ZADACI 11

1. $z = -50000$, hedging portfelj $= (x, y, z)$

$T = \frac{90}{365}$, $X = 1.8$, $S(0) = 1.82$, $\sigma = 0.14$, $r = 5\%$

$\Rightarrow p^E = 0.031648$ (iz B-S formule)

$$\frac{\partial p^E}{\partial S} = -N(-d_1) = -0.355300$$

$$\Rightarrow x = 50000 \cdot -0.355300 = -17765$$

$$\Rightarrow V(S) = x \cdot S + y + z \cdot p^E = 0$$

$$-17765 \cdot 1.82 + y - 50000 \cdot 0.031648 = 0$$

$$\Rightarrow y = 33914.69$$

$$\Rightarrow (x, y, z) = (-17765, 33914.69, -50000)$$

<u>Za 1 dan</u> tj. u $t = \frac{1}{365}$ put opcija će vrijediti (BS formula):

$$p^E = 0.038885$$

$\Rightarrow$ vrijednost portfelja će biti:

$$V = -17765 \cdot 1.82 + 33914.69 \cdot e^{0.05 \cdot \frac{1}{365}} - 50000 \cdot 0.038885 =$$

$$= -357.2138338$$

(2.) Iz put - call pariteta se dobije:

$$\text{delta}_{pE} = N(d_1) - 1 = \text{delta}_{c^E} - 1 = -N(-d_1)$$

$$\text{gamma}_{pE} = \frac{\partial(\text{delta}_{pE})}{\partial S} = \frac{\partial(\text{delta}_{c^E} - 1)}{\partial S} = \text{gamma}_{c^E}$$

$$\text{theta}_{pE} = -\frac{S\sigma}{2\sqrt{2\pi T}} e^{-\frac{d_1^2}{2}} + rX \cdot e^{-rT} \cdot N(-d_2)$$

$$\text{vega}_{pE} = \text{vega}_{c^E}$$

$$\text{rho}_{pE} = -TX e^{-rT} \cdot N(-d_2)$$

(3.) $T = \frac{30}{365}$, $X = 60$, $z = -1000$

$S(0) = 60$, $\sigma = 0.3$, $r = 0.08$

$\hat{T} = \frac{60}{365}$, $\hat{X} = 65$ iz

portfelj: (x, y, z, d)

- x – broj dionica
- y – kol. nerizične imovine
- z – broj opcija, $T = \frac{30}{365}$
- d – broj opcija, $\hat{T} = \frac{60}{365}$

<u>U $t=0$</u> vrijednost portfelja u ovisnosti o cijeni dionice je

$$V(S) = x \cdot S + y + z \cdot C^E + d \cdot \hat{C}^E = 0$$

$$\Rightarrow \text{delta portfelja} = \frac{\partial V(S)}{\partial S} = x + z \cdot \frac{\partial C^E}{\partial S} + d \cdot \frac{\partial \hat{C}^E}{\partial S} = 0$$

$$\text{gamma portfelja} = \frac{\partial^2 V(S)}{\partial S^2} = z \cdot \frac{\partial^2 C^E}{\partial S^2} + d \cdot \frac{\partial^2 \hat{C}^E}{\partial S^2} = 0 \quad (*)$$

Kako je $z = -1000$, imamo sustav tri jednadžbe s 3 nepoznanice (x, y i α)

Iz BS formule: $C^E = 4.14452$, $\frac{\partial C^E}{\partial S} = N(d_1) = 0.581957$

$\hat{C}^E = 1.37826$, $\frac{\partial \hat{C}^E}{\partial S} = N(\hat{d}_1) = 0.312373$

$$\Rightarrow \quad x - 1000 \cdot 0.581957 + \alpha \cdot 0.312373 \qquad (**)$$

Uvrštavanjem odgovarajućih vrijednosti u formulu za gammu call opcije dobije se:

$$\frac{\partial^2 C^E}{\partial S^2} = 0.043688, \quad \frac{\partial^2 \hat{C}^E}{\partial S^2} = 0.048502$$

iz $(*) \Rightarrow$ $-1000 \cdot 0.043688 + \alpha \cdot 0.048502 = 0$

$$\Rightarrow \boxed{\alpha = 300.76}$$

uvrštavanjem u $(**)$ dobivamo

$$\boxed{x = 300.58}$$

Da bi u $t=0$ $V(S)$ bilo jednako 0, (iz prve jednadžbe sustava) slijedi da mora biti $\boxed{y = -15131.77}$,

$$\Rightarrow \boxed{\text{portfelj } (x, y, z, \alpha) = (300.58, -15131.77, -1000, 300.76)}$$

Rješenje:

u $t=0$ $\quad V(S) = x \cdot S + y + z \cdot C^E + \alpha \cdot \hat{C}^E = 0$

$$\left.\begin{array}{l} \text{delta } V = x + z \, \text{delta}_{C^E} + \alpha \cdot \text{delta}_{\hat{C}^E} = 0 \\ \text{vega } V = \dfrac{\partial V}{\partial \sigma} = z \cdot \underbrace{\dfrac{\partial C^E}{\partial \sigma}}_{\text{vega}_{C^E}} + \alpha \cdot \underbrace{\dfrac{\partial \hat{C}^E}{\partial \sigma}}_{\text{vega}_{\hat{C}^E}} = 0 \end{array}\right\} (*)$$

Kako je $\text{vega}_{C^E} = \dfrac{S\sqrt{T}}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}}$, dobivamo

$$\text{vega}_{C^E} = 11.634305$$
$$\text{vega}_{\hat{C}^E} = 8.610687$$

Rješavanjem sustava $(*)$ uz $z = -1000$, dobivamo

$\alpha = 1351.15$, $\quad x = 153.83$, $\quad y = -7311.12$

$$\Rightarrow \boxed{(x, y, z, \alpha) = (153.83, -7311.12, -1000, 1351.15)}$$

(5.) $S(0) = K$, $\quad K$ izvršna cijena europske put opcije

$\dfrac{P^E}{S(0)} < 0.05$, $\quad \text{delta}_{P^E} = -0.364$, $\quad r = 0.012$, $\quad T = 1$

$\sigma = ?$

$$\text{delta}_{P^E} = \frac{\partial P^E}{\partial S} = -N(-d_1) = -0.364$$

$$\frac{P^E}{S(0)} = \frac{\cancel{K} \cdot e^{-rT} \cdot N(-d_2) - \cancel{S(0)} \cdot N(-d_1)}{\cancel{S(0)}} = \underbrace{e^{-rT} \cdot N(-d_2) - N(-d_1)}_{< 0.05}$$

$$-N(-d_1) = -0.364$$

$$\Rightarrow N\left(-\frac{\overbrace{\ln\frac{S(0)}{K}}^{=0} + (r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) = 0.364$$

$$\Rightarrow N\left(-\frac{0.012+\frac{1}{2}\sigma^2}{\sigma}\right) = 0.364$$

$$+\frac{0.012+\frac{1}{2}\sigma^2}{\sigma} = N^{-1}(0.364) = +0.347787204$$

$$\frac{1}{2}\sigma^2 - 0.347787204\,\sigma + 0.012 = 0 \quad / \cdot 2$$

$$\sigma^2 - 0.695574409\,\sigma + 0.024 = 0$$

$$\sigma_{1,2} = \frac{0.695574409 \pm \sqrt{0.387823758}}{2}$$

$$\sigma_1 = 0.659164691 \qquad \sigma_2 = 0.036409717$$

uvjet: $$e^{-rT}\cdot N(-d_2) - N(-d_1) < 0.05$$

$$e^{-0.012}\cdot N\left(-\frac{0.012-\frac{1}{2}\sigma^2}{\sigma}\right) < 0.05 + 0.364$$

$$N\left(-\frac{0.012-\frac{1}{2}\sigma^2}{\sigma}\right) < 0.418997927$$

za $\sigma = \sigma_1$ imamo

$$L.S. = N(+0.640959832) = 0.739226 > 0.418997927 \quad ↯$$

za $\sigma = \sigma_2$ imamo

$$L.S. = N(-0.311377489) = 0.377757 < 0.418997927 \quad ✓$$

$$\Rightarrow \boxed{\sigma = 0.036409717 = 3.64\%}$$

6. $S(0) > 80, \quad C = 2.34, \ delta_C = -0.81, \ gamma_C = 0.035$

$S(T) = 86, \quad S(0) = ?$

$$\varepsilon = 86 - S(0)$$

$$C(S+\varepsilon) \approx C(S) + delta_C \cdot \varepsilon + \frac{1}{2} gamma_C \cdot \varepsilon^2$$

$$2.21 = 2.34 - 0.81(86 - S(0)) + \frac{1}{2} \cdot 0.035 \cdot (86 - S(0))^2$$

supstitucije $86 - S(0) = x$

$$\Rightarrow \quad 0.0175 x^2 - 0.81x + 0.13 = 0$$

$$x_{1,2} = \frac{0.81 \pm \sqrt{0.6561 - 0.0091}}{0.035}$$

$x_1 = 0.16105 \qquad x_2 = 46.12$

$\Rightarrow S_1(0) = 85.83895 > 80 \checkmark \qquad S_2(0) = \cancel{3}9.875 < 80$

$$\Rightarrow \boxed{S(0) = 85.83895}$$