2. CIKLUS

PRIMJER 1,29.

$\frac{41}{47}$ RAZVITI U JEDNOSTAVNI VERIŽNI RAZLOMAK

$47 = 41 \cdot 1 + 6$
$41 = 6 \cdot 6 + 5$
$6 = 5 \cdot 1 + 1$
$5 = 1 \cdot 5$

$\frac{47}{41} = [1; 6, 1, 5] \quad \frac{47}{41} = 1 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{5}}}$

$\frac{41}{47} = [0; 1, 6, 1, 5] \quad \frac{41}{47} = \ldots + \cfrac{1}{\frac{47}{41}}$

PRIMJER 1,30.

$\sqrt{15}$ U VERIŽNI RAZLOMAK

$\sqrt{15} = \frac{0 + \sqrt{15}}{1} \qquad d = 15$

- $s_0 = 0, \ t_0 = 1, \ a_0 = \lfloor \sqrt{15} \rfloor = 3$

$$\boxed{s_{i+1} = a_i t_i - s_i, \quad t_{i+1} = \frac{d - s_{i+1}^2}{t_i}, \quad a_i = \left\lfloor \frac{s_i + a_0}{t_i} \right\rfloor}$$

- $s_1 = 3, \quad t_1 = \frac{15 - 3^2}{1} = 6, \quad a_1 = \left\lfloor \frac{3+3}{6} \right\rfloor = 1$
- $s_2 = 1 \cdot 6 - 3 = 3, \quad t_2 = \frac{15 - 3^2}{6} = 1, \quad a_2 = \left\lfloor \frac{3+3}{1} \right\rfloor = 6$
- $s_3 = 6 \cdot 1 - 3 = 3, \quad t_3 = \frac{15 - 3^2}{1} = 6, \quad a_3 = \left\lfloor \frac{3+3}{6} \right\rfloor = 1$

$(s_1, t_1) = (s_3, t_3)$
ISTO $\rightarrow$ KRAJ ALGORITMA

GLEDAMO a-OVE

$\sqrt{15} = [3; \overline{1, 6}]$

PRIMJER 1.31.

NAĐITE SVA RJEŠENJA JEDNADŽBI

$x^2 - 15y^2 = -1$

$x^2 - 15y^2 = 1$

ZA KOJE VRIJEDI $1 < x < 1000$

KORISTIMO PRETHODNI PRIMJER

$\sqrt{15} = [3; \overline{1, 6}]$ $\quad l = 2 \Rightarrow$ JEDNADŽBA $x^2 - 15y^2 = -1$ NEMA RJEŠENJA

$nL - 1 =$
$1 \cdot 2 - 1 = 1$
$2 \cdot 2 - 1 = 3$
$3 \cdot 2 - 1 = 5$

n	-1	0	1	2	3	4	5	6	7	8
a_i		3	1	6	1	6	1	6	1	6
p_i	1	3	4	27	31	213	245	1683		
q_i	0	1	1	7	8	55	63			

$1683 > 1000$ PA STANEMO

$1 + 3 \cdot 1$

$0 + 1 \cdot 1$

RJEŠENJA

$(p_1, q_1), (p_3, q_3), (p_5, q_5)$

$(4, 1), (31, 8), (245, 63)$

$$\left.\begin{array}{l} 4 + \sqrt{15} \\ 31 + 8\sqrt{15} \\ 245 + 63\sqrt{15} \end{array}\right\} \text{RJEŠENJA}$$

PRIMJER 1.32.

$x^2 - 29y^2 = -1$

$x^2 - 29y^2 = 1$

NAĆI NAJMANJA RJEŠENJA JEDNADŽBI U PRIRODNIM BROJEVIMA (AKO POSTOJE)

$s_0 = 0 \quad t_0 = 1 \quad a_0 = \lfloor 29 \rfloor = 5$

$s_1 = 5, \quad t_1 = \dfrac{29 - 5^2}{1} = 4, \quad a_1 = \left\lfloor \dfrac{5+5}{4} \right\rfloor = 2$

$s_2 = 2 \cdot 4 - 5 = 3, \quad t_2 = \dfrac{29 - 3^2}{4} = 5, \quad a_2 = \left\lfloor \dfrac{3+5}{5} \right\rfloor = 1$

$s_3 = 2, \quad t_3 = \dfrac{29 - 2^2}{5} = 5, \quad a_3 = \left\lfloor \dfrac{2+5}{5} \right\rfloor = 1$

$s_4 = 3, \quad t_4 = \dfrac{29 - 3^2}{5} = 4, \quad a_4 = \left\lfloor \dfrac{3+5}{4} \right\rfloor = 2$

$s_5 = 5, \quad t_5 = \dfrac{29 - 5^2}{4} = 1, \quad a_5 = \left\lfloor \dfrac{5+5}{1} \right\rfloor = 10$

$s_6 = 5, \quad t_6 = \dfrac{29 - 5^2}{1} = 4$

$\sqrt{29} = [5; \overline{2, 1, 1, 2, 10}]$

$l = 5$

$x^2 - 29y^2 = -1 \quad (p_4, q_4)$

$x^2 - 29y^2 = 1 \quad (p_9, q_9)$

n	-1	0	1	2	3	4	5	6	7	8	9
a_n		5	2	1	1	2	10	2	1	1	2
p_n	1	5	11	16	27	70	727	1524	2251	3375	9801
q_n	0	1	2	3	5	13	135	283	418	701	1820
						(p_4, q_4)					(p_9, q_9)

$(p_4, q_4) = (70, 13) \quad 70 + 13\sqrt{29}$

$(p_9, q_9) = (9801, 1820) \quad 9801 + 1820\sqrt{29}$

ZADATAK.

$x^2 - 183y^2 = 1$

$s_0 = 0, \quad t_0 = 1, \quad a_0 = \lfloor\sqrt{183}\rfloor = 13$

$s_1 = 13, \quad t_1 = \frac{183 - 13^2}{1} = 14, \quad a_1 = \left\lfloor \frac{13+13}{14} \right\rfloor = 1$

$s_2 = 1, \quad t_2 = \frac{183 - 1^2}{14} = 13, \quad a_2 = \left\lfloor \frac{1+13}{13} \right\rfloor = 1$

$s_3 = 12, \quad t_3 = 3, \quad a_3 = 8$

$s_4 = 12, \quad t_4 = 13, \quad a_4 = 1$

$s_5 = 1, \quad t_5 = 14, \quad a_5 = 1$

$s_6 = 13, \quad t_6 = 1, \quad a_6 = 26$

$s_7 = 13, \quad t_7 = 14$

$\sqrt{183} = [13; \overline{1, 1, 8, 1, 1, 26}]$

$\ell = 6 \quad \ell$ JE PARAN $\Rightarrow (p_5, q_5)$

n	-1	0	1	2	3	4	5
a_n		13	1	1	8	1	1
p_n	1	13	14	27	230	257	487
q_n	0	1	1	2	17	19	36

$(p_5, q_5) = (487, 36)$

$487 - 36\sqrt{183} = 1$

7. DZ

10. RAZVITI U VERIŽNI RAZLOMAK

$\frac{146}{177}$

$177 = 146 \cdot 1 + 31$
$146 = 31 \cdot 4 + 22$
$31 = 22 \cdot 1 + 9$
$22 = 9 \cdot 2 + 4$
$9 = 4 \cdot 2 + 1$
$4 = 1 \cdot 4$

$\frac{177}{146} = [1; 4, 1, 2, 2, 4]$

$\frac{146}{177} = [0; 1, 4, 1, 2, 2, 4]$

$\frac{341}{129} = 2 + \frac{83}{129}$

$129 = 83 \cdot 1 + 46$
$83 = 46 \cdot 1 + 37$
$46 = 37 \cdot 1 + 9$
$37 = 9 \cdot 4 + 1$
$9 = 1 \cdot 9$

$\frac{129}{83} = [1; 1, 1, 4, 9]$

$\frac{83}{129} = [0; 1, 1, 1, 4, 9]$

$\frac{341}{129} = [2; 0, 1, 1, 1, 4, 9]$

11. RAZVITI U VERIŽNI RAZLOMAK

a) $\sqrt{58}$

$s_0 = 0$, $t_0 = 1$, $a_0 = 7$

$s_1 = 7$, $t_1 = 9$, $a_1 = 1$

$s_2 = 2$, $t_2 = 6$, $a_2 = 1$

$s_3 = 4$, $t_3 = 7$, $a_3 = 1$

$s_4 = 3$, $t_4 = 7$, $a_4 = 1$

$s_5 = 4$, $t_5 = 6$, $a_5 = 1$

$s_6 = 2$, $t_6 = 9$, $a_6 = 1$

$s_7 = 7$, $t_7 = 1$, $a_7 = 14$

$s_8 = 7$, $t_8 = 9$

$\sqrt{58} = [7; \overline{1, 1, 1, 1, 1, 1, 14}]$

b) $\sqrt{89}$

$s_0 = 0 \quad t_0 = 1 \quad a_0 = 9$

$s_1 = 9 \quad t_1 = 8 \quad a_1 = 2$

$s_2 = 7 \quad t_2 = 5 \quad a_2 = 3$

$s_3 = 8 \quad t_3 = 5 \quad a_3 = 3$

$s_4 = 7 \quad t_4 = 8 \quad a_4 = 2$

$s_5 = 9 \quad t_5 = 1 \quad a_5 = 18$

$s_6 = 9 \quad t_6 = 8$

$\sqrt{89} = [9; \overline{2, 3, 3, 2, 18}]$

c) $\sqrt{173}$

$s_0 = 0 \quad t_0 = 1 \quad a_0 = 13$

$s_1 = 13 \quad t_1 = 4 \quad a_1 = 6$

$s_2 = 11 \quad t_2 = 13 \quad a_2 = 1$

$s_3 = 2 \quad t_3 = 13 \quad a_3 = 1$

$s_4 = 11 \quad t_4 = 4 \quad a_4 = 6$

$s_5 = 13 \quad t_5 = 1 \quad a_5 = 26$

$s_6 = 13 \quad t_6 = 4$

$\sqrt{173} = [13; \overline{6, 1, 1, 6, 26}]$

d)

$\sqrt{185}$

$s_0 = 0$	$t_0 = 1$	$a_0 = 13$
$s_1 = 13$	$t_1 = 16$	$a_1 = 1$
$s_2 = 3$	$t_2 = 11$	$a_2 = 1$
$s_3 = 8$	$t_3 = 11$	$a_3 = 1$
$s_4 = 3$	$t_4 = 16$	$a_4 = 1$
$s_5 = 13$	$t_5 = 1$	$a_5 = 26$
$s_6 = 13$	$t_6 = 16$	

$$\sqrt{185} = [13; \overline{1, 1, 1, 1, 26}]$$

14. a) $\alpha = [3; 2, 1]$ $\quad \alpha = 3 + \frac{1}{2 + \frac{1}{1}} = 3 + \frac{1}{3} = \frac{10}{3} = 3{,}333\dot{3}$

b) $\alpha = [3; \overline{2, 1}]$ → PERIODIČNO

$$s = [\overline{2, 1}] \Rightarrow s = 2 + \frac{1}{1 + \frac{1}{s}}$$

$$s = 2 + \frac{1}{\frac{s+1}{s}}$$

$$s = 2 + \frac{s}{s+1}$$

$$s = \frac{2s + 2 + s}{s+1} \quad / \cdot s+1$$

$$s^2 + s = 3s + 2$$
$$s^2 - 2s - 2 = 0 \qquad s_{1,2} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$
$$= 1 + \sqrt{3}$$

$$\alpha = 3 + \frac{1}{s} = 3 + \frac{1}{1+\sqrt{3}} = \frac{3 + 3\sqrt{3} + 1}{1 + \sqrt{3}} = \frac{4 + 3\sqrt{3}}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{4 - 4\sqrt{3} + 3\sqrt{3} - 9}{1 - 3} =$$

$$= \frac{-5 - \sqrt{3}}{-2} = \boxed{\frac{5+\sqrt{3}}{2}} \qquad \boxed{\alpha = \frac{5+\sqrt{3}}{2}} \quad ✓$$

13. a)

$s_i \quad \frac{5+\sqrt{17}}{4} \quad a_0 \qquad 4 \mid 17-5^2=-8$ ✓

$t_i \qquad \sqrt{17} = 4,123105\ldots$

$d=17$

$s_0 = 5 \quad t_0 = 4 \quad a_0 = \left\lfloor \frac{5+4}{4} \right\rfloor = 2$

$s_1 = 3 \quad t_1 = 2 \quad a_1 = 3$

$s_2 = 3 \quad t_2 = 4 \quad a_2 = 1$

$s_3 = 1 \quad t_3 = 4 \quad a_3 = 1$

$s_4 = 3 \quad t_4 = 2$

$$\frac{5+\sqrt{17}}{4} = [2; \overline{3,1,1}]$$

b) $\frac{1+\sqrt{13}}{5} \qquad 5 \nmid 13-1^2 = 12$

$$\frac{1+\sqrt{13}}{5} \cdot \frac{5}{5} = \frac{5+\sqrt{325}}{25} \qquad \sqrt{325} = 18,0277\ldots$$

$s_0 = 5 \quad t_0 = 25 \quad a_0 = \left\lfloor \frac{5+18}{25} \right\rfloor = 0$

$s_1 = 0\cdot 25-5 = -5 \quad t_1 = 12 \quad a_1 = \left\lfloor \frac{-5+18}{12} \right\rfloor = 1$

PONAVLJA SE

$s_2 = 17 \quad t_2 = 3 \quad a_2 = 11$

$s_3 = 16 \quad t_3 = 23 \quad a_3 = 1$

$s_4 = 7 \quad t_4 = 12 \quad a_4 = 2$

$s_5 = 17 \quad t_5 = 3$

$$\frac{1+\sqrt{13}}{5} = [0; 1, \overline{11, 1, 2}]$$ ✓

15. ODREDITE NAJMANJA RJEŠENJA (AKO POSTOJE) U SKUPO PRIRODNIH BROJEVA SLJEDEĆIH JEDNADŽBI

a) $x^2 - 57y^2 = \pm 1$

$\sqrt{57} = 7{,}54983\ldots$

$s_0 = 0 \quad t_0 = 1 \quad a_0 = 7$

$s_1 = 7 \quad t_1 = 8 \quad a_1 = 1$

$s_2 = 1 \quad t_2 = 7 \quad a_2 = 1$

$s_3 = 6 \quad t_3 = 3 \quad a_3 = 4$

$s_4 = 6 \quad t_4 = 7 \quad a_4 = 1$

$s_5 = 1 \quad t_5 = 8 \quad a_5 = 1$

$s_6 = 7 \quad t_6 = 1 \quad a_6 = 14$

$s_7 = 7 \quad t_7 = 8$

$\sqrt{57} = [7; \overline{1, 1, 4, 1, 1, 14}]$

$L = 6$ PARAN

$x^2 + 57y^2 = -1 \Rightarrow$ NEMA RJEŠENJA

$x^2 + 57y^2 = 1 \Rightarrow (p_5, q_5)$

n	-1	0	1	2	3	4	5
a_n		7	1	1	4	1	1
p_n	1	7	8	15	68	83	151
q_n	0	1	1	2	9	11	20

(p_5, q_5)

$151 - 20\sqrt{57}$

b) $x^2 - 95y^2 = \pm 1$ $\sqrt{95} = 9{,}746794\ldots$

$s_0 = 0$ $t_0 = 1$ $a_0 = 9$

$s_1 = 9$ $t_1 = 14$ $a_1 = 1$

$s_2 = 5$ $t_2 = 5$ $a_2 = 2$

$s_3 = 5$ $t_3 = 14$ $a_3 = 1$

$s_4 = 9$ $t_4 = 1$ $a_4 = 18$

$s_5 = 9$ $t_5 = 14$

$\sqrt{95} = [9; \overline{1, 2, 1, 18}]$

$L = 4 \Rightarrow$ PARAN

$x^2 - 95y^2 = -1$ NEMA RJEŠENJA

$x^2 - 95y^2 = 1$ (p_3, q_3) $nL-1$

n	-1	0	1	2	3
a_n		9	1	2	1
p_n	1	9	10	29	39
q_n	0	1	1	3	4

$(p_3, q_3) = 39 - 4\sqrt{95}$

14. c) $\alpha = [6; \overline{2, 2, 12}]$

$\beta = [\overline{2, 2, 12}]$ $\quad \beta = 2 + \cfrac{1}{2 + \cfrac{1}{12 + \frac{1}{\beta}}}$

$$\beta = 2 + \cfrac{1}{2 + \cfrac{1}{\frac{12\beta+1}{\beta}}}$$

$$\beta = 2 + \cfrac{1}{2 + \frac{\beta}{12\beta+1}}$$

$$\beta = 2 + \cfrac{1}{\frac{25\beta+2}{12\beta+1}}$$

$$\beta = 2 + \frac{12\beta+1}{25\beta+2}$$

$$\beta = \frac{62\beta+5}{25\beta+2} \quad / \cdot 25\beta+2$$

$$25\beta^2 + 2\beta = 62\beta + 5$$
$$25\beta^2 - 60\beta - 5 = 0 \quad /:5$$
$$5\beta^2 - 12\beta - 1 = 0$$

$$\beta_{1,2} = \frac{12 \pm \sqrt{144+20}}{10} = \frac{12 \pm \sqrt{164}}{10} = \frac{12 \pm 2\sqrt{41}}{10} =$$
$$= \frac{6 \pm \sqrt{41}}{5}$$

$$\alpha = 6 + \frac{1}{\beta} = 6 + \frac{5}{6+\sqrt{41}} = \frac{41 + 6\sqrt{41}}{6+\sqrt{41}} \cdot \frac{6-\sqrt{41}}{6-\sqrt{41}} = \frac{\cancel{246} - 41\sqrt{41} + 36\sqrt{41} - \cancel{246}}{36-41} =$$

$$= \frac{-5\sqrt{41}}{-5} = \sqrt{41}$$

$$\boxed{\alpha = \sqrt{41}} \quad \checkmark$$

8. DZ

(1.) NA SKUPU R DEFINIRANA JE BINARNA OPERACIJA $*$ NA SLJEDEĆI NAČIN:

$$x * y = \sqrt[3]{x^3+y^3}, \quad x, y \in R$$

DOKAZATI DA JE $(R, *)$ ABELOVA GRUPA

POTREBNO JE DOKAZATI:

1) ZATVORENOST

$x, y \in R \qquad x * y = \sqrt[3]{x^3+y^3} \in R$

2) ASOCIJATIVNOST

$(x * y) * z = \sqrt[3]{x^3+y^3} * z = \sqrt[3]{x^3+y^3+z^3}$

$x * (y * z) = x * \sqrt[3]{y^3+z^3} = \sqrt[3]{x^3+y^3+z^3}$ ✓

3) NEUTRALNI ELEMENT

$\boxed{x * e = x}$ ← MORA VRIJEDITI

$x * e = \sqrt[3]{x^3+e^3} = x \quad /^3$

$x^3+e^3 = x^3$

$e^3 = 0$

$\boxed{e = 0}$

4) INVERZ

$x * y = e$

$x * y = 0$

$\sqrt[3]{x^3+y^3} = 0 \quad /^3$

$x^3+y^3 = 0$

$y^3 = -x^3 \quad /\sqrt[3]{}$

$\boxed{y = -x}$

5) KOMUTATIVNOST

$x * y = y * x$

$x * y = \sqrt[3]{x^3+y^3} \qquad y * x = \sqrt[3]{y^3+x^3}$

ISTO

ABELOVA GRUPA ✓✓

10. ODREDITI RED ELEMENTA:

a) $\frac{1+i}{\sqrt{2}}$ U GRUPI $(\mathbb{C}^*, \cdot)$

$$\left(\frac{1+i}{\sqrt{2}}\right)^2 = \frac{1}{2} \cdot (1+i)^2 = \frac{1}{2}(\cancel{1}+2i-\cancel{1}) = \frac{2i}{2} = i$$

$$\left(\frac{1+i}{\sqrt{2}}\right)^8 = \frac{\overset{2i}{(1+i)^2} \cdot \overset{2i}{(1+i)^2} \cdot \overset{2i}{(1+i)^2} \cdot \overset{2i}{(1+i)^2}}{16} = \frac{16}{16} = 1$$

$\boxed{\text{RED JE } 8}$

b) 10 U GRUPI $(\mathbb{Z}_{15}, +_{15})$

$10x \equiv 0 \pmod{15}$ $\quad (10, 15) = 5$

$2x \equiv 0 \pmod 3$

$x = 3$ $\quad$ RED JE 3

PROVJERA:
$15 \cdot 3 = 45$
$\frac{45}{15} = 3$ (OSTATAK 0)

c) 10 U GRUPI $(\mathbb{Z}_{17}, +_{17})$

$10x \equiv 0 \pmod{17}$

RED JE 17

d) 10 U GRUPI $(\mathbb{Z}^*_{13}, \cdot_{13})$

$10^x \equiv 1 \pmod{13}$

$14 \equiv 1 \pmod{13}$
$27 \equiv 1 \pmod{13}$
$40 \equiv 1 \pmod{13}$

	10^0	10^1	10^2	10^3	10^4
(mod 13)	1	10	9	12	3

$27 = 3 \cdot 9$

$10^2 \cdot 10^4 = 10^6$ $\quad 10^6 \equiv 1 \pmod{13}$

$\boxed{\text{RED JE } 6}$

e) 10 U GRUPI $(\mathbb{Z}^*_{17}, \cdot_{17})$

$10^x \equiv 1 \pmod{17}$

$18 \equiv 1 \pmod{17}$
$35 = 5 \cdot 7$
$52 = 4 \cdot 13$
$69 = 3 \cdot 23$
$86 = 2 \cdot 43$

10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7
1	10	15	14	4	6	9	5

10^8	10^9
16	7

$10^7 \cdot 10^9 = 10^{16}$ $\quad$ RED JE 16 ✓

11. a) ODREDITE SVE ELEMENTE REDA 12 U GRUPI Z_{12}

$Z = \{0,1,2,3,4,5,6,7,8,9,10,11\}$

UZIMAMO ONE ELEMENTE ZA KOJE VRIJEDI
$nzd(\text{ELEMENT}, 12) = 1$

$\{1,5,7,11\}$ ✓

b) –||– REDA 12 U GRUPI $Z_3 \times Z_4$

(x, y)

$x \in \{0,1,2\}$ $y \in \{0,1,2,3\}$

$(1,1)$	$(2,1)$	$(3,1)$
$\cancel{(1,2)}$	$\cancel{(2,2)}$	$(3,2)$
$(1,3)$	$(2,3)$	$(3,3)$
$\cancel{(1,4)}$	$\cancel{(2,4)}$	$\cancel{(3,4)}$

UZIMAMO ONE ELEMENTE ZA KOJE VRIJEDI

$nzd(x, 3) = 1 \to$ BROJEVI 1,2 $nzd(y, 4) = 1 \to 1,3$

$$\boxed{\begin{matrix}(1,1) & (2,1)\\ (1,3) & (2,3)\end{matrix}}$$

12. a) ODREDITE PODGRUPU OD $(R^*, \cdot)$ GENERIRANU ELEMENTOM -1

$(-1)\cdot(-1) = 1$

$-1 \cdot 1 = -1$

$\{-1, 1\}$

b) –||– $(Z_7, +_7)$ GENERIRANU ELEMENTOM 4

$4 +_7 4 = 1$
$4 +_7 1 = 5$
$5 +_7 1 = 6$
$6 +_7 1 = 0$
$0 +_7 1 = 1$
$1 +_7 1 = 2$
$2 +_7 1 = 3$
$3 +_7 1 = 4$

$Z_7 = \{0,1,2,3,4,5,6\}$

$\{0,1,2,3,4,5,6\}$ ✓

1 c) -II- $(Z_8, +_8)$ GEN. EL 6

$Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$

$6 +_8 6 = 4$
$6 +_8 4 = 2$
$6 +_8 2 = 0$
$4 +_8 2 = 6$

$\{0, 2, 4, 6\}$

GREŠKA U ZADATKU!

NAPOMENA!
KOD VELIKIH BROJEVA
UZETI
$nzd(6,8) = 2$
UZETI SVE ELEMENTE (UKLJUČUJUĆI I NEUTRALNU NULU) KOJI SU DJELJIVI S DVA

ILI

$6r \equiv 0 \pmod 8$ /:2
$3r \equiv 0 \pmod 4$

RED JE 4

$6 \cdot 1 \pmod 8 = 6$
$6 \cdot 2 \pmod 8 = 4$
$6 \cdot 3 \pmod 8 = 2$
$6 \cdot 4 \pmod 8 = 0$

4 RETKA

$\{0, 2, 4, 6\}$

1 d) -II- $(Z_{17}^*, \cdot_{17})$ GEN. EL. 13

$13^1 \pmod{17} = 13$
$13^2 \pmod{17} = 16$
$13^3 \pmod{17} = 4$
$13^4 \pmod{17} = 1$ JEDINICA ⇒ STOP!

RED JE 4

$\{1, 4, 13, 16\}$

13. a) NAPIŠITE SVE ELEMENTE SIMETRIČNE GRUPE S_3 STUPNJA 3

"BROJEVI" SU $\{1, 2, 3\}$ IMA $3! = 6$ ELEMENATA GRUPE

$$S_3 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \right\}$$

(14) ODREDITE PODGRUPU OD S_7 GENERIRANU

a) CIKLUSOM $\kappa = (1234)$. KOLIKI JE RED CIKLUSA κ?

1. POTENCIRANJE $\begin{pmatrix} 1&2&3&4&5&6&7 \\ 2&3&4&1&5&6&7 \end{pmatrix} \cdot \begin{pmatrix} 1&2&3&4&5&6&7 \\ 2&3&4&1&5&6&7 \end{pmatrix} = \begin{pmatrix} 1&2&3&4&5&6&7 \\ 3&4&1&2&5&6&7 \end{pmatrix} = (13)(24)$

2. POTENCIRANJE $\begin{pmatrix} 1&2&3&4&5&6&7 \\ 2&3&4&1&5&6&7 \end{pmatrix} \cdot \begin{pmatrix} 1&2&3&4&5&6&7 \\ 3&4&1&2&5&6&7 \end{pmatrix} = \begin{pmatrix} 1&2&3&4&5&6&7 \\ 4&1&2&3&5&6&7 \end{pmatrix} = (1432)$

3. POTENCIRANJE $\begin{pmatrix} 1&2&3&4&5&6&7 \\ 2&3&4&1&5&6&7 \end{pmatrix} \cdot \begin{pmatrix} 1&2&3&4&5&6&7 \\ 4&1&2&3&5&6&7 \end{pmatrix} = \underbrace{\begin{pmatrix} 1&2&3&4&5&6&7 \\ 1&2&3&4&5&6&7 \end{pmatrix}}_{id}$

PODGRUPA $\overbrace{\{ (1234), (13)(24), (1432), id \}}^{4}$

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b) PERMUTACIJOM $\pi = (123)(57)$

$\begin{pmatrix} 1&2&3&4&5&6&7 \\ 2&3&1&4&7&6&5 \end{pmatrix} \cdot \begin{pmatrix} 1&2&3&4&5&6&7 \\ 2&3&1&4&7&6&5 \end{pmatrix} = \begin{pmatrix} 1&2&3&4&5&6&7 \\ 3&1&2&4&5&6&7 \end{pmatrix} = (132)$

$\begin{pmatrix} 1&2&3&4&5&6&7 \\ 2&3&1&4&7&6&5 \end{pmatrix} \cdot \begin{pmatrix} 1&2&3&4&5&6&7 \\ 3&1&2&4&5&6&7 \end{pmatrix} = \begin{pmatrix} 1&2&3&4&5&6&7 \\ 1&2&3&4&7&6&5 \end{pmatrix} = (57)$

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PODGRUPA $\{ (123)(57), (132), (123), (57), (132)(57), id \}$

RED PERMUTACIJE JE 6 ✓

NAPRAVITI SVE MOGUĆE
PERMUTACIJE SA: (123)
(57)
(132)