1.4 SILA NA NABOJ U GIBANJU

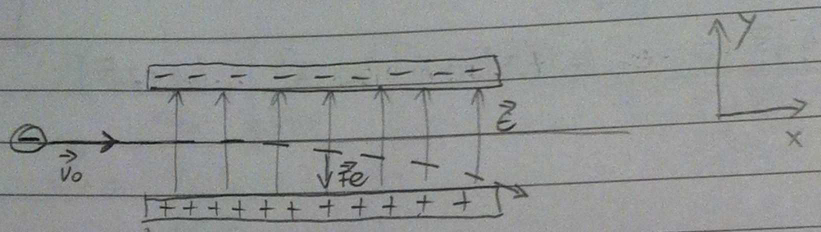
Lorentzova sila $\vec{F} = \underbrace{Q\vec{E}}_{\vec{F}_e} + \underbrace{Q(\vec{v}\times\vec{B})}_{\vec{F}_m} = Q(\vec{E} + \vec{v}\times\vec{B})$

$\vec{F}_e$ - električna komponenta

$\vec{F}_m$ - magnetska komponenta

SLAJDOVI:

1. Elektron upada u područje u kojem vlada el. polje $\vec{E}$ s početnom brzinom $\vec{v}_0$ prema slici. Odredi putanju elektrona, ako se zanemari gravitacijska sila.



- ravnotežna sila $m\vec{a} = (-)e\vec{E}$
  ↳ jer se radi o elektronu, privlači ga (+) nabijena ploča kondenzatora

$m \cdot \vec{a} = \vec{F}_e = -e \cdot \vec{E}$

$m \frac{d\vec{v}}{dt} = -e \cdot \vec{E}$

$\vec{v} = v_x \cdot \vec{a}_x + v_y \cdot \vec{a}_y = \frac{dx}{dt}\vec{a}_x + \frac{dy}{dt}\vec{a}_y$

$\vec{E} = \vec{E}_y = E \cdot \vec{a}_y$

$v(t=0) = v_{x0} = v_0 \rightarrow$ u $t=0$ elektron je u ishodištu koord. sustava

$\rightarrow m \cdot \frac{dv_x}{dt}\vec{a}_x + m \cdot \frac{dv_y}{dt}\vec{a}_y = -e \cdot E \cdot \vec{a}_y$

$m\frac{dv_x}{dt} = 0 \rightarrow \frac{dv_x}{dt} = 0 \Rightarrow \boxed{v_x = \text{konstanta} = v_{x_0}} \rightarrow$ početna brzina

$m\frac{dv_y}{dt} = -e \cdot E \;/ : m \,/ \int$ u $t=0$ $v_{y0} = 0$

$v_y = -\frac{e \cdot E}{m} \cdot t + \text{konst} = -\frac{e \cdot E}{m} \cdot t + v_{y0} \Rightarrow \boxed{v_y = -\frac{e \cdot E}{m} \cdot t}$

$v_x = \frac{dx}{dt} \quad / \int$

$x = \int v_x dt = v_{x_0} \cdot t + konst = v_{x_0} \cdot t + x_0$

$x_0 = x(t=0) = 0$

$\hookrightarrow \boxed{x(t) = v_{x_0} \cdot t} \rightarrow$ linearno

$v_y = \frac{dy}{dt} \quad / \int$

$\rightarrow y = \int v_y dt = \int -\frac{e \cdot E}{m} \cdot t\, dt = -\frac{e \cdot E}{m} \cdot \frac{t^2}{2} + konst = -\frac{eE}{m} \cdot \frac{t^2}{2} + y_0$

$y_0 = y(t=0) = 0$

$\hookrightarrow \boxed{y(t) = -\frac{eE}{m} \cdot \frac{t^2}{2}}$

(1) $x(t) = v_{x_0} \cdot t \rightarrow t = \frac{x(t)}{v_{x_0}}$

(2) $y(t) = -\frac{eE}{m} \cdot \frac{t^2}{2}$

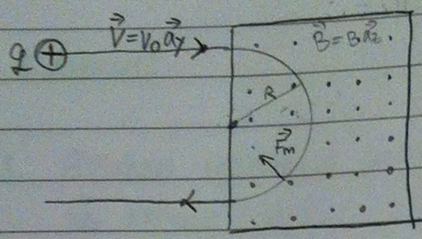
$\boxed{y(t) = -\frac{eE}{2m} \cdot \frac{x^2(t)}{v_{x_0}^2}} \rightarrow$ parabola

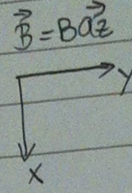
zinom

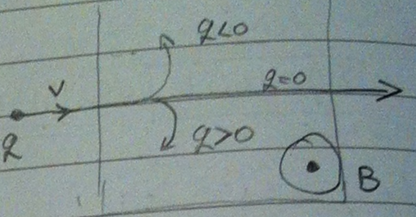
cijstha sila

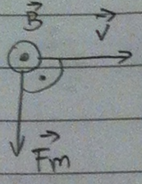
deviatora

2. Čestica mase m i naboja $+q$ upada u područje u kojem vlada magnetsko polje $\vec{B}$ s poč. brzinom $\vec{v}_0$ prema slici. Odredi putanju čestice.

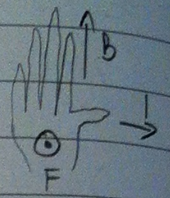








- Pravilo desne ruke: palac u smjeru struje
B u smjeru prstiju
Fm okomito iz dlana



$$\vec{B} = B\,\vec{a}_z \qquad \vec{v} = v_0 \cdot \vec{a}_y$$

$$t=0 \qquad \vec{F}_m = q\,(\vec{v}\times\vec{B}) = q \cdot v_0 \cdot B \cdot \vec{a}_x \qquad (v \perp B,\ \sin(90°)=1)$$

$$\vec{v} = v_x\,\vec{a}_x + v_y\,\vec{a}_y + v_z\,\vec{a}_z$$

$$\vec{a} = \frac{dv_x}{dt}\,\vec{a}_x + \frac{dv_y}{dt}\,\vec{a}_y + \frac{dv_z}{dt}\,\vec{a}_z$$

$$m\cdot\vec{a} = q\,(\vec{v}\times\vec{B})$$

$$m\left(\frac{dv_x}{dt}\,\vec{a}_x + \frac{dv_y}{dt}\,\vec{a}_y + \frac{dv_z}{dt}\,\vec{a}_z\right) = q\left[\left(v_x\vec{a}_x + v_y\vec{a}_y + v_z\vec{a}_z\right)\times B\cdot\vec{a}_z\right]$$

$$v\times B = \begin{vmatrix} a_x & a_y & a_z \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} = B\cdot v_y\,\vec{a}_x - B\cdot v_x\,\vec{a}_y$$

$$= qB\left(v_y\cdot\vec{a}_x - v_x\cdot\vec{a}_y\right)$$

(1) $m\cdot\frac{dv_z}{dt}\,\vec{a}_z = 0 \Rightarrow \frac{dv_z}{dt} = 0 \ \Big/\int \Rightarrow \boxed{v_z = \text{konst} = v_{z0} = 0}$

(2) $m\,\frac{dv_x}{dt}\cdot\vec{a}_x = qB\cdot v_y \longrightarrow \frac{dv_x}{dt} = \frac{qB}{m}\cdot v_y$

(3) $m\,\frac{dv_y}{dt}\cdot\vec{a}_y = -q\cdot B\cdot v_x \ \Big/\frac{d}{dt} \qquad m\cdot\frac{d^2v_y}{dt^2} = -q\cdot B\cdot\frac{dv_x}{dt}$

netsko

(2) i (3) $m\frac{d^2v_y}{dt^2} = -\frac{q^2B^2}{m} \cdot v_y \quad / : m$

$$\frac{d^2v_y}{dt^2} + \underbrace{\frac{q^2B^2}{m^2}}_{\to \omega_c^2} v_y = 0 \longrightarrow \boxed{\frac{d^2v_y}{dt^2} + \omega_c^2 v_y = 0}$$

opće rj: $\boxed{v_y(t) = A\sin\omega_c t + B\cos\omega_c t}$

(4) $\boxed{v_y(t) = A\sin(\omega_c t) + B\cos(\omega_c t)}$

(3) $v_x(t) = -\frac{m}{qB}\frac{dv_y}{dt} = -\frac{1}{\omega_c}\frac{dv_y}{dt}$

(4) u (3) $v_x(t) = -\frac{1}{\omega_c} \cdot \left[\cancel{\omega_c} \cdot A \cdot \cos(\omega_c t) - \cancel{\omega_c} B\sin(\omega_c t)\right]$

(5) $\boxed{v_x(t) = -A\cos(\omega_c t) + B\sin(\omega_c t)}$

za $t=0$ $\quad v_x(0) = 0 \quad v_y(0) = v_0$

(4) $v_y(0) = v_0 = B$

(5) $v_x(0) = 0 = A$

KONAČNE BRZINE

(5) $v_x(t) = v_0\sin(\omega_c t)$

(4) $v_y(t) = v_0\cos(\omega_c t)$

PUTANJA:

$v_x = \frac{dx}{dt} \rightarrow x(t) = \int v_x\, dt = -\frac{v_0}{\omega_c}\cos(\omega_c t) + konst$

$x(t=0) = 0 = -\frac{v_0}{\omega_c} + konst \rightarrow konst = \frac{v_0}{\omega_c}$

$$\boxed{x(t) = \frac{v_0}{\omega_c}\left(1 - \cos(\omega_c t)\right)}$$

$v_y = \frac{dy}{dt} \rightarrow y(t) = \int v_y\, dt = \frac{v_0}{\omega_c}\sin(\omega_c t) + konst$

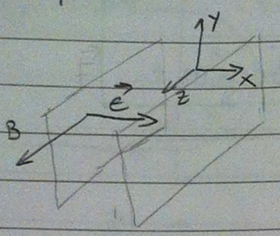
$y(t=0) = 0 = 0 \rightarrow konst = 0$

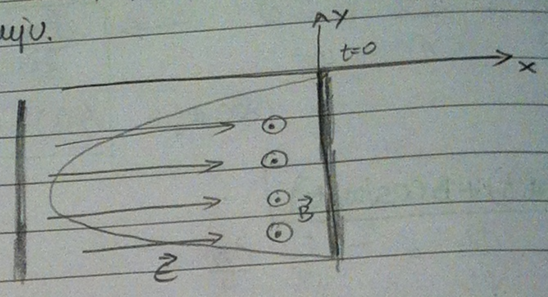
$$\boxed{y(t) = \frac{v_0}{\omega_c}\sin(\omega_c t)}$$

$x(t) = x_0 - R\cos(\omega_c t) \qquad x_0 = \frac{v_0}{\omega_c}$

$y(t) = R\sin(\omega_c t) \qquad R = \frac{v_0}{\omega_c}$

3. Elektron se iz stanja mirovanja emitira iz ploče pločastog kondenzatora unutar kojeg vlada homogeno električno polje. Kond. se nalazi u hom. magn. polju prema slici. Odredi putanju.





$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}) = m \cdot \vec{a}$$

$$\vec{v} = -v_x \vec{a_x} - v_y \vec{a_y} + v_z \vec{a_z}$$

$$\vec{a} = -\frac{d^2x}{dt^2}\vec{a_x} - \frac{d^2y}{dt^2}\vec{a_y} + \frac{d^2z}{dt^2}\vec{a_z}$$

$$\vec{E} = E\,\vec{a_x} \qquad \vec{v} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ -v_x & -v_y & v_z \\ 0 & 0 & B \end{vmatrix} = -B v_y \vec{a_x} + B \cdot v_x \vec{a_y}$$

$$\vec{B} = B\,\vec{a_z}$$

$$\vec{F} = Q \cdot E \cdot \vec{a_x} - Q \cdot B \cdot v_y \vec{a_x} + Q \cdot B \cdot v_x \vec{a_y} = m \cdot \vec{a}$$

(1) $m\frac{d^2z}{dt^2} = 0 \;/\int \;\rightarrow\; \frac{dz}{dt} = C_1 = v_z(0) = 0 \;/\int \quad z = C_2 = z(0) = 0$

(2) $-m\frac{d^2y}{dt^2} = +Q \cdot B \cdot \frac{dx}{dt} \;/\int \quad \frac{dy}{dt} = -\frac{Q \cdot B \cdot x}{m} + \underbrace{v_y(0)}_{0} = -\frac{Q \cdot B \cdot x}{m}$

(3) $-m\frac{d^2x}{dt^2} = Q \cdot E - Q \cdot B \cdot \frac{dy}{dt} \;/:(-m) \;\rightarrow\; \frac{d^2x}{dt^2} = -\frac{QE}{m} + \frac{QB}{m}\frac{dy}{dt}$

(2) u (3) $\quad \frac{d^2x}{dt^2} = -\frac{QE}{m} + \frac{QB}{m} \cdot \left(-\frac{Q \cdot B \cdot x}{m}\right) = -\frac{QE}{m} - \frac{Q^2B^2}{m^2}x$

Laplace $\quad s^2X(s) + \frac{Q^2B^2}{m^2}X(s) = -\frac{Q \cdot E}{m} \cdot \frac{1}{s}$ → ne zaboravi $\frac{1}{s}$!

$$X(s)\left(\frac{m^2s^2 + Q^2B^2}{m^2}\right) = -\frac{QE}{m} \cdot \frac{1}{s} \quad /:m^2 \quad /:m^2$$

$$X(s) = -\frac{QE}{m} \cdot \frac{1}{s} \cdot \frac{1}{s^2 + \underbrace{\frac{Q^2B^2}{m^2}}_{\omega^2}}$$

unutar
polju

$$\frac{1}{s(s^2+\omega^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+\omega^2} \quad / \cdot naz$$

$$1 = As^2 + A\omega^2 + Bs^2 \rightarrow A\omega^2 = 1 \Rightarrow A = \frac{1}{\omega^2} \qquad B = -\frac{1}{\omega^2}$$

$$\frac{1}{s(s^2+\omega^2)} = \frac{1}{\omega^2 s} - \frac{1}{\omega^2} \cdot \frac{s}{s^2+\omega^2}$$

$$X(s) = -\frac{QE}{m} \cdot \frac{1}{\omega^2}\left(\frac{1}{s} - \frac{s}{s^2+\omega^2}\right)$$

(b) $$\boxed{X(t) = \frac{-QE}{m} \cdot \frac{m^2}{Q^2B^2}\left(1 - \cos\left(\frac{QB}{m}t\right)\right)}$$

(4) U(2) $$\frac{dy}{dt} = \cancel{\frac{-Q \cdot x}{m}} \cdot \cancel{\frac{Q \cdot E}{m}} \cdot \frac{m^2}{Q^2B^2}\left(1 - \cos\left(\frac{QB}{m}t\right)\right)$$

$$\frac{dy}{dt} = \frac{E}{B}\left(1 - \cos\left(\frac{QB}{m}t\right)\right) \quad / \int \quad / \frac{d}{dt}$$

$$\boxed{y(t) = \frac{E}{B}\left(t - \frac{m}{QB}\sin\left(\frac{QB}{m}t\right)\right)} + C_4$$

$$C_4 = y(t=0) = 0$$

$$x(t_2) = 0 = \frac{-Em}{QB^2}\left(1 - \cos\frac{QBt}{m}\right) \rightarrow \cos\frac{QB}{m}t = 1 \Rightarrow \boxed{t = \frac{2\pi m}{QB}}$$

$$y(t_2) = \frac{E}{B}\left(\frac{2\pi m}{QB} - \frac{m}{QB}\sin\left(\frac{QB}{m} \cdot \frac{2\pi m}{QB}\right)\right) = \boxed{\frac{E \cdot 2\pi m}{QB^2}} = \frac{-E 2\pi m}{e \cdot B^2}$$

Berberović

1.4.2.

$\vec{E} = -5\vec{a_z}$ V/m

$\vec{B} = 2\vec{a_z}$

$v_0 = 3\vec{a_x} + 5\vec{a_y} + 10\vec{a_z}$ m/s

$\vec{v}(t) = ?$

Laplace: $f'(t) \rightarrow s \cdot F(s) - f(0)$

$f''(t) \rightarrow s^2 F(s) - s \cdot f(0) - f'(0)$

$f^n(t) \rightarrow s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}$

$\cos(\omega t) u(t) \circ\!\!-\!\!\bullet \frac{s}{s^2+\omega^2}$

$\sin(\omega t) u(t) \circ\!\!-\!\!\bullet \frac{\omega}{s^2+\omega^2}$

$$F = Q(\vec{E} + \vec{v} \times \vec{B}) = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$v \times B = \begin{vmatrix} a_x & a_y & a_z \\ v_x & v_y & v_z \\ 0 & 0 & 2 \end{vmatrix} = 2v_y\vec{a_x} - 2v_x\vec{a_y}$$

$$\vec{v} = v_x\vec{a_x} + v_y\vec{a_y} + v_z\vec{a_z}$$

$$\vec{E} = E\vec{a_z} \rightarrow E = -5$$

$$\vec{B} = B\vec{a_z} \rightarrow B = 2$$

$$F = \underset{(1)}{-5Q\vec{a_z}} + \underset{(3)}{2Q \cdot v_y\vec{a_x}} \underset{(2)}{- 2Q v_x\vec{a_y}} = m \cdot \frac{dv_x}{dt}\underset{(3)}{\vec{a_x}} + m\frac{dv_y}{dt}\underset{(2)}{\vec{a_y}} + m\frac{dv_z}{dt}\underset{(1)}{\vec{a_z}}$$

(1) $-5Q = m\frac{dv_z}{dt} \quad / \cdot \frac{dt}{m} \rightarrow -\frac{5Q\,dt}{m} = dv_z \quad / \int \rightarrow -\frac{5Qt}{m} + v_z(0) = v_z$

$$\boxed{v_z(t) = -\frac{5Qt}{m} + 10}$$

(2) $-2Q \cdot v_x(t) = m\frac{dv_y(t)}{dt}$

$$-2Q \cdot V_x(s) = m \cdot [s \cdot V_y(s) - v_y(0)] \Rightarrow V_x(s) = -\frac{m}{2Q}[sV_y(s) - v_y(0)]$$

$$= -\frac{m \cdot s}{2Q}V_y(s) + \frac{m}{2Q} \cdot v_y(0)$$

(3) $2Q v_y(t) = m \cdot \frac{dv_x}{dt}$

$$2Q \cdot V_y(s) = m \cdot [s \cdot V_x(s) - v_x(0)] \Rightarrow V_y(s) = \frac{m \cdot s}{2Q}V_x(s) - \frac{m}{2Q}v_x(0)$$

(2) ∪ (3) $2Q \cdot V_y(s) = m\left[-\frac{ms^2}{2Q}V_y(s) + \frac{ms}{2Q}\underbrace{v_y(0)}_{5} - \underbrace{v_x(0)}_{3}\right]$

$$V_y(s)\left[2Q + \frac{m^2 s^2}{2Q}\right] = \frac{5m^2 s}{2Q} - 3m$$

$$V_y(s)\left[\frac{4Q^2 + m^2 s^2}{\cancel{2Q}}\right] = \frac{5m^2 s - 6Qm}{\cancel{2Q}}$$

$$V_y(s) = \frac{5m^2 s - 6Qm \quad /:m^2}{4Q^2 + m^2 s^2 \quad /:m^2} = \frac{5s - \frac{6Q}{m}}{\frac{4Q^2}{m^2} + s^2} \rightarrow \omega^2 = \frac{4Q^2}{m^2}$$

$$\omega = \frac{2Q}{m}$$

$$\frac{5s}{\frac{4Q^2}{m^2} + s^2} \circ\!\!-\!\!\bullet \; 5\cos\left(\frac{2Q}{m}t\right)$$

$$\frac{-\frac{6Q}{m}}{\frac{4Q^2}{m^2} + s^2} \circ\!\!-\!\!\bullet \; -3\sin\left(\frac{2Q}{m}t\right)$$

$$V_y(t) = 5\cos\left(\frac{2Q}{m}t\right) - 3\sin\left(\frac{2Q}{m}t\right)$$

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(3) U (2) $$V_x(s) = -\frac{ms}{2Q}\left[\frac{m \cdot s}{2Q}V_x(s) - \frac{m}{2Q}\underbrace{V_x(0)}_{3}\right] + \frac{m}{2Q}\underbrace{V_y(0)}_{5}$$

$$V_x(s)\left(1 + \frac{m^2 s^2}{4Q^2}\right) = \frac{3m^2 s}{4Q^2} + \frac{5m}{2Q}$$

$$V_x(s)\left(\frac{4Q^2 + m^2 s^2}{\cancel{4Q^2}}\right) = \frac{3m^2 s + 10Qm}{\cancel{4Q^2}}$$

$$V_x(s) = \frac{3m^2 s + 10Qm \quad /:m^2}{4Q^2 + m^2 s^2 \quad /:m^2} = \frac{3s + \frac{10Q}{m}}{s^2 + \frac{4Q^2}{m^2}}$$

$$\frac{3s}{s^2 + \frac{4Q^2}{m^2}} \circ\!\!-\!\!\bullet \; 3\cos\left(\frac{2Q}{m}t\right)$$

$$\frac{\frac{10Q}{m}}{s^2 + \frac{4Q^2}{m^2}} \circ\!\!-\!\!\bullet \; 5\sin\left(\frac{2Q}{m}t\right)$$

$$\vec{V}(t) = \left[3\cos\left(\frac{2Q}{m}t\right) + 5\sin\left(\frac{2Q}{m}t\right)\right]\vec{a_x} + \left[5\cos\left(\frac{2Q}{m}t\right) - 3\sin\left(\frac{2Q}{m}t\right)\right]\vec{a_y}$$

$$+ \left[-\frac{5Qt}{m} + 10\right]\vec{a_z}$$