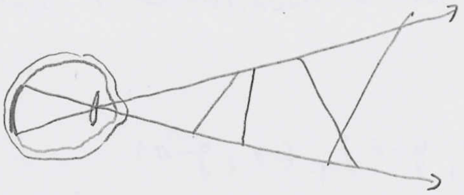
3.1 Perspektivna projekcija je projekcija 3D scene u 2D sliku
- svaka točka u 3D prostoru se preslikava u jednu točku u 2D slici (gubi se 3D informacija)
  - jedna točka na retini može nastati od beskonačno puno točaka u prostoru (uzduž jednog pravca)
  - jedna linija na retini može biti rezultat više mogućih linija u 3D prostoru

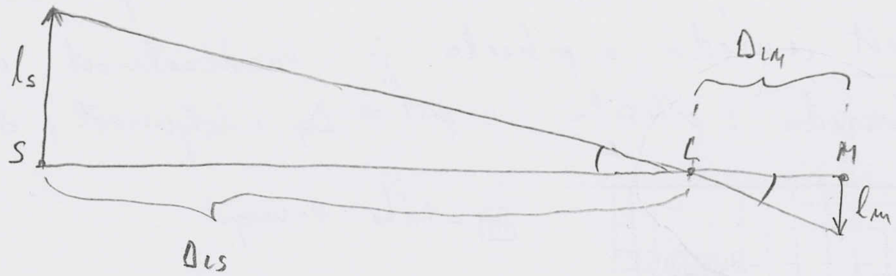


$D_{LS} = 100\,m$ (od leće do slike)

$D_{LM} = 17\,mm$ (od leće do mrežnice

$l_S = 15\,m$

$l_M = ?$



$$\frac{l_M}{l_S} = \frac{D_{LM}}{D_{LS}}$$

$$l_M = \frac{D_{LM}}{D_{LS}} \cdot l_S$$

$$l_M = \frac{17}{100\,000} \cdot 15\,000$$

$$\boxed{l_M = 2.55\,mm}$$

4. 4-susjedstvo

Točka p na lokaciji (x,y) ima četiri horizontalna i vertikalna susjeda na koordinatama:

N_4: $(x+1, y), (x-1, y), (x, y+1), (x, y-1)$

Taj skup se naziva 4-susjedstvo točke (piksela) p i označava se s $N_4(p)$

8-susjedstvo

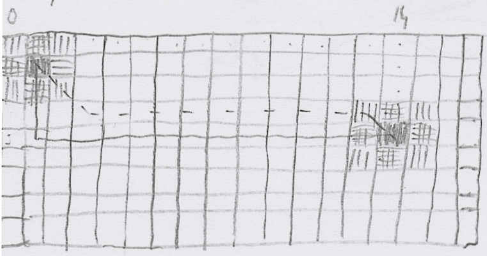
Točka p na lokaciji (x,y) ima četiri horizontalna i vertikalna susjeda i 4 dijagonalnih susjeda

$N_4(p)$ $(x+1, y), (x-1, y), (x, y+1), (x, y-1)$

$D_8(p)$ $(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$

Ovaj skup se naziva 8-susjedstvo točke (piksela) p i označava se s $N_8(p)$

- Put između 2 piksela je niz susjednih piksela (zavisno o korištenim relacijama susjedstva, možemo imati 4-put ili 8-put)
- Udaljenost između 2 piksela je kardinalnost najkraćeg mogućeg puta između 2 piksela (4-put ⇒ D_4 udaljenost, 8-put ⇒ D_8 udaljenost)



- točka - 8-susjed
- točka - 4 susjed
- točka - i 4 i 8-susjed

——— 4-putanja

- - - 8-putanja

⇒ najkraća putanja putanja između 2 točke ne treba biti jedinstvena, najčešće ih ima više

za $p(1,2), q(14,5)$:

$D_4(p,q) = |x-s| + |y-t| = |1-14| + |2-5| = 13 + 3 = 16$

$D_8(p,q) = \max(|x-s|, |y-t|) = \max(|1-14|, |2-5|) = \max(13, 3) = 13$

6.1 Definicija funkcije udaljenosti :

- Neka su p, q i z pikseli s koordinatama (x,y), (s,t) i (v,w)
- D je funkcija udaljenosti ako vrijedi :

1. $D(p,q) \geq 0$ $\quad (D(p,q) = 0 \Leftrightarrow p = q)$
2. $D(p,q) = D(q,p)$
3. $D(p,z) \leq D(p,q) + D(q,z)$

---

a) $d_4(p,q) = |p_1 - q_1| + |p_2 - q_2|$

1.) $p = q$

$d_4(p,p) = |\cancel{p_1} - \cancel{p_1}| + |\cancel{p_2} - p_2| = 0$ ✓

2.) $d_4(p,q) = |p_1 - q_1| + |p_2 - q_2|$

$d_4(q,p) = |p_2 - q_2| + |p_1 - q_1| \Rightarrow d_4(p,q) = d_4(q,p)$ ✓

3.)

$d_4(p,z) \leq d_4(p,q) + d_4(q,z)$

$|p_1 - z_1| + |p_2 - z_2| \leq |p_1 - q_1| + |p_2 - q_2| + |q_1 - z_1| + |q_2 - z_2|$

- npr. za $q(0,0)$:

$|p_1 - z_1| + |p_2 - z_2| \leq |p_1| + |p_2| + |z_1| + |z_2|$ ✓

---

b) $d_e(p,q) = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2}$

1.) $p = q$

$d_e(p,p) = \sqrt{(p_1 - p_1)^2 + (p_1 - p_1)^2} = 0$ ✓

2.) $d_e(p,q) = d_e(q,p)$

$\sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2} = \sqrt{(q_1 - p_1)^2 - (q_2 - p_2)^2} \quad /^2$

$(\cancel{p_1 - q_1})^2 + (\cancel{p_2 - q_2})^2 = (-1)^2(\cancel{p_1 - q_1})^2 - (-1)^2(\cancel{p_2 - q_2})^2$

$0 = 0$ ✓

3.)

3.) $d_e(p,z) \leq d_e(p,q) + d_e(q,z)$

$$\sqrt{(p_1-z_1)^2+(p_2-z_2)^2} \leq \sqrt{(p_1-q_1)^2+(p_2-q_2)^2} + \sqrt{(q_1-z_1)^2+(q_2-z_2)^2} \quad /^2$$

$$(p_1-z_1)^2+(p_2-z_2)^2 \leq (p_1-q_1)^2+(p_2-q_2)^2+(q_1-z_1)^2+(q_2-z_2)^2 + 2\sqrt{\dots}\sqrt{\dots}$$

$\Updownarrow$ $\qquad$ $\parallel$ pozitivni broj

slično kao u d_1, neda mi se raspisivat

=> uglavnom s desne strane nejednakosti još je veći broj nego u d_1 zbog člana $2\sqrt{\dots}\sqrt{\dots}$

=> svojstvo vrijedi ✓

---

c) $d_\infty(p,q) = \max\{|p_1-q_1|, |p_2-q_2|\}$

1.) $p=q$

$$d_\infty(p,p) = \max\{|p_1-p_1|, |p_2-p_2|\} = \max\{0,0\} = 0 \ ✓$$

2.) $d_\infty(p,q) = d_\infty(q,p)$

$$\max\{|p_1-q_1|, |p_2-q_2|\} = \max\{|q_1-p_1|, |q_2-p_2|\} \ ✓$$

3.) $d_\infty(p,z) \leq d_\infty(p,q) + d_\infty(q,z)$

$$\max\{|p_1-z_1|, |p_2-z_2|\} \leq \max\{|p_1-q_1|, |p_2-q_2|\} + \max\{|q_1-z_1|, |q_2-z_2|\}$$

slučajevi:
1. $|p_1-z_1| \leq |p_1-q_1| + |q_1-z_1|$
2. $|p_1-z_1| \leq |p_2-q_2| + |q_1-z_1|$
3. $|p_1-z_1| \leq |p_1-q_1| + |q_2-z_2|$
4. $|p_1-z_1| \leq |p_2-q_2| + |q_1-z_1|$
5.
6. $|p_2-z_2|\left\{\begin{array}{l}\leq\\ \leq\\ \leq\\ \leq\end{array}\right.$ — " —
7.
8.

...

8.) $p=(p_1,p_2)$ i $q=(q_1,q_2)$

$$d_8(p,q) \le d_e(p,q) \le d_4(p,q) \le 2d_8(p,q), \quad \forall p,q \in \mathbb{R}^2$$

$$\max\{|p_1-q_1|,|p_2-q_2|\} \le \sqrt{(p_1-q_1)^2+(p_2-q_2)^2}$$

1.) $|p_1-q_1| \le \sqrt{(p_1-q_1)^2+(p_2-q_2)^2} \;/^2$

$$\cancel{(p_1-q_1)^2} \le \cancel{(p_1-q_1)^2}+(p_2-q_2)^2$$

$$(p_2-q_2)^2 \ge 0 \checkmark$$

2.) $|p_2-q_2| \le \sqrt{(p_1-q_1)^2+(p_2-q_2)^2}$

⋮ slično kao i 1.)

$$(p_1-q_1)^2 \ge 0 \checkmark$$

$$\Rightarrow d_8(p,q) \le d_e(p,q)$$

$$\sqrt{(p_1-q_1)^2+(p_2-q_2)^2} \le |p_1-q_1|+|p_2-q_2| \;/^2$$

$$\cancel{(p_1-q_1)^2}+\cancel{(p_2-q_2)^2} \le \cancel{(p_1-q_1)^2}+\cancel{(p_2-q_2)^2}+2|(p_1-q_1)(p_2-q_2)| \;/\cdot\tfrac{1}{2}$$

$$|(p_1-q_1)(p_2-q_2)| \ge 0 \checkmark$$

$$\Rightarrow d_e(p,q) \le d_4(p,q)$$

$$|p_1-q_1|+|p_2-q_2| \le 2\max\{|p_1-q_1|,|p_2-q_2|\}$$

1. za $|p_1-q_1| \ge |p_2-q_2|$

$$|p_1-q_1|+|p_2-q_2| \le 2|p_1-q_1|$$

$$|p_2-q_2| \le |p_1-q_1| \checkmark$$

2. za $|p_1-q_1| \le |p_2-q_2|$

$$|p_1-q_1|+|p_2-q_2| \le 2|p_2-q_2|$$

$$|p_1-q_1| \le |p_2-q_2| \checkmark$$

$$\Rightarrow d_4(p,q) \le 2d_8(p,q)$$

$$\Rightarrow d_8(p,q) \le d_e(p,q) \le d_4(p,q) \le 2d_8(p,q) \checkmark$$

[9.] za lijevu sliku ⇒ objekt nije ni 4-povezan ni 8-povezan
⇒ pozadina je 8-povezana i

za desnu sliku ⇒ objekt je 8-povezan
⇒ pozadina je 4-povezana

desna slika $O = \{i, k, l, f\}$, $Q = \{j\}$

$\partial(O, Q) = ?$

$$\partial(O,Q) = \{(c,d) \mid c \in O,\ d \in Q,\ (c,d) \in W_2\}$$

4-susjedi

$$\partial(O,Q) = \{(k,j), (i,j)\}$$

stavak 1.1.:

1. $O \in I$ i $Q \in E$
2. $\partial(O,Q) = \partial(I,E)$
3. $I \cup E = \mathbb{Z}^2$ i $I \cap E = \emptyset$
4. I je α_1 povezan podskup od $\mathbb{Z}^2$
5. svaki w_2-put koji povezuje element iz I s elementom iz E prolazi kroz granicu $\partial(O,Q)$

stavak vrijedi $I = \{i, k, l, f\}$, $E = \{j\}$

svaki član iz I ima w_2 put prolazi kroz $\partial(Q,Q)$ da dođe do članova E

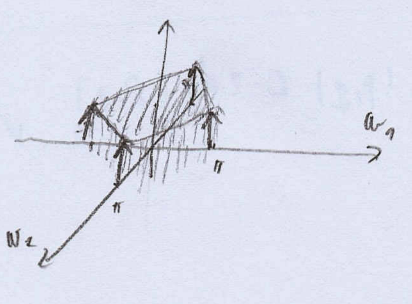
[10. a)] $f(x,y) = \underbrace{\mathrm{sinc}(x)}_{f_x(x)} \underbrace{\mathrm{sinc}(y)}_{f_y(y)}$

$$f_x(x) = \mathrm{sinc}(x)$$

$$F_x(w_1) = \int_{-\infty}^{\infty} \mathrm{sinc}(x) e^{-jw_1 x} dx = \mathrm{rect}\left(\frac{w}{2\pi}\right)$$

$$F(w_1, w_2) = \mathrm{rect}\,\frac{w_1}{2\pi} \cdot \mathrm{rect}\,\frac{w_2}{2\pi}$$

Amplitudni spektar $|F(w_1, w_2)| = \mathrm{rect}\,\frac{w_1}{2\pi} \cdot \mathrm{rect}\,\frac{w_2}{2\pi}$



$\boxed{11.}$ $f(x,y): \mathbb{R}^2 \to \mathbb{R}$

ako je $f(x,y) = f_1(x) f_2(y)$

tada $f(x,y) \circ\!\!-\!\!\bullet F(w_1, w_2) = F_1(w_1) F_2(w_2)$

$$f(x,y) = f_1(x) f_2(y)$$

$$F(w_1,w_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j(w_1 x + w_2 y)} dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(x) f_2(y) \cdot e^{-jw_1 x} \cdot e^{-jw_2 y}\, dx\,dy$$

$$= \int_{-\infty}^{\infty} f_1(x) e^{-jw_1 x} dx \int_{-\infty}^{\infty} f_2(y) e^{-jw_2 y}\, dy$$

$$= F_1(w_1) \cdot F_2(w_2)$$

$\boxed{14.}$ 2D linearna konvolucija

$$(f * g)(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\, g(x-\alpha, y-\beta)\, d\alpha\, d\beta$$

vrijedi:

$$f(x,y) ** g(x,y) \xleftrightarrow{F} F(u,v)\, G(u,v)$$

$$f(x,y)\, g(x,y) \xleftrightarrow{F} F(u,v) ** G(u,v)$$

(F je kontinuirana F.T.)

$f(x,y) ** g(x,y) \longleftrightarrow F_1(u) F_2(v) \cdot G_1(u) G_2(v)$

ne znam izračunati $FT(a^{-x})$??

$\boxed{17.}$ $f(x,y) = f_1(x)\, f_2(y)$

$g(x,y)$

$$f(x,y) ** g(x,y) = (f_1(x) * g(x,y)) * f_2(y) = (f_2(y) * g(x,y)) * f_1(x)$$

$$f(x,y) ** g(x,y) \circ\!\!-\!\!\bullet\ F(x,y) \cdot G(x,y) = F_1(x) \cdot F_2(y) \cdot G(x,y)$$

$$= F_1(x) \cdot G(x,y) \cdot F_2(y) \quad (1)$$

$$= F_2(x) \cdot G(x,y) \cdot F_1(x) \quad (2)$$

$$(1) \bullet\!\!-\!\!\circ\ (f_1(x) * g(x,y)) * f_2(y)$$

$$(2) \bullet\!\!-\!\!\circ\ (f_2(x) * g(x,y)) * f_1(x)$$

✓

$\boxed{18.d.}$ $S(f(x,y)) = 2f(x,y) + f(y,x) = g(x,y)$

(1) $S(f(x+x_0, y+y_0)) = 2f(x+x_0, y+y_0) + f(y,x)$

$g(x+x_0, y+y_0) = 2f(x+x_0, y+y_0) + f(y+y_0, x+x_0)$

$S(f(x+x_0, y+y_0)) \neq g(x+x_0, y+y_0)$

$\Rightarrow$ sustav S je prostorno promijenjiv

⑧

$\boxed{19.c,}$ $S(f(x,y)) = \int_0^x \int_0^y f(\chi, \upsilon)\, d\chi\, d\upsilon$

$S(a f(x,y)) = \int_0^x \int_0^y f(\chi, \upsilon)\, d\chi\, d\upsilon$

$S(a \cdot f(x,y)) \neq a \cdot S(f(x,y)) \Rightarrow$ sustav nije linearan

---

$S(f(x+x_0, y+y_0)) = \int_0^x \int_0^y f(\chi, \upsilon)\, d\chi\, d\upsilon$

$g(x+x_0, y+y_0) = \int_0^{x+x_0} \int_0^{y+y_0} f(\chi, \upsilon)\, d\chi\, d\upsilon$

$S(f(x+x_0, y+y_0)) \neq g(x+x_0, y+y_0)$

$\Rightarrow$ sustav S je prostorno promijenjiv

---

sustav je memorijski zbog integrala.

21. a) $S(f(x,y)) = \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty} f(i,j) = g(x,y)$

$S(a \cdot f(x,y)) = S(f(x,y)) \neq a \cdot S(f(x,y))$ $\Rightarrow$ sustav <u>nije</u> linearan

$S(f(x+x_0, y+y_0)) = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} f(i,j)$

$g(x+x_0, y+y_0) = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} f(i,j)$ $\Rightarrow$ sustav je prostorno <u>nepromijenjiv</u>

b) $S(f(x,y)) = \sum_{i=-x}^{0} \sum_{j=-y}^{0} f(i,j)$

$S(a \cdot f(x,y)) = \sum_{i=-x}^{0} \sum_{j=-y}^{0} f(i,j) = S(f(x,y)) \neq a \cdot S(f(x,y))$

$\Rightarrow$ sustav <u>nije</u> linearan

$S(f(x+x_0, y+y_0)) = \sum_{i=-x}^{0} \sum_{j=-y}^{0} f(i,j)$

$g(x+x_0, y+y_0) = \sum_{i=-x-x_0}^{0} \sum_{j=-y-y_0}^{0} f(i,j)$

$S(f(x+x_0, y+y_0)) \neq g(x+x_0, y+y_0)$ $\Rightarrow$ sustav je prostorno <u>PROMJENJIV</u>

22. a) $h(x,y) = \begin{Bmatrix} 1 & 8 & 1 \\ 8 & \underline{62} & 8 \\ 1 & 8 & 1 \end{Bmatrix}$

$h(-1,1) = 1$ $\quad h(0,1) = 8$ $\quad h(1,1) = 1$

$h(-1,0) = 8$ $\quad h(0,0) = 62$ $\quad h(1,0) = 8$

$h(-1,-1) = 1$ $\quad h(0,-1) = 8$ $\quad h(1,-1) = 1$

- da impulsni odziv bude separabilan mora vrijediti:

$$\begin{Bmatrix} 1 & 8 & 1 \\ 8 & \underline{62} & 8 \\ 1 & 8 & 1 \end{Bmatrix} = \begin{Bmatrix} h_x(-1)h_y(1) & h_x(0)h_y(1) & h_x(1)h_y(1) \\ h_x(-1)h_y(0) & \underline{h_x(0)h_y(0)} & h_x(1)h_y(0) \\ h_x(-1)h_y(-1) & h_x(0)h_y(-1) & h_x(1)h_y(-1) \end{Bmatrix}$$

$\Rightarrow$ nema rješenja

$\Rightarrow$ impulsni odziv $h(x,y)$ <u>nije</u> separabilan

$\rightarrow$

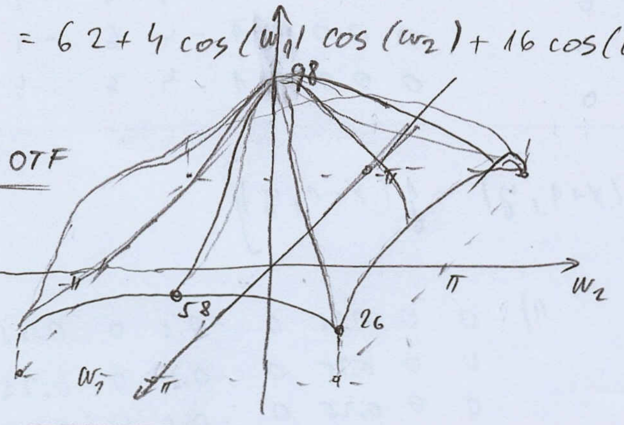
$$OTF = \frac{H(w_1, w_2)}{H(0,0)}, \qquad MTF = |OTF|$$

$$H(w_1, w_2) = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} h(x,y)\, e^{-jw_1 x} e^{-jw_2 y}$$

$$H(w_1, w_2) = 1e^{jw_1} e^{-jw_2} + 8e^{-jw_2} + 1e^{-jw_1} e^{-jw_2} +$$
$$+ 8e^{jw_1} + 62 + 8e^{-jw_1} +$$
$$+ 1e^{jw_1} e^{jw_2} + 8e^{jw_2} + 1e^{-jw_1} e^{jw_2}$$

$$= 62 + 4\cos(w_1)\cos(w_2) + 16\cos(w_2) + 16\cos(w_1)$$

$MTF = OTF$



23.a.

a) $\frac{\partial}{\partial x} f(x,y) \approx f(x+1, y) - f(x,y)$

$f(x,y)$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	1	0	0	0	0
1	0	0	0	0	1	0	0	0	0
2	0	0	0	0	1	0	0	0	0
3	0	0	0	0	1	0	0	0	0

$\frac{\partial}{\partial x} f(x,y)$

0	0	0	1	-1	0	0	0	0
0	0	0	1	-1	0	0	0	0
0	0	0	1	-1	0	0	0	0
0	0	0	1	-1	0	0	0	0

$\boxed{24.}$ $g(x,y) = S_1(f(x,y)) = f(x,y) - f(x-1,y)$

I)

$$\begin{matrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{matrix}$$

II)

$$\begin{matrix} 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \end{matrix}$$

III)

$$\begin{matrix} 0 & 0 & 0 & 0 & 1 & -3 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & 3 & -1 & 0 \end{matrix}$$

IV)

$$\begin{matrix} 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 \end{matrix}$$

$g(x,y) = S_2(f(x,y)) = \frac{1}{2}\left[f(x+1,y) - f(x-1,y)\right]$

I)

$$\begin{matrix} 0 & 0 & 0 & 0{,}5 & 0 & -0{,}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}5 & 0 & -0{,}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}5 & 0 & -0{,}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}5 & 0 & -0{,}5 & 0 & 0 & 0 \end{matrix}$$

II)

$$\begin{matrix} 0 & 0 & 0.25 & 0 & -0.5 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & -0.5 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & -0.5 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & -0.5 & 0 & 0.25 & 0 & 0 \end{matrix}$$

III)

$$\begin{matrix} 0 & 0.125 & 0 & -0.375 & 0 & 0.375 & 0 & -0.125 & 0 \\ 0 & 0.125 & 0 & -0.375 & 0 & 0.375 & 0 & -0.125 & 0 \\ 0 & 0.125 & 0 & -0.375 & 0 & 0.375 & 0 & -0.125 & 0 \\ 0 & 0.125 & 0 & -0.375 & 0 & 0.375 & 0 & -0.125 & 0 \end{matrix}$$

IV)

$$\begin{matrix} 0.0625 & 0 & -0.25 & 0 & 0.375 & 0 & -0.25 & 0 & 0.0625 \\ 0.0625 & 0 & -0.25 & 0 & 0.375 & 0 & -0.25 & 0 & 0.0625 \\ 0.0625 & 0 & -0.25 & 0 & 0.375 & 0 & -0.25 & 0 & 0.0625 \\ 0.0625 & 0 & -0.25 & 0 & 0.375 & 0 & -0.25 & 0 & 0.0625 \end{matrix}$$

---

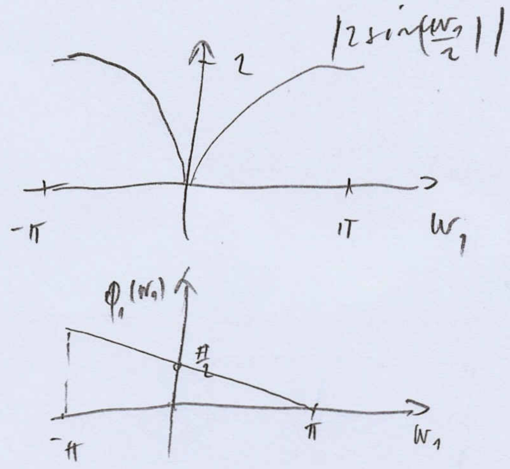
vyjet da ne dodo do pomaka ?

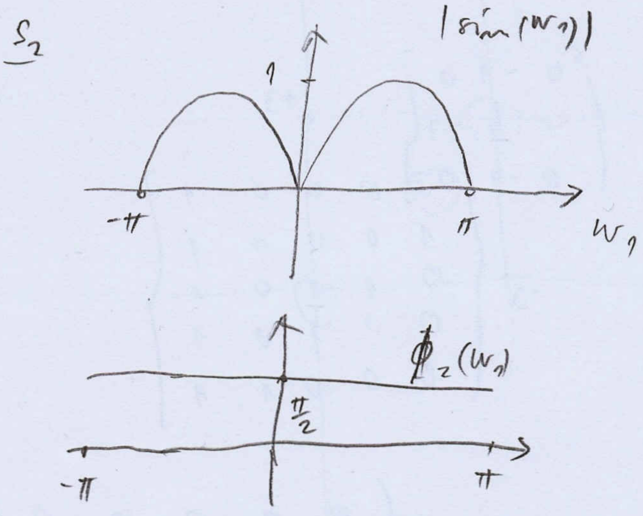
35. $h_1(x,y) = \{\underline{1}, -1\} = h_{1x}(x)\,\delta(y)$

$h_2(x,y) = \frac{1}{2}\{1, \underline{0}, -1\} = h_{2x}(x)\,\delta(y)$

$$H_1(w_1, w_2) = \sum_{x=0}^{1} h_{1x}(x)\, e^{-jw_1 x} = 1 - e^{-jw_1} = e^{-j\frac{w_1}{2}} \cdot j \cdot 2\sin\left(\frac{w_1}{2}\right)$$

$$H_2(w_1, w_2) = \frac{1}{2}\sum_{x=-1}^{1} h_{2x}(x)\, e^{-jw_1 x} = j\sin w_1$$





$$-\frac{d}{dw_1}\phi_1(w_1) = \frac{1}{2}$$

$$-\frac{d}{dw_1}\phi_2(w_1) = 0$$

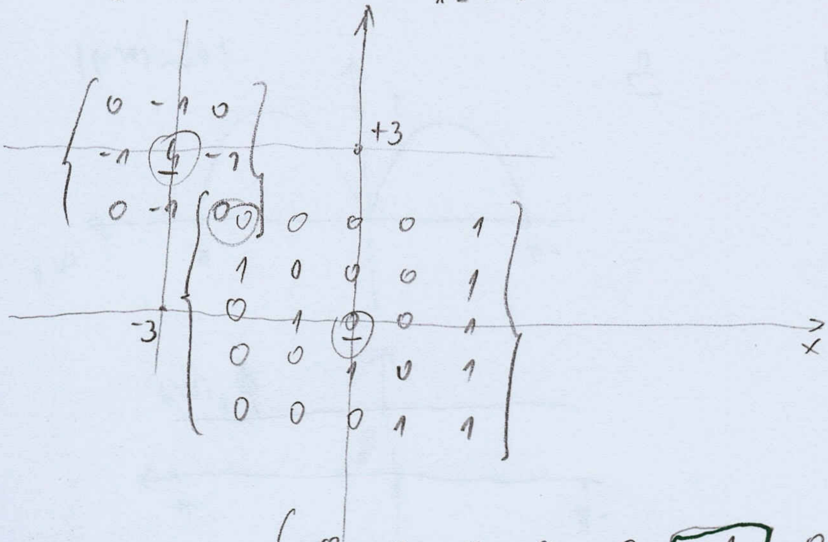
$\Rightarrow$ sustav S_1 ima generaliziranu linearnu fazu

27.a.

$$f(x,y) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \qquad h(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

2D-konvolucija:

$$f(x,y) ** h(x,y) = \sum_{\chi=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} f(\chi,\nu)\, h(x-\chi, y-\nu) = g(x,y)$$



$$g(x,y) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 3 & -1 \\ -1 & \boxed{4} & -2 & 0 & -1 & 2 & -1 \\ 0 & -2 & \boxed{4} & -2 & -1 & 2 & -1 \\ 0 & 0 & -2 & \boxed{4} & -3 & 2 & -1 \\ 0 & 0 & 0 & -2 & 3 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

30. $f(x,y) = \sin(w_1 x + \theta_1)\cos(w_2 y + \theta_2) + 1$, $x \in [0,4]$
$y \in [0,4]$

$w_1 = w_2 = \frac{\pi}{2}$

$\theta_1 = \theta_2 = 0$, $f_{sx} = f_{sy} = 1$

Nyquist Shannonov teorem

$$\boxed{\begin{aligned} w_{sx} &\geq 2w_1 \\ w_{sy} &\geq 2w_2 \end{aligned}}$$

$w_{sx\,min} = 2\cdot w_1 = \pi$

$w_{sy\,min} = 2w_2 = \pi$

$w_{sx} = 2\pi \cdot f_{sx} = 2\pi$, $w_{sx} > w_{sx\,min}$

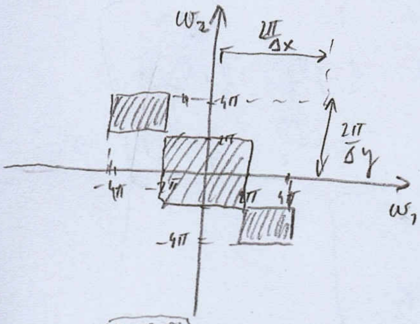
$w_{sy} = 2\pi f_{sy} = 2\pi$, $w_{sy} > w_{sy\,min}$

$\Rightarrow$ ne dolazi do preklapanja spektra

31. a) $F_d(\Omega_1, \Omega_2) = \frac{1}{\Delta x \Delta y} \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} F_k\left(\underbrace{\frac{\Omega_1 + 2\pi i}{\Delta x}}_{w_1}, \underbrace{\frac{\Omega_2 + 2\pi j}{\Delta y}}_{w_2}\right)$ - diskretni signal

$F_k(w_1, w_2)$ - kontinuirani signal

$\Delta x = \frac{1}{2}$, $\Delta y = \frac{1}{2}$

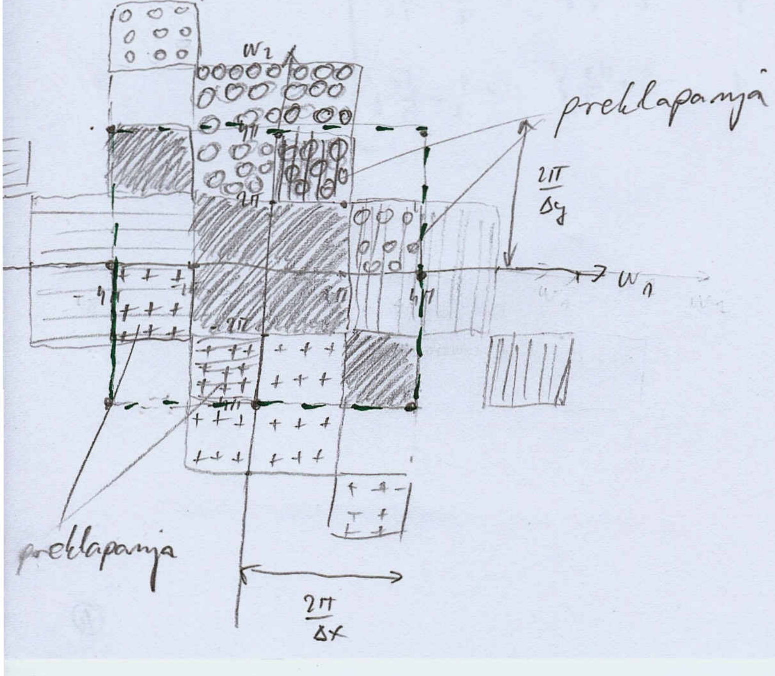


$w_{sx\,min} = 2\cdot 4\pi = 8\pi$, $\Delta x_{max} = \frac{2\pi}{w_{sx\,min}} = \frac{2\pi}{8\pi} = \frac{1}{4}$

$w_{sy\,min} = 2\cdot 4\pi = 8\pi$, $\Delta y_{max} = \frac{2\pi}{8\pi} = \frac{1}{4}$

$\Delta x = \Delta y = \frac{1}{2} > \Delta x_{max} = \Delta y_{max}$

$\Rightarrow$ dolazi do preklapanja



$F_d(\Omega_1, \Omega_2) = \frac{1}{\frac{1}{2}\cdot\frac{1}{2}}(1+1) = 4\cdot 2 = 8$

32.c.

$$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{bmatrix}$$

matrica je unitarna ako vrijedi $U \cdot U^{-1} = U^{-1} \cdot U = I$

$$W_3 \cdot W_3^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \end{bmatrix} * \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{6} + j\frac{\sqrt{3}}{6} & -\frac{1}{6} - \frac{\sqrt{3}}{6} \\ \frac{1}{3} & -\frac{1}{6} - j\frac{\sqrt{3}}{6} & -\frac{1}{6} + \frac{\sqrt{3}}{6} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ izračunao s matlabom

$\Rightarrow$ matrica W_3 jest unitarna

33.a. $W_N F W_M^T$

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad N = M = 3$$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{bmatrix} \qquad W_3^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + \frac{\sqrt{3}}{2}j \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - \frac{\sqrt{3}}{2}j \end{bmatrix}$$

$$W_N F W_M^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - \frac{\sqrt{3}}{2}j & -\frac{1}{2} + \frac{\sqrt{3}}{2}j \\ 1 & -\frac{1}{2} + \frac{\sqrt{3}}{2}j & -\frac{1}{2} + \frac{\sqrt{3}}{2}j \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - \frac{\sqrt{3}}{2}j & -\frac{1}{2} + \frac{\sqrt{3}}{2}j \\ 1 & -\frac{1}{2} + \frac{\sqrt{3}}{2}j & -\frac{1}{2} - \frac{\sqrt{3}}{2}j \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$$

33.d)

$$F^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad N=6,\ M=3$$

$$W_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.5-\frac{\sqrt3}{2}j & -0.5-\frac{\sqrt3}{2}j & -1 & -0.5+\frac{\sqrt3}{2}j & 0.5+\frac{\sqrt3}{2}j \\ 1 & -0.5-\frac{\sqrt3}{2}j & -0.5+\frac{\sqrt3}{2}j & 1 & -0.5-\frac{\sqrt3}{2}j & -0.5+\frac{\sqrt3}{2}j \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.5+\frac{\sqrt3}{2}j & -0.5-\frac{\sqrt3}{2}j & 1 & -0.5+\frac{\sqrt3}{2}j & -0.5-\frac{\sqrt3}{2}j \\ 1 & 0.5+\frac{\sqrt3}{2}j & -0.5+\frac{\sqrt3}{2}j & -1 & -0.5-\frac{\sqrt3}{2}j & 0.5-\frac{\sqrt3}{2}j \end{bmatrix}$$

$W_6 \cdot F \cdot W_3^T =$ ... računao sam s MATLABOM

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1.5+2.59j & 0 \\ 0 & 0 & -1.5-2.59j \\ 3 & 0 & 0 \\ 0 & -1.5+2.59j & 0 \\ 0 & 0 & -1.5-2.59j \end{bmatrix}$$

34.

$G = W_N F W_M^T$

$F = c\,(W_N^H)\,G\,W_M^H$

$c = ?$

W_M i W_N su simetrične matrice $\Rightarrow W_M^T = W_M$, $W_N^T = W_N$

$\Rightarrow G = W_N F W_M$

$\Rightarrow F = W_N^{-1} G W_M^{-1}$

W_M i W_N matrice su unitarne $\Rightarrow W_N^{-1} = W_N^H$, $W_M^{-1} = W_M^H$

$\Rightarrow F = W_N^H G W_M^H$

$c\cancel{(W_N^H)}\cancel{G W_M^H} = W_N^H \cancel{G W_M^H}$

$\underline{\underline{c = 1}}$

37.

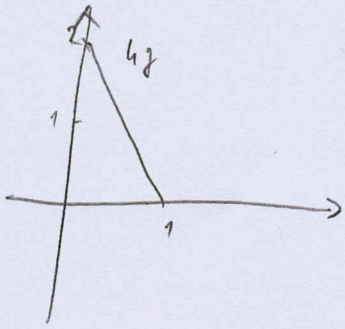
$$f(x,y) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 3 \\ 3 & 4 & 3 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

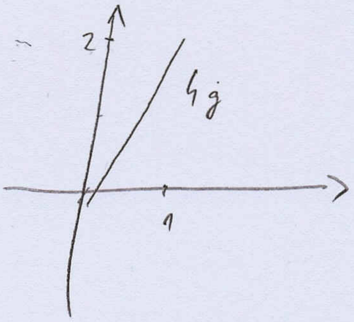
$$v(m,n) = \begin{pmatrix} 2 & 3 & 3 & 4 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 4 & 3 & 3 & 2 \end{pmatrix}$$

$\boxed{38.}$

$$h_f(i) = \begin{cases} 2 - 2i, & i \in \langle 0, 1 \rangle \\ 0, & \text{inače} \end{cases}$$

$$h_g(i') = \begin{cases} 2i', & i \in \langle 0, 1 \rangle \\ 0, & \text{inače} \end{cases}$$





$$g(x,y) = f(x,y) f(x,y)$$