2

a) ISTE DNJA

1) k)

8

$u(n) = 2^n u(n)$

$y(n) - 4y(n-1) + 4y(n-2) = u(n)$

$y(z) - 4z^{-1}y(z) + 4z^{-2}y(z) = u(z)$

$H(z) = \frac{y(z)}{u(z)} = \frac{1}{1 - 4z^{-1} + 4z^{-2}} = \frac{z^2}{z^2 - 4z + 4}$

b) $z^2 - 4z + 4 = 0$ nule

$p_{1,2} = 2$ $n_{1,2} = 0$

nestabilan

c) $H(z) \circ\!\!-\!\!\bullet\, h(n)$

$H(z) = \frac{z^2}{z^2 - 4z + 4}$

$U(z) = \frac{z^2}{(z-2)^2}$

$h(n) = (n+1)2^n$

d) $y(z) = H(z)\, u(z)$

$H(z) = \frac{z^2}{(z-2)^2}$ $u(z) = \frac{z}{z-1}$

$\frac{y(z)}{z} = \frac{z^2}{(z-2)^2(z-1)} = \frac{Az+B}{(z-2)^2} + \frac{C}{z-1}$

$$(Az+B)(z-1)+C(z-2)^2 = z^2$$

$$Az^2+Bz-Az-B+Cz^2-4zC+4C=z^2$$

$$A+C=1 \qquad -8c+c=1 \qquad c=-\frac{1}{7}$$

$$B-A-4C=0 \qquad -4c-A-4c=0$$

$$B+4c=0 \qquad A=-8c$$

$$B=-4c \qquad =\frac{8}{7}$$

$$=\frac{4}{7}$$

$$\frac{Y(z)}{z} = \frac{1}{7}\left(\frac{8z+4}{(z-2)^2} - \frac{1}{(z-1)}\right)$$

$$y(z) = \frac{8}{7}\frac{z^2}{(z-2)^2} + \frac{4}{7}\frac{z}{(z-2)^2} - \frac{1}{7}\frac{z}{z-1}$$

$$y(n) = \left(\frac{8}{7}(n+1)2^n + \frac{4}{7}n(n+1)2^n - \frac{1}{7}1^n\right)\mu(n)$$

$$u(n) = 2^n\mu(n) \Rightarrow U(z) = \frac{z}{z-2}$$

$$Y(z) = H(z)\cdot U(z) = \frac{z^3}{(z-2)^3}$$

$$y(n) = \left(\frac{(n+1)(n+2)}{2!}2^n\right)\mu(n)$$

3) t)

$u(t) = e^{2t}\mu(t)$

s)

$y''(t) - 5y'(t) + 6y(t) = u(t)$

$s^2 y(s) - 5sy(s) + 6y(s) = u(s)$

$H(s) \quad \frac{1}{s^2 - 5s + 6}$

b) nule

$n_{1,2} = \infty$

pobovi

$s^2 - 5s + 6 = 0$

$s_1 = 3 \quad s_2 = 2$

c) $H(s) = \frac{1}{(s-3)(s-2)}$

$a = -3$

$b = -2$

$h(t) = \frac{1}{-2+3}\left(e^{3t} - e^{2t}\right)\mu(t)$

d) $u(t) = \mu(t)$

$u(s) = \frac{1}{s}$

$y(t) = \left(\frac{1}{3}e^{3t} - \frac{1}{2}e^{2t} + \frac{1}{6}\right)\mu(t)$

$y(s) = h(s) \cdot u(s) = \frac{1}{(s-3)(s-2)s}$

$= \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s}$

$A(s-2)s + B(s-3)s + C(s-3)(s-2) = 1$

$As^2 - 2As + Bs^2 - 3Bs + Cs^2 - 5Cs + 6C = 1$

$A + B + C = 0 \quad / 2$

$-2A - 2B - 2C = 0$

$-3A - 3B - 5C = 0$

$6C = 1$

$C = \frac{1}{6}$

$-B - 3C = 0$

$-B = 3C \quad B = -\frac{1}{2}$

$A = -B - C = \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{1}{3}$

i)

$$u(t) = e^{2t}\mu(t)$$

$$u(s) = \frac{1}{s-2}$$

$$y(s) = \frac{1}{(s-2)^2(s-3)}$$

$$(As+B)(s-3) + C(s-2)^2 = 1$$

$$As^2 + Bs - 3As - 3B + Cs^2 - 4Cs + 4C = 1$$

$$A + C = 0 \qquad -3B + 4C = 1$$

$$A = -C$$

$$B - 3A - 4C = 0 \qquad C = 1 = B$$

$$B - C = 0 \qquad A = -1$$

$$B = C$$

$$y(t) = \left(te^{2t} + e^{3t}\right)\mu(t)$$