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Mass 2citlvs

$$\vec{B} = \frac{a}{2x+y}\vec{a_x}$$

Najosnovnije formule

$$\vec{B} = \vec{H}\mu_0\mu_r$$

$$\nabla\vec{B} = 0 \quad - \oiint \vec{B}\,d\vec{s} = 0$$

$$\nabla \times \vec{B} = \mu_0\vec{J}$$

$$I = \iint \vec{J}\,d\vec{s}$$

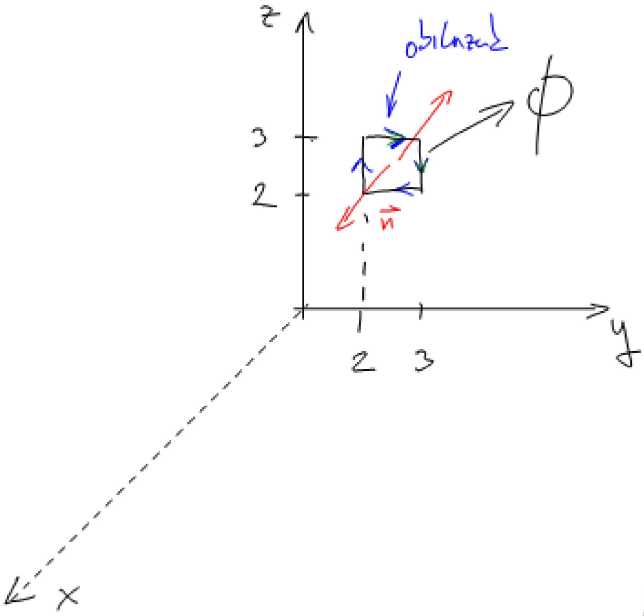
$$\phi = \iint \vec{B}\,d\vec{s}$$

$$B = \nabla \times \vec{A}$$

$$\vec{A} = \frac{1}{x+2y+3}\vec{a_z}$$

$$\vec{B}(1,2,3) = ?$$

$$B = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{1}{x+2y+3} \end{vmatrix} = \vec{a_x}\frac{\partial}{\partial y}\left(\frac{1}{x+2y+3}\right) - \vec{a_y}\frac{\partial}{\partial x}\left(\frac{1}{x+2y+3}\right)$$



Kako ne kažu vektor smjera uzimamo apsolutne vrijednosti

$$\Phi = \left|\iint \vec{B}\, d\vec{S}\right| = \left|\oint \vec{A}\, d\vec{l}\right|$$

$$\vec{n} = \vec{a}_x$$

$$d\vec{S} = \vec{a}_x\, dy\, dz$$

$$d\vec{l} = dy\, \vec{a_y} + dz\, \vec{a_z}$$

$$= \begin{cases} dy\, \vec{a_y}, & z=2,\ x=0,\ y\in[2,3] \\ dz\, \vec{a_z}, & x=0,\ y=3,\ z\in[2,3] \\ -dy\, \vec{a_y}, & z=3,\ x=0,\ y\in[2,3] \\ -dz\, \vec{a_z}, & x=0,\ y=2,\ z\in[2,3] \end{cases}$$

jer smanjujemo y-lon

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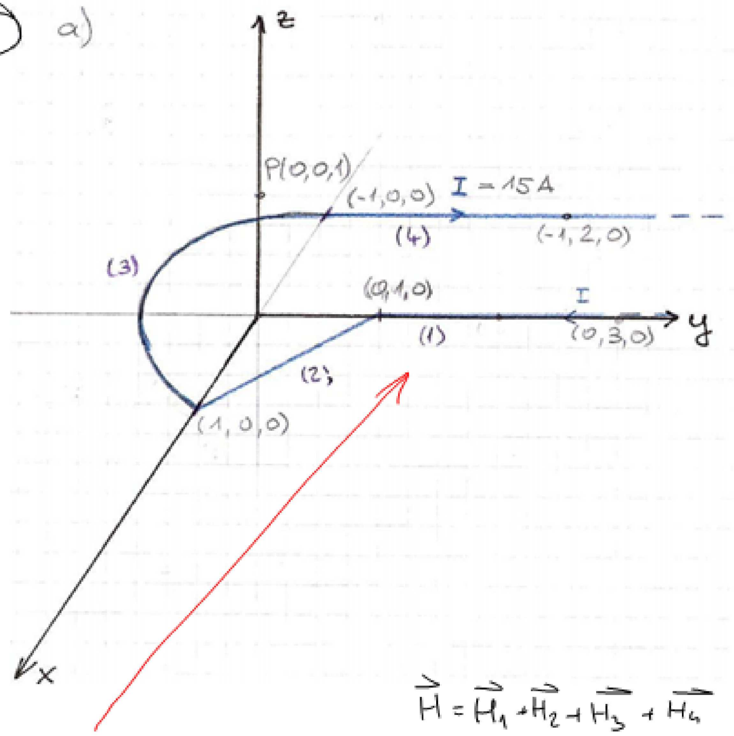
Polje oko tanke strujnice

→ odrediti sve komponente jakosti polja u točki P

2 a)



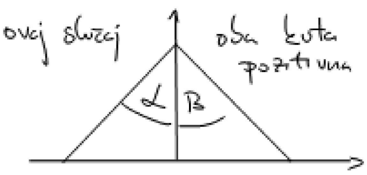
$$\vec{H} = \vec{H_1} + \vec{H_2} + \vec{H_3} + \vec{H_4}$$

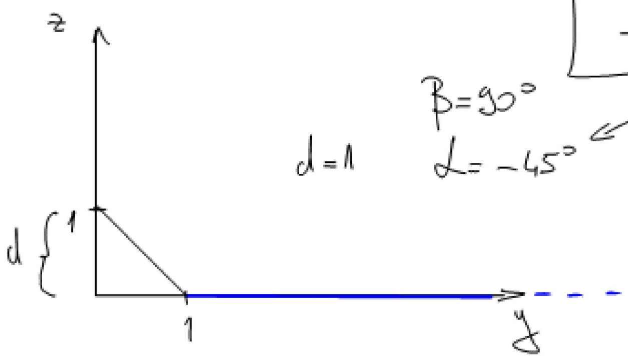
(1) dio

$$H = \frac{I}{4\pi d} \left(\sin\alpha + \sin\beta \right)$$

$\vec{n_0}$ $\quad \vec{H} = H \vec{n_0}$

ovaj slučaj oba kuta pozitivna

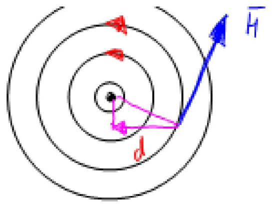




$\beta = 90^\circ$

$d = 1 \quad \alpha = -45^\circ$ ← negativan jer gleda u istom smjeru kao i β

$$H_1 = \frac{15}{4\pi \cdot 1} \left(\sin(-45^\circ) + \sin(90^\circ) \right)$$

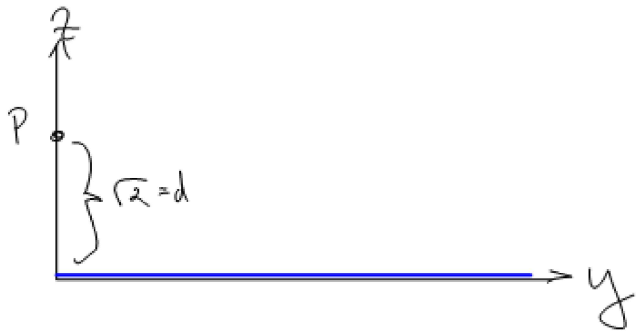


(0, 0, 1)
(0, 1, 0)
(0, 3, 0)

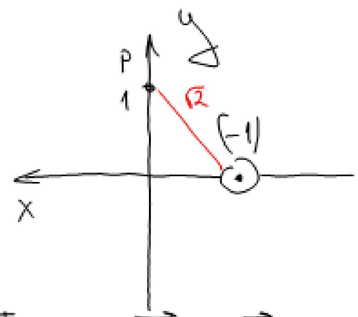
$$\vec{n}_0 = \frac{\vec{n}}{|n|} = -\vec{a}_x$$

$$\vec{n} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ & & \\ & & \end{vmatrix} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0 & -1 & 1 \\ 0 & -2 & 0 \end{vmatrix} = 2\vec{a}_x$$

(4) dio



(0, 0, 0)
(−1, 0, 0)
(−1, 2, 0)

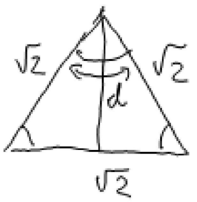


$$\vec{n} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 1 & 0 & 1 \\ 0 & -2 & 0 \end{vmatrix} = 2\vec{a}_x - 2\vec{a}_z$$

$$\vec{H}_4 = \frac{I}{4\pi\sqrt{2}} \left(\frac{2\vec{a}_x - 2\vec{a}_z}{\sqrt{2^2 + 2^2}} \right) = 0{,}59\,\vec{a}_x - 0{,}59\,\vec{a}_z$$

Dio 2.

$$d = \frac{\sqrt{6}}{2}$$



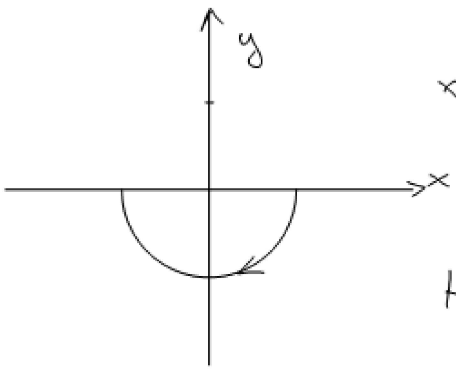
$$H_2 = \frac{2I}{4\pi\sqrt{6}}\left(\sin 30^\circ + \sin 30^\circ\right)$$

$$\vec{n} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \vec{a_x} + \vec{a_y} + \vec{a_z}$$

$$\vec{H_2} = H_2 \left(\frac{-\vec{a_x} - \vec{a_y} - \vec{a_z}}{\sqrt{3}}\right)$$

(3)

$$\vec{H_3} = \frac{I R}{4\pi\sqrt{(R^2+h^2)}^3} \int_{2\pi}^{\pi} h\cos\varphi \, d\varphi \, \vec{a_x} + h\sin\varphi \, d\varphi \, \vec{a_y} + R\,dy\,\vec{a_z}$$



$xy \quad \varphi \in [0, 2\pi]$

$$H_3 = 0.422\,\vec{a_y} - 1.33\,\vec{a_z}$$

$$\vec{H} = \vec{H_1} + \vec{H_2} + \vec{H_3} + \vec{H_4} = -0.32\,\vec{a_x} - 0.138\,\vec{a_y} - 2.49\,\vec{a_z}$$

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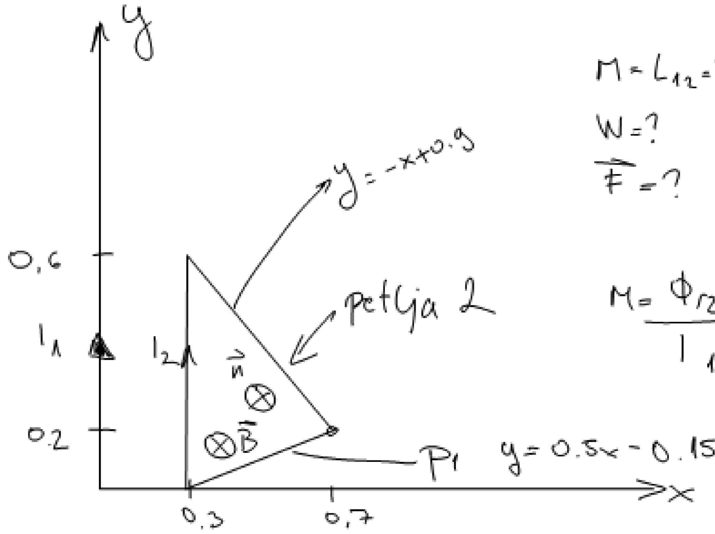

$M = L_{12} = ?$

$W = ?$

$\vec{F} = ?$

$$M = \frac{\Phi_{12}}{I_1}$$

$$\Phi_{12} = \iint_{S_2} \vec{B}_1 \, d\vec{S}$$

$$B_1 = \frac{\mu_0 I_1}{4\pi d}\left(\sin 90^\circ + \sin 90^\circ\right)$$

$$\vec{B}_1 = \frac{I_1 \mu_0}{2\pi x}\left(-\vec{a_z}\right)$$

$$d\vec{S} = -\vec{a_z}\, dx\, dy$$

$$\Phi_{12} = \iint \frac{I_1 \mu_0}{2\pi x} \, dx\, dy = \frac{I_1 \mu_0}{2\pi} \iint \frac{1}{x} \, dx\, dy$$

$$M = \frac{\mu_0}{2\pi} \int_{0.3}^{0.7} \frac{1}{x} \, dx \int_{0.5x-0.15}^{-x+0.9} dy = 57.8\,\mathrm{nH}$$

$I_1 = 3A \qquad I_2 = 5A$

$W = M \cdot I_1 \cdot I_2$

$$\vec{F} = I_2 \oint_c d\vec{L}_2 \times \vec{B}_1 \qquad\qquad d\vec{L} = \int dx\,\vec{a_x} + dy\,\vec{a_y}$$

$$d\vec{L}_2 \times \vec{B}_1 = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ dx & dy & 0 \\ 0 & 0 & \frac{-I_1 \mu_0}{2\pi x} \end{vmatrix} = \frac{-I_1 \mu_0}{2\pi x} dy\,\vec{a_x} + \frac{I_1 \mu_0}{2\pi x} dx\,\vec{a_y}$$

$$\vec{F} = F_x\,\vec{a_x} + F_y\,\vec{a_y} + \cancel{F_z}\,\vec{a_z}$$

$$F_x = I_2 \int \frac{-I_1 \mu_0}{2\pi x}\, dy = \frac{-I_1 \cdot I_2 \cdot \mu_0}{2\pi x}\left[\int_0^{0.6} \frac{dx}{0.3} + \int_{0.6}^{0.2} \frac{dy}{0.9-y} + \int_{0.2}^{0} \frac{dy}{2y+0.3}\right]$$

$$= -4{,}04\,\mu N$$

$$F_y = I_2 \int \frac{I_1 \mu_0}{2\pi x}\, dx = \frac{I_1 I_2 \mu_0}{2\pi}\left[\cancel{\int_{0.3}^{0.7} \frac{dx}{x}} + \cancel{\int_{0.7}^{0.3} \frac{dx}{x}}\right] = 0$$

$$F = -4{,}04\,\vec{a_x} \quad [\mu N]$$

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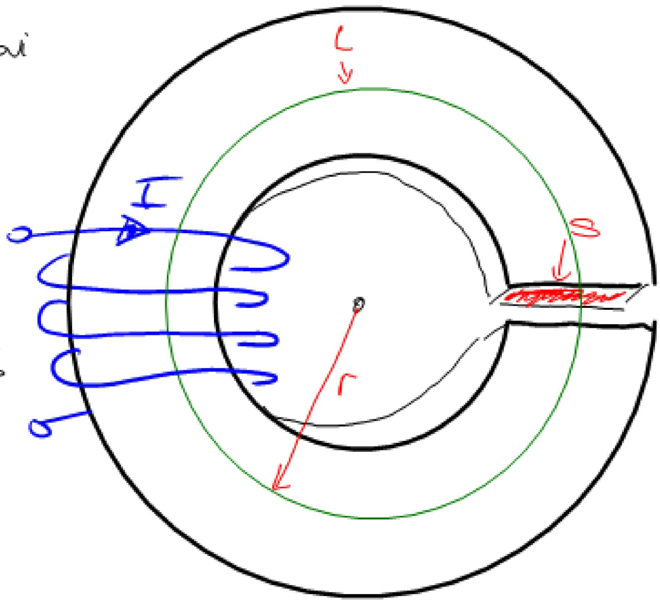
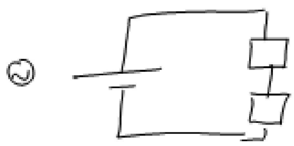
Magnetski krugovi

$$\Phi = \frac{\Theta}{R_m}$$

$$\Theta = I N$$

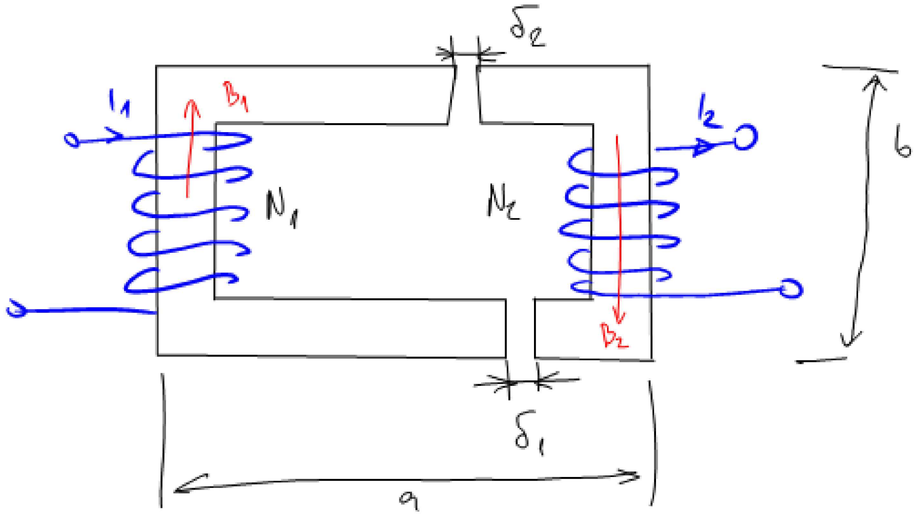
$$R_m = \frac{L}{\mu_0 \mu_r S}$$

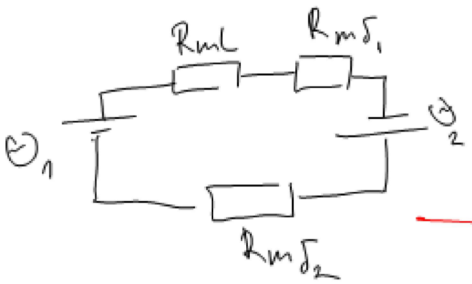




R_{mL} – otpor prstena

R_{mS} – zračni raspor





$a = 20\ mm$
$b = 40\ mm$
$\delta_1 = 2\ mm$
$\delta_2 = 5\ mm$

$S = 5\ mm^2$
$I_1 = 5\ A$
$N_1 = 1000$
$N_2 = 600$

$W_{\delta_1} = 3\ J$

$I_2 = ?$

$$\Theta_1 + \Theta_2 = \phi\left(R_{mL} + R_{m\delta_1} + R_{m\delta_2}\right)$$

$$L = 2a + 2b - \delta_1 - \delta_2 = 213\ mm$$

$$W = \frac{1}{2}\iiint BH\ dV$$

konst — $\phi = \iint \vec{B}\, d\vec{S}$ — konst

$\phi = BS \quad \Rightarrow B \rightarrow$ konst.

$B = \mu_r \mu_0 H \quad \Rightarrow H \rightarrow$ konst,

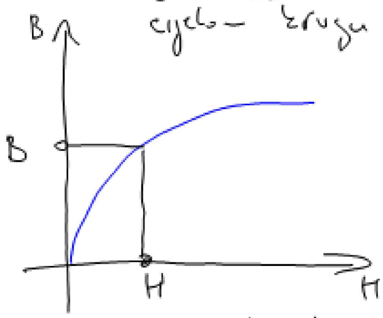
$H = \dfrac{B}{\mu_0 \mu_r}$ $\quad \mu_r = 1$

$$W = \frac{BH}{2}\iiint dV$$

$$W_{\delta_1} = \frac{1}{2} B H_{\delta_1} \cdot S \cdot \delta_1 = \frac{B^2 S \delta_1}{2\mu_0}$$

$B = \ldots\ldots$

B – konst. u cijelom krugu

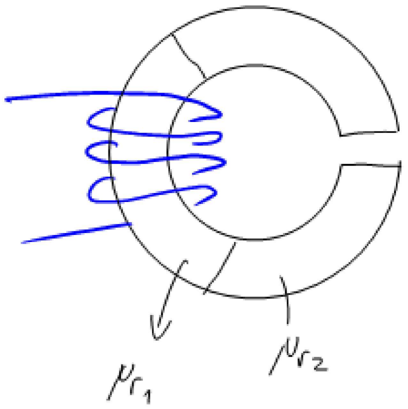


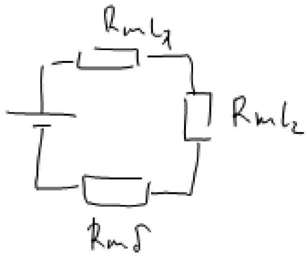
grafički odredimo H_L pomoću poznatog B

$$H_\delta = \frac{B}{\mu_0}$$

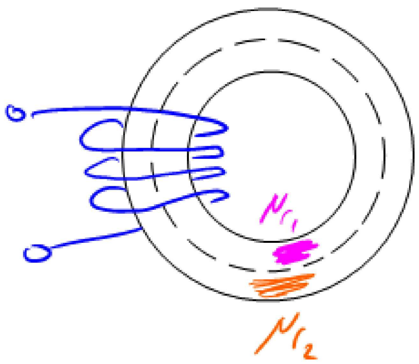
$$\frac{B}{\mu_0 \mu_r} = H \qquad \Phi \left[\frac{L}{\mu_0 \mu_r S} + \frac{\delta_1}{\mu_0 S} + \frac{\delta_2}{\mu_0 S} \right] = \Theta_1 + \Theta_2$$

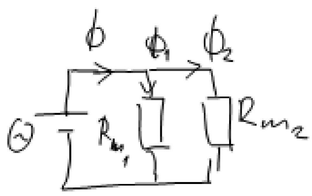
$$= H_L \cdot L + H_\delta \cdot \delta_1 + H_\delta \cdot \delta_2 = I_1 N_1 + I_2 N_2$$

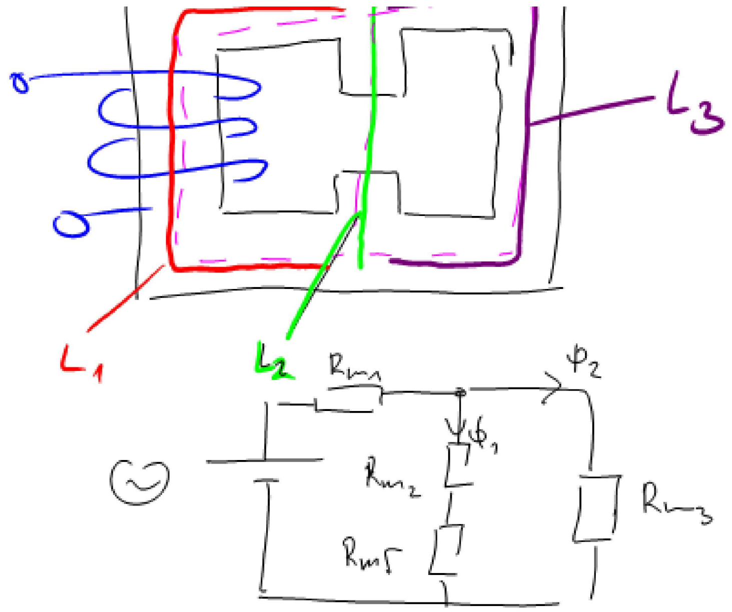




B-wti







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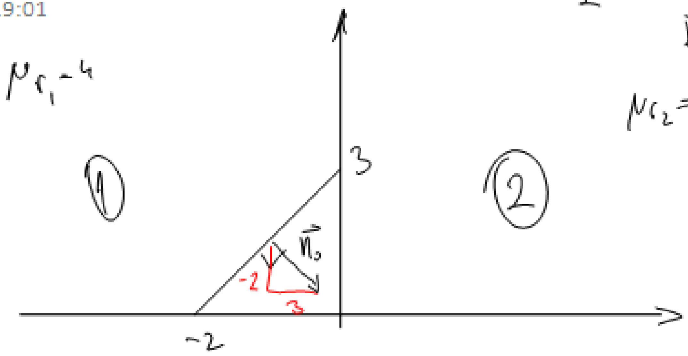
$\mu_{r_1}=4$

$\vec{H}_2 = 3\vec{a}_x - 7\vec{a}_y + 2\vec{a}_z$

$\vec{K}_S = 4\vec{a}_x - 5\vec{a}_z$

$\mu_{r_2}=2$



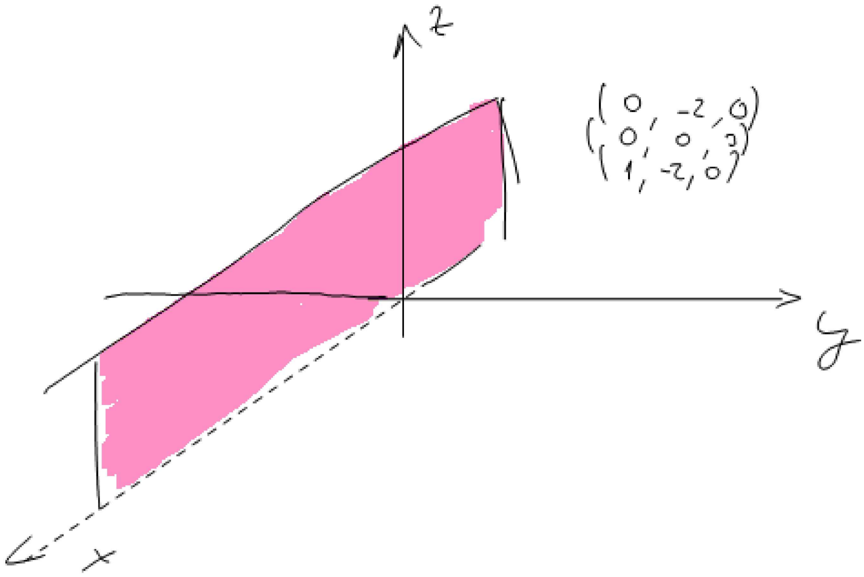
$$\vec{n}_0 = \frac{3\vec{a}_y - 2\vec{a}_z}{\sqrt{9+4}}$$

$$\vec{n}_0 \times (\vec{H}_2 - \vec{H}_1) = \vec{K}_S$$

$$\vec{n}_0 \cdot (\vec{B}_2 - \vec{B}_1) = \vec{K}_S$$

$$\vec{B}_2 = \mu_0 \mu_{r_2} H_2$$

$$\vec{H}_1 = H_x \vec{a}_x + H_y \vec{a}_y + H_z \vec{a}_z$$



$$\begin{pmatrix} 0, & -2, & 0 \end{pmatrix}$$
$$\begin{pmatrix} 0, & 0, & 3 \end{pmatrix}$$
$$\begin{pmatrix} 1, & -2, & 0 \end{pmatrix}$$

$$4x + 3y - 9z = 6$$

$$n_0 = \frac{4\vec{a_x} + 3\vec{a_y} - 9\vec{a_z}}{\sqrt{4^2 + 9^2 + 3^2}}$$

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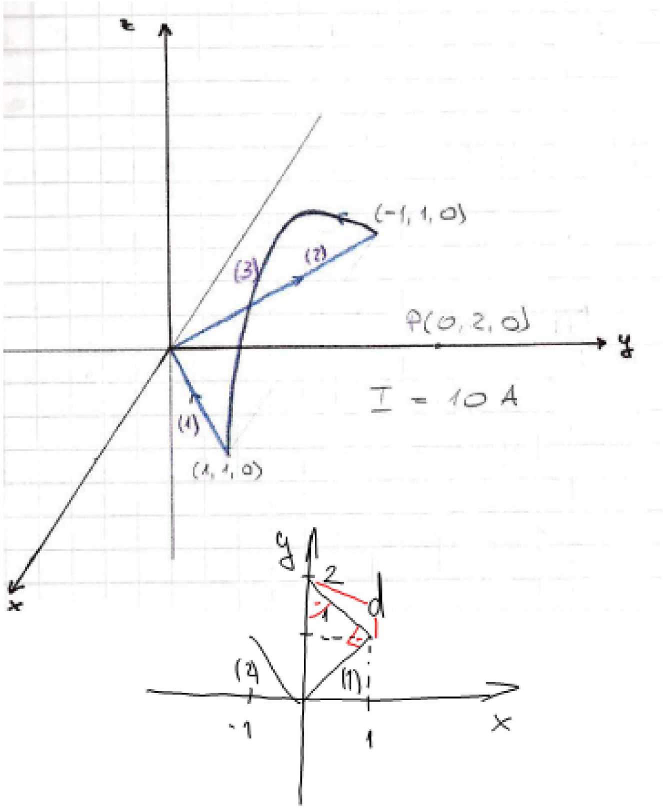


$$H_1 = H_2 = \frac{I}{4\pi\sqrt{2}}\left[\sin 0^\circ + \sin 45^\circ\right] = \frac{I}{8\pi}$$

(1) dio

(0, 0, 0)
(1, 1, 0)
(0, 2, 0)

$$\vec{n} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ -1 & -1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = 2\vec{a_z}$$

$$H_1 = \frac{-I}{8\pi} a$$

dio 2:

(0, 0, 0)
(-1, 1, 0)
(0, 2, 0)

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 1 & -1 & 0 \\ -1 & -1 & 0 \end{vmatrix} = -2\vec{a_z}$$

$$H_2 = \frac{-I}{} \vec{a}$$

$$I_2 = \frac{\quad}{8\pi} \quad z$$

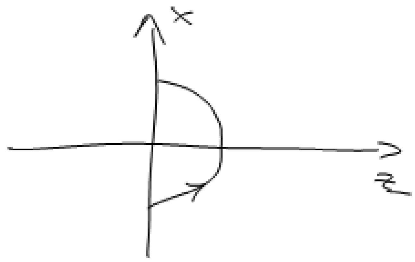
Kruzni dio

xy z

z x y

$$H_3 = \frac{1\ R}{4\pi\sqrt{R^2+h^2}^3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} h\cos\varphi\, d\varphi \vec{a_z} + h\sin\varphi\, d\varphi \vec{a_x} + R\, d\varphi \vec{a_y}$$

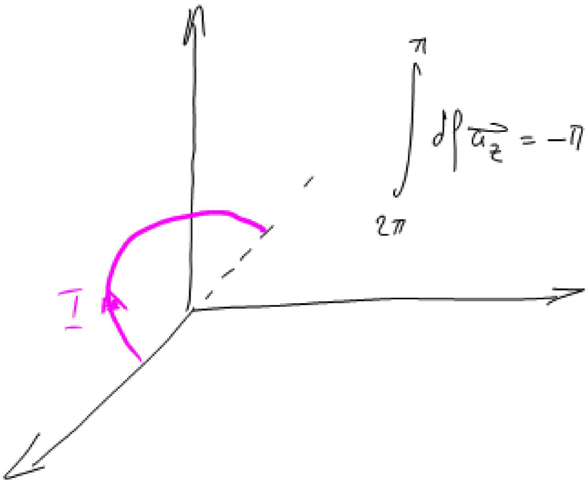
$R=1,\ h=1$



$$\int R\, d\varphi \vec{a_y} = \int d\varphi \vec{a_y}$$ – ako stavimo $\int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} \Rightarrow (-)$ Ne!

$$\rightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \vec{a_y} = \pi \vec{a_y}$$

1. Zadatak



$$\int_{2\pi}^{\pi} d\varphi\, \vec{a_z} = -\pi\, \vec{a_z}$$