EMP – 3. auditorne (2014./2015.)

3 h , pripreme za MI

1. 1. MI 2009./2010. 5. zad ili trkulja 2.57.

Cilindrični kondenzator, 2 sloja dielektrika

$$\rho = \begin{cases} 0, & r < R_1 \\ \frac{1}{r^4+81}, & R_1 \le r \le R_2 \\ 0, & r > R_2 \end{cases} \quad \left[\frac{C}{m^3}\right]$$

$\varepsilon_{r1} = 4$ $\quad R_1 = 1\,cm \quad R_6 = 20\,cm$

$\varepsilon_{r2} = 2$ $\quad R_2 = 3\,cm$

$R_3 = 4{,}5\,cm$

$R_4 = 5\,cm$

$R_5 = 10\,cm$

---

a) $|\vec{E}|(\underset{r_1}{9\,cm}) = ?$

$9\,cm \in \langle R_4, R_5 \rangle$

Gaussov zakon: $\nabla \cdot \vec{D} = \rho_s \,/ \iiint_V$

$$\iiint_V \nabla \cdot \vec{D}\, dV = \iiint_V \rho_s\, dV$$

$$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho_s\, dV = Q$$

$$r_1 = 9\,cm \Rightarrow \oiint \vec{D} \cdot d\vec{S} = Q = \iiint_V \rho_s\, dV = \left| dV = 2r\pi \cdot l \cdot dr \right| \Rightarrow \iiint_V \Rightarrow \int_r$$

$$\vec{D} \cdot \vec{S}_{ZATVORENO} = \iiint_V \rho_s\, 2r\pi l \cdot dr$$

$$D \cdot 2r\pi \cdot l = 2\pi l \int_r \rho_s \cdot r\, dr \,/ : 2\pi l$$

$$D \cdot r_1 = \int_r \rho_s\, r\, dr \,/ : r$$

$$D = \frac{\int_r \rho_s \cdot r\,dr}{r} = \frac{\int_{r=R_1}^{r_2} \frac{1}{r^4+81} \cdot r\,dr}{r} \qquad \vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_r \varepsilon_0 \vec{E} \quad (*)$$

$$D = \frac{\int_{0{,}01}^{0{,}03} \frac{r}{r^4+81} \cdot dr}{r} = \frac{4{,}94 \cdot 10^{-6}}{r} = D \qquad r \in \langle R_4, R_5 \rangle$$

$$(*) \Rightarrow E = \frac{D}{\varepsilon_0 \varepsilon_{r1}} = \frac{4{,}94 \cdot 10^{-6}}{r \cdot \varepsilon_0 \cdot \varepsilon_{r1}} = E(r) \qquad r \in \langle R_4, R_5 \rangle \quad (A)$$

$$|\vec{E}|(r_1 = 9\,cm) = \frac{4{,}94 \cdot 10^{-6}}{0{,}09 \cdot \varepsilon_0 \cdot 4}$$

$\varepsilon_0 \Rightarrow$ shift, 7, 32

$$|\vec{E}|(r_1 = 9\,cm) = 1{,}55 \cdot 10^{6} \frac{V}{m} = \boxed{1550 \frac{kV}{m} = |\vec{E}|(r_1 = 9\,cm)}$$

b) $|\vec{P}|(r_2 = 12\,cm) = ?$

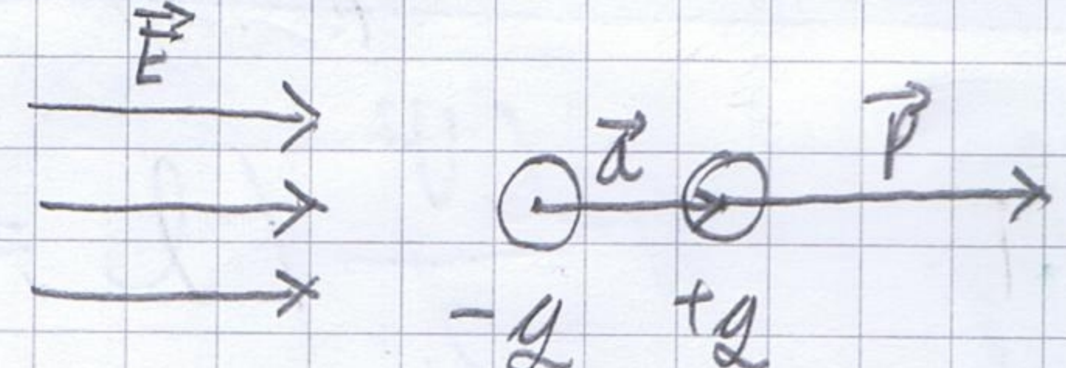
$$\vec{P} = \vec{D} - \varepsilon_0 \vec{E} = \varepsilon_r \varepsilon_0 \vec{E} - \varepsilon_0 \vec{E} = (\varepsilon_r - 1) \cdot \varepsilon_0 \cdot \vec{E}$$

$r_2 \in \langle R_5, R_6 \rangle$

D – gustoća električnog toka
E – jačina električnog polja

D esencijalan $\Rightarrow$ $\vec{P} = \vec{D} - \varepsilon_0 \cdot \frac{\vec{D}}{\varepsilon_r \varepsilon_0} = \boxed{\frac{\varepsilon_r - 1}{\varepsilon_r} \cdot \vec{D} = \vec{P}}$

$\vec{P} = \frac{d\vec{p}}{dV}$ $\quad \vec{p}$ ... dipolni moment



$\vec{p} = q\vec{a}$

Gauss: $\nabla \cdot \vec{D} = \rho_s \,/ \iiint_V$

$$\oint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho_s\,dV$$

$$r_2 = 12\,cm \Rightarrow D \cdot S_{ZATVORENO,\ DOPRINOS} = \int_r 2\pi r l\,dr = Q$$

$$D = \frac{4{,}94 \cdot 10^{-6}}{r} = |r = 12\,cm| = \frac{4{,}94 \cdot 10^{-6}}{0{,}12} = 41{,}15 \cdot 10^{-6} \frac{C}{m^2}$$

$$P = \frac{\varepsilon_{r2} - 1}{\varepsilon_{r2}} \cdot D = \frac{2-1}{2} \cdot 41{,}15 \cdot 10^{-6} = \boxed{20{,}6\,\mu C/m^2 = |\vec{P}|(r_2 = 12\,cm)}$$

c) $W = ?$ $$W = \frac{CU^2}{2} = \frac{1}{2}\iiint_V (\vec{D}\cdot\vec{E})\,dV = \frac{1}{2}\iiint_V \frac{|\vec{D}|^2}{\varepsilon}\,dV$$

$$W = \frac{1}{\cancel{2}}\int_r \frac{|\vec{D}|^2}{\varepsilon}\cdot \cancel{2}\pi r l\,dr = \pi l \int_r \frac{|\vec{D}|^2}{\varepsilon} r\,dr \;/:l$$

$$\frac{W}{l} = w_{gust.} = \pi\int_r \frac{|\vec{D}|^2}{\varepsilon} r\,dr = \pi\int_{r=R_4}^{R_6} \frac{|\vec{D}|^2}{\varepsilon} r\,dr = \pi\left[\int_{r=R_4}^{R_5} \frac{|\vec{D}|^2}{\varepsilon_0\varepsilon_{r1}} r\,dr + \right.$$

$$\left. + \int_{r=R_5}^{R_6} \frac{|\vec{D}|}{\varepsilon_0\cdot\varepsilon_{r2}} r\,dr\right] = \pi\left[\int_{r=0,05}^{0,1} \frac{(4,94\cdot10^{-6})^2}{\varepsilon_0\cdot 4\cdot r}\,dr + \int_{r=0,1}^{0,2} \frac{(4,94\cdot10^{-6})^2}{\varepsilon_0\cdot 2\cdot r}\,dr\right]$$

$$w_{gust.} = \frac{\pi}{\varepsilon_0}(4,94\cdot10^{-6})^2\left[\int_{r=0,05}^{0,1}\frac{dr}{r}\cdot\frac{1}{4} + \int_{r=0,1}^{0,2}\frac{dr}{r}\cdot\frac{1}{2}\right] =$$

$$= \frac{\pi\cdot(4,94\cdot10^{-6})^2}{\varepsilon_0}\cdot\left[\frac{1}{4}\cdot \underbrace{\ln\left(\frac{0,1}{0,05}\right)}_{\ln 2} + \frac{1}{2}\underbrace{\ln\left(\frac{0,2}{0,1}\right)}_{\ln 2}\right] =$$

$$= \frac{\pi(4,94\cdot10^{-6})^2}{\varepsilon_0}\cdot\ln(2)\cdot\frac{3}{4} = \boxed{4,5\,\frac{J}{m} = w_{gust.}}$$

---

d) $\frac{C}{l} = ?$ $$W = \frac{CU^2}{2} \;/:l \Rightarrow w_{gust} = \frac{\frac{C}{l}\cdot U^2}{2}$$

$$U_{AB} = -\int_B^A \vec{E}\cdot d\vec{l} = \int_A^B \vec{E}\cdot d\vec{l}$$

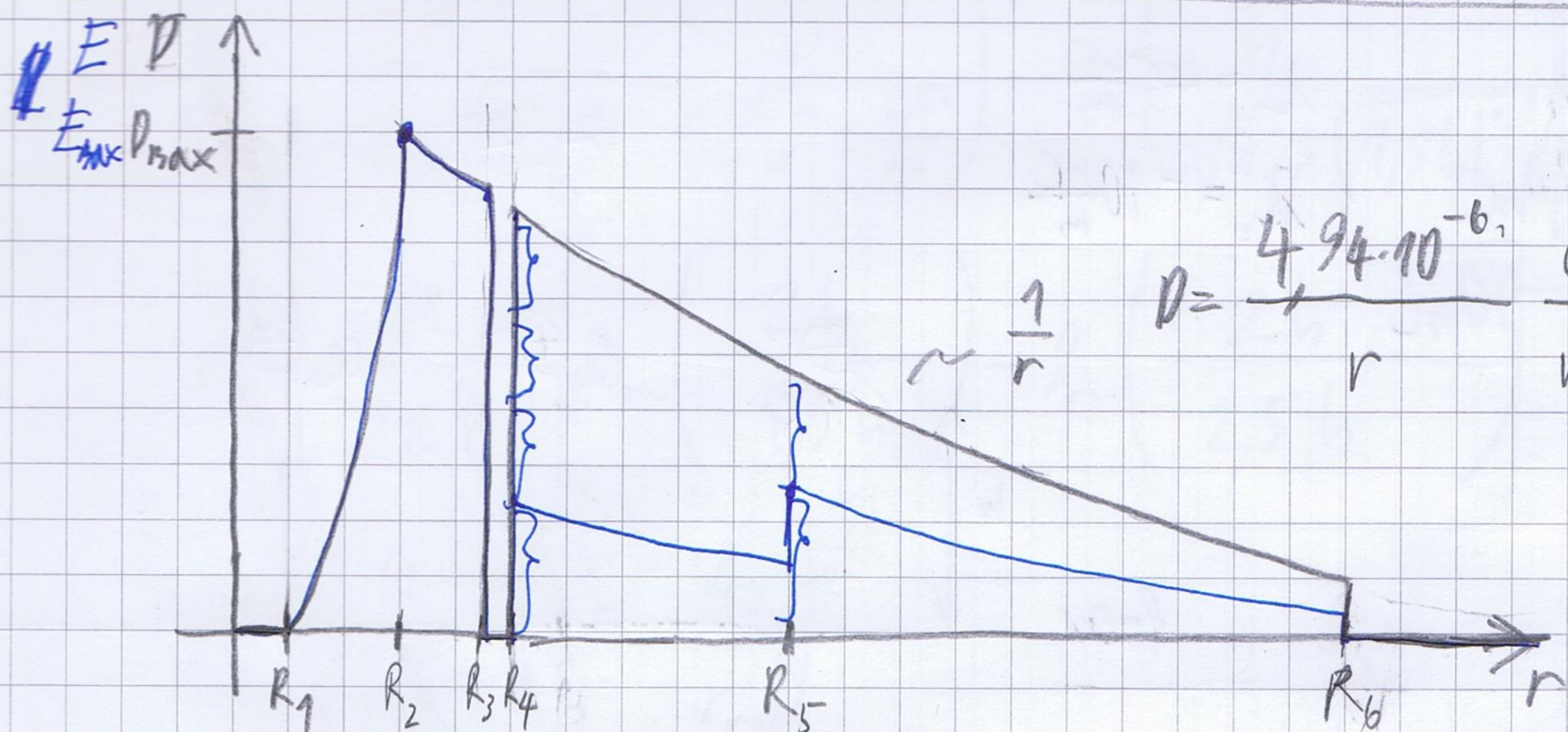
$$U = \int_{R_4}^{R_6} \vec{E}\cdot d\vec{l} = \int_{R_4}^{R_6} \frac{\vec{D}}{\varepsilon_0\varepsilon_r}\cdot d\vec{l} = \int_{R_4}^{R_5} \frac{4,94\cdot10^{-6}}{\varepsilon_{r1}\cdot\varepsilon_0\cdot r}\,dr + \int_{R_5}^{R_6} \frac{4,94\cdot10^{-6}}{\varepsilon_{r2}\cdot\varepsilon_0\cdot r}\,dr =$$

$$= \frac{4,98\cdot10^{-6}}{\varepsilon_0}\cdot\left[\frac{1}{4}\ln\frac{0,1}{0,05} + \frac{1}{2}\ln\frac{0,2}{0,1}\right] = \frac{4,98\cdot10^{-6}}{\varepsilon_0}\cdot\frac{3}{4}\ln(2)$$

$$U = 289943,225\,V \Rightarrow \frac{C}{l} = \frac{2w_{gust}}{U^2} = \frac{2\cdot4,5}{(289943,225)^2} = 1,07\cdot10^{-10}\,F/m$$

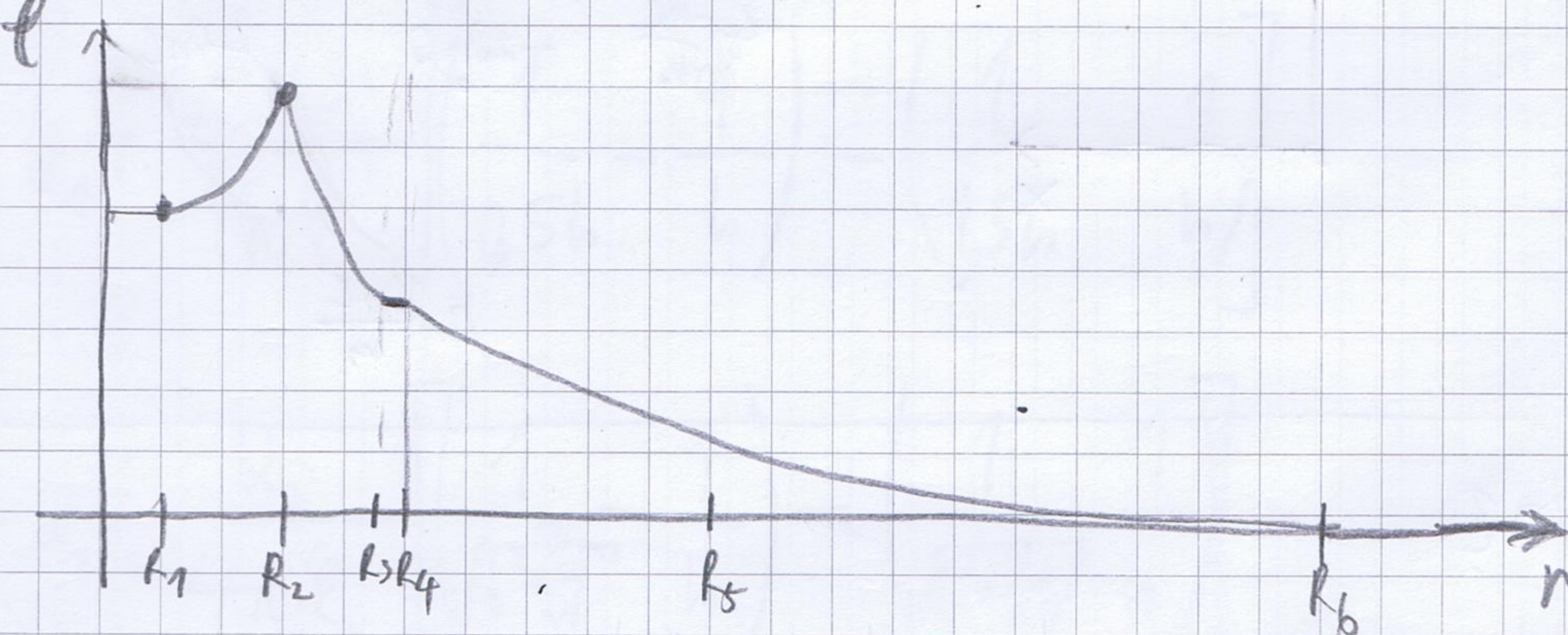
$$\boxed{C/l = 107\,\frac{pF}{m}}$$

električna indukcija = gustoća magnetskog polja ⇒ $\vec{D}$!



$\vec{D} = \varepsilon_0 \varepsilon_r \cdot \vec{E}$

$\sim \frac{1}{r}$ $\quad D = \frac{4{,}94 \cdot 10^{-6}}{r} \frac{C}{m^2}$



$E = -\nabla \cdot \varphi$

~~2.58.~~ $|E|(r) \quad r \in \langle R_1, R_2 \rangle \qquad \oiint \vec{D}\, d\vec{S} = \iiint_V \rho_s\, dV = \int_r \rho_s \cdot 2\pi r \cdot l \cdot dr$

$$D \cdot \cancel{2\pi} r \cdot \cancel{l} = \cancel{2\pi l} \int_r \rho_s \cdot r\, dr$$

$$D = \frac{\int_r \rho_s \cdot r\, dr}{r}$$

$$D = \frac{\int_{r=R_1}^{r} \frac{r}{r^{4+8}}\, dr}{r} \quad \ldots\ldots$$

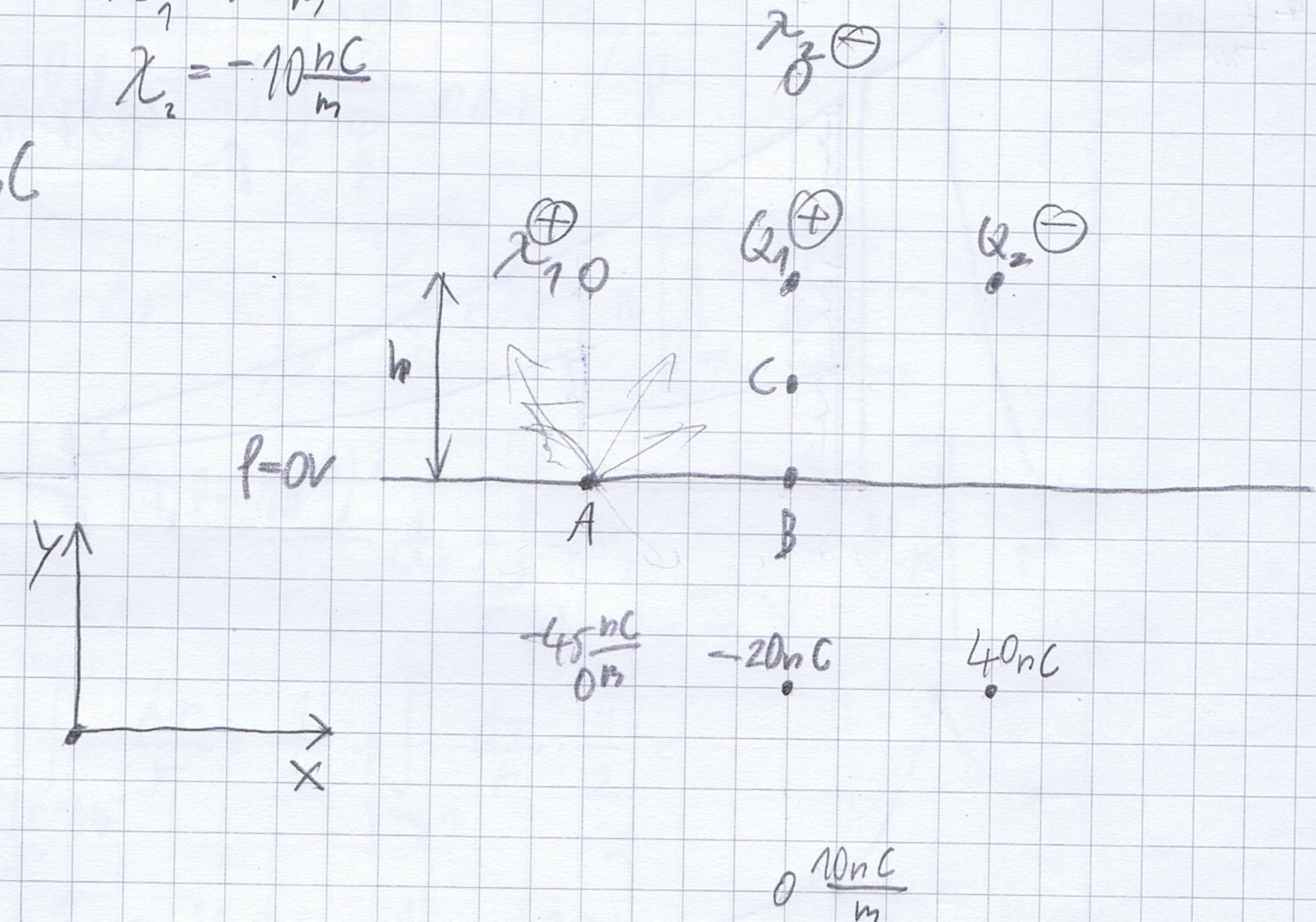
② (2.59.), MI 2011, 2. ② 2.54. zbirka

$\varphi = 0\,V$ $h = 3\,m$ $\lambda_1 = 45\,\frac{nC}{m}$

$Q_1 = 20\,nC$ $\lambda_2 = -10\,\frac{nC}{m}$

$Q_2 = -40\,nC$

$\lambda = \frac{Q}{l} = \frac{\rho_V}{V \cdot l}$



a) $|E|(\text{točka } A) = ?$

$$\oint\limits_S \vec{E}\cdot d\vec{S} = \iiint\limits_V \frac{\rho_V}{\varepsilon_0}\, dV$$

$$\vec{E}\cdot 2r\pi \cdot l = \frac{\lambda}{\varepsilon_0}\cdot l \Rightarrow \boxed{\vec{E} = \frac{\lambda}{2r\pi\varepsilon_0}} \text{ štap} \qquad \text{točka } E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

$$\lambda_1: \vec{E} = \cancel{2}\cdot\frac{\lambda_1}{\cancel{2}\cdot h\cdot\pi\varepsilon_0}\cdot(-\hat{a}_y) = \frac{|\lambda_1|}{h\pi\varepsilon_0}(-\hat{a}_y)$$

$$Q_1: \vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{|Q_1|}{(\sqrt{2}\cdot h)^2}\cdot\left[\left(\frac{\cancel{-\hat{a}_x}-\hat{a}_y}{\sqrt{2}}\right)+\left(\frac{\cancel{\hat{a}_x}-\hat{a}_y}{\sqrt{2}}\right)\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{|Q_1|}{2h^2}\cdot\frac{2}{\sqrt{2}}\cdot(-\hat{a}_y)$$

$$\lambda_2: \vec{E} = \frac{|\lambda_2|}{2\pi\varepsilon_0\cdot\sqrt{h^2+4h^2}}\left[\frac{\cancel{\hat{a}_x}+2\hat{a}_y}{\sqrt{5}}+\frac{\cancel{-\hat{a}_x}+2\hat{a}_y}{\sqrt{5}}\right] = \frac{|\lambda_2|}{2\pi\varepsilon_0\cdot\sqrt{5}\cdot h}\cdot\frac{4}{\sqrt{5}}(\hat{a}_y)$$

$$Q_2: \vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{|Q_2|}{5h^2}\left[\frac{\cancel{2\hat{a}_x}+\hat{a}_y}{\sqrt{5}}+\frac{\cancel{-2\hat{a}_x}+\hat{a}_y}{\sqrt{5}}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{|Q_2|}{5h^2}\cdot\frac{2}{\sqrt{5}}(\hat{a}_y)$$

c) potencijal točke C $\varphi_C = ?$ $\quad \boxed{\varphi(r) = \frac{\lambda}{2\pi\varepsilon} \ln\left(\frac{r_{ref}}{r}\right)}$ štap

$$\lambda_1: \frac{\lambda_1}{2\pi\varepsilon}\left[\ln\left(\frac{h}{\sqrt{h^2+\frac{h^2}{4}}}\right) - \ln\left(\frac{h}{\sqrt{h^2+(1{,}5h)^2}}\right)\right]$$

$$\lambda_2: \frac{\lambda_2}{2\pi\varepsilon}\left[\ln\left(\frac{2h}{1{,}5h}\right) - \ln\left(\frac{2h}{2{,}5h}\right)\right]$$

$$\varphi(r) = \frac{Q}{4\pi\varepsilon_0}\cdot\left(\frac{1}{r} - \frac{1}{r_{ref}}\right)$$

$$Q_1: \frac{Q_1}{4\pi\varepsilon_0}\left[\left(\frac{1}{0{,}5h} - \frac{1}{h}\right) - \left(\frac{1}{1{,}5h} - \frac{1}{h}\right)\right]$$

$$Q_2: \frac{Q_2}{4\pi\varepsilon_0}\left[\left(\frac{1}{\sqrt{h^2+\left(\frac{h}{2}\right)^2}} - \frac{1}{h}\right) - \left(\frac{1}{\sqrt{h^2+\left(\frac{3}{2}h\right)^2}} - \frac{1}{h}\right)\right]$$

$$\varphi_u = \sum_{i=1}^{4} \varphi_i \ldots \quad \boxed{\varphi_C = 334\,V}$$

d) $Q = 15\,pC$

$$W_{AC} = ? = Q\cdot U_{CA} = Q\cdot\left[\varphi_C - \varphi_A\right] = 15\cdot 10^{-12}\left[334 - 0\right] = \boxed{5\,nJ = W_{AC}}$$