{J}(\underline{X})$;

RIJEŠI $\underline{J}\,\underline{\Delta X} = -\underline{G}$; // nađi ΔX

$\underline{X} = \underline{X} + \underline{\Delta X}$;

DOK JE $\Delta \underline{X} > \underline{\varepsilon}$;

RJŠ. $= \underline{X}$;

Modifikacija algoritma – koristi se konst. Jakobijeva matrica izrač. u npr. početnoj točki

IZRAČUNAJ $\underline{J}(\underline{X})$ izračunamo samo na početku za točku X_0

POČETAK $(\underline{X}_0, \underline{\varepsilon})$
$\underline{X} = \underline{X}_0$;
IZRAČUNAJ $\underline{J}(\underline{X})$;
IZRAČUNAJ $\underline{J}^{-1}(X)$;

PONAVLJAJ

IZRAČUNAJ $\underline{G}(\underline{X})$;
$\Delta X = -\underline{J}^{-1} \cdot \underline{G}$
$\underline{X} = X + \Delta X$;

DOK JE $\Delta X > \varepsilon$

RJŠ. $= X$;

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Problem – moguća je lošija stabilnost postupka (divergencija)

Zadatak

$$x_1^2 + x_2^2 - 1 = 0$$
$$x_1 - 2x_2 = 0$$

$\underline{x_0} = (1,1)$

$$J = \begin{bmatrix} 2x_1 & 2x_2 \\ 1 & -2 \end{bmatrix}$$

$$J \cdot \Delta X = -G \Rightarrow \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\left.\begin{aligned} 2\Delta_1 + 2\Delta_2 &= 1 \\ \Delta_1 - 2\Delta_2 &= -1 \end{aligned}\right\} \Rightarrow \begin{aligned} \Delta x_1 &= 0 \\ \Delta x_2 &= -1/2 \end{aligned}$$

$\underline{x_1} = (1, \frac{1}{2})$

$$\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \Delta x = \begin{bmatrix} 1/4 \\ 0 \end{bmatrix} \Rightarrow \Delta x = \begin{bmatrix} -1/10 \\ -1/20 \end{bmatrix}$$

$x_2 = (0.9, 0.45)$

$x_3 = (0.894, 0.4472)$

$$G(x_3) = \begin{bmatrix} 3 \cdot 10^{-5} \\ 0 \end{bmatrix}$$

ITD.

Pronalaženje opt. više varijabli s ograničenjem

Rj. problema s ograničenjima imamo 2 pristupa

① Matematički transformirati probl. s ogr. u probl. bez ograničenja i primijeniti neki od postojećih postupaka optimiranja

② Upotrijebiti prilagođeni postupak optimiranja koji uzima ograničenja u obzir

Zadatak:

Zadana je f-ja cilja dvije varijable uz dva ograničenja u obliku jdn.

$F(\underline{x})$

$$a_1 x_1 + b_1 x_2 = c_1 \quad (h_1)$$
$$a_2 x_1 + b_2 x_2 = c_2 \quad (h_2)$$

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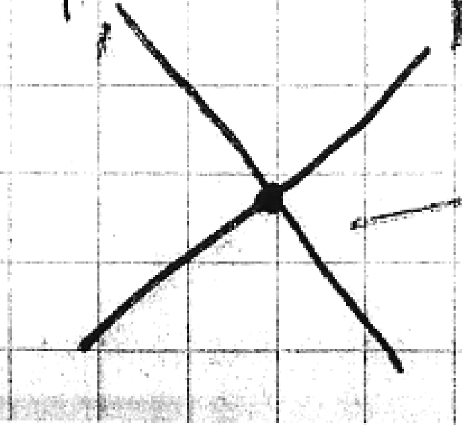
Navedite uvjet postojanja rj. koje zadovoljava sva ograničenja i f-ju cilja koja uzima ograničenja u obzir



→ uvjet postojanja rj. je da se pravci sijeku

$$h_1: \quad x_2 = \frac{-a_1}{b_1} x_1 + \frac{c_1}{b_1}$$

$$h_2: \quad x_2 = \frac{-a_2}{b_2} x_1 + \frac{c_2}{b_2}$$

UVJET:

$$\frac{a_1}{b_1} \neq \frac{a_2}{b_2}$$ (ne smiju biti paralelni)

$$U(\underline{x}, t) = F(\underline{x}) + t\left(h_1^2(\underline{x}) + h_2^2(\underline{x})\right) = F(\underline{x}) + t\left[(a_1x_1 + b_1x_2 - c_1)^2 + (a_2x_1 + b_2x_2 - c$$

zadatak

Transformacijom problema s ograničenjem na mješoviti način dobiva se pomoćna f-ja

$$U(\underline{x}, t) = F(\underline{x}) - t\left[\ln(x_1 - x_2) + \ln(2 + x_2)\right] + \frac{1}{t}(x_1 + 4)^2$$

Nađite ograničenja ovog problema

$$x_1 - x_2 \geq 0$$

$$2 + x_2 \geq 0$$

$$x_1 + 4 = 0$$

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$F(\underline{x})$ - f-ja

Postupak po Box-u

$F(\underline{x})$ - f-ja cilja (n-dim)

$\underline{X}_d \leq \underline{X} \leq \underline{X}_g$

$g(\underline{x}) \geq \underline{0}$

Podaci:
- skup od $2n$ točaka $x[1] \dots x[2n]$
- centroid $\underline{X}_c$, T. D. I. $\underline{X}_R$, F. R. $\alpha = 1{,}3$
- x_i - i-ta komponenta od $\underline{X}$
- Rješenje: $\underline{X}_c$
- Uvjet zaustavljanja: $|x_i[h] - x_{ci}| < \varepsilon \;, \forall i$

Zadatak:

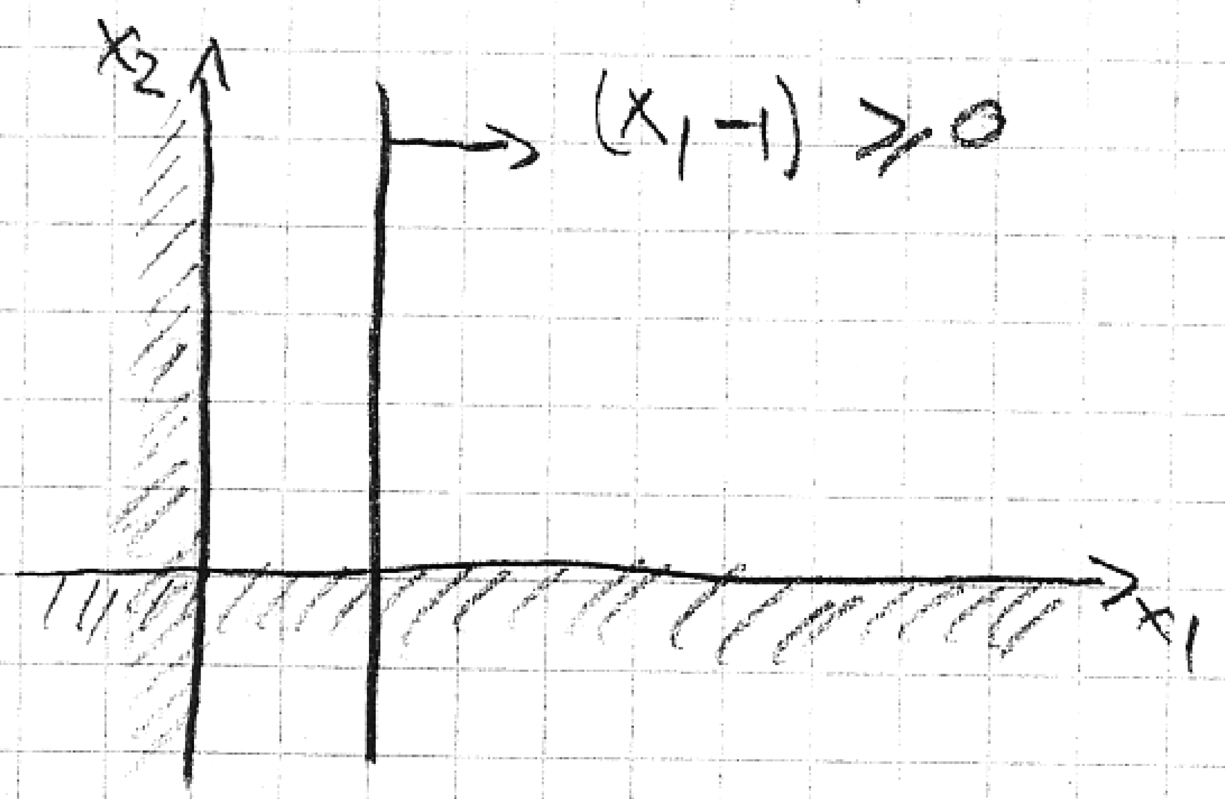
$F(\underline{x}) = x_1^2 + (x_2 - 1)^2$

$x_1, x_2 \in [0, \infty\rangle$ eksplicitno ograničenje

$x_1 - 1 \geq 0$ implicitno ograničenje

Trenutni skup točaka: $(1,0), (2,1), (2,3), (1,3)$

$\alpha = 2$, provesti 2 iteracije postupka po Box-u



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1)

$\underline{x}$	1,0	2,1	2,3	1,3
$f(\underline{x})$	2	4	8 x_h	5

$x_c = \left(\frac{4}{3}, \frac{4}{3}\right) = \left(\frac{1+2+1}{3}, \frac{0+1+3}{3}\right)$

$\underline{X}_R = (1+\alpha)\underline{X}_c - \alpha \underline{X}_R = (4,4) - (4,6) = (0,-2)$

EXPL.: $\Rightarrow$ pomaknemo se na granicu ograničenja $\underline{X}_R^{I} = (0,0)$

IMPL.: $\Rightarrow x_1 - 1 = -1 \ngeq 0 \Rightarrow x_R^{II} = \frac{1}{2}(\underline{X}_R^{I} + \underline{X}_c) = \left(\frac{2}{3}, \frac{2}{3}\right)$ NE

$\underline{X}_R^{III} = \frac{1}{2}(2,2) = (1,1)$ ✓

$f(\underline{x}_e''') = 1$ ✓ (nije najlošija)

2)

$\underline{x}$	1,0	2,1	1,1	1,3
$f(\underline{x})$	2	4	1	5

najlošija x_h

centroid bez najlošije točke $\underline{x}_c = \left(\frac{4}{3}, \frac{2}{3}\right)$

$\underline{x}_e = (4,2) - (2,6) = (2,-4)$

EXPL.: $\underline{x}_e' = (2,0)$

IMPL.: ✓

$f(\underline{x}_e') = 5$ (i dalje je to najlošija točka)

$\Rightarrow \underline{x}_e'' = \frac{1}{2}(\underline{x}_e' + \underline{x}_c) = \left(\frac{5}{3}, \frac{1}{3}\right)$

3)

x	1,0	2,1	1,1	5/3, 1/3
$f(x)$	2	4	1	29/9

zadatak: (BOX)

$f(\underline{x}) = -x_1 \cdot x_2 \cdot |x_1 - x_2|$

$x_1 + x_2 - 8 \leq 0$ implicit. ogr. $\alpha = 2$

trenutni skup točaka: (0,0), (1,3), (2,1), (3,2)

Provesti 1 iteraciju po BOX-u

$\underline{x}$	0,0	1,3	2,1	3,2
$f(\underline{x})$	0	-6	-2	-6

$\underline{x}_h$ najlošija točka

$\underline{x}_c = \left(\frac{1+2+3}{3}, \frac{3+1+2}{3}\right) = (2,2)$

$\underline{x}_e = (1+\alpha)\underline{x}_c - \alpha \underline{x}_h = (6,6) - (0,0) = (6,6)$

IMPLICITNA: $6+6-8 = 4 > 0$ NE

$\underline{x}_e' = \frac{1}{2}(8,8) = (4,4)$ ✓

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$f(x_2'') = 0$ (i dalje najbolja)

$x_2'' = \frac{1}{2}(6,6) = (3,3)$

2)

x	3,3	1,3	2,1	3,2
$f(x)$	0	-6	-2	-6

opet najbolja točka

$x_c = (2,2)$

Pogreške u postupcima analize

Prikaz brojeva po IEEE 754 standardu

z	e	f

e → eksponent

f → frakcija

k - broj bitova za eksponent

p - broj bitova za frakciju

Vrijednost exp. se def. do predstavljeno e ali sa posmakom za a

$e - \underbrace{(2^{k-1}-1)}_{a \text{ posmak}}$

Signifikand: 1.f ili 0.f

Baza $B=2$

e	f	vrijednost
$\emptyset$	$\emptyset$	$(-1)^z \cdot 0 = \pm 0$
$\emptyset$	$1 \le f \le 2^p-1$	$(-1)^z \cdot 2^{a+1}(0.f)$ → denormirani signifikand
$1 \le e \le 2^k-2$	$0 \le f \le 2^p-1$	$(-1)^z \cdot 2^{e-a}(1.f)$ → normalno radno područje
2^{k-1}	$\emptyset$	$\pm\infty$
2^{k-1}	$f \neq 0$	NaN

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Apsolutna pogreška $|rd(x) - x|$

x → stvarna veličina koju želimo prik.

$rd(x)$ → vrijednost prikazana kodom

Max. aps. pogr. - na najvećem području se dobiva $= 2^{a-p-1}$

zadatak:

IEEE 754 : $k=3$; $p=5$; $(B=2)$

a) Najmanja i max vrijednost koja se ovdje može zapisati

MIN = 000 | 00001 ; $a = 2^{k-1} - 1 = 3$

$2^{1-3} \cdot 0.00001$

$= 2^{-2} \cdot 0.00001$

$= 0.0078125_{(10)}$

MAX = $\overset{\text{max exp. koji može ići}}{110}$ | 11111 $\Rightarrow 2^{6-3} \cdot 1.11111 = 2^3 \cdot 1.96875_{(10)}$

$= 15.75_{(10)}$

b)

"0" ≡ 000 : 00000

"∞" ≡ 111 : 00000

c) Max aps. pog. provjera:

$MAP = 2^{a-p-1} = 2^{3-5-1} = 2^{-3} = 0.125$

prva sljedeća manja vrijed koju možemo zapisati

$= 2^3 \cdot 1.11110 = 15.5$

$15.75 - 15.5 = 0.25 / 2 = 0.125$

3 1

Zadatak 9, -0.1875

$9 = 2^3 \cdot 1.001 \Rightarrow$ MIN 3 FR
MIN 3 EXP
(posmak=3)

$-0.1875 = 0.0011$ MIN exp (3 bita)
$= 2^{-3} \cdot 1.1$ "001" $\Rightarrow 2^{-2} \cdot 1. \ldots$
$= 2^{-2} \cdot 0.11$

denormirani signifikant = "000" $\Rightarrow 2^{-2} \cdot 0. \ldots$

$9 \equiv$ "0 | 110 | 001"

$-0.1875 \equiv$ "1 | 000 | 110"

zbrajanje

(9) $2^3 \cdot 1.001$
$(-0.1875) - 2^{3}\ 0.000\boxed{0011} \Rightarrow$ FR NA 7 bita

Zadatak

Za prikaz po uzoru na
1 bit za predznak, neodređeni exp i FR
koliko je min. bitova potrebno za exp i frakciju kako bi vrijednosti
24.7 i 22.25 mogli predstaviti uz grešku ne veću od 0.25
Predstavite brojeve -15.5 i 8.75 u tom zapisu i provedite operaciju
zbrajanja

EXP: 3 BITA $\Rightarrow$ 16
4 BITA $\Rightarrow$ 256, posmak $= 7 = 2^{k-1} - 1$

FRAKCIJA:

$22.25 = 10110.01$
$24.7 = 11000.1010\ldots\ldots$

jer greška 0.25 $22 = 2^4\ 1.0110$

$24.5 = 2^4 \cdot \underbrace{1.1000 1} \Rightarrow 5$ bitova za frakciju (binarno najteže)

X $25 = 2^4 \cdot 1.1001$

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$-15.5 = -2^3 \cdot 1.11110 \equiv$ "1 | 1010 | 11110"

$8.75 = 2^3 \cdot 1.00011 \equiv$ "0 | 1010 | 00011"

$15.5 = 2^3\ 1.11110$
$-8.75 = -2^3\ 1.00011$

$= 2^3 \cdot 0.11011$
$= -2^2\ 1.10110 = -6.75$

zadatak
Koliko je
bitova

min. bitova potrebno ako bez pogreške želimo predstaviti 31.25 i 504 ; 9.5, -25.25 u tom zapisu i provesti operaciju zbrajanja

$31{,}25 = 2^4 \cdot 1{,}111101 \Rightarrow$ FR : 6 BITA
EXP : 4 BITA

uzimamo teže zahtjeve

$504 = 2^8 \cdot 1.11111 \Rightarrow$ FR : 5 BITA
EXP : 5 BITA
(posmak $= 2^{5-1} - 1 = 15$)

$9.5 = 2^3 \cdot 1.0011 \quad \equiv$ "0 | 10010 | 001100"
$-25.25 = 2^4 \cdot 1.100101 \quad \equiv$ "1 | 10011 | 100101"

33

a

e

27.4.2008

1 ZZ - 28.4./19:00 h

(-matrice)

- LU/LUP

- PNJ

- PPA

sve zaključno s današnjim predavanjem

→ PPA zadaci ?

zad

- 1 bit predznak, 4 bita eksponent, 4 bita frakcija (po uzoru na IEEE 4 standard)

Ako je br. u tom prikazu napisan kao "100001100" odredite njegovu vrijednost u domeni realnih br.

1|0000|1100

z=1 -negativan broj

EXP=Ø - područje denormiranog signifikanda 0....

POMAK $a = 2^{k-1} - 1 = 7$

vrijednost $EXP = 2^{-6}$ $(= 2^{-a+1})$

vrijednost

$VR. = -2^{-6} \cdot 0.1100 = -2^{-6} \cdot 0.75 = 0.01171875$

1|0001|1100 → npr. da je ovo što onda ?

$VR.EXP = 2^{1-7} = 2^{-6}$

$VR. = -2^{-6} \cdot 1.1110$

Zad

- 1 b predznak, nepoznat br. bitova za EXP i FRAKCIJU
- Odredite nepoznate veličine ako je broj 5.625 u toj pokretnoj predstavljen kao: "0 1001 01101"

$5.625 = 101.101 = 1.\underbrace{01101}_{\text{FRAKCIJA}} \cdot 2^2$

VR. EXP $= 2^2$

EXP $= 1001 = 9$

POSMAK $= a = 7$ ✓ $9 - 7 = 2$

→ 4 bita za EXP, 5 bita za FR.

Zad

- 1 b predz, 3 b EXP, 4 b FR.

a) $(4.75 + 0.25) + 10$

b) $4.75 + (0.25 + 10)$

A: $4.75 = 2^2 \cdot 1.\overbrace{0011}^{4\ FR.} = $ "0 101 0011"

B: $0.25 = 2^{-2} \cdot 1.0000$

C: $10 = 2^3 \cdot 1.0100$

$a = 2^{3-1} - 1 = 3$

a)

$$
\begin{array}{lrl}
A & & 2^2 \cdot 1.0011 \\
B & + & 2^2 \cdot 0.0001 \\
\hline
& & 2^2 \cdot 1.0100 \\
& & 2^3 \cdot 0.1010 \\
C & + & 2^3 \cdot 1.0100 \\
\hline
& & 2^3 \cdot 1.1110 \quad = 15 ✓
\end{array}
$$

kad se dekodira

b)

$$
\begin{array}{lrl}
B & & 2^3 \cdot 0.0000|1 \quad \text{ovo se gubi} \\
C & + & 2^3 \cdot 1.0100 \\
\hline
& & 2^3 \cdot 1.0100 \\
A & + & 2^3 \cdot 0.1001|1 \\
\hline
& & 2^3 \cdot 1.1101 \quad = 14 ✓
\end{array}
$$

⇒ NE VRIJEDI ASOCIJATIVNOST !

zad

- 1 b PR, 4 EXP, 5 FR.
- Predstaviti broj -249.5 i 25.25 u tom zapisu, provesti operac. zbrajanja, dekodirati rezultat i utvrditi kolika je greška nastala.

zaokružiti na 0 ili 1

$249.5 = 11111001.1 = 2^7 \cdot 1.\underbrace{11110}011$

imamo samo 5 bitova za signifikand pa radimo zaokruživanje

bliži pa uzimamo

$248 = 2^7 \cdot 1.11110$

$252 = 2^7 \cdot 1.11111$

$25.25 = 2^4 \cdot 1.\underbrace{10010}1$

$25.0 = 2^4 \cdot 1.0010$ } ~~uzimamo bilo što, isto je greška~~

$25.5 = 2^4 \cdot 1.0011$ } uzmemo npr. 25.0

- zbrajanje :

(25 - 248)

$2^7 \cdot 0.00110|0101$

$- 2^7 \cdot 1.11110$

10.01000 → mijenja se predznak (negativne br prikazujemo kao dvojni komplement sa vodećom 1)

(2k) 1.10111 } dvojni komplement

$+ \quad 1$

$- 2^7 \cdot 1.11000 \Rightarrow -224$

(248 - 25) - pa onda staviti - → možemo i tako!

$25.25 - 249.5 = -224.25$

→ pogreška je 0.25

Zad

- 1 b PR, 4 EXP, 5 FR.
- Predstaviti broj −249.5 i 25.25 u tom zapisu, provesti operac. zbrajanja, dekodirati rezultat i utvrditi kolika je greška nastala.

zaokružiti na 0 ili 1

$249.5 = 11111001.1 = 2^7 \cdot 1.\underbrace{11110}011$

imamo samo 5 bitova za signifikand pa radimo zaokruživanje

bliži pa uzimamo

$248 = 2^7 \cdot 1.11110$

$252 = 2^7 \cdot 1.11111$

$25.25 = 2^4 \cdot 1.\underbrace{10010}1$

$25.0 = 2^4 \cdot 1.0010$

$25.5 = 2^4 \cdot 1.0011$

~~uzimamo bilo što, isto je greška~~ uzmemo npr. 25.0

- zbrajanje :

$(25 - 248)$

$2^7 \cdot 0.00110\,|\,0101$

$- 2^7 \cdot 1.11110$

$\boxed{1}0.01000$

→ mijenja se predznak (negativne br prikazujemo kao dvojni komplement sa vodećom 1)

(2k) 1.10111

$+ \quad 1$

dvojni komplement

$-2^7 \cdot 1.11000 \Rightarrow -224$

$(248 - 25)$ - pa onda staviti − → možemo i tako!

$25.25 - 249.5 = -224.25$

→ pogreška je 0.25