~~Digitalna~~ vježba

Mat - masovne

DEF.

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{x=0}^{n} x^i = a_1 \cdot \frac{x^{n+1}-1}{x-1}$$

1) $\lim_{\to\infty} (a_n \pm b_n) = \lim_{\to\infty} a_n \pm \lim_{\to\infty} b_n$

2) $\lim_{\to\infty} (a_n \cdot b_n) = \lim_{\to\infty} a_n \cdot \lim_{\to\infty} b_n$

1. $\lim_{n\to\infty} \frac{3n+1}{4n^2+1} \begin{matrix} /n^2 \\ /n^2 \end{matrix} = \lim_{n\to\infty} \frac{\frac{3}{n}+\frac{1}{n^2}}{4+\frac{1}{n^2}} = \frac{0}{4} = 0$

2. $\lim_{n\to\infty} \frac{n^3+n^2+1}{n^2+1} \begin{matrix} /:n^3 \\ /:n^3 \end{matrix} = \frac{1+\frac{1}{n}+\frac{1}{n^3}}{\frac{1}{n}+\frac{1}{n^3}} = \frac{1}{0} = \infty$

3. $\lim_{n\to\infty} \frac{n^2+1}{n+\sqrt{n^3+1}} \begin{matrix} /:n^2 \\ /:n^2 \end{matrix} = \frac{1+\frac{1}{n^2}}{\frac{1}{n}+\sqrt{\frac{1}{n}+\frac{1}{n^4}}} = \frac{1}{0} = \infty$

4. $\lim_{n\to\infty} \frac{3^n+4^n}{2^n+5^n} \begin{matrix} /:5^n \\ /:5^n \end{matrix} = \frac{\left(\frac{3}{5}\right)^n+\left(\frac{4}{5}\right)^n}{\left(\frac{2}{5}\right)^n+1} = \frac{0}{1} = 0$

→ bitno je da su ovi razlomci < 1 !

5. $\lim_{n\to\infty} \left(n - \frac{(n+1)^2}{(n+1)^3}\right) = \lim_{n\to\infty} \left(\frac{n(n^3+3n^2+3n+1)-n^2+2n+1}{(n+1)^3}\right) = \infty$

6. $\lim_{n\to\infty} \frac{1+2^2+3^2+\ldots+n^2}{n^3} = \lim_{n\to\infty} \frac{\boxed{\frac{n\cdot(n+1)(2n+1)}{6}}}{n^3}$! FORMULA

$= \lim_{n\to\infty} \frac{2n^3+n^2+\ldots}{6n^3} \begin{matrix} /:n^3 \\ /:n^3 \end{matrix} = \frac{2}{6} = \frac{1}{3}$

7.

$$\boxed{\lim_{n\to\pm\infty}\left(1+\frac{1}{n}\right)^n = e}$$ DEF.

7. $\lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^{2n+1} = \lim_{n\to\infty}\left(1+\frac{-1}{\frac{n+2}{1}}\right)^{(2n+1)\left(\frac{n+2}{n+2}\right)} = e^{-2}$

8. $\lim_{n\to\infty}\left(\frac{3n}{3n+1}\right)^{n} = \lim_{n\to\infty}\left(\frac{3n+1-1}{3n+1}\right)^{n} = \lim_{n\to\infty}\left(1-\frac{1}{3n+1}\right)^{n\cdot\frac{3n+1}{3n+1}} = e^{-\frac{1}{3}}$

10.

11.

9. $a_n = \sqrt{2\sqrt{2\ldots\sqrt{2}}}$ → odredi je li konv. ako je, nađi limes

$a_1 = \sqrt{2}$

$a_2 = \sqrt{2\sqrt{2}} = 1.68 > a_1$ ⇒ niz je rastući

pretp. da je cijeli niz $< 2 \to a_n < 2$

$a_{n+1} = \sqrt{2a_n}$

$L = \lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$

$\lim_{n\to\infty}\sqrt{2a_n} = \sqrt{2\underbrace{\lim_{n\to\infty} a_n}_{L}} = \underbrace{\lim_{n\to\infty} a_{n+1}}_{L}$ 12.

$\sqrt{2L} = L \quad /^2$

$2L = L^2$

$L = 2 \qquad L \neq 0$

Def:

$$\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$$! bitno za ispit (dokaz)

$$\lim_{n\to 0} \frac{\sin(n)}{n} = 1$$

10. $$\lim_{n\to\infty} \frac{n\sin(n)}{n^2+n+1} \begin{matrix} /:n^2 \\ /:n^2 \end{matrix} = \lim_{n\to\infty} \frac{\frac{\sin(n)}{n}}{1+\frac{1}{n}+\frac{1}{n^2}} = \frac{0}{1}$$

$\frac{\sin(n)}{n} \to 0$ po definiciji

11. $$\lim_{n\to\infty} \left(\sqrt[4]{n^4+n^3+n^2+1} - n\right) \cdot \frac{\sqrt[4]{n^4+n^3+n^2+1} + n}{\text{---}\,||\,\text{---}}$$ → racionalizacija

$$= \lim_{n\to\infty} \frac{\sqrt{n^4+n^3+n^2+1} - n^2}{\sqrt[4]{n^4+n^3+n^2+1} + n} \cdot \frac{\sqrt{n^4+n^3+n^2+1} + n^2}{\text{---}\,||\,\text{---}}$$ → opet racionaliz.

→ razlika kvadrata

$$= \lim_{n\to\infty} \frac{\cancel{n^4}+n^3+n^2+1-\cancel{n^4}}{\left(\sqrt[4]{n^4+n^3+n^2+1}+n\right)\cdot\left(\sqrt{n^4+n^3+n^2+1}+n^2\right)} \begin{matrix} /:n^3 \\ /:n^3 \end{matrix}$$

$$= \frac{1}{4}$$

$\frac{\infty}{\infty}$ → prijc. dijeljenja s n^3

→ sve smo racionalizirali jer imamo oblik $\infty - \infty$ koji je nedefiniran

12. traženje konvergencije:

$a_1 = 1$

$$a_{n+1} = 3 - \frac{1}{a_n+3}$$

$$a_{n+1} = 3 - \frac{1}{4} = \frac{11}{4} > a_1$$ → rastući niz

pretp: $a_n < 3$

$$L = \lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$$

$$L = 3 - \frac{1}{L+3}$$

$$L = 2\sqrt{2}$$ → odgovara pretpostavci

7. knjižica

$0^-, 0^+$ oko 0; $4^-, 4^+$ oko 4 → ista stvar vrijedi za sve brojeve

1. $$\lim_{x\to-\infty}\frac{2x^2+1}{3x^2+3x+4} = \lim_{x\to+\infty}\frac{2x^2+1}{3x^2-3x+4}\ \begin{matrix}/:x^2\\ /:x^2\end{matrix} = \frac{2}{3}$$

na neparne potencije djeluje ⊖

2. $$\lim_{x\to-\infty}\frac{2x+1}{\sqrt{4x^2+3}} = \lim_{x\to\infty}\frac{-2x+1}{\sqrt{4x^2+3}}\ \begin{matrix}/:x\\ /:x\end{matrix} = -1$$

3. $$\lim_{x\to-\infty}\frac{\sqrt{x^4+x^3+1}-x^2}{x+1} = \lim_{x\to\infty}\frac{\sqrt{x^4-x^3+1}-x^2}{-x+1}$$ → racionalizacija jer je $\infty-\infty$

$$= \lim_{x\to\infty}\frac{\sqrt{x^4-x^3+1}-x^2}{-x+1}\cdot\frac{\sqrt{x^4-x^3+1}+x^2}{-\|-}$$

$$= \lim_{x\to\infty}\frac{\cancel{x^4}-x^3+1-\cancel{x^4}}{-x\sqrt{x^4-x^3+1}-x^3+\sqrt{x^4-x^3+1}+x^2} = \lim_{x\to\infty}\frac{-x^2+1}{\cdots\cdots}\ \begin{matrix}/:x^3\\ /:x^3\end{matrix}$$

$$= \frac{-1}{-1-1} = \frac{1}{2}$$

4. $$\lim_{x\to-2^-}\frac{x+1}{x+2} = \lim_{x\to-2^-}\frac{-2-0+1}{-2-0+2} = \lim_{x\to-2^-}\frac{-1}{0^-} = \infty$$

$0^- = -0$

$x\to-2^-$ znači da teži -2 s lijeve strane

kada je broj s lijeve strane pišemo $x-0$

tj: $-2-0$

kada je s desne:

$x+0$

tj. $2+0$

$$5.\ \lim_{x\to 1^+} \frac{x^2-3x+2}{x^2-2x+1} = \lim_{x\to 1^+} \frac{x-2}{x-1} = \lim_{x\to 1^+} \frac{1+0-2}{1+0-1} = \frac{-1}{0^+} = -\infty$$

$x^2-3x+2 \to (x-1)(x-2)$

$x^2-2x+1 \to (x-1)^2$

$$6.\ \lim_{x\to 2^+} = \operatorname{arc\,tg}\left(\frac{x+1}{x-2}\right) = \lim_{x\to 2^+} \operatorname{arc\,tg}\left(\frac{3}{2^+-2}\right)$$

$$= \operatorname{arc\,tg}(\infty) = \frac{\pi}{2}$$

$$7.\ \lim_{x\to 0} \frac{\arcsin(2x)}{\arcsin(3x)} = \lim_{x\to 0} \frac{2x}{3x} = \frac{2}{3}$$

$\to$ aproksimacija

$$\underbrace{\frac{\arcsin 2x}{\cancel{2x}}}_{=1} \cdot \cancel{2x} = 2x$$

$$8.\ \lim_{x\to 0} \frac{\sin(5x)\cdot \sin 3x}{\sin^2(4x)} = \lim_{x\to 0} \frac{15x^2}{16x^2} = \frac{15}{16}$$

$\to$ aproks