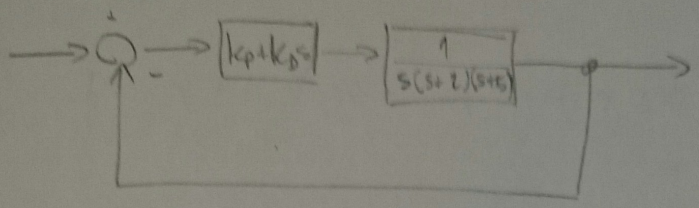
1.



$$\zeta = 0.707 \qquad \omega_n = \frac{4.6}{\zeta \cdot t_{1\%}} = 2.828$$

$$s^2 + 2\zeta\omega_n + \omega_n^2$$

$$\Rightarrow P_{1,2} = -2 \pm j2$$

$$G_0 = \frac{k_P + k_D s}{s(s+2)(s+5)}$$

$$1 + G_0 = s^3 + 7s^2 + 10s + K_D s + k_P = 0$$

$$1 + k_D \frac{s + \frac{k_P}{k_D}}{s^3 + 7s^2 + 10s} = 0 \qquad \frac{k_P}{k_D} \leftarrow k'$$

Fazni uvjet: $(s = P_1)$

$$\arg(s + k') - \underbrace{\arg(s^3 + 7s^2 + 10s)}_{-101.3^\circ} = 180^\circ$$

$$\Rightarrow \operatorname{atan}\frac{2}{k' - 2} = 78.7^\circ \Rightarrow \boxed{k' = \frac{12}{5}}$$

Amplitudni uvjet: $(s = P_1)$

$$\left| \frac{s + \frac{k_P}{k_D}}{s^3 + 7s^2 + 10s} \right| = \frac{1}{k_D} \Rightarrow \boxed{k_D = 10} \xRightarrow{k' = \frac{k_P}{k_D}} \boxed{k_P = 24}$$

2.

a) $G_L G_P = \frac{1}{(s+1)(s+2)} K \qquad 1 + G_0 = 0 \Rightarrow s^2 + 3s + 2 + K = 0$

$$s_{pi} = -\frac{3}{2} \pm \frac{1}{2}\sqrt{1-4K} \; ; \quad \frac{\partial s_{pi}}{\partial K} = \mp \frac{1}{\sqrt{1-4K}} = \frac{-1}{2s_{pi}+3}$$

$$S_K^{s_{pi}} = \frac{K}{s_{pi}} \frac{\partial s_{pi}}{\partial K} = \frac{-(s_{pi}+1)(s_{pi}+2)}{s_{pi}} \frac{-1}{2s_{pi}+3}$$

$$\boxed{S_K^{s_{pi}} = \frac{(s_{pi}+1)(s_{pi}+2)}{s_{pi}(2s_{pi}+3)}}$$

b)

$$G_P = \frac{1}{s+1} K \Rightarrow Y = \frac{G}{1+G} R \; ;$$

$$\frac{\partial Y}{\partial G} = \frac{1}{(1+G)^2} R \; ;$$

$$\boxed{S_G^Y = \frac{G}{Y} \frac{\partial Y}{\partial G} = \frac{1}{1+G}} \; ;$$

$$S_G^Y = \frac{s+1}{s+2} \Rightarrow |S_G^Y(jw)| = \frac{(w^2+2)^2 + w^2}{(w^2+4)^2}$$

$$|S_G^Y(jw)| \leq 0.9$$

$$\Rightarrow 0.1 w^4 - 1.48 w^2 - 8.96 = 0$$

$$\boxed{w \leq 3.435}$$

3.

$$G = \frac{-3+2z}{2-3z+z^2}$$

$$A_m = (z-0.5)^2 \qquad B_m = -3+2z$$

$$A \cdot R + B \cdot S = P \Longrightarrow (2-3z+z^2)R + (-3+2z)S = z^2+z+0.25$$

$$R = r_0 \qquad S = s_1 z + s_0$$

$$\left.\begin{aligned} r_0 + 2s_1 &= 1 \\ 2s_0 - 3s_1 - 3r_0 &= 1 \\ 2r_0 - 3s_0 &= 0.25 \end{aligned}\right\} \begin{aligned} r_0 &= -16 \\ s_1 &= 8.5 \\ s_0 &= 10.75 \end{aligned}$$

$$1 = T(1) = \frac{B(1)}{P(1)} t_0 \Longrightarrow \boxed{t_0 = -2.25}$$

$$\boxed{R = -16} \qquad \boxed{S = 8.5z + 10.75}$$

4.

$$x(k+1) = Ax(k) + Bu(k) \qquad A \in \mathbb{R}^{n \times n} \qquad B \in \mathbb{R}^{n \times 1}$$

a)

$$\|x(k+1)\| = (x^T A^T + u^T B^T)(Ax + Bu)$$

$$= x^T A^T A x + u^T B^T B u + u^T B^T A x + x^T A^T B u$$

$$\frac{\partial \|x(k+1)\|}{\partial u} = \frac{\partial}{\partial u}(u^2 B^T B) + \frac{\partial}{\partial u}(u B^T A x) + \frac{\partial}{\partial u}(x^T A^T B u)$$

$$0 = 2u B^T B + B^T A x + x^T A^T B \leftarrow \text{Minimum}$$

$$\boxed{u = -\frac{(B^T A x)^T + B^T A x}{2 B^T B}}$$

b)

$$x(k+1) = Ax(k) - B \cdot \left(\frac{(B^T A x(k))^T + B^T A x(k)}{2 B^T B} \right)$$

($(B^T A x(k))^T + B^T A x(k)$: Skalar)

$$= Ax(k) - B \cdot \frac{2 B^T A x(k)}{2 B^T B}$$

$$\boxed{x(k+1) = \left(I - \frac{B B^T}{B^T B} \right) A x(k)}$$

c)

$$x_1(k+1) = F\,x(k) \qquad F = \left(1 - \frac{BB^T}{B^TB}\right)A$$

$$A = \begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \boxed{F = \frac{1}{2}\begin{pmatrix} 0 & 3 \\ 0 & -3 \end{pmatrix}}$$

$$x_1(k+1) = \frac{1}{2}\begin{pmatrix} 0 & 3 \\ 0 & -3 \end{pmatrix} x(k) \qquad \det(zI - F) = \det\begin{pmatrix} z & -\frac{3}{2} \\ 0 & z+\frac{3}{2} \end{pmatrix} = z^2 + \frac{3}{2}z + \frac{3}{2}$$

$$x(k+1) = \begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix} x(k) \qquad \det(zI - A) = \det\begin{pmatrix} z & -3 \\ 0 & z \end{pmatrix} = z^2$$

sustav sa F je mestabilan

d) Nevalja, dead-beat?

5. $\tau(k+1) = 0.9\,\tau(k) + 0.8\,q_u(k)$

a) $J = \sum_0^\infty 10\,\tau^2(k) + q_u^2(k) \qquad Q = 10 \qquad R = 1$

$$S_\infty = Q + A^T S_\infty A - A^T S_\infty B G_\infty$$

$$G_\infty = (R + B^T S_\infty B)^{-1} B^T S_\infty A = \frac{0.72\,S_\infty}{1 + 0.64\,S_\infty}$$

$$A^T S_\infty A = 0.81\,S_\infty; \qquad A^T S_\infty B = 0.72\,S_\infty$$

$$S_\infty = 10 + 0.81\,S_\infty - 0.72\,S_\infty \frac{0.72\,S_\infty}{1 + 0.64\,S_\infty}$$

$$\boxed{S_\infty = 11.11} \leftarrow \text{jer mora biti } S_\infty > 0$$

$$\Rightarrow \boxed{G_\infty = 0.986}$$

$$u(k) = -0.986\,x(k)$$

b) $\det(z - 0.9 + 0.986) \Rightarrow z_1 = -0.086 \leftarrow$ stabilan