$\boxed{ZZV \quad 11}$

Zad 1

1) $x(2x^2+y^2)+y(x^2+2y^2)y'=0$ $\qquad P'_y=2xy \quad Q'_x=2xy$

$\underbrace{(2x^3+xy^2)}_{P}dx+\underbrace{(yx^2+2y^3)}_{Q}dy=0$ $\qquad$ Egzaktna

$$\int_0^x(2x^3+\cancel{xy^2})dx+\int_0^y(yx^2+2y^3)dy=C$$

$\frac{1}{2}x^4+\frac{1}{2}x^2y^2+\frac{1}{2}y^4=C \quad /\cdot 2 \qquad 2C \downarrow C$

$$\boxed{x^4+x^2y^2+y^4=C}$$

2) $\underbrace{\left(2x+\frac{1}{y}+\frac{y}{x^2}\right)}_{P}dx+\underbrace{\left(-\frac{x}{y^2}-\frac{1}{x}\right)}_{Q}dy=0$ $\qquad P'_y=-\frac{1}{y^2}+\frac{1}{x^2}$ $\quad Q'_x=-\frac{1}{y^2}+\frac{1}{x^2}$ $\qquad$ Egzaktna

$$\int_1^x\left(2x+\frac{1}{y}+\frac{y}{x^2}\right)dx+\int_1^y\left(-\frac{x}{y^2}-\frac{1}{x}\right)dy=C_1$$

$$\left(x^2+x-\frac{1}{x}\right)\Big|_1^x+\left(\frac{x}{y}-\frac{y}{x}\right)\Big|_1^y=C_1$$

$$x^2+\cancel{x}-\cancel{\frac{1}{x}}-1-\cancel{1}+\cancel{1}+\frac{x}{y}-\frac{y}{x}-\cancel{x}+\cancel{\frac{1}{x}}=C_1$$

$x^2+\frac{x}{y}-\frac{y}{x}=C \quad /\cdot xy$

$$\boxed{x^3y+x^2-y^2=Cxy}$$

3) $\underbrace{(3x^2-(x-y))}_{P}dx+\underbrace{(2y-x+3y^2)}_{Q}dy=0$ $\qquad P'_y=-1 \quad Q'_x=-1 \Rightarrow$ Egzaktna

$$\int_0^x(3x^2-2x-\cancel{x})dx+\int_0^y(2y-x+3y^2)dy=C$$

$$(x^3-x^2)\Big|_0^x+(y^2-xy+y^3)\Big|_0^y=C$$

$$\boxed{x^3-x^2+y^2-xy+y^3=C}$$

4.) $\underbrace{\frac{2x}{y^3}}_{P}dx + \underbrace{\left(\frac{1}{y^2} - 3\frac{x^2}{y^4}\right)}_{Q}dy = 0, \quad y(1) = \boxed{1}$ $\quad P'_y = -6\frac{x}{y^4} \quad Q'_x = -6\frac{x}{y^4}$

Egzaktna

$$\int_1^x \left(\frac{2x}{y^3}\right)dx + \int_1^y \left(\frac{1}{y^2} - 3\frac{x^2}{y^4}\right)dy = 0$$

$$x^2\Big|_1^x + \left(-\frac{1}{y} + \frac{x^2}{y^3}\right)\Big|_1^y = 0$$

$$\cancel{x^2} - 1 - \frac{1}{y} + \frac{x^2}{y^3} + 1 - \cancel{x^2} = 0$$

$$\frac{x^2}{y^3} = \frac{1}{y} \Rightarrow x^2 = y^2 \Rightarrow y = \pm x \quad 1 = \pm 1 \Rightarrow \boxed{y = x}$$

5.) $\underbrace{(3x^2y + y^3)}_{P}dx + \underbrace{(x^3 + 3xy^2)}_{Q}dy = 0$ $\quad P'_y = 3x^2 + 3y^2 \quad Q'_x = 3x^2 + 3y^2$

Egzaktna

$$\int_0^x \cancel{(3x^2y + y^3)}dx + \int_0^y (x^3 + 3xy^2)dy = C_1$$

$$x^3y + xy^3 = C$$

$$\boxed{xy(x^2 + y^2) = C}$$

6.) $\underbrace{\left(\frac{\sin 2x}{y} + x\right)}_{P}dx + \underbrace{\left(y - \frac{\sin^2 x}{y^2}\right)}_{Q}dy = 0$ $\quad P'_y = -\frac{\sin 2x}{y^2} \quad Q'_x = -\frac{2\sin x \cos x}{y^2}$

Egzaktna

$$\int_0^x \left(\frac{\sin 2x}{y} + x\right)dx + \int_1^y \left(y - \cancel{\frac{\sin^2 x}{y^2}}\right)dy = C_1$$

$$\left(-\frac{1}{2y}\cos 2x + \frac{1}{2}x^2\right)\Big|_0^x + \frac{1}{2}y^2\Big|_1^y = C_1$$

$$-\frac{1}{2y}\cos 2x + \frac{1}{2}x^2 + \frac{1}{2y} + \frac{1}{2}y^2 - \frac{1}{2} = C_1$$

$$\frac{1}{2y}(1 - \cos 2x) + \frac{1}{2}(x^2 + y^2) = C_2$$

$$\boxed{\frac{\sin^2 x}{y} + \frac{x^2 + y^2}{2} = C_2}$$

Podsjetnik: Eulerovi multiplikatori

1) $\mu = \mu(x)$ Uvjet egzaktnosti: $\boxed{[\mu P]'_y = [\mu Q]'_x}$

$$\cancel{\mu(x)'_y \cdot P} + \mu(x) \cdot P'_y = \mu(x)'_x \cdot Q + \mu(x) \cdot Q'_x$$

$$\mu(x)\left[P'_y - Q'_x\right] = \mu' \cdot Q$$

$$\frac{\mu'}{\mu} = \frac{1}{Q}\left[P'_y - Q'_x\right] \Rightarrow \int \frac{d\mu}{\mu} = \int \frac{1}{Q}\left[P'_y - Q'_x\right] dx$$

$$\ln \mu(x) = \int \underbrace{\frac{1}{Q}\left[P'_y - Q'_x\right]}_{\text{mora biti fja samo od } x \text{ !!!}} dx$$

2) $\mu = \mu(y)$ $[\mu P]'_y = [\mu Q]'_x$

$$\mu(y)'_y \cdot P + \mu(y) \cdot P'_y = \cancel{\mu(y)'_x \cdot Q} + \mu(y) \cdot Q'_x$$

$$\mu' \cdot P = -\mu(y)\left[P'_y - Q'_x\right]$$

$$\int \frac{d\mu}{\mu} = -\int \frac{1}{P}\left[P'_y - Q'_x\right] dy$$

$$\ln \mu = -\int \underbrace{\frac{1}{P}(P'_y - Q'_x)}_{\text{mora biti fja samo od } y \text{ !!!}} dy$$

Zad 2

1) $\underbrace{(2xy^2 - 3y^3)}_{P}dx + \underbrace{(7 - 3xy^2)}_{Q}dy = 0$

(1.) $P'_y = 4xy - 9y^2 \quad Q'_x = -3y^2$

$P'_y - Q'_x = 4xy - 6y^2$

$= 2y(2x - 3y)$

$-\frac{1}{P}\left(P'_y - Q'_x\right) = \frac{2y(2x-3y)}{-y^2(2x-3y)}$

(2.)

$\ln\mu = \int -\frac{2}{y}\,dy$

$\ln\mu = \ln\frac{1}{y^2} \Rightarrow \mu = \frac{1}{y^2}$

$\int_0^x (2x - 3y)\,dx + \int_1^y \left(\frac{7}{y^2} - \cancel{3x}\right)dy = C_1$

$x^2 - 3yx - \frac{7}{y}\Big|_1^y = C_1$

$x^2 - 3xy - \frac{7}{y} + 7 = C_1$

$$\boxed{x^2 - 3xy - \frac{7}{y} = C_2}$$

2.) $\underbrace{(2+y^2)}_{P}dx + \underbrace{(xy + 2y + y^3)}_{Q}dy = 0$

$P'_y = 2y \quad Q'_x = y$

$P'_y - Q'_x = y$

$\ln\mu = -\int \frac{y}{2+y^2}\,dy = -\frac{1}{2}\int \frac{d(2+y^2)}{2+y^2} = -\frac{1}{2}\ln(2+y^2)$

$\mu = \frac{1}{\sqrt{2+y^2}}$

$\int_0^x \sqrt{2+y^2}\,dx + \int_0^y \frac{\cancel{xy} + 2y + y^3}{\sqrt{2+y^2}}\,dy = C_1$

$x\sqrt{2+y^2} + \int_0^y \frac{y(2+y^2)}{\sqrt{2+y^2}}\,dy = C_1$

$x\sqrt{2+y^2} + \int_0^y y\sqrt{2+y^2}\,dy = C_1$

$x\sqrt{2+y^2} + \frac{1}{2}\int_0^y \sqrt{2+y^2}\,d(2+y^2) = C_1$

$x\sqrt{2+y^2} + \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}(2+y^2)^{\frac{3}{2}} = C_1$

$$\boxed{x\sqrt{2+y^2} + \frac{1}{3}\sqrt{(2+y^2)^3} = C_1}$$

3) $\underbrace{(1-x^2y)}_{P}dx + \underbrace{x^2(y-x)}_{Q}dy = 0 \qquad P'_y = -x^2 \qquad Q'_x = 2xy - 3x^2$

$P'_y - Q'_x = 2x^2 - 2xy = 2x(x-y)$

$\ln\mu(x) = \int \frac{1}{x^2(y-x)} \cdot 2x(x-y) = -\int \frac{2}{x} \Rightarrow \mu = \frac{1}{x^2}$

$\int_1^x \left(\frac{1}{x^2} - \cancel{y}\right)dx + \int_0^y (y-x)\,dy = C_1$

$-\frac{1}{x}\Big|_1^x + \frac{1}{2}y^2 - xy = C_1$

$-\frac{1}{x} + 1 + \frac{1}{2}y^2 - xy = C_1 \Rightarrow \boxed{\frac{1}{2}y^2 - \frac{1}{x} - xy = C_2}$

4) $\underbrace{(xy\cos xy + 1)}_{P}dx + \underbrace{(x^2\cos xy)}_{Q}dy = 0 \qquad P'_y = x\cos xy - x^2y\sin xy$

$Q'_x = 2x\cos xy - x^2y\sin xy$

$\ln\mu(x) = \int \frac{-x\cos xy}{x^2\cos xy}dx \qquad P'_y - Q'_x = \cos xy\,(x-2x) = -x\cos xy$

$\mu = \frac{1}{x}$

$\int_1^x \left(y\cancel{\cos xy} + \frac{1}{x}\right)dx + \int_0^y x\cos xy\,dy = C_1$

$\ln x\Big|_1^x + \cancel{\frac{x}{x}}\sin y\Big|_0^y = C_1$

$\ln x - 0 + \sin y - 0 = C_1$

$\boxed{\ln x + \sin y = C}$

5) $\underbrace{x}_{P}dx + \underbrace{(e^{2y} - x^2)}_{Q}dy = 0 \qquad P'_y = 0 \qquad Q'_x = -2x \qquad P'_y - Q'_x = 2x$

$\ln\mu(y) = -\int \frac{2x}{x}dy = -2y \Rightarrow \mu = e^{-2y}$

$\int_0^x x e^{-2y}dx + \int_0^y \left(1 - \cancel{x^2e^{-2y}}\right)dy = C_1$

$\frac{1}{2}x^2e^{-2y} + y = C_1 \quad /\cdot 2$

$\boxed{x^2e^{-2y} + 2y = C_2}$

6)

$$\underbrace{(e^x\cos^2 y)}_{P}dx + \underbrace{(2y\cos y - \tfrac{1}{2}e^x\sin 2y)}_{Q}dy = 0 \qquad P'_y = -2e^x\cos y\sin y = -e^x\sin 2y$$

$$Q'_x = -\frac{1}{2}e^x\sin 2y$$

$$\ln\mu(y) = -\int \frac{-\frac{1}{2}e^x\sin 2y}{e^x\cos^2 y}dy \qquad P'_y - Q'_x = -\frac{1}{2}e^x\sin 2y$$

$$= \frac{1}{2}\int\frac{2\sin y\cos y}{\cos^2 y}dy = -\int\frac{d(\cos y)}{\cos y} = -\ln(\cos y) \Rightarrow \boxed{\mu = \frac{1}{\cos y}}$$

$$\int_{-\infty}^{x}(e^x\cos y)dx + \int_0^y\left(2y - \frac{1}{2}\cancel{e^x\frac{2\sin y\cos y}{\cos y}}\right)dy = C_1$$

$$= e^x\cos y\Big|_{-\infty}^{x} + y^2\Big|_0^y = C_1$$

$$e^x\cos y - 0 + y^2 = C_1 \Rightarrow \boxed{e^x\cos y + y^2 = C_1}$$

7)

$$\underbrace{(x\cos y - y\sin y)}_{Q}dy + \underbrace{(x\sin y + y\cos y)}_{P}dx = 0 \qquad P'_y = x\cos y + \cos y - y\sin y$$

$$Q'_x = \cos y$$

$$\ln\mu(x) = \int\frac{x\cos y - y\sin y}{x\cos y - y\sin y}dx$$

$$\boxed{\mu = e^x} \qquad P'_y - Q'_x = x\cos y - y\sin y$$

$$\int^x\left(\cancel{e^x x\sin y + e^x y\cos y}\right)dx + e^x\int_0^y(x\cos y - y\sin y)dy = C$$

$$\int y\sin y\,dy = \begin{vmatrix} u = y & du = dy \\ dv = \sin y\,dy & v = -\cos y\end{vmatrix} = -y\cos y + \int\cos y\,dy = -y\cos y + \sin y$$

$$= e^x\left[x\sin y + y\cos y - \sin y\right]\Big|_0^y = C$$

$$\boxed{e^x(x-1)\sin y + e^x y\cos y = C}$$

8) $y' = -\frac{(x+y)y}{xy+1} \Rightarrow \underbrace{dy(xy+1)}_{Q} + \underbrace{dx(xy+y^2)}_{P} = 0$

$P'_y = x+2y \quad Q'_x = y \quad P'_y - Q'_x = x+y$

$\ln\mu(y) = -\int \frac{\cancel{x+y}}{y(\cancel{x+y})}\,dy \Rightarrow \mu = \frac{1}{y}$

$\int_0^x (x+1)\,dx + \int_1^y \left(\cancel{x} + \frac{1}{y}\right)dy = C$

$\boxed{\frac{1}{2}x^2 + xy + \ln y = C}$

9) $\underbrace{(2x+y)}_{P}dx + \underbrace{(x^2+xy+x)}_{Q}dy = 0 \qquad P'_y = 1 \quad Q'_x = (2x+y)+1$

$P'_y - Q'_x = -(2x+y)$

$\ln\mu(y) = -\int \frac{-(2x+y)}{2x+y}\,dx = y \Rightarrow \boxed{\mu = e^y}$

$\int_0^x (2e^y x + e^y y)\,dx + \int_0^y (\cancel{x^2e^y} + \cancel{xye^y} + \cancel{xe^y})\,dx = C$

$\boxed{x^2e^y + xe^y \cdot y = C}$

10) $x^2y^2y' + xy^3 = 1 \Rightarrow \underbrace{(x^2y^2)}_{Q}dy + \underbrace{(xy^3-1)}_{P}dx = 0$

$P'_y = 3xy^2 \quad Q'_x = 2xy^2 \quad P'_y - Q'_x = xy^2$

$\ln\mu(x) = \int \frac{\cancel{xy^2}}{x^2\cancel{y^2}}\,dx \Rightarrow \mu = x$

$\int_0^x (x^2y^3 - x)\,dx + \int_0^y (\cancel{x^3y^2})\,dy = C$

$\boxed{\frac{1}{3}x^3y^3 - \frac{1}{2}x^2 = C}$

11) $y' + \frac{\sin x}{\cos x} + \frac{e^y}{\cos x} = 0 \;/\cdot \cos x$

$\cos x \, y' + \sin x + e^y = 0 \Rightarrow \underbrace{(\sin x + e^y)}_{P} dx + \underbrace{(\cos x)}_{Q} dy = 0$

$P'_y = e^y \qquad Q'_x = -\sin x \qquad P'_y - Q'_x = e^y + \sin x$

$\ln \mu(y) = -\int \frac{e^y + \sin x}{e^y + \sin x} dy \Rightarrow \mu = e^{-y}$

$\int_{\frac{\pi}{2}}^{x} (e^{-y} \sin x + 1) dx + \int_0^y e^{-y} \cancel{\cos x} \, dy = C$

$(-e^{-y} \cos x + x)\Big|_{\frac{\pi}{2}}^{x} = C$

$-e^{-y} \cos x + x - \frac{\pi}{2} = C \Rightarrow \boxed{x - e^{-y} \cos x = C_1}$

12) To je 10 :-)

Zad 3 / Zad 5 Krivi je oblik u (3) !!!!

$\mu = \mu(xy)$, $\alpha = xy$ $\quad \mu'_y = \mu'_\alpha \alpha'_y = x \mu'_\alpha$, $\quad \mu'_x = \mu'_\alpha \alpha'_x = y \mu'_\alpha$

$[\mu P]'_y = [\mu Q]'_x \Rightarrow \mu'_y P + \mu P'_y = \mu'_x Q + \mu Q'_x$

$\mu'_\alpha [xP - yQ] = \mu [Q'_x - P'_y]$

$\frac{\mu'}{\mu} = \frac{Q'_x - P'_y}{xP - yQ}$ Q.E.D.

$\underbrace{(2x^3y^2 - y)}_{P} dx + \underbrace{(2x^2y^3 - x)}_{Q} dy = 0 \qquad P'_y = 4x^3y - 1 \qquad Q'_x = 4xy^3 - 1$

$Q'_x - P'_y = 4xy(y^2 - x^2)$

$xP - yQ = 2x^4y^2 - 2x^2y^4$

$= -2x^2y^2(y^2 - x^2)$

$\int_1^x \left(2x - \frac{1}{x^2 y}\right) dx + \int_1^y \left(2y - \frac{1}{x y^2}\right) dy = C$

$\left(x^2 + \frac{1}{xy}\right)\Big|_1^x + \left(y^2 + \frac{1}{y}\right)\Big|_1^y = C$

$x^2 + \frac{1}{xy} - 1 - \cancel{\frac{1}{y}} + y^2 + \cancel{\frac{1}{y}} - 1 - 1 = C \qquad \ln \mu(xy) = -2 \int \frac{\cancel{4xy}(\cancel{y^2 - x^2})}{+2x^2y^2(y^2 - x^2)} d(xy)$

$\boxed{x^2 + y^2 + \frac{1}{xy} = C} \qquad \mu = \frac{1}{x^2y^2}$

Zad 4 $\mu = \mu(x+y^2)$ $\alpha = x+y^2$ $\mu'_x = \mu'_\alpha \cdot \alpha'_x = \mu'_\alpha$

$\mu'_y = \mu'_\alpha \cdot \alpha'_y = 2y\mu'_\alpha$

$$[\mu P]'_y = [\mu Q]'_x \Rightarrow \mu'_y \cdot P + \mu P'_y = \mu'_x \cdot Q + \mu Q'_x$$

$$2y\mu' P - \mu' Q = \mu[Q'_x - P'_y]$$

$$\frac{\mu'}{\mu} = \frac{Q'_x - P'_y}{2yP - Q} \Rightarrow \boxed{\ln \mu(x+y^2) = \int \frac{Q'_x - P'_y}{2yP - Q}\, d(x+y^2)}$$

$$(\underbrace{3x + 2y + y^2}_{P})dx + (\underbrace{x + 4xy + 5y^2}_{Q})dy = 0$$

$P'_y = 2 + 2y$ $Q'_x = 1 + 4y$ $Q'_x - P'_y = 2y - 1$

$$2y \cdot P = 6xy + 4y^2 + 2y^3$$

$$2yP - Q = 6xy + 4y^2 + 2y^3 - x - 4xy - 5y^2$$

$$= 2xy - y^2 + 2y^3 - x = x(2y-1) + y^2(2y-1)$$

$$= (x+y^2)(2y-1)$$

$$\ln \mu(x+y^2) = \int \frac{\cancel{2y-1}}{(x+y^2)\cancel{(2y-1)}} \qquad \boxed{\mu = x+y^2}$$

Zad 6 $\mu = \mu(x^2+y^2)$ $\alpha = x^2+y^2$ $\mu'_x = \mu'_\alpha \cdot \alpha'_x = 2x\mu'_\alpha$

$\mu'_y = \mu'_\alpha \cdot \alpha'_y = 2y\mu'_\alpha$

$$[\mu P]'_y = [\mu Q]'_x \Rightarrow \mu'_y \cdot P + \mu P'_y = \mu'_x Q + \mu \cdot Q'_x$$

$$2y\mu'_\alpha \cdot P - 2x\mu'_\alpha \cdot Q = \mu[Q'_x - P'_y]$$

$$\frac{\mu'}{\mu} = \frac{Q'_x - P'_y}{2yP - 2xQ}$$

Q.E.D

Zad 7.

1) $xy'^2 - yy' + 1 = 0 \Rightarrow y = xy' + \frac{1}{y'}$ Clairaut $\varphi(u)$

O.R. $\boxed{y = xC + \frac{1}{C}}$ S.R. $y' = p$

$x = -\varphi'(p) = \frac{1}{p^2} \Rightarrow p = \pm\frac{1}{\sqrt{x}}$

$y = x \cdot p + \frac{1}{p} = \frac{xp^2+1}{p} = \frac{\frac{1}{x}+1}{\pm\frac{1}{\sqrt{x}}} = \boxed{\pm 2\sqrt{x}}$

2.) $y' = \ln(xy' - y)$ $\quad e^{y'} = xy' - y \Rightarrow y = xy' - e^{y'}$ Clairaut

O.R. $\quad$ SR

$y = Cx - e^C$ $\quad x = e^p \quad y = xp - e^p$

$\boxed{y = x\ln x - x}$

3.) $xy'^2 - yy' - y' + 1 = 0 \Rightarrow y = xy' - 1 + \frac{1}{y'}$ Clairaut

O.R. $\boxed{y = Cx - 1 + \frac{1}{C}}$ S.R. $x = \frac{1}{p^2}$ $\quad p = \frac{1}{\pm\sqrt{x}}$

$y = xp - 1 + \frac{1}{p} = 2xp - 1$

$\boxed{y = \pm 2\sqrt{x} - 1}$

4.) $y = y'^2 - xy' + \frac{1}{2}x^2$ $\quad y' = p(x)$ $\quad y = p^2 - xp + \frac{1}{2}x^2 \;/\frac{d}{dx}$

$p = 2pp' - p - xp' + x$

$2p = 2pp' - xp' + x$

$(2p - x) = p'(2p - x)$

$(p' - 1)(2p - x) = 0$

① $\quad$ ②

① $p' = 1$

$y' = p = x + C$

$y = \frac{1}{2}x^2 + Cx + K$

$\cancel{\frac{1}{2}x^2} + \cancel{Cx} + K = \cancel{x^2} + \cancel{2Cx} + C^2$

$- \cancel{x^2} - \cancel{xC} + \cancel{\frac{1}{2}x^2}$

$K = C^2$

$\boxed{y = \frac{1}{2}x^2 + Cx + C^2}$ O.R.

2) $2p - x = 0$

$p = \frac{1}{2}x$

$y = \frac{1}{4}x^2 + C$

$\boxed{y = \frac{1}{4}x^2}$ S.R.

$\cancel{\frac{1}{4}x^2} + C$

$= \cancel{\frac{1}{4}x^2} - \cancel{\frac{1}{2}x^2}$

$- Cx + \cancel{\frac{1}{2}x^2}$

$C = Cx$

$C = 0$

Zad 8

1) $x = a + \frac{y}{2y'} - \frac{1}{2}yy'$ $y' = p(y)$ $\frac{dy}{dx} = p$ $dy = p \cdot dx$

$x = a + \frac{y}{2p} - \frac{1}{2}yp \quad / \frac{d}{dy}$

$0 = 0 + \frac{1}{2}\left(\frac{p - yp'}{p^2}\right) - \frac{1}{2}(p - yp') \quad / \cdot 2$

$(p - yp')\left(\frac{1}{p^2} - 1\right) = 0$ $p - yp' = 0$

O.R. S.R. $p = yp' = y \frac{dp}{dy}$

$\frac{dy}{y} = \frac{dp}{p}$ $p = C \cdot y$

$x = a + \frac{\cancel{y}}{2C\cancel{y}} - \frac{1}{2}y \cdot C \cdot y = a + \frac{1}{2C} - \frac{1}{2}Cy^2$ $C \to \frac{1}{C}$

$$\boxed{x = a + \frac{y^2}{2C} - \frac{C}{2}}$$

2) $y = xy'^2 + y'^2$ $y' = p(x)$

$y = xp^2 + p^2$

$y = p^2(x+1)$

$p = \pm\sqrt{\frac{y}{x+1}}$

$\int \frac{dy}{\sqrt{y}} = \pm\int \frac{dx}{\sqrt{x+1}}$

$\cancel{\frac{1}{2}}\sqrt{y} = \pm\cancel{\frac{1}{2}}\sqrt{x+1} + C$

$$\boxed{\sqrt{y} = \pm\sqrt{x+1} + C}$$

3) $\frac{y'^2}{4} + \frac{y^2}{x^2} = \frac{y'y}{x} + x^2y$ $\frac{1}{4}y'^2 - \frac{y}{x}y' + \frac{y^2}{x^2} - x^2y = 0$

$y' = \frac{\frac{y}{x} \pm \sqrt{\frac{y^2}{x^2} - \frac{1}{4}\cdot 4\left(\frac{y^2}{x^2} - x^2y\right)}}{2 \cdot \frac{1}{4}} = 2\frac{y}{x} \pm 2\sqrt{x^2y}$

$y' - 2\frac{y}{x} = \pm 2x\sqrt{y}$ $/ : \sqrt{y}$

$\frac{1}{\sqrt{y}} y' - 2\frac{1}{x}\sqrt{y} = \pm 2x$ Bernoulli

$t = \sqrt{y}$ $t' = \frac{1}{2\sqrt{y}}y'$ $\cancel{2}t' - \cancel{2}\frac{1}{x}t = \pm 2x$ linearna $t' - \frac{t}{x} = 0$

$t = x \cdot C$

$C'x + \cancel{C} - \cancel{C} = \pm x$ $C' = \pm 1$ $C(x) = \pm x + C$ $t = x(C \pm x)$

$x \cdot t^2 =$

$$\boxed{\sqrt{y} = Cx \pm x^2}$$

4) $y'^2 + 2xy' + 2y = 0$

5) $y\,dx + (2x - x^2y^2)\,dy = 0 \quad /:dx$

1) $y + (2x - x^2y^2)\,y' = 0$

$$y' = \frac{y}{x^2y^2 - 2x} \qquad x = x(y)$$

$$x' = \frac{x^2y^2 - 2x}{y}$$

$$x' = x^2y - 2\frac{x}{y}$$

$$x' + \frac{2}{y}x = x^2y \;/ : x^2$$

$$\frac{1}{x^2}x' + \frac{2}{y}\cdot\frac{1}{x} = y \quad \text{Bernulli}$$

2) $t = \frac{1}{x} \qquad t' = -\frac{1}{x^2}x'$

$$-t' + \frac{2}{y}t = y \quad \text{linearna}$$

$$t' - \frac{2}{y}t = 0$$

$$\frac{dt}{t} = 2\frac{dy}{y}$$

$$t = y^2 \cdot C$$

$$-C'y^2 - \cancel{2yC} + \cancel{2yC} = y$$

$$C' = -\frac{1}{y} \Rightarrow C(y) = -\ln y - \ln C$$

$$t = -y^2\ln(Cy)$$

$$x = \frac{-1}{y^2\ln Cy} \Rightarrow \boxed{xy^2\ln(Cy) = -1}$$

6) $(x^4y^3 - x)\,y' = y$

1) $y' = \frac{y}{x^4y^3 - x}, \quad x = x(y)$

$$x' = \frac{x^4y^3 - x}{y}$$

$$x' = x^4 \cdot y^2 - \frac{x}{y}$$

$$x' + \frac{x}{y} = x^4y^2 \;/ : x^4$$

$$\frac{1}{x^4}x' + \frac{1}{y}\cdot\frac{1}{x^3} = y^2 \quad \text{Bernulli}$$

2) $t = \frac{1}{x^3} \qquad t' = -\frac{3}{x^4}x'$

$$-\frac{1}{3}t' + \frac{1}{y}t = y^2 \quad \text{Linearna}$$

$$-\frac{1}{3}t' + \frac{1}{y}t = 0$$

$$t' = \frac{3}{y}t$$

$$\frac{dt}{t} = 3\frac{dy}{y}$$

$$t = y^3 C(y)$$

$$-\frac{1}{3}C'y^3 - \cancel{y^2C} + \cancel{y^2C} = y^2$$

$$C' = -3\frac{1}{y} \qquad C(y) = -3\ln yC$$

$$t = -3y^3\ln yC$$

$$-3y^3\ln yC = \frac{1}{x^3}$$

$$-3x^3y^3\ln yC - 1 = 0$$

$$\boxed{3x^3y^3\ln yC + 1 = 0}$$

Zad 9

1) $y = 2xy' + \sin y'$ $\quad y' = p \quad x = x(y)$

$2xp = y - \sin p \quad \frac{d}{dy}$

$2x'p + 2xp' = p - p'\cos p$

$\frac{2\cancel{x'}}{\cancel{x'}} + 2xp' = p - p'\cos p$

2) $e^x = \dfrac{y^2 + y'^2}{2y'}$ $\qquad y' = p(y)$

$e^x = \dfrac{y^2 + p^2}{2p} = \ldots$

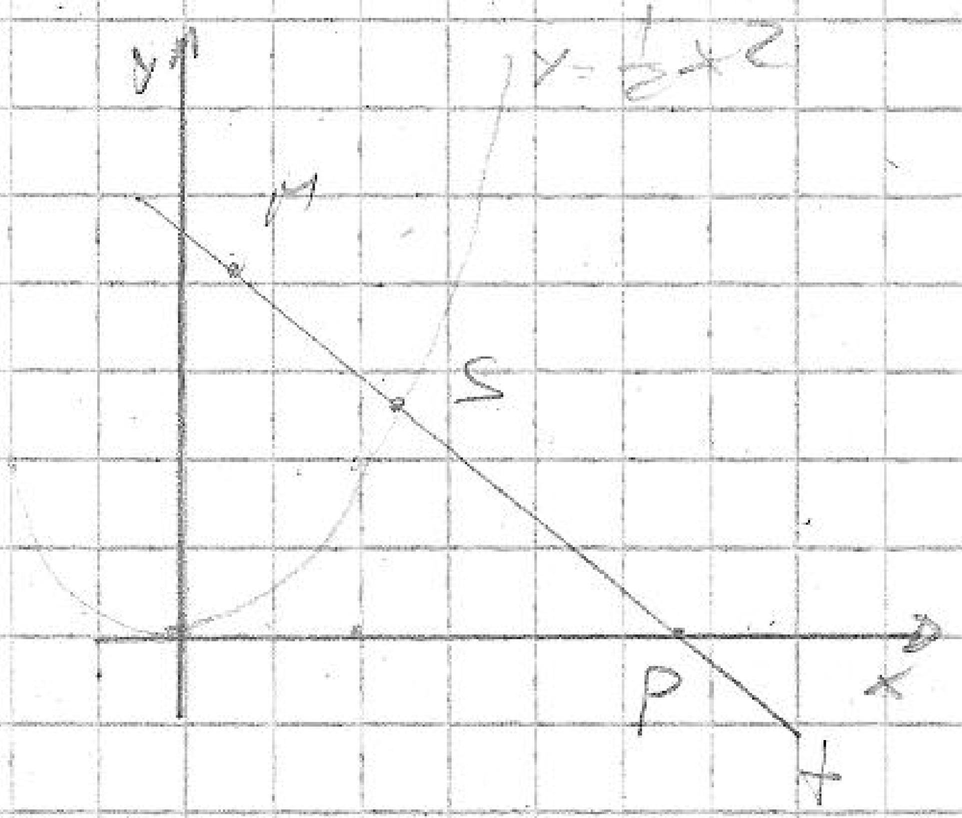
$2e^x = \dfrac{y^2}{p} + p \quad \Big/ \dfrac{d}{dx}$

$0 = \dfrac{2yp - y^2p'}{p^2} + p' \quad \Big/ \cdot p^2$

$0 = 2yp - y^2p' + p^2p'$

$0 = p'(p^2 - y^2) + 2yp$

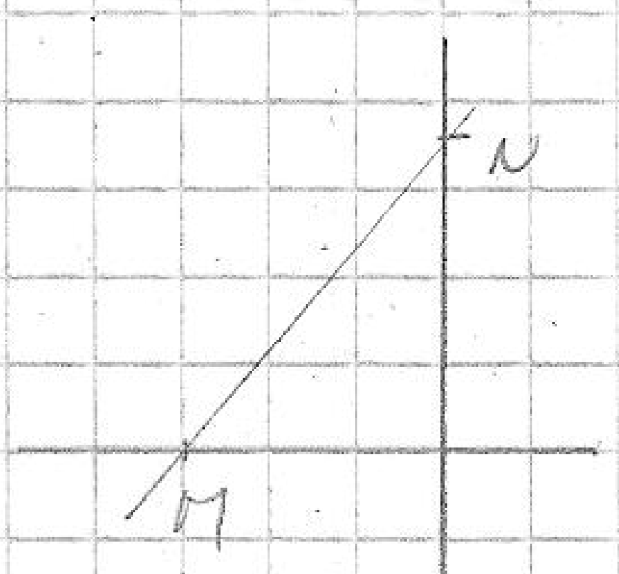
Zad 10



Pogledati izvode o ZZV 10

$$M_t = x - \frac{y}{y'} \qquad M_n = x + yy'$$

$$N_t = y - y'x \qquad N_n = y + \frac{x}{y'}$$



$$P(M_t, 0) \qquad M(x,y)$$

$$S\left(\frac{M_t + x}{2}, \frac{0+y}{2}\right) \in y = \frac{1}{2}x^2$$

$$\Rightarrow \frac{y}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{1}{2}(M_t + x)\right)^2$$

$$y = \frac{1}{4}\left(2x - \frac{y}{y'}\right)^2 \qquad x = x(y)$$

$$4y = (2x - yx')^2$$

$$\pm 2\sqrt{y} = 2x - yx'$$

$$x'y - 2x = \mp 2\sqrt{y} \quad / : y$$

$$x' - \frac{2}{y}x = \mp 2 \cdot \frac{1}{\sqrt{y}} \qquad \text{linearna}$$

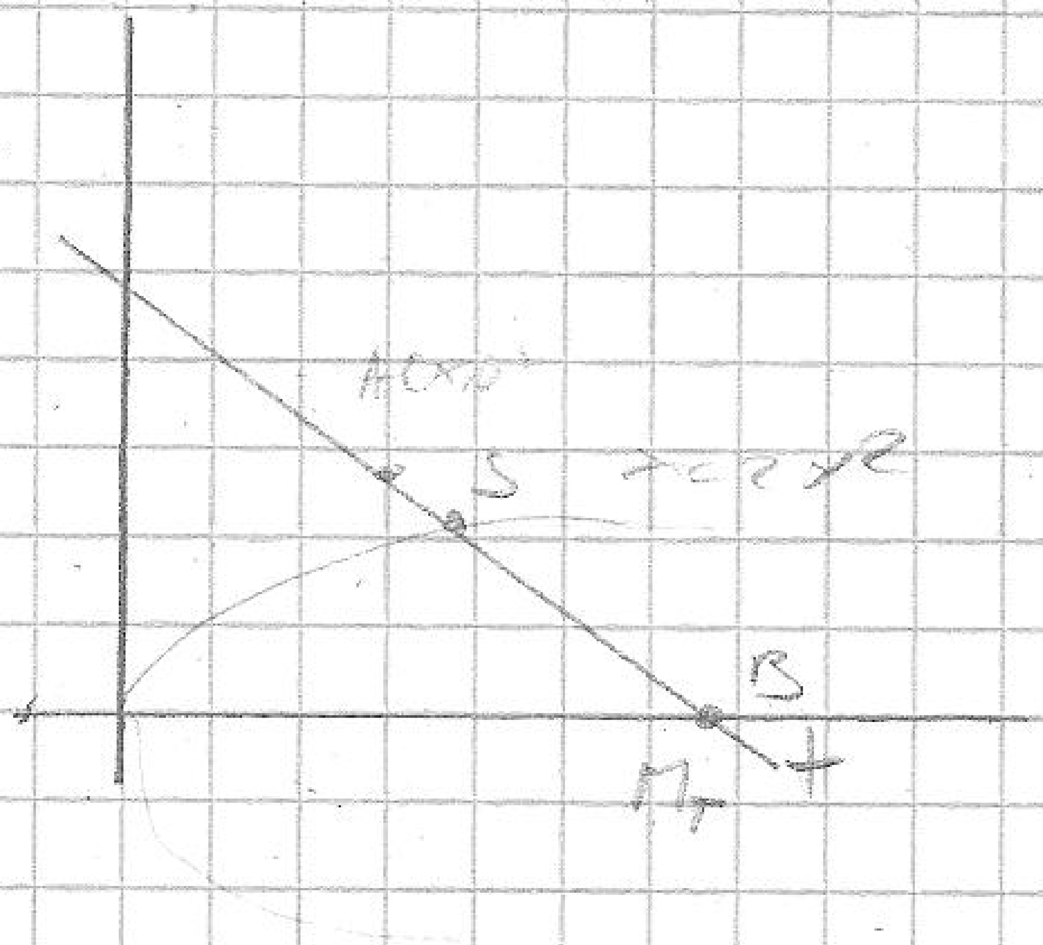
$$x' - \frac{2}{y}x = 0 \qquad \frac{dx}{x} = \frac{2dy}{y} \qquad x = Cy^2$$

$$C'y^2 + \cancel{2Cy} - \cancel{2Cy} = \pm\frac{2}{\sqrt{y}}$$

$$C' = 2 \pm y^{-\frac{5}{2}} \qquad C = 2 \cdot \frac{2}{3} \pm y^{-\frac{3}{2}} + C$$

$$\boxed{x = \frac{4}{3} \pm \sqrt{y} + Cy^2}$$

Zad 11. $T(0,1)$



$A(x,y) \qquad B(M_T, 0)$

$S\left(\frac{x+M_T}{2}, \frac{y+0}{2}\right)$

$\Rightarrow \frac{1}{2}(x+M_T) = 2\left(\frac{1}{2}y\right)^2$

$2x - \frac{y y}{y'} = y^2$

$(2x - y^2)y' = y$

$\underbrace{(2x-y^2)}_{Q}dy + \underbrace{(-y)}_{P}dx = 0$

$P'_y = -1 \qquad Q'_x = 2 \qquad P'_y - Q'_x = -3$

$\ln\mu(y) = -\int \frac{-3}{-y}\,dy = -3\ln y \qquad \mu = \frac{1}{y^3}$

$\int_0^x \left(-\frac{1}{y^2}\right)dx + \int_1^y \left(\cancel{\frac{2x}{y^3}} - \frac{1}{y}\right)dy = C$

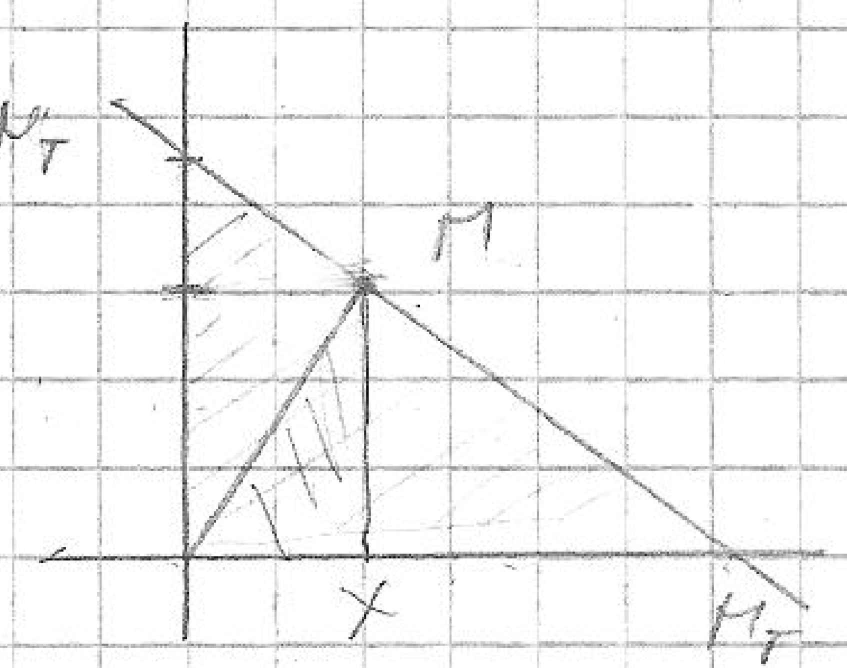
$-\frac{1}{y^2}x\Big|_0^x \quad (-\ln y)\Big|_1^y = C$

$-\frac{x}{y^2} - \ln y = C \qquad T(0,1)$

$0 - 0 = C \Rightarrow C = 0$

$\boxed{x + y^2\ln y = 0}$

Zad 12 $\quad A(2,3)$



$P_1 = \bigcirc \qquad \frac{1}{2}(M_T \cdot M_T) = 4 \cdot \frac{1}{2}\, x \cdot y$

$P_2 = \bigcirc \qquad \left(x - \frac{y}{y'}\right)\left(y - y'x\right) = 4xy$

$(xy' - y)(y - y'x) = 4xyy'$

$-(xy' - y)^2 = -4xyy'$

$x^2y'^2 - 2xyy' + y^2 = -4xyy'$

$x^2y'^2 + 2xyy' + y^2 = 0$

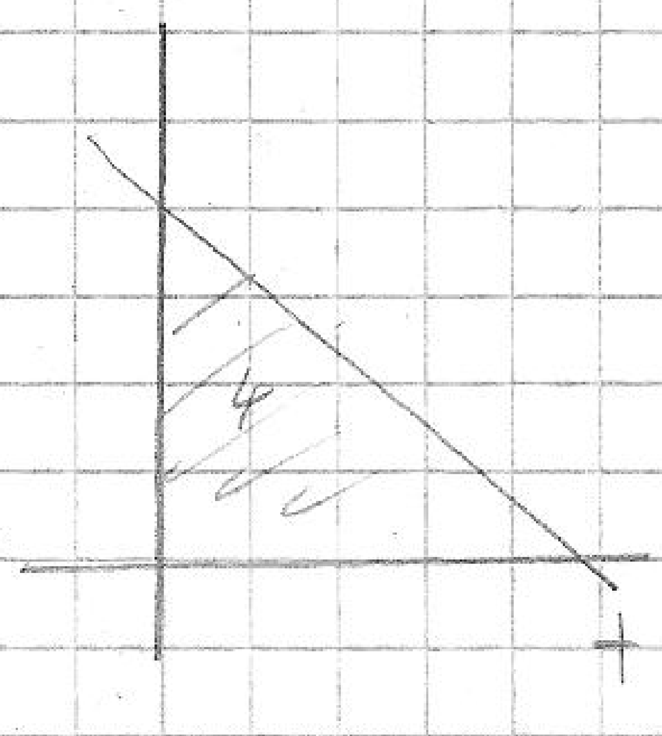
$(xy' + y)^2 = 0$

$xy' + y = 0 \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

Opet je pogrešan text zadatka!

$\ln y = \ln \frac{c}{x} \Rightarrow y = \frac{c}{x} \Rightarrow 3 = \frac{c}{2} \Rightarrow c = 6 \Rightarrow \boxed{y = \frac{6}{x}}$

Zad 13



$\frac{1}{2}|M_T \cdot M_T| = 4 \Rightarrow \left|(y - y'x)\left(x - \frac{y}{y'}\right)\right| = 8$

$(y - y'x)(y - y'x) = -8$

$\left| -\frac{(y - y'x)^2}{y'} \right| = 8$

$y - y'x = \pm\sqrt{8y'}$

$y = xy' \pm \sqrt{8y'} \qquad$ Clairaut

O.R [illegible] $\qquad$ S.R

$y = xp \pm \sqrt{8p} \qquad x = -\left(\pm\sqrt{8p}\right)'$

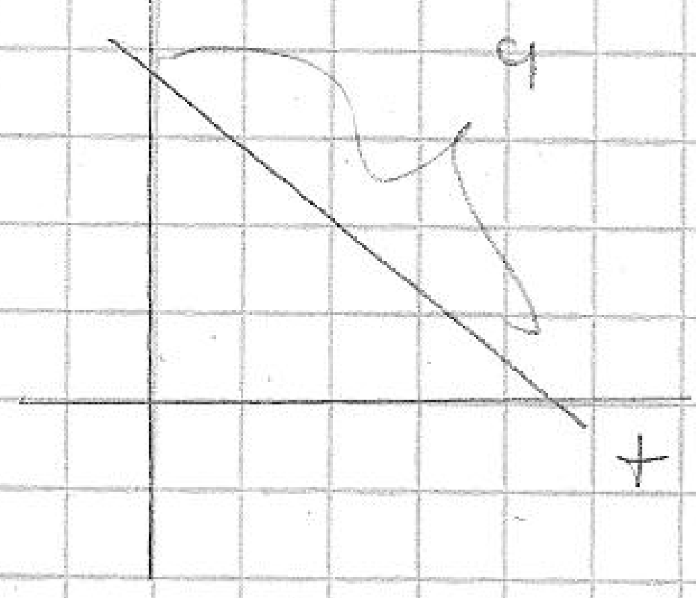
$= -\left(\pm \frac{1}{2}\frac{1}{\sqrt{8p}} \cdot 8\right) = \mp \frac{4}{\sqrt{8p}}$

$x = \mp \frac{4}{\sqrt{8p}}$

$8p = \frac{16}{x^2} \qquad p = \frac{2}{x^2}$

$y = x \cdot \frac{2}{x^2} \pm \frac{4}{x} = \frac{2}{x} \pm \frac{4}{x} = \boxed{\pm \frac{2}{x}}$

Zad 14 / Zad 26 [illegible]



$$M_T^2 + N_T^2 = a^2$$

$$\left(x - \frac{y}{y'}\right)^2 + (y - y'x)^2 = a^2$$

$$\left(\frac{y - xy'}{y'}\right)^2 + (y - y'x)^2 = a^2$$

$$(y - xy')^2 (1 + y'^2) = a^2 y'^2$$

$$y - xy' = \pm \frac{a y'}{\sqrt{1 + y'^2}}$$

$$y = x y' \pm \frac{a y'}{\sqrt{1 + y'^2}} \qquad \text{Clairaut}$$

C.h.

$$y = x p \pm \frac{a p}{\sqrt{1 + p^2}} \qquad x = -\left[\pm \frac{a p}{\sqrt{1 + p^2}}\right]'$$

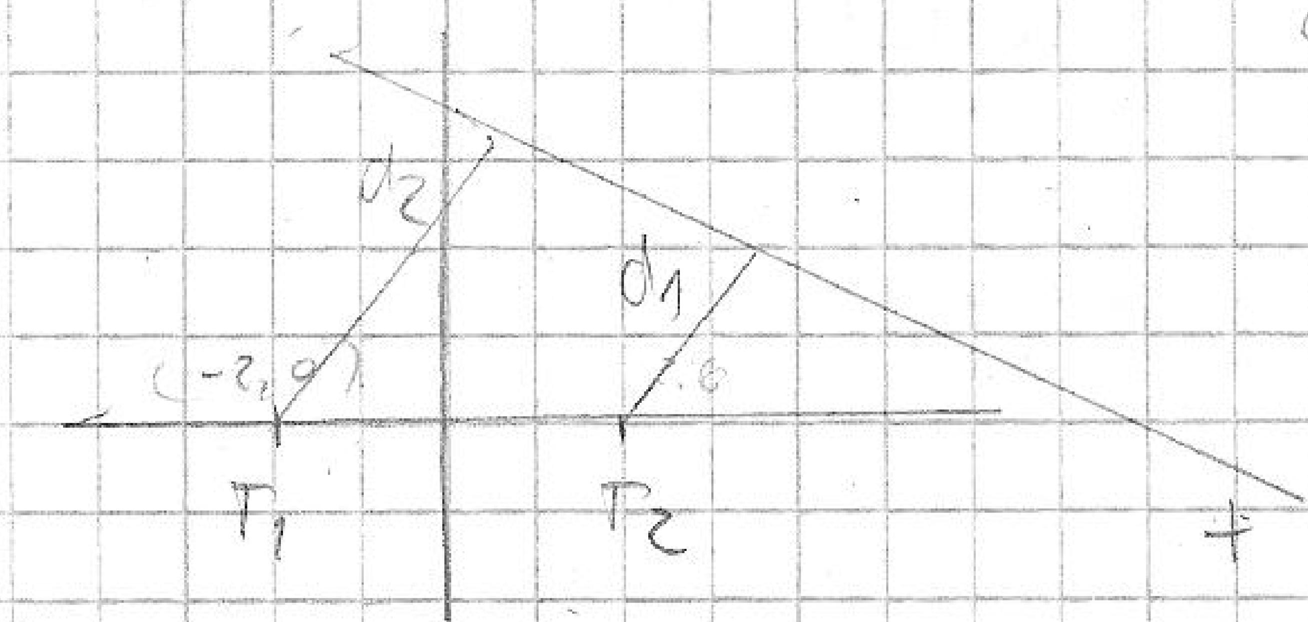
$$x = \mp \left[\frac{a\sqrt{1+p^2} - ap \cdot \frac{1}{2\sqrt{1+p^2}} \cdot 2p}{1 + p^2}\right] = \mp \left[\frac{\frac{a(1+p^2) - ap^2}{\sqrt{1+p^2}}}{1+p^2}\right]$$

$$\boxed{x = \mp \left[\frac{a}{(1+p^2)\sqrt{1+p^2}}\right]}$$

$$y = \mp \frac{a}{\sqrt{1+p^2}\,(1+p^2)} \pm \frac{a p}{\sqrt{1+p^2}} = \mp \frac{a}{\sqrt{1+p^2}} \left[\frac{p}{(1+p^2)} - p\right]$$

$$= \mp \frac{a}{\sqrt{1+p^2}} \left[\frac{\cancel{p} - \cancel{p} - p^3}{(1+p^2)}\right] = \boxed{\mp \frac{a p^3}{(1+p^2)\sqrt{1+p^2}}}$$

Zad 15.



$d_1 \cdot d_2 = 1$ t: $\beta - y = y'(\alpha - x)$

$$\underbrace{1}_{A}\beta \underbrace{- y'}_{B}\alpha \underbrace{+ y'x - y}_{C} = 0$$

$$d_1 = \frac{|-2\cdot 1 + 0\cdot y' + y'x - y|}{\sqrt{1+y'^2}} = \frac{|y'x - y - 2|}{\sqrt{1+y'^2}}$$

$$d_2 = \frac{|2\cdot 1 + 0\cdot y' + y'x - y|}{\sqrt{1+y'^2}} = \frac{|y'x - y + 2|}{\sqrt{1+y'^2}}$$

$d_1 \cdot d_2 = 1$

$|y'x - y - 2|\,|y'x - y + 2| = 1 + y'^2$

$|(y'x - y)^2 - 4| = 1 + y'^2$

$[\mp(y - y'x)]^2 = 4 \pm (1 + y'^2)$

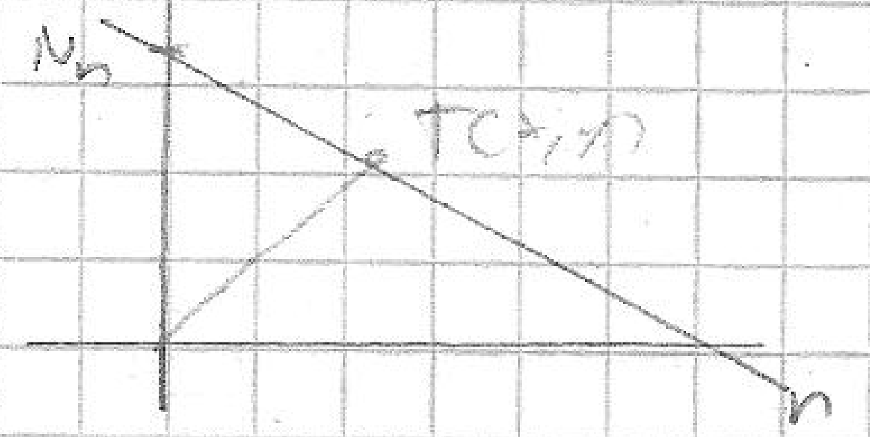
$y = xy' \pm \sqrt{4 \pm (1 + y'^2)}$ (Clairaut)

S.R. $y = xp \pm \sqrt{4 \pm (1 + p^2)}$

$$x = -\left[\pm\sqrt{4 \pm (1+p^2)}\right]' = \mp\left(\frac{1}{2\sqrt{4 \pm (1+p^2)}} \cdot (\pm 2p)\right) = \boxed{\frac{p}{\sqrt{4 \pm (1+p^2)}}}$$

$$y = \frac{p^2}{\sqrt{4 \pm (1+p^2)}} \pm \sqrt{4 \pm (1+p^2)} = \boxed{\frac{p^2 \pm [4 \pm (1+p^2)]}{\sqrt{4 \pm (1+p^2)}}}$$

Zad 16.



$$|N_n| = \sqrt{x^2+y^2} \Rightarrow \left|y + \frac{x}{y'}\right| = \sqrt{x^2+y^2}$$

$$|x + yy'| = y'\sqrt{x^2+y^2} \quad /^2$$

$$x^2 + 2xyy' + \cancel{y^2 y'^2} = y'^2 x^2 + \cancel{y^2 y'^2}$$

$$x^2 y'^2 - 2xyy' - x^2 = 0 \quad /:x^2$$

$$y' = \frac{2xy \pm \sqrt{4x^2y^2 + 4x^4}}{2x^2} = \frac{y}{x} \pm \sqrt{\frac{y^2}{x^2} + 1}$$

Homogeny $\quad p = \frac{y}{x} \quad y' = p + p'x$

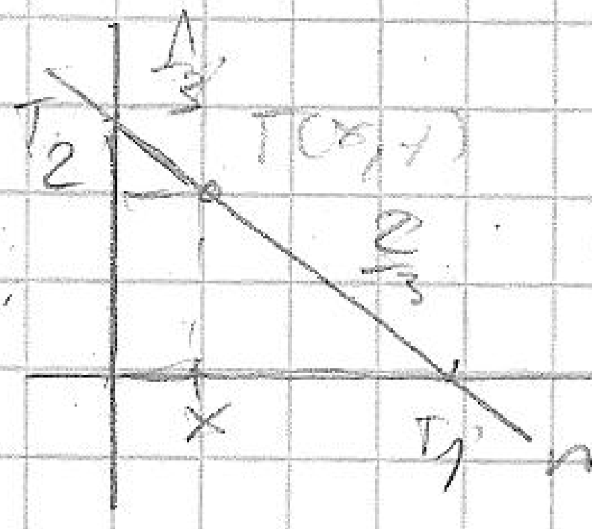
$$\cancel{p} + p'x = \cancel{p} \pm \sqrt{p^2+1}$$

$$\int \frac{dp}{\sqrt{p^2+1}} = \pm \int \frac{dx}{x}$$

$$\ln\left|p + \sqrt{p^2+1}\right| = \pm \ln x + \ln C$$

$$\boxed{\frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2 + 1} = C \cdot x} \qquad \boxed{\frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2 + 1} = \frac{C}{x}}$$

Zad 17



$$x = \frac{1}{3}|T_1 T_1| \quad 3x = |x + yy'|$$

$$2x = yy' \quad \frac{1}{2}y^2 = x^2 + C$$

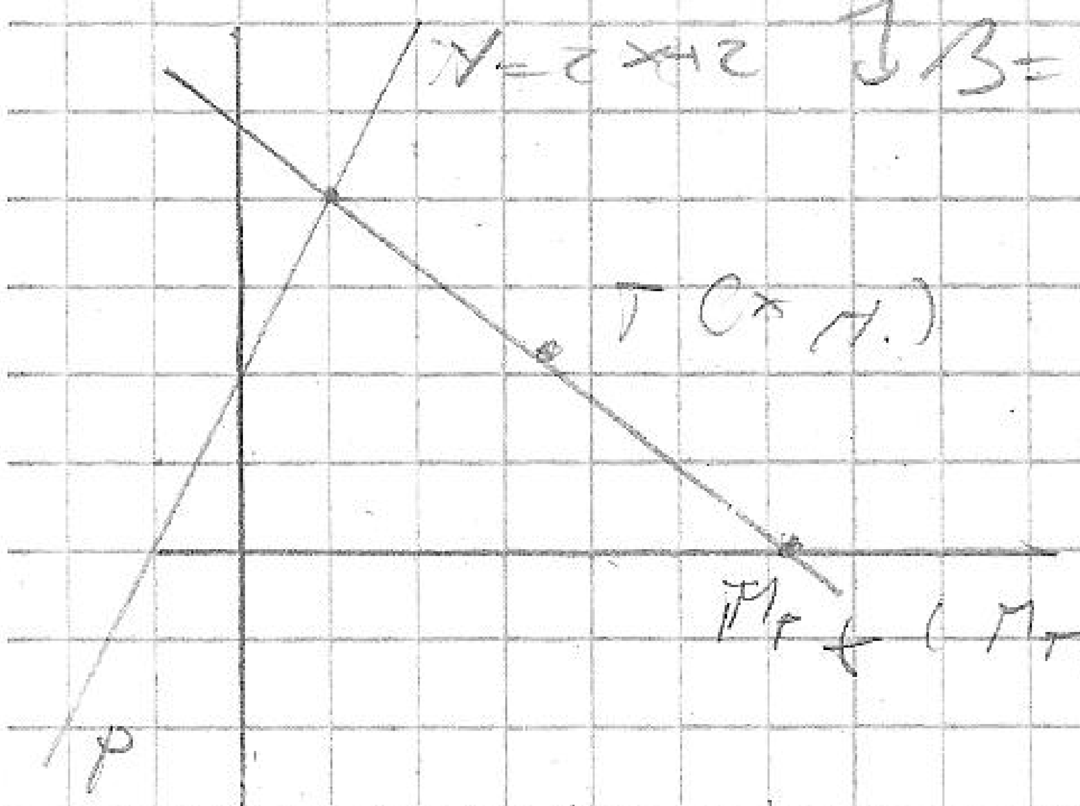
$$y^2 = 2x^2 + C$$

$$-4x = yy' \quad \frac{1}{2}y^2 = -2x^2 + C$$

$$y^2 = -4x + C$$

$$\boxed{y^2 = 2x + C} \qquad \boxed{y^2 = -4x + C}$$

Zad 13.



$\downarrow\ \beta = 2\alpha + 2$

$t:\ \beta - y = y'(\alpha - x)$

$t \cap p$

$2\alpha + 2 - y = y'\alpha - y'x$

$\alpha(2 - y') = y - 2 - y'x$

$\alpha = \boxed{\dfrac{y'x + 2 - y}{y' - 2}}$

$\beta = 2\alpha + 2 = \dfrac{2y'x + 4 - 2y + 2y' - 4}{y' - 2}$

$= \boxed{\dfrac{2y'x - 2y + 2y'}{y' - 2}}$

$x = \dfrac{\alpha + M_T}{2}$ $\quad \boxed{y = \dfrac{\beta + 0}{2}}$

$x(y' - 2) = y'x - y + y'$

$yy' - 2y = y'x - y + y'$

$y'(y - x - 1) = y$

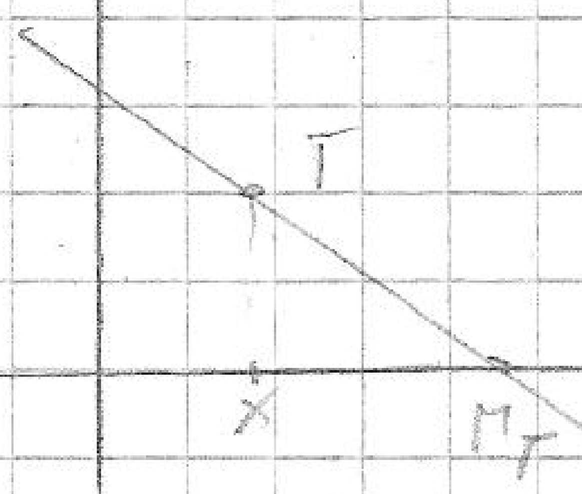
$dy(y - x - 1) - (y)\,dx = 0$ $\quad$ Egzakt. $\to P'_y = -1,\ Q'_x = -1$

$-\cancel{\int_1^x 0\,dx} + \int_0^y (y - x - 1)\,dy = C$

$\boxed{\frac{1}{2}y^2 - xy - y = C}$

Zad 14

$P(3,1)$



$|M_T - x| = x + y$

$|\cancel{x} + \frac{y}{y'} - \cancel{x}| = x + y$

$y' = \pm \dfrac{y}{x+y}$, $x = x(y)$

$x' = \pm\left(\dfrac{x+y}{y}\right) = \pm\left(\dfrac{x}{y} + 1\right)$ Homog.

$p = \dfrac{x}{y}$ $\quad x' = p + yp'$

$p + yp' = \pm p \pm 1$

(1) (2)

(1) $\cancel{p} + yp' = \cancel{p} + 1$

$dp = \frac{1}{y}\,dy$

$p = \ln y + C$

$x = y\ln y + Cy \Rightarrow \boxed{x = y\ln y + 3y}$

$3 = 0 + C \quad C = 3$

(2) $p + yp' = -p - 1$

$yp' = -2p - 1$

$\dfrac{dp}{2p+1} = -\dfrac{dy}{y}$

$\frac{1}{2}\ln(2p+1) = \ln\frac{C}{y}$

$\sqrt{2p+1} = \dfrac{C}{y}$

$C = \sqrt{12}$

$\boxed{\sqrt{2\frac{x}{y} + 1} = \dfrac{\sqrt{12}}{y}}$

CB

Zad 20

$$(2x+3y-5)\,dx+(3x+2y-5)\,dy=0$$ [illegible]

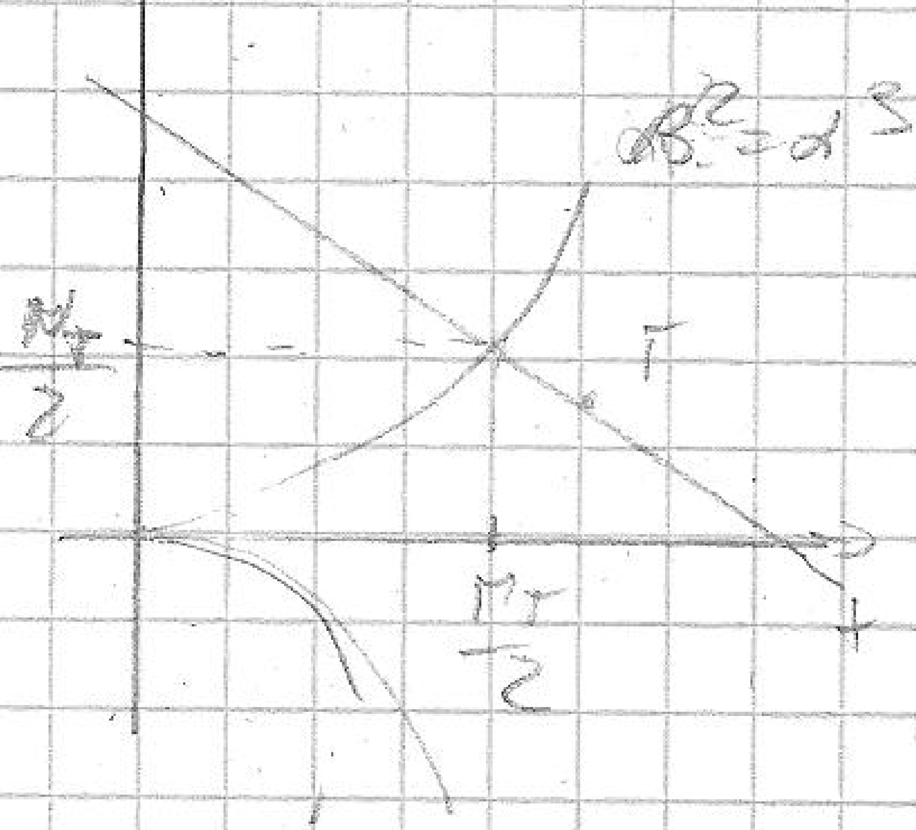
$$\int_0^x (2x+3y-5)\,dx+\int_0^y (\cancel{3x}+2y-5)\,dy=C \qquad T(1,0)$$

$$x^2+3xy-5x+y^2-5y=C$$

$$1+0-5+0+0=C \Rightarrow C=-4$$

$$\boxed{x^2+3xy-5x+y^2-5y=-4}$$

Zad 21.



$$\Rightarrow \left(\frac{N_T}{2}\right)^2=\left(\frac{M_T}{2}\right)^3$$

$$\frac{1}{4}(y-y'x)^2=\frac{1}{8}\left(x-\frac{y}{y'}\right)^3 \quad /\cdot 8$$

$$2(y'x-y)^2=\frac{1}{y'^3}(y'x-y)^3$$

$$(y'x-y)^2=0 \Rightarrow y'x=y \quad y=C\cdot x \quad \text{prosta}$$

$$2y'^3=(y'x-y)$$

$$y=xy'-2y'^3 \quad \text{Clairaut} \quad O.R \Rightarrow \text{prosta}$$

Sol: $y=xp-2p^3$

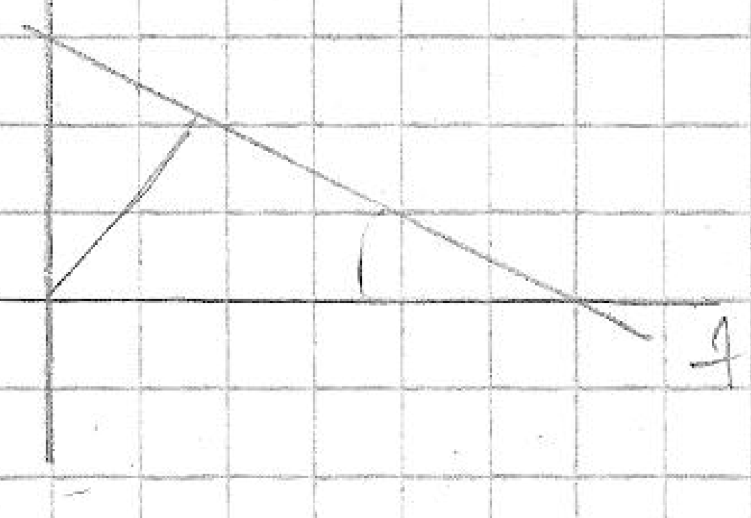
$$x=6p^2$$

$$y=p(x-2p^2)=p\left(x-\frac{1}{3}x\right)=\frac{2}{3}x\cdot p \quad /^2$$

$$y^2=\frac{4}{9}p^2\cdot x^2=\frac{4}{9}\cdot\frac{1}{6}x\cdot x^2=\frac{2}{27}x^3$$

$$\boxed{y^2=\frac{2}{27}x^3}$$

Zad 22.



koeficijent $= y'$ $t \ldots \; \beta - y = y'(\alpha - x)$

$$\underbrace{1\cdot\beta}_{A} \underbrace{-y'\cdot\alpha}_{B} \underbrace{+xy'-y}_{C}$$

$$d(0,t) = \frac{|0\cdot 1 + 0\cdot y' + xy' - y|}{\sqrt{1+y'^2}} = \frac{|xy'-y|}{\sqrt{1+y'^2}}$$

$$y' = d(0,t) \Rightarrow |xy'-y'| = y'\sqrt{1+y'^2}$$

$$xy' - y' = \pm y'\sqrt{1+y'^2}$$

$$y' = xy' \pm y'\sqrt{1+y'^2} \quad \text{Clairaut}$$

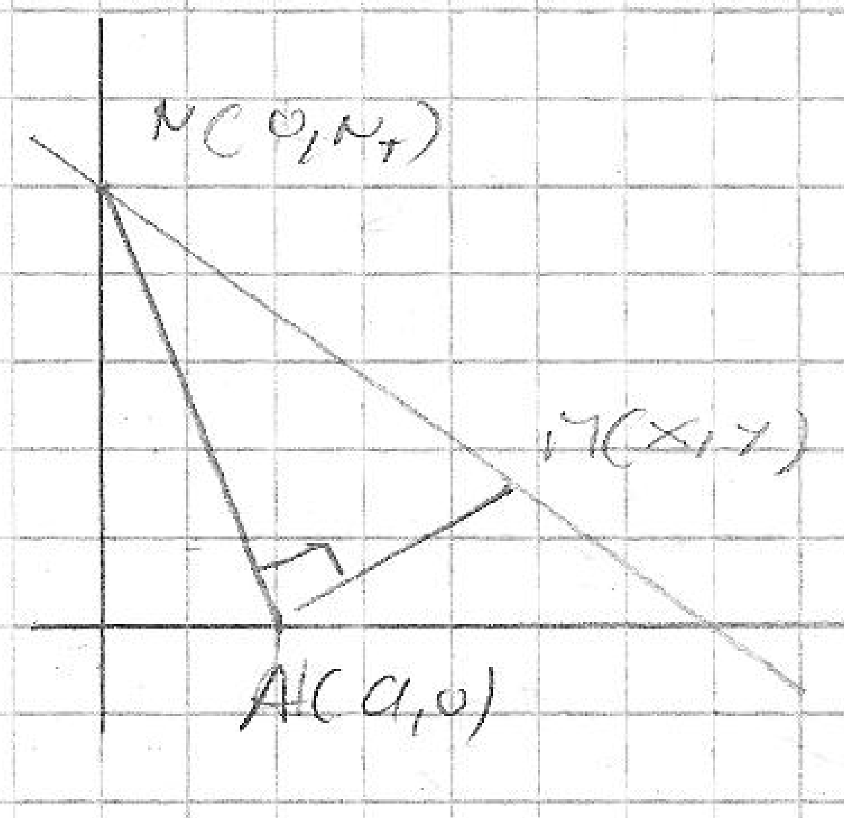
$$y = xp \pm p\sqrt{1+p^2} \qquad x = \mp\left[p\sqrt{1+p^2}\right]' = \mp\left[\sqrt{1+p^2} + \frac{p}{2\sqrt{1+p^2}}\cdot 2p\right]$$

$$= \mp\left[\frac{1+p^2+p^2}{\sqrt{1+p^2}}\right] = \boxed{\mp\frac{2p^2+1}{\sqrt{1+p^2}}}$$

$$y = p\left[\mp\frac{2p^2+1}{\sqrt{1+p^2}} \pm \sqrt{1+p^2}\right] = \mp p\left[\frac{2p^2+1}{\sqrt{1+p^2}} - \sqrt{1+p^2}\right]$$

$$= \mp p\left[\frac{\cancel{2p^2+1-1-p^2}}{\sqrt{1+p^2}}\right] = \boxed{\mp\frac{p^3}{\sqrt{1+p^2}}}$$

Zad 23.



$$d(A,M)^2 + d(A,N)^2 = d(N,M)^2$$

$$(a-x)^2 + y^2 + a^2 + n_T^2 = x^2 + (y - n_T)^2$$

$$a^2 - 2ax + \cancel{x^2} + \cancel{y^2} + a^2 + \cancel{n_T^2} = \cancel{x^2} + \cancel{y^2} - 2yn_T + \cancel{n_T^2}$$

$$2a^2 - 2ax = -2y^2 - 2xyy'$$

$$xyy' + ax - y^2 - a^2 = 0 \quad /:x$$

$$yy' - \frac{1}{x}y^2 = -a + a^2\cdot\frac{1}{x} \quad \text{Bernoulli}$$

$$t = y^2 \quad t' = 2yy' \quad \frac{1}{2}t' - \frac{1}{x}t = -a + a^2\cdot\frac{1}{x} \quad \text{Linear}$$

$$\frac{1}{2}t' - \frac{1}{x}t = 0 \qquad \frac{1}{2}c'x^2 + \cancel{xc} - \cancel{xc} = -a + a^2\cdot\frac{1}{x}$$

$$t = cx^2$$

$\Downarrow$

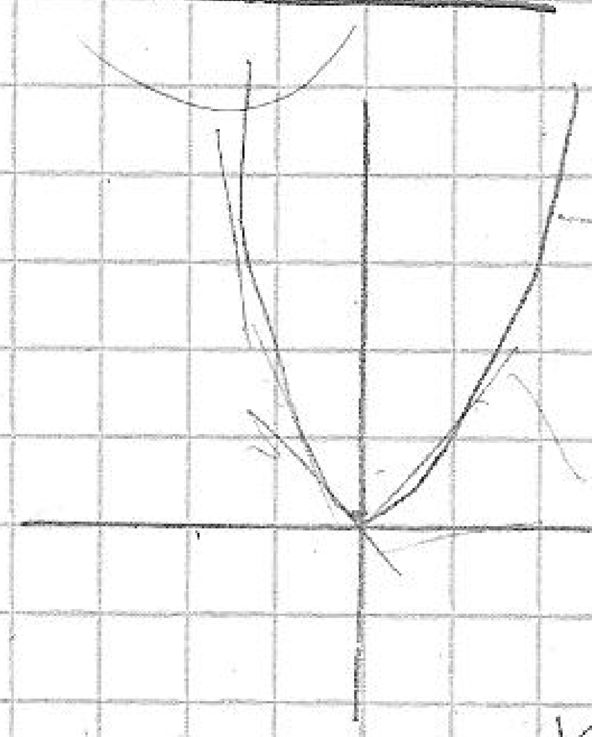
$$\frac{1}{2}C'x^2 = -a + \frac{1}{x}a^2 \Rightarrow C' = -\frac{2a}{x^2} + \frac{2a^2}{x^3}$$

$$C(x) = \frac{1}{x}2a - \frac{a^2}{x^2} + C$$

$$t = 2ax - a^2 + Cx^2$$

$$\boxed{y^2 - Cx^2 - 2ax + a^2 = 0}$$

Zad 24



$y = x^2$ $\quad y(2) = 1$

O.R. je familija pravaca $\Rightarrow$ D.J. je Clairautova

$y = xp + f(p)$

$y - x^2 = 0$ $\quad$ S.R.: $x = -f'(p)$

$y' - 2x = 0$

$p - 2x = 0$

$p + 2f'(p) = 0$

$2\frac{df}{dp} = -p$

$2df = -p\,dp$

$2f = -\frac{1}{2}p^2$

$f = -\frac{1}{4}p^2$

$$\boxed{y = xy' - \frac{1}{4}y'^2}$$

O.R. $\quad y = xC - \frac{1}{4}C^2$

$1 = 2C - \frac{1}{4}C^2 \quad / \cdot (-4)$

$C^2 - 8C + 4 = 0$

$C_{1,2} = \frac{8 \pm \sqrt{64-16}}{2} = 4 \pm \sqrt{16-4}$

$= 4 \pm \sqrt{12}$

$= 4 \pm 2\sqrt{3}$

$= 2(2 \pm \sqrt{3})$

$$\boxed{y = 2x(2 \pm \sqrt{3}) - (2 \pm \sqrt{3})^2}$$

Zad 25 riješen kao Zad 17 u ZZV 10 na str. 12.