PRIKAZ SUSTAVA U PROSTORU STANJA

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$sX - x(0) = Ax(s) + Bu(s)$$

$$y(s) = Cx(s) + Du(s)$$

$$(sI - A)X(s) = x(0) + BU(s) \quad / \ (sI-A)^{-1}$$

jedinična matrica

$$X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}BU(s)$$

$$(sI-A)^{-1} = \phi(s)$$

$$Y(s) = C\left[\phi(s)x(0) + \phi(s)BU(s) + DU(s)\right]$$

$$Y(s) = \underbrace{C\phi(s)x(0)}_{\text{NEPOPUDENI ODZIV}} + \underbrace{\left[C\phi(s)B + D\right]U(s)}_{H(s)}$$

MIRNI ODZIV

PRIJENOSNA FUNK.

$$x(t) = \phi(t)x(0) + \int_0^t \phi(t-\tau)Bu(\tau)\,d\tau$$

$$y(t) = C\phi(t)x(0) + \int_0^t C\phi(t-\tau)Bu(\tau)\,d\tau + Du(t)$$

ZAD 6.3

$$A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D \neq 0 \quad 1$$

$$u(t) = -\begin{bmatrix} 2\delta(t) \\ 5(t) \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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1. izračunavanje matrice karakt. frekvencija

$$\phi(s) = (sI - A)^{-1}$$

$$\phi(s) = \frac{\text{adj}\,(sI - A)}{\det(sI - A)}$$

$$\phi(s)^{-1} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} s & 2 \\ -1 & s+3 \end{bmatrix}$$

$$\det \phi(s)^{-1} = s(s+3) + 2 = s^2 + 3s + 2$$

$$= (s+1)(s+2)$$

$$\text{adj}\,(sI - A) = \begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix}$$

1. elemente na gl. dij. zamijenimo
2. elementima na sporednoj dijagonali promijenimo predznak

$$\phi(s) = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix}$$

$$H(s) = C\,\Phi(s)\,B + D$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{(s+1)(s+2)} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)} \begin{bmatrix} 2s+6 & s^2+3s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{s+1} + \frac{-2}{s+2} & \frac{-2}{s+1} + \frac{2}{s+2} + 1 \end{bmatrix}$$

$$h(t) = \begin{bmatrix} 4e^{-t} - 2e^{-2t} & -2e^{-t} + 2e^{-2t} + \delta(t) \end{bmatrix} \quad t \geq 0$$

$$y(s) = C\,\Phi(s)\,x(0) + \left(C\,\Phi(s)\,B + D\right) U(s)$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$+ \frac{1}{(s+1)(s+2)} \begin{bmatrix} 2s+6 & s^2+3s \end{bmatrix} \begin{bmatrix} \frac{2}{s} \\ \frac{1}{s} \end{bmatrix}$$

$$y(t) = 12e^{-t} - 6e^{-2t}$$

ZAD

$$y_1'(t) + 3y_2(t) = u_1(t)$$
$$3y_1(t) + y_2'(t) = u_2(t)$$

- MIRNI SUSTAV
- POČ. UVJETI = 0

$$u(t) = \begin{bmatrix} 3\,\mu(t) \\ \delta(t) \end{bmatrix}$$

$$x' = Ax(t) + Bu(t)$$
$$y = Cx(t) + Du(t)$$

OPĆENITO

$$y_1 = x_1 \qquad x_1' = y_1'$$
$$y_2 = x_2 \qquad x_2' = y_2'$$

$$x_1' = -3x_2 + u_1$$
$$x_2' = -3x_1 + u_2$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$\phi(1)$

$$H(1) = C\,\phi(1)\,B + D$$

$$\phi(1) = (sI - A)^{-1} = \frac{adj\,(sI - A)}{det\,(sI - A)}$$

$$\Phi(s) = \frac{1}{(s+3)(s-3)} \begin{bmatrix} s & -3 \\ -3 & s \end{bmatrix}$$

B i C se eliminišu

$$H(s) = \frac{1}{(s+3)(s-3)} \begin{bmatrix} s & -3 \\ -3 & s \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{2}\left(\frac{1}{s+3} + \frac{1}{s-3}\right) & \frac{1}{2}\left(\frac{1}{s+3} - \frac{1}{s-3}\right) \\ \frac{1}{2}\left(\frac{1}{s+3} - \frac{1}{s-3}\right) & \frac{1}{2}\left(\frac{1}{s+3} + \frac{1}{s-3}\right) \end{bmatrix}$$

$$h(t) = \begin{bmatrix} \frac{1}{2}\left(e^{-3t} + e^{3t}\right) & \frac{1}{2}\left(e^{-3t} - e^{3t}\right) \\ \frac{1}{2}\left(e^{-3t} - e^{3t}\right) & \frac{1}{2}\left(e^{-3t} + e^{3t}\right) \end{bmatrix}$$

$$Y(s) = U(s)\ H(s)$$

$$= \frac{1}{(s+3)(s-3)} \begin{bmatrix} s & -3 \\ -3 & s \end{bmatrix} \begin{bmatrix} \frac{3}{s} \\ 1 \end{bmatrix}$$

$$= \frac{1}{(s+3)(s-3)} \begin{bmatrix} 0 \\ -\frac{9}{s} + s \end{bmatrix} = \frac{1}{(s+3)(s-3)} \begin{bmatrix} 0 \\ \frac{s^2-9}{s} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \frac{1}{s} \end{bmatrix}$$

$$Y(s) = \begin{bmatrix} 0 \\ u(t) \end{bmatrix}$$

DISKRETNI SUSTAVI

ZAD

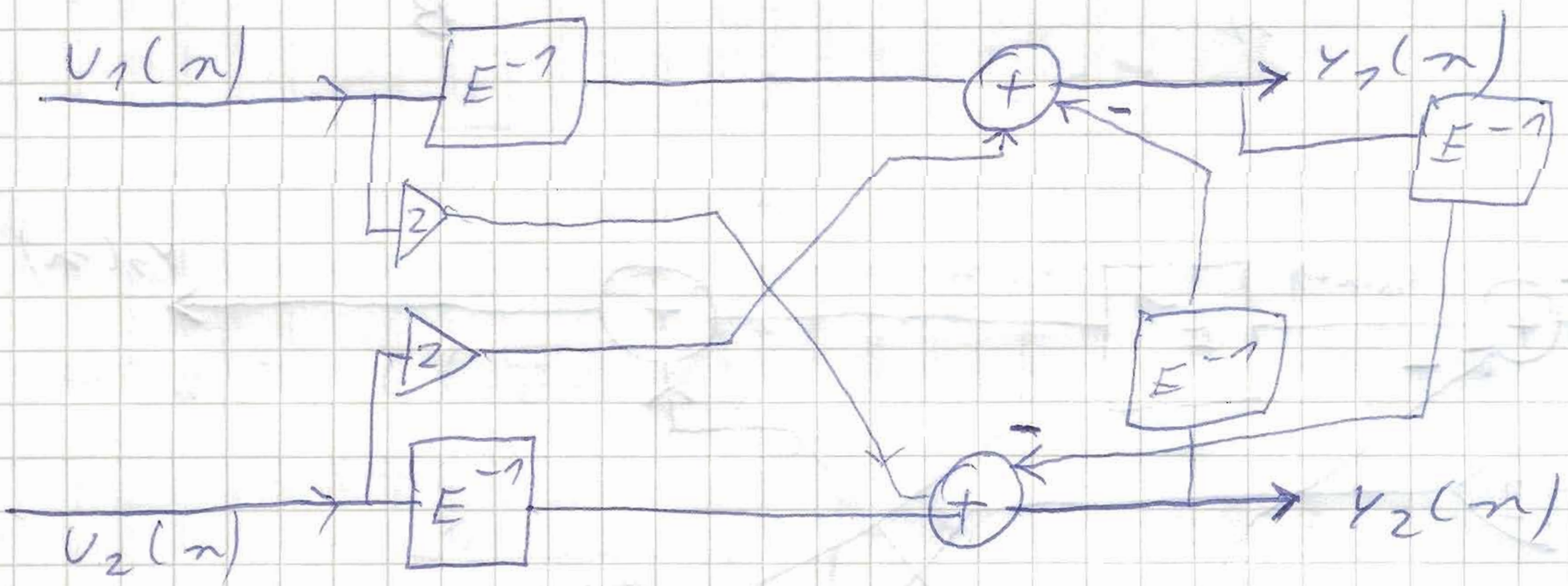
$$y_1(n) + y_2(n-1) = u_1(n-1) + 2u_2(n)$$
$$y_1(n-1) + y_2(n) = 2u_1(n) + u_2(n-1)$$
$$x[0] = 0$$

$$u(n) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \delta(n) \\ u(n) \end{bmatrix}$$

E^{-1} zbog kašnjenja

$$y_1(n) = -y_2(n-1) + u_1(n-1) + 2u_2(n)$$

$$y_2(n) = -y_1(n-1) + 2u_1(n) + u_2(n-1)$$



$$x[n+1] = Ax[n] + Bu(n)$$
$$y(n) = Cx[n] + Du(n)$$

OPĆENITO

$n = n+1$

$$x_1(n) = -y_2(n-1) + u_1(n-1)$$
$$x_2(n) = -y_1(n-1) + u_2(n-1)$$

$$\Rightarrow \quad x_1(n+1) = -y_2(n) + u_1(n)$$
$$x_2(n+1) = -y_1(n) + u_2(n)$$

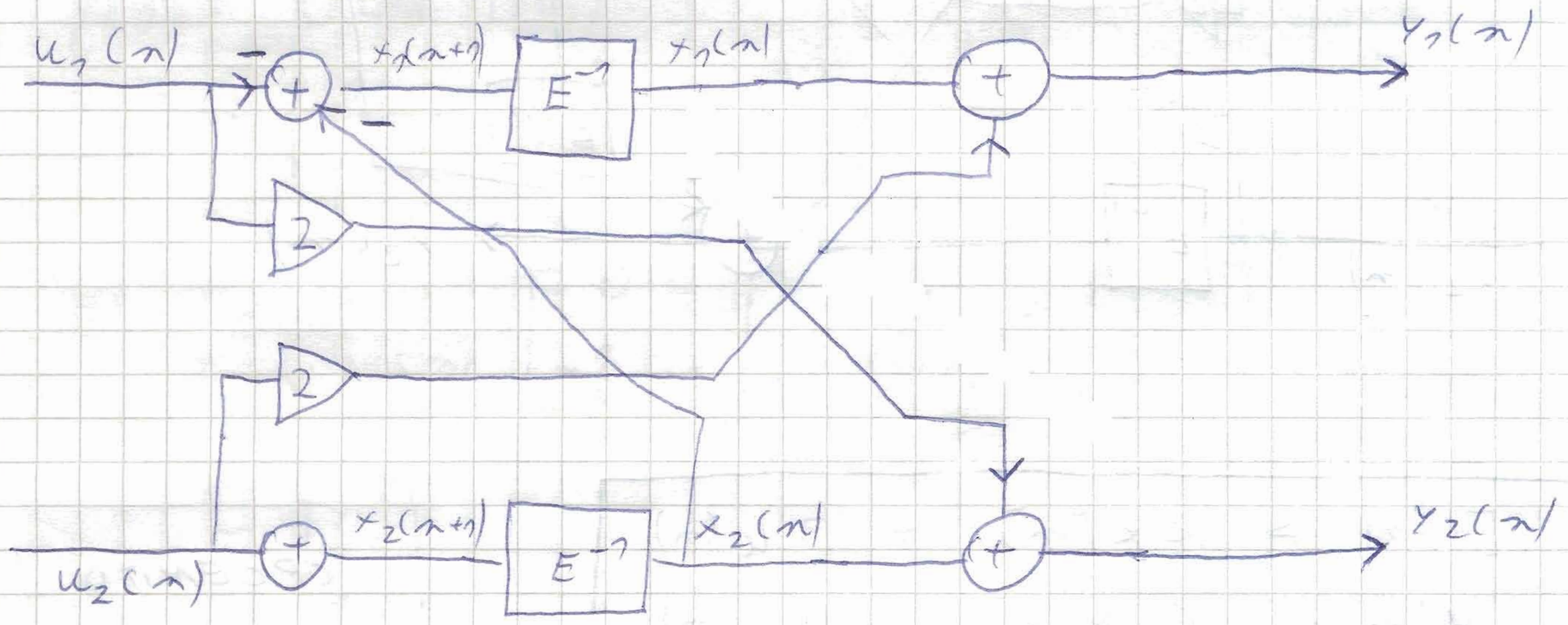
$$y_1(n) = x_1(n) + 2u_2(n)$$
$$y_2(n) = x_2(n) + 2u_1(n)$$

$$\begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}}_{D} \begin{bmatrix} u_1(n) \\ u_2(n) \end{bmatrix}$$

$$x_1(n+1) = -x_2(n) - u_1(n)$$

$$x_2(n+1) = -x_1(n) - u_2(n)$$

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}_{B} \begin{bmatrix} u_1(n) \\ u_2(n) \end{bmatrix}$$



$$zX(z) - zX(0) = AX(z) + BU(z)$$

$$Y(z) = CX(z) + DU(z)$$

$$Y(z) = C\phi(z)X(0) + \underbrace{\left(C(zI-A)^{-1}B + D\right)}_{H(z)}U(z)$$

$$\phi(z) = z(zI-A)^{-1}$$

$$(zI-A)^{-1} = \frac{\text{adj}(zI-A)}{\det(zI-A)}$$

$$= \frac{1}{(z+1)(z-1)}\begin{bmatrix} z & -1 \\ -1 & z \end{bmatrix}$$

$$H(z) = C(zI-A)^{-1}B + D$$

$$= \frac{1}{(z+1)(z-1)}\begin{bmatrix} -z & 2z^2-1 \\ 2z^2-1 & -z \end{bmatrix}$$

$$Y(z) = H(z)U(z)$$

$$= \frac{1}{(z+1)(z-1)}\begin{bmatrix} \frac{z^2(2z-1)}{z-1} \\ \frac{2z^3-3z^2+z+1}{z-1} \end{bmatrix}$$

DIREKTNA REALIZACIJA

najčešća kad. u najčešća mnogi odgovarati prijenosnoj funkciji

$$H(s) = \frac{s^3 + 2s^2 + 3s + 4}{s^3 + 2s^2 + 5s + 6} = \frac{P(s)}{Q(s)}$$

$$Y(s) = H(s)\,U(s) = \frac{P(s)}{Q(s)}\,U(s) = P(s)\,\frac{U(s)}{Q(s)}$$

$$Z(s) = \frac{U(s)}{Q(s)}$$

$$Y(s) = P(s)\,Z(s)$$

$$Q(s)\,Z(s) = U(s)$$

$$(s^3 + 2s^2 + 5s + 6)\,Z(s) = U(s)$$

$$z'''(t) + 2z''(t) + 5z'(t) + 6z(t) = u(t)$$

$$x_1(t) = z(t)$$
$$x_2(t) = z'(t)$$
$$x_3(t) = z''(t)$$

broj varijabli stanja = red sustava

$$x_1'(t) = x_2$$

$$x_2'(t) = x_3$$

$$x_3'(t) = z'''(t)$$

$$x_3'(t) = u(t) - 6x_1(t) - 5x_2(t) - 2x_3(t)$$

jednadžba manjkalli stanja

$$Y(s) = P(s)\, z(s)$$

$$= (s^3 + 2s^2 + 3s + 4)\, z(s)$$

$$y(t) = z'''(t) + 2z''(t) + 3z'(t) + 4z(t)$$

$$y(t) = u(t) - 6x_1(t) - 5x_2(t) - 2x_3(t)$$

$$+ 2x_1(t) + 3x_2(t) + 4x_1(t)$$

$$= u(t) - 2x_1(t) - 2x_2(t)$$

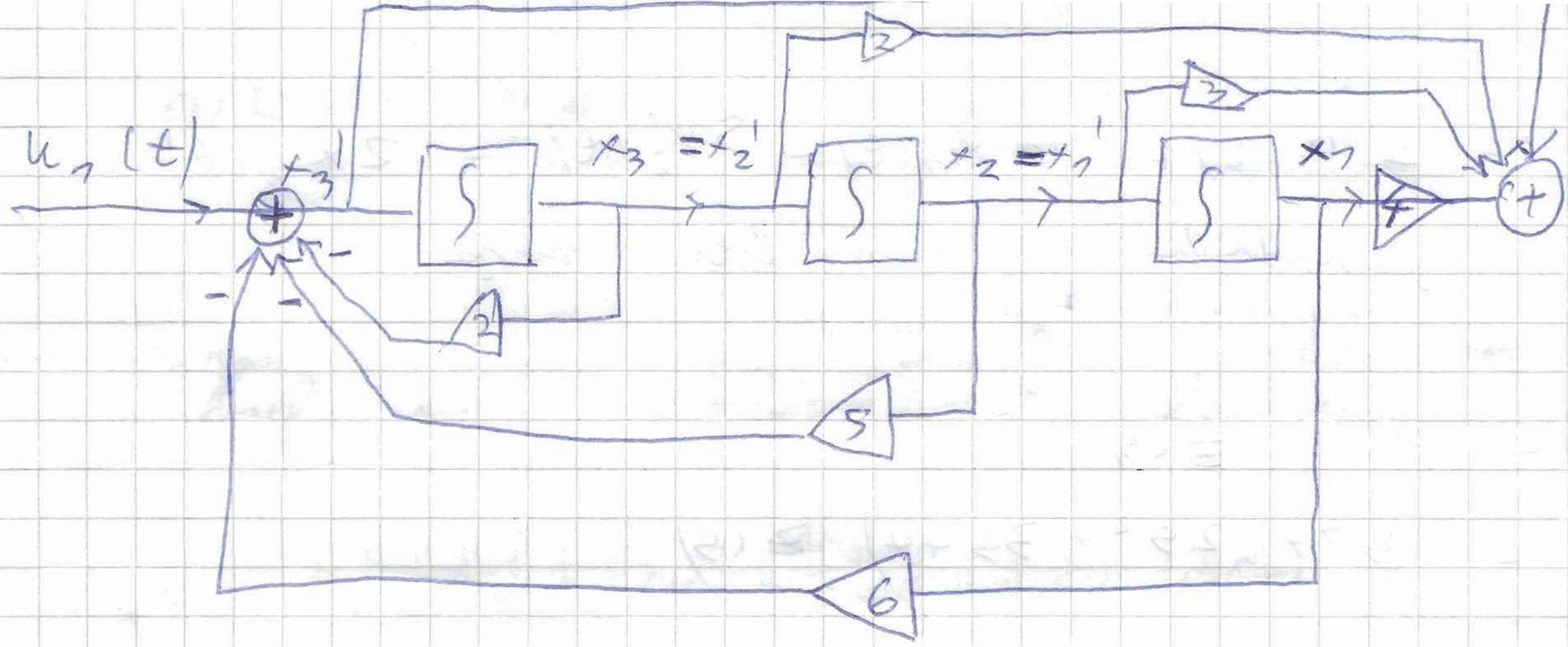
$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -5 & -2 \end{bmatrix}}_{A} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u(t)$$

$$y(t) = \underbrace{\begin{bmatrix} -2 & -2 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{D} u(t)$$

$$H(s) = \frac{s^3 + 2s^2 + 3s + 4}{s^3 + 2s^2 + 5s + 6}$$

$$Y(s) = (s^3 + 2s^2 + 3s + 4)\, z(s)$$

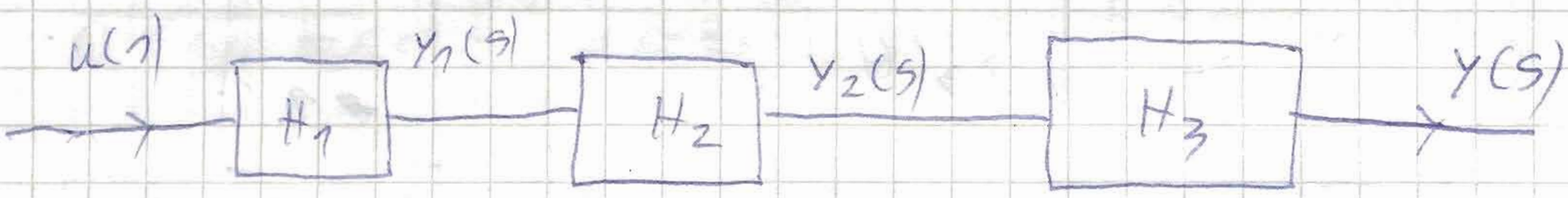
sustav je 3. reda $\Rightarrow$ 3 integratora



KASKADNA REALIZACIJA

$$Z(s) = \frac{(s+2)(s-1)s}{(s+4)(s^2+s+3)(s-2)}$$

red matrika = 4



- prijemnome funk. in poreznne
- UMNOŽAK najenostavnejših podsistemov

$$Z(s) = \frac{(s+2)}{s+4} \cdot \frac{s-1}{s^2+s+3} \cdot \frac{s}{s-2}$$

H_1 H_2 H_3

1)

$$Y_1(s) = H_1(s)\, U(s) = \frac{s+2}{s+4}\, U(s) = (s+2)\, X_1(s)$$

VARIJABLA STANJA

$$(s+4)\, X_1(s) = U(s)$$

$$x_1'(t) + 4x_1(t) = u(t)$$

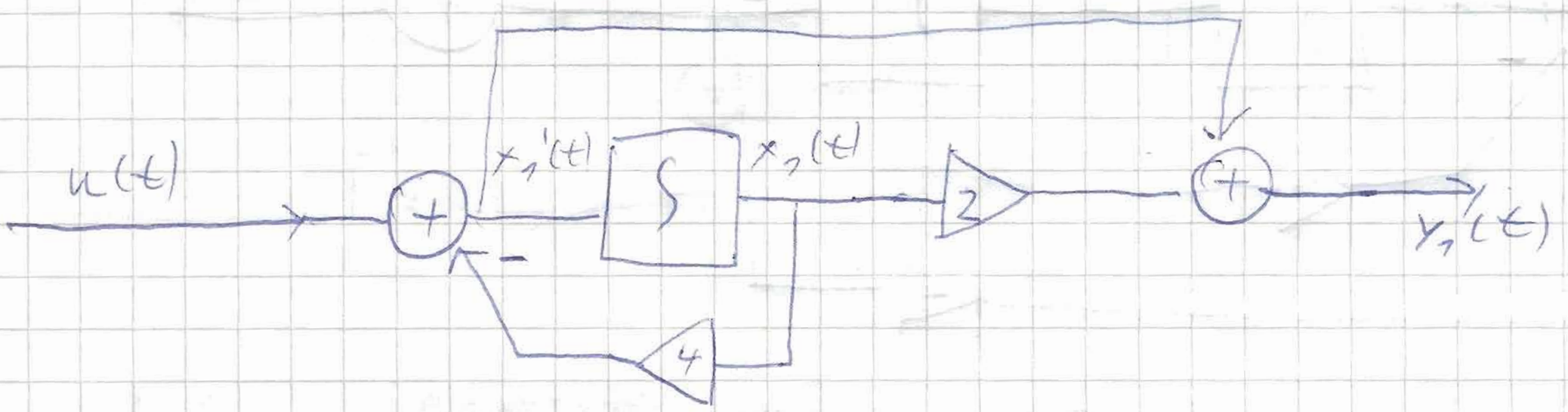
$$x_1'(t) = -4x_1(t) + u(t)$$

$$Y_1(s) = (s+2)\, X_1(s)$$

=

$$y_1(t) = x_1'(t) + 2x_1(t)$$

realizacija pomoću podsustava



$$Y_2(s) = H_2(s)\, Y_1(s) = \frac{s-1}{s^2+s+3}\, Y_1(s)$$

$$= (s-1)\, X_2(s)$$

$$X_2(s) \cdot (s^2+s+3) = Y_1(s)$$

$$x_2''(t) + x_2'(t) + 3x_2(t) = y_1(t)$$

u pitanju je matrica za matricama se ne mogu pojavljivati druge derivacije

$$x_2' = x_3 \quad \Rightarrow \quad x_2'' = x_3'$$

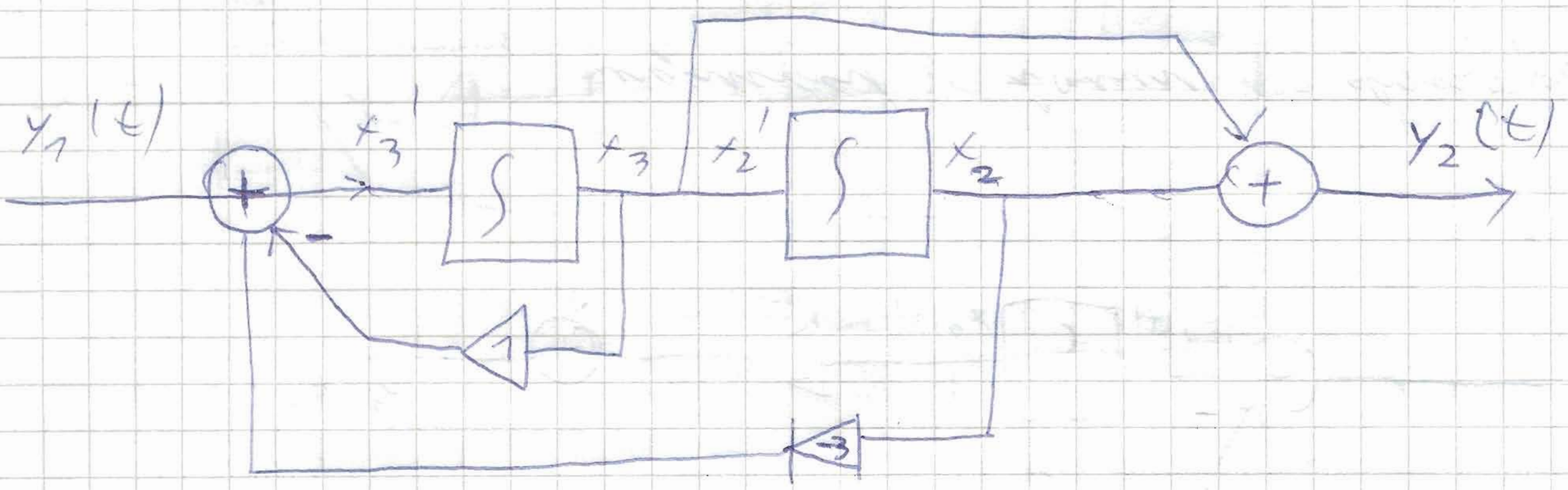
$$x_3'(t) = y_1(t) - x_3(t) - 3x_2(t)$$

$$Y_2(s) = (s-1)\, X_2(s)$$

$$y_2(t) = x_2'(t) - x_2(t)$$

$$= x_3(t) - x_2(t)$$

realizacija drugog poretka



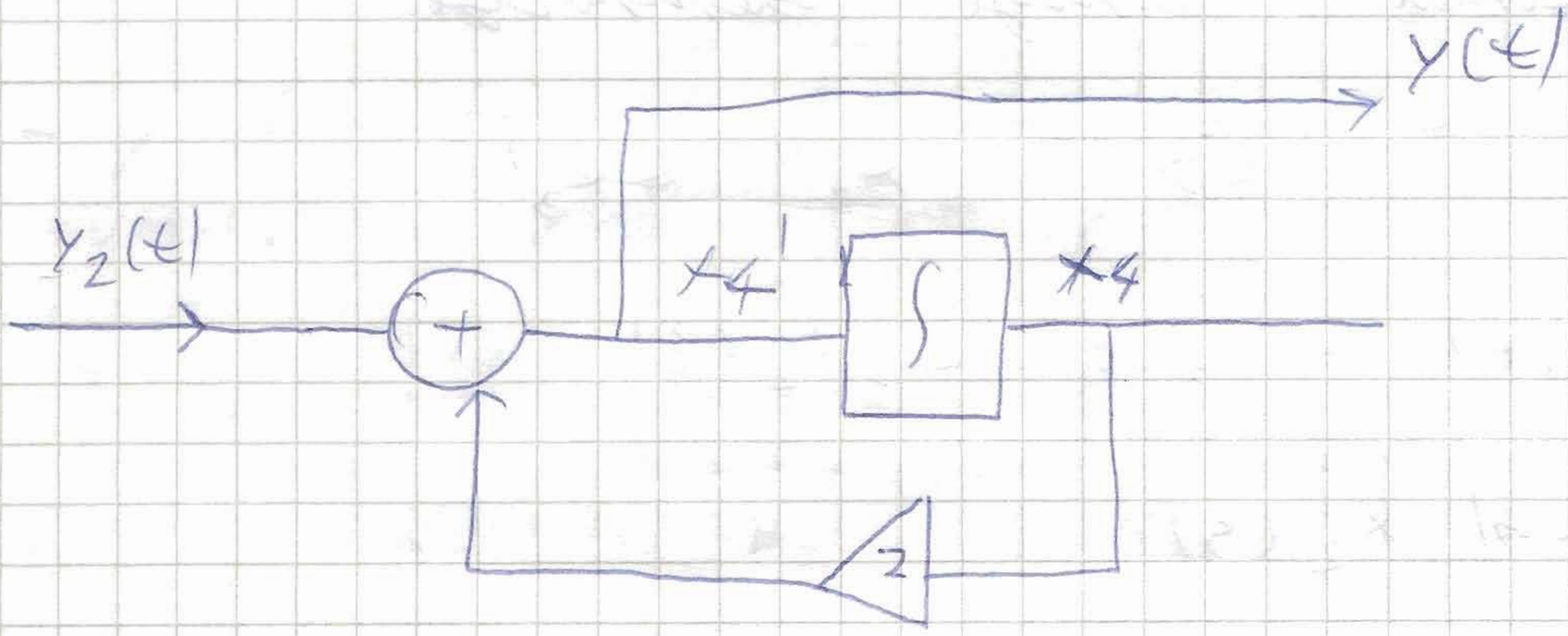
3) $Y(s) = A_3(s)\ Y_2(s)$

$= \frac{s}{s-2}\ Y_2(s) = s\,x_4(s)$

$(s-2)\,x_4(s) = Y_2(s)$ $\qquad$ $y(s) = s\,x_4(s)$

$x_4'(t) - 2x_4(t) = Y_2(t)$ $\qquad$ $y(t) = x_4'(t)$

$x_4'(t) = 2x_4(t) + Y_2(t)$

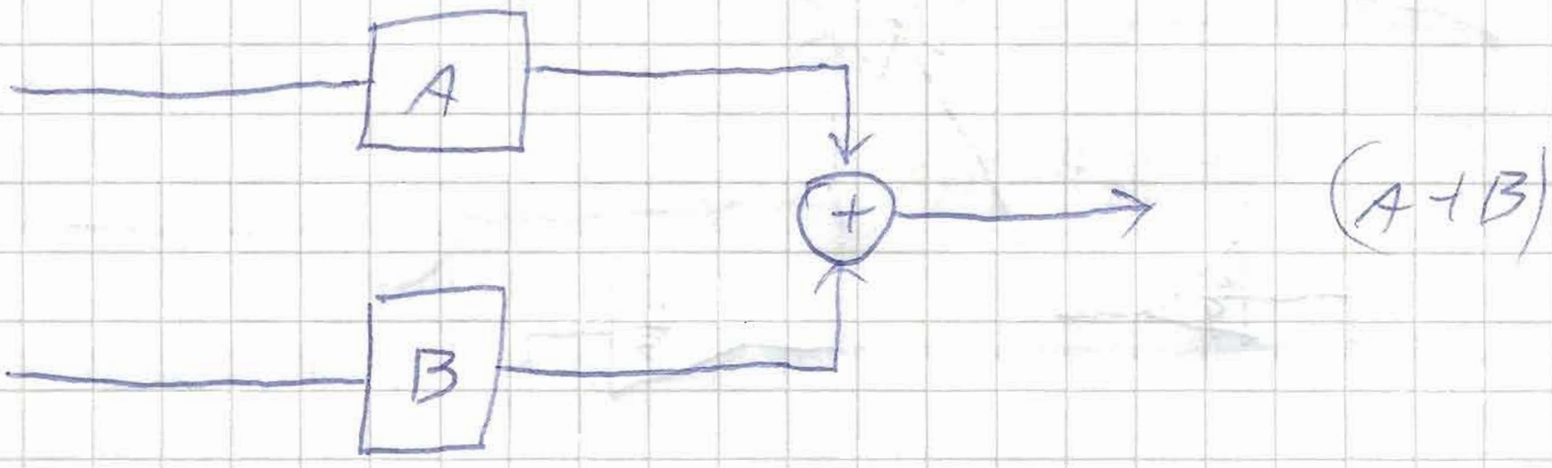


prijelaz iz podudarne u konačno rješenje

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & -3 & -1 & 0 \\ 0 & -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]\,u(t)$$

PARALELNA REALIZACIJA



ZBROJ pojačalničnih pojačanja

$$H(s) = \frac{s+7}{(s-7)^2(s+2)} = \frac{A}{s+2} + \frac{B}{(s-7)^2} + \frac{C}{s-7}$$

$$= -\frac{1}{9}\cdot\frac{1}{s+2} + \frac{2}{3}\frac{1}{(s-7)^2} + \frac{1}{9}\frac{1}{s-7}$$

$$Y(s) = \underbrace{-\frac{1}{9}\frac{U(s)}{s+2}}_{x_1} + \underbrace{\frac{2}{3}\frac{U(s)}{(s-7)^2}}_{x_2} + \underbrace{\frac{1}{9}\frac{U(s)}{s-7}}_{x_3}$$

$$Y(s) = -\frac{1}{9}x_1 + \frac{2}{3}x_2 + \frac{1}{9}x_3$$

$$x_1(s) = \frac{U(s)}{s+2}$$

$$x_1'(s) + 2x_1(s) = u(t)$$

$$x_1'(s) = -2x_1(s) + u(t)$$

$$x_2(s) = \frac{U(s)}{(s-7)^2} = \frac{x_3}{s-7}$$

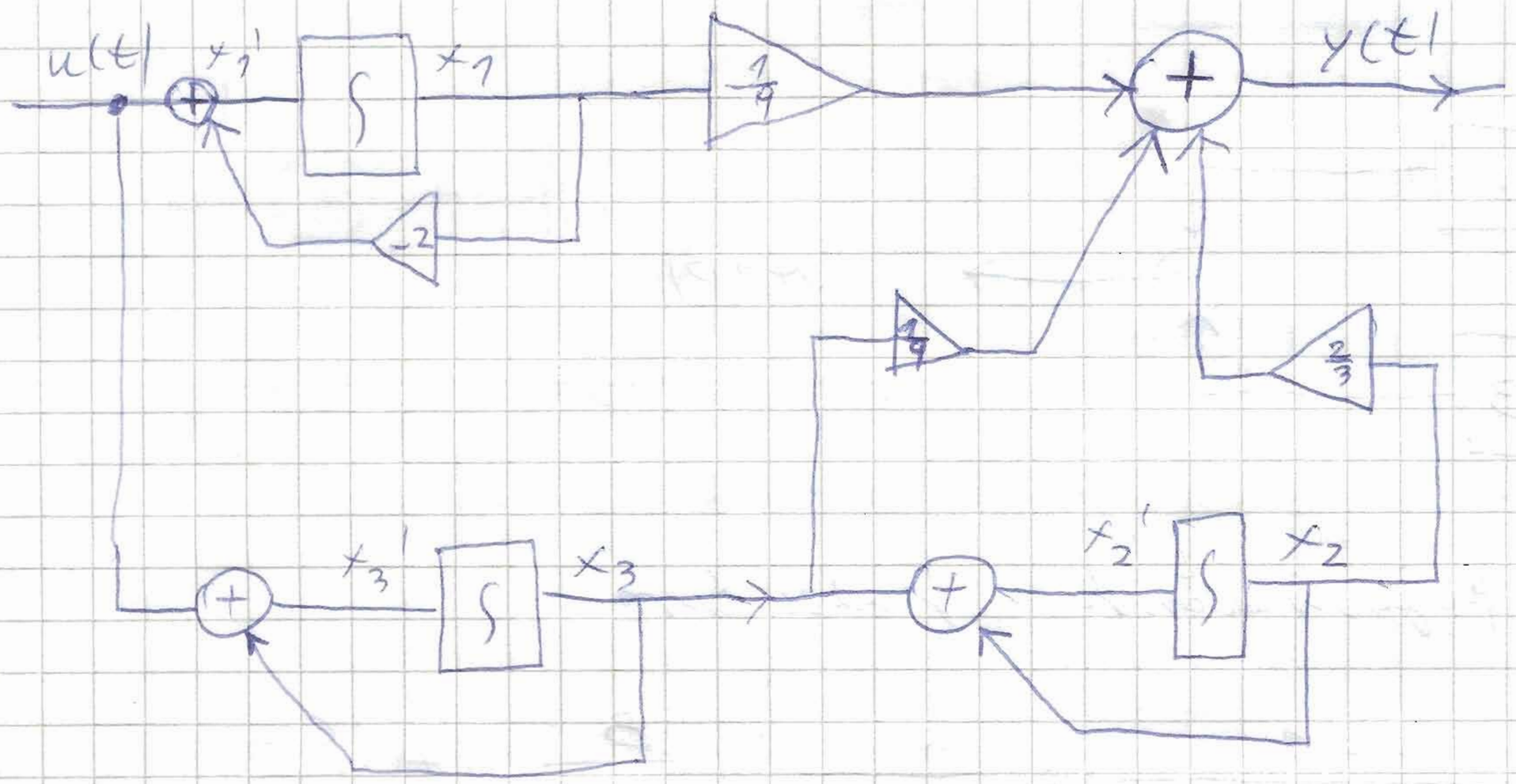
ne može biti više nego što je red sustava

$$x_2'(s) - x_2(s) = x_3(s)$$

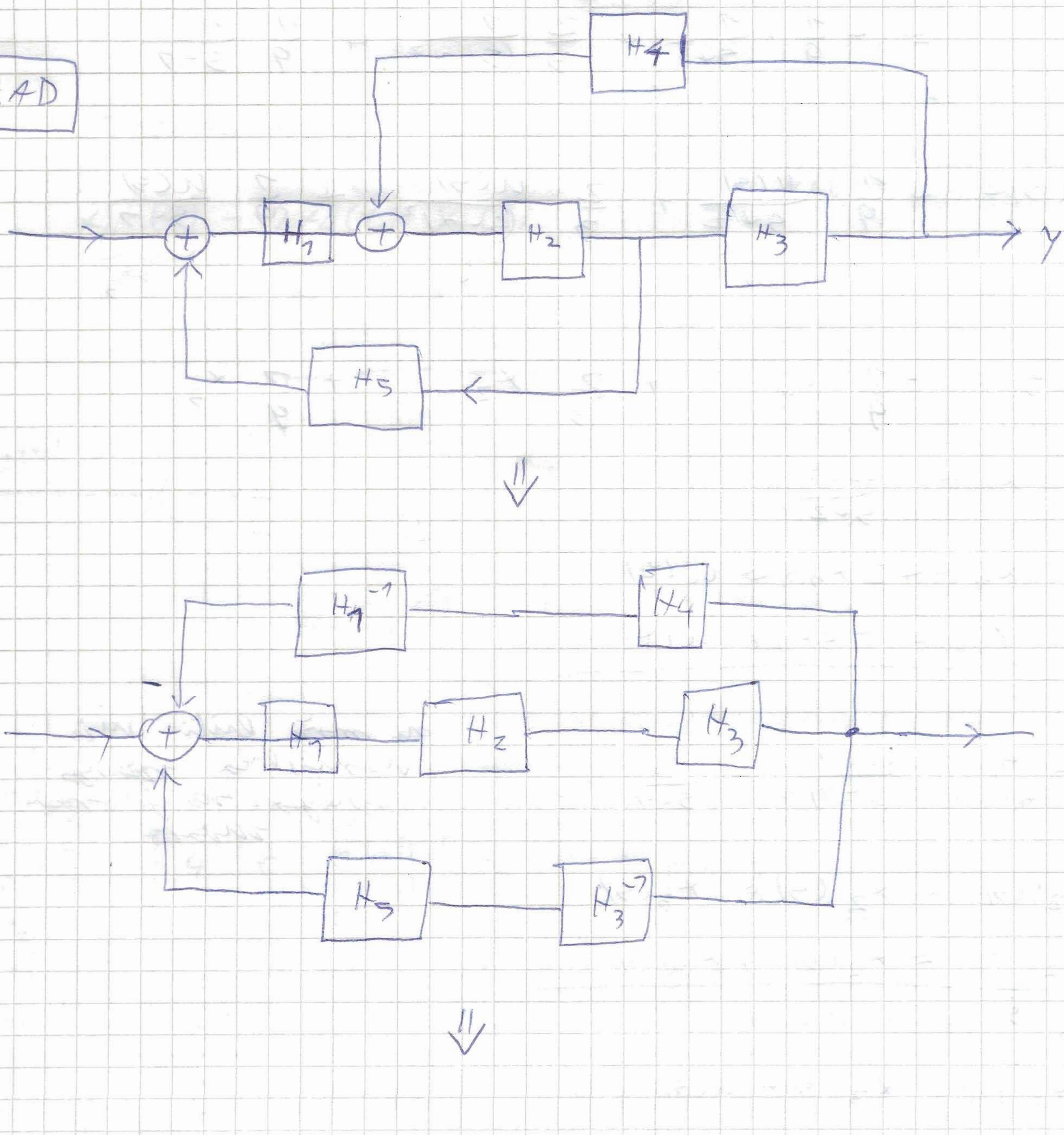
$$x_2'(s) = x_2(s) + x_3(s)$$

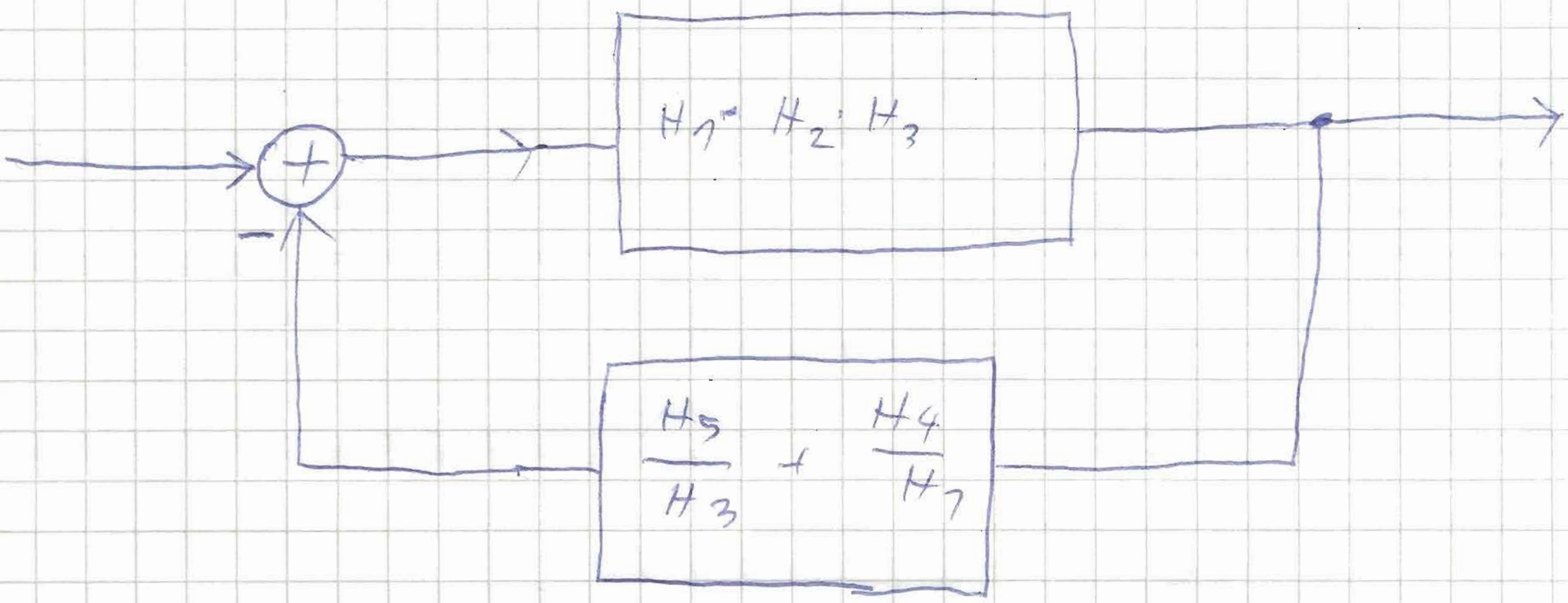
$$x_3(s)(s-7) = U(s)$$

$$x_3'(s) = x_3(s) + u(s)$$



ZAD





POVRATNA VEZA

$$H(s) = \frac{H_1 H_2 H_3}{1 + H_2 H_3 H_4 + H_1 H_2 H_5}$$