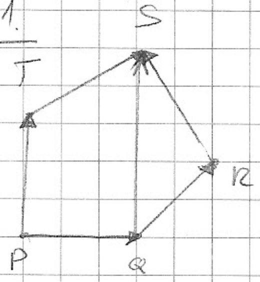
3.10. Zadatci za vježbu

Zad 1.



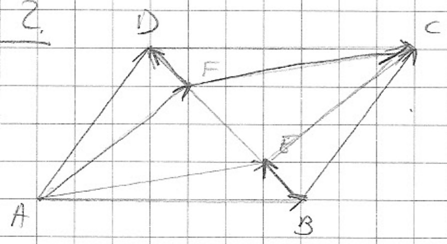
$\overrightarrow{QR}+\overrightarrow{RS}=\overrightarrow{PT}$ T: $\overrightarrow{PQ}=\overrightarrow{TS}$

$\overrightarrow{QR}+\overrightarrow{RS}=\overrightarrow{QS}=\overrightarrow{PT}$

$-\overrightarrow{PQ}+\cancel{\overrightarrow{PT}}+\overrightarrow{TS}=\overrightarrow{QS}=\cancel{\overrightarrow{PT}}$

$\overrightarrow{TS}=\overrightarrow{PQ}$

Zad 2.



$\overrightarrow{AB}=\overrightarrow{DC}$

$\overrightarrow{BC}=\overrightarrow{AD}$

$\overrightarrow{BE}=\overrightarrow{FD}$

$\overrightarrow{AB}+\overrightarrow{BE}=\overrightarrow{AE}$

$\overrightarrow{FD}+\overrightarrow{DC}=\overrightarrow{FC}$

$\overrightarrow{BE}+\overrightarrow{AB}=\overrightarrow{FC}$

$\overrightarrow{AE}=\overrightarrow{FC}$

$\overrightarrow{AF}+\overrightarrow{FD}=\overrightarrow{AD}$

$\overrightarrow{AF}=\overrightarrow{AD}-\overrightarrow{FD}$

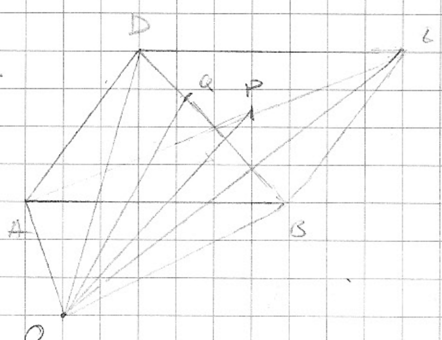
$\overrightarrow{BE}+\overrightarrow{EC}=\overrightarrow{BC}$

$\overrightarrow{EC}=\overrightarrow{BC}-\overrightarrow{BE}$

$\overrightarrow{EC}=\overrightarrow{AD}-\overrightarrow{FD}$

$\overrightarrow{EC}=\overrightarrow{AF}$

Zad 3



$\overrightarrow{AB}=\overrightarrow{DC}$ P – polovište $\overline{AC}$

$\overrightarrow{BC}=\overrightarrow{AD}$ Q – polovište $\overline{BD}$

$\overrightarrow{OQ}=\frac{1}{2}(\overrightarrow{OB}+\overrightarrow{OD})$ $\overrightarrow{OP}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OC})$

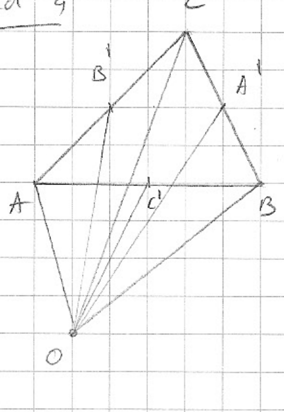
$\overrightarrow{OA}+\overrightarrow{AD}=\overrightarrow{OD}$ $\overrightarrow{OB}+\overrightarrow{BC}=\overrightarrow{OC}$

$\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$

$\overrightarrow{OQ}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{OA}+\overrightarrow{AD})$ $\overrightarrow{OP}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{BC})$

$\overrightarrow{OQ}=\frac{1}{2}(2\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC})=\overrightarrow{OP}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC})$

Zad 4



$\overrightarrow{OA'}=\frac{1}{2}(\overrightarrow{OB}+\overrightarrow{OC})$

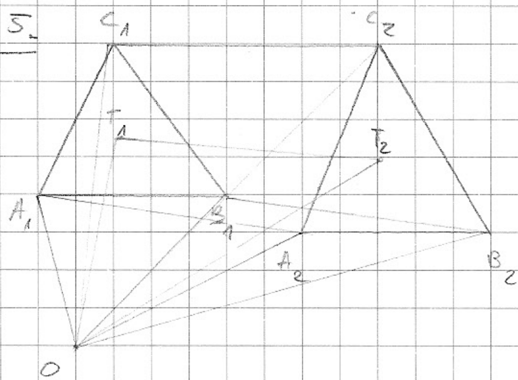
$\overrightarrow{OB'}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OC})$

$\overrightarrow{OC'}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OB})$ / +

$\overrightarrow{OA'}+\overrightarrow{OB'}+\overrightarrow{OC'}=\frac{1}{2}(2\overrightarrow{OB}+2\overrightarrow{OC}+2\overrightarrow{OA})$

$=\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$

Zad 5.



$\vec{v}_{T_1} = \frac{1}{3}(\vec{v}_{A_1} + \vec{v}_{B_1} + \vec{v}_{C_1})$

$\vec{v}_{T_2} = \frac{1}{3}(\vec{v}_{A_2} + \vec{v}_{B_2} + \vec{v}_{C_2})$

$\overrightarrow{A_1A_2} = \vec{v}_{A_2} - \vec{v}_{A_1}$

$\overrightarrow{B_1B_2} = \vec{v}_{B_2} - \vec{v}_{B_1}$

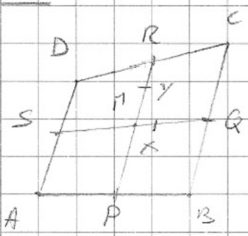
$\overrightarrow{C_1C_2} = \vec{v}_{C_2} - \vec{v}_{C_1}$

+

$\vec{v}_{A_2} + \vec{v}_{B_2} + \vec{v}_{C_2} - (\vec{v}_{A_1} + \vec{v}_{B_1} + \vec{v}_{C_1})$

$= 3 \cdot \vec{v}_{T_2} - 3\vec{v}_{T_1} = 3(\vec{v}_{T_2} - \vec{v}_{T_1}) = 3\overrightarrow{T_1T_2}$

Zad 6.



1) formula za sjecište

$\vec{v}_p = \frac{1}{2}(\vec{v}_a + \vec{v}_b)$ x - polovište $\overline{SQ}$ $\vec{v}_x = \frac{1}{2}(\vec{v}_s + \vec{v}_q)$

$\vec{v}_q = \frac{1}{2}(\vec{v}_b + \vec{v}_c)$ y - polovište $\overline{PR}$ $\vec{v}_y = \frac{1}{2}(\vec{v}_p + \vec{v}_r)$

$\vec{v}_r = \frac{1}{2}(\vec{v}_c + \vec{v}_d)$ $= \frac{1}{2}\left(\frac{1}{2}\vec{v}_a + \frac{1}{2}\vec{v}_b + \frac{1}{2}\vec{v}_c + \frac{1}{2}\vec{v}_d\right)$

$\vec{v}_s = \frac{1}{2}(\vec{v}_d + \vec{v}_a)$ $= \frac{1}{2}(\vec{v}_s + \vec{v}_q) = \vec{v}_x$

2.) ostatak zadatka

$x \equiv y \equiv M \to \vec{v}_M = \frac{1}{2}(\vec{v}_p + \vec{v}_s)$

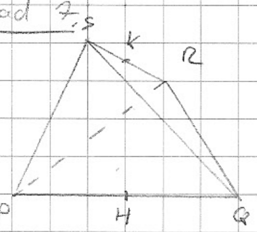
$= \boxed{\frac{1}{4}(\vec{v}_a + \vec{v}_b + \vec{v}_c + \vec{v}_d)}$

$\vec{v}_M = \frac{1}{4}(\vec{v}_a + \vec{v}_b - \vec{v}_c + \vec{v}_d)$ $\overrightarrow{AE} + \overrightarrow{BF} + \overrightarrow{CG} + \overrightarrow{DH}$

$\vec{v}_N = \frac{1}{4}(\vec{v}_e + \vec{v}_f + \vec{v}_g + \vec{v}_h)$ $= \vec{v}_e - \vec{v}_a + \vec{v}_f - \vec{v}_b + \vec{v}_g - \vec{v}_c + \vec{v}_h - \vec{v}_d$

$= 4\vec{v}_n - 4\vec{v}_m = 4 \cdot \overrightarrow{MN}$

Zad 7.



$\vec{v}_K = \frac{1}{2}(\vec{v}_r + \vec{v}_s)$

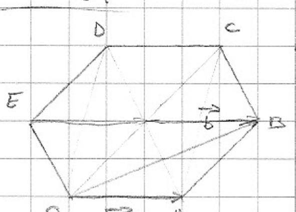
$\vec{v}_H = \frac{1}{2}(\vec{v}_p + \vec{v}_q)$

$\overrightarrow{PR} + \overrightarrow{QR} + \overrightarrow{PS} + \overrightarrow{QS} = \vec{v}_r - \vec{v}_p + \vec{v}_r - \vec{v}_q + \vec{v}_s - \vec{v}_p + \vec{v}_s - \vec{v}_q$

$= 2(\vec{v}_r + \vec{v}_s) - 2(\vec{v}_p + \vec{v}_q)$

$= 4(\vec{v}_K - \vec{v}_H) = 4\overrightarrow{HK}$

Zad 8.



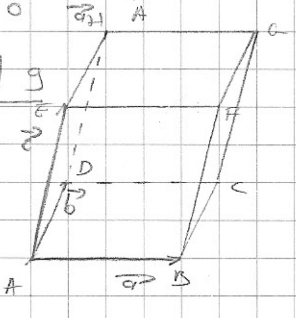
$\vec{a} + \overrightarrow{AB} = \vec{b}$ $\overrightarrow{OE} = \overrightarrow{BC} = \vec{b} - 2\vec{a}$ $\overrightarrow{OD} = \cancel{\vec{a}} + \vec{b} - \cancel{\vec{a}} + \vec{b} - 2\vec{a} - \vec{a}$

$= 2\vec{b} - 3\vec{a}$

$\overrightarrow{AB} = \vec{b} - \vec{a}$ $\overrightarrow{OC} = \vec{b} + \vec{b} - 2\vec{a}$

$= 2\vec{b} - 2\vec{a}$

Zad 9



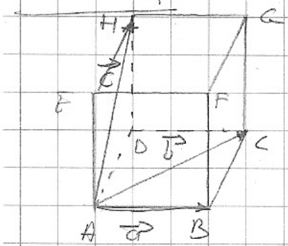
$\overrightarrow{AG} = \vec{a} + \vec{c} + \vec{b}$

$\overrightarrow{BH} = \vec{c} - \vec{a} + \vec{b} = \vec{b} + \vec{c} - \vec{a}$

$\overrightarrow{CE} = \vec{c} - \vec{a} - \vec{b}$ $\overrightarrow{EC} = \vec{a} + \vec{b} - \vec{c}$

$\overrightarrow{DF} = \vec{a} + \vec{c} - \vec{b}$

Zad 10.

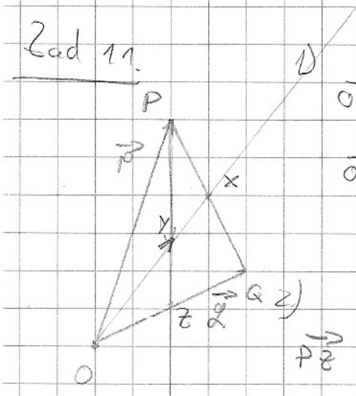


$\vec{AD} = \vec{b} - \vec{a}$

$\vec{AE} = \vec{c} - \vec{AD} = \vec{a} + \vec{c} - \vec{b}$

$\vec{CE} = \vec{AE} - \vec{b} = \vec{a} + \vec{c} - 2\vec{b}$

Zad 11.



$\vec{OY} = 2\vec{YX}$

$\vec{OX} = \frac{1}{2}(\vec{p} + \vec{q}) = \vec{OY} + \frac{1}{2}\vec{OY}$

$= \frac{3}{2}\vec{OY} \rightarrow \vec{OY} = \frac{1}{3}(\vec{p} + \vec{q})$

$\vec{p} + \vec{PY} = \vec{OY}$

$\vec{PY} = \vec{OY} - \vec{p}$

$= \frac{1}{3}\vec{q} - \frac{2}{3}\vec{p}$

$\vec{PZ} = k\,\vec{PY}$

$\vec{p} + \vec{PZ} = \vec{OZ} \rightarrow \vec{OZ} = \vec{p} + k\cdot\vec{PY} = \vec{p} + \frac{1}{3}k\vec{q} - \frac{2}{3}k\vec{p}$

$\vec{OZ} = (1 - \frac{2}{3}k)\vec{p} + \frac{1}{3}k\cdot\vec{q}$

$\alpha\cdot\vec{OZ} = \vec{q}$

$\vec{q} = \alpha(1 - \frac{2}{3}k)\cdot\vec{p} + \alpha\cdot\frac{1}{3}k\cdot\vec{q}$

$\vec{0} = \alpha(1 - \frac{2}{3}k)\cdot\vec{p} + (\alpha\cdot\frac{1}{3}k - 1)\vec{q}$

$\alpha\cdot\frac{1}{3}k - 1 = 0$

$\alpha\cdot\frac{1}{3}k = 1$

$\alpha = \frac{3}{k}$

$\alpha = \frac{3}{\frac{3}{2}} = 2$

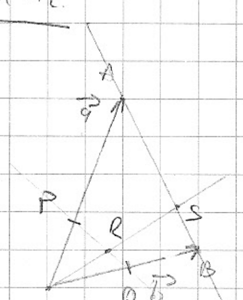
$\frac{3}{k}(1 - \frac{2}{3}k) = 0$

$\frac{3}{k} - 2 = 0$

$\frac{3}{k} = 2 \rightarrow k = \frac{3}{2}$

$\vec{OZ} = \frac{1}{2}\vec{q}$

Zad 12.



1.

$3\vec{OP} = \vec{OA} = \vec{a}$

$3\vec{OQ} = 2\vec{OB} = 2\vec{b}$

$2\vec{PR} = \vec{RQ}$

$\vec{AS} = k\cdot\vec{AB}$

$\vec{OS} = l\cdot\vec{OR}$

2.

$\vec{OP} = \frac{1}{3}\vec{a}$

$\vec{OQ} = \frac{2}{3}\vec{b}$

$\vec{PR} = \frac{1}{2}\vec{RQ}$

$\vec{AB} = \vec{b} - \vec{a}$

$\vec{SB} = \vec{AB} - \vec{AS}$

$= \vec{b} - \vec{a} - k(\vec{b} - \vec{a})$

$= (\vec{b} - \vec{a})(1 - k)$

3.

$\vec{OP} + \vec{PQ} = \vec{OQ}$

$\vec{PQ} = \vec{OQ} - \vec{OP} = \frac{2}{3}\vec{b} - \frac{1}{3}\vec{a}$

$\vec{PQ} = \vec{PR} + \vec{RQ} = \frac{1}{2}\vec{RQ} + \vec{RQ} = \frac{3}{2}\vec{RQ}$

$\frac{3}{2}\vec{RQ} = \frac{2}{3}\vec{b} - \frac{1}{3}\vec{a}$

$\vec{RQ} = \frac{4}{9}\vec{b} - \frac{2}{9}\vec{a}$

$\vec{PR} = \frac{2}{9}\vec{b} - \frac{1}{9}\vec{a}$

4.

$\vec{OR} + \vec{RQ} = \vec{OQ}$

$\underline{\vec{OR}} = \vec{OQ} - \vec{RQ}$

$= \frac{2}{3}\vec{b} - \frac{4}{9}\vec{b} + \frac{2}{9}\vec{a}$

$\vec{OR} = \frac{2}{9}\vec{b} + \frac{2}{9}\vec{a}$

5.

$\vec{a} + \vec{AS} = \vec{OS}$

$\vec{OS} + \vec{SB} = \vec{b}$

$\vec{a} + k(\vec{b} - \vec{a}) = l\cdot\frac{2}{9}(\vec{b} + \vec{a})$

$l\cdot\frac{2}{9}(\vec{b} + \vec{a}) + (1 - k)(\vec{b} - \vec{a}) = \vec{b}$

$\vec{OS} = \frac{9}{4}\cdot\frac{2}{9}(\vec{b} + \vec{a}) = \frac{1}{2}(\vec{b} + \vec{a})$

6.

$\vec{a}(1 - k - \frac{2}{9}l) + \vec{b}(k - \frac{2}{9}l) = \vec{0}$

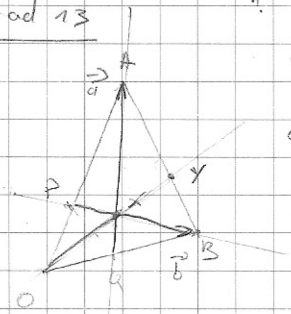
$\vec{a}(\frac{2}{9}l - (1 - k)) + \vec{b}(\frac{2}{9}l + 1 - k - 1) = \vec{0}$

$1 - k - \frac{2}{9}l = 0 \quad k - \frac{2}{9}l = 0$

$1 - \frac{4}{9}l = 0 \rightarrow \boxed{l = \frac{9}{4}}$

$\boxed{k = \frac{1}{2}}$

Zad 13



1.

$\vec{OP} = \frac{1}{3}\vec{a}$

$\vec{OQ} = \frac{2}{3}\vec{b}$

$\vec{PQ} = \frac{2}{3}\vec{b} - \frac{1}{3}\vec{a}$

$\vec{AB} = \vec{b} - \vec{a}$

2.

$\vec{OP} + \vec{PB} = \vec{b}$

$\vec{PB} = \vec{b} - \frac{1}{3}\vec{a}$

$\vec{OQ} + \vec{QA} = \vec{a}$

$\vec{QA} = \vec{a} - \frac{2}{3}\vec{b}$

3.

$\vec{XB} = \beta \cdot \vec{PB}$

$\vec{XA} = \alpha \cdot \vec{QA}$

$\vec{XA} + \vec{AB} = \vec{XB}$

$\vec{PQ} + \vec{QX} = \vec{PX}$

$\vec{PX} + \vec{XB} = \vec{PB}$

$\vec{PX} = \vec{PB} - \vec{XB}$

$\vec{QX} + \vec{XA} = \vec{QA}$

$\vec{QX} = \vec{QA} - \vec{XA}$

4.

$\vec{XA} + \vec{AB} = \vec{XB}$

$\alpha \cdot (\vec{a} - \frac{2}{3}\vec{b}) + (\vec{b} - \vec{a}) = \beta \cdot (\vec{b} - \frac{1}{3}\vec{a})$

$\vec{a}(\alpha - 1 + \frac{1}{3}\beta) + \vec{b}(-\frac{2}{3}\alpha + 1 - \beta) = \vec{0}$

$\alpha - 1 + \frac{1}{3}\beta = 0 \qquad -\frac{2}{3}\alpha + 1 - \beta = 0$

$\alpha - 1 + \frac{1}{3}(1 - \frac{2}{3}\alpha) = 0 \qquad \beta = 1 - \frac{2}{3}\alpha$

$\alpha - 1 + \frac{1}{3} - \frac{2}{9}\alpha = 0$

$\frac{7}{9}\alpha = \frac{2}{3}$

$\alpha = \frac{6}{7} \qquad \beta = 1 - \frac{2}{3} \cdot \frac{6}{7} = \frac{3}{7}$

5.

$\vec{XB} = \frac{3}{7} \cdot (\vec{b} - \frac{1}{3}\vec{a}) = \frac{3}{7}\vec{b} - \frac{1}{7}\vec{a}$

$\vec{XA} = \frac{6}{7}(\vec{a} - \frac{2}{3}\vec{b}) = \frac{6}{7}\vec{a} - \frac{4}{7}\vec{b}$

$\vec{PX} = \vec{b} - \frac{1}{3}\vec{a} - \frac{3}{7}\vec{b} + \frac{1}{7}\vec{a}$

$= \frac{4}{7}\vec{b} - \frac{4}{21}\vec{a}$

$\vec{QX} = \vec{a} - \frac{2}{3}\vec{b} - \frac{6}{7}\vec{a} + \frac{4}{7}\vec{b}$

$= \frac{1}{7}\vec{a} - \frac{2}{21}\vec{b}$

$\vec{OQ} + \vec{QX} = \vec{OX}$

$\vec{OX} = \frac{2}{3}\vec{b} + \frac{1}{7}\vec{a} - \frac{2}{21}\vec{b} = \boxed{\frac{1}{7}\vec{a} + \frac{4}{7}\vec{b}}$

6.

$\vec{AY} = r \cdot \vec{AB} = r(\vec{b} - \vec{a}) \qquad \vec{XY} = \delta \cdot \vec{OX}$

$\vec{XA} + \vec{AY} = \vec{XY}$

$\frac{6}{7}\vec{a} - \frac{4}{7}\vec{b} + r\vec{b} - r\vec{a} = \delta \cdot \frac{1}{7}\vec{a} + \delta \cdot \frac{4}{7}\vec{b}$

$\vec{a}(\frac{6}{7} - r - \frac{1}{7}\delta) + \vec{b}(-\frac{4}{7} + r - \frac{4}{7}\delta) = \vec{0}$

$\frac{6}{7} - r - \frac{1}{7}\delta = 0 / \cdot 7 \qquad -\frac{4}{7} + r - \frac{4}{7}\delta = 0 / \cdot 7$

$6 - 7r - \delta = 0 \qquad -4 + 7r - 4\delta = 0$

$6 - \delta = 7r \qquad -4 + 6 - \delta - 4\delta = 0$

$2 - 5\delta = 0$

$\delta = \frac{2}{5}$

7.

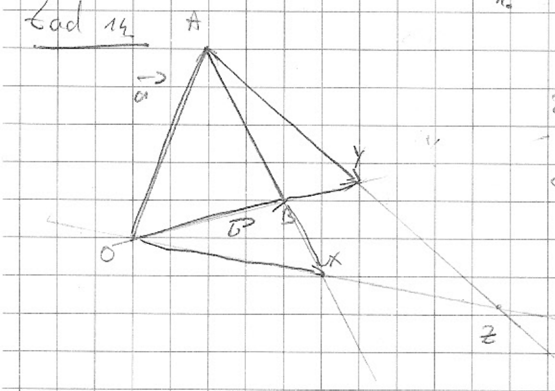
$\vec{OY} = \vec{OX} + \vec{XY}$

$= \vec{OX} + \frac{2}{5}\vec{OX}$

$= \frac{7}{5}\vec{OX} = \frac{7}{5}(\frac{1}{7}\vec{a} + \frac{4}{7}\vec{b})$

$= \boxed{\frac{1}{5}\vec{a} + \frac{4}{5}\vec{b}}$

Zad 14.



1. $\vec{OA}=\vec{a}$, $\vec{OB}=\vec{b}$

$2\vec{AX}=3\vec{AB} \Rightarrow \boxed{\vec{AX}=\frac{3}{2}(\vec{b}-\vec{a})}$

$\vec{OB}=\vec{BY}$ $\boxed{\vec{BY}=\vec{b}}$ $\boxed{\vec{BX}=\frac{1}{2}(\vec{b}-\vec{a})}$

$\vec{OX}=\vec{a}+\frac{3}{2}(\vec{b}-\vec{a})$

$\boxed{\vec{OX}=\frac{3}{2}\vec{b}-\frac{1}{2}\vec{a}}$

$\vec{AY}=\vec{AB}+\vec{BY}$

$=\vec{b}-\vec{a}+\vec{b}$

$\boxed{\vec{AY}=2\vec{b}-\vec{a}}$

2. $\vec{OZ}=\alpha\cdot\vec{OX}$

$\vec{AZ}=\beta\cdot\vec{AY}$

$\vec{OA}+\vec{AZ}=\vec{OZ}$

$\vec{a}+\beta\cdot(2\vec{b}-\vec{a})=\alpha\cdot(\frac{3}{2}\vec{b}-\frac{1}{2}\vec{a})$

$\vec{a}(1-\beta+\frac{1}{2}\alpha)+\vec{b}(2\beta-\frac{3}{2}\alpha)=0$

$1+\frac{1}{2}\alpha=\beta$ $\quad 2\beta-\frac{3}{2}\alpha=0$

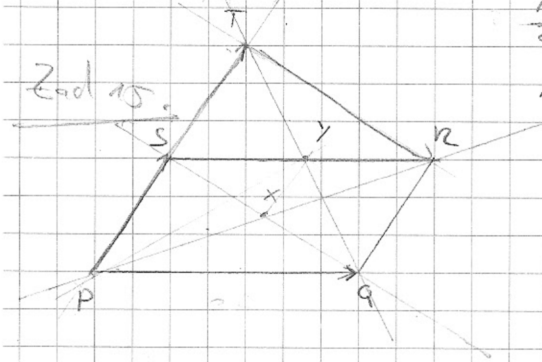
$2+\alpha-\frac{3}{2}\alpha=0$

$\frac{1}{2}\alpha=2 \Rightarrow \underline{\alpha=4}$

$\vec{OZ}=4\cdot\vec{OX}=4(\frac{3}{2}\vec{b}-\frac{1}{2}\vec{a})$

$=\underline{6\vec{b}-2\vec{a}}$

Zad 15.



1. $\vec{PQ}=2\vec{a}$

$\vec{PS}=\vec{b} \Rightarrow \vec{QR}=\vec{b}$

$\vec{PT}=2\vec{b}$ $\quad \vec{SQ}=\vec{TR}$

$\vec{PS}+\vec{SQ}=\vec{PQ}$

$\vec{SQ}=\vec{PQ}-\vec{PS}$

$\boxed{\vec{SQ}=2\vec{a}-\vec{b}}=\vec{TR}$

2. Dijagonale se raspolavljaju

$\vec{QY}=\frac{1}{2}\vec{QT}=\frac{1}{2}(\vec{PT}-\vec{PQ})$

$=\frac{1}{2}(2\vec{b}-2\vec{a})$

$\boxed{\vec{QY}=\vec{b}-\vec{a}}$

$\vec{PQ}+\vec{QX}=\vec{PX}$

$\vec{PX}=2\vec{a}+\vec{b}-\vec{a}$

$\boxed{\vec{PX}=\vec{a}+\vec{b}}$

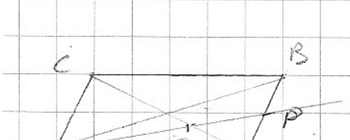
$\vec{XS}+\vec{SY}=\vec{XY}$

$\vec{XY}=\frac{1}{2}\vec{QS}+\frac{1}{2}\vec{SR}$

$=\frac{1}{2}(\vec{b}-2\vec{a})+\frac{1}{2}(2\vec{a})$

$=\frac{1}{2}\vec{b}$ $\quad \boxed{\vec{XY}=\frac{1}{2}\vec{b}}$

Zad 16.



$\vec{AP}=\vec{PB}$

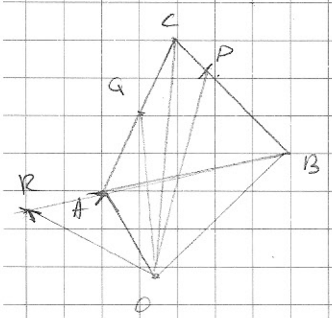
$\vec{OQ}=2\vec{QP}$

Točka Q je težište trokuta OAC, a budući da se dijagonale paralelograma raspolavljaju onda je i na pravcu AC jedna od težišnica te ona mora prolaziti točkom Q.

Točka Q dijeli dio dužine $\overline{AC}$ unutar △OAC u omjeru 2:1 te stoga dijeli cijelu dužinu u omjeru $2\cdot\frac{1}{2} : (1+3)\cdot\frac{1}{2}$

$2:4 = \underline{1:2}$

Zad 17.



1.

$\vec{BP} = 3\vec{PC}$ $\quad \vec{AB} = \vec{v_B} - \vec{v_A}$ $\quad \vec{BC} = \vec{v_C} - \vec{v_B}$

$\vec{RA} = \frac{1}{2}\vec{AB}$ $\quad \vec{RA} = \frac{1}{2}(\vec{v_B} - \vec{v_A})$ $\quad \vec{v_C} - \vec{PC} = \vec{v_P}$

$\boxed{\vec{v_Q} = \frac{1}{2}(\vec{v_A} + \vec{v_C})}$ $\quad \vec{v_R} = \vec{v_A} - \vec{RA}$ $\quad \vec{v_P} = \vec{v_C} - \frac{1}{4}(\vec{v_C} - \vec{v_B})$

$\vec{v_R} = \vec{v_A} - \frac{1}{2}\vec{v_B} + \frac{1}{2}\vec{v_A}$ $\quad \boxed{\vec{v_P} = \frac{3}{4}\vec{v_C} + \frac{1}{4}\vec{v_B}}$

$\vec{BC} = 4\vec{PC}$ $\quad \boxed{\vec{v_R} = \frac{3}{2}\vec{v_A} - \frac{1}{2}\vec{v_B}}$

$\vec{PC} = \frac{1}{4}\vec{BC}$

2.

$\vec{v_R} = \alpha \vec{v_Q} + \beta \vec{v_P}$

$\frac{3}{2}\vec{v_A} - \frac{1}{2}\vec{v_B} = \frac{1}{2}\alpha \cdot \vec{v_A} + \frac{1}{2}\alpha\vec{v_C} + \frac{3}{4}\beta\vec{v_C} + \frac{1}{4}\beta\vec{v_B}$ $\quad \boxed{\vec{v_R} = 3\vec{v_Q} - 2\vec{v_P}}$

$\frac{1}{2}\alpha + \frac{3}{4}\beta = 0$ $\quad \frac{1}{2}\alpha = \frac{3}{2}$ $\quad \frac{1}{4}\beta = -\frac{1}{2}$

$2\alpha = -3\beta$ $\quad \boxed{\alpha = 3}$ $\quad \boxed{\beta = -2}$

$0 = 0$ ✓

3. T: $\vec{PR} = \mu \vec{QR}$ $\quad \vec{v_P} + \vec{PR} = \vec{v_R}$ $\quad \vec{v_Q} + \vec{QR} = \vec{v_R}$

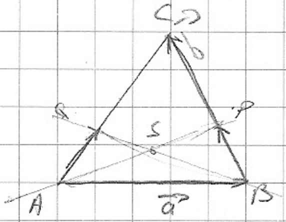
$\vec{PR} = \vec{v_R} - \vec{v_P}$ $\quad \vec{QR} = \vec{v_R} - \vec{v_Q}$

$= 3\vec{v_Q} - 3\vec{v_P}$ $\quad = 2\vec{v_Q} - 2\vec{v_P}$

$\Downarrow$

$\vec{PR} = \frac{3}{2} \cdot \vec{QR}$

Zad 18.



1.

$\vec{AB} = \vec{a}$, $\vec{BC} = \vec{b}$ $\quad \vec{AP} = \vec{a} + \frac{1}{3}\vec{b}$

$\vec{AC} = \vec{a} + \vec{b}$, $\vec{BP} = \frac{1}{3}\vec{b}$ $\quad \vec{BQ} = -\vec{a} + \frac{1}{3}(\vec{a} + \vec{b})$

$\vec{AQ} = \frac{1}{3}(\vec{a} + \vec{b})$ $\quad = \frac{1}{3}\vec{b} - \frac{2}{3}\vec{a}$

2.

$\vec{AS} = \alpha \cdot \vec{AP}$

$\vec{SQ} = \beta \cdot \vec{BQ}$

3. $\vec{AS} + \vec{SQ} = \vec{AQ}$

$\alpha \cdot (\vec{a} + \frac{1}{3}\vec{b}) + \beta(\frac{1}{3}\vec{b} - \frac{2}{3}\vec{a}) = \frac{1}{3}\vec{a} + \frac{1}{3}\vec{b}$ $\quad \vec{AS} = \frac{3}{5}(\vec{a} + \frac{1}{3}\vec{b})$

$\vec{a}(\alpha - \frac{2}{3}\beta - \frac{1}{3}) + \vec{b}(\frac{1}{3}\alpha + \frac{1}{3}\beta - \frac{1}{3}) = \vec{0}$ $\quad = \boxed{\frac{3}{5}\vec{a} + \frac{1}{5}\vec{b}}$

$3\alpha - 2\beta - 1 = 0$ $\quad \alpha + \beta - 1 = 0$

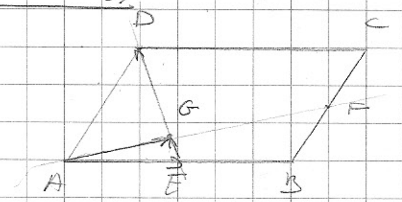
$\beta = 1 - \alpha$

$\Rightarrow 3\alpha - 2 + 2\alpha - 1 = 0$

$5\alpha = 3$

$\alpha = \frac{3}{5}$

Zad 19.



1.

$\vec{AE} = \vec{EB}$ $\qquad \vec{AG} = \alpha \vec{AF}$

$\vec{BF} = \vec{FC}$ $\qquad \vec{EG} = \beta \vec{ED}$

$\vec{AB} = \vec{a}$

$\vec{BC} = \vec{b}$

2.

$\vec{AB} + \vec{BF} = \vec{AF}$

$\vec{AE} + \vec{EG} = \vec{AG}$

$\vec{AG} + \vec{GD} = \vec{AD}$

$\vec{AE} + \vec{ED} = \vec{AD}$

3.

$\vec{a} + \frac{1}{2}\vec{b} = \vec{AF}$

$\frac{1}{2}\vec{a} + \beta \cdot \vec{ED} = \alpha \cdot \vec{AF}$

$\alpha \cdot \vec{AF} + (1-\beta)\vec{ED} = \vec{b}$

$\frac{1}{2}\vec{a} + \vec{ED} = \vec{b} \rightarrow \vec{ED} = \vec{b} - \frac{1}{2}\vec{a}$

$\vec{GD} = \vec{ED} - \vec{EG}$

$= \vec{ED} - \beta\vec{ED}$

$= (1-\beta)\vec{ED}$

3.

$\frac{1}{2}\vec{a} + \beta(\vec{b} - \frac{1}{2}\vec{a}) = \alpha(\vec{a} + \frac{1}{2}\vec{b})$

$\vec{a}(\frac{1}{2} - \frac{1}{2}\beta - \alpha) + \vec{b}(\beta - \frac{1}{2}\alpha) = \vec{0}$

$1 - \beta - 2\alpha = 0 \leftarrow 2\beta = \alpha$

$1 - \beta - 4\beta = 0$

$\boxed{\beta = \frac{1}{5}}$ $\qquad \boxed{\alpha = \frac{2}{5}}$

4.

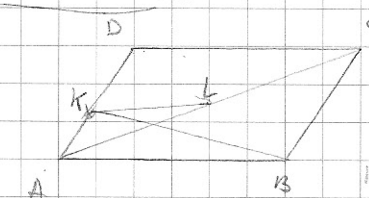
$\vec{AG} = \frac{2}{5}\vec{AF}$ $\qquad \vec{EG} = \frac{1}{5}\vec{ED}$

$\vec{AG} = \frac{2}{5}(\vec{AG} + \vec{GF})$ $\qquad \vec{EG} = \frac{1}{5}(\vec{EG} + \vec{GD})$

$\frac{3}{5}\vec{AG} = \frac{2}{5}\vec{GF}$ $\qquad \frac{4}{5}\vec{EG} = \frac{1}{5}\vec{GD}$

$\vec{AG} = \frac{2}{3}\vec{GF}$ $\qquad \vec{EG} = \frac{1}{4}\vec{GD}$

Zadatak 20.



1.)

$\vec{AK} = \frac{1}{4}\vec{AD} = \frac{1}{4}\vec{b}$

$\vec{AL} = \frac{1}{5}\vec{AC} = \frac{1}{5}(\vec{a}+\vec{b})$

$\vec{DC} = \vec{AB} = \vec{a}$

$\vec{AD} = \vec{BC} = \vec{b}$

$\vec{AC} = \vec{a} + \vec{b}$

T: $\vec{KB} = \alpha \cdot \vec{KL}$

2.)

$\vec{AK} + \vec{KB} = \vec{AB}$

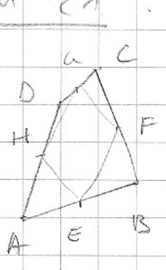
$\boxed{\vec{KB} = \vec{a} - \frac{1}{4}\vec{b}}$

$\vec{KL} + \frac{4}{5}\vec{AC} + \vec{CD} = \frac{3}{4}\vec{AD}$

$\vec{KL} = \frac{3}{4}\vec{b} - \frac{4}{5}(\vec{a}+\vec{b}) + \vec{a}$

$\vec{KL} = \boxed{\frac{1}{5}\vec{a} - \frac{1}{20}\vec{b}} \rightarrow \alpha = \frac{1}{5}$ Q.E.D.

Zad 21.



T: $\vec{EF} = \vec{HG}$

$\vec{FG} = \vec{EH}$

1.

$\vec{v_E} = \frac{1}{2}(\vec{v_A} + \vec{v_B})$

$\vec{v_F} = \frac{1}{2}(\vec{v_B} + \vec{v_C})$

$\vec{v_G} = \frac{1}{2}(\vec{v_C} + \vec{v_D})$

$\vec{v_H} = \frac{1}{2}(\vec{v_D} + \vec{v_A})$

2.

$\vec{EF} = \vec{v_F} - \vec{v_E} = \frac{1}{2}(\cancel{\vec{v_B}} + \vec{v_C} - \vec{v_A} - \cancel{\vec{v_B}})$ =

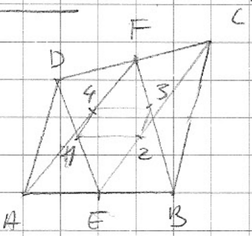
$\vec{HG} = \vec{v_G} - \vec{v_H} = \frac{1}{2}(\vec{v_C} + \cancel{\vec{v_D}} - \cancel{\vec{v_D}} - \vec{v_A})$

$\vec{FG} = \vec{v_G} - \vec{v_F} = \frac{1}{2}(\cancel{\vec{v_C}} + \vec{v_D} - \vec{v_B} - \cancel{\vec{v_C}})$ =

$\vec{EH} = \vec{v_H} - \vec{v_E} = \frac{1}{2}(\vec{v_D} + \cancel{\vec{v_A}} - \cancel{\vec{v_A}} - \vec{v_B})$

Q.E.D.

Zad 22.



T: dužine ... istih vektora

1. $\vec{v_E} = \frac{1}{2}(\vec{v_A} + \vec{v_B})$ $\quad \vec{v_1} = \frac{1}{2}(\vec{v_D} + \vec{v_E}) = \frac{1}{2}\vec{v_D} + \frac{1}{4}\vec{v_A} + \frac{1}{4}\vec{v_B}$

$\vec{v_F} = \frac{1}{2}(\vec{v_C} + \vec{v_D})$ $\quad \vec{v_2} = \frac{1}{2}(\vec{v_C} + \vec{v_E}) = \frac{1}{2}\vec{v_C} + \frac{1}{4}\vec{v_A} + \frac{1}{4}\vec{v_B}$

$\vec{v_3} = \frac{1}{2}(\vec{v_B} + \vec{v_F}) = \frac{1}{2}\vec{v_B} + \frac{1}{4}\vec{v_C} + \frac{1}{4}\vec{v_D}$

$\vec{v_4} = \frac{1}{2}(\vec{v_A} + \vec{v_F}) = \frac{1}{2}\vec{v_A} + \frac{1}{4}\vec{v_C} + \frac{1}{4}\vec{v_D}$

2. $\vec{12} = \vec{v_2} - \vec{v_1} = \frac{1}{2}(\vec{v_C} - \vec{v_D}) \to$ dijagonala

$\vec{13} = \vec{v_3} - \vec{v_1} = \frac{1}{4}\vec{v_B} + \frac{1}{4}\vec{v_C} - \frac{1}{4}\vec{v_D} - \frac{1}{4}\vec{v_A}$

$\vec{14} = \vec{v_4} - \vec{v_1} = \frac{1}{4}\vec{v_A} + \frac{1}{4}\vec{v_C} - \frac{1}{4}\vec{v_D} - \frac{1}{4}\vec{v_B}$

$\vec{23} = \vec{v_3} - \vec{v_2} = \frac{1}{4}\vec{v_B} - \frac{1}{4}\vec{v_C} + \frac{1}{4}\vec{v_D} - \frac{1}{4}\vec{v_A} = -\vec{14} \to \boxed{\vec{23} = \vec{41}}$

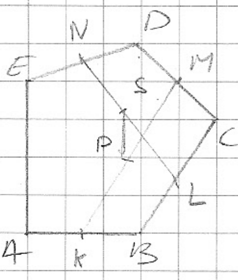
$\vec{24} = \vec{v_4} - \vec{v_2} = \frac{1}{4}\vec{v_A} - \frac{1}{4}\vec{v_C} + \frac{1}{4}\vec{v_D} - \frac{1}{4}\vec{v_B} = -\vec{13} \to \boxed{\vec{24} = \vec{31}}$

stranice

$\vec{34} = \vec{v_4} - \vec{v_3} = \frac{1}{2}(\vec{v_A} - \vec{v_B}) \to$ dijagonala

Q.E.D

Zad 23.



T: $\vec{PS} = \alpha \cdot \vec{AE}$

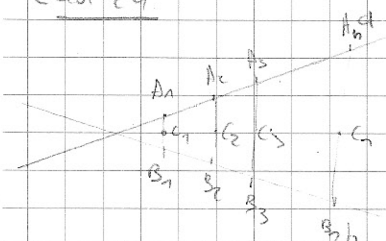
1. $\vec{v_K} = \frac{1}{2}(\vec{v_A} + \vec{v_B})$ $\quad \vec{v_P} = \frac{1}{2}(\vec{v_K} + \vec{v_M}) = \frac{1}{4}(\vec{v_A} + \vec{v_B} + \vec{v_C} + \vec{v_D})$

$\vec{v_M} = \frac{1}{2}(\vec{v_C} + \vec{v_D})$ $\quad \vec{v_S} = \frac{1}{2}(\vec{v_L} + \vec{v_N}) = \frac{1}{4}(\vec{v_B} + \vec{v_C} + \vec{v_D} + \vec{v_E})$

$\vec{v_L} = \frac{1}{2}(\vec{v_B} + \vec{v_C})$ $\quad$ 2.) $\vec{PS} = \vec{v_S} - \vec{v_P}$

$\vec{v_N} = \frac{1}{2}(\vec{v_D} + \vec{v_E})$ $\quad = \frac{1}{4}(\vec{v_E} - \vec{v_A}) = \frac{1}{4}\vec{AE}$ Q.E.D

Zad 24



$a \cap b = \emptyset$

$\frac{|A_1A_2|}{|B_1B_2|} = \frac{|A_2A_3|}{|B_2B_3|} = \dots = \frac{|A_{n-1}A_n|}{|B_{n-1}B_n|} = k$

$\vec{A_1A_2} = k \cdot \vec{A_2A_3}$

$\vec{B_1B_2} = k \cdot \vec{B_2B_3}$

Dokažimo za prve tri

1. $\vec{v_{C_1}} = \frac{1}{2}(\vec{v_{A_1}} + \vec{v_{B_1}})$

$\vec{v_{C_2}} = \frac{1}{2}(\vec{v_{A_2}} + \vec{v_{B_2}})$

$\vec{C_1C_2} = \vec{v_{C_2}} - \vec{v_{C_1}} = \frac{1}{2}(\vec{v_{A_2}} - \vec{v_{A_1}}) + \frac{1}{2}(\vec{v_{B_2}} - \vec{v_{B_1}})$

$= \frac{1}{2}\vec{A_1A_2} + \frac{1}{2}\vec{B_1B_2}$

$= \frac{k}{2}(\vec{A_2A_3} + \vec{B_2B_3}) = k \cdot \vec{C_2C_3}$

2. Točke C_1, C_2, C_3 su kolinearne

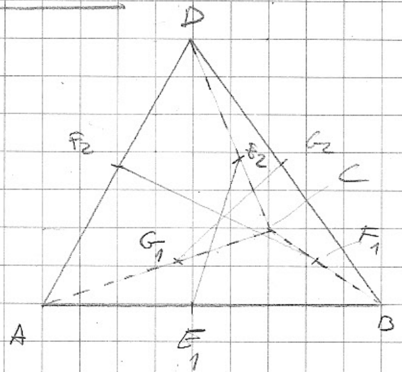
Mat ind $\quad \vec{C_{n-2}C_{n-1}} = k\,\vec{C_{n-1}C_n}$

T: $\vec{C_{n-1}C_n} = k \cdot \vec{C_nC_{n+1}}$

$\vec{C_{n-1}C_n} = \frac{1}{2}\vec{A_{n-1}A_n} + \frac{1}{2}\vec{B_{n-1}B_n} = \frac{k}{2}(\vec{A_nA_{n+1}} + \vec{B_nB_{n+1}}) = k \cdot \vec{C_nC_{n+1}}$

Q.E.D

Zad 25.



Odaberemo na spojnicama $\overline{E_1E_2}, \overline{F_1F_2}, \overline{G_1G_2}$ točke polovišta tih spojnica S_E, S_F, S_G

$\vec{r}_{E_1}=\frac{1}{2}(\vec{r}_A+\vec{r}_B)$ $\quad \vec{r}_{F_1}=\frac{1}{2}(\vec{r}_B+\vec{r}_C)$ $\quad \vec{r}_{G_1}=\frac{1}{2}(\vec{r}_A+\vec{r}_C)$

$\vec{r}_{E_2}=\frac{1}{2}(\vec{r}_C+\vec{r}_D)$ $\quad \vec{r}_{F_2}=\frac{1}{2}(\vec{r}_A+\vec{r}_D)$ $\quad \vec{r}_{G_2}=\frac{1}{2}(\vec{r}_B+\vec{r}_D)$

$$\vec{r}_{S_E}=\frac{1}{2}(\vec{r}_{E_1}+\vec{r}_{E_2})=\frac{1}{4}(\vec{r}_A+\vec{r}_B+\vec{r}_C+\vec{r}_D)$$

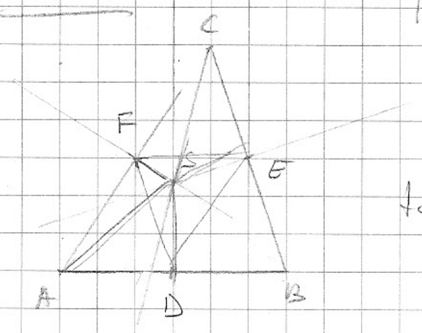
$$\vec{r}_{S_F}=\frac{1}{2}(\vec{r}_{F_1}+\vec{r}_{F_2})=\frac{1}{4}(\vec{r}_A+\vec{r}_B+\vec{r}_C+\vec{r}_D)$$

$$\vec{r}_{S_G}=\frac{1}{2}(\vec{r}_{G_1}+\vec{r}_{G_2})=\frac{1}{4}(\vec{r}_A+\vec{r}_B+\vec{r}_C+\vec{r}_D)$$

vektori su jednaki, te kako imaju iste početne točke moraju imati i isto završ.

Q.E.D

Zad 26.



Neka je S sjecište simetrala stranica $\overline{AB}$ i $\overline{BC}$

tj. $\vec{SD}\perp\vec{AB}$ i $\vec{SE}\perp\overline{BC}$ tj. $\vec{SD}\cdot\vec{AB}=0$, $\vec{SE}\cdot\vec{BC}=0$

tada je $\vec{SF}\cdot\vec{AC}=\vec{SF}\cdot(\vec{AB}+\vec{BC})=\vec{SF}\cdot\vec{AB}+\vec{SF}\cdot\vec{BC}$

$=(\vec{SD}+\vec{DF})\cdot\vec{AB}+(\vec{SE}+\vec{EF})\cdot\vec{BC}$

$=\vec{DF}\cdot\vec{AB}+\vec{EF}\cdot\vec{BC}$

$=\frac{1}{2}\vec{BC}\cdot\vec{AB}-\frac{1}{2}\vec{BC}\cdot\vec{AB}=0.$

Q.E.D.

$\vec{AD}+\vec{DF}=\vec{AF}$

$\vec{DF}=\frac{1}{2}\vec{AC}-\frac{1}{2}\vec{AB}$

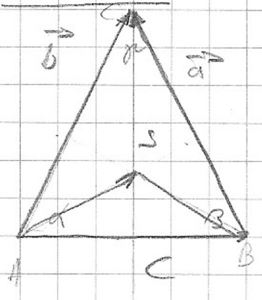
$=\frac{1}{2}\vec{BC}$

$\vec{EF}+\vec{FC}=\vec{EC}$

$\vec{EF}=\frac{1}{2}\vec{BC}-\frac{1}{2}\vec{AC}$

$=\frac{1}{2}\vec{BA}$

Zad 27.



S presjecište simetrala α i β

$\vec{a}=\vec{b}-\vec{c}$ $\qquad \vec{AB}+\vec{BS}+\vec{SA}=0$

$\vec{AS}=\lambda(\hat{b}+\hat{c})$ $\qquad \vec{c}+\mu\left(\frac{\vec{a}}{|\vec{a}|}-\frac{\vec{c}}{|\vec{c}|}\right)-\lambda\left(\frac{\vec{b}}{|\vec{b}|}+\frac{\vec{c}}{|\vec{c}|}\right)=0$

$\vec{BS}=\mu(\hat{a}-\hat{c})$

$$\vec{c}+\mu\left(\frac{\vec{b}}{|\vec{a}|}-\frac{\vec{c}}{|\vec{a}|}-\frac{\vec{c}}{|\vec{c}|}\right)-\lambda\left(\frac{\vec{b}}{|\vec{b}|}+\frac{\vec{c}}{|\vec{c}|}\right)=0$$

$\frac{\mu}{|\vec{a}|}-\frac{\lambda}{|\vec{b}|}=0$ $\qquad 1-\frac{\mu}{|\vec{a}|}-\frac{\mu}{|\vec{c}|}-\frac{\lambda}{|\vec{c}|}=0$

$\lambda=\frac{|\vec{b}|}{|\vec{a}|}\mu$ $\qquad 1=\mu\left(\frac{1}{|\vec{a}|}+\frac{1}{|\vec{c}|}+\frac{|\vec{b}|}{|\vec{c}||\vec{a}|}\right)$

$\mu=\frac{|\vec{a}|\cdot|\vec{c}|}{|\vec{a}|+|\vec{b}|+|\vec{c}|}$ $\qquad \lambda=\frac{|\vec{b}||\vec{c}|}{|\vec{a}|+|\vec{b}|+|\vec{c}|}$

$\Downarrow$

S_1 presjecište simetrala β i γ

$\vec{BS_1} = \chi(\hat{a} - \hat{c})$ $\qquad \vec{CS_1} + \vec{S_1B} + \vec{BC} = 0$

$\vec{CS_1} = -\psi(\hat{a} + \hat{b})$ $\qquad -\psi\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right) - \chi\left(\frac{\vec{a}}{|\vec{a}|} - \frac{\vec{c}}{|\vec{c}|}\right) + \vec{a} = 0$

$\vec{a} = \vec{b} - \vec{c}$

$$-\psi\left(\frac{\vec{b}}{|\vec{a}|} - \frac{\boxed{\vec{c}}}{|\vec{a}|} - \frac{\vec{b}}{|\vec{b}|}\right) - \chi\left(\frac{\vec{b}}{|\vec{a}|} - \frac{\boxed{\vec{c}}}{|\vec{a}|} - \frac{\boxed{\vec{c}}}{|\vec{c}|}\right) + \vec{b} - \boxed{\vec{c}} = 0$$

$$1 - \frac{\psi}{|\vec{a}|} - \frac{\psi}{|\vec{b}|} - \frac{\chi}{|\vec{a}|} = 0 \qquad -1 + \frac{\psi}{|\vec{a}|} + \frac{\chi}{|\vec{a}|} + \frac{\chi}{|\vec{c}|} = 0$$

$$\chi = |\vec{a}| - \psi - \frac{|\vec{a}|}{|\vec{b}|}\psi \qquad 0 = \cancel{-1} + \cancel{\frac{\psi}{|\vec{a}|}} + \cancel{1} - \cancel{\frac{\psi}{|\vec{a}|}} - \frac{\psi}{|\vec{b}|} + \frac{|\vec{a}|}{|\vec{c}|} - \frac{\psi}{|\vec{c}|} - \frac{|\vec{a}|}{|\vec{b}||\vec{c}|}\psi$$

$$\frac{\psi}{|\vec{b}|} + \frac{\psi}{|\vec{c}|} + \psi\frac{|\vec{a}|}{|\vec{b}||\vec{c}|} = \frac{|\vec{a}|}{|\vec{c}|}$$

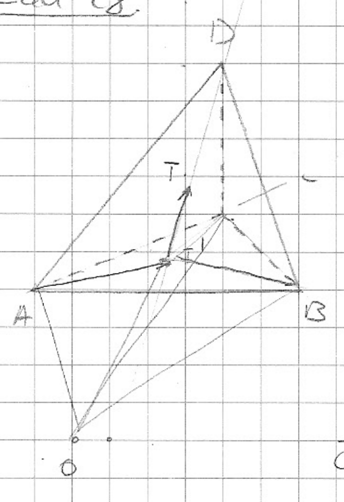
$$\psi \cdot \frac{|\vec{a}| + |\vec{b}| + |\vec{c}|}{|\vec{b}|\cancel{|\vec{c}|}} = \frac{|\vec{a}|}{\cancel{|\vec{c}|}} \Rightarrow \psi = \frac{|\vec{a}| \cdot |\vec{b}|}{|\vec{a}| + |\vec{b}| + |\vec{c}|}$$

$$\chi = |\vec{a}| - \frac{|\vec{a}| \cdot |\vec{b}|}{|\vec{a}| + |\vec{b}| + |\vec{c}|} - \frac{|\vec{a}|}{\cancel{|\vec{b}|}} \cdot \frac{|\vec{a}|\cancel{|\vec{b}|}}{|\vec{a}| + |\vec{b}| + |\vec{c}|}$$

$$= \frac{\cancel{|\vec{a}|^2} + \cancel{|\vec{a}||\vec{b}|} + |\vec{a}||\vec{c}| - \cancel{|\vec{a}||\vec{b}|} - \cancel{|\vec{a}|^2}}{|\vec{a}| + |\vec{b}| + |\vec{c}|} = \frac{|\vec{a}||\vec{c}|}{|\vec{a}| + |\vec{b}| + |\vec{c}|}$$

$$\vec{BS_1} = \frac{|\vec{a}||\vec{c}|}{|\vec{a}| + |\vec{b}| + |\vec{c}|} = \vec{BS} \Rightarrow S = S_1 \quad \underline{Q.E.D}$$

Zad 28.



$$\vec{AT'} = \alpha\vec{AB} + \beta\vec{AC}$$

$$= \alpha\vec{OB} - \alpha\vec{OA} + \beta\vec{OC} - \beta\vec{OA}$$

$$= \vec{OA}(-\alpha - \beta) + \alpha\vec{OB} + \beta\vec{OC} = \vec{OT'} - \vec{OA}$$

$$\vec{OT'} = \vec{OA}(1 - \alpha - \beta) + \alpha\vec{OB} + \beta\vec{OC}$$

$$\vec{T'T} = \delta \cdot \vec{T'D}$$

$$\vec{OT} - \vec{OT'} = \delta\vec{OD} - \delta\vec{OT'}$$

$$\vec{OT} = \delta\vec{OD} + (1-\delta)\vec{OT'}$$

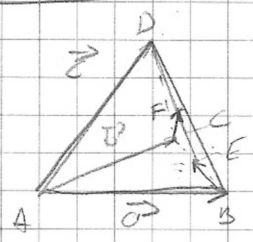
$$= \delta\vec{OD} + (1-\delta)(1-\alpha-\beta)\vec{OA} + (1-\delta)\alpha\vec{OB} + (1-\delta)\beta\vec{OC}$$

$$\delta + (1-\delta)(1-\alpha-\beta) + (1-\delta)\alpha + (1-\delta)\beta =$$

$$= \cancel{\delta} + 1 - \cancel{\alpha} - \cancel{\beta} - \cancel{\delta} + \cancel{\alpha\delta} + \cancel{\beta\delta} + \cancel{\alpha} - \cancel{\alpha\delta} + \cancel{\beta} - \cancel{\beta\delta} = \underline{\underline{1}}$$

Q.E.D

Zad 29.



$\vec{BC} = \vec{b} - \vec{a}$

$\vec{BE} = \frac{1}{2}(\vec{b} - \vec{a})$

$\vec{a} + \vec{BE} + \vec{ED} = \vec{c}$

$\vec{ED} = \vec{c} - \vec{a} - \frac{1}{2}(\vec{b} - \vec{a})$

$= \vec{c} - \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a}$

$\vec{EF} = \frac{1}{2}\vec{ED} = \frac{1}{2}\vec{c} - \frac{1}{4}\vec{b} - \frac{1}{4}\vec{a}$

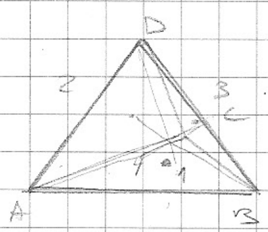
$\vec{AB} + \vec{BE} + \vec{EF} - \vec{CF} - \vec{AC} = 0$

$\vec{CF} = \vec{a} + \frac{1}{2}(\vec{b} - \vec{a}) + \frac{1}{2}\vec{c} - \frac{1}{4}\vec{b} - \frac{1}{4}\vec{a}$

$- \vec{b}$

$\vec{CF} = \frac{1}{4}\vec{a} - \frac{3}{4}\vec{b} + \frac{1}{2}\vec{c}$

Zad 30.



Odaberemo na težišnicama točke T_x tako da su im udaljenosti od vrha 3 puta veća od udaljenosti od plohe

$\vec{AT_A} = \frac{3}{4}\vec{AT_3} = \frac{3}{4}\cdot(\vec{OT_3} - \vec{OA}) = \frac{3}{4}\left(\frac{1}{3}(\vec{OB} + \vec{OC} + \vec{OB}) - \vec{OA}\right)$

$\vec{OT_A} - \vec{OA} = \frac{1}{4}(\vec{OB} + \vec{OC} + \vec{OD}) - \frac{3}{4}\vec{OA}$

$\vec{OT_A} = \frac{1}{4}(\vec{OB} + \vec{OC} + \vec{OD} + \vec{OA})$

$\vdots$

$\vec{OT_A} = \vec{OT_B} = \vec{OT_C} = \vec{OT_D} \Rightarrow T_A \equiv T_B \equiv T_C \equiv T_D$ Q.E.D.

Zad 31.

$|\vec{a} - \vec{b}| \geq ||\vec{a}| - |\vec{b}||$ $\quad L = \sqrt{(\vec{a}-\vec{b})^2} = \mathrm{sqrt}(\vec{a}^2 - 2\vec{a}\cdot\vec{b} + \vec{b}^2)$

$= \mathrm{sqrt}(|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cdot\cos\varphi) \ /^2$

$D = ||\vec{a}| - |\vec{b}|| \ /^2$

$D^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$

$L^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\varphi$ $\quad \varphi \in [0, \pi] \Rightarrow \cos\varphi \in [-1, 1]$ $\quad$ Q.E.D.

$L = D \Rightarrow \cos\varphi = 1 \Rightarrow \vec{a} = \lambda\cdot\vec{b} \quad \lambda > 0$

Zad 32.

$\vec{a} = 24\vec{i} - 7\vec{j} \quad |\vec{b}| = 15 \quad |\vec{a} + \vec{b}| \quad \sqrt{24^2 + 7^2} = 25$

$|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b})^2 = |\vec{a}|^2 + 2|\vec{a}||\vec{b}|\cdot\cos\varphi + |\vec{b}|^2$

$= 625 + 750\cos\varphi + 225$

$= 850 + 750\cos\varphi \quad \cos\varphi \in [-1, 1]$

$|\vec{a} + \vec{b}|^2 \in [100, 1600]$

$|\vec{a} + \vec{b}| \in [10, 40]$

Zad 33. $\vec a = \vec p - 2\vec q$, $|\vec p| = 2$, $|\vec q| = 3$, $\sphericalangle(\vec p, \vec q) = \frac{\pi}{6}$, $\cos\frac{\pi}{6} = \frac{\sqrt3}{2}$

$$|\vec a|^2 = \vec a^2 = (\vec p - 2\vec q)(\vec p - 2\vec q) = \vec p^2 - 4\vec p\cdot\vec q + 4\vec q^2$$
$$= 4 - 4\cdot\cancel{2}\cdot 3\cdot\frac{\sqrt3}{\cancel{2}} + 4\cdot 9$$
$$= 40 - 12\sqrt3 \Rightarrow |\vec a| = \sqrt{40-12\sqrt3} = 2\sqrt{10-3\sqrt3}$$

Zad 34. $\vec d = \hat m - \hat n$, $\vec e = 5\hat m - 4\hat n$, $\sphericalangle(\hat m, \hat n) = 30^\circ$, $\sin 30^\circ = \frac12$

$\vec a + \vec b = \vec d$
$\vec a - \vec b = \vec e$ $\quad |+$

$2\vec a = 4\vec m - 5\hat n$

$\boxed{\vec a = 2\vec m - \frac52\vec n}$

$\vec b = \vec d - \vec a$
$= \hat m - \hat n - 2\hat m + \frac52\hat n$
$= \boxed{\frac32\hat n - \hat m}$

$P_{\square} = |\vec a\times\vec b|$ $\quad \vec a\times\vec b = (2\hat m - \frac52\hat n)\times(\frac32\hat n - \hat m) = 3\,\hat m\times\hat n - \frac52\hat m\times\hat n$
$= \frac12\hat m\times\hat n$

$= \frac12\cdot 1\cdot 1\cdot\frac12 = \frac14$

Zad 35.
$|\vec a| = 13$, $|\vec b| = 19$, $|\vec a+\vec b| = 24$, $|\vec a - \vec b|$

$|\vec a - \vec b|^2 = \vec a^2 - 2\vec a\cdot\vec b + \vec b^2$
$|\vec a + \vec b|^2 = \vec a^2 + 2\vec a\cdot\vec b + \vec b^2$ $\quad +$ $\quad 24^2 + |\vec a - \vec b|^2 = 2\cdot 13^2 + 2\cdot 19^2$
$|\vec a - \vec b|^2 = 484 \rightarrow |\vec a - \vec b| = 22$

Zad 36.
$|\vec a| = 11$, $|\vec b| = 23$, $|\vec a - \vec b| = 30$

$|\vec a + \vec b|^2 = 11^2 + 2\vec a\cdot\vec b + 23^2$
$30^2 = 11^2 - 2\cdot\vec a\cdot\vec b + 23^2$ $\quad |+$

$|\vec a + \vec b|^2 = 2\cdot 11^2 + 2\cdot 23^2 - 30^2 = 400$
$|\vec a + \vec b| = 20$

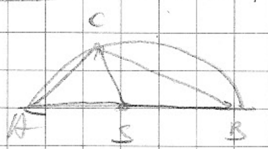
Zad 37. $\vec a + \vec b + \vec c = \vec 0$, $|\vec a| = 3$, $|\vec b| = 1$, $|\vec c| = 4$, $\vec a\cdot\vec b + \vec b\cdot\vec c + \vec c\cdot\vec a$

$(\vec a + \vec b + \vec c)^2 = \vec a^2 + \vec b^2 + \vec c^2 + 2(\vec a\cdot\vec b + \vec b\cdot\vec c + \vec c\cdot\vec a) = 0$

$26 + 2(\vec a\cdot\vec b + \vec a\cdot\vec c + \vec b\cdot\vec c) = 0$

$\vec a\cdot\vec b + \vec a\cdot\vec c + \vec b\cdot\vec c = -13$

Zad 38.



$|\vec{SA}| = |\vec{SB}| = |\vec{SC}|$ $\quad \vec{AC}\cdot\vec{CB} = (\vec{SC}-\vec{SA})(\vec{SB}-\vec{SC})$

$\vec{AC} = \vec{SC} - \vec{SA}$ $\quad \vec{SB} = -\vec{SA}$ $\quad = (\vec{SC}-\vec{SA})(\vec{SC}+\vec{SA})\cdot -1$

$\vec{CB} = \vec{SB} - \vec{SC}$ $\quad = \left[|\vec{SC}|^2 - |\vec{SA}|^2\right]\cdot -1 = 0$

$\vec{AC} \perp \vec{CB}$ Q.E.D.

Zad 39

$\vec{a}, \vec{b}, \vec{c} \neq \vec{0}$

$\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$

$\vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} = 0$

$\vec{b} \neq \vec{c}$

$\vec{a}\cdot(\vec{b}-\vec{c}) = 0 \Rightarrow \vec{a} \perp \vec{b}-\vec{c}$

Q.E.D

Zad 40.

$\vec{a}\cdot\vec{c} = 3 \quad \vec{b}\cdot\vec{c} = 4$

$\vec{d} = \vec{a} + \lambda\vec{b} \quad \perp \vec{c}$

$\vec{d}\cdot\vec{c} = 0$

$(\vec{a} + \lambda\vec{b})\cdot\vec{c} = 0$

$\vec{a}\cdot\vec{c} + \lambda\,\vec{b}\cdot\vec{c} = 0$

$\lambda = -\frac{3}{4}$ //

Zad 41.

$\vec{a}, \vec{b}, \vec{c}$ komplanarni $\quad \vec{c} = \alpha\vec{a} + \beta\vec{b}$

$|\vec{a}| = 2 \quad |\vec{b}| = 1$

$\vec{a}\cdot\vec{b} = 1 \quad \vec{a}\cdot\vec{c} = 5 \quad \vec{b}\cdot\vec{c} = -1$

$\vec{a}\cdot(\alpha\vec{a} + \beta\vec{b}) = 5 \quad \vec{b}\cdot(\alpha\vec{a} + \beta\vec{b}) = -1$

$\alpha\cdot\vec{a}^2 + \beta\,\vec{a}\cdot\vec{b} = 5 \quad \alpha\,\vec{a}\cdot\vec{b} + \beta\,\vec{b}^2 = -1$

$4\alpha + \beta = 5 \quad \alpha + \beta = -1$

$\beta = -1 - \alpha$

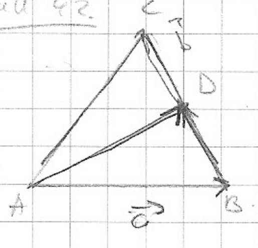
$4\alpha - 1 - \alpha = 5$

$3\alpha = 6$

$\alpha = 2$ $\quad \vec{c} = 2\vec{a} - 3\vec{b}$

$\beta = -3$

Zad 42.



$\vec{AB} = \vec{a}$

$\vec{BC} = \vec{b}$

$\vec{AC} = \vec{a} + \vec{b}$

$\vec{BD} = \frac{1}{2}\vec{b}$

$\vec{AD} = \vec{a} + \frac{1}{2}\vec{b}$

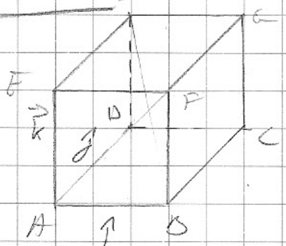
$|\vec{AB}|^2 + |\vec{AC}|^2 = |\vec{a}|^2 + |\vec{a}+\vec{b}|^2$

$= 2|\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$

$|\vec{AD}|^2 + |\vec{BD}|^2 = |\vec{a}| + \vec{a}\cdot\vec{b} + \frac{1}{2}|\vec{b}|^2$ $\quad \cdot 2$

Q.E.D

Zad 43.

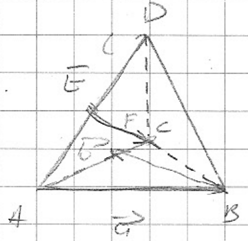


$\vec{AG} = \vec{i} + \vec{j} + \vec{k}$

$\vec{BH} = -\vec{i} + \vec{j} + \vec{k}$

$\cos\varphi = \dfrac{\vec{AG}\cdot\vec{BH}}{|\vec{AG}||\vec{BH}|} = \dfrac{-1+1+1}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{1}{3} \Rightarrow \varphi = 70,5^\circ$

Zad 44.



$\measuredangle(\vec{a},\vec{b}) = 60^\circ$

$\measuredangle(\vec{b},\vec{c}) = 60^\circ$

$\measuredangle(\vec{a},\vec{c}) = 60^\circ$

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\cos 60^\circ = \frac{1}{2}$

1.

$\vec{a} + \vec{BF} = \frac{1}{2}\vec{b}$

$\vec{BF} = \frac{1}{2}\vec{b} - \vec{a}$

$\frac{1}{2}\vec{c} + \vec{EC} = \vec{b}$

$\vec{EC} = \vec{b} - \frac{1}{2}\vec{c}$

$|\vec{EC}|^2 = (\vec{b} - \frac{1}{2}\vec{c})^2$
$= \vec{b}^2 - \vec{b}\cdot\vec{c} + \frac{1}{4}\vec{c}^2$
$= 1 - \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

2.

$\cos\varphi = \dfrac{\vec{BF}\cdot\vec{EC}}{|\vec{BF}|\cdot|\vec{EC}|}$

$|\vec{BF}|^2 = (\frac{1}{2}\vec{b} - \vec{a})^2$
$= \frac{1}{4}\vec{b}^2 - \vec{a}\cdot\vec{b} + \vec{a}^2$
$= \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$

$\vec{BF}\cdot\vec{EC} = (\frac{1}{2}\vec{b} - \vec{a})(\vec{b} - \frac{1}{2}\vec{c})$
$= \frac{1}{2}\vec{b}^2 - \frac{1}{4}\vec{b}\cdot\vec{c} - \vec{a}\cdot\vec{b} + \frac{1}{2}\vec{a}\cdot\vec{c}$
$= \frac{1}{2} - \frac{1}{4}\cdot\frac{1}{2} - \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}$
$= \frac{1}{8}$

$\cos\varphi = \dfrac{\frac{1}{8}}{\frac{3}{4}} = \dfrac{4}{3\cdot 8} = \dfrac{1}{6}$

Zad 45.

$\vec{a} = 2\vec{i} - 3\vec{j} + 5\vec{k}$

$\vec{b} = -\vec{i} + 4\vec{j} + 2\vec{k}$

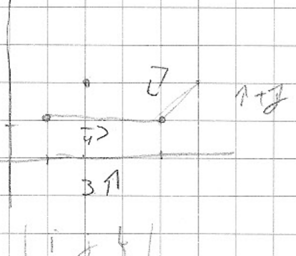
$\vec{c} = 2\vec{i} + 3\vec{j}$

$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} -1 & 4 & 2 \\ 2 & 3 & 0 \\ 2 & -3 & 5 \end{vmatrix} = \begin{vmatrix} -1 & 4 & 2 \\ 0 & 11 & 4 \\ 0 & 5 & 9 \end{vmatrix} = -\begin{vmatrix} 11 & 4 \\ 5 & 9 \end{vmatrix}$
$= -(99-20) = -79$

$\vec{b}\cdot(\vec{a}\times\vec{c}) = \begin{vmatrix} 2 & -3 & 5 \\ 2 & 3 & 0 \\ -1 & 4 & 2 \end{vmatrix} = -\begin{vmatrix} -1 & 4 & 2 \\ 2 & 3 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 79$

Zad 46

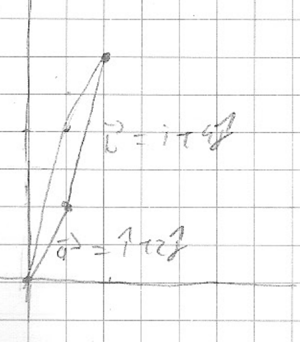
a)



$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0\vec{i} + 0\vec{j} + 3\vec{k}$

$|\vec{a}\times\vec{b}| = 3$

b)



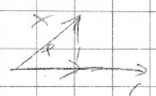
$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 1 & 4 & 0 \end{vmatrix} = 2\vec{k}$

$P = 2$

Zad 47. a) $\vec{a}=(3,-6,1)$, $\vec{b}=(1,4,-5)$, $\vec{c}=(3,4,-12)$

$\vec{x}=\vec{a}+\vec{b}=(4,-2,-4)$

$\vec{x}\cdot\vec{c}=|\vec{x}|\cdot|\vec{c}|\cdot\cos\varphi=$



$=x_c\cdot|\vec{c}|$

$\vec{x}\cdot\vec{c}=12-8+4\cdot 12$

$\cos\varphi=\frac{x_c}{|x|}$

$|\vec{c}|=\sqrt{9+16+12^2}=13$

$=52$

$a_b=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\vec{a}\cdot\vec{b_0}$

$x_c=\frac{52}{13}=4$

$\frac{\vec{a_b}}{a_b}=\vec{b_0} \Rightarrow \vec{a_b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\cdot\vec{b}=\vec{a}\cdot\vec{b_0}^2$

$x_c=4$

$\vec{x_c}=\frac{4}{13}\vec{c}$

b) $\vec{a}=(1,-3,4)$ $\vec{b}=(3,-4,2)$, $\vec{c}=(-1,1,4)$

$\vec{x}=\vec{b}+\vec{c}=(2,-3,6)$

$\vec{a_x}=\frac{\vec{a}\cdot\vec{x}^2}{|\vec{x}|^2}$

$|\vec{x}|=\sqrt{4+9+36}=7$

$\vec{a}\cdot\vec{x}=2+9+24=35$

$\vec{a_x}=\frac{35}{7}\cdot\vec{x}=\frac{5\vec{x}}{7}$

c) $(\vec{a}+\vec{b})_c \overset{\text{uklitersh}}{=} \vec{a_c}+\vec{b_c}$

$L=\frac{(\vec{a}+\vec{b})\cdot\vec{c}^2}{|\vec{c}|^2}=\frac{\vec{a}\cdot\vec{c}^2}{|\vec{c}|^2}+\frac{\vec{b}\cdot\vec{c}^2}{|\vec{c}|^2}=\vec{a_c}+\vec{b_c}=D$

Q.E.D.

d) $\vec{a}_{b+c}=\vec{a_b}+\vec{a_c}$ npr. $\vec{a}(1,0,0)$ $\vec{b}(0,1,0)$ $\vec{c}(1,0,1)$

$\vec{a_b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}=\vec{0}$

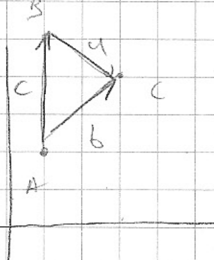
$\frac{\vec{a}\cdot(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|^2}(\vec{b}+\vec{c})=\frac{1}{\sqrt{3}}(\vec{b}+\vec{c})=\frac{1}{\sqrt{3}}(1,1,1)$

$\vec{a_c}=\frac{\vec{a}\cdot\vec{c}}{|\vec{c}|^2}\cdot\vec{c}=\frac{\vec{c}}{|\vec{c}|^2}=\frac{1}{\sqrt{2}}(1,0,1)$

$\neq$

Zad 48.

A)



$\vec{AB}=3\vec{j}$ $c=3$

$c^2=a^2+b^2-2ab\cos\gamma$

$\vec{AC}=2\vec{i}+2\vec{j}$ $b=\sqrt{4+4}=2\sqrt{2}$

$\cos\gamma=\frac{-4}{-4\sqrt{10}}=+\frac{1}{\sqrt{10}}$

$\vec{BC}=2\vec{i}-\vec{j}$ $a=\sqrt{4+1}=\sqrt{5}$

$\gamma=71,57^0$

$b^2=a^2$

$\beta=63,43^0$

$\alpha=45^0$

B) $A(-1,2,3)$ $B(2,1,2)$ $C(0,3,0)$

$\vec{a}=\vec{BC}=\vec{OC}-\vec{OB}=(-2,2,-2)$ $a=\sqrt{12}$

$\cos\alpha=\frac{\frac{1}{2}\sqrt{12}}{\sqrt{11}}$ $\alpha=58,52^0$

$\vec{c}=\vec{AB}=\vec{OB}-\vec{OA}=(3,-1,-1)$ $c=\sqrt{11}$

$\beta=58,25^0$

$\vec{b}=\vec{AC}=\vec{OC}-\vec{OA}=(1,1,-3)$ $b=\sqrt{11}$

$\gamma=62,96^0$

Zad 49 $\vec a = \hat\imath + 3\hat\jmath \quad \vec b = 2\hat\imath - \sqrt6\,\hat\jmath$

$$(\vec a + \vec b)(\vec a - \vec b) = \vec a^2 - \vec b^2 \quad \rightarrow \quad |\vec a| = |\vec b| \Rightarrow \vec a - \vec b \perp \vec a + \vec b$$

$$= |\vec a|^2 - |\vec b|^2$$

$$= \sqrt{1+9}^{\,2} - \sqrt{4+6}^{\,2} = 0$$

Zad 50.

$\vec a = (1, y, z) \quad \vec b = (1, -2, 1) \quad \vec c = (-1, 1, 2)$

$$\vec a = \alpha \vec b \times \vec c = \alpha \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 1 & -2 & 1 \\ -1 & 1 & 2 \end{vmatrix} = \alpha\left[\hat\imath \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} - \hat\jmath \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} + \hat k \begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix}\right]$$

$$= \alpha(-5\hat\imath - 3\hat\jmath - \hat k) \rightarrow \alpha = -\frac{1}{5}$$

$$= \hat\imath + \underbrace{\left(\frac{3}{5}\right)}_{x}\hat\jmath + \underbrace{\left(\frac{1}{5}\right)}_{z}\hat k$$

Zad 51

$\vec a = 4\hat\imath + 2\hat\jmath \qquad |\vec b| = \sqrt5 \qquad \vec b = \alpha\hat\imath + \beta\hat\jmath$

$\vec a \cdot \vec b = 0 \qquad 4\alpha + 2\beta = 0 \qquad \alpha^2 + \beta^2 = 5$

$2\alpha + \beta = 0 \qquad \alpha^2 + 4\alpha^2 = 5$

$\beta = -2\alpha \qquad \alpha^2 = 1$

$\alpha = \pm 1 \quad \beta = \mp 2$

Zad 52

A) $\vec a \perp \hat\imath \quad \vec a \perp \hat k$

$\vec a = \pm \hat k$

B) $\vec a = \alpha\hat\imath + \beta\hat\jmath + \gamma\hat k \qquad \vec a \cdot \vec b = 0 \qquad \vec a \cdot \vec c = 0$

$\vec b = \hat\imath + \hat\jmath + \hat k \qquad \alpha + \beta + \gamma = 0 \qquad \alpha = 0$

$\vec c = \hat\imath$

$\beta = -\gamma \qquad \vec a_1 = \beta\hat\jmath - \beta\hat k$

$\hat a_1 = \dfrac{\beta(\hat\jmath - \hat k)}{\sqrt2\,\beta} \qquad \vec a_2 = -\gamma\hat\jmath + \gamma\hat k$

$\hat a_1 = \frac{1}{\sqrt2}(\hat\jmath - \hat k) \qquad \hat a_2 = \frac{1}{\sqrt2}(-\hat\jmath + \hat k)$

c) $\vec a = \alpha\hat\imath + \beta\hat\jmath + \gamma\hat k$

$\vec b = \hat\imath + \hat\jmath \qquad \vec a \cdot \vec b = 0 \qquad \vec a \cdot \vec c = 0$

$\vec c = \hat\imath - \hat\jmath \qquad \alpha + \beta = 0 \qquad \alpha - \beta = 0$

$\beta = 0 \qquad \alpha = 0$

$\vec a = \gamma\hat k$

$\hat a = \pm\hat k$

d) $\vec a = \alpha\hat\imath + \beta\hat\jmath + \gamma\hat k \qquad \vec a \cdot \vec b = 0 \qquad \vec a \cdot \vec c = 0$

$\vec b = \hat\imath + \hat\jmath - \hat k \qquad \alpha + \beta - \gamma = 0 \qquad \beta + 2\gamma = 0$

$\vec c = \hat\jmath + 2\hat k \qquad \alpha = 3\gamma \qquad \beta = -2\gamma$

$\vec a = 3\gamma\hat\imath - 2\gamma\hat\jmath + \gamma\hat k$

$$\hat a = \frac{\gamma\cdot(3\hat\imath - 2\hat\jmath + \hat k)}{\mathrm{sgn}(\gamma)\sqrt{14}} = \pm\frac{1}{\sqrt{14}}(3\hat\imath - 2\hat\jmath + \hat k)$$

Zad 53. $A(-2,-5)$, $B(5,-1)$, $C(2,3)$ $D(-3,3)$

$\vec{AC} = 4\vec{i} + 8\vec{j}$ $\quad \vec{BD} = -8\vec{i} + 4\vec{j}$

$\vec{AC} \cdot \vec{BD} = -48 + 48 = 0$ Q.E.D.

Zad 54.

$A(3,1,\lambda)$ $B(2,-1,4)$ $C(1,2,3)$

$\vec{AC} = -2\vec{i} + \vec{j} + (3-\lambda)\vec{k}$ $\quad \vec{BC} = -\vec{i} + 3\vec{j} - \vec{k}$

$\vec{AC} \cdot \vec{BC} = 2 + \cancel{3} + \lambda - \cancel{3} = 0$

$\lambda = -2$

Zad 55.

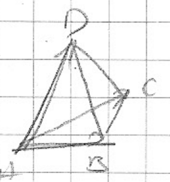
$A = (2,-1,5)$ $B(0,2,-1)$ $C(-2,4,3)$ $D(\alpha,\beta,\gamma)$

$\vec{AB} = \vec{DC}$ $\quad (-2,3,-6) = (-2-\alpha, 4-\beta, 3-\gamma)$

$-2-\alpha = -2$ $\quad 4-\beta = 3$ $\quad 3-\gamma = -6$

$\alpha = 0$ $\quad \beta = 1$ $\quad \gamma = 9$ $\quad D(0,1,9)$

Zad 56.



a) $A(1,1,1)$ $B(6,3,1)$ $C(3,6,1)$ $D(2,3,5)$

$\vec{AB} = 5\vec{i} + 2\vec{j}$

$\vec{AC} = 2\vec{i} + 5\vec{j}$

$\vec{AD} = \vec{i} + 2\vec{j} + 4\vec{k}$

$V = (\vec{AB} \times \vec{AC}) \cdot \vec{AD} = \begin{vmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 1 & 2 & 4 \end{vmatrix} = 4 \begin{vmatrix} 5 & 2 \\ 2 & 5 \end{vmatrix} = 4 \cdot (25-4) = 4 \cdot 21 = 84$

paralelepipeda

$V_T = \frac{1}{6} V = \frac{1}{6} \cdot 84 = 14$

b) $A(1,1,1)$ $B(2,3,1)$ $C(1,-1,2)$ $D(3,-1,1)$

$\vec{AB} = (1,2,0)$

$\vec{AC} = (0,-2,1)$

$\vec{AD} = (2,-2,0)$

$V_T = \frac{1}{6} \begin{vmatrix} 1 & 2 & 0 \\ 0 & -2 & 1 \\ 2 & -2 & 0 \end{vmatrix} = -\frac{1}{6} \begin{vmatrix} 1 & 2 \\ 2 & -2 \end{vmatrix} = -\frac{1}{6}(-2-4) = -\frac{1}{6} \cdot -6 = 1$

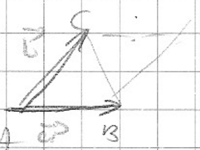
Zad 57 točke leže u istoj ravnini ako je volumen paralelepipeda = 0

$A(-3,2,4)$ $B(6,5,10)$ $C(9,1,4)$ $D(3,-1,0)$

$\vec{AB} = (9,3,6)$ $\quad \vec{AC} = (12,-1,0)$ $\quad \vec{AD} = (6,-3,-4)$

$V = \begin{vmatrix} 9 & 3 & 6 \\ 12 & -1 & 0 \\ 6 & -3 & -4 \end{vmatrix} = 9 \cdot \begin{vmatrix} 1 & 1 & 2 \\ 4 & -1 & 0 \\ 2 & -3 & -4 \end{vmatrix} = 9 \begin{vmatrix} 1 & 1 & 2 \\ 0 & -5 & -8 \\ 0 & -5 & -8 \end{vmatrix} = 0$

Zad 38.



a) $A(4,4,4)$

$B(2,4,2)$

$C(3,3,6)$

$P_\Delta = \frac{1}{2} P_{\square} = \frac{1}{2} |\vec a \times \vec b|$

$\vec a = \vec{AB} = (-2,0,-2)$

$\vec b = \vec{AC} = (-1,-1,2)$

$$\vec a \times \vec b = \begin{vmatrix} \vec i & \vec j & \vec k \\ -2 & 0 & -2 \\ -1 & -1 & 2 \end{vmatrix} = \vec i \begin{vmatrix} 0 & -2 \\ -1 & 2 \end{vmatrix} - \vec j \begin{vmatrix} -2 & -2 \\ -1 & 2 \end{vmatrix} + \vec k \begin{vmatrix} -2 & 0 \\ -1 & -1 \end{vmatrix}$$

$$= -2\vec i + 6\vec j - 2\vec k$$

$$\frac{1}{2}|\vec a \times \vec b| = \frac{1}{2}\sqrt{4+36+4} = \frac{1}{2}\sqrt{44} = \sqrt{11}$$

b) $A(4,-2,6)$

$B(6,-1,7)$

$C(5,0,5)$

$\vec a = (2,1,1)$

$\vec b = (1,2,-1)$

$$\vec a \times \vec b = \begin{vmatrix} \vec i & \vec j & \vec k \\ 2 & 1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = \vec i \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} - \vec j \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + \vec k \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= -3\vec i + 3\vec j + 3\vec k$$

$$\frac{1}{2}|\vec a \times \vec b| = \frac{1}{2}\sqrt{9+9+9} = \frac{3}{2}\sqrt{3}$$

Zad 59

$\vec a = (2,1,0)$ $\quad \vec a = \alpha \vec b + \beta \vec c + \gamma \vec d$

$\vec b = (1,-1,-2)$ $\quad (2,1,0) = (\alpha + 2\beta + 3\gamma,\ -\alpha + 2\beta + 8\gamma,\ -2\alpha - \beta + 5\gamma)$

$\vec c = (2,2,-1)$

$\vec d = (3,8,5)$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ -1 & 2 & 8 & 1 \\ -2 & -1 & 5 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 4 & 11 & 3 \\ 0 & 3 & 11 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 3 & 11 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 11 & 7 \end{array}\right]$$

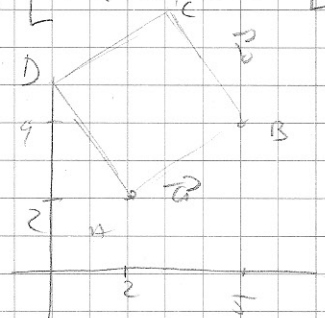
$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 23/11 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 7/11 \end{array}\right]$$

$\vec b$ ...

$$\left[\begin{array}{ccc|c} 2 & 2 & 3 & 1 \\ 1 & 2 & 8 & -1 \\ 0 & -1 & 5 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & -2 & -13 & 3 \\ 1 & 2 & 8 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 18 & -5 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & -23 & 5 \end{array}\right] \ldots$$

... itd.

Zad 60.



$A(2,2)$ $\quad B(5,4)$

$C(3,7)$ $\quad D(0,5)$

$\vec{AB} = (3,2)$ $\quad |\vec{AB}| = \sqrt{13}$ $\quad P = 13$

$\vec a = 3\vec i + 2\vec j$

$\vec a \cdot \vec b = 0$

$3\alpha + 2\beta = 0$

$\alpha = -\frac{2}{3}\beta$

$\vec b = \alpha \vec i + \beta \vec j$

$|\vec b| = |\vec a| = \sqrt{13}$

$\alpha^2 + \beta^2 = 13$

$\frac{4}{9}\beta^2 + \beta^2 = 13$

$\frac{13}{9}\beta^2 = 13$

$\beta^2 = 9$

$\beta = \pm 3$

$\alpha = \mp 2$

$\vec b = -2\vec i + 3\vec j$

$C = (5-2,\ 4+3)$

$= (3,7)$

$D = (3-3,\ 7-2) = (0,5)$

Zad 61 a) $(\vec{i}, \vec{j}, \vec{k})$ $(\vec{j}, \vec{k}, \vec{i})$ $(\vec{k}, \vec{i}, \vec{j})$

$$[\vec{i}, \vec{j}, \vec{k}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1 > 0 \rightarrow \text{desno}$$

$$[\vec{j}, \vec{k}, \vec{i}] = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 > 0 \text{ desno}$$

$$[\vec{k}, \vec{i}, \vec{j}] = \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 > 0 \text{ desno}$$

b) $(\vec{i}, \vec{j}, \vec{k})$, $(\vec{j}, \vec{i}, \vec{k})$

$$[\vec{i}, \vec{j}, \vec{k}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 > 0 \rightarrow \text{desno} \qquad [\vec{j}, \vec{i}, \vec{k}] = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 < 0$$
$\rightarrow$ lijevo

c) $(\vec{a}, \vec{b}, \vec{c})$ $(\vec{a}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c})$

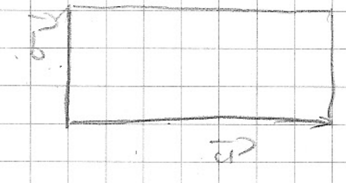
$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \qquad [\vec{a}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = \begin{vmatrix} a_x & a_y & a_z \\ a_x+b_x & a_y+b_y & a_z+b_z \\ a_x+b_x+c_x & a_y+b_y+c_y & a_z+b_z+c_z \end{vmatrix}$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \qquad \text{q.e.D}$$

Zad 62

$$(\vec{a}\cdot\vec{b})^2 + (\vec{a}\times\vec{b})^2 = \vec{a}^2\cdot\vec{b}^2$$

$$= \left[|\vec{a}||\vec{b}|\cos\varphi\right]^2 + \left[|\vec{a}||\vec{b}|\sin\varphi\right]^2 = |\vec{a}|^2|\vec{b}|^2 = \vec{a}^2\cdot\vec{b}^2$$



$\vec{a}\cdot\vec{b} = 0$ $\quad P = |\vec{a}|\cdot|\vec{b}| = \sqrt{(\vec{a}\cdot\vec{b})^2 + (\vec{a}\times\vec{b})^2}$

$= \sqrt{|\vec{a}|^2\cdot|\vec{b}|^2 \pm 0}$

$= \sqrt{a^2 b^2 - (\vec{a}\cdot\vec{b})^2}$ ...

Zad 63

a) $(\vec{a}+\vec{b})\cdot\left[(\vec{a}+\vec{c})\times\vec{b}\right] = -[\vec{a}, \vec{b}, \vec{c}]$

$$L = (\vec{a}+\vec{b})\cdot\left[\vec{a}\times\vec{b} + \vec{c}\times\vec{b}\right] = \underbrace{\vec{a}\cdot(\vec{a}\times\vec{b})}_{0} + \underbrace{\vec{a}\cdot(\vec{c}\times\vec{b})}_{-\vec{a}\cdot(\vec{b}\times\vec{c})} + \underbrace{\vec{b}\cdot(\vec{a}\times\vec{b})}_{0} + \underbrace{\vec{b}\cdot(\vec{c}\times\vec{b})}_{0}$$

$$-\vec{a}\cdot(\vec{b}\times\vec{c}) = -[\vec{a}, \vec{b}, \vec{c}]$$

$\downarrow$

b) $(\vec a + 2\vec b - \vec c)\,[(\vec a-\vec b)\times(\vec a-\vec b-\vec c)] = 3[\vec a,\vec b,\vec c]$

$L = (\vec a+2\vec b-\vec c)\cdot[\underbrace{(\vec a-\vec b)\times(\vec a-\vec b)}_{0} - (\vec a-\vec b)\times\vec c]$

$=(\vec a+2\vec b-\vec c)\,[\vec c\times(\vec a-\vec b)] = (\vec a+2\vec b-\vec c)\cdot(\vec c\times\vec a - \vec c\times\vec b)$

$= \underbrace{\vec a\cdot\vec c\times\vec a}_{0} + 2\,\vec b\cdot\vec c\times\vec a - \underbrace{\vec c\cdot\vec c\times\vec a}_{0} - \vec a\cdot\vec c\times\vec b + 0 + 0$

$= 2\cdot\vec b\cdot(\vec c\times\vec a) + \vec a\cdot(\vec b\times\vec c)$

$= 2[\vec c,\vec a,\vec b] + [\vec a,\vec b,\vec c] = 3[\vec a,\vec b,\vec c] = D$ Q.E.D.

c) $[(\vec a-\vec b)\times\vec c]\times[(\vec a+\vec b)\times\vec c] = 2[\vec a,\vec b,\vec c]\cdot\vec c$

$L = [\vec a\times\vec c - \vec b\times\vec c]\times[\vec a\times\vec c+\vec b\times\vec c] =$

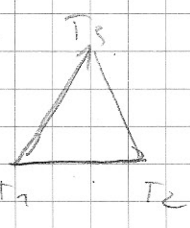
$= \underbrace{(\vec a\times\vec c)\times(\vec a\times\vec c)}_{0} + (\vec a\times\vec c)\times(\vec b\times\vec c) - (\vec b\times\vec c)\times(\vec a\times\vec c) + 0$

$= 2\,(\vec a\times\vec c)\times(\vec b\times\vec c)$

$= 2\cdot[\vec c\cdot\vec a\cdot(\vec b\times\vec c) - \vec a\cdot\vec c\cdot\underbrace{(\vec b\times\vec c)}_{0}] = D$

$\boxed{(\vec a\times\vec b)\times\vec c = (\vec a\cdot\vec c)\cdot\vec b - (\vec b\cdot\vec c\cdot\vec a)}$ → dvostruki umnožak

Zad 65.



$D = \frac{1}{2}\,|\vec{T_1T_2}\times\vec{T_1T_3}|$

$T_1(x_1, y_1)$

$T_2(x_2, y_2)$

$T_3(x_3, y_3)$

$\vec{T_1T_2}\times\vec{T_1T_3} = \begin{vmatrix} \vec i & \vec j & \vec k\\ x_2-x_1 & y_2-y_1 & 0\\ x_3-x_1 & y_3-y_1 & 0\end{vmatrix}$

$\vec{T_1T_2} = (x_2-x_1,\ y_2-y_1)$

$\vec{T_1T_3} = (x_3-x_1,\ y_3-y_1)$

$= \vec k\cdot\begin{vmatrix} x_2-x_1 & y_2-y_1\\ x_3-x_1 & y_3-y_1\end{vmatrix}$

$\frac{1}{2}\,|\vec{T_1T_2}\times\vec{T_1T_3}| = \frac{1}{2}\,\underbrace{|\vec k|}_{1}\,\begin{vmatrix} x_2-x_1 & y_2-y_1\\ x_3-x_1 & y_3-y_1\end{vmatrix}$

$\begin{vmatrix} x_2-x_1 & y_2-y_1\\ x_3-x_1 & x_3-y_1\end{vmatrix} = \begin{vmatrix} 0 & 0 & 1\\ x_2-x_1 & y_2-x_1 & 1\\ x_3-x_1 & y_3-x_1 & 1\end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$ Q.E.D.

$1 \quad \cdot x_1 \quad \cdot y_1 \quad +$