Signali i sustavi

Završni ispit (grupa A) – 1. srpnja 2009.

1. Zadan je kontinuirani LTI sustav opisan jednadžbom $y''(t) + 3y'(t) + 2y(t) = 3u(t)$.

a) Odredite prijenosnu funkciju sustava.

b) Izračunajte inverznu Laplaceovu transformaciju prijenosne funkcije! Što ona predstavlja?

c) Odredite polove i nule sustava te ispitajte stabilnost.

d) Odredite frekvencijsku karakteristiku te pomoću nje izračunajte odziv sustava u stacionarnom stanju na pobudu $u(t) = \sin(3t)$.

2. Kontinuirani kauzalni LTI sustav opisan jednadžbom $y'(t) + 6y(t) = u'(t) + 3u(t)$ pobuđen je signalom $u(t) = 2\,\mu(t)$. Poznat je početni uvjet $y(0^-) = 3$.

a) Odredite početni uvjet $y(0^+)$ u trenutku $t = 0^+$.

b) Izračunajte odziv sustava na zadanu pobudu za $t > 0$.

3. Zadan je diskretni kauzalni LTI sustav opisan jednadžbom $y(n) + y(n-2) = 2u(n)$.

a) Postupkom u vremenskoj domeni odredite impulsni odziv sustava.

b) Odredite prijenosnu funkciju sustava.

c) Odredite polove i nule sustava te ispitajte stabilnost.

d) Što je rezonancija? Odredite odziv mirnog sustava na pobudu $u(n) = \cos(\frac{n\pi}{2})\,\mu(n)$ (bilo kojim postupkom)!

4. Vremenski DISKRETNI sustav s više ulaza i izlaza (MIMO) opisan je matricama

$$\mathbf{A} = \begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{i} \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

a) Odredite prijenosnu matricu sustava.

b) Odredite matricu impulsnog odziva sustava.

5. Zadan je signal $x(t) = 4\cos(3t) + 6\sin(6t)$.

a) Izračunajte i skicirajte amplitudni i fazni spektar zadanog signala. Koju Fourierovu transformaciju ste koristili i zašto?

b) Koja je najmanja frekvencija otipkavanja potrebna da ne dođe do preklapanja spektra (eng. *aliasing*)?

c) Odredite otipkani signal za frekvenciju otipkavanja $f_s = \frac{12}{\pi}$.

d) Izračunajte i skicirajte amplitudni i fazni spektar dobivenog otipkanog signala. Koju Fourierovu transformaciju ste koristili i zašto?

e) Izračunajte snage oba signala.

$\frac{2\pi}{\frac{2\pi}{3}} = 3$

ZAVRŠNI ISPIT 2008./2009. GRUPA (A)

(1.) $y''(t) + 3y'(t) + 2y(t) = 3u(t)$

(a) $y(t) \circ\!\!-\!\!\bullet\ Y(s) \quad y'(t) \circ\!\!-\!\!\bullet\ sY(s) \quad y''(t) \circ\!\!-\!\!\bullet\ s^2Y(s)$

$u(t) \circ\!\!-\!\!\bullet\ U(s)$

$s^2Y(s) + 3sY(s) + 2Y(s) = 3U(s)$

$Y(s)(s^2 + 3s + 2) = 3U(s)$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{3}{s^2+3s+2}$$

(b) $H(s) = \dfrac{3}{(s+1)(s+2)} = \dfrac{3}{s+1} + \dfrac{-3}{s+2}$

$\bullet\!\!-\!\!\circ\ h(t) = (3e^{-t} - 3e^{-2t})\mu(t)$ – IMPULSNI ODZIV

(c) POLOVI: $s_{p1} = -1$ i $s_{p2} = -2$ → STABILAN SUSTAV

NULE: NEMA!

(d) $H(j\omega) = H(s)\Big|_{s=j\omega} \rightarrow H(j\omega) = \dfrac{3}{2-\omega^2+3j\omega}$

$$|H(j\omega)| = \frac{3}{\sqrt{(2-\omega^2)^2 + (3\omega)^2}}$$

$$\measuredangle H(j\omega) = -\mathrm{arctg}\frac{3\omega}{2-\omega^2}$$

$u(t) = 1\cdot\sin(3t) \rightarrow y_p(t) = K\sin(3t+\varphi)$

$K = 1\cdot|H(j\omega)|\Big|_{\omega=3} = \dfrac{3}{\sqrt{130}} \qquad \varphi = 0^\circ + \measuredangle H(j\omega)\Big|_{\omega=3} = -127.87^\circ$

$$y_p(t) = \frac{3}{\sqrt{130}}\sin(3t - 127.87^\circ)\mu(t)$$

(2) $y'(t) + 6y(t) = u'(t) + 3u(t)$

$u(t) = 2\mu(t) \quad y(0^-) = 3$

(a) $y'(t) + a_1 y(t) = b_0 u'(t) + b_1 u(t)$

$a_1 = 6 \quad b_0 = 1 \quad b_1 = 3$

$y(0^+) - y(0^-) = b_0 u(0^+) \rightarrow y(0^+) = 2 + 3 = 5$

(b) $y(t) \circ\!\!-\!\!\bullet\ Y(s) \qquad y'(t) \circ\!\!-\!\!\bullet\ sY(s) - y(0^-)$

$u(t) \circ\!\!-\!\!\bullet\ U(s) = \frac{2}{s}$

$sY(s) - 3 + 6Y(s) = sU(s) + 3U(s)$

$Y(s)(s+6) = U(s)(s+3) + 3$

$Y(s) = U(s)\frac{s+3}{s+6} + \frac{3}{s+6}$

$Y(s) = \frac{2s+6+3s}{s(s+6)} = \frac{5s+6}{s(s+6)} = \frac{1}{s} + \frac{4}{s+6}$

$\bullet\!\!-\!\!\circ\ y(t) = (1 + 4e^{-6t})\mu(t)$

3. $y(n) + y(n-2) = 2u(n)$

(a) $y_h(n) = Cq^n$

$$Cq^{n-2}(q^2+1) = 0 \qquad q_{1,2} = \pm j = 1e^{\pm j\frac{\pi}{2}}$$

$$y_h(n) = A\cos(n\tfrac{\pi}{2}) + B\sin(n\tfrac{\pi}{2})$$

$$h(n) = y_h(n) \qquad y(-2) = y(-1) = 0 \qquad u(n) = \delta(n)$$

$$y(0) = 2u(0) - y(-2) = 2$$
$$y(1) = 2u(1) - y(-1) = 0$$

$$\left.\begin{array}{l} h(0) = A = 2 \\ h(1) = B = 0 \end{array}\right\} h(n) = 2\cos(n\tfrac{\pi}{2})\,\mu(n)$$

(b) $y(n) \multimap Y(z) \qquad y(n-2) \multimap z^{-2}Y(z)$
$u(n) \multimap U(z)$

$$Y(z) + z^{-2}Y(z) = 2U(z)$$

$$Y(z)(1+z^{-2}) = 2U(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{2}{1+z^{-2}} = \frac{2z^2}{z^2+1}$$

(c) POLOVI: $z_{P1} = j \qquad z_{P2} = -j \rightarrow$ MARGINALNO STABILAN

NULE: $z_{N1,2} = 0$

(d) REZONANCIJA - POJAVA KADA FREKV. POBUDE POGODI KARAKT. FREKV. SUSTAVA

$$u(n) = \cos(n\pi/2)\mu(n)$$

$$y_p(n) = \left(K_1\cos(n\pi/2) + K_2\sin(n\pi/2)\right)n$$

$$y_p(n) + y_p(n-2) = 2u(n)$$

$$K_1 n\cos(n\pi/2) + K_2 n\sin(n\pi/2) - K_1(n-2)\cos(n\pi/2) - K_2(n-2)\sin(n\pi/2) = 2\cos(n\pi/2)$$

$$2K_1 n\cos(n\pi/2) + 2K_2\sin(n\pi/2) = 2\cos(n\pi/2)$$

$$K_1 = 1 \quad K_2 = 0 \rightarrow y_p(n) = n\cos(n\pi/2)$$

$$y_h(n) = A\cos(n\pi/2) + B\sin(n\pi/2)$$

$$y_m(n) = A\cos(n\pi/2) + B\sin(n\pi/2) + n\cos(n\pi/2)$$

$$y(-2) = y(-1) = 0$$

$$y(0) = 2u(0) - y(-2) = 2$$

$$y(1) = 2u(1) - y(-1) = 0$$

$$\left.\begin{array}{l} y_m(0) = A = 2 \\ y_m(1) = B = 0 \end{array}\right\} y_m(n) = (2+n)\cos(n\pi/2)\mu(n)$$

(4.)

$$A = \begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix} \qquad B = C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(a) $$H(z) = C(zI-A)^{-1}B + D = (zI-A)^{-1}$$

$$(zI-A) = \begin{bmatrix} z & 4 \\ 4 & z \end{bmatrix}$$

$$H(z) = (zI-A)^{-1} = \frac{1}{(z-4)(z+4)}\begin{bmatrix} z & -4 \\ -4 & z \end{bmatrix} = \begin{bmatrix} \frac{z}{(z-4)(z+4)} & \frac{-4}{(z-4)(z+4)} \\ \frac{z}{(z-4)(z+4)} & \frac{-4}{(z-4)(z+4)} \end{bmatrix}$$

(b) $$A(z) = \frac{z}{(z-4)(z+4)}, \quad A_1(z) = \frac{A(z)}{z} = \frac{1}{(z-4)(z+4)} = \frac{\frac{1}{8}}{z-4} + \frac{-\frac{1}{8}}{z+4}$$

$$A(z) = \frac{1}{8}\frac{z}{z-4} - \frac{1}{8}\frac{z}{z+4} \circ\!\!-\!\!\bullet \left(\frac{1}{8}4^n - \frac{1}{8}(-4)^n\right)\mu(n)$$

$$B(z) = \frac{-4}{(z-4)(z+4)}, \quad B_1(z) = \frac{B(z)}{z} = \frac{-4}{z(z-4)(z+4)}$$

$$B_1(z) = \frac{\frac{1}{4}}{z} + \frac{-\frac{1}{8}}{z-4} + \frac{-\frac{1}{8}}{z+4}$$

$$B(z) = \frac{1}{4} - \frac{1}{8}\frac{z}{z-4} - \frac{1}{8}\frac{z}{z+4} \circ\!\!-\!\!\bullet \frac{1}{4}\delta(n) - \left(\frac{1}{8}4^n + \frac{1}{8}(-4)^n\right)\mu(n)$$

$$h(n) = \begin{bmatrix} \frac{1}{8}\left(4^n - (-4)^n\right)\mu(n) & \frac{1}{4}\delta(n) - \frac{1}{8}\left(4^n + (-4)^n\right)\mu(n) \\ \frac{1}{4}\delta(n) - \frac{1}{8}\left(4^n + (-4)^n\right)\mu(n) & \frac{1}{8}\left(4^n - (-4)^n\right)\mu(n) \end{bmatrix}$$

(5.) $x(t) = 4\cos(3t) + 6\sin(6t) = 4\cos(3t) + 6\cos(6t - \frac{\pi}{2})$

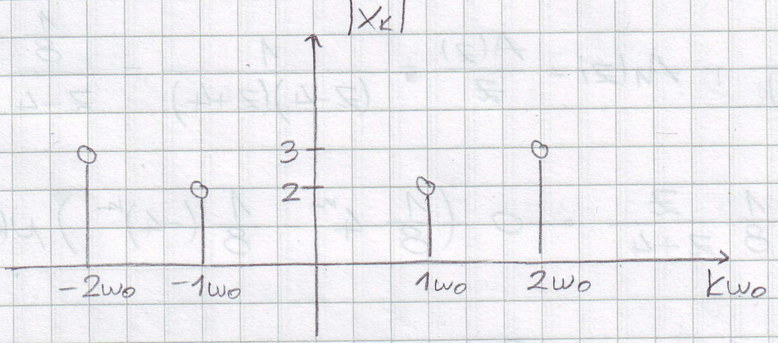
(a) $\omega_1 = 3,\ \omega_2 = 6 \rightarrow \omega_0 = 3$

$$x(t) = 4 \cdot \frac{1}{2}\left(e^{j3t} + e^{-j3t}\right) + 6 \cdot \frac{1}{2}\left(e^{j(6t-\frac{\pi}{2})} + e^{-j(6t-\frac{\pi}{2})}\right)$$

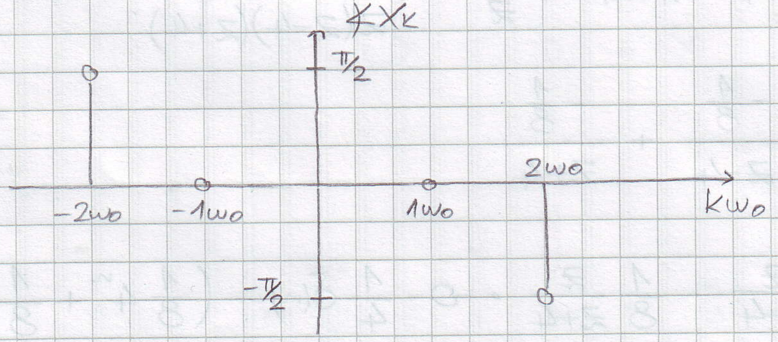
$$x(t) = 2e^{j3t} + 2e^{-j3t} + 3e^{-j\frac{\pi}{2}}e^{j6t} + 3e^{j\frac{\pi}{2}}e^{-j6t}$$

$$x(t) = X_1 e^{j1\omega_0} + X_{-1}e^{j(-1)\omega_0} + X_2 e^{j2\omega_0} + X_{-2}e^{j(-2)\omega_0}$$

$$X_1 = 2 \quad X_{-1} = 2 \quad X_2 = 3e^{j(-\frac{\pi}{2})} \quad X_{-2} = 3e^{j\frac{\pi}{2}}$$



CTFS – ZA PERIODIČNE, KONTINUIRANE SIGNALE



(b) $\Omega_S \geq 2\,\Omega_{max}$ $\quad \Omega_S \geq 12\,s^{-1} \rightarrow 2\pi f_S \geq 12\,s^{-1}$

$$f_S \geq \frac{\pi}{6}\ Hz$$

(c) $f_S = \frac{12}{\pi} \rightarrow \omega_S = 24 s^{-1}$

$$x(t) = A\cos(\omega_0 t + \phi)$$

$$x(nT_S) = A\cos(\omega_0 T_S n + \phi)$$

$$x(nT_S) = A\cos\left(2\pi \frac{\omega_0}{\omega_S} n + \phi\right)$$

$$x(nT_S) = 4\cos\left(\frac{\pi}{4}n\right) + 6\cos\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)$$

(d) $x(nT_S) = 4 \cdot \frac{1}{2}\left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right)$

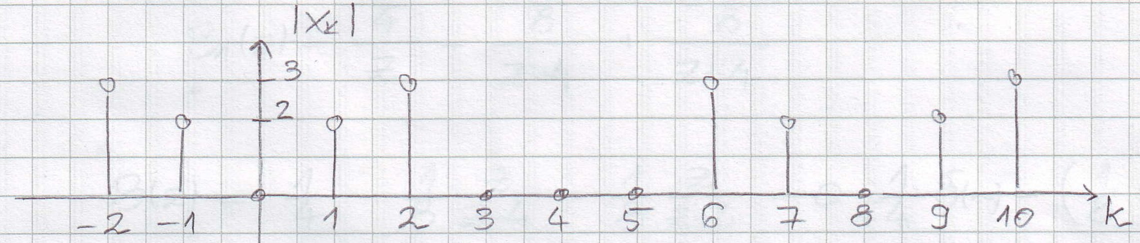
$$+ 6 \cdot \frac{1}{2}\left(e^{j\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)} + e^{-j\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)}\right)$$

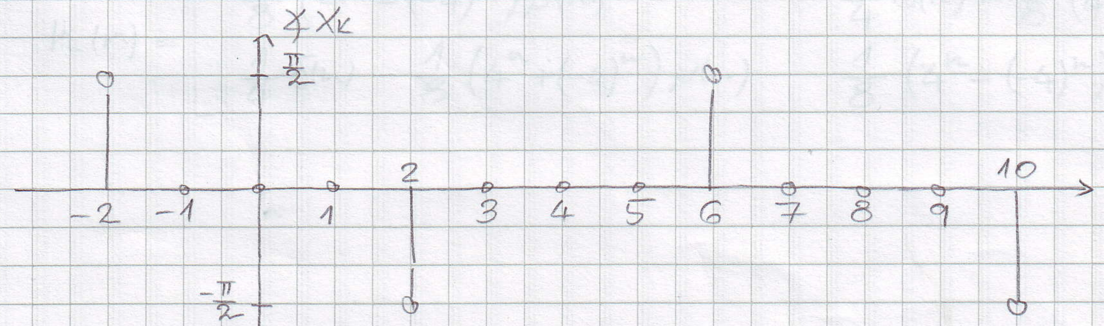
$$x(nT_S) = 2e^{j\frac{\pi}{4}n} + 2e^{-j\frac{\pi}{4}n} + 3e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}n} + 3e^{j\frac{\pi}{2}} e^{-j\frac{\pi}{2}n}$$

$$\omega_1 = \frac{\pi}{4} \quad \omega_2 = \frac{\pi}{2} \rightarrow \omega_0 = \frac{\pi}{4}$$

$$x(n) = X_1 e^{j1\omega_0 n} + X_{-1} e^{j(-1)\omega_0 n} + X_2 e^{j2\omega_0 n} + X_{-2} e^{j(-2)\omega_0 n}$$

$$X_1 = 2 \quad X_{-1} = 2 \quad X_2 = 3e^{-j\frac{\pi}{2}} \quad X_{-2} = 3e^{j\frac{\pi}{2}}$$





(d) $P = \sum_{k=-\infty}^{\infty} |X_k|^2$ – KONTINUIRANI

$$P_{KONT.} = 3^2 + 2^2 + 2^2 + 3^2 = 26$$

$P = \sum_{k=0}^{N-1} |X_k|^2$ – DISKRETNI

$$P_{DISKT.} = 2^2 + 3^2 + 3^2 + 2^2 = 26$$

$$P_{KONT.} = P_{DISKT.} \; !$$