

$$1. \quad y'(t) + 5y(t) + 2 = u(t) \rightarrow y'(t) = u(t) - 5y(t) - 2$$

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y'_1(0) = u_1(0) - 2$$

$$y'_2(0) = u_2(0) - 2$$

$$y'(0) = u(0) - 2$$

$$y'(0) = \alpha u_1(0) + \beta u_2(0) - 2$$

$$y'(0) = \alpha y'_1(0) + \beta y'_2(0) + 2 \rightarrow \text{SUSTAV NIJE LINEARAN.}$$

U SLUČAJU DA JE SUSTAV LINEARAN, I JOS K
TOMU I VREMENSKI NEPROMJENJIV (LTI), MOGLI BISMO
PRONACI ODZIV NA BILO KOJU POBUDU u_2 KOJA
SE MOŽE PRIKAZATI KAO LINEARNA KOMBINACIJA
IZ PRVOG SIGNALA u_1 (MORAMO ZNATI ODZIV NA
POBUDU u_1)

$$2. \quad y'''(t) - y''(t) + y'(t) + 39y(t) = u''(t) + 2u(t)$$

- TRAŽENJE VLASTITIH FREKVENCIJA

$$y_n(t) = Ce^{st}$$

$$Ce^{st} (s^3 - s^2 + s + 39) = 0, \quad Ce^{st} \neq 0$$

$$s^3 - s^2 + s + 39 = 0$$

JEDNO RJEŠENJE MORAMO POGODITI, A DRUGA
DVA ČEMO DOBITI RJEŠAVANJEM KVADRATNE
JEDNADŽBE KOJU DOBIJEMO SUJEDECIM POSTUPKOM

$$(s^3 - s^2 + s + 39) : (s - s_0) = a_0 s^2 + a_1 s + a_2$$

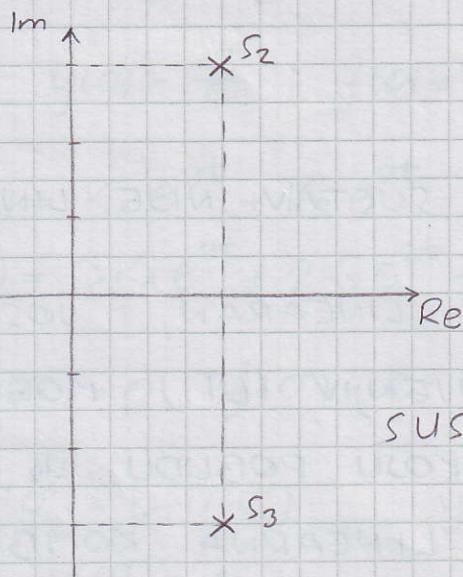
POGOĐENO RJEŠENJE: $s_1 = -3$

$$\begin{array}{r} (s^3 - s^2 + s + 39) : (s + 3) = s^2 - 4s + 13 \\ \hline -s^3 + 3s^2 \\ -4s^2 + s \\ \hline +4s^2 + 12s \\ 13s + 39 \\ -13s - 39 \\ \hline 0 \end{array}$$

$$s^2 - 4s + 13 = 0 \quad s_{2,3} = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$s_{2,3} = \frac{4 \pm j\sqrt{36}}{2}$$

$$s_2 = 2 + j3, \quad s_3 = 2 - j3$$



SUSTAV JE NESTABILAN.

3. $y'(t) + y(t) = u'(t) + 2u$

$$u(t) = 3N(t)$$

- OPĆA HOMOGENA JEDNAĐEŽBA

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s+1) = 0, \quad Ce^{st} + 0$$

$$s+1=0 \rightarrow s=-1$$

$$y_h(t) = C_1 e^{-t}$$

- PARTIKULARNA JEDNAĐEŽBA

$$u(t) = 3N(t) \rightarrow y_p(t) = K$$

$$K = 6, \quad t > 0 \rightarrow y_p(t) = 6, \quad t > 0$$

$$y(t) = y_n(t) + y_p(t)$$

$$y(t) = C_1 e^{-t} + 6, t \geq 0$$

$$(a) \quad y(0^-) = 9$$

$$y(0^+) = b_0 u(0^+) + y(0^-)$$

$$y(0^+) = 12$$

$$y(0) = C_1 + 6 = 12$$

$$C_1 = 6$$

$$y(t) = (6e^{-t} + 6) n(t)$$

$$(b) \quad y(0^+) = 9$$

$$y(0) = C_1 + 6 = 9$$

$$C_1 = 3$$

$$y(t) = (3e^{-t} + 6) n(t)$$

$$4. \quad y_n(t) = \frac{1}{2} e^{3t} + \frac{5}{3} e^{2t}$$
$$u(t) = \frac{5}{2} e^{2t}, t \geq 0$$

$$\sigma_1 = 3, \quad C_1 = \frac{1}{2}$$

$$y''(t) + a_1 y'(t) + a_0 y(t) = b_2 u(t)$$

$$\sigma_2 = 2, \quad C_2 = \frac{5}{3}$$

$$(s-3)(s-2) = 0$$

$$s^2 - 5s + 6 = 0 \rightarrow a_1 = -5$$

$$a_0 = 6$$

$$y''(t) - 5y'(t) + 6y(t) = u(t)$$

$$u(t) = \frac{5}{2} e^{2t} \rightarrow y_p(t) = A + e^{2t}$$

$$y_p'(t) = Ae^{2t}(1+2t)$$

$$y_p''(t) = Ae^{2t}(4+4t)$$

$$Ae^{2t}(4+4t-5-10t+6t) = \frac{5}{2}e^{2t}$$

$$A = -\frac{5}{2} \rightarrow y_p(t) = -\frac{5}{2}te^{2t}$$

$$y(t) = \frac{1}{2}e^{3t} + \frac{5}{3}e^{2t} - \frac{5}{2}te^{2t} - \text{TOTALNI ODZIV}$$

$$y'(t) = \frac{3}{2}e^{3t} + \frac{10}{3}e^{2t} - \frac{5}{2}e^{2t}(1+2t)$$

$$y(0) = \frac{1}{2} + \frac{5}{3} = \frac{13}{6}$$

$$y'(0) = \frac{3}{2} + \frac{10}{3} - \frac{5}{2} = \frac{7}{3}$$

$$y(0) = \frac{13}{6}, \quad y'(0) = \frac{7}{3}$$

$$y_m(t) = K_1 e^{3t} + K_2 e^{2t} - \frac{5}{2}te^{2t}$$

$$y'_m(t) = 3K_1 e^{3t} + 2K_2 e^{2t} - \frac{5}{2}e^{2t}(1+2t)$$

$$y(0^-) = 0, \quad y'(0^-) = 0$$

$$y(0^+) = b_0 u(0^+) \quad y'(0^+) = b_0 u'(0^+) + b_1 u(0^+)$$

$$y(0^+) = 0 \quad y'(0^+) = 0$$

$$\begin{aligned} y_m(0) &= K_1 + K_2 = 0 \\ y'_m(0) &= 3K_1 + 2K_2 - \frac{5}{2} = 0 \end{aligned} \quad \left. \right\} \quad K_1 = \frac{5}{2}, \quad K_2 = -\frac{5}{2}$$

$$y_m(t) = \frac{5}{2}e^{3t} - \frac{5}{2}e^{2t} - \frac{5}{2}te^{2t}, \quad t \geq 0$$

$$y(t) = \frac{1}{2}e^{3t} + \frac{5}{3}e^{2t} - \frac{5}{2}te^{2t}, \quad t \geq 0$$

5. $y'(t) + 2y(t) = u(t)$

$$u(t) = A \cos(\omega_0 t) \cdot n(t)$$

$$y(0^-) = 0, \quad b_0 = 0, \quad b_1 = 1$$

$$y(0^+) = b_0 u(0^+) + y(0^-)$$

$$y(0^+) = 0$$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(t) = C e^{st}$$

$$C e^{st} (s+2) = 0, \quad C e^{st} + 0$$

$$s+2 = 0 \rightarrow s = -2$$

$$y_h(t) = C_1 e^{-2t}$$

- PARTIKULARNA JEDNADŽBA

$$u(t) = A \cos(\omega_0 t) \rightarrow y_p(t) = K_1 \cos(\omega_0 t) + K_2 \sin(\omega_0 t)$$

$$y_p'(t) = -\omega_0 K_1 \sin(\omega_0 t) + \omega_0 K_2 \cos(\omega_0 t)$$

$$-\omega_0 K_1 \sin(\omega_0 t) + \omega_0 K_2 \cos(\omega_0 t) + 2K_1 \cos(\omega_0 t) + 2K_2 \sin(\omega_0 t) = A \cos(\omega_0 t)$$

$$\cos(\omega_0 t) (\omega_0 K_2 + 2K_1) + \sin(\omega_0 t) (-\omega_0 K_1 + 2K_2) = A \cos(\omega_0 t)$$

$$\left. \begin{array}{l} \omega_0 K_2 + 2K_1 = A \\ -\omega_0 K_1 + 2K_2 = 0 \end{array} \right\} \quad K_1 = \frac{2}{4+\omega_0^2} A, \quad K_2 = \frac{\omega_0}{4+\omega_0^2} A$$

$$y_p(t) = \frac{2}{4+\omega_0^2} A \cos(\omega_0 t) + \frac{\omega_0}{4+\omega_0^2} A \sin(\omega_0 t)$$

• TOTALNI ODZIV

$$y(t) = y_n(t) + y_p(t)$$

$$y(t) = C_1 e^{-2t} + \frac{2}{4+w_0^2} A \cos(w_0 t) + \frac{w_0}{4+w_0^2} A \sin(w_0 t)$$

$$y(0) = C_1 + \frac{2}{4+w_0^2} A = 0$$

$$C_1 = -\frac{2}{4+w_0^2} A$$

$$y(t) = \frac{-2}{4+w_0^2} A e^{-2t} + \frac{2}{4+w_0^2} A \cos(w_0 t) + \frac{w_0}{4+w_0^2} A \sin(w_0 t)$$

• ODZIV NA POBUDU $u(t) = B \cos(w_0(t-1)) \nu(t-1)$

KORISTIMO ČINJENICU DA JE SUSTAV LTI.

$$y_1(t) = y(t-1)$$

$$y_1(t) = \frac{-2}{4+w_0^2} B e^{-2(t-1)} + \frac{2}{4+w_0^2} B \cos(w_0(t-1)) + \frac{w_0}{4+w_0^2} A \sin(w_0(t-1))$$

6. $y'(t) - y(t) = -u(t)$

$$u(t) = \nu(-t) \rightarrow u(t) = 1, t \leq 0$$

• OPĆA HOMOGENA JEDNADŽBA

$$y_n(t) = C e^{st}$$

$$C e^{st} (s-1) = 0, C e^{st} \neq 0$$

$$s-1=0 \rightarrow s=1$$

$$y_n(t) = C_1 e^t$$

• PARTIKULARNA JEDNADŽBA

$$u(t) = \nu(-t) \rightarrow y_p(t) = K$$

$$K=1, t < 0 \rightarrow y_p(t) = 1, t < 0$$

$$y(t) = y_n(t) + y_p(t)$$

$$y(t) = C_1 e^t + 1, \quad t < 0$$

$$y(t) = y_n(t)$$

$$y(t) = C_1 e^t, \quad t > 0$$

$$(a) \quad y(0^+) = 0 \rightarrow y(0^-) = y(0^+) = 0$$

$$y(0^-) = C_1 + 1 = 0$$

$$C_1 = -1$$

$$y(t) = -e^t + 1, \quad t < 0$$

KONACNO:

$$y(0^+) = C_1 = 0$$

$$C_1 = 0$$

$$y(t) = 0, \quad t > 0$$

$$(b) \quad y(0^+) = 1 \rightarrow y(0^-) = y(0^+) = 1$$

$$y(0^-) = C_1 + 1 = 1$$

$$C_1 = 0$$

$$y(t) = 1, \quad t < 0$$

KONACNO:

$$y(0^+) = C_1 = 1$$

$$y(t) = e^t + (1-e^t) \nu(-t)$$

$$C_1 = 1$$

$$y(t) = e^t, \quad t > 0$$

$$7. \quad y'(t) + 2y(t) = u'(t) + u(t)$$

$$h(t) = -e^{-2t} \nu(t) + \delta(t)$$

$$h'(t) = e^{-2t} (-\delta(t) + 2\nu(t)) + \delta'(t)$$

$$-e^{-2t} \delta(t) + 2e^{-2t} \nu(t) + \delta'(t) - 2e^{-2t} \nu(t) + 2\delta(t) = \delta'(t) + \delta(t)$$

$$\delta(t) (2 - e^{-2t}) + \delta'(t) = \delta(t) + \delta'(t)$$

$$\cup \quad t=0 \rightarrow \delta(t) + \delta'(t) = \delta(t) + \delta'(t)$$

$$8. \quad y''(t) + 2y'(t) + y(t) = u(t)$$

$$y(0^-) = y'(0^-) = 0$$

$$y(0^+) = y'(0^+) = 0$$

$$y'(0^+) = y'(0^-) = 0$$

$$y''(t) = u(t) - 2y'(t) - y(t)$$

$$y''(0) = u(0) - 2y'(0) - y(0)$$

$$y''(0) = u(0)$$

$$u(t) = \mathcal{L}^{-1} u_1(t) + \beta u_2(t)$$

$$y_1''(0) = \mathcal{L} u_1(0), \quad y_2''(0) = \beta u_2(0)$$

$$y''(0) = \mathcal{L} u_1(0) + \beta u_2(0)$$

$$y''(0) = \mathcal{L} y_1''(0) + \beta y_2''(0) \rightarrow \text{SUSTAV JE LINEARAN}$$

• OPĆA HOMOGENA JEDNADŽBA

$$y_n(t) = C e^{st}$$

$$C e^{st} (\sigma^2 + 2\sigma + 1) = 0, \quad C e^{st} \neq 0$$

$$\sigma^2 + 2\sigma + 1 = 0 \rightarrow \sigma_{1,2} = -1$$

$$y_n(t) = (C_1 + C_2 t) e^{-t}$$

$$(a) \quad u(t) = t \nu(t) \rightarrow u(t) = t, \quad t \geq 0$$

$$y_p(t) = K_0 + K_1 t, \quad y_p'(t) = K_1, \quad y_p''(t) = 0$$

$$2K_1 + K_0 + K_1 t = t$$

$$\left. \begin{array}{l} 2K_1 + K_0 = 0 \\ K_1 = 1 \end{array} \right\} \quad K_1 = 1, \quad K_0 = -2$$

$$y_p(t) = -2 + t, \quad t \geq 0$$

$$y(t) = y_n(t) + y_p(t)$$

$$y(t) = (C_1 + C_2 t) e^{-t} - 2 + t$$

$$y'(t) = (-C_1 + C_2 - C_2 t) e^{-t} + 1$$

$$y(0) = C_1 - 2 = 0$$

$$C_1 = 2$$

$$y'(0) = -C_1 + C_2 + 1 = 0$$

$$C_2 = 1$$

$$y(t) = (2 + t) e^{-t} - 2 + t, \quad t \geq 0$$

$$(b) \quad u_b(t) = \mu(t)$$

$$u_a(t) = t \nu(t) \rightarrow u_a'(t) = \nu(t) + t \delta(t)$$

$$t \delta(t) = 0, \quad \forall t$$

$$u_a'(t) = \nu(t) = u_b(t)$$

ZBOG LINEARNOSTI SUSTAVA MOŽEMO PISATI

$$y_b(t) = y_a'(t)$$

$$y(t) = (-1 - t) e^{-t} + 1, \quad t \geq 0$$

$$(c) \quad u_c(t) = \delta(t)$$

$$u_b(t) = \nu(t) \rightarrow u_b'(t) = \delta(t) = u_c(t)$$

$$y_c(t) = y_b'(t)$$

$$y(t) = t e^{-t}, \quad t \geq 0$$

$$(d) \quad u_d(t) = t \nu(t) + \nu(t) + \delta(t)$$

$$u_d(t) = u_a(t) + u_b(t) + u_c(t) \rightarrow y_d(t) = y_a(t) + y_b(t) + y_c(t)$$

$$y(t) = (1 + t) e^{-t} - 1 + t, \quad t \geq 0$$

$$9. \quad y'(t) + ay(t) = u(t)$$

$$y(0^-) = 0 \rightarrow y(0^+) = 0$$

• OPĆA HOMOGENA JEDNADŽBA

$$y_n(t) = Ce^{st}$$

$$Ce^{st}(s+a) = 0, \quad Ce^{st} \neq 0$$

$$s+a=0 \rightarrow s=-a$$

$$y_n(t) = C_1 e^{-at}$$

$$(a) \quad u(t) = \delta(t)$$

$$y'(t) + a_1 y(t) = b_0 u'(t) + b_1 u(t)$$

$$a_1 = a, \quad b_1 = 1$$

$$b_0 = 0$$

$$h_A'(t) + a h_A(t) = \delta(t)$$

$$h_A(0^+) = 1$$

$$h_A'(t) + a h_A(t) = 0$$

$$Ce^{st}(s+a) = 0, \quad Ce^{st} \neq 0$$

$$s+a=0 \rightarrow s=-a$$

$$h_A(t) = C_1 e^{-at}$$

$$h_A(0^+) = C_1 = 1$$

$$C_1 = 1$$

$$h_A(t) = e^{-at}$$

$$h(t) = \sum_{m=0}^M (b_{N-m} D^m) h_A(t) \quad n=0, \quad N=1$$

$$h(t) = e^{-at}, \quad t \geq 0$$

$$(b) \quad u(t) = n(t) \rightarrow y_p(t) = A$$

$$y_p'(t) = 0$$

$$aA = 1$$

$$A = \frac{1}{a}, \quad t \geq 0 \rightarrow y_p(t) = \frac{1}{a}, \quad t \geq 0$$

$$y(t) = y_n(t) + y_p(t)$$

$$y(t) = C_1 e^{-at} + \frac{1}{a}$$

$$y(0^+) = C_1 + \frac{1}{a} = 0$$

$$C_1 = -\frac{1}{a}$$

$$y(t) = \frac{1}{a} (1 - e^{-at}), \quad t \geq 0$$

$$(c) \quad u(t) = n(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) u(t-\tau) d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-a\tau} n(\tau) \cdot n(t-\tau) d\tau$$

$$y(t) = \left\{ \int_0^t e^{-a\tau} d\tau \right\} n(t)$$

$$y(t) = \frac{1}{a} (1 - e^{-at}) n(t)$$

$$(d) \quad u_d(t) = \delta(t)$$

$$u_b(t) = n(t) \rightarrow u_d(t) = u_b'(t)$$

SUSTAV JE LINEARAN TE STOGA MOŽEMO PISATI

$$y_d(t) = y_b'(t)$$

$$y(t) = e^{-at} n(t)$$

$$10. \quad y'(t) + 2y(t) = u'(t) + u(t)$$

$$N = M = 1$$

$$a_1 = 2, \quad b_0 = b_1 = 1$$

$$u(t) = \delta(t)$$

$$h_A'(t) + 2h_A(t) = \delta'(t) + \delta(t)$$

$$h_A'(t) + 2h_A(t) = 0$$

$$Ce^{st}(s+2) = 0, \quad Ce^{st} \neq 0$$

$$s+2 = 0 \rightarrow s = -2$$

$$h_A(t) = C_1 e^{-2t}, \quad h_A(0^+) = 1$$

$$h_A(0^+) = C_1 = 1$$

$$C_1 = 1$$

$$h_A(t) = e^{-2t}, \quad t \geq 0$$

$$h(t) = \sum_{m=0}^M (b_{N-m} D^m) h_A(t) + \delta(t)$$

$$h(t) = b_1 D^0 h_A(t) + b_0 D^1 h_A(t) + \delta(t)$$

$$h(t) = h_A'(t) + h_A(t) + \delta(t)$$

$$h(t) = -e^{-2t} + \delta(t), \quad t \geq 0$$

$$11. \quad y''(t) + 6y'(t) + 13y(t) = u'(t) + 4u(t)$$

$$N=2, \quad M=1$$

$$a_1=6, \quad a_2=13$$

$$b_0=0, \quad b_1=1, \quad b_2=4$$

$$h_A''(t) + 6h_A'(t) + 13h_A(t) = \delta'(t) + 4\delta(t)$$

$$h_A(0^+) = 0, \quad h_A'(0^+) = 1$$

$$h_A''(t) + 6h_A'(t) + 13h_A(t) = 0$$

$$Ce^{st}(s^2 + 6s + 13) = 0, \quad Ce^{st} \neq 0$$

$$s^2 + 6s + 13 = 0 \rightarrow s_{1,2} = -3 \pm j2$$

$$s_1 = -3 + j2, \quad s_2 = -3 - j2$$

$$h_A(t) = Ae^{-3t}e^{+j2t} + Be^{-3t}e^{-j2t}$$

$$h_A(t) = e^{-3t}(Ae^{+j2t} + Be^{-j2t})$$

$$h_A(t) = e^{-3t}(C_1 \cos(2t) + C_2 \sin(2t))$$

$$h_A'(t) = e^{-3t}(C_1(-3\cos(2t) - 2\sin(2t)) + C_2(-3\sin(2t) + 2\cos(2t)))$$

$$\left. \begin{array}{l} h_A(0) = C_1 = 0 \\ h_A'(0) = 2C_2 = 1 \end{array} \right\} \quad C_1 = 0, \quad C_2 = \frac{1}{2}$$

$$h_A(t) = \frac{1}{2}e^{-3t}\sin(2t)$$

$$h_A'(t) = e^{-3t}(\cos(2t) - \frac{3}{2}\sin(2t))$$

$$h(t) = b_2 h_A(t) + b_1 h_A'(t)$$

$$h(t) = e^{-3t}(\cos(2t) + \frac{1}{2}\sin(2t)), \quad t \geq 0$$

$$12. \quad u_1(t) = \nu(t)$$

$$y_1(t) = \cos(\omega_0 t) \nu(t)$$

$$u_2(t) = \delta(t)$$

$$u_2(t) = u_1'(t) \xrightarrow{LT} y_2(t) = y_1'(t)$$

$$y_2(t) = -\omega_0 \sin(\omega_0 t) \nu(t) + \delta(t)$$

$$h(t) = -\omega_0 \sin(\omega_0 t) \nu(t) + \delta(t)$$

$$13. \quad y(t) = \int_{-\infty}^t u(\tau) \delta(\tau) d\tau$$

$$(a) \quad u(t) = \delta(t)$$

$$h(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

$$h(t) = \nu(t)$$

$$(b) \quad y'(t) = u(t)$$

$s=0 \rightarrow$ RUBNO STABILAN

$$15. \quad y(t) = \int_0^1 u(t-h) dh, \quad \forall t \in \mathbb{R}$$

SUPSTITUCIJA:

$$t-h=\tau \rightarrow h=t-\tau$$

$$dh = -d\tau$$

GRANICE:

$$h=1 \rightarrow t-\tau=1$$

$$h=0 \rightarrow t-\tau=0$$

$$\tau=t-1$$

$$\tau=t$$

$$y(t) = \int_t^{t-1} -u(\tau) d\tau$$

$$y(t) = \int_{t-1}^t u(\tau) d\tau$$

$$(a) \quad u(t) = \delta(t)$$

$$h(t) = \int_{t-1}^t \delta(\tau) d\tau = \int_0^t \delta(\tau) d\tau + \int_0^{t-1} \delta(\tau) d\tau$$
$$h(t) = \int_0^t \delta(\tau) d\tau - \int_0^{t-1} \delta(\tau) d\tau$$

$$h(t) = N(t) - N(t-1)$$

$$(b) \quad u(t) = \sin\left(\frac{\pi}{2}t\right), \quad \forall t \in \mathbb{R}$$

$$y(t) = \int_{t-1}^t \sin\left(\frac{\pi}{2}\tau\right) d\tau$$

$$y(t) = \frac{2}{\pi} \cos\left(\frac{\pi}{2}\tau\right) \Big|_{t-1}^t$$

$$y(t) = \frac{2}{\pi} \left(\cos\left(\frac{\pi}{2}t - \frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}t\right) \right)$$

$$y(t) = \frac{2}{\pi} \left(\sin\left(\frac{\pi}{2}t\right) - \cos\left(\frac{\pi}{2}t\right) \right)$$

15. Zadan je kontinuirani sustav $y''(t) + 5y'(t) + 6y(t) = u(t)$. Odredite odziv sustava u vremenskoj domeni ako je sustav pobuđen signalom $u(t) = (12t + 16)\mu(t)$ te ako su početni uvjeti $y(0^-) = 3, y'(0^-) = -8$. Odredite prirodni i prisilni odziv, te odziv mirnog i nepobuđenog sustava. Odredite impulsni odziv sustava. Komentirajte stabilnost sustava.

Rješenje:

Homogeno rješenje je: $y_h(t) = C_1 e^{-2t} + C_2 e^{-3t}$

Partikularno rješenje je: $y_p(t) = 1 + 2t$

Prisilni odziv je: $y_{prisilni}(t) = (1 + 2t)\mu(t)$

Prirodni odziv je: $y_{tot}(t) = (-4e^{-2t} + 6e^{-3t})\mu(t)$

Totalni odziv je: $y_{tot}(t) = (-4e^{-2t} + 6e^{-3t} + 1 + 2t)\mu(t)$

Impulsni odziv sustava: $h(t) = (e^{-2t} - e^{-3t})\mu(t)$

Sustav je stabilan.

16. Zadan je kontinuirani sustav $y''(t) + 5y'(t) + 4y(t) = u(t)$. Odredite odziv mirnog sustava u vremenskoj domeni ako je sustav pobuđen signalom $u(t) = (4 + \cos(4t))\mu(t)$. Nađite impulsni odziv sustava. Komentirajte stabilnost sustava.

Rješenje:

Impulsni odziv:

$$h(t) = \left(-\frac{1}{3}e^{-4t} + \frac{1}{3}e^{-t} \right) \mu(t)$$

Kada $t \rightarrow \infty$ impulsni odziv $h(t) \rightarrow 0$, pa je sustav stabilan.

Ili na temelju karakteristične jednadžbe: rješenja karakteristične jednadžbe su -4 i -1 . Kako su oba rješenja negativna – sustav je stabilan.

Odziv mirnog sustava:

$$y_{mirni}(t) = \left(\frac{7}{30}e^{-4t} - \frac{11}{15}e^{-t} + \frac{1}{2} + \frac{1}{10}\sin(2t) \right) \mu(t)$$