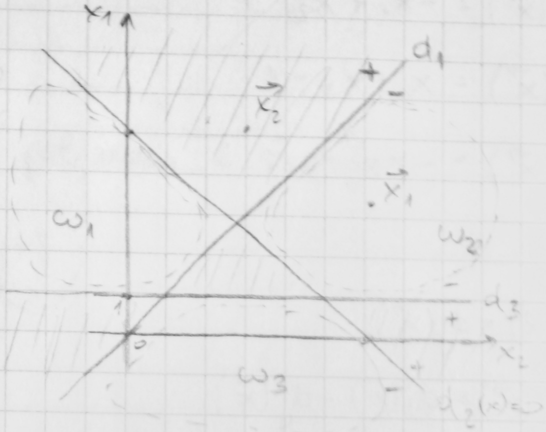
UEU /

PROBLEM VISE OD 2 RAZREDA

(1) $d_i(x) > 0 \Rightarrow \vec{x} \in \omega_i$



$d_1(\vec{x}) = -x_1 + x_2$

$d_2(\vec{x}) = x_1 + x_2 - 5$

$d_3(\vec{x}) = -x_2 + 1$

$d_1(\vec{x}_1) < 0$

$d_2(\vec{x}_2) > 0 \quad \Rightarrow \quad \vec{x}_1 \in \omega_2$

$d_3(\vec{x}_3) < 0$

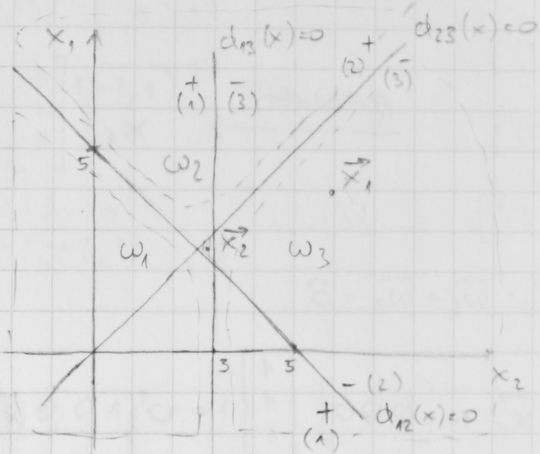
$d_1(\vec{x}_2) > 0$

$d_2(\vec{x}_2) > 0 \quad \Rightarrow \quad ?$

$d_3(\vec{x}_2) < 0$

(2) $\vec{x} \in \omega_i$ ako $d_{ij} > 0 \quad \forall j \neq i$

$d_{ij} = -d_{ji}$



$d_{12}(x) = -x_1 - x_2 + 5$

$d_{13}(x) = -x_1 + 3$

$d_{23}(x) = -x_1 + x_2$

$$\left.\begin{array}{l} d_{12}(\vec{x}_1) < 0 \Rightarrow d_{21}(\vec{x}_1) > 0 \\ d_{13}(\vec{x}_1) < 0 \Rightarrow d_{31}(\vec{x}_1) > 0 \\ d_{23}(\vec{x}_1) < 0 \Rightarrow d_{32}(\vec{x}_1) > 0 \end{array}\right\} \vec{x}_1 \in \omega_3$$

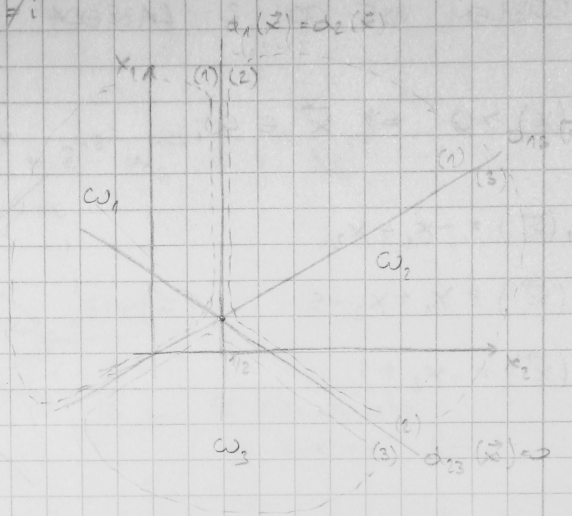
$$\left.\begin{array}{l} d_{12}(\vec{x}_2) < 0 \Rightarrow d_{21}(\vec{x}_2) > 0 \\ d_{13}(\vec{x}_2) > 0 \Rightarrow d_{31}(\vec{x}_2) < 0 \\ d_{23}(\vec{x}_2) < 0 \Rightarrow d_{32}(\vec{x}_2) > 0 \end{array}\right\} \vec{x}_2 \in ?$$

③ $\vec{x} \in \omega_i$ ako $d_i > d_j \quad \forall j \neq i$

$d_1(\vec{x}) = -x_1 + x_2$

$d_2(\vec{x}) = x_1 + x_2 - 1$

$d_3(\vec{x}) = -x_2$



$d_i(\vec{x}) = d_j(\vec{x})$

$d_{ij}(\vec{x}) = d_i(\vec{x}) - d_j(\vec{x})$

$\Rightarrow \quad d_{12}(\vec{x}) = d_1(\vec{x}) - d_2(\vec{x})$

$d_{13}(\vec{x}) = d_1(\vec{x}) - d_3(\vec{x}) = -x_1 + 2x_2$

$d_{23}(\vec{x}) = d_2(\vec{x}) - d_3(\vec{x}) = x_1 + 2x_2 - 1$

→ nema nedefiniranih područja (osim samog sjecišta)

Primjer SLUČAJ ③ - PERCEPTRON

$\omega_1 = \{(1,1)^T\}$

$\omega_2 = \{(0,0)^T\}$ $\xrightarrow{\text{proširiti}}$ $x_1 = \begin{bmatrix}1\\1\\1\end{bmatrix} \quad x_2 = \begin{bmatrix}0\\0\\1\end{bmatrix} \quad x_3 = \begin{bmatrix}-1\\-1\\1\end{bmatrix}$

$\omega_3 = \{(-1,-1)^T\}$

$c = 1 \qquad \vec{w}_1 = \vec{w}_2 = \vec{w}_3 = \vec{0}$

$\vec{w}_1^T \cdot \vec{x}_1 = [0\,0\,0]\begin{bmatrix}1\\1\\1\end{bmatrix} = 0 \qquad \vec{w}_1 = \vec{w}_1' + c\vec{x}_1 = \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$

$\vec{w}_2^T \cdot \vec{x}_2 = [0\,0\,0]\begin{bmatrix}0\\0\\1\end{bmatrix} = 0 \qquad \vec{w}_2 = \vec{w}_2' - c\vec{x}_1 = \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}-1\\-1\\-1\end{bmatrix} = \begin{bmatrix}-1\\-1\\-1\end{bmatrix}$

$\vec{w}_3^T \cdot \vec{x}_3 = [0\,0\,0]\begin{bmatrix}-1\\-1\\1\end{bmatrix} = 0 \qquad \vec{w}_3 = \vec{w}_3' - c\vec{x}_1 = \begin{bmatrix}0\\0\\0\end{bmatrix} - \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}-1\\-1\\-1\end{bmatrix}$

$\vec{w}_1^T \vec{x}_2 = [1\,1\,1]\begin{bmatrix}0\\0\\1\end{bmatrix} = 1 \qquad \vec{w}_1 = \vec{w}_1' - c\vec{x}_2 = \begin{bmatrix}1\\1\\1\end{bmatrix} - \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}1\\1\\0\end{bmatrix}$

$\vec{w}_2^T \vec{x}_2 = [-1\,-1\,-1]\begin{bmatrix}0\\0\\1\end{bmatrix} = -1 \qquad \vec{w}_2 = \vec{w}_2' + c\vec{x}_2 = \begin{bmatrix}-1\\-1\\-1\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-1\\-1\\0\end{bmatrix}$

$\vec{w}_3^T \vec{x}_2 = -1 \qquad \vec{w}_3 = \vec{w}_3' - c\vec{x}_3 = \begin{bmatrix}-1\\-1\\-1\end{bmatrix} - \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-1\\-1\\-2\end{bmatrix}$

$\vec{w}_1^T \vec{x}_3 = [1\,1\,0]\begin{bmatrix}-1\\-1\\1\end{bmatrix} = -2 \qquad \vec{w}_3 = \vec{w}_3' + c\vec{x}_3 = \begin{bmatrix}-1\\-1\\-2\end{bmatrix} + \begin{bmatrix}-1\\-1\\1\end{bmatrix} = \begin{bmatrix}-2\\-2\\-1\end{bmatrix}$

$\vec{w}_2^T \vec{x}_3 = [-1\,-1\,0]\begin{bmatrix}-1\\-1\\1\end{bmatrix} = 2 \qquad \vec{w}_2 = \vec{w}_2' - c\vec{x}_3 = \begin{bmatrix}-1\\-1\\0\end{bmatrix} - \begin{bmatrix}-1\\-1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$\vec{w}_3^T \vec{x}_3 = [-1\,-1\,-2]\begin{bmatrix}-1\\-1\\1\end{bmatrix} = 0 \qquad \vec{w}_1 = \begin{bmatrix}1\\1\\0\end{bmatrix}$ - ostaje isti jer $0 > -2$

POOPĆENE DECIZIJSKE FUNKCIJE

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} \Rightarrow \vec{x}^* = \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

polinom 2. stupnja

$\omega_1 = \{ (0,1)^T, (-1,-2)^T, (1,-2)^T \}$

$\omega_2 = \{ (0,0)^T \}$

$c = 1$, $\vec{w}_1 = 0$

$$\vec{x}^* = \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix} \Rightarrow \vec{x}_1^* = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad \vec{x}_2^* = \begin{bmatrix} 1 \\ 4 \\ 2 \\ -1 \\ -2 \\ 1 \end{bmatrix} \quad \vec{x}_3^* = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 1 \\ -2 \\ 1 \end{bmatrix} \quad \vec{x}_4^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ +1 \end{bmatrix}$$

$\vec{w}_1^T \vec{x}_1^* = 0$

$\vec{w}_2 = \vec{w}_1 + c\vec{x}_1^* = (0,4,0,0,2,1)^T$

$\vec{w}_2^T \cdot \vec{x}_2^* = (0,4,0,0,2,1)(1,4,2,-1,-2,1)^T = 13$

$\vec{w}_3 = \vec{w}_2$

$\vec{w}_3^T \vec{x}_3^* = (0,4,0,0,2,1)(1,4,-2,1,-2,1) = 13$

$\vec{w}_4 = \vec{w}_3$

$\vec{w}_4^T \vec{x}_4^* = (0,4,0,0,2,1)(0,0,0,0,0,1)^T = -1$

$\vec{w}_5 = \vec{w}_4 + 1 \cdot \vec{x}_1^* = (0,4,0,0,2,0)$

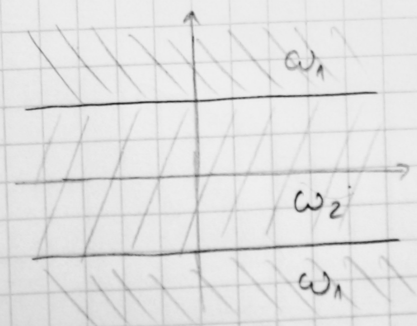
!

$\vec{w} = (0,4,0,0,2,-1)^T \Rightarrow d(x) = 4x_2^2 + 2x_2 - 1$

$4x_2^2 + 2x_2 + \frac{1}{4} - \frac{5}{4} = 0$

$\left(2x_2 + \frac{1}{2}\right)^2 - \frac{5}{4} = 0$

$\Rightarrow x_2 = \frac{1}{4} \pm \frac{\sqrt{5}}{2}$



GRADIJENTNI SPUST

$$j(\vec{w}, \vec{x}, b) = \frac{1}{8\|\vec{x}\|^2}\left[(w^T\vec{x} - b) - |w\vec{x} - b|\right]^2$$

$$\vec{w}(k+1) = \vec{w}(k) - c\left\{\frac{\partial j(\vec{w}, \vec{x}, b)}{\partial \vec{w}}\right\}_{\vec{w} = \vec{w}(k)}$$

$$\frac{\partial j}{\partial \vec{w}} = \frac{1}{8\|\vec{x}\|^2}\cdot 2\left[(\vec{w}\vec{x} - b) - |\vec{w}^T\vec{x} - b|\right]\left[\vec{x} - sgn(\vec{w}^T\vec{x} - b)\cdot\vec{x}\right] =$$

$$= \frac{1}{4\|\vec{x}\|^2}\Big[(\vec{w}^T\vec{x} - b)\vec{x} - \overbrace{(\vec{w}^T\vec{x} - b)\, sgn(\vec{w}^T\vec{x} - b)}^{= |\vec{w}^T\vec{x} - b|}\vec{x} - |\vec{w}^T\vec{x} - b|\vec{x} -$$

$$+ \underbrace{|\vec{w}^T\vec{x} - b|\, sgn(\vec{w}^T\vec{x} - b)}_{= (\vec{w}^T\vec{x} - b)}\vec{x}\Big] =$$

$$= \frac{1}{4\|\vec{x}\|^2}\left[2(\vec{w}^T\vec{x} - b)\vec{x} - 2|\vec{w}^T\vec{x} - b|\vec{x}\right] =$$

$$= \frac{1}{2\|\vec{x}\|^2}\left[(\vec{w}^T\vec{x} - b) - |\vec{w}^T\vec{x} - b|\right]\vec{x}$$

$$\Rightarrow \vec{w}(k+1) = \vec{w}(k) - \frac{c}{2\|x\|^2}\left[(\vec{w}^T\vec{x} - b) - |\vec{w}^T\vec{x} - b|\right]\vec{x}$$

$$\vec{w}(k+1) = \begin{cases} \vec{w}(k), & \vec{w}^T\vec{x} \geq b \\ \vec{w}(k) - \frac{c}{\|x\|^2}(\vec{w}^T\vec{x} - b)\vec{x}, & \vec{w}^T\vec{x} < b \end{cases}$$