MINI-MASOVNE MATBE/R

FURIEROV RED

npr. $[-1, 1]$ simetričan interval $T = b - a$ npr. $[-3, 7]$

1.) Parna

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$$

2.) neparna

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

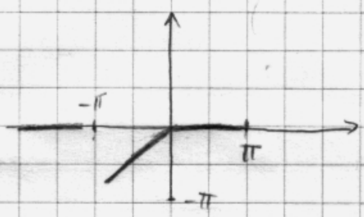
$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi x}{T} + b_n \sin\frac{2n\pi x}{T}\right)$$

$$a_0 = \frac{1}{T}\int_a^b f(x)\,dx$$

$$a_n = \frac{1}{T}\int_a^b f(x)\cos\frac{2n\pi x}{T}\,dx$$

$$b_n = \frac{1}{T}\int_a^b f(x)\sin\frac{2n\pi x}{T}\,dx$$

zadaci: 1.



nije ni parna ni neparna (simetrične s y osi i ishodištem)

$T = b - a$
$a = -\pi$
$b = \pi$

$f(x) = x$

$$a_0 = \frac{1}{T}\int_a^b f(x)\,dx$$

$$a_0 = \frac{2\cdot 1}{2\pi}\int_{-\pi}^{0} x\,dx = \frac{1}{\pi}\frac{x^2}{2}\Big|_{-\pi}^{0} = \frac{1}{\pi}\left(-\frac{(-\pi)^2}{2}\right) = \frac{-\pi}{2}$$

$\frac{2n\pi x}{2\pi} = nx$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} x\cos(nx)\,dx = \left|\begin{matrix} u = x & dv = \cos nx\,dx \\ du = dx & v = \frac{1}{n}\sin(nx)\end{matrix}\right| \quad \left\{ u\cdot v - \int v\,du \right.$$

$\sin n\pi = 0$

$$= \frac{1}{\pi}\left(\underbrace{\frac{x\sin nx}{n}\Big|_{-\pi}^{0}}_{=0} - \int_{-\pi}^{0}\frac{\sin(nx)}{n}\,dx\right) =$$

$$= \frac{-1}{n\pi}\cdot\frac{1}{n}\left(-\cos nx\right)\Big|_{-\pi}^{0}$$

$$= \frac{\cos nx}{n^2\pi}\Big|_{-\pi}^{0} = \frac{1 - \cos(-n\pi)}{n^2\pi} \qquad \cos(-n\pi) = \cos(n\pi)$$

$$= \frac{1 - (-1)^n}{n^2\pi} \quad \begin{matrix} \nearrow a_{2n} = 0 \\ \searrow a_{2n+1} = \frac{2}{(2n+1)^2\pi}\end{matrix}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx = \left|\begin{matrix} u = x & dv = \sin(nx)\,dx \\ du = dx & v = \frac{-\cos(nx)}{n}\end{matrix}\right|$$

$$= \frac{1}{\pi}\left(\frac{-x\cos nx}{n}\Big|_{-\pi}^{0} + \int_{-\pi}^{0}\frac{\cos nx\,dx}{n}\right) =$$

$$= \frac{1}{\pi}\left(-\frac{(-\pi)\cos n\pi}{n} + \underbrace{\frac{1}{n}\cdot\frac{1}{n}\sin nx\Big|_{-\pi}^{0}}_{=0}\right)$$

$\cos n\pi = (-1)^n$ $\quad 1.) = \frac{-1}{\pi} \cdot \frac{\pi}{n} (-1)^n = \frac{(-1)^{n+1}}{n}$

$$f(x) = -\frac{\pi}{4} + \sum_{n=0}^{\infty} \frac{2}{(2n+1)^2\pi} \cos(2n+1)\cdot x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx)$$

2.) $f(x) = x^2 \quad \langle -\pi, \pi \rangle$, simetrična , parna $(f(-x) = f(x))$

$a_0, a_n, b_n = 0$

$$a_0 = \frac{2}{L} \int_0^L f(x)dx = \frac{2}{\pi} \int_0^{\pi} x^2 dx = \frac{2}{\pi} \frac{x^3}{3}\Big|_0^{\pi} = \frac{2}{\pi} \cdot \frac{\pi^3}{3} = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x^2 \cos nx \, dx = \begin{vmatrix} u = x^2 & dv = \cos nx dx \\ du = 2x dx & v = \frac{1}{n} \sin nx \end{vmatrix}$$

$$= \frac{2}{\pi} \Big(\underbrace{\frac{x^2 \sin nx}{n}\Big|_0^{\pi}}_{=0} + \frac{1}{n} \int_0^{\pi} x \sin nx \, dx \Big)$$

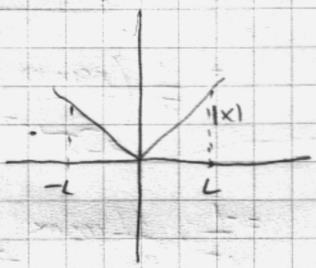
$$= \frac{-4}{n\pi} \cdot \int_0^{\pi} x \sin nx \, dx = \begin{vmatrix} u = x & dv = \sin nx dx \\ du = dx & v = \frac{1}{n}(-\cos nx) \end{vmatrix}$$

$$= \frac{-4}{n\pi} \Big(\frac{-x \cos nx}{n}\Big|_0^{\pi} + \underbrace{\frac{1}{n} \int_0^{\pi} \cos nx \, dx}_{=0} \Big) =$$

$$= \frac{-4}{n\cancel{\pi}} \Big(\frac{-\cancel{\pi}(-1)^n}{n} \Big) = \frac{4(-1)^n}{n^2}$$

3.) $f(x) = |x| \quad [-1, 1] \rightarrow$ parna, simetrična

$$\underbrace{1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots}_{\frac{1}{(2n+1)^2}} = ?$$



$$\frac{1}{1} \int_{-1}^{1} |x| dx = \frac{1}{2} \Big(\int_0^1 x dx + \int_{-1}^{0} -x dx \Big)$$

$$a_0 = \frac{2}{L} \int_0^L f(x) dx$$

$$= \frac{2}{1} \int_0^1 x$$

na $\int_0^1$ je $|x|$ pozitivan

$$a_0 = \frac{2}{1} \int_0^1 x dx = \cancel{2} \frac{x^2}{\cancel{2}}\Big|_0^1 = 1$$

$$a_n = 2 \int_0^1 x \cos(n\pi x) dx = \begin{vmatrix} u = x & dv = \cos(n\pi x) dx \\ du = dx & v = \frac{1}{n\pi} \sin(n\pi x) \end{vmatrix}$$

$$= 2 \Big(\underbrace{\frac{x \sin n\pi x}{n\pi}\Big|_0^1}_{=0} - \frac{1}{n\pi} \int_0^1 \sin(n\pi x) dx = -\frac{2}{n\pi} \cdot \frac{1}{n\pi} \cdot (-\cos n\pi x)\Big|_0^1 = \frac{2 \cos n\pi x}{n^2\pi^2} = \frac{2}{n^2\pi^2} \Big(\underbrace{\cos n\pi}_{(-1)^n} - \underbrace{\cos 0}_{1} \Big)$$

$a_{2n}=0$

$a_{2n+1}=\frac{2}{(2n+1)^2\cdot\pi^2}\cdot(-2)=\frac{4}{(2n+1)^2\pi^2}$

$S(x)=\frac{1}{2}+\sum_{n=0}^{\infty}\frac{-4}{(2n+1)^2\pi^2}\cdot\cos(2n+1)\pi x$

$f(x)=S(x)$ {uštimavamo x-eve da dobimo početnu formulu}

$f(0)=0$

$S(0)=\frac{1}{2}-\frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}$ } $\quad 0=\frac{1}{2}-\frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\rightarrow\sum_{n=0}^{\infty}=\frac{\frac{1}{2}}{\frac{4}{\pi^2}}=\frac{\pi^2}{8}$

Parsevalova jednadžba (37. str - knjižica)

$$\boxed{\frac{1}{2}a_0^2+\sum_{n=1}^{\infty}a_n^2+\sum_{n=1}^{\infty}b_n^2=\frac{2}{T}\int_a^b|f(x)|^2dx}$$

1.) $f(x)=x^2\ [-\pi,\pi]$ $\quad S(x)=\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx$

$\sum_{n=1}^{\infty}\frac{1}{n^4}=?$

$\Downarrow$

$\frac{a_0}{2}\rightarrow a_0=\frac{2}{3}\pi^2\ /^2$

$a_0^2=\frac{4}{9}\pi^4$

$\frac{1}{2}\cdot\frac{4\pi^4}{9}+\sum_{n=1}^{\infty}\frac{16}{n^4}=\frac{2\pi^4}{5}$

$16\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{2\pi^4}{5}-\frac{4\pi^4}{18}$

$\sum_{n=1}^{\infty}=\frac{\frac{2\pi^4}{5}-\frac{4\pi^4}{18}}{16}=\frac{\pi^4}{90}$

MI $\quad f(x)=\frac{\pi}{4}-\frac{x}{2}\quad\langle 0,\pi\rangle$, po sinus funkcijama

2. zad 2k6

$a_0, a_n=0,\ b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

$=\frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi}{4}-\frac{x}{2}\right)\sin(nx)dx$

$=\frac{2}{\pi}\left(\frac{\pi}{4}\int_0^{\pi}\sin(nx)dx-\frac{1}{2}\int_0^{\pi}x\sin nx\,dx\right)=\left|\begin{matrix}u=x & dv=\sin nx\,dx\\ du=dx & v=\frac{1}{n}(-\cos(nx))\end{matrix}\right|$

$=\frac{1}{2}\cdot\frac{1}{n}\cdot(-\cos nx)\Big|_0^{\pi}-\frac{1}{\pi}\left(-\frac{x\cos(xn)}{n}\Big|_0^{\pi}+\frac{1}{n}\int_0^{\pi}\cos nx\,dx\right)$

$=\frac{\cos nx}{2n}\Big|_0^{\pi}+\frac{\cancel{\pi}\overbrace{\cos n\pi}^{(-1)^n}}{n\cancel{\pi}}-\underbrace{\frac{1}{n\pi}\cdot\frac{1}{n}\sin nx\Big|_0^{\pi}}_{=0}=$

$=\frac{-\cos(n\pi)-(-\cos 0)}{2n}+\frac{(-1)^n}{n}$

$=\frac{(-1)^{n+1}+1}{2n}+\frac{(-1)^n}{n}=\frac{(-1)^{n+1}+1+2(-1)^n}{2n}=\frac{(-1)^n+1}{2n}$ $\quad b_{2n}=\frac{1}{2n}$

$b_{2n+1}=0.$

$$s(x)=\sum_{n=1}^{\infty}\frac{\sin(2nx)}{2n}\qquad s(3\pi)=\sum_{n=1}^{\infty}\frac{\sin 6\pi n}{2n}=0$$

Ako se u zadatku traži po sin ili cos f-ja, uvijek ajeli interval L

2. zad 2007 $f(x)=\begin{cases}1, & \langle 0,\frac{1}{2}\rangle\\ -1, & \langle \frac{1}{2},1\rangle\end{cases}$ po kosinus funkcijama

$b_n=0$

Kak se zove potencija koja daje $\emptyset$?

(nupotencija =)

$$a_0=\frac{2}{L}\int_0^L f(x)dx=2\left(\int_0^{1/2}dx+\int_{1/2}^{1}-dx\right)$$

$$=2\left(x\Big|_0^{1/2}-x\Big|_{1/2}^{1}\right)=2\left(\frac{1}{2}-1+\frac{1}{2}\right)=0$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}=$$

$$=2\left(\int_0^{1/2}\cos(n\pi x)dx-\int_{1/2}^{1}\cos(n\pi x)dx\right)=2\left(\frac{1}{n\pi}\sin(n\pi x)\Big|_0^{1/2}-\frac{1}{n\pi}\sin(n\pi x)\Big|_{1/2}^{1}\right)=$$

$$=2\left(\frac{\sin\frac{n\pi}{2}}{n\pi}+\frac{1}{n\pi}\sin\frac{n\pi}{2}\right)=\frac{4\sin\frac{n\pi}{2}}{3\pi}$$

$$\left\{\ s(x)=\sum_{n=1}^{\infty}\frac{4\sin\left(\frac{n\pi}{2}\right)}{n\pi}\cos(n\pi x)\ \right\}$$

$a_{2n}=0$

$$a_{2n+1}=\frac{4(-1)^n}{(2n+1)\pi}\qquad s(x)=\sum_{n=0}^{\infty}\frac{4(-1)^n}{(2n+1)\pi}\cos(2n)$$

$$\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}=?$$

$$\sum_{n=0}^{\infty}a_{2n+1}^2=\frac{2}{T}\int_a^b|f(x)|^2dx=\sum_{n=1}^{\infty}\frac{16}{(2n+1)^2\pi^2}=\frac{2}{1}\cdot\int_0^1 1^2dx=2x\Big|_0^1=2$$

$$\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}=\frac{\pi^2}{8}$$

FURIEROV INTEGRAL

$$f(x) = \int_0^\infty (A(\lambda)\cos\lambda x + B(\lambda)\sin\lambda x)\,d\lambda$$

$$A(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(\varphi)\cos\lambda\varphi\,d\varphi$$

$$B(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(\varphi)\sin\lambda\varphi\,d\varphi$$

PARNA: $f(\varphi) = \int_0^\infty A(\lambda)\cos\lambda\varphi\,d\lambda$

$$A(\lambda) = \frac{2}{\pi}\int_0^\infty f(\varphi)\cos\lambda\varphi\,d\varphi$$

Neparna: $f(x) = \int_0^\infty B(x)\sin\lambda x\,d\lambda$

$$B(\lambda) = \frac{2}{\pi}\sin\lambda\varphi\,d\varphi$$

Parna f-ja $f(x) = e^{-px}$ $\langle 0, \infty \rangle$

$$f(x) = \int_0^\infty A(\lambda)\cos\lambda x\,d\lambda$$

$$A(\lambda) = \frac{2}{\pi}\int_0^\infty e^{-p\varphi}\cos\lambda\varphi\,d\varphi$$

$$\int e^{-p\varphi}\cos\lambda\varphi\,d\varphi = \left| \begin{matrix} u = e^{-p\varphi} & dv = \cos\lambda\varphi\,d\varphi \\ du = -pe^{-p\varphi}d\varphi & v = \frac{1}{\lambda}\sin\lambda\varphi \end{matrix} \right|$$

$$= \frac{e^{-p\varphi}\sin\lambda\varphi}{x} + \frac{p}{\lambda}\int \sin\lambda\varphi \cdot e^{-p\varphi}d\varphi = \left| \begin{matrix} dv = \sin\lambda\varphi\,d\varphi \\ u = \frac{-\cos\lambda\varphi}{\lambda} \end{matrix} \right| = \frac{e^{-p\varphi}\sin\lambda\varphi}{\lambda} + \frac{p}{\lambda}\left(-\frac{e^{-p\varphi}\cos\lambda\varphi}{\lambda} - \int -\frac{\cos\lambda\varphi}{\lambda}\cdot(-pe^{-p\varphi})d\varphi\right)$$

$$\int e^{-p\varphi}\cos\lambda\varphi\,d\varphi \cdot \left(1 + \frac{p^2}{\lambda^2}\right) = \frac{x e^{-p\varphi}\sin\varphi\lambda - pe^{-p\varphi}\cos\lambda\varphi}{x^{-2}}$$

LAPLACE

$$F(s) := \int_0^\infty e^{-st} f(t)\,dt$$

↓ slika ↓ original

$t = ?$ $f(t) \circ\!\!-\!\!\bullet F(s)$ i $F(s) \bullet\!\!-\!\!\circ f(t)$

$$F(s) = \int_0^\infty e^{-st}\, t\,dt = \left| \begin{matrix} u=t & dv = e^{-st}dt \\ du = dt & v = -\frac{1}{s}e^{-st}\end{matrix}\right| =$$

$$= \underbrace{\frac{-t}{s}e^{-st}\Big|_0^\infty}_{0} - \int -\frac{1}{s} e^{-st}\, dt = \frac{1}{s}\cdot\left(-\frac{1}{s}\right)e^{-st}\Big|_0^\infty = \frac{1}{s^2}$$

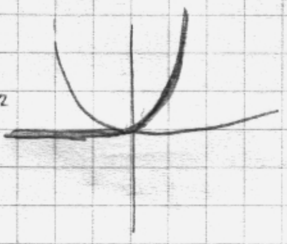
$$\boxed{t \circ\!\!-\!\!\bullet \frac{1}{s^2}}$$

Linearnost $\alpha f(t) + \beta g(t) \circ\!\!-\!\!\bullet \alpha F(s) + \beta G(s)$

$$\frac{1}{3} f(t) \circ\!\!-\!\!\bullet \frac{1}{3}F(s)$$

Step funkcija

→npr. x^2



$$\boxed{u(t) \circ\!\!-\!\!\bullet \frac{1}{s}}$$

Eksponencijalna funkcija $f(t) = e^{\alpha t}$ $\boxed{e^{\alpha t} \circ\!\!-\!\!\bullet \frac{1}{s-\alpha}}$

$\sin\omega t \circ\!\!-\!\!\bullet \frac{\omega}{s^2+\omega^2}$	$\cos\omega t \circ\!\!-\!\!\bullet \frac{s}{s^2+\omega^2}$
$\mathrm{sh}\,\omega t \circ\!\!-\!\!\bullet \frac{\omega}{s^2-\omega^2}$	$\mathrm{ch}\,\omega t \circ\!\!-\!\!\bullet \frac{s}{s^2-\omega^2}$

npr. $\frac{1}{s^2+3^2} = \frac{1}{\sqrt3}\cdot\frac{\sqrt3}{s^2+(\sqrt3)^2} = \frac{1}{\sqrt3}\cdot\sin(\sqrt3 t)$

$f(t) = t^4$ $\boxed{t^n \circ\!\!-\!\!\bullet \frac{n!}{s^{n+1}}}$

$F(s) =$

$\lim_{t\to\infty} e^{-at} f(t)$

$\frac{1}{t}$ $\lim_{t\to 0^-} = -\infty$, $\lim_{t\to 0^+} = +\infty$ } nije prekid 1. vrste, moraju biti konačni limesi

npr. (5, 16) ✓

$t^2 \sin 2t \to \lim_{t\to\infty} t^2 \sin 2t$

TEOREM O PRIGUŠENJU

$$e^{-at} f(t) \circ\!-\!\bullet F(s+a)$$

pr. $e^{2t} t^5 \circ\!-\!\bullet \frac{120}{(s-2)^6}$

$t^5 \circ\!-\!\bullet \frac{5!}{s^6} = \frac{120}{s^6}$

$e^{-2t} u(t) = \frac{1}{s+2}$

$u(t) \circ\!-\!\bullet \frac{1}{s}$

Teorem o pomaku orginala

$$f(t-a)\cdot u(t-a) \circ\!-\!\bullet e^{-as} F(s)$$

npr. $\sin(t-2)\, u(t-2) \circ\!-\!\bullet e^{-2s} \frac{1}{s^2+1}$

step fja

$\sin(t-2)$

↓

$\sin t = \frac{1}{s^2+1}$ $F(s) = \frac{1}{s^2+1}$

step fja

$$\boxed{f(t-a)\, u(t-a) \circ\!-\!\bullet e^{-as}\cdot F(s)}$$

$(t-2)^2 u(t-2) \circ\!-\!\bullet e^{-2s}\cdot \frac{2}{s^3}$

↓

$(t-2)^2$

↓

$t^2 \circ\!-\!\bullet \frac{2}{s^3}$

Pr 5 06/07 $f(t) = (t-2)^3 e^{-t} u(t-2)$ $e^{-2(s+1)} \frac{6}{(s+1)^4}$

↓

$(t-2)^3 u(t-2) \circ\!-\!\bullet e^{-2s} \frac{6}{s^4}$

↓

$(t-2)^3$

↓

$t^3 = \frac{6}{s^4}$

Gate fja

– sve izvan $\langle a, b\rangle = 0$

$$\boxed{g_{[a,b]}(t) = u(t-a) - u(t-b)}$$

Pr. $f(t) \begin{cases} 3, & 0<t<2 \\ -1, & 2<t<4 \\ 0, & t>4 \end{cases}$

$f(t) = 3\cdot g_{[0,2]}(t) - 1\, g_{[2,4]}(t)$

$= 3\big(u(t) - u(t-2)\big) - u(t-2) + u(t-4)$

$= 3u(t) - 3u(t-2) - u(t-2) + u(t-4)$

$\circ\!-\!\bullet\ 3\cdot\frac{1}{s} - 4\frac{e^{-2s}}{s} + \frac{e^{-4s}}{s}$

$\Big\{\ u(t-2)\, u(t-2) \circ\!-\!\bullet e^{-2s}\frac{1}{s}$

↓

$u(t) \circ\!-\!\bullet \frac{1}{s}$

Teorem o deriviranju orginala

$$f^{(n)}(t) \circ\!\!-\!\bullet\, s^n F(s) - s^{n-1} f(0) - s^{n-1} f'(0) - \dots - f^{(n-1)}(0)$$

$f(t) = \sin t$

$\sin t \circ\!\!-\!\bullet\, F(s) \; /'$

$\cos t \circ\!\!-\!\bullet\, sF(s) - f(0) = sF(s)$

$-\sin t \circ\!\!-\!\bullet\, s^2F(s) - sf(0) - f'(0) = s^2F(s) - 1$

$\sin t \circ\!\!-\!\bullet\, 1 - s^2F(s)$

$F(s) = 1 - s^2F(s)$

$F(s)(1+s^2) = 1$

$F(s) = \dfrac{1}{s^2+1}$

T. o der. slike

$\boxed{t^n f(t) \circ\!\!-\!\bullet\, (-1)^n F^{(n)} s}$ $= (-1)\left(\dfrac{1}{s^2-2s+2}\right)' = (-1)\cdot\left(\dfrac{-(2s-2)}{(s^2-2s+2)^2}\right) = \dfrac{2s-2}{(s^2-2s+2)^2}$

$te^t \sin t \circ\!\!-\!\bullet\, ?$

$e^t \sin t \circ\!\!-\!\bullet\, \dfrac{1}{(s-1)^2+1} = \dfrac{1}{s^2-2s+2}$

$\sin t \circ\!\!-\!\bullet\, \dfrac{1}{s^2+1}$

Tm o integriranju slike

1.) $f(t) \circ\!\!-\!\bullet\, F(s)$

2.) $\dfrac{f(t)}{t} \circ\!\!-\!\bullet\, \int_s^\infty F(s)\,ds$

$\dfrac{\sin t}{t}$? $\sin t \circ\!\!-\!\bullet\, \dfrac{1}{s^2+1}$

$$= \int_s^\infty \frac{ds}{s^2+1} = \frac{1}{a}\operatorname{arctg}\frac{s}{a}\Big|_s^\infty = \operatorname{arctg} s\Big|_s^\infty = \frac{\pi}{2} - \operatorname{arctg} s$$

Pr2 $\displaystyle\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\,dt = ?$

$F(s) = \int_0^\infty e^{-st} f(t)\,dt$

$F(1) = \int_0^\infty e^{-t} \dfrac{\sin^2 t}{t}\,dt$

$\dfrac{\sin^2 t}{t}$ 1.) $\sin^2 t \circ\!\!-\!\bullet\, ?$

$\dfrac{1}{2}(1-\cos 2t) \circ\!\!-\!\bullet\, \dfrac{1}{2}\left(\dfrac{1}{s} - \dfrac{s}{s^2+4}\right)$

$\dfrac{\sin^2 t}{t} \circ\!\!-\!\bullet\, \dfrac{1}{2}\int_s^\infty \left(\dfrac{1}{s} - \dfrac{s}{s^2+4}\right) ds$

$= \dfrac{1}{2}\left(\ln|s| - \dfrac{1}{2}\ln|s^2+4|\right)\Big|_s^\infty$

$= \dfrac{1}{2}\left(2\cdot\dfrac{1}{2}\ln|s| - \dfrac{1}{2}\ln|s^2+4|\right)\Big|_s^\infty$

$= \dfrac{1}{4}\ln\dfrac{s^2}{s^2+4}\Big|_s^\infty = -\dfrac{1}{4}\ln\dfrac{s^2}{s^2+4} = \underbrace{\dfrac{1}{4}\ln\dfrac{s^2+4}{s^2}}_{F(s)}$

$F(1) = \dfrac{1}{4}\ln 5$

TM o int. or.

$$\int_0^t f(\tau)d\tau \circ\!\!-\!\bullet \frac{F(s)}{s} \qquad \circ\!\!-\!\bullet \frac{\left(\frac{\pi}{2} - \mathrm{arctg}\, s\right)}{s}$$

Pr. $\int_0^t \frac{\sin u}{u} du$ $\qquad \frac{\sin u}{u} \circ\!\!-\!\bullet ?$

$F(s) = ?$

$$\frac{\sin u}{u} \circ\!\!-\!\bullet \int_s^\infty \frac{1}{s^2+1} ds = \mathrm{arctg}\, s \Big|_s^\infty = \left(\frac{\pi}{2} - \mathrm{arctg}\, s\right)$$

Konvolucija

$$f_1(t) * f_2(t) \circ\!\!-\!\bullet F_1(s) F_2(s)$$

$$(f_1 * f_2)(t) = \int_0^t f_1(\tau) f_2(t-\tau)\, d\tau$$

Pr. $\frac{1}{s(s^2+1)} = \frac{1}{s} \cdot \frac{1}{(s^2+1)}$

$$\left.\begin{array}{l} \frac{1}{s} \circ\!\!-\!\bullet u(t) \\ \frac{1}{s^2+1} \circ\!\!-\!\bullet \sin t \end{array}\right\} u(t) * \sin(t) = \int_0^t \sin\tau\, u(t-\tau)\, d\tau$$

zanemarujemo step f-ju ispod integrala

$$= \int_0^t \sin\tau\, d\tau = -\cos\tau \Big|_0^t = (1-\cos t) u(t)$$

MI $\int_0^\infty e^{-t} t^{100} dt = ?$

$$F(s) = \int_0^\infty e^{-st} f(t) dt \qquad t^{100} \circ\!\!-\!\bullet \frac{100!}{s^{101}}$$

$$F(1) = \int_0^\infty e^{-t} (t^{100})\, dt \qquad F(1) = \frac{100!}{1^{101}} = 100!$$

$s = 1$

$$\int_0^\infty e^{-\frac{x}{2}} x^2 \cos x\, dx = ? \qquad x^2 \cos x \circ\!\!-\!\bullet ?$$

$$\int_0^\infty e^{-st} f(t) dt \qquad t^n f(t) \circ\!\!-\!\bullet (-1)^n F^{(n)}(s)$$

$s = \frac{1}{2}$ $\qquad n = 2 \qquad \cos x \circ\!\!-\!\bullet \frac{s}{s^2+1}$

$$x^2 \cos x \circ\!\!-\!\bullet \left(\frac{s}{s^2+1}\right)'' = \left(\frac{1-s^2}{(s^2+1)^2}\right)' = \frac{-2s(s^2+1)^2 - 4s(s^2+1)(s^2-1)}{(s^2+1)^4}$$

$$F\left(\frac{1}{2}\right) = \frac{-2 \cdot \frac{1}{2}\left(\frac{1}{4}+1\right)^2 + 4 \cdot \frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{4}-1\right)}{\left(\frac{1}{4}+1\right)^4}$$

$= -$ ___