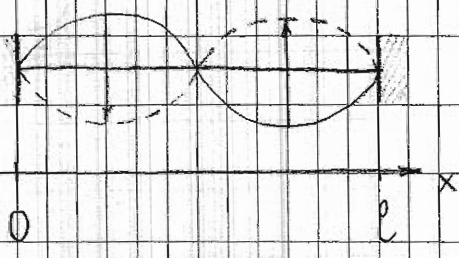
7. Stojni valovi na užetu



$$y(t,x) = y_{\rightarrow}(t,x) + y_{\leftarrow}(t,x)$$
$$= A\cos(\omega t - kx) + A\cos(\omega t + kx + \pi)$$
$$= \dots = 2A\sin(\omega t)\sin(kx)$$

* Rubni uvjeti

$$y(t, x=0) = y(t, x=l) = 0$$

$$\sin k\emptyset = \underbrace{\sin kl}_{} = 0$$

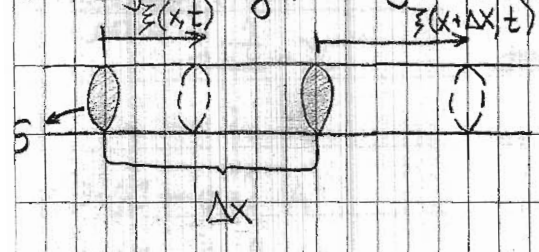
$$kl = n\pi \text{ tj. } k_n = \frac{n\pi}{l}, \quad \lambda_n = \frac{2\pi}{k_n} = \frac{2l}{n}, \quad \omega_n = k_n v = \frac{n\pi}{l} v = n\omega_1, \quad f_n = \frac{nv}{2l}$$

$n \in \mathbb{Z}$

2. CIKLUS

8. Longitudinalni valovi (zvuk)

Medij: $E = \frac{\sigma}{\delta}$, $\sigma = \frac{F}{S}$, $\delta = \frac{\Delta l}{l}$



Sila uslijed $\frac{\Delta \xi}{\Delta x}$?

Ym.

$$\rightarrow ES\Delta\left(\frac{\partial \xi}{\partial x}\right) = ES\left(\frac{\partial^2 \xi}{\partial x^2}\right)\Delta x$$

Newton. jedn. gib. $\Delta m \cdot a = \Delta F$

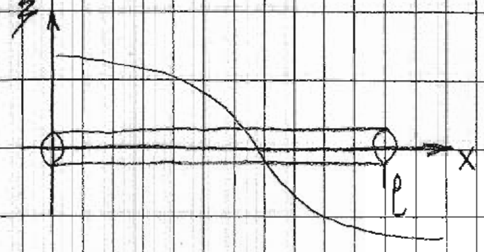
$$S\cancel{\Delta x}\rho \cdot \frac{\partial^2 \xi}{\partial t^2} = E \cdot S \frac{\partial^2 \xi}{\partial x^2}\cancel{\Delta x}$$

$$\boxed{\frac{\partial^2 \xi}{\partial x^2} - \frac{\rho}{E}\frac{\partial^2 \xi}{\partial t^2} = 0}$$

trans. long.

$$v^2 = \frac{T}{\mu} = \frac{E}{\rho}$$

9. Long. valovi u štapovima



$$\xi(t,x)=\xi_{\rightarrow}+\xi_{\leftarrow}=\xi_0\cos(\omega t-kx)+\xi_0\cos(\omega t+kx)=2\xi_0\cos(\omega t)\cos(k\ldots$$

* Rubni uvjeti

$$\xi(t,x=0)=2\xi_0\cos\omega t$$
$$\xi(t,x=l)=\pm 2\xi_0\cos\omega t$$

$\cos kl=\pm 1 \qquad k_n=\frac{n\pi}{l}$ (kao i u (7))

Dodatni rubni uvjet (ako štap držimo na sredini):

$\xi(t,x=\frac{l}{2})=0 \qquad k_n=\frac{2n\pi}{l}$

27.10.2008.

10. Longitudinalni valovi u plinovima

$$v_{long.}=\sqrt{\frac{E}{\rho}}=\sqrt{\frac{\varkappa\cdot p}{\rho}}=\sqrt{\frac{B}{\rho}}$$

$\varkappa$ → adijabatska konstanta; p → tlak; B → volumni modul stlačivosti

$\varkappa=\frac{f+2}{f}$ — f: br. stupnjeva slobode kod plina

Plin općenito: $pV=NkT=nRT$

Adijabata: $pV^{\varkappa}=\text{const.}$

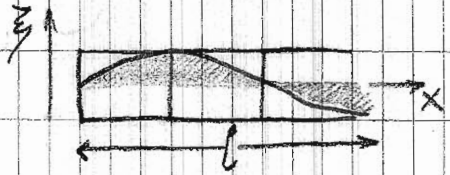
1A: $f=3 \rightarrow \varkappa=5/3$

2A: $f=5 \rightarrow \varkappa=7/5$

Stojni valovi u stupcu plina: a) otvoreni stupac

(Izvod je ekvivalentan izvodu u (9))

b) poluotvoreni stupac



rubni uvjeti: $\sin kl = \pm 1$

$$k_n l = \frac{2n-1}{2}\pi$$

$$k_n = \frac{2n-1}{2l}\pi$$

$$\lambda_n = \frac{4l}{2n-1}$$

$$\omega_n = k_n v \qquad \omega_n = (2n-1)\omega_1$$

c) zatvoreni stupac

(Izvod jednak kao u (7), stojni valovi na užetu)

11.) Intenzitet zvuka u plinovima

- stupac plina titra prema zakonu $\xi(x,t) = \xi_0 \cos(kx - \omega t)$

Energija titranja:

$$\Delta E = \frac{1}{2}\Delta m\, \dot{\xi}(x,t)^2_{max} = \frac{1}{2}\Delta m\, \xi_0^2 \omega^2 \underbrace{\sin^2(kx-\omega t)}_{1} = \frac{1}{2}\Delta m\, \overbrace{\xi_0^2\omega^2}^{(\text{maksimum brzine})^2} = \frac{1}{2}\rho S \Delta x\, \xi_0^2 \omega^2$$

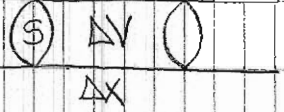
Snaga:

$$\frac{\Delta E}{\Delta t} = \frac{\Delta E}{\Delta x} v = \frac{1}{2}\rho v S \omega^2 \xi_0^2$$

Intenzitet: $$I = \frac{P}{S} = \frac{\Delta E / \Delta t}{S} = \frac{1}{2}\rho v \omega^2 \xi_0^2$$

Razina buke: $D = 10 \log_{10} \frac{I}{I_0}$ [dB] referentni intenzitet $I_0 = 10^{-12} \frac{W}{m^2}$

(12) Amplituda tlaka



$\xi = \xi_0 \cos(\omega t - kx)$

Volumni modul stlačivosti: $B = -\frac{\sigma}{\delta_V} = -\frac{\Delta F/S}{\Delta V/V} = -\frac{dp}{dV/V} = -V\frac{dp}{dV} = \varkappa p$

Adijabatski proces: (ne stigne se ohladiti tj. predati toplinu okolini)

$pV^{\varkappa} = const.$

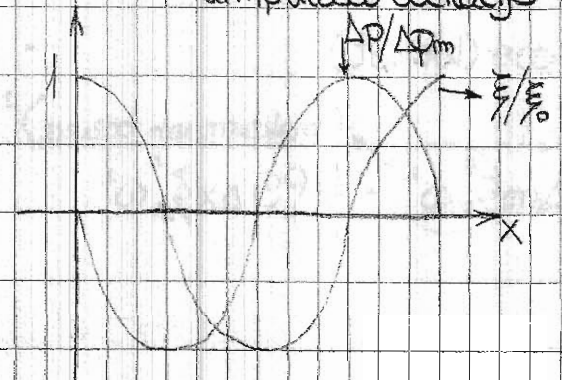
$dp V^{\varkappa} + p\varkappa V^{\varkappa - 1} dV = 0 \Rightarrow \frac{dp}{dV} = -\frac{\varkappa p}{V}$

Tlak: $\Delta p = -B\frac{\Delta V}{V} = -B\frac{S\left(\frac{\partial \xi}{\partial x}\right)\Delta x}{S\Delta x} = -B\frac{\partial \xi}{\partial x} = -Bk\xi_0 \sin(\omega t - kx)$

koristimo $v = \sqrt{\frac{B}{\rho}}$

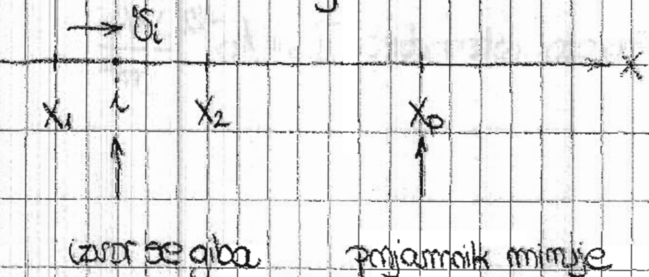
$\rightarrow \quad \Delta p = -\underbrace{v^2 \rho k \xi_0}_{\Delta p_{max}} \sin(\omega t - kx) = -\Delta p_m \cos\left(\omega t - kx + \frac{\pi}{2}\right)$

→ amplituda oscilacije tlaka



(13) Dopplerov efekt

Jednostavan slučaj:



$x_1 = x_i(t_1)$

$x_2 = x_i(t_2)$

- i odašilje zvuk frekvencije f_i, signal putuje brzinom v_0

- prijamnik čuje signale u t_1' i t_2'

$$t_{1,2}' = t_{1,2} + \frac{x_0 - x_{1,2}}{v_0}$$

$$t_2' - t_1' = t_2 - t_1 - \frac{x_2 - x_1}{v_0} = t_2 - t_1 - \frac{1}{v_0} v_i (t_2 - t_1) = (t_2 - t_1)\left(1 - \frac{v_i}{v_0}\right)$$

$$\frac{t_2' - t_1'}{t_2 - t_1} = \frac{\Delta t'}{\Delta t} = 1 - \frac{v_i}{v_0} \Longrightarrow \frac{f'}{f} = \frac{1}{1 - \frac{v_i}{v_0}} = \frac{v_0}{v_0 - v_i}$$

Općenito (za zadatke): $\frac{f_P}{f_i} = \frac{v_0 - \hat{r}_0 \cdot \vec{v}_P}{v_0 - \hat{r}_0 \cdot \vec{v}_i}$

v_0 - brzina zvuka, $\hat{r}_0$ - jedinični vektor $i \to p$

$\vec{v}_P$ - brzina prijamnika, $\vec{v}_i$ - brzina izvora

"nabla"

$$\vec{\nabla} = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}, \quad \underset{\text{vekt. polje}}{\vec{A}} = \vec{\nabla}\psi = \text{grad}\,\psi$$

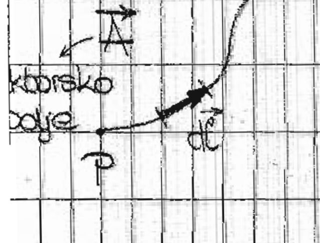
$$\vec{\nabla}\vec{A} = \vec{i}\frac{\partial A_x}{\partial x} + \vec{j}\frac{\partial A_y}{\partial y} + \vec{k}\frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \vec{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

2. DDZ - dokazati identitet br. 20

Teorem o gradijentu:



$$\int_P^Q \vec{A}\,d\vec{l} = \int_P^Q \vec{\nabla}\psi\,d\vec{l} = \psi(Q) - \psi(P)$$

$$\oint_C \vec{A}\,d\vec{l} = \oint_C \vec{\nabla}\psi\,d\vec{l} = 0$$

Gaussov teorem (teorem o divergenciji): 28.10.2008.

$$\int_V \vec{\nabla}\vec{A}\, d^3x = \oint_S \vec{A}\,\vec{n}\, da$$ → komadić površine

$\vec{\nabla}\vec{A}$ → vekt. polje

$\vec{n}$ → jedin. vektor okomit na da

Stokesov teorem (ploha omeđena krivuljom C):

$$\int_S (\vec{\nabla}\times\vec{A})\cdot\vec{n}\, da = \oint_C \vec{A}\cdot d\vec{l}$$

Maxwellove jednadžbe:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$c^2\vec{\nabla}\times\vec{B} = \frac{\vec{j}}{\varepsilon_0} + \frac{\partial\vec{E}}{\partial t}$$

$\vec{E}, \vec{B}$ ... električno i magnetsko polje

ρ ... gustoća električnog naboja, j ... gustoća električne struje

Lorentzova sila: $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

3.11.2008.

IV ELEKTROMAGNETIZAM

1) El. polje

Coulombov zakon

Q_1 $\vec{r}_{12}$ Q_2

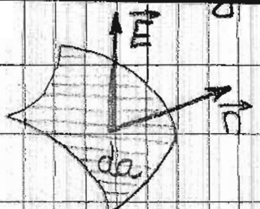
$$\underbrace{\vec{F}_{12}}_{\text{sila na } Q_2} = k\,\frac{Q_1 Q_2}{|\vec{r}_{12}|^2}\,\hat{r}_{12} = Q_2 \underbrace{\vec{E}_1}_{\text{el. polje naboja 1}} \quad k=\frac{1}{4\pi\varepsilon_0}$$

$$= k\,\frac{Q_1 Q_2}{|\vec{r}_2-\vec{r}_1|^2}\cdot\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|} = k\,\frac{Q_1 Q_2}{|\vec{r}_2-\vec{r}_1|^3}\,(\vec{r}_2-\vec{r}_1)$$

Električno polje:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\sum_i \frac{Q}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i)$$

2.) Tok električnog polja



$$d\vec{a} = da\cdot\vec{n}$$

$$\underbrace{d\Phi}_{\text{tok}} = \vec{E}\cdot\vec{n}\,da = \vec{E}\,d\vec{a}$$

3.) Gaussov zakon

I. Maxwellova jednadžba

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\int_V \vec{\nabla}\cdot\vec{E}\,dV = \int_V \frac{\rho}{\varepsilon_0}\,dV = \frac{1}{\varepsilon_0}\int_V \rho\,dV = \frac{1}{\varepsilon_0}Q$$

Gaussov teorem

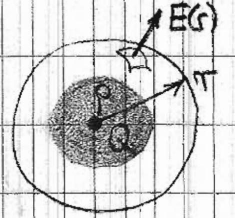
$$\int_V \vec{\nabla}\cdot\vec{E}\,dV = \int_S \vec{E}\cdot\vec{n}\,da$$

Gaussov zakon $\int \vec{E}\cdot\vec{n}\,da = \frac{Q}{\varepsilon_0}$

$$\underbrace{\boxed{\Phi = \frac{Q}{\varepsilon_0}}}_{\text{tok kroz zatvorenu plohu}}$$

- točkasti naboj

PRIMJERI:

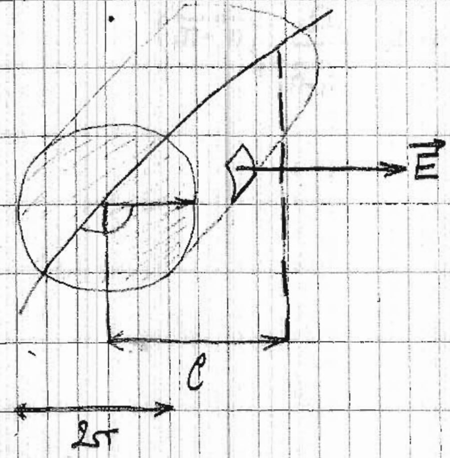


$$\Phi = 4r^2\pi E(r) = \frac{Q}{\varepsilon_0} \Rightarrow E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\Phi = 4r^2\pi E(r),\ Q = \rho \cdot \frac{4}{3} r^3 \pi,\ \Phi = \frac{Q}{\varepsilon_0} \rightarrow E(r) = \frac{1}{\varepsilon_0} \frac{\rho}{3} r$$

$$4r^2\pi E(r) = \frac{1}{\varepsilon_0} \rho \cdot \frac{4}{3} r^3 \pi$$

- beskonačna žica



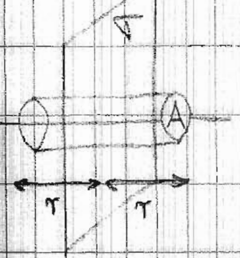
$$dQ = \lambda\, dl$$

$$\Phi = 2r\pi l\, E(r)$$

$$Q = \lambda l$$

$$\Phi = \frac{Q}{\varepsilon_0} \longrightarrow E(r) = \frac{1}{\varepsilon_0}$$

- beskonačna ploha



$$dQ = \sigma\, da$$

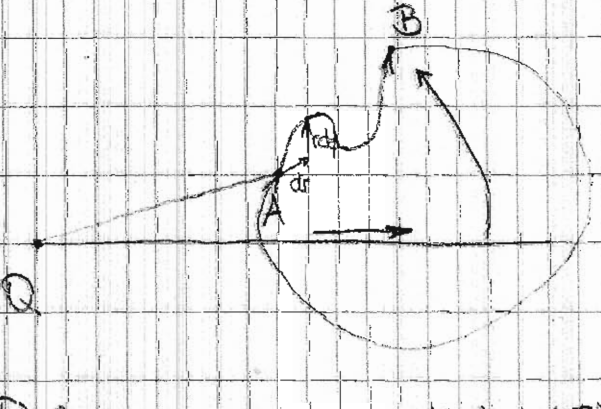
$$Q = \sigma A$$

$$\Phi = 2E(r)A = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{1}{2A}\frac{\sigma A}{\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}$$

4. Električni potencijal

Polje točkastog naboja



$$\int_A^B \vec{E}\, d\vec{l} = \frac{Q}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$\oint \vec{E}\, d\vec{l} = 0 \Longrightarrow \vec{E} \text{ je konzervativno polje}$$

$$\vec{\nabla} \times \vec{E} = 0$$

Def. razliku potencijala: $V(\vec{r}_B) - V(\vec{r}_A) = -\int_A^B \vec{E}\, d\vec{l}$ na osnovu teorema o gradijentu

$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})$ obično je $\vec{r}_A \to \infty$ $\boxed{V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\, d\vec{l}}$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{|\vec{r}_i - \vec{r}|} = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}\,')}{|\vec{r}\,' - \vec{r}|}d^3r' = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(\vec{r}\,')}{|\vec{r}\,' - \vec{r}|}d^2r' = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(\vec{r}\,')\,dl}{|\vec{r}\,' - \vec{r}|}$$ ← zapisi

(5.) Rad u električnom polju

(vanjska sila)

$$W_{A\to B} = -\int_A^B \underbrace{Q\vec{E}}_{\vec{F}}\, d\vec{l} = Q\,[V(B)-V(A)]$$

Pot. energija: donesemo Q u P

$$W_{\infty\to P} = -\int_\infty^P Q\vec{E}\cdot d\vec{l} = QV(P)$$

5.11.2008.

(6.) Poissonova i Laplaceova jednadžba

Max: $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

Pot: $\vec{E} = -\vec{\nabla}V$

$$\nabla^2 V = \vec{\nabla}\cdot(\vec{\nabla}V) = \vec{\nabla}\cdot(-\vec{E}) = -\frac{\rho}{\varepsilon_0}$$ Poissonova jednadžba

Ako $\rho = 0$ $\nabla^2 V = 0$ ... Laplaceova jednadžba

(7.) Polarizacija

el. dipol

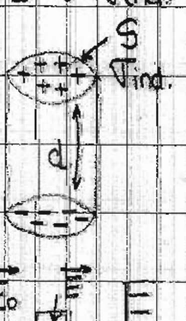
- u polju $\vec{E_0}$

dvopolni moment:

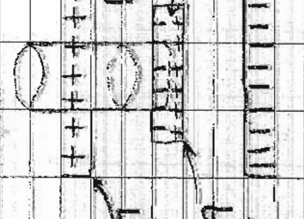
$\vec{p} = q\vec{\delta}$

vektor polarizacije: $\vec{P} = \frac{N}{V}\vec{p}$

Veza P i $\sigma_{ind.}$



$$|\vec{P}| = \frac{1}{Sd} S\sigma_{ind.} d = \sigma_{ind} \quad (1)$$ → površinska gustoća naboja



$$E_0 = \frac{\sigma_0}{\varepsilon_0} \quad (2)$$

$$|\vec{E}| = \frac{\sigma_0 - \sigma_{ind.}}{\varepsilon_0} \quad (3)$$

Zbog polarizacije $|\vec{E}| < |\vec{E_0}|$ (istog su smjera)

Definira se $\vec{E} = \frac{1}{\varepsilon_r}\vec{E_0}$ (4)

ε_r → relativna permeabilnost

$\varepsilon_r > 1$

Def: $\vec{E} \equiv \vec{E}_0 + \vec{E}_{ind.}$ (5)

→ inducirano polje (suprotnog predznaka u odnosu na $\vec{E}$ & $\vec{E}_0$)

(2)(3) → (4) $\quad \sigma_{ind.} = \sigma_0 \left(1 - \frac{1}{\varepsilon}\right)$

(2)(3) → (5) $\quad |\vec{E}_{ind.}| = |\vec{E}_0| - |\vec{E}| = \frac{\sigma_{ind.}}{\varepsilon_0}$

uz (1) $\quad \vec{E}_{ind.} = -\frac{1}{\varepsilon_0} \vec{P}$

$$Q_0 = \varepsilon_0 \Phi = \varepsilon_0 \int \vec{E}\, d\vec{a} = \varepsilon_0 \int (\vec{E} - \vec{E}_{ind})\, d\vec{a} = \int (\varepsilon_0 \vec{E} + \vec{P})\, d\vec{a} \equiv \vec{D}$$

I Maxwellov

$\vec{\nabla} \cdot \vec{D} = \rho_0$

$Q_0 = \int \vec{D}\, d\vec{a}$

slobodni naboj

polje dielektričnog pomaka

Polarizabilnost

$\vec{p} \equiv \alpha \vec{E}_0$ at. pol.

$\vec{P} = n\vec{p} = \chi_e \varepsilon_0 \vec{E}$ električna susceptibilnost

Veza: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E} = \varepsilon_0 (1 + \chi_e) \vec{E}$

$\vec{D} = \varepsilon_0 \vec{E}_0 = \varepsilon_0 \varepsilon_r \vec{E}$

$\boxed{\varepsilon_r = 1 + \chi_e}$

10.11.2008.

8 El. struja

$\Delta Q = \Delta N q = n \Delta V q = n S \Delta x q$

el. naboj č.

broj čestica

- neka se čestice gibaju brzinom v_d

$\Delta x = v_d \Delta t$

$\Delta Q = n S v_d \Delta t\, q$

Def: $I = \frac{\Delta Q}{\Delta t} = n S v_d q$

Gustoća struje: $j = \frac{I}{S} = n v_d q = \sigma E = \sigma \frac{V}{L}$

vodljivost

potencijal

$I = \frac{S\sigma}{L} V = \frac{1}{R} V$

Energija, snaga

razlika potencijala

$$P = \frac{dW}{dt} = \frac{\Delta V\, dq}{dt} = UI = \frac{U^2}{R} = I^2 R$$

$$W = \int P\,dt = I^2 R\, t = \frac{U^2}{R} t$$

(9.) Očuvanje el. naboja

$$\frac{\partial \rho}{\partial t} = \varepsilon_0 \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E}) = \varepsilon_0 \vec{\nabla}\cdot\left(\frac{\partial \vec{E}}{\partial t}\right) = \varepsilon_0 \cdot \vec{\nabla}\cdot\left(c^2 \vec{\nabla}\times\vec{B} - \frac{\vec{j}}{\varepsilon_0}\right) = \varepsilon_0 c^2 \underbrace{\vec{\nabla}(\vec{\nabla}\times\vec{B})}_{=0} - \vec{\nabla}\vec{j} = -\vec{\nabla}\vec{j}$$

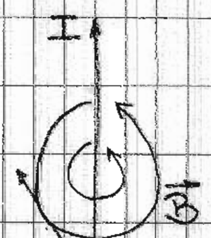
...jednadžba kontinuiteta za el. naboj

$$\int \frac{\partial \rho}{\partial t}\, dV = \int (-\vec{\nabla}\cdot\vec{j})\, dV$$

$$\frac{\partial}{\partial t}\int \rho\, dV = -\int \vec{j}\cdot d\vec{a}$$

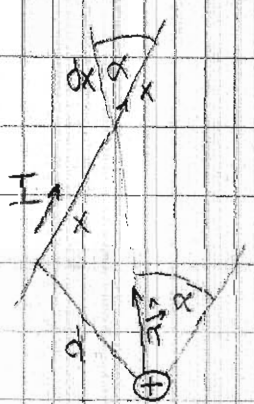
$$\frac{\partial}{\partial t} Q = -\int \vec{j}\, d\vec{a}$$ ...zakon očuvanja el. naboja

(10.) Oerstedov pokus i Biot-Savartov zakon



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l}\times\vec{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}\times\vec{r}}{r^2}$$



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{x}\times\vec{r}}{r^2}$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi} I \frac{dx}{r^2}\sin\alpha = \frac{\mu_0}{4\pi} I \frac{d\; dx}{(x^2+d^2)^{3/2}}$$

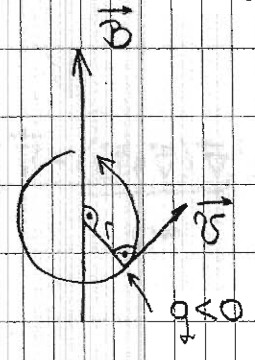
$$|\vec{B}| = \frac{\mu_0}{4\pi} I d \int_{-\infty}^{\infty} \frac{dx}{(x^2+d^2)^{3/2}} = \dots = \frac{\mu_0 I}{2\pi d}$$

$$\boxed{\mu_0 = \frac{1}{\varepsilon_0 c^2} \text{ ... za vakuum}}$$

11. Lorentzova sila

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

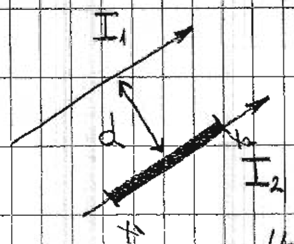
Primjer: kružno gibanje naboja u homogenom magn. polju



$$|\vec{F}| = qvB = \frac{mv^2}{r}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$ ... Larmorova frekvencija

Primjer: sila koja djeluje na vodič



$$d\vec{F} = dq\, \vec{v} \times \vec{B} = I\, d\vec{l} \times \vec{B}$$

$$F = -I \int \vec{B} \times d\vec{l}$$

$$F_2 = -I \frac{\mu_0 I_1}{2\pi d} L$$

$$(1N) = \frac{\mu_0}{2\pi} \frac{(1A)}{(1m)} (1m)$$

12. Hallov efekt

13. Magnetski tok

(analogno toku el. polja)

$$d\Phi_M = \vec{B}\cdot\vec{n}\,da = \vec{B}\,d\vec{a}$$

$$\Phi_M = \int_S \vec{B}\cdot d\vec{a}$$

Ampere i Ampere-Maxwellov zakon

$$\underbrace{c^2\vec{\nabla}\times\vec{B} = \underset{\substack{\uparrow \\ A}}{\frac{\vec{j}}{\varepsilon_0}} + \frac{\partial\vec{E}}{\partial t}}_{A\text{-}M}$$

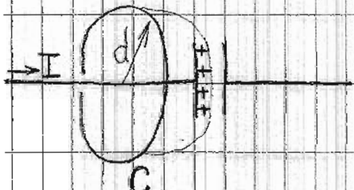
Amperov zakon: $c^2\vec{\nabla}\times\vec{B} = \frac{\vec{j}}{\varepsilon_0}$

$$\int c^2(\vec{\nabla}\times\vec{B})\,d\vec{a} = \int\frac{\vec{j}}{\varepsilon_0}\,d\vec{a}$$

$$c^2\oint_C \vec{B}\,d\vec{l} = \frac{1}{\varepsilon_0}\underbrace{\int\vec{j}\,d\vec{a}}_{I}$$

→ integral magn. polja duž krivulje C

Ampere-Maxwellov:

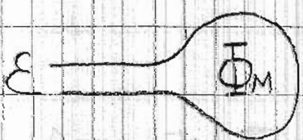


$$c^2\oint_C \vec{B}\cdot d\vec{l} = \frac{1}{\varepsilon_0}\underbrace{\int\vec{j}\,d\vec{a}}_{I} + \frac{\partial}{\partial t}\underbrace{\int\vec{E}\cdot d\vec{a}}_{\Phi_E}$$

$$\frac{\partial}{\partial t}\Phi_E = \frac{\partial}{\partial t}ES \overset{\text{površ. kond.}}{=} \frac{\partial}{\partial t}\frac{\sigma}{\varepsilon_0}S = \frac{\partial}{\partial t}\frac{Q/S}{\varepsilon_0 S} = \frac{\partial}{\partial t}\frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon}\frac{\partial Q}{\partial t} = \frac{1}{\varepsilon}I$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{d}$$

14. El. magnetska indukcija



$$\mathcal{E} = -\frac{\partial}{\partial t}\Phi_M$$

$$\mathcal{E} = \oint_C \vec{E}\cdot d\vec{l} = \int_S(\vec{\nabla}\times\vec{E})\,d\vec{a} = \int_S\left(-\frac{\partial\vec{B}}{\partial t}\right)\cdot d\vec{a} = -\frac{\partial}{\partial t}\int_S\vec{B}\,d\vec{a} = -\frac{\partial}{\partial t}\Phi_M$$

(15) Valna jednadžba za $\vec{E}$ i $\vec{B}$

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\cdot\frac{\partial^2}{\partial t^2}\right)\Phi(x,t)=0$$

$$\frac{\partial^2}{\partial x^2} \to \nabla^2$$

$$\Phi(x,t)=f(x\pm vt)$$

M_1 $\vec{\nabla}\cdot\vec{E}=0$ $\qquad$ M_3 $\vec{\nabla}\cdot\vec{B}=0$

M_2 $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$ $\qquad$ M_4 $\vec{\nabla}\times\vec{B}=\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

$$\nabla^2\vec{E}=\vec{\nabla}\cdot\underbrace{(\vec{\nabla}\cdot\vec{E})}_{=0}-\vec{\nabla}\times(\vec{\nabla}\times\vec{E})=-\vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)=\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})=\frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\right)=\frac{1}{c^2}\frac{\partial^2}{\partial t}$$

$$\boxed{\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}=0}$$

$$\nabla^2\vec{B}=\vec{\nabla}(\vec{\nabla}\vec{B})-\vec{\nabla}\times(\vec{\nabla}\times\vec{B})$$
$$=-\vec{\nabla}\times\left(\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\right)=-\frac{1}{c^2}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})=-\frac{1}{c^2}\frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right)=\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\boxed{\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B}=0}$$

Sredstvo: $\frac{1}{c^2}\longrightarrow\frac{1}{v^2}=\mu_r\mu_0\varepsilon_r\varepsilon_0=\frac{n^2}{c^2}$ → indeks loma

Rješenje:

$$\vec{E}=\vec{E}(x,t)=\vec{j}\,\underbrace{E_{0y}\cos(kx-\omega t)}_{E_y(x,t)}$$

$$\vec{\nabla}\times\vec{E}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ 0 & E_y(x,t) & 0\end{vmatrix}=\vec{k}\left(-kE_{y0}\sin(kx-\omega t)\right)=-\frac{\partial}{\partial t}\vec{B}$$

$$B_z(x,t)=\int kE_{y0}\sin(kx-\omega t)\,dt=\frac{k}{\omega}E_{y0}\cos(kx-\omega t)\to\vec{B}(x,t)=\vec{k}\,\frac{1}{c}E_{y0}\cos(kx-\omega$$

Komentari:

$\vec{E}$ i $\vec{B}$, transv. valovi, $\vec{E}$ i $\vec{B}$ okomiti, 12.11.2008.

16. Poyntingov teorem

$$dW = \vec{F}\cdot d\vec{l} = dq(\vec{E}+\vec{v}\times\vec{B})\cdot d\vec{l}$$
$$= dq(\vec{E}+\vec{v}\times\vec{B})\cdot\vec{v}\,dt$$
$$= dq\,\vec{E}\cdot\vec{v}\,dt = \vec{E}\cdot\vec{j}\,dV\,dt$$

gustoća struje

$$dq\cdot\vec{v} = \rho\,dV\,\vec{v} = \vec{j}\,dV$$

Snaga $P = \frac{dW}{dt} = \int_V \vec{E}\,\vec{j}\,dV$

$$\vec{E}\cdot\vec{j} = \vec{E}\cdot\left(\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right) = \frac{1}{\mu_0}\vec{E}(\vec{\nabla}\times\vec{B}) - \varepsilon_0\vec{E}\frac{\partial\vec{E}}{\partial t} = \frac{1}{\mu_0}\left[\vec{B}(\vec{\nabla}\times\vec{E}) - \vec{\nabla}(\vec{E}\times\vec{B})\right] - \varepsilon_0\vec{E}\frac{\partial\vec{E}}{\partial t}$$
$$= \frac{1}{\mu_0}\left[\vec{B}\left(-\frac{\partial\vec{B}}{\partial t}\right) - \vec{\nabla}(\vec{E}\times\vec{B})\right] - \varepsilon_0\vec{E}\frac{\partial\vec{E}}{\partial t} = -\frac{1}{\mu_0}\vec{B}\frac{\partial\vec{B}}{\partial t} - \varepsilon_0\vec{E}\frac{\partial\vec{E}}{\partial t} - \frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B})$$
$$= -\frac{1}{2}\frac{\partial}{\partial t}\left(\frac{1}{\mu_0}B^2 + \varepsilon_0E^2\right) - \frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B})$$

$$P = \int_V \vec{E}\vec{j}\,dV = -\int \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{1}{\mu_0}B^2+\varepsilon_0E^2\right)dV - \int_V\frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B})\,dV$$
$$= \underbrace{-\frac{1}{2}\frac{\partial}{\partial t}\int_V\left(\frac{1}{\mu_0}B^2+\varepsilon_0E^2\right)dV}_{①\quad ②} - \underbrace{\frac{1}{\mu_0}\int_S(\vec{E}\times\vec{B})\,d\vec{a}}_{②}$$

$$P = \frac{dW}{dt} = \underbrace{-\frac{\partial}{\partial t}W_{em}}_{①} - \int_S\vec{S}\,d\vec{a}$$

① → EM energija $W_{em} = \int w_{em}\,dV$

→ gustoća elektromagn. energije $w_{em} = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu_0}B^2\right)$

$\vec{S}$ ... Poyntingov vektor, $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

$$\boxed{\vec{\nabla}\cdot\vec{S} = -\frac{\partial}{\partial t}(e_{meh.} + w_{em})}$$

17. Fotometrija

Veličina		Fotometrijska jedinica	Energijska jedinica	
Jakost svjetlosti	$I = \frac{d\Phi}{d\Omega}$	cd	W / sr	(steradijan) prostorni kut
Tok svjetlosti	Φ	lm = cd·sr	W	
Količina svjetl.	Q	lm·s	J	
Osvijetljenost površine	$E_{ds} = \frac{d\Phi}{} = \frac{I\,d\Omega}{} = I\frac{dS}{r^2}$	cd/m²	W/m²	
Sjaj izvora	L $= \frac{I}{r^2}\cos\vartheta$	cd/m²	W/m²sr	
Svjetljenje	M	lm/m²	W/m²	

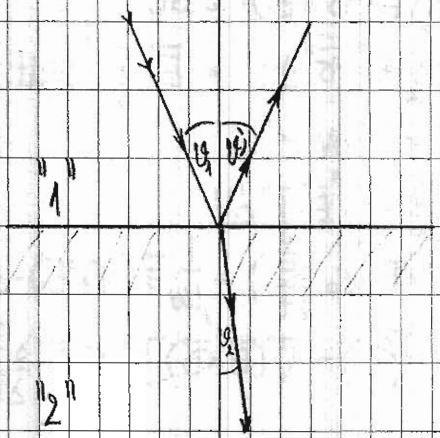
$\hookrightarrow = \frac{\Delta\Phi}{\Delta S} = L\pi$

17.11.2008

OPTIKA

1. Zakoni geom. optike

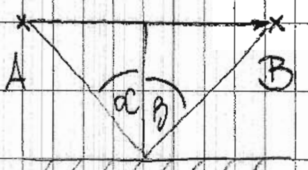
1. Zakon pravocrtnog širenja
2. Zakon nezavisnosti snopova
3. Zakon refleksije "$\vartheta_1' = \vartheta_1$"
4. Zakon loma $\frac{\sin\vartheta_1}{\sin\vartheta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$



$n = \sqrt{\varepsilon_r \mu_r}$

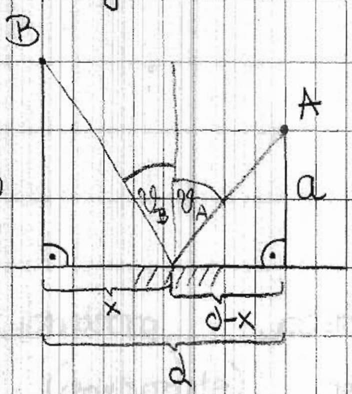
2. Fermatov princip

Ako zraka može stići iz A u B, ona odabire onaj put duž kojeg će putovanje najduže ili najkraće trajati (u odnosu na bliske putanje).



$$t_{AB} = \int_A^B \frac{dl}{v} = \int_A^B \frac{n}{c}\, dl$$

Ravno ogledalo

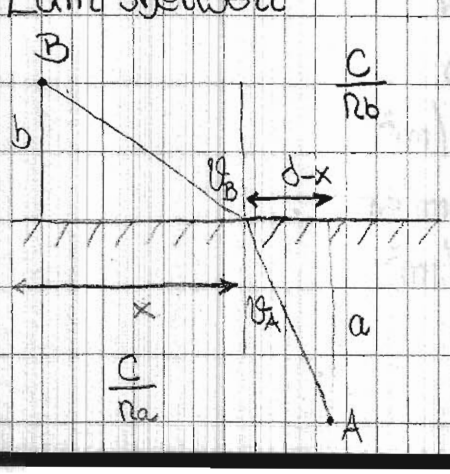


$$t_{AB} = \frac{1}{c}\sqrt{a^2 + (d-x)^2} + \frac{1}{c}\sqrt{b^2 + x^2}$$

$$\frac{dt_{AB}}{dx} = \frac{1}{c}\frac{1}{2\sqrt{\ }}\left[-2(d-x)\right] + \frac{1}{c}\frac{1}{2\sqrt{\ }}2x = 0$$

$$\Rightarrow \frac{d-x}{\sqrt{a^2+(d-x)^2}} = \frac{x}{\sqrt{b^2+x^2}}, \quad \sin\vartheta_A = \sin\vartheta_B$$

Lom svjetlosti



$$t_{AB} = \frac{n_a}{c}\sqrt{a^2+(d-x)^2} + \frac{n_a}{b}\sqrt{b^2+x^2}$$

$$\frac{dt_{AB}}{dx} = \frac{n_a}{c}\frac{1}{2\sqrt{\ }}\left[-2(d-x)\right] + \frac{n_b}{c}\frac{1}{2\sqrt{\ }}2x = 0$$

$$n_a \frac{d-x}{\sqrt{a^2+(d-x)^2}} = n_b \frac{x}{\sqrt{b^2+x^2}}$$

$$n_a \sin\vartheta_A = n_b \sin\vartheta_B \qquad \frac{n_a}{n_b} = \frac{\sin\vartheta_B}{\sin\vartheta_A}$$

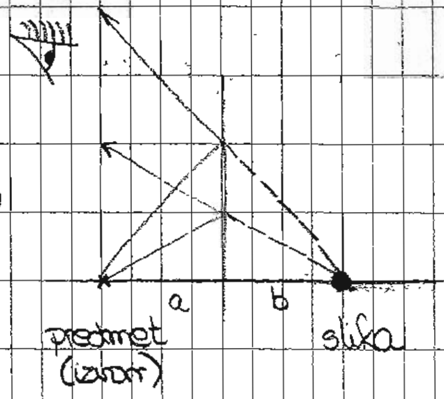
(3) Zrcala

Ravno zrcalo

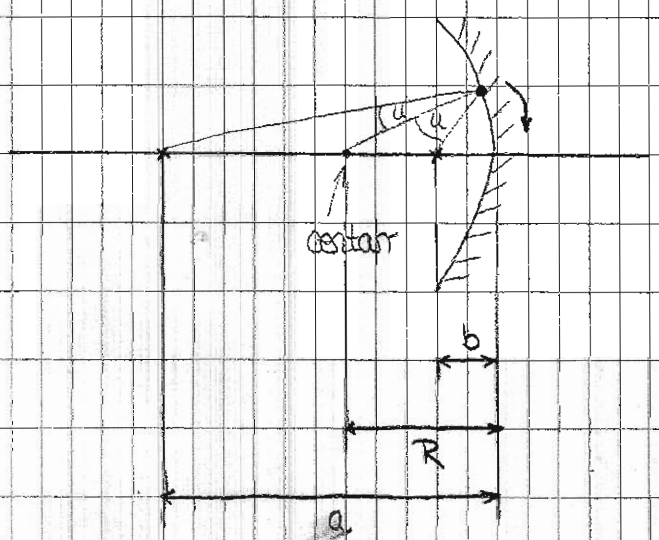
a - udaljenost izvora (predmeta)

b - udaljenost slike

$a = b$



Sferno zrcalo



- može se pokazati kada $u \to 0$

vrijedi $\frac{1}{a} + \frac{1}{b} \to \frac{2}{R}$

kao jednadžbu uzimamo $\frac{1}{a} + \frac{1}{b} = \frac{2}{R}$

Predmetna žarišna duljina:

$a \to f_a$ tako da $b \to \infty \Rightarrow f_a = \frac{R}{2}$

- za sferno zrcalo: $f_a = f_b = f = \frac{R}{2}$

$\frac{f_a}{a} + \frac{f_b}{b} = 1$, za sf. $\frac{f}{a} + \frac{f}{b} = 1$

- povećanje $m = \frac{y}{y'} = -\frac{b}{a}$