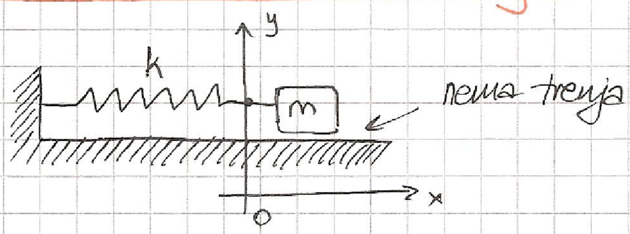
Jednostavno harmoničko titranje



Jednadžba gibanja:

$$F = -kx$$

$$m \cdot a = -kx$$

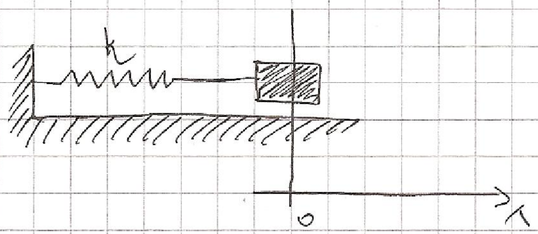
$$m \cdot \frac{d^2x}{dt^2} = -kx$$

$$m \cdot \ddot{x} + kx = 0 \quad /:m$$

$$\ddot{x} + \frac{k}{m} x = 0$$

$$\ddot{x} + \omega_0^2 x = 0 \qquad \omega_0^2 = \frac{k}{m} > 0$$

Dobivanje jednadžbe gibanja preko E = konst.



$$E = E_{kin} + E_{pot}$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$$

$$E = \text{konst.} \Rightarrow \frac{dE}{dt} = 0$$

$$\frac{dE}{dt} = m \dot{x} \ddot{x} + k \dot{x} x$$

$$= \dot{x} (m \ddot{x} + kx)$$

$$= m \dot{x} \underbrace{\left(\ddot{x} + \frac{k}{m} x \right)}_{=0}$$

ako je ovo 0 onda je u ravnotežnom položaju

$$\ddot{x} + \frac{k}{m} x = 0$$

Rješavanje jednadžbe gibanja:

$$\ddot{x} + \omega_0^2 x = 0$$

Početni uvjeti u $t = t_0$:

$$x(t_0) = x_0 \quad , \quad \dot{x}(t_0) = \dot{x}_0 = v_0$$

„Ansatz" rješenje: $x[t] = e^{\lambda t}$

$$\lambda^2 e^{\lambda t} + \omega_0^2 e^{\lambda t} = 0$$

$$\lambda^2 + \omega_0^2 = 0$$

$$\lambda_{1,2} = \pm i\omega_0$$

Opće rješenje (linearna kombinacija):

$$x[t] = a_1 e^{i\omega_0 t} + a_2 e^{-i\omega_0 t}$$

Fizikalni zahtjev: $x[t] \in \mathbb{R}$ $\qquad e^{i\phi} = \cos\phi + i\sin\phi$

$$a_1 = \bar{a}_2 \equiv \frac{A}{2} e^{i\phi}$$

$$x[t] = \frac{A}{2} e^{i(\omega_0 t + \phi)} + \frac{A}{2} e^{-i(\omega_0 t + \phi)}$$

$$= \frac{A}{2}\left(\cos[\omega_0 t + \phi] + i\sin[\omega_0 t + \phi] + \cos[-(\omega_0 t + \phi)] + i\sin[-(\omega_0 t +\right.$$

$$= \frac{A}{2}\left(2\cos(\omega_0 t + \phi) + 0\right)$$

$$= A\cos[\omega_0 t + \phi]$$

Energija titranja

$$E = E_{kin} + E_{pot}$$

$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$$

$$= \frac{1}{2} m (\omega_0 A)^2 \sin^2[\omega_0 t + \phi] + \frac{1}{2} k A^2 \cos^2[\omega_0 t + \phi]$$

$$= \frac{1}{2} \cancel{m} \frac{k}{\cancel{m}} \cdot A^2 \sin^2[\,] + \frac{1}{2} k A^2 \cos^2[\,]$$

$$= \frac{1}{2} A^2 k \left(\sin^2[\,] + \cos^2[\,]\right)$$

$$= \frac{A^2 k}{2}$$

//

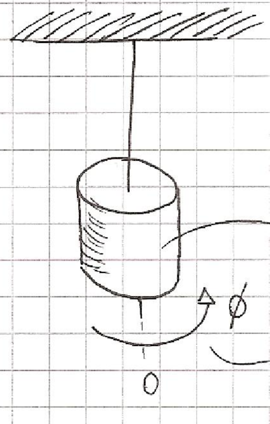
Srednja kinetička energija

$$\overline{E_{kin}} = \frac{1}{T}\int_0^T E_{kin}[t]\,dt$$

$$E_{kin} = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m(\omega_0 A)^2 \sin^2[\omega_0 t + \phi]$$

$$\overline{E_{kin}} = \frac{m(\omega_0 A)^2}{4} = \overline{E_{pot}}$$

TORZIONO "NJIHALO"



Konstanta torzije za šipku $D = \frac{G\pi R^4}{2L}$

moment tromosti I

kut zakreta $\dot{\phi} = \omega$

Jednadžba gibanja

$$L = I \cdot \omega = I \cdot \dot{\phi} \qquad \frac{dL}{dt} = M$$

$$M = -D \cdot \phi$$

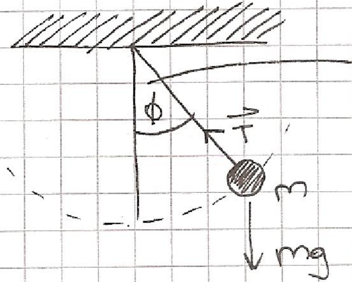
$$I\ddot{\phi} = -D\phi$$

$$I\ddot{\phi} + D\phi = 0 \quad /:I$$

$$\ddot{\phi} + \omega_0^2\phi = 0$$

$$\omega_0^2 = \frac{D}{I}$$

MATEMATIČKO „NJIHALO"



kut otklona ϕ

$\omega = \dot{\phi}$

Vrtnja oko objesišta: $\qquad L = I\omega,\ \omega = \dot{\phi},\ I = ml^2$

$$M = -mgl\sin\phi$$

Jednadžba gibanja: $\frac{dL}{dt} = M$

$$\to ml^2\ddot{\phi} = -mgl\sin\phi \quad /:ml^2$$

$$\ddot{\phi} + \frac{g}{l}\sin\phi = 0$$

za $\phi \ll 1$ $\quad \sin\phi \simeq \phi + \dots$

$$\ddot{\phi} + \omega_0^2\phi = 0$$

$\hookrightarrow \frac{g}{l}$

Preko ZOE:

$$E = E_{kin} + E_{pot}$$

$$= \frac{1}{2}I\omega^2 + mgh$$

$$= \frac{1}{2}ml^2\dot{\phi}^2 + mgl(1-\cos\phi)$$

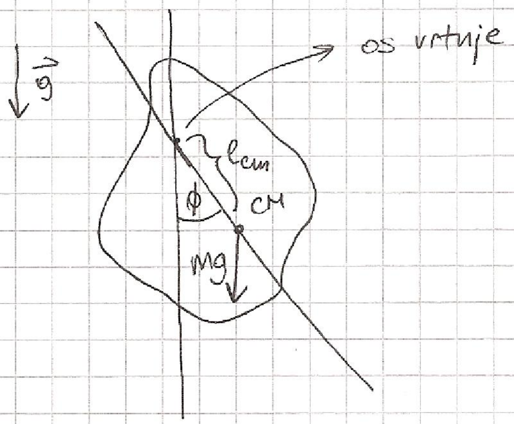
za $\phi \ll 1$ $\quad \cos\phi \simeq 1 - \frac{\phi^2}{2}$

$$E = \frac{1}{2}ml^2\dot{\phi}^2 + mgl\frac{\phi^2}{2}$$

$$\frac{dE}{dt} = 0 = ml^2\dot{\phi}\ddot{\phi} + mgl\phi\dot{\phi}$$

$$= ml^2\dot{\phi}\Big(\underbrace{\ddot{\phi} + \frac{g}{l}\phi}_{\ddot{\phi} + \omega_0^2\phi = 0}\Big)$$

FIZIČKO NJIHALO



Jednadžba gibanja : $\frac{dL}{dt} = M$

$$I\ddot{\phi} = -mgl_{CM}\sin\phi$$

$$\ddot{\phi} + \frac{mgl_{cm}}{I}\sin\phi = 0$$

$$\phi \ll 1 \qquad \sin\phi \approx \phi$$

$$\ddot{\phi} + \frac{mgl_{cm}}{I}\phi = 0$$

Steinerov tm. $\Rightarrow$ $I = I_{cm} + ml_{cm}^2$

$$\omega_0^2 = \frac{mgl_{cm}}{I_{cm} + ml_{cm}^2}$$

Jednadžba gibanja preko E = konst :

$$E = E_{kin} + E_{pot}$$

$$= \frac{1}{2}I\omega^2 + mgh$$

$$= \frac{1}{2}I\dot{\phi}^2 + mgl_{cm}(1 - \cos\phi)$$

$$\phi \ll 1, \quad \cos\phi \cong 1 - \frac{\phi^2}{2}$$

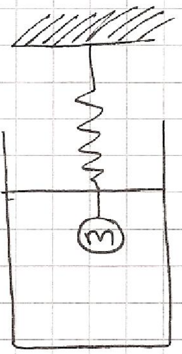
$$E = \frac{1}{2}I\dot{\phi}^2 + mgl_{cm}\frac{\phi^2}{2}$$

$$\frac{dE}{dt} = 0$$

$$I\dot{\phi}\ddot{\phi} + mgl_{cm}\phi\dot{\phi} = 0$$

$$I\dot{\phi}\left(\ddot{\phi} + \frac{mgl_{cm}}{I}\phi\right) = 0$$

PRIGUŠENI OSCILATOR



Sila otpora:

$$F_{otp} : -bv$$

brzina

sila viskoznog člana

Jednadžba gibanja:

$$m\ddot{x} = -kx - b\dot{x} \;/ : m$$

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0$$

$$2\delta = \frac{b}{m}$$

Rješavanje jednadžbe gibanja:

„Ansatz" rješenje : $e^{\alpha t}$

$$\alpha^2 e^{\alpha t} + 2\delta\alpha e^{\alpha t} + \omega_0^2 e^{\alpha t} = 0 \;/ : e^{\alpha t}$$

$$\alpha^2 + 2\delta\alpha + \omega_0^2 = 0$$

$$\alpha_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

SLABO PRIGUŠENJE $(\delta < \omega_0)$

$$\alpha_{1,2} = -\delta \pm i\omega$$

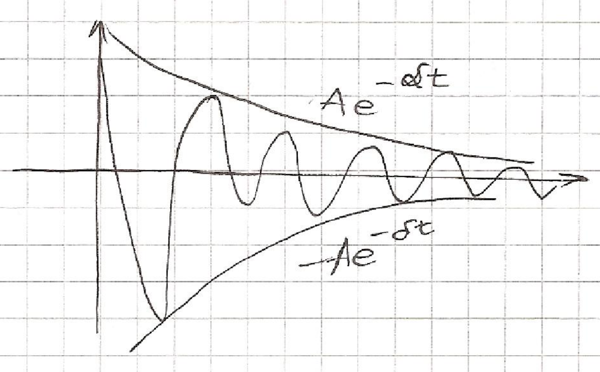
$$\omega \equiv \sqrt{\omega_0^2 - \delta^2}$$

$$x[t] = a_1 e^{\alpha_1 t} + a_2 e^{\alpha_2 t}$$

$$x[t] \in \mathbb{R}! \Rightarrow a_1 = \overline{a_2} = \frac{A}{2} e^{i\phi}$$

rješenje:

$$x[t] = A \cdot e^{-\delta t} \cos[\omega t + \phi]$$



KRITIČNO PRIGUŠENJE $\quad \omega_0 = \delta$

$$x[t] = (a_1 + a_2 t)e^{-\delta t}$$

početni uvjeti: $t=0$

$$x_0 = x[0] \qquad v_0 = \dot{x}[0]$$

rješenje:

$$x[t] = e^{-\delta t}(x_0 + (v_0 + x_0\delta)t)$$

SNAŽNO (NADKRITIČNO) PRIGUŠENJE $\quad (\delta > \omega_0)$

$$\alpha_{1,2} = -\delta \pm \varrho \quad , \quad \varrho = \sqrt{\delta^2 - \omega_0^2}$$

$$x[t] = a_1 e^{\alpha_1 t} + a_2 e^{\alpha_2 t}$$

$$= e^{-\delta t}\left[b_1 \operatorname{ch}(\varrho t) + b_2 \operatorname{sh}(\varrho t)\right]$$

Početni uvjeti:

$$x[0] = x_0 \, , \quad \dot{x}[0] = v_0$$

Rješenje:

$$x[t] = e^{-\delta t}\left(x_0 \operatorname{ch}(\varrho t) + \frac{v_0 + x_0\delta}{\varrho} \operatorname{sh}(\varrho t)\right)$$

MJERE JAKOSTI PRIGUŠENJA (samo za $\delta < \omega_0$)!

* logaritamski dekrement prigušenja

$$\lambda = \ln \frac{x[t]}{x[t+\tau]} = \ln e^{\delta\tau} = \delta\tau = \delta\frac{2\pi}{\omega}$$

$$\tau = \frac{2\pi}{\omega} \qquad\qquad \simeq 2\pi\frac{\delta}{\omega_0}$$

* Q faktor (faktor kvalitete)

$$Q^{-1} = -\frac{1}{2\pi}\left(\frac{\Delta E}{E}\right)$$

$$Q = -2\pi \frac{1}{\left(\frac{\Delta E}{E}\right)} = \dots = \frac{\omega}{2\delta} \simeq \frac{\omega_0}{2\delta}$$

PRISILNO TITRANJE

$\omega_0^2 = \frac{k}{m} \Rightarrow$ vlastita frekvencija oscilatora

amplituda

Vanjska sila $F_p[t] = F_p \cos[\omega_p t]$

Jednadžba gibanja:

$$m \cdot \ddot{x} + b\dot{x} + kx = F_p \cos[\omega_p t]$$

$$2\delta = \frac{b}{m} \qquad \omega_0^2 = \frac{k}{m} \qquad f_p = \frac{F_p}{M}$$

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = f_p \cos[\omega_p t]$$

Očekujemo rješenje oblika $x[t] = A\cos[\omega_p t - \phi]$

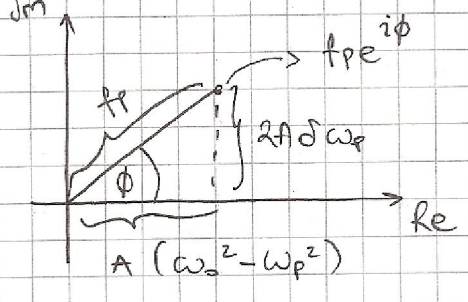
pišemo: $\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = f_p e^{i\omega t}$

„Ansatz" rješenje: $x[t] = A e^{i(\omega_p t - \phi)}$

$$(-\omega_p^2 + i\omega_p 2\delta + \omega_0^2) A \cancel{e^{i\omega_p t}} e^{-i\phi} = f_p \cancel{e^{i\omega_p t}}$$

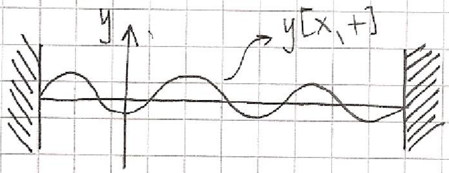
$$[(\omega_0^2 - \omega_p^2) + i2\delta\omega_p] A = f_p e^{i\phi}$$

$$A[\omega_p] = \frac{f_p}{\sqrt{(\omega_0^2 - \omega_p^2) - (2\delta\omega_p)^2}}$$



$$\mathrm{tg}\,\phi = \frac{2\delta\omega_p}{\omega_0^2 - \omega_p^2}$$

TRANSVERZALNO TITRANJE NAPETOG UŽETA

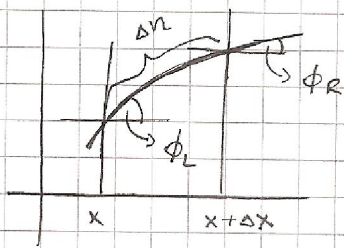


M - masa
L - duljina
T - napetost

Jednadžba gibanja za el. uže

$\dot{y}$ - derivacija po t

y' - derivacija po x



$$\Delta m \ddot{y}[x,t] = T_R \sin\phi_R + T_L \sin\phi_L \simeq$$
$$\simeq (tg\phi_R - tg\phi_L) \simeq$$
$$\simeq T(y'[x+\Delta x,t] - y'[x,t])$$

$$\Delta m = \mu \Delta x, \quad \mu = \frac{M}{L}, \quad \ddot{y}[x,t] = \frac{T}{\mu} \underbrace{\frac{y'[x+\Delta x,t] - y'[x,t]}{\Delta x}}_{\Delta x \to 0 \ = y''[x,t]}$$

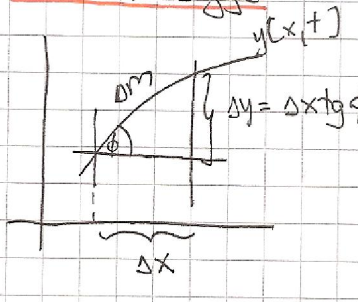
$$\Rightarrow \ddot{y} - \frac{T}{\mu} y'' = 0$$

$$Rj: \quad y[x,t] = f[x \pm vt] \qquad v \equiv \sqrt{\frac{T}{\mu}}$$

valna jednadžba $\ddot{y} - v^2 y'' = 0$

$$Rj: \quad y[x,t] = f[x \pm vt]$$
$$\ddot{y} = f''[x \pm vt] \cdot (\pm v^2)^2 = v^2 f''$$
$$y'' = f''[x \pm vt](1)^2 = f''$$

Preko energije



jer je došlo do produljenja žice ukoso, a ne ravno

$$\Delta E = \Delta E_{kin} + \Delta E_{pot}$$

linijska gustoća mase

$$\Delta E_{kin} = \frac{1}{2} \Delta m v^2 = \frac{1}{2} \mu \Delta x \dot{y}^2$$

brzine u y smjeru u kojem naš sustav titra.

$$\Delta E_{pot} = \Delta W = T(\Delta l - \Delta x)$$

$$\Delta l = \sqrt{(\Delta x)^2 + (\Delta x \, tg\phi)^2}$$
$$= \Delta x \sqrt{1 + (tg\phi)^2}$$

$$= \Delta x \left(1 + \frac{1}{2} tg^2\phi + \ldots\right)$$
$$= \Delta x \left(1 + \frac{1}{2} y'^2\right)$$
$$\Delta E_{pot} = \frac{1}{2} T \Delta x \, y'^2$$

$$\frac{\Delta E}{\Delta x} = \frac{1}{2}\mu \dot{y}^2 + \frac{1}{2} T y'^2$$

za harmonički val $y[x,t] = A\cos[\omega t \pm kx]$

$$\frac{\Delta E}{\Delta x} = \left(\frac{1}{2}\mu\omega^2 + \frac{1}{2}Tk^2\right)A^2 \sin^2[\omega t \pm kx]$$

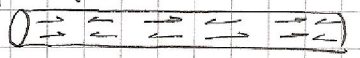
$$= A^2 \mu \omega^2 \sin^2[\omega t \pm kx]$$

$$\overline{\frac{\Delta E}{\Delta x}} = \frac{1}{2}A^2\mu\omega^2$$

$$\overline{P} = \overline{\frac{\Delta E}{\Delta x}} \cdot v$$

LONGITUDINALNO (VALNO) GIBANJE

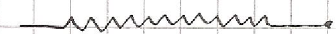
štap ili fluid:



L, S, M, E ili B ↗ za fluid

↳ Youngov modul za štap

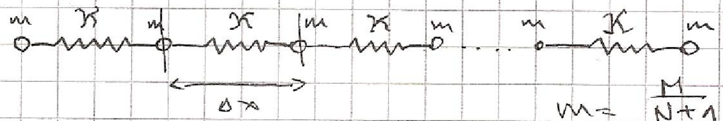
ekvivalentna opruga iste duljine i mase



$L, M,$ $k = \frac{SE}{L}$ za štap

$k = \frac{SB}{L}$ za fluid

ekvivalentni niz masa i opruga



$$m = \frac{M}{N+1} \approx \frac{M}{N}$$

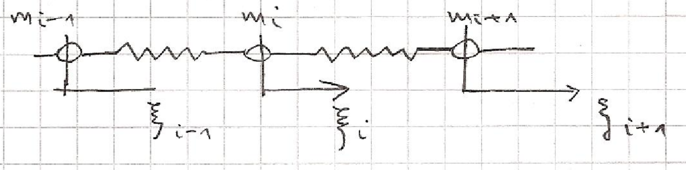
$$K = Nk$$

$$\Delta x = \frac{L}{N}$$

$$m = \frac{M}{L}\Delta x = \mu \Delta x$$

$$K = \frac{Lk}{\Delta x}$$

Jednadžba gibanja



$$m\ddot{\xi}_i = -\varkappa(\xi_i - \xi_{i+1}) - \varkappa(\xi_i - \xi_{i-1})$$

→ produljenje opruge u odnosu na ravnotežnu duljinu

uvodimo $\xi[x,t]$

$$(\xi_i = \xi[i\Delta x, t])$$

$$\mu\Delta x\,\ddot{\xi}[x,t] = Lk\,\frac{\xi[x+\Delta x,t]-\xi[x,t]}{\Delta x} - Lk\,\frac{\xi[x,t]-\xi[x-\Delta x,t]}{\Delta x}$$

$$= \frac{Lk}{\Delta x}\left(\xi'[x+\tfrac{\Delta x}{2},t] - \xi'[x-\tfrac{\Delta x}{2},t]\right)$$

$$= Lk\,\xi''[x,t]$$

$$\ddot{\xi}[x,t] - \frac{Lk}{\mu}\xi''[x,t] = 0$$

$$\ddot{\xi} - v^2\xi'' = 0$$

$$v^2 = \frac{Lk}{\mu} = \frac{k}{\mu}\cdot\frac{SE}{k} = \frac{SE}{\mu} = \frac{E}{\rho}$$

$$\downarrow$$

$$\frac{B}{\rho}$$

Energija

$$\Delta E = \frac{1}{2}m\dot{\xi}^2 + \frac{1}{2}\varkappa(\xi_{i+1}-\xi_i)^2$$

$$= \frac{1}{2}\mu\Delta x\,\dot{\xi}^2 + \frac{1}{2}\frac{Lk}{\Delta x}(\xi'\Delta x)^2$$

$$\frac{\Delta E}{\Delta x} = \frac{1}{2}\mu\dot{\xi}^2 + \frac{1}{2}k\frac{SE}{k}\cdot\xi'^2$$

za harmonički val $\xi[x,t] = A\cos[\omega t \pm kx]$

$$\frac{\Delta E}{\Delta x} = \frac{1}{2}\mu\omega^2A^2\sin^2[\omega t\pm kx] + \frac{1}{2}SEk^2A^2\sin^2[\omega t\pm kx]$$

$$k^2 = \left(\frac{\omega}{v}\right)^2 = \omega^2\frac{\rho}{E} \qquad S\rho = \mu$$

$$\frac{\Delta E}{\Delta x} = \mu\omega^2A^2\sin^2[\omega t\pm kx]$$

$$\overline{\frac{\Delta E}{\Delta x}} = \frac{1}{2}\mu\omega^2A^2 \Rightarrow \text{linijska gustoća energije}$$

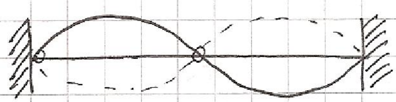
$$= \frac{1}{2}S\rho\omega^2A^2$$

$$\frac{\overline{\Delta E}}{\Delta V} = \frac{1}{S}\frac{\overline{\Delta E}}{\Delta x} = \frac{1}{2}\rho\omega^2A^2 \Rightarrow \text{volumna gustoća energije}$$

$$\overline{P} = \frac{\overline{\Delta E}}{\Delta x}\cdot v \Longrightarrow \text{srednja snaga}$$

Intenzitet $I = \frac{\overline{P}}{S}$ (srednja snaga po površini)

STOJNI VAL



$$\xi[x,t] = a\cos[\omega t - kx] + a\cos[\omega t + kx]$$
$$= \underbrace{2a\cos[kx]}_{\text{„amplituda"}}\cos\omega t$$

→ titrajući član

Općenito: $\xi[x,t] = A\cos[kx+\phi]\cos[\omega t+\varphi]$

čvorovi – točke u kojima vrijedi $\xi[x,t]=0$
brijegovi – točka za koji vrijedi $\xi[x,t] = -A\cos[\omega t+\varphi]$

* transverzalni val na užetu

$y[x,y] = A\sin[kx]\cos[\omega t]$

$\omega = kv$, $v = \sqrt{\frac{T}{\mu}}$

za osnovni mod

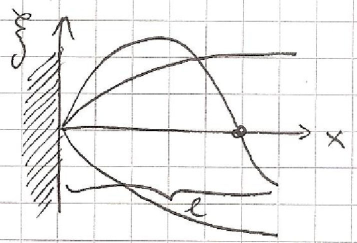
$$k = \frac{2\pi}{\lambda}, \quad \lambda = 2l$$
$$\Rightarrow k = \frac{\pi}{\lambda}, \quad \omega = \frac{\pi}{l}\sqrt{\frac{T}{\mu}}$$

osnovni mod

općenito $n = 1, 2, \ldots$

$$l = n\frac{\lambda}{2}$$
$$k_p = \frac{2\pi}{\lambda_n} = \frac{n\pi}{l}$$
$$\omega = k_n v = \frac{n\pi}{l}\sqrt{\frac{T}{\mu}}$$

* situacija : čvrst - slobodan



osnovni mod

$$l = \frac{\lambda}{4}$$

za $n \geq 1$

$$l = \frac{\lambda}{4} + (n-1)\frac{\lambda}{2}$$

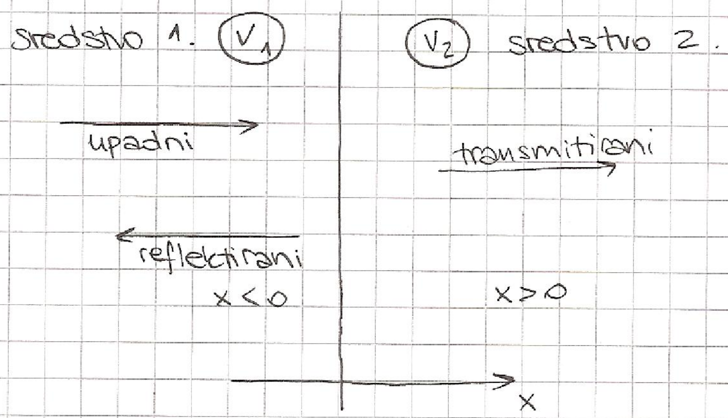
$$= \frac{2n-1}{4}\lambda_n \qquad \lambda_n = \frac{4l}{2n-1}$$

$$k_n = \frac{2\pi}{\lambda_n} = 2\pi \cdot \frac{2n-1}{4l}$$

$$k_n = \frac{2n-1}{2l}\pi$$

$$\omega_n = k_n \cdot v = \frac{(2n-1)}{2l}\pi\sqrt{\frac{E}{\rho}}$$

REFLEKSIJA I TRANZMISIJA VALA



μ_1 $\qquad T \qquad$ μ_2 $\qquad v = \sqrt{\frac{T}{\mu}}$

$$v_1 = \sqrt{\frac{T}{\mu_1}} \qquad v_2 = \sqrt{\frac{T}{\mu_2}}$$

$$\xi_{upad}[x,t] = a_{upad} \cdot \cos[\omega t - k_1 x]$$

$$\xi_{ref}[x,t] = a_{ref} \cdot \cos[\omega t + k_1 x]$$

$$\xi_{tran}[x,t] = a_{tran} \cdot \cos[\omega t - k_2 x]$$

UVJET 1: spojenost

$$\xi_{upad} + \xi_{ref} = \xi_{tran} \quad u \; x=0$$

$$\boxed{a_u + a_r = a_t}$$

UVJET 2: "glatka" spojenost

$$\xi_u' + \xi_r' = \xi_t' \quad u \quad x=0$$

$$\boxed{k_1 a_u - k_1 a_r = k_2 a_t}$$

$$a_r = \frac{k_1 - k_2}{k_1 + k_2} a_u \qquad a_t = \frac{2k_1}{k_1 + k_2} a_u$$

ZVUK

* oscilacija tlaka

$$p = p_{atm} + (\Delta p)_{max} \sin[\omega t \pm kx]$$

↳ amplituda oscilacije tlaka

$$= BkA = B\frac{\omega}{v}A$$

$$\frac{(\Delta p)^2_{max}}{2v\rho} = \frac{B^2\frac{\omega^2}{v^2}A^2}{2v\rho} = \frac{B\omega^2A^2}{2v} = \frac{B\omega^2A^2\rho}{2\rho v} = \frac{\rho\omega^2A^2v}{2}$$

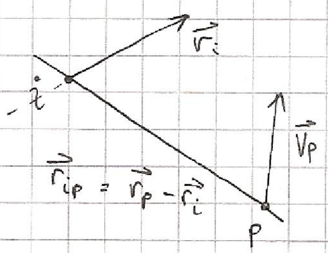
* intenzitet

$$I = \frac{\bar{P}}{S} = \frac{1}{2}\rho\omega^2A^2v = \frac{(\Delta p)^2_{max}}{2v\rho}$$

* Razina buke

$$L_{dB} = 10\log_{10}\frac{I}{I_0} \qquad I = 10^{-12}\frac{W}{m^2}$$

* Dopplerov efekt



i = izvor
p = prijemnik

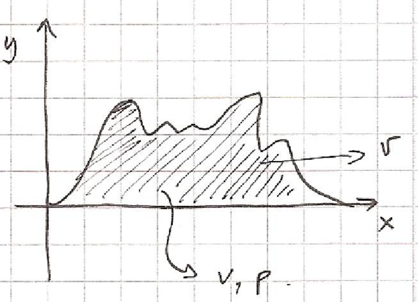
$$\frac{f_p}{f_i} = \frac{v_z - \hat{\vec{r}}_{ip}\cdot\vec{v}_p}{v_z - \hat{\vec{r}}_{ip}\cdot\vec{v}_i}$$

$\hat{\vec{r}}$ → ide uvijek od izvora prema prijemniku!

$\hat{\vec{r}}_{ip}\cdot\vec{v}_p$, $\hat{\vec{r}}_{ip}\cdot\vec{v}_i$ → skalarni umnožak → $v_p \cdot \cos\sphericalangle(v_p, r)$

$v_i \cdot \cos\sphericalangle(v_i, r)$

* Disperzija vala



1. nedisperzivno sredstvo

$$\ddot{y} - v^2 x'' = 0$$

rj. $y[x,t] = f[x \pm vt]$

↳ valni paket
ne mijenja oblik

superpozicija, $\omega_1 \approx \omega_2$

$$y[x,t] = A\cos[\omega_1 t - k_1 x] + A\cos[\omega_2 t - k_2 x]$$

$$= 2A\cos[\omega_- t - k_- x]\cos[\omega_+ t - k_+ x]$$

$$\left[\omega \pm = \frac{\omega_1 \pm \omega_2}{2} \qquad k\pm = \frac{k_1 \pm k_2}{2}\right]$$

$$= \underbrace{2A\cos\left[k_-\left(x - \frac{\omega_-}{k_-}t\right)\right]}_{\text{sporotitrajući faktor}} \underbrace{\cos\left[k_+\left(x - \frac{\omega_+}{k_+}t\right)\right]}_{\text{brzotitrajući faktor}}$$

2. disperzivno sredstvo

superpozicija:

$$y[x,t] = 2A\cos\left[k_-\left(x - \frac{\omega_-}{k_-}t\right)\right]\cos\left[k_+\left(1 - \frac{\omega_+}{k_+}t\right)\right.$$

$$\left.\begin{aligned} v_- &= \frac{\omega_-}{k_-} = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk} \\ v_+ &= \frac{\omega_+}{k_+} = \frac{\omega_{1,2}}{k_{1,2}} \end{aligned}\right\} \neq$$

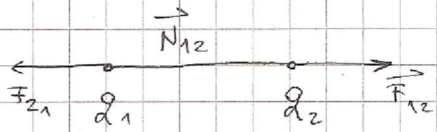
$$\frac{d\omega}{dk} < \frac{\omega}{k} \implies v_- < v_+$$

↳ fazna brzina
↳ grupna brzina

ELEKTROMAGNETIZAM

Električno polje:

Coulombov zakon



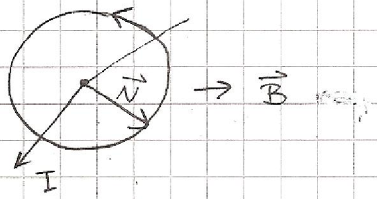
$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$r_{12} = |\vec{r}_{12}| \cdot |\vec{r}_1 - \vec{r}_2|$$

$$\hat{r}_{12} = \frac{r_{12}}{|r_{12}|}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} \cdot \vec{r}_{12} = -\vec{F}_{21}$$

Magnetsko polje:

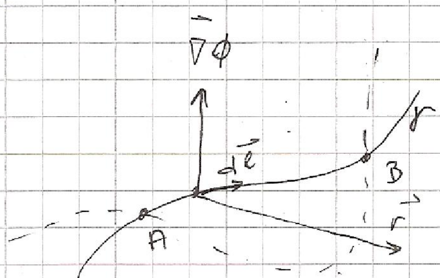


$$I = \frac{dq}{dt}$$

$$|\vec{B}| = \frac{1}{2\pi\varepsilon_0 c^2} \int \frac{I}{r} = \frac{\mu_0}{2\pi} \cdot \frac{I}{r_0}$$

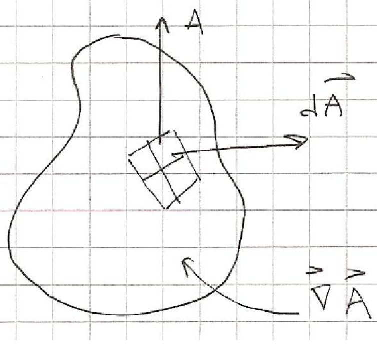
Tm o gradijentu

$$\int_{r=r_A}^{r_B} \vec{\nabla}\phi \cdot d\vec{l} = \phi[\vec{r}_B] - \phi[\vec{r}_A]$$



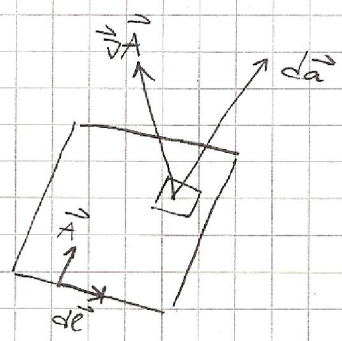
Tm o divergenciji (Gaussov tm)

$$\int_V (\vec{\nabla} \cdot \vec{A})\, dV = \oint_{S=\partial V} \vec{A}$$



Tm o rotaciji (Stokesov tm)

$$\int_S (\vec{\nabla}\times\vec{A})\cdot d\vec{a} = \oint_{\Gamma=\partial S} \vec{A}\cdot d\vec{l}$$



1. Maxwellova jednadžba

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

→ primjena Gaussovog teorema

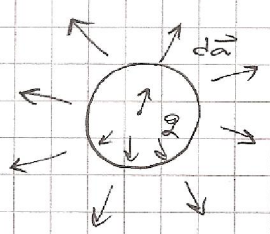
lijeva strana: $\int_V \vec{\nabla}\vec{E}\,dV = \oint_{\Gamma=\partial V} \vec{E}\cdot d\vec{A} = \Phi_E$

desna strana: $\int_V \frac{\rho}{\varepsilon_0}\,dV = \frac{1}{\varepsilon_0}\int_V \rho\,dV = \frac{q}{\varepsilon_0}$

Gaussov zakon za $\vec{E}$ polje

$$\Phi_E = \frac{1}{\varepsilon_0} Q$$

· Primjena Gaussovog zakona za $\vec{E}$ polje



"Gaussova ploha"

zbog simetrije $\vec{E}$ je radijalno polje (okomito na S)

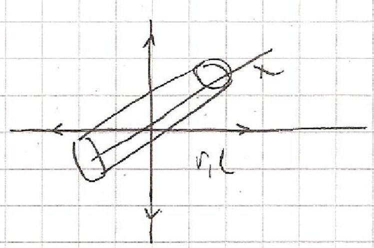
i svugdje na S ono je jednake jakosti

$$\Phi_E = \int_S \vec{E}\,d\vec{a} = E\cdot 4r^2\pi = \frac{1}{\varepsilon_0} Q$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$$ //

→ $\vec{E}$ polje beskonačnog linijskog naboja

$$\lambda = \frac{dq}{dl}$$



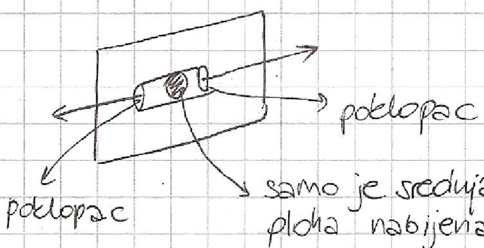
$$\Phi_E = \oint \vec{E}\, d\vec{a} = E \cdot 2r\pi \cdot l$$

$$= \frac{1}{\varepsilon_0} \cdot Q = \frac{1}{\varepsilon_0} \cdot l \cdot \lambda$$

$$\boxed{E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{r}}$$

→ $\vec{E}$ polje uniformno nabijene ravnine

$$\sigma = \frac{dq}{da}$$



$$\Phi_E = \oint_S \vec{E} \cdot d\vec{a} = 2E r^2 \pi$$

$$= \frac{1}{\varepsilon_0} \cdot Q = \frac{1}{\varepsilon_0} \cdot r^2 \pi\, \sigma$$

$$\boxed{E = \frac{1}{2\varepsilon_0} \cdot \sigma}$$

2. Maxwellova jednadžba

$$\nabla \cdot \vec{B} = 0$$

$$\downarrow$$

$$\Phi_B = 0$$

3. Maxwellova jednadžba

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

rotacija el. polja = − vremenska promjena magnetskog polja

Primjena Stokesa:

lijeva strana 3. MJ:

$$\int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = \oint_{\gamma = \partial S} \vec{E} \cdot d\vec{e} = \mathcal{E}$$

$\downarrow$

EMS!

desna strana 3. MJ:

$$\int_S \left(-\frac{\partial \vec{B}}{\partial t}\right) \cdot d\vec{a} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{a} = -\frac{d\Phi_B}{\partial t}$$

Faradayev zakon indukcije

$$\mathcal{E} = -\frac{\partial}{\partial t} \Phi_B$$

4. Maxwellova jednadžba

$$c^2 \vec{\nabla} \times \vec{B} = \frac{\vec{J}}{\varepsilon_0} + \frac{\partial \vec{E}}{\partial t}$$

c^2 (rotacija mag. polja) = $\frac{1}{\varepsilon_0}$ (gustoća el. struje) + (vremenska promjena el. polja)

Primjena Stokesova teorema:

lijeva strana: $\int_S c^2(\vec{\nabla}\times\vec{B})d\vec{a} = c^2\oint_{\gamma=\partial S} \vec{B}\cdot d\vec{l}$

desna strana:

$$\int_S \left(\frac{\vec{J}}{\varepsilon_0}+\frac{\partial \vec{E}}{\partial t}\right)\cdot d\vec{a} = \frac{1}{\varepsilon_0}\underbrace{\int_S \vec{J}\cdot d\vec{a}}_{I} + \frac{\partial}{\partial t}\underbrace{\int_S \vec{E}\cdot d\vec{a}}_{\Phi_E}$$

$$= \frac{1}{\varepsilon_0} I + \frac{\partial}{\partial t}\Phi_E$$

$$\Longrightarrow \quad c^2\oint_{\partial S} \vec{B}\cdot d\vec{l} = \frac{1}{\varepsilon_0} I + \frac{\partial}{\partial t}\Phi_E$$

$$\underbrace{\underbrace{c^2\begin{pmatrix}\text{Int } \vec{B} \text{ po}\\ \text{zatvorenoj}\\ \gamma=\partial S\end{pmatrix} = \frac{1}{\varepsilon_0}\begin{pmatrix}\text{struja}\\ \text{kroz}\\ S\end{pmatrix}}_{\text{AMPEROV ZAKON} \to \text{netočan!}} + \underbrace{\frac{\partial}{\partial t}\begin{pmatrix}\text{tok } E\\ \text{kroz } S\end{pmatrix}}_{\text{Maxwellov član}}}_{\text{AMPER-MAXWELLOV ZAKON (točno!)}}$$

* Jedn. kontinuiteta za električni naboj (očuvanje naboja)

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}}$$

$\rho \to$ gustoća naboja

$\vec{\nabla}\cdot\vec{J} \to$ gradijent gustoće struje

Interpretacija:

$$\int_V \vec{\nabla}\vec{J}\, dV = \oint \vec{J}\cdot d\vec{a}$$

$$\int_V \left(-\frac{\partial \rho}{\partial t}\right) dV = -\frac{\partial}{\partial t}\int_V \rho\, dV = -\frac{\partial}{\partial t} Q$$

$$\Rightarrow \underbrace{\oint_{S=\partial V} \vec{J}\cdot d\vec{a}}_{\text{el. tok kroz plohu}} = -\frac{\partial Q}{\partial t}$$

$\to$ vremenska promjena naboja unutar volumena

Izvod iz MJ:

$$\frac{\partial \rho}{\partial t} \overset{(1MJ)}{=} \frac{d}{dt}\left(\varepsilon_0\vec{\nabla}\cdot\vec{E}\right) = \varepsilon_0\vec{\nabla}\cdot\frac{\partial \vec{E}}{\partial t}$$

$$\overset{(4MJ)}{=} \varepsilon_0\vec{\nabla}\left(c^2\vec{\nabla}\times\vec{B} - \frac{\vec{J}}{\varepsilon_0}\right)$$

$$= -\vec{\nabla}\vec{J} \qquad \left(\text{zato jer } \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0\right)$$

* Elektrostatika

$$\frac{\partial \vec{E}}{\partial t} = -\frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

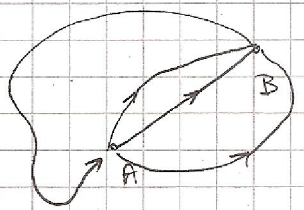
$$\boxed{\vec{\nabla} \times \vec{E} = 0}$$

„konzervativno polje"

$$\oint_\gamma \vec{E} \cdot d\vec{l} = 0$$

(stokes)

$$c^2 \vec{\nabla} \times \vec{B} = \frac{\vec{J}}{\varepsilon_0}$$



→ svi integrali su jednaki nuli

$$\int_A^B \vec{E} \cdot d\vec{l}$$

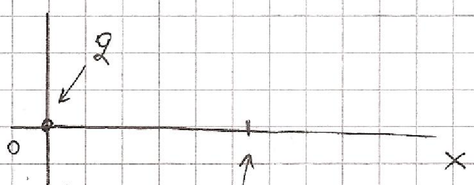
ne ovisi o putu

$$V[\vec{r}] \equiv -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\, d\vec{l}$$

$$\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{l} = \left(-V[\vec{r}]\right) - \left(-V[\vec{r}_0]\right)$$

Primjenom tm o gradijentu:

$$\vec{E} = -\vec{\nabla} V$$



$$V[x] = -\int_{+\infty}^{x} E_x\, dx$$

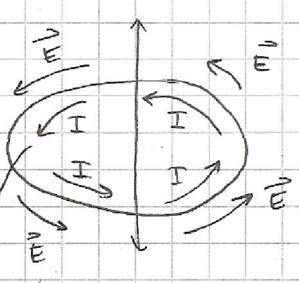
$$= -\int_{+\infty}^{x} \frac{1}{4\pi\varepsilon_0} \frac{Q}{x'^2}\, dx' = \frac{Q}{4\pi\varepsilon_0} \int_{x}^{+\infty} \frac{dx'}{x'^2} =$$

$$= \frac{Q}{4\pi\varepsilon_0} \left(-\frac{1}{x}\right)\Big|_x^{\infty} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{x}$$

$$V[\vec{r}] = \frac{1}{4\pi\varepsilon_0} \frac{Q_i}{|\vec{r} - \vec{r}_i|}$$

Za sve naboje: $V[\vec{r}] = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{Q_i}{|\vec{r} - \vec{r}_i|} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\, dV'}{|\vec{r} - \vec{r}_i|}$

LENZOVO PRAVILO



$\vec{\nabla} \times \vec{E}$ – rotacija el. polja

3. MJ : $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\frac{\partial \vec{B}}{\partial t}$ ~ promjena magnetskog toka

– po Stokesu će $\vec{E}$ imati bar 1 komponentu
će kružiti naokolo

→ to polje izaziva struju u istom smjeru – pravilo
dobijemo smjer mag. polja (palac pokazuje u
smjeru struje).