FER 2 Električni krugovi
Aktivni i pasivni filtri

Napisao:
Sharp_Shooter

PDF pripremio i uredio:
Sharp_Shooter

PASIVNI FILTERI

① NISKOPROPUSNI

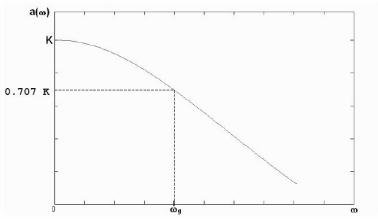
- područje propuštanja: $0 < \omega < \omega_g$

NP FILTAR 1. REDA

$$H(s) = K \cdot \frac{\omega_g}{s + \omega_g}$$

$s_p = -\omega_g$

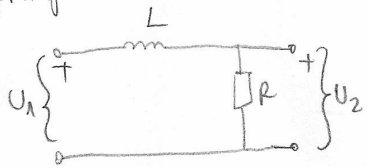
$s_0 \to \infty$



$$a(\omega) = |H(j\omega)| = \frac{K}{\sqrt{1 + \left(\frac{\omega}{\omega_g}\right)^2}} \Rightarrow$$ AMPLITUDNO-FREKVENCIJSKA KARAKTERISTIKA

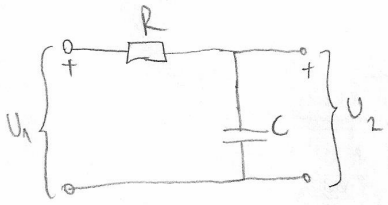
$$a(\omega_g) = \frac{K}{\sqrt{2}} = 0.707\,K$$

- primjer:



$$H(s) = \frac{\frac{R}{L}}{s + \frac{R}{L}}$$

$$\omega_g = \frac{R}{L}, \quad K = 1$$



$$H(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

$$\omega_g = \frac{1}{RC}, \quad K = 1$$

NP FILTAR 2. REDA

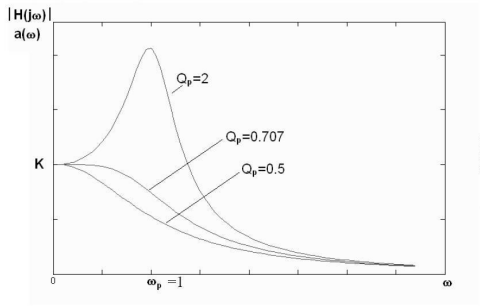
$$H(s) = K \cdot \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

$$s_{p1,2} = \sigma_p \pm j\,\tilde{\omega}_p = \frac{-\omega_p}{2Q_p} \pm j\,\omega_p \sqrt{1 - \frac{1}{4Q_p^2}}$$

$s_{0\,1,2} \longrightarrow \infty$

FREKVENCIJA POLOVA: $\omega_p = \sqrt{\sigma_p^2 + \tilde{\omega}_p^2}$

$$Q_p = -\frac{\omega_p}{2\sigma_p} \Rightarrow$$ FAKTOR KVALITETE

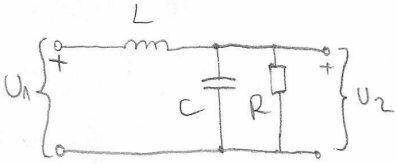


$$a(\omega) = |H(j\omega)| = K \cdot \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_p}\right)^2\right)+\left(\frac{1}{Q_p}\cdot\frac{\omega}{\omega_p}\right)^2}}$$

- za: $\omega \to 0 \Rightarrow a(\omega) \to K$

$\omega \to \infty \Rightarrow a(\omega) \to 0$

- primjer:



$$H(s) = \frac{\frac{1}{LC}}{s^2+\frac{s}{RC}+\frac{1}{LC}}$$

(2.) VISOKOPROPUSNI

- propušta $\omega > \omega_g$

VP FILTAR 1. REDA

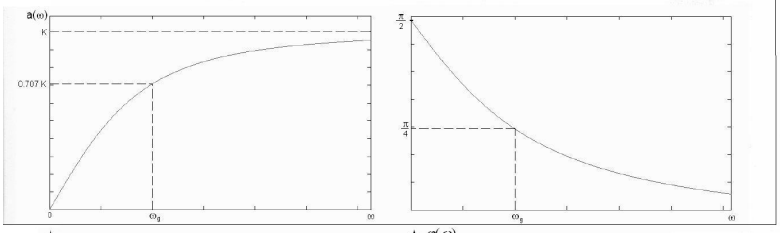
$$H_{VP}(s) = \frac{K \cdot s}{s+\omega_g}$$

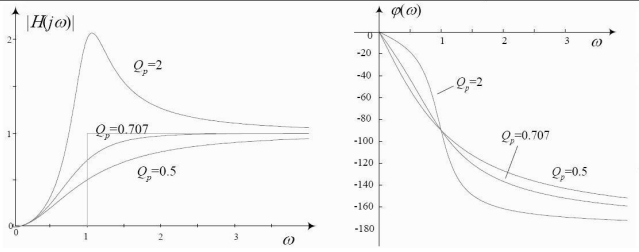
$s_p = -\omega_g$, $s_0 = 0$

$$a(\omega) = |H(j\omega)| = K \cdot \frac{\omega}{\sqrt{\omega^2+\omega_g^2}}$$

$$\varphi(\omega) = \frac{\pi}{2} - \operatorname{arctg}\left(\frac{\omega}{\omega_g}\right)$$

- realizacije: RL i RC četveropolom





VP FILTAR 2. REDA

$$H(s) = K \cdot \frac{s^2}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$

$$s_{PA2} = -\frac{\omega_p}{Q_p} \pm j\omega_p\sqrt{1-\frac{1}{4Q_p^2}}$$

$s_{0A/2} = 0$

$$s_{PA2} = \sigma_p \pm j\tilde{\omega}_p$$

$$\omega_p = \sqrt{\sigma_p^2+\tilde{\omega}_p^2}$$

$$Q_p = \frac{-\omega_p}{2\sigma_p}$$

- realizacija: RLC četveropolom (kao kod NP-filtara)

$$a(\omega) = |H(j\omega)| = K \cdot \frac{\omega^2}{\sqrt{\left(\omega_p^2-\omega^2\right)^2+\left(\frac{\omega\omega_p}{Q_p}\right)^2}}$$

$$\varphi(\omega) = \pi - \operatorname{arctg}\frac{\omega_p \cdot \frac{\omega}{Q_p}}{\omega_p^2-\omega^2}$$

3. POSASNO – PROPUSNI FILTRI

- propušta: $\omega_d < \omega < \omega_g$

- za PP karakteristiku, filtar mora biti NAJMANJE 2. REDA

PP FILTAR 2. REDA

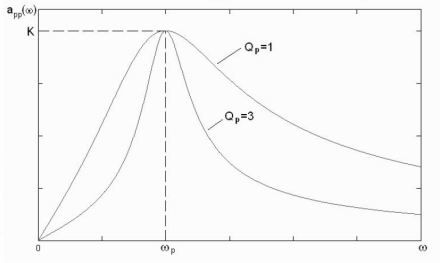
$$H_{pp}(s) = K \cdot \frac{s \cdot \frac{\omega_p}{Q_p}}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

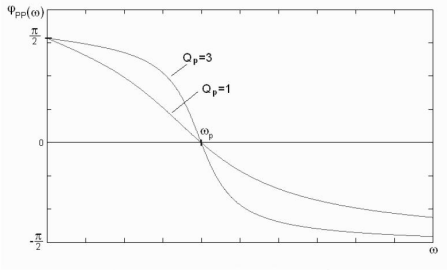
$$s_{p1,2} = -\frac{\omega_p}{2Q_p} \pm j\omega_p \sqrt{1 - \frac{1}{4Q^2}}$$

$s_{01} = 0$, $s_{02} \to \infty$

$$a_{pp}(\omega) = \frac{K}{\sqrt{1 + Q_p^2 \cdot \left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right)^2}}$$

$$\varphi_{pp}(\omega) = -\operatorname{arctg}\left(Q\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right)\right)$$





- za: $\omega = \omega_p \Rightarrow a(\omega) \to K$ (MAXIMUM)

  $\omega \to 0 \Rightarrow a(\omega) \to 0$

  $\omega \to \infty \Rightarrow a(\omega) \to 0$

- propušta signale s frekv oko ω_p
- povećanjem Q, karakteristika se sužava

$$a_{pp}(\omega_d) = a_{pp}(\omega_g) = \frac{K}{\sqrt{2}}$$

$$\omega_{g/d} = \omega_p \cdot \sqrt{1 + \frac{1}{4Q_p^2}} \pm \frac{\omega_p}{2Q_p}$$

$$B = \omega_g - \omega_d = \frac{\omega_p}{Q_p}$$

$$\omega_p = \omega_c = \sqrt{\omega_d \cdot \omega_g}$$

$\Downarrow$

CENTRALNA FREKVENCIJA

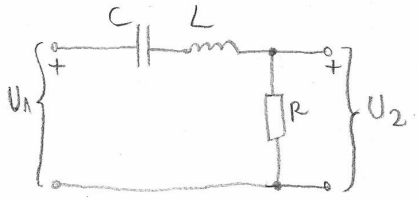
- uskopojasni PP filtri: $B \ll \omega_c$

$$Q = \frac{\omega_c}{B} \geq 10$$

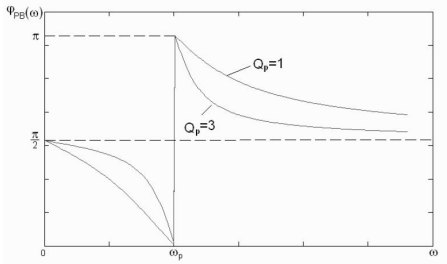
- tada je: $\omega_{g1d} = \omega_p \pm \frac{1}{2} B$

- realizacija:

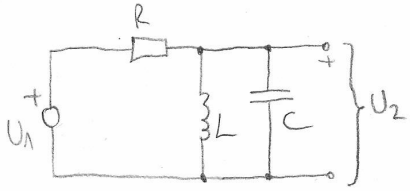
1. SERIJSKIM RLC KRUGOM



$$H(s) = \frac{s \cdot \frac{R}{L}}{s^2 + s \cdot \frac{R}{L} + \frac{1}{LC}}$$

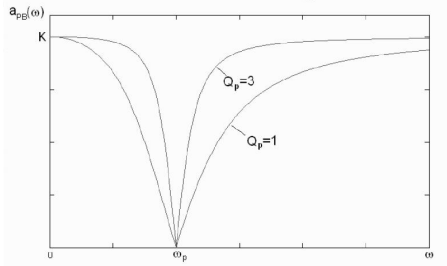


2. PARALELNIM RLC KRUGOM



$$H(s) = \frac{s \cdot \frac{1}{RC}}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$$

4. POSAŠNA BRANA



- propušta: $\omega > \omega_g$ i $\omega < \omega_d$
- prijenosna funkcija PB mora biti NAJMANJE 2. REDA

POSAŠNA BRANA 2. REDA

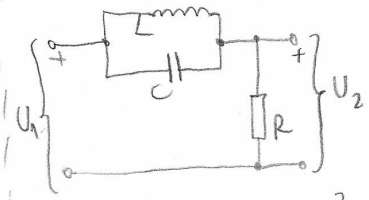
$$H(s) = K \cdot \frac{s^2 + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

- za: $\omega = \omega_p \Rightarrow a(\omega) = 0$
  $\omega \to 0 \Rightarrow a(\omega) \to K$
  $\omega \to \infty \Rightarrow a(\omega) \to K$

$$s_{P1,2} = -\frac{\omega_p}{2Q_p} \pm j\omega_p \sqrt{1 - \frac{1}{4Q^2}}$$

$$s_{O1,2} = \pm j\omega_p$$

- realizacija: sa RLC četveropolom



$$H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$$

$$a_{PB}(\omega) = K \cdot \frac{Q_p \cdot \left| \frac{\omega_p}{\omega} - \frac{\omega}{\omega_p} \right|}{\sqrt{1 + Q_p^2 \left(\frac{\omega_p}{\omega} - \frac{\omega}{\omega_p} \right)^2}}$$

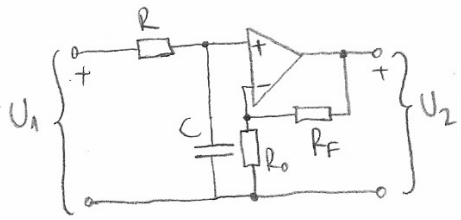
$$\varphi_{PB}(\omega) = \pi \cdot S(\omega - \omega_p) - \operatorname{arctg}\left(Q_p \cdot \left(\frac{\omega_p}{\omega} - \frac{\omega}{\omega_p} \right) \right)$$

$$a_{PB}(\omega_d) = a_{PB}(\omega_g) = \frac{K}{\sqrt{2}}$$

AKTIVNI FILTERI

① AKTIVNI FILTRI 1. REDA

1. NP FILTAR



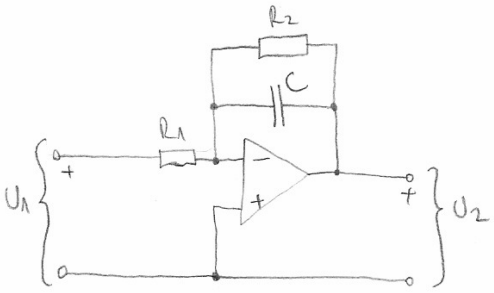
$$H_{NP}(s) = k \cdot \frac{\omega_g}{s+\omega_g}$$

$$k = 1 + \frac{R_F}{R_0}$$

$$\omega_g = \frac{1}{RC}$$

$$a(\omega) = |H(j\omega)| = k \cdot \frac{\omega_g}{\sqrt{\omega^2+\omega_g^2}}$$

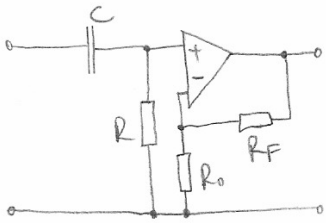
- moguća realizacija sa invertirajućim pojačalom:



$$k = -\frac{R_2}{R_1}$$

$$\omega_g = \frac{1}{R_2 C}$$

2. VP FILTAR

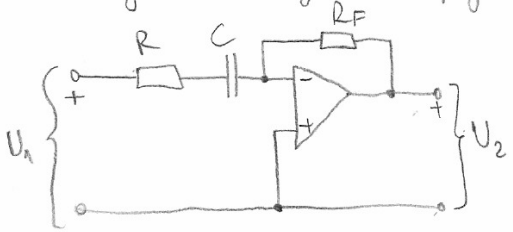


$$H_{VP}(s) = k \cdot \frac{s}{s+\omega_g}$$

$$k = 1 + \frac{R_F}{R_0} > 1$$

$$\omega_g = \frac{1}{RC}$$

- realizacija sa negativnim pojačanjem:

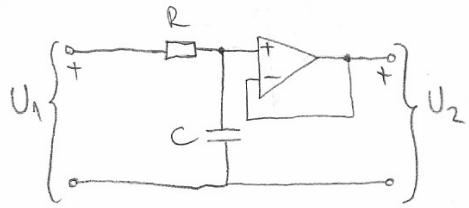


$$k = -\frac{R_F}{R}$$

$$\omega_g = \frac{1}{RC}$$

REALIZACIJA VP I NP FILTARA SA JEDINIČNIM POJAČALOM

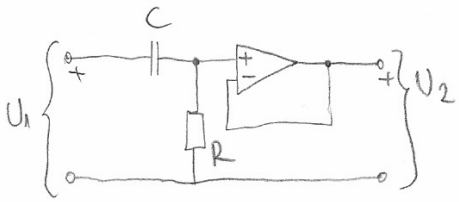
- NP:



$$k = 1$$

$$\omega_g = \frac{1}{RC}$$

- VP



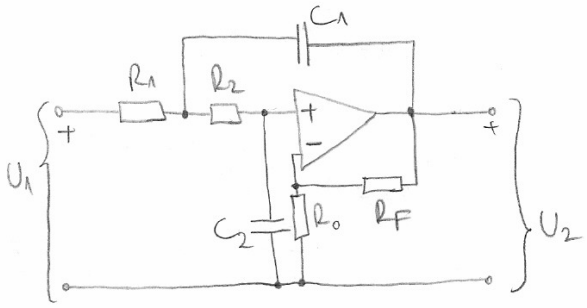
$$k = 1$$

$$\omega_g = \frac{1}{RC}$$

(2.) AKTIVNI FILTRI 2. REDA

1. NP FILTAR

$$H_{NP}(s) = k \cdot \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

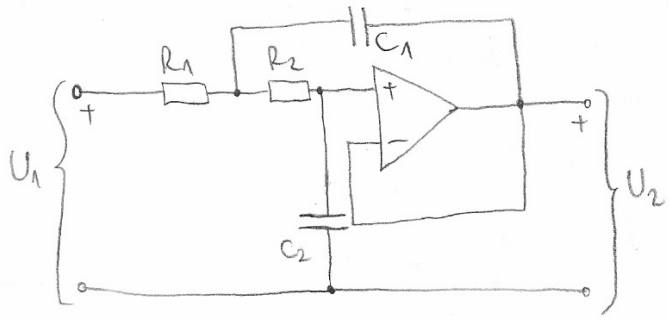


$$k = 1 + \frac{G_0}{G_F}$$

$$\omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$$

$$Q_p = \frac{\sqrt{\frac{C_1 G_1}{C_2 G_2}}}{1 + \frac{G_1}{G_2} + (1-k) \cdot \frac{C_1}{C_2}}$$

- realizacija sa jediničnim pojačalom:



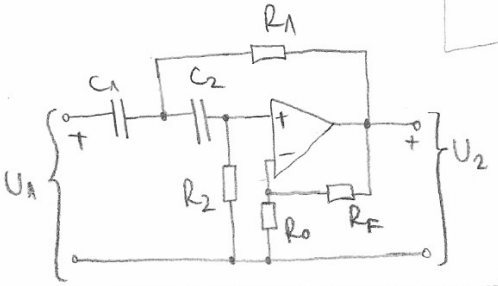
$$k = 1$$

$$\omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$$

$$\frac{\omega_p}{Q_p} = \frac{G_1 + G_2}{C_1}$$

2. VP FILTAR

$$H_{VP}(s) = k \cdot \frac{s^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

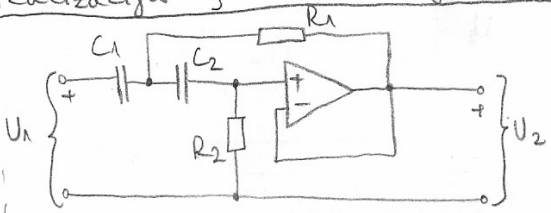


$$k = 1 + \frac{G_0}{G_F}$$

$$\omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$$

$$Q_p = \frac{\sqrt{\frac{C_1 G_1}{C_2 G_2}}}{1 + \frac{C_1}{C_2} \cdot \frac{G_1}{G_2} \cdot \frac{G_0}{G_F}}$$

- realizacija jediničnim pojačalom:

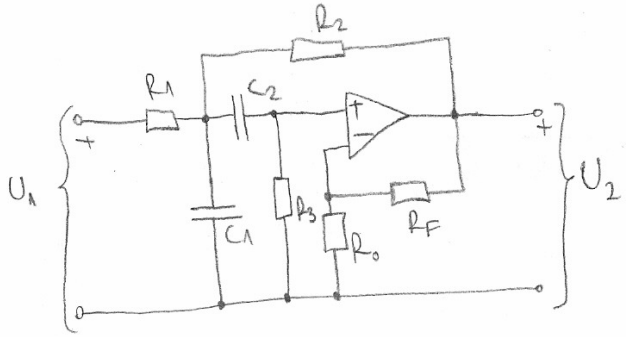


$$k = 1$$

$$\omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$$

$$Q_p = \frac{\sqrt{\frac{C_1 G_1}{C_2 G_2}}}{1 + \frac{C_1}{C_2}}$$

3. PP FILTAR



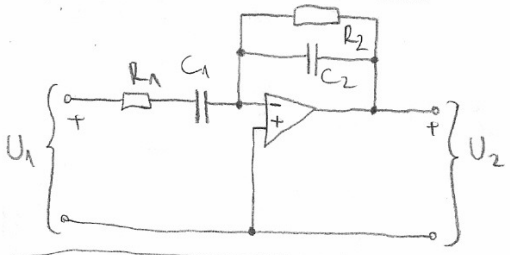
$$H_{PP}(s) = k \cdot \frac{\frac{\omega_p}{Q_p} \cdot s}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$$

$$k = 1 + \frac{R_F}{R_0}$$

$$\omega_p = \sqrt{\frac{(G_1 + G_2) \cdot G_3}{C_1 C_2}}$$

$$Q_p = \frac{\sqrt{\frac{G_3}{G_1+G_2} \cdot \frac{C_2}{C_1}}}{\frac{G_3}{G_1+G_2} \cdot \left(1 + \frac{C_2}{C_1}\right) + \frac{C_2}{C_1} - \left(1 + \frac{G_0}{G_F}\right) \cdot \frac{G_2}{G_1+G_2}}$$

ŠIROKOPOJASNI FILTAR



$$\omega_p = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

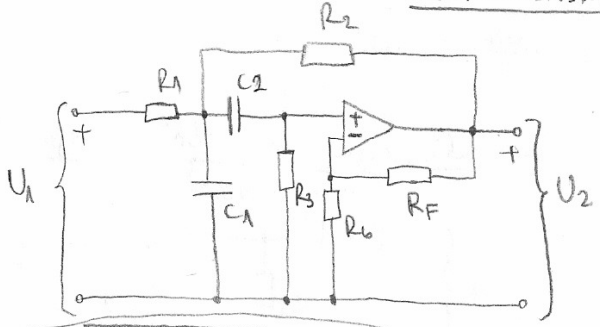
$$k = \frac{1}{\frac{C_2}{C_1} + \frac{R_1}{R_2}}$$

$$S_{p1} = -\frac{1}{R_1 C_1} = -\omega_2$$

$$S_{p2} = -\frac{1}{R_2 C_2} = -\omega_1$$

$$Q_p = \frac{1}{\sqrt{\frac{R_1 C_1}{R_2 C_2}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}}} \leq 0.5$$

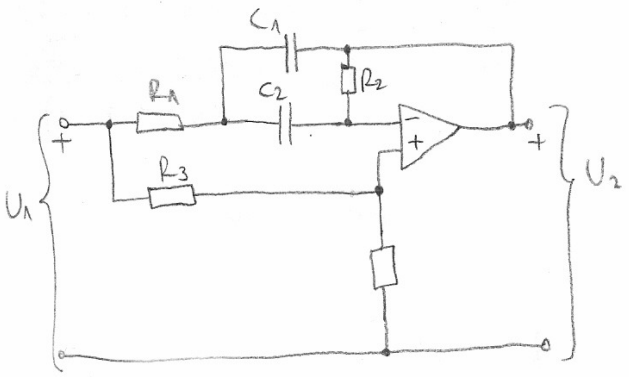
USKOPOJASNI FILTAR



$$\omega_p = \sqrt{\frac{(G_1+G_2) \cdot G_3}{C_1 C_2}}$$

$$Q_p = \frac{\omega_p}{\frac{G_3}{C_1} + \frac{G_3}{C_2} + \frac{G_1+G_2}{C_1} - \left(1 + \frac{G_0}{G_F}\right) \cdot \frac{G_2}{C_1}}$$

4. PODAŠNA BRANA



$$H_{PB}(s) = k \cdot \frac{s^2 + \frac{\omega_p}{Q_z} \cdot s + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

$Q_z \to \infty$

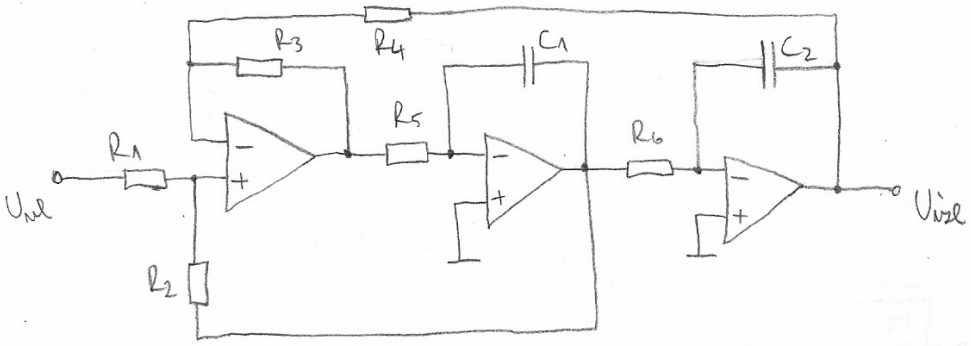
- ovaj uvjet je moguće postići ako je:

$$G_2 \cdot \left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{G_1 \cdot G_4}{C_1 G_3}$$

$$k = \frac{G_3}{G_3 + G_4}$$

$$\omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}} \qquad Q_p = \frac{\omega_p}{G_2} \cdot \frac{C_1 C_2}{C_1 + C_2}$$

5. UNIVERZALNA FILTARSKA SEKCIJA



- analiza sekcija:

$$U_{iz1}(s) = -\frac{1}{R_6 \cdot C_2 \cdot s} \cdot U_{iz3}(s)$$

$$U_{iz3}(s) = \frac{-1}{R_5 \cdot C_1 \cdot s} \cdot U_{iz2}(s)$$

$$U_{iz2}(s) = \left(1 + \frac{R_3}{R_4}\right) \cdot \frac{R_2 \cdot U_{ul}}{R_1 + R_2} + \left(1 + \frac{R_3}{R_4}\right) \cdot \frac{R_1 \cdot U_{iz3}}{R_1 + R_2} - \frac{R_3}{R_4} U_{iz1}$$

- NP prijenosna funkcija:

$$H_{NP}(s) = \frac{U_{iz1}}{U_{ul}} = \frac{\frac{G_1(G_4+G_3)\cdot G_5\cdot G_6}{G_3\cdot(G_1+G_2)\cdot C_1\cdot C_2}}{s^2 + s\cdot\left(\frac{G_2\cdot G_5\cdot(G_4+G_3)}{G_3\cdot(G_1+G_2)\cdot C_1}\right) + \frac{G_4\cdot G_5\cdot G_6}{G_3\cdot C_1\cdot C_2}}$$

U_{iz1} – izlazni napon; U_{ul} – ulazni napon

- VP prijenosna funkcija:

$$H_{VP}(s) = \frac{U_{iz2}}{U_{ul}} = \frac{\frac{G_1\cdot(G_4+G_3)}{G_3\cdot(G_1+G_2)}\cdot s^2}{s^2 + s\cdot\left(\frac{G_2\cdot G_5(G_4+G_3)}{G_3\cdot(G_1+G_2)\cdot C_1}\right) + \frac{G_4 G_5 G_6}{G_3 C_1 C_2}}$$

- PP prijenosna funkcija:

$$H_{PP}(s) = \frac{U_{iz3}}{U_{ul}} = \frac{-\frac{G_1\cdot(G_4+G_3)\cdot G_5}{G_3\cdot(G_1+G_2)\cdot C_1}\cdot s}{s^2 + s\cdot\left(\frac{G_2\cdot G_5\cdot(G_4+G_3)}{G_3\cdot(G_1+G_2)\cdot C_1}\right) + \frac{G_4\cdot G_5\cdot G_6}{G_3\cdot C_1\cdot C_2}}$$

- općenito vrijedi:

$$\omega_p = \sqrt{\frac{G_4\cdot G_5\cdot G_6}{G_3\cdot C_1\cdot C_2}}$$

$$Q_p = \sqrt{\frac{C_1}{C_2}}\cdot\frac{\sqrt{G_4 G_6}}{\sqrt{G_3 G_5}}\cdot\frac{1+\frac{G_1}{G_2}}{1+\frac{G_4}{G_3}}$$

$$k_{NP} = \frac{G_1(G_4+G_3)}{G_4(G_1+G_2)}$$

$$k_{PP} = \frac{G_1}{G_2}$$

$$k_{VP} = \frac{G_1(G_4+G_3)}{G_3(G_1+G_2)}$$

- broj pasivnih elemenata filtra je 8, a broj uvjeta koje treba ispuniti 3 $\Rightarrow$ 5 elemenata je moguće proizvoljno odabrati
- uobičajeno:

$G_5 = G_6 = G$, $C_1 = C_2 = C$ $\Rightarrow$

$$\omega_p = \sqrt{\frac{G_4}{G_3}}\cdot\frac{G}{C} \qquad Q_p = \sqrt{\frac{G_4}{G_3}}\cdot\frac{1+\frac{G_1}{G_2}}{1+\frac{G_4}{G_3}}$$