BURIĆ SRIJEDA 10.6.15

13.1 -> TEORIJA uvijek na ispitu

13.2, 13.3, 13.4 Linearne dif. jdžbe višeg reda s konst. koef.

$$y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y = f(x)$$

O.R. $y = y_h + y_p$

homogeni dio $y^{(n)} + \dots + a_1 y' + a_0 y = 0$

-> pripadna karakt. jdžba:

$$r^n + \dots + a_1 r + a_0 = 0 \qquad \boxed{y^{(n)} \to r^n}$$

-> n nultočki

- ako su r_i različita: $e^{r_1 x}, e^{r_2 x}, e^{r_3 x} \dots$
- ako su r_i ista: $e^{r_1 x}, x e^{r_1 x}, x^2 e^{r_1 x}, \dots$
- ako su $r_{ij} = \alpha \pm i\beta$: $e^{\alpha x} \cos \beta x, e^{\alpha x} \sin \beta x$

zad) $y^{IV} - 3y''' + 3y'' - y' = 0$

$r^4 - 3r^3 + 3r^2 - r = 0$

$r(r^3 - 3r^2 + 3r - 1) = 0$

$r(r-1)^3 = 0$

$r_1 = 0 \quad r_{2,3,4} = 1$ <- pojavljuje se 3 puta!

$$y = C_1 e^0 + C_2 e^{1x} + C_3 x e^x + C_4 x^2 e^x$$

[illegible]

21-14-8)

c) $y''' + 2y'' + 3y' + 6y = 0$

$r^3 + 2r^2 + 3r + 6 = 0$

$r^2(r+2) + 3(r+2) = 0$

$(r+2)(r^2+3) = 0$

$r_1 = -2 \qquad r^2 = -3$

$r_{23} = \pm i\sqrt{3}$

$$y = C_1 e^{-2x} + C_2 \cos\sqrt{3}x + C_3 \sin\sqrt{3}x$$

$\alpha = 0, \ \beta = \sqrt{3}$

Dokažite da su rj. lin. nez.

BEZ KONST. UNOSIMO U W
→ konst nisu bitne

$$W(y_1, y_2, y_3) = \begin{vmatrix} e^{-2x} & \cos\sqrt{3}x & \sin\sqrt{3}x \\ -2e^{-2x} & -\sqrt{3}\sin\sqrt{3}x & \sqrt{3}\cos\sqrt{3}x \\ 4e^{-2x} & -3\cos\sqrt{3}x & -3\sin\sqrt{3}x \end{vmatrix} \cdot 3$$

$$= \begin{vmatrix} e^{-2x} & \cos\sqrt{3}x & \sin\sqrt{3}x \\ -2e^{-x} & -\sqrt{3}\sin\sqrt{3}x & \sqrt{3}\cos\sqrt{3}x \\ 7e^{-2x} & 0 & 0 \end{vmatrix} = 7e^{-2x}\left(\sqrt{3}\cos^2\sqrt{3}x + \sqrt{3}\sin^2\sqrt{3}x\right) =$$

$= 7\sqrt{3}e^{-2x} \neq 0$

$\Rightarrow y_1, y_2, y_3$ su linearno nez.

ALGORITAM ZA ODREĐIVANJE PART. RJ!

1. Ako je $f(x)=e^{\alpha x}(p_1(x)\cos\beta x + P_2(x)\sin\beta x)$, tada je

$\boxed{y_p = e^{\alpha x}(R_1(x)\cos\beta x + R_2(x)\sin\beta x)x^m}$ pri čemu su

R_1 i R_2 polinomi stupnja max od stupnja P_1 i P_2

a m je kratnost nultočke $r_{1,2}=\alpha+i\beta$ u karakt. jdžbi

2. Ako je $f(x)=f_1(x)+\dots+f_n(x)$, tada je

$$y_p = y_{p_1}+y_{p_2}+\dots+y_{p_n}$$

3. Ako $f(x)$ nije oblika iz (1) tada radimo metodu varijacije konstanti

Npr. a) $f(x)=x^3+1 \rightarrow y_p = Ax^3+Bx^2+Cx+D$
$(\alpha=0, \beta=0)$
Provjera dal je $r=0$ rj. homog.?

b) $f(x)=\pi \sin 2x \rightarrow y_p = A\sin 2x + B\cos 2x$
$\alpha=0, \beta=2$
provjera dal je $r=\pm 2i$ rj karakt.

c) $f(x)=-e^{5x} \rightarrow y_p = Ae^{5x}$
$\alpha=5, \beta=0$ provjera za $r=5$

3 DZ-(a,b) $y'''-y''+y'-y=x^2+x$

1. $r^3-r^2+r-1=0$

$r^2(r-1)+r-1=0$

$(r-1)(r^2+1)=0$

$r_1=1 \quad r_{2,3}=\pm i \quad \alpha=0, \beta=1$

$y_h = C_1e^x + C_2\cos x + C_3\sin x$

2. $y_p = Ax^2 + Bx + C$ $\alpha = 0$? → nije rj. karakt. jedžbe pa nemamo x^m

$\alpha = 0$ kod cijele rj. ne odnosi se samo na onaj $\alpha = 0$ u 1. dijelu

$y' = 2Ax + B$

$y'' = 2A$ $\quad 0 - 2A + 2Ax + B - Ax^2 - Bx - C = x^2 + x$

$y''' = 0$ $\quad -Ax^2 = x^2 \Rightarrow A = -1$

$2Ax - Bx = x \rightarrow 2A - B = 1 \rightarrow B = 2A - 1 = -3$

$-2A + B - C = 0 \rightarrow C = -2A + B \Rightarrow C = -1$

$y_p = -x^2 - 3x - 1$

Konačno rj. $y = C_1 e^x + C_2 \cos x + C_3 \sin x - x^2 - 3x - 1$

21-09-7) $y'' + 3y' - 4y = (10x + 2)e^x$

1.) $y'' + 3y' - 4y = 0$!

$r^2 + 3r - 4 = 0$

$r_1 = 1, \; r_2 = -4$

$y_h = C_1 e^x + C_2 e^{-4x}$

→ ISTA FJA S RAZL. KONSTANTAMA

2.) $y_p = (Ax + B)e^x \cdot x$ $\quad \alpha = 1$? → rj. karakt je → 1 put → množimo s x

$= (Ax^2 + Bx)e^x$

$y' = (2Ax + B)e^x + (Ax^2 + Bx)e^x = e^x(2Ax + B + Ax^2 + Bx)$

$y'' = (2A + 2Ax + B)e^x + (2Ax + B + Ax^2 + Bx)e^x$

→ uvrštavamo $\quad = e^x(2A + 4Ax + 2B + Bx + Ax^2)$

$\cancel{e^x}(2A + 4Ax + 2B + \cancel{Bx} + \cancel{Ax^2}) + \cancel{e^x}(6Ax + 3B + \cancel{3Ax^2} + \cancel{3Bx}) + \cancel{e^x}(\cancel{-4Ax^2} - \cancel{4Bx})$

$= (10x + 2)\cancel{e^x}$

$10Ax + 2A + 5B = 10x + 2$

$10A = 10 \rightarrow A = 1$ $\quad y_p = (1x^2 + C)e^x = x^2 e^x$

$2A + 5B = 2 \rightarrow B = 0$ $\quad \Rightarrow \boxed{y = C_1 e^x + C_2 e^{-4x} + x^2 e^x}$

13 DZ - 9 B) $y'' + y = 4x\cos x$

1.) $y'' + y = 0$

$r^2 + 1 = 0$

$r_{1,2} = \pm i$

$y_h = C_1 \cos x + C_2 \sin x$

cijeli polinom! polinom istog stupnja + sin

2.) $y_p = \left[(Ax + B)\cos x + (Cx + D)\sin x\right] x$

$\left.\begin{matrix} \alpha = 0 \\ \beta = 1 \end{matrix}\right\} \pm i \rightarrow$ je li rj? DA $x^1 \Rightarrow x$

$(Ax^2 + Bx)\cos x + (Cx^2 + Dx)\sin x$

$y' = (2Ax + B)\cos x + (Ax^2 + Bx)(-\sin x) + (2Cx + D)\sin x + (Cx^2 + Dx)\cos x$

$= \cos x(2Ax + B + Cx^2 + Dx) + \sin x(2Cx + D - Ax^2 - Bx)$

$y'' = -\sin x(2Ax + B + Cx^2 + Dx) + \cos x(2A + 2Cx + D) + \cos x(2Cx + D - Ax^2 - Bx) +$
$\sin x(2C - 2Ax - B)$ → ne može se ništa ovdje kratiti nikad tek poslije !!

$\cos x(2A + 4Cx + 2D - \cancel{Ax^2} - \cancel{Bx}) + \sin x(2C - 4Ax - 2B - \cancel{Cx^2} - \cancel{Dx}) +$
$\cos x(\cancel{Ax^2} + \cancel{Bx}) + \sin x(\cancel{Cx^2} + \cancel{Dx}) = 4x\cos x$

ovdje kraćenje mora biti MORA!

$2A + 4Cx + 2D = 4x$ $\qquad$ $2C - 4Ax - 2B = 0$

$4C = 4 \Rightarrow C = 1$ $\qquad$ $-4A = 0 \Rightarrow A = 0$

$2A + 2D = 0$ $\qquad$ $2C - 2B = 0 \Rightarrow B = 1$

$D = 0$

Rj. $y = C_1 \cos x + C_2 \sin x + x\cos x + x^2 \sin x$

21-13-5) $y'' - 2y' + y = -\frac{1}{x^2} e^x$, $y(1) = -e$, $y'(1) = 0$

nije polinom!

→ mora biti analog oblik stoga

METODA VAR. KONST!

1. $r^2 - 2r + 1 = 0$

$r_{1,2} = 1$

$y_h = C_1 e^x + C_2 x e^x$

općenito

$y_h = C_1(x) y_1 + \dots + C_n(x) y_n$

$C_1'(x) y_1 + \dots + C_n'(x) y_n = 0$

$C_1'(x) y_1' + \dots + C_n'(x) y_n' = 0$

$\vdots$

$C_1'(x) y^{(n-1)} + \dots + C_n'(y_n)^{(n-1)} = f(x)$

SVE 0 OSIM ZADNJE

2.) $y = C_1(x) e^x + C_2(x) x e^x$

$$- \begin{cases} C_1'(x) e^x + C_2'(x) x e^x = 0 \\ C_1'(x) e^x + C_2'(x)(e^x + x e^x) = -\frac{1}{x^2} e^x \end{cases}$$

$-C_2'(x) \cancel{e^x} = \frac{1}{x^2} \cancel{e^x}$

$C_2'(x) = -\frac{1}{x^2}$

uvrstimo u 1 => $C_1'(x) + \frac{-1}{x^2} x = 0$

$C_1'(x) = \frac{1}{x}$

integriramo

$C_2(x) = -\frac{1}{x^2}$ $\qquad$ $C_1'(x) = \frac{1}{x}$

$C_2(x) = \frac{1}{x} + D_2$ $\qquad$ $C_1(x) = \ln(x) + D_1$

O.R. $y = (\ln x + D_1) e^x + (\frac{1}{x} + D_2) x e^x$

uvrštavamo početne uvjete

$D_1 e + (1 + D_2) e = -e \quad /:e$

$D_1 + 1 + D_2 = -1$

$D_1 + D_2 = -2$

$y' = (\frac{1}{x}) e^x + (\ln x + D_1) e^x + (-\frac{1}{x^2}) x e^x + (\frac{1}{x} + D_2)(e^x + x e^x)$

uvrstimo 1 => $\cancel{e} + D_1 e - \cancel{e} + (1 + D_2)(e + e) = 0 \quad /:e$

$D_1 + D_2 = -2$ $\qquad$ $D_1 + (1 + D_2) \cdot 2 = 0$

$D_1 + 2D_2 = -2$ $\qquad$ $D_1 + 2D_2 = -2$

$-D_2 = 0$

$D_2 = 0$ $\quad$ $D_1 = -2$

KONAČNO => $y = e^x (\ln x - 1)$

BURIĆ PETAK 12.06.2015

12 DZ - 11 B) $y'' + 4y' = x + e^{-4x}$ DVA RAZL. PART. RJ

1. $y'' + 4y' = 0$

$r^2 + 4r = 0$

$r_1 = 0, \quad r_2 = -4$

$y_h = C_1 + C_2 e^{-4x}$

2. $f_1(x) = x$ $\quad \alpha = 0 \rightarrow$ je li rj. hom

$\rightarrow$ DA $\rightarrow$ koliko puta

$\rightarrow$ 1 put $\rightarrow$ množimo s x

$y_{p_1} = (Ax + B)x$

$y_{p_1} = Ax^2 + Bx$

$\rightarrow$ uvrstimo u poč.

$y' = 2Ax + B$

$y'' = 2A$

$\rightarrow 2A + 8Ax + 4B = x$

$8A = 1 \rightarrow A = \frac{1}{8}$

$2A + 4B = 0$

$B = -\frac{1}{16}$

$y_{p_1} = \frac{1}{8}x^2 - \frac{1}{16}x$

$f_2(x) = e^{-4x} \rightarrow y_{p_2} = Ae^{-4x} \cdot x$

$\alpha = -4 \rightarrow$ je li rj $\rightarrow$ DA $\rightarrow$ s x !

$y' = Ae^{-4x} - 4Axe^{-4x}$

$y'' = -4Ae^{-4x} - 4Ae^{-4x} + 16Axe^{-4x}$

$\rightarrow$ u početnu

$-8A\cancel{e^{-4x}} + \cancel{16Axe^{-4x}} + 4A\cancel{e^{-4x}} - \cancel{16Axe^{-4x}} = \cancel{e^{-4x}}$

$-4A = 1 \qquad A = -\frac{1}{4}$

$y_{p_2} = -\frac{1}{4}xe^{-4x}$

KONAČNO

$\rightarrow$

$y = y_h + y_{p_1} + y_{p_2} = C_1 + C_2 e^{-4x} + \frac{1}{8}x^2 - \frac{1}{16}x - \frac{1}{4}xe^{-4x}$

Obavezno napisati zadnje rj.

ZI - 11-12) $y^{IV} - y'' = e^x \sin x$

1. $y^{IV} - y'' = 0$

$r^4 - r^2 = 0$

$r^2(r^2-1) = 0$

$r_{1,2} = 0 \quad r_{3,4} = \pm 1$

$y_h = C_1 + C_2 x + C_3 e^{1x} + C_4 e^{-x}$

2. $y_p = e^x(A\sin x + B\cos x)$ - polinom istog stupnja i e^x ispred njega

$\alpha = 1, \beta = 1 \} 1 \pm 1i$

→ je li vj. homogene → nije (nema ei kom nigdje sinusa, znači neće se ponavljati!, ne moramo množiti s x)

$y' = e^x(A\sin x + B\cos x) + e^x(A\cos x - B\sin x)$

$= e^x(A\sin x + B\cos x + A\cos x - B\sin x)$ ← odma izlučujemo

$y'' = e^x(A\sin x + B\cos x + A\cos x - B\sin x + A\cos x - B\cos x - B\sin x - A\sin x)$

$y'' = e^x(2A\cos x - 2B\sin x)$

$y''' = e^x(2A\cos x - 2B\sin x - 2A\sin x - 2B\cos x)$

$y^{IV} = e^x(\cancel{2A\cos x} - \cancel{2B\sin x} - 2A\sin x - 2B\cos x - 2A\sin x - 2B\cos x - \cancel{2A\cos x} + \cancel{2B\sin x}$

u početku!

$\cancel{e^x}(-4A\sin x - 4B\cos x) - \cancel{e^x}(2A\cos x - 2B\sin x) = \cancel{e^x}\sin x$

→ da smo množili s x ovdje bi se krotilo

$\sin x(\underbrace{-4A + 2B}_{=1}) + \cos x(\underbrace{-4B - 2A}_{=0}) = \sin x$

$-4A + 2B = 1 \quad / \cdot 2$

$-2A - 4B = 0$

$-10A = 2$

$\boxed{A = -\frac{1}{5}} \quad \boxed{B = \frac{1}{10}}$

$\boxed{y = C_1 + C_2 + C_3 e^x + C_4 e^{-x} + e^x\left(-\frac{1}{5}\sin x + \frac{1}{10}\cos x\right)}$

- 12-10)

$$y''' + y' = \frac{1}{\cos^2 x}$$

$$y''' + y' = 0$$

$$r^3 + r = 0$$

$$r(r^2+1) = 0$$

$$r_1 = 0 \quad r_{2,3} = \pm i$$

$$y_h = C_1 e^0 + C_2 \cos x + C_3 \sin x \qquad e^0 = 1$$

$$y_h = C_1 + C_2 \cos x + C_3 \sin x$$

$$y = C_1(x) + C_2(x)\cos x + C_3(x)\sin x$$

$$C_1'(x) + C_2'(x)\cos x + C_3'(x)\sin x = 0$$

$$\cancel{C_1'(x)}\, 0 + C_2'(x)(-\sin x) + C_3'(x)\cos x = 0 \qquad \cdot \sin x$$

$$\cancel{C_1'(x)}\, 0 + C_2'(x)(-\cos x) + C_3'(x)(-\sin x) = \frac{1}{\cos^2 x} \qquad \cdot \cos x \quad \Big] \oplus$$

$$-C_2'(x)\sin^2 x - C_2'(x)\cos^2 x = \frac{1}{\cos x} \Rightarrow \boxed{C_2'(x) = -\frac{1}{\cos x}}$$

uvistimo C_2 u 2.

$$-\frac{1}{\cos x}(-\sin x) + C_3'(x)\cos x = 0 \rightarrow C_3'(x) = -\frac{\sin x}{\cos^2 x}$$

$\boxed{1. + 3.}$ $$C_1'(x) = \frac{1}{\cos^2 x}$$

$$C_1(x) = \int \frac{dx}{\cos^2 x} = \operatorname{tg} x + D_1$$

$$C_2(x) = -\int \frac{dx}{\cos x} = -\ln\left|\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + D_2$$

$$C_3(x) = \int \frac{\sin x}{\cos^2 x}\,dx = \left| \begin{matrix} \cos x = t \\ \dots \end{matrix} \right| = -\frac{1}{\cos x} + D_3$$

$$y = (\operatorname{tg} x + D_1) + \left(-\ln\left|\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + D_2\right)\cos x + \left(-\frac{1}{\cos x} + D_3\right)\sin x$$

$$y = D_1 \underbrace{- \ln\left|\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|}_{y_p} + \underbrace{D_2 \cos x + D_3 \sin x}_{y_h}$$

Zad 7) - primjer kamu flaže

$$\dots = 2\cos^2 x \sin^2 x - 3\cos^2 x - 1 = \frac{3}{2}(1+\cos 2x) - 1$$

$$f(x) = \frac{1}{2} + \frac{3}{2}\cos 2x \quad \alpha = 0, \ \beta = 2$$

POKUŠATI NAMJESTITI

$$y_{P_1} = A \qquad y_{P_2} = A\cos 2x + B\sin 2x$$

...

13.5 → preskačemo [zadnja 4. u DZ] } ne treba
6, 7, 8, 9, → ZZV

13.6 Rješavanje dif. jdžbi pomoću redova

IR - 2012 - 10) $(1-x^2)y'' - 2xy' + 20y = 0, \quad y(0) = 1, \ y'(0) = 0$

→ LINEARNA

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} a_n \cdot n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2}$$

$$\sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} - \sum_{\cancel{n=2}\ 0}^{\infty} a_n n(n-1) x^n - 2\sum_{\cancel{n=1}\ 0}^{\infty} a_n \cdot n \cdot x^n + 20\sum_{\cancel{n=1}\ 0}^{\infty} a_n x^n = 0$$

Namjestiti da kreće od 0 i ima x^n!

$$\sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} \overset{\substack{=n \text{ gornji} \\ n \to n+2}}{\Rightarrow} \sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n$$ → pomaknе se sve

$$\Rightarrow \sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n - \sum_{n=0}^{\infty} a_n n(n-1)x^n - 2\sum_{n=0}^{\infty} a_n n x^n + 20\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \Big[\underbrace{a_{n+2}(n+2)(n+1) - a_n n(n-1) - 2a_n \cdot n + 20 a_n}_{=0}\Big] x^n = 0$$

$$a_{n+2}(n+2)(n+1) - a_n(n^2 - n + 2n - 20) = 0$$

$$a_{n+2}(n+2)(n+1) = a_n(n^2 + n - 20) \qquad n_1 = -5, \ n_2 = 4$$

$$a_{n+2} = \frac{(n-4)(n+5) \cdot a_n}{(n+2)(n+1)}, \quad n \geq 0$$

1. član $n=0$ $a_2 = \frac{-4\cdot 5}{2\cdot 1} a_0 = -10 a_0 = -10$

$n=1$ $a_3 = \frac{-3\cdot 6}{3\cdot 2} a_1 = 0$

Početni

$a_0 = 1$

$a_1 = 0$

} 1. i 2. koeficijent

$n=2$ $a_4 = \frac{-2\cdot 7}{4\cdot 3} a_2 = -\frac{7}{6}\cdot(-10) = \frac{35}{3}$

svi neparni su 0

$n=3$: $a_5 = 0$

$n=4$: $a_6 = 0$ ... svi ostali su 0, a_8 ide preko a_6 ...

Rj. $a_1 = a_0 + \cancel{a_1 x} + a_2 x^2 + \cancel{a_3 x^3} + a_4 x^4 + \cancel{a_5 x^5}$.

$y = 1 - 10x^2 + \frac{35}{3} x^4$

Z1-12-8) $y'^2 - y'\sin y\cos y - xy' + x\sin y\cos y = 0$

$y'(y'-x) - \sin y\cos y\,(y'-x) = 0$

$(y'-x)(y'-\sin y\cos y) = 0$

$y' = x$

$y = \frac{x^2}{2} + C_1$

$y' = \sin y\cos y = \frac{1}{2}\sin 2y$

$\frac{dy}{\sin 2y} = \frac{1}{2}dx$

$\cancel{\frac{1}{2}}\ln|\tg y| = \cancel{\frac{1}{2}}x + C_2$

$x = \ln|\tg y| + C_2$

Rj: $(y - \frac{x^2}{2} + C_1)(\ln|\tg y| - x + C_2) = 0$

Z1-11-6) Prikladnom supst. riješi: $\frac{y'}{y} + 2\ln y = x$, $y(0) = 1$

$p = \ln y$

$p' = \frac{1}{y}y'$

(po x der.)

$p' + 2p = x$

Linearna: $\frac{dp}{dx} = -2p$

$\ln p = -2x + C$

$p = Ce^{-2x}$

$C'(x)e^{-2x} - \cancel{2C(x)e^{-2x}} + \cancel{2C(x)e^{-2x}} = x$

$C'(x) = xe^{2x} \rightarrow \left| \begin{matrix} u = x & dv = e^{2x}dx \\ du = dx & v = \frac{1}{2}e^{2x} \end{matrix} \right|$

$C(x) = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

$p = (\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C)e^{-2x}$

$\Rightarrow \ln y = \frac{1}{2}x - \frac{1}{4} + Ce^{-2x}$ $(C = \frac{1}{4})$

Z1-14-6) $y = xy'^2 - 2y'^3$ (opće i singularno)

$y = xp^2 - 2p^3 \quad /'$

$p = p^2 + 2pp'x - 6p^2p' \quad /:p$

$p' = \frac{dp}{dx}$

$p + 2p'x - 6pp' - 1 = 0$

$p'(2x - 6p) = 1 - p$

$p' = \frac{1-p}{2x-6p}$

$x' = \frac{2x-6p}{1-p}$

singularno: $p = 1$

$\Rightarrow y = x - 2$

$x' - \frac{2}{1-p}x = \frac{-6p}{1-p}$

$x' + \frac{2}{p-1}x = \frac{6p}{p-1}$

Linearna: $\frac{dx}{x} = \frac{-2}{p-1}dp$

$\ln x = -2\ln|p-1| + \ln C$

$x = \frac{C}{(p-1)^2}$

$\frac{C'(p)}{(p-1)^2} - \cancel{\frac{2C(p)}{(p-1)^3}} + \cancel{\frac{2}{p-1}\frac{C(p)}{(p-1)^2}} = \frac{6p}{p-1}$

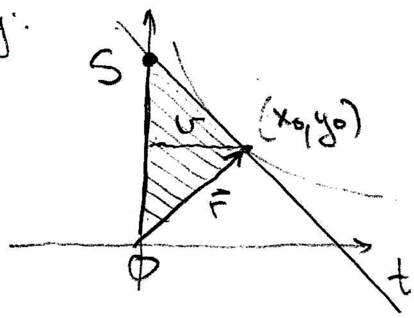
$C'(p) = 6p(p-1) = 6p^2 - 6p$

$C(p) = 2p^3 - 3p^2 + D$

$\Rightarrow \begin{cases} x(p) = \frac{2p^3 - 3p^2 + D}{(p-1)^2} \\ y(p) = xp^2 - 2p^3 \end{cases}$

21-12-7) Nađite krivulju koja prolazi točkom (2,2) i za koju je površina trokuta određenog osi y, radij-vektorom dirališta tangente i tangentom konstantne veličine A.

Rj:



$t \ldots \quad y - y_0 = y'(x_0)(x - x_0)$

$S(0, y_S): \quad y_S = y'(x_0)(-x_0) + y_0$

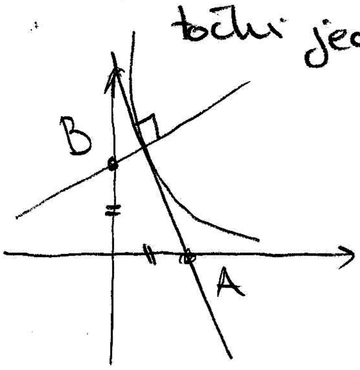
$$P_\Delta = \frac{1}{2} y_S \cdot x_0 = A$$

$$\left(y'(x_0)(-x_0) + y_0\right) \cdot x_0 = 2A$$

$$\Rightarrow \left.\begin{aligned} (-x y' + y) \cdot x &= 2A \\ y' - \frac{y}{x} &= \frac{-2A}{x^2} \end{aligned}\right\} \text{ linearna!}$$

$$y = \frac{A}{x} + \frac{4-A}{4} \cdot x$$

21-13-3) Odredite sve krivulje za koje je odsječak tangente na x-osi u svakoj točki jednak odsječku normale na y-osi.



$t \ldots \quad y - y_0 = y'(x_0)(x - x_0)$

$A(x_A, 0): \quad 0 - y_0 = y'(x_0)(x_A - x_0)$

$$x_A = x_0 - \frac{y_0}{y'(x_0)}$$

$n \ldots \quad y - y_0 = -\frac{1}{y'(x_0)}(x - x_0)$

$B(0, y_B): \quad y_B = \frac{x_0}{y'(x_0)} + y_0$

$=$

$$\Rightarrow \quad x - \frac{y}{y'} = \frac{x}{y'} + y$$

$$y' = \frac{x+y \,/:x}{x-y\,/:x} = \frac{1 + \frac{y}{x}}{1 - \frac{y}{x}}, \qquad z = \frac{y}{x}, \quad y' = z + xz'$$

$$z + xz' = \frac{1+z}{1-z}$$

$$\cancel{z} - z^2 + xz' - xzz' = 1 + \cancel{z}$$

$$xz'(1-z) = 1 + z^2$$

$$\int \frac{1-z}{1+z^2}\, dz = \int \frac{dx}{x}$$

$$\operatorname{arctg} z - \frac{1}{2} \ln|1+z^2| = \ln x + C$$

$$\operatorname{arctg} \frac{y}{x} - \frac{1}{2} \ln\left|1 + \frac{y^2}{x^2}\right| = \ln x + C$$