

11.) NOVČIĆ I KOCKA

$$A = \{ \text{pojavili se grb i 6-ica} \} \quad P(A) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} = 0,083$$

$$B = \{ \text{pojavili se grb ili 6-ica} \} \quad P(B) = \frac{1}{2} + \frac{1}{6} = 0,66$$

$$C = \{ \text{na kocki broj veći od 4} \} \quad P(C) = \frac{2}{2} \cdot \frac{2}{6} = \frac{1}{3} = 0,3$$

bilo koji ishod na kocki

$$12.) \text{ STRIJELAC POGADA} = 0,6 \rightarrow \text{STRIJELAC PROMASUJE} = 0,4$$

$$A = \{ \text{meta pogodena u 3. polovnjaju} \} \quad P(A) = 0,4 \cdot 0,4 \cdot 0,6 = 0,096$$

$$B = \{ \text{meta pogodena u prva 3 polovnjaju} \} \quad P(B) = 0,6 + 0,4 \cdot 0,6 + 0,4 \cdot 0,4 \cdot 0,6 = 0,936$$

$$C = \{ \text{meta pogodena nakon 5. polovnjaju} \} \quad P(C) = 1 - 0,4^5 = 0,9897$$

$$13.) 2 \text{ KOCKE}$$

$$A = \{ \text{barem jedna 1} \} \quad P(A) = \frac{2 \cdot 6}{36} = \frac{1}{3}$$

$$B = \{ \text{dva različita broja} \} \quad P(B) = \frac{36-6}{36} = \frac{5}{6}$$

$P(AB) \rightarrow \text{barem jedna 1 i dva različita broja}$

$$\text{NEZAVISNOST: } P(AB) = P(A) \cdot P(B) \Rightarrow \frac{1}{6} \neq \frac{1}{3} \cdot \frac{5}{6} = \frac{5}{18} \rightarrow \underline{\text{NISU NEZAVISNI}}$$

14.) 1, 2, 3, 4, 5

slučajni odabir, vj. da je drugi broj paran

$$3 \text{ neparna} \rightarrow 1, 3, 5 \quad H_1 = \{ 1. \text{ je paran} \} \quad P(H_1) = \frac{2}{5}$$

$$2 \text{ parna} \rightarrow 2, 4 \quad H_2 = \{ 1. \text{ je neparan} \} \quad P(H_2) = \frac{3}{5}$$

$$A = \{ 2. \text{ je paran} \} \quad P(A|H_1) = \frac{1}{4}, \quad P(A|H_2) = \frac{2}{4}$$

$$P(A) = \sum_i P(H_i)P(A|H_i) = \frac{2}{5} \cdot \frac{1}{4} + \frac{3}{5} \cdot \frac{2}{4} = \underline{0,4}$$

15.) 10 STRIJELACA

$$\begin{array}{c} \downarrow \text{ODLIČNA} \\ \checkmark \end{array} \quad \begin{array}{c} \downarrow \text{DOBRIH} \\ 6 \end{array}$$

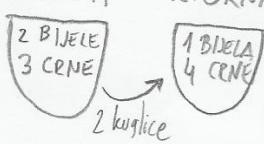
POGODAK = 0,9 POGODAK = 0,7

$$P(H_1) = \frac{6}{10} \quad P(A|H_1) = 0,7$$

$$P(H_2) = \frac{4}{10} \quad P(A|H_2) = 0,9$$

$$P(A) = \frac{6}{10} \cdot 0,7 + \frac{4}{10} \cdot 0,9 = \frac{78}{100} = \underline{0,78}$$

16.) 1. URNA



$$H_1 = \{ \text{iz 1. urne izvučene 2 bijele} \} \quad P(H_1) = \frac{2}{5} \cdot \frac{1}{4} \quad P(A|H_1) = \frac{3}{7}$$

$$H_2 = \{ \text{iz 1. urne izvučene 1 bijela i 1 crna} \} \quad P(H_2) = \frac{2}{5} \cdot \frac{3}{4} \cdot 2 \rightarrow \text{poredale} \quad P(A|H_2) = \frac{2}{7}$$

$$H_3 = \{ \text{iz 1. urne izvučene 2 crne} \} \quad P(H_3) = \frac{3}{5} \cdot \frac{2}{4} \quad P(A|H_3) = \frac{1}{4}$$

$$A = \{ \text{iz 2. izvučena bijela} \} \quad P(A) = \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{3}{7} + 2 \cdot \frac{2}{5} \cdot \frac{3}{4} \cdot \frac{2}{7} + \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{4} = \underline{0,25714}$$

17.)

| | | |
|----------|----------|-----------|
| 2 CRNE | 2 CRNE | 1 CRNA |
| 2 BIJELE | 2 BIJELE | 5 BIJELIH |

 \rightarrow izvučena je bijela kuglica $A = \{\text{izvučena bijela iz 3.-pregrade}\}$

$$\begin{aligned} P(\text{bijela iz 1.}) &= \frac{2}{4} = \frac{1}{2} \\ P(\text{bijela iz 2.}) &= \frac{1}{2} \\ P(\text{bijela iz 3.}) &= \frac{5}{6} \end{aligned}$$

$$P(A) = \frac{P(\text{bijela iz 3.})}{P(\text{bijela iz 1.}) + P(\text{bijela iz 2.}) + P(\text{bijela iz 3.})} = \frac{\frac{5}{6}}{\frac{1}{2} + \frac{1}{2} + \frac{5}{6}} = \frac{\frac{5}{6}}{\frac{11}{6}} = \underline{\underline{0,45}}$$

18.)

| |
|---------|
| 1000 |
| KOCKICA |

, 999 ISPEAVNIH, 1IMA SVE STRANE 6-ICE

 \rightarrow izvučena je kockica i bacena 4 puta \rightarrow sva 4 puta je pala na broj 6 \Rightarrow uj-daju li to neispravna?

$$\begin{aligned} P(\text{izvučena je neispravna}) &= \frac{1}{1000} \quad \rightarrow P(\text{pala je na 6-icu 4 puta}) = 1 \\ P(\text{izvučena je ispravna}) &= \frac{999}{1000} \quad \rightarrow P(\text{pala je na 6-icu 4 puta}) = \frac{1}{6^4} \end{aligned}$$

$$P(\text{neispravna}) = \frac{1 \cdot \frac{1}{1000}}{1 \cdot \frac{1}{1000} + \frac{1}{6^4} \cdot \frac{999}{1000}} = \underline{\underline{0,56}}$$

23.) 3 KOCKE

$$A = \{\text{pala su tri različita broja}\} \quad P(A) = \frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} = \frac{6 \cdot 5 \cdot 4}{6^3} = \frac{5}{9} = \underline{\underline{0,55}}$$

$$B = \{\text{pala je barem jedna 6-ica}\} \quad P(B) = 1 - \text{šestica} = 1 - \frac{5^3}{6^3} = \frac{91}{216} = \underline{\underline{0,421}}$$

 $P(AB) \Rightarrow$ presjek A i B događaja $\{\text{pala su tri različita broja od kojih je jedna 6-ica}\}$

$$P(AB) = \frac{\binom{3}{1} \cdot 1 \cdot 5 \cdot 4}{6^3} = \frac{5}{18} = \underline{\underline{0,27}}$$

biramo na koju od
3 kocke će biti 6-ica!

oprez: TOČNO JEDNA !! JER MORAU BITI
SVI BROJEVI RAZLIČITI

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{5}{18}}{\frac{91}{216}} = \frac{1080}{1638} = \frac{60}{91} = \underline{\underline{0,659}}$$

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{5}{18}}{\frac{5}{9}} = \frac{1}{2} = \underline{\underline{0,5}}$$

24.) 6 KOCKI

$$A = \{ \text{pale su tri različita broja} \} \quad P(A) = \frac{\binom{6}{2} \cdot \binom{4}{2} \cdot \binom{2}{2} \cdot 4^2}{6^6} = \underline{\underline{0,0309}}$$

$$B = \{ \text{svi brojevi manji od } 4 \} \quad P(B) = \frac{3^6}{6^6} = \left(\frac{1}{2}\right)^6 = \underline{\underline{0,015625}}$$

$$C = \{ \text{zbroj brojeva na kockama manji od } 9 \}$$

POVOĐUJU IŠHODI:

| | | | |
|----|-----------------------|------------------|-------------------|
| a) | $1+1+1+1+1+1 = 6 < 9$ | (1 kombinacija) | $\frac{6!}{5!}$ |
| b) | $1+1+1+1+1+2 = 7 < 9$ | (6 kombinacija) | $\frac{6!}{5!}$ |
| c) | $1+1+1+1+1+3 = 8 < 9$ | (6 kombinacija) | $\frac{6!}{5!}$ |
| d) | $1+1+1+1+2+2 = 8 < 9$ | (15 kombinacija) | $\frac{6!}{4!2!}$ |

$$P(C) = \frac{28}{6^6} = \underline{\underline{0,0006}} \quad = 28 \text{ kombinacija}$$

$$P(AB) = \{ \text{tri različita broja i da su sva tri manja od } 9 \} \quad P(AB) = \frac{90}{6^6} = \underline{\underline{0,001929}}$$

$$P(B|A) = \frac{P(AB)}{P(A)} = \underline{\underline{0,123456}} \quad ? \quad (\text{MOŽDA JE KRIVO})$$

25.) 4 KOCKE

$$A = \{ \text{pale su dvoje 1-ice i jedna 2} \}$$

$$P(A) = \frac{\binom{4}{2} \cdot 1 \cdot \binom{2}{1} \cdot 1 \cdot 4}{6^4} = \underline{\underline{0,037}}$$

$$B = \{ \text{svi brojevi manji od } 4 \}$$

$$P(B) = \frac{3^4}{6^4} = \underline{\underline{0,0625}}$$

$$C = \{ \text{zbroj brojeva veći od } 6 \}$$

$$P(C) = 1 - \frac{\text{brojevi manji od } 6}{\text{celokomb. } (\leq 6)} = 1 - \frac{15}{6^4} = \underline{\underline{0,9884}}$$

| | | |
|----|----------------------|----------|
| a) | $1+1+1+1 = 4 \leq 6$ | (1 komb) |
| b) | $1+1+1+2 = 5 \leq 6$ | (4 komb) |
| c) | $1+1+1+3 = 6 \leq 6$ | (4 komb) |
| d) | $1+1+2+2 = 6 \leq 6$ | (6 komb) |

15 komb

$$P(CB) = \{ \text{zbroj brojeva veći od } 6 \text{ i sv. brojevi manji od } 4 \}$$

SVE KOMBINACIJE

$$\begin{aligned} 2221 &= 7 > 6 \quad (4 \text{ komb}) \\ 2222 &= 8 > 6 \quad (1 \text{ komb}) \\ 3333 &\rightarrow (1 \text{ komb}) \\ 3332 &\rightarrow (4 \text{ komb}) \\ 3322 &\rightarrow (6 \text{ komb}) \\ 3222 &\rightarrow (4 \text{ komb}) \\ 3331 &\rightarrow (4 \text{ komb}) \\ 3311 &\rightarrow (6 \text{ komb}) \\ 3211 &\rightarrow (12 \text{ komb}) \\ 3221 &\rightarrow (12 \text{ komb}) \\ 3321 &\rightarrow (12 \text{ komb}) \\ &\hline 66 \text{ komb} \end{aligned}$$

$$P(CB) = \frac{66}{6^4} = \underline{\underline{0,0509}}$$

$$P(C|B) = \frac{P(CB)}{P(B)} = \frac{22}{27} = \underline{\underline{0,8148}}$$

26.)

| |
|-----------|
| 4 PLAVE |
| 5 BIJELIH |
| 6 CRNIH |

UKUPNO: 15 KUGLICA
IZVLAČIMO 3 $A = \{ \text{sve tri različite boje} \}$

$$P(A) = \frac{\binom{4}{1} \cdot \binom{5}{1} \cdot \binom{6}{1}}{\binom{15}{3}} = \underline{\underline{0,2637}}$$

 $B = \{ \text{prva kuglica je bijela} \} P(B) = \frac{5}{15} = \underline{\underline{0,3}}$ $C = \{ \text{prva dve kuglice su različitih boja} \} P(C) = \frac{\binom{4}{1} \cdot \binom{5}{1} + \binom{4}{1} \cdot \binom{6}{1} + \binom{5}{1} \cdot \binom{6}{1}}{\binom{15}{2}} = \underline{\underline{0,704}}$

$P(A|B) = P(A) = \underline{\underline{0,2637}}$

$P(AC) = P(A) \rightarrow P(A|C) = \frac{P(A)}{P(C)} = \frac{1260}{3367} = \underline{\underline{0,3742}}$

$P(A) \cdot P(B) = P(AB) \rightarrow 0,1856 \neq 0,2637 \rightarrow \text{NEZAVISNI SU!}$

27.) ZAMISLIMO PRVO SAMO 2 ZNAMENKE

a) $\begin{array}{c} 12 \\ 21 \end{array} \quad \left\{ \begin{array}{l} 2 \text{ komb., samo 1 odg. ("12") } \\ \rightarrow P = \frac{1}{2} \end{array} \right.$

b) $\begin{array}{c} 123 \\ 132 \\ 213 \\ 231 \\ 312 \\ 321 \end{array} \quad \left\{ \begin{array}{l} 6 \text{ komb., samo 2 odg. ("12") } \\ \rightarrow P = \frac{2}{6} = \frac{1}{3} \end{array} \right.$

c) $n \rightarrow \text{znamenki} \rightarrow P = \frac{1}{n}$

28.) ZADATAK JE POTPUNO IDENTIČAN ZADATKU BROJ 14.

29.) 6 KOCAKA

 $\rightarrow 2 \text{ ŽUTE}, 4 \text{ BIJELE}$ $A = \{ \text{ŽUTE 6-ice} > \text{BIJELE 6-ice} \}$ izbor koja kostka će
pokazati brojica

bijele kostice

scenariji: 1 ŽUTA 6-ica > 0 BIJELIH $\rightarrow \frac{\binom{2}{1} \cdot 1 \cdot 5}{6^2} \cdot \frac{5^4}{6^4}$

2 ŽUTE 6-ice > 0 BIJELIH $\rightarrow \frac{1}{6^2} \cdot \frac{5^4}{6^4}$

2 ŽUTE 6-ice > 1 BIJELIH $\rightarrow \frac{1}{6^2} \cdot \frac{\binom{4}{1} \cdot 1 \cdot 5^3}{6^4}$

$P(A) = \frac{\binom{2}{1} \cdot 5}{6^2} \cdot \frac{5^4}{6^4} + \frac{1}{6^2} \cdot \frac{5^4}{6^4} + \frac{1}{6^2} \cdot \frac{\binom{4}{1} \cdot 5^3}{6^4} = \underline{\underline{0,1581}}$

30.) 10 KOVANICA

$$1 \text{ dinar} = 6 \text{ kovanica}$$

$$2 \text{ dinara} = 4 \text{ kovanice}$$

Izvlačimo 2 PA 2 kovanice, sume jednake?

$$H_1 = \{\text{suma}=2, \text{ tj. } 1 \text{ dinar} + 1 \text{ dinar}\} = \frac{6}{10} \cdot \frac{5}{9}$$

$$P(A|H_1) = \frac{4}{8} \cdot \frac{3}{7}$$

$$H_2 = \{\text{suma}=3, \text{ tj. } 1 \text{ dinar} + 2 \text{ dinara}\} = \binom{2}{1} \cdot \frac{6}{10} \cdot \frac{4}{9}$$

$$P(A|H_2) = \binom{2}{1} \cdot \frac{5}{8} \cdot \frac{3}{7}$$

$$H_3 = \{\text{suma}=4, \text{ tj. } 2 \text{ dinara} + 2 \text{ dinara}\} = \frac{4}{10} \cdot \frac{3}{9}$$

$$P(A|H_3) = \frac{2}{8} \cdot \frac{1}{7}$$

$$A = \{\text{suma jednaka u 2 izvlačenju}\}$$

$$P(A) = \sum_j P(H_j) \cdot P(A|H_j) = \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} \cdot \frac{3}{7} + \frac{2}{10} \cdot \frac{6}{9} \cdot \frac{2}{8} \cdot \frac{5}{7} + \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} \cdot \frac{1}{7} = 0,3619$$

32.) CRAPS

Sume za 2 kocke:

$$P(2) = \frac{1}{36}$$

$$P(3) = \frac{2}{36}$$

$$P(4) = \frac{3}{36}$$

$$P(5) = \frac{4}{36}$$

$$P(6) = \frac{5}{36}$$

$$P(7) = \frac{6}{36}$$

$$P(8) = \frac{5}{36}$$

$$P(9) = \frac{4}{36}$$

$$P(10) = \frac{3}{36}$$

$$P(11) = \frac{2}{36}$$

$$P(12) = \frac{1}{36}$$

1. BACANJE

→ POBJEDA U SLUČAJU 7 ILI 11:

$$P(A) = P(7) + P(11) = \frac{8}{36}$$

→ PORAZ U SLUČAJU 2, 3, 12

$$P(B) = P(2) + P(3) + P(12) = \frac{4}{36}$$

→ OSTALI SLUČAJEVU: (4, 5, 6, 8, 9, 10)

→ POBJEDA AKO SE PONOVI OPET TAJ ISTI BROJ,
PORAZ AKO SE POJAVI 7, OSTALI SLUČAJEVU KOCKE SE
BACAJU PONOVNO

npr. slučaj 4:

$$P(\text{ukupno za 4}) = \sum_{n=0}^{\infty} P(4) \cdot (1 - P(4) - P(7))^n \cdot P(4)$$

$$P(\text{ukupno za 4}) = P(4) \cdot \sum_{n=0}^{\infty} (1 - P(4) - P(7))^n = P(4)^2 \cdot \frac{1}{1 - (1 - P(4) - P(7))}$$

$$= \frac{P(4)^2}{P(4) + P(7)}$$

↳ sve isto vrijedi i za ostale brojove 5, 6, 8, 9, 10

$$P(\text{ukupno za } i) = \frac{P(i)^2}{P(i) + P(7)}$$

$$P(\text{pobjeda}) = \frac{8}{36} + \frac{P(4)^2}{P(4) + P(7)} + \frac{P(5)^2}{P(5) + P(7)} + \frac{P(6)^2}{P(6) + P(7)} + \frac{P(8)^2}{P(8) + P(7)} + \frac{P(9)^2}{P(9) + P(7)} + \frac{P(10)^2}{P(10) + P(7)} = \frac{244}{495} = 0,4929$$

odmat u 1. bacanju

33.) DVOBOJ U TROJE \rightarrow POGLEDATI POD LITERATUROM!

34.) KOCKA, TOČNO I PETICA U DVA BACANJA,
2. BACANJE DRUGO KOCAKA KOJI JE KOCKA
POKAZALA U 1. BACANJU $A = \{\text{točno jedna 5. petica u dva bacanja}\}$

1. bacanje:

$$P(1) = \frac{1}{6}$$

$$P(A|1) = \frac{1}{6}$$

$$P(2) = \frac{1}{6}$$

$$P(A|2) = \frac{\binom{2}{1} \cdot 1 \cdot 5}{6^2}$$

$$P(3) = \frac{1}{6}$$

$$P(A|3) = \frac{\binom{3}{1} \cdot 1 \cdot 5^2}{6^3}$$

$$P(4) = \frac{1}{6}$$

$$P(A|4) = \frac{\binom{4}{1} \cdot 1 \cdot 5^3}{6^4}$$

$$P(5) = \frac{1}{6}$$

$$P(A|5) = \frac{5^5}{6^5}$$

$$P(6) = \frac{1}{6}$$

$$P(A|6) = \frac{\binom{6}{1} \cdot 1 \cdot 5^5}{6^6}$$

$$P(A) = \sum_{j=1}^6 P(j) \cdot P(A|j) = 0,027 + 0,0463 + 0,0578 + 0,0643 + 0,067 + 0,067 \\ = \underline{\underline{0,3302}}$$

35.) $A = \{\text{barem 3 pisma}\}$

HIPOTEZE \rightarrow BEZ GRBOVA NAKON 1. BACANJA: (5 NOVČICA)

$$P(\emptyset \text{ grbova}) = \frac{1^5}{2^5} \leftarrow 5 \text{ pisma}$$

$$P(A|0) = 1 \quad \leftarrow \text{jer smo već dobili 5 pisma, a to je } \geq 3$$

$$P(1 \text{ grb}) = \frac{\binom{5}{1} \cdot 1^4}{2^5} \leftarrow 4 \text{ pisma}$$

$$P(A|1) = 1 \quad -11-$$

$$P(2 \text{ grba}) = \frac{\binom{5}{2} \cdot 1^3}{2^5} \leftarrow 3 \text{ pisma}$$

$$P(A|2) = 1 \quad -11-$$

$$P(3 \text{ grba}) = \frac{\binom{5}{3} \cdot 1^2}{2^5} \leftarrow 2 \text{ pisma}$$

$$P(A|3) = \frac{\binom{3}{1} \cdot 1^2 + \binom{3}{2} \cdot 1 + \binom{3}{3}}{2^3} \quad \begin{matrix} \leftarrow & \text{imamo veći} \\ & 2 \text{ pisma pa} \\ & nai je dvojno} \\ & još 1,2 ili 3 \\ & pisma \end{matrix}$$

$$P(4 \text{ grba}) = \frac{\binom{5}{4} \cdot 1}{2^5} \leftarrow 1 \text{ pismo}$$

$$P(A|4) = \frac{\binom{4}{2} \cdot 1^2 + \binom{4}{3} \cdot 1 + \binom{4}{4}}{2^4}$$

$$P(5 \text{ grbova}) = \frac{\binom{5}{5}}{2^5} = \frac{1}{2^5} \leftarrow 0 \text{ pisma}$$

$$P(A|5) = \frac{\binom{5}{3} \cdot 1^2 + \binom{5}{4} \cdot 1 + \binom{5}{5}}{2^5}$$

$$P(A) = \sum_{i=1}^5 P(i) \cdot P(A|i) = 0,03125 + 0,15625 + 0,3125 + 0,2734 + 0,10742 + 0,015625$$

$$\underline{\underline{P(A) = 0,8964}}$$

36.) 4 KOCKE

\rightarrow 2 BIJELE, 2 CRVENE

\rightarrow NAKON 1. BACANJA BACAJU SE OPET ONE BIJELE KOCKE KOJE POKAZUJU PARAN BROJ 1
ONO CRVEJE KOJE POKAZUJU BROJ 6.

\rightarrow U. DA SU PONOVNO BACENE KOCKE SVE POKAZUJU 5-ICU?

BIJELA KOCKA DA POKAZUJE 5: $\frac{1}{6} + \frac{3}{6} \cdot \frac{1}{6}$

CRVENA KOCKA DA POKAZUJE 5: $\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}$

$$P(A) = \left(\frac{1}{6} + \frac{3}{6} \cdot \frac{1}{6} \right)^2 \cdot \left(\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} \right)^2 = \underline{\underline{0,002363}}$$

37.) 32 KARTE, 2 SNOPA

$\frac{1}{2}$ A SNOPA IZVLAČIMO 2 KARTE

$\frac{1}{2}$ B SNOPA IZVLAČIMO 1 KARTU

$$A = \left\{ \begin{array}{l} 3 \text{ KARTE IZMEĐJAMO 1 IZVLAČIMO DVije, } \\ \text{VS. DA SU TO ASEVI?} \end{array} \right\}$$

1. slučaj \rightarrow 2 AS-a od 3 karte

a) $A \rightarrow 2 \text{ AS-a}, B \rightarrow \emptyset \text{ asevi}$

$$P(a) = \frac{\binom{4}{2}}{\binom{32}{2}} \cdot \frac{\binom{28}{1}}{\binom{32}{1}} = \underline{\underline{0,01058}}$$

\nearrow A snop \nwarrow B snop

b) $A \rightarrow 1 \text{ AS}, 1 \text{ obična karta}$

$B \rightarrow 1 \text{ AS}$

$$P(b) = \frac{\binom{4}{1}\binom{28}{1}}{\binom{32}{2}} \cdot \frac{\binom{4}{1}}{\binom{32}{1}} = \underline{\underline{0,02822}}$$

$$P(1. \text{ slučaj}) = \frac{2}{3} \cdot \frac{1}{2} \cdot (0,01058 + 0,02822) = \underline{\underline{0,012935}}$$

\hookrightarrow da od 3 konačne karte izvučemo 2 AS-a

2. slučaj \rightarrow 3 AS-a od 3 karte

$$P(2. \text{ slučaj}) = 1 \cdot \left(\frac{\binom{4}{3}}{\binom{32}{3}} \cdot \frac{\binom{1}{1}}{\binom{32}{1}} \right) = \underline{\underline{0,001512}}$$

\nearrow od 3 karte izvučemo 3 AS-a

$$P(A) = P(1. \text{ slučaj}) + P(2. \text{ slučaj}) = \underline{\underline{0,01444}}$$

38.) 52 KARTE, 2 SNOPA PO 2 KARTE, OD 4 IZVUĆENE OTKRIJEMO DULJE → DA BUDU ASEVI?

1. slučaj 2 AS-a

oba as-a iz 1. spila

1. as je A sp. t., drugi je B

$$P(1. \text{slučaj}) = \frac{2}{4} \cdot \frac{1}{3} \cdot \left[2 \cdot \frac{\binom{4}{2}}{\binom{52}{2}} \cdot \frac{\binom{48}{2}}{\binom{52}{2}} + 4 \cdot \frac{\binom{4}{1} \cdot \binom{48}{1}}{\binom{52}{2}} \cdot \frac{\binom{4}{1} \cdot \binom{48}{1}}{\binom{52}{2}} \right] =$$

od 4 karte da
izvemo 2 as-a

$$= \frac{1}{6} \cdot [0,00769 + 0,02096] = \underline{\underline{0,004776}}$$

2. slučaj 3 AS-a

$$P(2. \text{slučaj}) = \frac{3}{4} \cdot \frac{2}{3} \cdot \left[\frac{\binom{4}{2} \cdot \binom{4}{1} \cdot \binom{48}{1}}{\binom{52}{2}} \cdot 2 \right] = \underline{\underline{0,000655187}}$$

3. slučaj 4 AS-a

$$P(3. \text{slučaj}) = \frac{\binom{4}{2}}{\binom{52}{2}} \cdot \frac{\binom{4}{1}}{\binom{52}{1}} = \underline{\underline{0,000020475}}$$

$$P(A) = P(1. \text{slučaj}) + P(2. \text{slučaj}) + P(3. \text{slučaj}) = \underline{\underline{0,005453}}$$

39.) 52 KARTE, 2 SNOPA, 2 KARTE DA NIJE AS

! ali gledamo slučajevе kada je as:

1. slučaj: 1 AS od 2 karte

$$P(1. \text{slučaj}) = \frac{1}{2} \cdot \left[2 \cdot \frac{\binom{4}{1}}{\binom{52}{1}} \cdot \frac{\binom{48}{1}}{\binom{52}{1}} \right] = \underline{\underline{0,071}}$$

2. slučaj 2 AS-a

$$P(2. \text{slučaj}) = \frac{\binom{4}{2}}{\binom{52}{2}} \cdot \frac{\binom{4}{1}}{\binom{52}{1}} = \underline{\underline{0,005917}}$$

$$P(\text{ukupno}) = P(1) + P(2) = 0,0769$$

$$P(\text{NIJE AS}) = 1 - 0,0769 = \underline{\underline{0,923}}$$

40.) 2 SNOPA PO 52 KARTE, 1Z JEDNOG SNOPA 1 KARTA, 1Z DRUGOG 2 KARTE

W. DA OTKRIVENA KARTA JE AS

Ukupno tri karte:

$$H_0 = \{0 \text{ asa}\}$$

$$H_1 = \{1 \text{ as}\}$$

$$H_2 = \{2 \text{ asa}\}$$

$$H_3 = \{3 \text{ asa}\}$$

$$A = \{\text{nac kraj 1 as}\}$$

$$P(H_0) = \frac{48}{52} \cdot \frac{\binom{48}{2}}{\binom{52}{2}} \quad \begin{matrix} 1. \text{spil} \\ \swarrow \\ 2. \text{spil} \end{matrix}$$

$$P(A|H_0) = \emptyset$$

$$P(H_1) = \frac{4}{52} \cdot \frac{\binom{48}{1}}{\binom{52}{1}} + \frac{48}{52} \cdot \frac{\binom{4}{1} \cdot \binom{48}{1}}{\binom{52}{2}}$$

$$P(A|H_1) = \frac{1}{3}$$

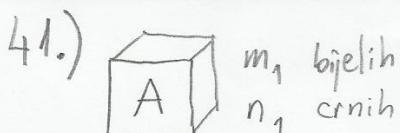
$$P(H_2) = \frac{48}{52} \cdot \frac{\binom{4}{1}}{\binom{52}{1}} + \frac{4}{52} \cdot \frac{\binom{4}{1} \cdot \binom{48}{1}}{\binom{52}{2}}$$

$$P(A|H_2) = \frac{2}{3}$$

$$P(H_3) = \frac{4}{52} \cdot \frac{\binom{4}{2}}{\binom{52}{2}}$$

$$P(A|H_3) = 1$$

$$P(A) = \sum_{j=0}^3 P(H_j) \cdot P(A|H_j) = \frac{1}{13} = \underline{\underline{0,0769}}$$



→ iz svake kutije izvlačimo po jednu kuglicu
→ od te 2 da izvučemo bijelu?

$$H_0 = \{1 \text{ bijela od } 2 \text{ kugl}\} = \frac{m_1}{n_1+m_1} \cdot \frac{n_2}{n_2+m_2} + \frac{n_1}{n_1+m_1} \cdot \frac{m_2}{n_2+m_2} \quad P(A|H_0) = \frac{1}{2}$$

$$H_1 = \{2 \text{ bijele}\} = \frac{m_1}{n_1+m_1} \cdot \frac{m_2}{n_2+m_2} \quad P(A|H_1) = 1$$

$$A = \{\text{od 2 kuglice izvučena bijela na kraju}\}$$

$$P(A) = \frac{1}{2} \left(\frac{m_1 n_2}{(n_1+m_1)(n_2+m_2)} + \frac{n_1 m_2}{(n_1+m_1)(n_2+m_2)} \right) + 1 \cdot \frac{m_1 m_2}{(n_1+m_1)(n_2+m_2)}$$

$$P(A) = \frac{1}{2} \frac{(m_1 n_2 + n_1 m_2)}{(n_1+m_1)(n_2+m_2)} + \frac{m_1 m_2}{(n_1+m_1)(n_2+m_2)} = \frac{m_1 n_2 + n_1 m_2 + 2m_1 m_2}{2(n_1+m_1)(n_2+m_2)}$$

42.)

| | |
|----------|----------|
| 2 BIJELA | 1 BIJELA |
| 3 CRNE | 4 CRNE |

prebacimo 2

iz 1. pregrade:

$$H_0 = \{\text{obe bijele}\} \quad P(H_0) = \frac{2}{5} \cdot \frac{1}{4}$$

$$P(A|H_0) = \frac{3}{7}$$

$$H_1 = \{\text{bijela i crna}\} \quad P(H_1) = 2 \cdot \frac{2}{5} \cdot \frac{3}{4}$$

$$P(A|H_1) = \frac{2}{7}$$

$$H_2 = \{\text{obe crne}\} \quad P(H_2) = \frac{3}{5} \cdot \frac{2}{4}$$

$$P(A|H_2) = \frac{1}{7}$$

$$A = \{\text{iz druge pregrade izvučena bijela}\}$$

$$P(A) = \frac{1}{10} \cdot \frac{3}{7} + \frac{3}{5} \cdot \frac{2}{7} + \frac{3}{10} \cdot \frac{1}{7} = \underline{\underline{0,25714}}$$

43.) 10 STRUVELACA

4 ODUĆNA 6 DOBRA
POGOĐAK 0,9 POGODAK 0,7
→ IZVUČEMO 2 STRUVELACA
1 DA OBА POGODE METU

$$P(A) = \frac{\binom{4}{2}}{\binom{10}{2}} \cdot 0,9 \cdot 0,9 + 2 \cdot \frac{\binom{4}{1} \binom{6}{1}}{\binom{10}{2}} \cdot 0,9 \cdot 0,7 + \frac{\binom{6}{2}}{\binom{10}{2}} \cdot 0,7 \cdot 0,7$$

↓ ↑ ↓
 2 odlična 1 odličan + 1 dobar 2 dobra

$$P(A) = \underline{\underline{0,6073}}$$

44.) 8 STRUVELACA

3 odlična 5 dobra
pogodak: 0,9 pogodak: 0,6

$$H_1 = \{\text{oba odlična}\} \quad P(H_1) = \frac{3}{8} \cdot \frac{2}{7} \quad P(A|H_1) = 0,9 \cdot 0,9 + 2 \cdot 0,9 \cdot 0,1 = 0,99$$

$$H_2 = \{\text{odličan + dobar}\} \quad P(H_2) = 2 \cdot \frac{3}{8} \cdot \frac{5}{7} \quad P(A|H_2) = 0,9 \cdot 0,6 + 0,9 \cdot 0,4 + 0,1 \cdot 0,6 = 0,96$$

$$H_3 = \{\text{oba dobra}\} \quad P(H_3) = \frac{5}{8} \cdot \frac{4}{7} \quad P(A|H_3) = 0,6 \cdot 0,6 + 2 \cdot 0,6 \cdot 0,4 = 0,84$$

$$A = \{\text{meta pogodena barem 1}\}$$

$$P(A) = \frac{3}{28} \cdot 0,99 + \frac{15}{28} \cdot 0,96 + \frac{5}{14} \cdot 0,84 = \underline{\underline{0,9202}}$$

47.) 1 bijela
1 crvena dodajemo još jednu bijelu ukoliko ne izvučemo crvenu, te vratimo bijelu.

$A \Rightarrow$ UJ. DA NEĆE BITI PONOVLJENJA?

1. slucaj

$$\begin{matrix} 1 \text{ BIJELA} \\ 1 \text{ CRVENA} \end{matrix} \quad P(A) = \frac{1}{2}$$

2. slucaj

$$\begin{matrix} 2 \text{ BIJELA} \\ 1 \text{ CRVENA} \end{matrix} \quad P(A) = \frac{1}{3}$$

3. slucaj

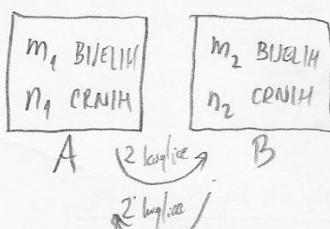
$$\begin{matrix} 3 \text{ BIJELA} \\ 1 \text{ CRVENA} \end{matrix} \quad P(A) = \frac{1}{4}$$

n. slucaj

$$\begin{matrix} n \text{ BIJELIH} \\ 1 \text{ CRVENA} \end{matrix} \quad P(A) = \frac{1}{n}$$

48.) POTPUNO IDENTIČNI ZADATAK KAO 42.)

49.)



1. kutija
 $H_1 = \{\text{obe bijele}\}$
 $H_2 = \{1 \text{ bijela}, 1 \text{ crna}\}$
 $H_3 = \{\text{obe crne}\}$

$$P(H_1) = \frac{\binom{m_1}{2}}{\binom{n_1+m_1}{2}}$$

$$P(H_2) = \frac{2 \cdot \binom{m_1}{1} \binom{n_1}{1}}{\binom{n_1+m_1}{2}}$$

$$P(H_3) = \frac{\binom{n_1}{2}}{\binom{n_1+m_1}{2}}$$

$$P(A|H_1) = \frac{\binom{m_2+2}{2}}{\binom{n_2+m_2+2}{2}}$$

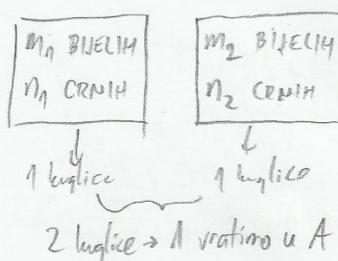
$$P(A|H_2) = \frac{2 \cdot \binom{m_2+1}{1} \binom{n_2+1}{1}}{\binom{n_2+m_2+2}{2}}$$

$$P(A|H_3) = \frac{\binom{n_2+2}{2}}{\binom{n_2+m_2+2}{2}}$$

$A = \{\text{da u } A \text{ bude ponovno } m_1 \text{ bijelih i } n_1 \text{ crnih}\}$

$$P(A) = \frac{\binom{m_1}{2} \binom{m_2+2}{2} + \binom{m_1}{1} \binom{n_1}{1} \binom{m_2+1}{1} \binom{n_2+1}{1} + \binom{n_1}{2} \binom{m_2+2}{2}}{\binom{n_1+m_1}{2} \cdot \binom{n_2+m_2+2}{2}} \quad (?) *$$

50.)



$H_1 = \{\text{obe bijele}\}$

$$P(H_1) = \frac{m_1}{n_1+m_1} \cdot \frac{m_2}{n_2+m_2}$$

$$P(A|H_1) = 1$$

$H_2 = \{1 \text{ bijela}, 1 \text{ crna}\}$

$$P(H_2) = \frac{2 \cdot \frac{m_1}{n_1+m_1} \cdot \frac{n_2}{n_2+m_2}}{2(n_1+m_1)(n_2+m_2)}$$

$$P(A|H_2) = \frac{1}{2}$$

$H_3 = \{2 \text{ crne}\}$

$$P(H_3) = \frac{n_1}{n_1+m_1} \cdot \frac{n_2}{n_2+m_2}$$

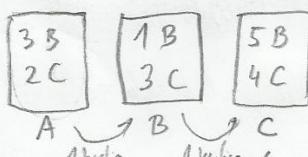
$$P(A|H_3) = 1$$

$A = \{\text{da u } A \text{ bude ponovno } m_1 \text{ bij. i } n_1 \text{ crnih}\}$

$$P(A) = \frac{m_1 n_1}{(n_1+m_1)(n_2+m_2)} + \frac{1}{2} \cdot \frac{2 m_1 n_2}{(n_1+m_1)(n_2+m_2)} + \frac{n_1 n_2}{(n_1+m_1)(n_2+m_2)} = \frac{2 m_1 n_1 + 2 m_1 n_2 + 2 n_1 n_2}{2(n_1+m_1)(n_2+m_2)}$$

$$P(A) = \frac{m_1 n_1 + m_1 n_2 + n_1 n_2}{(n_1+m_1)(n_2+m_2)} \quad \leftarrow ? \quad (\text{ne slazi se s rješenjima}) *$$

51.)



1. slucaj (bijela kugla)

$$P(1. \text{ slucaj}) = \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{6}{10} = 0,144$$

2. slucaj (crna kuglica)

$$P(2. \text{ slucaj}) = \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{5}{10} = 0,16$$

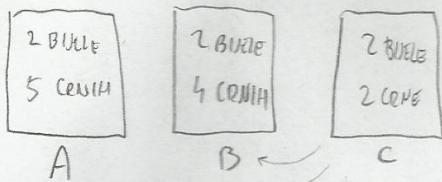
poznato da je broj bijelih/crnih ne promjenjen

$A = \{\text{iz kutije } B \text{ u } C \text{ prebacili crnu kuglicu}\}$

$A \cap B \cap C$

$$P(A) = \frac{P(2)}{P(1)+P(2)} = \frac{0,16}{0,144+0,16} = 0,526$$

56.)



→ odabirano na sreću 2 kuglice iz kutije C te probacujeno u A, a onda preostale dvije u B!

SCENARIJI

$$\text{iz } C \text{ u } A: 1.) \text{ dve bijele } P(1) = \frac{2}{4} \cdot \frac{1}{3} = \underline{0,16} \quad \text{iz } C \text{ u } B: \text{ dve crne } P(1B) = \frac{2}{2} \cdot \frac{1}{1} = 1$$

$$2.) 1 \text{ bijela, 1 crna } P(2) = 2 \cdot \frac{2}{4} \cdot \frac{2}{3} = \underline{0,66} \quad \text{iz } C \text{ u } B: 1 \text{ bijela, 1 crna } P(2B) = 1$$

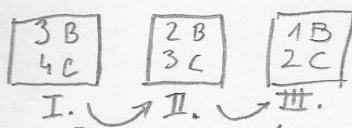
$$3.) \text{ dve crne } P(3) = \frac{2}{4} \cdot \frac{1}{3} = \underline{0,16} \quad \text{iz } C \text{ u } B: \text{ dve bijele } P(3B) = 1$$

→ 2 kuglice → ISTOBOJNE:

$$\text{Imamo: } 1.) A \rightarrow \begin{pmatrix} 4B \\ 5C \end{pmatrix}, B \rightarrow \begin{pmatrix} 2B \\ 6C \end{pmatrix} \quad 2.) A \rightarrow \begin{pmatrix} 3B \\ 5C \end{pmatrix}, B \rightarrow \begin{pmatrix} 3B \\ 4C \end{pmatrix} \quad 3.) A \rightarrow \begin{pmatrix} 2B \\ 7C \end{pmatrix}, B \rightarrow \begin{pmatrix} 4B \\ 4C \end{pmatrix}$$

$$P(A) = 0,16 \left(\frac{4}{9} \cdot \frac{2}{8} + \frac{5}{9} \cdot \frac{6}{8} \right) + 0,66 \left(\frac{3}{8} \cdot \frac{3}{7} + \frac{5}{8} \cdot \frac{4}{7} \right) + 0,16 \left(\frac{2}{9} \cdot \frac{4}{8} + \frac{7}{9} \cdot \frac{4}{8} \right) = \underline{\underline{0,57}}$$

57.)



-da u I. ostane nepronjenjen broj, ako iz I. sačemo bijelu, moramo iz III.-r I. dobiti bijelu natrag (isto za crnu kuglicu)

$$P(A) = P(1) + P(2) + P(3) + P(4) = \underline{\underline{0,541}}$$

POVOLJNI SCENARIJI

$$\text{sačemo bijelu: } \begin{matrix} \text{I.-II.} \\ \checkmark \end{matrix} \begin{matrix} \text{I.-III.} \\ \checkmark \end{matrix} \begin{matrix} \text{III.-I.} \\ \checkmark \end{matrix} \quad 1.) B \ B \ B = \frac{3}{7} \cdot \frac{3}{6} \cdot \frac{2}{4} = \underline{\underline{0,10714}}$$

$$2.) B \ C \ B = \frac{3}{7} \cdot \frac{3}{6} \cdot \frac{1}{4} = \underline{\underline{0,05357}}$$

$$3.) C \ C \ C = \frac{4}{7} \cdot \frac{4}{6} \cdot \frac{3}{4} = \underline{\underline{0,1857}}$$

$$4.) C \ B \ C = \frac{4}{7} \cdot \frac{2}{6} \cdot \frac{2}{4} = \underline{\underline{0,0952}}$$

59.) 3 KOCE

$X \rightarrow$ zbroj kocaka

$A \rightarrow$ barem 1 šestica

$P(X=11) \rightarrow$ kombinacije:

- 344 → 3 komb.
 - 641 → 6 komb.
 - 551 → 3 komb.
 - 542 → 6 komb.
 - 632 → 6 komb.
 - 353 → 3 komb.
- 27 komb.

$P(X=12) \rightarrow$ kombinacije:

- 444 → 1 komb.
 - 552 → 3 komb.
 - 651 → 6 komb.
 - 543 → 6 komb.
 - 633 → 3 komb.
 - 624 → 6 komb.
- 25 komb.

$$P(X=11) = \frac{27}{6^3} = \frac{27}{216}$$

$$P(X=12) = \frac{25}{6^3} = \frac{25}{216}$$

$$P(A|X=12) = \frac{\frac{651}{633} \binom{6 \text{ komb}}{3 \text{ komb}}}{\frac{624}{15 \text{ komb}}} = \frac{15}{25} = \underline{\underline{0,6}}$$

60.) 2 kocke

ZBEOJ JE 8,

POJAVA IMA SE 6-ICA?

kombinacije

| | |
|---------|----------|
| $4+4=8$ | (1 komb) |
| $6+2=8$ | (2 komb) |
| $5+3=8$ | (2 komb) |

5 komb

$$P(A) = \frac{2}{5}$$

61.) 2 igrača

1.IGRAČ DOBIO VEĆI BROJ OD 2.
VI. DA JE TO 6-ICA?

| | | 1.igrac | 2.igrac | 3 | 4 | 5 |
|---------|----|---------|---------|---|---|---|
| 1.igrac | 2. | - | - | - | - | - |
| 2 | - | - | - | - | - | - |
| 3 | 3 | - | - | - | - | - |
| 4 | 4 | 4 | - | - | - | - |
| 5 | 5 | 5 | 5 | - | - | - |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

15 ishoda

$$P(A) = \frac{5}{15} = \frac{1}{3} = 0,33$$

63.) n studenata

godine k = 1, 2, 3, 4

→ izabrana 2 studenta, 1-stučić više godine
vjeđa je 4. godina. ($x_1 > x_2$)

$n_1 \rightarrow$ 1-god. student

$n_2 \rightarrow$ student 2-god

$n_3 \rightarrow$ student 3-god

$n_4 \rightarrow$ student 4-god

$$P(A) = \frac{n_4(n_1+n_2+n_3)}{n_1n_1 + n_2(n_1+n_2) + n_3(n_1+n_2+n_3)}$$

SCENARIJI:

1-student | 2-student

2.GODINA | 1.GODINA

3.GODINA | 1. + 2.GOD.

4.GODINA | 1. + 2. + 3.GOD.

$$\Rightarrow n_2 \cdot n_1$$

$$\Rightarrow n_3 \cdot (n_1+n_2)$$

$$\Rightarrow n_4 \cdot (n_1+n_2+n_3)$$

64.) 3 IGRAČA, 1.IGRAČ POBLO NAJVEĆI BROJ,
A = VI. DA JE TAJ BROJ > 4?

→ ispišemo sve kombinacije:

1.igrac | 2.igrac | 3.igrac

| | | |
|---|---|---|
| 2 | 1 | 1 |
| 3 | 1 | 1 |
| 3 | 2 | 2 |
| 3 | 2 | 1 |
| 3 | 1 | 2 |
| 4 | 1 | 1 |
| 4 | 1 | 2 |
| 4 | 2 | 2 |
| 4 | 3 | 1 |
| 4 | 1 | 3 |
| 4 | 3 | 2 |
| 4 | 2 | 3 |
| 4 | 3 | 3 |
| 5 | 1 | 1 |
| 5 | 1 | 2 |
| 5 | 2 | 1 |
| 5 | 3 | 1 |
| 5 | 1 | 3 |
| 5 | 2 | 2 |
| 5 | 3 | 2 |
| 5 | 2 | 3 |
| 5 | 3 | 3 |
| 5 | 4 | 1 |
| 5 | 1 | 4 |
| 5 | 4 | 2 |
| 5 | 2 | 4 |

1.igrac | 2.igrac | 3.igrac

| | | |
|---|---|---|
| 5 | 4 | 3 |
| 5 | 3 | 4 |
| 5 | 4 | 4 |
| 6 | 1 | 1 |
| 6 | 2 | 2 |
| 6 | 3 | 3 |
| 6 | 4 | 4 |
| 6 | 5 | 5 |
| 6 | 2 | 1 |
| 6 | 3 | 3 |
| 6 | 4 | 1 |
| 6 | 1 | 3 |
| 6 | 3 | 2 |
| 6 | 2 | 3 |
| 6 | 4 | 1 |
| 6 | 1 | 4 |
| 6 | 3 | 4 |
| 6 | 4 | 2 |
| 6 | 5 | 1 |
| 6 | 1 | 5 |
| 6 | 2 | 5 |
| 6 | 5 | 2 |
| 6 | 5 | 3 |
| 6 | 3 | 5 |

1.igrac | 2.igrac | 3.igrac

| | | |
|---|---|---|
| 6 | 4 | 5 |
| 6 | 5 | 4 |

Ukupno: 55 kombinacija

> 4: 41 kombinacija

$$P(A) = \frac{41}{55} = 0,745$$

65.) 4 KOCKE

A) Vj. da je pala barem jedna 6-ka
ako je poznato da su pala tačno 2
jednaka broja?

oblik: $AABC \rightarrow 4$ odabira
6 odabira \downarrow 5 odabira

oblik: 1123, 2261, 2253, ...

$$\text{UKUPNI BROJ KOMB.} = \frac{6 \cdot 5 \cdot 4 \cdot \binom{4}{2}}{6^4}$$

$$\text{BEZ } 6\text{-ICA} : = \frac{5 \cdot 4 \cdot 3 \cdot \binom{4}{2}}{6^4}$$

$$P(A) = 1 - \text{prob. nesu} = 1 - \frac{5 \cdot 4 \cdot 3 \cdot \binom{4}{2}}{6 \cdot 5 \cdot 4 \cdot \binom{4}{2}} = 1 - 0,5 = 0,5$$

66.) CRVENA, BUELA I PLAVA KOČKA ZBEDO JE 13, UDA JE BUELA PALA NA 4?

$$\begin{aligned} 4+4+5 &= 13 \quad (3 \text{ komb.}) \\ 6+6+1 &= 13 \quad (3 \text{ komb.}) \\ 6+5+4 &= 13 \quad (6 \text{ komb.}) \\ 3+4+6 &= 13 \quad (6 \text{ komb.}) \\ 5+5+3 &= 13 \quad (3 \text{ komb.}) \\ &\hline 21 \text{ komb.} \end{aligned}$$

$$\text{BUELA NA 4} \rightarrow \begin{aligned} 4+4+5 &\Rightarrow 2 \text{ komb.} \\ 6+5+4 &\Rightarrow 2 \text{ komb.} \\ 3+4+6 &\Rightarrow 2 \text{ komb.} \end{aligned}$$

$$P(A) = \frac{6}{21} \quad ? \text{ (takvo rješ. je } \frac{5}{21} \text{)}$$

68.) 10 PREGRADADA, U PRVIM 9 → 2 CRNE 2 BUELE BULIGICE, U 10. PREGRADI → 5 BUELIM 1 CRNA

$$\begin{aligned} H_1 &= \{1. \text{ pregrada}\} \quad P(H_1) = \frac{1}{10} \quad P(A|H_1) = \frac{4}{2} \\ H_2 &= \{2. \text{ pregrada}\} \quad P(H_2) = \frac{1}{10} \quad P(A|H_2) = \frac{1}{2} \\ H_{10} &= \{10. \text{ pregrada}\} \quad P(H_{10}) = \frac{1}{10} \quad P(A|H_{10}) = \frac{1}{2} \\ A &= \{\text{izvučena bijela buliga}\} \quad P(A|H_{10}) = \frac{5}{6} \end{aligned}$$

$$P(H_{10}|A) = \frac{\frac{1}{10} \cdot \frac{5}{6}}{9 \cdot \frac{1}{10} \cdot \frac{1}{2} + \frac{1}{10} \cdot \frac{5}{6}} = \underline{0,15625}$$

69.) 4 STRUČLCA

$$\begin{aligned} 1. &\rightarrow \text{POGODAK } 0,4 \\ 2. &\rightarrow \text{POGODAK } 0,6 \\ 3. &\rightarrow \text{POGODAK } 0,7 \\ 4. &\rightarrow \text{POGODAK } 0,8 \end{aligned}$$

$$A = \{\text{meta je pogodena}\}$$

$$P(A) = 1 - \frac{\text{metanje}}{\text{pogodena}} = 1 - \frac{0,6 \cdot 0,4 \cdot 0,3 \cdot 0,2}{0,9856}$$

B = {meta pogodena → 3 metka, a promocija 4. trijekse}

$$P(B) = \frac{0,4 \cdot 0,6 \cdot 0,7 \cdot 0,2}{0,6 \cdot 0,6 \cdot 0,7 \cdot 0,8 + 0,4 \cdot 0,6 \cdot 0,7 \cdot 0,8 + 0,4 \cdot 0,6 \cdot 0,3 \cdot 0,8 + 0,4 \cdot 0,6 \cdot 0,7 \cdot 0,2}$$

$$P(B) = \underline{0,0879}$$

$$70.) A \rightarrow 0,6$$

$$B \rightarrow 0,7$$

$$C \rightarrow 0,8$$

$$D \rightarrow 0,9$$

meta pogodena 2 puta.

A = {meta pogodila A i B}

$$P(A) = \frac{0,6 \cdot 0,7 \cdot 0,2 \cdot 0,1}{0,6 \cdot 0,7 \cdot 0,2 \cdot 0,1 + 0,6 \cdot 0,3 \cdot 0,8 \cdot 0,1 + 0,6 \cdot 0,3 \cdot 0,2 \cdot 0,9 + 0,6 \cdot 0,7 \cdot 0,8 \cdot 0,1 + 0,4 \cdot 0,7 \cdot 0,2 \cdot 0,9 + 0,4 \cdot 0,3 \cdot 0,8 \cdot 0,9}$$

$$P(A) = \frac{0,0084}{0,2144} = \underline{0,03917}$$

71.) 1. strjelac $\rightarrow 0,8$

2. strjelac $\rightarrow 0,7$

3. strjelac $\rightarrow 0,6$

4. strjelac $\rightarrow 0,5$

meta pogodena 2 puta,

$A = \{1. \text{strjelac nije pogodio}\}$

$$P(A) = \frac{0,2 \cdot 0,7 \cdot 0,6 \cdot 0,5 + 0,2 \cdot 0,7 \cdot 0,5 \cdot 0,5 + 0,2 \cdot 0,3 \cdot 0,6 \cdot 0,5}{0,8 \cdot 0,7 \cdot 0,6 \cdot 0,5 + 0,8 \cdot 0,3 \cdot 0,6 \cdot 0,5 + 0,8 \cdot 0,3 \cdot 0,5 \cdot 0,5 + 0,2 \cdot 0,7 \cdot 0,6 \cdot 0,9 + 0,2 \cdot 0,7 \cdot 0,4 \cdot 0,9 + 0,2 \cdot 0,3 \cdot 0,6 \cdot 0,9}$$

$$P(A) = \frac{0,088}{0,32} = 0,275$$

72.) 4 TOPA

1.TOP $\rightarrow 0,3$

2,3,4.TOP $\rightarrow 0,2$

$A = \{\text{cijeli uništen}\}$

$$H_1 = \{1. \text{top gadao}\} - P(H_1) = \frac{1}{4} \quad P(A|H_1) = 0,3 \cdot 0,7 \cdot \binom{2}{1} + 0,3^2$$

$$H_2 = \{2. \text{top gadao}\} - P(H_2) = \frac{1}{4} \quad P(A|H_2) = 0,2 \cdot 0,8 \binom{2}{1} + 0,2^2$$

$$H_3 = \{3. \text{top gadao}\} - P(H_3) = \frac{1}{4} \quad P(A|H_3) = 0,2 \cdot 0,8 \binom{2}{1} + 0,2^2$$

$$H_4 = \{4. \text{top gadao}\} - P(H_4) = \frac{1}{4} \quad P(A|H_4) = 0,2 \cdot 0,8 \binom{2}{1} + 0,2^2$$

$$P(H_1|A) = \frac{P(A|H_1)P(H_1)}{\sum_i P(H_i)P(A|H_i)} = \frac{0,1275}{0,3975} = 0,32$$

73.) m bijelih

n crnih

2 izvučena, 1 je bijela, vjedno je druga bijela: $P(A) = \frac{m-1}{m-1+n}$

74.)

| | |
|-----|-----|
| 3 B | 3 B |
| 3 P | 4 C |

$$H_1 = \{1. \text{pregrada}\} \quad P(H_1) = \frac{1}{2} \quad P(A|H_1) = \frac{3}{6} \cdot \frac{2}{5} \cdot 2 \quad \leftarrow 2 \text{ bijele ili 2 plave}$$

$$H_2 = \{2. \text{pregrada}\} \quad P(H_2) = \frac{1}{2} \quad P(A|H_2) = \frac{3}{7} \cdot \frac{2}{6} + \frac{4}{7} \cdot \frac{3}{6} \quad \leftarrow 2 \text{ bijele ili 2 crvene}$$

-dvoje kuglice izvucićemo

$A = \{\text{kuglice iste boje}\}$

B = {kuglice plave boje}

↳ samo su u 1. pregradi:

$$P(A) = P(H_1)P(A|H_1) + P(H_2)P(A|H_2) = 0,41428$$

75.)

| | |
|----------|----------|
| 1B 3C | 2B 2C |
|----------|----------|

$$P(A) = \frac{1}{4} \cdot \frac{2}{4}$$

$A = \{\text{druga kuglica je crvena}\}$

$$P(2) = \frac{1}{4} \cdot \frac{2}{4}$$

$$P(A) = \frac{P(2) + P(3)}{P(1) + P(2) + P(3)} = 0,8$$

$$P(3) = \frac{3}{4} \cdot \frac{2}{4}$$

→ ne može biti jedna bijela

76.)

| |
|-------------------------------|
| 5 BIJELIH ili 5 CRVENIH |
|-------------------------------|

$$H_1 = \{1. \text{bijela}\} \rightarrow P(H_1) = \frac{1}{5}, \quad P(A|H_1) = \left(\frac{1}{5}\right)^3$$

$$H_2 = \{2. \text{bijele}\} \rightarrow P(H_2) = \frac{1}{15}, \quad P(A|H_2) = \left(\frac{2}{5}\right)^3$$

$$H_3 = \{3. \text{bijele}\} \rightarrow P(H_3) = \frac{1}{15}, \quad P(A|H_3) = \left(\frac{3}{5}\right)^3$$

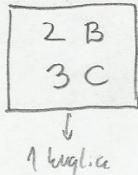
$$H_4 = \{4. \text{bijele}\} \rightarrow P(H_4) = \frac{1}{15}, \quad P(A|H_4) = \left(\frac{4}{5}\right)^3$$

$$H_5 = \{5. \text{bijelih}\} \rightarrow P(H_5) = \frac{1}{15}, \quad P(A|H_5) = 1$$

$$P(A) = \sum_{i=1}^5 P(H_i)P(A|H_i) = 0,36$$

$$\rightarrow P(H_5|A) = \frac{P(H_5) \cdot P(A|H_5)}{P(A)} = 0,556$$

78.)



jedna je bijela, vj.-da je druga crvena?

$$\begin{array}{l} \text{SCENARIJI:} \\ \begin{array}{ll} BB & \rightarrow (1) \\ BC & \rightarrow (2) \\ CB & \rightarrow (3) \end{array} \end{array} \quad P(1) = \frac{2}{5} \cdot \frac{1}{4}$$

$$P(2) = \frac{2}{5} \cdot \frac{3}{4}$$

$$P(3) = \frac{3}{5} \cdot \frac{1}{4}$$

$$P(A) = \frac{P(2)+P(3)}{P(1)+P(2)+P(3)} = \underline{\underline{0,81}}$$

79.) 5 KUGLICA

↳ može biti crvena ili plava

↳ na sreću izabrana je crvena

↳ vj.-za najveći broj plavih?

mogući scenariji:

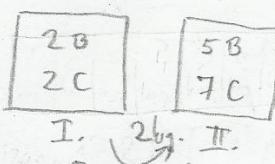
da imamo: 1, 2, 3 ili 4 plavih

radi se u 3. cijelini !!

najviši broj (očekivanje) *

$$E(X) = \frac{1+2+3+4}{5} = \underline{\underline{2 \text{ plave kuglice}}}$$

82.)



- u I. kugli su ostale istobojne kug.

↳ znači da smo poslali dve bijele

ili 2 crne, pa iz II. u I. smo

vratili 2 kug. suprotne boje!!

1. slučaj: (stali 2 bijele - vratili 2 crne)

$$P(1) = \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{7}{14} \cdot \frac{6}{13} = 0,03846$$

2. slučaj (stali 2 crne - vratili 2 bijele)

$$P(2) = \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{5}{14} \cdot \frac{4}{13} = 0,0183$$

$$P(A) = \frac{P(1)}{P(1)+P(2)} = \underline{\underline{0,67758}}$$

84.) 1 i 2 pokvaren

$$1 \rightarrow 0,2, 2 \rightarrow 0,3, 3 \rightarrow 0,1$$

| POKVAREN | RADI |
|----------|------|
| 12 | 3 |
| 13 | 2 |
| 23 | 1 |

$$P(123) = 0,2 \cdot 0,4 \cdot 0,7$$

$$P(132) = 0,2 \cdot 0,3 \cdot 0,6$$

$$P(231) = 0,3 \cdot 0,1 \cdot 0,8$$

$$P(A) = \frac{P(123)}{P(123)+P(132)+P(231)}$$

$$P(A) = \underline{\underline{0,2978}}$$

87.) AAAA, BBBB, CCCC
0,3 0,4 0,3

$$A, B, C \rightarrow 0,6$$

$$\text{POGREŠKA} \rightarrow 0,1$$

primili: ABCA, treba AAAA?

$$A = \{ABC\}$$

$$P(\text{AAAA}) = 0,3 \quad P(A|\text{AAAA}) = 0,6 \cdot 0,2 \cdot 0,2 \cdot 0,1$$

$$P(\text{BBBB}) = 0,4 \quad \begin{array}{l} \text{obzir A kugla} \\ \text{takao B kugla} \\ \text{takao C kugla} \\ \text{takao D kugla} \end{array}$$

$$P(A|\text{BBBB}) = 0,2 \cdot 0,6 \cdot 0,2 \cdot 0,2$$

$$P(\text{CCCC}) = 0,3 \quad P(A|\text{CCCC}) = 0,2 \cdot 0,2 \cdot 0,6 \cdot 0,2$$

$$P(A) = \frac{P(\text{AAAA}) + P(A|\text{AAAA})}{P(\text{AAAA}) + P(\text{BBBB}) + P(\text{CCCC})} = \frac{9}{16} = \underline{\underline{0,5625}}$$

44.) 2 TENKA, 3 RAZLIČITA CIJENA, 1 METAK

VJEĆOVATNOST P, TUČNO 1 CIJENI S BAREM 1 METKOM

→ obrnuta situacija: ni jedan pogoden ili 2 razlicita pogodena : $1 - P(1.\text{situacija}) - P(2.\text{situacija})$

1. pogodena 2 razlicita

odabir 2 razlicita : $P = \frac{6}{9} = \frac{2}{3}$, šanse da pogode tenkove $p \cdot p$

$$P(1.\text{situacija}) = \frac{2}{3} p^2$$

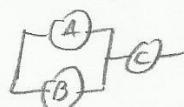
2. nijedan pogoden

tenk promašuje $(1-p) = g$, oba tenka promašjuju $g \cdot g$

$$P(2.\text{situacija}) = g^2 = (1-p)^2$$

$$P(A) = 1 - \frac{2}{3} p^2 - (1-p)^2 = 1 - \frac{2}{3} p^2 - 1 + 2p - p^2 = 1 - 1 + 2p - \frac{5}{3} p^2 = 2p - \frac{5}{3} p^2 = \frac{1}{3} p (6 - 5p)$$

83.) $p=0,2$, samo C dijelio?

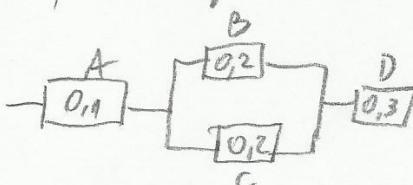


| | A | B | C |
|----|---|---|---|
| 1. | + | + | - |
| 2. | + | - | - |
| 3. | - | + | - |
| 4. | - | - | - |
| 5. | - | - | + |

$$\begin{aligned} P(1) &= 0,8 \cdot 0,2 \cdot 0,2 \\ P(2) &= 0,8 \cdot 0,2 \cdot 0,2 \\ P(3) &= 0,2 \cdot 0,8 \cdot 0,2 \\ P(4) &= 0,2 \cdot 0,2 \cdot 0,2 \\ P(5) &= 0,2 \cdot 0,2 \cdot 0,8 \end{aligned}$$

$$P(\text{Samo } C \text{ ne radi}) = \frac{P(1)}{P(1)+P(2)+P(3)+P(4)+P(5)} = \underline{\underline{0,55172}}$$

85.) $A = \{ \text{medu pokvarenim : } B \text{-dij}\}$



| | A | B | C | D |
|-----|---|---|---|---|
| 1. | + | + | + | - |
| 2. | - | + | + | + |
| 3. | + | + | - | - |
| 4. | + | - | + | - |
| 5. | - | + | + | - |
| 6. | - | - | + | + |
| 7. | - | + | - | + |
| 8. | + | - | - | + |
| 9. | - | - | - | + |
| 10. | - | - | + | - |
| 11. | - | + | - | - |
| 12. | + | - | - | - |
| 13. | - | - | - | - |

$$P_1 = 0,9 \cdot 0,8 \cdot 0,8 \cdot 0,3$$

$$P_2 = 0,1 \cdot 0,8 \cdot 0,8 \cdot 0,7$$

$$P_3 = 0,9 \cdot 0,1 \cdot 0,8 \cdot 0,3$$

$$P_4 = 0,9 \cdot 0,2 \cdot 0,8 \cdot 0,3$$

$$P_5 = 0,1 \cdot 0,8 \cdot 0,8 \cdot 0,3$$

$$P_6 = 0,1 \cdot 0,2 \cdot 0,8 \cdot 0,7$$

$$P_7 = 0,1 \cdot 0,1 \cdot 0,8 \cdot 0,2 \cdot 0,7$$

$$P_8 = 0,9 \cdot 0,1 \cdot 0,8 \cdot 0,7$$

$$P_9 = 0,1 \cdot 0,2 \cdot 0,2 \cdot 0,7$$

$$P_{10} = 0,1 \cdot 0,2 \cdot 0,8 \cdot 0,3$$

$$P_{11} = 0,1 \cdot 0,8 \cdot 0,2 \cdot 0,3$$

$$P_{12} = 0,9 \cdot 0,2 \cdot 0,2 \cdot 0,3$$

$$P_{13} = 0,1 \cdot 0,2 \cdot 0,2 \cdot 0,3$$

$B \rightarrow$ nije radio : slučajevi $\{4, 6, 8, 9, 10, 12, 13\}$

$$P(A) = \frac{P(4)+P(6)+P(8)+P(9)+P(10)+P(12)+P(13)}{\sum_{n=1}^{13} P_n}$$

$$P(A) = \frac{0,0952}{0,3952} = \underline{\underline{0,251012}}$$

SLUŽBENA RJEŠENJA:

§2. Uvjetna vjerojatnost

1. Ne.

9. $r \leq 0$, $r \geq \frac{2}{3}$ te $r = \frac{1}{3}$.

19. $A \cap B; A \cap D; A \cap E; A \cap F; B \cap C; B \cap E;$
 $B \cap F; C \cap D; C \cap E; C \cap F; D \cap E; D \cap F; E \cap F.$

20. $\frac{1}{3}$, zavisni su.

21. $\frac{91}{216}, \frac{5}{9}, \frac{1}{2}$, ne.

22. Zavisni su.

23. $\frac{5}{9}, \frac{91}{216}, \frac{60}{91}, \frac{1}{2}$.

24. $0.0309, 0.5^6, \frac{7}{11664}, \frac{1}{16}$.

25. $P(A) = \frac{1}{27}, P(B) = \frac{1}{16}, P(C) = \frac{1281}{64},$
 $P(C|B) = \frac{22}{27}.$

26. $P(A) = P(A|B) = \frac{24}{91}, P(A|C) = \frac{1260}{3367}.$

$A \cap B$ su nezavisni.

27. $\frac{1}{n}$.

28. $\frac{2}{5}$.

29. 0.1581.

30. $\frac{38}{105}$.

31. Vjerojatnost je $1/n$, za svaku osobu.

32. $\frac{244}{495}$.

33. Prvi! Vjerojatnost za njega je $\frac{47}{90}$, za drugog $\frac{16}{90}$, a za trećeg $\frac{27}{90}$. Optimalna strategija za prvog strijelca jest (ukoliko su preostala dva još uvijek nazočna) da gađa u zrak.

34. 0.3302.

35. 0.896.

36. 0.00236

37. $\frac{43}{2976}$.

38. 0.00416.

39. $\frac{12}{13}$.

40. 0.0769.

41. $\frac{2m_1m_2 + m_1n_2 + m_2n_1}{2(m_1 + n_1)(m_2 + n_2)}$.

42. $\frac{9}{35}$.

43. 0.607.

44. $\frac{1}{3}p(6 - 5p)$.

45. $\frac{1}{2} - p_1q_1q_2^2 - q_1^2p_2q_2 - \frac{1}{2}q_1^2q_2^2 + \frac{1}{2}p_1^2q_2^2 + \frac{1}{2}p_2^2q_1^2 + p_1^2p_2q_2 + p_1p_2^2q_1$.

46. 0.9204.

47. $\frac{1}{n}$.

48. $\frac{9}{35}$.

49. $\frac{C_{m_1}^2 C_{m_2+2}^2 + C_{m_1}^1 C_{n_1}^1 C_{m_2+1}^1 C_{n_2+1}^1 + C_{n_1}^2 C_{n_2+2}^2}{C_{m_1+n_1}^2 C_{m_2+n_2+2}^2}$.

50. $\frac{1}{2} + \frac{m_1m_2 + n_1n_2}{2(m_1 + n_1)(m_2 + n_2)}$

51. $\frac{10}{19}$.

52. $\frac{12}{17}$.

53. m/n u svim slučajevima

54. $\frac{m}{m+k}$.

55. $\frac{1}{2}, \left(1 + \frac{n}{2}\right) \frac{2^{n-3}}{3^{n-1}}$.

56. $\frac{124}{243}$.

57. $\frac{13}{24}$

58. $\frac{2n^2 - 3n + 2}{n^2(n-1)}.$

59. $P\{X = 12\} = \frac{25}{216}, P\{X = 11\} = \frac{27}{216},$
 $P(A | X = 12) = 0.6.$

60. $\frac{2}{5}.$

61. $\frac{1}{3}.$

62. 0.615.

63. $\frac{n_4(n_1+n_2+n_3)}{n_2n_1 + n_3(n_1+n_2) + n_4(n_1+n_2+n_3)}.$

64. $\frac{41}{55}.$

65. $\frac{1}{2}$

66. $\frac{4}{21}$

67. $\frac{1}{3}.$

68. $\frac{5}{32}.$

69. $p_1 = 0.9856, p_2 = 0.0879.$

70. 0.0392.

71. $\frac{11}{40}$

72. $\frac{17}{53}$

73. $\frac{m-1}{m+n-1}.$

74. 0.41; 0.24.

75. $\frac{4}{5}.$

76. 0.556.

77. $1/\sum_{i=0}^n \binom{n}{i} \left(\frac{i}{n}\right)^k.$

78. $\frac{37}{72}.$

79. Dvije plave kuglice.

80. 0.25.

81. $\frac{n-k}{n+m-k},$ baš kao da uzorka i nema.

82. $\frac{21}{31}.$

83. 0.552.

84. 0.298.

85. 0.251.

86. $\frac{(1-t)p}{1-tp}.$

87. $\frac{9}{16}.$

88. $\frac{q(1-\alpha)^2}{p\alpha^2 + (1-\alpha)^2 q}.$

89. 0.756.

91. $\sum_{k=0}^5 \binom{1000}{k} 0.005^k 0.995^{1000-k}.$

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(2 i 3 su za zadatke 83. I 85.)