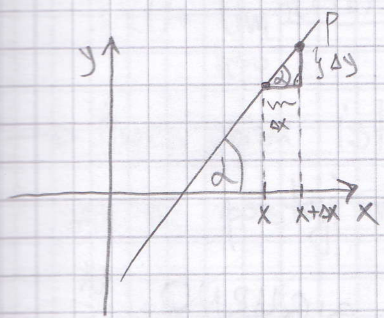
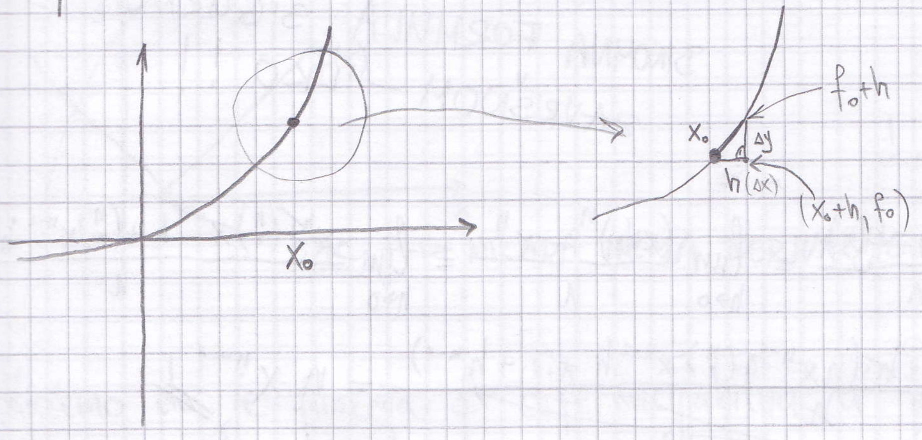
8. DIFERENCIJALNI RAČUN

8.1. MOTIVACIJA I DEFINICIJA DERIVACIJE



$p \ldots y = kx + l$

$k = \operatorname{tg} \alpha = \dfrac{\Delta y}{\Delta x}$ „brzina promjene y ovisno o x"



Def. Derivaciju funkcije f u točki x_0 definiramo sa

$$\boxed{f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}}$$

→ još koristimo oznaku $f'(x_0) = \dfrac{df(x_0)}{dx}$

$df(x) = f'(x) \cdot dx$ zovemo diferencijal funkcije f

$\Delta f(x) = f(x+\Delta x) - f(x)$ zovemo prirast funkcije f

$$\boxed{f'(x) = \lim_{h \to 0} \frac{\Delta f(x)}{\Delta x} = \frac{df(x)}{dx}}$$

1.) $f(x)=c$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{c-c}{h}=\lim_{h\to 0}\frac{0}{h}=0 \,//$$

$$\boxed{c'=0}$$

2.) $f(x)=x^n,\ n\in\mathbb{N}$

$$\boxed{f'(x)=nx^{n-1}}$$

BINOMNA FORMULA SIGURNO U ZAVRŠNOM – ILKO !!

!!! VAŽNO ZA ISPIT: !!

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}=\lim_{h\to 0}\frac{\cancel{x^n}+\binom{n}{1}x^{n-1}h+\binom{n}{2}x^{n-2}h^2+\dots+h^n\cancel{-x^n}}{h}$$

$$=\left(\frac{0}{0}\right)=\lim_{h\to 0}\frac{\cancel{h}\cdot\left(nx^{n-1}+\binom{n}{2}x^{n-2}h+\dots+h^{n-1}\right)}{\cancel{h}}=nx^{n-1} \,//$$

vrijedi i za $n\in\mathbb{R}$ wow !!

2. MI-2009-4)

$$f(x)=\sqrt[3]{x} \longrightarrow (x^{1/3})'=\frac{1}{3}x^{\frac{1}{3}-1}=\frac{1}{3}x^{-\frac{2}{3}}=\frac{1}{3\sqrt[3]{x^2}}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt[3]{(x+h)}-\sqrt[3]{x}}{h}=\left(\text{ZA DZ. } \sqrt[6]{x}\right)=$$

$$=\lim_{h\to 0}\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}\cdot\frac{\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)\cdot x}+\sqrt[3]{x^2}}{-\,||\,-}=$$

$$=\lim_{h\to 0}\frac{\cancel{x+h-x}}{\cancel{h}\cdot\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)\cdot x}+\sqrt[3]{x^2}\right)}=\lim_{h\to 0}\frac{1}{3\sqrt[3]{x^2}}=\frac{1}{3\sqrt[3]{x^2}}$$

ZAVRŠNI → 8 ZAD PO 5 BODOVA IZ DRUGOG DIJELA

2 ZAD PO 5 BODOVA IZ PRVOG DIJELA (vjerojatno matrice) } 50

= 10 ZAD × 5 BOD, 2.5 h ISPIT

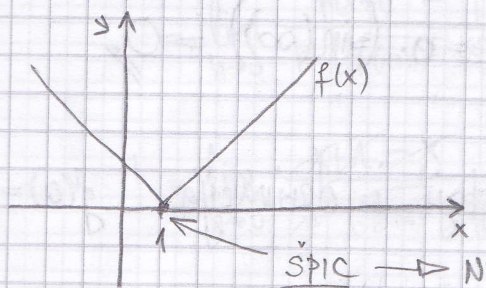
IM-08-4.)

$f(x) = |x-1|$. Postoji li $f'(1)$?

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{|1+h-1| - |0|}{h} = \lim_{h\to 0} \frac{|h|}{h} = \begin{cases} \lim\limits_{h\to 0^+} \frac{h}{h} = 1 \\ \lim\limits_{h\to 0^-} \frac{-h}{h} = -1 \end{cases}$$

→ limesi nisu isti ⇒ ne postoji limes $h \to 0$

⇒ ne postoji ni derivacija



ŠPIC → NIKADA NEMA DERIVACIJE U TOJ TOČKI !!!

def. Kažemo da je funkcija f diferencijabilna u točki x ako postoji derivacija funkcije u x.
Još kažemo da je f GLATKA u x.

⇒ !! Ako je funkcija neprekinuta ne mora značiti da je diferencijabil.

$\boxed{\text{TM}}$ Ako je funkcija diferencijabilna u točki x onda je neprekinuta u toj točki.

Dokaz: !!! VAŽNO ZA ISPIT !!

$$\lim_{h\to 0}[f(x+h) - f(x)] =$$

$$= \lim_{h\to 0} \frac{h \cdot [f(x+h) - f(x)]}{h} = 0 \cdot f'(x) = 0$$

$f'(x)$ → postoji po pretpostavci TM

⇒ $\lim\limits_{h\to 0} f(x+h) = f(x)$ pa je f neprekinuta !!

* podsjetnik neprekinutosti: $\lim\limits_{x\to a} f(x) = f(a)$

NAPOMENA: OBRAT $\boxed{\text{TM}}$ NE VRIJEDI !!! (Rečenica prije, ako ima špic)

npr. $f(x) = |x|$ nije diferenc. u $x = 0$

8.DZ-7.) a) Postoji li $f'(0)$ za $f(x) = x \sin\frac{1}{x}$, $x \neq 0$, $f(0) = 0$?

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h\sin\frac{1}{h} - 0}{h} = \lim_{h\to 0} \sin\frac{1}{h} = (\underbrace{\sin(\infty)}_{\to [-1,1]}) \Rightarrow \text{LIMES}$$

NE POSTOJI PA NE POSTOJI $f'(0)$!

b)

$g(x) = x^2 \sin\frac{1}{x}$, $x \neq 0$, $g(0) = 0$

0 · nešto omeđeno

$$g'(0) = \lim_{h\to 0} \frac{g(0+h) - g(0)}{h} = \lim_{h\to 0} \frac{h^2 \sin\frac{1}{h} - 0}{h} = 0 \cdot (\underbrace{\sin(\infty)}_{[-1,1]}) = 0$$

→ postoji limes, jednak je 0, pa postoji i derivacija! $g'(0) = 0$.

$\boxed{\text{TM}}$ Vrijedi: 1) $(c \cdot f'(x)) = c \cdot f'(x)$

2) $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

3) $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g(x)$

4) $$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Dokazi: (VAŽNI ZA ISPIT !!!)

1. ubaciti u definiciju → TRIVIJALNO

2. $$(f(x) \pm g(x))' = \lim_{h\to 0} \frac{f(x+h) \pm g(x+h) - f(x) - g(x)}{h} =$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \pm \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = f'(x) \pm g'(x)$$

3. $$(f(x) \cdot g(x))' = \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x) - f(x)\cdot g(x+h) + f(x)\cdot g(x+h)}{h} =$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \cdot g(h+x) + \lim_{h\to 0} f(x) \cdot \frac{g(x+h) - g(x)}{h} = f'(x)\cdot g(x) - f(x)\cdot g'(x$$

4. DOMA! Zad. za DZ: a) $f(x) = (3x^2+1)\sin x$ b) $f(x) = \frac{5x+1}{4\cos x}$

EŽBA:

$$f'(x_0)=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}=\frac{d f(x_0)}{dx} \longrightarrow \boxed{d f(x)=f'(x)\cdot dx}$$

Burić:

$$f(x)=\sqrt[6]{x}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt[6]{(x+h)}-\sqrt[6]{x}}{h}\cdot\frac{\left(\sqrt[6]{(x+h)^2}+\sqrt[6]{(x+h)\cdot x}+\sqrt[6]{(x)^2}\right)}{\left(\sqrt[6]{(x+h)^2}+\sqrt[6]{(x+h)\cdot x}+\sqrt[6]{(x)^2}\right)}$$

$$=\lim_{h\to 0}\frac{\sqrt{(x+h)}-\sqrt{x}}{h\cdot\left(\sqrt[6]{(x+h)^2}+\sqrt[6]{(x+h)\cdot x}+\sqrt[6]{x^2}\right)}\cdot\frac{\sqrt{(x+h)}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=$$

$$=\lim_{h\to 0}\frac{\cancel{x}+\cancel{h}-\cancel{x}}{\cancel{h}\cdot\left(\sqrt[6]{(x+h)^2}+\sqrt[6]{(x+h)\cdot x}+\sqrt[6]{x^2}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}=$$

$$=\lim_{h\to 0}\frac{1}{\left(\sqrt[3]{x}+\sqrt[3]{x}+\sqrt[3]{x}\right)\left(2\sqrt{x}\right)}=\frac{1}{3\underbrace{\sqrt[3]{x}}_{x^{1/3}}\cdot 2\underbrace{\sqrt{x}}_{x^{1/2}}}=\frac{1}{6\sqrt[6]{x^5}}$$

$$\frac{1}{3}+\frac{1}{2}=\frac{2+3}{6}=\frac{5}{6}$$

provjera:

$$f(x)=x^{1/6}$$

$$f'(x)=\frac{1}{6}\cdot x^{\frac{1}{6}-1}=\frac{1}{6\sqrt[6]{x^5}}=\frac{1}{6}\cdot x^{-\frac{5}{6}}$$

o.k.

TM Ako je funkcija diferencijabilna onda je neprekinuta u toj točki.

Obrat ne mora vrijediti!

Ako je funkcija neprekinuta u točki ne mora značiti da je i diferencijabilna u toj točki!

Dokaz:

* podsjetnik neprekinutosti: $\lim_{x\to a} f(x)=f(a)$

$$\lim_{h\to 0}\left[f(x+h)-f(x)\right]=\lim_{h\to 0}\frac{h\cdot\left[f(x+h)-f(x)\right]}{h}=\lim_{h\to 0}h\cdot f'(x)=0\cdot f'(x)=0$$

↓ po pretpostavci TM je diferenc.

$$\rightarrow \lim_{h\to 0} f(x+h)=f(x)$$ → neprekinuta

Dokazi:

2.) $$\left(f(x) \pm g(x)\right)' = \lim_{h\to 0} \frac{f(x+h) \pm g(x+h) - f(x) - g(x)}{h} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \pm \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) \pm g'(x)$$

3.) $$\left(f(x)\cdot g(x)\right)' = \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x) - f(x)\cdot g(x+h) + f(x)\cdot g(x+h)}{h} =$$

$$= \lim_{h\to 0} \underbrace{\frac{f(x+h) - f(x)}{h}}_{f'(x)} \cdot g(x+h) + \lim_{h\to 0} \underbrace{\frac{g(x+h) - g(x)}{h}}_{g'(x)} \cdot f(x) =$$

$$= f'(x)\cdot g(x+0) + g'(x)\cdot f(x) = f'(x)\cdot g(x) + g'(x)\cdot f(x)$$

BUREK DZ:

1.) $f(x) = (3x^2+1)\sin x$

$f'(x) = 6x\cdot \sin x + (3x^2+1)\cdot \cos x$

$$\boxed{\begin{aligned}(\sin x)' &= \cos x \\ (\cos x)' &= -\sin x\end{aligned}}$$

2.) $f(x) = \dfrac{5x+1}{4\cos x}$

$$f'(x) = \frac{5\cdot 4\cos x + (5x+1)4\cdot \sin x}{16\cos^2 x} = \frac{20\cos x + 4(5x+1)\sin x}{16\cos^2 x}$$

$f(x)$	$f'(x)$
c	0
x	1
x^n	nx^{n-1}
e^x	e^x
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \ln a}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\mathrm{tg}\, x$	$\frac{1}{\cos^2 x}$
$\mathrm{ctg}\, x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$
$\mathrm{arctg}\, x$	$\frac{1}{x^2+1}$
$\mathrm{arcctg}\, x$	$\frac{-1}{1+x^2}$
$\mathrm{sh}\, x$	$\mathrm{ch}\, x$
$\mathrm{ch}\, x$	$\mathrm{sh}\, x$
$\mathrm{th}\, x$	$\frac{1}{\mathrm{ch}^2 x}$
$\mathrm{cth}\, x$	$\frac{-1}{\mathrm{sh}^2 x}$
$\mathrm{arsh}\, x$	$\frac{1}{\sqrt{1+x^2}}$
$\mathrm{arch}\, x$	$\frac{1}{\sqrt{x^2-1}}$
$\mathrm{arth}\, x$	$\frac{1}{1-x^2}$
$\mathrm{arcth}\, x$	$\frac{1}{1-x^2}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\frac{1}{x}$	$-\frac{1}{x^2}$

DOKAZI:

1) $f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h\to 0} \frac{2\sin\frac{h}{2}\cos\left(x+\frac{h}{2}\right)}{h} =$$

!! $\sin\alpha - \sin\beta$

$$= \lim_{h\to 0} \frac{\sin\frac{h}{2}\cos\left(x+\frac{h}{2}\right)}{\frac{h}{2}} = \cos(x+0) = \cos x$$

2) $f(x) = \cos x$

$f'(x) =$ analogno

3.

$$f(x) = \sin(x^2) \Rightarrow f'(x) = \cos(x^2)\cdot 2x$$

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h)^2 - \sin(x)^2}{h} = \left| \begin{array}{l} u = x^2 \\ \Delta u = (x+h)^2 - x^2 \\ \Delta u \to 0 \\ (x+h)^2 = x^2 + \Delta u = u + \Delta u \end{array} \right| =$$

$$= \lim_{\Delta u\to 0} \frac{\sin(u+\Delta u) - \sin u}{h} \cdot \frac{(x+h)^2 - x^2}{\underbrace{(x+h)^2 - x^2}_{\Delta u}} =$$

$$= \underbrace{\lim_{\Delta u\to 0} \frac{\sin(u+\Delta u) - \sin u}{\Delta u}}_{\cos u} \cdot \underbrace{\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}}_{2x} = \cos x^2 \cdot 2x$$

MOŽE DOĆI NA ISPITU !! BUREK PREPORUKA ! HE HE

TM Neka je $f\circ g$ definirana u točki x, te neka je g diferencijabilna u x, a f diferencijabilna u $g(x)$. Tada $f\circ g$ ima derivaciju u x i vrijedi:

$$\boxed{(f\circ g)'(x) = f'(g(x))\cdot g'(x)}$$

DOKAZ:

$$(f\circ g)'(x) = \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{h} \cdot \frac{g(x+h)-g(x)}{g(x+h)-g(x)} = \left| \begin{array}{l} g(x+h) = u \\ g(x+h) - g(x) = \Delta u \\ \Delta u \to 0 \end{array} \right| =$$

Pretp. da postoji okoliš od x takav da za sve točke iz okoliša je $g(a) \neq g(x)$

$$= \lim_{\Delta u\to 0} \frac{f(u+\Delta u) - f(u)}{\Delta u} \cdot \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = f'(u)\cdot g'(x) = f'(g(x+h))\cdot g'(x)$$

Primjedba! „LANČANO PRAVILO"

npr. $$(f\circ g\circ h)'(x) = f'(g(h(x)))\cdot g'(h(x))\cdot h'(x)$$!

Pr.) $$f(x) = (x^3+\pi)^{314} \longrightarrow f'(x) = 314\,(x^3+\pi)^{313}\cdot 3x^2$$

Pr. 2) $$f(x) = \frac{1}{x^5+\sin(3x+14)} = -\frac{1}{(x^5+\sin(3x+14))^2}\cdot(5x^4+\cos(3x+14)\cdot 3)$$

* $\left(\frac{1}{x}\right)' = (x^{-1})' = -\frac{1}{x^2}$ *

• DOKAZ DERIVACIJE KVOCIJENTA:

$$\left(\frac{f}{g}\right)' = \left(f\cdot\frac{1}{g}\right)' = f'\cdot\frac{1}{g} + f\cdot\left(\frac{1}{g}\right)' = f'\cdot\frac{1}{g} + f\cdot\frac{-1}{g^2}\cdot g' =$$

NE ZABORAVI

$$= \frac{f'g - fg'}{g^2}$$

• DERIVIRANJE INVERZNE FUNKCIJE:

$$(f\circ f^{-1})(y) = y \quad /\,' \text{ (po } y\text{)}$$

$$f'(f^{-1}(y))\cdot(f^{-1}(y))' = 1$$

$$\boxed{(f^{-1}(y)) = \frac{1}{f'(f^{-1}(y))}}$$

$$y' = \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{x'} \Longrightarrow x'(y) = \frac{1}{y'(x)}$$

$y = f(x)$

$x = f^{-1}(y)$

3.) $f(x) = \arcsin x$

$$f'(x) = (\arcsin x)' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

arcsinx

$$f(x) = \operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$f'(x) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} \quad //$$

$$f(x) = \operatorname{arctg} x$$

$$\left.\begin{array}{l} y = \operatorname{arctg} x \\ x = \operatorname{tg} y \end{array}\right\} (\operatorname{arctg} x)' = \frac{1}{\frac{1}{\cos^2 y}} = \frac{1}{\frac{\sin^2 y + \cos^2 y}{\cos^2 y}} = \frac{1}{\operatorname{tg}^2 y + 1} = \frac{1}{x^2+1} \quad //$$

$$f(x) = \ln x$$

$$f'(x) = \lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h} = \lim_{h \to 0} \frac{\ln \frac{x+h}{x}}{h} = \lim_{h \to 0} \frac{\ln\left(1 + \overset{\to 0}{\frac{h}{x}}\right) \sim \frac{h}{x}}{h} = \lim_{h \to 0} \frac{\frac{\cancel{h}}{x}}{\frac{\cancel{h}}{1}} = \frac{1}{x} \quad //$$

$$f(x) = e^x$$

$$\left.\begin{array}{l} y = e^x \\ x = \ln y \end{array}\right\} (e^x)' = \frac{1}{\frac{1}{y}} = y = e^x$$

$$f(x) = \log_a x = \frac{\ln x}{\ln a}$$

$$f'(x) = \frac{1}{x \ln a} \quad //$$

$$f(x) = \operatorname{sh} x$$

!/

$$(\operatorname{sh} x)' = \left(\frac{1}{2}(e^x - e^{-x})\right)' = \frac{1}{2}\left(e^x - e^{-x} \cdot (-x)'\right) = \frac{1}{2}(e^x + e^{-x}) = \operatorname{ch} x \quad //$$

$$f(x) = \operatorname{arsh} x$$

$$\left.\begin{array}{l} y = \operatorname{arsh} x \\ x = \operatorname{sh} y \end{array}\right\} (\operatorname{arsh} x)' = \frac{1}{\operatorname{ch} y} = \frac{1}{\sqrt{1 + \operatorname{sh}^2 y}} = \frac{1}{\sqrt{1+x^2}} \quad //$$

- FUNKCIJA OPĆE POTENCIJE : $f(x) = x^c,\ c \in \mathbb{R}$

$$f(x) = e^{c \ln x}$$

$$f'(x) = e^{c \ln x} \cdot c \frac{1}{x} = x^c \cdot \frac{c}{x} = c \cdot x^{c-1}$$

2. MI-2010.-4.) $y = x \cdot \operatorname{arctg}(e^{-x^2})$

NE ZABORAVI ! !!!

$$y' = 1 \cdot \operatorname{arctg}(e^{-x^2}) + x \cdot \frac{1}{1+(e^{-x^2})^2} \cdot e^{-x^2} \cdot (-2x)$$

Zad.) $f(x) = \ln(\operatorname{sh}(\frac{\pi}{2}x)),\ f''(x) = ?$

$$f'(x) = \frac{1}{\operatorname{sh}(\frac{\pi}{2}x)} \cdot \operatorname{ch}(\tfrac{\pi}{2}x) \cdot \frac{\pi}{2} = \frac{\pi}{2} \cdot \operatorname{cth}(\tfrac{\pi}{2}x)$$

$$f''(x) = \frac{\pi}{2} \cdot \frac{-1}{\operatorname{sh}^2(\frac{\pi}{2}x)} \cdot \frac{\pi}{2} = \frac{-\pi^2}{4\operatorname{sh}^2(\frac{\pi}{2}x)}$$

$$f''(1) = \frac{-\pi^2}{4\operatorname{sh}^2(\frac{\pi}{2})}$$ → ako imaš $f''(1)$ PRVO DVODERIVIRAJ PA UVRSTI 1 !! !

8ZZV-25.) $f(x) = \ln(ax+b),\ f^{(n)}(x) = ?$

$$f'(x) = \frac{1}{ax+b} \cdot a$$

$$f''(x) = \frac{-a}{(ax+b)^2} \cdot a = \frac{-1a^2}{(ax+b)^2}$$ * prema $\frac{1}{x} = \frac{-1}{x^2} \cdot x'$

$$f'''(x) = \left\{ \left(\frac{1}{x}\right)^2 = (x^{-1})^2 = x^{-2} = \frac{1}{x^2} = -2x^{-3} = \frac{-2}{x^3} \right\} = \frac{2a^3}{(ax+b)^3}$$

1 = 1
2 = 1·2
6 = 1·2·3

$$f^{IV}(x) = \frac{-6a^4}{(ax+b)^4}$$

$$\boxed{f^{(n)}(x) = (-1)^{n+1} \cdot \frac{(n-1)!\, a^n}{(ax+b)^n}}$$

DOKAZATI IND. !

1. baza $n=1$

$$f'(x)=\frac{a}{(ax+b)} \quad ✓$$

2. pretpostavka da je $f^{(n)}(x)=(-1)^{n+1}\frac{(n-1)!\,a^n}{(ax+b)^n}$ za neki $n\in\mathbb{N}$

3. korak indukcije, za $n+1$

$$f^{(n+1)}(x)=\frac{(-1)^{n+1}\cdot(n-1)!\cdot a^n\cdot(-n)\cdot a}{(ax+b)^{n+1}}=\frac{(-1)^{n+2}\,n!\,a^{n+1}}{(ax+b)^{n+1}}$$

"$(f^n(x))'$"
pretp.

Ex.)

$$f(x)=\underbrace{(1+x)^{\sin 2x}}_{\text{!! UVIJEK } e^{y\ln x}\text{ !!}}=e^{\sin 2x\ln(1+x)}$$

UVIJEK! $x^y=e^{y\ln x}$

PAZI!!

$$f'(x)=e^{\sin 2x\ln(1+x)}\cdot\left[\cos 2x\cdot 2\cdot\ln(1+x)+\sin 2x\cdot\frac{1}{1+x}\cdot 1\right]$$

→ AUDITORNE VJEŽBE ←

$$\boxed{(\sqrt{x})'=\frac{1}{2\sqrt{x}}}$$

1)

$$f(x)=\sqrt[3]{x}+\sqrt[4]{x}+\sqrt[6]{x}$$

$$f'(1)=?$$

PRVO DERIVIRATI PA UVRSTITI 1 !!

$$f'(x)=(x^{\frac{1}{3}})'+(x^{\frac{1}{4}})'+(x^{\frac{1}{6}})'=$$

$$=\frac{1}{3}x^{-\frac{2}{3}}+\frac{1}{4}x^{-\frac{3}{4}}+\frac{1}{6}x^{-\frac{5}{6}}$$

$$f'(1)=\frac{1}{3}\cdot 1^{-\frac{2}{3}}+\frac{1}{4}\cdot 1^{-\frac{3}{4}}+\frac{1}{6}\cdot 1^{-\frac{5}{6}}=\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{3}{4}$$

2)

$$f(x)=x\cdot\sin x+\operatorname{tg}x$$

$$f'(x)=(x\sin x)'+(\operatorname{tg}x)'=$$

$$=1\cdot\sin x+x\cdot\cos x+\frac{1}{\cos^2 x}$$

3.) $$f(x) = \frac{\log_{10} x}{x}$$

$$f'(x) = ?$$

$$f(x) = [\log_{10} x] \cdot x^{-1}$$

$$f'(x) = (\log_{10} x)' \cdot x^{-1} + \log_{10} x \cdot (x^{-1})' =$$

$$= \frac{1}{x \ln 10} \cdot \frac{1}{x} + \log_{10} x \cdot \left(-\frac{1}{x^2}\right)$$

4.) $$f(x) = \frac{x \operatorname{arctg} x}{x^2+1}$$

$$f'(1) = ?$$

$$f'(x) = \frac{(x \operatorname{arctg} x)' \cdot (x^2+1) - (x \cdot \operatorname{arctg} x) \cdot (x^2+1)'}{(x^2+1)^2} =$$

$$= \frac{\left(\operatorname{arctg} x + x \cdot \frac{1}{1+x^2}\right)(x^2+1) - 2x^2 \operatorname{arctg} x}{(x^2+1)^2} =$$

$$= \frac{(x^2+1) \operatorname{arctg} x + x - 2x^2 \operatorname{arctg} x}{(x^2+1)^2} =$$

$$= \frac{(1-x^2) \operatorname{arctg} x + x}{(x^2+1)^2}$$

$$f'(1) = \frac{1}{2^2} = \frac{1}{4} \,//$$

$f(x) = 2^{\frac{1}{x}}$

$f'(x) = ?$

$\boxed{(a^x)' = a^x \ln a} = a^x \ln a \cdot (x)'$

PAZI! !!

$f'(x) = 2^{\frac{1}{x}} \cdot \ln 2 \cdot \left(\frac{1}{x}\right)'$

$= 2^{\frac{1}{x}} \ln 2 \cdot \frac{-1}{x^2}$

$f'(x) = -\frac{2^{\frac{1}{x}}}{x^2} \cdot \ln 2$

$f(x) = x \cdot e^{-x} \cdot \sin(\pi x)$

$= e^{-x} \cdot \sin(\pi x) + x \cdot e^{-x} \cdot (-x)' \cdot \sin(\pi x) +$

$+ x \cdot e^{-x} \cdot \cos(\pi x) \cdot \pi$

$= (e^{-x} - x e^{-x}) \cdot \sin(\pi x) + x \cdot e^{-x} \cdot \pi \cdot \cos(\pi x)$

$(f \cdot g \cdot h)' = (f \cdot g)' h + (f \cdot g) \cdot h'$

$= (f' \cdot g + f \cdot g') h + f \cdot g \cdot h'$

$= f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$

$f(x) = \frac{x^2}{\sqrt{x^4+1}}$

PAZI!

$f'(x) = \frac{2x \cdot \sqrt{x^4+1} - x^2 \cdot \frac{1}{2} \cdot (x^4+1)^{-\frac{1}{2}} \cdot (x^4+1)'}{x^4+1} =$

$= \frac{2x \cdot \sqrt{x^4+1} - x^2 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x^4+1}} \cdot 4x^3}{x^4+1} = \frac{2x \cdot \sqrt{x^4+1} - \frac{2x^5}{\sqrt{x^4+1}}}{x^4+1}$

$= \frac{\frac{2x \cdot \sqrt{x^4+1} \cdot \sqrt{x^4+1} - 2x^5}{\sqrt{x^4+1}}}{x^4+1} = \frac{\frac{2x \cdot (x^4+1) - 2x^5}{\sqrt{x^4+1}}}{x^4+1} = \frac{2x}{\sqrt{x^4+1} \cdot (x^4+1)} =$

$(x^4+1)^{\frac{1}{2}} \cdot (x^4+1)^1 = (x^4+1)^{\frac{3}{2}}$

$= \frac{2x}{\sqrt{(x^4+1)^2 (x^4+1)}} = \frac{2x}{\sqrt{(x^4+1)^3}}$

8.) $f(x) = sh^4(10x)$

$$f'(x) = 4\,sh^3(10x)\cdot\left(sh(10x)\right)' =$$

$$= 4\,sh^3(10x)\cdot\left(ch(10x)\cdot(10x)'\cdot(x')\right)$$

$$= 4\,sh^3\,10x\cdot ch\,10x\cdot 10$$

$$f'(x) = 40\,sh^3(10x)\cdot ch(10x) \quad //$$

9.) $f(x) = e^{-\cos^2(5x)}$

$$f'(x) = e^{-\cos^2(5x)}\cdot\left(-\cos^2(5x)\right)' =$$

$$= -e^{-\cos^2(5x)}\cdot\left(2\cos(5x)\cdot(\cos(5x))'\right) =$$

$$= -e^{-\cos^2(5x)}\cdot\left(2\cos(5x)\cdot(-\sin(5x)\cdot(5x)')\right) =$$

$$= \boxed{2}\,e^{-\cos^2 5x}\cdot\boxed{\cos 5x\cdot\sin 5x}\cdot 5 =$$

$$= 5\,e^{-\cos^2 5x}\cdot\sin 10x \quad //$$

10.) $f(x) = \underbrace{x^x}_{g(x)} + \underbrace{\left(\frac{x}{x+1}\right)^x}_{h(x)}$

$f'(1) = ?$

$$f'(x) = g'(x) + h'(x)$$

$$f'(x) = x^x(\ln x + 1) + \left(\frac{x}{x+1}\right)^x\left(\ln\frac{x}{x+1} + \frac{1}{x+1}\right)$$

$$f'(1) = 1 + \frac{1}{2}\left(\ln\frac{1}{2} + \frac{1}{2}\right)$$

$$f'(1) = 1 + \frac{1}{2}\ln\frac{1}{2} + \frac{1}{4} =$$

$$= \frac{5}{4} + \frac{1}{2}\ln\frac{1}{2}$$

$$g(x) = x^x$$

$$g(x) = e^{x\ln x} \quad /'$$

$$g'(x) = e^{x\ln x}\cdot(x\ln x)' =$$

$$= e^{x\ln x}\cdot\left(\ln x + x\cdot\frac{1}{x}\right) =$$

$$= e^{x\ln x}\cdot(\ln x + 1) = x^x\cdot(\ln x + 1)$$

$$h(x) = e^{x\ln\frac{x}{x+1}} \quad /'$$

$$h'(x) = e^{x\ln\frac{x}{x+1}}\cdot\left(x\ln\frac{x}{x+1}\right)' =$$

$$= e^{x\ln\frac{x}{x+1}}\cdot\left(\ln\frac{x}{x+1} + x\cdot\left(\frac{1}{\frac{x}{x+1}}\right)\cdot\left(\frac{x}{x+1}\right)'\right)$$

$$\cancel{x}\cdot\frac{x+1}{\cancel{x}}$$

$$= e^{x\ln\frac{x}{x+1}}\cdot\left(\ln\frac{x}{x+1} + \cancel{(x+1)}\cdot\frac{\cancel{x}+1-\cancel{x}}{(x+1)^{\cancel{2}}}\right) =$$

$$= e^{x\ln\frac{x}{x+1}}\cdot\left(\ln\frac{x}{x+1} + \frac{1}{x+1}\right) = \left(\frac{x}{x+1}\right)^x\cdot\left(\ln\frac{x}{x+1} + \ldots\right)$$

$f(-2) = ?$

$f(x) = \arcsin^2\left(\frac{1}{x}\right)$

$f(x)' = x \cdot \ln(x \cdot \operatorname{ch} x)$

$f'' = ?$ $\quad f(x) = \cos(\operatorname{arctg} x)$

D.Z.

DERIVIRAJ SRCE:

$x = 16\sin^3 t$

$y = 13\cos t - 5\cos 2t - 2\cos 3t - \cos 4t$

$♡' = ?$ u π

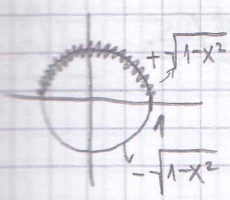
8.3. IMPLICITNO I PARAMETARSKI ZADANE FUNKCIJE

$F(x, y) = 0$

$x^2 + y^2 = 1 \longrightarrow$ kružnica

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$



IMPLICITNO DERIVIRANJE:

$x^2 + y^2 = 1 \quad |'$

$2x + 2y \cdot y' = 0 \longrightarrow$ svugdje gdje je y još y' !!!

SVA MUDROST:

$$\boxed{y' = \frac{-x}{y}}$$

...-16.) $\quad x \cdot y^2 + e^{x+y} = 2 \quad |'$

y', y'' u $T(1,-1) = ?$

UMNOŽAK

$1 \cdot y^2 + x \cdot 2y \cdot y' + e^{x+y} \cdot (1 + y') = 0$

$y'(x \cdot 2y + e^{x+y}) = -y^2 - e^{x+y}$

$$y' = \frac{-y^2 - e^{x+y}}{2xy + e^{x+y}}$$

vrstiti

$$y'\Big|_{T(1,-1)} = 2$$

kao umnožak

$$y''=\frac{\left(-2y\cdot y'-e^{x+y}\cdot(1+y')\right)\cdot\left(2xy+e^{x+y}\right)-\left(-y^2-e^{x+y}\right)\cdot\left(2y+2xy'+e^{x+y}(1+y')\right)}{\left(2xy+e^{x+y}\right)^2}$$

vrstiti

$$y''\Big|_{T(1,-1)} = 9$$

o 2.MI-2008-6.)

$$x^y+e^{x-1}=\sqrt{y}+1 \quad /'$$ $$x^y=e^{y\ln x}$$

$$e^{y\ln x}\cdot\left(y'\cdot\ln x+y\cdot\frac{1}{x}\right)+e^{x-1}\cdot 1=\frac{1}{2\sqrt{y}}\cdot y'$$

$$y'\left(x^y\ln x-\frac{1}{2\sqrt{y}}\right)=-e^{x-1}-x^y\cdot\frac{y}{x}$$

$$y'=\frac{-e^{x-1}-x^y\cdot\frac{y}{x}}{x^y\ln x-\frac{1}{2\sqrt{y}}}$$

$$y'\Big|_{T(1,1)}=4$$

* PARAMETARSKI ZADANE FUNKCIJE :

10.

$$\left.\begin{array}{l} x=x(t)\\ y=y(t)\end{array}\right\}$$

$$y'=\frac{dy}{dx}\cdot\frac{dt}{dt}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\dot{y}}{\dot{x}}$$

$$\boxed{y'=\frac{\dot{y}}{\dot{x}}}$$

$\dot{y}$ = derivacija y

$$y''=\frac{dy'}{dx}\cdot\frac{dt}{dt}=\frac{\frac{dy'}{dt}}{\frac{dx}{dt}}=\frac{\frac{\ddot{y}\dot{x}-\dot{y}\ddot{x}}{(\dot{x})^2}}{\dot{x}}=\frac{\ddot{y}\dot{x}-\dot{y}\ddot{x}}{(\dot{x})^3}$$

RAZ!

$$\boxed{y'' = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{(\dot{x})^3}}$$

7.)

$$x = t + \ln t$$
$$y = t^2 + \ln 2t$$

y', y'' za $t=1 \rightarrow ?$

$$y' = \frac{2t + \frac{1}{2t}\cdot 2}{1+\frac{1}{t}} = \frac{2t^2+1}{t+1}$$

za $t=1 \rightarrow y'\Big|_{t=1} = \frac{3}{2}$ //

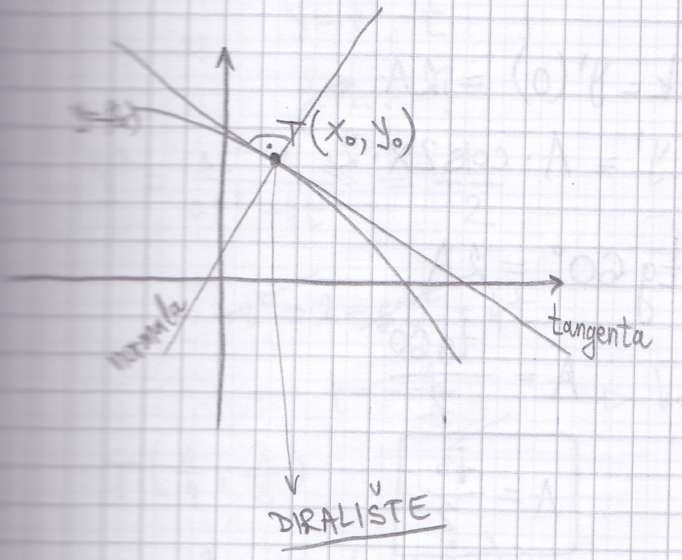
$\dot{y}' =$ (nemoj preko formule sa točkicama, nego još jednom y')

$$y'' = \frac{\frac{4t(t+1) - (2t^2+1)}{(t+1)^2}}{1+\frac{1}{t}}$$

$1+\frac{1}{t} \rightarrow \dot{x} = x$ derivirano !! PAZI !!

NE ZABORAVI !!

8.4. TANGENTA I NORMALA



$$\boxed{k = f'(x_0)}$$

k ..... koeficijent smjera tangente u točki x_0

$$y - y_0 = k(x - x_0)$$

$$\boxed{y - y_0 = f'(x_0)(x - x_0)}$$ .... tangenta

$$\boxed{y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)}$$ ...... normala

8.DZ.-9.) $y = x\cdot\sqrt[3]{x-1}$

Odredi tangentu u točki s apcisom u : a) 0 b) $\frac{3}{4}$ c) 1

$T(0,0)$ $T\left(\frac{3}{4}, -\frac{3}{4\sqrt[3]{4}}\right)$ $T(1,0)$

$y' = 1\cdot\sqrt[3]{x-1} + x\cdot\frac{1}{3}(x-1)^{-\frac{2}{3}}\cdot 1$

a) za $x=0$

$T(0,0)$, $y' = \sqrt[3]{x-1} + x\cdot(x-1)^{-\frac{2}{3}}$

$y - 0 = -1\cdot(x-0)$

$\boxed{y = -x}$

b) $T\left(\frac{3}{4}, -\frac{3}{4\sqrt[3]{4}}\right)$ → uvrštavamo u y'

$y + \frac{3}{4\sqrt[3]{4}} = 0\cdot\left(x - \frac{3}{4}\right)$

$\boxed{y = -\frac{3}{4\sqrt[3]{4}}}$

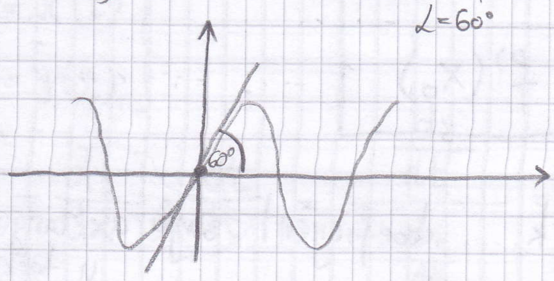
c) $T(1,0)$

$y' = \frac{1}{\sqrt[3]{(x-1)^3}} = \frac{1}{0} = \infty \rightarrow k = "\infty"$

$y - 0 = "\infty"(x-1)$

$\Rightarrow \boxed{x=1}$

2.MI-06-5.) $y = A\sin 2x$, $A = ?$



$k = y'(0) = 2A$

$y' = A\cdot\cos 2x\cdot 2$

$\mathrm{tg}\,60^\circ = 2A$

$A = \frac{\mathrm{tg}\,60^\circ}{2}$

$\boxed{A = \frac{\sqrt{3}}{2}}$

[illegible] $a = ?$ da se diraju $f_1(x) = \ln x$ i $f_2(x) = ax^2$

— [illegible] imati isti koeficijent smjera

— [illegible] : $T(x_0, y_0)$, parametar $a = ?$

$\underline{\ln x_0 = a x_0^2}$

$$\underbrace{f_1'(x) = \frac{1}{x} \qquad f_2'(x) = a \cdot 2x}$$

$\frac{1}{x_0} = 2a x_0 \longrightarrow$ koef. isti u toj točki x_0

$\rightarrow a = \frac{1}{2x_0^2}$

$\rightarrow \ln x_0 = \frac{1}{2\cancel{x_0^2}} \cdot \cancel{x_0^2}$

$\ln x_0 = \frac{1}{2}$

$x_0 = e^{\frac{1}{2}} \rightarrow x_0 = \sqrt{e} \rightarrow \boxed{a = \frac{1}{2e}}$

[illegible])

$$\left.\begin{aligned} x &= 2t^3 + 2t^2 \\ y &= t^3 - t \end{aligned}\right\} \quad t \parallel p \ldots\ldots x - 2y - 1 = 0$$

$t \ldots\ldots ?$

$-2y = 1 - x$

$y = \frac{x}{2} - \frac{1}{2} \rightarrow k = \frac{1}{2}$

$y' = \frac{3t^2 - 1}{6t^2 + 4t} = \frac{1}{2}$

$\cancel{6t^2} - 2 = \cancel{6t^2} + 4t$

$t = -\frac{1}{2} \longrightarrow$ koordinate dirališta: $x = 2 \cdot \left(-\frac{1}{2}\right)^3 + 2 \cdot \left(-\frac{1}{2}\right)^2$

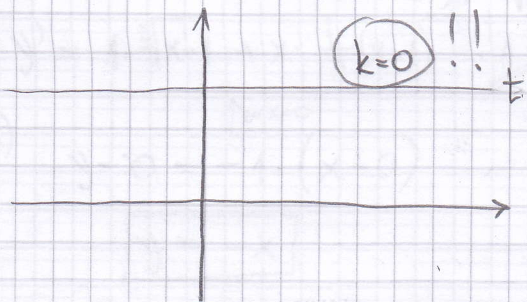
$y = \left(-\frac{1}{2}\right)^3 + \frac{1}{2}$

$T\left(\frac{1}{4}, \frac{3}{8}\right)$

$y - \frac{3}{8} = \frac{1}{2}\left(x - \frac{1}{4}\right)$

$\boxed{y = \frac{1}{2}x + \frac{1}{4}}$

2.MI-2010-5.) Odredi sve točke na krivulji $3x^2y-2y^3+2x^3=2$ u kojima je tangenta okomita na y os.



$\underline{k=0} \longrightarrow k=f'(x)$

$3x^2y-2y^3+2x^3=2 \quad /'$

$6xy+3x^2y'-6y^2\cdot y'+6x^2=0$

$y'(3x^2-6y^2)=-6xy-6x^2$

$y'=\dfrac{-6x^2-6xy}{3x^2-6y^2}=0$

$-6x^2-6xy=0 \quad /:(-6)$

$x^2+xy=0$

$x(x+y)=0$

$\boxed{x=0} \qquad \boxed{y=-x}$

I $\quad \underline{x=0}$

$0-2y^3+0=2$

$y^3=-1$

$\boxed{y=-1}$

$\boxed{T_1(0,-1)}$

II $\quad y=-x$

$-3x^3+2x^3+2x^3=2$

$x^3=2$

$x=\sqrt[3]{2} \longrightarrow y=-\sqrt[3]{2}$

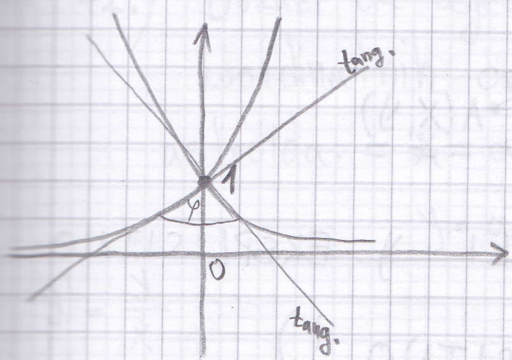
$\boxed{T_2(\sqrt[3]{2},-\sqrt[3]{2})}$

DOĐI LAKO !!!

DZ-20.)

$$y=e^{2x}$$
$$y=e^{-3x}$$

kut između tih krivulja?



$\underline{\underline{D(0,1)}}$ $\quad e^{2x}=e^{-3x} \quad /\ln$

$2x=-3x$

$\boxed{x=0} \longrightarrow \boxed{y=1}$ (sa slike

→ kut između krivulja je kut između tangenti u točki u kojoj se sijeku !!!

$$\left.\begin{aligned} y' &= e^{2x}\cdot 2 \\ y' &= e^{-3x}\cdot(-3) \end{aligned}\right\} \text{ u točki } D(0,1) \begin{cases} k_2 = 2 \\ k_1 = -3 \end{cases}$$

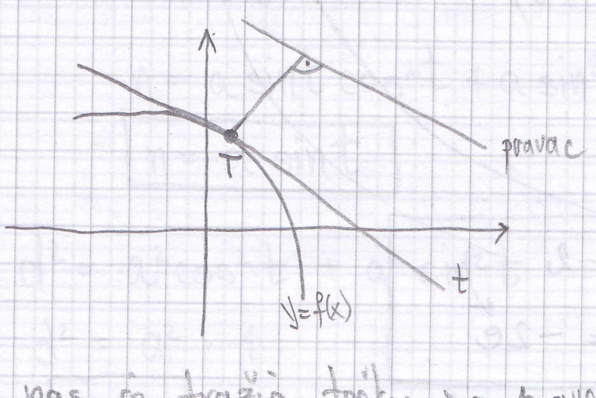
za k_2 uvijek veći k

! $\boxed{\operatorname{tg}\varphi = \dfrac{k_2-k_1}{1+k_1 k_2}}$!

$$\operatorname{tg}\varphi = \frac{2+3}{1-6} = -1 \longrightarrow \boxed{\varphi = \frac{3\pi}{4}}$$

→ UVIJEK NA $[0, 2\pi]$!

DZ-14.) Odredi točku na paraboli $y=x^2+x-2$ koja je najbliže pravcu $y=5x-10$



$y'=2x+1=5$

$\boxed{x=2} \longrightarrow \boxed{y=4}$

$T(2,4)$

udaljenost dvije točke:

$$\boxed{d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

→ da nas je tražio točku na pravcu

– odredili bi normalu i našli presjek normale i pravca

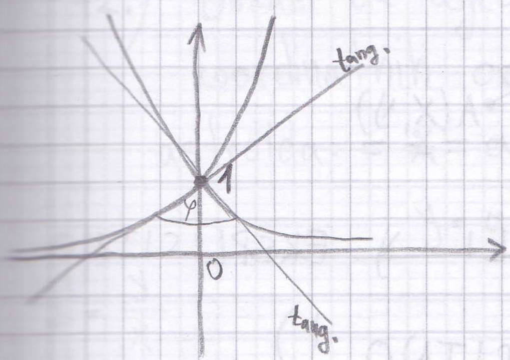
NE DOĐI LAKO !!!

ZAD-13.)

$y = e^{2x}$

$y = e^{-3x}$

kut između tih krivulja ?



$\underline{\underline{D(0,1)}}$ $\quad e^{2x} = e^{-3x} \quad / \ln$

$2x = -3x$

$\boxed{x=0} \rightarrow \boxed{y=1}$ (sa slike

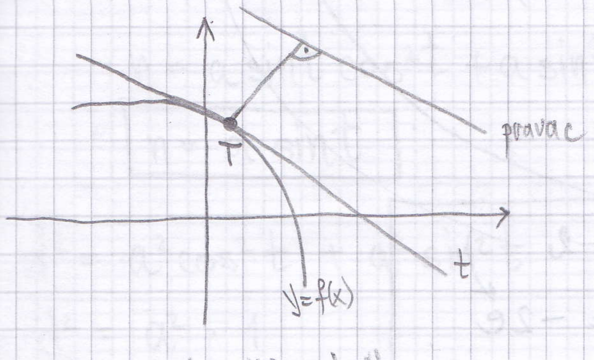
→ kut između krivulja je kut između tangenti u točki u kojoj se sijeku !!!

$\left.\begin{array}{l} y' = e^{2x} \cdot 2 \\ y' = e^{-3x} \cdot (-3) \end{array}\right\}$ u točki $D(0,1)$ → $k_2 = 2$, $k_1 = -3$ za k_2 uvijek veći k

$$\boxed{\operatorname{tg} \varphi = \frac{k_2 - k_1}{1 + k_1 k_2}}$$!

$\operatorname{tg} \varphi = \frac{2+3}{1-6} = -1 \longrightarrow \boxed{\varphi = \frac{3\pi}{4}}$ → UVIJEK NA $[0, 2\pi]$!

ZAD-14.) Odredi točku na paraboli $y = x^2 + x - 2$ koja je najbliže pravcu $y = 5x - 10$



$y' = 2x + 1 = 5$

$\boxed{x=2} \rightarrow \boxed{y=4}$

$T(2,4)$

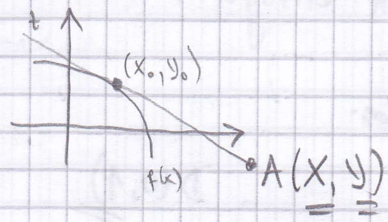
udaljenost dvije točke:

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

→ da nas je tražio točku na pravcu

– odredili bi normalu i našli presjek normale i pravca zad.) Iz točke $A(-1,0)$ povučene su tangente na krivulju $2\ln\frac{y}{x}+x=0$. Odredi njihove jednadžbe.

→ to nije točka dirališta !



→ PAZI !! UVIJEK POGLEDAJ MOŽEŠ LI y EKSPLICITNO IZVUĆI !! (IAKO IZGLEDA IMPLICITNO !)

$$2\ln\frac{y}{x}+x=0$$

$$\ln\frac{y}{x}=-\frac{x}{2} \quad / \; e$$

$$\frac{y}{x}=e^{-\frac{x}{2}}$$

A(-1,0)

$$y=x\cdot e^{-\frac{x}{2}} \longrightarrow \boxed{y_0=x_0 e^{-\frac{x_0}{2}}}$$

$$y'=e^{-\frac{x}{2}}+x\cdot e^{-\frac{x}{2}}\cdot\frac{-1}{2}=e^{-\frac{x}{2}}\left(1-\frac{x}{2}\right)$$

$$t \dots \quad \underline{\underline{y}}-y_0=e^{-\frac{x_0}{2}}\left(1-\frac{x_0}{2}\right)(\underline{\underline{x}}-x_0)$$

UVRŠTAVAMO y, x, a ne x_0, y_0

$$-y_0=e^{-\frac{x_0}{2}}\left(1-\frac{x_0}{2}\right)(-1-x_0)$$

$$x_0\cancel{e^{-\frac{x_0}{2}}}=\cancel{e^{-\frac{x_0}{2}}}\left(1-\frac{x_0}{2}\right)(-1-x_0)$$

$$x_0=1+x_0-\frac{x_0}{2}-\frac{x_0^2}{2}$$

$$x_0^2+x_0-2=0$$

$$\boxed{\begin{aligned}x_1&=1\\ y_1&=e^{-\frac{1}{2}}\end{aligned}} \qquad \boxed{\begin{aligned}x_2&=-2\\ y_2&=-2e\end{aligned}}$$

$y - e^{-\frac{1}{2}} = e^{-\frac{1}{2}} \cdot \frac{1}{2}(x-1)$

$y + 2e = e \cdot 2(x+2)$

7.

12.) Dokaži da osječak tangente na astroidu između koordinatnih osi ima konstantnu duljinu.

astroida: $x = a\cos^3 t$

$y = a\sin^3 t$

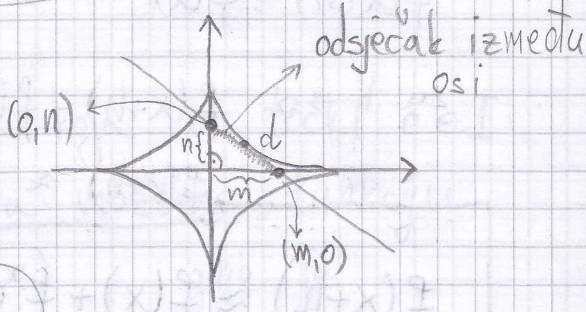
$y' = \frac{\dot{y}}{\dot{x}}$

$y' = \frac{\cancel{a} \cdot \cancel{3}\sin^2 t \cdot \cancel{\cos t}}{\cancel{a} \cdot \cancel{3}\cos^2 t \cdot \cancel{(-\sin t)}} = -\frac{\sin t}{\cos t} = -\operatorname{tg} t$ //

$y - y_0 = -\operatorname{tg} t\,(x - x_0)$

točke na krivulji (dirališće)

$y - a\sin^3 t = -\operatorname{tg} t\,(x - a\cos^3 t)$



za $(m,0)$:

$0 - a\sin^3 t = -\operatorname{tg} t\,(m - a\cos^3 t) \quad / : \operatorname{tg} t \quad a\cos^3 t$

$d^2 = n^2 + m^2$

$\boxed{m = a\cos t}$

za $(0,n)$:

$n - a\sin^3 t = -\operatorname{tg} t\,(0 - a\cos^3 t)$

$n = a\sin t\cos^2 t + a\sin^3 t = a\sin t\,(\underbrace{\cos^2 t + \sin^2 t}_{1})$

$\boxed{n = a\sin t}$

$d^2 = a^2\cos^2 t + a^2\sin^2 t$

$d^2 = a^2 \cdot 1$

$\boxed{d = a} \longrightarrow$ ne ovisi o koordinati dirališta!