(15) $y''(t)+5y'(t)+6y(t)=u(t)$

a) TOTALNI ODZIV $[y_{TOTALNI}(t)=\underbrace{y_{HOMOGENO}(t)}_{y_H(t)}+\underbrace{y_{PARTIKULARNO\ ILI\ PRISILNO}(t)}_{y_P(t)}]$

pobuda: $u(t)=(12t+16)\mu(t)$

početni uvjeti u 0^-: $y(0^-)=3$, $y'(0^-)=-8$

homogena jed: $y''(t)+5y'(t)+6y(t)=0$ (prirodni odziv)

$Ce^{\lambda t}(\lambda^2+5\lambda+6)=0 \quad /:Ce^{\lambda t}$

$y_H(t)=Ce^{\lambda t}$

$y_H'(t)=C\lambda e^{\lambda t}$

$y_H''(t)=C\lambda^2 e^{\lambda t}$

$\lambda^2+5\lambda+6=0$

$m=1 \quad m+n=5 \quad n=6$

$m\cdot n=1\cdot 6=6 \lessdot \begin{matrix}2\\3\end{matrix}$

$m+n=5$

$2\cdot 3=6$ ✓

$2+3=5$ ✓

$\lambda^2+3\lambda+2\lambda+6=0$

$\lambda(\lambda+3)+2(\lambda+3)=0$

$(\lambda+3)(\lambda+2)=0$

$\lambda_1=-3,\ \lambda_2=-2$

$y_H(t)=C_1e^{-3t}+C_2e^{-2t}$

$y_H(t) = C_1 e^{\quad} + C_2 e^{\quad}$

partikularno rj.: $u(t) = (12t+16)\mu(t) \Rightarrow y_p(t) = K_0 + K_1 t$

y_H (prisilni odziv)

y_H

$y_p'(t) = 0 + K_1$

$y_p''(t) = 0$

$0 + 5(0 + K_1) + 6(K_0 + K_1 t) = 12t + 16$

$5K_1 + 6K_0 + 6K_1 t = 12t + 16 \Rightarrow 5K_1 + 6K_0 = 16 \Rightarrow 5 \cdot 2 + 6K_0 = 16$

$6K_1 = 12 \Rightarrow K_1 = 2$ $\qquad 6K_0 = 6$

$K_0 = 1$

$y_p(t) = 1 + 2t$

totalni odziv:

$y_T(t) = y_H(t) + y_p(t) = C_1 e^{-3t} + C_2 e^{-2t} + 1 + 2t$

*

računanje početnih uvjeta u 0^+ iz 0^- (radi određivanja C_1 i C_2):

sustav (2.) reda $y^{(II)}(t) \Rightarrow$ treba nam $y(0^+)$ i $y'(0^+)$ $\Longrightarrow$

$y(0^+)$ se računa iz šalabahtera iz formule:

$$\Delta y = b_0 u(0^+)$$

(pri čemu: $y''(t) + 5y'(t) + 6y(t) = 0 \cdot u''(t) + 0 \cdot u'(t) + u(t)$

$a_0 = 1 \quad a_1 = 5 \quad a_2 = 6 \quad b_0 = 0 \quad b_1 = 0 \quad b_2 = 1$)

$$\underbrace{y(0^+) - y(0^-)}_{\Delta y} = \underbrace{0}_{b_0} \cdot u(0^+) \Rightarrow y(0^+) = y(0^-)$$

$$y(0^+) = 3$$

$y'(0^+)$ se računa iz: $\Delta y^{(1)}$ (1 - red derivacije) $+ a_1 \Delta y = b_0 u^{(1)}(0^+) + b_1 u(0^+)$

$$\underbrace{y'(0^+) - y'(0^-)}_{\Delta y^{(1)}} + a_1 \underbrace{(y(0^+) - y(0^-))}_{\Delta y} = \underbrace{b_0}_{0} u'(0^+) + \underbrace{b_1}_{0} u(0^+)$$

$$y'(0^+) = y'(0^-) \qquad y'(0^+) = -8$$

$$y_T(0^+) = 3 = C_1 e^{-3 \cdot 0} + C_2 e^{-2 \cdot 0} + 1 + 2 \cdot 0 = C_1 + C_2 + 1 = 3 \Rightarrow C_1 = 2 - C_2 = 2 - (-4) = 6$$

$$\left(y_T'(t) = -3C_1 e^{-3t} - 2C_2 e^{-2t} + 0 + 2 \right)$$

$$y_T'(0^+) = -8 = -3C_1 e^{-3 \cdot 0} - 2C_2 e^{-2 \cdot 0} + 2 = -3C_1 - 2C_2 + 2 = -8$$

$$-3(2 - C_2) - 2C_2 + 2 = -8$$

$$-6 + 3C_2 - 2C_2 + 2 = -8 \Rightarrow C_2 = -4$$

$$y_{TOT}(t) = \left[6e^{-3t} - 4e^{-2t} + 1 + 2t \right] \mu(t)$$

b) MIRNI SUSTAV $\Rightarrow$ početni uvjeti $y(0^+)$ i $y'(0^+)=0$

$$y_T(t)=C_1e^{-3t}+C_2e^{-2t}+1+2t$$

$$y(0^+)=0=C_1e^{-3\cdot0}+C_2e^{-2\cdot0}+1-2\cdot0=C_1+C_2+1=0 \Rightarrow C_1=-1-C_2=-1-(-5)=4$$

$$y'(0^+)=0=-3C_1e^{-3\cdot0}-2C_2e^{-2\cdot0}+\underbrace{0}_{1'}+2=-3C_1-2C_2+2=0$$

$$-3(-1-C_2)-2C_2+2=0$$

$$3+3C_2-2C_2+2=0 \Rightarrow C_2=-5$$

$$y_{MIRNI}(t)=\left[4e^{-3t}-5e^{-2t}+1+2t\right]u(t)$$

c) NEPOBUDENI SUSTAV $\Rightarrow u(t)=0$

$$y''(t)+5y'(t)+6y(t)=\underbrace{0}_{u(t)}$$

$$y_H(t)=y_{NEP}(t)=C_1e^{-3t}+C_2e^{-2t}$$

$$y_H'(t)=-3C_1e^{-3t}-2C_2e^{-2t}$$

tj. $y''(t)+5y'(t)+6y(t)=0\cdot u''(t)+0\cdot u'(t)+0\cdot u(t)$

$a_0=1$ $a_1=5$ $a_2=6$ $b_0=0$ $b_1=0$ $b_2=0$

odredivanje poč. uv. lijevo u c)

$$\Delta y=b_0u(0^+)$$

$$y(0^+)-y(0^-)=0\cdot u(0^+)$$

$y_H(t) = y_{NEP}(t) = C_1 e^{-3t} + C_2 e^{-2t}$

$y_H'(t) = -3C_1 e^{-3t} - 2C_2 e^{-2t}$

$\Delta y = b_0 u(0^+)$

$y(0^+) - y(0^-) = 0 \cdot u(0^+)$

$y(0^+) = y(0^-) = 3$

$y_H(0^+) = 3 = C_1 e^{-3\cdot 0} + C_2 e^{-2\cdot 0} = C_1 + C_2 = 3 \Rightarrow C_1 = 3 - C_2$

$\Delta y' + a_1 \Delta y = b_0 u'(0^+) + b_1 u(0^+)$

$y'(0^+) - y'(0^-) + 5(y(0^+) - y(0^-)) = 0 \cdot u'(0) + 0 \cdot u(0^+)$

$3 - 3 = 0$

$y_H'(0^+) = -8 = -3C_1 e^{-3\cdot 0} - 2C_2 e^{-2\cdot 0} =$

$= -3C_1 - 2C_2 = -8$

$y'(0^+) = y'(0^-) = -8$

$-3(3 - C_2) - 2C_2 = -8$

$-9 + 3C_2 - 2C_2 = -8 \Rightarrow C_2 = 1$

$C_1 = 3 - 1 = 2$

$y_{NEP}(t) = \left[2e^{-3t} + e^{-2t}\right] \mu(t)$

* prorjeni: $y_{TOTALNI}(t) = y_{HOMOGENO}(t) + y_{PARTIKULARNO}(t)$, ali vrijedi i
(PRIRODNI) (PRISILNI)

$y_{TOTALNI}(t) = y_{MIRNI}(t) + y_{NEPOBUDENI}(t)$

a) $y_{TOT}(t) = \left[6e^{-3t} - 4e^{-2t} + 1 + 2t\right] \mu(t)$

b) + c) $\Rightarrow y_{TOT}(t) = y_M(t) + y_N(t) = \left[4e^{-3t} - 5e^{-2t} + 1 + 2t\right] \mu(t) + \left[2e^{-3t} + e^{-2t}\right] \mu(t) =$

$= \left[6e^{-3t} - 4e^{-2t} + 1 + 2t\right] \mu(t)$ ✓

d) IMPULSNI ODZIV $\Rightarrow$ pobuda $u(t)=\delta(t)$ (Diracov impuls)

$$y''(t)+5y'(t)+6y(t)=\delta(t)$$

prvo se traži imp. odz. "temeljnog sustava" ($h_A(t)$), a zatim i imp. odz. zadanog sustava

$$y(t) \longrightarrow h_A(t)$$

homogena jed: $h_A''(t)+5h_A'(t)+6h_A(t)=0 \quad (h_A(t)=Ce^{\lambda t})$

vidi a) $\Rightarrow h_A(t)=y_H(t)=\left[C_1e^{-3t}+C_2e^{-2t}\right]\mu(t)$

(za sustav drugog reda (ili kad imamo C_1 i C_2 u $h_A(t)$), uvrjeli uvrjedi $h_A'(0^+)=1$, $h_A(0^+)=0$

tj. kad imamo: $h_A(t)=C_1e^{\lambda t} \Rightarrow h_A(0^+)=1$

$=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t} \Rightarrow h_A'(0^+)=1,\ h_A(0^+)=0$)

$h_A(0^+) = 0 = C_1 e^{-3\cdot 0} + C_2 e^{-2\cdot 0} = \underbrace{C_1 + C_2 = 0} \Rightarrow C_1 = -C_2 = -1$

$\left(h_A'(t) = -3C_1 e^{-3t} - 2C_2 e^{-2t}\right)$

$h_A'(0^+) = 1 = -3C_1 e^{-3\cdot 0} - 2C_2 e^{-2\cdot 0} = \underbrace{-3C_1 - 2C_2 = 1}$

$-3(-C_2) - 2C_2 = 1$

$3C_2 - 2C_2 = 1$

$C_2 = 1$

$h_A(t) = \left[-e^{-3t} + e^{-2t}\right]\mu(t)$

Iz "temelj. sust." $h_A(t)$, kup. odz. $h(t)$ dobije se formulom:

$$h(t) = b_0 h_A''(t) + b_1 h_A'(t) + b_2 h_A(t)$$

a naša je jednadžba: $y''(t) + 5y'(t) + 6y(t) = 0 \cdot u''(t) + 0 \cdot u'(t) + 1 \cdot u(t)$

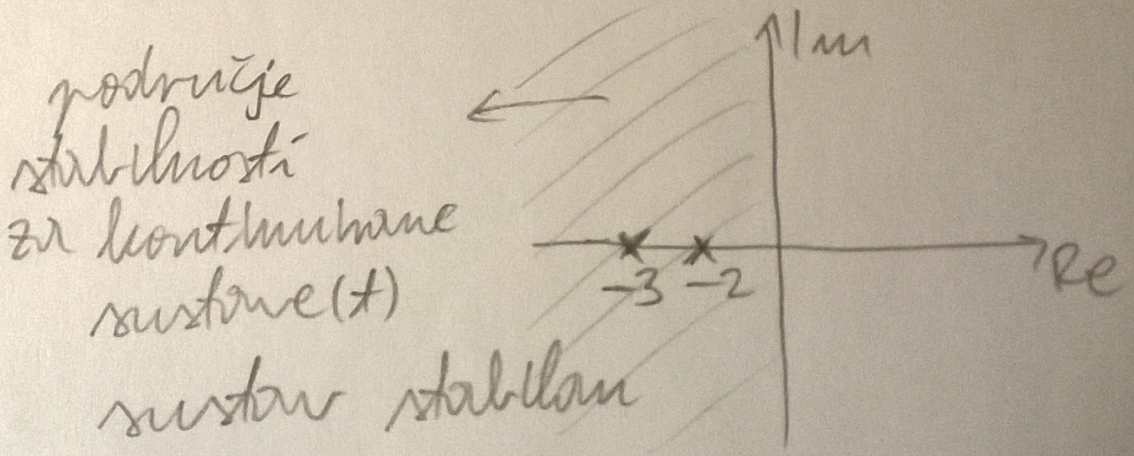
(b_0, b_1, b_2)

$h(t) = 0 \cdot h_A''(t) + 0 \cdot h_A'(t) + 1 \cdot h_A(t) = \left[e^{-2t} - e^{-3t}\right]\mu(t)$

stabilnost slijedi iz podatka o realnom dijelu nultočaka nazivnika prijenosne funkcije:

$$H(s)=\frac{b_2 s^0}{a_0 s^2+a_1 s^1+a_2 s^0}=\frac{1}{s^2+5s+6}=\frac{1}{(s+3)(s+2)}$$

"polovi": nazivnik $=0 \Rightarrow (s+3)(s+2)=0 \Rightarrow s_1=-3,\ s_2=-2$



6. a) MIRNI SUSTAV $\Rightarrow y(0^+)=y'(0^+)=0$