

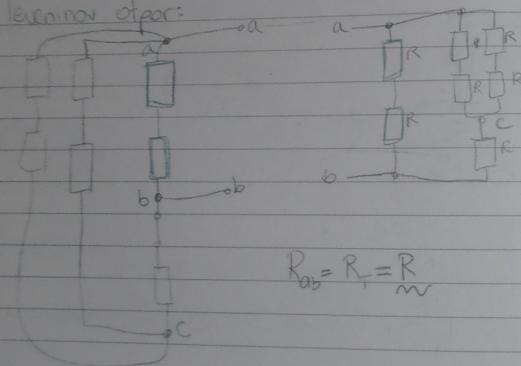
Nortonov teorem

-7-
nort struje - a i b su kratko spijeni

izmj struje u rezonanciji → samo nortonov teorem

1 punjer -10-

Terninov otpor:



-11-

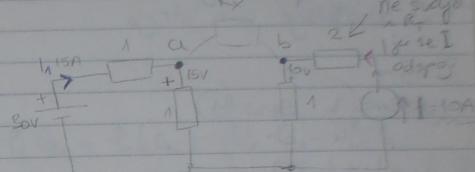
$$R_1 = R_2 = R_3 = 1 \Omega$$

$$R_4 = 2 \Omega$$

$$U = 30V$$

$$I = 10A$$

gornja elementarne sekvenci



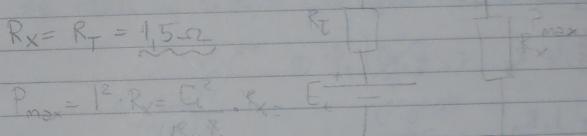
$$R_T = 0,5 + 1 = 1,5 \Omega$$

- dva nekorisna kruža

$$U_{ab} = -U_3 + U_2 = \frac{5}{m} V = E_T$$

$$U_{ab} = -1 \cdot R_3 + \frac{U}{(R_1+R_2)} \cdot R_2 = -10 + \frac{30}{2} \cdot 1 = -10 + 15 = 5V$$

$$R_X = R_T = 1,5 \Omega$$



$$P_{max} = I^2 \cdot R_T = E_T^2 \cdot \frac{1}{R_T} = \frac{E_T^2}{R_T}$$

$$= \frac{5^2}{4 \cdot 1,5} = \frac{25}{4 \cdot 1,5} = 4,17 W$$

Metoda potencijala čvorova

- za aktivirane smj. struje

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- uzemljivi smj. (4)

$$\textcircled{1} \quad I_1 + I_0 - I_4 = 0$$

$$\textcircled{2} \quad I_5 - I_1 - I_2 = 0$$

$$\textcircled{3} \quad I_2 - I_0 - I_3 = 0$$

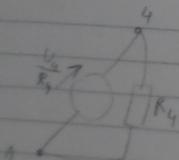
$$I_4 = I_2$$

or suprotne smj. struje
 $I_1 = I_4 + I_4 \cdot R_4 - U_4$

$$I_2 = I_4 - I_5 \cdot R_5 \Rightarrow ?$$

$$I_3 = I_0 + I_1 R_1$$

$$I = \frac{I_1 + U_4}{R_4} = (\varphi_1 + U_4) G_{45}$$



$$+\varphi_1 (G_0 + G_1 + G_4) - \varphi_2 G_1 - \varphi_3 G_0 = +U_0 \cdot G_2 - U_4 G_4$$

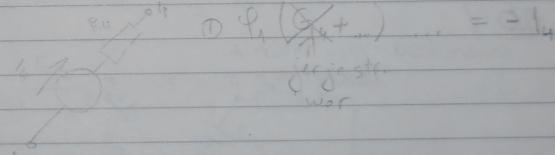
valjivosti
koje su oko tog
čvora

$$\textcircled{1} \quad +\varphi_1 (G_0 + G_1 + G_4) - \varphi_2 G_1 - \varphi_3 (G_0) = +U_0 \cdot G_2 - U_4 \cdot G_4$$

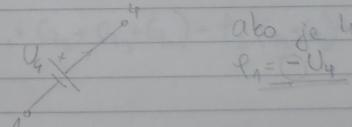
$$\textcircled{2} \quad -\varphi_1 (G_1) + \varphi_2 (G_1 + G_2 + G_5) - \varphi_3 (G_2) = 0$$

$$\textcircled{3} \quad -\varphi_1 G_0 - \varphi_2 (G_2) + \varphi_3 (G_3 + G_2 + G_0) = -U_0 G_0 - U_3 G_3$$

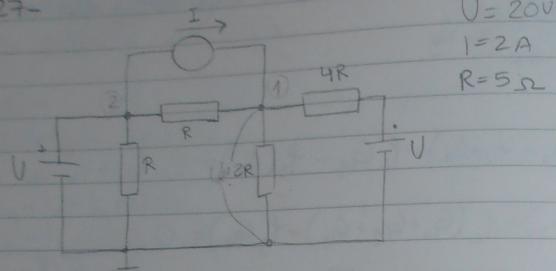
- u slučaju strujnog izvora:
 ne ulazi u izračun (valjivo izm 2. čvora)



- ako izm (1)(4) imamo samo U_4



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$$\varphi_2 = U = 20V$$

$$\varphi_1 \left(G + 0.5G + 0.25G \right) - \frac{U}{R} = 1 + \frac{U}{4R}$$

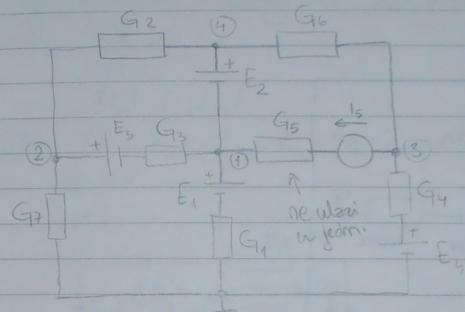
$$\varphi_1 = 20V$$

$$U = 20V$$

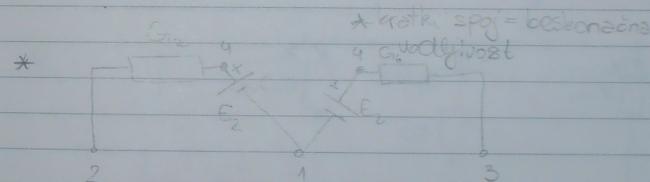
$$I = 2A$$

$$R = 5\Omega$$

Primjer:
Napisati jedn. za čvor (1):



$$\varphi_1(G_3 + G_4 + G_5) - \varphi_2(G_3) - \varphi_3(G_4) =$$



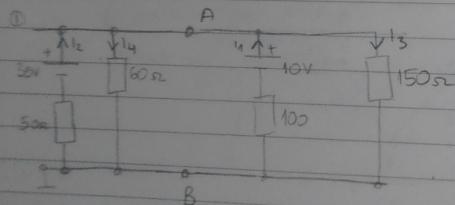
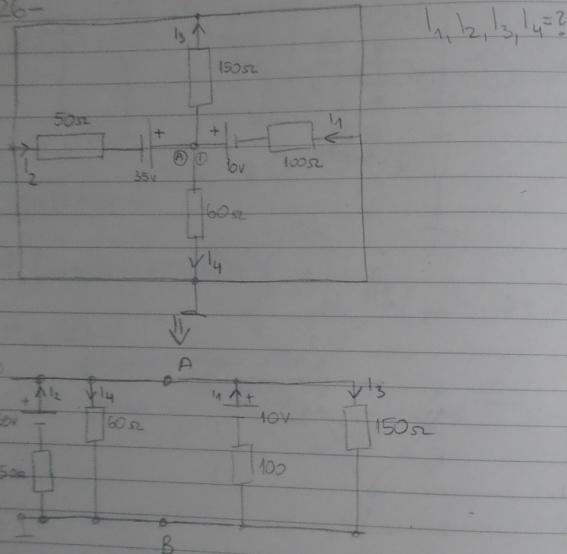
$$\varphi_1(G_1 + G_2 + G_3 + G_6) - \varphi_2(G_3 + G_2) - \varphi_3(G_6) = E_1 G_4 - E_3 G_3 + E_2 (G_2 + G_4)$$

Suma struja krajem

Millmanov teoreem

-sie nepravste pretv. u strjine

-26-



$$U_{AB} = \frac{35 + 10}{50 + 100} = \frac{70 + 10}{100} = \frac{240}{160} = 15 \text{ V}$$

$$I_4 = \frac{15}{60} \quad I_3 = \frac{15}{150}$$

$$\varphi_A = \varphi_B - I_2 \cdot 50 + 35$$

$$\varphi_A = \varphi_B - I_2 \cdot 100 + 15$$

$$25 = -I_2 \cdot 100 + 15$$

$$I_2 = -\frac{10}{100} = -0.1 \text{ A}$$

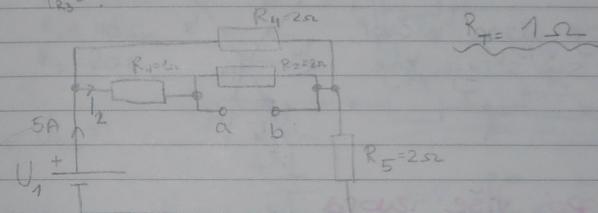
$$I_2 \cdot 50 = -15 + 35$$

$$I_2 = \frac{20}{50} = 0.4 \text{ A}$$

-28- $2R_1 = R_2 = 2R_3 = R_4 = R_5 = 2 \Omega$

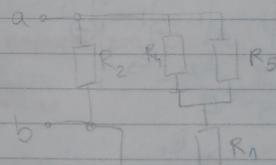
$$U_1 = 16 \text{ V}$$

$$P_{R_2} = ?$$



$$R_T :$$

$$\text{Napon } U_T = \text{napon na } R_2$$

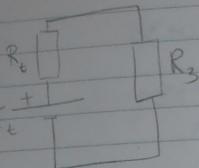


$$I_{2, \text{vora}} = \frac{U_1}{R_{0, \text{vora}}} = \frac{16}{3 \Omega} = 5 \text{ A}$$

$$I_2 = \frac{U_1}{R_{\text{vora}}} \cdot \frac{R_4}{R_4 + R_3 + R_2} = \frac{16}{3 \Omega} \cdot \frac{2}{2 + 1 + 2} = 2 \text{ A}$$

1. základní metodou
2. základní metodou
3. základní metodou

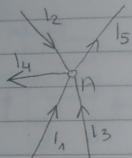
$$E_t = I_2 \cdot R_2 = 4V$$



$$P_t = \frac{E_t^2}{(R_2 + R_3)^2} \cdot R_3 = 4W$$

1. KZ - Struje

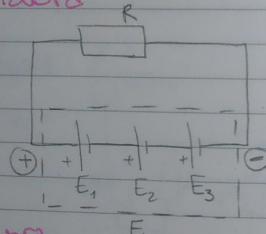
zvanično
- struja koje ulaze u čvor jednake su strujama koje izlaze



Serijski spoj više izvora

$$E = E_1 + E_2 + E_3$$

- struje su jednake



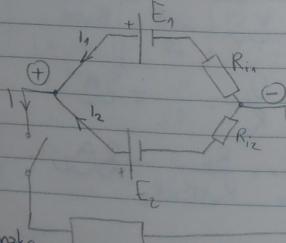
Paralelni spoj više izvora

$$I = I_1 + I_2 \\ V = V_1 = V_2$$

2. KZN

$$\rightarrow \sum E = \sum IR$$

- učrtati konturne struje i
fiksne tocke stavljeti predznake
u jednadžbine



THEVENINOV TEOREM

$$1.170.) I_1 = I_2 = 0,1A$$

$$E_1 = 25V$$

$$E_2 = 10V$$

$$E_3 = 11V$$

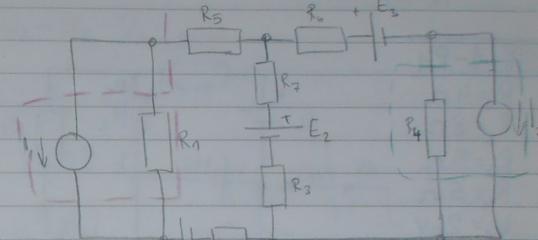
$$R_1 = R_2 = 10\Omega$$

$$R_3 = R_4 = 30\Omega$$

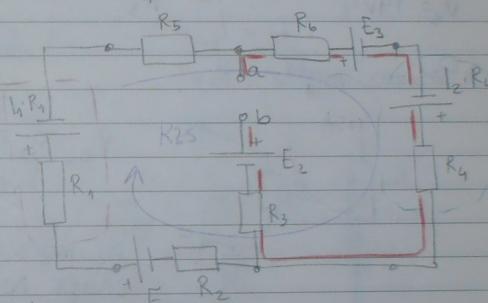
$$R_5 = 25\Omega$$

$$R_6 = 40\Omega$$

$$R_7 = 20\Omega$$



$$E_T = V_{ab} =$$



Pred napona izm. a i b:

$$E_T = V_{ab} =$$

$$- E_2 + |R_3 + |R_4 - |I_2 R_4 + E_3 + |R_6$$

struja kroz tu granu ne teče

$$|R_2 + |R_1 + |R_5 + |R_6 + |R_4 = E_1 - |I_1 R_1 - E_3 + |I_2 R_4$$

$$|I = \frac{E_1 - |I_1 R_1 - E_3 + |I_2 R_4}{R_1 + R_2 + R_4 + R_5 + R_6} = 0,14A$$

$$E_t = 7,74V \quad R_t = R_3 + \frac{(R_2 + R_4 + R_5)(R_6 + R_7)}{R_2 + R_4 + R_5 + R_6 + R_7} = 57,39\Omega$$

$I_7 = \frac{E_1}{R_1 + R_2} = 0,1A$, teče u smjeru A \rightarrow B

X.I-P4

$R_4 = 4\Omega$

$R_2 = 8\Omega$

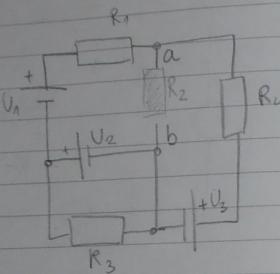
$R_3 = 5\Omega$

$R_4 = 5\Omega$

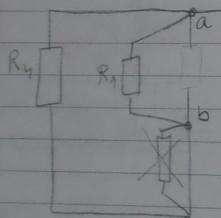
$U_A = 26V$

$U_2 = 30V$

$U_3 = 14V$



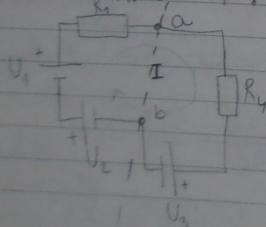
• Pasivna mreža $R_t = ?$



$R_t = \frac{R_1 R_2}{R_1 + R_2} = \frac{20}{9} = 2,22\Omega$

R_2 mora biti $2,22\Omega$ da bi snaga na njemu bila maksimalna.

• Aktivna mreža $U_T = ?$



$U_{AB} = U_2 + U_1 - I R_1 = 37,3V$

• Stroja $I = ?$

$I \cdot (R_1 + R_4) = U_1 + U_2 - U_3$

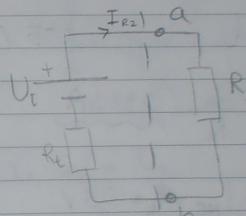
$I = \frac{U_1 + U_2 - U_3}{R_1 + R_4} = 4,67A$

• Snaga na $R_2 = ?$

$E = \frac{U_T}{R_t + R_2} = \frac{3,65}{\sim} A$

$U_{R_2} = I \cdot R_2 = 29,22V$

$P = U_{R_2} \cdot I_{R_2} = 106,67W$



X.I.P5

$U_1 = 1,2V$

$U_2 = 1,2V$

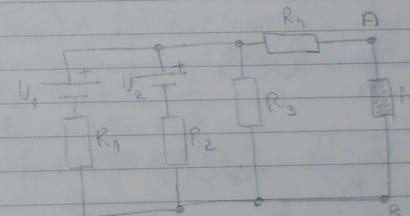
$R_1 = R_2 = 0,8\Omega$

$R_3 = 6\Omega$

$R_4 = 1,5\Omega$

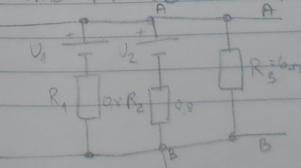
$U_{NE} = 0,5V$

$I_1, I_2, I_3, I_4 = ?$



• Aktivna mreža $U_T = ?$

$U_T = \frac{U_1 + U_2}{R_1 + R_2} = \frac{1,2 + 1,2}{0,8 + 0,8} = 1,5V$



Uvijek $\frac{U_1 + U_2}{R_1 + R_2} = \frac{2,4}{1,6} = 1,5V$

$U_T = \frac{U_1 + U_2}{R_1 + R_2 + R_3} = \frac{1,2 + 1,2}{0,8 + 0,8 + 6} = 0,22V$

• Passiva mitte R_4 :

$$R_{R4} = R_4 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = 1,875 \Omega$$

$$\bullet U_{R_E} = U_{AB} - U_{NE} = 0,625 V$$

$$I_{NE} = \frac{U_{R_E}}{R_E} = 0,333 A$$

$$U_{R4} = I_{NE} \cdot R_4 = 0,5 V$$

$$U_{R_5} = U_{NE} + U_{R_4} = 1 V$$

$$I_{R_5} = \frac{U_{R_5}}{R_5} = 0,167 A$$

$$I_{R_2} = \frac{U_{R_5} - U_2}{R_2} = 0,25 A$$

$$I_{R_1} = \frac{U_{R_5} - U_2}{R_1} = 0,25 A$$

X.1-P3

$$R_1 = 6 \Omega$$

$$R_2 = 2 \Omega$$

$$R_3 = 3 \Omega$$

$$R_4 = 2 \Omega$$

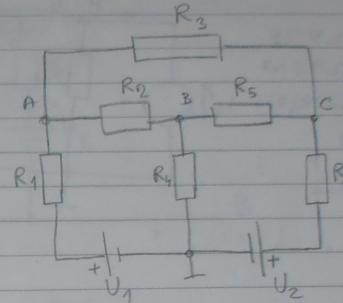
$$R_5 = 2 \Omega$$

$$R_6 = 12 \Omega$$

$$U_A = 12 V$$

$$U_2 = 24 V$$

$$I_4 = ?$$



$$\varphi_A (G_1 + G_2 + G_3) - \varphi_B (G_2) - \varphi_C (G_3) = \frac{U_1}{R_1}$$

$$-\varphi_A (G_2) + \varphi_B (G_2 + G_5 + G_6) - \varphi_C (G_5) = 0$$

$$-\varphi_A (G_3) - \varphi_B (G_5) + \varphi_C (G_3 + G_5 + G_6) = \frac{U_2}{R_6}$$

$$\varphi_A - 0,5 \varphi_B - 0,33 \varphi_C = 2$$

$$-0,5 \varphi_A + 1,5 \varphi_B - 0,5 \varphi_C = 0$$

$$-0,333 \varphi_A - 0,5 \varphi_B + 0,317 \varphi_C = 2$$

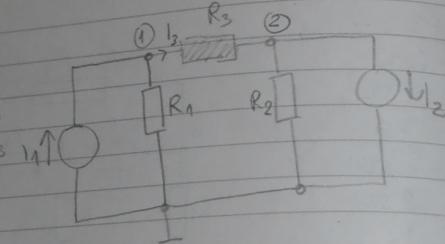
$$\varphi_A = 6,61 V \quad \varphi_B = 4,56 V \quad \varphi_C = 7,1 V$$

X.1.-1.

$$R_1 = 2 \Omega \Rightarrow 0,5$$

$$R_2 = 6 \Omega \Rightarrow 0,167$$

$$R_3 = 12 \Omega \Rightarrow 0,083$$



$$I_1 = 4A$$

$$I_2 = 2A$$

$$\varphi_1, \varphi_2 = ?$$

$$U_{1,2} = ?$$

$$I_3 = ?$$

$$\varphi_1(G_1 + G_3) - \varphi_2(G_3) = I_1$$

$$-\varphi_1(G_3) + \varphi_2(G_3 + G_2) = -I_2$$

$$0,583\varphi_1 - 0,083\varphi_2 = 4$$

$$-0,083\varphi_1 + 0,25\varphi_2 = -2$$

$$\varphi_1 = 6V \quad \varphi_2 = -6V$$

$$U_{1,2} = \varphi_1 - \varphi_2 = 12V \Rightarrow I_3 = \frac{12}{12} = 1A \text{ u smjeru } 1 \rightarrow 2$$

X.1.-2.

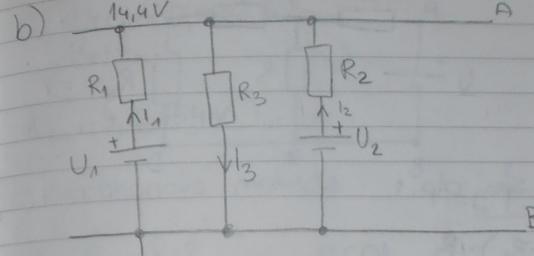
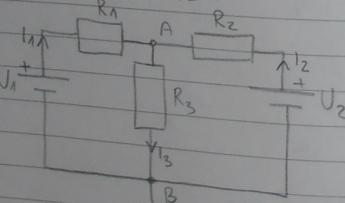
$$R_1 = 3 \Omega$$

$$R_2 = 3 \Omega$$

$$R_3 = 6 \Omega$$

$$U_1 = 24V$$

$$U_2 = 12V$$



$$U_{AB} = \frac{\frac{24}{3}}{\frac{1}{3} + \frac{1}{3} + \frac{1}{6}} + \frac{\frac{12}{3}}{\frac{1}{3} + \frac{1}{3} + \frac{1}{6}} = 14,4V$$

$$I_1 = 3,2A$$

$$I_2 = 0,8A$$

$$I_3 = 2,4A$$

b) Kirchhoffovi zakoni

$$U_1 = I_1 R_1 + I_3 R_3$$

$$U_2 = I_2 R_2 + I_3 R_3$$

$$I_1 + I_2 = I_3 \Rightarrow I_1 = I_3 - I_2$$

$$U_1 = I_3 \cdot R_1 - R_1 \cdot I_2 + I_3 R_3$$

$$U_2 = I_2 \cdot R_2 + I_3 R_3$$

$$24 = I_3 \cdot 9 - 3I_2 \quad | :3$$

$$12 = I_2 \cdot 6 + I_3 \cdot 3 \quad | :3$$

$$I_3 = 2,4A \quad I_2 = -0,8A \quad I_1 = 3,2A$$

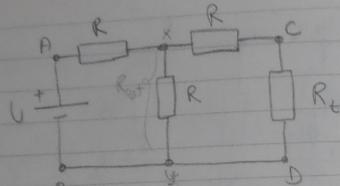
X.1-3

$$U = 12 \text{ V}$$

$$R_t = 100 \Omega$$

$$R = 57,735 \Omega$$

$$U_{ad} / U_{ab} = ?$$



a) kao mješoviti spoj otp.:
8

$$R_{vt} = R + \frac{(R+R_t) \cdot R}{2R+R_t} = 100 \Omega$$

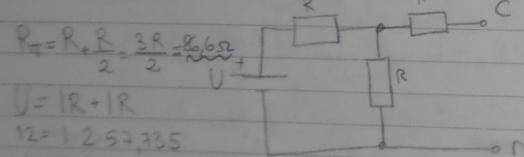
$$R_{ost} = \frac{R(R+R_t)}{2R+R_t} = 42,265 \Omega$$

$$U_{xy} = U \cdot \frac{R_{ost}}{R+R_{ost}} = 5,07 \text{ V}$$

$$U_{cd} = U_{xy} \cdot \frac{R_t}{R+R_t} = 3,2154 \text{ V}$$

$$\frac{U_{cd}}{U_{ab}} = 0,268 = 26,8\%$$

b) Theveninova metoda \rightarrow svi otpori osim isključenog!



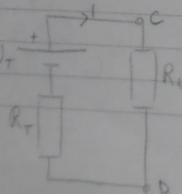
$$U = IR + IR$$

$$I_2 = 1,257,335$$

$$I = 0,039 \text{ A}$$

$$I_1 = I - I_2 = 0,039 - 1,257,335 = -1,257,296 \text{ A}$$

$$U = I_1 \cdot R = 0,039 \cdot 86,62 = 3,36 \text{ V}$$



$$I = \frac{U}{R_t + R_vt} = 0,032 \text{ A}$$

$$U_{cd} = 3,2154 \text{ V}$$

c) Northonove metoda

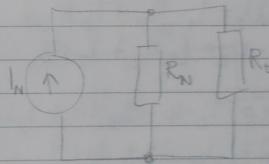
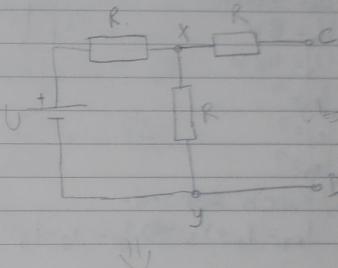
$$R_N = R_vt = \frac{R \cdot R_t}{2R} = 86,62 \Omega$$

$$I = \frac{U}{R_N} = 0,13856$$

$$I_{Nz} = \frac{1}{2} = 0,06933$$

$$I_{Nz} = I_N \cdot \frac{R_N}{R_N + R_t} = 32,09 \text{ mA}$$

$$U_t = I_{Nz} \cdot R_t = 3,215 \text{ V}$$



$$I_1 \cdot R_1 = I_2 \cdot R_2$$

$$I_2 \cdot R_1 + I_2 \cdot R_2 = I_2 \cdot R_2$$

$$I_2 (R_1 + R_2) = I_2 \cdot R_2$$

$$I_2 = \frac{I_2 \cdot R_2}{R_1 + R_2}$$

X. 1-4

$$R_1 = 16\Omega$$

$$R_2 = 4\Omega$$

$$R_3 = 24\Omega$$

$$U_1 = 72V$$

$$U_2 = 8V$$

$$U_3 = 48V$$

$$I_{R1} = ?$$

$$I_{R2} = ?$$

$$I_{R3} = ?$$

a) Kirchhoff's law:

$$I_{R2} = I_1 + I_3 \Rightarrow I_1 = I_2 - I_3$$

$$\textcircled{1} U_1 - U_2 = I_1 \cdot R_1 + I_2 \cdot R_2$$

$$\textcircled{2} U_3 - U_2 = I_3 \cdot R_3 + I_2 \cdot R_2$$

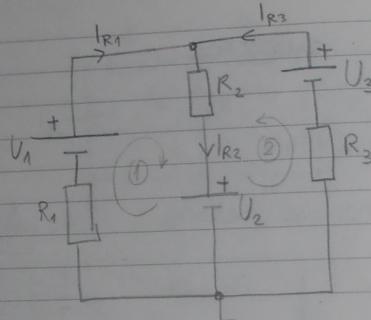
$$64 = I_2 \cdot 16 - I_3 \cdot 16 + I_2 \cdot 4$$

$$40 = I_3 \cdot 24 + I_2 \cdot 4$$

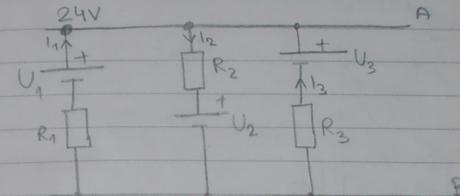
$$20I_2 - 16I_3 = 64$$

$$4I_2 + 24I_3 = 40$$

$$I_2 = 4A \quad I_3 = 1A \quad I_1 = 3A$$



b) Millmanova metoda



$$U_{AB} = \frac{\frac{72}{16} + \frac{8}{4} + \frac{48}{24}}{\frac{1}{16} + \frac{1}{4} + \frac{1}{24}} = \frac{4,5 + 2 + 2}{0,3542} = \frac{8}{0,3542} = 24V$$

$$I_1 = 3A \quad I_2 = \frac{U_2 - 24}{R_2} = \frac{48 - 24}{4} = 4A \quad I_3 = 1A$$

X. 1-9

$$R_1 = 20\Omega$$

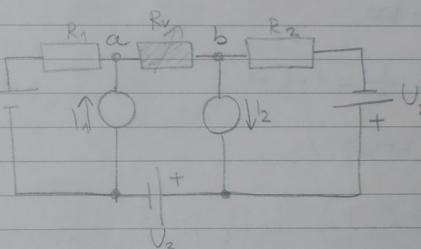
$$R_2 = 10\Omega$$

$$U_A = U_3 = 20V$$

$$U_2 = 30V$$

$$I_1 = I_2 = 1A$$

$$P_{max} = ?$$

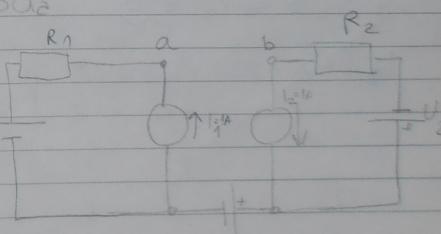


Theveninova metoda

$$R_T = R_1 + R_2 = 30\Omega$$

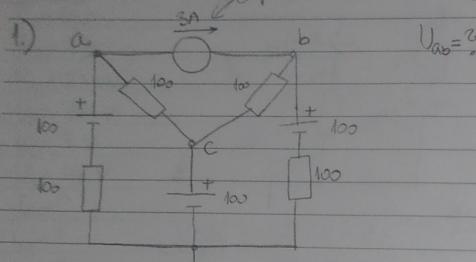
• aktivne mreža

$$\begin{aligned} & \text{fješljivo na str. } \\ & -36- \end{aligned}$$



Podaci:

Otpor bestončića \rightarrow vrednost 0



stoga je išta je fa

$$\varphi_A \left(\frac{1}{100} + \frac{1}{100} \right) - \varphi_B \left(\frac{1}{100} \right) = -3 + \frac{100}{100}$$

$$\varphi_C = 100$$

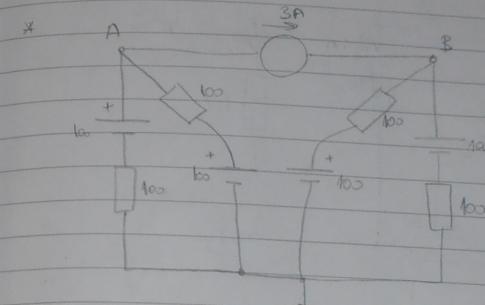
$$\varphi_A \cdot \frac{1}{50} = 1 - 3 + 1$$

$$\varphi_A = -50 \text{ V}$$

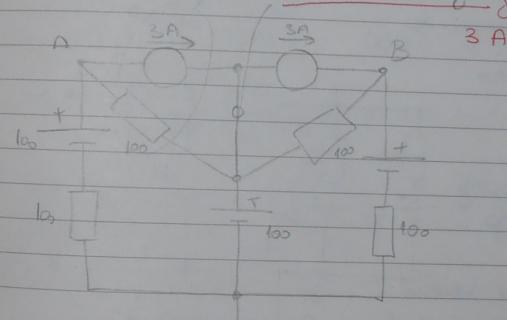
$$\varphi_B \left(\frac{1}{100} + \frac{1}{100} \right) - \varphi_C \left(\frac{1}{100} \right) = 3 + \frac{100}{100}$$

$$\varphi_B \cdot \frac{1}{50} = 5 \Rightarrow \varphi_B = 250$$

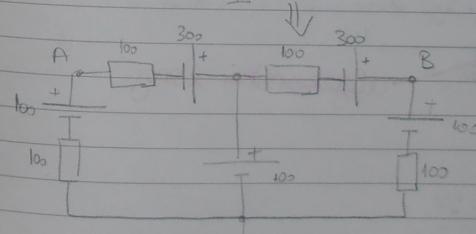
$$U_{AB} = -50 - 250 = -300 \text{ V}$$



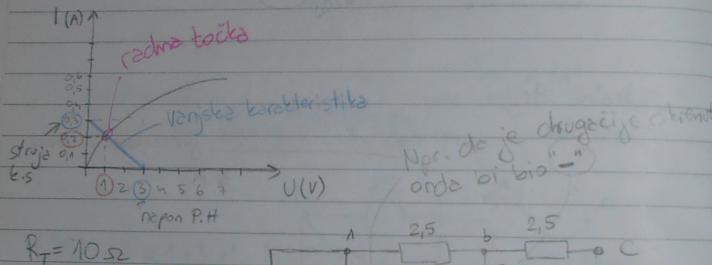
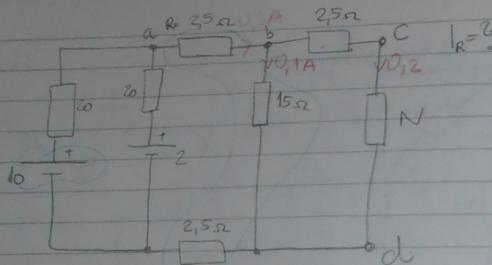
* idealni strujni izvor ne možemo transformirati u nepomski
možemo pretvoriti u nepomski
ne leže struja jer utezi i izlazi



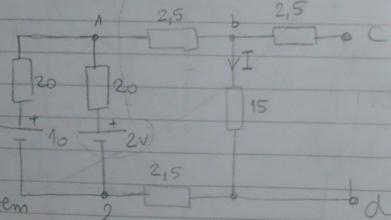
← 2ašto
struja
jer utezi
i izlazi



2)



$U_{AB} = \text{možemo primj. metodu superp.}$



$U_{12} = \text{millmanov teorem}$

sumu struje koja ulaze u (1) / sumu vodljivosti

$$U_{12} = \frac{\frac{10}{20} + \frac{2}{20}}{\frac{1}{20} + \frac{1}{20} + \frac{1}{20}} = 4V$$

algebarski zbroj

$$I = \frac{4}{20} = 0,2A$$

$$E_T = 0,2 \cdot 15 = 3V$$

25

1V napon na neelinearnom elementu
0,2A struje na neelinearnom elementu

$$U_{34} = 1,5V$$

$$I_R = I_N + 0,1 = 0,3A$$

* primjenom superpozicije

$$R'_1 = 30\Omega$$

$$I' = \frac{10}{30} \cdot \frac{1}{2} = \frac{1}{6} A$$

2 isti otvar

$$I'' = \frac{2}{30} \cdot \frac{1}{2} = \frac{1}{30} A$$

$$I = I' + I'' = \frac{1}{6} + \frac{1}{30} = \frac{1}{5} A = 0,2A$$

26

$$U_{22} = i_2 \cdot Z_2 \pm i_1 \cdot Z_M$$

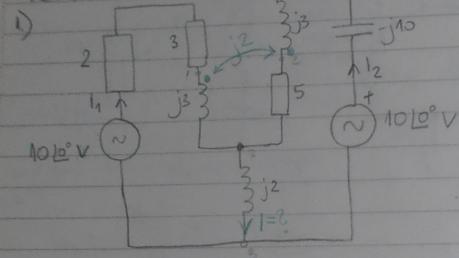
$$U_{12} = i_1 \cdot z_1 \pm (i_2 - i_1) \quad Z_m = i_1 (z_1 \mp z_m) \pm i_2 \cdot z_m$$

$$U_{23} = |_2 \cdot \underline{z}_2 \pm (|_3 - |_2) \cdot \underline{z}_m = |_2 (\underline{z}_2 + \underline{z}_m) \pm |_2 \underline{z}_m$$

Pomier

$$M=10$$

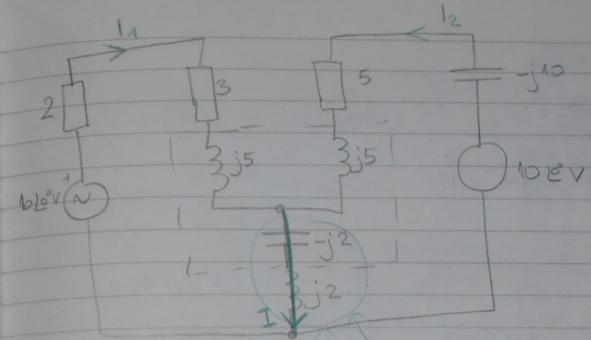
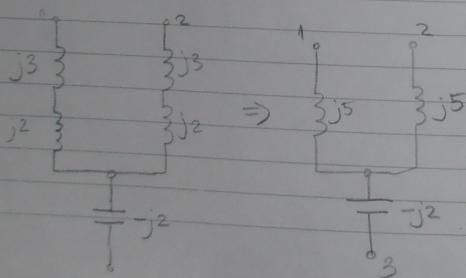
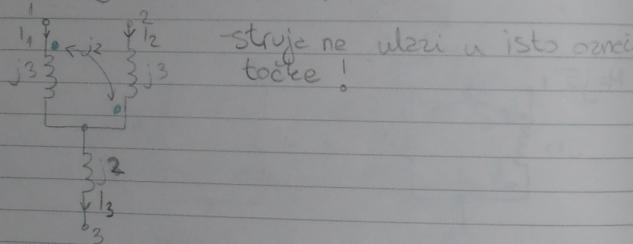
zadaci



$$i = i_1 + i_2$$

Primjena transf. Induktivitete:

- Ovo ne smijemo kod top. dijagrama



rezonancija
ketke spaj!

$$i = i_1 + i_2 = \frac{10}{5+j} + \frac{10}{5-j} = \frac{5+5j}{25+25} + \frac{5-5j}{25-25} = \frac{10j}{50} = 2 \text{ mA}$$

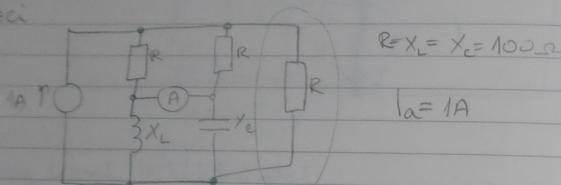
$$10 = i_1 (5+j)$$

$$10 = i_2 (5-j)$$

1113-2005/2006

$$7) U_{AB} = \frac{\frac{200}{j100} - 2}{\frac{1}{100} + \frac{1}{j100} - \frac{1}{j100}} = -20 - j200 \quad (\text{A})$$

Zadaci



↑ bez otpora $R \rightarrow$ zadatak
je redefiniran jer je
otpornost konstantan

$$\underline{X-15} \\ U_f = 72V \\ I_R = ?$$

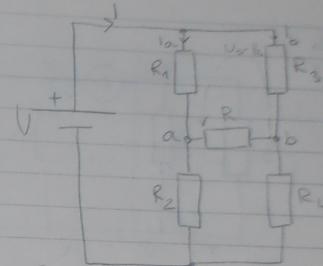
$$R_1 = 5 \Omega$$

$$R_f = 6 \Omega$$

$$R_2 = 3 \Omega$$

$$R_3 = 12 \Omega$$

$$R_4 = 4 \Omega$$



Theveninova metoda

$$R_T = 2 + 3 = 5 \Omega$$

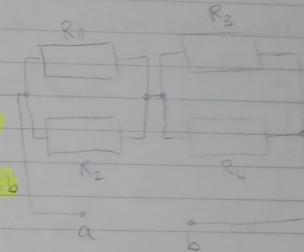
Napon $U_{ab} = ?$

- izracunati struje kroz granice
gdje su sterezljive a,b

- bitno je dobiti potencijale a,b

$$I_a = \frac{U_f}{R_1 + R_2} = 8A$$

$$I_b = \frac{U_f}{R_3 + R_4} = 4.5A$$

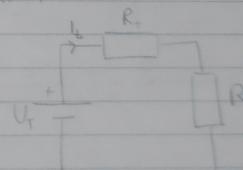


$$U_{ab} = U_f = U_o - U_a = I_b R_3 - I_a R_1 = 6V$$

$$I_R = \frac{U_f}{R_f + R} = 0.6A$$

Kada je otpor $R = 0$

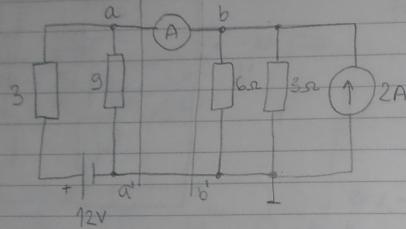
$$I_R = \frac{U_f}{R} = 1.2A$$



Da je struja $I = 0A$:

$$R_1 \cdot R_4 = R_2 \cdot R_3 \Rightarrow R_1 = \frac{R_2 \cdot R_3}{R_4} = 9 \Omega$$

X16

 $I_a = ?$ 

a) Theveninova metoda - spoj podijeliti po projektu

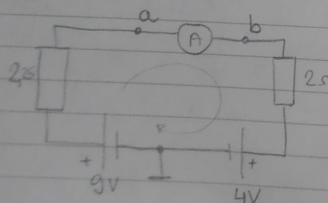
$$R_{aa'} = \frac{3 \cdot 9}{12} = 2,25 \Omega = R_{ia}$$

$$I_{ba} = \frac{12}{3+9} = 1A$$

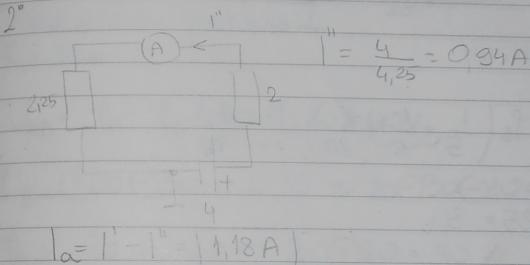
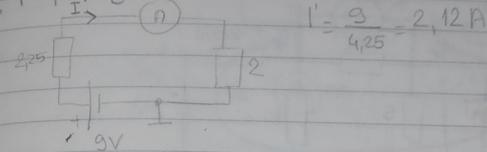
$$U_{aa'} = I \cdot R_{ia} = 1 \cdot 2,25 = 2,25V$$

$$R_{bb'} = \frac{6 \cdot 3}{9} = 2\Omega$$

$$U_{tb} = R_{tb} \cdot I = 2 \cdot 2A = 4V$$



superpozicija:



$$I_a = I' - I'' = 1,18A$$

-kada bi umjesto

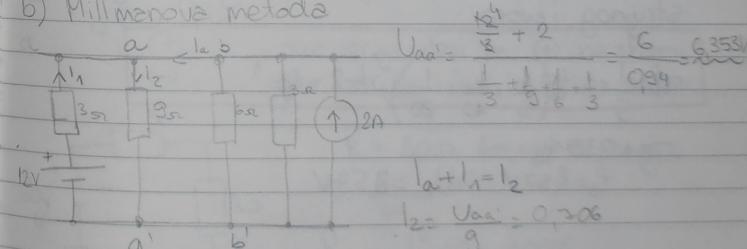
strujnog bio
naponski izvor

$$R_t = 0,32$$

Potencijalni stupnjevi ampermetra

$$\varphi_a = \varphi_b = 4V + 1 \cdot 2\Omega = 4 + 1 \cdot 2 = 6,36V$$

b) Millmanova metoda



$$12 - I_1 \cdot 3 \cdot 2 = 6,36$$

$$I_1 = 1,882A$$

$$I_a = I_2 - I_1 = -1,176A$$

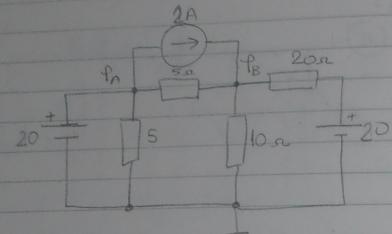
X.1.5.

$$R = 5\Omega$$

$$I = 2A$$

$$U = 20V$$

$$U_{ba} = ?$$



$$\varphi_A = 10V$$

$$-\varphi_A \left(\frac{1}{5}\right) + \varphi_B \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{20}\right) = 2 + \frac{20}{20}$$

$$-4 + \varphi_B \cdot 0,35 = 3$$

$$0,35 \varphi_B = 7$$

$$\varphi_B = 20V \Rightarrow U_{ba} = 20V$$

b) napon na 10Ω ako se promjeni smjer struje strujnog izvora

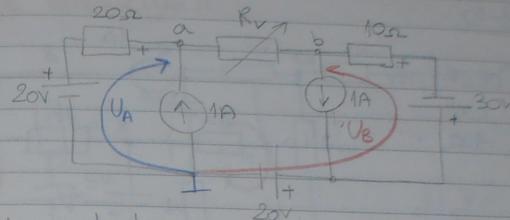
$$-\varphi_A \frac{1}{5} + \varphi_B \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{20}\right) = -2 + \frac{20}{20}$$

$$-4 + 0,35 \varphi_B = -1$$

$$\varphi_B = 8,57V \Rightarrow U_{ba} = 8,57V$$

X.1.9.

$$P_{max} = ?$$



Theveninova metoda

$$R_t = 30\Omega$$

$$U_{AB} = \varphi_A - \varphi_B = 40 - (-20) = 60V$$

$$U_{AB} = +20 - 30 = 1 \cdot 10 \Rightarrow 20 - 30 - 10 = -20V$$

$$\varphi_A = 20 = -1 \cdot 20 \Rightarrow 20 + 20 = 40V$$

PREDZNACI:

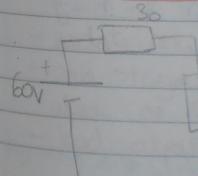
1. Postaviti uređenje

2. Računati φ_A na način da se uzme 1 grana i predznaci se postave na elemente te u jednadžbu se sve ukloni tako da je izvor!

jednadžbe:



akao struje izlazi iz + na shemi
akao struje izlazi iz - na shemi



$$R = 30\Omega \text{ za max snagu}$$

$$I = \frac{60}{60} = 1A$$

$$P_{max} = 1^2 \cdot 30 = 30W$$

X.1-10

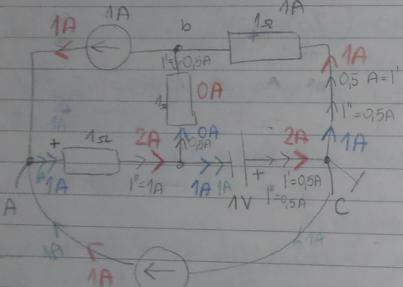
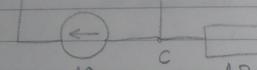
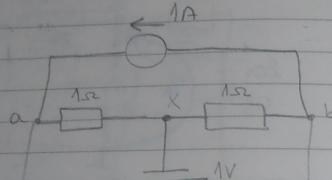
a) Theveninov

aic

$$R_t = 1\Omega$$

$$U_{AC} = \varphi_a - \varphi_c$$

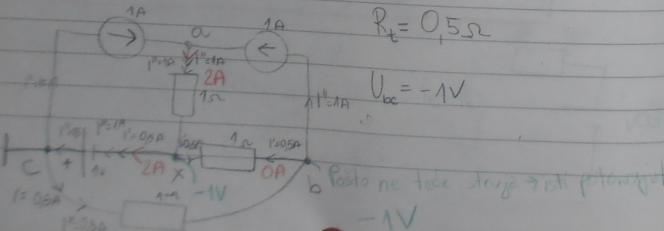
između točaka aic



$$U_{AC} = U_{TA} = \frac{1}{m} U + \frac{1}{m} R = 1V$$

6. Predznaci

Između točaka bic



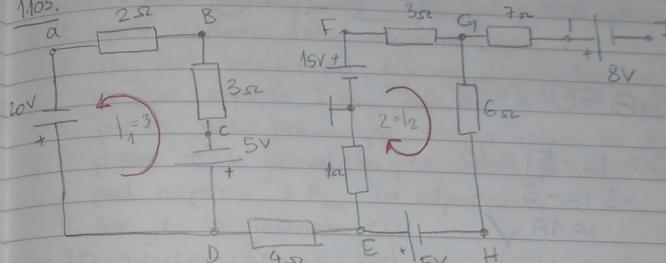
$$R_t = 0,5\Omega$$

$$U_{BC} = -1V$$

$$-1V$$

PONAVLJANJE POTENCIJALA

1.103.



$$U_{BD}, U_{DB}, U_{CA}, U_{AH}, U_{JC} = ?$$

- iz sheme se zaključi da kroz 4Ω i 7Ω ne teče struja \Rightarrow nema pada napona
- struje I_1 i I_2 su neovisne jedna o drugoj

$$I_1 = \frac{20-5}{2+3} = 3A$$

$$I_2 = \frac{15+5}{12} = 2A$$

$$\varphi_F = 15V$$

$$\varphi_E = \varphi_F - I_2 \cdot R = 15 - 2 \cdot 3 = 9V$$

$$\varphi_G = \varphi_F - I_1 \cdot R = 15 - 3 \cdot 3 = 6V$$

$$\varphi_H = \varphi_G + 5 = 11V$$

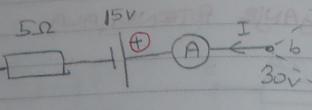
$$\varphi_I = \varphi_H - 8 = 3V$$

- zadati neki smjer struje i ravnati se po njemu

- ako struja ući je $(+)$ onda u jedn. ide $-$, ako ulazi na $(-)$ onda u jedn. $+$

- kod otpornika uvijek $-$

1.106. $I_A = 10V$
 $V_B = 30V$



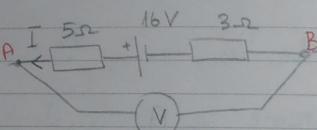
$$V_B - 15V = I \cdot R \Rightarrow V_A = I \cdot 5\Omega$$

$$30 - 15 - 5I = 10$$

$$-5I = -5$$

$$I = 1A \quad \checkmark$$

1.107. $U_V = 0V$



$$-1.5 + 16 - 1 \cdot 3 = 0$$

$$-8I = -16$$

$$I = 2A \quad \checkmark$$

ZAKLJUČAK:

- zadamo pravljivo neki smjer struje

- 1.106. želimo izračunati potencijel φ_A (premda ga već imamo), to radimo na način:

1. krećemo od prveg poznatog potencijela (φ_B)

2. nailazimo na φ_A → ovdje struja "neprirodno" ulazi na $(+)$, te u jednadžbi pišemo minus "-"

3. nailazimo na otpornik → to nije išvor te nikako ne može biti "+" u jednadžbi, tj. kod otpornika je UVJEK "-"!

4. i sada dolazimo do φ_A + j. sve ovo prije je jednako φ_A

- 1.107. feli nam struja, no sada nemamo potencijele točke veće nego napon (zadan kao rezl. potencijale = 0V)

1. opet zadamo neki smjer struje

2. ovej put krećem od lijevog otpornika, ne njenu je pad naponsa $I \cdot R_{sa}$, ali "-"

3. pošto naša pravljiva struja izlazi iz $(+)$, znači, pišem "+" u jednadžbi, te imam ponovno neki pad naponsa $I \cdot R_{sa}$ na otporniku

4. sve je to jednako 0V

→ 2. bunjujuće djeluje sad ovo... (0V), mogla-sam to sve zapisati na način da je $\varphi_A = 3V$ i $\varphi_B = 3V \Rightarrow$ Razlike ta dva potencijela bilo bi 0V (što mjeri voltmeter), ali jednadžba bi izgledala isto kao i u 1.106. zadatka:

$$\varphi_B - 1.5 + 16 - 1 \cdot 3 = \varphi_A$$

$$-1.5 + 16 - 1 \cdot 3 = \varphi_A - \varphi_B = 0$$

1.108. $U_{AB} = 10V$

$$I = ?$$

$$\varphi_B - 2.5 + 5 = \varphi_x$$

$$10 - 10 + 5 = \varphi_x$$

$$\varphi_x = 5V$$

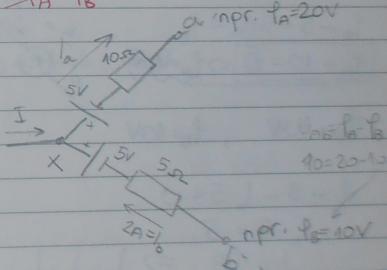
$$\varphi_x - 5 - 10 \cdot I_a = \varphi_a$$

$$5 - 10 \cdot I_a = 20$$

$$I_a = -2A$$

$$I + I_b = I_a$$

$$I = I_a - I_b = -2 - 2 = -4A$$



A.1.10. $I_A = 0,5A$ $U_{AB}?$
 $I_B = 2A$

$\varphi_x - I_1 \cdot 5 - 10 = 0$

$\varphi_x = 10 + 2,5$

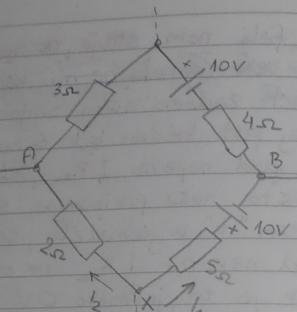
$\varphi_x = 12,5V$

$\varphi_x - I_2 \cdot 2 = \varphi_A$

$12,5 - 4 = \varphi_A$

$\varphi_A = 8,5V$

$U_{AB} = \varphi_A - \varphi_B = 8,5V$



A.1.12.

$I_A = 1A$

$I_B = 2A$

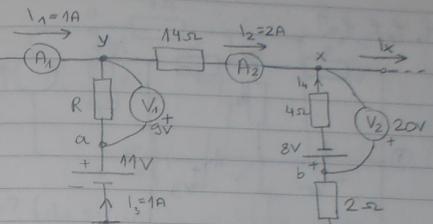
$U_{in} = 9V$

$U_{out} = 20V$

$\text{a)} R = \frac{U_{out}}{I_B} = 9\Omega$

$\text{b)} I_x?$

$\text{c)} U_{AB}?$



a) - Záleží na tom, že je $I_3 = 1A$ iž $I_2 = I_1 + I_3 \Rightarrow I_2 = I_3 - I_1 = 1A$

$- \frac{U_{out}}{I_3} = R = \underline{\underline{9\Omega}}$

b) $20 = 8 + I_4 \cdot 4$

$12 = 4I_4$

$- \underline{\underline{I_4 = 3A}}, \Rightarrow I_x = 3A + 2A = \underline{\underline{5A}}$

c) $\varphi_A = 11V$

$\varphi_B = -I_4 \cdot 2 = -6V$

$U_{AB} = \varphi_A - \varphi_B = 11 - (-6) = 11 + 6 = 17V$

A.1.11. $I_A = 1A$

$U_{in} = 10V$

$U_{AB}?$

$\varphi_A - 5 - 5 \cdot I_a = \varphi_x$

$\varphi_A - 10 = 0 \Rightarrow \varphi_A = 10V$

$\varphi_B = 10V, \varphi_B = 10V$

$\varphi_x - I_b \cdot 5 = 0$

$-I_b \cdot 5 = -5$

$I_b = 1A \Rightarrow I_c = I_b + I_a = 2A$

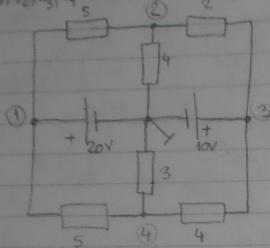
$\varphi_x - I_c \cdot 5 = \varphi_c$

$0 - 10 = -\varphi_c$

$\varphi_c = -10V$

$U_{AB} = \varphi_c - \varphi_A = -10 - 10 = -20V$

X.I-11 $\varphi_1, \varphi_2, \varphi_3, \varphi_4 = ?$



$$\varphi_1 = 20V, \varphi_3 = 10V, \varphi_2 = 9.47V$$

$$\varphi_2 \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{2} \right) - \varphi_1 \left(\frac{1}{5} \right) - \varphi_3 \left(\frac{1}{2} \right) = 0$$

$$\varphi_2 0.95 - 4 - 5 = 0$$

$$0.95 = 9$$

$$\varphi_2 = 9.47V$$

struje = strujama (grane s izvorima)

$$\varphi_4 \left(\frac{1}{5} + \frac{1}{3} + \frac{1}{3} \right) - \varphi_3 \left(\frac{1}{4} \right) - \varphi_1 \left(\frac{1}{3} \right) = 0$$

$$0.783 \varphi_4 = 6.5$$

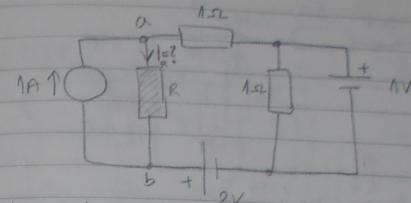
$$\varphi_4 = 8.3V$$

- kod "potencijela" s lijeve strane je potencijel i
vodećost. (suma) "+" i ne crtež koj računamo SVI ostali imaju
UVJET "-" (to su "struje")

- s desne strane isto moramo imati struje ali nanačin.
de uvelimenu obzir smo one grane u kojima je
reparativni izvor i otpornik.

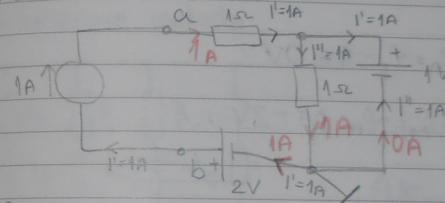
43) Pisati još s desne strane

X.I-12



$$I_R^2 = ? \quad I_R = ? \quad (\text{da strujni izvor prav. smjer})$$

a) Théveninova metoda



$$R_{Th} = 1\Omega$$

$$V_{AB} = \varphi_A - \varphi_B = 2 - 2 = 0V \Rightarrow I_R = 0 \text{ A}$$

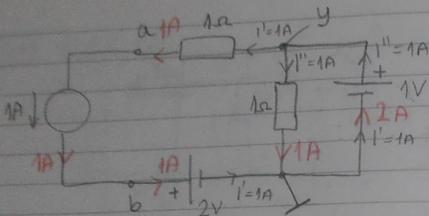
$$\varphi_A = 0 + 1 \cdot 1 + 1 \cdot 1 = 2V$$

$\varphi_B = 2V$ krećemo od I i vidimo da struja ide
prema mesi, što znači da se padovi napona
ne otvaraju! ODN. ovako sam pre
pisale → UVJEK SAM KRETALA IZ STRUJNOG ISTOŠ

$$\varphi_A - 1 \cdot R - 1 \cdot R = 0$$

$$\varphi_A = 1 \cdot R + 1 \cdot R = 2V$$

b) da se strujni izvor prekrene:



$$R_t = 1 \Omega$$

$$\varphi_y - 1 \cdot R = 0$$

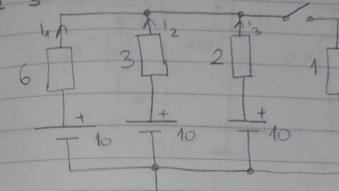
$$\varphi_y = 1V \Rightarrow \varphi_A = \varphi_y - 1 = 0V$$

$$\varphi_B = 2V \Rightarrow U_{AB} = \varphi_A - \varphi_B = -2V = U_T$$

$$I_t = I_R = \frac{U_T}{R} = \frac{-1V}{1\Omega} = -1A$$

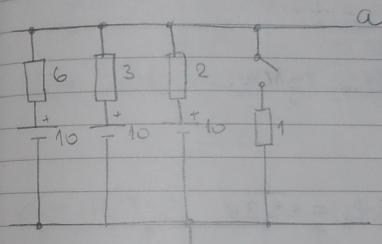
- pošto ne piše učinak $R=1\Omega$
li se U_{AB} ili U_B , ovo
je dobar rezultat

$$\begin{aligned} & X_1 - X_3 \\ & U_1 = U_2 = U_3 = 10V \end{aligned}$$



$\varphi_A = ?$ (prije zatvarača
i poslije)

Millmanova metoda



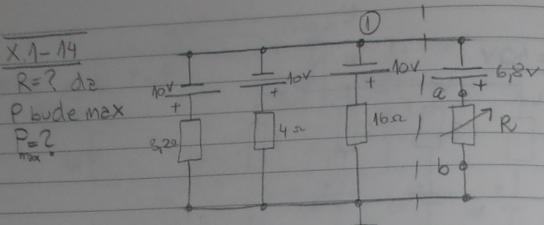
$$\text{prije: } U_{AB} = \frac{\text{alg } \Sigma I}{\Sigma G} = \frac{\frac{10}{6} + \frac{10}{3} + \frac{10}{2}}{\frac{1}{6} + \frac{1}{3} + \frac{1}{2}} = \frac{10}{1} = 10V \Rightarrow \varphi_A = 10V$$

φ_B je uzemljeno
Uvijek "+" SVE vodljivost/otpornici
Prezenaci

Poslije:

$$U_{AB} = \frac{\text{alg } \Sigma I}{\Sigma G} = \frac{10}{\frac{1}{6} + \frac{1}{3} + \frac{1}{2}} = 5V$$

Svi otpornici
između ova 2 "grane" tj.
potencijala A i B



- metoda potencijala čvorova

$$(R_t = 1.6 \Omega) = R^*$$

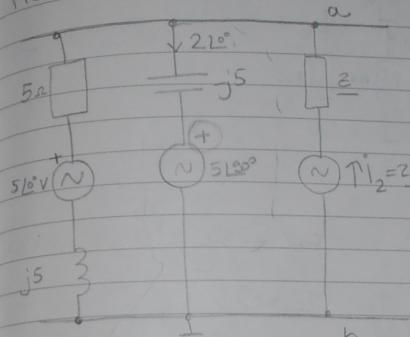
$$\Psi_1 = U_{AB} - \frac{-10}{32} - \frac{10}{4} - \frac{10}{16} = -10V$$

$$U_{AB} = \varphi_A - \varphi_B = \varphi_A - 0 = \varphi_A = -3,2V$$

$$V_A = V_0 + \underline{I} \cdot R = -3,2 \text{ V}$$

$I = 1 \text{ A}$

da je izvor od
6,8 V bio drugačije
okrenut tako da bi bio u jedn. "-"

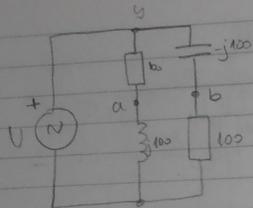


$$a \cup_{ab} = \frac{3U}{4Z} - \frac{3U}{4Z} = 0V$$

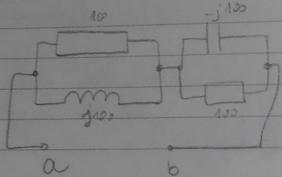
-26-

E_t ?

Z_t ?



Z_t :



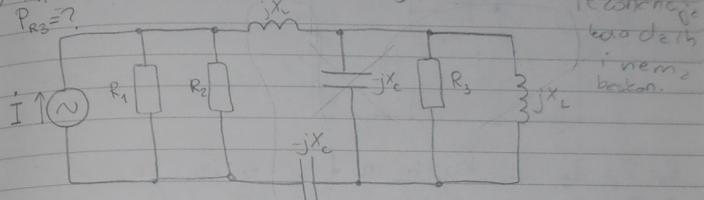
$$Z_t = 50 + 50j + 50 - 50j = 100\Omega$$

$$E_t = 0V$$

Zadaci

1) $R_1 = R_2 = R_3 = X_L = X_C = 50\Omega \quad I = j6A$

$P_{R3} = ?$



rezonancija
korak dath
i nema
reakcija.

d) Northonov teorem

-odspojiti R3

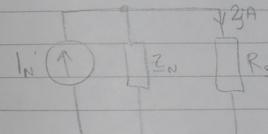
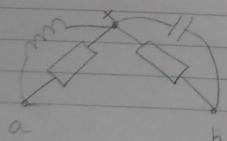
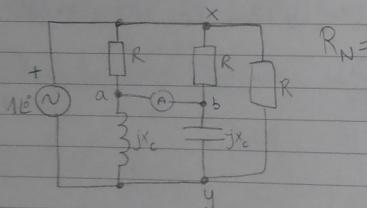
u ser. rezon. kada da ih
nema - rezonans

$$Z_t = Z_N = \frac{R_1 R_2}{R_1 + R_2} = 25\Omega$$

$$I_N = I = j6A$$

$$P = I^2 R_3 = 4 \cdot 50 = 200W$$

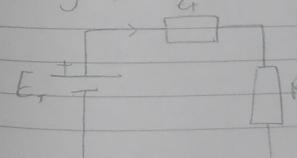
-27- $I_a = ?$



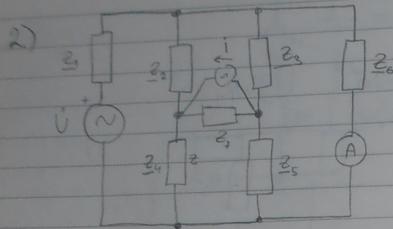
b) Theveninova metoda

$$E_T = j150V$$

$$I_{R3} = \frac{j150}{75} = j2$$



$$P = I^2 R = 4 \cdot 50 = 200W$$



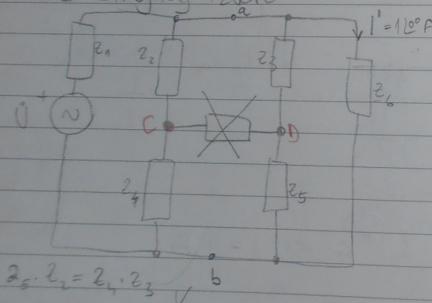
$$Z_1 = Z_3 = Z_5 = Z_7 = 100 \angle 60^\circ \Omega = 50 + 50\sqrt{3}j \Omega$$

$$Z_2 = Z_4 = Z_6 = 100 \angle -60^\circ \Omega = 50 - 50\sqrt{3}j \Omega$$

$$U = 150 \angle 0^\circ V \quad I = 110^\circ A$$

SUPERPOZICIJA

1º bez strujnog izvora

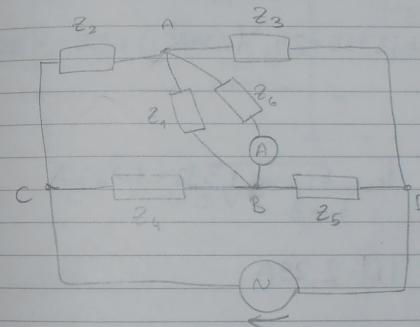


$$I = \frac{U_{AB}}{Z}$$

$$U_{AB} = \frac{U}{Z_1} \cdot \frac{1}{Z_1 + Z_2 + Z_4 + Z_6} + \frac{1}{Z_3 + Z_5} \cdot \frac{1}{Z_3 + Z_5 + Z_6}$$

$$I = \frac{U}{2Z_1 + 2Z_2} = \frac{2}{3} \cdot \frac{1}{Z_1} \cdot \frac{1}{Z_2} \cdot \frac{U}{Z_1 + Z_2} = \frac{2}{3} \frac{U}{Z_1 + Z_2}$$

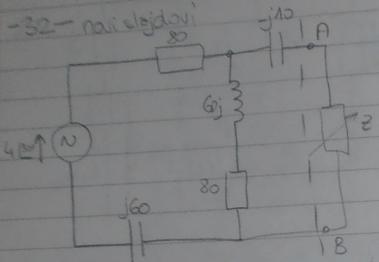
$$= \frac{\Delta}{3} \cdot \frac{150}{700} = 110^\circ A$$



most u ravnotezi

$$U_{AB} = 0 \Rightarrow I_A = 0$$

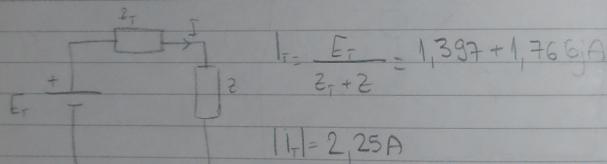
$$I_A = I' + I'' = I' = 110^\circ A \Rightarrow I = 1 A$$



$$Z_T = 8\Omega + 50j \Rightarrow Z_{max} = 8\Omega + 50j$$

$$E_T = 4\angle 0^\circ \cdot (8\Omega + 50j) = (3,195 + 24j)(80 + 50j) = 135,2 + 352,8j$$

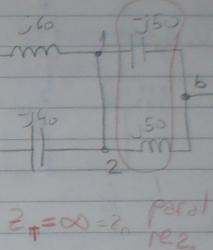
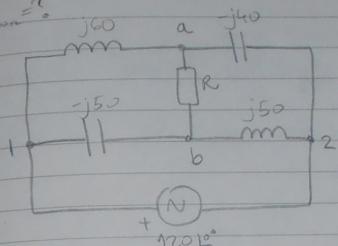
$$= 40\sqrt{89} \angle 69^\circ V$$



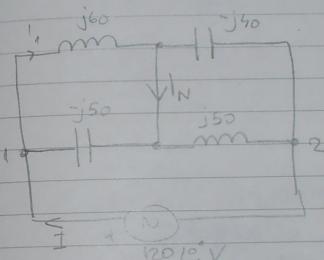
$$P_{max} = \left(\frac{E_T}{2 \operatorname{Re}\{Z\}} \right)^2 \cdot \operatorname{Re}\{Z\} = 500W$$

-33-

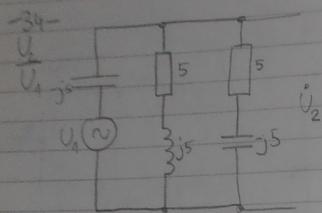
$$P_{max} = ?$$



$$Z_T = \infty = Z_1 \text{ paralel } Z_2$$



-ako je mogni spoj
onda je $I_N = 0A$



$$U_{ab} = \frac{U_1}{\frac{1}{j5} + \underbrace{\frac{1}{5} + \frac{1}{5}}_{5}} = U_2 = U_1 \cdot \frac{j}{1+j}$$

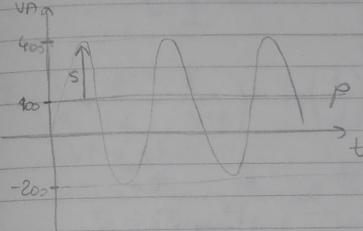
$$\frac{U_2}{U_1} = 0,5 + j0,5$$

prijenosna funkcija
Millmanov teorem deje interpol

$$P_A = 1 (-j5) - 5 \cdot 0 \Rightarrow P_A = 5i - 10j = -5j$$

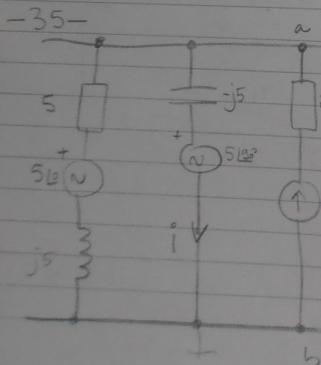
2.) Max. trenutna snaga trošila iznosi 400 VA, a minimalna -200 VA, kolika je fazovz snaga?

$p(t)$ - max. tren. snaga trošila



$$400 + 200 = 600 \text{ VA} \quad | : 2$$

300 VA, srednje snaga po def. \bar{P}



$$U_{a5} = 5 L 30^\circ + i(-j5) = -j5V$$

$$U_{ab} = \frac{\frac{5}{5+j5} - 1 + i_2}{1 + \frac{1}{j5}} = -j5 \quad (3)$$

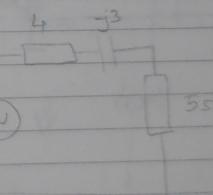
$$i_2 = 1 L 0^\circ A$$

$$Q = \sqrt{S^2 - P^2} = 2,82,$$

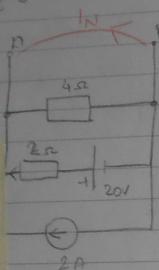
$$3) U = 10 L 30^\circ V$$

$$Z_1 = 4 - j3 \Omega$$

Da bi snaga bila
max

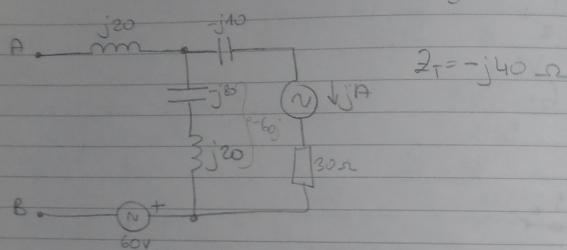
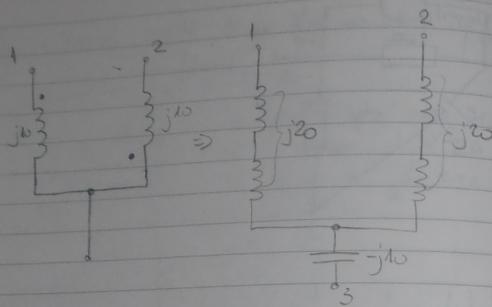


- samo smijemo mjenj. R, znači gledamo modul imped.



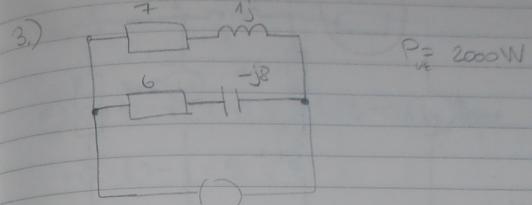
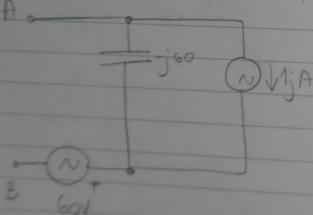
In krečka spajj svel grene
-strujni izvor daje 2A
- a napravški $\frac{20}{2}$, kroz 4 říz
ne teče struj

$$M = 10^{\circ}$$



$$U_{AB} =$$

$$E_1 = U_{AB} = 60 - [j \cdot (-j60)] = 60V$$

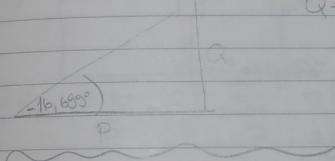


$$Z_{AB} = \frac{(7+j)(6-j)}{13-j} = \frac{500}{109} = \frac{150}{109} j$$

$$\tan \varphi = \frac{15}{109} \Rightarrow \varphi = 16,689^\circ$$

$$Q = \text{base} \tan 16,689^\circ \cdot 2000 = 600 \text{ VA}$$

✓



$$I_1^2 \cdot 7 + I_2^2 \cdot 6 = 2000$$

$$I_1 = \frac{U}{Z_1}, \quad I_2 = \frac{U}{Z_2}$$

$$Z_1 = \sqrt{50}, \quad Z_2 = 10$$

$$I_1^2 \cdot 7 + \frac{I_1^2}{2} \cdot 6 = 2000$$

$$I_1^2 = 200, \quad I_2^2 = 100$$

$$Q = I_1^2 \cdot 1 - I_2^2 \cdot j8 = -600 \text{ VA}$$

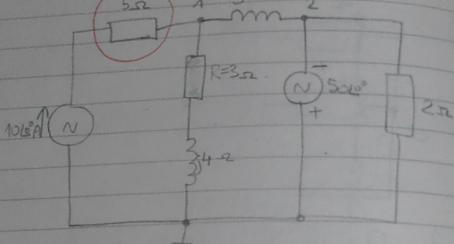
Element u seriji sa str. i u kom ne ulazi u jedan!

XI. P-4

$P_2 = ?$

$I = 10 \angle 0^\circ A$

$U = 50 \angle 0^\circ V$



$$f_2 = -U = -50V$$

$$\varphi_1 \left(\frac{1}{3+4j} + \frac{1}{j5} \right) + 5\Omega \cdot \frac{1}{j5} = 10$$

$$\varphi_1 \left(\frac{3}{25} - \frac{9}{25}j \right) = 10 + 10j$$

$$\varphi_1 = -50 + \frac{100}{3}j = 37,27 \angle 116,57^\circ V$$

Straža od φ_1 do \perp je:

$$I = \frac{\varphi_1}{3+4j} = \underline{3,33 + 6,67j}$$

XI. 1-P5

U = 1 V

Z = 1 + 1j Ω

$0,3 - 0,15j$

$0,3 - 0,15j$

$0,6 + 0,3j$

A

$\frac{1}{3+2j}$

$0,3 - 0,15j$

$0,3 - 0,15j + 1j$

$0,16 - 0,3j$

V

69

$$U_{AB} = ?$$

$$f_0 = 0$$

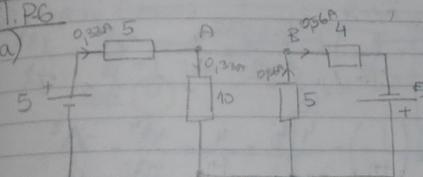
$$f_A = 0 + 10 - (0,6 - 0,3j)(1,33 + 0,66j) =$$

$$= 0 + 10 - 1 = 9 V$$

$$U_{AB} = 9 V$$

XI. P-6

a)



$$R_{AB} = R_T = \frac{5 \cdot 10}{15} = \frac{20}{5} = 5 \Omega \quad 5,556 \Omega$$

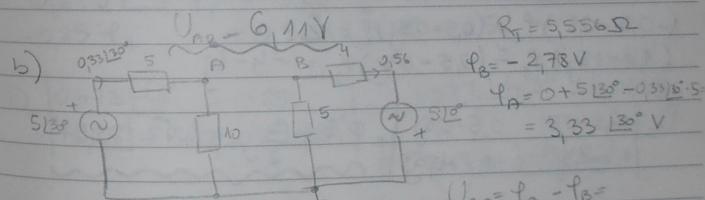
$$\varphi_B - 0,56 \cdot 4 + 5 = 0$$

$$\varphi_B = -2,76 V$$

$$0 + 5 - 0,33 \cdot 5 = \varphi_A$$

$$\varphi_A = 3,33 V$$

b)



$$R_T = 5,556 \Omega$$

$$\varphi_B = -2,78 V$$

$$\varphi_A = 0 + 5 \angle 0^\circ - 0,33 \angle 0^\circ \cdot 5 = 3,33 \angle 30^\circ V$$

$$U_{AB} = \varphi_A - \varphi_B = -4,67 \angle 30^\circ + 2,76$$

XI. 1-4.

$$\varphi_3 = 4 - j4 \text{ V}$$

$$-\varphi_1 \left(\frac{1}{Z_1} \right) + \varphi_2 \left(\frac{1}{Z_1} + \frac{1}{Z_0} + \frac{1}{Z_2} \right) - \varphi_3 \frac{1}{Z_2} = \frac{U_1}{Z_1} - \frac{U_0 + U_2}{Z_0 + Z_2}$$

$$\varphi_1 \left(\frac{1}{Z_1} + \frac{1}{Z_0} + \frac{1}{Z_2} \right) - \varphi_2 \left(\frac{1}{Z_1} \right) - \varphi_3 \frac{1}{Z_2} = \frac{U_1}{Z_1} + \frac{U_2}{Z_0} - \frac{U_0}{Z_2}$$

$$\left\{ \begin{array}{l} \varphi_1 - \frac{1}{1+j} + \varphi_2 \left(\frac{1}{1+j} + \frac{1}{j^2} + \frac{1}{-j^2} \right) - \frac{4-jj}{j^2} = \frac{4}{1+j} - \frac{-jj^2}{j^2} + \frac{j8^2}{j^2} \\ \varphi_1 \left(\frac{1}{1+j} + \frac{1}{j^2} + \frac{1}{-j^2} \right) - \varphi_2 \left(\frac{1}{1+j} \right) - \frac{4-jj}{j^2} = \frac{4}{1+j} + \frac{j8^2}{j^2} - \frac{-jj^2}{j^2} \end{array} \right.$$

$$\frac{1}{\sqrt{2}} \underline{135^\circ} \varphi_1 + \varphi_2 \frac{1}{\sqrt{2}} \underline{-45^\circ} = 2\sqrt{2} \underline{45^\circ} + 2\sqrt{2} \underline{-5^\circ} - 2$$

$$2,12|3| \underline{-45^\circ} \varphi_1 - \varphi_2 \underline{0,707} \underline{-45^\circ} = -4 - j4 + 2 - 2j - 4 + 2$$

$$(-0,5 - 0,5j)\varphi_1 + (0,5 - 0,5j)\varphi_2 = 2$$

$$(15 - 15j)\varphi_1 - (0,5 - 0,5j)\varphi_2 = -4 - 6j$$

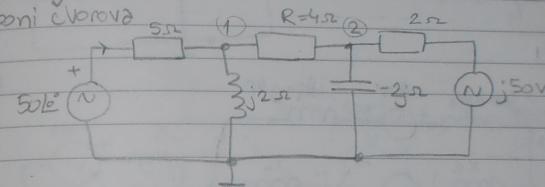
$$D = \begin{vmatrix} -0,5 - 0,5j & 0,5 - 0,5j \\ 15 - 15j & -0,5 + 0,5j \end{vmatrix} = 0,5 + \frac{3}{2}j = 10\sqrt{2} \underline{71,57^\circ}$$

$$D_{11} = \begin{vmatrix} 2 & 0,5 - 0,5j \\ -4 - 6j & -0,5 + 0,5j \end{vmatrix} = -1 + i - (-5 - i) = 4 + 2i$$

$$\varphi_1 = \frac{D_{11}}{D} = \underline{2 - 2j}$$

XI. 1-3

a) Naponi článková



$$\varphi_1 \left(\frac{1}{5} + \frac{1}{j^2} \right) - \varphi_2 \left(\frac{1}{4} \right) = \frac{50}{5}$$

$$-\varphi_1 \frac{1}{4} + \varphi_2 \left(\frac{1}{4} + \frac{1}{j^2} + \frac{1}{2} \right) = \frac{150}{2}$$

$$0,673 \underline{-48^\circ} \varphi_1 - 0,25 \varphi_2 = 10$$

$$-0,25 \varphi_1 + 0,803 \underline{33,69^\circ} = 25j$$

$$D = \begin{vmatrix} 0,673 \underline{-48^\circ} & -0,25 \\ -0,25 & 0,803 \underline{33,69^\circ} \end{vmatrix} = 0,6077 \underline{14,31^\circ} - 0,0625 = 0,5474 \underline{-15,927^\circ}$$

$$D_{11} = \begin{vmatrix} 10 & -0,25 \\ 25 & 0,903 \underline{33,69^\circ} \end{vmatrix} = 9,03 \underline{33,69^\circ} + 6,25 = 14,65 \underline{20^\circ}$$

$$\varphi_1 = \frac{D_{11}}{D} = \underline{26,7623 \underline{35,922^\circ}}$$

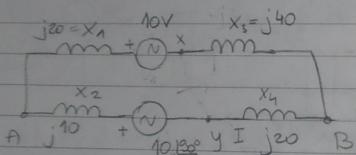
$$D_2 = \begin{vmatrix} 0,6731 & -48^\circ & 10 \\ 0,25 & 25 \end{vmatrix} = 16,825 \angle 42^\circ - 2,5 = 15,06 \angle 48,377^\circ$$

$$\varphi_2 = \frac{15,06 \angle 48,377^\circ}{0,5474 \angle -15,972^\circ} = 27,51 \angle 64,35^\circ V$$

$$U_{AB} = \varphi_1 - \varphi_2 = 26,7629 \angle 35,927^\circ - 27,51 \angle 64,35^\circ = 13,344 \angle -42,98^\circ$$

$$I = \frac{U_{AB}}{4} = 3,336 \angle -42,98^\circ A$$

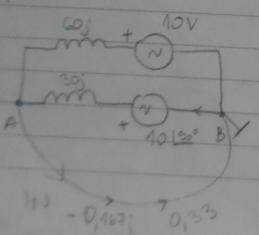
XI.1-4



- možní spoj může se X_C zanemazat "ist gleich"

$$R_F = \frac{20}{\frac{60}{3} + \frac{30}{3}} = 20j$$

Nort.:



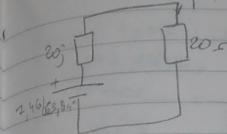
$$\varphi_A \left(\frac{1}{30j} + \frac{1}{60j} \right) = 0 = \frac{10}{60j} + \frac{10j20}{30j}$$

$$-0,05j \quad \varphi_A = -0,167j + 0,333$$

$$\varphi_A = 3,34 + 6,667j$$

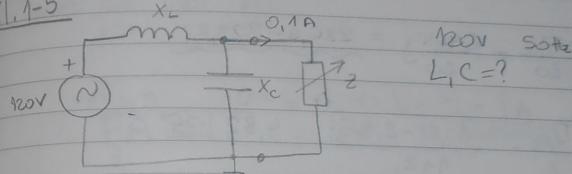
$$= 7,46 \angle 63,389^\circ V = U_{AB}$$

$$I_N = 0,336385 \angle -26,84^\circ A$$

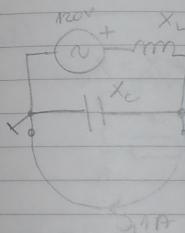


$$I = \frac{7,46 \angle 63,389^\circ}{20+2j} = 0,261 \angle 12.8^\circ$$

XI.1-5



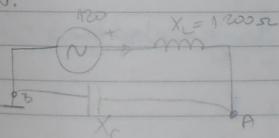
Nort.



$$I_N = \frac{120}{X_L} \Rightarrow X_L = \frac{120}{0,1} = 1200 \Omega$$

$$X_L = 2\pi f L \Rightarrow L = 3,82 H$$

Theu.



Resonancia

$$X_C = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_L} = \frac{1}{2\pi f \cdot 1200} F$$

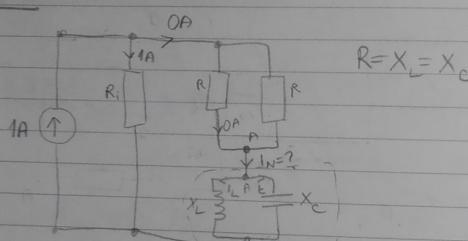
XI.1-7

$$\frac{U_{AB}}{Z_1} = \frac{\frac{U_1 + U_2}{Z_1 + Z_2}}{1+j} = \frac{\frac{10 + 5-8j}{1-j}}{100^{-20}} = \frac{10 + 5-8j}{1-j} = \frac{100^{-20}}{6-2j} =$$

$$= \frac{5732}{20} - \frac{6464}{20}j = \underline{286,6 - 323,2j}$$

$$I_{Z_2} = \frac{U_{AB}}{Z_2} = \frac{10,58 - 2,165j}{1+2j} = 4,83 \angle -75^\circ A$$

XI.1-5



$Z_{AS} = \frac{X_L + X_C - j\omega}{X_L - X_C}$ kao da je odspojeno

te sva struja teče kroz R_i :

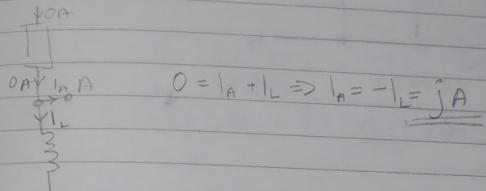
$$U_c = U_g = U_{R_i}$$

Uree. kao da je odspojeno i zato je pad naponata jednako U_{R_i}

prema tome, ako su naponi jednaki onda je struja jednaka samo je fazi kroz drugaciji

$$I_a = -jA \quad I_b = jA$$

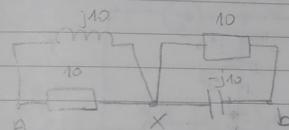
Jedn. 22 čvor A:



$$0 = I_a + I_L \Rightarrow I_a = -I_L = \underline{jA}$$

XI.1-10)

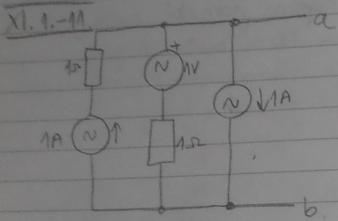
$$R_f = 10 \Omega$$



$$5+5j+5-5j = \underline{10 \Omega}$$

$$U_f = 0V \leftarrow \text{mosni spoj}$$

mosni spoj
mosni spoj
mosni spoj

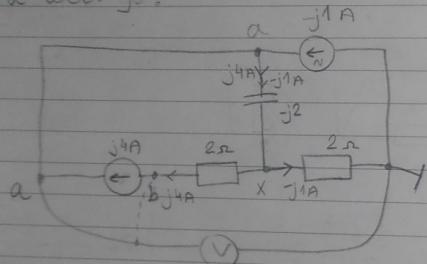


$$Z_T = 1 \Omega$$

$$U_T = U_{AB} = \frac{1+1-1}{0+1+0} = 1 V$$

XI. 1-12

u točki Y:



$$f_x - (-j \cdot 2) = 0$$

$$f_x = -2j V$$

$$f_x - (j4A \cdot 2\Omega) = 0$$

Ne zna se nepon izvora
(stavljenog) te se mora "dugim
putem" doći do Φ_A

$$-2j - 8j = f_B \Rightarrow f_B = -10j \text{ mjeri } 10V$$

$$\begin{aligned} f_x + 2j &= 0 \\ f_A - 3j(-2j) &= f_x \end{aligned}$$

$$\Phi_A = 6 - 2j = 2\sqrt{10} \angle 184.3^\circ \text{ mjeri: } 2\sqrt{10}$$

XI. 1-13

$$Z_T = (15 + 5j)(5 + 5j) = \frac{50 + 100j}{20 + 10j} = \frac{4 + 3j}{2} = 5 \angle 36.87^\circ$$

$$U_{AB} = Z_T \cdot 1 = \frac{25}{2} + 22.5j = \underline{25 / 66.87^\circ V} = -f_x$$

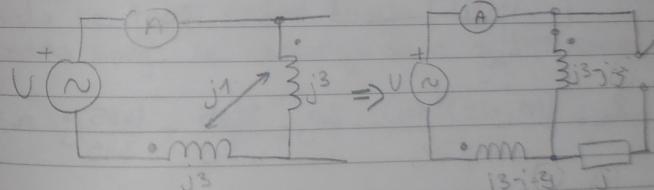
Strujno djelilo:

$$f_x = 1 \cdot \frac{5 + 5j}{20 + 10j} = 1.05 + 1.18j = \frac{10}{\sqrt{2}} \angle 68.44^\circ = 1.58 \angle 68.44^\circ$$

Naponsko djelilo

$$U_{AB} = U_{x_5} \cdot \frac{5 + 5j}{15 + 5j} = \underline{11.18 \angle 33.44^\circ}$$

XI. 1-16



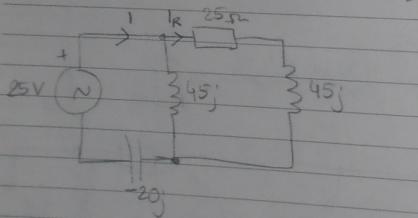
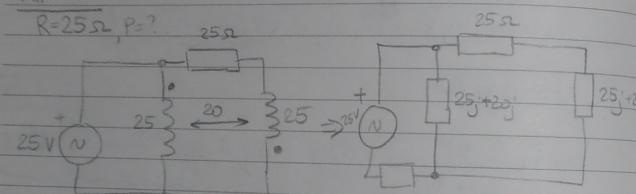
Kad su obje točkice dajte od čvora ili obje blize čvoru onde se ne nadomj. shemi od tih istih imped. oduzima međuinduktivitet, ales npr. jedna je blize, a druga daje onde se ne njih dodaje među onoj "dodatnoj" je uviska suprotno!

$$I_{A_{12}} = \frac{U}{Z} = \frac{1}{\frac{-8}{3j}} = \frac{1}{\frac{-8}{3j}} = \frac{1}{\frac{8}{3j}} = \frac{2}{3} V$$

$$2j + \frac{-2}{3j} = \frac{-6-2}{3j} = \frac{-8}{3j}$$

XI.1-17

$$R = 25 \Omega, P = ?$$



$$Z_{AB} = \frac{(45j+25)(45j)}{25+50j} - 20j = \frac{2025}{349} + \frac{1435}{349} j \Omega =$$

$$= 5,8 + 4,11j \Omega = 7,11 | 35,32^\circ \Omega$$

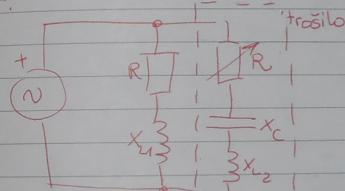
$$I = \frac{25}{7,11 | 35,32^\circ} = \frac{2025}{706} - \frac{1435}{706} j = 2,87 - 2,03j = 3,51 | -25,32^\circ$$

$$I_R = I \cdot \frac{45j}{25+50j} = \frac{1125}{706} - \frac{405}{706} j = 1,59 - 0,57j = 1,69 | -13,8^\circ$$

$$P_R = I^2 R = 55,25 - 45,7j = 71,707 | -39,6^\circ W$$

Q) radna
Maksimalna snaga:

- algebarski zbroj svih impedancija osim "trežene"
npr.



$$R = \sqrt{R^2 + (X_{L1} + X_{L2} - X_C)^2}$$

- kada se traži radna snaga ???

- radna snaga se ostvaruje samo na OTORNIKU!

Q)

Polifazni sustavi

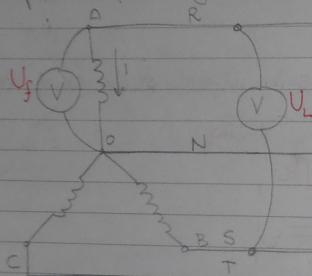
simetričan

- jednaka amplituda i frekvencija, ali pomak u fazi (120°)

Pojmovi:

FAZNI NAPON

- napon 1 faze generatora tj. napon na 1 fazni točki



FAZNA STRUJA

- struja koja prolazi kroz narmot jedne faze generatora

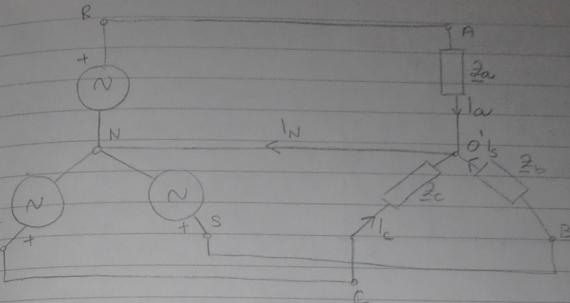
- struja koja prolazi kroz otpor jedne faze na trošila

LINIJSKI NAPON (medufazni napon)

- napon izm 2 vrsta trofazne linije, različitih pola

LINIJSKA STRUJA

- prolazi jednim vodičem trofazne linije



$U_{TR}, U_{RS}, U_{ST} \rightarrow$ linijski naponi

$U_{RN}, U_{SN}, U_{TN} \rightarrow$ fazni naponi

Linijska struja između točaka Ra, Tc, Sb

$$U_L = ?$$

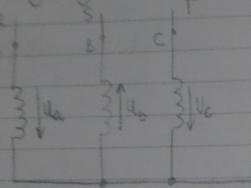
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$U_L = \sqrt{3} \cdot U_f$$

$$U_L = \sqrt{3} U_f \quad I_L = I_f$$

$$U_f = 220 \text{ V} \Rightarrow U_L = 370 \text{ V}$$

-ako je jedan svitak drugacije spojen

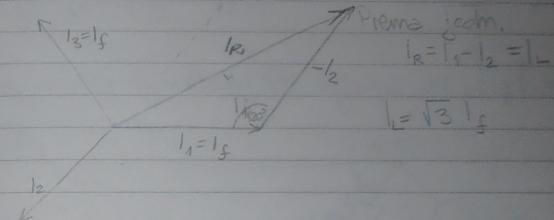


$$I_R = I_a - I_c = I_1 - I_2$$

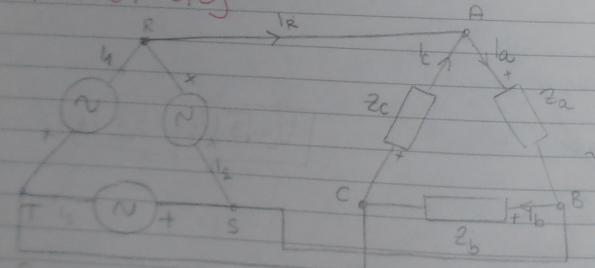
sa str. trošila
sa str. generatora

-prijenos energije vrši se samo trifaznim linijom RST

$$I_L = \sqrt{3} I_f \quad U_L = U_f$$

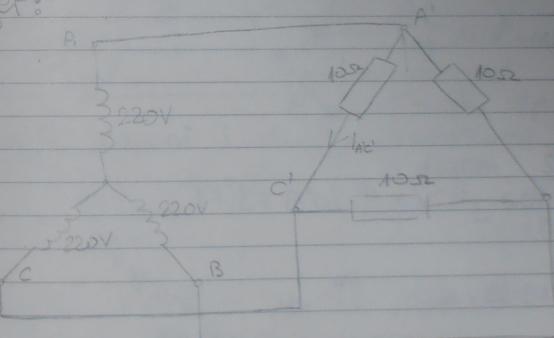


TROKUT SPOJ



-fazni napisi trošila jednaki su linijskim napisima trošila

Prijer:



Fazni napon generatora jednake je

$$U_L = \sqrt{3} U_f$$

$$U_L = \sqrt{3} \cdot 220 = \underline{\underline{380 \text{ V}}}$$

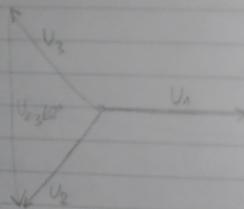
$$I_L = I_{AC} = \frac{U_L}{R_L} = 38 \text{ A} \quad \leftarrow \text{linijska struja}$$

$$I_L = I_f \quad u \text{ spajju } 2 \text{ vijezda}$$

XII 1-P1

$$U_1 = ? \text{ kut?}$$

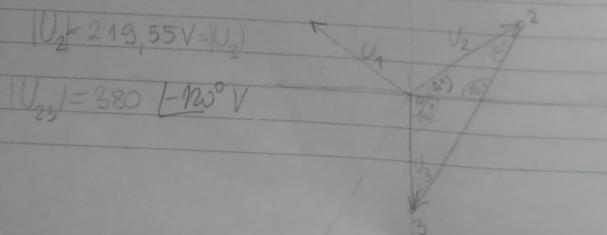
U_1 ima kut 90°



XII 1-P2

$$U_2 = 190 + j110 \text{ V} \Rightarrow \varphi = 30^\circ$$

$$U_{23} = ?$$



$$|U_2 - 219,55 \text{ V} - U_3|$$

$$|U_{23}| = 380 \text{ } \underline{-120^\circ \text{ V}}$$

XIII 1-P3

$$U_{12} = 190 + j320 \text{ V} \Rightarrow \varphi = 60^\circ$$

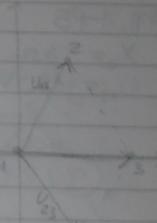
$$U_{23} = 190 - j320 \text{ V} \Rightarrow \varphi = -60^\circ$$

$$U_{31} = ?$$

$$|U_{12}| = 380,8 \text{ V}$$

$$U_{12} = U_{23} + U_{31} = 0$$

$$U_{31} = -380 = 380 \text{ } \underline{+180^\circ \text{ V}}$$



XII 1-P4



- svaki četvrtnik
- linijski napon i struja
- smatramo jednom fazom

$$I_f = \frac{U_f}{Z_{eq}} = \frac{220}{22 \text{ sL}} = 17,27 \text{ A}$$

- linijska je struja za $\sqrt{3} \cdot 10\%$ od fazne
u prekut spajalj

$$I_L = \sqrt{3} \cdot I_f = 30 \text{ A}$$

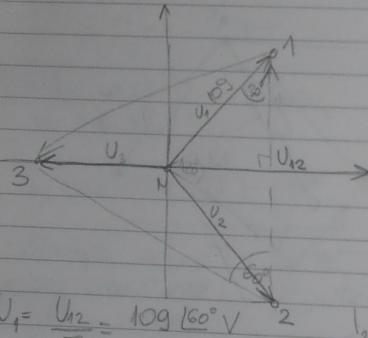
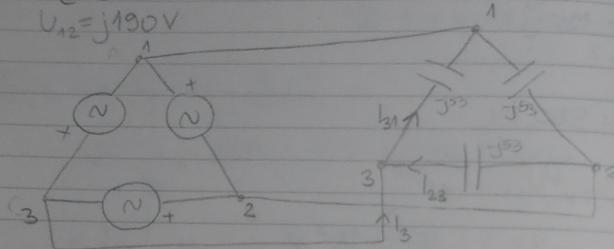
XII. 1-P5

$$X_c = 53 \Omega$$

$$I_{23} = ?$$

$$I_3 = ?$$

$$U_{12} = j190V$$



$$U_1 = \frac{U_{12}}{\sqrt{3}} = 109 \angle 60^\circ V$$

$$I_{23} = \frac{U_{23}}{X_c} = 3,59 \angle 60^\circ A$$

$$U_2 = 109 \angle -60^\circ V$$

$$I_3 = ?$$

$$U_{23} = 130 \angle -30^\circ V$$

$$U_{31} = 130 \angle 150^\circ V$$

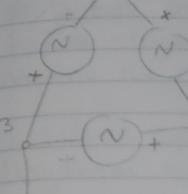
$$I_{31} = \frac{130 \angle -150^\circ}{X_c} = 3,58 \angle -60^\circ A$$

$$I_3 = I_{31} - I_{23}$$

$$I_3 = 6,21 \angle -30^\circ A$$

XII. 1-P6

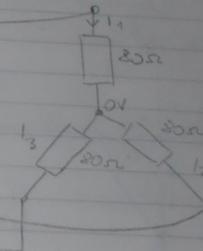
$$\varphi_1 = 220 \angle 0^\circ V$$



$$U_{12} = 380 \Rightarrow U_1 = 220 \angle 0^\circ V = \varphi_1$$

$$U_2 = 220 \angle 120^\circ V$$

$$U_3 = 220 \angle 240^\circ V$$

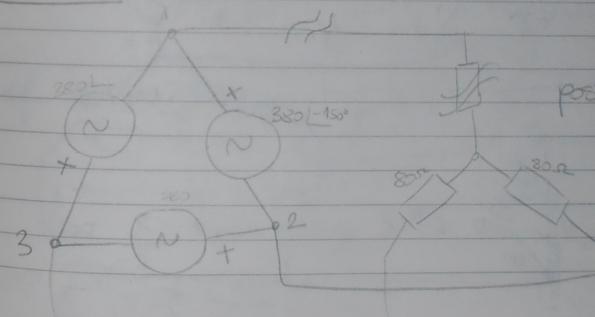


$$I_1 = \frac{220}{8\Omega} = 2,75 \angle 0^\circ A$$

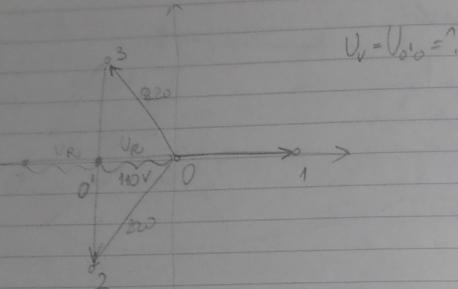
$$I_2 = \frac{220 \angle 120^\circ}{8\Omega} = 2,75 \angle 120^\circ A$$

$$I_3 = 2,75 \angle 240^\circ A$$

XII. 1-P7



postaje nesim.



$$U_V = U_{R1} = ?$$

XII. 1 - P8

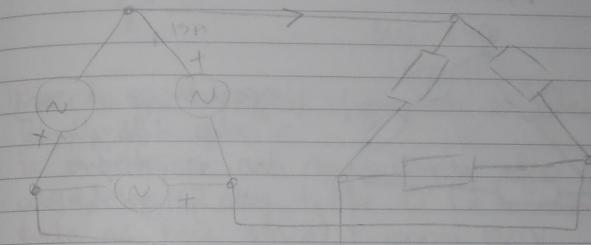
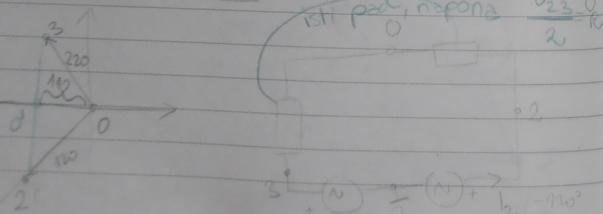
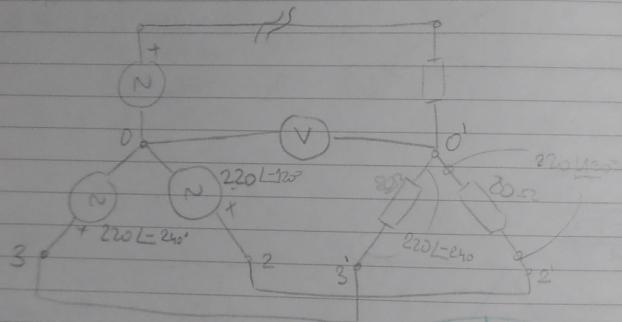
- struja kroz otpor trošila
fazna struja \rightarrow ovisi o
naponu na otporniku



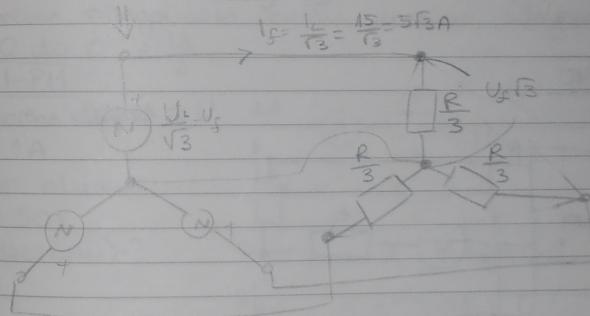
- u toku spoja trošila je taj napon jednako
linijskom naponu izvora

Moj način:

- transformirati izvor u spoj A



$$I_S = \frac{U_1}{R_1} = \frac{15}{\sqrt{3}} = 5\sqrt{3} A$$

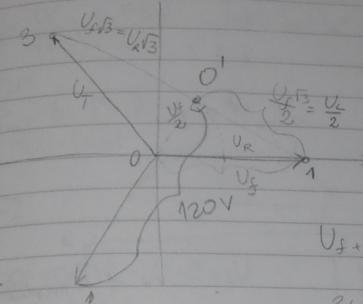
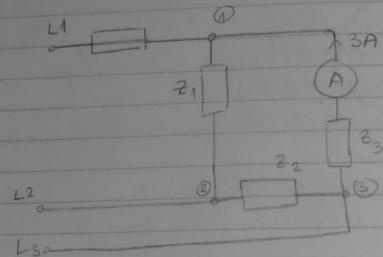


Kroz \underline{R} teče struja $5\sqrt{3} A$ na \underline{R} je napon $\frac{U_12}{\sqrt{3}}$

XII. 1-P9

$$I_A = 3A$$

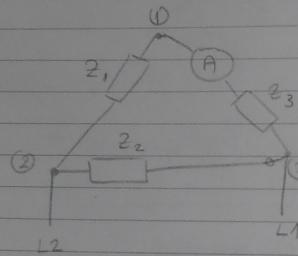
$$I_A = 2$$



$$U_L f + \frac{U_L f}{2} = 120 \quad U_E = 80\sqrt{3} V$$

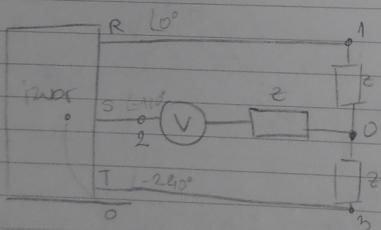
$$3U_f = 240$$

$$U_f = 80V$$



Impedancija u seriji
s amper. dodjiva pola
lin. napona \Rightarrow
polu struje $1.5A$

XII. 1-P10



$$U_V = 120V$$

XII. 1-P11

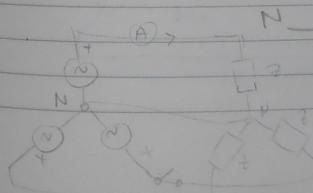
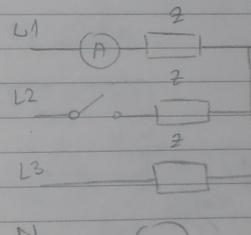
Zatvorenje sklopke:

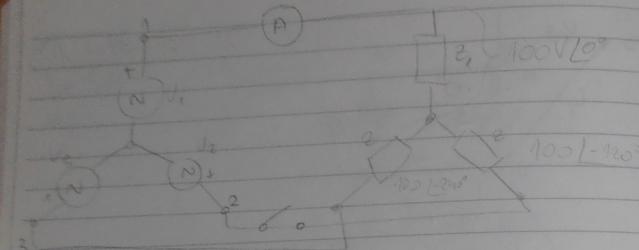
$$I_A = 1A$$

Voltm. otvorena

$$U_V = 50V$$

U_V (zatvorena) = ?





$$a) U_f = \frac{U_L}{\sqrt{3}} = 6,93 V$$

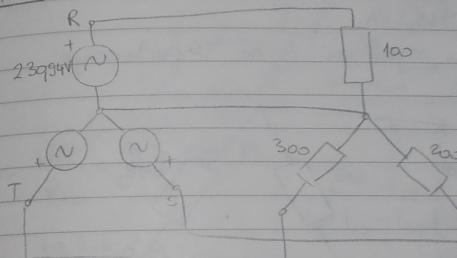
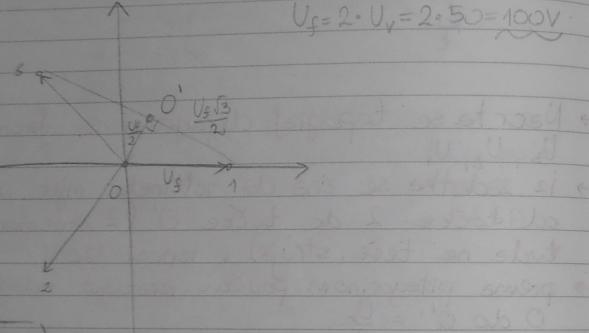
$$b) I_f = I_L = \frac{U_f}{Z} = \frac{6,93}{100} = 69,3 \text{ mA}$$

$$d) OA$$

$$e) P_{me} = 3 \cdot 1^2 \cdot 2 = 1,44 W$$

XII.1-2

$$U_f = 2 \cdot U_v = 2 \cdot 50 = 100 V$$



$$U_E = 400 V \Rightarrow$$

$$U_f = 230,94 V$$

$$I_{Rf} = \frac{230,94}{100} = 2,310^\circ A$$

$$I_{Ls} = \frac{230,94 \angle -120^\circ}{200} = 1,15 \angle -120^\circ A$$

$$I_{LT} = \frac{230,94 \angle -240^\circ}{300} = 0,77 \angle -240^\circ A$$

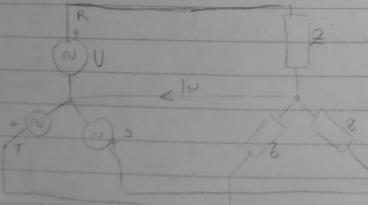
$$I_N = I_{Rf} + I_{Ls} + I_{LT} = 1,38 \angle -13,8^\circ A$$

$$P_{UE} = U_f^2 \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{300} \right) = \frac{230,94^2 \cdot 11}{600} = 977,78 W$$

XII.1-1

$$U_f = 12 V$$

$$Z = 100 \angle 0^\circ \Omega$$



$$a) U_f = ?$$

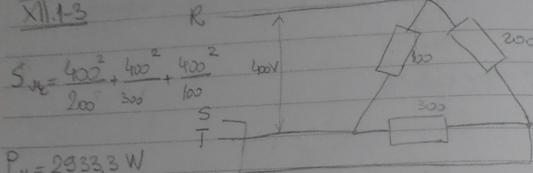
$$b) I_f = ?$$

$$c) I = ?$$

$$d) I_r = ?$$

$$e) P_e = ?$$

XII.1.3

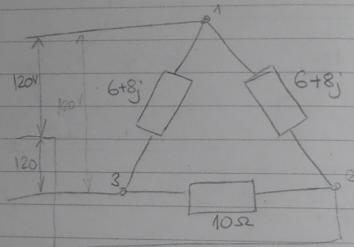


$$P_{UE} = 2333,3 \text{ W}$$

$$S_{UE} = \sqrt{P_{UE}^2 + Q_{UE}^2} = 2333,3 \text{ VA}$$

$Q_{UE} = 0 \text{ VA}$ \leftarrow jer su otpornici, a ne impedancije

XII.1.4



$$U_L = 120 \text{ V}$$

$$\text{a)} I_{12}, I_{23}, I_{31} = ?$$

$$\text{b)} P, Q = ?$$

$$U_{L12} = 120 \angle 0^\circ \text{ V}$$

$$U_{L23} = 120 \angle -120^\circ \text{ V}$$

$$U_{L31} = 120 \angle 240^\circ \text{ V}$$

$$I_{12} = \frac{120 \angle 0^\circ}{6+8j} = 7,2 - 9,6j = 12 \angle -53,13^\circ \text{ A}$$

$$I_{23} = \frac{120 \angle -120^\circ}{6+8j} = 12 \angle -173,18^\circ \text{ A}$$

$$I_{31} = \frac{120 \angle 240^\circ}{10} = 12 \angle -240^\circ \text{ A}$$

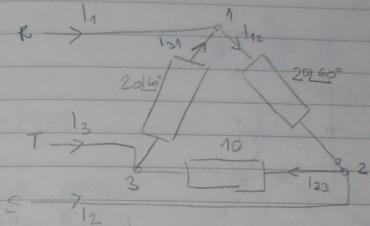
$$S_{UE} = U_L^2 \left(\frac{1}{Z_{12}} + \frac{1}{Z_{23}} + \frac{1}{Z_{31}} \right) = \frac{120^2}{3167} \angle 36,03^\circ = \underline{\underline{3162 - 2304j}}$$

$$P_{UE} \quad Q_{UE}$$

XII.1.5

$$U_{12} = 120 \angle 0^\circ \text{ V}$$

$$I_{11}, I_{21}, I_{31} = ?$$



$$I_{12} = \frac{U_{12}}{Z_{12}} = 9,51 \angle 60^\circ \text{ A}$$

$$I_{23} = 12 \angle -120^\circ \text{ A}$$

$$I_{31} = 9,51 \angle 60^\circ \text{ A}$$

$$I_1 = I_{12} - I_{31} = 16,45 \angle 30^\circ \text{ A}$$

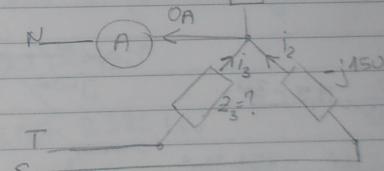
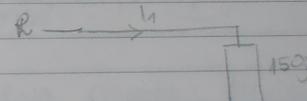
$$I_2 = I_{23} - I_{12} = 16,45 \angle -150^\circ \text{ A}$$

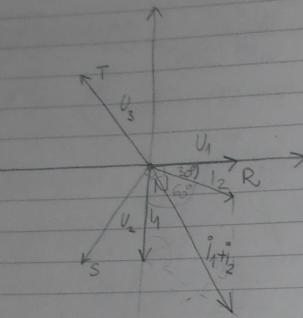
$$I_3 = I_{31} - I_{23} = 28,5 \angle 60^\circ \text{ A}$$

XII.1.6

$$I_1 + I_2 + I_3 = 0$$

$$\frac{U_F}{150} + \frac{U_S}{-j150} + \frac{U_T}{23} = 0$$





$$I_1 + I_2 = \sqrt{3} I_1 = \sqrt{3} I_2$$

- iz slike se može zaključiti da bi ukupna struja bila OA treća struja (I_3) mora biti u fazici napona $U_3 \Rightarrow$ radni otpor

- struja $I_3 = \sqrt{3} I_1 \Rightarrow Z_3 = R_3 = \frac{21}{\sqrt{3}}$ (je putove veće struje, ne je konstanta, \rightarrow otpor je manji)

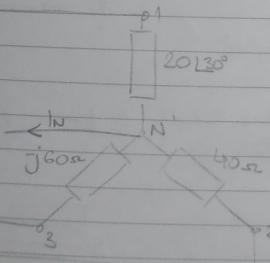
XII.1-7

$$U_f = 230V$$

$$I_s = ?$$

$$I_s = \frac{230}{20 \angle 30^\circ} = 11,5 \angle -30^\circ$$

$$I_2 = \frac{230 \angle -120^\circ}{40} = 5,75 \angle -120^\circ$$



$$I_3 = \frac{230 \angle -240^\circ}{-j60} = 3,83 \angle -30^\circ A$$

$$I_N = I_1 + I_2 + I_3 = 16,38 \angle -59,56^\circ A$$

XII.1-8

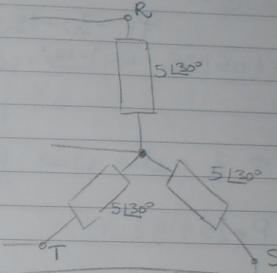
$$U_f = 380V \Rightarrow U_f = 220V$$

a) amperiar mjeri snagu između točaka R i T

$$P_R = 220 \angle 10^\circ$$

$$P_T = 220 \angle 240^\circ$$

$$U_{RT} = P_R - P_T = 381,05 \angle -30^\circ V$$



struja kroz strujne sterejlke je:

$$I_{fs} = \frac{U_{fs}}{5 \angle 30^\circ} = \frac{220 \angle -120^\circ}{5 \angle 30^\circ} = 44 \angle -150^\circ = 44 \angle 210^\circ$$

Razlika kutova naponi i struje 3

$$-30 - (-150) = 120^\circ$$

$$P_1 = U_{Rf} \cdot I_s \cdot \cos 120^\circ = 8323,1 W$$

$$b) I_s = 44 \angle -150^\circ A$$

$$U_{ST} = \varphi_s - \varphi_f = 220 \angle -120^\circ - 220 \angle -240^\circ = 381,05 \angle 90^\circ$$

$$\varphi_s = 220 \angle -120^\circ$$

$$\varphi = -90 + 150 = 60^\circ$$

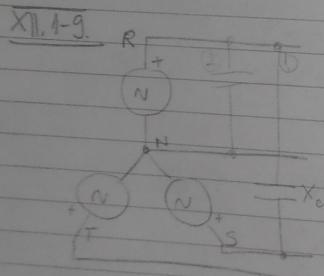
$$P = 381,05 \angle 90^\circ \cdot 44 \angle -150^\circ \cos 60 = 8383,1 \text{ W}$$

c) $P_{vt} \rightarrow 3$ puta vecia jer je trošilo simetrično

$$P_{vt} = 3 \cdot P = 25149,3 \text{ W}$$

$$S = U \cdot I =$$

$$Q = S - P_{vt} = 14519,8 \text{ VAR}$$



$$\Delta I = 7,59 \text{ A}$$

$$U_L = ?$$

$$X_c = \frac{1}{31,450 \cdot 10^{-6}} = 21,23 \Omega$$

$$|f - I| = 7,59 \text{ A}$$

$$U_L = \sqrt{3} U_f$$

$$I_1 = \frac{U_L}{X_C}$$

$$I_2 = \frac{U_f}{X_C}$$

$$I_1 = I_2 + \Delta I$$

$$\frac{U_L}{X_C} = \frac{U_f}{X_C} + \Delta I \quad | \cdot X_C$$

$$U_L = U_f + X_C \Delta I$$

$$U_L = \frac{U_f}{\sqrt{3}} + \Delta I \cdot X_C$$

$$U_L = 381 \text{ V}$$

XII.1-10

$$U_L = 100 \text{ V} \Rightarrow U_S = 57,74 \text{ V}$$

$$R_1 = R_2 = R_3 = X = Y = 10 \Omega$$

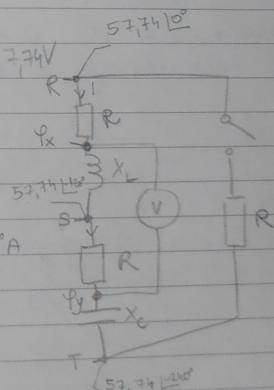
Pojed zatv.v.:

$$U_{RS} = 100 \angle 30^\circ \text{ V}$$

$$I_{RS} = 100 \angle 30^\circ = 70,71 \angle -15^\circ \text{ A}$$

$$10 + 10$$

$$\varphi_x = 21,12 \angle 120^\circ$$



$$U_s = 57,74 | -120^\circ - 57,74 | -240^\circ = 100 | -90^\circ$$

$$I_{\text{sum}} = 100 | -120^\circ \rightarrow 7,07 | -45^\circ \text{ A}$$

$$\dot{\varphi}_x = 57,74 | -120^\circ - (7,07 | -45^\circ \cdot 10) = 78,86 | -180^\circ \text{ V}$$

$$U_v = \dot{\varphi}_x - \dot{\varphi}_y = 70,7 | 15^\circ \text{ V}$$

Snage:

$$P_1 = \frac{1}{b} \cdot R = 499,85 \text{ W}$$

$$P_2 = \frac{1}{b} \cdot R = 499,85 \text{ W} \quad P_{\text{ve}} = P_1 + P_2 = 1000 \text{ W}$$

$Q = 0 \text{ VAR}$ jer je spoj u rezonanciji

Kad je se sklopka zatvori pokazivanje je: olaye jednako, ali rečna snaga se 2 puta poveća

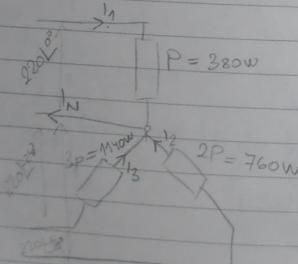
XI.1-11

$$\cos \varphi = 1$$

$$U_L = 580 \text{ V} > U_s = 220 \text{ V}$$

$$P = 380 \text{ W}$$

$$I = ?$$



$$I_1 = \frac{P_1}{U_s} = \frac{380}{220} = 1,727 \text{ A}$$

$$I_2 = \frac{760}{220 | -120^\circ} = 3,455 | 120^\circ \text{ A}$$

$$I_2 = 5,18 | 240^\circ \text{ A}$$

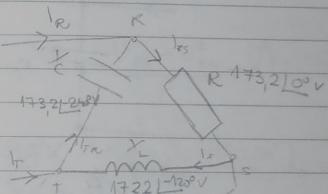
$$I_N = I_1 + I_2 + I_3 = 3 | -150^\circ$$

XII.1-12

a) $U_f = 100 \text{ V}$

$U_L = 173,2 \text{ V}$

$10 \Omega = R = X_c = X_L$

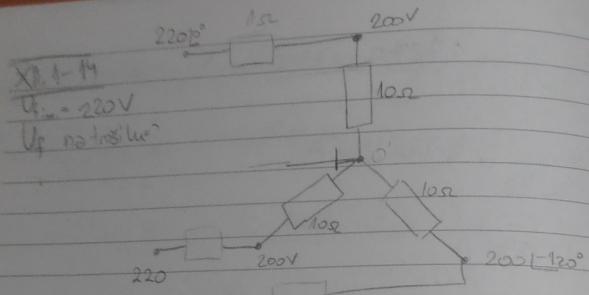


$$I_{\text{RS}} = \frac{173,2}{10} = 17,32 \text{ A} \Rightarrow P_{\text{ve}} = 3000 \text{ W}$$

- islova snaga = 0 VAR, ali za proujere

$$Q_1 = (173,2 | -120^\circ)^2 \cdot 10 | 120^\circ = 299982,4 | -150^\circ \text{ VAR}$$

$$Q_2 = (173,2 | -240^\circ)^2 \cdot 10 | -90^\circ = 299982,4 | 150^\circ \text{ VAR}$$



$$U_{fT} = U_f \cdot \frac{10}{11} = \underline{\underline{200V}}$$

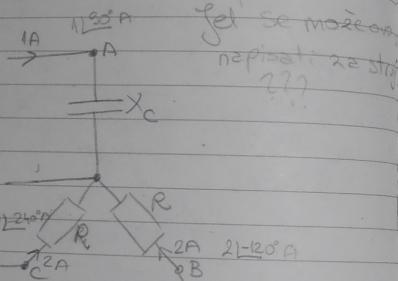
XII.1-15)

$I_{A1} = 1A$

$I_{B1} = 2A$

$P = 690W$

$R?$



$$\text{ba} - \frac{U_f}{X_C} = 1 \Rightarrow U_f = X_C \cdot 1$$

$$\text{ba} - \frac{U_f}{R} = 2 \Rightarrow U_f = R \cdot 2$$

$$X_C = 2R$$

$$U_{0,0} = U_f \cdot \left(\frac{1}{j^2} + 1 - 120^\circ + 1 \right) \underline{\underline{U_f \cdot (-0,412 + j0,353)}}$$

Naponi na otpornicima

$$U_{RB} = U_f \cdot 1 - 120^\circ - U_{0,0} = U_f \left(1 - 120^\circ + 0,412 - j0,353 \right) = \underline{\underline{1,22 \cdot 1 - 59,13^\circ V}} \quad U_f$$

$$U_{RC} = U_f \cdot 1 - 240^\circ - U_{0,0} = U_f \underline{\underline{0,52 \cdot 188,73^\circ V}}$$

$$P = \frac{U_{RB}^2}{R} + \frac{U_{RC}^2}{R} \Rightarrow R = \frac{U_{RB}^2 + U_{RC}^2}{P} = \frac{U_f^2 \cdot (1,22 \cdot 1 - 59,13^\circ)^2 + U_f^2 \cdot (0,52 \cdot 188,73^\circ)^2}{690}$$

$$R = \frac{U_f^2 \cdot (1,4884 \cdot 171,74^\circ + 0,2704 \cdot 160,59^\circ)}{690} =$$

$$R = \frac{1,73 \cdot 175,9 \cdot U_f^2}{690}$$

$$1 \quad \underline{\underline{U_f = 2,51 \cdot 15^3 \cdot 175,9 \cdot U_f^4}}$$

$$U_f = \underline{\underline{196,85 V}}$$

Pada se prelaze nul vod (millmanov teorem)

$$U_{0,0} = \frac{U_f + U_f \cdot 1 - 120^\circ + U_f \cdot 1 - 240^\circ}{j2R} = \frac{U_f}{j2R} + \frac{U_f \cdot 1 - 120^\circ}{R} + \frac{U_f \cdot 1 - 240^\circ}{R} = \frac{1}{j2R} + \frac{1}{R} + \frac{1}{R}$$

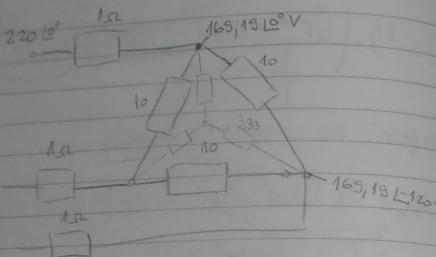
$$R = \frac{U_f^2 \cdot 1,754}{690} = \underline{\underline{98 \Omega}}$$

XII.1-14

$$U_f = 220 \text{ V}$$

$$U_L = 380 \text{ V}$$

$$U_B = 293,05 \text{ V}$$



XII.1-16

$$5 \angle 30^\circ + 5 \angle 240^\circ + 5 \angle 120^\circ =$$

$$I_N = -5 + 5j \text{ A}$$

Uz struju se zaključuje da je $Y_c = R = R_s$



$$U_{B_0} = \frac{U_f}{R} + \frac{U_f \angle -120^\circ}{R} + \frac{U_f \angle 240^\circ}{R} = U_f \left(\frac{-1}{R} + \frac{1 \angle -120^\circ + 1 \angle 240^\circ}{R} \right) =$$

$$\frac{1}{jX} + \frac{1}{R} + \frac{1}{jX}$$

$$= U_f \left(\frac{-1+1}{2+j} \right) = U_f (0,2 + 0,6j) = 0,63 |108,4| \cdot U_f$$

Prednosti

- ista snaga s manjim brojem polova
- pojenos snage koja je u imenu konstantna (ft) konstanta
- stvarne rotac. mag. pojeve

SNAGA

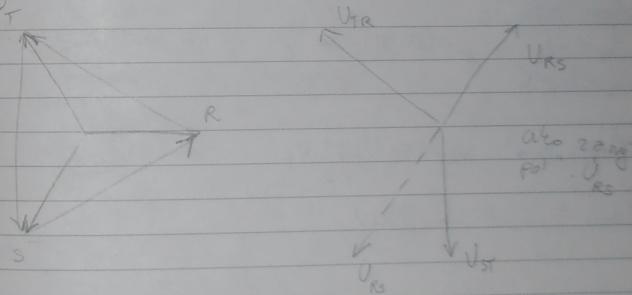
$$P_{UB} = \sum_{i=1}^3 P_i = \sum_{i=1}^3 U_i I_i \cos \phi_i$$

formula za trosita

$$Q_{UB} = \sum_{i=1}^3 Q_i = \sum_{i=1}^3 U_i I_i \sin \phi$$

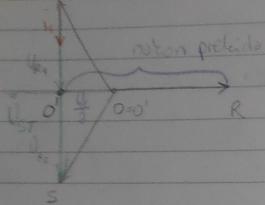
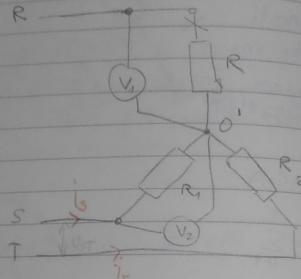
- trenutna snaga je konstantna

Prijava - se okreuti izvor



Zadaci (niz tava)

za voltmetri
prekidačem
11. 220V



Prije

$$U_m = U_f = U_p, \quad U_{re} = U_f = U_s$$

Nakon prekida

$$I_s = -I_T \quad \text{to vrijedi jer nema vodice } O \dots ??$$

$$I_s + I_T = 0$$

$$\rightarrow U_{re} \text{ mjeri } U_{re} = U_f + \frac{1}{2} U_f = 220 + 110 = 330 \text{ V}$$

$$\rightarrow U_{re} = U_{so} = \frac{U_f}{2} = 110 \text{ V}$$

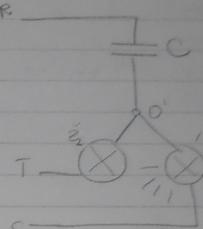
2) Spoj za odred. faze

$$B_c = G_{z1} = G_{z2} = G$$

- tomo gdje žarulja ječe zvježđu

5

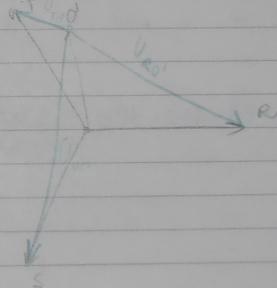
Hillmannov teorem



$$U_{re} = B_c \cdot \bar{U}_R + G_{z2} \bar{U}_T + G_{z1} \bar{U}_S = j \cdot G \bar{U}_R + G (\bar{U}_T + \bar{U}_S) = \frac{j \cdot G \bar{U}_R + G (\bar{U}_T + \bar{U}_S)}{jG + 2G} =$$

$$= \frac{j \bar{U}_R - \bar{U}_R}{jG + 2G} = \frac{j \bar{U}_R - \bar{U}_R}{j+2} = \frac{\bar{U}_R (j-1)}{j+2} = -\bar{U}_R \frac{1-j}{2j} =$$

$$= 0,63 \angle 108,4^\circ \bar{U}_R$$



ČESTO NA USMENOM !!!

→ Izveninov:

- isključena Z_1 , pa je

- impedancija RST \Rightarrow kretko spajani R_p

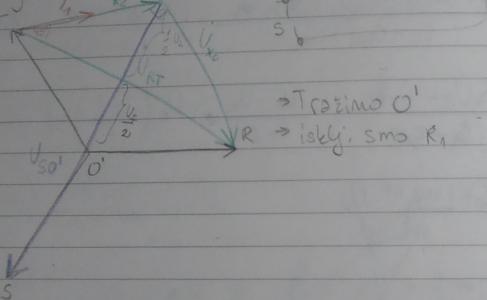
$$U_{SO} = ?$$

- isklj. R_1

$$Z_{T1} = jX_C \cdot R = Z_{T2}$$

$$R - jX_C : i_2 = i_1$$

$$T = \frac{R}{R - jX_C}$$



$$U_{SO} = U_f + \frac{U_f}{2} + \frac{1}{2} U_b = 220 + 110 + 150 = 520 \text{ V}$$

$$\bar{U}_{TO} = \bar{T}_2 \cdot \frac{U_L}{2} = 268 \text{ V}$$

→ kada isključimo R_2

$$Z_T = Z_{T1}$$

T

i_1

i_2

i_3

i_4

i_5

i_6

i_7

i_8

i_9

i_{10}

i_{11}

i_{12}

i_{13}

i_{14}

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$$16) P_L = 3 \cdot U_f \cdot I_3 = 3 \frac{U_f^2}{R}$$

$$P_L = \frac{U_f^2}{2R} = \frac{3 \cdot U_f^2}{2R} = \frac{1}{2} P_L$$

Dekompleksni rečnik
5.) $U_f = 120\sqrt{3}V$

$$P_{L2} = ?$$

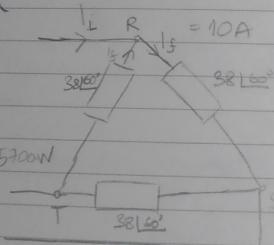
$$U_L = 360V$$

$$P = \frac{360^2}{100} = 1296W$$

$$10) I_L = 17.3A$$

$$I_f = 10A$$

$$P_{L2} = 3 \cdot 10^2 \cdot R_{L2} = 3 \cdot 100 \cdot 15 = 5700W$$



22.1.2007.

$$9) P_{L2} [Z] = 22 \cdot \frac{\sqrt{3}}{2} = 11\sqrt{3}$$

$$U_f = 380V \Rightarrow U_f$$

$$\begin{aligned} P &= \sqrt{3} \cdot U_f \cdot I_L \cdot \cos \varphi = \\ &= \sqrt{3} \cdot 380 \cdot 1 \cdot \frac{\sqrt{3}}{2} = 570W \end{aligned}$$

$$10) \cos \varphi = 1 \leftarrow \text{samo otpar}$$

$$U_L = 380V \Rightarrow U_f = 220V$$

$$I_N = ?$$

$$I_4 = \frac{P}{U} = 1.73 \cdot 10^\circ = \sqrt{3} 10^\circ$$

$$I_2 = 3.45 142^\circ$$

$$I_3 = 5.18 125^\circ$$

$$I_N = I_1 + I_2 + I_3 = 3 1 - 150^\circ A$$

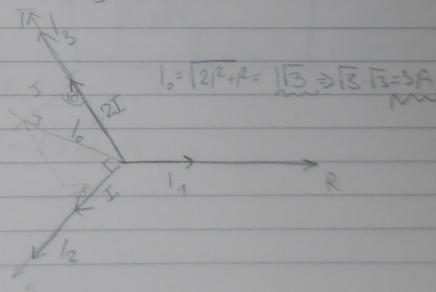
Drugi nacin:

$$P_1 : P_2 : P_3 = 1 : 2 : 3$$

$$I_1 = 1$$

$$I_2 = 2$$

$$I_3 = 3$$



2012

4) $I_A = 1A$

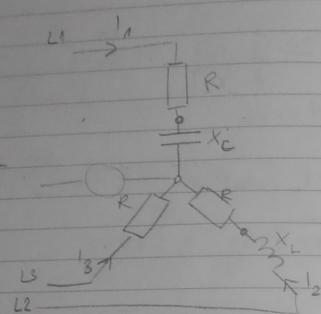
$U_L = ?$

$R = \sqrt{3} X_L = \sqrt{3} X_C = 200\Omega$

$R = 200\Omega$

$X_C = \frac{200}{\sqrt{3}} \Omega$

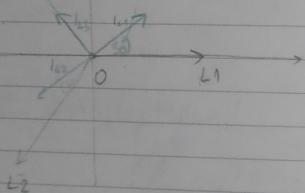
$X_C = -j \frac{200}{\sqrt{3}} \Omega$



$I_F = 1A = I_3 \Rightarrow I_L = 1| -240^\circ A$

$I_A \cdot R = 200V = U_F \cdot L_3$

$U_L = 200\sqrt{3} = 346V$



8.) $U_V = 380V = U_L$

$I_A = I_S = 1A$

$U_L \cdot I_S \cdot \cos \varphi = P_{face}$

$R = \sqrt{3} X_L$

$\text{and } \tan \varphi = \frac{X_L}{\sqrt{3} X_R} = \frac{1}{\sqrt{3}} \Rightarrow 30^\circ L_3$

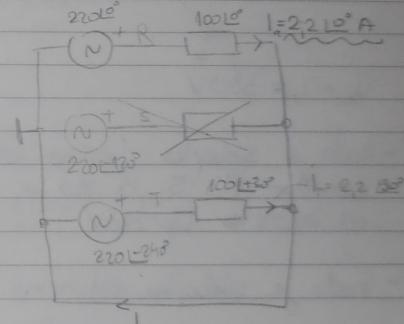
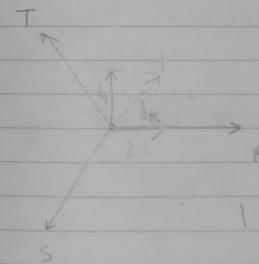
$P_{face} = U_L \cdot I_S \cdot \cos 30^\circ = \frac{\sqrt{3}}{2} \cdot 380 \cdot 1 = 150\sqrt{2} = 329,1$

$P = 987 = 3 \cdot P_{face}$

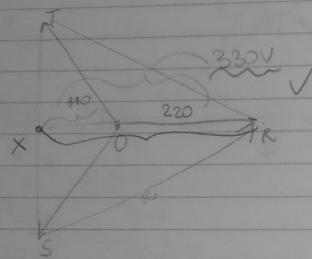
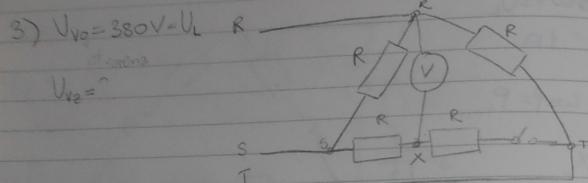
2007.

8.) $I_B = ?$

2 u predictor



$I = 2,2\sqrt{2} A$

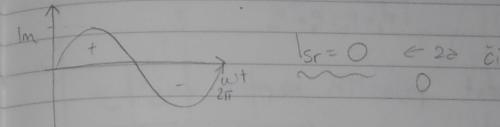


$$U_{Rx} = U_V = 330V$$

- kada je sklopka otvorena voltmeter pokazuje napon između R i S , a to je ujedno i lininski napon od $380V \Rightarrow U_f = 220V$

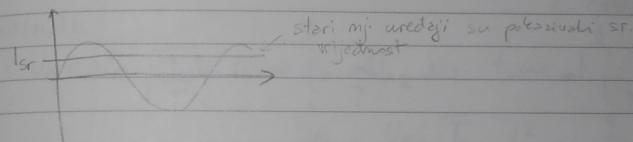
- kada je sklopka zatvorena voltmeter pokazuje

$$i(t) = I_{Sk} + I_m \cdot \sin \omega t$$

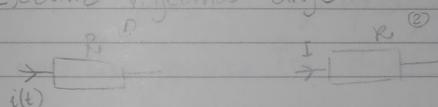


$I_{Sr} = 0$ $\leftarrow 2a$ čiste izmjenične ujet

Elektrolitska sr. vrijednost



Efektivna vrijednost struje



$$W_1 = W_2 \quad \text{u vremenu } T$$

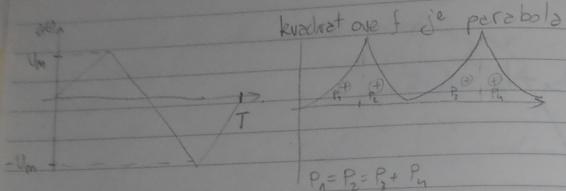
$$\int_0^T i^2(t) \cdot R \, dt = I^2 \cdot R \cdot T$$

$i(t)$

$$\int_0^T i^2(t) \, dt = I^2 T$$

$$I = \sqrt{\frac{1}{T} \int_0^T i^2(t) \, dt}$$

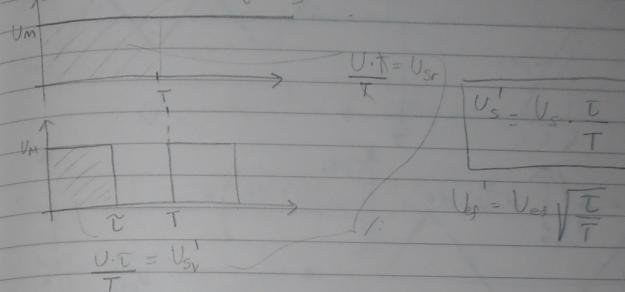
Kvadratni



Defektivna vrijednost pilastih napona uvijek je
o jednakoj $\frac{U_m}{\sqrt{3}}$

Periodički niz · impulsa

srednja vrijednost



Poluvalbro :

$$I_s = \frac{1}{T} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_m \sin \omega t \cdot d\omega t = \frac{I_m}{T} \int_0^{\frac{\pi}{2}} \sin \omega t \cdot d\omega t = \quad \left\{ d(\omega t) = \omega dt \right.$$

$$= \frac{I_m}{T} \int_0^{\frac{\pi}{2}} \sin \omega t \cdot \omega d(\omega t) = \frac{I_m}{\omega T} \int_0^{\frac{\pi}{2}} \sin \omega t \cdot d(\omega t) =$$

$$= \frac{I_m}{\omega T} \cdot (1 + 1) = \frac{I_m}{\omega T}$$

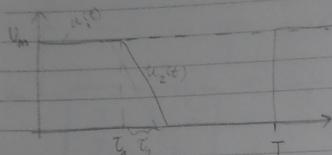
$$U_{pr} = U_m \cdot \sqrt{\frac{T}{T}} \quad U_m = U_{m2}$$

$$U_{sin} = \frac{U_m}{\sqrt{2}}$$

$$\frac{U_m}{\sqrt{2}} = \frac{U_{m2}}{\sqrt{2}} \cdot \sqrt{\frac{T}{T}} \quad \frac{1}{2} = \frac{T}{T} \Rightarrow \frac{1}{2} = \frac{T}{T}$$

T mora biti polovica T-a da bi efektivne vrijednosti bile iste

Složeni valni oblici



$$U_{\text{ef}}^2 = \frac{1}{T} \left(\int_0^{T_1} u_1^2(t) dt + \int_{T_1}^{T_1+T_2} u_2^2(t) dt \right) = \frac{U_{\text{ef},1}^2}{T_1} + \frac{U_{\text{ef},2}^2}{T_2}$$

$$U_{\text{ef}} = \sqrt{U_{\text{ef},1}^2 + U_{\text{ef},2}^2}$$

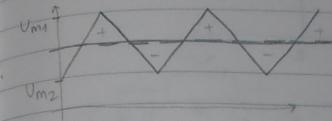
Dodavanje istosmjerne komponente

$$u(t) = U_0 + U_m \sin \omega t$$

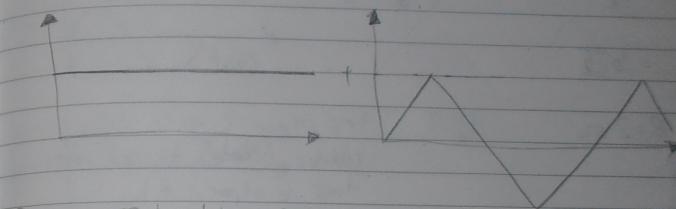
$$U_{\text{ef}}^2 = \frac{1}{T} \int_0^T u^2(t) dt = \frac{1}{T} \int_0^T (U_0 + U_m \sin \omega t)^2 dt =$$

$$= \frac{1}{T} \left[\int_0^T U_0^2 dt + 2 \int_0^T U_0 U_m \sin \omega t dt + \int_0^T (U_m \sin \omega t)^2 dt \right] = U_{\text{ef},0}^2 + U_{\text{ef}}^2$$

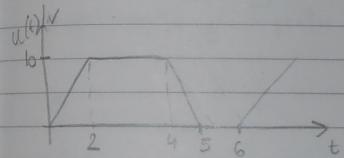
Primjer 2



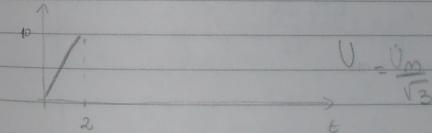
\Downarrow



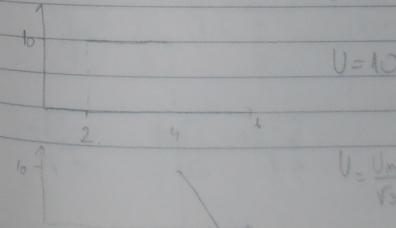
Primjer: Odredi efektivnu vrijednost slj. valnog oblika



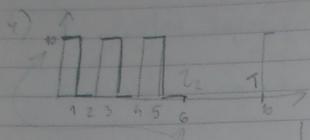
$$U = \sqrt{100 + \frac{200}{3}} = \frac{500}{\sqrt{3}}$$



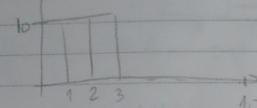
$$U = \frac{U_m}{\sqrt{3}}$$



$$U = \frac{U_m}{\sqrt{3}}$$



$$U_{sr} = \sqrt{\frac{U_m^2 + U_0^2}{2}} \cdot \frac{6}{10}$$



NESINUSOIDNE PERIODIČKE VELIČINE

SREDNA VRIJEDNOST

$$I_{sr} = \frac{1}{T} \int i(t) dt$$

paršina → neobj
 kada se podjeli
 s vremenom to
 je stvrtka

$$U_{sr} = \frac{1}{T} \int u(t) dt$$

- za "čiste" veličine = 0

- istosmjerna komponenta el. struje

ELEKTROLITSKA VRIJEDNOST

- srednje apsolutna vrijednost (pumovalni ispravljač)

$$I_{esr} = \frac{1}{T} \int |i(t)| dt$$

$$U_{esr} = \frac{1}{T} \int |u(t)| dt$$

- negativni djelovi postaju pozitivni

EFIKTIVNA VRIJEDNOST

$$I_{ef} = \sqrt{\frac{1}{T} \int i^2(t) dt}$$

$$U_{ef} = \sqrt{\frac{1}{T} \int u^2(t) dt}$$

- toplina koju razvija periodički promjenjiva struja mora biti jednaka vrijednosti istosmjerne struje koja razvija istu tu toplinu u istom periodu T.

Izvod:

$$W = \int_0^T i^2(t) \cdot R dt$$

$$W = I_{ef}^2 \cdot R \cdot T$$

$$I_{ef}^2 = \frac{1}{T} \int_0^T i^2(t) dt / T$$

$$I_{ef} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

Tjemeni faktor - izobličenje veličine u odnosu na sinusni oblik

$$\phi = 1m \Rightarrow \text{za sinusni } \sqrt{2}$$

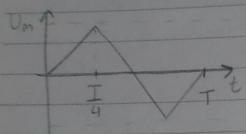
Faktor oblike - omogućava određivanje efektivnih vrijednosti u odnosu na srednje

$$f = \frac{I_{ef}}{I_{sr}}$$

SINUSNI VALNI OBLIK

$$U_{sr} = \frac{U_m}{\sqrt{3}}, \quad T = 2\pi, \quad \omega = \frac{2\pi}{T}$$

PILASTI VALNI OBLIK

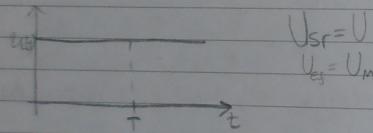


$$-3 \text{ pravca } [0, I_1], [I_1, \frac{3T}{2}], [\frac{3T}{2}, T]$$

$$U_{sr} = 0 \quad U_{es} = \frac{U_m}{\sqrt{3}}$$

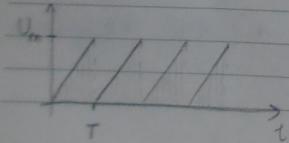
$$U_{es} = \frac{U_m}{\sqrt{3}}$$

PARAMETRI KONSTANTNOG VALNOG OBLIKA



zakonit i jednostavni

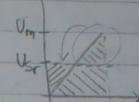
PILASTI VALNI OBLIK (2)



ekstremno jednostavni

Srednja vrijednost

$$U_{sr} = \frac{U_m}{2}$$



Faktor oblike:

$$\gamma_p = \frac{U}{U_{sr}} = \frac{\frac{U_m}{\sqrt{3}}}{\frac{U_m}{2}} = \frac{2}{\sqrt{3}}$$

Efektivna vrijednost: ≈ 22 sive pilaste $U_m/\sqrt{3}$

$$U = \frac{U_m}{\sqrt{3}}$$

Periodički niz impulsa

T - period trajanja

T - vrijeme trajanja impulsa

- dobijemo tako da se osnovni valni oblik impulsa gasi u nečetnim vrem. intervalima



$$Y_{es} = Y_{osnovni} \cdot \sqrt{\frac{T}{T}}$$

$$Y_{sr} = Y_{osnovni} \cdot \frac{T}{T}$$

Primjer $\frac{I}{T} = 0,4$

$$I_{sr} = ?$$

$$I_{sr} = ?$$

$$I_{sr} = ? \quad R = 4\Omega$$

$$I_m = 2A$$

$$I_{es} = I_{esm} \cdot \sqrt{\frac{C}{T}} = \frac{2}{12} \cdot \sqrt{0,4}$$

$$I_{sr} = I_{srn} \cdot \frac{T}{T} = 0$$

Primjer: $\frac{D}{T} = 0,33$

$$I_{es} = I_{esm} \cdot \sqrt{\frac{C}{T}} = I_m \cdot \sqrt{\frac{T}{T}} = I_m \cdot 0,575$$

$$I_{sr} = I_{srn} \cdot \frac{T}{T} = I_{srn} \cdot 0,33$$

Primjer $\frac{D}{T} = 0,667$

$$U_{es} = U_{esm} \cdot \sqrt{\frac{C}{T}} = \frac{U_m}{13} \sqrt{0,667} = U_m \cdot 0,17$$

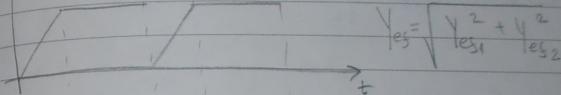
$$U_{sr} = U_{srn} \cdot \frac{T}{T} = U_m \cdot 0,667$$

SLOŽENI VALNI OBLICI

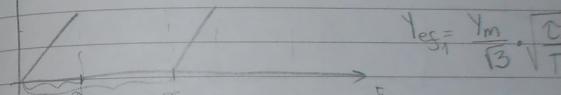
- zbroj više valnih oblika \rightarrow komponenta

- složeni, složeni koji se mogu predstaviti zbrojem po vremenu neprekidnih impulsa ili dodavanjem istosmjernih komponente

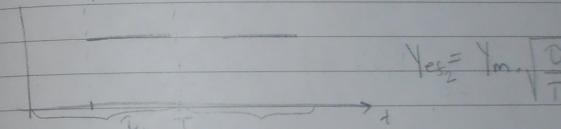
Primjer



$$I_{es} = \sqrt{I_{es1}^2 + I_{es2}^2}$$

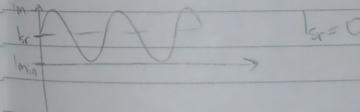


$$I_{es} = \frac{Y_m}{13} \cdot \sqrt{\frac{C}{T}}$$



$$I_{es} = Y_m \cdot \sqrt{\frac{C}{T}}$$

Primjer: dodavanje istosmjerne komponente



$$I_{sr} = 0$$

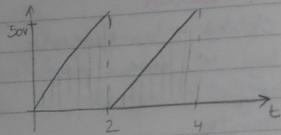
XIII.1.P1

$$T=2s$$

$$U_m = 50V$$

$$U_{sr} = ?$$

$$U_{es} = ?$$



$$U_{sr} = \frac{50}{2} = 25V$$

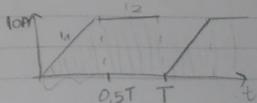
$$U_{es} = \frac{50}{\sqrt{3}}$$

XIII.1-P4

$$I_{sr} = ?$$

$$I_{es} = ?$$

$$I_m = 10A$$



$$I_{res} = I_{osn.es} \cdot \sqrt{\frac{T}{T}} = \frac{10}{\sqrt{3}} \cdot \frac{20}{\sqrt{3}} = 4,08A$$

$$I_{2es} = I_{osn.es} \cdot \sqrt{\frac{T}{T}} = \frac{10}{\sqrt{2}} = 7,07A$$

$$I_{es} = \sqrt{\frac{1^2}{4\pi} + \frac{1^2}{4\pi}} = 8,16A$$

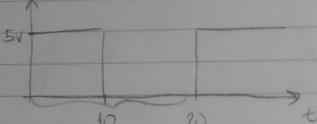
XIII.1.P2

$$T = 10ms$$

$$T = 20ms$$

$$U_m = 5V$$

$$U_{sr}, U_{es} = ?$$



$$U_{sr} = U_{osn.es} \cdot \frac{T}{T} = 5 \cdot \frac{10}{20} = 2.5V$$

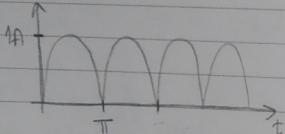
$$U_{es} = U_{esn.es} \cdot \sqrt{\frac{T}{T}} = \frac{5}{\sqrt{2}} V$$

XIII.1.P3

$$R = 10\Omega$$

$$I_m = 1A$$

$$P = ?$$



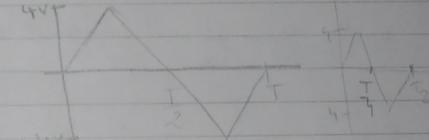
$$I_{sr} = \frac{1}{\pi} \int_0^{\pi} I_m \cdot \sin(\omega t) dt = I_m \cdot \left(-\frac{\cos(\omega t)}{\omega} \right) \Big|_0^{\pi} = -I_m \cdot \frac{1}{\omega \pi}$$

XIII.1-H

$$U_{pp} = 8V$$

$$U_{es} = 0$$

$$I_i = \frac{I}{2}$$



$$a) U_{es} = \frac{4}{\sqrt{3}}$$

$$b) U_{es} = \frac{4}{\sqrt{3}} \cdot \sqrt{\frac{\pi_2}{\frac{1}{2}}} = \frac{4}{\sqrt{6}}$$

XIII.1-2

$$R = 25\Omega$$

$$P = 400W$$

$$I_m = ?$$

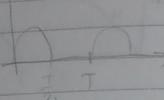
$$P = I^2 \cdot R \Rightarrow I_{es} = \sqrt{\frac{P}{R}} = 4A$$

$$a) I_m = I_{es} \cdot \sqrt{2} = 5,66A$$

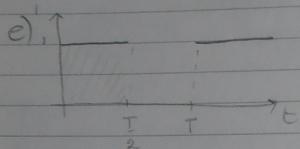
$$b) I_m = I_{es} \cdot \sqrt{3} = 6,93A$$

$$c) I_m = I_{es} \cdot \sqrt{3} = 6,93A$$

$$d) I_{ef} = I_{es \cos} \sqrt{\frac{T}{T}} = \frac{4}{\sqrt{2}}$$



$$I = \frac{I_m}{2} \Rightarrow I_m = 12.8 A$$



$$I_{ef} ? \quad I_{ef} = 4 A$$

$$I_{es}, I_{es \cos} \sqrt{\frac{T}{T}} \Rightarrow 4 = I_{es \cos} \cdot \frac{1}{\sqrt{2}} \Rightarrow 4\sqrt{2} = I_{es \cos} = I_m = 5.65 A$$

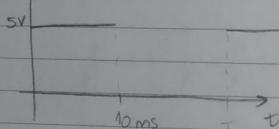
XIII.1-3

$$U = 5 V$$

$$t = 10 \text{ ms}$$

$$U_{es} = 3,536$$

$$f = ?$$

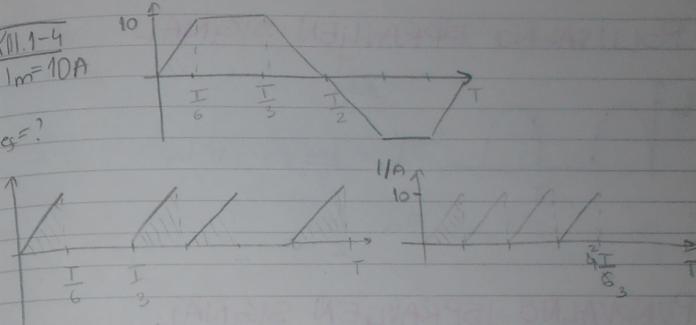


$$U_{ef} = U_{es \cos} \sqrt{\frac{T}{T}} = \sqrt{3,536} = 5\sqrt{10} \Rightarrow T = 20 \text{ ms}$$

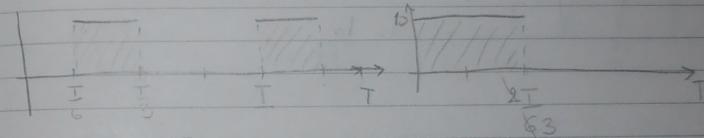
$$f = \frac{1}{T} = \frac{1}{20 \text{ ms}} = 50 \text{ Hz}$$

XIII.1-4
I_m = 10 A

$$I_{ef} ?$$



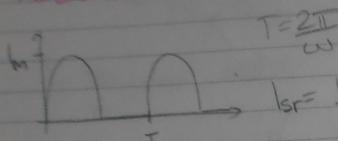
$$I_{ef_1} = \frac{10}{\sqrt{3}} \cdot \sqrt{\frac{2\frac{5}{6}}{T}} = \frac{10\sqrt{2}}{3} A$$



$$I_{ef_2} = 10 \cdot \sqrt{\frac{\frac{1}{3}}{T}} = \frac{10}{\sqrt{3}} A$$

$$I_{ef_{tot}} = \sqrt{\frac{100}{2} + \frac{100}{3}} = 7,45 A$$

POLUVALNO ISPRAVLJEN SIGNAL

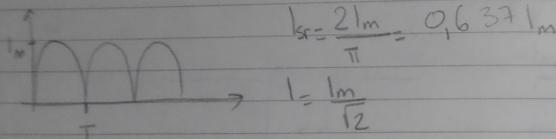


$$T = \frac{2\pi}{\omega}$$

$$I_{sr} = \frac{I_m}{\pi} = 0,318 I_m$$

$$I_{es} = \frac{I_m}{2}$$

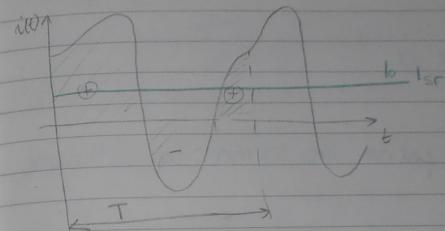
PUNOVALNO ISPRAVLJEN SIGNAL



$$I_{sr} = \frac{2I_m}{\pi} = 0,637 I_m$$

$$I = \frac{I_m}{\sqrt{2}}$$

Fourijerova analiza



$$i(t) = I_0 + i_1 + i_2 + i_3 + i_4 + \dots$$

↑
konstantno

$$i_1(t) = I_{m1} \sin(\omega t + \varphi_1), \quad k=1,2,3,\dots$$

$$i_2(t) = I_{m2} \sin(\omega t + \varphi_2)$$

$$i_n(t) = I_{mn} \sin(\omega t + \varphi_n)$$

22 besjed \leftarrow islobanje kopp

$$i_k(t) = I_{mk} \sin(\omega_k t + \varphi_k)$$

$$= I_{mk} \cos \varphi_k \sin(\omega_k t) + I_{mk} \sin \varphi_k \cos(\omega_k t)$$

$$i_o(t) = \frac{A_0}{2} + \frac{1}{T} \int i(t) dt \Rightarrow \begin{array}{l} \text{SREDNA VRJEDNOST} \\ \text{FUNKCIJE} \end{array}$$

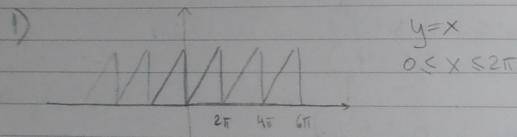
$$A_k = \frac{2}{T} \int_0^T f(t) \sin k\omega t dt$$

$$B_k = \frac{2}{T} \int_0^T f(t) \cos k\omega t dt$$

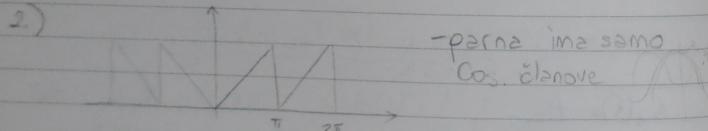
→ Ako je funkcija parna ima samo cos članove,
A_k nema

→ Ako je funkcija neparna ima samo sinusne
članove,

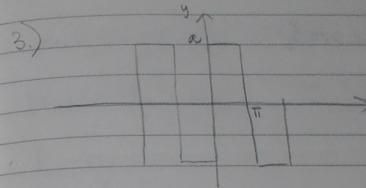
Primer:



$$y = \pi - 2 \left(\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \dots \right)$$



$$y = \frac{\pi}{2} - \frac{4}{\pi} \left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots \right)$$



$$y = \frac{4a}{\pi} \left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right)$$

$$i(t) = i_0 + i_1 + i_2 + \dots + i_n + \dots$$

$$|i|^2 = \frac{1}{T} \int_0^T i^2(t) dt = \frac{1}{T} \int_0^T (i_0 + i_1 + i_2 + \dots + i_n)^2 dt$$

$$= \frac{1}{T} \left[\int_0^T i_0^2 dt + \int_0^T i_1^2 dt + \dots + \int_0^T i_n^2 dt + \dots + \sum \int_0^T 2i_i i_j dt \right]$$

$$= \frac{1}{4} |i_0|^2 + \frac{1}{1} |i_1|^2 + \frac{1}{2} |i_2|^2 + \dots + \frac{1}{n} |i_n|^2$$

Ojer su
čas. frekv.

$$|i|^2 = \frac{|i_0|^2}{2} + \frac{|i_1|^2}{2} + \frac{|i_2|^2}{2} + \dots + \frac{|i_n|^2}{2}$$

srednja vrijednost

$$u(t) = u_0 + u_1 + u_2 + \dots + u_n + \dots$$

$$i(t) = i_0 + i_1 + i_2 + \dots + i_n + \dots$$

$$p(t) = i(t) \cdot u(t)$$

$$P_o = \frac{1}{T} \int_{t_0}^{t_0+T} u_o \cdot i_o dt = U_o \cdot I_o \cdot \cos \varphi_L$$

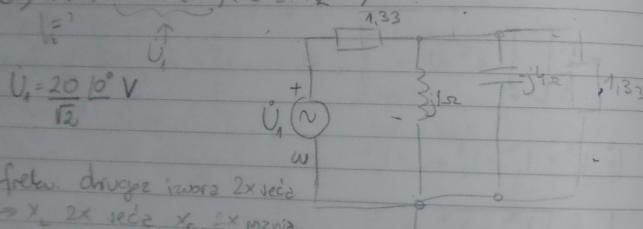
$$P_{out} = \sum_{n=1}^N P_n$$

Primjer 1 - 42-

$$U_{eff} = \sqrt{U_o^2 + U_{res}^2 + U_{seis}^2} =$$

Pokarski

$$20) \text{ ult} = 20 \sin(\omega t) + 8 \sin(2\omega t)$$



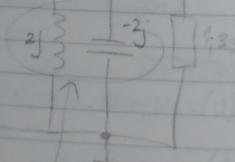
-frekv. struje raste 2x veće
→ X_L 2x veće X_C ex maza

$$I_o = 8$$

$$I_2 = 2,66$$

↑ otporni

$$\frac{8}{R} + U_2$$



Paralelna → ne teče struja

-kut struje nije bitan, samo ef. vrijednost gledamo

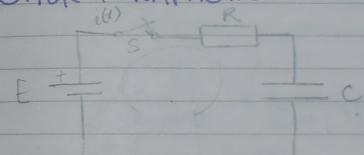
$$|Z_{ekv}| \equiv |Z_1 + Z_p| \quad \leftarrow 1. \text{ slika}$$

$$I_{1,33} = \frac{U_1}{|Z_{ekv}|}$$

$$I = \sqrt{I_1^2 + I_2^2} = 7A \quad \leftarrow 1. \text{ rezultati!!!}$$

Prijelazne pojave

OTPOR I KAPACITET



$t=0^+$ ← početak zatvara

$$E = U_R + U_C$$

→ u 1. trenutku kapacitet je prazen ($U_C = 0$),
čitav E je na otporu $I = \max$

→ te struja nabija kond. $\rightarrow U_C raste$, $U_C pada$,
struja I_{max} R opada

→ nakon što se nabije, $U_C = 0$, $U_C = E \rightarrow$ str. struje

$$U_R = i \cdot R$$

$$i = C \cdot \frac{du(t)}{dt}$$

$$u_c = \frac{1}{C} \int_0^t i(t) dt$$

naboj na kond. / kapacitet = $\rho \beta n$

$$du_c = \frac{1}{C} i(t) dt$$

$$E = R \cdot C \cdot \frac{du_c(t)}{dt} + u_c(t)$$

nehomogenac
linearni koef. 1. reda

Terivacije te funkcije

homogenac:

$$RC \cdot \frac{du_c(t)}{dt} + u_c(t) = 0 \quad \text{homogenac}$$

- prisilna funkc. definira što će biti u stan. stanj

$$u_c = U_p + U_g$$

pristno opće rješenje

$$u_c(t) = E \left(1 - e^{-\frac{t}{RC}} \right)$$

najanju tvaru
vrijeme trenutak
pri. log.

$$C = RC$$

4-5^o funkcijske poprimi:
dolazimo u stan. stanje

$$E = U_r + U_c$$

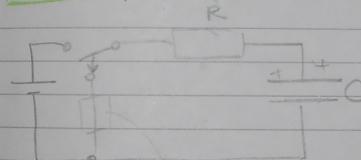
$$U_r(t) = E \cdot e^{-\frac{t}{T}}$$

$$i = \frac{U_c(t)}{R} = \frac{E}{R} \cdot e^{-\frac{t}{T}}$$

→ Sto je T manji funkcija je brža (to je puno veće od D_s)

$$W = E^2 C \quad \frac{1}{2}$$

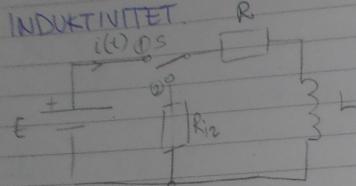
→ kod nabijanja kond. 50% energije UVJEK OSTANE
NA OTPORU bez obzira na vrijednost otpora



- kod izbijanja:

$$u_c = E \cdot e^{\frac{t}{RC}}$$

$T_{izbijanje}$ → pošto tog otpora se mijenja C u 2. sljedećem



- 2. a) Lenzova zakona u $t=0^+$ induktivitet se poveća polaskom struje
- u $t=0^+$ je struja je 0 \Rightarrow napon = 0, čime u napon na induktivitetu
- izvor će pospješiti rast struje da bzbjedi indukt.
- ne kraju je energija max., a na poc. 0

$$N_m = \frac{1}{2} L \cdot i^2$$

$$u_R(t) = E(1 - e^{-\frac{t}{\tau}})$$

$$u_L(t) = E e^{-\frac{t}{\tau}}$$

→ da bi se održalo sl. stanje, da bi ga održao trebamo davati toplinu oz otpor

→ otpor manji \Rightarrow en. manja, $\tau \rightarrow$ veci

2a ② položaj

"izbijanje"

→ mag. en. se izbjija, struja opada

→ I. struja je max.

→ nakon nekog vremena je 0 \leftarrow vrijeme ovisi o τ

→ otpor izbijanja je puno veci od R .

$$\Delta E = u_R + u_L$$

$$E = iR + L \frac{di}{dt} \quad | : R$$

$$\frac{E}{R} = \frac{i}{R} + \frac{L}{R} \frac{di}{dt} \quad \leftarrow \text{eina. jedn. 1. reda nehomog.}$$

\uparrow
 τ vrem. konstanta

$$i(t) = \frac{E}{R} \left(1 - e^{-\frac{t}{\tau}} \right) \quad | : R \quad \tau = \frac{L}{R}$$