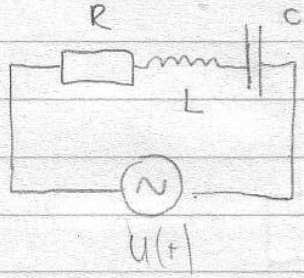
# 05

$u(t) = U_m \sin(\omega t + \alpha_u)$

$\omega \to 2\pi f$

$U_{max} = \sqrt{2}\, U_{ef}$



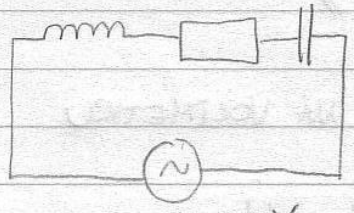

$\dot{X}_C = \frac{1}{\omega C} \angle -90^\circ$ kasni napon za 90° $= \frac{-j}{\omega C}$

$\dot{X}_L = \omega L \angle 90^\circ$ prethodi struji za 90° $= j\omega L$

$\underline{U} = \dot{U} = U_{ef} \angle \alpha_u$ */

$= U_{ef} \cdot \cos\alpha_u + jU_{ef} \cdot \sin\alpha_u$

$Re\{U\}$ $Im\{U\}$




$R = 4\,\Omega$

$X_L = 10\,\Omega$

$X_C = 7\,\Omega$

$\dot{Z}_{UK} = \dot{R} + \dot{X}_L + \dot{X}_C \Rightarrow \dot{Z}_{UK} = 4 + j10 - j7 =$

$= R + jX_L - jX_C \quad = 4 + j3 =$

$= Z_{UK} \angle \varphi$

$= \sqrt{4^2 + 3^2} \angle \operatorname{arctg} \frac{3}{4}$

$= 5 \angle 36{,}87\,\Omega$

$$\dot{I} = I\angle 0^\circ$$

$$I\angle 0^\circ = \frac{\dot{U}}{\dot{Z}} = \frac{100\angle \varphi_u}{5\angle 36{,}87^\circ}$$

$$I = \frac{100}{5} = 20\,A$$

$$0^\circ = \varphi_u - 36{,}87^\circ$$

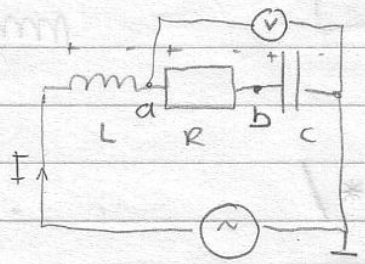
$$\varphi_u = 36{,}87^\circ$$

$$* \quad \frac{5\angle 30^\circ}{10\angle 45^\circ} = 0.5\angle -15^\circ$$

$$\dot{U}_R = \dot{I}\cdot R$$

$$= 20\angle 0^\circ \cdot 4\angle 0^\circ$$

$$= 80\angle 0^\circ$$




$$\dot{U}_L = \dot{I}\cdot X_L = 20\angle 0^\circ \cdot 10\angle 90^\circ = 200\angle 90^\circ$$

$$\dot{U}_C = \dot{I}\cdot X_C = 20\angle 0^\circ \cdot 7\angle -90^\circ = 140\angle -90^\circ$$

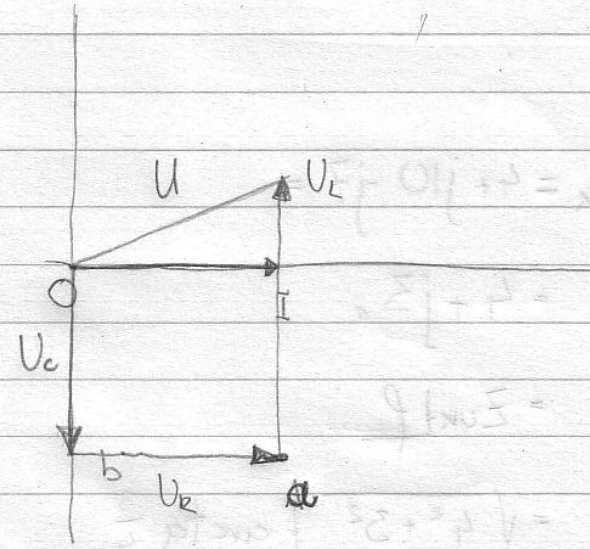
NAPON NA VOLTMETRU

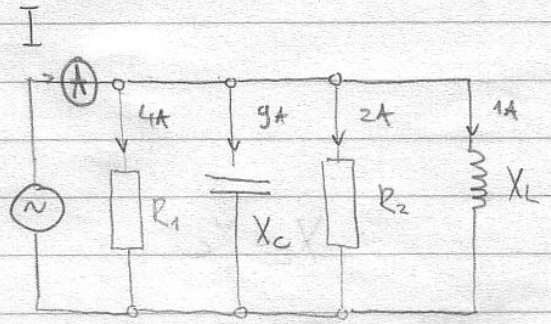
$$\dot{U}_V = \dot{U}_R + \dot{U}_C =$$

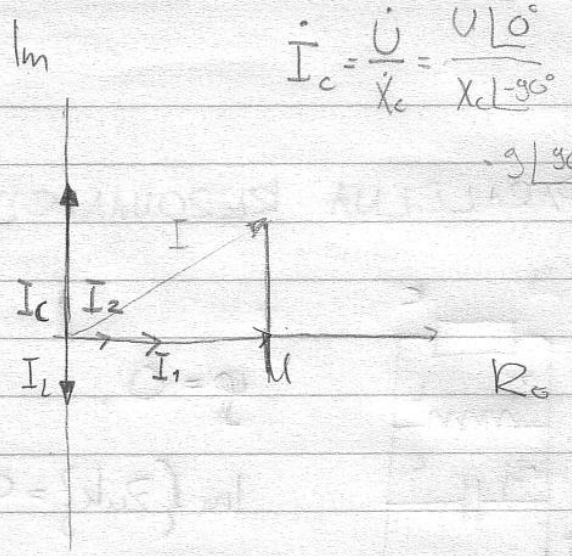
$$= 80\angle 0^\circ + 140\angle -90^\circ$$

$$= 80 - j\,140$$

$$U_V = \sqrt{80^2 + 140^2} =$$





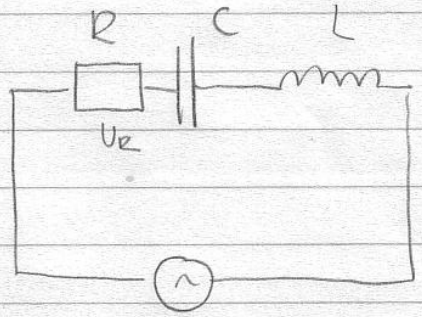

$$\dot{I}_c = \frac{\dot{U}}{\dot{X}_c} = \frac{U\angle 0^\circ}{X_c\angle -90^\circ} = \frac{U}{X_c}\angle 0-(-90^\circ)$$

$$9\angle 90^\circ = j9A$$

STRUJA KROZ OTPORNIK JE U FAZI S NAPONOM

$$\dot{I} = 4 + 9j + 2 - j = 6 + 8j = 10\angle 53^\circ$$

$$I = \sqrt{6^2 + 8^2} = 10A$$




$$\gamma = 0^\circ \qquad X_C = X_L$$

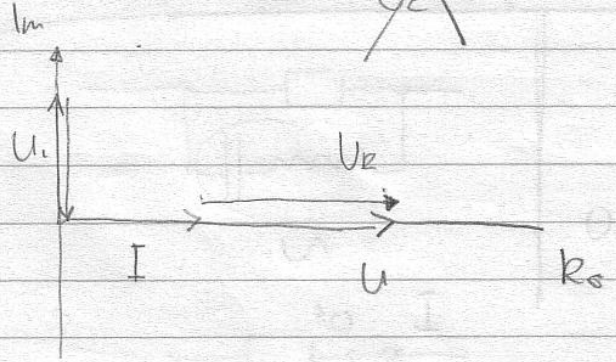
$$\mathrm{Im}\{Z_{uk}\} = 0 \qquad \frac{1}{\omega C} = \omega L$$

$$\omega = \frac{1}{\sqrt{LC}}$$

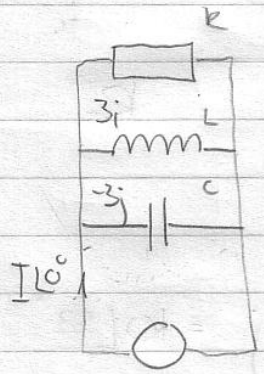
$$\dot{U} = U\angle 0^\circ \qquad U = U_R$$

$$\dot{Z}_{uk} = R$$

$$\dot{I} = I\angle 0^\circ \qquad U_L = 0 \qquad U_C = 0$$



# PARALELNA REZONANCIJA




$\varphi = 0$

$X_L = X_C$

$\mathrm{Im}\{Z_{uk}\} = 0$

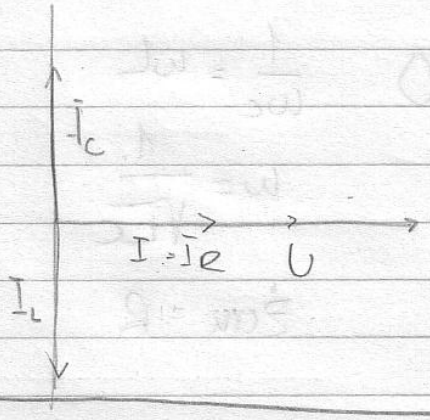
$I_L = 0$

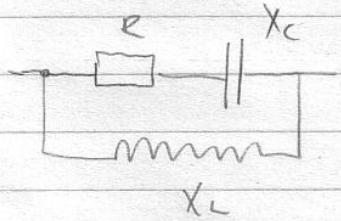
$I_C = 0$

$$\frac{1}{\dot{Z}_{uk}} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$\dot{Z}_{uk} = R$$

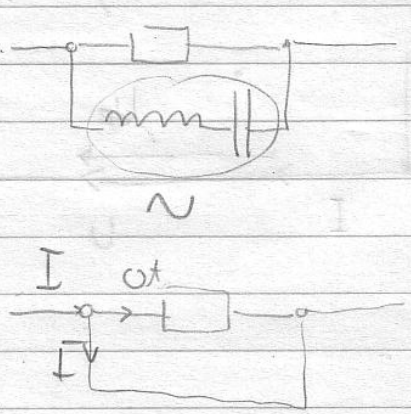
$$\dot{Z}_{LC} = \frac{\dot{X}_L \cdot \dot{X}_C}{\dot{X}_L + \dot{X}_C} = \frac{3j(-3j)}{3j - 3j} = \infty$$




$\varphi = 0$

$\mathrm{Im}\{Z_{uk}\} = 0$



KUT IMPEDANCIJE

Samo je jedan Mali Jurica

$$\dot{S} = \dot{U} \cdot (\dot{I})^* = U \cdot I \angle \varphi \ VA$$

Ukupna (prividna) snaga

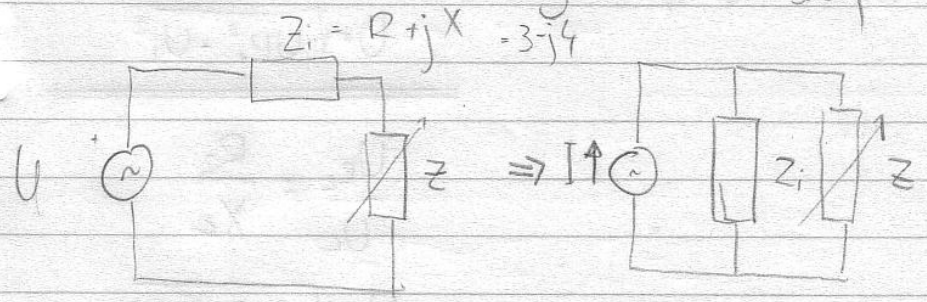
$$= U \cdot I \cdot \cos\varphi + j \cdot U \cdot I \cdot \sin\varphi$$

$P$ ; $jQ$

$Q > 0$ ind.

$[W]$ radna snaga ; $[VAr]$ jalova snaga

$Q < 0$ kap



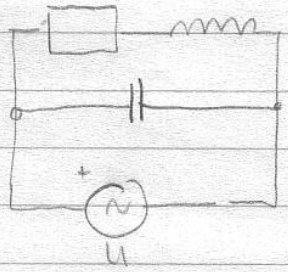

Za max snagu na Z:

$$Z = (Z_i)^* = 3 + j4$$

$Z \rightarrow$ radni otpor $(R)$

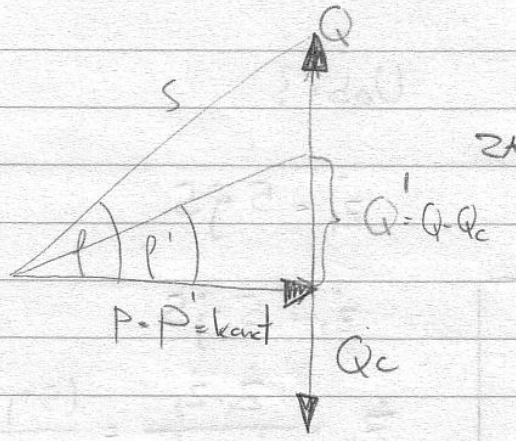
$$R = |Z_i| = \sqrt{(Re)^2 + (Im)^2}$$

$$= \sqrt{3^2 + 4^2} = 5\,\Omega$$

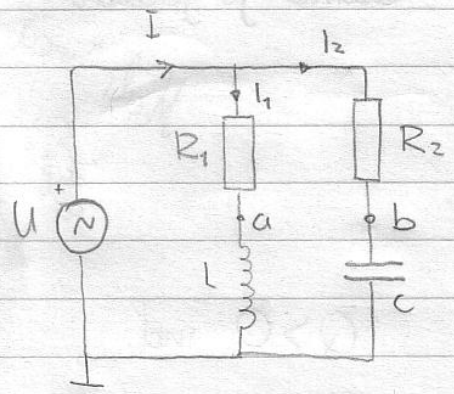


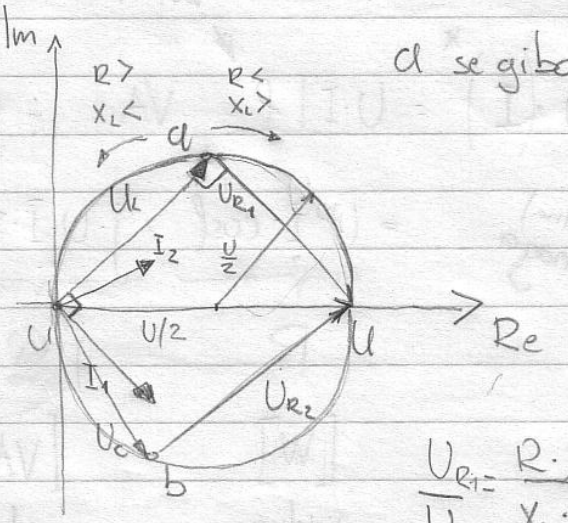

$P, S, \cos\varphi$

$C = ? \Rightarrow \cos\varphi$




ZA KOMPENZACIJU JALOVE SNAGE KONDENZATOR IDE PARALELNO




d se giba po kružnici

$U_{ab} = ?$

$\frac{U_{R_1}}{U_L} = \frac{R \cdot I_1}{X_L \cdot I_1} = \frac{R}{X_L}$

$I_1 \rightarrow$ RL (nalazi se između 0° i 90°)

$U = \sqrt{U_{R_1}^2 + U_L^2}$

I na R → 0°

na L → -90°

na C → 90°

$\frac{U_{R_2}}{U_C} = \frac{R_2}{X_C}$

$U = \sqrt{U_{R_2}^2 + U_C^2}$

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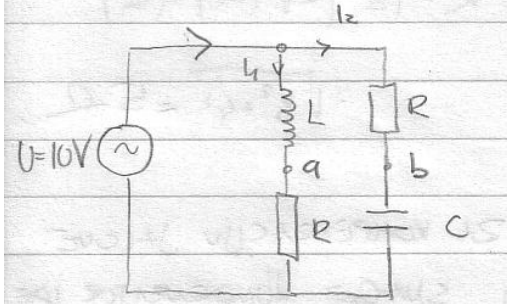


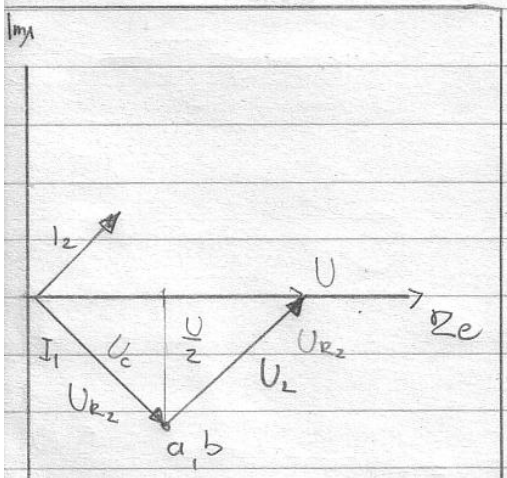

$R = X_L = X_C = 5\,\Omega$

$U_{ab} = ?$

$\dot{z}_1 = 5 + j5$

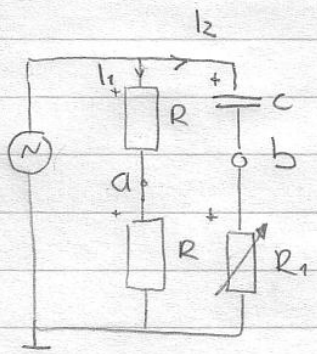
$\dot{z}_2 = 5 - j5$

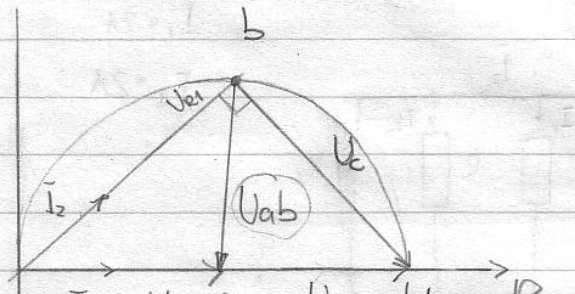
$\dot{Z}_{uk} = \frac{\dot{z}_1 \cdot \dot{z}_2}{\dot{z}_1 + \dot{z}_2} = \frac{(5+j5)(5-j5)}{5+j5+5-j5} = 5\,\Omega$

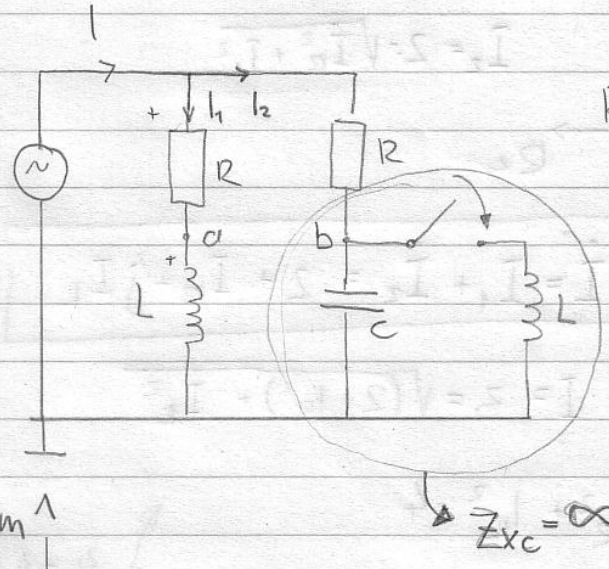



$\dot{U}_{ab} = \dot{U}_{R2} - \dot{U}_L$

$= -\dot{U}_C + \dot{U}_{R1}$



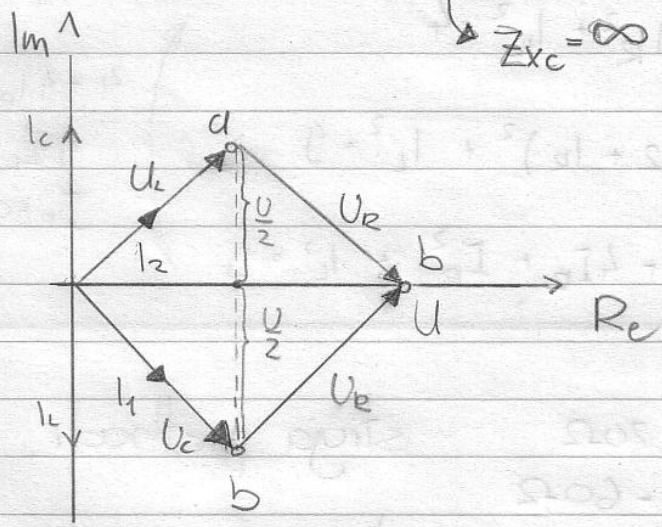

$R = X_L = X_C = 10\,\Omega$

$U_{ab} = ?$ (kako se promijeni kad se sklopka zatvori)

$\underline{I_1} = \underline{|-45^\circ}$ ⇒ jer su R i $X_L$ jednaki

$\underline{I_2} = \underline{|45^\circ}$ ⇒ jer su R i $X_C$ jednaki

$Z_{XC} = \infty$

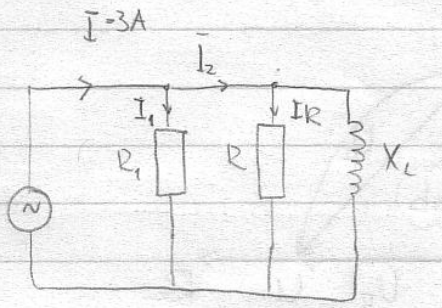



$U_{ab} = U$

$U_{ab} = \frac{U}{\sqrt{2}}$

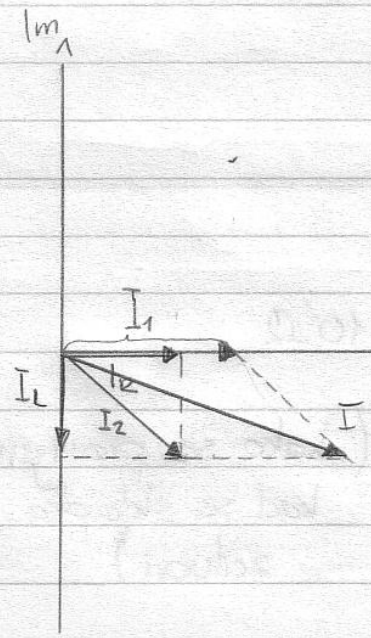
$I_C = \frac{U}{X_L} = 10A$

na LC ⇒ U




$I_1 = 2A$

$I_2 = 2A$




$$\dot{I}_2 = I_R - jI_L$$

$$I_2 = 2 = \sqrt{I_R^2 + I_L^2}$$

$$\boxed{\dot{I} = I_1 + \dot{I}_2 = 2 + I_R - jI_L}$$

$$I = 3 = \sqrt{(2+I_R)^2 + I_L^2}$$

$$I_R^2 + I_L^2 = 4$$

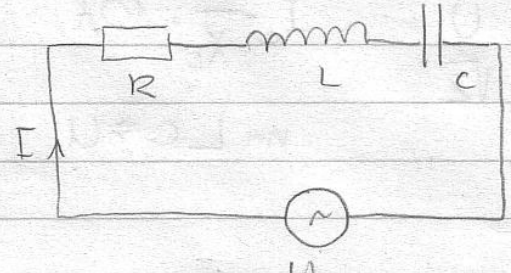
$$(2+I_R)^2 + I_L^2 = 9$$

$$4 + 4I_R + I_R^2 + I_L^2 = 9$$

$$4 + 4I_R = 9 - 4$$

$$4I_R = 1$$

$$I_R = 0{,}25A$$




$X_L = 20\Omega$

$X_C = 60\Omega$

struja prethodi ukupnom naponu za $30°$

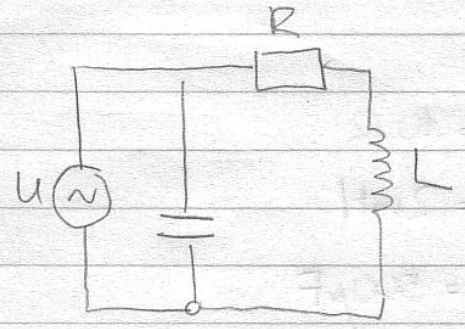
$\dot{I} = I \angle \delta_i$

$\dot{U} = U \angle \delta_u$

$\delta_i = 30° + \delta_u$

$$\dot{Z} = \frac{\dot{U}}{\dot{I}} = \frac{U \angle \delta_u}{I \angle 30° + \delta_u} = Z \angle -30°$$

$$\dot{Z} = R + jX_L - jX_C = R - j40 \Rightarrow \varphi = \operatorname{arctg} \frac{-40}{R}$$




$R = 30\,\Omega$

$L = 2\,mH$

$C = 45\,\mu F$

$P = ?$

→ REZONANCIJA

$P = 0$

$\mathrm{Im}\{Z_{uk}\} = 0$

$Q = 0$

U, I u fazi

$$Z_{uk} = RL \| C = \frac{(R + jX_L)(-jX_C)}{R + jX_L - jX_C} = \frac{X_L X_C - jRX_C}{R + j(X_L - X_C)} \cdot \frac{R - j(X_L - X_C)}{R - j(X_L - X_C)}$$

$$= \frac{RX_L X_C - RX_C(X_L - X_C)}{R^2 + (X_L - X_C)^2} - j\,\frac{R^2 X_C + X_L X_C (X_L - X_C)}{R^2 + (X_L - X_C)^2}$$

$= 0$

$$R^2 X_C + X_L X_C (X_L - X_C) = 0 \quad / : X_C$$

$$30^2 + \omega L \left(\omega L - \frac{1}{\omega C}\right) = 0$$

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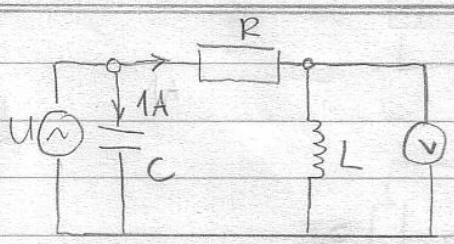



$I_C = 1A$

$U_V = 6V$

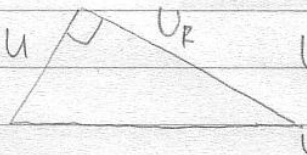
$U = 10V$

$X_L = ?$

$\mathrm{Im}\{Z_{uk}\} = 0$

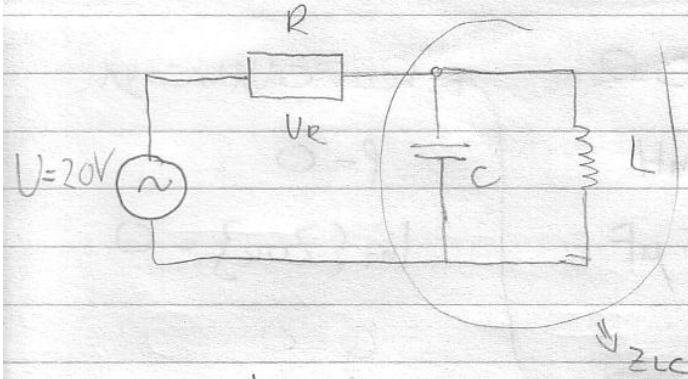
$R^2 + X_L(X_L - X_C) = 0$

$$X_C = \frac{U}{I_C} = 10\,\Omega$$




$U_R = \sqrt{U^2 - U^2} = 8V$

$$\frac{U_R}{U_L} = \frac{I_{RL} \cdot R}{I_{RL} \cdot X_L} = \frac{R}{X_L} = \frac{8}{6}$$




$R = 10\,\Omega$

$L = 2\,mH$

$C = 500\,\mu F$

$\omega = 1000\ rad/s$

$X_L = \omega L = 2\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{10^3 \cdot 0{,}5 \cdot 10^{-3}} = 2\,\Omega$

$Z_{LC} = \frac{jX_L(-X_C)}{jX_L - jX_C} = \infty$

$Z_{uk} = R + Z_{LC} = \infty\,\Omega$

$I_{UK} = 0$

$U_R = 0V$

$\dot{U}_{RC} = 20$

$\dot{I}_L = \frac{\dot{U}_{LC}}{jX_L} = \frac{20}{j2} = -j10\,A$

$\dot{I}_R = \frac{\dot{U}_{LC}}{-jX_C} = j10\,A$

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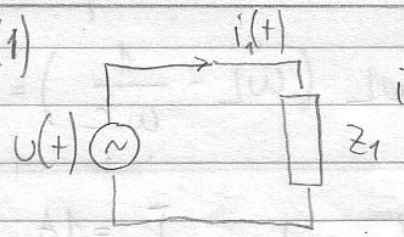
$u(t) = 10\sin(\omega t + \frac{\pi}{4})$

$i(t) = 2\cos(\omega t)$

$i_2(t)$ $\quad Z_2 = 20\angle 30°$

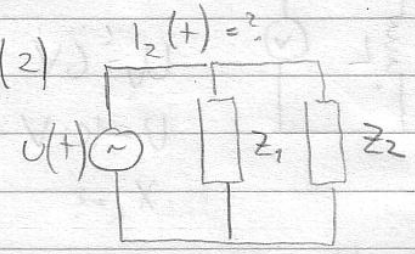
$\dot{U} = \frac{10}{\sqrt{2}}\angle 45°$ | $\dot{I} = \frac{2}{\sqrt{2}}\angle 90°$

(1)




$i(t) = 2\sin(\omega t + \frac{\pi}{2})$

(2)



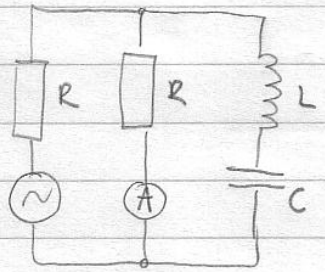

$\dot{Z} = \frac{\dot{U}}{\dot{I}} = \frac{\frac{10}{\sqrt{2}}\angle 45°}{\frac{2}{\sqrt{2}}\angle 90°} = 5\angle -45°$

$\dot{Z}_{uk} = \frac{\dot{Z}_1 \cdot \dot{Z}_2}{\dot{Z}_1 + \dot{Z}_2} = \frac{100\angle -15°}{3{,}53 - j3{,}53 + 17{,}32 + j10} = \frac{}{21 + j6{,}5} = \frac{100\angle -15°}{22\angle 17°} = 4{,}5\angle -32°$

$$\dot{I}_2 = \frac{\dot{U}}{\dot{Z}_{ok}} = \frac{\frac{10}{\sqrt{2}}\angle 45^\circ}{\frac{4,5}{1}\angle -32^\circ} = 1.57\angle 77^\circ$$

$$i_2(t) = 1.57\sqrt{2}\,\sin(\omega t + 77^\circ)$$

RADIJANI!




$$u(t) = 10\sqrt{2}\,\sin(314t)\ V$$

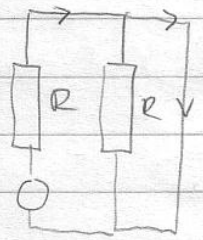
$R = 100\ \Omega$

$X_L = 10\ \Omega$

$C = ?$ $\quad X_L = X_C$

$I_A = 0$ $\quad X_C = 10\ \Omega$

$$\frac{1}{\omega C} = 10$$

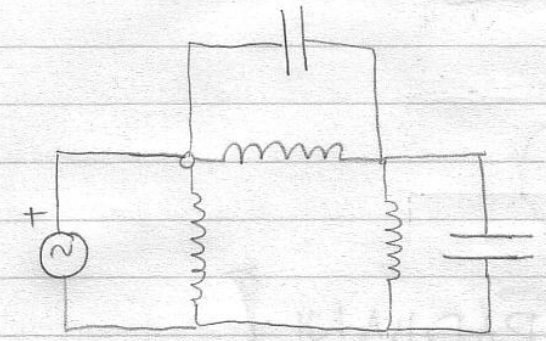



$Z_{uk} = R$

$$I_{uk} = \frac{10}{100} = 0.1\ A$$

$$U_C = I X_C = 1\ V //$$

TYWINA ZA PREDSJEDNIKA!



$L = 4\,mH$

$C = 20\,\mu F$

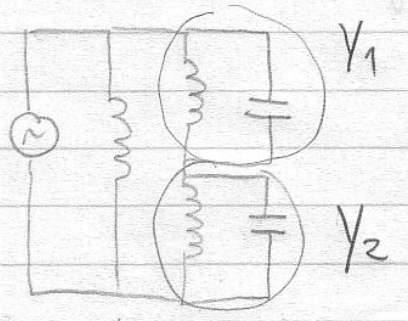
$Z_{uk} = \frac{U}{I} = \infty$

$f = ?$

$I = 0$

$Y_{uk} = 0 \qquad G = \frac{1}{R}$

$\dot{X}_C = \frac{-j}{\omega C} = \frac{1}{B_C}$




$B_C = j\omega C$

$\dot{X}_L = j\omega L = \frac{1}{B_L}$

$B_L = \frac{-j}{\omega L}$

$\dot{Y}_1 = \dot{B}_C + \dot{B}_L = j(B_C - B_L)$

$\dot{Y}_2 = \dot{Y}_1$

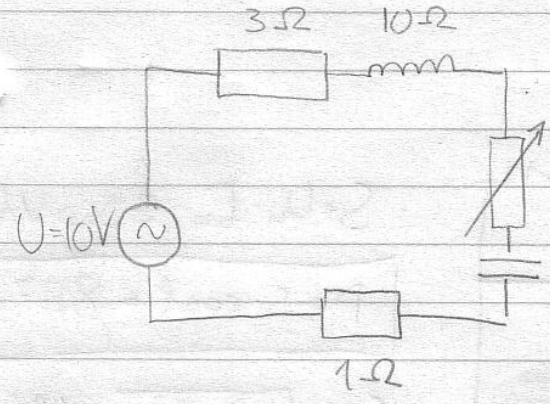
$$\dot{Y}_{12} = \frac{\dot{Y}_1 \cdot \dot{Y}_2}{\dot{Y}_1 + \dot{Y}_2} = \frac{j(B_C - B_L) \cdot j(B_C - B_L)}{2 \cdot j(B_C - B_L)} = j\frac{B_C - B_L}{2}$$

$\dot{Y}_{uk} = \dot{Y}_{12} + \dot{B}_L =$

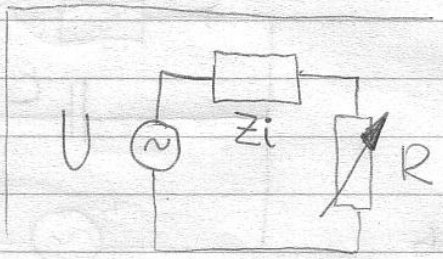
$= j\frac{B_C}{2} - j\frac{B_L}{2} - j\frac{B_L}{2}$

$B_C = 3B_L$

$\omega C = \frac{3}{\omega L}$




$P_{MAX}$ na $R = ?$




$$Z_i = 3 + j10 - j7 + 1$$

$$= 4 + j3$$

$$R = |Z_i| = \sqrt{4^2 + 3^2} = 5\,\Omega$$

$$\dot{Z}_{uk} = \dot{Z}_i + R$$

$$= 4 + j3 + 5$$

$$= 9 + j3$$

$$Z_{uk} = \sqrt{9^2 + 3^2} = \sqrt{90}$$

$$I = \frac{U}{Z} = \frac{10}{\sqrt{90}}$$

$$P = U_R \cdot I$$

$$= I \cdot R \cdot I = I^2 R$$

$$= \frac{100}{90} \cdot \sqrt{90}\ W$$

2° slučaj: sve isto; umjesto ~~R~~ je Z

$$Z = Z_i^* = 4 - j3\,\Omega \qquad I = \frac{U}{Z_{uk}} = \frac{10}{8}$$

$$Z_{uk} = Z + \dot{Z}_i$$

$$= 8\,\Omega$$

$$P_Z = I^2 \cdot Re\{Z\} = \frac{100}{64} \cdot 4\ W$$

$$Q = I^2 \cdot Im\{Z\} = \frac{100}{64} \cdot (-3)\ VA_r$$

$$\dot{S} = P + jQ$$

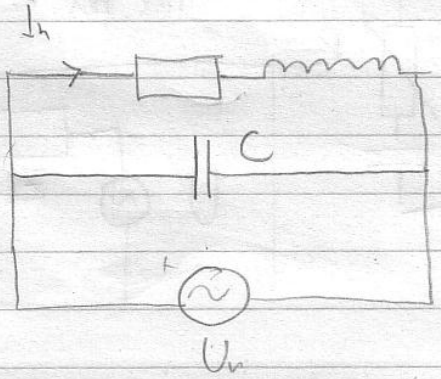
$$S_Z = \sqrt{P_Z^2 + Q_Z^2}$$

$U_n = 220\,V$

$f = 50\,Hz$

$I_n = 4{,}5\,A$

$\cos\varphi = 0{,}866$




$S = U_n \cdot I_n = 990\,VA$

$\boxed{P = S \cdot \cos\varphi = 857\,W}$

$Q = \sqrt{S^2 - P^2} = 496\,VAr$

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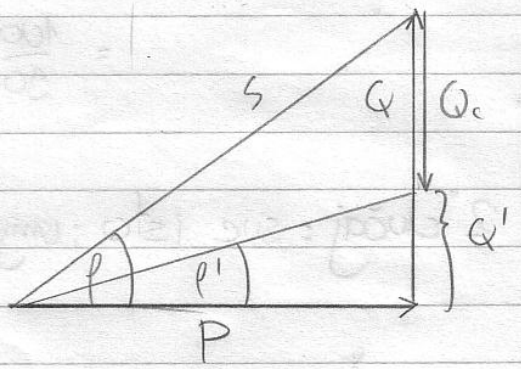
sa C

$P = S' \cdot \cos\varphi \Rightarrow S' = 902\,VA$

$Q' = \sqrt{S'^2 - P^2}$

$= 281\,VAr$




$Q - Q_c = Q'$

$Q_c = Q - Q'$

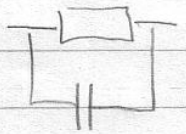
$= 215\,VAr$

$= U \cdot I_c = \frac{U_n^2}{X_c}$

$\frac{1}{\omega C} = X_c = \frac{U_n^2}{Q_c}$

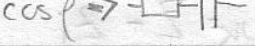
$\cos\varphi - \frac{P}{S} = \frac{P}{\sqrt{P^2+Q^2}}$

RC



$\cos\varphi = 0{,}5$

$U_c = U_R$

$\cos\varphi' \Rightarrow$ 

$\cos\varphi = \frac{P}{\sqrt{P^2+Q^2}}$

ZADATAK SA PROŠLOGODIŠNJEG MEĐUISPITA

$\frac{U}{Z}$

$\dot{S} = \dot{U}(\dot{I})^* = U \cdot I \angle\varphi$

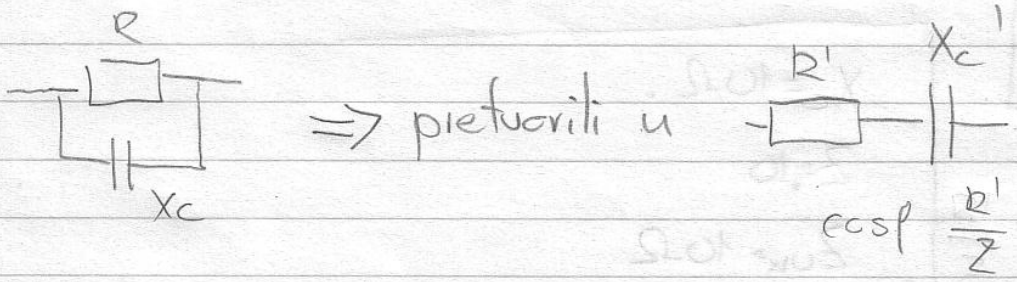
$= \frac{U^2}{Z}\cos\varphi + j\frac{U^2}{Z}\sin\varphi$

P Q

## II. NAČIN ZA RJEŠAVANJE (pretvaranje paralele u seriju)

$$Z_{uk} = \frac{-jRX_c}{R-jX_c} = \frac{RX_c^2 - jR^2X_c}{R^2+X_c^2}$$

$$= \underbrace{\frac{RX_c^2}{R^2+X_c^2}}_{R'} - j\underbrace{\frac{R^2X_c}{R^2+X_c^2}}_{X_c'}$$

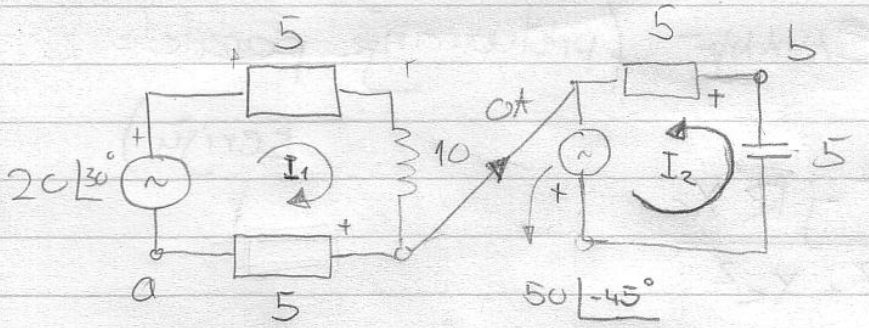



$\varphi = 60°$

$$tg\varphi = \frac{X_c'}{R'} = \frac{R^2X_c}{RX_c^2} = \frac{R}{X_c}$$

$$tg\varphi' = \frac{X_c}{R} \Rightarrow \varphi'$$

$$\Rightarrow \cos\varphi'$$




$U_{ab} = -5\dot{I}_2 - 5\dot{I}_1$

$U_{ab} = |-5\dot{I}_2 - 5\dot{I}_1|$

$U_{ab} = ?$

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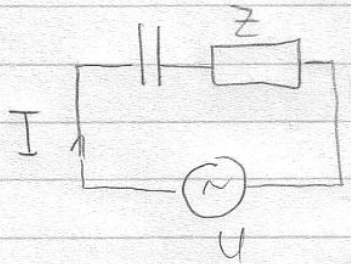
VII 1-14.

$I = 10\,A$

$U_c = U_Z = U = 100\,V$




$X_c = 10\,\Omega$

$Z = 10$

$Z_{UK} = 10\,\Omega$

$\dot{Z} = R + jX$ $\quad\Rightarrow\quad Z = 10 = \sqrt{R^2 + X^2}$

$\dot{Z}_{UK} = R + jX - jX_c$ $\quad\Rightarrow\quad \dot{Z}_{UK} = 10 = \sqrt{R^2 + (X-10)^2}$

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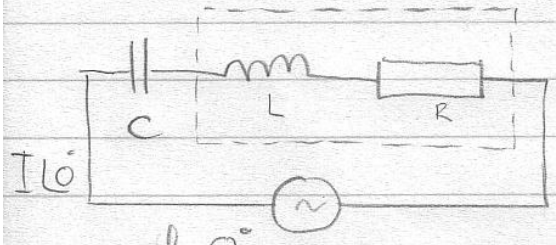



$U = 120\,V$

$U_c = 160\,V$

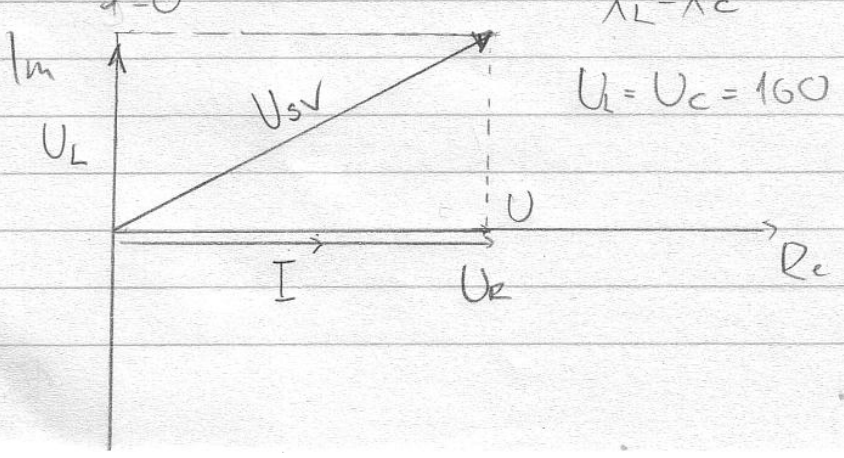
$X_L = X_c$

$U_L = U_c = 160$

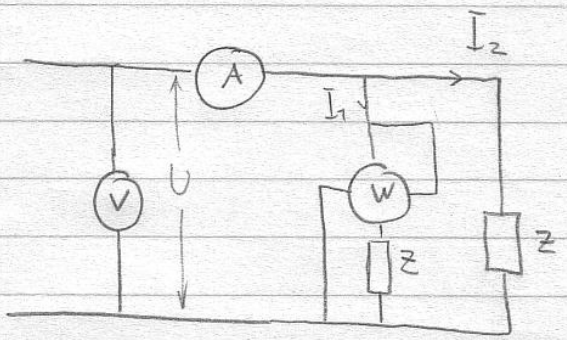
$\dot{U} = \dot{U}_R + j\dot{U}_L - j\dot{U}_c$

$= \dot{U}_R$

$U_R = 120\,V$




$U_{SV} = \sqrt{U_R^2 + U_L^2}$




$I = 10\,A$

$U = 100\,V$

$\dot{Z} = R + jX_L$

$R = \frac{1}{4} Z$

$I_1 = 5A$

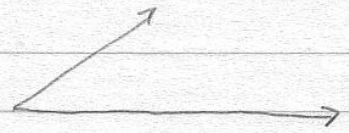
$P = U \cdot I_1 \cdot \cos\varphi$

$Z_{UK} = \frac{Z}{2} = 10\Omega$

$Z = 20\Omega$

$R = 5\Omega$

$X = \sqrt{Z^2 - R^2}$



$P = I_1^2 \cdot R = 125\,W$