

$$1. \quad S_1[u(n)] = 3u(n) + 5$$

$$S_2[u(t)] = \min(\pi t) u(t)$$

A

a) $S[\alpha u_1(n) + \beta u_2(n)] = \alpha S[u_1(n)] + \beta S[u_2(n)] \quad , \quad \forall \alpha, \beta \in \mathbb{R}$

$$S[\alpha u_1(t) + \beta u_2(t)] = \alpha S[u_1(t)] + \beta S[u_2(t)]$$

b) $u_1(n) \rightarrow y_1(n) = 3u_1(n) + 5 \rightarrow \alpha y_1(n) = 3\alpha u_1(n) + 5\alpha$
 $u_2(n) \rightarrow y_2(n) = 3u_2(n) + 5 \rightarrow \beta y_2(n) = 3\beta u_2(n) + 3\beta$
 $\rightarrow y_3(n) = \alpha y_1(n) + \beta y_2(n) = 3\alpha u_1(n) + 3\beta u_2(n) + 5\alpha + 3\beta$

$u(n) = \alpha u_1(n) + \beta u_2(n) \rightarrow y(n) = 3\alpha u_1(n) + 3\beta u_2(n) + 5$
 $y_3(n) \neq y(n) \quad \text{NJE LINEARAN}$

c) $u_1(t) \rightarrow y_1(t) = \sin(\pi t) u_1(t) \rightarrow \alpha y_1(t) = \alpha \sin(\pi t) u_1(t)$
 $u_2(t) \rightarrow y_2(t) = \sin(\pi t) u_2(t) \rightarrow \beta y_2(t) = \beta \sin(\pi t) u_2(t)$
 $y_3(t) = \alpha y_1(t) + \beta y_2(t) = \alpha \sin(\pi t) u_1(t) + \beta \sin(\pi t) u_2(t)$
 $u(t) = \alpha u_1(t) + \beta u_2(t) \rightarrow y(t) = \sin(\pi t) \cdot (\alpha u_1(t) + \beta u_2(t))$
 $y_3(t) = y(t) \quad \text{SUSTAV JE LINEARAN}$

d) $u(n) \rightarrow u_1(n) = u(n-N) \rightarrow y(n) = 3u_1(n) + 5$
 $= 3u(n-N) + 5$

$u(n) \rightarrow y(n) = 3u(n) + 5 \rightarrow y(n-N) = 3u(n-N) + 5$
 $y_1(n) = y(n-N) \quad \text{SUSTAV JE VREMENSKI STACAN}$

e) $u(t) \rightarrow u_1(t) = u(t-T) \rightarrow y_1(t) = \sin(\pi t) u_1(t)$
 $= \sin(\pi(t-T)) u(t-T)$
 $u(t) \rightarrow y(t) = \sin(\pi t) u(t) \rightarrow y(t-T) = \sin(\pi(t-T)) u(t-T)$
 $y_1(t) \neq y(t-T) \quad \text{SUSTAV JE VREMENSKI PROMJENJIV.}$

[A]

$$2. \quad y(n) - \frac{3}{4} y(n-1) = u(n)$$

$$u(n) = \left(\frac{3}{2}\right)^n \mu(n)$$

$$a) \quad 2 - \frac{3}{4} = 0$$

$$2 = \frac{3}{4}$$

$$h(n) = C \left(\frac{3}{4}\right)^n$$

$$h(0) = C = 1$$

$$\boxed{h(n) = \left(\frac{3}{4}\right)^n \mu(n)}$$

$$y(n) = u(n) + \frac{3}{4} h(n-1)$$

$$y(0) = 0(0) = 1$$

$$b) \quad u(n) = \left(\frac{3}{2}\right)^n \mu(n)$$

$$y(n) = \sum_{m=-\infty}^{\infty} h(n-m) u(m) = \sum_{m=0}^{\infty} \left(\frac{3}{4}\right)^{n-m} \mu(n-m) \left(\frac{3}{2}\right)^m \mu(m)$$

$$= \sum_{m=0}^n \left(\frac{3}{4}\right)^{n-m} \left(\frac{3}{2}\right)^m = \left(\frac{3}{4}\right)^n \sum_{m=0}^n \left(\frac{4}{3} \cdot \frac{3}{2}\right)^m = \left(\frac{3}{4}\right)^n \sum_{m=0}^n 2^m$$

$$= \left(\frac{3}{4}\right)^n \frac{1 - 2^{n+1}}{1 - 2} = \left(\frac{3}{4}\right)^n (2^{n+1} - 1) = \left(\frac{3}{2}\right)^n / 2^{n+1} - 1$$

$$\boxed{y(n) = 2 \left(\frac{3}{2}\right)^n - \left(\frac{3}{4}\right)^n}$$

$$c) \quad y_u(n) = C \left(\frac{3}{4}\right)^n$$

$$y_p(n) = K \left(\frac{3}{2}\right)^n$$

$$K \left(\frac{3}{2}\right)^n - \frac{3}{4} K \left(\frac{3}{2}\right)^{n-1} = \left(\frac{3}{2}\right)^n$$

$$\frac{3}{2} K - \frac{3}{4} K = \frac{3}{2}$$

$$K = 2$$

$$y_p(n) = 2 \left(\frac{3}{2}\right)^n$$

$$y(n) = u(n) + \frac{3}{4} y(n-1)$$

$$y(0) = u(0) + \frac{3}{4} y(-1) = 1$$

$$\stackrel{\text{TOTAAL}}{y(n)} = C \left(\frac{3}{4}\right)^n + 2 \left(\frac{3}{2}\right)^n$$

$$y(0) = C + 2 = 1$$

$$C = -1$$

$$\boxed{y(n) = \left[-\left(\frac{3}{4}\right)^n + 2 \left(\frac{3}{2}\right)^n\right] \mu(n)}$$

$$3. \quad y''(t) + 12y'(t) + 45y(t) = u(t)$$

A

$$u(t) = 48 \cos(15t) \mu(t)$$

$$y(0^-) = y'(0^-) = 0$$

$$a) \quad s^2 y(s) + 12s y(s) + 45y(s) = U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s}{s^2 + 12s + 45}$$

$$b) \quad \text{mule} \quad n_1=0, \quad n_2=\infty$$

$$\text{polovi} \quad s^2 + 12s + 45 = 0$$

$$s_{1,2} = \frac{-12 \pm \sqrt{144 - 4 \cdot 45}}{2} = \frac{-12 \pm \sqrt{-36}}{2} = -6 \pm 3j$$

c) Kriterij umutnešuje stabilitet sustava:

asimtotički stabilan sustav ako je $\operatorname{Re}\{s_i\} < 0, \forall i$
 stabilno-marginalno stabilan ako je $\operatorname{Re}\{s_i\} \leq 0, \forall i$
 nestabilan - ako postoji $\operatorname{Re}\{s_i\} > 0$

d) $\operatorname{Re}\{s_1\} < 0 \quad \text{z} \quad \text{stabilan sustav}$
 $\operatorname{Re}\{s_2\} < 0$

$$e) \quad y(s) = H(s) \cdot U(s)$$

$$u(t) = 48 \cos(15t) \mu(t) \rightarrow U(s) = 48 \frac{s}{s^2 + 15^2}$$

$$y(s) = \frac{s}{s^2 + 12s + 45} \cdot \frac{48s}{s^2 + 15^2} = \frac{As + B}{s^2 + 12s + 45} + \frac{Cs + D}{s^2 + 225}$$

$$\underline{As^3 + 225As^2 + Bs^2} + \underline{Bs^2 + 225B} + \underline{Cs^3 + C \cdot 12s^2} + \underline{45Cs} + \underline{Ds^2 + 12Ds + 45D} = 48s$$

$$A + C = 0 \rightarrow A = -C$$

$$B + 12C + D = 48 \rightarrow B + 12(-C) - 5B = 48 \rightarrow -4B + 12C = 48 \rightarrow -B + 3C = 12$$

$$225A + 45C + 12D = 0 \rightarrow -225C + 45C + 12D = 0 \rightarrow -180C + 12D = 0$$

$$225B + 45D = 0 \rightarrow 5B + D = 0 \quad \begin{aligned} -15C + D = 0 \\ -15C - 5B = 0 \\ 3C + B = 0 \end{aligned}$$

$$y(s) = \frac{-2s - 6}{s^2 + 12s + 45} + \frac{2s + 30}{s^2 + 225}$$

$$B = -3C = -6$$

$$D = 30$$

$$6C = 12$$

$$C = 2$$

$$A = -2$$

$$= -2 \frac{s+6-3}{(s+6)^2+9} + 2 \frac{s+15}{s^2+225}$$

$$= -2 \frac{s+6}{(s+6)^2+9} + 2 \frac{3}{(s+6)^2+9} + 2 \frac{s}{s^2+225} + 2 \frac{15}{s^2+225}$$

$$y(t) = \boxed{(-2 e^{-6t} \cos(3t) + 2 e^{-6t} \sin(3t) + 2 \cos(15t) + 2 \sin(15t)) \mu(t)}$$

$$4. \quad y(n) + \frac{3}{5}y(n-1) + \frac{2}{25}y(n-2) = u(n) - u(n-1)$$

A

$$u(n) = \{5, 2, 5, 2, \dots\}$$

$$y(-1) = \Sigma$$

$$y(-2) = -\frac{25}{4}$$

$$a) \quad y(z) + \frac{3}{5}z^{-1}y(z) + \frac{2}{25}z^{-2}y(z) = U(z) + z^{-1}U(z)$$

$$H(z) = \frac{y(z)}{U(z)} = \frac{1+z^{-1}}{1+\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}} = \frac{z^2+z}{z^2+\frac{3}{5}z+\frac{2}{25}} = \frac{z(z+1)}{(z+\frac{2}{5})(z+\frac{1}{5})}$$

$$b) \quad H(e^{j\omega}) = \frac{1+e^{-j\omega}}{1+\frac{3}{5}e^{-j\omega}+\frac{2}{25}e^{-2j\omega}}$$

$$c) \quad u(n) = \{5, 2, 5, 2, \dots\}$$

$$\begin{aligned} U(z) &= 5 + 2z^{-1} + 5z^{-2} + 2z^{-3} + \dots \\ &= 5(1+z^{-2}+z^{-4}+\dots) + 2z^{-1}(1+z^{-2}+z^{-4}+\dots) \\ &= (5+2z^{-1})(1+z^{-2}+z^{-4}+\dots) \\ &= (5+2z^{-1}) \cdot \sum_{n=0}^{\infty} (z^{-2})^n = (5+2z^{-1}) \cdot \frac{1}{1-z^{-2}} = \frac{5z+2}{z} \cdot \frac{z^2}{z^2-1} \end{aligned}$$

$$U(z) = \frac{5z(z+\frac{2}{5})}{(z-1)(z+1)}$$

$$d) \quad y(z) + \frac{3}{5}z^{-1}y(z) + \frac{2}{25}y(z)z^{-2} + \frac{2}{25}z^{-1}y(-1) + \frac{2}{25}y(-2) \\ = U(z) + z^{-1}U(z) + u(-1)$$

$$y(z) \left(1 + \frac{3}{5}z^{-1} + \frac{2}{25}z^{-2} \right) = U(z) \left(1 + z^{-1} \right) - \frac{3}{5}y(-1) - \frac{2}{25}z^{-1}y(-1) - \frac{2}{25}y(-2)$$

$$\begin{aligned} y(z) &= \frac{1+z^{-1}}{1+\frac{3}{5}z^{-1}+\frac{2}{25}z^{-2}} \cdot U(z) - \frac{z(z+\frac{1}{5})}{(z+\frac{2}{5})(z-\frac{1}{5})} \\ &= \frac{(z+\frac{1}{5})z}{(z+\frac{2}{5})(z-\frac{1}{5})} \cdot \frac{5z(z+\frac{2}{5})}{(z-1)(z+1)} - \frac{z}{z+\frac{2}{5}} \end{aligned}$$

$$\frac{5z}{(z+\frac{2}{5})(z-1)} = \frac{A}{z+\frac{1}{5}} + \frac{B}{z-1} \rightarrow A = \frac{5}{6}, \quad B = \frac{25}{6}$$

$$y(z) = \frac{5}{6} \frac{z}{z+\frac{1}{5}} + \frac{25}{6} \frac{z}{z-1} - \frac{z}{z+\frac{2}{5}}$$

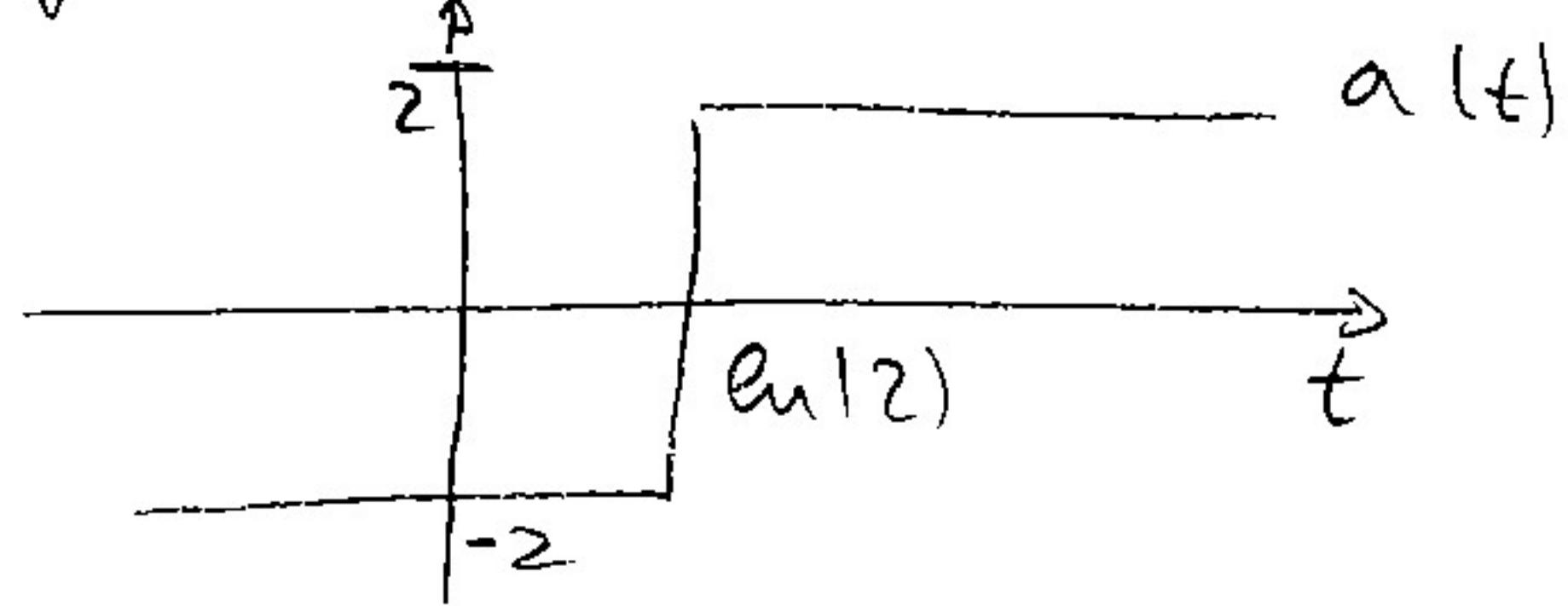
$$y(n) = \left[\frac{5}{6} \left(-\frac{1}{5} \right)^n - \left(\frac{2}{5} \right)^n + \frac{25}{6} \right] u(n)$$

5.

$$y'(t) + \alpha(t) y(t) = 8u(t) - u'(t)$$

$$u(t) = \mu(t)$$

$$y(0^-) = 0$$



$$y(0^+) - y(0^-) = -u(0^+)$$

$$y(0^+) = -1$$

A

a) ze $t > 0$ i $t < \ln(2)$

$$\begin{aligned} y'(t) + 2y(t) &= -u'(t) + 8u(t) \\ &= -\delta(t) + 8\mu(t) \end{aligned}$$

$$y'(t) + 2y(t) = 8\mu(t)$$

homogen

$$s = 2$$

$$y_u(t) = C e^{2t}$$

partikular

$$y_p(t) = k$$

$$-2k = 8$$

$$k = -4$$

totalmi

$$y_t(t) = C e^{2t} - 4$$

$$y_t(0^+) = C - 4 = -1$$

$$C = -1 + 4 = 3$$

$$y_t(t) = 3e^{2t} - 4$$

ne krajnje intervala

$$y_t(\ln(2)) = 3e^{2\ln 2} - 4 = 12 - 4 = 8$$

ze $t \geq \ln(2)$

$$y'(t) + 2y(t) = 8\mu(t)$$

$$s = -2$$

homogen

$$y_u(t) = C e^{-2t}$$

partikular

$$y_p(t) = k$$

$$2k = 8$$

$$k = 4$$

$$y_p(t) = 4$$

totalmi

$$y_t(t) = C e^{-2t} + 4$$

$$y_t(\ln(2)) = C e^{-2\ln 2} + 4$$

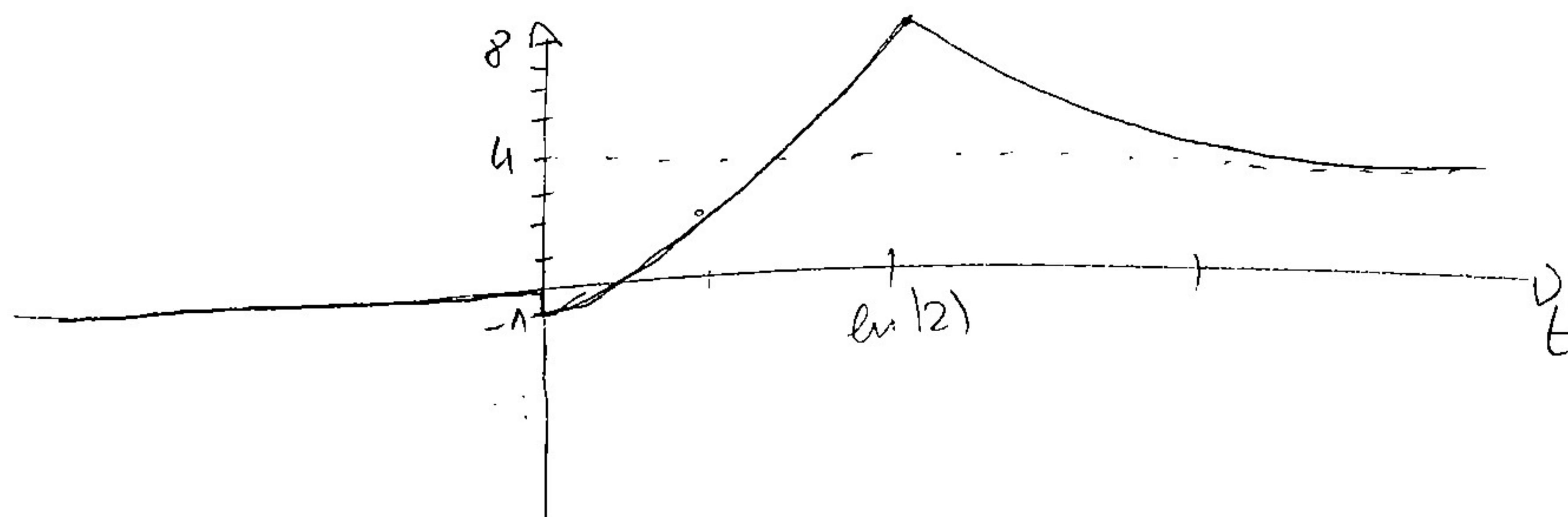
$$= C(-2) + 4 = 8$$

$$-2C = 8 - 4 = 4$$

$$C = -2$$

$$y_t(t) = 2e^{-2t} + 4$$

b)



$$1. \quad S_1\{u\}(n) = 5u(n) + 3$$

$$S_2\{u\}(t) = \cos(\pi t) u(t)$$
(B)

a) $S\{\alpha u_1(n) + \beta u_2(n)\} = \alpha S\{u_1(n)\} + \beta S\{u_2(n)\}$, $\alpha, \beta \in \mathbb{R}$

$$S\{\alpha u_1(t) + \beta u_2(t)\} = \alpha S\{u_1(t)\} + \beta S\{u_2(t)\}$$

b) $u_1(n) \rightarrow y_1(n) = 5u_1(n) + 3 \rightarrow \alpha y_1 = 5\alpha u_1(n) + 3\alpha$

$$u_2(n) \rightarrow y_2(n) = 5u_2(n) + 3 \rightarrow \beta y_2 = 5\beta u_2(n) + 3\beta$$

$$y_3(n) = \alpha y_1(n) + \beta y_2(n) = 5\alpha u_1(n) + 3\alpha + 5\beta u_2(n) + 3\beta$$

$$= 5\alpha u_1(n) + 5\beta u_2(n) + 3(\alpha + \beta)$$

$$u(n) = \alpha u_1(n) + \beta u_2(n) \rightarrow y(n) = 5(\alpha u_1(n) + \beta u_2(n)) + 3$$

$y_3(n) \neq y(n)$ SUSTAN JE LINEAR

c) $u_1(t) \rightarrow y_1(t) = \cos(\pi t) u_1(t) \rightarrow \alpha y_1(t) = \alpha \cos(\pi t) u_1(t)$

$$u_2(t) \rightarrow y_2(t) = \cos(\pi t) u_2(t) \rightarrow \beta y_2(t) = \beta \cos(\pi t) u_2(t)$$

$$y_3(t) = \alpha y_1(t) + \beta y_2(t) = \alpha \cos(\pi t) u_1(t) + \beta \cos(\pi t) u_2(t)$$

$$u(t) = \alpha u_1(t) + \beta u_2(t) \rightarrow y(t) = \cos(\pi t) (\alpha u_1(t) + \beta u_2(t))$$

$y_3(t) = y(t)$ SUSTAN JE LINEAR

d) $u(n) \rightarrow u_1(n) = u(n-N) \rightarrow y_1(n) = 5u_1(n) + 3 = 5u(n-N) + 3$

$$u(n) \rightarrow y(n) = 5u(n) + 3 \rightarrow y(n-N) = 5u(n-N) + 3$$

$y_1(n) = y(n-N)$ SUSTAN JE VEEHENSCH. STAKN

e) $u(t) \rightarrow u_1(t) = u(t-T) \rightarrow y_1(t) = \cos(\pi(t-T)) u(t)$

$$= \cos(\pi t) u(t-T)$$

$$u(t) \rightarrow y(t) = \cos(\pi t) u(t) \rightarrow y(t-T) = \cos(\pi(t-T)) u(t-T)$$

$y_1(t) \neq y(t-T)$ SUSTAN JE VEEHENSCH. PROBLEMNIV

$$2. \quad y(n) - \frac{2}{5}y(n-1) = u(n)$$

(b)

$$u(n) = \left(\frac{4}{5}\right)^n \mu(n)$$

$$a) \quad g - \frac{2}{5}g = 0$$

$$g = \frac{2}{5}$$

$$h(n) = C \left(\frac{2}{5}\right)^n$$

$$h(n) = S(n) + \frac{2}{5}h(n-1)$$

$$h(0) = 1$$

$$h(0) = C = 1$$

$$\boxed{h(n) = \left(\frac{2}{5}\right)^n \mu(n)}$$

$$b) \quad u(n) = \left(\frac{4}{5}\right)^n \mu(n)$$

$$\begin{aligned} y(n) &= \sum_{m=-\infty}^{\infty} h(n-m) u(m) = \sum_{m=-\infty}^{\infty} \left(\frac{2}{5}\right)^{n-m} \mu(n-m) \left(\frac{4}{5}\right)^m \mu(m) \\ &= \sum_{m=0}^n \left(\frac{2}{5}\right)^{n-m} \left(\frac{4}{5}\right)^m = \left(\frac{2}{5}\right)^n \sum_{m=0}^n \left(\frac{5}{2} \cdot \frac{4}{5}\right)^m = \left(\frac{2}{5}\right)^n \sum_{m=0}^n 2^m \\ &= \left(\frac{2}{5}\right)^n \frac{1-2^{n+1}}{1-2} = \left(\frac{2}{5}\right)^n (2^{n+1}-1) = \underline{-\left(\frac{2}{5}\right)^n + 2\left(\frac{4}{5}\right)^n} \end{aligned}$$

$$c) \quad y_h(n) = C \left(\frac{2}{5}\right)^n$$

$$y_p(n) = K \left(\frac{4}{5}\right)^n$$

$$K \left(\frac{4}{5}\right)^n - \frac{2}{5}K \left(\frac{4}{5}\right)^{n-1} = \left(\frac{4}{5}\right)^n$$

$$\frac{4}{5}K - \frac{2}{5}K = \frac{4}{5}$$

$$\frac{2}{5}K = \frac{4}{5}$$

$$K = 2$$

$$y_p(n) = 2 \left(\frac{4}{5}\right)^n$$

TOTAWI

$$y(n) = u(n) + \frac{2}{5}y(n-1)$$

$$y(-1) = 0 \quad \rightarrow \quad y(0) = 1$$

$$y(n) = C \left(\frac{2}{5}\right)^n + 2 \left(\frac{4}{5}\right)^n$$

$$y(0) = C + 2 = 1$$

$$C = -1$$

$$\boxed{y(n) = \left[-\left(\frac{2}{5}\right)^n + 2\left(\frac{4}{5}\right)^n\right] y(n)}$$

$$3. \quad y''(t) + 8y'(t) + 20y(t) = u(t)$$

(b)

$$u(t) = 32 \cos(10t) \mu(t)$$

$$y|_{t=0} = y'|_{t=0} = 0$$

$$\text{a)} \quad s^2 y(s) + 8sy(s) + 20y(s) = U(s)$$

$$H(s) = \frac{y(s)}{U(s)} = \frac{s}{s^2 + 8s + 20}$$

$$\text{b)} \quad \text{NVL}_D \quad n_1 = 0, \quad n_2 = \infty$$

$$\text{POLOVI} \quad s^2 + 8s + 20 = 0$$

$$s_{1,2} = \frac{-8 \pm \sqrt{64 - 80}}{2} = -4 \pm 2j$$

c) Kriterij numeričke stabilnosti:

asimptotki stabilan sustav ako je $\operatorname{Re}\{s_i\} < 0, \forall i$

stabilan (marginelno stabilan) ako je $\operatorname{Re}\{s_i\} \leq 0, \forall i$

nestabilan - ako postoji $\operatorname{Re}\{s_i\} > 0$

$$\text{d)} \quad \begin{aligned} \operatorname{Re}\{s_1\} &< 0 \\ \operatorname{Re}\{s_2\} &< 0 \end{aligned} \quad \left. \begin{array}{l} \text{stabilan sustav} \end{array} \right.$$

$$\text{e)} \quad u(t) = 32 \cos(10t) \mu(t) \rightarrow U(s) = 32 \frac{s}{s^2 + 100}$$

$$y(s) = H(s) \cdot U(s) = \frac{s}{s^2 + 8s + 20} \cdot \frac{32s}{s^2 + 100} = \frac{As + B}{s^2 + 8s + 20} + \frac{Cs + D}{s^2 + 100}$$

$$A\underline{s^3} + 100A + B\underline{s^2} + 100B + Cs^3 + 8Cs^2 + 20Cs + Ds^2 + 8Ds + 20D = 32s^2$$

$$A + C = 0 \rightarrow A = -C$$

$$B + 8C + D = 32 \rightarrow B + 8(-C) - 5B = 32 \rightarrow -4B + 8C = 32$$

$$100A + 20C + 8D = 0 \rightarrow -100C + 20C + 8D = 0 \rightarrow -80C - 40B = 0$$

$$100B + 20D = 0 \rightarrow 5B + D = 0 \quad D = -5B \quad 2C + B = 0$$

$$D = 20$$

$$B = -2C$$

$$B = -4$$

$$4C = 8$$

$$C = 2$$

$$A = -2$$

$$y(s) = \frac{-2s - 4}{s^2 + 8s + 20} + \frac{2s + 20}{s^2 + 100}$$

$$= -2 \frac{s+4}{(s+4)^2 + 4} + 2 \frac{s+10}{s^2 + 100} =$$

$$= -2 \frac{s+4}{(s+4)^2 + 4} + 2 \frac{2}{(s+4)^2 + 4} + 2 \frac{s}{s^2 + 100} + 2 \frac{10}{s^2 + 100}$$

$$y(t) = [-2 \cdot e^{-4t} \cos 2t + 2 e^{-4t} \sin 2t + 2 \cos 10t + 2 \sin 10t] \mu(t)$$

9.

$$y(n) + \frac{1}{14} y(n-1) + \frac{1}{7} y(n-2) = u(n) + u(n-1)$$

(B)

$$u(n) = \{ 7, 2, 7, 2, \dots \}$$

$$y(-1) = \frac{7}{2}$$

$$y(-2) = -\frac{49}{4}$$

a) $y(z) + \frac{1}{14} z^{-1} y(z) + \frac{1}{7} z^{-2} y(z) = U(z) + z^{-1} U(z)$

$$H(z) = \frac{y(z)}{U(z)} = \frac{1+z^{-1}}{1 + \frac{1}{14} z^{-1} + \frac{1}{7} z^{-2}} = \frac{z/z+1}{z^2 + \frac{1}{14} z + \frac{1}{7}}$$

b) $H(e^{iz}) = \frac{1+e^{-iz}}{1 + \frac{1}{14} e^{-iz} + \frac{1}{7} e^{-2iz}}$

c) $u(n) = \{ 7, 2, 7, 2, \dots \}$

$$\begin{aligned} U(z) &= 7 + 2z^{-1} + 7z^{-2} + 2z^{-3} + \dots \\ &= 7(1+z^{-2}+z^{-4}+\dots) + 2z^{-1}(1+z^{-2}+z^{-4}+\dots)_{\infty} \\ &= (7+2z^{-1})(1+z^{-2}+z^{-4}+\dots) = (7+2z^{-1}) \cdot \sum_{n=0}^{\infty} (z^{-2})^n \\ &= (7+2z^{-1}) \frac{1}{1-z^{-2}} = \frac{z(7z+2)}{z^2-1} = \frac{7z(z+\frac{2}{7})}{z^2-1} \end{aligned}$$

d) $y(z) + \frac{1}{14} z^{-1} y(z) + \frac{1}{14} y(-1) + \frac{1}{7} z^{-2} y(z) + \frac{1}{7} z^{-1} y(-1) + \frac{1}{7} y(-2)$
 $= U(z) + z^{-1} U(z) + \cancel{u(-1)}$

$$y(z)(1 + \frac{1}{14} z^{-1} + \frac{1}{7} z^{-2}) = U(z)(1+z^{-1}) - \frac{1}{14} y(-1) - \frac{1}{7} y(-2) - \frac{1}{7} z^{-1} y(-1)$$

$$\begin{aligned} y(z) &= \frac{1+z^{-1}}{1 + \frac{1}{14} z^{-1} + \frac{1}{7} z^{-2}}, \quad \frac{7z(z+\frac{2}{7})}{z^2-1} - \frac{z+\frac{1}{2}}{(z+\frac{2}{7})(z+\frac{1}{2})} \\ &= \frac{z(z+1)}{z^2 + \frac{1}{14} z + \frac{1}{7}} - \frac{7z(z+\frac{2}{7})}{(z-1)(z+1)} - \frac{1}{(z+\frac{2}{7})} \\ &= \frac{7z^2(z+\frac{2}{7})}{(z+\frac{2}{7})(z+\frac{1}{2})(z-1)} - \frac{1}{z+\frac{2}{7}} \end{aligned}$$

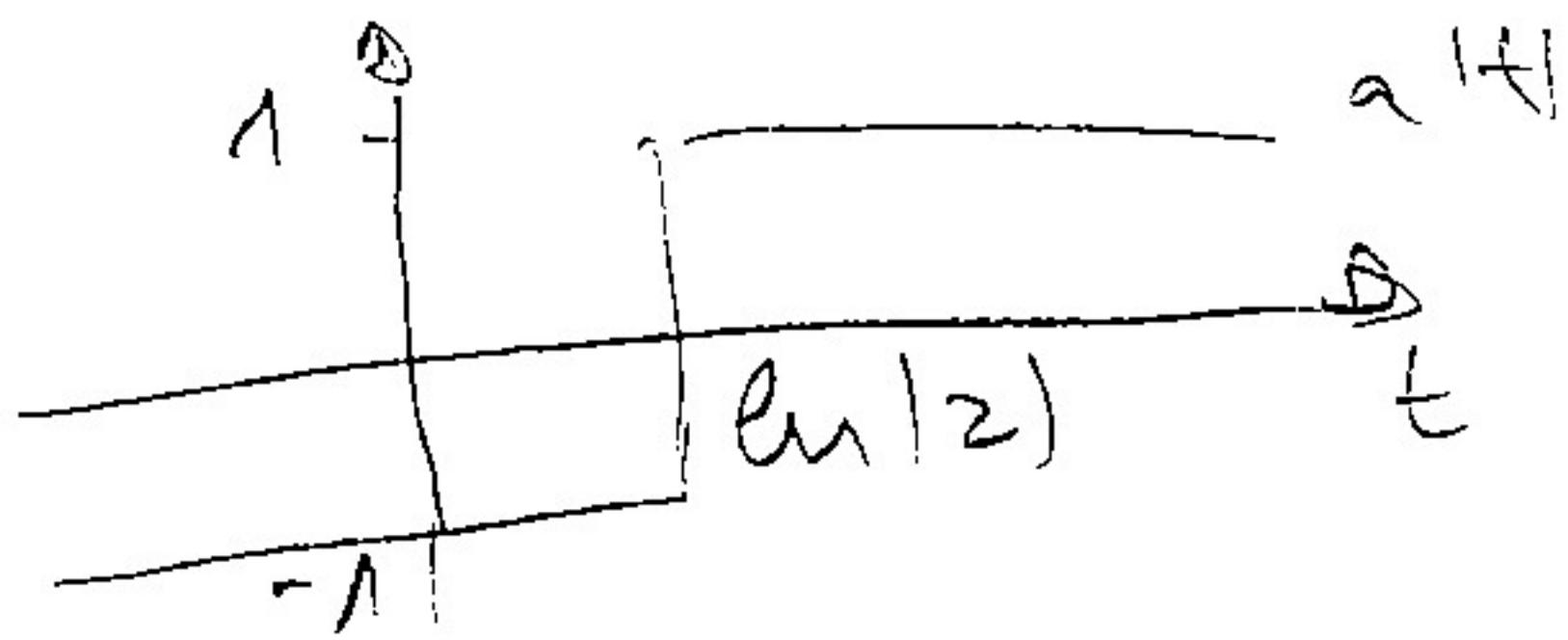
$$\frac{7z}{(z+\frac{1}{2})(z-1)} = \frac{A}{z-1} + \frac{B}{z+\frac{1}{2}} = \frac{14}{3} \frac{1}{z-1} + \frac{7}{3} \frac{1}{z+\frac{1}{2}}$$

$$y(z) = \frac{14}{3} \frac{z}{z-1} + \frac{7}{3} \frac{z}{z+\frac{1}{2}} - \frac{1}{z+\frac{2}{7}}$$

$$y(n) = \left[\frac{14}{3} + \frac{7}{3} \left(-\frac{1}{2} \right)^n - \left(-\frac{2}{7} \right)^n \right] u(n)$$

$$5. \quad y'(t) + a(t)y(t) = -u(t) + 2u(t)$$

B



$$u(t) = \mu(t)$$

$$y(0^+) = 0$$

POČETNÍ COND.

$$y(0^+) - y(0^-) = -u(0^+)$$

$$y(0^+) = -2 + 0 = -1$$

$\Rightarrow t > 0 \text{ if } t \leq \ln(2) \rightarrow a(t) = -1$

$$y'(t) - y(t) = -u(t) + 2u(t)$$

$$y'(t) - y(t) = -\delta(t) + 2\mu(t)$$

HOMOGENA

$$s - 1 = 0$$

$$y_h(t) = Ce^t$$

$$s = 1$$

IMPULSNI

$$h(t) = Ce^t$$

$$h(0^+) = C = 1$$

$$h(t) = e^t$$

$$h(t) = -e^{-t}$$

PARTIČNÍ

$$y_p(t) = K$$

$$-K = 2$$

$$K = -2$$

$$y_p(t) = -2$$

TOTALNÍ

$$y(t) = Ce^t - e^t - 2$$

$$y(0^+) = C - 1 - 2 = C - 3 = -1$$

$$C = 2$$

$$\boxed{y(t) = e^t - 2}$$

$$\boxed{y(0^+) = 2 - 2 = 0}$$

$\Rightarrow t > \ln(2) \rightarrow a(t) = 1$

$$y'(t) + y(t) = 2\mu(t)$$

$$y'(t) + y(t) = 2\mu(t) \rightarrow \text{for } t > \ln(2) \text{ same like impulse}$$

HOMOGENA

$$s + 1 = 0$$

$$y_h(t) = Ce^{-t}$$

$$s = -1$$

PARTIČNÍ

$$y_p(t) = K$$

$$K = 2$$

$$y_p(t) = 2$$

TOTALNÍ

$$y(t) = Ce^{-t} + 2$$

$$y(t| \ln(2)) = Ce^{-\ln(2)} + 2 = 0 \\ = C \cdot \frac{1}{2} + 2 = 0$$

$$\boxed{y(t) = -4e^{-t} + 2}$$

b)

