M1 - 2012/2013

1. ?

2. obojeni šum

$$S_{xx} = \begin{cases} 3 & za \quad 0 < |\omega| < 2\pi \\ 0 & izvan\ pojasa \end{cases}$$

a) izvediti $R_{xx}(\tau) = ?$

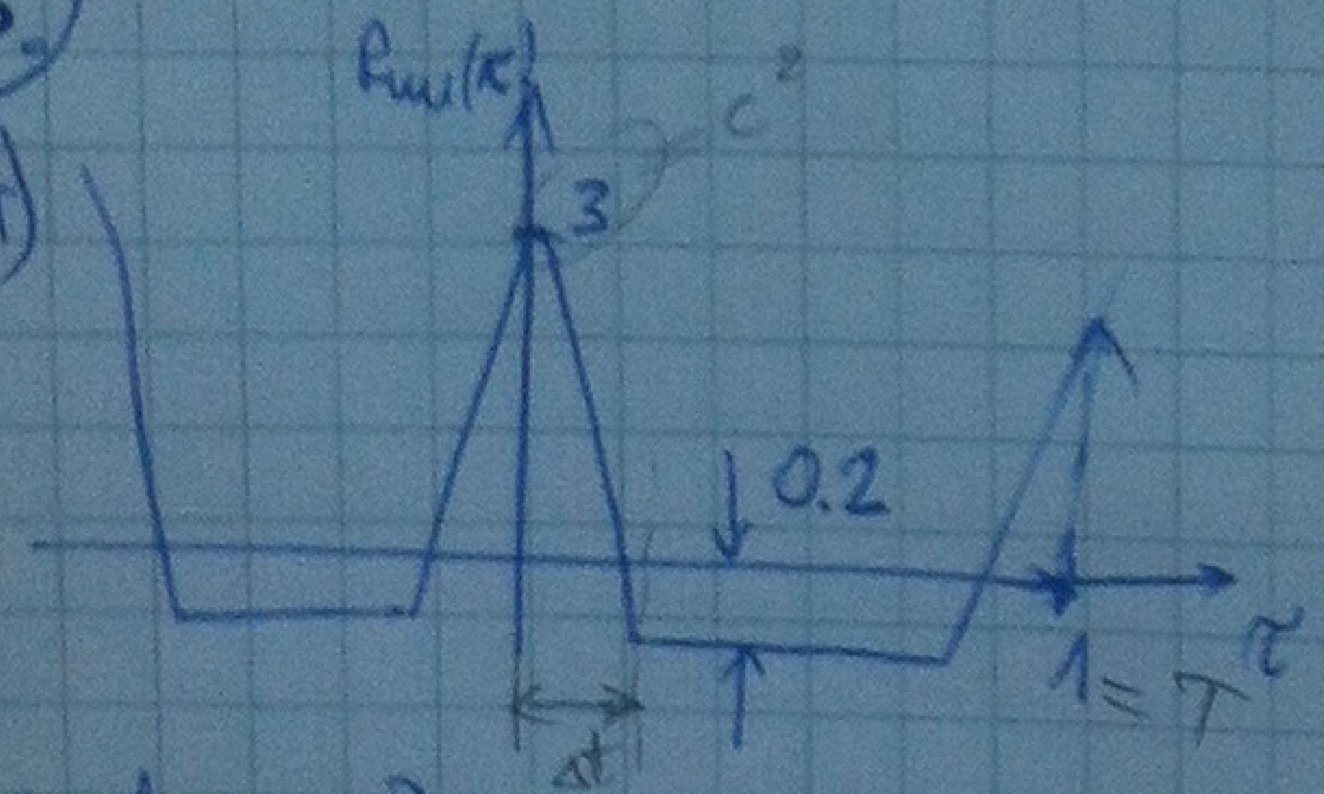
$$\boxed{R_{xx}(\tau) = \frac{1}{\pi}\int_0^{\infty} S_{xx}(\omega)\cos(\omega\tau)\,d\omega} = \frac{1}{\pi}\int_0^{2\pi} 3\cdot\cos(\omega\tau)\,d\omega = \frac{3}{\pi}\cdot\frac{\sin(\omega\tau)}{\tau}\Big|_0^{2\pi} =$$

$$= \frac{3}{\pi\tau}\cdot\left(\sin(2\pi\tau) - \cancel{\sin(0\cdot\tau)}^{0}\right) = \frac{3}{\pi\tau}\cdot\sin(2\pi\tau)$$

b) $\overline{P}_x = ?$

$$\boxed{\overline{P}_x = R_{xx}(0)} = \frac{3\cdot 2\cancel{\pi}\cdot\cos(2\pi\tau)}{\cancel{\pi}} = 6$$

3. a)




$c, \Delta t, N = ?$

$c = \sqrt{3}$

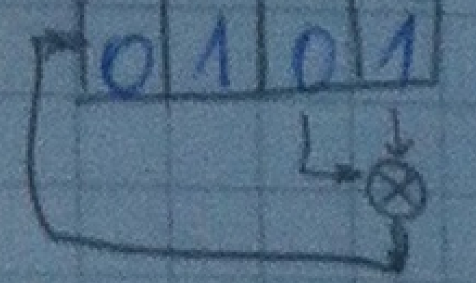
$$\boxed{R_{uu}(\tau) = \frac{-c^2}{N}}$$

$$N = -\frac{c^2}{R_{uu}(\tau)} = -\frac{3^2}{-0{,}2} = \boxed{15}$$

$$T = N\cdot\Delta t$$

$$\Delta t = \frac{T}{N} = \frac{1}{15} = 0{,}0\dot{6}$$

b)

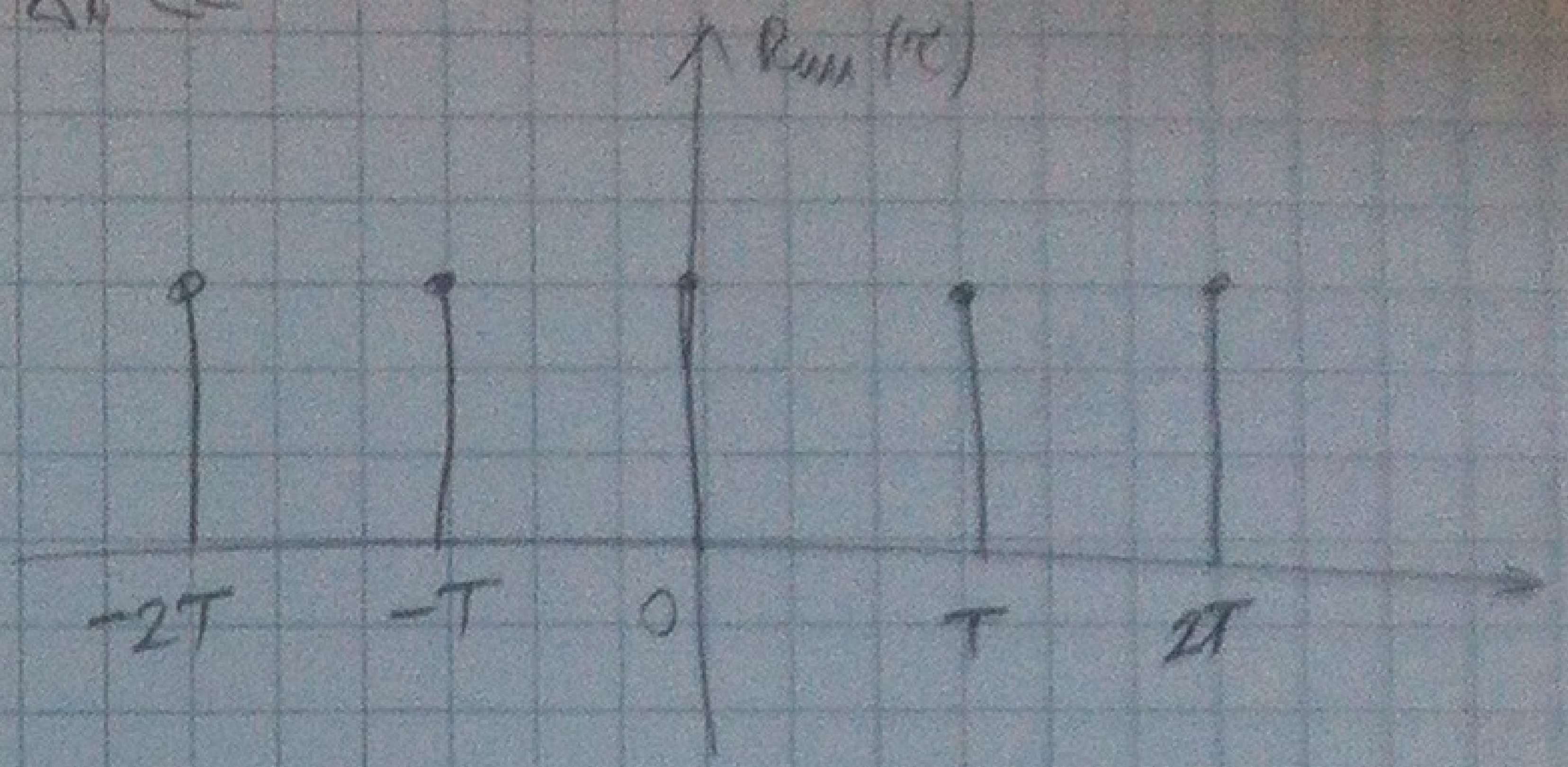



1 0 1 0
1 1 0 1
1 1 1 0
1 1 1 1
0 1 1 1
0 0 1 1
0 0 0 1

c) $N >>$ , $\Delta t <<$

$R_{uy}(\tau) = ?$ izraz

za $\Delta t <<$




$$\boxed{R_{uy}(\tau) = \int_0^{\infty} R_{uu}(\tau - \sigma)\cdot g(\sigma)\, d\sigma}$$

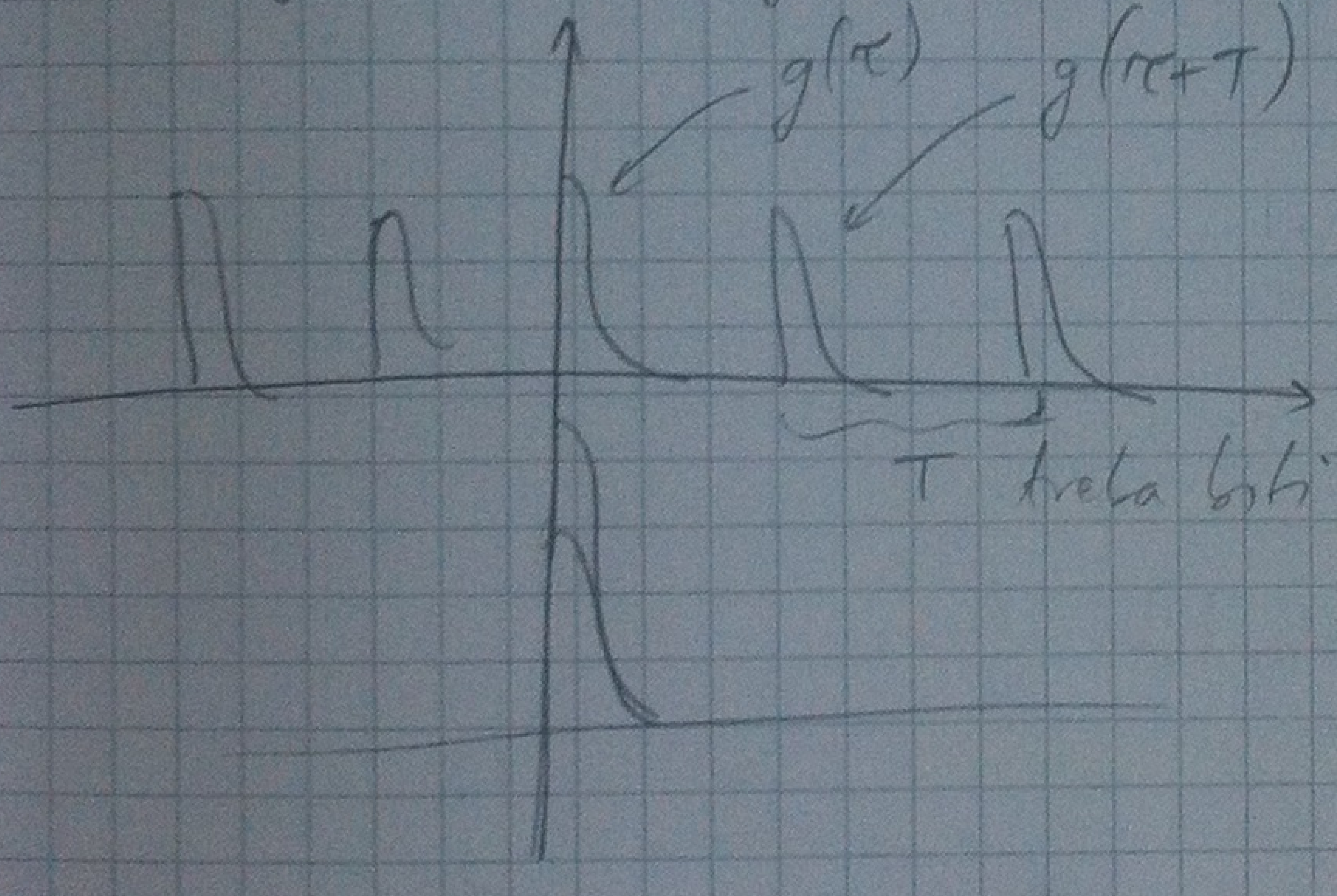
$$\boxed{R_{uu}(\tau) = c^2 \Delta t \sum_{n=-\infty}^{\infty} \delta(\tau - nT)}$$

$$R_{uy}(\tau) = c^2 \Delta t \sum_{n=-\infty}^{\infty} \int_0^{\infty} g(\sigma)\cdot\delta(\tau - \sigma - nT)\, d\sigma = c^2 \Delta t \sum_{n=-\infty}^{\infty} g(\tau - nT) =$$

$$= c^2 \cdot \Delta t \left( g(\tau + \infty T) + \ldots + g(\tau + T) + g(\tau) + \cancel{g(\tau - T)} + \ldots + \cancel{g(\tau - \infty T)} \right)$$

$\tau < T$, $g(\tau) = 0$, $\forall \tau$

$$\Rightarrow R_{uy}(\tau) = c^2 \Delta t \cdot \left( g(\tau) + g(\tau + T) + \ldots \right)$$




T treba biti dovoljno veliki da se ne bi preklapali signali ...

4. $f(t) = \lambda e^{-\lambda t}$ , $t \geq 0$

ML metodom odrediti estimat $\lambda$

$$L = \prod_{i=1}^{N} \lambda e^{-\lambda t_i} = \lambda^N \cdot e^{-\lambda \sum_i^N t_i} \quad / \ln$$

$$\ln L = N \cdot \ln \lambda + (-\lambda) \sum_i^N t_i$$

tražimo max.

$$\frac{\partial L}{\partial \lambda} = N \cdot \frac{1}{\lambda} - \sum_i^N t_i = 0$$

$$\boxed{\lambda = \frac{N}{\sum_i^N t_i}}$$

(5) $y(k) = ay(k-1) + bu(k-1) + v(k)$

$v(k) = 0{,}3\,u(k) + \varepsilon(k)$

tablica: $k \mid u(k) \mid y(k)$

$$\boxed{Y = \Phi \cdot \Theta + \varepsilon}$$

a) $\hat{a} = ?$, $\hat{b} = ?$, LS metodom.

$y(k) = ay(k-1) + bu(k-1) + 0{,}3u(k) + \varepsilon(k)$

$y(k) - 0{,}3u(k) = ay(k-1) + bu(k-1) + \varepsilon(k)$

$$\underbrace{\begin{bmatrix} y(1)-0{,}3u(1) \\ y(2)-0{,}3u(2) \\ y(3)-0{,}3u(3) \\ y(4)-0{,}3u(4) \\ y(5)-0{,}3u(5) \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} ay(0)+bu(0) \\ ay(1)+bu(1) \\ ay(2)+bu(2) \\ ay(3)+bu(3) \\ ay(4)+bu(4) \end{bmatrix}}_{\Phi\Theta} + \underbrace{\begin{bmatrix} \varepsilon(1) \\ \varepsilon(2) \\ \varepsilon(3) \\ \varepsilon(4) \\ \varepsilon(5) \end{bmatrix}}_{\varepsilon}$$

$Y$ – vektor izlaznog signala

$\Phi$ – matrica podataka

$\Theta$ – vektor parametara

$\varepsilon$ – vektor pogreške

$\hat{\Theta} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$, $\hat{\Theta} = (\Phi^T \cdot \Phi)^{-1} \cdot \Phi^T \cdot Y$

$$\Phi = \begin{bmatrix} y(0) & u(0) \\ y(1) & u(1) \\ y(2) & u(2) \\ y(3) & u(3) \\ y(4) & u(4) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0{,}76 & 1 \\ 0{,}86 & -1 \\ 0{,}02 & -1 \\ -0{,}72 & -1 \end{bmatrix}, \quad Y = \begin{bmatrix} 0{,}76-0{,}3 \\ 0{,}86+0{,}3 \\ 0{,}02+0{,}3 \\ -0{,}72+0{,}3 \\ -0{,}85-0{,}3 \end{bmatrix} = \begin{bmatrix} 0{,}46 \\ 1{,}16 \\ 0{,}32 \\ -0{,}42 \\ -1{,}15 \end{bmatrix}$$

$$\hat{\Theta} = \left( \begin{bmatrix} 1 & 0{,}76 & 0{,}86 & 0{,}02 & -0{,}72 \\ -1 & 1 & -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 0{,}76 & 1 \\ 0{,}86 & -1 \\ 0{,}02 & -1 \\ -0{,}72 & -1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 0{,}76 & 0{,}86 & 0{,}02 & -0{,}72 \\ -1 & 1 & -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0{,}46 \\ 1{,}16 \\ 0{,}32 \\ -0{,}42 \\ -1{,}15 \end{bmatrix}$$

$$= \begin{bmatrix} 2{,}836 & -0{,}4 \\ -0{,}4 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2{,}4364 \\ 1{,}95 \end{bmatrix} = \begin{bmatrix} 0{,}3566 & 0{,}0285 \\ 0{,}0285 & 0{,}2022 \end{bmatrix} \cdot \begin{bmatrix} 2{,}4364 \\ 1{,}95 \end{bmatrix} = \begin{bmatrix} 0{,}924395 \\ 0{,}463727 \end{bmatrix}$$

b) konzistentni?

?

c) IV-metodom

$y_H(k) = u(k-1)$

$W = ?$, $\Phi = ?$, $Y = ?$, izraz za identifikaciju param IV metodom.

$$\hat{\Theta} = (W^T \cdot \Phi)^{-1} \cdot \Phi^T \cdot Y$$

$$W = \begin{bmatrix} y_H(0) & u(0) \\ \vdots & \\ y_H(4) & u(4) \end{bmatrix} = \begin{bmatrix} u(0) & u(1) \\ u(1) & u(2) \\ u(2) & u(3) \\ u(3) & u(4) \end{bmatrix}, \quad \Phi = \begin{bmatrix} y(1) & u(1) \\ \vdots & \\ y(4) & u(4) \end{bmatrix}, \quad Y = ?$$

6.

a) $$\boxed{R_{xy}(\tau) = \frac{1}{N} \sum_{k=0}^{N-1} x(k)\, y(k+\tau)}$$

$N = 4$ (period)

$$R_{xy}(0) = \frac{1}{4}\big(x(0)y(0) + x(1)y(1) + x(2)y(2) + x(3)y(3)\big) = 1$$

$$R_{xy}(1) = \frac{1}{4}\big(x(0)y(1) + x(1)y(2) + x(2)y(3) + x(3)y(4)\big) = 1$$

$$R_{xy}(2) = \frac{1}{4}\big(x(0)y(2) + x(1)y(3) + x(2)y(4) + x(3)y(5)\big) = 1$$

$$R_{xy}(3) = \frac{1}{4}\big(x(0)y(3) + x(1)y(4) + x(2)y(5) + x(3)y(6)\big) = 1$$

$\vdots$

b) $R_{yx}(-317) = R_{xy}(317) = 1$

7.

$$S_{uu}(\omega) = \frac{4+25\omega^2}{\omega^2+9}, \quad S_{yy}(\omega) = \frac{16}{\omega^2+9}$$

a) $|G(j\omega)| = ?$

$$\boxed{|G(j\omega)|^2 = \frac{S_{yy}(\omega)}{S_{uu}(\omega)}} = \frac{16}{4+25\omega^2}$$

$$|G(j\omega)| = \frac{4}{\sqrt{4+25\omega^2}}$$

b) $G(j\omega) = \dfrac{a}{a+bj\omega}$, $a>0$, $b>0$

$\Rightarrow G(j\omega) = \dfrac{4}{2+j5\omega}$

$$\boxed{S_{uy}(j\omega) = S_{uu} \cdot G(j\omega)}$$

$$S_{uy}(j\omega) = \frac{4+25\omega^2}{\omega^2+9} \cdot \frac{4}{2+j\omega 5}$$

8. $y(k) = p\,y(k-1) + (1-p)\,u(k-1) + \varepsilon(k) - 0{,}2\,\varepsilon(k-1)$

a) ARMAX model

$$A(z^{-1}) \cdot y(z) = B(z^{-1}) \cdot u(z) + C(z^{-1}) \cdot \varepsilon(z)$$

$$y(k) - p\,y(k-1) = (1-p)\,u(k-1) + \varepsilon(k) - 0{,}2 \cdot \varepsilon(k-1)$$

$A(z^{-1}) = 1 - p z^{-1}$ !?

$B(z^{-1}) = (1-p)\,z^{-1}$

$C(z^{-1}) = 1 - 0{,}2\,z^{-1}$

b) ?