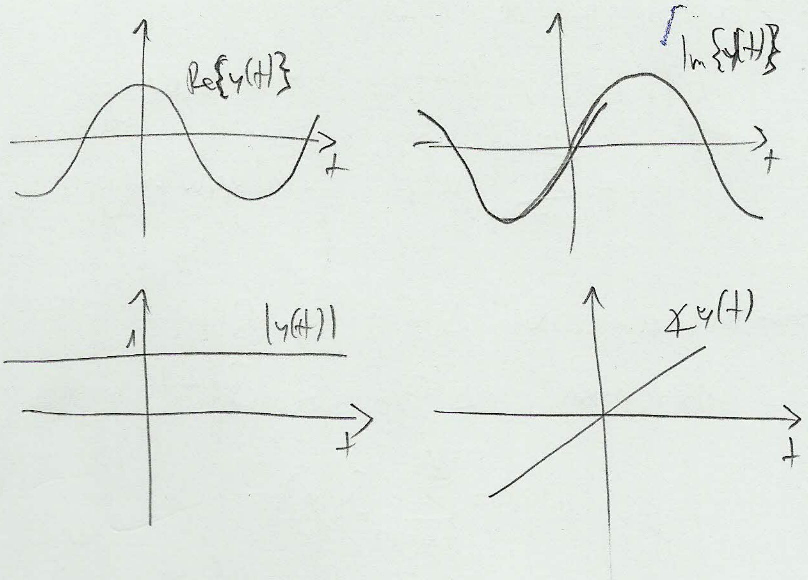
SIGNALI I SUSTAVI

SAŽETAK BY Cope

- prema prezentacijama (dopune) i predavanjima prof. Sović (većinom)

212



$$y(t)=e^{jt}=\cos t+j\sin t$$

$$|y(t)|=\sqrt{Re^2+Im^2}=\sqrt{\sin^2 t+\cos^2 t}=1$$

$$\measuredangle y(t)=\operatorname{arctg}\frac{\sin t}{\cos t}=t$$

→ $y(t_1^-)\neq y(t_1^+)$ → diskontinuitet

→ $\int_{-\infty}^{+\infty}|y(t)|^2dt<\infty$ → kvad. integrabilan

$\int_{-\infty}^{+\infty}|y(t)|\,dt<\infty$ → aps. integrabilan (zbog $|\cdot|$)

→ $E=\int_{t_1}^{t_2}|y(t)|^2dt \qquad P=\frac{1}{L}\int_{t_1}^{t_2}|y(t)|^2dt, \quad L=t_2-t_1$

→ $E_{tot}=\int_{-\infty}^{+\infty}|y(t)|^2dt \qquad P_{tot.sr.}=\lim_{L\to\infty}\frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}}|y(t)|^2dt$

$$\boxed{\sum_{n=0}^{\infty}q^n=\frac{1}{1-q}}$$

→ za $|q|<1$

$$\boxed{\sum_{n=0}^{N}q^n=\frac{1-q^{N+1}}{1-q}}$$

$P_{period.}=\frac{1}{T_0}\int_{t_0}^{t_0+T_0}|y(t)|^2dt$

→ diferencija: uzlazna $\Delta u(n)=u(n+1)-u(n)$

silazna $\Delta u(n)=u(n)-u(n-1)$

→ parne i neparne:

$$u_p(t)=\frac{u(t)+u(-t)}{2},\quad u_n(t)=\frac{u(t)-u(-t)}{2}$$

→ konjugirano kompleksni:

simetrični $u(t)=u^*(-t)$ $\qquad u_{ks}(t)=\frac{u(t)+u^*(-t)}{2}$

nesimetrični $u(t)=-u^*(-t)$ $\qquad u_{kn}(t)=\frac{u(t)-u^*(-t)}{2}$

OSNOVNI SIGNALI

-> odnos koraka i stepa:

$$\mu(n)=k(n+1)-k(n) \quad \text{- diferencija} \qquad \mu(t)=\frac{dk(t)}{dt}$$

$$k(n)=\sum_{m=-\infty}^{N-1}\mu(m) \quad \text{- akumulacija} \qquad k(t)=\int_{-\infty}^{t}\mu(\tau)\,d\tau$$

-> Kroneckerova Δ fja $-\ \delta(n)$

Diracova Δ fja $-\ \delta(t)$

$$\mu(n)=\sum_{m=-\infty}^{N}\delta(m) \qquad \mu(t)=\int_{-\infty}^{t}\delta(\lambda)\,d\lambda$$

$$\delta(n)=\mu(n)-\mu(n-1) \qquad \delta(t)=\frac{d\mu(t)}{dt} \qquad \mu(t)'=\delta(t)$$

$$\text{comb}(n)=1=\sum_{m=-\infty}^{\infty}\delta(n-m)$$

-> $\int_{-\infty}^{\infty}f(t)\,\delta'(t)\,dt=-f'(0) \quad \Rightarrow \quad \boxed{\int_{-\infty}^{\infty}f(t)\,\delta^{(n)}(t)\,dt=(-1)^n f^{(n)}(0)}$

-> $f(t)*\delta(t-t_1)=\int_{-\infty}^{\infty}f(\tau)\,\delta(t-t_1-\tau)\,d\tau=f(t-t_1)$

$\tau=t-t_1$

-> očitavanje:

$$2\omega\le\omega_s \quad , \quad 2f\le f_s$$

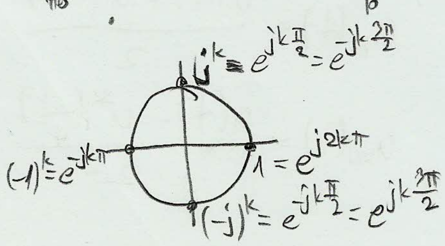
-> $\boxed{\cos(\omega t)=\frac{e^{j\omega t}+e^{-j\omega t}}{2}}$, $\boxed{\sin(\omega t)=\frac{e^{j\omega t}-e^{-j\omega t}}{2j}}$ $=\frac{1}{2}e^{-j\frac{\pi}{2}}\left(e^{j\omega t}-e^{-j\omega t}\right)$

$$\frac{1}{2j}\cdot\frac{j}{j}=-\frac{1}{2}j=\frac{1}{2}e^{-j\frac{\pi}{2}}$$

CTFS

-> Parseval

$$P=\frac{1}{T_0}\int_{T_0}x(t)\,x^*(t)\,dt=\frac{1}{T_0}\int_{T_0}\left[x(t)\sum_{k=-\infty}^{\infty}X_k^*e^{-jk\omega_0 t}\right]dt=\sum_{k=-\infty}^{\infty}X_k^*\underbrace{\frac{1}{T_0}\int_{T_0}x(t)\,e^{-jk\omega_0 t}\,dt}_{=X_k}=\sum_{k=-\infty}^{\infty}X_k X_k^*$$



CTFT

- Parseval:

$$E=\int_{-\infty}^{\infty}|x(t)|^2\,dt=\int_{-\infty}^{\infty}x(t)x^*(t)\,dt=\int_{-\infty}^{\infty}x(t)\cdot\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}\,d\omega\right]^*dt=$$

$$=\int_{-\infty}^{\infty}x(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}X^*(j\omega)e^{-j\omega t}\,d\omega\right]dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}X^*(j\omega)\left[\int_{-\infty}^{\infty}x(t)e^{-j\omega t}\,dt\right]d\omega=$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}X^*(j\omega)\cdot X(j\omega)\,d\omega$$

-> faza umnoška = zbroj faza

faza kvocijenta = razlika faza

- vrem -> frekv:

$$g(t)=1 \qquad -\frac{\tau}{2}<\tau\le\frac{\tau}{2}\ \rightarrow\ G(j\omega)=\tau\,\frac{\sin(\omega\frac{\tau}{2})}{\omega\frac{\tau}{2}}=\tau\,\mathrm{sinc}(\omega\tfrac{\tau}{2})$$

frekv -> vrem:

$$g(-\omega)=2\pi \qquad -\frac{\tau}{2}\le\omega\le\frac{\tau}{2}\ \rightarrow\ G(jt)=\tau\,\mathrm{sinc}(t\tfrac{\tau}{2})$$

DTFS

pravokutni $\xleftrightarrow{CFT}$ sinc -> svojstvo dualnosti

$$\sum_{n=0}^{N-1}q^n=\frac{1-q^n}{1-q} \qquad X_{k+N}=X_k$$

signal		spektar
KONT	$\xleftrightarrow{CT}$	APER
DISK	$\xleftrightarrow{DT}$	PER
APER	$\xleftrightarrow{FT}$	KONT
PER	$\xleftrightarrow{FS}$	DISK

DTFT

$$X(e^{j(\omega+2k\pi)})=X(e^{j\omega})$$

-> period. signali: $F(j\omega)=\sum_{k=-\infty}^{\infty}2\pi F_k\,\delta(\omega-k\omega_0)$

(1. CTFS, 2. CTFT)

-> linearnost: $a.f(n)+b.g(n)\xleftrightarrow{DTFT}aF(e^{j\omega})+bG(e^{j\omega})$

-> simetričnost: $F^*(j\omega)=F(-j\omega)$

-> REKONSTRUKCIJE: bilježnica

-> KONVOLUCIJA: $f(n)*g(n)=\sum_{m=-\infty}^{\infty}f(m)g(n-m)$

$$DTFT(f(n)*g(n))=\sum_{n=-\infty}^{\infty}(f(n)*g(n))e^{-j\omega n}=\sum_{n=-\infty}^{\infty}\left(\sum_{m=-\infty}^{\infty}f(m)g(n-m)\right)e^{-j\omega n}=\sum_{m=-\infty}^{\infty}f(m)\sum_{n=-\infty}^{\infty}g(n-m)e^{-j\omega n}=$$

$$=\left|\begin{matrix}n-m=r\\ n=r+m\\ n=-\infty\rightarrow r=-\infty\\ n=+\infty\rightarrow r=+\infty\end{matrix}\right|=\sum_{m=-\infty}^{+\infty}f(m)\cdot\sum_{r=-\infty}^{+\infty}g(r)e^{-j\omega(r+m)}=\sum_{m=-\infty}^{\infty}f(m)e^{-j\omega m}\cdot\sum_{r=-\infty}^{\infty}g(r)e^{-j\omega r}=$$

$$=F(e^{j\omega})\cdot G(e^{j\omega})$$

DFT

$$\boxed{\begin{aligned} x(n) &= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn} \\ X(k) &= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} \end{aligned}}$$

SUSTAVI

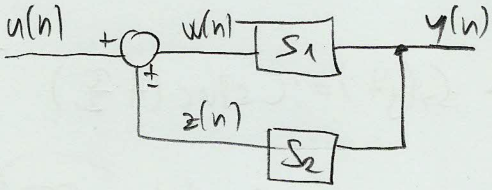
- Inverz fje:

$w = 2u+1 \rightarrow u = 2w+1$

$2w = u-1$

$w = \frac{u-1}{2} \rightarrow$ Inverz od $w = 2u+1$

- povratna veza:



$y(n) = S_1(w(n)) = S_1(u(n) \pm z(n)) =$

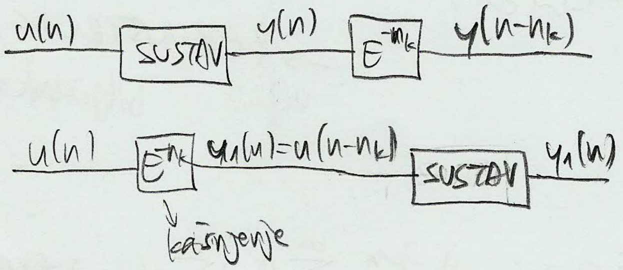
$= S_1(u(n) \pm S_2(y(n)))$

SVOJSTVA

- memorija: bezmemorijski – ovise o sadašnjosti
  memorijski – ovise o budućnosti i prošlosti
- početni uvjeti: $y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau = \boxed{y(t_0) + \int_{t_0}^{t} u(\tau)\,d\tau}$

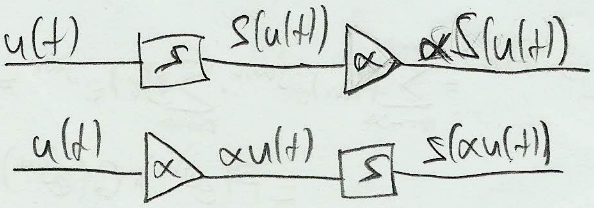
  ako nema ovog → NEPOBUĐEN SUSTAV

  MIRAN SUSTAV
- kauzalnost: kauzalan – ovisi o sadašnjosti i prošlosti
  nekauzalan – – " – – " – , – " – i budućnosti
- vremenska stalnost:



→ ako su $\boxed{y(n-n_k) = y_1(n)}$ → vrem. stalan, inače vrem. promjenjiv

- homogenost:



→ uvjet homogenosti:

$$\boxed{\alpha S(u(t)) = S(\alpha u(t))}$$

- adititvnost: $\boxed{S(u_1(t)+u_2(t)) = S(u_1(t)) + S(u_2(t))}$

- linearnost = homogenost + aditivnost

- linearnost integratora - $y(t) = \int_{-\infty}^{t} u(\tau)d\tau$ → linearan

$$y(t) = \int_{-\infty}^{t} u(\tau)d\tau = y(t_0) + \int_{t_0}^{t} u(\tau)d\tau$$ → nije linearan

- BIBO stabilnost (Bounded input bounded output)

  Sustav je BIBO stabilan ako je za svaki omeđen ulaz njegov izlaz također omeđen. $|u(t)| \le M_u \le \infty \rightarrow |y(t)| \le M_y \le \infty$

- impulsni odziv vrem. disk. sustava:

$\delta(n)$ → $\boxed{S}$ → $y(n) = ?$

$h(n)$ - impulsni odziv

poč. uvjeti = 0

$$y(n) - 0{,}75\,y(n-1) = u(n) = \delta(n)$$

$$h(n) = \left\{ \begin{matrix} (0{,}75)^n, & n \ge 0 \\ 0, & n < 0 \end{matrix} \right\} = (0{,}75)^n \mu(n)$$

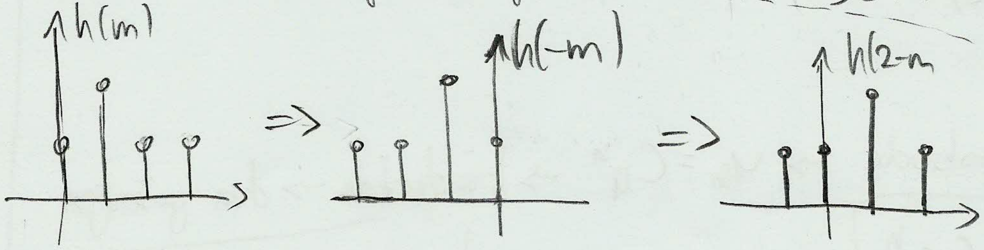
LTI - linearni vrem. nepromjenjivi sustav

- možemo tražiti odziv za svaku pobudu posebno

$$y(n) = \sum_{m=-\infty}^{\infty} u(m)h(n-m)$$

↳ za kauzalne: $\boxed{y(n) = \sum_{m=0}^{n} u(m)h(n-m)}$

zapravo konvolucija



- svojstva konvolucije

  - asocijativnost: $y(n) = (u(n) * h_1(n)) * h_2(n) = u(n) * (h_1(n) * h_2(n))$
  - distributivnost: $y(n) = u(n) * (h_1(n) + h_2(n)) = u(n) * h_1(n) + u(n) * h_2(n)$
  - pomak: $y(n) = x_1(n) * x_2(n)$

    $y_1(n) = x_1(n-p) * x_2(n-q) = y(n-p-q)$
  - impuls: $x(n) * \delta(n) = x(n)$, $x(n) * \delta(n-p) = x(n-p)$
  - broj uzoraka nakon konvolucije: $L_1 + L_2 - 1$

- Impulsni odziv vrem. kont. signala

$$y(t)=\int_{-\infty}^{t}u(\tau)d\tau = y(0^-)+\int_{0^-}^{t}u(\tau)d\tau$$

$$h(t)=0+\int_{0^-}^{t}\delta(\tau)d\tau = 1\cdot\mu(t)$$

$$h'(t)=\delta(t)\longrightarrow\boxed{\int}\longrightarrow h(t)$$

$h'(0^-)=0 \quad h(0^-)=0$

$h'(0^+)=0 \quad h(0^+)=1$

$$\boxed{y(t)=\int_{-\infty}^{\infty}u(\tau)h(t-\tau)d\tau}$$

partikularno rješenje:

$u(n)$	$y_p(n)$
r^n	Kr^n
$A\cos(\omega_0 n)$	$K_1\cos(\omega_0 n)+K_2\sin(\omega_0 n)$
$A\sin(\omega_0 n)$	— " —
A	K
r^n, za $r=q$	$K\cdot n\cdot r^n$

kad je frekv. pobude jednake frekv. u part. rješenju množimo sa n

DIFERENCIJSKE JEDNADŽBE

$$y(n)+a_1y(n-1)+a_2y(n-2)=b_0u(n)+b_1u(n-1)+b_2u(n-2)$$

-> realizacija:

I. $b_0u(n)+b_1u(n-1)+b_2u(n-2)=w(n)$, $y(n)=w(n)-a_1y(n-1)-a_2y(n-2)$

II. $y_a(n)+a_1y_a(n-1)+a_2y_a(n-2)=u(n)$

$y_a(n)=u(n)-a_1y_a(n-1)-a_2y_a(n-2)$

$u(n)\to y_a(n)$

$u(n-1)\to y_a(n-1) \quad\Rightarrow y(n)=b_0y_a(n)+b_1y_a(n-1)+b_2y_a(n-2)$

$b_2u(n-2)\to b_2y_a(n-2)$

-> rješavanje:

=> homogena -> nema pobude -> $y_h=Cq^n$ -> kvadratna -> dva rješenja

-> $\boxed{y_h(n)=C_1q_1^n+C_2q_2^n}$ -> ako su $q_1=q_2$ -> $\boxed{y_h(n)=C_1q^n+C_2nq^n}$

- za kompl. brojeve: $\boxed{q_{1kompl}=|q_1|e^{j\measuredangle q_1}}$

$$y_h(n)=\frac{C}{2}e^{j\theta}\left(|q_1|e^{j\measuredangle q_1}\right)^n+\frac{C}{2}e^{-j\theta}\left(|q_2|e^{\measuredangle q_2}\right)^n=C\cdot|q|^n\cdot\cos(\beta n+\theta)$$

=> partikularno: pretpostavimo tj: $u(n)=Ar^n$ -> $y_p(n)=Kr^n$ -> izrač. K

=> totalno: $y_T(n)=y_h(n)+y_p(n)$

- preko poč. uvjeta -> nađemo $y(0)$ i $y(1)$ -> nađemo c_1 i c_2

→ odziv linearnog sustava kao zbroja odziva nepobuđenog i mirnog

⇒ nepobuđeni – $u(n)=0$ → homogena jedn.

$y_h(n) = y_0(n)$

– iz poč. uvjeta → C_1 i C_2 → rješenje

– stabilnost:

$|q|>1$ nestabilan sustav

$|q|=1$ granično stabilan

$|q|<1$ stabilan

⇒ miran sustav – poč. uvjeti $=0$

– iz njih → $y(0)$ i $y(1)$ ⇒ C_1, C_2 → rješenje $y_m(n)$

⇒ totalni odziv: $y_T(n) = y_0(n) + y_m(n)$

IMPULSNI ODZIV

1. način: $u(n) = \delta(n)$ → tražimo $h(n)$, ne $y(n)$

homogena uz iz poč. uvjeta → $h(0), h(1), h(2) \to C_1, C_2$

– rješenje: $h(n) = \underbrace{(C_1 q_1^n + C_2 q_2^n)\mu(n-1)}_{\text{za } n>0} + \underbrace{\delta(n)}_{\text{za } n=0}$, $n \geq 0$

2. način – neću, teže je

DIFERENCIJALNE JEDNADŽBE

$$y''(t) + a_1 y'(t) + a_2 y(t) = \underbrace{b_0 u''(t) + b_1 u'(t) + b_2 u(t)}_{} \quad (1)$$

⇒ homogeno: nema u-ova, tj. $=0$

$$\left.\begin{aligned} y_h(t) &= Ce^{st} \\ y_h'(t) &= Cse^{st} \\ y_h''(t) &= Cs^2e^{st} \end{aligned}\right\} \quad \begin{aligned} &Cs^2e^{st} + a_1 Cse^{st} + a_2 Ce^{st} = 0 \\ &Ce^{st}\underbrace{(s^2 + a_1 s + a_2)}_{s_{1,2}} = 0 \end{aligned}$$

→ za kompleksne $s_{1,2}$ → $y_h(t) = \frac{C}{2} e^{j\theta} e^{s_1 t} + \frac{C}{2} e^{-j\theta} e^{s_2 t} = Ce^{-t}\cos(at + \theta)$

⇒ određivanje poč. uvjeta: (1) integriramo $\int_{0^-}^{+}$ toliko puta koliki je red jednadžbe

→ signal kauzalan: $u(0^-) = u'(0^-) = u''(0^-) = 0$

$y(0^+) - y(0^-) = b_0 u(0^+)$

→ za svevremenske pobude (npr. sinus) – odziv poslije $t=0$ je partikularno rješenje jer je homogeno iščezlo u $t=-\infty$ → to nam treba za $y_P(0^-)$

→ stabilnost: $s=\alpha+j\beta$ → $\alpha<0$ → $\lim_{t\to\infty} y_0(t)=0$ – stabilan

$\alpha=0$ → – marginalno stabilan

$\alpha>0$ → $\lim_{t\to\infty} y_0(t)=\infty$ – nestabilan

IMPULSNI ODZIV

$u(t)=\delta(t)$

→ homogena → $s_{1,2}$ → homogeno rješenje $h_a(t)=C_1e^{s_1t}+C_2e^{s_2t}$

→ poč. uvjeti uvjet $\boxed{\begin{matrix} h_a(0^+)=0 \\ h_a'(0^+)=1 \end{matrix}}$ → ubacimo u $h_a(t)$ i $h_a'(t)$ → C_1, C_2

dodajemo kad je broj derivacija izlaza na $h(t)$ jednak broju der. ulaza

→ $h(t)=h_a''(t)+h_a'(t)+h_a(t)+b_0\delta(t)$ (ako je desna strana jednadžbe $=u''(t)+u'(t)+u(t)$)

→ deriviramo što treba i dobijemo $h(t)$ = impulsni odziv

$$\boxed{y(t)=u(t)*h(t)}$$

POBUDA EKSPONENCIJALOM

$u(t)=e^{st}$

$$y(t)=u(t)*h(t)=e^{st}*h(t)=\int_{-\infty}^{\infty}h(\tau)\underbrace{e^{s(t-\tau)}}_{e^{st}\cdot e^{-s\tau}}d\tau=e^{st}\underbrace{\int_{-\infty}^{\infty}h(\tau)e^{-s\tau}d\tau}_{H(s)}$$

$y(t)=H(s)e^{st}$

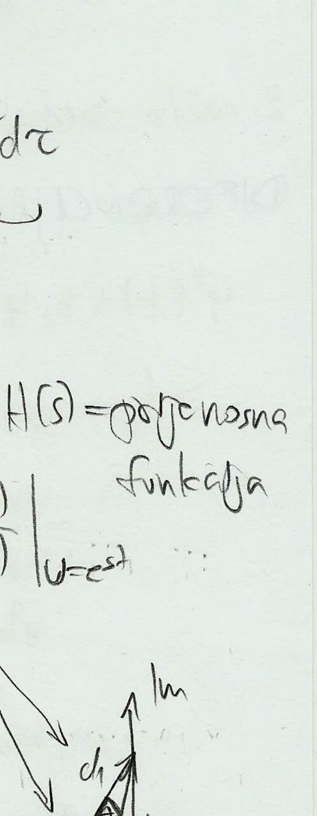
$$\left.\begin{matrix} u(t)=Ue^{st} \\ y(t)=Ye^{st} \end{matrix}\right\} Y=\underbrace{\frac{b_0s^N+b_1s^{N-1}+\dots+b_N}{s^N+a_1s^{N-1}+\dots+a_N}}_{H(s)}\cdot U$$

$\boxed{Y=H(s)U}$ → $H(s)$ = prijenosna funkcija

$$H(s)=\left.\frac{y_p(t)}{u(t)}\right|_{u=e^{st}}$$

$H(s)=\frac{\boxed{}}{\boxed{}}$ → nule, → polovi

$$|H(j\omega)|=\frac{d_1}{d_2}$$



- $s=\sigma+j\omega$ → vekt. zbroj frekv. pobude i pola → $H(s)$
- $|H(j\omega)|$ → ampl.-frekv. karakteristika

$\measuredangle H(j\omega)$ → fazno.-frekv. — " — → za $\boxed{s=j\omega}$

→ $\measuredangle=\text{arctg}\frac{Im}{Re}$

$$H(j\omega)=\frac{1}{(j\omega)^2+2j\omega+2}=\dots=\frac{1}{|j\omega-(-1+j)|e^{j\measuredangle(-1+j)}\cdot|j\omega-(-1-j)|e^{j\measuredangle(-1-j)}}=\frac{1}{d_1d_2e^{j\theta_1}e^{j\theta_2}}$$

POČ. UVJETI KOD DIFERENCIJALNIH

$\rightarrow$ ako imamo $y(0^-)$ i $y'(0^-)$ a trebamo za 0^+

1. odredimo $-$ a_1, a_2, b_0, b_1, b_2
2. $a_0 \Delta y = b_0 \Delta u$

$$y(0^+) - y(0^-) = b_0\left(u(0^+) - u(0^-)\right) \Rightarrow y(0^+)$$

$$a_0 \Delta y' + a_1 \Delta y = b_0 \Delta u' + b_1 \Delta u$$

$$y'(0^+) - y'(0^-) + a_1\left(y(0^+) - y(0^-)\right) = \underbrace{0}_{\text{ako su npr } b_0 \text{ i } b_1 = 0} \Rightarrow y'(0^+)$$

3. nakon toga možemo odrediti C_1 i C_2
4. za mirni odziv – PONOVO odrediti poč. uvjete – za $y(0^-) = y'(0^-) = 0$

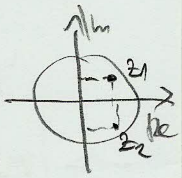
EKSP. POBUDA ZA DISKRETNE

– primjer: $y(n) - y(n-1) + \frac{1}{2} y(n-2) = u(n)$

$$Y\left(1 \cdot z^0 - 1 \cdot z^{-1} + \frac{1}{2} z^{-2}\right) = 1 \cdot z^0 U$$

$$H(z) = \frac{Y}{U} = \frac{1 \cdot z^0}{1 \cdot z^0 - 1 \cdot z^{-1} + \frac{1}{2} z^{-2}} \Big/ \cdot z^2 = \frac{z^2}{z^2 - z + \frac{1}{2}}$$

$\rightarrow z_{01} = z_{02} = 0$



$z^2 - z + \frac{1}{2} = 0$

$\rightarrow z_{1,2} = \frac{1}{2} \pm \frac{j}{2} \rightarrow$ stabilan sustav

$$H(z) = \frac{(z-0)^2}{\left(z - \left(\frac{1}{2} + \frac{j}{2}\right)\right)\left(z - \left(\frac{1}{2} - \frac{j}{2}\right)\right)}$$

$$H(e^{j\Omega}) = \frac{(e^{j\Omega})^2}{(e^{j\Omega})^2 - e^{j\Omega} + \frac{1}{2}} = \frac{\cos(2\Omega) + j\sin(2\Omega)}{\cos(2\Omega) + j\sin(2\Omega) - \cos(\Omega) - j\sin(\Omega) + \frac{1}{2}}$$

$\rightarrow$ modul $= 1$

$$\boxed{|H(e^{j\Omega})| = \frac{1}{\sqrt{Re^2 + Im^2}}}$$

$$\boxed{\measuredangle H(e^{j\Omega}) = \text{arctg}\frac{Im(\text{brojnik})}{Re(\text{brojnik})} - \text{arctg}\frac{Im(\text{naz.})}{Re(\text{naz.})}}$$

$\rightarrow$ pobuda: $u(n) = 2\cos\left(\frac{\pi}{6} n\right)\mu(n)$ $\rightarrow y_p(n) = 2\left|H(e^{j\frac{\pi}{6}})\right| \cos\left(\frac{\pi}{6} n + \measuredangle H(e^{j\frac{\pi}{6}})\right)$

Z-TRANSFORMACIJA

$$H(z)=\sum_{n=-\infty}^{+\infty} h(n) z^{-n}$$

$$h(n)=\{\underline{1},2,3,3,2,1\} \rightarrow H(z)=1\cdot z^{0}+2\cdot z^{-1}+3\cdot z^{-2}+3\cdot z^{-3}+2\cdot z^{-4}+z^{-5}=$$

$$=\frac{z^5+2z^4+3z^3+3z^2+2z+1}{z^5}$$

- primjer:

$$\boxed{x(n)=\alpha^{n}\mu(n)}$$

$$X(z)=\sum_{n=-\infty}^{+\infty}\alpha^{n}\mu(n)z^{-n}=\sum_{n=0}^{\infty}\alpha^{n}z^{-n}=\sum_{n=0}^{\infty}\left(\frac{\alpha}{z}\right)^{n}=\frac{1}{1-\frac{\alpha}{z}}=\boxed{\frac{z}{z-\alpha} \quad za\ |z|>|\alpha|}$$

$$\left|\frac{\alpha}{z}\right|<1 \rightarrow |\alpha|<|z|$$

$$|z|>|\alpha|$$

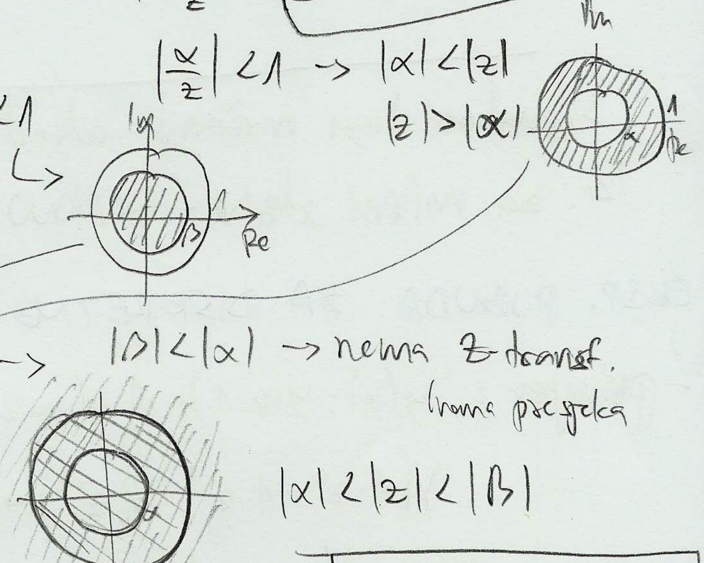
- za: $\boxed{x(n)=-\beta^{n}\mu(-n-1)}$ ○—● $\boxed{X(z)=\frac{z}{z-\beta} \quad |z|<|\alpha|}$, $\left|\frac{z}{\alpha}\right|<1$

- za: $x(n)=\alpha^{n}\mu(n)+\beta^{n}\mu(-n-1)$

$$X(z)=\frac{z}{z-\alpha}-\frac{z}{z-\beta} \rightarrow \text{presjek} \rightarrow |\beta|<|\alpha| \rightarrow \text{nema z-transf.}$$

(nema presjeka)

$$|\alpha|<|z|<|\beta|$$



→ primjena: diferencijske (pomak!)

$$Y(z)=\underbrace{H(z)U(z)}_{\text{mirni}}+\frac{\text{ostatak u vrem. domeni}}{\text{z-ovi}} \rightarrow \boxed{\text{nepobuđeni}}$$

$$\boxed{H(z) \rightarrow \text{poč. uvjeti}=0}$$

→ Inverzna: $Y_1(z)=\frac{Y(z)}{z}$ → parc. razlomci $\left(\frac{A}{\text{nešto}}+\frac{B}{\text{nešto}}+\frac{C}{\text{nešto}}\right.$, zajednički nazivnik, $A+B+C=$ koeficijenti od polje $\rightarrow A,B,C)$

$$y(z)=Y_1(z)\cdot z$$

inverzna transf. → $y(n)$

→ OPREZ! $\boxed{H(z)=\frac{Y(z)}{U(z)}=\frac{\text{ono uz } U}{\text{ono uz } Y}}$

- Impulsni odziv u vremenu je prijenosna funkcija u z-domeni
  - mirni sustav - ono uz $U(z)$, nepobuđeni - odziv
  - konvolucija u vrem. domeni odgovara množenju u z-domeni

L-TRANSFORMACIJA (LAPLACEOVA)

- frekv. pomak: $x(t)e^{s_0 t}$ ○—● $X(s-s_0)$

- derivacija:

$$\boxed{\begin{aligned} x'(t) &\circ\!\!-\!\!\bullet\ sX(s)-x(0^-) \\ x''(t) &\circ\!\!-\!\!\bullet\ s^2X(s)-sx(0^-)-x'(0^-)\end{aligned}}$$

→ partikularna:

$u(t)$	$y_p(t)$
r^t	Kr^t
r^t, za $r=s$	Ktr^t
$A\cos(\omega_0 t)$	$K_1\cos(\omega_0 t) + K_2\sin(\omega_0 t)$
$A\sin(\omega_0 t)$	— " —
A	K

ovaj dio je dodatak, kao reki moji tutorijalčići

diferencijalne! tutorijal

- homogeno ($=$prirodni): nema pobude } → za svevrem. pobude → izdikralo ($u \to -\infty$) za $t<0$ pa ga nema tamo
- partikularno ($=$prisilni): pretpostavimo } totalno
- nepobuđeni: homogeno rješenje
- mirni: totalno, ali poč. uvjeti $=0$ (za 0^- vrijede, pa nam treba 0^+)
- postupak:
  1. napiši si $a_0, a_1, \dots, b_0, b_1, \dots$
  2. s-ovi → kvadratna → homogeno sa C-ovima
  3. partikularno pretp.
  4. ↳ to deriviramo, i pobudu, ubacimo u početnu i dobijemo K
  6. y_p W
  7. početni uvjeti
  8. $y_{tot}(0^+)$ i $y'_{tot}(0^+)$ → dobijemo C-ove → pazi, ako ti je npr pobuda e^{-5t} a uz jedan C je isto to, onda je partikularno Kte^{-5t} – vidi tablicu gore
  9. y_{tot} W

- mirni: 1. iz $y(0^-) = y'(0^-) = 0 \to y(0^+), y'(0^+)$
  2. dobijemo C-ove
  3. y_{tot} W
- nepobuđeni 1. uvjeti $0^+ = 0^-$
  2. ubacimo uvjete u $y_h(t)$ i $y_h'(t)$
  3. dobijemo C-ove
  4. $y_h = y_h$ W

- diferencijske ; npr. $y(n) - \frac{1}{6}y(n-1) = 2u(n)$

diferencijske tutorial

1. pardlikamo s K
2. ubacit u početnu, dobit K
3. $y_p(n)$ W – prisilni
4. $q - \frac{1}{6} = 0 \rightarrow q = \frac{1}{6} \rightarrow y(n) = Cq^n + y_p(n)$

→ ako imamo $y(n-2)$ → onda ide kvadratna

npr. $y(n) - \frac{1}{6}y(n-1) + 2y(n-2) = 2u(n)$

ako $q_1 = q_2$; $y(n) = C_1 q^n + C_2 n q^n$

$q^2 - \frac{1}{6}q + 2 = 0 \rightarrow q_{1,2} \rightarrow y(n) = C_1 q_1^n + C_2 q_2^n + y_p(n)$

5. početni uvjeti → $y(0)$ i $y(1)$ → iz početne → ubacimo u odziv → C-ovi
6. $y_h(n)$ W – prirodni
7. gotovo

→ mirni i nepobuđeni → kao kod diferencijalnih

- impulsni odziv – diskretni – z-domena ;

$H(z) \circ\!\!-\!\!\circ\, h(n)$

1. $H(z)$ izračunamo pa podijelimo sa z → $\frac{H(z)}{z} = \frac{A}{\sim} + \frac{B}{\sim}$ → parcijalni → riješimo
2. $\frac{H(z)}{z}$ riješeni pomnožimo sa z → $H(z)$ → vratimo u vremensku domenu

- impulsni odziv – kont. – vremenska

1. homogena $= h_a(t)$
2. $h_a(t), h_a'(t), h_a''(t) \ldots$ LTI sustav → $h_a(0^+) = 0$, $h_a'(0^+) = 1$, $h_a''(0^+) = 0$
3. dobijemo C-ove (ubacimo)
4. $h(t) = h_a(t)$ → ako je $u(t)$

$= h_a'(t) + h_a(t)$ → ako je $u'(t) + u(t)$ itd...

$= h_a'(t) + h_a(t) + b_0\delta$ → kad je broj derivacija isti s obje strane

$\rightarrow y(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau$

$y(n) = h(n) * u(n) = \sum_{m=0}^{n} h(m)u(n-m)$