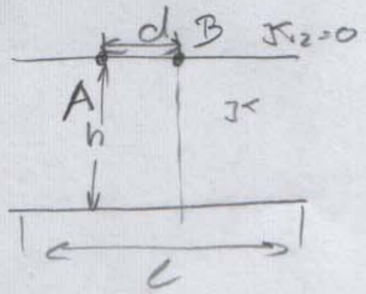
by: Anaconnda

Statičko strujno polje

- analogija statičkog strujnog i električnog polja

1. Ravni vodič polumjera $r_0 = 1cm$, $l = 5m$, ukopan u zemlju provodnosti $\kappa = 0.01\, S/m$ na dubini $h = 1m$ paralelno s površinom zemlje. Odrediti otpor rasprostiranja vodiča i razliku potencijala između točaka A i B na površini zemlje razmaknutih za $d = 1m$ ako iz vodiča istječe struja $I = 100A$. (2.2)



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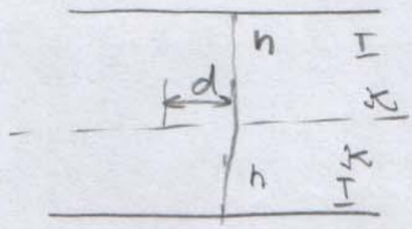


$$\varphi(r,z) = \frac{\lambda}{2\pi\varepsilon} \ln \left\{ \frac{z+d+\sqrt{(z+d)^2+r^2}}{z-d+\sqrt{(z-d)^2+r^2}} \right.$$

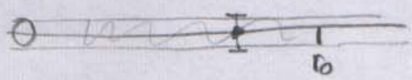
$$i = \frac{I}{l}$$

$$\varphi(r,z) = \frac{i}{2\pi\kappa} \ln \left\{ \frac{z+d+\sqrt{(z+d)^2+r^2}}{z-d+\sqrt{(z-d)^2+r^2}} \right\}$$

otpor

na sredini štapa $z=0$, $r=0$

potencijal na liniji



pa tražimo potencijal na površini tog cilindra gdje je $z=0$ $r=r_0$

potencijal te točke na površini vodiča je:

$$\varphi_v = \frac{I}{2\pi\kappa\ell}\ln\left\{\frac{\frac{\ell}{2}+\sqrt{\left(\frac{\ell}{2}\right)^2+r_0^2}}{-\frac{\ell}{2}+\sqrt{\left(\frac{\ell}{2}\right)^2+r_0^2}}\right\}+$$

$$+\underbrace{\frac{I}{2\pi\kappa\ell}\ln\left\{\frac{\frac{\ell}{2}+\sqrt{\left(\frac{\ell}{2}\right)^2+(2h)^2}}{-\frac{\ell}{2}+\sqrt{\left(\frac{\ell}{2}\right)^2+(2h)^2}}\right\}}_{\text{doprinos drugog vodiča}} = \dots$$

→ uvrstimo brojke i dobijemo

$$\dots = \frac{I}{4\pi\cdot 0.01\cdot 5}\cdot 14.5244$$

otpor: $R = \frac{\varphi_v}{I} = 23.116\,\Omega$

potencijal u točki $A(-d, h) \rightarrow \varphi(A) = 1006.1\,V$

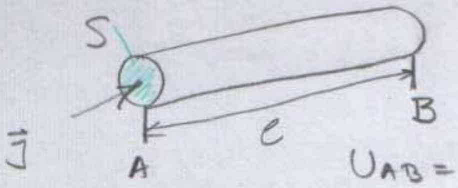
$B(0, h) \rightarrow \varphi(B) = 1048.66\,V$

$$U_{AB} = \varphi(A) - \varphi(B) = -42.65\,V$$

Ohmov zakon i električna vodljivost

- odnos između gustoće struje i jakosti el. polja

$$\vec{J} = \kappa \vec{E}$$ Linearna ovisnost



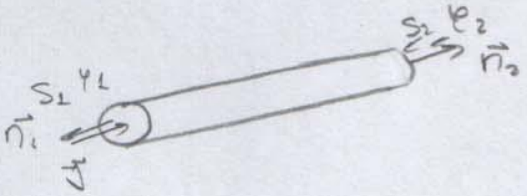
$$I = J S = \kappa \cdot E \cdot S \Rightarrow E = \frac{I}{\kappa S}$$

$$U_{AB} = \varphi_{AB} = -\int_B^A \vec{E}\, d\vec{l} = El = \underbrace{\frac{I}{\kappa S} \cdot l}_{\text{veza napona i struje}}$$

$$= \frac{I}{\kappa S}\, l$$

$$U = I \cdot R \rightarrow \boxed{R = \frac{1}{\kappa} \cdot \frac{l}{S}}$$

- vodič se grije zbog titranja slobodnih elektrona u vodiču, što uzrokuje gubitak energije/snage



$$P = -\oint\!\!\oint_S \varphi \vec{J} \cdot \vec{n}\, dS = I\varphi_1 - I\varphi_2 = UI = I^2 R$$ Joulesov zakon

Elektromotorna sila

- tu energiju ne može osigurati statičko el. polje jer

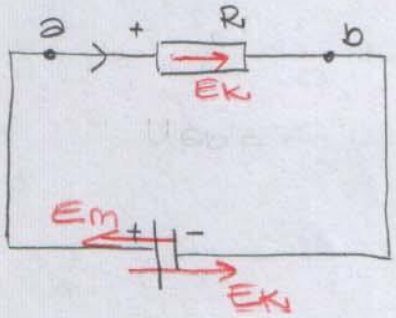
$$\oint_C \vec{E}_k d\vec{l} = 0$$

- da bi se održao stalni tok struje potrebni su izvori električno polje koje nije konzervativno (baterije ...)

EMS (Elektromotorna sila)

$$EMS = \oint_C (\vec{E}_k + \vec{E}_m) d\vec{l} = \oint_C \vec{E}_m \cdot d\vec{l} = \oint_C \frac{\vec{J}}{\kappa} d\vec{l} = \frac{I}{S} \cdot \frac{l}{\kappa} = I \cdot R$$

II. Kirchhoffov zakon



$U_{ab} = I \cdot R$

Statičko magnetsko polje u vakuumu

Biot-Savartov zakon

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \vec{R}}{|\vec{R}|^3} \qquad \vec{R} = \vec{r} - \vec{r}\,'$$

$$\mu_0 = 4\pi \cdot 10^{-7}\ [Vs/Am = H/m]$$

- on opisuje odnos vektora duljine i struje

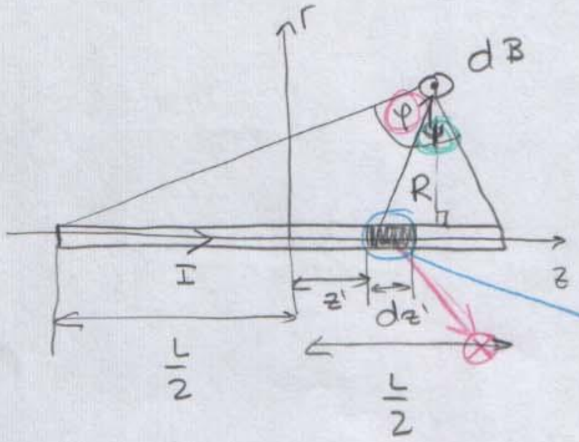
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{\ell} \times \vec{R}}{|\vec{R}|^3}$$

analogno coulombovom zakonu

B ima tangencijalan smjer s obzirom na struju

tu je $\vec{E}$ radijalan

1. Odrediti magnetsku indukciju kratke ravne strujnice duljine L kojom protječe istosmjerna struja I



jednadžba kontinuiteta

$$\nabla \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$= 0$ za istosmjerne struje

digresija

elementarni $d\vec{\ell}$ ima smjer $+$ osi z

po Biot Savartovu zakonu $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{R}}{|\vec{R}|^3}$

$\vec{R} = \vec{r} - \vec{r}'$

$\vec{r}'$ → vektor položaja izvora

$\vec{r}$ → vektor položaja točke

$$\left.\begin{aligned} \vec{r} &= z\vec{a}_z + r\vec{a}_r \\ \vec{r}' &= z\vec{a}_z \end{aligned}\right\} \vec{R} = (z - z')\vec{a}_z + r\vec{a}_r$$

$$d\vec{\ell} = d\vec{z}'\vec{a}_z \Rightarrow d\vec{\ell} \times \vec{R}$$

$$d\vec{\ell} \times \vec{R} = dz'\,\vec{a}_z \times \left[(z - z')\vec{a}_z + r\vec{a}_r\right] = dz' \cdot r \cdot \vec{a}_\alpha$$

$$|R| = \sqrt{(z - z')^2 + r^2}$$

$$d\vec{B} = \frac{\mu_0 I \; d\vec{\ell} \times \vec{R}}{4\pi \cdot |R|^3} = \vec{a}_\alpha \frac{\mu_0 I}{4\pi} \cdot \frac{r\,dz'}{\left[r^2 + (z - z')^2\right]^{3/2}}$$

- ukupni $\vec{B}$ dobijemo integriranjem $d\vec{B}$:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \vec{a}_\alpha \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{r\,dz'}{\left[r^2 + (z - z')^2\right]^{\frac{3}{2}}} =$$

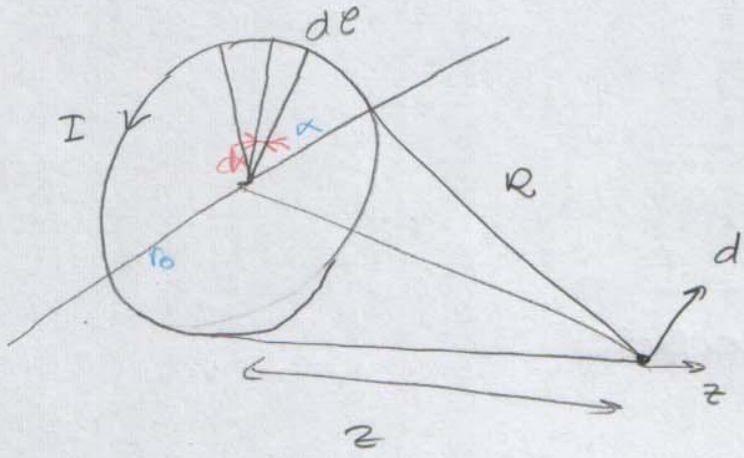
$$= \vec{a}_\alpha \frac{\mu_0 I}{4\pi r} \left[\underbrace{\frac{\frac{L}{2} + z}{\sqrt{\left(\frac{L}{2} + z\right)^2 + r^2}}}_{\sin\varphi} + \underbrace{\frac{\frac{L}{2} - z}{\sqrt{\left(\frac{L}{2} - z\right)^2 + r^2}}}_{\sin\psi} \right] =$$

$$= \vec{a}_\alpha \frac{\mu_0 I}{4\pi r} (\sin\varphi + \sin\psi)$$

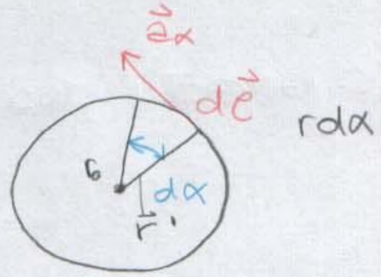
- kad bi imali beskonačno dugu strujnicu:

$$\ell \to \infty \qquad \left.\begin{aligned} \varphi &\to \\ \psi &\to \end{aligned}\right\} \varphi = \psi \to \frac{\pi}{2} \qquad \boxed{\vec{B} = \vec{a}_\alpha \frac{\mu_0 I}{2\pi r}} \quad L \to \infty$$

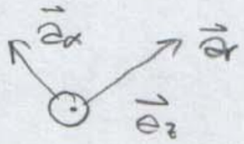
2. odredite magnetsku indukciju na osi kružne strujnice polumjera r_0 kojom teče istosmjerna struja I



$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I \, d\vec{l} \times \vec{R}}{|\vec{R}|^3}$$



$$\left.\begin{array}{l} d\vec{l} = r_0 \, d\alpha \cdot \vec{a}_\alpha \\ \left.\begin{array}{l} \vec{r} = z \cdot \vec{a}_z \\ r' = r_0 \vec{a}_r \end{array}\right\} \vec{R} = \vec{r} - \vec{r}' = z \cdot \vec{a}_z - r_0 \vec{a}_r \end{array}\right\} d\vec{l} \times \vec{R}$$



$$d\vec{l} \times \vec{R} = z \cdot r_0 d\alpha \, \vec{a}_r + r_0^2 d\alpha \cdot \vec{a}_z$$

$\downarrow$

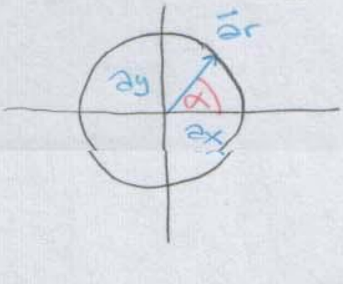
$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I(z \cdot r_0 d\alpha \vec{a}_r + r_0^2 \, d\alpha \, \vec{a}_z)}{|\vec{R}|^3}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{z r_0 \vec{a}_r + r_0^2 a_z}{(r_0^2 + z^2)^{3/2}} \, d\alpha =$$

$$= \frac{\mu_0 I}{4\pi} \left[\frac{z \; r_0}{\sqrt{r_0^2 + z^2}} \cdot \int_0^{2\pi} \vec{a}_r \cdot d\alpha + \frac{r_0^2}{\sqrt{r_0^2 + z^2}} \int_0^{2\pi} \vec{a}_z \, d\alpha \right]$$

$$\vec{a}_r = \cos\alpha \, \vec{a}_x + \sin\alpha \, \vec{a}_y$$

$$\int_0^{2\pi} \vec{a}_r d\alpha = \int_0^{2\pi} [\cos\alpha \, \vec{a}_x + \sin\alpha \, \vec{a}_y] = \int_0^{2\pi} \cos\alpha \, \vec{a}_x \, d\alpha + \vec{a}_y \int_0^{2\pi} \sin\alpha \, d\alpha$$



$$\vec{a}_r = \cos\alpha \, \vec{a}_x + \sin\alpha \, \vec{a}_y$$

$$B = \frac{\mu_0 I}{2} \cdot \frac{r_0^2}{(r_0^2 + z^2)^{3/2}} \cdot \vec{a}_z$$

Sila na strujni element u magnetskom polju

$$\vec{J} = \rho \cdot \vec{v}$$

$$dF = dq(\vec{v} \times \vec{B}) = (\vec{J} \times \vec{B})\, dV$$

Sila između dva duga vodiča

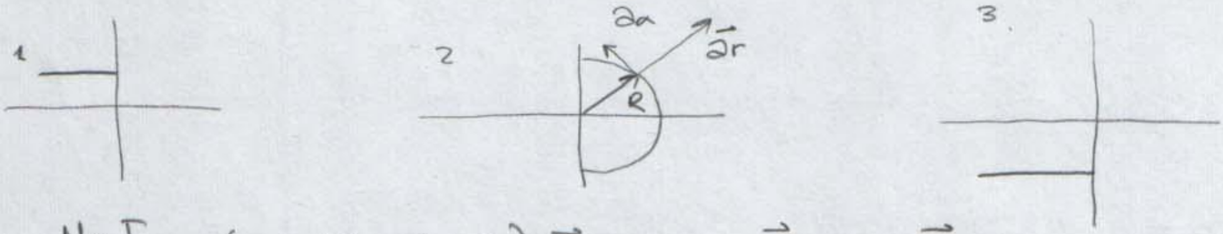
$$\vec{B}_1 = \mu_0 \frac{I}{2\pi r} \vec{a}_\alpha$$

$$\vec{F}_2 = I' \int_l d\vec{l}_1 \times \vec{B}_1$$ prema vodiču

$$\vec{F}_2 = l I' \cdot B_1$$

$$\frac{F_2}{l} = I' \cdot B = \frac{\mu_0 I \cdot I'}{2\pi r}$$

BLITZ



① $B = \dfrac{\mu_0 I}{4\pi R}(\sin\varphi + \sin\psi)\vec{a}_\alpha$ $\qquad \vec{a}_\alpha = -\vec{a}_z$

$\varphi = \frac{\pi}{2} \quad \psi = 0 \qquad B = \dfrac{-\mu_0 I}{4\pi R}\vec{a}_z$

③ $B = \dfrac{\mu_0 I}{4\pi R}(\sin\varphi - \sin\psi)\vec{a}_\alpha$ $\qquad \vec{a}_\alpha = -\vec{a}_z$

$\varphi = 0$ od početka se mjeri $\quad \psi = \frac{\pi}{2}$ od kraja $\quad B = \dfrac{-\mu_0 I}{4\pi R} \cdot \vec{a}_z$

② $dB = \dfrac{\mu_0}{4\pi} \dfrac{I \cdot d\vec{l} \times \vec{R}}{|R|^3}$ $\qquad B = \dfrac{\mu_0 I}{4\pi}\displaystyle\int_{-\pi/2}^{\pi/2} \frac{1}{R}\, d\alpha = \frac{1}{R}\frac{\mu_0 I}{4\pi}\int_{-\frac{\pi}{2}}^{\pi/2} d\alpha = \frac{1}{R}\frac{\mu_0 I}{4} \cdot (-\vec{a}_z)$

$r = 0 \quad r' = R \cdot \vec{a}_r$

$r - r' = -R \cdot \vec{a}_r$

$d\vec{l} = -\vec{a}_\alpha \cdot R\, d\alpha$

Vektorski magnetski potencijal

$\nabla \cdot \vec{B} = 0$ $\nabla dB = -\mu_0 I$

$\nabla \cdot (\nabla \cdot \vec{A}) \equiv 0$

$B = \nabla \times \vec{A}$ → vektorski magnetski potencijal (kontinuirana funkcija, bez skokova)

Kad se opisuje $\nabla \vec{A}$ divergens od $\vec{A}$ zove se BAŽDARENJE

$\nabla \vec{A} \equiv 0$ Coulombovo baždarenje

$\nabla \times (\nabla \times \vec{A}) = \mu_0 \vec{J}$

$\nabla (\nabla \vec{A}) - \Delta \vec{A} = \mu_0 \vec{J}$

$\Delta \vec{A} = -\mu_0 \vec{J}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV$$

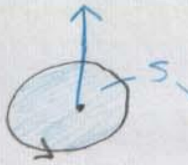
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_C \frac{\vec{J}(\vec{r}) d\ell}{|\vec{r} - \vec{r}'|}$$

$$\phi = \iint_S \underbrace{\vec{B}}_{\nabla \times \vec{A}} \vec{n} \, dS = \iint_S \underbrace{(\nabla \times \vec{A})}_{\text{stokes}} \vec{n} \, dS$$

$$\phi = \oint_C \vec{A} \, d\vec{\ell}$$

Materijali u magnetskom polju

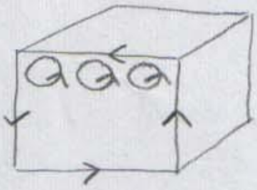
- magnetski dipol i magnetizacija



$\vec{m} = \vec{n}\cdot S \cdot I$ → dipolni moment

$$\boxed{\vec{M} = \frac{d\vec{m}}{dV}}$$

MAKROSKOPSKI PRIMJER



$$d\vec{m} = \vec{M}dV = \vec{M}\,dl\,dS = (\vec{M}\cdot d\vec{l})\,\vec{n}\,dS = dI\,\vec{n}\,dS$$

$$dI_a = \vec{M}\,d\vec{l}$$

$$I_a = \oint_C \vec{M}\,d\vec{l} = \iint_S \vec{J}_i\,\vec{n}\,dS = \iint_S (\nabla \times \vec{M})\,\vec{n}\,dS$$

$\vec{H}$ - jakost magnetskog polja

$$\vec{B} = \vec{B}_0 + \vec{B}_a \Rightarrow \nabla \times \vec{B} = \mu(\vec{J}_s + \vec{J}_i) \Rightarrow \nabla \times \vec{B} = \mu_0(\vec{J}_s + \nabla \times \vec{M}) \quad /:\mu_0$$

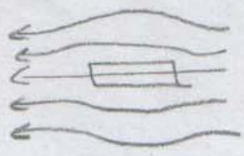
$$\nabla \times \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)}_{H} = \vec{J}_s$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

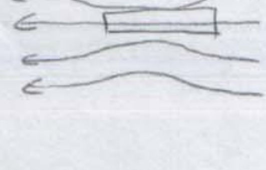
$$\nabla \times \vec{H} = \vec{J}_s$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \overbrace{\mu_0 \underbrace{(1 + \chi_m)}_{\mu_r}}^{\text{permabilnost}} \cdot \vec{H}$$

DIJEMAGNETI $\chi_m < 0$ $(-10^{5}]$



PARAMAGNETI $\chi_m > 0$ $(10^{-2} - 10^{-6})$



FEROMAGNETI

$(10^{2} - 10^{6})$

BLITZ. 6.

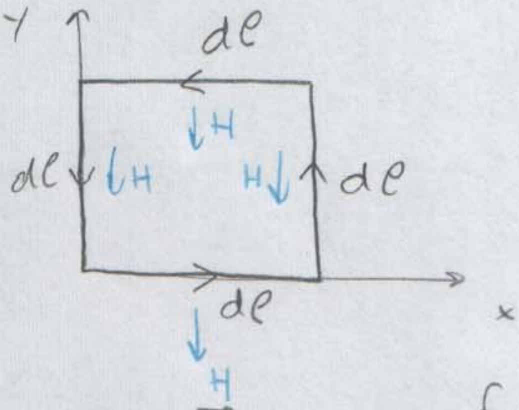
$$\vec{J}_s = \nabla \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{1}{x+2} & 0 \end{vmatrix} =$$

$$= \vec{a}_x \left[\frac{\partial}{\partial y} \cdot 0 - \frac{\partial}{\partial z} \left(\frac{1}{x+2} \right) \right] - a_y \left[\frac{\partial}{\partial x} 0 + \frac{\partial}{\partial z} \cdot 0 \right] + a_z \left[\frac{\partial}{\partial x} \cdot 0 \right] =$$

$$= -\vec{a}_z \cdot \frac{1}{(x+2)^2}$$

$$\vec{J}(0) = -\vec{a}_z = \frac{1}{4} \ A/m^2$$

$$\oint \vec{B} d\vec{l} = \sum \mu I_s = \mu \int \frac{1}{x+2} \, \vec{a}_y$$

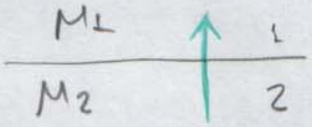


$H \cdot dl = 0$

$$\nabla \times \vec{H} = \vec{J}_s \rightarrow \oint \vec{H} dl = \sum I$$

$$\nabla \vec{B} = \emptyset \rightarrow \oint\!\!\oint_S \vec{B} \, \vec{n} \, dS = 0$$

$$\vec{n}(\vec{B}_2 - \vec{B}_1) = 0$$



$$\vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{k}$$

targ komponenta

1. PRI (22 [illegible].

$\vec{B}_1 = 0.01\vec{a}_x + 0.02\vec{a}_z$

$k_s = 5000\,\vec{a}_x \quad A/M$

$\vec{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \qquad \vec{n} = \vec{a}_z$

$\vec{n} \times (\vec{H}_2 - \vec{H}_1) = 0$

$\vec{B}_2 = B_{2x} \cdot \vec{a}_x + B_{2y}\,\vec{a}_y + B_{2z}\vec{a}_z$

$\vec{a}_z \cdot \left(B_{2x} \cdot \vec{a}_x + B_{2y}\,\vec{a}_y + B_{2z}\,\vec{a}_z - 0.01\vec{a}_x - 0.02\vec{a}_z\right) = 0$

$B_{2z} - 0.02 = 0$

$B_{2z} = 0.02T$

$\vec{n} \times (\vec{H}_2 - \vec{H}_1)\,\vec{k}_s$

$\vec{a}_z \times \left[\vec{a}_x(H_{2x} - H_{1x}) + \vec{a}_y(H_{2y} - H_{1y}) + \vec{a}_z(H_{2z} - H_{1z})\right] = \vec{a}_x k_s$

$\vec{a}_z \times \vec{a}_x = \vec{a}_y \qquad \vec{a}_z \times \vec{a}_y = \vec{a}_x \qquad \vec{a}_z \times a_z = 0$

$\vec{a}_y\,(H_{2x} - H_{1x}) - \vec{a}_x(H_{2y} - H_{1y}) = \vec{a}_x k_s$

$H_{1x} = \dfrac{B_{1x}}{\mu_0} \qquad H_{1y} = 0 \qquad H_{2x} = H_{1x} = \dfrac{B_{1x}}{\mu_0}$

$B_{2x} = \mu_0 \mu_r H_{2x} = \mu_r \cdot B_{1x} = 1T$

$H_{2y} = -k_s = -5000\,\frac{A}{M} \Rightarrow B_2 = \mu_0 \mu_r H_{2y} = -0.628T$

$\vec{B}_2 = 1\vec{a}_x - 0.628\,\vec{a}_y + 0.02\vec{a}_z \ [T]$

Energija pohranjena u statičkom magnetskom polju

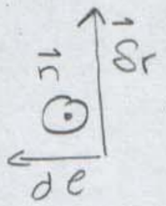
$$d\vec{F}_v = -d\vec{F}_m = -I(d\vec{\ell}\times\vec{B})$$

$$dW = d\vec{F}_v \cdot \vec{\delta}_r = I(d\vec{\ell}\times\vec{B})\,\vec{\delta} r = -I\vec{B}\cdot(\vec{\delta}_r\times d\vec{\ell}) = -I\vec{B}\vec{n}\,dS$$

$$\vec{A}(\vec{B}\times\vec{R}) = \vec{C}(\vec{A}\times\vec{B})$$

plašt

$$\delta\Phi_{Pl} = -\vec{B}\vec{n}\,dS \Rightarrow I\,d\Phi_{Pl} = dW$$



$P = d\vec{\ell}\,\vec{\delta}_r$

$$dW = I(\Phi_2 - \underbrace{\Phi_1}_{0}) \quad \rightarrow \quad W = I\Phi_2 = I\Phi$$

- za više(2) strujnica

$$\left.\begin{aligned} W_{12} &= I_1\Phi_{12} \\ W_{21} &= I_2\Phi_{21} \end{aligned}\right\} \Rightarrow dW = I_1\Phi_{12} + I_2\Phi_{21} = \frac{1}{2}\left(\Phi_{12}I_2 + \Phi_{21}I_1\right)$$

$$W = \frac{1}{2}\sum_{i=1}^{n} I_i \underbrace{\sum_{\substack{k=1\\k\neq i}}^{n}\Phi_{ki}}_{\Phi_i} = \frac{1}{2}\sum_{i=1}^{n} I_i\Phi_i$$

- za vis

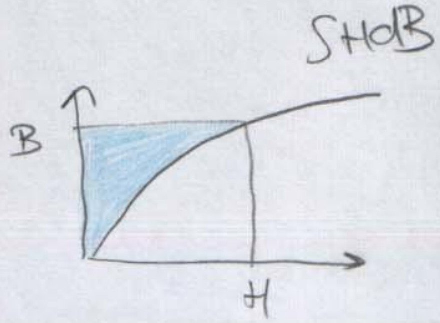
$$I_i = \iint_{S_i} \vec{J}_i\vec{n}_i\,dS_i$$

$$\Phi = \iint \vec{B}\vec{n}\,dS = \oint_C \vec{A}_i\,d\vec{\ell}$$

$$W = \frac{1}{2}\sum_{i=1}^{n} \underbrace{\oiint \vec{J}\vec{n}\,dS}_{I}\ \overbrace{\left(\sum_{j=1}^{n}\oint \vec{A}_i\,d\ell_j\right)}^{\Phi}$$

$$W = \frac{1}{2}\iiint_V \vec{J}\vec{A}\,dV \quad \rightarrow \quad W = \frac{1}{2}\iiint_V \vec{B}\vec{H}\,dV$$

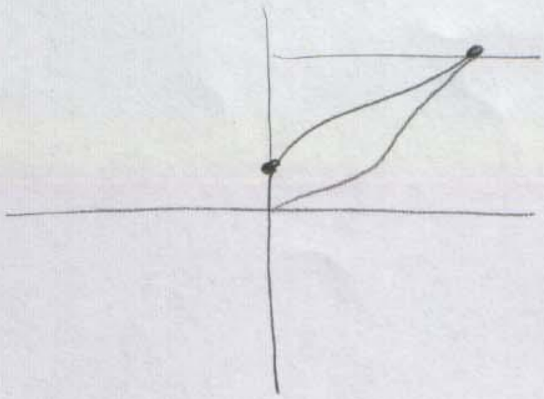
u linearnim materijalima $\underline{\mu = konst}$



$$\frac{\vec{B}\vec{H}}{2} = \frac{\mu\vec{H}\cdot\vec{H}}{2} = \frac{\mu|\vec{H}|^2}{2} = \frac{|\vec{B}|^2}{2\mu}$$

nelinearni materijali $\quad W = \iiint\left\{\int_{B=0}^{B}\vec{H}\,d\vec{B}\right\}dV$

Gubitci zbog histereze



BLITZ

$B = 0.6\sqrt[4]{H}$

$|\vec{H}| = 39{,}29$

$B = 0.\cancel{6}\sqrt[4]{H} = 1.5$

$\downarrow$

72 0.5

$W = \int_0^B H\,dB = \int_0^{1.75} \left(\frac{B}{0.05}\right)^4$

Induktiviteti

L - induktivitet [H]

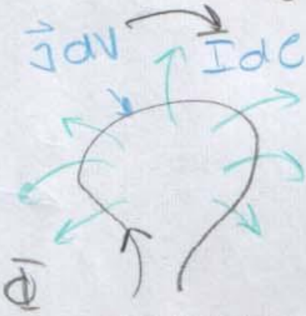
$$W = \frac{1}{2}\iiint_V \vec{H}\vec{B}dV = \frac{1}{2}LI^2 \rightarrow L = \frac{1}{I^2}\iiint_V \vec{H}\vec{B}dV$$

$$L = \frac{1}{I^2}\iiint_V \vec{J}\vec{A}dV$$

$$W = \frac{1}{2}LI^2 \qquad B \sim H \sim I$$
$$\vec{B}\cdot\vec{H} \sim I^2$$

vektorski magnetski potencijal

Induktivitet tankih strujnih petlji



za tanke petlje vrijedi

$$\vec{J}dV = Id\vec{l} \Rightarrow L = \frac{\oint \vec{A}dl}{I} = \frac{\Phi}{I}$$

pa se induktivitet računa kao

$$L = \frac{\oint \vec{A}d\vec{l}}{I}$$ → tok koji žica obuhvaća

$$L = \frac{\Phi}{I}$$

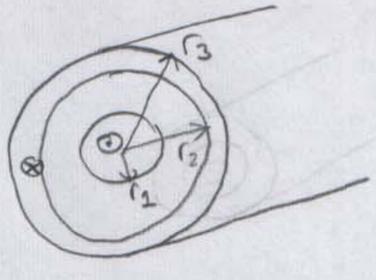
- žice se motaju da bi se više energije moglo pohraniti

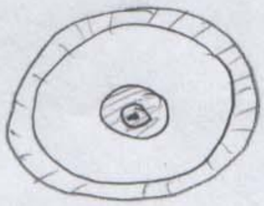
radi se o ulančanom toku i on se računa

$$\Psi = N\Phi \rightarrow L = \frac{N\Phi}{I} = \frac{\Psi}{I}$$

Z-1. Odrediti unutarnji i vanjski induktivitet po jedinici duljine koaksijalnog kabela prema slici.

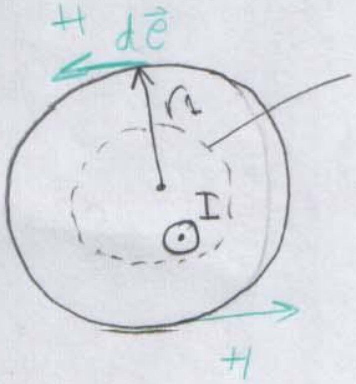
signal koji ide po unutrašnjem vodiču nema nikakvih smetnji





- možemo preko energije ili preko toka
- idemo preko ENERGIJE

$$W = \frac{1}{2}\,\frac{\vec{B}\cdot\vec{H}}{1} = \frac{\vec{B}\cdot\vec{B}}{2\cdot\mu} = \frac{B^2}{2\mu}$$



$$\oint \vec{H}\, d\vec{\ell} = \sum I$$

$\vec{H}$ ima tangencijalan smjer

$$\vec{H}\, d\vec{\ell} = |\vec{H}|\, d\vec{\ell}$$

$$\oint \vec{H}\, d\vec{\ell} = H\, 2r\pi$$

$$\vec{J} = \frac{I}{r_1^2 \pi} \qquad \text{i površ } r^2\pi$$

$$\frac{I}{r_1^2 \cancel{\pi}} \cdot r^2 \cancel{\pi} = \frac{I r^2}{r_1^2} \qquad \vec{B} = \mu_0 \frac{I r}{2 r_1^2 \pi}$$

$$H = \frac{I r}{2 r_1^2 \pi} \qquad dV = \ell\, dS$$

ljuske $dS = 2r\pi\, dr$

energija po jedinici duljine

$$W' = \frac{W}{\ell} = \frac{1}{2\mu_0 \ell}\int_V B^2 dV = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r_1}\right)^2 \int_0^{r_1} r^2\, 2r\pi\, dr =$$

$$= \frac{\mu_0 I^2}{16\pi}$$

$$L'_0 = \frac{2W'}{I^2} = \frac{\mu_0}{8\pi}$$

gustoća magnetskog toka

$$r_1 \le r \le r_2 \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\vec{a}_\alpha$$

$$\downarrow$$

$$dV = 2r\pi dr \cdot \ell$$

$$W' = \frac{1}{2\mu_0 \ell}\int_V B^2 dV = \frac{1}{\mu_0}\left(\frac{\mu_0 I}{2\pi}\right)^2 \int_{r_1}^{r_2} \frac{2\pi \cancel{r} dr}{r^{\cancel{2}}} = \frac{\mu_0 I^2}{4\pi} \ln\frac{r_2}{r_1}$$

↓

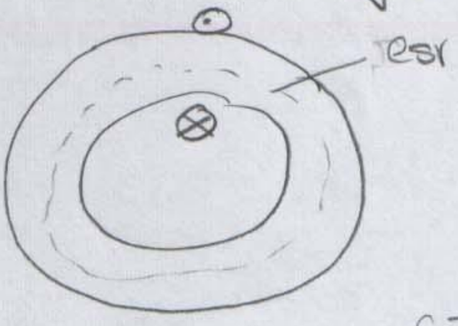
energija po jedinici duljine

$$L'_v = \frac{2W'}{I^2} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$$

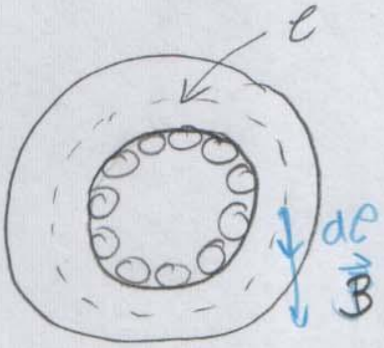
↓

vanjski induktivitet po jed. dužine

Z-2. Odrediti induktivitet torusne zavojnice sred. opsega l_{sr} namotane s N zavoja na jezgru načinjenu od materijala s $\mu = const$, kl. pres S, protjecane strujom I



Amperovim zakonom



$$\oint_c \vec{H} d\vec{l} = \sum I$$

$$\vec{B} d\vec{l} = |\vec{B}| dl$$

$$\oint \vec{B} d\vec{l} = \mu \sum I$$

$$B l_s = \mu N I$$

$$B = \mu \frac{NI}{l_{sr}}$$

$$\Phi = BS = \mu \frac{NI}{l_{sr}} S$$

obuhvaćeni tok $\Psi = N\Phi = \mu \frac{N^2 I}{l_{sr}} S$

$$L = \frac{\Psi}{I} = N^2 \frac{\mu S}{l_{sr}}$$

Z-3 Odrediti magnetsku energiju sadržanu u jezgri torus. zavojnice pravokutnog poprečnog presjeka ako je

a) $\mu_r = 5000$

b) krivulja magnetiziranja aprox. jedn.

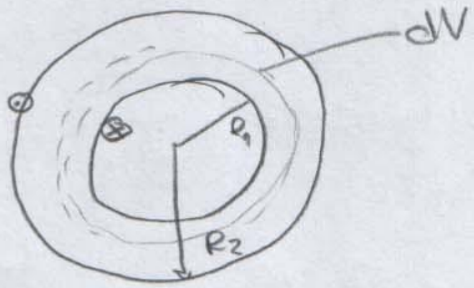
$$B = k\sqrt{H} \qquad k = 0.1\,\mathrm{VsA^{-1/2}\,m^{-3/2}}$$

$R_1 = 5\,\mathrm{cm}$

$a = 2\,\mathrm{cm} \qquad b = 1\,\mathrm{cm}$

$N = 100$

$I = 1\,\mathrm{A}$

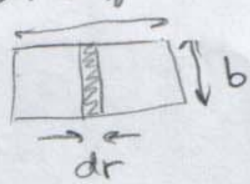


a)

$$\oint_C \vec{H}\,d\vec{l} = \sum I = NI$$

$$H\,2r\pi \Rightarrow H = \frac{NI}{2r\pi}$$

$$W = \frac{1}{2}\mu\int_V H^2\,dV \qquad dV = 2r\pi\,dr\,b$$



$$W = \frac{1}{2}\mu\int_{R_1}^{R_2}\left(\frac{NI}{2r\pi}\right)^2\frac{1}{r^2}\,2r\pi\,dr =$$

$$= \frac{(NI)^2}{4\pi}\,\mu b \ln\left|\frac{R_2}{R_1}\right| = \dots = 16.82\,\mathrm{mJ}$$

b) razlika je samo u μ (s obzirom na slučaj a)

H nađemo preko Biot-Savarovog zakona

$$H = \frac{NI}{2\pi r} \qquad \cancel{B = \mu H}$$

nego

$$B = k\sqrt{H} \Rightarrow H = \frac{B^2}{k^2}$$

$$W = \int_0^B H dB = \int_0^B \frac{B^3}{k^2} dB = \frac{B^3}{3k^2} =$$

$$W = \frac{1}{3k^2} k^3 H^{\frac{3}{2}}$$

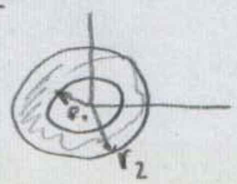
$$W = \int_V \left\{ \int_0^B \vec{H} d\vec{B} \right\} dV = \int_0^B H dB \int_V dV = \int_{R_1}^{R_2} \underbrace{\frac{k}{3}\left(\frac{NI}{2\pi}\right)^{\frac{3}{2}} r^{-3/2}}_{HdB} \underbrace{2r\pi dr b}_{dV}$$

$dV = 2\pi r dr b$

$$= \frac{k}{3}\left(\frac{NI}{2\pi}\right)^{3/2} 2\pi b + \int_{R_1}^{R_2} r^{-\frac{1}{2}} dr =$$

$$= \frac{k}{3}\left(\frac{NR}{2\pi}\right)^{3/2} 4\pi b\left(\sqrt{R_2} - \sqrt{R_1}\right) = 1089 mJ$$

BLITZ



A. u točki $r = \frac{r_1}{2}$

B. u $r = (r_1 + r_2)/2$

C u $r = r_1 + r_2$

po Amperovom kružnom zakonu

$$\oint_C \vec{H} d\vec{l} = \iint_S \vec{J} \vec{n} ds$$

$$\oint_C \vec{H} d\vec{l} = 2r\pi H \qquad J = \frac{I}{(r_2^2 - r_1^2)\pi}$$

A) $H = 0$ ništa nije obuhvaćeno

B) $$2\frac{r_1 + r_2}{2}\pi H = \frac{I}{(r_2^2 - r_1^2)\pi}\left[\left(\frac{r_1 + r_2}{2}\right)^2 - r_1^2\right]\pi$$

$$H = \frac{\frac{I}{r_2^2 - r_1^2}\left[\left(\frac{r_1 + r_2}{2}\right)^2 - r_2^2\right]}{}$$

Međuinduktivitet

sustav 2 struje I_1 i I_2

linearni materijal

superpozicija $\vec{H} = \vec{H}_1 + \vec{H}_2$

$\vec{B} = \vec{B}_1 + \vec{B}_2$

L_{12} međuinduktivitet

L_{11}, L_{22} samoinduktiviteti

$$W = \frac{1}{2}\iiint_V \underbrace{\vec{H}}_{\vec{H}_1+\vec{H}_2}\cdot\underbrace{\vec{B}}_{\vec{B}_1+\vec{B}_2}\, dV$$

nastojimo prebaciti u linijski → tanke žice

MEĐUINDUKTIVITET L_{12}

utjecaj 12 $\quad L_1 \quad L_2$

$$= \vec{H}_1\vec{B}_1 + \vec{H}_2\vec{B}_1 + \vec{H}_1\vec{B}_2 + \vec{H}_2\vec{B}_2 = \quad (*)$$

Energija pohranjena od struje I_1 ... I_2

$H_1 \sim I_1$

$B_2 \sim I_2$

(*)

$$= \frac{1}{2}L_{11}I_1^2 + \frac{1}{2}L_{22}I_2^2 + L_{12}I_1I_2$$

ovaj član može dobiti negativni predznak, vidjeti ćemo poslije o čemu se radi

S volumne na linijsku:

$$\vec{J}dV \rightarrow I d\vec{l}$$

$$L_{12} = L_{21} = \frac{1}{I_1 I_2}\iiint_{V_2} \underbrace{\vec{J}_2 A_1 dV}_{I_2 dl} = \frac{1}{I_1}\oint \underbrace{\vec{A}d\vec{l}_2}_{\text{tok}} = \frac{\Psi_{21}}{I_1}$$

druga def: preko ulančanog toka

$\boxed{\text{min pohr. en } L_{12} \leq \sqrt{L_{11}L_{22}}}$

minimalna pohranjena energija

$$\boxed{L_{12} \leq \sqrt{L_{11} L_{22}}}$$

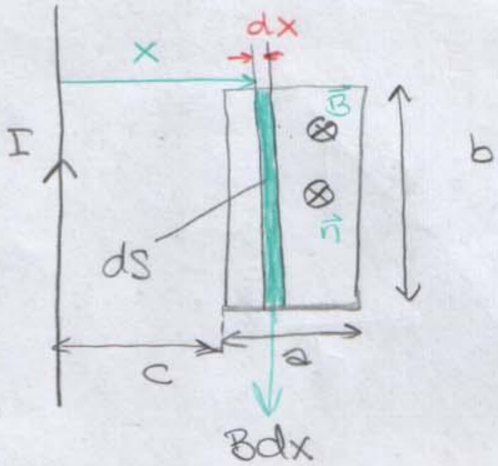
ovo slijedi iz ENERGIJA ≥ 0

Z-4 (sa slideova)

odrediti međuinduktivitet ravne, beskonačno duge stvujnice i pravokutne vodljive petlje prema slici.

$$L_{12} = \frac{\Phi}{I}$$

$$\Phi = \iint_S \underbrace{\vec{B}\vec{n}}_{\frac{\mu_0 I}{2\pi x}} ds = \iint_{\underbrace{S}_{\text{površina pravokutnika}}} \frac{\mu_0 I}{2\pi x} ds = *$$



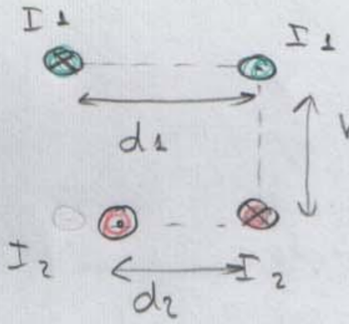
$$* = \int_c^{c+a} \frac{\mu_0 I}{2\pi x} b dx = \frac{\mu_0 I b}{2\pi} \ln \frac{c+a}{c}$$

$$L_{12} = \frac{\Phi}{I}$$

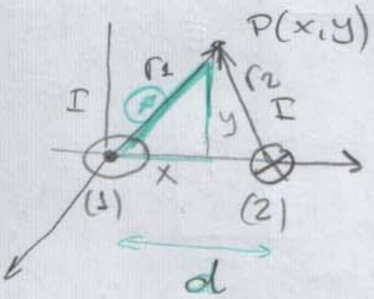
$$L_{12} = \frac{\mu_0 b}{2\pi} \ln \frac{c+a}{c}$$

Z - 5.

Odrediti međuinduktivitet po jedinici duljine dvožičnog voda prema slici



rješenje preko $\vec{A}$ vektorski magnetski potencijal



$$\vec{A} = \frac{\mu_0 I}{4\pi} \int_C \frac{d\vec{l}}{R}$$

$$(1) \rightarrow \quad d\vec{l} = \vec{a}_z \, dz'$$

$$\vec{r} = x\vec{a}_x + y\vec{a}_y$$

$$\vec{r}\,' = z'\vec{a}_z$$

$$\vec{R} = \vec{r} - \vec{r}\,' = x\vec{a}_x + y\vec{a}_y - z'\vec{a}_z$$

$$|\vec{R}| = \sqrt{\underbrace{x^2 + y^2}_{r_1^2} + (z')^2}$$

$$\vec{A}_{(1)} = \frac{\mu_0 I}{4\pi} \vec{a}_z \int_{-\infty}^{\infty} \frac{dz'}{\sqrt{r^2 + z'^2}} =$$

$$= 2\,\frac{\mu_0 I}{4\pi}\,\vec{a}_z \int_0^{\infty} \frac{dz'}{\sqrt{r_1^2 + z'^2}} = \vec{a}_z \frac{\mu_0 I}{2\pi} \ln\left(z' + \sqrt{r_1^2 + z'^2}\right)\Big|_0^{\infty}$$

$$(2) \quad d\vec{l} = -\vec{a}_z dz' \qquad \vec{r} = x\vec{a}_x + y\vec{a}_y \qquad \vec{r}\,' = \vec{a}_x \cdot d + z'\vec{a}_z$$

$$\vec{R} = (x-d)\,\vec{a}_x + y\,\vec{a}_y + z'\vec{a}_z$$

sve isto samo se predznak mijenja (skoro sve)

$$|\vec{R}| = \sqrt{\underbrace{(x-d)^2 + y^2}_{r_2^2} + z'^2}$$

dalje sve iste pa dobijemo : $\vec{A}_{(2)} = -\vec{a}_z \frac{\mu_0 I}{2\pi} \ln\left(z' + \sqrt{r_2^2 + z'^2}\right)\Big|_{z'=0}^{\infty}$

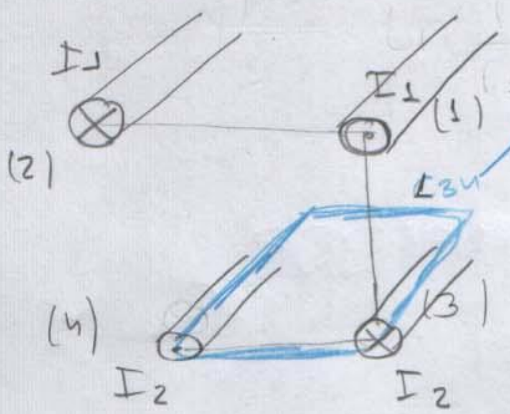
$$\vec{A} = \vec{A}_1 + \vec{A}_{(2)} = +\,\vec{a}_z \frac{\mu_0 I}{2\pi} \ln\left(\frac{z' + \sqrt{r_1^2 + z'^2}}{z' + \sqrt{r_2^2 + z'^2}}\right)\Bigg|_{z'=0}^{\infty} = \longrightarrow$$

$$= -\vec{a}_z \frac{\mu_0 I}{2\pi} \ln\left(\frac{z' + \sqrt{r_1^2 + z'^2}}{z' + \sqrt{r_2^2 + z'^2}}\right)^{/z'}\Bigg|_{z'=0}^{\infty} =$$

$$= \vec{a}_z \frac{\mu_0 I}{2\pi}\left[\underbrace{\ln \frac{1 + \sqrt{\frac{r_1^2}{z'^2} + 1}}{1 + \sqrt{\frac{r_2^2}{z'^2} + 1}}}_{=0} - \ln\frac{r_1}{r_2}\right]$$

$$= \vec{a}_z \frac{\mu_0 I}{2\pi} \ln \frac{r_2}{r_1}$$

krivulja integracije



određujemo međuinduktivitet, ali preko toka kojeg onda podijelimo sa strujom I

$$\Phi_{34} = \int_{C_{34}} \vec{A}_{12}\, d\vec{l}_{33}$$

$$A_{12} = \vec{a}_z \frac{\mu_0 I_1}{2\pi} \underbrace{\ln \frac{r_2}{r_1}}$$

uopće ne ovisi o koordinati z

L_{34}

$= 0$

$-d x\vec{a}_x$

$d\vec{l}_4 = dz\vec{a}_z$

$= 1$

$d\vec{l}_3 = -dz\vec{a}_z$ $= 1$

$\vec{a}_z \vec{a}_x = 0$

(4) (3)

$dx\vec{a}_x = 0$

pa otpadaju doprinosi na poprečnim plohama krivulje

$$\Phi_{34} = -\int_{L_3} \frac{\mu_0 I_1}{2\pi} \ln\frac{r_{23}}{r_{13}} + \int_{L_4} \frac{\mu_0 I}{2\pi} \ln \frac{r_{24}}{r_{14}} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_{13} \cdot r_{24}}{r_{23} \cdot r_{14}}\right)$$

$r_{13} = h$ (iz slike)

$r_{24} = \sqrt{(d_1 - d_2)^2 + h^2}$

$r_{23} = \sqrt{d_1^2 + h^2}$

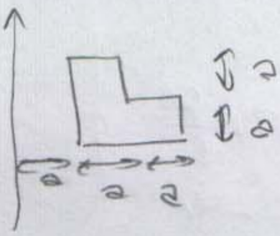
$r_{14} = \sqrt{d^2 + h^2}$

$$M = \frac{\Phi_{34}}{I_1} = \ldots = -0.161\,\mu H/m$$

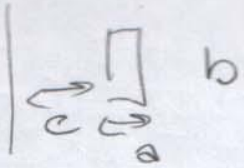
ali on je i dobija $+0.161\,\mu H/m$

blitz

1. odrediti međuinduktivitet



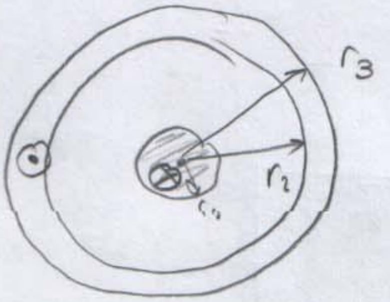
$$M = \frac{\Psi}{I} = \frac{\mu_0 b}{2\pi} \ln\left(\frac{c+a}{a}\right)$$



Određivanje sile pomoću energije

1. odrediti silu po jedinici duljine na vanjski vodič koaksijalnog kabela

$$L = \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$



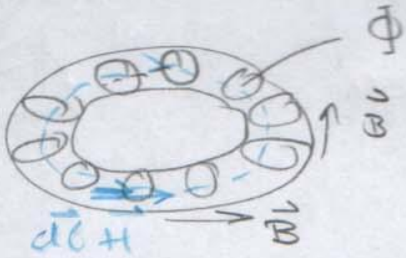
$$\vec{F}_{12} = \vec{a}_{r2} \cdot \frac{1}{2} I^2 \frac{\partial L}{\partial r_2} = \vec{a}_r \frac{1}{2} I^2 \frac{\mu_0}{2\pi} \frac{1}{r_2}$$

Magnetski krugovi

- za zatvorenu krivulju c kroz os silocijevi:

$$\oint_c \vec{H} d\vec{l} = \sum I = NI$$

silocijev



$\vec{H} d\vec{l} = |\vec{H}| dl$

$\oint_c \vec{H} d\vec{l} = \sum I$ po Amperovom (kružnom) zakonu

$$\oint_c \vec{H} d\vec{l} = \oint_c H dl = \oint_c \frac{B}{\mu} dl = \oint_c \frac{Bds}{\mu ds} dl = d\Phi$$

dio toka koji silocijev obuhvaća

kako je to konst po def. ide ispred ∫

magnetomotorna sila

analogno

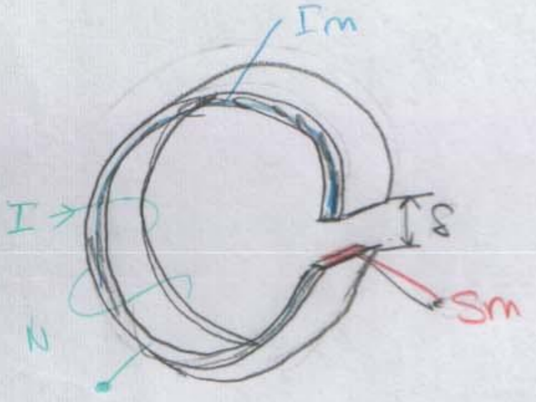
$$d\Phi = \frac{NI}{\oint_c \frac{dl}{\mu ds}} \sim I = \frac{U}{R}$$

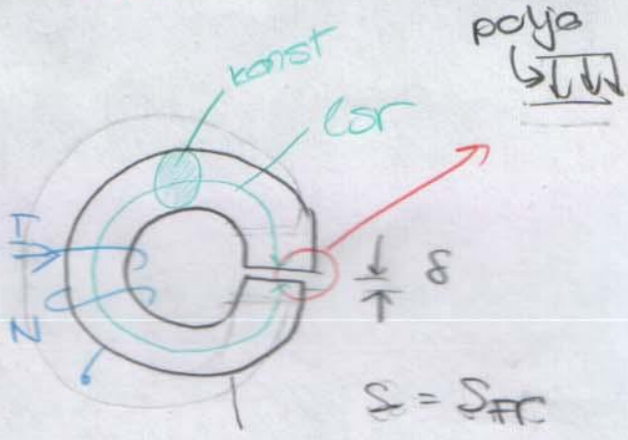
analogno

analogno

magnetski otpor

Feromagnetski torus sa znatnim rasporom





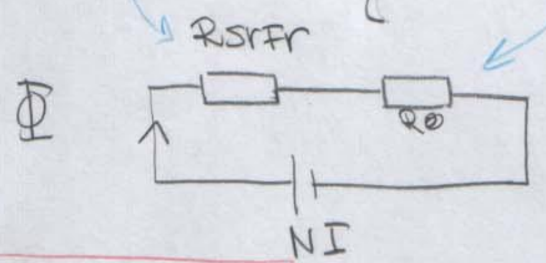
$S = S_{Fe}$ zbog malog raspora

$$\oint_C \vec{H}\,d\vec{l} = NI = \oint_C \frac{B}{\mu}\,dl = \int_{l_{sr}} \frac{B}{\mu_{Fe}}\,dl + \int_{\delta} \frac{B}{\mu_0}\,dl \;\Big/ S =$$

Φ tok = konst l_{sr}

$$= \int_{l_{sr}} \frac{BS}{\mu_{Fe} S}\,dl + \int_{\delta} \frac{B \cdot S_0}{\mu_0 S_0}\,dl =$$

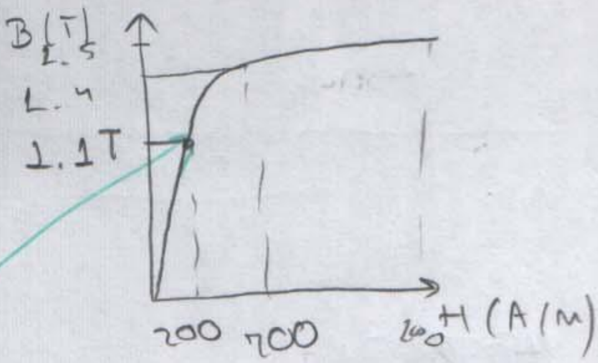
$$= \Phi \underbrace{\left[\frac{l_{sr}}{\mu_{Fr} \cdot S} + \frac{\delta}{\mu_0 S} \right]}_{\text{serijski spoj}} = NI$$



$$\boxed{\Phi = \frac{NI}{\frac{1}{\mu} \frac{l_{sr}}{S_{Fe}} + \frac{1}{\mu_0} \cdot \frac{\delta}{S_0}}}$$

1. Prsten od silicijskog čelika ima sr. prom 80mm, a oko njega zavojnica s 1200 zav. U prstenu je zračni raspor $\delta = 3mm$. Odrediti potrebnu ...

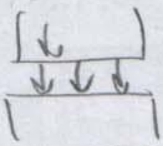
krivulja magnetiziranja



a) $B_\delta = 1.1T$

b) $B_\delta = 1.4T$

a) $B_\delta = 1.1T \rightarrow \Phi_M = \Phi_0 \Rightarrow B_{Fe} \cancel{S_{nFe}} = B_0 \cancel{S_0}$



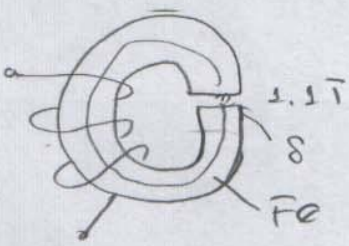
$\Phi_0 = \Phi_{Fe}$ $\quad B_{Fe} = B_0$

$S_0 = S_{Fe}$

krivulja

$B_{Fe} = 1.1T \Rightarrow H_{Fe} = 200\ A/M$

$B_0 = 1.1T \Rightarrow H_0 = \frac{B_0}{\mu_0} = 875\ 350\ \frac{A}{M}$



$NI = \oint_C \vec{H} d\vec{l} = H_{Fe} l_{Fe} + H_0 \delta = 2626A$

$l_{Fe} \rightarrow D_{sr}\pi - \delta$

$1200 I = 2626 \Rightarrow I = 2.23A$

b) sve isto samo druga indukcija:

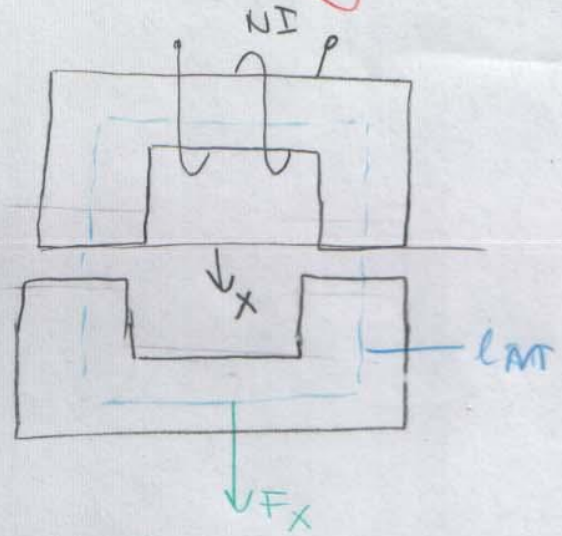
$B_{Fe} = 1.4T \quad H_{Fe} = 700 \frac{A}{M}$

$B_0 = 1.4T \quad H_0 = \frac{B_0}{\mu_0} = 1\ 114\ 080 \frac{A}{M}$

$NI = H l_{sr} + H_0 \delta = 3516A$

$I = \frac{3516}{1200} = 2.93A$

Elektromagnet



serijski krug

$$\sim \quad \boxed{\Phi = \frac{NI}{\frac{1}{\mu}\cdot\frac{l_m}{S_m} + 2\cdot\frac{1}{\mu_0}\frac{\delta}{S_0}}}$$

$$\boxed{S_0 = S_m}$$

Energija pohranjena u magnetu

$$W = \frac{1}{2} N I \Phi = \frac{1}{2} N^2 \cancel{I^2} \frac{1}{\frac{1}{\mu}\cdot\frac{l_m}{S_m} + \frac{2}{\mu_0}\frac{\delta}{S_m}} = \frac{1}{2} L \cancel{I^2}$$

$$\rightarrow \boxed{L = \frac{N^2}{\frac{1}{\mu}\frac{l_m}{S_m} + \frac{2}{\mu_0}\frac{\delta}{S_m}}}$$

- sila može djelovati samo u x smjeru, mijenja se raspor pa onda čuda... :)

$$\vec{F}_\delta = \vec{a}_\delta \frac{\partial W}{\partial \delta} = \vec{a}_\delta \frac{1}{2} N^2 I^2 \frac{\partial}{\partial \delta}\left(\frac{1}{\frac{1}{\mu}\frac{l_m}{S_m} + \frac{2}{\mu_0}\frac{\delta}{S_m}}\right)$$

jer je $I = konst$

smjer u kojem uzrokuje djeluje δ prom

$$= -\vec{a}_\delta \frac{\Phi^2}{\mu_0 S_m} = -\vec{a}_\delta \frac{B_\delta \cdot B_\delta \cdot S_m^2}{\mu_0 \cdot S_m} =$$

$$= -\vec{a}_\delta H_\delta \cdot B_\delta \cdot S_m = -\vec{a}_\delta \frac{B_\delta^2}{\mu_0}\cdot S_m$$

privlačna sila

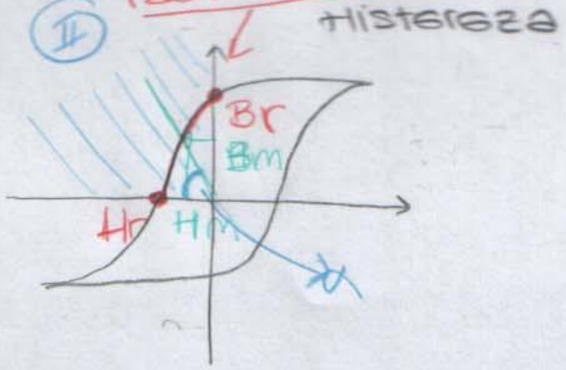
<u>npr</u> $\left.\begin{matrix} B_\delta = 1T \\ S_m = 4cm^2 \end{matrix}\right\} F_\delta = 318N$

Permanentni magnet

- isto primjenjujemo Amperov kružni zakon

$$\oint_c \vec{H}\, d\vec{l} = 0 \longrightarrow \text{nema nikakvih struja}$$

radna točka je u II kvad.



$$H_m l_m + H_\delta \delta = 0$$

$$H_m = -H_\delta \cdot \frac{\delta}{l_m}$$

$$H_\delta = \frac{B_\delta}{\mu_0}$$

$$H_m = -\frac{B_\delta}{\mu_0} \cdot \frac{\delta}{l_m}$$

$$H_m = -B_m \frac{\delta}{\mu_0 l_m}$$

$$\operatorname{tg}\alpha = \frac{B_m}{H_m} = -\mu_0 \frac{l_m}{\delta}$$