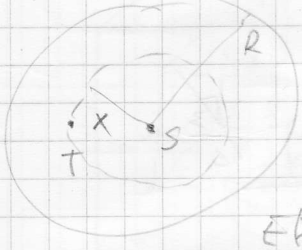
ZZV 18



$$P(X<x) = \frac{x^2\pi}{R^2\pi} = \frac{x^2}{R^2} = F_X(x)$$

$$f(x) = \left(\frac{x^2}{R^2}\right)' = \frac{2x}{R^2}$$

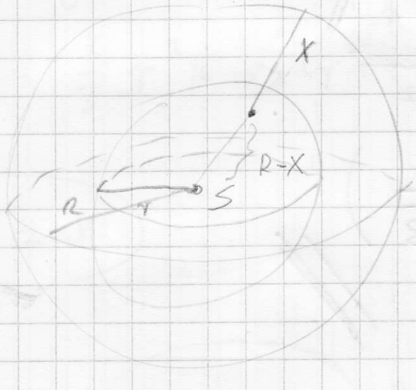
$$E(X) = \int_0^R x \cdot \frac{2x}{R^2} = \frac{2x^3}{3R^2}\Big|_0^R = \frac{2}{3}R$$

$x_{MAX} = R$

ZZV 20

$$P(X<x) = 1 - \frac{\cancel{\frac{4}{3}}(R-x)^3\cancel{\pi}}{\cancel{\frac{4}{3}}r^3\cancel{\pi}}$$

$P(X>x)$



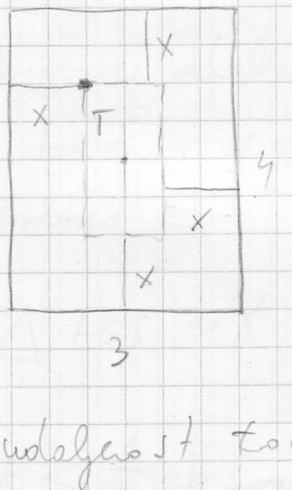
$$F_X(x) = 1 - \frac{(R-x)^3}{r^3} \qquad 0<X<R$$

Točka T unutar manje kugl

ZZV 21.

$12 - 6x - 8x + 4x^2$

$$P(X<x) = \frac{12 - (3-2x)(4-2x)}{12}$$



$$F_X(x) = \frac{4x^2 - 14x}{12}$$

$$f(x) = \frac{2}{3}x - \frac{7}{6}$$

X ~ udaljenost točke T do najbliže stranice

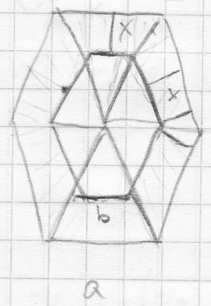
$$E(X) = \int_0^{1.5} \frac{2}{3}x^2 - \frac{7}{6}x$$

$$= 0.5625 = \frac{9}{16}$$

$X \sim (0, 1.5)$

(23.)

$$P(X < x) = \frac{P_{VELIKI} - P_{MALI}}{P_{VELIKI}}$$

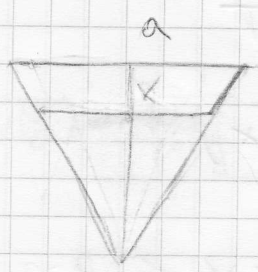


$$F_X(x) = 1 - \frac{6 \cdot \left(a - \frac{2}{\sqrt{3}}x\right)^2 \cancel{\frac{\sqrt{3}}{4}}}{\cancel{6} \cdot \frac{a^2 \cancel{\sqrt{3}}}{\cancel{4}}}$$

X ~ „udaljenost do najbliže strane"

$$f(x) = \left(1 - \frac{a^2 - \frac{4a}{\sqrt{3}}x + \frac{4}{3}x^2}{a^2}\right)'$$

$$P_{VELIKI} = 6 \cdot \frac{a^2\sqrt{3}}{4}$$



$$= \frac{-\frac{4}{3} \cdot 2x}{a^2} + \frac{\frac{4}{\sqrt{3}}a}{a^2}$$

$$= \frac{4}{\sqrt{3}a}\left(1 - \frac{2x}{\sqrt{3}}\right)$$

$$v_{VELIKI} = \frac{a\sqrt{3}}{2}$$

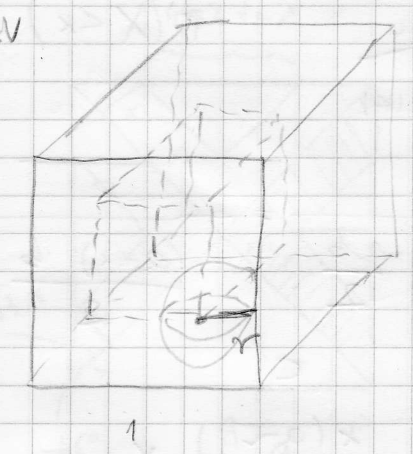
$$v_{MALI} = \frac{a\sqrt{3}}{2} - x = \frac{b\sqrt{3}}{2} \Rightarrow \frac{a\sqrt{3} - 2x}{\sqrt{3}} = b$$

$$0 < x < \frac{a\sqrt{3}}{2}$$

$$b = a - \frac{2}{\sqrt{3}}x$$

$$RJ: f_X(x) = \frac{4}{a\sqrt{3}}\left(1 - \frac{2x}{a\sqrt{3}}\right)$$

(25.) ZZV



$$F(x) = P(X < x)$$

$$= \frac{1 - (1-2r)^3}{1}$$

$$F_X(x) = 1 - \left(1 - 2\sqrt[3]{\frac{3x}{4\pi}}\right)^3$$

T unutar kocke

$X \sim$ volumen kugle koja se upisuje

$$X = \frac{4}{3} r^3 \pi \Rightarrow r = \sqrt[3]{\frac{3X}{4\pi}}$$

$$0 < X < \frac{4}{3}\left(\frac{1}{2}\right)^3 \pi$$

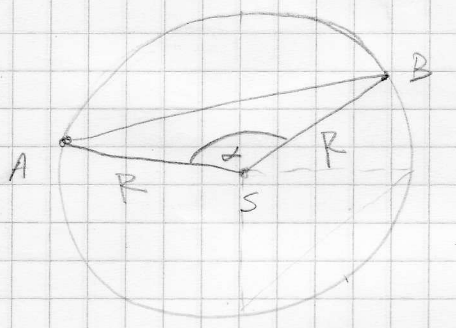
$$0 < x < \frac{\pi}{6}$$

27. ZZV

$\alpha \in (0, \pi)$ u $[0, \pi]$

$$f(\alpha) = \frac{1}{\pi}$$

$$F(x) = P(X < x)$$



$$g_\alpha(\varphi) = f(\alpha) \left| \frac{d\left(\arcsin \frac{2x}{R^2}\right)}{dx} \right|$$

$$g_\alpha = \frac{1}{\pi} \cdot \frac{1}{\sqrt{1 - \frac{4x^2}{R^4}}} \cdot \frac{2}{R^2} = \frac{2}{\pi\sqrt{R^4 - 4x^2}}$$

→ na kružnici odaberemo dvije točke i spojimo sa središtem

→ pov $\triangle ABS \sim X = \frac{1}{2} R^2 \sin\alpha \Leftrightarrow \alpha = \arcsin \frac{2X}{R^2}$

$\varphi(\alpha)$

$$0 < x < \frac{1}{2} ab \sin\alpha$$

$$0 < x < \frac{1}{2} R^2 \sin\frac{\pi}{2}$$

$$0 < x < \frac{R^2}{2}$$

$$E(X) = \int_0^\pi \frac{1}{2} R^2 \sin\alpha \cdot f(\alpha)\, d\alpha$$

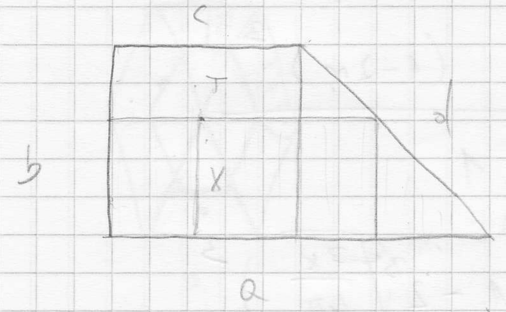
$$= \frac{1}{2} R^2 \cdot \left(-\cos\alpha \Big|_0^\pi\right) \cdot \frac{1}{\pi}$$

$$= \frac{R^2}{\pi}$$

$E(X) = \frac{R^2}{\pi}$ RJ: $\frac{4}{\pi\sqrt{R^4 - 4x^2}} = f(x)$

29.

$$F_x(X) = 1 - \frac{P_{MALI}}{P_{VELIKI}} \qquad P(X<x)$$



$$F_x(x) = 1 - \frac{\quad}{c^2 + \frac{a \cdot b}{2}} = \frac{\frac{\quad}{2}}{\frac{(a+c)\cdot b}{2}}$$

c'

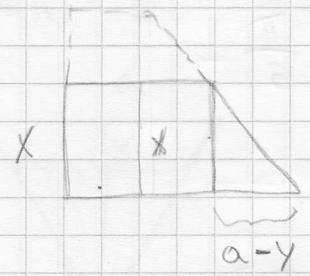
$$\frac{(a+c)\cdot \overset{b}{v}}{2}$$

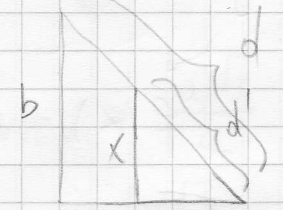
T unutar trapeza

X udaljenost do stranice a

$$F_x(x) = 1 - \frac{\left(a + a - \frac{x(a-c)}{b}\right)\cdot x}{(a+c)\cdot b}$$

$$F_x(x) = 1 - \frac{2abx - x^2(a-c)}{(a+c)b^2}$$





$$\frac{d}{b} = \frac{d'}{x} \Rightarrow d' = \frac{x \cdot d}{b}$$

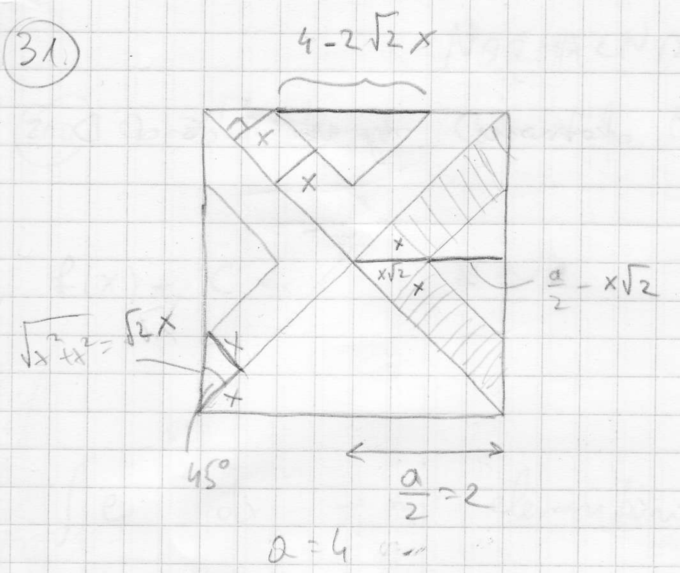
$$y = \sqrt{\frac{x^2 d^2}{b^2} - \frac{x^2 b^2}{b^2}}$$

$$y = \frac{x\sqrt{d^2 - b^2}}{b}$$

$$(a-c) = \sqrt{d^2 - b^2}$$

$$y = \frac{x \cdot (a-c)}{b}$$

31.



$$P(X<x) = F_X(x)$$

$$= 16 - \frac{4\cdot\left(\frac{a}{2}-x\sqrt{2}\right)\cdot\left(4-2\sqrt{2}x\right)}{4^2}$$

$$1 - P(X > x)$$

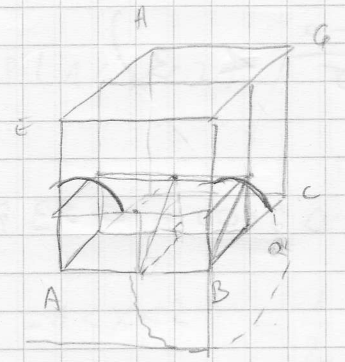
→ unutar kvadrata

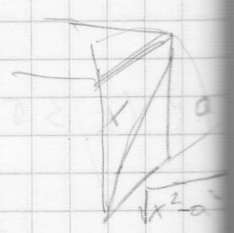
$X \sim$ udaljenost do bliže dijagonale

ZZV 33

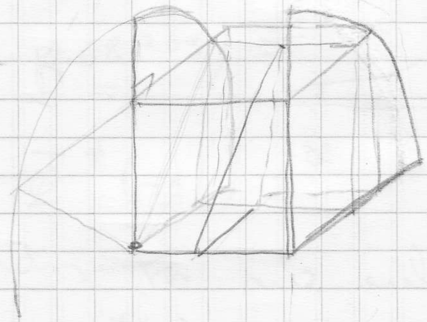
I

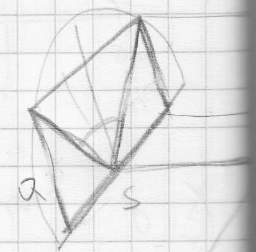
$$F(x) = P(X < x) = \frac{m(G_x)}{a^3}$$





II





$$0 < X < a\sqrt{2}$$

I $x \in [0, a]$

$$P(X<x) = \frac{\frac{1}{4} V_{VALJKA}}{a^3} = \frac{\frac{1}{4} \cdot x^2 \pi \cdot a}{a^3} = \frac{1}{4}\frac{x^2}{a^2} = F(x)$$

II $x \in [a, a\sqrt{2}]$

$$P(X<x) = \frac{\frac{1}{4} V_{VALJKA} - V_{\text{dio valjka van kocke}}}{a^3}$$

$$= \frac{1}{4}\frac{x^2\pi}{a^2} - \frac{1}{a^3}\left(\frac{1}{4}x^2\pi a - \sqrt{x^2-a^2}\cdot a\right)$$

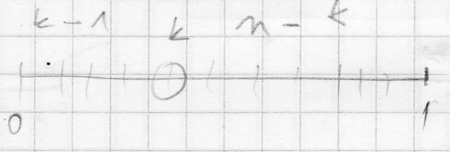
$$f(x) = \begin{cases} \dfrac{x\pi}{2a^2} & x \in [0, a] \\ \dfrac{x\pi}{2a^2} - \left(\dfrac{x^2\pi}{4a^2} - \dfrac{\sqrt{x^2-a^2}}{a^2}\right)' & \end{cases}$$

$$\frac{x\pi}{2a^2} - \frac{1}{a^2} \cdot \frac{1}{2\sqrt{x^2-a^2}} \cdot 2x$$

$$\frac{x\pi}{2a^2} - \frac{2x}{a^2}$$

Rj. $\frac{\pi}{2a^2}x - \frac{2x}{a^2}\arccos$

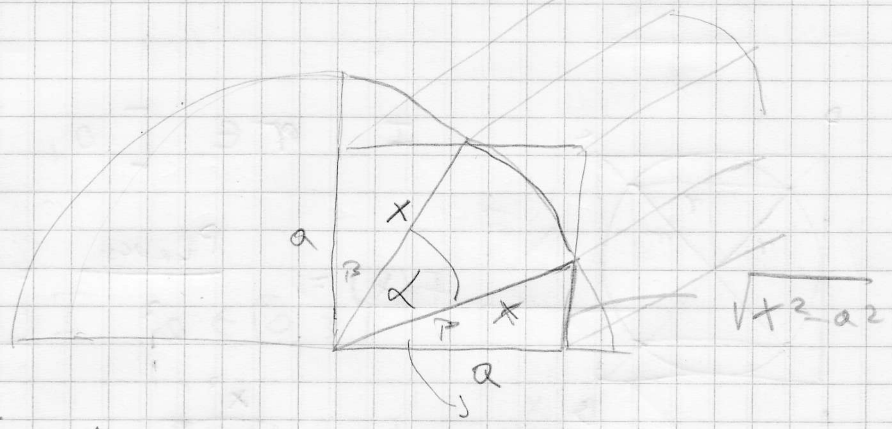
(37) n točaka na intervalu $[0,1]$. X_k je k-to točka s lijeva.
Odredi gustoću



$$f_x(x) = \binom{n}{1}\binom{n-1}{k-1}\cdot x^{k-1}(1-x)^{n-k}$$

mjesto za ovo mjesto

→ II $x \in [a, a\sqrt{2}]$



$$\alpha = \frac{\pi}{2} - 2\beta$$

$$= \frac{\pi}{2} - 2\arccos\frac{a}{x}$$

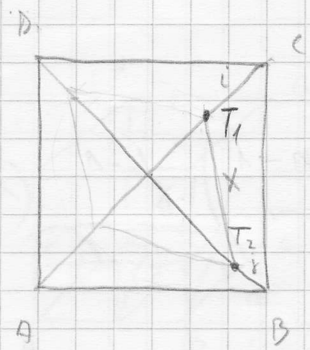
$$P(X<x) = \frac{\cancel{\alpha}\cdot\cancel{2}\cdot\frac{a\cdot\sqrt{x^2-a^2}}{\cancel{2}} + \frac{1}{2}x^2\alpha\cdot\cancel{a}}{a^{\cancel{3}\,2}} \qquad \cos\beta = \frac{a}{x}$$

$$= \frac{a\sqrt{x^2-a^2} + \frac{1}{2}x^2\cdot\frac{\pi}{2} - x^2\arccos\frac{a}{x}}{a^2}$$

$$f(x) = \frac{1}{a}\cdot\frac{1}{\cancel{2}\sqrt{x^2-a^2}}\cdot\cancel{2}x + \frac{x\pi}{2a^2} - \frac{2x}{a^2}\arccos\frac{a}{x} - \frac{x^2\frac{1}{-\sqrt{1-\frac{a^2}{x^2}}}\cdot\frac{-a}{\cancel{x^2}}}{a^2}$$

$$= \qquad -\frac{1}{a}\frac{\sqrt{x^2}}{\sqrt{x^2-a^2}}$$

43.



$a=2$

$$F(x) = P(X<x) = \frac{m(G_x)}{a^2}$$

$$F(x) = \frac{a^2 - P_{ROMBA}}{a^2}$$

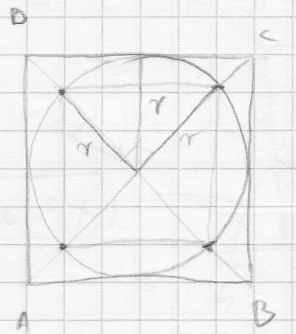
$$= 1 - \frac{\frac{e \cdot f}{2}}{a^2}$$

$$= 1 - \frac{(a\sqrt{2}-2i)\cdot(a\sqrt{2}-2j)}{a^2}$$

$$x = |T_1 - T_2|$$

$$X \in [0, a]$$

43.



$$\text{I} \quad r \in \left[0, \frac{a}{2}\right] \Rightarrow x \in [0, \sqrt{2}]$$

$$F(x) = \frac{P_{KRUGA}}{a^2} = \frac{r^2\pi}{a^2} = \frac{\frac{x^2}{2}\pi}{4}$$

$$= \frac{x^2}{8}\pi$$

$$X = \sqrt{r^2+r^2} =$$

$$x = \sqrt{2}\,r$$

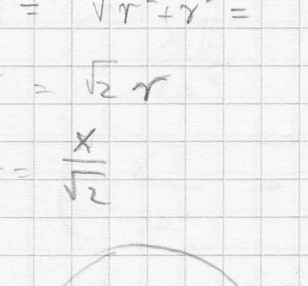
$$r = \frac{x}{\sqrt{2}}$$

$$f'(x) = \frac{x}{4}\pi$$

$2\sqrt{2}$

$\sqrt{\left(\frac{2\sqrt{2}}{2}\right)^2}$

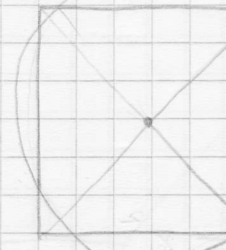
$$\text{II} \quad r \in \left[\frac{a}{2}, \frac{a\sqrt{2}}{2}\right] \Rightarrow x \in [\sqrt{2}, 2]$$



$$F(x) = \frac{\frac{x^2}{8}\pi - (\text{KRUŽNI ISJEČAK} + 2\cdot\text{TROKUTIĆ})}{a^2}$$

$$2\cdot\frac{1\cdot r}{2}$$

$$\frac{1}{2}\cdot r^2\alpha$$

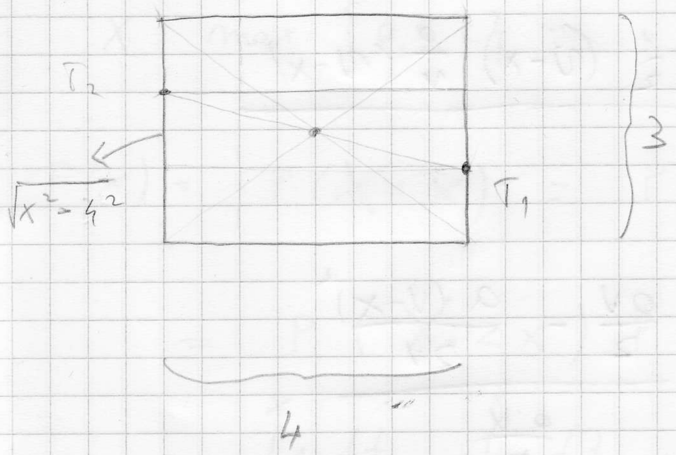


$$\cos\beta = \frac{1}{r} = \frac{\sqrt{2}}{x}$$

$$\beta = \arccos\frac{\sqrt{2}}{x}$$

$$\alpha = \frac{\pi}{2} - 2\beta$$

45.



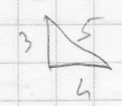
$$F(x) = P(X < x)$$

$$F(x) = \frac{2 \cdot 4 \sqrt{x^2 - 4^2}}{4 \cdot 3}$$

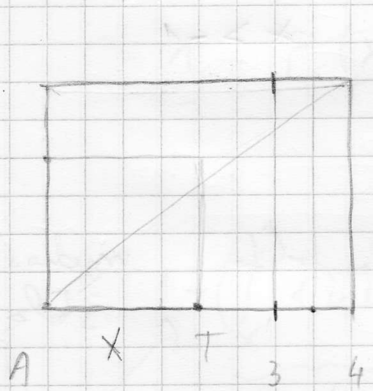
$$F(x) = \frac{2}{3}\sqrt{x^2 - 4^2}$$

$X \to$ udaljenost $|T_1 T_2|$

$x \in [4, 5]$



46.



I $\quad x \in [0, 3]$

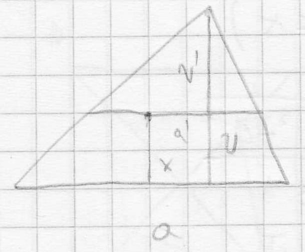
$$F(x) = P(X < x) = \frac{x^2}{4 \cdot 3}$$

II $\quad x \in [3, 4]$

$x \in [0, 5]$

udaljenost T do A

19.

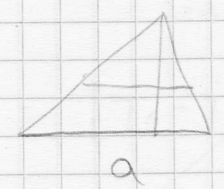


$$P(x<X) = F(x)$$

$$= \frac{\frac{a\cdot v}{2} - \frac{1}{2}(v-x)\cdot\frac{a}{v}(v-x)}{\frac{a\cdot v}{2}}$$

X ~ udaljenost do osnovice

$x \in [0, v]$



$$F(x) = \frac{\frac{av}{2} - \frac{a(v-x)^2}{2v}}{\frac{a\cdot v}{2}}$$

$$= \frac{1}{v} - 1 - \frac{2x}{v} + \frac{x^2}{v^2}$$

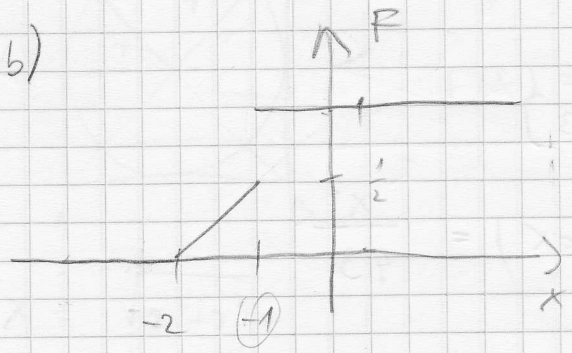
$$\frac{v}{a} = \frac{v-x}{a'}$$

$$a' = \frac{a(v-x)}{v}$$

$$f(x) = F'(x) = -\frac{2}{v} + \frac{2x}{v^2}$$

15.

b)



$$F(x) = \begin{cases} 0 & x \le -2 \\ \frac{1}{2}(x+2) & -2 < x \le -1 \\ 1 & x > -1 \end{cases}$$

PREKID

točka prekida

vrijednost skoka

F'(x)

$$E(X) = \int_{-\infty}^{-2} x\cdot 0 + \int_{-2}^{-1} x\cdot\frac{1}{2}\,dx + \int_{-1}^{\infty} x\cdot 0\cdot dx + -1\cdot\frac{1}{2}$$

$$= \frac{-5}{4}$$

(38) interval $[0,1]$, tri broja a, b, c

$X = \max\{a, b, c\}$ $\qquad \downarrow$ uniformno $[0,1]$ $f(x) = \frac{1}{1}$

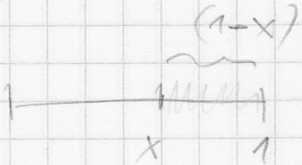
$$F(x) = P(X < x) = P(\max\{X_1, X_2, X_3\} < x)$$

$$= P(X_1 < x)\, P(X_2 < x)\, P(X_3 < x)$$

$$= \int_0^x 1 \cdot dt \cdot \int_0^x 1 \cdot dt \int_0^x 1 \cdot dt = x^3$$

$f(x) = 3x^2$

$$E(X) = \int_0^1 x \cdot 3x^2 = \frac{3}{4} \;//$$



$X = \min\{X_1, X_2, X_3\}$

$$F(x) = P(X < x) = P(\min\{X_1, X_2, X_3\} < x)$$

$$= 1 - P(X_1 > x) \cdot P(X_2 > x) \cdot P(X_3 > x)$$

$$P(\min\{X_1, X_2, X_3\} > x)$$

$$= 1 - \Big(\underbrace{\int_x^1 1\, dt}\Big)^3 = 1 - (1-x)^3$$

$$\downarrow$$

$$\text{ILI} \quad \left(1 - \int_0^x 1\, dt\right)^3$$