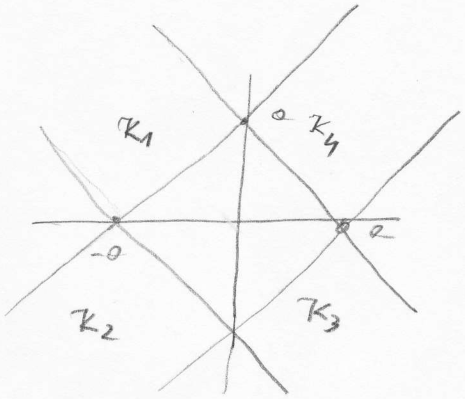
(1) $\int_K xy \, dS \qquad K : |x| + |y| = a \qquad a > 0$



$I_1 = \int_{K_1} t(t-a) \cdot \sqrt{1+1} \qquad K_1 : x = t$

$y = -t - a$

$t [-a, 0]$

$= -\sqrt{2} \int_{-a}^{0} (t^2 + ta) \, dt =$

$= \sqrt{2} \left(\frac{-a^3}{a} + a \frac{a^2}{2} \right)$

$I_2 = \int_{K_2} t(t-a) \cdot \sqrt{2} \, dt \qquad K_2 : x = t$

$y = t - a$

$t [0, a]$

$= \sqrt{2} \int_0^a t^2 - ta = \sqrt{2} \left(\frac{a^3}{3} - \frac{a^3}{2} \right)$

$I_3 = \int_{K_3} t(t-a) \sqrt{2} \qquad K_3 : x = t$

$y = -t + a$

$t [0, a]$

$= \sqrt{2} \int_0^a -t^2 + ta = \sqrt{2} \left(-\frac{a^3}{3} + \frac{a^3}{2} \right)$

$I_4 = \int_{K_4} t(t+a) \sqrt{2} \qquad K_4 : x = t$

$y = t + a \qquad t [-a, 0]$

$= \sqrt{2} \int_{-a}^{0} (t^2 + at) \, dt = \sqrt{2} \left(-\frac{a^3}{3} + \frac{a^2}{2} \right)$

$I = I_1 + I_2 + I_3 + I_4 = 0$ //

(2.) $\int y^2 ds$ K cikloida $x = a(t-\sin t)$

$y = a(1-\cos t)$ $t\,[0, 2\pi]$

$$= \int_0^{2\pi} a^2(1-\cos t)^2 \sqrt{(a-a\cos t)^2 + a^2\sin^2 t}\, dt$$

$a^2 - 2a^2\cos t + a^2\cos^2 t$

$$\int_a^b f(x(t), y(t), z(t)) \cdot \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$$

$$= \int_0^{2\pi} a^2(1-\cos t + \cos^2 t) \sqrt{2a^2 - 2a^2\cos t}$$

$\sin^2 \frac{t}{2} = \frac{1-\cos t}{2}$

$2\sin^2\frac{t}{2} = 1-\cos t$

$$= \int_0^{2\pi} \sqrt{2}\, a^3 \, \underbrace{(1 - 2\cos t + \cos^2 t)}_{\substack{(1-\cos t)^2 \\ (2\sin^2\frac{t}{2})^2}} \underbrace{\sqrt{1-\cos t}}_{2\sin^2\frac{t}{2}}$$

$$= \int_0^{2\pi} a^3 \cdot 8 \cdot \sin^5 \frac{t}{2} \quad \left| \begin{array}{l} u = \frac{t}{2} \\ du = \frac{dt}{2} \end{array} \right| \quad = a^3 8 \cdot 2 \int_0^{\pi} \sin^5 du$$

$0 \to \pi$

$$= a^3 \cdot 16 \cdot \frac{2}{3} \cdot 2 \cdot \frac{4}{5} = \frac{256}{15} a^3$$

$I_5 = \frac{n-1}{n} I_{n-2} = \frac{4}{5} \cdot \frac{4}{3}$

$I_2 = \frac{2}{3} I_1$ $\quad I_1 = \int_0^{\pi} \sin\, d$

$I_3 = \frac{4}{3}$ $\quad = 2$

(3) $\int_K (x^2+y^2)^2 ds$ K log. spirala $r = a e^{m\varphi}$ $m>0$

$A(a,0) \to O(0,0)$

$$\int_K (r^2\cos^2\varphi + r^2\sin^2\varphi)^2 \sqrt{(r(\varphi))^2+(r'(\varphi))^2}\, d\varphi$$

$x = a e^{m\varphi}\cos\varphi$ (with r written above $a e^{m\varphi}$)

$y = a e^{m\varphi}\sin\varphi$

$x'(\varphi) = a m e^{m\varphi}\cos\varphi - a e^{m\varphi}\sin\varphi$

$y'(\varphi) = a m e^{m\varphi}\sin\varphi + a e^{m\varphi}\cos\varphi$

$$= \int_{-\infty}^{0} r^4 \sqrt{a^2 e^{2m\varphi} + a^2 m^2 e^{2m\varphi}}$$

$$= \int_{-\infty}^{0} a^4 e^{4m\varphi} \cdot a e^{m\varphi}\sqrt{1+m^2}$$

$$= \sqrt{1+m^2}\int_{-\infty}^{0} a^5 \cdot e^{5m\varphi}$$

$$= a^5\sqrt{1+m^2}\cdot \frac{1}{5m} e^{5m\varphi}\Big|_{-\infty}^{0}$$

$$= \frac{a^5\sqrt{1+m^2}}{5m}\left(e^0 - \cancel{e^{-\infty}}^{\to 0}\right)$$

$$= \frac{a^5\sqrt{1+m^2}}{5m} \quad //$$

$A(a,0) = (a e^{m\varphi}\cos\varphi, a e^{m\varphi}\sin\varphi)$

$\cancel{a} = \cancel{a} e^{m\varphi}\cos\varphi$ $\quad$ $0 = a e^{m\varphi}\sin\varphi$

$1 = e^{m\varphi}\cos\varphi$ $\quad\Downarrow$

$\varphi = 0$ $\quad$ $\underline{\varphi = 0}$

$A(0,0) = (a e^{m\varphi}\cos\varphi, a e^{m\varphi}\sin\varphi)$

$0 = a e^{m\varphi}\cos\varphi$

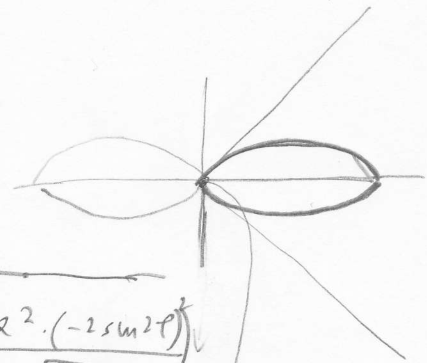
$\Downarrow$

$\lim_{\varphi\to-\infty} a e^{m\varphi} = 0$

$\Downarrow$

$\underline{\varphi = -\infty}$

(4.) $\int\limits_K (x+y)\,ds$ $\quad$ K desno latica lemniskate $\quad r^2 = a^2\cos 2\varphi$



$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\sqrt{a^2\cos 2\varphi}\,\cos\varphi + \sqrt{a^2\cos 2\varphi}\,\sin\varphi\right) \cdot$$

$$\sqrt{\left(\sqrt{a^2\cos 2\varphi}\right)^2 + \left(\frac{a^2\cdot(-2\sin 2\varphi)}{2\sqrt{a^2\cos 2\varphi}}\right)^2}$$

$r=0$

$\Downarrow$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{a^2\cos 2\varphi}\,(\cos\varphi+\sin\varphi)\sqrt{a^2\cos 2\varphi + \frac{4a^4\sin^2 2\varphi}{\cancel{4}a^2\cos 2\varphi}}$$

$a\sqrt{\cos 2\varphi}=0$

$\cos 2\varphi = 0$

$\varphi = \pm\frac{\pi}{4}$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \big(\underbrace{r\cos\varphi + r\sin\varphi}_{\sqrt{a^2\cos 2\varphi}}\big)\sqrt{\frac{a^2}{\cos 2\varphi}}$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r = \sqrt{a\cos 2\varphi}$

$$a^2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\cancel{\cos 2\varphi}}\cdot(\cos\varphi+\sin\varphi)\cdot\sqrt{\frac{1}{\cancel{\cos 2\varphi}}}\,d\varphi$$

$$= a^2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\cos\varphi+\sin\varphi)\,d\varphi$$

$$= a^2\left[\sin\varphi\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - \cos\varphi\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\right]$$

$$= a^2\left[\frac{\sqrt{2}}{2} + \cancel{\frac{\sqrt{2}}{2}} - \cancel{\frac{\sqrt{2}}{2}} + \frac{\sqrt{2}}{2}\right]$$

$$= a^2\sqrt{2}$$

//

5.

$$\int_K \frac{ds}{x^2+y^2+z^2}$$

K: prvi zavoj zavojnice

$x = a\cos t$

$y = a\sin t$

$z = bt$

$t\ [0, 2\pi]$

$$\int_0^{2\pi} \frac{1}{a^2\cos^2 t + a^2\sin^2 t + b^2 t^2}\sqrt{((a\cos t)')^2 + ((a\sin t)')^2 + (bt')^2}$$

$$\int_0^{2\pi} \frac{1}{a^2 + b^2 t^2}\sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2}\,dt$$

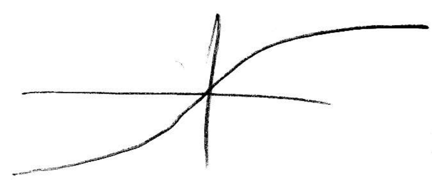
$$= \sqrt{a^2+b^2}\int_0^{2\pi} \frac{1}{a^2+b^2t^2}\,dt = \frac{1}{b^2}\sqrt{a^2+b^2}\int_0^{2\pi}\frac{1}{\frac{a^2}{b^2}+t^2}\,dt$$

$$= \frac{1}{b^{\cancel{2}}}\sqrt{a^2+b^2}\cdot\frac{\cancel{b}}{a}\operatorname{arctg}\left(\frac{bt}{a}\right)\Big|_0^{2\pi}$$

$$= \frac{\sqrt{a^2+b^2}}{ab}\operatorname{arctg}\left(\frac{2\pi b}{a}\right)$$

$\int \frac{dt}{t^2+a^2} =$

$\frac{1}{a}\operatorname{arctg}\frac{x}{a}$



6. $\int_K \sqrt{2y^2+z^2}\, ds$ $\quad z=\sqrt{a^2-2t^2}$

K kružnica kao presjecište sfere $x^2+y^2+z^2=a^2$ i $y=x$

$$\int_{K_1} \sqrt{2t^2+a^2-2t^2} \cdot \sqrt{\frac{2a}{a^2-2t^2}} + \int_{K_2} a \cdot \sqrt{\frac{2a^2}{a^2-2t^2}}\, dt$$

$$= \int_{-\frac{a}{\sqrt2}}^{\frac{a}{\sqrt2}} a \cdot \frac{a\sqrt2}{\sqrt{a^2-2t^2}}\, dt + \int_{-\frac{a}{\sqrt2}}^{\frac{a}{\sqrt2}} \frac{a^2\sqrt2}{\sqrt{a^2-2t^2}}\, dt$$

$$= 2 \cdot a^2\sqrt2 \int_{-\frac{a}{\sqrt2}}^{\frac{a}{\sqrt2}} \frac{1}{\sqrt{a^2-2t^2}} \cdot \frac{\frac{1}{\sqrt2}}{\frac{1}{\sqrt2}}\, dt =$$

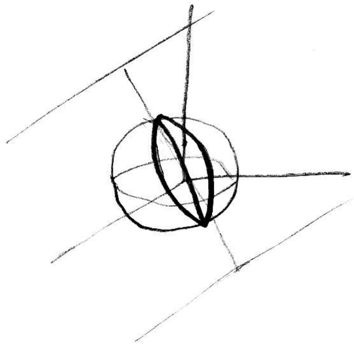
$$= 2a^2 \int_{-\frac{a}{\sqrt2}}^{\frac{a}{\sqrt2}} \frac{1}{\sqrt{\frac{a^2}{2}-t^2}}\, dt$$

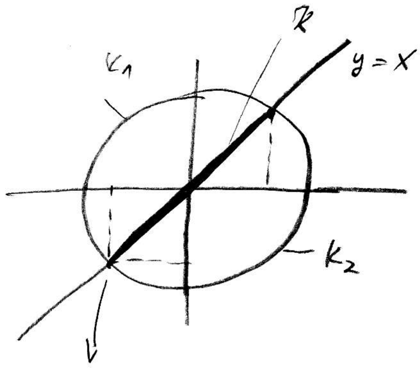
$$= 2a^2 \arcsin\left(\frac{\sqrt2}{a} t\right) \Big|_{-\frac{a}{\sqrt2}}^{\frac{a}{\sqrt2}}$$

$$= 2a^2 \left[\arcsin(1) - \arcsin(-1)\right]$$

$$= 2a^2 \left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$= 2a^2\pi$$





$x^2+y^2=a^2$

$y=x$

$2x^2=a^2 \to x=\sqrt{\frac{a^2}{2}}$

$K_1 \to$ gornja polovica

$x=t \quad y=t \quad z=\sqrt{a^2-t^2-t^2}$

$K_2 \to$ donja polovica

$x=t \quad y=t \quad z=-\sqrt{a^2-2t^2}$

$$ds = \sqrt{1+1+\left(\frac{1}{2\sqrt{a^2-2t^2}} \cdot -4t\right)^2}$$

$$ds = \sqrt{2+\frac{4t^2}{a^2+2t^2}}$$

$$ds = \sqrt{\frac{2a^2 \cancel{-4t^2} \cancel{+4t^2}}{a^2+2t^2}}$$

$$ds = \sqrt{\frac{2a^2}{a^2+2t^2}}$$

(7.) stožastu zavojnicu $x = ae^t \cos t$

$y = ae^t \sin t$ $O(0,0,0)$ $A(a,0,a)$

$z = ae^t$

$t = -\infty$ $t = 0$

$\lim_{t \to -\infty} ae^{-\infty}\cos t = 0$ $ae^0 \cos 0 = a$

$$l = \int_{-\infty}^{0} 1 \cdot \sqrt{(\underbrace{ae^t}_{A} \cos t - ae^t \sin t)^2 + (\underbrace{ae^t}_{A} \sin t + ae^t \cos t) + \underbrace{a^2e^{2t}}_{A^2}}\, dt$$

$$l = \int_{a}^{b} 1 \cdot \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$$

$$l = \int_{-\infty}^{0} \sqrt{A^2 (\cos^2 t - \cancel{2\cos t \sin t} + \sin^2 t + \sin^2 t \cancel{+ 2\sin t \cos t} + \cos^2 t + 1}$$

$$l = \int_{-\infty}^{0} \sqrt{A^2 (1+1+1}$$

$$l = a\sqrt{3} \int_{-\infty}^{0} e^t = a\sqrt{3} \int_{-\infty}^{0} e^t$$

$$l = a\sqrt{3} \, (e^0 - e^{-\infty})$$

$$l = a\sqrt{3}$$ //

(8.)

PRIMJENA ...

(9.)

PRIMJENA ...

(10.) $\int_{k} xy \, ds \qquad k: \ y = \sqrt{x^2+1} \qquad y+z=3 \qquad x \geq 0 \qquad z \geq 0$

$z = 3 - y$

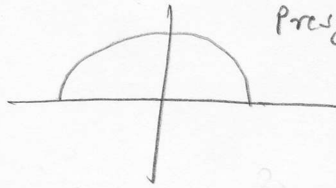
$\Downarrow$

$x = t$

$y = \sqrt{t^2+1}$

$z = 3 - \sqrt{t^2+1}$

Presjek



$x \geq 0 \quad t = 0$

$z \geq 0 \Rightarrow 3 - \sqrt{t^2+1} \geq 0$

$t = 2\sqrt{2}$

$t \, [0, 2\sqrt{2}]$

$$ds = \sqrt{1^2 + \left(-\frac{1}{2} \frac{1}{\sqrt{t^2+1}} 2t\right)^2 + \left(-\frac{1}{2} \frac{1}{\sqrt{t^2+1}} 2t\right)^2} \, dt$$

$$= \sqrt{1 + \frac{t^2}{t^2+1} + \frac{t^2}{t^2+1}} = \sqrt{\frac{3t^2+1}{t^2+1}}$$

$$\int_0^{2\sqrt{2}} t \cdot \cancel{\sqrt{t^2+1}} \cdot \frac{\sqrt{3t^2+1}}{\cancel{\sqrt{t^2+1}}} \, dt \qquad \left| \begin{array}{c} 3t^2+1 = u \\ 6t\,dt = du \\ t\,dt = \frac{du}{6} \end{array} \right|$$

$1 \longrightarrow 3(2\sqrt{2})^2 + 1$

25

$$\int_0^{25} \sqrt{u} \cdot \frac{1}{6} \, du$$

$$= \frac{1}{6} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^{25}$$

$$= \frac{1}{\cancel{6}_{3}} \cdot \frac{\cancel{2}}{3} \cdot (5^3 - 1)$$

$$= \frac{124}{9}$$

//

(11.) $\int_K x^2yz\,ds$ K presjek $5z = x^2+5(y-1)^2$ i $2y+z=2$

$z=2-2y$

eliptički elipsoid

$z=\frac{x^2}{5}+(y-1)^2$

$$ds=\sqrt{(\sqrt5\cos t)^2+(-\sin t)^2+(2\sin t)^2}$$

$$ds=\sqrt{5\cos^2 t+5\sin^2 t}=\sqrt5$$

$$I=\int_0^{2\pi}(\sqrt5\cos t)^2\cdot(\sin t)\cdot(2-2\sin t)\cdot\sqrt5$$

$5\sin^2 t\cdot\cos t\cdot 2(1-\cos t)$

$$I=10\sqrt5\int_0^{2\pi}\cos^2 t\,(\sin t-\sin^2 t)$$

$$=10\sqrt5\Big(\underbrace{\int_0^{2\pi}\cos^2 t\sin t}_{I_1}-\underbrace{\int_0^{2\pi}\cos^2 t\sin^2 t\,dt}_{I_2}\Big)$$

$$\left|\begin{array}{l}\cos t=u\\ -\sin t\,dt=du\end{array}\right|$$

$$I_1=-\int u^2du=-\frac{u^3}{3}\Big|_0^{a}=\frac{\cos^3 t}{3}\Big|_0^{2\pi}=0$$

$$I_2=\int_0^{2\pi}\cos^2 t\sin^2 t\,dt=\int_0^{2\pi}\frac{\sin^2 2t}{4}dt$$

$$I_2=\frac14\int_0^{2\pi}\frac{1-\cos 4t}{2}dt=\frac18 2\pi-\frac18\int_0^{2\pi}\cos 4t\,dt$$

$$I_2=\frac{\pi}{4}-\cancel{\frac18\cdot\frac14\sin 4t\Big|_0^{2\pi}}=\frac{\pi}{4}$$

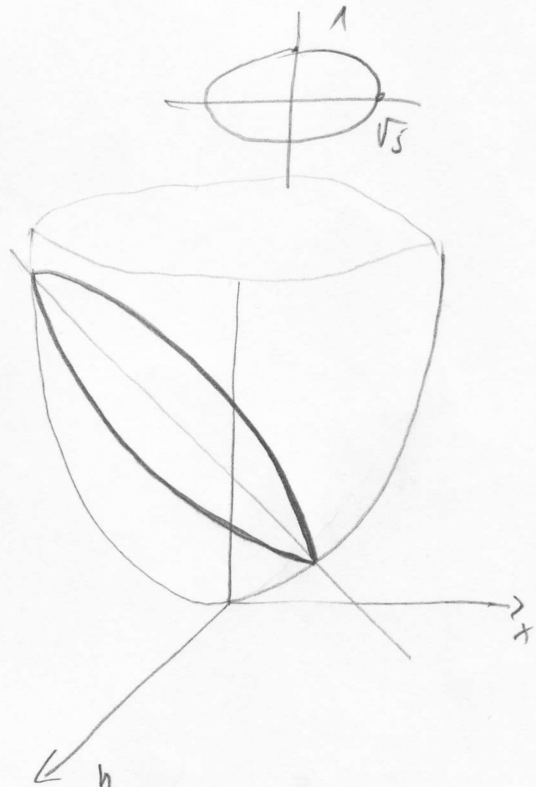
$$I=10\sqrt5\,\frac{\pi}{4}=\frac{5\sqrt5}{2}\pi$$

PRESJEK:

$$\frac{x^2}{5}+(y-1)^2=2-2y$$

$$\frac{x^2}{5}+y^2-\cancel{2y}+1=2-\cancel{2y}$$

$$\frac{x^2}{(\sqrt5)^2}+y^2=1$$



$x=\sqrt5\cos t$

$y=\sin t$

$z=2-2y=2-2\sin t$

$t\,[0,2\pi]$

12.

$\int_K (x^2+y^2)\,ds$ K je presjek $x^2+2y^2=4$ i $z=y$ $y\geq 1$

$\frac{x^2}{2^2}+\frac{y^2}{(\sqrt{2})^2}=1$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(4\cos^2 t + 2\sin^2 t\right) 2\cdot dt$$

$2\cos^2 t + 2\cos^2 t$

$$= 2\cdot \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (2\cos^2 t + 2)\,dt$$

$\cos^2 t = \frac{1+\cos 2t}{2}$

$2\cos^2 t = 1+\cos 2t$

$2\cos^2 t + 2 = 3+\cos 2t$

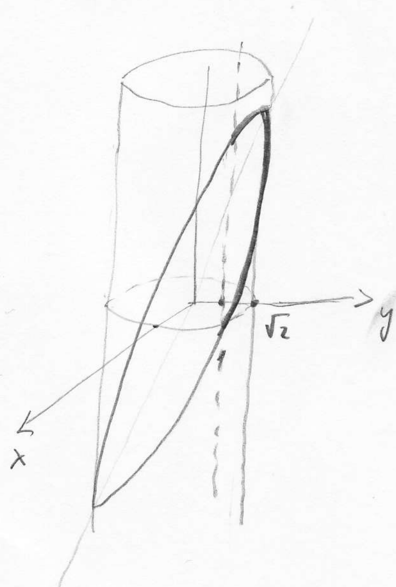
$$= 2\cdot \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (3+\cos 2t)\,dt$$

$$= 2\cdot \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (3+\cos 2t)\,dt =$$

$$= 6\cdot\left(\frac{3\pi}{4}-\frac{\pi}{4}\right) + 2\cdot\frac{1}{2}\sin 2t \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= 3\pi + \left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right) =$$

$$= 3\pi - 2$$



$x = 2\cos t$ $x' = -2\sin t$

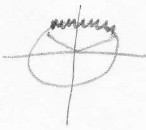
$y = \sqrt{2}\sin z$ $y' = \sqrt{2}\cos t$

$z = y = \sqrt{2}\sin t$

$y \geq 1$

$\sqrt{2}\sin t \geq 1$

$\sin t \geq \frac{1}{\sqrt{2}}$



$t\ \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$ds = \sqrt{(-2\sin t)^2 + (\sqrt{2}\cos t)^2 + (\sqrt{2}\cos t)^2}$

$ds = \sqrt{2^2(\cos^2 t + \sin^2 t)}$

$ds = 2$

13.

$$\int_K 2xy\,dx - x^2\,dy$$ po paraboli $y = \frac{x^2}{4}$ od $(0,0)$ do $(2,1)$

$x = t$ $\quad dx = dt$

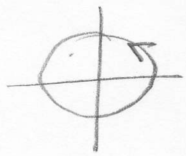
$y = \frac{t^2}{4}$ $\quad t\,[0,2]$ $\quad dy = \frac{t}{2}$

$$= \int_K 2 \cdot t \cdot \frac{t^2}{4} \cdot dt - t^2 \cdot \frac{t}{2}\,dt =$$

$$= \int_0^2 0\,dt = 0$$

14.

$$\oint_K \frac{(x+y)\,dx - (x-y)\,dy}{x^2+y^2}$$ po kružnici $x^2+y^2 = a^2$ u $(+)$ smjeru



$$\oint_K \frac{(a\cos t + a\sin t)\cdot(-a\sin t) - (a\cos t - a\sin t)(a\cos t)}{a^2}\,dt$$

$x = a\cos t$

$y = a\sin t$

$t\,[0, 2\pi]$

$dx = -a\sin t$

$dy = a\cos t$

$$\oint_K \frac{\cancel{-a^2\cos t\sin t} - a^2\sin^2 t - a^2\cos^2 t\ \cancel{+a^2\sin t\cos t}}{a^2}$$

$$\int_0^{2\pi} -1 = -2\pi$$

15.

$$\int_K 2xy\,dx + x^2\,dy$$ po $y = \frac{x}{2}$ $\overrightarrow{OA}$ $O(0,0)$ $A(2,1)$

$x = t$

$y = \frac{t}{2}$ $\quad t \in [0,2]$

$dx = dt$

$dy = \frac{1}{2}dt$

$$= \int_0^2 \cancel{2} \cdot t\,\frac{t}{\cancel{2}}\,dt + t^2 \cdot \frac{1}{2}\,dt$$

$$= \int_0^2 t^2 + \frac{1}{2}t^2 =$$

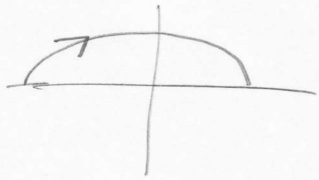
$$= \frac{\cancel{3}}{2} \cdot \frac{t^3}{\cancel{3}}\Big|_0^2 = 4$$

16. $\int_{K} y^2 dx + x^2 dy$ po gornjoj polovini elipse

$x = a\cos t$

$y = b\sin t$ u $\ominus$ smjeru

$$\int_{\pi}^{0} b^2 \sin^2 t \, (-a\sin t) + a^2\cos^2 t \cdot b\cos t$$



$$= \int_{\pi}^{0} -b^2 a \sin^3 t + a^3 b \cos^3 t =$$

$x = a\cos t$

$y = a\sin t$

$t\,[\pi, 0]$

$dx = -a\sin t\,dt$

$dy = b\cos t\,dt$

$\sin(1-\cos^2 t)$

$$= ab^2 \int_0^{\pi} \sin^3 t \; - a^2 b \int_0^{\pi} \cos^3 t$$

$$= ab^2 \left[\int_0^{\pi} \sin t\,dt + \int_0^{\pi} -\sin\,\cos^2 t \right] - a^2 b \left[\int_0^{\pi} \cos t - \int_0^{0} \cos t \sin^2 t \right]$$

$\begin{vmatrix} \cos t = u \\ du = -\sin t\,dt \end{vmatrix}$

$1 \to -1$

$\begin{vmatrix} \sin t = u \\ \cos t\,dt = du \\ 0 \to 0 \end{vmatrix}$

$$= ab^2 \left[-\cos t \Big|_0^{\pi} + \int_1^{-1} u^2\,du \right]$$

$$= ab^2 \left[-(-1-1) + \frac{u^3}{3}\Big|_1^{-1} \right] = ab^2 \left[2 + \left(\frac{-1}{3} - \frac{1}{3} \right) \right]$$

$$= \frac{4}{3} ab^2$$

REKURZIJA...

$$\int_0^{\pi} \sin^3 t$$

$I_3 = \frac{2}{3} I_1 = \frac{4}{3}$ $\qquad I_1 = \int_0^{\pi} \sin t =$

$= -\cos\pi + \cos 0 = 2$

$$\int_0^{\pi} \cos^3 t$$

$I_3 = \frac{2}{3}$ $\qquad I_1 = \int_0^{\pi} \cos t$

$= -0$

(17) $\oint \frac{xy\,(y\,dx - x\,dy)}{x^2+y^2}$ po desnoj latici od $r^2 = a^2\cos 2\varphi$

$a^2\cos 2\varphi\,(\sin^2\varphi + \cos^2\varphi)$

$$= \oint \frac{a^2\cancel{\cos 2\varphi}\sin\varphi\cos\varphi\left(\cancel{-a\sin 2\varphi\cos\varphi\sin\varphi} - a^2\cos 2\varphi\sin^2\varphi + \cancel{a\sin 2\varphi\sin\varphi\cos\varphi} - a^2\cos 2\varphi\cos^2\varphi\right)}{a^2\cancel{\cos 2\varphi}}$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} -a^2\cos 2\varphi \cdot \cos\varphi \cdot \sin\varphi \, d\varphi$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} -a^2\cos 2\varphi \cdot \sin 2\varphi \cdot \frac{1}{2}\, d\varphi$$

$$= -\frac{a^2}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin 4\varphi}{2}\, d\varphi$$

$$= -\frac{a^2}{4}\,(-)\frac{1}{4}\cos 4\varphi \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{a^2}{4}\left(\cos 2\pi - \cos 2\pi\right)$$

$$= 0$$

$2\varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

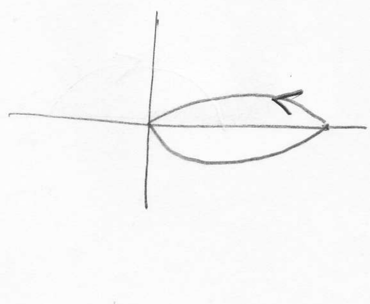
$\varphi \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$x = a\sqrt{\cos 2\varphi}\,\cos\varphi$

$y = a\sqrt{\cos 2\varphi}\,\sin\varphi$

$dx = \frac{a}{\cancel{2}\sqrt{\cos 2\varphi}} \cdot (-\sin 2\varphi)\cdot \cancel{2}\cos\varphi + a\sqrt{\cos 2\varphi}\cdot(-\sin\varphi)$

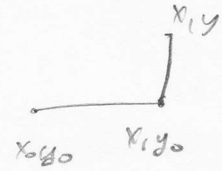
$dy = \frac{a}{\cancel{2}\sqrt{\cos 2\varphi}} \cdot (-\sin 2\varphi)\cdot \cancel{2}\sin\varphi + a\sqrt{\cos 2\varphi}\,\cos\varphi$



(18.) $$\int_{(1,2)}^{(2,1)} \frac{y\,dx - x\,dy}{y^2} = \int_{(1,2)}^{(2,1)} \underbrace{\frac{1}{y}}_{f_1} dx + \underbrace{\frac{x}{y^2}}_{f_2} dy$$

→ naći funkciju čiji je totalni diferencijal ove podintegralne funkc.

$$u(x,y) = \int_{x_0}^{x} f_1(x,y_0)\,dx + \int_{y_0}^{y} f_2(x,y)\,dy$$



za $(x_0,y_0) = (0,1)$ → da nemamo dijeljenja sa 0

$$u(x,y) = \int_0^x \frac{1}{y_0}\,dx + \int_1^y \frac{x}{y^2}\,dy$$

$$= \int_0^x 1\,dx - \int_1^y \frac{x}{y^2}\,dy$$

$$= x - x \cdot \frac{1}{-y}\Big|_1^y$$

$$= \cancel{x} + \frac{x}{y} - \cancel{x}$$

$$u(x,y) = \frac{x}{y} + C$$

$$I = u(2,1) - u(1,2) = 2 - \frac{1}{2} = \frac{3}{2}$$

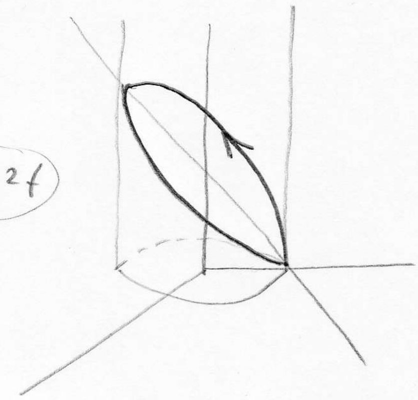
19.

$$\iint_K (y-z)\,dx + (z-x)\,dy + (x-y)\,dz$$

K presjek $x^2+y^2=1$ i $x+z=1$

$y \geq 0$

$$= \int_0^\pi (\sin t - 1 + \cos t)(-\sin t) + (1-\cos t-\cos t)\cdot\cos t + (\cos t-\sin t)\sin t$$



$$= \int_0^\pi -\sin^2 t + \cancel{\sin t\cos t} + \sin t + \cos t - 2\cos^2 t + \cancel{\cos t\sin t} - \sin^2 t$$

$$= \int_0^\pi (\sin t + \cos t - 2)\,dt$$

$$= 2 - 2\pi$$

$x = \cos t$

$y = \sin t$

$z = 1 - \cos t$

$dx = -\sin t$

$dy = \cos t$

$dz = \sin t$

$y \geq 0$

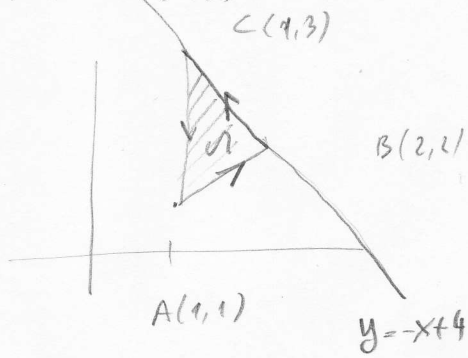
$\sin t \geq 0$

$\Downarrow$

$t\ [0, \pi]$

(21.) $\oint 2(x^2+y^2)\,dx + (x+y)^2\,dy$ K konturo trokuta (+) smjer

A(1,1) B(2,2) C(1,3)



GREEN: $\oint \vec{f}\,d\vec{r} = \oint f_1\,dx + f_2\,dy = \iint_\Omega \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dx\,dy$

$$I = \iint_\Omega \left[2(x+y) - 4y\right] dx\,dy$$

$$= \int_1^2 dx \int_x^{-x+4} (2x-2y)\,dy =$$

$$= \int_1^2 dx\,(-x+4-x) - \left((-x+4)^2 - x^2\right) dx = \int_1^2 -4x^2+8x+8x-16$$

($\cancel{x^2}-8x+16-\cancel{x^2}$)

$$= -\frac{4}{3}\cdot 7 + \frac{16}{2}\cdot 3 - 16 = \frac{-28}{3} + \frac{24}{3} = -\frac{4}{3} \;//$$

BEZ GREENA: $\int_{\overrightarrow{AB}} + \int_{\overrightarrow{BC}} + \int_{\overline{CA}} = \iint_A$

$\overrightarrow{AB}$:

$x=t \quad dx=dt \quad t[1,2]$

$y=t \quad dy=dt$

$$= \int_1^2 2\cdot(2t^2) + 4t^2\,dt$$

$$= \int_1^2 8t^2 = \frac{8}{3}\cdot 7 = \frac{56}{3}$$

$\overrightarrow{BC}$:

$x=t \quad t[2,1]$

$y=-t+4$

$$= \int_2^1 2\cdot(t^2+t^2-8t+16)\,dt + (t-t+4)^2$$

$$= \int_2^1 4t^2-16t+32-16$$

$$= -\frac{4}{3}$$

$\overline{CA}$:

$x=1 \quad dx=0$

$y=t \quad dy=dt \quad t[3,1]$

$$= \int_3^1 (1+t)^2\,dt$$

$$= \int_3^1 1+2t+t^2$$

$$= -\int_1^3 1+2t+t^2$$

$$= -\left(2+8+\frac{26}{3}\right)$$

$$= -\frac{56}{3}$$

$$\iint_\Omega = \cancel{\frac{56}{3}} - \frac{4}{3} - \cancel{\frac{56}{3}} = -\frac{4}{3} \;//$$

(22.)

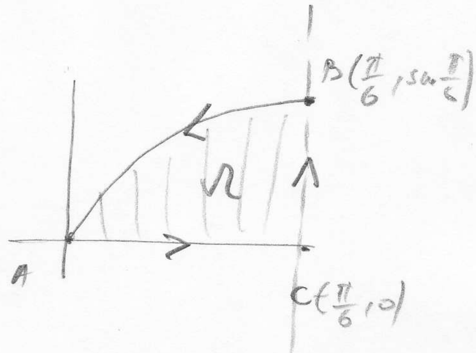
$$\int_K \underbrace{(y+\sin x \sin y)}_{f_1}\,dx - \underbrace{(\cos x \cos y)}_{f_2}\,dy$$

K luk sinusoide

$y = \sin x$

od $x=0$ do $x=\frac{\pi}{6}$

$$\int_{\overrightarrow{BA}} + \underset{I_3}{\underset{\parallel}{\int_{\overrightarrow{AC}}}} + \underset{I_2}{\underset{\parallel}{\int_{\overrightarrow{CB}}}} = \underset{I_1}{\underset{\parallel}{\iint_{\Omega}}}$$



$$I_1 = -\iint_\Omega \frac{\partial(\cos x \cos y)}{\partial x} - \iint \frac{\partial(y+\sin x \sin y)}{\partial y}$$

$$= \cancel{-\iint_\Omega -\sin x \cos y} - \iint_\Omega 1\,dx\,dy - \cancel{\iint_\Omega \sin x \sin y}$$

$$= -\int_0^{\frac{\pi}{6}} dx \int_0^{\sin x} 1\,dy = -\int_0^{\frac{\pi}{6}} \sin x = +\cos x \Big|_0^{\frac{\pi}{6}}$$

$$I_1 = \frac{\sqrt{3}}{2} - 1$$

$\to$ mi zatvorimo krivulju so CB pa možemo Greena primijeniti

$\to$ tražimo $\int_{\overrightarrow{BA}}$

$$I_2 = \int_{\overrightarrow{AC}} (\overset{0}{y} + \sin x \sin \overset{0}{y})\,dx - \cancel{\cos x \cos y\, dy}^{0}$$

$$= 0$$

$\overrightarrow{AC}$: $x = t$, $dx = dt$; $y = 0$, $dy = 0$; $t[0, \frac{\pi}{6}]$

$$I_3 = \int_{\overrightarrow{CB}} \cancel{(y+\sin x \sin y)\,dx}^{0} - \cos \overset{\frac{\pi}{6}}{x} \cos y\,dy$$

$$= \int_0^{\frac{1}{2}} -\cos\frac{\pi}{6} \cos y\,dy = -\frac{\sqrt{3}}{2} \cdot \sin y \Big|_0^{\frac{1}{2}}$$

$\overrightarrow{CB}$: $x = \frac{\pi}{6}$, $dx = 0$; $y = t$, $dy = dt$; $t[0, \frac{1}{2}]$

$$I = I_1 - I_2 - I_3 = \frac{\sqrt{3}}{2} - 1 + \frac{\sqrt{3}}{2} \sin\frac{1}{2} \quad //$$

25. $\int_{\mathcal{K}} \left(\frac{2x}{y} + \frac{y}{x}\right)dx + \left(\ln x - \frac{x^2}{y^2}\right)dy$ $\quad$ $\mathcal{K}$ luk krivulje

$P \qquad Q$

$x = e^t$

$y = t^2 + t + 1$

od A za $t=0$

do B za $t=1$

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \qquad \frac{1}{x} - \frac{2x}{y^2} = -\frac{2x}{y^2} + \frac{1}{x} \checkmark$$

POTENCIJALNO

$(x_0, y_0) = (1,1)$

$A(1,1)$

e^0

$B(e,3)$

$$\int_{x_0}^{x} \left(\frac{2x}{y} + \frac{y}{x}\right)dx + \int_{y_0}^{y} \ln x_0 - \frac{x_0}{y^2} =$$

$$= \int_1^x \frac{2x}{y} + \int_1^x \frac{y}{x}\,dx + \int_1^y 1\,dy - \int_1^y \frac{1}{y^2}\,dy$$

$$= \frac{x^2}{y} - \cancel{\frac{1}{y}} + y\cdot(\ln x - \cancel{\ln 1}^{0}) + y - \cancel{1} + \cancel{\frac{1}{y}} \cancel{+1}$$

$$= \frac{x^2}{y} + y\ln x + y + C$$

$$I = U(B) - U(A)$$

$$= \left(\frac{e^2}{3} + \cancel{3\ln 1}^{0} + 3\right) - \left(1 + \cancel{1\cdot\ln 1}^{0} + 1\right)$$

$$= \frac{e^2}{3} + 1$$