5. ZADACI ZA VJEŽBU (22.4.2016.)

1. tržišna cijena portfelja: $100 = C_K$

$D = 10$ god. (Macauleyeva)

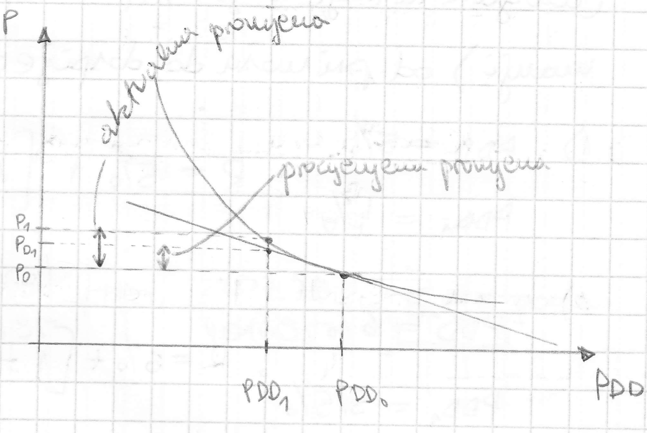
$N = 10\,000$ e

$PDD = y = 4\%$; $\Delta PDD = -3\%$

$\Delta y_1 = -3\% = 1\% = y_2$

a) procijenjena relativna promjena vrijednosti portfelja.

$$\frac{\Delta P}{P} \cong -\frac{1}{(1+PDD)} \cdot D \cdot \Delta PDD$$



$$\frac{\Delta P}{P} \cong -\frac{1}{(1+0{,}04)} \cdot 10 \cdot (0{,}03)$$

$$\frac{\Delta P}{P} \cong -0{,}2885 = \underline{-28.85\%}$$

– aktualna promjena je VEĆA od procijenjene

b) $t = 5$ god. (horizont ulaganja)

$PDD = 4\%$

$\Delta PDD = +2\%$

POVRAT procjenjujemo prema: $R = PDD + \left[1 - \frac{D}{t}\right] \cdot \Delta PDD$

R = ? (na kraju 5. godine) hoće li efekt reinvestiranja biti veći ili manji od efekta promjene cijene u portfelju?

$$R = 4\% + \left[1 - \frac{10}{5}\right] \cdot 2\% = \underline{2\%}$$

$D > t$, PDD ↑ → efekt reinvestiranja je MANJI od efekta promjene cijene

(2) $D = 6$ god. (Macauleyeva)

$C = 10\,000$ €

$PDD > 4.5\%$ $\qquad \Delta PDD < 0$

$t = 5$ god. (horizont ulaganja) $\qquad y_2 < y_1 \quad \Delta PDD = -x$

$$R = PDD + \left[1 - \frac{D}{t}\right]\Delta PDD \qquad \left[1 - \frac{6}{5}\right] = -0.2 \ ; \ \Delta PDD < 0$$

očekuje se smanjenje u kamatnim stopama, tada je <u>moguće</u> (moguće ili nije moguće) postići povrat koji je <u>veći</u> (veći ili manji) od prinosa do dospijeća PDD.

1) $PDD = 5\%$, $PDD_1 = 4\%$
$$R = 5\% + \left[1 - \frac{6}{5}\right] \cdot (4-5) = 5.2\%$$

2) $PDD = 6\%$, $PDD_1 = 5.5\%$
$$R = 6\% + \left[1 - \frac{6}{5}\right] \cdot (5.5-6) = 6.1\%$$

3) $PDD = 7\%$, $PDD_1 = 5\%$
$$R = 7\% + \left[1 - \frac{6}{5}\right](5-7) = 7.4\%$$

4) $PDD = 9\%$, $PDD_1 = 8.5\%$
$$R = 9\% + \left[1 - \frac{6}{5}\right](8.5-9) = 9.1\%$$

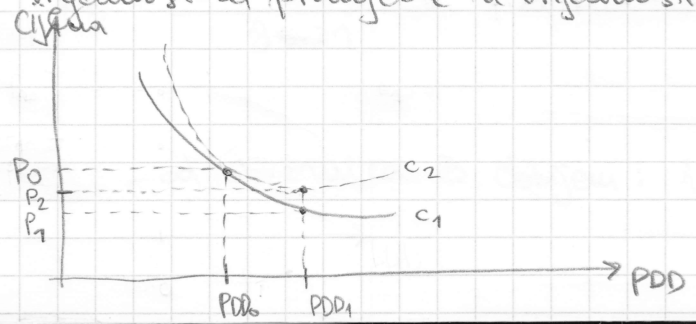
5) $PDD = 6\%$, $PDD_1 = 5\%$
$$R = 6\% + \left[1 - \frac{6}{5}\right](5-6) = 6.2\%$$

(3.) 2 PORTFELJA : P_1, P_2 $\qquad y_1 = x, \ y_2 = x + 2\%$

$D_1 = D_2 > 5$

$C_1 < C_2$ – konveksnost

tada će se vrijednost portfelja P_1 <u>smanjiti</u> (smanjiti ili povećati) za <u>veću</u> (veću ili manju) vrijednost od promjene u vrijednosti portfelja P_2.



④ PORTFELJ A: BKO → $N = 2000€$, $T = 1$ god. → $C_{BK_1} = 1809.67$ kn
BKO → $N = 6000€$, $T = 10$ god. → $C_{BK_2} = 2207.28$ kn } 4016.95 kn

PORTFELJ B: BKO → $N = 5000€$, $T = 5.95$ → $C_{BK} = 2757.81$ kn

$y = 10\%$

a) pokaži da oba portfelja imaju istu duraciju.

$$D_A = w_{BK_1} \cdot D_{BK_1} + w_{BK_2} \cdot D_{BK_2}$$

→ $D_{BK_1} = 1$ god.

→ $D_{BK_2} = 10$ god.

$$D_A = w_{BK_1} + 10 w_{BK_2}$$

$$D_A = \frac{C_{BK_1}}{C_A} + 10 \cdot \frac{C_{BK_2}}{C_A} = \frac{1809.67}{4016.95} + 10 \cdot \frac{2207.28}{4016.95} = 5.95 \text{ god.}$$

$D_B = 5.95$ god.

b) relativne promjene u vrijednosti oba portfelja ?
+0.1% u kamatnim stopama

$$\frac{\Delta P_A}{P_A} \cong -D_A \cdot \Delta y = -5.95 \cdot (0.1\%) = -0.595\%$$

$$\frac{\Delta P_B}{P_B} \cong -D_B \cdot \Delta y = -5.95\,(0.1\%) = -0.595\%$$

II. NAČIN $y_{NOVI} = 10 + 0.1 = 10.1\%$

$$C_A' = 2000\,e^{-0.101 \cdot 1} + 6000 \cdot e^{-0.101 \cdot 10} = 3993.18 \text{ kn}$$

$$\frac{\Delta C_A}{C_A} = \frac{3993.18 - 4016.95}{4016.95} \cdot 100\% = -0.59\%$$

$$C_B' = 5000\,e^{-0.101 \cdot 5.95} = 2741.45 \text{ kn}$$

$$\frac{\Delta C_B}{C_B} = \frac{2741.45 - 2757.81}{2757.81} = -0.59\%$$

c) relativna promjena vrijednosti oba portfelja

$\Delta y = +5\%$

$$C_A' = 2000e^{-0{,}15\cdot 1} + 6000e^{-0{,}15\cdot 10} = \underline{3060{,}197\text{ kn}}$$

$$C_B' = 5000e^{-0{,}15\cdot 5{,}95} = \underline{2048{,}152\text{ kn}}$$

$$\frac{\Delta C_A}{C_A} = \frac{3060{,}197 - 4016{,}95}{4016{,}95}\cdot 100\% = \underline{-23{,}82\%}$$

$$\frac{\Delta C_B}{C_B} = \frac{2048{,}152 - 2757{,}81}{2757{,}81} = -25{,}73\%$$

d) Tko ima veću konveksnost?

- manja relativna promjena VEĆA konveksnost

→ dakle portfelj A ima VEĆU konveksnost

(5.) $PDD = 3\%$

$D = 5$ god. (Macaulay)

$C = 99$ (tržišna cijena)

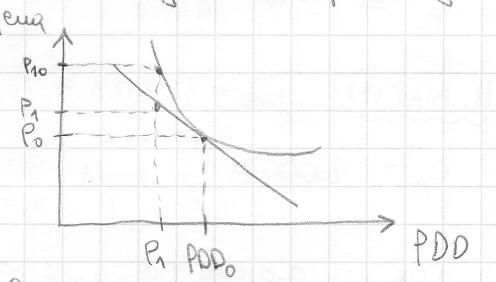
$N = 10\,000$

$\Delta PDD = -1\%$

a) procijenjena trenutna relativna promjena u vrijednosti portfelja

$$\frac{\Delta P}{P} \cong -D\cdot\frac{1}{1+PDD}\cdot\Delta PDD$$

$$\frac{\Delta P}{P} \cong -5\cdot\frac{1}{1+0{,}03}\cdot(-0{,}01) = \boxed{+4{,}85\%}$$



- aktualna promjena je VEĆA od procijenjene

b) $t = 3$ god

$\Delta PDD = -0{,}01 = -1\%$

$R = ?$

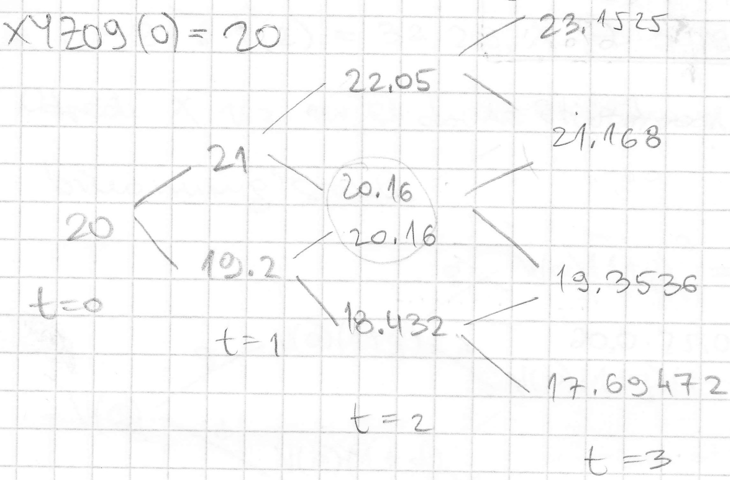
$$R = 3\% + \left[1 - \frac{5}{3}\right]\cdot(-1) = \boxed{+3{,}67\%}$$

- veći je od 3% jer je $t < D$, stoga će efekt cijene biti veći od efekta reinvestiranja

6. ZADACI ZA VJEŽBU (13.5.2016.)

(1.) XYZ09 +5% → skiciraj stablo za razvoj portfelja u iduća 3 dana trgovanja
−4%

XYZ09(0) = 20



• 4 različite cijene

• $2^n = 2^3 = 8$ različitih scenarija

(2.) XYZ09(0) = 45 ; POVRATI :

Scenarij	R(1)	R(2)	R(3)
ω_1	10%	5%	−10%
ω_2	5%	10%	10%
ω_3	5%	−10%	10%

povrati u svakom periodu

R(1) = R(0,1)

R(2) = R(1,2)

R(3) = R(2,3)

$45 \xrightarrow{+10\%} 49.5 \xrightarrow{+5\%} 51,975 \xrightarrow{-10\%} 46.7775$

$45 \xrightarrow{+5\%} 47,25 \xrightarrow{+10} 51,975 \xrightarrow{+10\%} 57.1725$

$45 \xrightarrow{+5\%} 47.25 \xrightarrow{-10\%} 42,525 \xrightarrow{+10\%} 46.775$

(3.) portfelj XYZ09

u trenutku n → V(n)

R(1) = R(2)

Odredi jednoperiodne povrate

Scenarij	V(0) ??	V(2)
ω_1	35	41
ω_2	35	32
ω_3	35	28

R(0,1) = R(1)

R(1,2) = R(2)

$w1: R(1) = R(2) = \frac{41-35}{35} = \underline{17.143\%}$

$w2: R(1) = R(2) = \frac{32-35}{35} = \underline{-8,571\%}$

$w3: R(1) = R(2) = \frac{28-35}{35} = \underline{-20\%}$

④ Gospodarski uvjeti

Gospodarski uvjeti	Vjerojatnost	Stopa povrata
Poboljšanje	25%	X %
Recesija	50%	-5%
Nema promjene	25%	6%

$E[R] = 6\%$ očekivana stopa povrata na vrijednost imovine

$$\boxed{E[R] = \sum_{i=1}^{3} P_i \cdot R_i}$$ očekivana stopa povrata

$$E[R_i] = P_1 \cdot X + P_2 \cdot R_2 + P_3 \cdot R_3$$

$$0.06 = 0.25 \cdot X + 0.5 \cdot (-0.05) + 0.25 \cdot 0.06$$

$$X = 0.28 \cdot 100\% = \underline{28\%}$$

⑤ portfelj XYZ09

Scenarij	V(1)	V(2)
ω_1	110	120
ω_2	105	100
ω_3	90	100

$V(0) = 100$

$p(\omega_1) = p(\omega_2) = 25\%$

$p(\omega_3) = 50\%$

$E[R(1)], E[R(2)], E[R(0,2)] = ?$

$$E[R(1)] = p(\omega_1) \cdot R_1(\omega_1) + p(\omega_2) \cdot R_1(\omega_2) + p(\omega_3) \cdot R_1(\omega_3)$$

$$E[R(1)] = 0.25 \cdot \frac{110-100}{100} + 0.25 \cdot \frac{105-100}{100} + 0.5 \cdot \frac{90-100}{100} = \underline{-1.25\%}$$

$$E[R(2)] = p(\omega_1) \cdot R_2(\omega_1) + p(\omega_2) \cdot R_2(\omega_2) + p(\omega_3) \cdot R_2(\omega_3)$$

$$E[R(2)] = 0.25 \cdot \frac{120-110}{110} + 0.25 \cdot \frac{100-105}{105} + 0.5 \cdot \frac{100-90}{90} = \underline{6.638\%}$$

$$E[R(0,2)] = p(\omega_1) \cdot R_1(0,2) + p(\omega_2) \cdot R_2(0,2) + p(\omega_3) \cdot R_3(0,2)$$

$$E[R(0,2)] = 0.25 \cdot \frac{120-100}{100} + 0.25 \cdot \frac{100-100}{100} + 0.5 \cdot \frac{100-100}{100} = \underline{5\%}$$

$$1 + E[R(0,2)] \stackrel{?}{=} (1 + E[R(1)])(1 + E[R(2)])$$

$$\underline{1.05 = 1.05}$$ ⇒ Zaključujemo da su jednoperiodni povrati NEZAVISNI

⑥ $r = 14\%$ (neprekidno) - mjesečne stope povrata

$V(0) = 22$ $\quad r = \frac{14}{12} = 1.167\%$ (zbog mjesečnih povrata) $= 0.01167$

$d = -0.01$

$d < r < g$ $\xrightarrow{\text{SLIJEDI}}$ $V(0)(1+d)(1+g) > V(0)(1+d)(1+r)$

$g > r$!

$$V(0)(1+d)(1+g) > 22 \cdot (1-0.01)(1+0.01167)$$

$$\underline{V(0)(1+d)(1+g) > 22.0342}$$

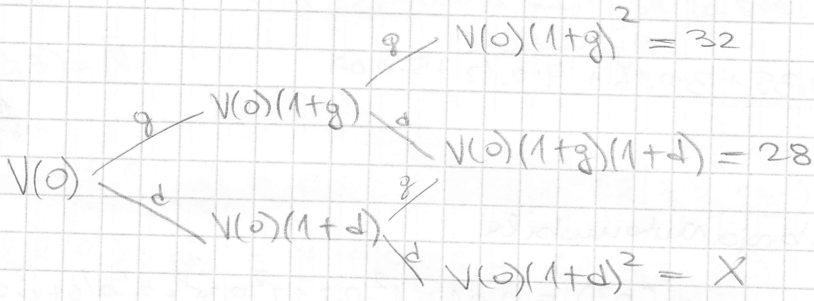
$V(0)(1+d)(1+g) = ?$

$$(1+g) > \frac{22.0342}{22\cdot(1-0.01)}$$

$$g > 0.01167125803$$

⑦ u $n=2$: $V(2) = 32, 28, X \rightarrow$ 3 jednadžbe

Odredi X uz uvjet da se vrijednost imovine kreće u skladu s modelom binomnog stabla



$$\left.\begin{array}{ll} V(0)\cdot(1+g)^2 = 32 & \rightarrow V(0) = \frac{32}{(1+g)^2} \\ V(0)(1+g)(1+d) = 28 & \rightarrow (1+d) = \frac{28}{V(0)(1+g)} \\ \underline{V(0)(1+d)^2 = X} & \end{array}\right\} \text{u 3) jednadžbu uvrstim}$$

$$\frac{32}{(1+g)^2}\cdot\frac{28^2}{\frac{32^2}{(1+g)^{4}}\cdot\cancel{(1+g)^2}} = X$$

$$\frac{1}{\cancel{(1+g)^2}}\cdot\frac{28^2\cdot\cancel{(1+g)^2}}{32} = X$$

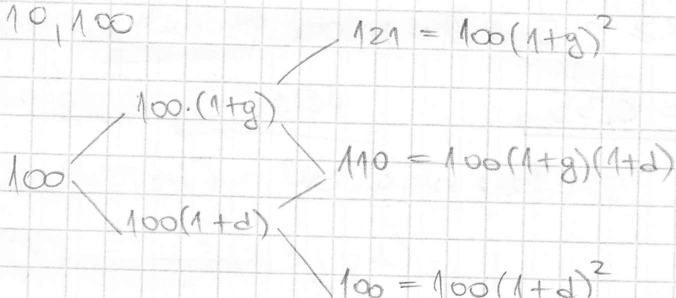
$$\boxed{X = 24.5}$$

⑧ model binomnog stabla

$n=2$: $V(2) = 121, 110, 100$

$\underline{V(0) = 100}$

$d, g = ?$



$121 = 100(1+g)^2 \rightarrow g = 0,095 = \underline{9,5\%}$

$100 = 100(1+d)^2 \rightarrow d = \underline{0\%}$

⑨ trgovac automobilima

Br. auta	0	1	2	3	4	5
Vjerojatnost	0.1	0.2	0.35	0.16	0.12	0.07

$\Sigma = 15$

a) očekivani broj automobila koji će biti prodani?

$$E[X] = \sum_{i=0}^{5} P_i \cdot R_i$$

$$E[X] = 0 \cdot 0.1 + 0.2 + 2 \cdot 0.35 + 3 \cdot 0.16 + 4 \cdot 0.12 + 5 \cdot 0.07$$

$$\underbrace{E[X] = 2.21}$$

b) standardna devijacija broja automobila

$$\sigma = \sqrt{E[R^2] - [E(R)]^2}$$

$$E(R^2) = 0 \cdot 0.1 + 1^2 \cdot 0.2 + 2^2 \cdot 0.35 + 3^2 \cdot 0.16 + 4^2 \cdot 0.12 + 5^2 \cdot 0.07$$

$$E[R^2] = 6.71$$

$$\underbrace{\sigma = 1.3513}$$

c) $C_{tjedna} = 250\$ + 300\$$ za svaki auto

$\overline{C}$ i $\sigma_C = ?$

$$\sigma_C = \sqrt{E[R^2] - E[R]^2}$$

$$\sigma_C =$$

$$\overline{C} = 250 + 300 \cdot 2.21 = \underbrace{913\$}$$

d) p da će mu plaća biti $> 1000\$$

$$250 + 300 \cdot x > 1000$$

$$300x > 750$$

$X \geq 2.5$ auta mora prodati

$$\underbrace{p = 0.35}$$

8. ~~7.~~ ZADACI ZA VJEŽBU (20.5.2016.)

(1)

$lR(1,3;g) = 0{,}26\%$ — $lR(2,3;gg) = 0{,}22\%$

$lR(1,3;g) = 0{,}26\%$ — $lR(2,3;gd) = y$

$lR(1,3;d) = x$ — $lR(2,3;dg) = 0{,}20\%$

$lR(1,3;d) = x$ — $lR(2,3;dd) = z$

T = 3 ~~god~~ mjeseca

$lR(3,3;gg) = 0.16\%$

$lR(3,3;gd) = 0.18\%$

$lR(3,3;dg) = 0.21\%$

$lR(3,3;dd) = 0{,}25\%$

a) $x, y, z = ?$

$B(0,3)$ $\qquad$ $lR(0,3;x)$ mora biti isti za sve scenarije

$B(3,3) = N$

$B(3,3;\tilde{s}) = 1$

(1) $lR(0,3;gg) = 0{,}22 + 0{,}26 + lR(3,3;gg)$

(2) $lR(0,3;gd) = y + 0{,}26 + lR(3,3;gd)$

(3) $lR(0,3;dg) = 0.20 + x + lR(3,3;dg)$

(4) $lR(0,3;dd) = z + x + lR(3,3;dd)$

(1) i (2) $\quad 0.22 + 0{,}26 + 0.16 = y + 0{,}26 + 0.18$

$$y = 0{,}20\%$$

(3) i (4) $\quad 0.2 + x + 0{,}21 = z + x + 0.25$

$$z = 0{,}16\%$$

(iz 1. i 3.) $\quad x = 0{,}23\%$

b) $lR(3,3,gg) = \ln B(3,3,gg) - \ln B(2,3;gg)$

$-(0.16\% - \ln B(3,3,gg)) = \ln B(2,3;gg)$

$$B(2,3;gg) = 0{,}9984$$

$lR(3,3;gd) = \ln B(3,3;gd) - \ln B(2,3;gd)$

$0.18\% = \ln(1) - \ln B(2,3;gd)$

$$B(2,3;gd) = 0.9982$$

$lR(3,3;dg) = \ln B(3,3;dg) - \ln B(2,3;dg)$

$$B(2,3;dg) = 0.9979$$

$lR(3,3;dd) = \ln B(3,3;dd) - \ln B(2,3;dd)$

$$B(2,3;dd) = 0{,}9975$$

$\ell R(2,3;gg) = \ln B(2,3;gg) - \ln B(1,3;g)$

$B(1,3;g) = \underline{0.9962}$

$\ell R(2,3;dg) = \ln B(2,3;dg) - \ln B(1,3;d)$

$B(1,3;d) = \underline{0.9959}$

$\ell R(1,3;g) = \ln B(1,3;g) - \ln B(0,3)$

$B(0,3) = \underline{0.9936}$

$$
\begin{tikzcd}[column sep=small]
 & & 0.9984 \\
 & 0.9962 \arrow[ur, dash] \arrow[dr, dash] & \\
 & & 0.9982 \\
0.9936 \arrow[uur, dash] \arrow[ddr, dash] & & \\
 & & 0.9979 \\
 & 0.9959 \arrow[ur, dash] \arrow[dr, dash] & \\
 & & 0.9975
\end{tikzcd}
$$

② Model binomnog stabla za cijene obveznica:

$$
\begin{tikzcd}[column sep=small]
 & & B(2,3;gg)=0.9905 \\
 & B(1,3;g)=0.9848 \arrow[ur] \arrow[dr] & \\
 & & B(2,3;gd)=0.9875 \\
B(0,3)=0.9726 \arrow[uur] \arrow[ddr] & & \\
 & & B(2,3;dg)=0.9908 \\
 & B(1,3;d)=0.9808 \arrow[ur] \arrow[dr] & \\
 & & B(2,3;dd)=0.9891
\end{tikzcd}
$$

$y(i,3;s_i),\ i=0,1,2,3=?$

$$\boxed{B(i,3;s_i) = Ne^{-\frac{y(i,3,s_i)}{12}\cdot(3-i)}} \quad ;\ N=1$$

$B(0,3) = e^{-\frac{y(0,3)}{12}\cdot 3} \rightarrow y(0,3) = \underline{11.11\%}$

$B(1,3;g) = e^{-\frac{y(1,3;g)}{12}\cdot 2} \rightarrow y(1,3;g) = \underline{9.19\%}$

$B(1,3;d) = e^{-\frac{y(1,3;d)}{12}\cdot 2} \rightarrow y(1,3;d) = \underline{11.63\%}$

$B(2,3;gg) = e^{-\frac{y(2,3;gg)}{12}\cdot 1} \rightarrow y(2,3;gg) = \underline{11.45\%}$

$B(2,3;gd) = e^{-\frac{y(2,3;gd)}{12}\cdot 1} \rightarrow y(2,3;gd) = \underline{15.09\%}$

$B(2,3;dg) = e^{-\frac{y(2,3;dg)}{12}\cdot 1} \rightarrow y(2,3;dg) = \underline{11.09\%}$

$B(2,3;dd) = e^{-\frac{y(2,3;dd)}{12}\cdot 1} \rightarrow y(2,3;dd) = \underline{13.15\%}$

$B(3,3;gg) = e^{-\frac{y(3,3;gg)}{12}}$

③ $S(t)$ = vrijednost rizične imovine ; x ⇒ pozicija u rizičnoj imovini

$A(t)$ - vrijednost nerizične imovine ; y ⇒ pozicija u nerizičnoj imovini

$A(0)=10 \qquad S(0)=10$

$A(1)=11 \qquad S(1)=13$ ili 9

JEDNOPERIODNE STRATEGIJE ? ⇒ Mora biti ZADOVOLJENO

$$\underline{V(0)\geq 0} \text{ i } \underline{V(1)\geq 0}$$

I. $10y+10x\geq 0$

II. $11y+13x\geq 0$, Rješenje je skup točaka :

III. $11y+9x\geq 0$

I. $y\geq -x$

II. $y\geq -\frac{13}{11}x$

III. $y\geq -\frac{9}{11}x$

$$\left\{ (x,y) : y \begin{cases} \geq -\frac{13}{11}x, \text{ za } x\leq 0 \\ \leq -\frac{9}{11}x, \text{ za } x\geq 0 \end{cases}, x\in\mathbb{R} \right\}$$

④ TRŽIŠTE: 1 dionica i NERIZIČNE IMOVINE

→ cijena dionice prati dinamiku kretanja modela binarnog stabla

→ SVAKI PUT kada cijena dionice NARASTE u idućem će trenu PASTI

$V(0)=0$ i $V(2)>0$ Odredi samofinancirajuću strategiju.

$\underline{t=0}$ - ne investiramo ništa $(x(1),y(1))=(0,0)$

u $\underline{t=1}$ $V(1)=0$

- ako je cijena dionice PALA u $t=1$ ne investiramo ništa, tj. $(x(2),y(2))=(0,0)$ i u tom slučaju je i $V(2)=0$

- ako je cijena dionice u $t=1$ PORASLA tada znamo da će u $t=2$ cijena pasti → isplati nam se ući u KRATKU POZICIJU:

tj. formiramo $\left(x(2),y(2)\right)=\left(-1,\frac{S(1)}{A(1)}\right)$ tada je

da ju prodamo u $t=2$

$V(2)=-S(2)+\frac{S(1)}{A(1)}\cdot A(2)>0 \qquad S(2)<S(1)$

$A(2)>A(1)$