APR mi

Bože pomozi :)

① RJEŠAVANJE SUSTAVA LIN. JEDNADŽBI

$$\boxed{\underline{A} \cdot \underline{x} = \underline{b}}$$

→ **LU**

$$\boxed{A = L \cdot U}$$

$$\underbrace{L \cdot U \cdot x}_{y} = b \quad \rightarrow \quad \boxed{\begin{array}{l} U \cdot x = y \\ L \cdot y = b \end{array}} \begin{array}{l} \rightarrow \text{supst. unatrag} \\ \rightarrow \text{supst. unaprijed} \end{array}$$

pr.

$$\begin{bmatrix} 4 & 3 & 2 & 1 \\ 4 & 6 & 1 & -1 \\ -8 & 3 & -5 & -6 \\ 12 & 12 & 7 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{4} & 3 & 2 & 1 \\ 1 & 3 & -1 & -2 \\ -2 & 9 & -1 & -4 \\ 3 & 3 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & \boxed{3} & -1 & -2 \\ -2 & 3 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 3 & -1 & -2 \\ -2 & 3 & 2 & 2 \\ 3 & 1 & 1 & 1 \end{bmatrix}$$

$$\rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 3 & 1 & 0 \\ 3 & 1 & 1 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

pr.

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 4 & 2 \end{bmatrix} \underline{x} = \begin{bmatrix} 25 \\ 10 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & -3 \\ 0 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & -3 \\ 0 & 4 & 14 \end{bmatrix}$$

$$\rightarrow LUx = b \Rightarrow L \cdot y = b \Rightarrow U \cdot x = y$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 10 \\ 10 \end{bmatrix} \rightarrow \begin{array}{l} y_1 = 25 \\ y_2 = -15 \\ y_3 = 70 \end{array}$$

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 14 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 25 \\ -15 \\ 70 \end{bmatrix} \Rightarrow \begin{array}{l} x_3 = 5 \\ x_2 = 0 \\ x_1 = 5 \end{array}$$

//

~~(LUP)~~

PIVOTIRANJE

→ POTPUNO → kada dođemo do k-tog el. gledamo cijelu matricu što nam je ostala pa radimo zamjenu redaka i stupaca

→ DJELOMIČNO

→ PO STUPCIMA

→ PO RETCIMA

LUP

→ LU dekompozicija uz djelomično pivotiranje po stupcima

„Kraj tebe zvijezda Danica guli svoj sjaj"

$$\boxed{P \cdot A = L \cdot U}$$

└→ permutacijska matrica

$$\boxed{L \cdot y = P \cdot b}$$

Zad.
$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 2 \\ 4 & 3 & 4 \end{bmatrix} \cdot \underline{x} = \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 1 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 & 4 \\ 1/4 & 5/4 & 1 \\ 3/4 & -1/4 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 1/4 & 5/4 & 1 \\ 3/4 & -1/5 & -9/5 \end{bmatrix}$$

$$Ly = Px \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ 3/4 & -1/5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 4 \end{bmatrix} \Rightarrow \begin{array}{l} y_1 = 8 \\ y_2 = 1 \\ y_3 = -\frac{9}{5} \end{array}$$

$$Ux = y \Rightarrow \begin{bmatrix} 4 & 3 & 4 \\ 0 & 5/4 & 1 \\ 0 & 0 & -9/5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ -9/5 \end{bmatrix}$$

$x_3 = 1$

$x_2 = 0$

$x_1 = 1$

② POSTUPCI NELINEARNE OPTIMIZACIJE

$$\underbrace{F(\underline{x})}_{\geq F(x_M)} = F(\underline{x_M}) + \underbrace{\nabla F(x_M)^T \cdot \Delta x}_{0} + \underbrace{\frac{1}{2} \Delta x^T \cdot H(x_M) \cdot \Delta x}_{\geq 0} + \ldots$$

→ x_M je min (uvjet) ako je $\nabla F |_{x_M} = 0$

→ H negativno definitna → x_M je maksimum

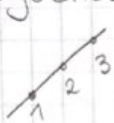
1. Optimiranje fje 1 var.

→ rješenje : interval $x \in [-,-]$

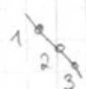
pr. $f(x_0 - h)$ $f(x_0)$ $f(x_0 + h)$
$\quad 1 \qquad 2 \qquad 3$

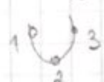
4 mogućnosti :

ALG ZA NALAŽENJE UNIMODALNOG INTERVALA

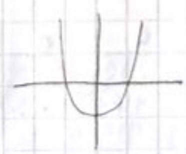
1) 
$f(x_0 - 2^k h)$ do $f(x_0 - 2^k h) > f(x_0 - 2^{k-1} h)$

$[x_0 - 2^k \cdot h, x_0 - 2^{k-2} \cdot h]$

2) 
$[x_0 + 2^{k-2} h, x_0 + 2^k h]$

3) 
$[x_0 - h, x_0 + h]$

pr// $f(x) = x^2 - 2$
$x_0 = 100$
$h = 1$



x	101	100	99	98	96	92	...
f(x)							

14:50
15:40
16:25

$f(x_0 - 2^i h) \Rightarrow$ $(100 - 2^4) = 84$
$(100 - 2^5) = 68$
$(100 - 2^6) = 36$
$(100 - 2^7) = -28$
$100 - 2^8 = -156$

ovaj interval

$x \in [-156, 36]$

pr. $g(x)$
$x_0 = 2$ $[6, 18]$
$h = 1$

$2, 2+2^0, 2+2^1, 2+2^2, 2+2^3, 2+2^4$

$2, 3, 4, 6, 10, 18$

x	2	3	4	6	10	18
$g(x)$	>	>	>	>	<	

a) $g(1) < g(2)$ NE
b) $g(5) > g(10)$ DA
c) $g(6) > g(18)$?
d) $g(11) < g(18)$ DA
e) $g(9) < g(18)$?

Zlatni rez

$$\boxed{k = 0.618}$$

$$\boxed{\begin{aligned} c &= b - k(b-a) \\ d &= a + k(b-a) \end{aligned}}$$

→ smanjujemo interval dok $\boxed{(b-a) < \varepsilon}$

pr. $f(x) = (x-4)^2$
$x_0 = 0$
$h = 1$
$\varepsilon \leq 1$

x	0	1	2	4	8
$f(x)$	>	>	>	<	

$x \in [2, 8]$

	a	c	d	b
1	2	4.292	5.708	8
2	2	3.416	4.292	5.708
3	3.416	4.292	4.832	5.708
4	3.416	3.957	4.292	4.832
5	3.416		3.957	4.232
6				
7				

$c = b - k(b-a)$
$d = a + k(b-a)$
$f(c) < f(d)$ → odbacujemo b

→ $b - a < 1$ ⟹ kraj!

$x \in [3.416, 4.232]$

pr. $[-100, 100]$ $\varepsilon = 0.001$

$I \cdot k^n \leq \varepsilon$ $n = ?$

$k^n \leq \frac{\varepsilon}{I}$

$n \geq \frac{\log \frac{\varepsilon}{I}}{\log k} \Rightarrow \frac{\log \frac{0.001}{200}}{\log 0.618} \Rightarrow 25.36$

$n = 26$ iteracija

Fibonacci

$$\boxed{\frac{I_0}{\varepsilon} \le \varphi_N}$$

$$\boxed{\begin{aligned} c &= b - \frac{\varphi_{i-1}}{\varphi_i}(b-a) \\ d &= a + \frac{\varphi_{i-1}}{\varphi_i}(b-a) \end{aligned}}$$

pr// $f(x) = (x-4)^2$ $[-2, 6]$
$\varepsilon = 1$

$1, 1, 2, 3, 5, 8, 13, 21, 34, \dots$

$\varphi_n \ge \frac{b-a}{\varepsilon} = 8$

	a	c	d	b
1	-2	1	3	6
2	1	3	4	6
3	3	4	5	6
4	3		4	5
5				

- ispitivanje vrijednosti fje u točki blizu sredine intervala

pr. 3.9 $\rightarrow x \in [3.9, 5]$

pr. 4.1 $\rightarrow x \in [3, 4.1]$

Hooke - Jeeves

x_0 Δx x_B x_P x_N $\qquad x_P = 2x_N - x_B$

pr// $F(x) = x_1^2 + x_2^2$
$x_0 = (7, 3)$
$\Delta x = 1$

x_B	x_P	x_N	uvjet	
7, 3	7, 3	6, 2	✓	
6, 2	5, 1	4, 0	✓	
4, 0	2, -2	1, -1	✓	
1, -1	-2, -2	-1, -1	✗	$\Delta x = 0.5$
1, -1	1, -1	0.5, -0.5	✓	
0.5, -0.5	0, 0	0, 0	✓	
0, 0	-0.5, 0.5	0, 0	✗	$\Delta x = 0.25$

Pr/ $F(x) = x_1^2 + x_2^2 + x_3^2$

$x_0 = (2,3,4)$

$\Delta x = 1$

$\varepsilon = 0.25$

$x_P = 2x_N - x_B$

	x_B	x_P	x_N	Uvjet	
1	2,3,4	2,3,4	1,2,3	✓	
2	1,2,3	0,1,2	0,0,1	✓	
3	0,0,1	-1,-2,-1	0,-1,0	✗	$\Delta x = 0.5$
4	0,0,1	0,0,1	0,0,0.5	✓	
	0,0,0.5	0,0,0	0,0,0	✓	
	$\boxed{0,0,0}$	0,0,-0.5	0,0,0	✗	$\Delta x = 0.25 \leq \varepsilon$ KRAJ

Rj: 0,0,0

Nelder & Mead

$\boxed{x[h]}$ → točka s max fom cije

$\boxed{x[l]}$ → min

$\boxed{x_c}$ → ar. sr. svih osim $x[h]$

3. operacije:

→ refleksija centroida

① Refleksija $\quad x_R = (1+\alpha)x_c - \alpha x[h]$

② Ekspanzija $\quad x_E = (1-\gamma)x_c + \gamma x_R$

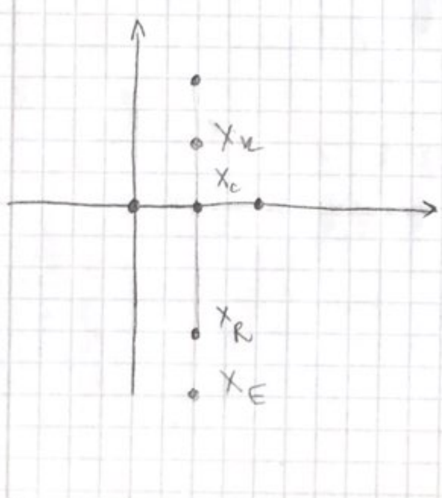
③ Kontrakcija $\quad x_K = (1-\beta)x_c + \beta x[h]$

→ na pola između x_c i $x[h]$

Pr/ (0,0)
(2,0)
(1,2)

$f(x) = x_1^2 + x_2^2$

$x_c = (1,0)$



Pr.// $F(x) = (x_1 - 2)^2 + (x_2 + 1)^2$ $\boxed{x_1 = x_0 + \lambda v}$

$x_0 = (0, 0)$

$v = (2, 1)$

$h = 1$

$\varepsilon < 0.5$

$$x = x_0 + \lambda v = (0,0) + (2\lambda, \lambda) = (2\lambda, \lambda)$$

$$F(x) = (2\lambda - 2)^2 + (\lambda + 1)^2 = f(\lambda)$$

– UNIMODALNI

λ	−1	0	1	2
$f(\lambda)$	16	5	4	13

$>$ $>$ $<$

$\lambda \in [0, 2]$

– ZLATNI REZ

a	c	d	b
0	0.764	1.236	2
0	0.472	0.764	1.236
0	0.292	0.472	0.764
0.292	0.472		0.764

$f(d) = 5.22$
$f(c) = 3.33$

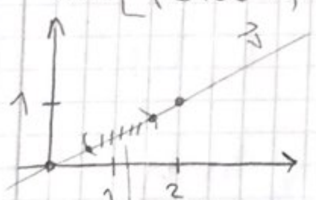
→ $f(c) = 3.28$ $f(d) = 3.33$

→ $f(c) = 3.67$ $f(d) = 3.28$ ×

→ $b - a < \varepsilon$

$$\lambda \in [0.292, 0.764]$$

$$x \in [(0.584, 0.292), (1.528, 0.764)]$$



→ zlatni rez

Powellov postupak

$$\boxed{\begin{array}{ll} V_i^T A \cdot v_j = 0 & za\ i \neq j \\ V_i^T A v_j > 0 & za\ i = j \end{array}}$$

pr// $f(x) = (x_1 - 2)^2 + (x_2 + 1)^2$

$V = (0, 1)$

npr. $T_1 = (0,0) \Rightarrow f(T_1 + \lambda \cdot V) = (0 + \lambda \cdot 0 - 2)^2 + (0 + \lambda \cdot 1 + 1)^2$

$$= 4 + \lambda^2 + 2\lambda + 1$$

$$= \lambda^2 + 2\lambda + 5$$

$$= f(\lambda)$$

$$\frac{\partial f}{\partial \lambda} = 2\lambda + 2 = 0 \Rightarrow \lambda = -1$$

1. minimum $= (0, -1)$

npr. $T_2 = (1,1) \Rightarrow f(\lambda) = (1 - \lambda \cdot 0 - 2)^2 + (1 + \lambda \cdot 1 + 1)^2$

$$= 1 + \lambda^2 + 4 + 4\lambda$$

$$= \lambda^2 + 4\lambda + 5$$

$$\frac{\partial f}{\partial \lambda} = 2\lambda + 4 \Rightarrow \lambda_2 = -2$$

2. minimum $(1, -1)$

Konjugirani smjer : $M_1 - M_2 = (-1, 0)$ ili $(1, 0)$

LI: $F(x) = x_1^2 - 4x_1 + 4 + x_2^2 + 2x_2 + 1 = x_1^2 - 4x_1 + x_2^2 + 2x_2 + 5$

$$= 5 + \begin{bmatrix} -4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

konjugirani su za

$$V_1^T A V_2 = 0$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix}$$

$$2V_{22} = 0$$

druga komponenta je 0, prva može bit bilo šta.

Zadaci

① $f(x) = (x+1)^2$ $[-2,2]$ ZLATNI REZ

$\varepsilon \leq 0.5$

	a	c	d	b
1	-2	-0.472	0.472	2
2	-2	-1.056	-0.472	0.472
3	-2	-1.416	-1.056	-0.472
4	-1.416	-1.056	-0.834	-0.472
5	-1.416	-1.194	-1.056	-0.834
6	-1.194	-1.056		-0.834

$\rightarrow \leq \varepsilon$ $x \in [-1.194, -0.834]$

$d = a +$
$c = b -$

$f(c) < f(d)$ ↘
$f(c) > f(d)$ ↙

② $x_0 = 0$
$h = 2$ $[-32, -8]$

x	2	0	-2	-4	-8	-16	-32	-64
g(x)		>	>	>	>	>	>	<

$x_0 - 2^k \cdot h$

a) $g(2) < g(-2)$ NE
b) $g(-5) > g(-10)$ DA
c) $g(-10) > g(10)$ NE
d) $g(0) < g(-30)$?

③ Riješen

④ $[0, 10]$ $I \cdot k^n \leq \varepsilon$

$\varepsilon = 0.01$ $n \geq \dfrac{\log \frac{\varepsilon}{I}}{\log k}$

$n \geq 14 \rightarrow n = 15$

⑤ $f(x) = (x-4)^2$
$x_0 = 20$ $\varepsilon \leq 1$
$h = 1$
granice unimodalnog intervala + zlatni rez

		0	1	2	3	4	5	
x	20	19	18	16	12	4	-12	-44
f(x)	>	>	>	>	>	<		

$x \in [-12, 12]$

	a	c	d	b
1	-12	-2.832	2.832	12
2	-2.832	2.832	6.334	12
3	-2.832	0.67	2.832	6.334
4	0.67	2.832	4.17	6.334
5	0.67	1.977	2.832	4.17
6	1.977	2.832	3.32	4.17
7	2.832	3.32	3.65	4.17
8	3.32	3.65		4.17

$\rightarrow x \in [3.32, 4.17]$

$I \cdot k^n \leq \varepsilon$

$n \geq \dfrac{\log \frac{\varepsilon}{I}}{\log k}$

$n = 7$

Negdi sam možda fali al mi se NEDA

⑥ n.R.
⑧ n.R.
⑨ n.

⑥ $I k^n \le \varepsilon$

$200 \cdot 0.618^{15} \le \varepsilon$ Zašto to nije u rj?

$\varepsilon \geq 0.1465$

⑪ $F(x) = (x_1-2)^2 + (x_2+1)^2$

$v = [2 \; 1]^T$

$x_0 = (0,0)$

$\varepsilon \le 0.5$

$\Delta\lambda = 1$

$x = x_0 + v\lambda$

$x = (2\lambda, \lambda)$

$F(\cancel{x}) = (2\lambda-2)^2 + (\lambda+1)^2$

$F(\lambda) = 4\lambda^2 - 8\lambda + 4 + \lambda^2 + 2\lambda + 1$
$= 5\lambda^2 - 6\lambda + 5$

λ	-1	0	1	2
$f(\lambda)$	16	5	4	13
	>	>	<	

$\lambda \in [0, 2]$

	a	c	d	b
1	0	0.764	1.236	2
2	0	0.472	0.764	1.236
3	0	0.292	0.472	0.764
	0.292	0.472		0.764

1) $f(c) = 3.334$
$f(d) = 5.22$

2) $f(c) = 3.28$

3) $f(c) = 3.67$

$\cancel{b} - a \le 0.5$

$\lambda \in [0.292, 0.764]$

$x \in [(0.584, 0.292), (1.528, 0.764)]$

⑫ $f(x) = 2 \cdot (x-18)^2$

$x_0 = 0$
$h = 1$
$\varepsilon \le 3$

x	0	1	2	4	8	16	32	64
$f(x)$	684	578	512	392	200	32	392	
	>	>	>	>	>	<		

$\frac{I}{\varepsilon} \le F_N$

$8 \le F_N \Rightarrow F_N = 8$ $x \in [8, 32]$

1, 1, 2, 3, 5, 8, 13

	a	c	d	b
1	8	17	23	32
2	8	14	17	23
3	14	17	20	23
4	14	17	17	20
5	17	17		20

$x \in [17, 20]$

(13) $f(x,y,z) = x^2+y^2+z^2$

$x_0 = (2,3,4)$
$\Delta x = 1 \quad /2$

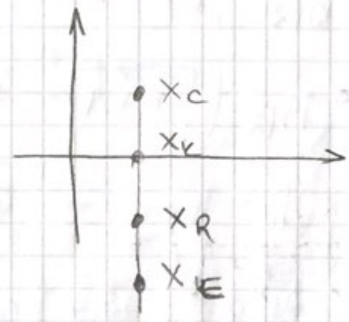
x_B	x_P	x_N	uvjet	
2,3,4	2,3,4	1,2,3	✓	
1,2,3	0,1,2	0,0,1	✓	
0,0,1	-1,-2,-1	0,-1,0	×	$\Delta x = 0.5$
0,0,1	0,0,1	0,0,0.5	✓	
0,0,0.5	0,0,0	0,0,0	✓	
0,0,0	0,0,-0.5	0,0,0	×	$\Delta x = 0.25$

$(0,0,0)$

(14) $(0,0)$
$(2,0)$
$(1,2)$

$F(x) = x_1^2 + x_2^2$

$x_c = (1,0)$



(15) $F(x) = (x_1-2)^2 + (x_2-1)^2$

$x_1 / x_2 \qquad x_0 = 5,5$

postupak traženja u smjerovima koord. osi

+ kaže da se nikad ne koristi samostalno ?

(15) $F(x) = x_1^2 + x_2^2 + x_3^2$

$(1,2,3) \qquad F(1,2,3) = 1+4+9 = 14$
$(0,2,4) \qquad F(0,2,4) = 0+4+16 = 20 \rightarrow$ najlošije
$(-2,0,3) \qquad F(-2,0,3) = 4+0+9 = 13$
$(-4,0,1) \qquad F(-4,0,1) = 16+0+1 = 17$

$$x_c = \left(\frac{1-2-4}{3}, \frac{2}{3}, \frac{3+3+1}{3}\right) = \left(-\frac{5}{3}, \frac{2}{3}, \frac{7}{3}\right)$$

(21) Konstrukcija pravilnog simpleksa

X_0 - poč. toč.

t - razmak između točaka

$$x_j = x_0 + d_j$$

$$d_1 = \begin{bmatrix} a_1 \\ a_2 \\ a_2 \\ \vdots \\ a_2 \end{bmatrix} \quad d_2 = \begin{bmatrix} a_2 \\ a_1 \\ a_2 \\ \vdots \\ a_2 \end{bmatrix} \quad d_n = \begin{bmatrix} a_2 \\ a_2 \\ a_2 \\ \vdots \\ a_1 \end{bmatrix}$$

$$a_1 = \frac{t}{n\sqrt{2}} \left(\sqrt{m+1} + m - 1 \right)$$

$$a_2 = \frac{t}{n\sqrt{2}} \left(\sqrt{m+1} - 1 \right)$$

$$\| x_j - x_0 \| = \| d_j \| = \sqrt{a_1^2 + (n-1) a_2^2}$$

$$a_1^2 + (n-1) a_2^2 = \frac{t^2}{2n^2} \left(n+1 + 2(n-1)\sqrt{n+1} + n^2 - 2n + 1 \right)$$

$$+ (n-1) \frac{t^2}{2n^2} \left(n+1 - 2\sqrt{n+1} + 1 \right)$$

$$= \frac{t^2}{2n^2} \left(n^2 \cancel{-n+2} \cancel{+2(n-1)\sqrt{n+1}} + n^2 \cancel{-1} \cancel{-2(n+1)\sqrt{n+1}} \cancel{+n-1} \right)$$

$$= \frac{t^2}{\cancel{2n^2}} \cdot \cancel{2n^2} = t^2$$

$$\| x_j - x_0 \| = t$$

$$\| x_j - x_i \| = \sqrt{2(a_1 - a_2)^2}$$

$$2(a_1 - a_2)^2 = 2 \left(\frac{t}{\cancel{n}\sqrt{2}} \cdot \cancel{n} \right)^2 = \frac{2t^2}{2} = t^2$$

$$\| x_j - x_i \| = \sqrt{t^2} = t$$

pr//

$n = 2$

$x_0 = 0$

$t = 1$

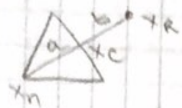
$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad a_1 = \frac{1}{2\sqrt{2}} \left(\sqrt{3} + 1 \right)$$

$$a_2 = \frac{1}{2\sqrt{2}} \left(\sqrt{3} - 1 \right)$$

$$x_1 = \begin{bmatrix} \frac{\sqrt{2}}{4}(\sqrt{3}+1) \\ \frac{\sqrt{2}}{4}(\sqrt{3}-1) \end{bmatrix} \qquad x_2 = \begin{bmatrix} \frac{\sqrt{2}}{4}(\sqrt{3}-1) \\ \frac{\sqrt{2}}{4}(\sqrt{3}+1) \end{bmatrix}$$

3. operacije:

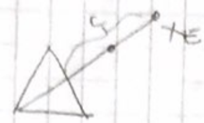
① REFLEKSIJA



$$x_C = x_n + a \cdot v \Longrightarrow v = \frac{1}{a}(x_C - x_n)$$
$$x_R = x_n + (a+b) \cdot v$$
$$x_R = x_n + \left(1 + \frac{b}{a}\right)(x_C - x_n)$$
$$x_R = \left(1 + \frac{b}{a}\right)x_C - \frac{b}{a}x_n \longrightarrow \alpha = \frac{b}{a}$$

KOEFICIJENT REFLEKSIJE

② EKSPANZIJA



$$x_R = x_C + b \cdot v \Longrightarrow v = \frac{1}{b}(x_R - x_C)$$
$$x_E = x_C + c \cdot v$$
$$x_E = x_C + \frac{c}{b}(x_R - x_C)$$
$$x_E = \left(1 - \frac{c}{b}\right)x_C + \frac{c}{b}x_R \longrightarrow \gamma = \frac{c}{b}$$

KOEF. EKSPANZIJE

③ KONTRAKCIJA

$$x_C = x_n + a \cdot v \longrightarrow v = \frac{1}{a}(x_C - x_n)$$
$$x_K = x_n + (a-d)v$$
$$x_K = x_n + \left(1 - \frac{d}{a}\right)(x_C - x_n)$$
$$x_K = \left(1 - \frac{d}{a}\right)x_C + \frac{d}{a}x_n \longrightarrow \beta = \frac{d}{a}$$

KOEF. KONTRAKCIJE

(28) $f(x,y) = x^2 + 4y^2$

$x_0 = 7,3$
$\Delta x = 1 \quad /2 \quad$ do 0.25

x_B	x_P	x_N	uvjet	
7,3	7,3	6,2	✓	
6,2	5,1	4,0	✓	
4,0	2,-2	1,-1	✓	
1,-1	-2,-2	-1,-1	✗	$\Delta x = 0.5$
1,-1	1,-1	0.5,-0.5	✓	
0.5,-0.5	0,0	0,0	✓	
0,0	-0.5,0.5	0,0	✗	$\Delta x = 0.25$

(0,0) //

(29) $f(x,y,z) = (x-1)^2 + y^2 + (z+2)^2$

$x_0 = 4,3,2$

Pitat za x_N

x_B	x_P	x_N		
4,3,2	4,3,2	3,2,1	✓	
3,2,1	2,1,0	1,0,-1	✓	
1,0,-1	-1,-2,-3	0,-1,-2	✗	$\Delta x = 0.5$
1,0,-1	1,0,-1	1,0,-2	✓	
1,0,-2	1,0,-3	1,0,-2.5	✗	$\Delta x = 0.25$

(1,0,-2)

(34)

$T_1(1,2,1)$
$T_2(2,1,1)$
$T_3(3,2,1)$
$T_4(-1,0,1)$

$d(T_1,T_2) = \sqrt{1^2+1^2+0^2} = \sqrt{2}$

$d(T_1,T_3) = \sqrt{2^2+0+0} = 2$

$d(T_1,T_4) = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$d(T_2,T_3) = \sqrt{1+1} = \sqrt{2}$

$d(T_2,T_4) = \sqrt{9+1+0} = \sqrt{10}$

$d(T_3,T_4) = \sqrt{16+2} = \sqrt{18} = 3\sqrt{2}$

⊳ OPTIMIRANJE FJE UZ POMOĆ DERIVACIJA

Najbrži spust

$$\boxed{v = -\frac{\nabla F}{\|\nabla F\|}}$$

pr// $f(x) = (x_1-4)^2 + 4(x_2-2)^2$

$x_0 = (0,0)$

(1 iteracija) $\left.\begin{aligned} \frac{\partial f(x)}{\partial x_1} &= 2(x_1-4) \\ \frac{\partial f(x)}{\partial x_2} &= 8(x_2-2) \end{aligned}\right\} \nabla F$

$$\nabla F(x_0) = \begin{bmatrix} -8 \\ -16 \end{bmatrix} \Rightarrow v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$f(x_0 + \lambda \cdot v) = (\lambda-4)^2 + 4(2\lambda-2)^2 = \lambda^2 - 8\lambda + 16 + 16\lambda^2 - 32\lambda + 16$$
$$= 17\lambda^2 - 40\lambda + 36$$

$$\frac{df}{d\lambda} = 34\lambda - 40 \Rightarrow \lambda = \frac{40}{34}$$

$$\lambda = 1.1765$$

$$x_1 = (1.1765, 2.353)$$

Newton - Raphson

$$f(x) = f(x_0 + \Delta x) = f(x_0) + f'(x_0)\cdot\Delta x + \frac{1}{2} f''(x_0)\cdot\Delta x^2$$

$$\frac{\partial f(x)}{\partial \Delta x} = f'(x_0) + f''(x_0)\,\Delta x = 0$$

$$\boxed{\Delta x = -\frac{f'(x_0)}{f''(x_0)}}$$

$$\boxed{x_{j+1} = x_j - \frac{f'(x_j)}{f''(x_j)}} \qquad \boxed{x_{j+1} = x_j - H^{-1}\big|_{x_j} \nabla F\big|_{x_j}}$$

→

pr// $F(x) = (x_1-4)^2 + 4(x_2-2)^2$

$$\left.\begin{array}{l} \frac{\partial F(x)}{\partial x_1} = 2(x_1-4) \\ \frac{\partial F(x)}{\partial x_2} = 8(x_2-2) \end{array}\right\} \nabla F$$

$$\nabla F = \begin{bmatrix} 2x_1-8 \\ 8x_2-16 \end{bmatrix} \quad ; \quad H = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix} \quad ; \quad H^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/8 \end{bmatrix}$$

$x_0 = (0,0) \qquad \nabla F = (-8,-16)$

$$\Delta x = -H^{-1} \cdot \nabla F = -\begin{bmatrix} 1/2 & 0 \\ 0 & 1/8 \end{bmatrix} \begin{bmatrix} -8 \\ -16 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$X_0' = (1,1), \qquad F(x+\Delta x) = (x_1'-3)^2 + 4(x_2'-1)^2$

$$\nabla F = \begin{bmatrix} 2x_1'-6 \\ 8x_2'-8 \end{bmatrix} \qquad \nabla F = \begin{bmatrix} -6 \\ -8 \end{bmatrix}$$

$$\Delta x = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/8 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

pr// $f(x) = \frac{x^2}{2} - \cos x \quad ; \quad f'(x) = x + \sin x$
$f''(x) = 1 + \cos x$

$\underline{\underline{x_0 = 1}} \qquad f'(x) = 1.841 \qquad f''(x) = 1.54$

$x_1 = x_0 - \frac{1.841}{1.54} = -0.195 \qquad f'(x) = -0.389 \qquad f''(x) = 1.981$

$x_2 = 8{,}6 \cdot 10^{-4}$

konvergira prema 0

pr// $f(x) = \frac{x_1^4}{4} - x_1^2 + 2x_1 + (x_2-1)^2$

$$\left.\begin{array}{l} \frac{\partial F}{\partial x_1} = x_1^3 - 2x_1 + 2 \\ \frac{\partial F}{\partial x_2} = 2(x_1-1) \end{array}\right\} \nabla F = \begin{bmatrix} x_1^3 - 2x_1 + 2 \\ 2x_1 - 2 \end{bmatrix} \qquad \nabla F = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

PITAT NEKOG !

Fletcher - Powell

→ matrica drugih parcijalnih derivacija se ne mora izračunati neposredno već se njezina inverzija aproksimira tijekom postupka, tj u j-toj iteraciji je

$$G_j \approx H_j^{-1}$$

Matrica G_j s gradijentom $\nabla F(x_j)$ određuje smjer v_{j+1}

$$v_{j+1} = - \frac{G_j \nabla F(x_j)}{\| G_j \nabla F(x_j) \|}$$

FJE VIŠE VARIJABLI S OGRANIČENJIMA

- EKSPLICITNA $\boxed{x_D \leq x \leq x_G}$

- IMPLICITNA : nejednadžbe $g_i(x) \geq 0$

  jednadžbe $h_i(x) = 0$

$$|h_i(x)| < \varepsilon \quad \begin{cases} -h_i(x) + \varepsilon \geq 0 \\ h_i(x) + \varepsilon \geq 0 \end{cases}$$

pr// $f(x) = (x-1)^2$
$g(x) = x^2 - 4 \geq 0$
...

→ Transformacija u problem bez ograničenja

$$\boxed{x = x_0 + (x_G - x_D) \cdot f(y)}$$

TRANSF.

1 pr// $G(x) = -t_1 t_2 - t_2 (x_1 - x_2^2)$ $\qquad G(x) = - \sum t_i g_i(x)$

$x_0 = (-2, -1)$

x_B	x_P	x_N	
-2, -1	-2, -1	-1, 0	✓
-1, 0	0, 1		

WTF?

PITAT

pr// $F(x) = x_1^2 + (x_2-1)^2$

$g_1(x) = x_1 - 1 \geq 0$

$h_1(x) = x_2 - x_1 = 0$

$$U(x,r) = F(x) - r \ln g_1(x) + \frac{1}{r} h_1(x)$$

$$= x_1^2 + (x_2-1)^2 - r \ln(x_1 - 1) + \frac{1}{r}(x_2 - x_1)^2$$

$$\frac{\partial U}{\partial x_1} = 2x_1 - \frac{r}{x_1 - 1} - \frac{1}{r} 2(x_2 - x_1)$$

$$\frac{\partial U}{\partial x_2} = 2(x_2 - 1) + \frac{2}{r}(x_2 - x_1) \Rightarrow x_1 = \frac{r}{2}\left(r\, x_2 - 1 + \frac{1}{r} x_2\right)$$

$$= r$$

$$x_{m_1}(r) = \frac{r+3+\sqrt{2r^3+2r^2+6r+1}}{2(r+2)}$$

$$x_{m_2}(r) = \frac{2r^2+5r+3+\sqrt{2r^3+7r^2+6r+1}}{2(r+2)(r+1)}$$

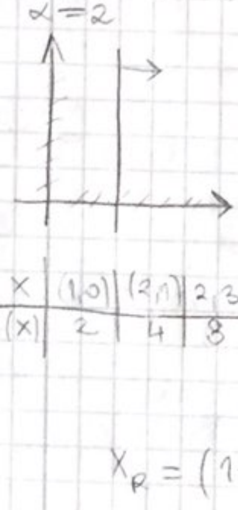
Postupak po Boxu

$F(x)$ $g(x) \geq 0$ $x_D \leq x \leq x_G$ X_c X_R

pr// $F(x) = x_1^2 + (x_2 - 1)^2$ $x_1, x_2 \in [0, +\infty\rangle$ $x_1 - 1 \geq 0$

$(1,0)$ $(2,1)$ $(2,3)$ $(1,3)$

$\alpha = 2$



2 iteracije

1.

x	$(1,0)$	$(2,1)$	$2,3$	$1,3$
$f(x)$	2	4	8	5

$x[h] = 2,3$

$$X_c = \left(\frac{4}{3}, \frac{4}{3}\right)$$

$$X_R = (1+\alpha)\, x_c - \alpha\, x[h] = (4,4) - (4,6) = (0,-2)$$

EKSPL. OGR $\rightarrow$ $x_R' = (0,0)$

IMPL. OGR $\rightarrow$ $X_R'' = \frac{1}{2}(X_c + X_R') = \left(\frac{2}{3}, \frac{2}{3}\right)$ X

$$X_R''' = (1,1)$$

$$F(X_R''') = 1 < F(x_h) = 8 \checkmark$$

2.

X	1,0	2,1	1,1	1,3
F(X)	2	4	1	5

$X_n = (1,3)$

$X_c = \left(\frac{4}{3}, \frac{2}{3}\right)$

$X_R = (4,2) - (2,6) = (2,-4)$

ekspl. ogr. $X_R' = (2,0)$ → kako se dobije?

impl. ogr. $X_R'' = \frac{1}{2}(X_c + X_R') = \frac{5}{3}, \frac{1}{3}$ ✓

$f(X_R') = 5$ $<$ $f(X_n) = 5$ NE!

3

X	1,0	2,1	1,1	$\frac{5}{3}, \frac{1}{3}$
F(X)	2	4	1	$\frac{29}{9}$

Pr// $F(X) = x_1^2 + x_2^2$ $x_1, x_2 \in [1, +\infty\rangle$

$x_1 + x_2 - 3 \geq 0$

$(1,4), (3,3), (1,2), (4,3)$ $\alpha = 2$

1.

X	1,4	3,3	1,2	4,3
F(X)	17	18	5	25

$X_n = (4,3)$

$X_c = \left(\frac{5}{3}, 3\right)$

$X_R = (5,9) - (8,6) = (-3,3)$

EKSPL. OGR. $X_R' = (1,3)$

IMPL. OGR $\cancel{X_R'' = \frac{1}{2}(X_R' + X_c) = \frac{1}{2}(-2,6) = (-1,3)}$

$F(X_R') = 10 < F(X_n) = 25$ ✓

2.

X	1,4	3,3	1,2	1,3
F(X)	17	18	5	10

$X_n = (3,3)$

$X_c = (1,3)$

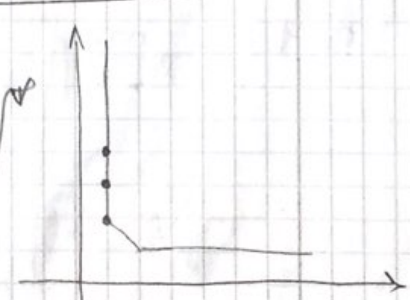
$X_R = (3,9) - (6,6) = (-3,3)$

EKS. $X_R' = (1,3)$

$f(X_R') = 10 < f(X_n) = 18$

3.

X	1,4	1,3	1,2	1,3
F(X)	17	10	5	10



konvergira u točki (1,2)

$$2x_1 - r\frac{1}{x_1-1} - \frac{1}{r}2(x_2-x_1) = 0$$

$$2(x_2-1) + \frac{1}{r}2(x_2-x_1) = 0 \rightarrow 2x_2 - 2 + \frac{2x_2}{r} - \frac{2x_1}{r}$$

$$\hookrightarrow x_1 = \frac{r}{2}\left(2x_2 - 2 + \frac{2x_2}{r}\right)$$

$$r\left(2x_2 - 2 + 2\frac{x_2}{r}\right) - \frac{r}{rx_2 - r + x_2 - 1} - \left(2\cancel{\frac{x_2}{r}} - 2x_2 + 2 - \cancel{\frac{2x_2}{r}}\right)$$

$$= 2x_2r - 2r + 2x_2 - \frac{r}{x_2r - r + x_2 - 1} + 2x_2 - 2 \quad \Big/ \cdot$$

$$= (2x_2r - 2r + 4x_2 - 2)(x_2r - r + x_2 - 1) - r$$

$$= 2x_2^2r^2 - \underline{\underline{2x_2r^2}} + 2x_2^2r - \underline{\underline{2x_2r}} - \underline{\underline{2x_2r^2}} + 2r^2 - \underline{\underline{2x_2r}} + 2r$$
$$+ 4x_2^2r - \underline{\underline{4x_2r}} + 4x_2^2 - \underline{\underline{4x_2}} - \underline{\underline{2x_2r}} + 2r - \underline{\underline{2x_2}} + 2 - r$$

$$= x_2^2(2r^2 + 2r + 4r + 4) - x_2(2r^2 + 2r + 2r^2 + 2r + 4r + 4 + 2r + 2)$$
$$+ (2r^2 + 2r + 2r + 2 - r)$$

$$= x_2^2(2r^2 + 6r + 4) - x_2(4r^2 + 10r + 6) + (2r^2 + 3r + 2)$$

$$x_{1,2} = \frac{4r^2 + 10r + 6 + \sqrt{(4r^2+10r+6)^2 - 4(2r^2+6r+4)(2r^2+3r+2)}}{2(2r^2+6r+4)}$$

$$r = 1 \qquad x_{2m} = \frac{20 + \sqrt{400 - 336}}{24} = \frac{20+8}{24}$$

$$= 1.167$$

FALA BOGU

VIŠE ?

Sustav nelinearnih jednadžbi

Gauss Newtonov postupak

$x_0 \to x$

$g_i(x) = g_i(x_0) + \nabla g^T(x_0)(x - x_0) + \dots$

$G(x) = G(x_0) + J(x_0)(x - x_0)$

pr/ $x_1^2 + x_2^2 - 1 = 0$
$x_1 - 2x_2 = 0$

$x_0 = (1,1)$

$J = \begin{bmatrix} 2x_1 & 2x_2 \\ 1 & -2 \end{bmatrix}$

$J \cdot \Delta x = -G \Rightarrow \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix} \cdot \Delta x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

~~$2\Delta x_1 + 2\Delta x_2 = -1 \Rightarrow 2\Delta x_2 = -1 - 2\Delta x_1$~~
~~$\Delta x_1 = 1 + 2\Delta x_2$~~
~~$\Delta x_1 = 1 - 1 - 2\Delta x_1$~~

$\begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$2\Delta x_1 + 2\Delta x_2 = -1$
$\Delta x_1 - 2\Delta x_2 = 1$

$2 + 4\Delta x_2 + 2\Delta x_2 = -1$

$\Delta x_1 = 0$

$6\Delta x_2 = -3$

$\Delta x_2 = -\frac{1}{2}$

$x_1 = (1, \frac{1}{2})$

$J \cdot \Delta x = -G \Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} -1/4 \\ 0 \end{bmatrix}$

$2\Delta x_1 + \Delta x_2 = -1/4$

$\Delta x_1 = 2\Delta x_2$

$4\Delta x_2 + \Delta x_2 = -\frac{1}{4} \Rightarrow \Delta x_2 = -\frac{1}{20} \quad \Delta x_1 = -\frac{1}{10}$

$x_2 = \left(\frac{9}{10}, \frac{9}{20}\right)$

$x_2 = (0.9, 0.45)$

$G(x_2) = \begin{bmatrix} 0.0125 \\ 0 \end{bmatrix}$

pr// $F(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$

$g_1(x) = 10(x_2 - x_1^2) = 0$
$g_2(x) = 1 - x_1 = 0$

$J = \begin{bmatrix} -20x_1 & 10 \\ -1 & 0 \end{bmatrix} \qquad x_0 = (0,0)$

$J \cdot \Delta x = -G$

$\begin{bmatrix} 0 & 10 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\Delta x_2 = 0 \qquad -\Delta x_1 = -1 \Rightarrow \Delta x_1 = 1$

$x_1 = (1,0)$

$\begin{bmatrix} -20 & 10 \\ -1 & 0 \end{bmatrix} \Delta x = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow \quad x_2 = (1,1)$

$\Delta x_1 = 0 \quad , \Delta x_2 = 1$

G-N za predefiniran sustav

$n \to$ varijabli

$m \to$ jednadžbi

$$\boxed{J^T J \, \Delta x = -J^T G}$$

pr// $y = f(t)$

(t_i, y_i) ; $(1,2), (0,4), (-1,4)$,

$f(t) = x_1 + x_2 t + x_3 t^3$

pr// $x + 1 = 0$
$2x^2 + x - 1 = 0$ $\qquad J = \begin{bmatrix} 1 \\ 4x+1 \end{bmatrix} \qquad J^T = \begin{bmatrix} 1 & 4x+1 \end{bmatrix}$

$x_0 \Rightarrow J^T J \cdot \Delta x = -J^T G(x_0) \qquad G(x_0) = [$

$J^T \cdot J = 16x^2 + 8x + 2$

$J^T G = \begin{bmatrix} 1 & 4x+1 \end{bmatrix} \begin{bmatrix} x+1 \\ 2x^2+x-1 \end{bmatrix} = x + \cancel{1} + 8x^3 + 6x^2 - 4x + x - \cancel{1}$
$= 8x^3 + 6x^2 - 2x$

$x_{k+1} = x_k - \dfrac{8x^3 + 6x^2 - 2x}{16x^2 + 8x + 2}$

$x_0 = 3 \Rightarrow x_1 = \longrightarrow 0.25$
$x_0 = -2 \longrightarrow -1$
$x_0 = 0 \longrightarrow 0$

Postupak Levenberg-Marquardt

$$\left.\boxed{\begin{array}{c} F(x) = \frac{1}{2}\sum g_i^2(x) \\ \hline \nabla F = J^T G(x) \end{array}}\right\} (J^T J + \mu I)\Delta x = -J^T G$$

MI 2018

①

ALGORITAM	FJA	KORACI
KO	kvadratna nekvadratna	1 korak >1 korak
GS	sve	>1 korak
NR	kvadratna nekvadratna	1 korak nepoznato
PW	kvadratna nekvadratna	1 korak n koraka
FP	sve	n koraka

$f(x) = 4\cdot(x_1-5)^2 + (x_2+2)^2$

a) FP $\geq$ PW b) KO = NR c) KO < GS d) FP > NR

e) KO < FP f) PW < GS g) NR $\leq$ PW h) KO $\leq$ PW

②

$$\begin{bmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{bmatrix} \to \begin{bmatrix} -8 & 9 & 3 \\ -5/8 & 3/8 & -1/8 \\ -1/2 & 3/2 & -1/2 \end{bmatrix} \to \begin{bmatrix} -8 & 9 & 3 \\ -1/2 & -1/2 & -1/2 \\ -5/8 & 3/4 & 1/4 \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$LU = PA$

$L \cdot U \cdot x = P \cdot E = P$

$L\, y_i = P\, e_i$ → unapr.

$U \cdot x_i = y_i$ → unatrag

① $\begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -5/8 & 3/4 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ → $y_1 = 0$, $y_2 = 1$, $y_3 = -3/4$

$\begin{bmatrix} -8 & 9 & 3 \\ 0 & -1/2 & -1/2 \\ 0 & 0 & 1/4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3/4 \end{bmatrix}$ $x_1 = 0$, $x_2 = 1$, → $x_3 = -3$

② $L \cdot y' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ → $y_1 = 0$, $y_2 = 0$, $y_3 = 1$

$U \cdot x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $x_1 = -3$, $x_2 = -4$, $x_3 = 4$

③ $L \cdot y = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ → $y_1 = 1$, $y_2 = 1/2$, $y_3 = 1/4$

$U \cdot x = \begin{bmatrix} 1 \\ 1/2 \\ 1/4 \end{bmatrix}$ $x_1 = -2$, $x_2 = -2$, $x_3 = 1$

3. $f(x) = (x_1-2)^2 + (x_2-3)^2$

$v_1 = [2 \ -1]^T$

$v_2 = ?$ (konj.) FIB v_2

$x_0 = (-13, -27)$ $\varepsilon \leq 3$

$h = 1$

~~$\frac{\partial f}{\partial x_1} = 2(x_1-2)$ $2(-15) = -30$~~

~~$\frac{\partial f}{\partial x_2} = 2(x_2-3)$ $2(-30) = -60$~~

$$F(x) = x_1^2 - 4x_1 + 4 + x_2^2 - 6x_2 + 9$$

$$= 13 + [x_1 \ x_2]\begin{bmatrix}-4\\-6\end{bmatrix} + \frac{1}{2}\cdot[x_1 \ x_2]\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

$v_i^T A v_j = 0$ $\quad [2 \ -1]\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}v_{21}\\v_{22}\end{bmatrix} = 0$

$$[4 \ -2]\begin{bmatrix}v_{21}\\v_{22}\end{bmatrix} = 0$$

$$4v_{21} - 2v_{22} = 0$$

$$4v_{21} = 2v_{22}$$

$$v_{21} = \frac{1}{2}v_{22}$$

$v_2 = \begin{bmatrix}1\\2\end{bmatrix}$

$$F(x_0 + \lambda v) = F(-13+\lambda, -27+2\lambda)$$

$$= (-13+\lambda-2)^2 + (-27+2\lambda-3)^2$$

$$= (\lambda-15)^2 + (2\lambda-30)^2$$

λ	-1	0	1	2	4	8	16	32
$f(\lambda)$	1280	1125	980	845	605	245	5	1445

> > > > > > <

$\lambda \in [8, 32]$

$\frac{I_0}{\varepsilon} \leq \varphi_N$

$\frac{24}{3} \leq \varphi_N \Rightarrow \varphi_N = 8$

a	c	d	b
8	17	23	32
8	14	17	23
8	11	14	17
11	14	14	17
14	14		17

$\Rightarrow \leq 3$

$\lambda \in [14, 17]$

4. a) $f(x) = (x+2)^2$

$x \in [-2, 5]$

Odstranjivanje ekspeicitnih ograničenja

$\rightarrow$ uvodi se nova varijabla

$x = x_D + (x_G - x_D) \cdot f(y) \rightarrow \sin^2 y_i, \frac{1}{1+e^{-ay_i}}$

$f(x) = (\cancel{-2} + \cancel{2} (5+2)\, \sigma(x))^2$

$f(x) = 49\, \sigma^2(x)$

b) $f(x) = (x_1+2)^2 + (x_2+7)^2$

$x_1 + x_2 - 3 = 0$

Pretvaranje u problem u obliku nejednakosti

$|h(x)| < \varepsilon$

$x_1 + x_2 - 3 + \varepsilon \geq 0 \qquad -x_1 - x_2 + 3 + \varepsilon \geq 0$

c) $f(x) = (x_1-2)^2 + (x_2+3)^2$

$x_1 + x_2 = 0$

$x_2 \leq 0 \rightarrow -x_2 \geq 0$

$x_1 + x_2 \leq 4 \quad 4 - x_1 - x_2 \geq 0$

1) Pretvaranje u problem bez ograničenja na mješovit način

$t = 0.5 \qquad t = \frac{1}{r} \rightarrow r = 2$

$U(x, r) = F(x) - r \sum \ln g_i(x) + \frac{1}{r} \sum h_j^2(x)$

$= (x_1-2)^2 + (x_2+3)^2 - 2\ln(-x_2) - 2\ln(4 - x_1 - x_2) + \frac{1}{2}(x_1+x_2)$

2) $(-1, 1)$

Može li bit poč točka? NE, ne zadovoljava g_2

3) $T = (-1, -1)$

$$\nabla F = \begin{bmatrix} 2(x_1-2) + (x_1+x_2) \\ 2(x_2+3) + (x_1+x_2) \end{bmatrix} = \begin{bmatrix} 3x_1 + x_2 - 4 \\ 3x_2 + x_1 + 6 \end{bmatrix}$$

$$\nabla F(T) = \begin{bmatrix} -8 \\ 2 \end{bmatrix} \rightarrow v = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$f(x + \lambda v) = (-1 + 4\lambda, -1 - \lambda)$

$f(\lambda) = (-3 + 4\lambda)^2 + (2 - \lambda)^2 + \frac{1}{2}(-2 + 3\lambda)^2$

$= 9 - 24\lambda + 16\lambda^2 + 4 - 4\lambda + \lambda^2 + 2 - 6\lambda + \frac{9}{2}\lambda^2$

$= 21.5\lambda^2 - 34\lambda + 15 \longrightarrow$

$21.5\lambda^2 - 34\lambda + 15$

$$\frac{\partial F(\lambda)}{\partial \lambda} = 43\lambda - 34 \Rightarrow \lambda = 0.7907$$

$x = (2.163,\ -1.7907)$

min na pravcu određen analitički

5. $f(x) = -(x_1-2)^2 - (x_2-5)^2 - (x_3+2)^2$

max ?

$(-1, 4, 2)$ H. J

$\Delta x = 1$

$/2$ do 0.25

max $= (2, 5, -2)$

x_B	x_p	x_N		
-1, 4, 2	-1, 4, 2	0, 5, 1	✓	
0, 5, 1	1, 6, 0	2, 5, -1	✓	
2, 5, -1	4, 5, -3	3, 5, -2	×	$\Delta x = 0.5$
2, 5, -1	2, 5, -1	2, 5, -1.5	✓	
2, 5, -1.5	2, 5, -2	2, 5, -2	✓	
2, 5, -2	2, 5, -2.5	2, 5, -2	×	$\Delta x = 0.25$ kraj

6.

x	1, 1	1, 3
$f(x)$	2	4

G.N. $f(x) = a^2 x_1^2 + b x_2 = 4x_1^2 + 2x_2$

$g_1 : a^2 + b - 2 = 0$

$g_2 : a^2 + 3b - 4 = 0$

$$\Delta x = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} \qquad -G(x_0) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

$$J = \begin{bmatrix} g_1'a & g_1'b \\ g_2'a & g_1'b \end{bmatrix}$$

$$\begin{bmatrix} 2a & 1 \\ 2a & 3b \end{bmatrix} \cdot \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = -\begin{bmatrix} a^2+b-2 \\ a^2+3b-4 \end{bmatrix}$$

$2a\Delta x_1 + \Delta x_2 = -(a^2+b-2) \Rightarrow 2a\Delta x_1 = -(a^2+b-2) - \Delta x$

$2a\Delta x_1 + 3b\Delta x_2 = a^2 + 3b - 4$

$\Rightarrow a^2 + b - 2 - \Delta x_2 + 3b\Delta x_2 = a^2 + 3b - 4$

$a = 2$
$b = 2$

$$\Delta x_2 = \frac{2b-6}{3b-1} = \frac{-2}{2} = -1$$

$4\Delta x_1 =$

$$\begin{bmatrix} 2a & 1 \\ 2a & 3 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = -G(x_0) = -\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$2a\Delta x_1 + \Delta x_2 = -4$$

$$2a\Delta x_1 + 3\Delta x_2 = -6$$

$$\rightarrow -4 - \Delta x_2 + 3\Delta x_2 = -6$$

$$2\Delta x_2 = -2$$

$$\Delta x_2 = -1$$

$$\Delta x_1 = -3/4$$

$$x_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -3/4 \\ -1 \end{bmatrix} = \begin{bmatrix} 5/4 \\ 1 \end{bmatrix}$$

II. iter.

$$J(x_1) \cdot \Delta x_2 = -G(x_1)$$

$$\begin{bmatrix} 5/2 & 1 \\ 5/2 & 3 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_{21} \\ \Delta x_{22} \end{bmatrix} = -\begin{bmatrix} a^2+b-2 \\ a^2+3b-4 \end{bmatrix} = \begin{bmatrix} -9/16 \\ -9/16 \end{bmatrix}$$

$$\frac{5}{2}\Delta x_{21} + \Delta x_{22} = -\frac{9}{16}$$

$$\frac{5}{2}\Delta x_{21} + 3\Delta x_{22} = -\frac{9}{16}$$

$$\Delta x_{22} = 0$$

$$\Delta x_{21} = -\frac{9}{40}$$

$$x_2 = \begin{bmatrix} 5/4 \\ 1 \end{bmatrix} + \begin{bmatrix} -9/40 \\ 0 \end{bmatrix} = \begin{bmatrix} 41/40 \\ 1 \end{bmatrix}$$

//

7. $f(x) = (x_1-1)^2 + x_2^2$

$(x_1-1)^2 + (x_2-1)^2 \geq 1$

$x_1 \in [1, +\infty\rangle \quad x_2 \in [1, +\infty\rangle$

$(1,2) \ (1,3) \ (1,4) \ (7,1) \quad \alpha = 2$

x	1,2	1,3	1,4	7,1
$f(x)$	4	9	16	37

$x_h = (7,1) \quad x_c = (1,3)$

$x_R = (3,9) - (14,2) = (-11,7)$

EKSPL. $\quad x_R' = (1,7) \quad X$

IMPL. $\quad \checkmark$

EKSPL $\quad x_R'' = \frac{1}{2}(x_c + x_R') = \frac{1}{2}((1,3) + (1,7))$

$= (1,5) \quad \checkmark$

II. iter.

x	1,2	1,3	1,4	1,5
$f(x)$	4	9	16	25

$x_h = (1,5)$

$x_c = (1,3)$

$x_R = (3,9) - (2,10) = (1,-1)$

EKSP. $\quad x_R' = (1,1)$

IMPL. $\quad x_R'' = \frac{1}{2}((1,3) + (1,1)) = (1,2) \quad \checkmark$

III. iter

x	1,2	1,3	1,4	1,2
$f(x)$	4	9	16	4

$x_h = (1,4)$

$x_c = (1, \frac{7}{3})$

MI 2015/2016

(2) LUP

$$\begin{bmatrix} 2 & 2 & 4 \\ 4 & 2 & 2 \\ 2 & 3 & 11 \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 4 \\ 4 & 2 & 2 \\ 2 & 3 & 11 \end{bmatrix} \to \begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 4 \\ 2 & 3 & 11 \end{bmatrix} \Bigg| \begin{bmatrix} 6 \\ 2 \\ -3 \end{bmatrix} \to \begin{bmatrix} 4 & 2 & 2 \\ 1/2 & 1 & 3 \\ 1/2 & 2 & 10 \end{bmatrix} \to \begin{bmatrix} 4 & 2 & 2 \\ 1/2 & 2 & 10 \\ 1/2 & 1 & 3 \end{bmatrix} \Bigg| \begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix}$$

$$\to \begin{bmatrix} 4 & 2 & 2 \\ 1/2 & 2 & 10 \\ 1/2 & 1/2 & -2 \end{bmatrix} \qquad L \cdot U \cdot x = Pb$$

$$L \cdot y = Pb \implies \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 1/2 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} \to \begin{matrix} y = 6 \\ y_2 = -6 \\ y_3 = 2 \end{matrix}$$

$$U \cdot x = y \implies \begin{bmatrix} 4 & 2 & 2 \\ 0 & 2 & 10 \\ 0 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 2 \end{bmatrix} \implies \begin{matrix} x_1 = 1 \\ x_2 = 2 \\ x_3 = -1 \end{matrix}$$

* $\det(A) = \det(P) \cdot \det(L) \cdot \det(U)$

$= (+1) \cdot 1 \cdot (4 \cdot 2 \cdot -2) = -16$

(3) $f(x) = (x_1 - 1)^2 + (x_2 + 5)^2$

Zlatni rez, pronaći min na pravcu odr. s $\vec{v} = [1,1]$, $x_0 = (2,2)$

$h = 1$
$\varepsilon = 0.25$

$$f(x + \lambda v) = (x_1 + \lambda - 1)^2 + (x_2 + \lambda + 5)^2$$
$$= (\lambda + 1)^2 + (\lambda + 7)^2$$

λ	2	1	0	-1	-2	-4	-8	-16
$f(\lambda)$	90	76	50	36	26	25	50	

$\lambda \in [-8, -2]$

18.17

	a	c	d	b
$f(c) > f(d)$	-8	-5.708	-4.292	-2
$f(c) < f(d)$	-5.708	-4.292	-3.416	-2
$f(c) > f(d)$	-5.706	-4.831	-4.292	-3.416
$f(c) > f(d)$	-4.831	-4.292	-3.956	-3.416
$f(c) < f(d)$	-4.292	-3.956	-3.75	-3.416
$f(c) < f(d)$	-4.292	-4.085	-3.956	-3.75
$f(d) > f(d)$	-4.292	-4.164	-4.085	-3.956
	-4.164	-4.085		-3.956

$\to$ 19.38
$\to$ 18.004 18,1243
$\to$ 18.01 $(f(c))$
$\to$ $f(c) = 18.05$

$\to \lambda \in [-4.164, -3.956]$

4. $f(x) = (x_1-3)^2 + (x_2+1)^2 + (x_3-5)^2$

$X_0 = (-3, 0, -5)$
$\Delta x = 1$
$\varepsilon = 0.25$

min $(3, -1, 5)$

H.J.

X_B	X_P	X_N	uvjet
-3, 0, -5	-3, 0, -5	-2, -1, -4	✓
-2, -1, -4	-1, -2, -3	0, -1, -2	✓
0, -1, -2	2, -1, 0	3, -1, 1	✓
3, -1, 1	6, -1, 4	5, -1, 5	✓
5, -1, 5	7, -1, 9	6, -1, 8	×
5, -1, 5	5, -1, 5	4.5, -1, 5	✓
4.5, -1, 5	4, -1, 5	3.5, -1, 5	✓
3.5, -1, 5	2.5, -1, 5	3, -1, 5	✓
3, -1, 5	2.5, -1, 5	3, -1, 5	×

$\Delta x = 0.5$

$\Delta x = 0.25$

5. $f(x) = (x_1-2)^2 + (x_2+1)^2$

$x_1 - x_2 = -2$
$x_1 + 2x_2 + 1 > 0$
$t = 0.5 \longrightarrow r = 2$
+ G.S. $X_0 = (0, 0)$

$(x_1-x_2+2)(x_1-x_2+2)$
$= x_1^2 - x_1x_2 + 2x_1 - x_1x_2 + x_2^2 - 2x_2 + 2x_1 - 2x_2 + 4$
$= x_1^2 + x_2^2 - 2x_1x_2 + 4x_1 - 4x_2 + 4$

$U(x,r) = F(x) - 2\ln(x_1+2x_2+1) + \frac{1}{2}(x_1-x_2+2)^2$

$= x_1^2 - 2x_1 + x_2^2 + 2x_2 + 5 + \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 - x_1x_2 + 2x_1 - 2x_2 + 2 - \nabla$

(NEDAMISE RASPISIVAT KAKO SAM IMALA
sve skupa
NAJBOLJU VOLJU, RIP VOLJO.)

$$\nabla F = \begin{bmatrix} 2(x_1-2) - \frac{2}{x_1+2x_2+1} + (x_1-x_2+2) \\ 2(x_2+1) - \frac{4}{x_1+2x_2+1} - x_1+x_2-2 \end{bmatrix}$$

$X_0 = (0,0)$

$$\nabla F(X_0) = \begin{bmatrix} -4-2+2 \\ 2-4-2 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \Rightarrow v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$f(X_0 + \lambda \cdot v) = (\lambda-2)^2 + (\lambda+1)^2 - 2\ln(\lambda+2\lambda+1) + \frac{1}{2}(\cancel{\lambda}-\cancel{\lambda}+2)^2$$
$$= (\lambda-2)^2 + (\lambda+1)^2 - 2\ln(3\lambda+1) + 2$$

$$\frac{\partial f}{\partial \lambda} = 2(\lambda-2) + 2(\lambda+1) - \frac{6}{3\lambda+1}$$
$$\rightarrow 4\lambda - 4 + 2\lambda + 2 \Rightarrow (\text{bez } \ln) \quad 6\lambda = 2 \rightarrow \lambda = \frac{1}{3}$$

$X = \left(\frac{1}{3}, \frac{1}{3}\right)$

6. $f(x) = x_1^2 + (x_2-2)^2$

$x_1 - x_2 \geq 0$

$x_1, x_2 \in [0, 10]$

Box: $(1,0), (2,1), (3,2)\ (4,0.5)$

$\alpha = 2$

x	1,0	2,1	3,2	4,0.5
$f(x)$	5	5	9	18.25

$x_h = (4, 0.5)$

$x_c = (2,1)$

$x_R = (6,3) - (8,1) = (-2,2)$

EKSP $x_R' = (0,2)$

IMPL $x_R'' = \frac{1}{2}((0,2) + (2,1)) = (1, 1.5)$ ✓

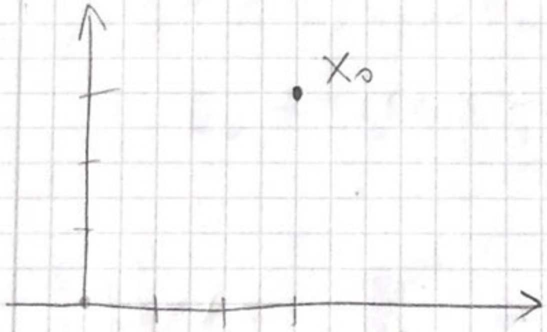
$x_R'' = \frac{1}{2}((2,1) + (1, 1.5)) = (1.5, 1.25)$

+ FALI KONF. ZA KOJU NE KONVERGIRA

7. $f(x) = x_1^2 + 2x_2^2$

$x_0 = 3,3$

KO + GS + graf.



$$U(x,r) = F(x) + \frac{1}{2}\underbrace{(x_1 - x_2 + 2)^2 + \frac{1}{2}(x_1 + 2x_2 + 1)^2}$$

$$= (x_1 - 2)^2 + (x_2 + 1)^2 + \text{—}$$

$$\nabla F = \begin{bmatrix} 2x_1 - 4 + x_1 - x_2 + 2 + x_1 + 2x_2 + 1 \\ 2x_2 + \cancel{2} - x_1 + x_2 - \cancel{2} + 2x_1 + 4x_2 + 2 \end{bmatrix}$$

$$\nabla F = \begin{bmatrix} 4x_1 + x_2 - 1 \\ 7x_2 + x_1 + 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \rightarrow v = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$f(x + \lambda v) = (\lambda - 2)^2 + (1 - 2\lambda)^2 + \frac{1}{2}(2 + 3\lambda)^2 + \frac{1}{2}(1 - 3\lambda)^2$$

$$\frac{\partial f}{\partial \lambda} = 2(\lambda - 2) - 4(1 - 2\lambda) + 3(2 + 3\lambda) - 3(1 - 3\lambda)$$

$$2\lambda - 4 - 4 + 8\lambda + 6 + 9\lambda - 3 + 9\lambda = 0$$

$$28\lambda = 5$$

$$\lambda = \frac{5}{28}$$

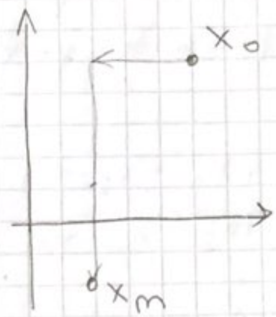
<u>Zadaci što riješila nisam</u>

19, 20 | 23 | – | 40 | 16 | 34 | 21 | 27, 28 | 42 | 22, ~~[illegible]~~ | 41 | 18 | ~~15~~ | 29 | 20

31 | 35, 36 | 39 | 30 | 38 | 26 | 25 | 33 | 32 |

15. $F(x) = (x_1 - 2)^2 + (x_2 + 1)^2$

$x_0 = (5, 5)$



$$F[(5,5) + \lambda(1,0)] = F(5+\lambda, 5) = (3+\lambda)^2 + 36$$

$$= 9 + 6\lambda + \lambda^2 + 36$$

$$= \lambda^2 + 6\lambda + 45$$

$$\frac{\partial F}{\partial \lambda} = 2\lambda + 6 \Rightarrow \lambda = -3$$

$$x_1 = (2, 5)$$

$$F[(2,5) + \lambda(0,1)] = F(2, 5+\lambda) = (6+\lambda)^2$$

$$\frac{\partial F}{\partial \lambda} = 12 + 2\lambda \Rightarrow \lambda_2 = -6$$

$$x_2 = (2, -1)$$

16. $F(x,y) = |(x-y)(x+y)| + \sqrt{(x^2+y^2)}$

N.S.

min $(0,0)$

$\to X_0 = (2,0)$

$$\frac{\partial F}{\partial x} = x \cdot \left(\frac{2(x-y)(x+y)}{|(x-y)(x+y)|} + \frac{1}{\sqrt{x^2+y^2}} \right)$$

$$\frac{\partial F}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} - \frac{2y(x-y)(x+y)}{|(x-y)(x+y)|}$$

$$\nabla F|_{x_0} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$F(x_0 + \lambda v) = (2+\lambda, 0) = |(2+\lambda)^2| + \sqrt{(2+\lambda)^{\cancel{2}}} = 4 + 4\lambda + \lambda^2 + 2 + \lambda$$

$$= \lambda^2 + 5\lambda + 6$$

$$\frac{\partial F}{\partial \lambda} = 2\lambda + 5 \Rightarrow \lambda = -\frac{5}{2}$$

$$x_1 = (2,0) + \left(-\frac{5}{2}, 0\right) = \left(-\frac{1}{2}, 0\right)$$

$$\nabla F|_{x_1} = \begin{bmatrix} -5/4 \\ 0 \end{bmatrix} \to v =$$

NEDAMISE, al neće doć do rj. jer λ bude 0 u drugoj iteraciji i algoritam zapne.

18. Neće se dobit rješenje jer zapne na rubu.

22. ① KO

② HJ

③ POW

④ NM

⑥ ?!

Kvantna računala (SI) Spregnuta stanja

28 ① Suma kvadrata

② Težinska suma kvadrata

③ —||— parnih potencija

④ Maksimalna apsolutna vrijednost

25. $f(x) = x\ln(x)$

N.R

$x_0 = 1$

$f'(x) = \ln(x) + 1$

$f''(x) = \frac{1}{x}$

$\Delta x = -\frac{f'(x_0)}{f''(x_0)} = -\frac{1}{1} = -1$

$x_1 = 1 - 1 = 0$

30. $F(x) = (x_1 - 1)^2 + (x_2 + 3)^2$

N.S.

$x_0 = (0, 0)$

$\left.\begin{aligned} \frac{\partial F}{\partial x_1} &= 2x_1 - 2 \\ \frac{\partial F}{\partial x_2} &= 2x_2 + 6 \end{aligned}\right\}$ $\nabla F = \begin{bmatrix} 2x_1 - 2 \\ 2x_2 + 6 \end{bmatrix}$ $\nabla F|_{x_0} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$F(x_0 + \lambda v) = (\lambda, -3\lambda) = (\lambda - 1)^2 + (3 - 3\lambda)^2$

$\frac{\partial F}{\partial \lambda} = 2\lambda - 2 + 6 - 6\lambda \implies \lambda = 1$

$x_1 = (1, -3)$

jedna iteracija postupka

31. $F(x) = (x_1+3)^2 + (x_2-2)^2$

K.O

$x_0 = (1,1)$

$F\left[(1,1) + \lambda(1,0)\right] = F(1+\lambda, 1) = (\lambda+4)^2 + 1$

$= \lambda^2 + 8\lambda + 17$

$\frac{\partial F}{\partial \lambda} = 2\lambda + 8 \Rightarrow \lambda = -4$

$x_1 = (-3,1)$

$F\left[(-3,1) + \lambda(0,1)\right] = F(-3, 1+\lambda) = (\lambda-1)^2$

$\frac{\partial F}{\partial \lambda} = 2\lambda - 2 \Rightarrow \lambda = 1$

$x_2 = (-3,2)$

32. $F(x) = a(x_1-1)^2 + b(x_2+3)^2$

N.S

$x_0 = 2,0$

$\left.\begin{aligned} \frac{\partial F}{\partial x_1} &= 2a(x_1-1) \\ \frac{\partial F}{\partial x_2} &= 2b(x_2+3) \end{aligned}\right\} \nabla F = \begin{bmatrix} 2a(x_1-1) \\ 2b(x_2+3) \end{bmatrix} \cdot \nabla F|_{x_0} = \begin{bmatrix} 2a \\ 6b \end{bmatrix}$

$V = \begin{bmatrix} a \\ 2b \end{bmatrix}$

33. $F(x) = (x_1-2)^2 + (x_2+1)^2$

$V = [0\ 1]^T$

$T = (0,0)$ $F(0,\lambda) = 4 + (\lambda+1)^2$ $\frac{\partial F}{\partial \lambda} = 2\lambda + 2 = 0 \Rightarrow \lambda = -1$

$x_1 = (0,-1)$

$T = (2,1)$ $F(2, 1+\lambda) = (\lambda+2)^2$ $\frac{\partial F}{\partial \lambda} = 2\lambda = -4 \Rightarrow \lambda = -2$

$x_2 = (2,-1)$

$V_2 = M_2 - M_1 = (2,-1) - (0,-1) = (2,0) \rightarrow V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

35. $F(x) = -x_1 x_2 |x_1 - x_2|$

$x_1 + x_2 - 8 \leq 0 \longrightarrow -x_1 - x_2 + 8 \geq 0$

$x_1, x_2 \in [-3, 3]$

$\alpha = 2$

x	0,0	1,3	2,1	3,2
$F(x)$	0	-6	-2	-6

$x_h = (0,0)$

$x_c = (2,2)$

$x_R = (6,6)$ ✓

EKSPL. ✓

IMPL. NE

$x_R' = \frac{1}{2}(6,6) = (3,3)$ ✓

Rj: (3,3), (1,3), (2,1), (3,2)

36. nj. na predavanju

37. —||—

38. $F(x) = (x_1 - 4)^2 + 4(x_2 - 2)^2$

$x_2 - x_1 \geq 0$
$2 - x_1 \geq 0$
$x_1 + x_2 - 4 = 0$

$U(x, r) = F(x) - r\ln(x_2 - x_1) - r\ln(2 - x_1) + \frac{1}{r}(x_1 + x_2 - 4)$

39. $F(x) = x_1 + x_2 - x_1 x_2$

$|x_1 x_2| - 8 \leq 0 \longrightarrow x_1 x_2 - 8 \leq 0 \quad -x_1 x_2 - 8 \leq 0$

$x_1, x_2 \in [-10, 10]$, $\alpha = 2$

x	2,4	2,0	4,2	1,1
$f(x)$	-2	2	-2	1

$x_h = 2,0$

$x_c = \left(\frac{7}{3}, \frac{7}{3}\right) \quad x_R = (7,7) - (4,0) = (3,7)$

EKSPL ✓

IMPL ✗ $x_R' = \left(\frac{8}{3}, \frac{14}{3}\right)$ ✗ $\quad x_R'' = \left(\frac{5}{2}, \frac{7}{2}\right)$ ✗

$x_R''' = \left(\frac{29}{12}, \frac{35}{12}\right)$ ✓ Rj: $(2,4), \left(\frac{29}{12}, \frac{35}{12}\right), (4,2), (1,1)$

40. $U(x,t) = F(x) - t\left[\ln(x_1 - x_2) + \ln(2 + x_2)\right] + \frac{1}{t}(x_1+4)^2$

$g_1 \Rightarrow x_1 - x_2 \geq 0$

$g_2 \Rightarrow 2 + x_2 \geq 0$

$h_1 \Rightarrow x_1 + 4 = 0$

41. $x_i = x_G - y_i^2$

$x_i = x_G - \text{abs}(y_i)$

$x_i = x_G - e^{y_i}$

42. $F(x) = x_1^2 + (x_2 - 1)^2$

$|x_1 x_2| - 8 \leq 0 \longrightarrow -|x_1 x_2| + 8 > 0$

$x_1, x_2 \in [-10, 10]$

$\alpha = 2$

x	2,4	1,0	4,2	0,2
$f(x)$	13	2	17	1

$x_h = (4,2)$

$x_c = (1,2)$

$x_R = (3,6) - (8,4) = (-5,2)$

EKSPL ✓

IMPL ✗ $x_R' = \frac{1}{2}(-4,4) = (-2,2)$

x	2,4	1,0	-2,2	0,2
$f(x)$	13	2	5	1

$x_h = 2,4$

$x_c = \left(-\frac{1}{3}, \frac{4}{3}\right)$

$x_R = (-1,4) - (4,6) = (-5,-2)$

EKSP ✓

IMPL. ✗ $x_R' = \frac{1}{2}\left(-\frac{16}{3}, -\frac{2}{3}\right) = \left(-\frac{8}{3}, -\frac{1}{3}\right)$ ✓

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7. $f(x) = x_1^2 + 2x_2^2$

$x_0 = 3,3$

KO GS

$$f(x_0 + \lambda(1,0)) = (3+\lambda, 3) = (3+\lambda)^2 + 18$$

$$\frac{\partial f}{\partial \lambda} = 6 + 2\lambda \Rightarrow \lambda = -3$$

$$x_1 = (0,3)$$

$$f(x_1 + \lambda(0,1)) = (0, 3+\lambda) = 0 + 2(3+\lambda)^2$$

$$\frac{\partial f}{\partial \lambda} = 12 + 4\lambda \rightarrow \lambda = -3$$

$$x_2 = (0,0) \text{ //}$$

$$\left.\begin{aligned} \frac{\partial F}{\partial x_1} &= 2x_1 \\ \frac{\partial F}{\partial x_2} &= 4x_2 \end{aligned}\right\} \nabla F = \begin{bmatrix} 2x_1 \\ 4x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix} \Rightarrow V = \begin{bmatrix} +1 \\ +2 \end{bmatrix}$$

$$f(x + \lambda V) = f(3+\lambda, 3+2\lambda) = (3+\lambda)^2 + 2(3+2\lambda)^2$$

$$\frac{\partial f}{\partial \lambda} = 6 + 2\lambda + 24 + 16\lambda \rightarrow 18\lambda = -30$$

$$\lambda = -\frac{5}{3}$$

$$x = \left(3 - \frac{5}{3}, 3 - \frac{10}{3}\right) = \left(\frac{4}{3}, -\frac{1}{3}\right)$$

$$f(x + \lambda V) = f\left(\frac{4}{3} + \lambda, -\frac{1}{3} + 2\lambda\right) = \left(\frac{4}{3} + \lambda\right)^2 + 2\left(2\lambda - \frac{1}{3}\right)^2$$

$$\frac{\partial f}{\partial \lambda} = \cancel{\frac{8}{3}} + 2\lambda + 16\lambda - \cancel{\frac{8}{3}} \rightarrow 18\lambda = 0$$

$$\lambda = 0$$