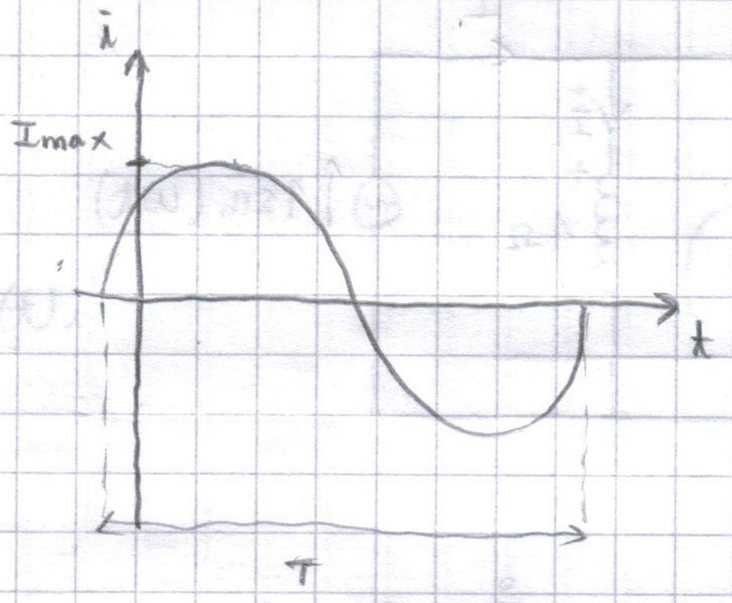
2. MASS iz Osnova elektrotehnike

By:
Tywin
&
hmp

Sadržaj:

Izmjenična struja 1
RLC krugovi 3
Frekvencijske ovisnosti 5

IZMJENIČNA STRUJA ⇒



$$i(t) = I_{max} \cdot \sin(\omega t + \alpha_i)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

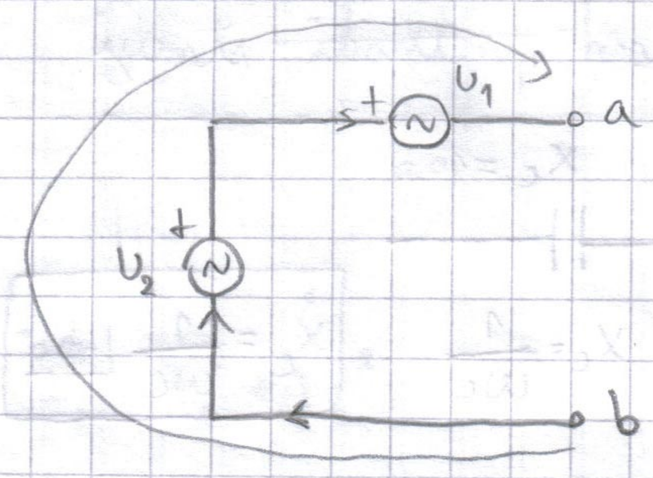
$$\dot{I} = |I| \angle \alpha_i$$

$$|I| = I_{ef} = \frac{I_{max}}{\sqrt{2}}$$

↳ uvijek koristimo efektivnu vrijednost

$$\boxed{\dot{U} \equiv \underline{U}}$$ ⇒ KOMPLEKSNI BROJ

1)



$$U_{ab}(t) = U_2 - U_1$$

$$\dot{U} = U_{ef} \angle \varphi$$

$$U_1(t) = 14{,}1 \cdot \sin(400t + 30^\circ)$$

$$U_2(t) = 10 \cdot \cos(400t - 45^\circ)$$

$$U_2(t) = 10 \sin(400t - 45^\circ + 90^\circ)$$

$$\dot{U}_1 = \frac{14{,}1}{\sqrt{2}} \angle 30^\circ = 10 \angle 30^\circ$$

↳ POLARNI OBLIK → u ovom obliku ne možemo zbrajati

$$\dot{U}_1 = 10 \cdot [\cos 30^\circ + j \sin 30^\circ] = 8{,}66 + j5$$

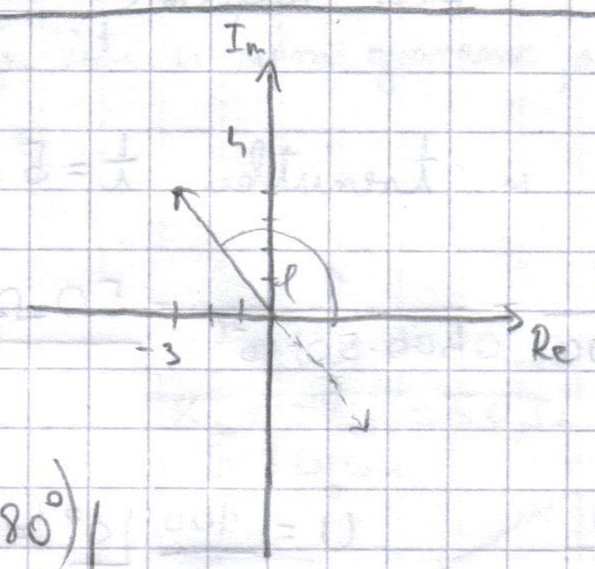
$$\dot{U}_2 = \frac{10}{\sqrt{2}} \angle 45^\circ = 5 + j5$$

$$\boxed{\dot{U}_{ab} = 3{,}66 \angle 180^\circ}$$

$$\dot{U}_{ab} = \dot{U}_2 - \dot{U}_1 = 5 + j5 - 8{,}66 - j5 = -3{,}66$$

$$U_{ab} = 3{,}66 \cdot \sqrt{2} \cdot \sin(400t + 180^\circ)$$

$$\dot{Z} = -3 + j4 = |\dot{Z}| \angle \varphi$$

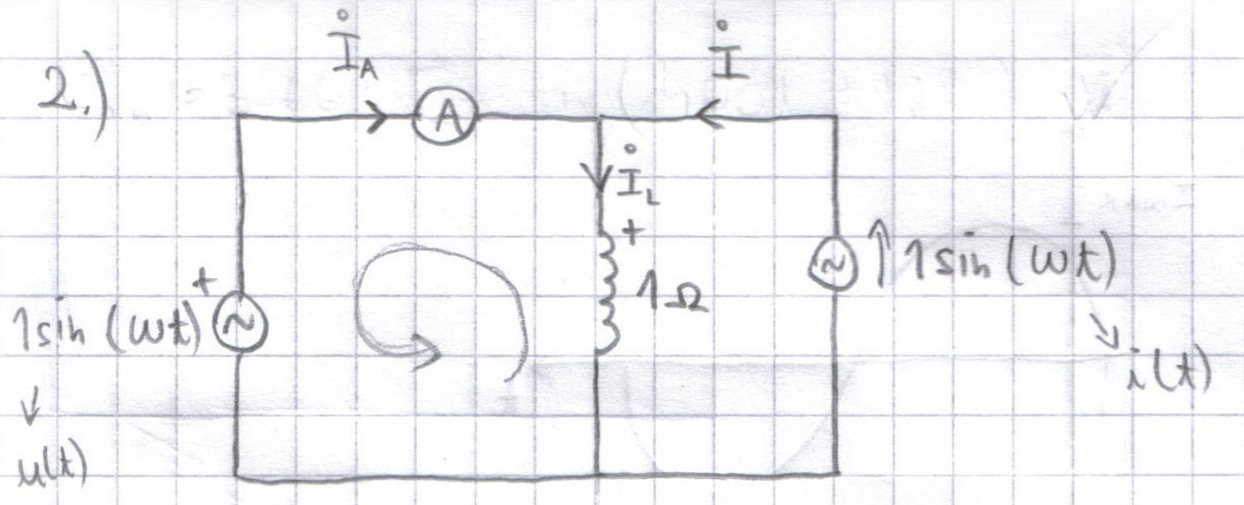


$$|\dot{Z}| = \sqrt{3^2 + 4^2} = 5$$

$$\varphi = \operatorname{arctg} \frac{Im}{Re} = \operatorname{arctg}\left(\frac{4}{-3}\right) = -53{,}13^\circ (+180^\circ)$$

$$Z = 5 \angle 126{,}87^\circ$$

2.)



$\dot{U} = \frac{1}{\sqrt{2}} \angle 0°$ $\qquad$ $\dot{I} = \frac{1}{\sqrt{2}} \angle 0°$

$\dot{I}_a + \dot{I} = \dot{I}_L$

$\dot{I}_a = \dot{I}_L - \dot{I}$

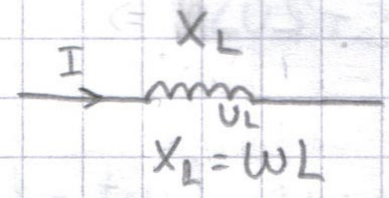
$\dot{I}_L \cdot \dot{X}_L - \dot{U} = 0$

$\dot{I}_L = \frac{\dot{U}}{\dot{X}_L} = \frac{\frac{1}{\sqrt{2}} \angle 0°}{1 \angle 90°}$

$\dot{I}_L = \frac{1}{\sqrt{2}} \angle -90° = -j \frac{1}{\sqrt{2}}$

$\dot{I}_a = -j \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$|I_a| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1A$ → dobijemo EFEKTIVNU vrijednost



$X_L = \omega L$

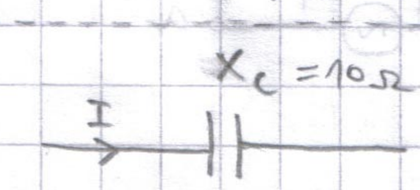
$\boxed{\dot{X}_L = \omega L \angle 90°} \Rightarrow j\omega L$

$\dot{U}_L = \dot{I}_L \cdot \dot{X}_L$

npr. $\dot{U}_L = 10 \angle 0°$ $\qquad$ $\dot{I} = \frac{10 \angle 0°}{5 \angle 90°} = 2 \angle -90°$

$\dot{X}_L = 5 \angle 90°$

→ U je na 0°, a I na -90° → struja ima manji kut od napona, znači da kasni za naponom, odnosno napon prethodi struji



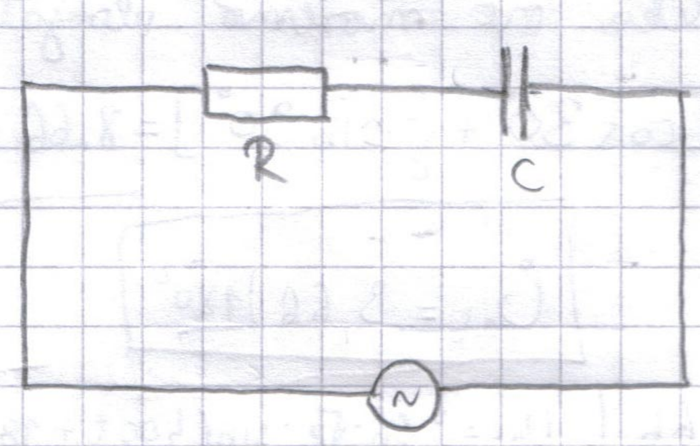
$X_C = \frac{1}{\omega C}$ $\qquad$ $\boxed{\dot{X}_C = \frac{1}{\omega C} \angle -90°} \Rightarrow \frac{-j}{\omega C}$

$\dot{I} = \frac{\dot{U}}{\dot{X}_C} = 1 \angle 90°$

→ struja prethodi naponu za 90°, odnosno napon kasni za strujom

$\dot{R} = R \angle 0° = R$

3.)



$R = 50\Omega$

$C = 50\mu F$

$u(t) = 100 \cdot \sin(400t)$

$U_C = ?$ u trenutku $t = 5$ ms

$X_C = \frac{1}{\omega C} = \frac{1}{400 \cdot 50 \cdot 10^{-6}} = 50\Omega$

$\dot{Z} = \dot{R} + \dot{X}_C = 50 \angle 0° + 50 \angle -90°$

$\dot{Z} = 50 - j50$

$\dot{Z} = 70{,}7 \angle -45°$

$\dot{I} = \frac{\dot{U}}{\dot{Z}}$ $\qquad$ $\dot{U} = \frac{100}{\sqrt{2}} \angle 0° = 70{,}7 \angle 0°$

$\dot{I} = \frac{70{,}7 \angle 0°}{70{,}7 \angle -45°} = 1 \angle 45° A$

$\dot{U}_C = \dot{I} \cdot \dot{X}_C = 1 \angle 45° \cdot 50 \angle -90° = 50 \angle -45°$

$u_C(t) = 50 \cdot \sqrt{2} \cdot \sin(400t - 45°)$

$u_C(t = 5ms) = 70{,}7 \cdot \sin(2 - 45°)$

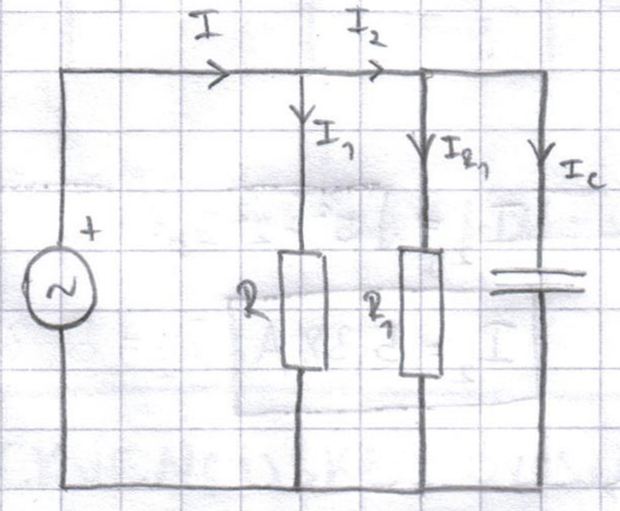
radijani · stupnjevi

$2\,rad = 2 \cdot \frac{180^\circ}{\pi} = 114.59^\circ$

$u_c(t = 5ms) = 70{,}7 \cdot \sin(69{,}59^\circ)$

RLC KRUGOVI

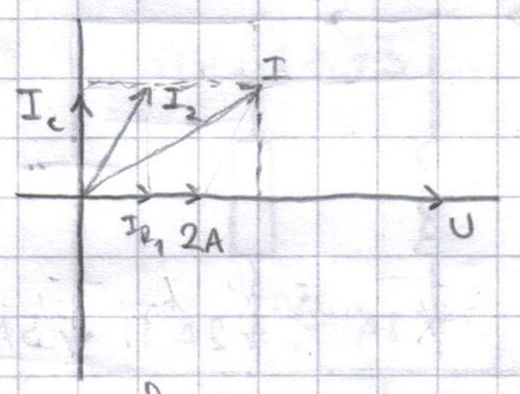
1.)



$I = 3A$ $\quad R_1 = 4\,\Omega$

$I_1 = 2A$

$I_2 = 2A$



struja je pod kutem jer se dijeli na I_{R_1} i I_c koja prethodi naponu za 90°

$I_2^2 = I_c^2 + I_{R_1}^2$

$$+\begin{cases} I^2 = \underbrace{(I_{R_1} + I_1)}_{\text{struje koje su u fazi s naponom}}{}^2 + I_c^2 \\ -I_2^2 = -I_c^2 - I_{R_1}^2 \end{cases}$$

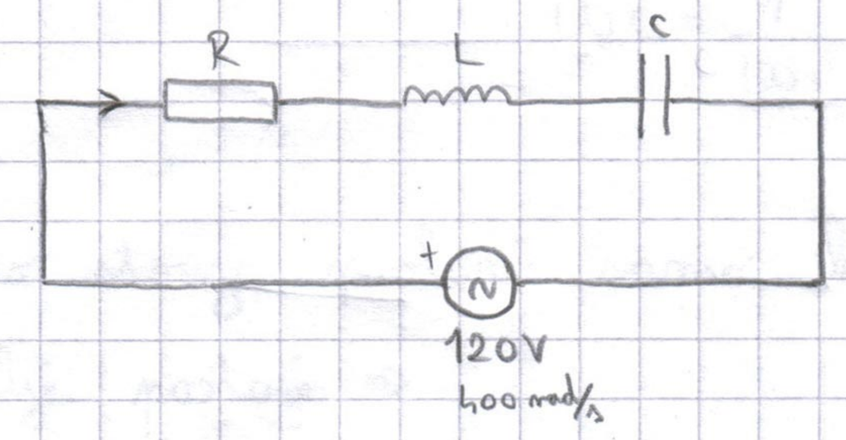
$I^2 - I_2^2 = 2 I_{R_1} I_1 + I_1^2$

$I_{R_1} = \frac{I^2 - I_2^2 - I_1^2}{2 I_1} = \frac{1}{4} A = \underline{0{,}25A}$

$U_{R_1} = U = I_{R_1} \cdot R_1 = \underline{1V}$

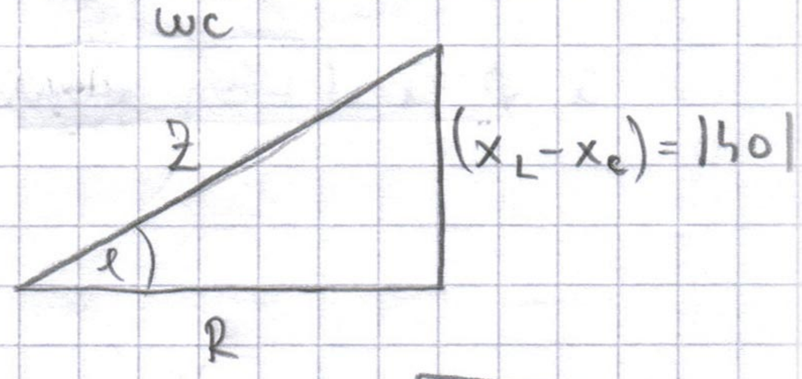
$\Rightarrow$ u paraleli postavljamo napon na 0° jer je isti u svim granama, a u seriji ćemo struju postaviti na 0° (u DIJAGRAMIMA)

2.) (1-9)



$L = 25\,mH$
$C = 50\,\mu F$
$\varphi = 63{,}4^\circ$

$X_L = \omega L = 10\,\Omega$
$X_c = \frac{1}{\omega C} = 50\,\Omega$

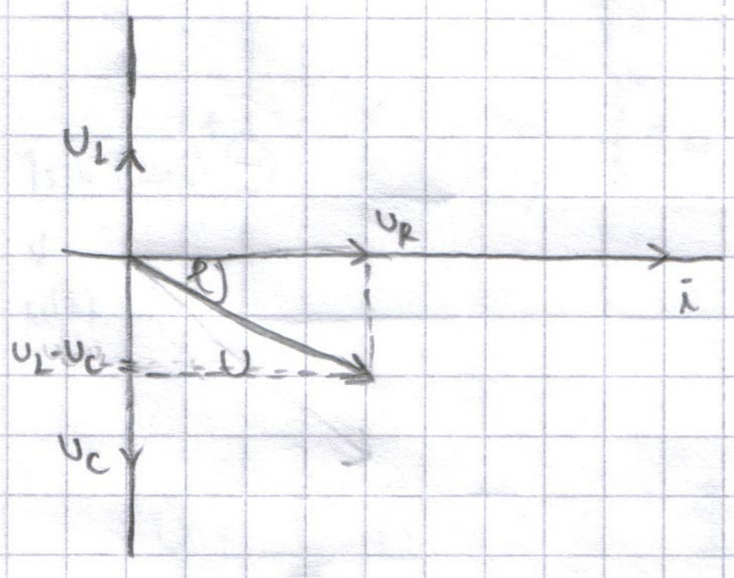


$\dot{X}_L = j10 \quad \dot{X}_c = -j50 \quad R = 20$

$\dot{Z}_{UK} = 20 + j10 - j50 = \underline{20 - j40}$

$tg\,\varphi = \frac{40}{R}$ $\quad \boxed{R = 20\,\Omega}$

$Z_{UK} = 20 - j40$



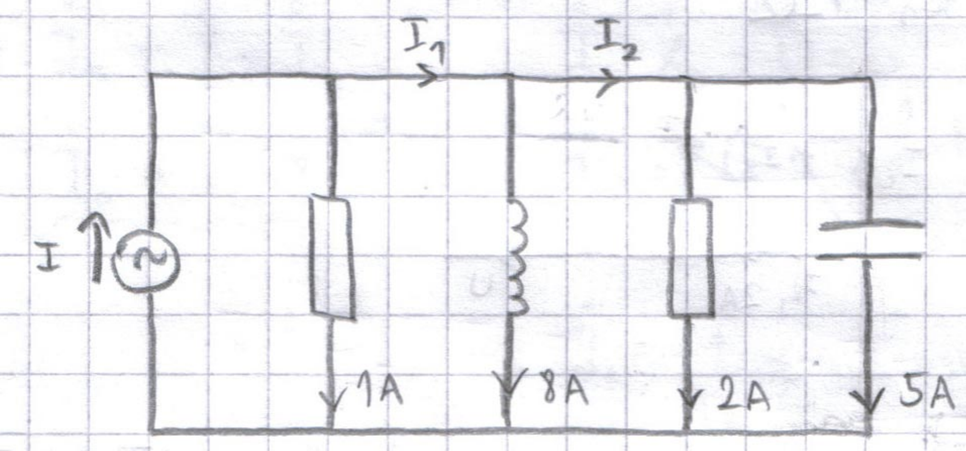
$\cos\varphi = \frac{U_R}{U}$ $\quad U_R = \cos\varphi \cdot U = \underline{53{,}73\,V}$

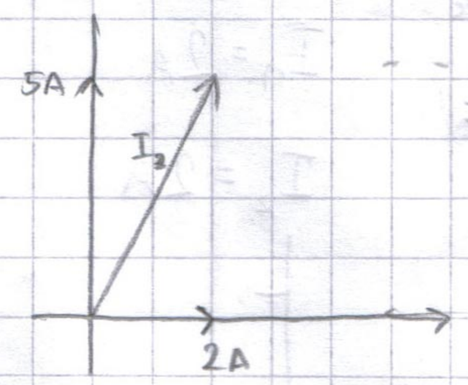
$I = \frac{U_R}{R} = \underline{2{,}68\,A}$

$U_L = I \cdot X_L = 26.8\,V$

$U_C = I \cdot X_C = 134\,V$

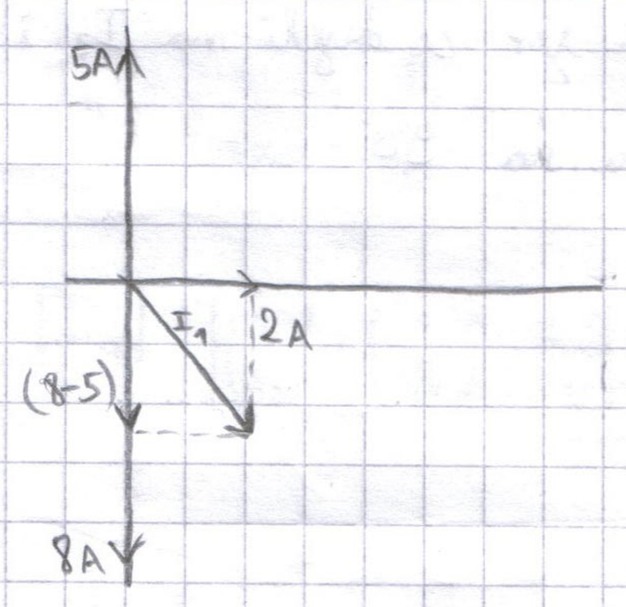
3.)





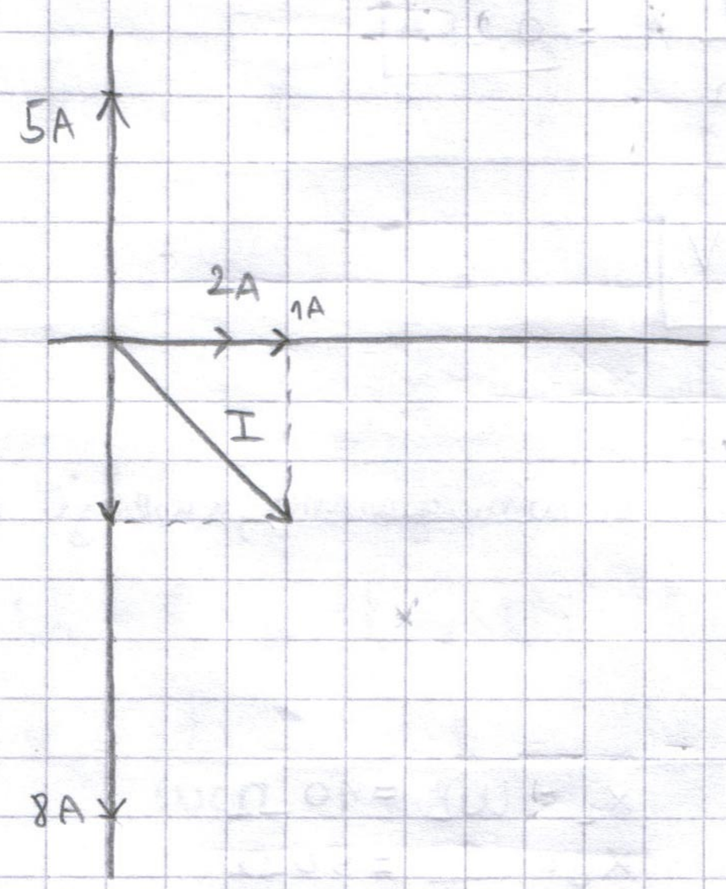
$I_2 = \sqrt{5^2 + 2^2}$

$\boxed{I_2 = 5{,}38\,A}$



$I_1 = \sqrt{(8-5)^2 + 2^2} = \sqrt{13}$

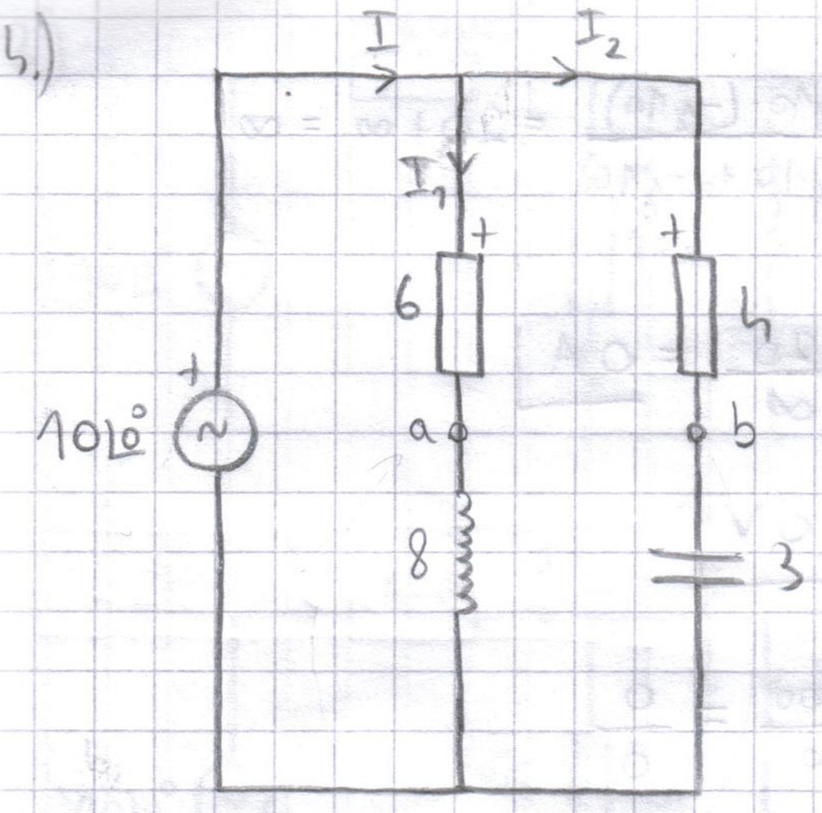
$\boxed{I_1 = 3{,}6\,A}$



$I = \sqrt{(8-5)^2 + (2+1)^2} = \sqrt{18} = 3\sqrt{2}\,A$

$\boxed{I = 4{,}24\,A}$

4.)



$\dot{U}_{ab} = 4 \cdot \dot{I}_2 - 6 \cdot \dot{I}_1$

$\dot{Z}_1 = 6 + j8$

$\dot{I}_1 = \frac{\dot{U}}{\dot{Z}_1} = \frac{10}{6+j8} = 1\angle -53.13 = 0,6 - j0,8$

$\dot{Z}_2 = 4 - j3$

$\dot{I}_2 = \frac{\dot{U}}{\dot{Z}_2} = 1,6 + j1,2$

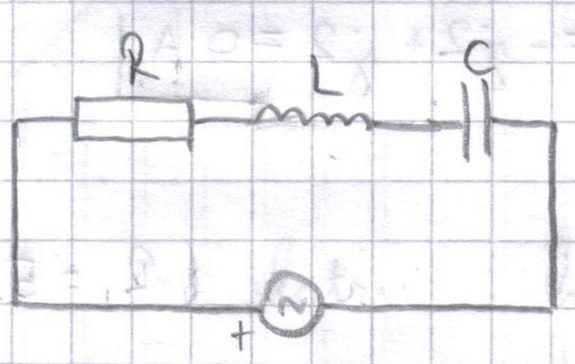
$\dot{U}_{ab} = 6,4 + j4,8 - 3,6 + j4,8 = 2,8 + j9,6 = 10\angle 73.73°$

FREKVENCIJSKE OVISNOSTI

- REZONANCIJA ⇒

$\mathrm{Im}\{Z_{UK}\} = 0$!

↳ imaginarni dio ukupne impedacije jednak je 0.



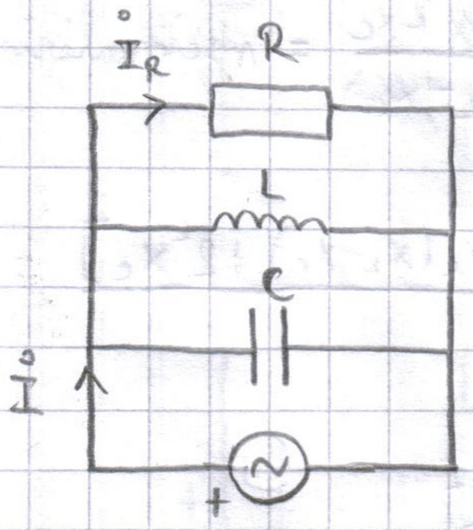
$Z_{UK} = R + jX_L - jX_C$

$\mathrm{Im}\{Z_{UK}\} = X_L - X_C = 0$

$X_L = X_C \rightarrow \omega L = \frac{1}{\omega C}$

$\dot{I} = \frac{\dot{U}}{\dot{Z}_{UK}} = \frac{\dot{U}}{R}$

$\boxed{\omega = \frac{1}{\sqrt{LC}}}$



$\frac{1}{\dot{Z}_{UK}} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$

VODLJIVOST ⇒ $\frac{1}{Z}$ $(\dot{Y})$

$\dot{Y}_{UK} = G - jB_L + jB_C = \underline{G}$

$\mathrm{Im}\{\dot{Y}_{UK}\} = 0$ ⇒ rezonancija

↳ $B_C = B_L$

$\omega C = \frac{1}{\omega L}$

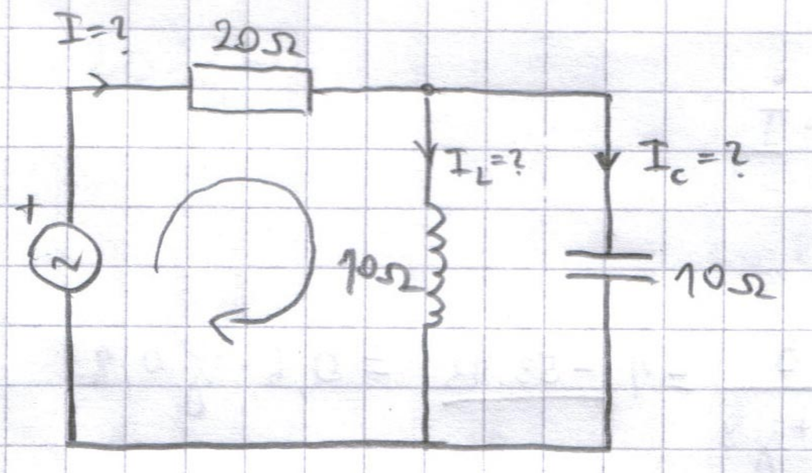
$\dot{I} = \frac{\dot{U}}{\dot{Z}_{UK}} = \frac{\dot{U}}{R}$

$\dot{I}_R = \frac{\dot{U}}{R}$

→ iako struja prolazi samo kroz otpornik, struje na L i C i dalje postoje ⇒

$\dot{I}_L = \frac{\dot{U}}{jX_L}$ $\qquad$ $\dot{I}_C = \frac{\dot{U}}{-jX_C}$

1.)



$$\dot{U} = 20\angle 0^\circ$$

$$\dot{I}_c = \frac{\dot{U}_P}{-jX_c} = \frac{20}{-j10} = \underline{j2}$$

$$\dot{I}_L = \frac{\dot{U}_P}{jX_L} = \frac{20}{j10} = \underline{-j2}$$

$$\dot{I} = \dot{I}_L + \dot{I}_c = -j2 + j2 = \underline{0\,A}$$

$$\dot{Z}_{UK} = 20 + \frac{j10\cdot(-j10)}{j10 + (-j10)} = 20 + \infty = \infty$$

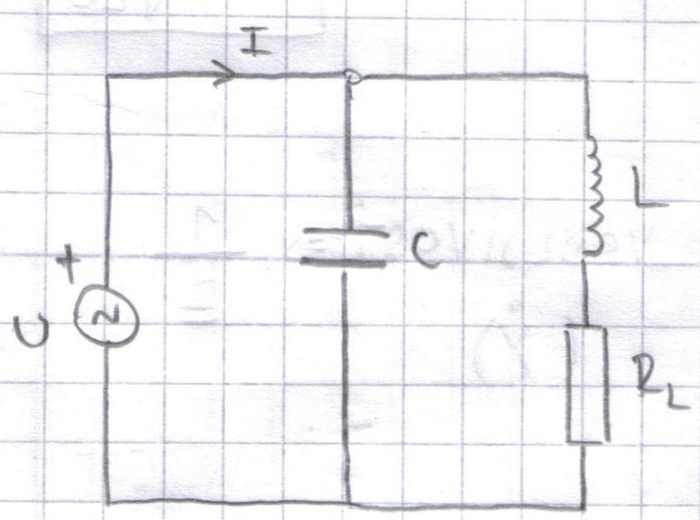
$$\dot{I} = \frac{\dot{U}}{\dot{Z}_{UK}} = \frac{20}{\infty} = \underline{0\,A}$$

$$\dot{U}_R = I\cdot R = \underline{0\,V}$$

$$\dot{U}_P = I\cdot\frac{100}{0} = \frac{0}{0}$$

$$\dot{U}_P = \dot{U} = \underline{20\angle 0^\circ}$$

2.) Priključimo realni svitak ($R_L = 5\,\Omega$, $L = 20\,mH$) u paralelu s kondenzatorom ($C = 300\,\mu F$), odredi frekvenciju na kojoj su struja i napon u fazi (pod istim kutom ⇒ spoj je u rezonanciji ⇒ $Im(Z_{UK}) = 0$)



$$\dot{Z}_{UK} = \frac{(R + jX_L)(-jX_c)}{R + jX_L - jX_c} = \frac{X_L X_c - jRX_c}{R + j(X_L - X_c)} = \text{racionaliziramo}$$

$$= \frac{RX_LX_c - RX_c(X_L - X_c) - j\left[X_LX_c(X_L - X_c) + R^2X_c\right]}{R^2 + (X_L - X_c)^2}$$

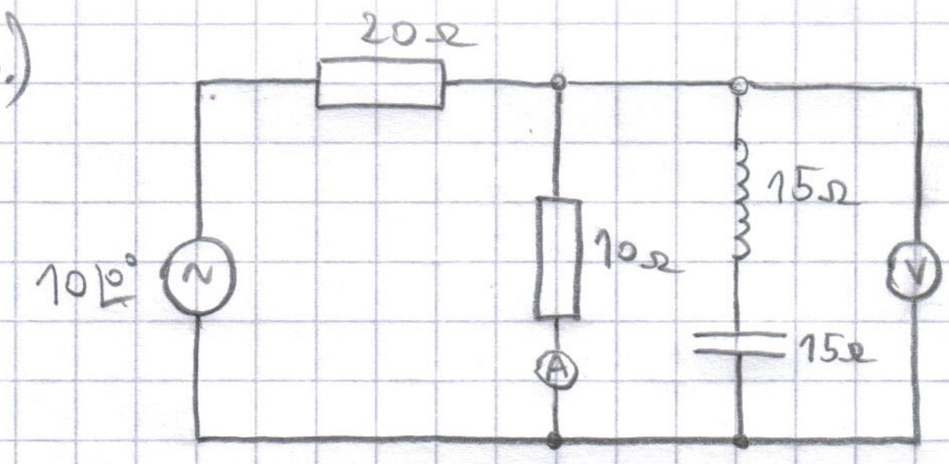
$$Im\{Z_{UK}\} = \frac{X_LX_c(X_L - X_c) + R^2X_c}{R^2 + (X_L - X_c)^2} = 0 \qquad -X_L^2 + X_LX_c = R^2$$

$$X_L^2 = X_LX_c - R^2$$

$$(\omega L)^2 = \cancel{\omega} L\cdot\frac{1}{\cancel{\omega} C} - R^2$$

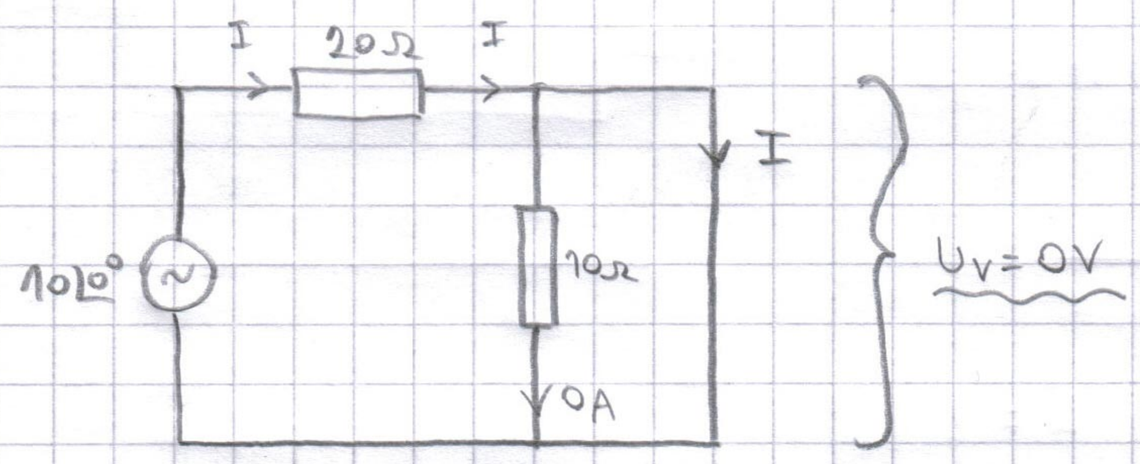
$$\boxed{\omega^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

3.)



$$\dot{U}_L = \dot{I} \cdot j\dot{X}_L = 0{,}5\angle 0^\circ \cdot 15\angle 90^\circ =$$

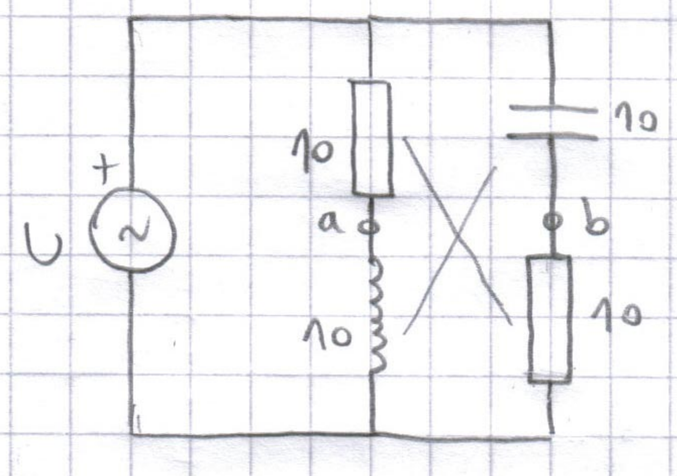
$$= 7{,}5\angle 90^\circ = \underline{j\,7{,}5\,V}$$



$$\dot{Z}_{VK} = 20\,\Omega$$

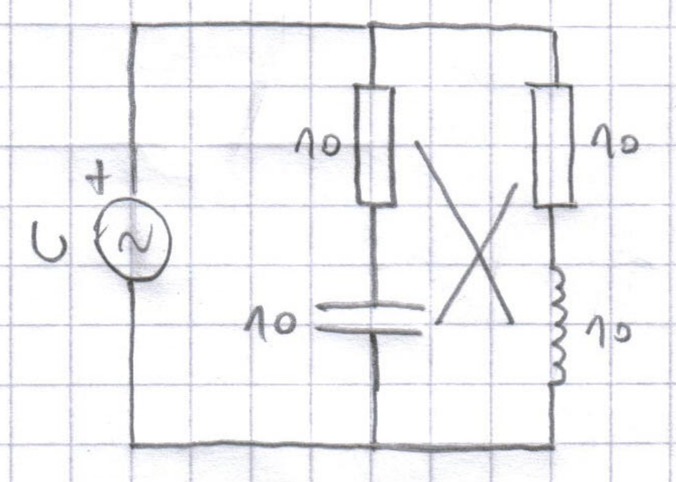
$$\dot{I} = \frac{\dot{U}}{\dot{Z}_{VK}} = \frac{10\angle 0^\circ}{20} = \underline{0{,}5\angle 0^\circ\,A}$$

4.)



$10 \cdot 10 = j10 \cdot (-j10)$ $\quad 100 = 100$

$\underline{U_{ab} = 0\,V}$



$10 \cdot j10 = 10 \cdot (-j10)$ $\quad j100 = -j100$

$\cancel{U_{ab} = 0}$